



Andrea Bracciali



Ronald de Haan



# Decentralization in Open Quorum Systems


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university of  
 groningen




UNIVERSITY OF AMSTERDAM

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## Stellar | Move Money Across Borders Quickly, Reliably, And For Fractions Of A Penny.

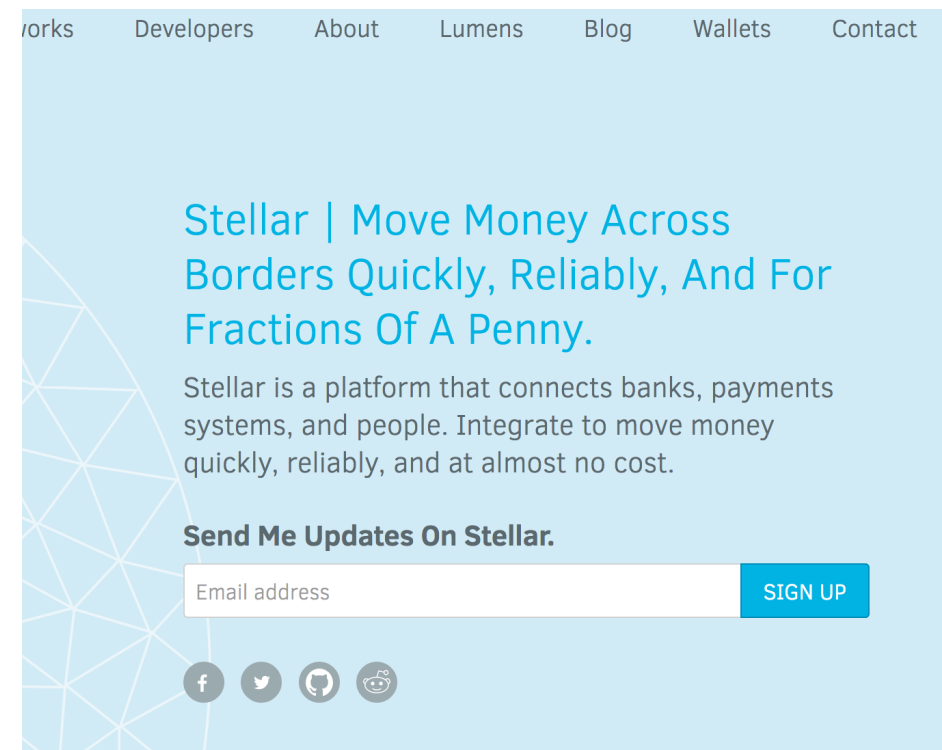
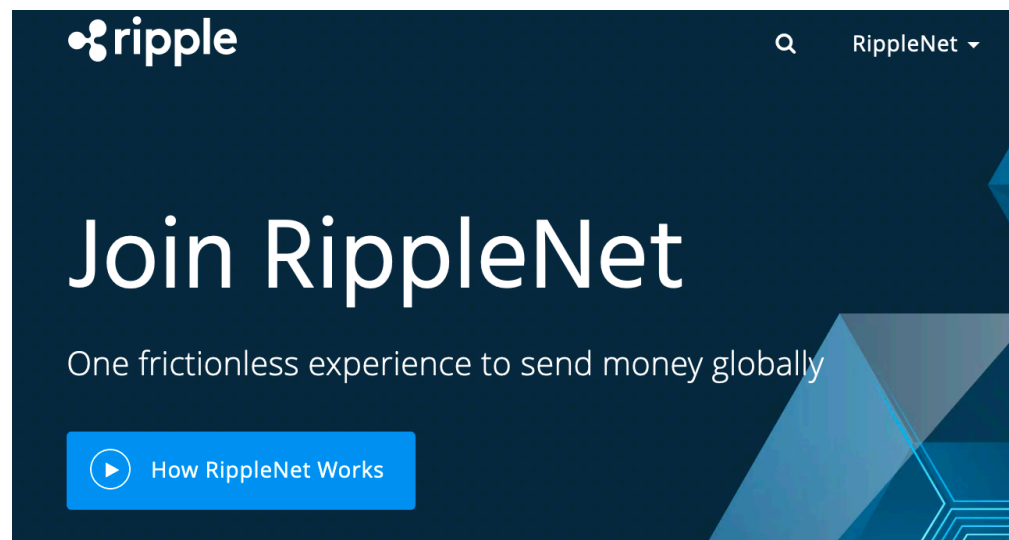
Stellar is a platform that connects banks, payments systems, and people. Integrate to move money quickly, reliably, and at almost no cost.

**Send Me Updates On Stellar.**

[SIGN UP](#)

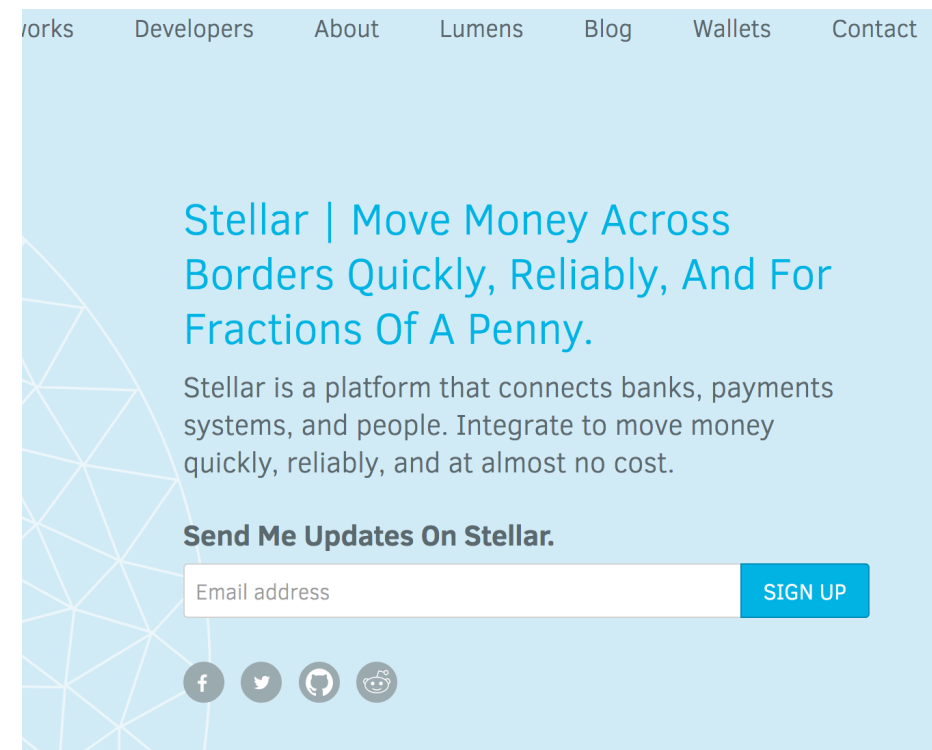
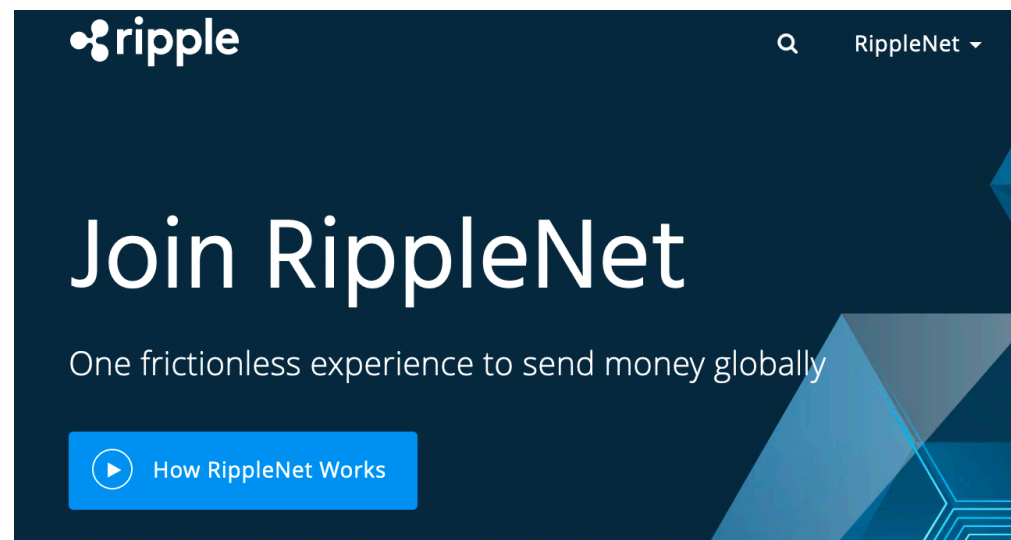
   





