Economic Valuation and Financial Management of an Insurance Firm

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Should insurers invest in liquid risky assets? YES! NO!

- Only few academic papers on the topic, e.g.
  - Froot & Stein (Journal of Finance, 1998), Froot (Journal of Risk and Insurance, 2007) say: **NO**.
  - Azcue & Muler (Annals of Probability, 2010) say: **YES**.
- Most insurers **do**, e.g. Warren Buffett says:
  "Insurers receive premiums upfront and pay claims later. This collect-now, pay-later model leaves insurance companies holding large sums – money we call **float** *– that will eventually go to others. Meanwhile, insurers get to invest this float for their benefit."*

We study this problem in a simple setting

- Limited-liability insurer with a **diffuse** shareholder base.
- Discrete-time setting with **infinite** time horizon.
- Financial frictions reflected in a **carry cost of capital**, **cost of raising capital** and a **minimum capital requirement** imposed by a regulator.
- **Financial policies** at the firm manager's disposal:
  - → liquidation/continuation,
  - → dividend payments,
  - → recapitalization, and
  - → investment.

A word of warning

in an attempt to be "light" i will adopt a slightly "casual" attitude towards technical statements and assumptions! The paper contains all necessary details.

The Financial Market, insurance risk and the Valuation Measure

The firm's objective is to maximize "value"

... which is "discounted expected cash flows to shareholders":

$$\sum_{n \geq 0} \frac{\mathbb{E}_{\mathbb{Q}}[\text{Dividends-Casflow Injections at date } n]}{(1+i)^n}$$

where $\mathbb{Q}$ is a "valuation measure" on the probability space $\left(\Omega, \mathcal{F}, P\right)$ used to model the possible states of our uncertain economy and i is a suitable "interest rate".

... BUT:

$\rightarrow$ Expectations w.r.t. which probability measure $\mathbb{Q}$?

$\rightarrow$ Discounting w.r.t. which interest rate i?

Two critical requirements on the valuation rule

- **Indifference to idiosyncratic risk.** A diffuse shareholder base implies that, when cash flows to shareholders do **not** depend on market prices, managers should act as if they were risk neutral and the valuation rule should use EXPECTATIONS W.R.T. THE "PHYSICAL" PROBABILITY MEASURE AND THE RISK-FREE DISCOUNT RATE.
- **Market-consistency.** When valuing cash flows stemming from market instruments, the valuation rule should reproduce their market prices and, hence, use EXPECTATIONS W.R.T. A "RISK-ADJUSTED" PROBABILITY MEASURE AND THE RISK-FREE DISCOUNT RATE.

The underlying (financial) market is of Black-Scholes type

Even though the insurer makes decisions at discrete dates, markets trade at a much higher frequency and we work with a continuous-time market of Black-Scholes type $(\boldsymbol{B}, \boldsymbol{S})$, where

$$B_t = e^{t\hat{r}} \quad \text{and} \quad S_t = s_0 \exp\Big\{\sigma W_t + \big(\mu - \frac{1}{2}\sigma^2\big)t\Big\}.$$

are defined on $\big(\Omega, \mathcal{F}, P\big)$.

The flow of information in the market is described by the *market filtration* $\mathbb{F}^W = \big(\mathcal{F}^W_t, t \geq 0\big)$. Set $\mathcal{F}^W := \mathcal{F}^W_\infty$.

Unique prices for cash flows that depend only on the market

For every finite maturity $T > 0$, the market is *complete and arbitrage-free*: An $\mathcal{F}_T^W$-measurable cash flow X matures at date T has a unique, well-defined **market price** $\pi(X)$ at date 0.

Result. There is a unique **pricing measure** $\mathbb{P}^*$, defined on $(\Omega, \mathcal{F}^W)$, such that *market consistency* holds, i.e. for every $\mathcal{F}_T^W$-measurable cash flow X maturing at date T

$$\pi(X) = \mathbb{E}_{\mathbb{P}^*}\left[e^{-T\hat{r}} X\right].$$

$\mathbb{P}^*$ is not equivalent to $\mathbb{P}$!