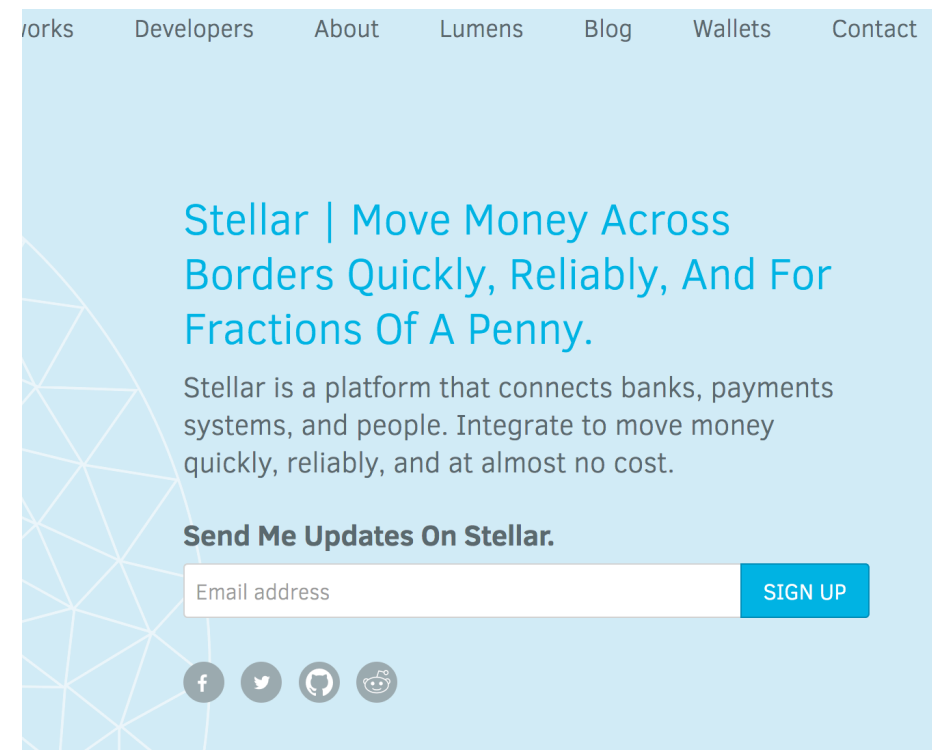
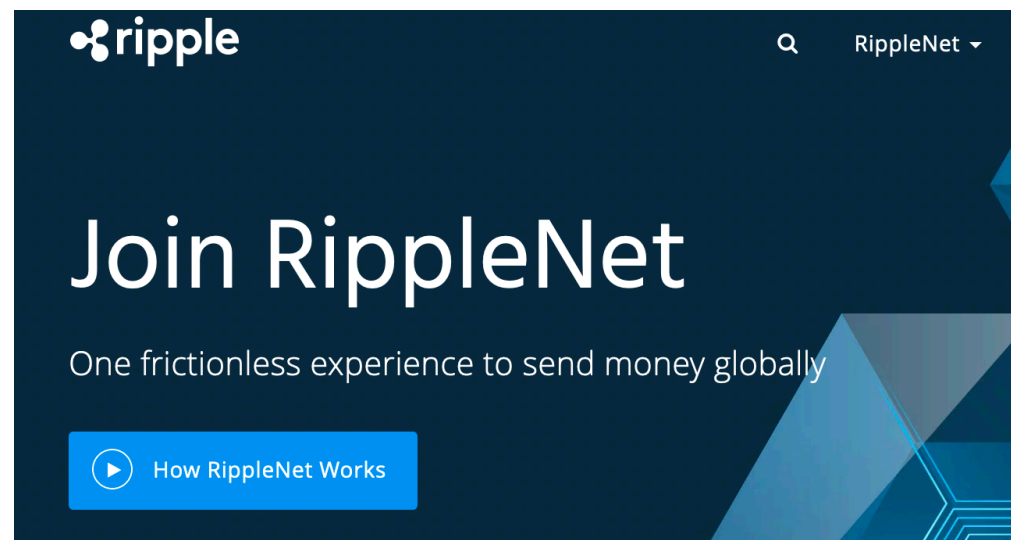
□ Ripple & Stellar





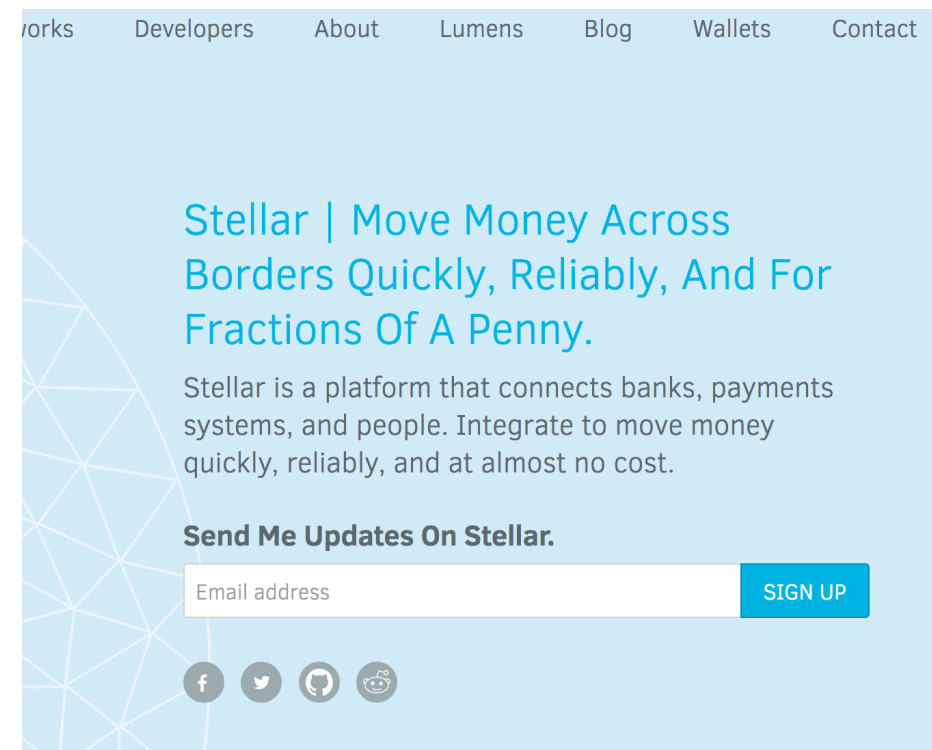
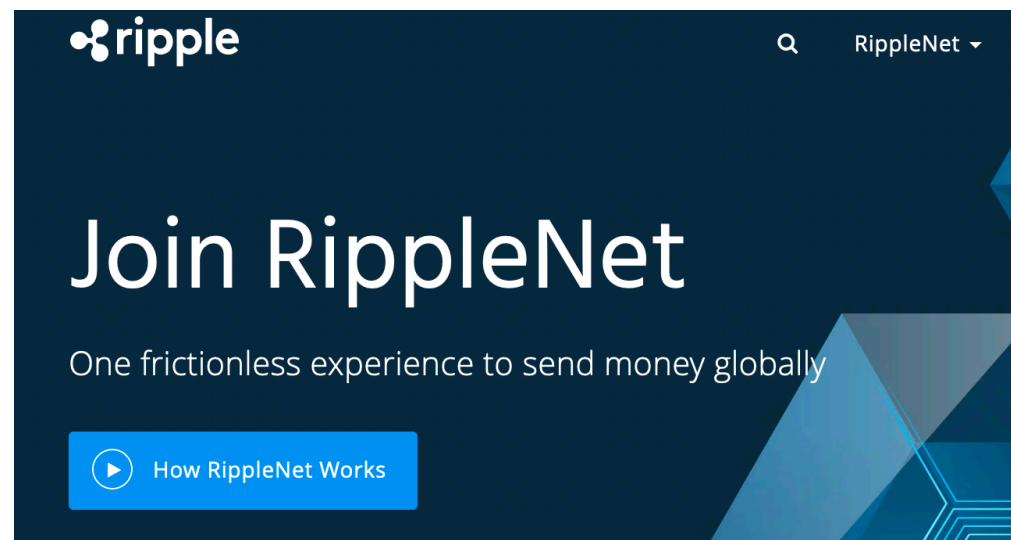
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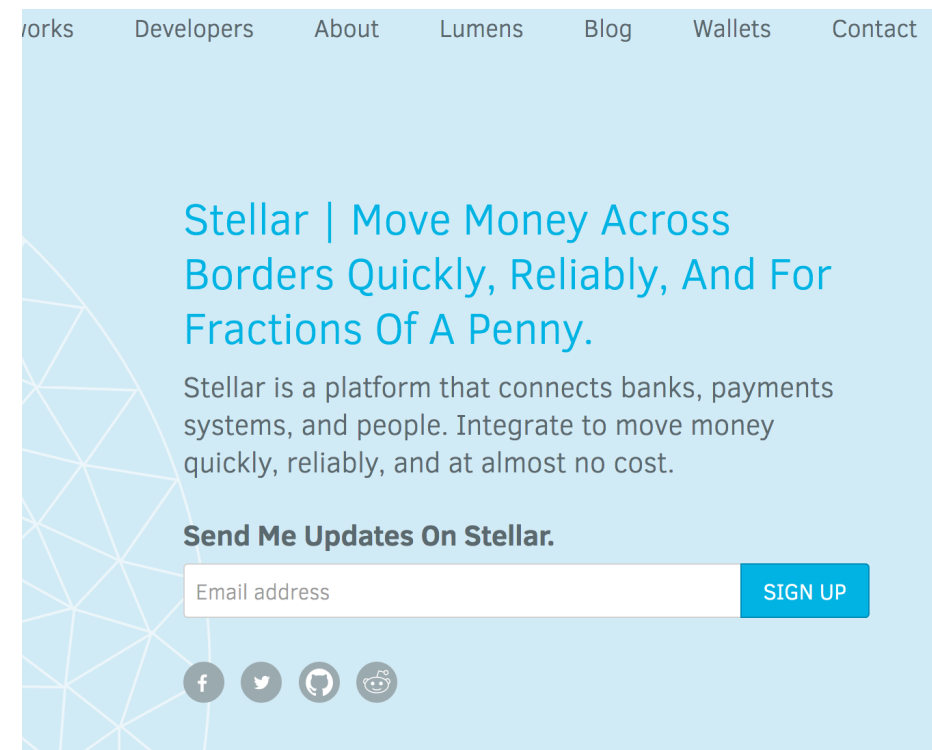
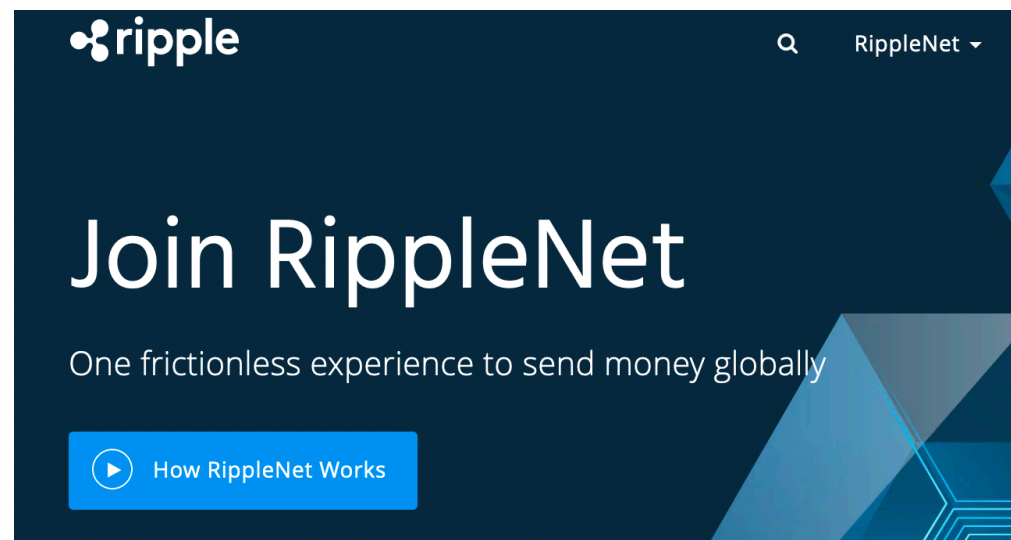
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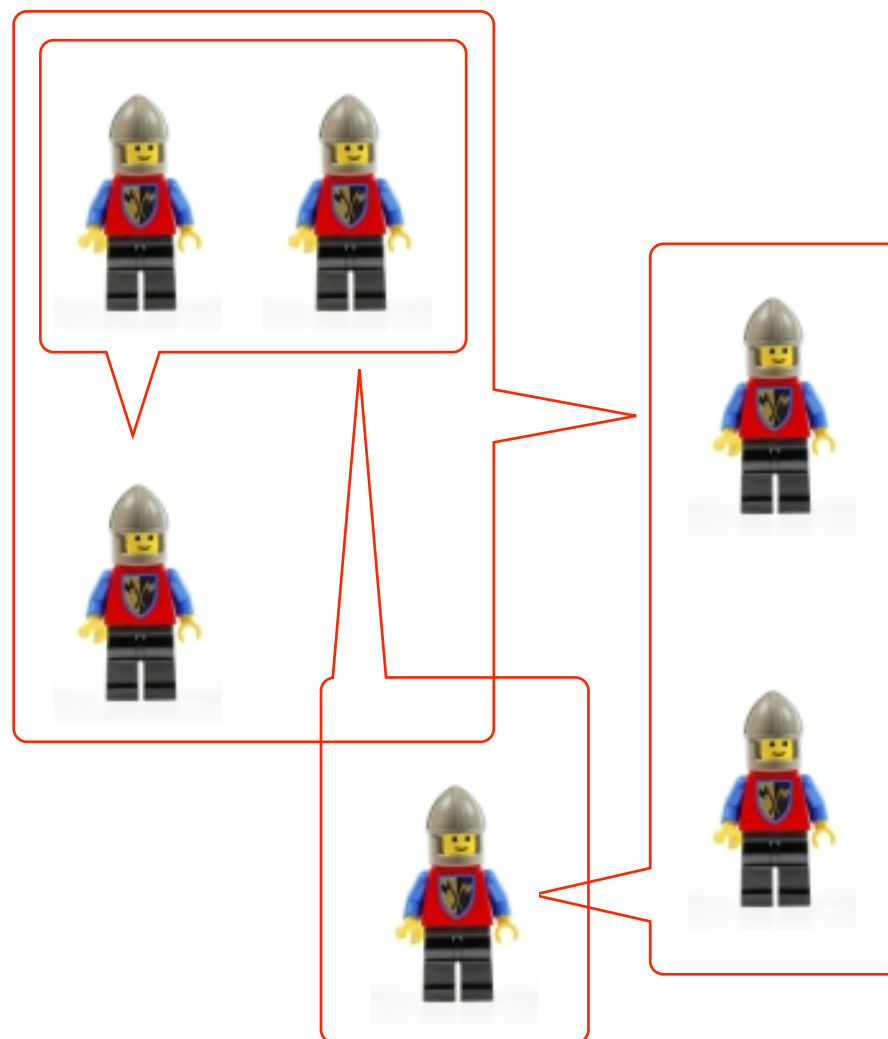
**Are there inherent limitations to decentralisation in this form of consensus?**

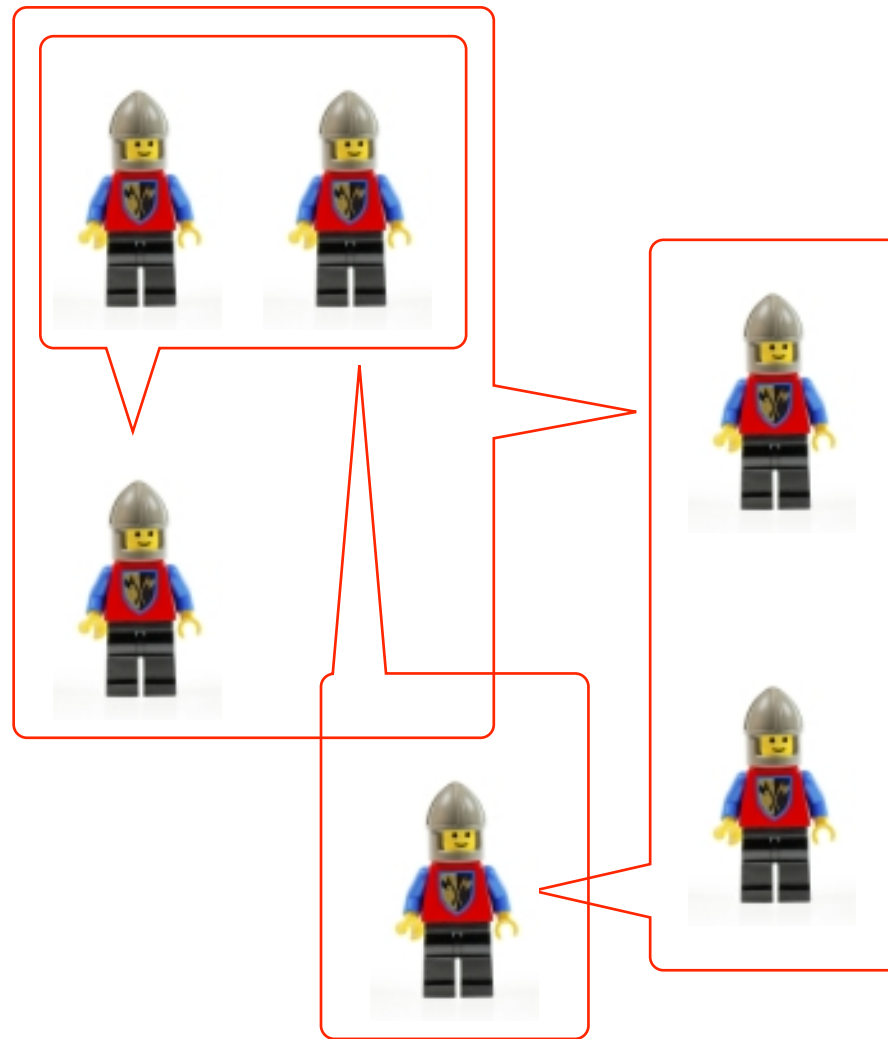


# PART I

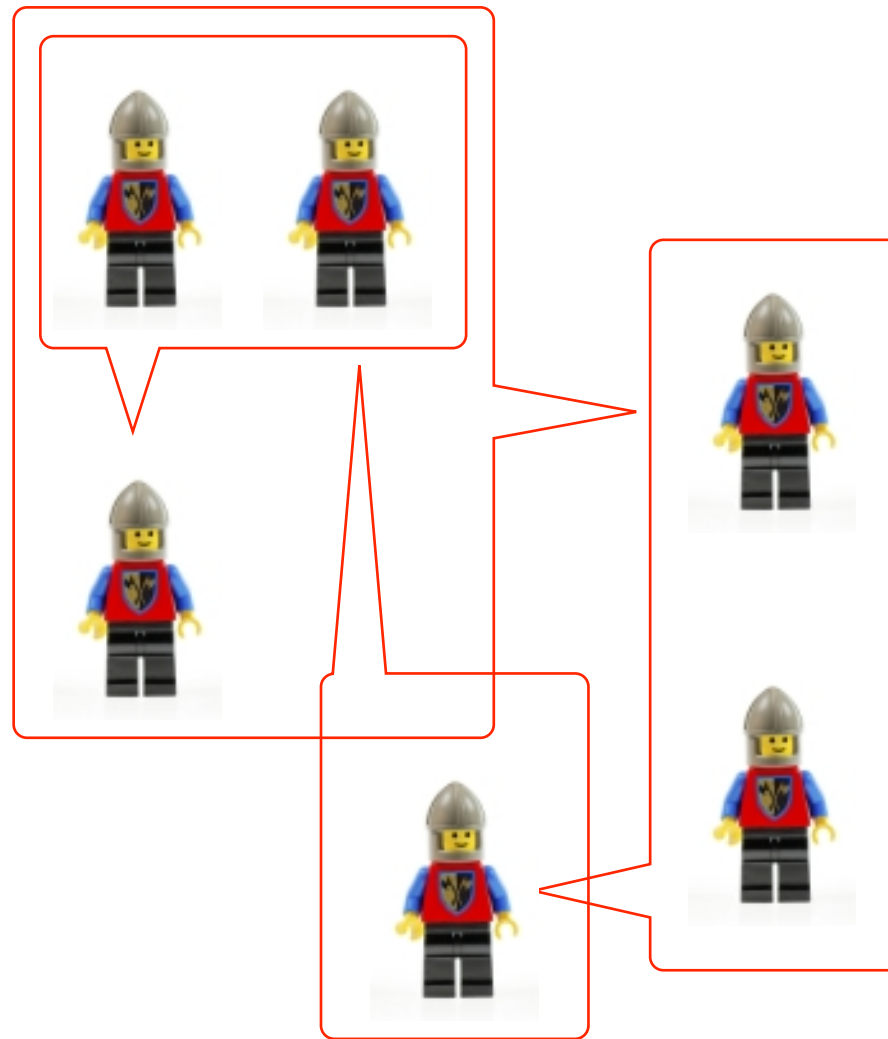
## P2P Trust Networks



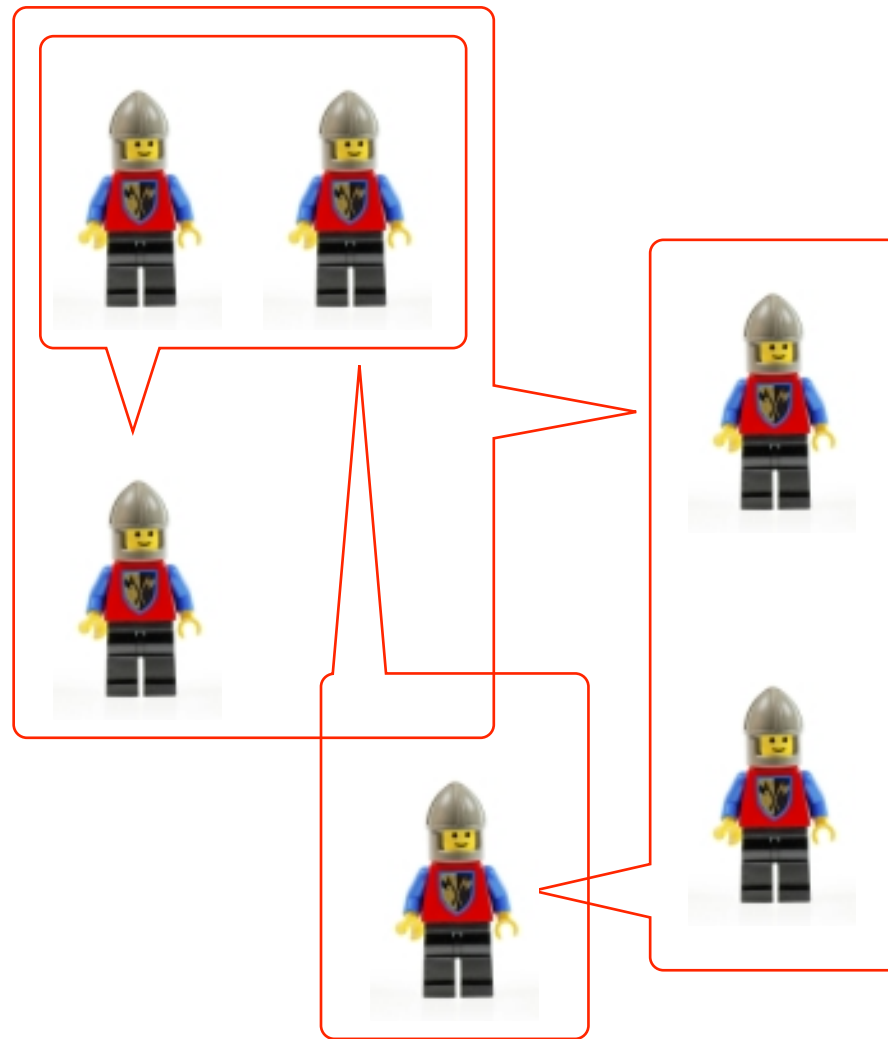




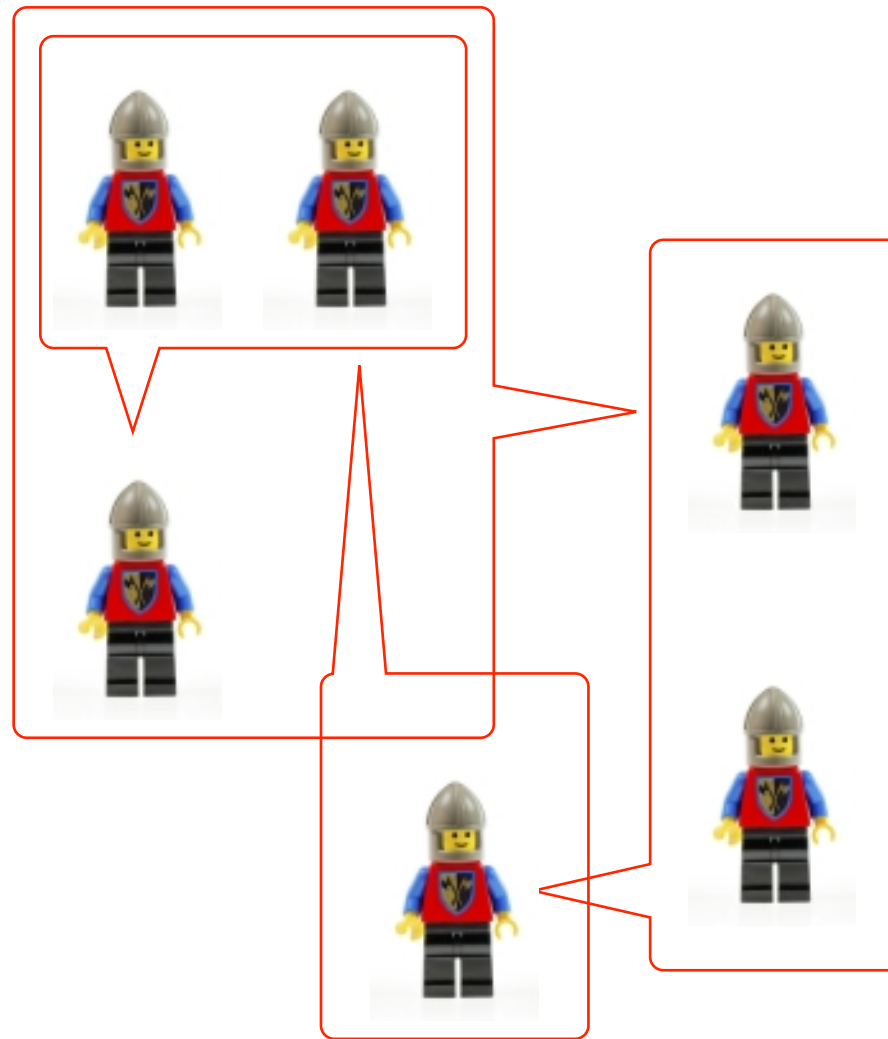
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- ☐ **CONSENSUS** = all honest nodes ***agree stably***