The insurer has more information than just the market's

We consider an insurance firm that sells a fixed-size portfolio of one-period policies with i.i.d. loss process

$$\boldsymbol{L} = (L_n,\ n = 0, 1, \ldots)$$

that is independent of $(\mathcal{F}_n^W,\ n = 0, 1, \ldots)$. The random variables

$$R_n := \mathbb{E}_{\mathbb{P}}[L_n] - L_n, \quad n = 0, 1, \ldots,$$

represent the **insurance risk**.

The **insurer's filtration** $\mathbb{F} = \big(\mathcal{F}_t, t \geq 0\big)$ contains the information on BOTH realized insurance losses AND financial market prices. We set $\mathcal{F} := \mathcal{F}_\infty$. Cash flows to shareholders are $\mathbb{F}$ rather than $\mathbb{F}^W$ adapted!

The "extended" market is incomplete and there are infinitely many market-consistent measures ...

We need to "value" cash flows to shareholders that are adapted to $\mathbb{F}$ but, while "extending" the market from $\mathbb{F}^W$ to $\mathbb{F}$ preserves arbitrage freedom, it introduces INCOMPLETENESS!

Consequence. There is an infinite number of *market-consistent valuation measures*, i.e. of probability measures $\mathbb{Q}$ defined on $\mathcal{F}$ such that

$$\pi(X) = \mathbb{E}_{\mathbb{Q}}\left[e^{-(T-t)\hat{r}} X\right]$$

for every $\mathcal{F}_T$-measurable, replicable cash flow X.

… luckily only one "right" shareholder valuation measure exists

Result. There is a unique **shareholder valuation measure** $\mathbb{Q}^*$, defined on $(\Omega, \mathcal{F})$, featuring, for every $\mathcal{F}_T$-measurable cash flow maturing at date T,

(i) Indifference to idiosyncratic risk:

$$\mathbb{E}_{\mathbb{Q}^*}\left[X\right] = \mathbb{E}_{\mathbb{P}^*}\left[\mathbb{E}_{\mathbb{P}}[X|\mathcal{F}_T^W]\right].$$

(ii) Market consistency:

$$\pi(X) = \mathbb{E}_{\mathbb{Q}^*}\left[e^{-(T-t)\hat{r}}X\right],$$

whenever X is replicable. $\mathbb{Q}^*$ is not equivalent to $\mathbb{P}$!

The manager should use the shareholder valuation measure

If $\boldsymbol{C} = (C_n,\ n = 0, 1, \ldots)$ denotes cash flows to shareholders[†], their value is given by

$$\begin{aligned}\Pi(\boldsymbol{C}) &= \sum_{n=0}^{\infty} \frac{1}{(1+r)^n} \mathbb{E}_{\mathbb{Q}^*}\left[C_n\right] \\ &= \sum_{n=0}^{\infty} \frac{1}{(1+r)^n} \mathbb{E}_{\mathbb{P}^*}\left[\mathbb{E}_{\mathbb{P}}[C_n | \mathcal{F}_n^W]\right].\end{aligned}$$

where $1 + r := e^{\hat{r} \cdot 1}$.

[†]... with suitable integrability conditions.

The manager's choices

Liquidation, continuation and financing operations

At each date n the manager "inherits" an amount of capital M_n from the previous period and decides to

- continue operations or liquidate the firm

$$\delta_n = 1 \text{ (continue) or } \delta_n = 0 \text{ (liquidate/default).}$$

- pay capital back as dividends and/or raise capital

$$\kappa_n = \text{ amount to raise} \quad z_n = 0 \text{ amount to pay back.}$$

→ If $\delta_n = 0$, then $\kappa_n = 0$, and $z_n = \max\{M_n, 0\}$.

→ If $\delta_n = 1$, then capital after "financing operations" satisfies $M_n - z_n + \kappa_n \geq M_{\text{reg}}$, where $M_{\text{reg}} \geq 0$ is the minimum regulatory capital requirement.

Insurance portfolio and investment strategy when $\delta_n = 1$

The firm sells the standardized insurance portfolio for the premium

$$\underbrace{\frac{1}{(1+r)}\mathbb{E}[L_n]}_{=:\boldsymbol{l}\text{ (fair premium)}} + \underbrace{\quad \boldsymbol{q} \quad}_{\text{("margin")}} .$$

Equity capital is subject to a **carry cost** $1-\gamma \in [0,1]$, so the total amount for investment is $\gamma\big(M_n - z_n + \kappa_n + \boldsymbol{q}\big) + \boldsymbol{l}$.