# Byzantine Trust Networks (BTNs)

$$\mathcal{T} = \langle N, H, L_i, q_i \rangle$$



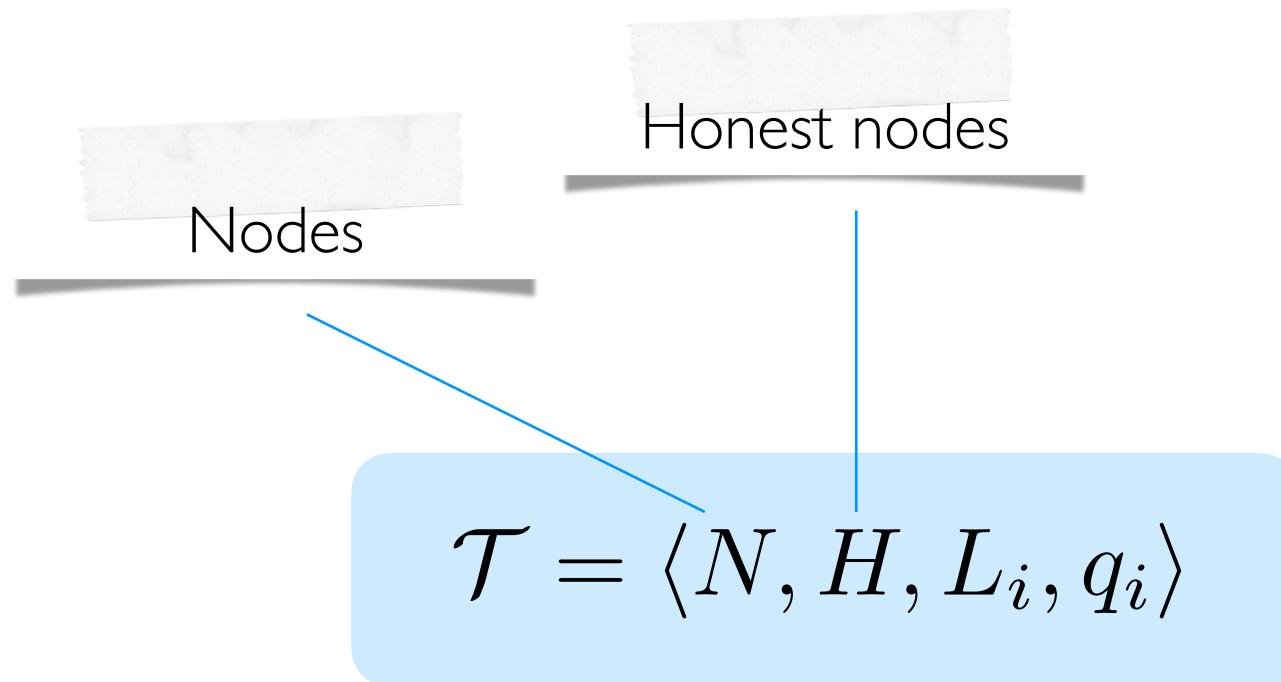
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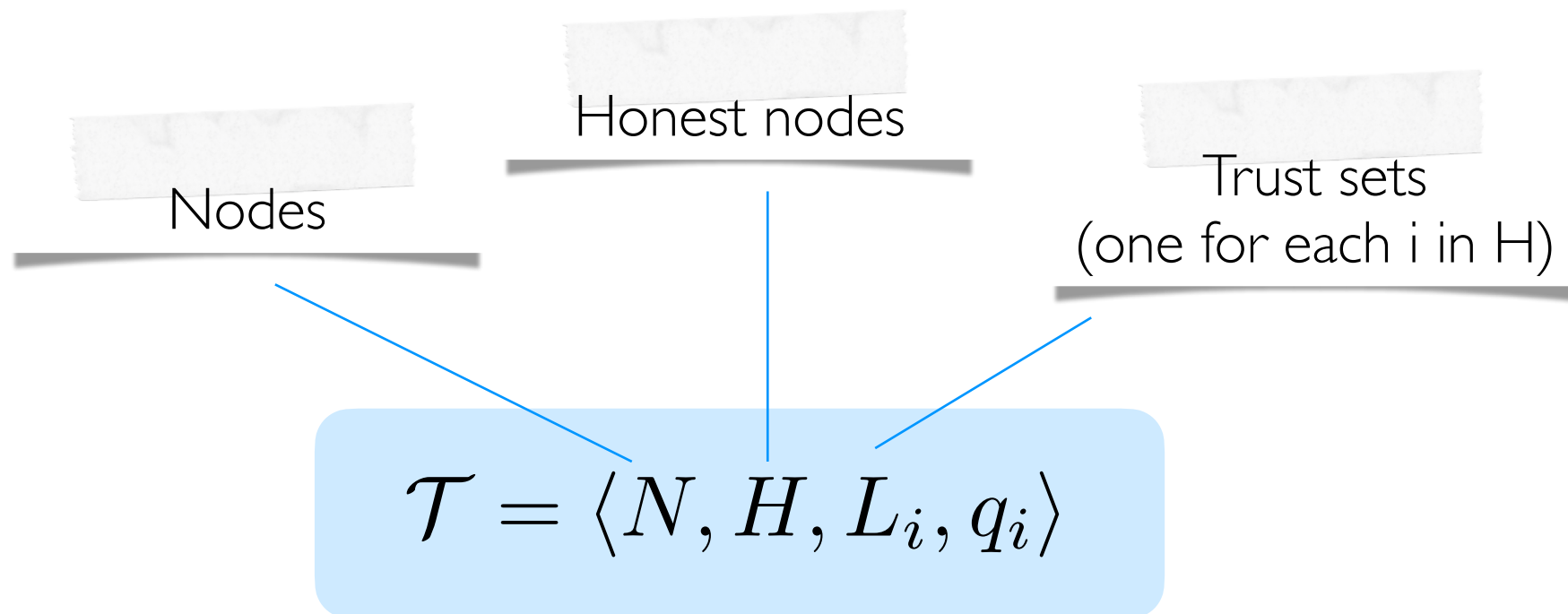


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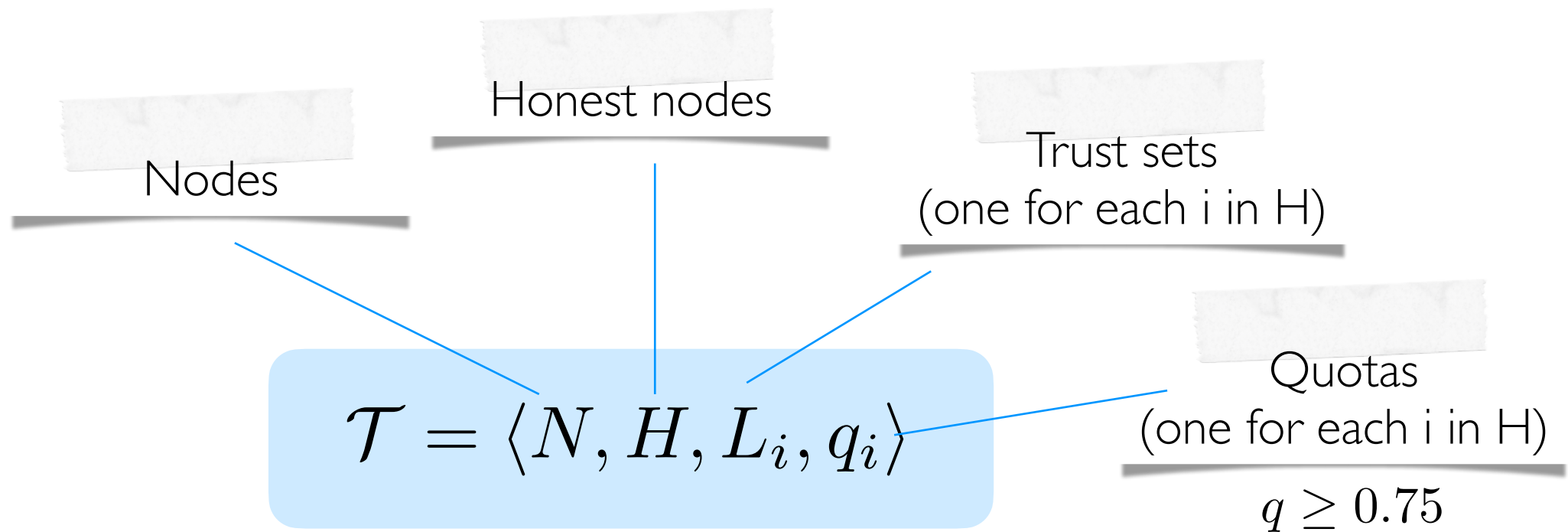




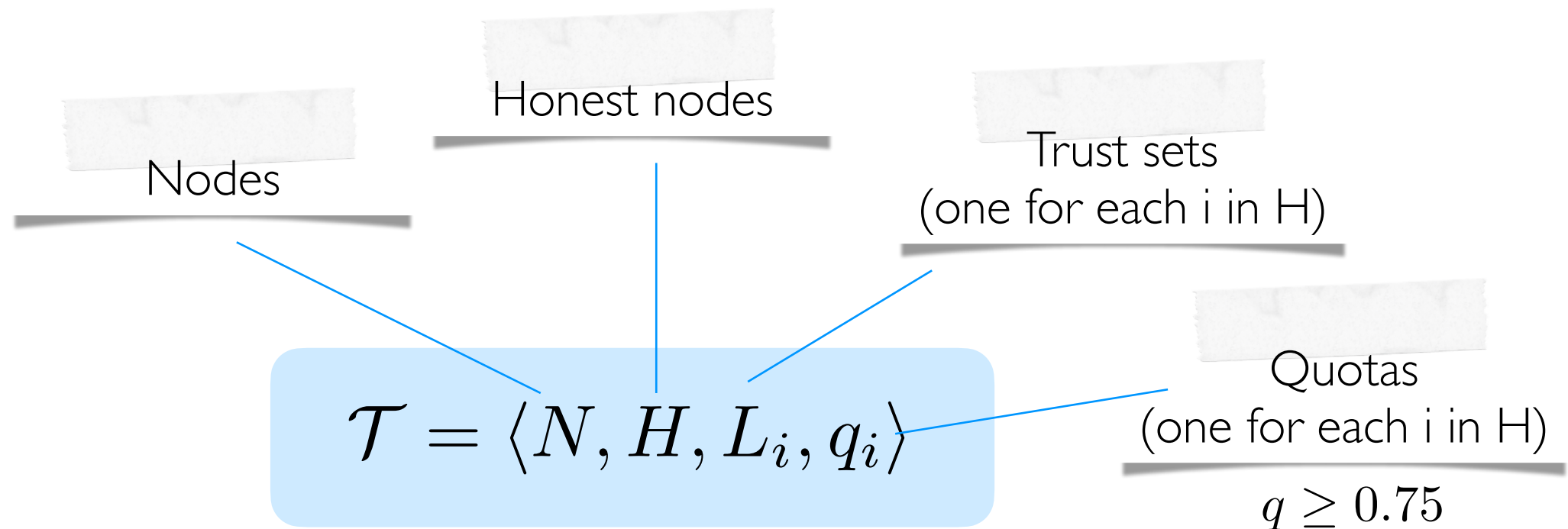
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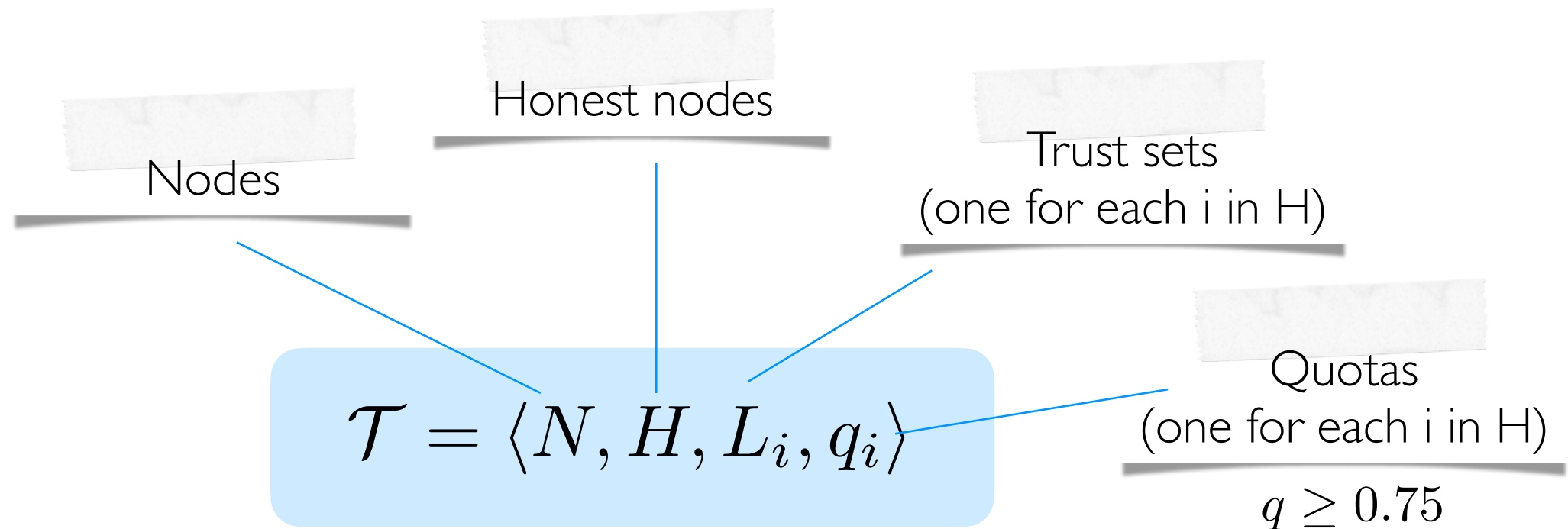
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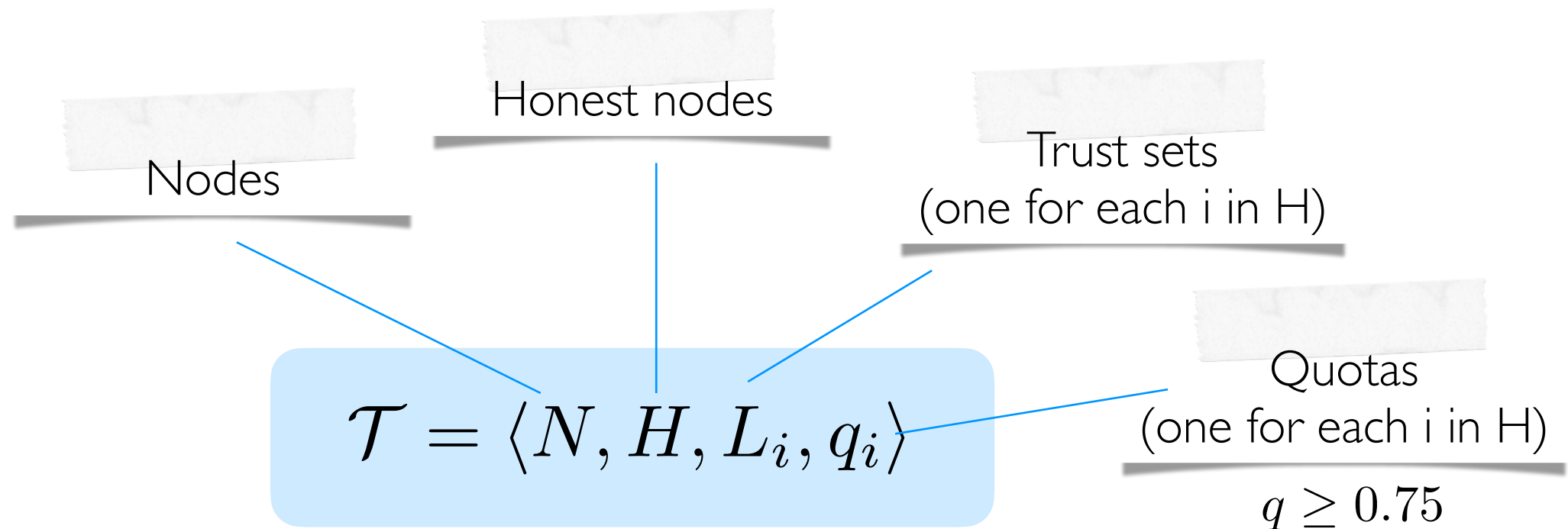
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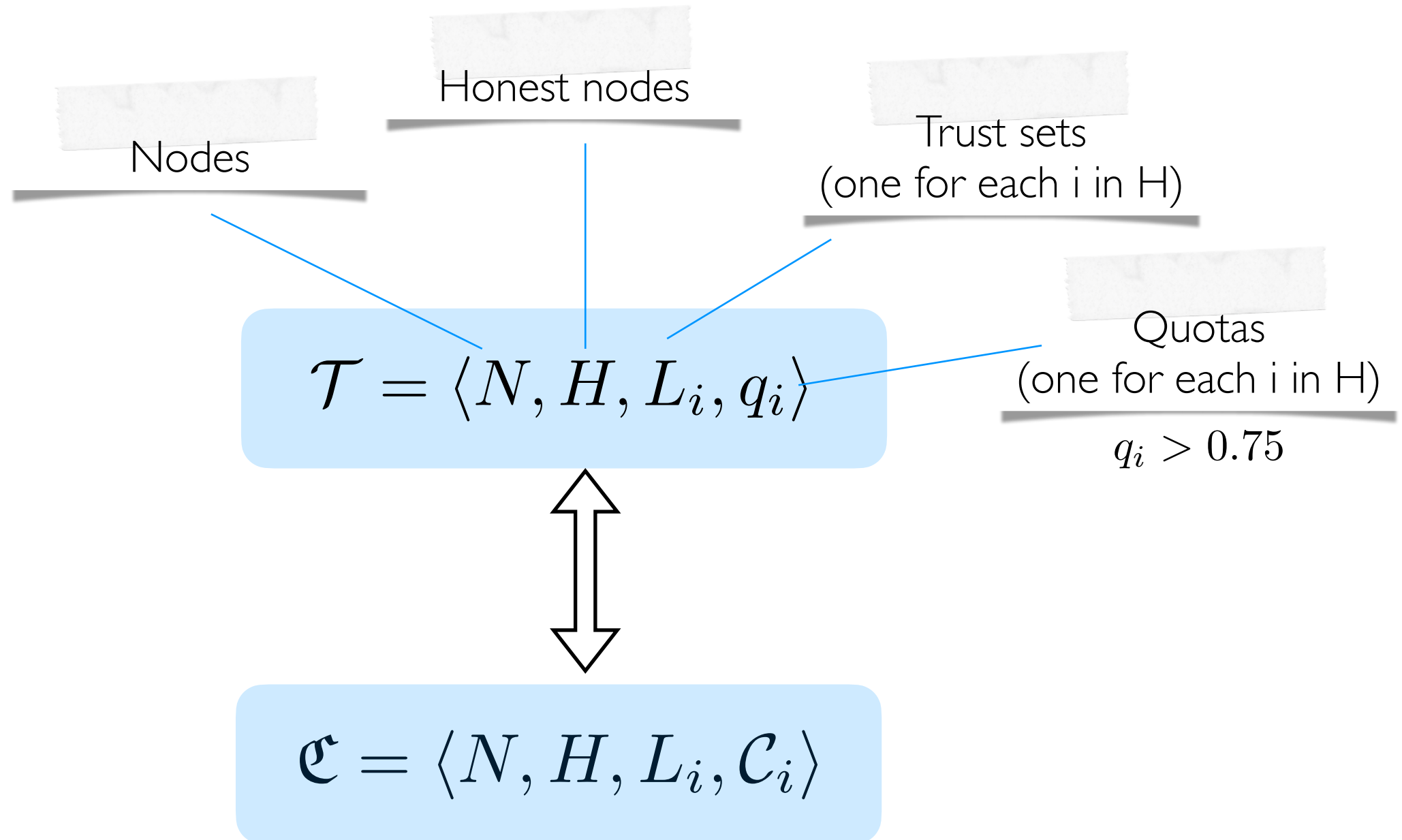