Assuming the fair premium $\boldsymbol{l}$ is invested in the risk-free asset, the amount that can be invested in the risky asset is

$$\gamma\big(M_n - z_n + \kappa_n + \boldsymbol{q}\big)$$

The investment decision and end-of-period capital when $\delta_n = 1$

The managers decides on the proportion $\lambda_n \in [0,1]$ of

$$\gamma\big(M_n - z_n + \kappa_n + \boldsymbol{q}\big)$$

to be invested in the risky asset.

The capital position at the end of the period is

$$M_{n+1} = \gamma\big(M_n - z_n + \kappa_n + \boldsymbol{q}\big)\big(1 + r + \lambda_n \rho_{n+1}\big) + R_{n+1},$$

where $\rho_{n+1} := (S_{n+1}/S_n) - (1+r)$ is the *one-period excess return* of the risky asset.

The manager chooses from the set of "admissible" policies

For an initial level of capital m, a policy is an $\mathbb{F}$-adapted process

$$\boldsymbol{P} = \big((\delta_n, z_n, \kappa_n, \lambda_n),\ n = 0, 1, \dots\big)$$

that generates the capital process

$$\boldsymbol{M}^{m,\boldsymbol{P}} = \big(M_n^{m,\boldsymbol{P}},\ n = 0, 1, \dots\big).$$

Admissible Policies satisfy the following "commandments":

(1) Never raise capital and distribute dividends simultaneously.

(2) Only raise capital when capital is below M_{reg}.

(3) Never hold capital in excess of $\frac{C}{(1-\gamma)(1+r)} + M_{\text{reg}}$.

$\mathcal{P}(m)$ = set of admissible policies for initial capital m.

Firm value and added value

Valuing cash flows to shareholders induced by a policy

Policy $\boldsymbol{P}$ induces at date $n \geq 0$ the cash flow to shareholders:

$$z_n - \kappa_n - C(\kappa_n),$$

where there is fixed **cost of raising capital** $C \geq 0$ and

$$C(\kappa_n) = \begin{cases} C, & \text{if } \kappa_n > 0; \\ 0, & \text{if } \kappa_n = 0. \end{cases}$$

The economic value of cash flows to shareholders is

$$\Pi(m; \boldsymbol{P}) := \mathbb{E}_{\mathbb{Q}^*}\left[\sum_{n=0}^{\infty} \frac{1}{(1+r)^n}\big(z_n - \kappa_n - C(\kappa_n)\big)\right].$$

Firm value requires finding an optimal policy ...

... and, for a given initial capital m, is defined as

$$V(m) := \sup_{\boldsymbol{P} \in \mathcal{P}(m)} \Pi(m; \boldsymbol{P}).$$

The mapping $m \mapsto V(m)$ is strictly increasing!

A policy $\boldsymbol{P}^*$ is **optimal** for capital level m whenever

$$V(m) = \Pi(m; \boldsymbol{P}^*).$$

Firm value depends only on future cash flows $\implies$ decisions should depend only on starting capital.

Decision functions and stationary policies

A **one-step decision function** is a mapping[†]

$$m \mapsto D(m) = \big(z(m), \kappa(m), \lambda(m), \delta(m)\big)$$

such that setting $M_0^{m,D} = m$ and

$$M_{n+1}^{m,D} = \gamma\big(M_n^{m,D} - z(M_n^{m,D}) + \kappa(M_n^{m,D}) + \boldsymbol{q}\big)\big(1 + r + \lambda(M_n^{m,D})\rho_{n+1}\big) + R_{n+1},$$

the **associated policy $\boldsymbol{P}^D$**

$$P_n^D = (z_n, \kappa_n, \lambda_n, \delta_n) = \big(z(M_n^{m,D}), \kappa(M_n^{m,D}), \lambda(M_n^{m,D}), \delta(M_n^{m,D})\big)$$

is admissible, i.e. $\boldsymbol{P}^D \in \mathcal{P}(m)$. Thus generated policies are called **stationary**.