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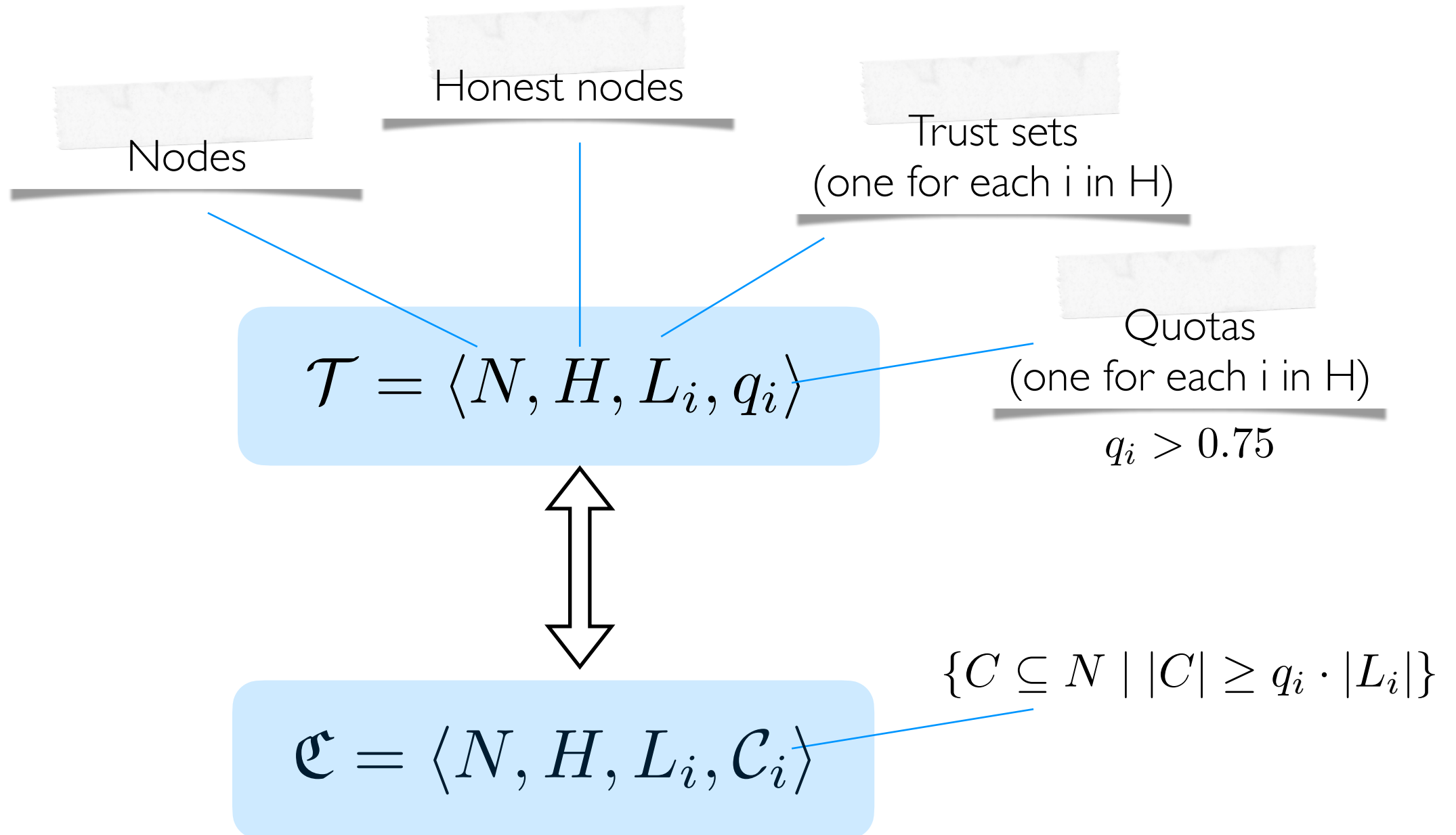
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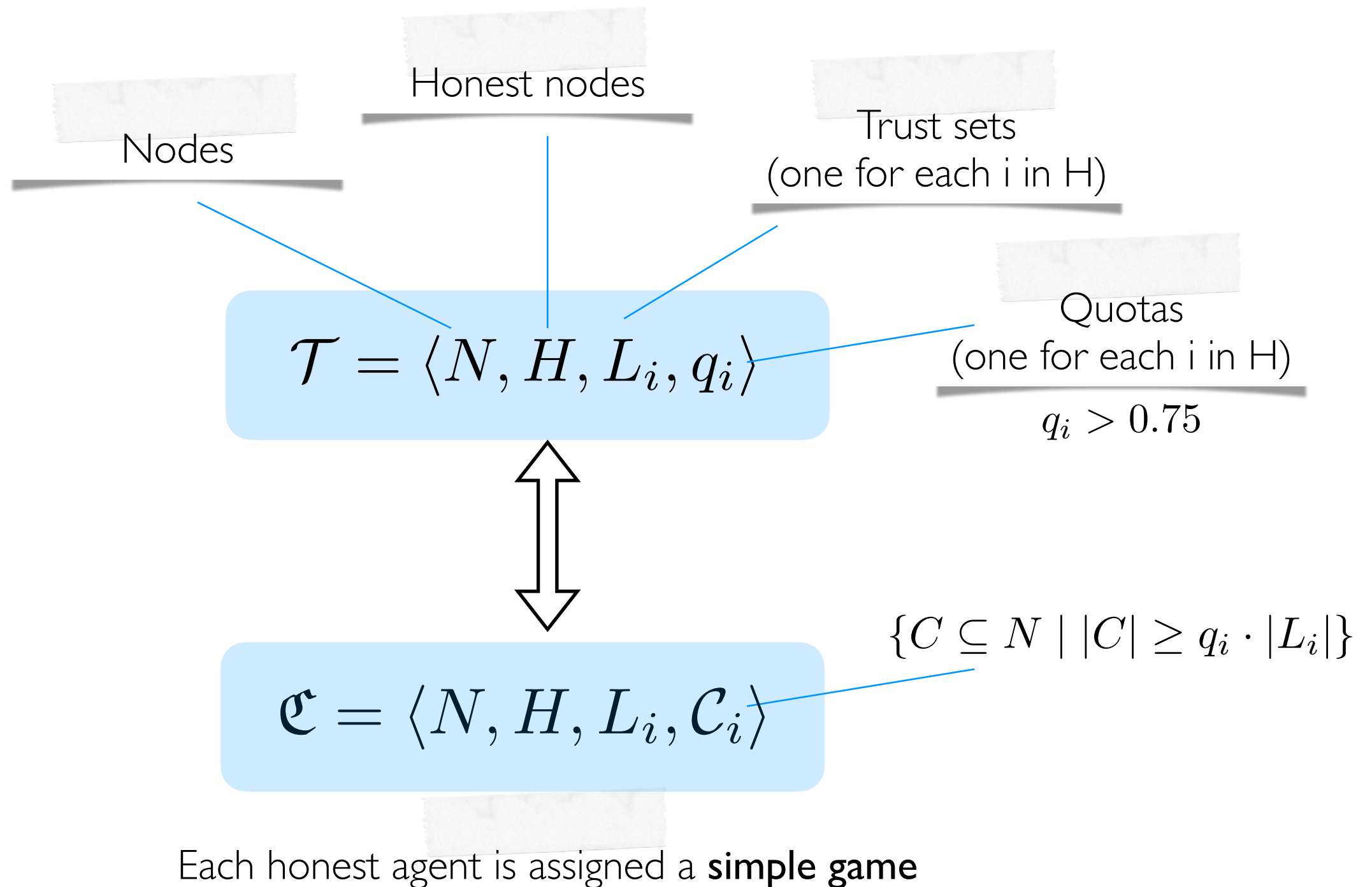
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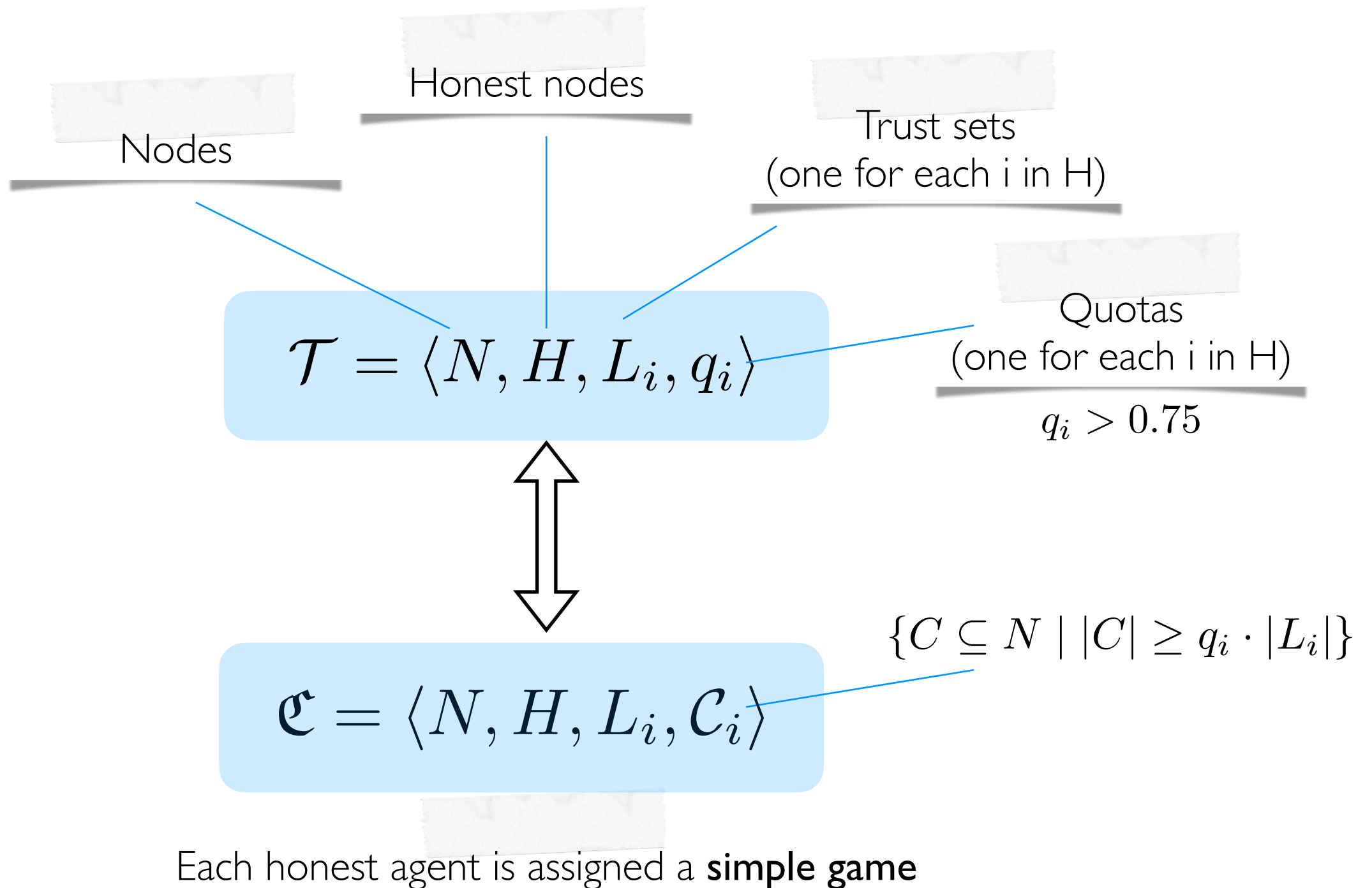


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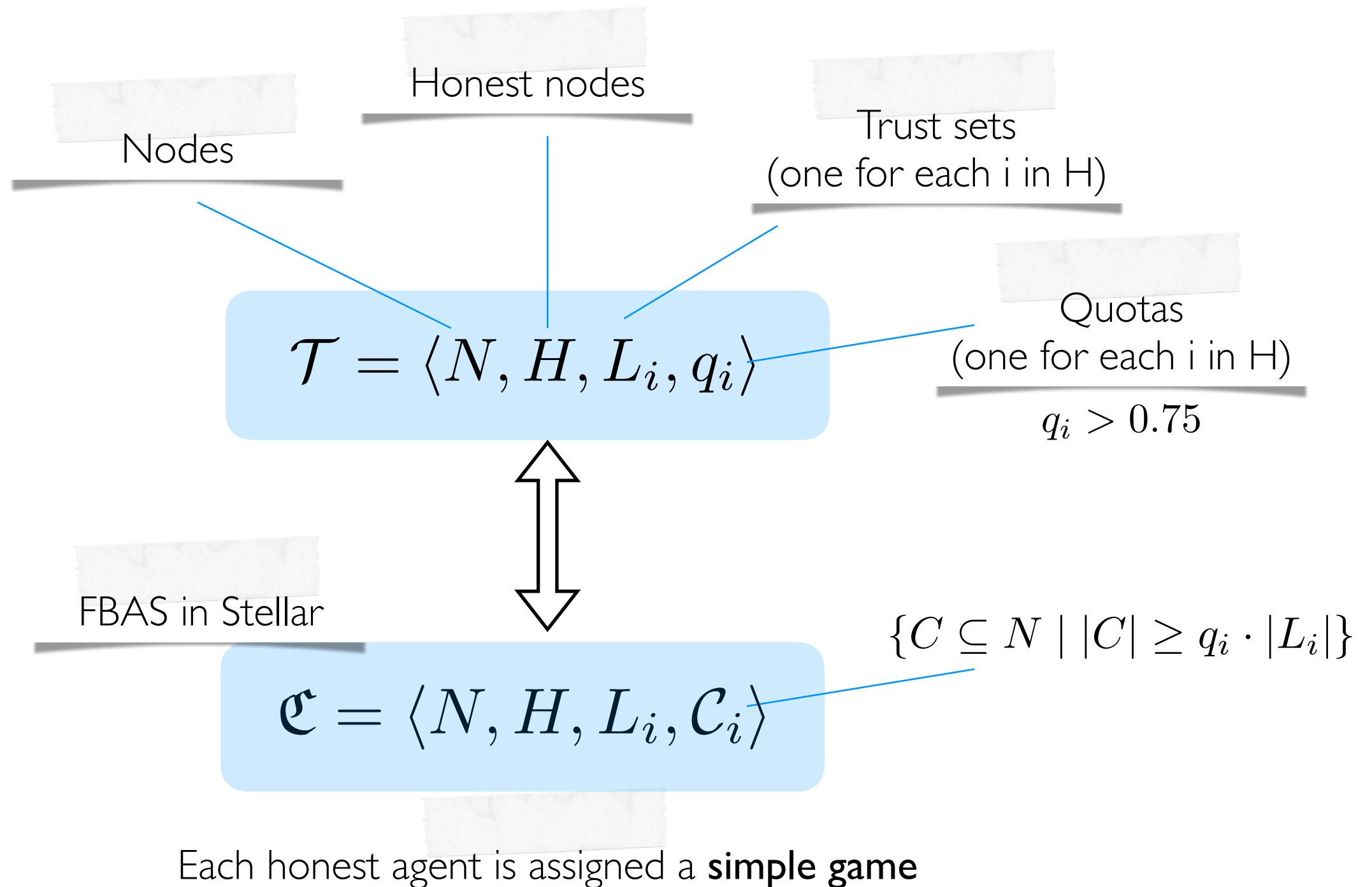