† ... sufficiently regular

"Optimal" decision functions always exist

Using dynamic programming techniques we have:

Result. There exists a decision function D^* such that the associated policy $\boldsymbol{P}^* = \boldsymbol{P}^{D^*}$ is optimal.[†] The corresponding evolution of capital is denoted by $\boldsymbol{M}^* = (M_n^*,\, n = 0, 1, \ldots)$.

[†] We omit the dependence of $\boldsymbol{P}^*$ on m.

Added value drives the firm's decisions

At a given capital level m, **added value** is defined as

$$AV(m) := \underbrace{V\left(m - z_0^* + \kappa_0^*\right) - \left(m - z_0^* + \kappa_0\right)}_{\substack{\text{added value of capital put to} \\ \text{productive use within the firm}}} - \underbrace{\left(\max\{-m, 0\} + C\left(\kappa_0^*\right)\right)}_{\substack{\text{cost of reaching the capital level} \\ m - z_0^* + \kappa_0^*}}$$

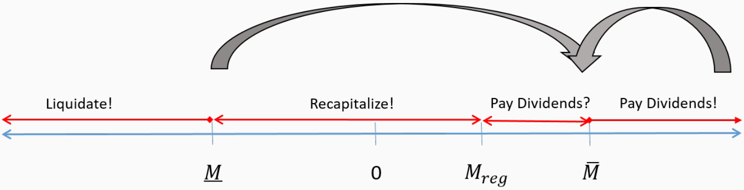
The mapping $m \mapsto AV(m)$ is increasing but not strictly increasing!

Golden rule. Capital that is not adding value should be paid back to shareholders and capital should only be raised if added value is positive!

Don't hold too much capital and, sometimes, liquidate!

1. There is and **upper-dividend barrier** $\overline{M} \geq M_{\text{reg}}$ such that AV is constant on $[\overline{M}, \infty)$ and $z(m) = m - \overline{M}$ for all $m \geq \overline{M}$.

2. There is a **liquidation barrier** $\underline{M} \leq M_{\text{reg}}$ such that $\delta(m) = 0$ if and only if $m < \underline{M}$, in which case $z(m) = \max\{m, 0\}$.



The if's and why's of investment in risky assets

The three sources of added value

$$EP(m) := \boldsymbol{q} - \underbrace{(1-\gamma)(m - z^* + \kappa^* + \boldsymbol{q})}_{\text{carry cost}}$$

$$- \underbrace{C(\kappa^*) + \min\{m, 0\}}_{\substack{\text{cost of reaching the capital level} \\ m - z^* + \kappa^*}} \qquad \textbf{(Economic Profit)}$$

$$DO(m) := \frac{1}{1+r}\mathbb{E}_{\mathbb{Q}^*}\left[\max\{-M_1^*, 0\}\right] \qquad \textbf{(Default Option)}$$

$$FV(m) := \frac{1}{1+r}\mathbb{E}_{\mathbb{Q}^*}\left[AV(M_1^*)\right] \qquad \textbf{(Franchise Value)}$$

$$AV(m) = EP(m) + DO(m) + FV(m).$$

What drives the investment strategy?

Investment risk

- has **no** impact on economic profit *EP*.
- typically has a **positive** impact on the default option *DO*.
- typically has a **negative** impact on the franchise value *FV*.

The optimal amount of investment risk **trades off** the changes in *DO* and *FV*.

Why the divergent academic opinions?

- Froot & Stein (1998) and Froot (2007) ignore *DO* but do capture the negative impact of investment risk on *FV*; therefore, they conclude that

  taking investment risk is **never** *optimal.*

- Azcue & Muler (2010) use a market-inconsistent setting, creating a bias towards risky investments; therefore, they conclude that

  taking some investment risk is **always** *optimal.*

So should insurers take investment risk?...it depends!

For $m \geq M_{\text{reg}}$, $\Lambda^*(m)$ is set of optimal investment strategies.

Assume non-trivial insurance risk ($L_n \not\equiv 0$. Then)

- $\rightarrow$ If $C = 0$ and insurance losses are "not too large", then $\Lambda^*(m) = [0, 1]$
- $\rightarrow$ If $C = 0$ and insurance losses are "sufficiently large", then $\Lambda^*(m) = \{1\}$.
- $\rightarrow$ If $C > 0$, then $\Lambda^*(m)$ is case specific.

Many thanks!