X. Hu and L. Shapley. On authority distributions in organizations: Controls. Games and Economic Behavior, 45:153–170, 2003.

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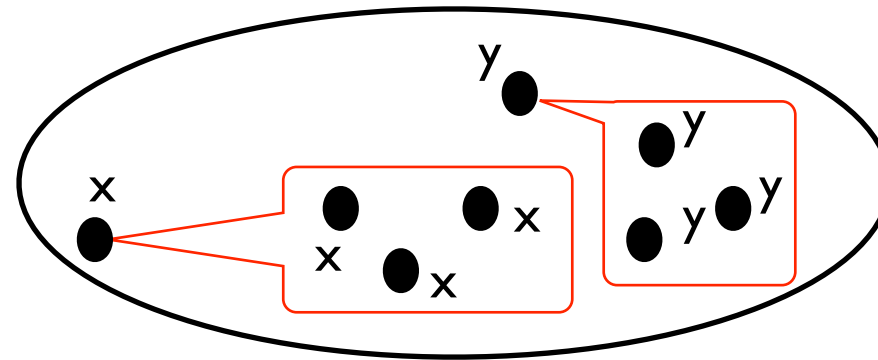


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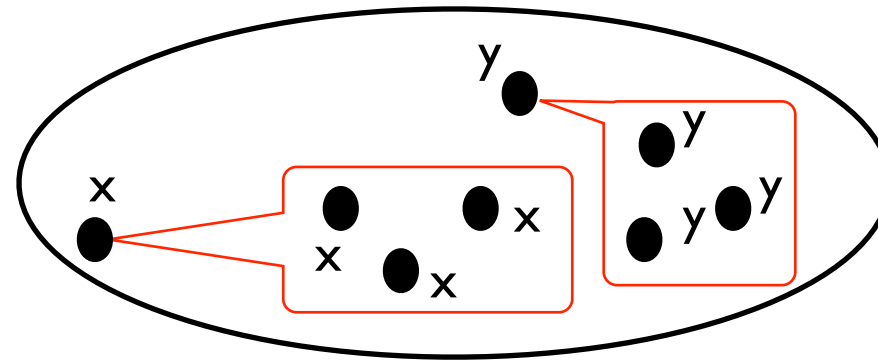


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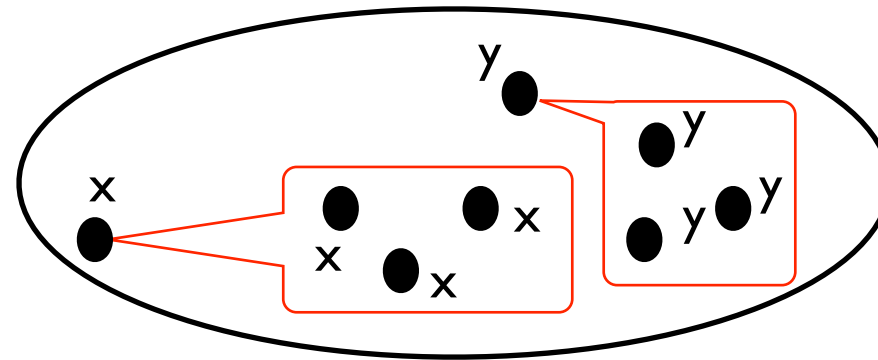
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- ☐ NOTE: safety is protocol-independent
- ☐ **QUESTION:** what are necessary structural conditions for safety?



# PART II

## Decentralization

# Safety & Decentralization in uniform BTNs

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- Safety implies any two trust sets should overlap for at least  $(1-q)/q$  of their combined size
- If all pairs of trust sets overlap for at least 0.25 of their combined size, then the intersection of all trust sets is non-empty (i.e., there are nodes trusted by all nodes)
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Fully decentralised  
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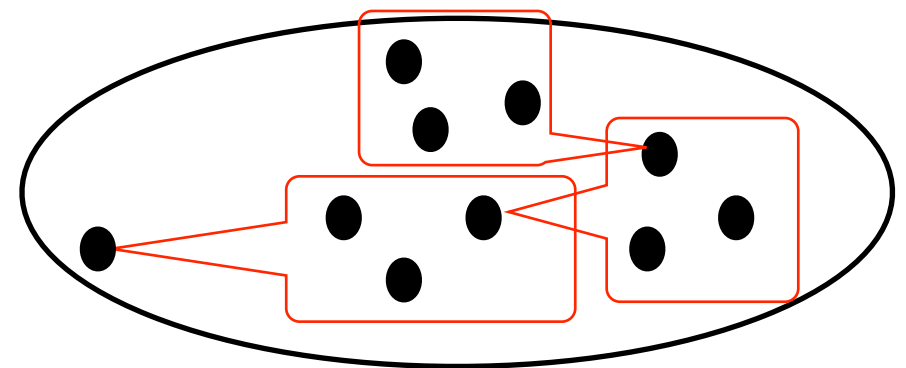


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# PART III

## Influence

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- ☐ What does this mean concretely in terms of the influence that nodes have on consensus?
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- ☐ It is trickier for consensus based on trust networks



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Influence matrix  
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