Dynamics of cash holdings and learning about profitability.

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This paper is about

Corporate cash management of firms that do not fully know their long-term profitability and face important external financing constraints (high-tech firms conducting intensive R&D, innovative activities)

- Almost entirely financed with cash flow or public share issues
- information problems, lack of collateral value of intangible assets (no debt)
- Cash reserves are specially important. When cash flow is exhausted, firms turn to new shares issues.
- Learning about profitability is key (better profitability prospects facilitate financing)

Brown, Fazzari and Petersen (2009), Hall and Lerner (2010), Graham and Leary (2018)

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Main issues

- What cash target levels ? What about their dynamics ?
- What to do with cash above cash target levels ? what payout ratios ?
- When to issue new shares, when to liquidate ?
- Dynamics of the trade-off between the benefits and the costs of holding cash in a changing environment.

Corporate life-cycle theory

- Young firms tend to hold more cash, pay little dividend, have limited access to external resources.
- Mature firms with well established profitability are more prone to pay dividends and have access to capital markets. Cash holdings decline when the firm goes towards maturity.

DeAngelo, DeAngelo and Stulz (2006, 2010), Drobetz, Halling and Schroder (2015).

Most of theoretical models

- remain silent on these stylized facts
- focus on liquidity issues assuming complete information about the firm's characteristics
- Little is known on the relationships between dynamics of cash holdings, dynamics of profitability prospects, dynamics of firm value.

Our model

- Dynamic model of corporate cash management with three key frictions: imperfect information, financing frictions, cost of cash
- Two concerns
  - Profitability concern (the risk to running a project that is not profitable)
  - liquidity concern (the risk to having to liquidate a profitable project).
- Characterize an optimal corporate policy that specifies at all times whether to continue the project, the issuance policy, the payout policy.
- Two-dimensional bayesian adaptive control problem.

Some take away results

- The model results in a two life-cycle stages for the firm. A "probation stage" in which the firm has no access to the capital markets and a ''mature stage", in which the firm has access to the capital markets.
- Cash target levels: continuous non-monotonic function of profitability prospects
- Payout ratio: non continuous, non monotonic function of profitability prospects.

Learning

- Cumulative cash flow process $(R_t)_{t\geq 0}$

$$dR_t = Y\,dt + \sigma\,dB_t.$$

- Firm's profitability prospects at time t

$$Y_t = \mathbb{E}[Y \mid \mathcal{F}_t^R].$$

- Y takes either of the two values $-\mu < 0 < \mu$.

$$dY_t = \frac{1}{\sigma}(\mu^2 - Y_t^2)dW_t, \qquad dW_t = \frac{1}{\sigma}(dR_t - Y_t dt).$$

Learning

- Present value of future cash flows:

$$\mathbb{E}\left[\int_0^\infty e^{-rs}\, dR_s\right] = \mathbb{E}\left[\int_0^\infty e^{-rs} Y_s\, ds\right] \leq \frac{\mu}{r}.$$

- The cumulative cash flow process is a sufficient statistic for the Bayesian updating.

$$dR_t = d\phi(Y_t),$$

where ϕ is the increasing function defined on $(-\mu, \mu)$ by $\phi(y) = \dfrac{\sigma^2}{2\mu} \ln\left(\dfrac{\mu + y}{\mu - y}\right)$.

The shareholders' problem

- Cash reserves process $(X_t)_{t\geq 0}$

$$dX_t = dR_t + \frac{dI_t}{p} - dL_t$$

- The shareholders' problem: For all $(x, y) \in [0, \infty) \times (-\mu, \mu)$

$$V(x, y) = \sup_{I,L\in\mathcal{A}} \mathbb{E}\left[\int_0^{\tau_0} e^{-rs}\,(dL_s - dI_s)\right].$$

$$X_t = \phi(Y_t) - \phi(y) + x + \frac{I_t}{p} - L_t$$

$$Y_t = y + \frac{1}{\sigma}\int_0^t (\mu^2 - Y_s^2)\,dW_s$$

$$\tau_0 = \inf\{t \geq 0 : X_t = 0\}.$$

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$\tau_0 = \inf\{t \geq 0 \,:\, X_t = 0\}$.

Benchmark 1: no financing frictions ($p = 1$)

$$dX_t = Y_t dt + \sigma dW_t + dI_t - dL_t$$

The shareholders' problem:

$$\hat{V}(x, y) = x + \sup_{\tau \in \mathcal{T}^R} \mathbb{E}\left[\int_0^\tau e^{-rs} Y_s ds\right].$$

$$\tau^* = \inf\{t \geq 0 \,:\, Y_t \leq y^*\}.$$

Benchmark 2: complete information ($Y = \mu$)

$$dX_t = \mu dt + \sigma dW_t + \frac{dI_t}{p} - dL_t$$

The shareholders' problem:

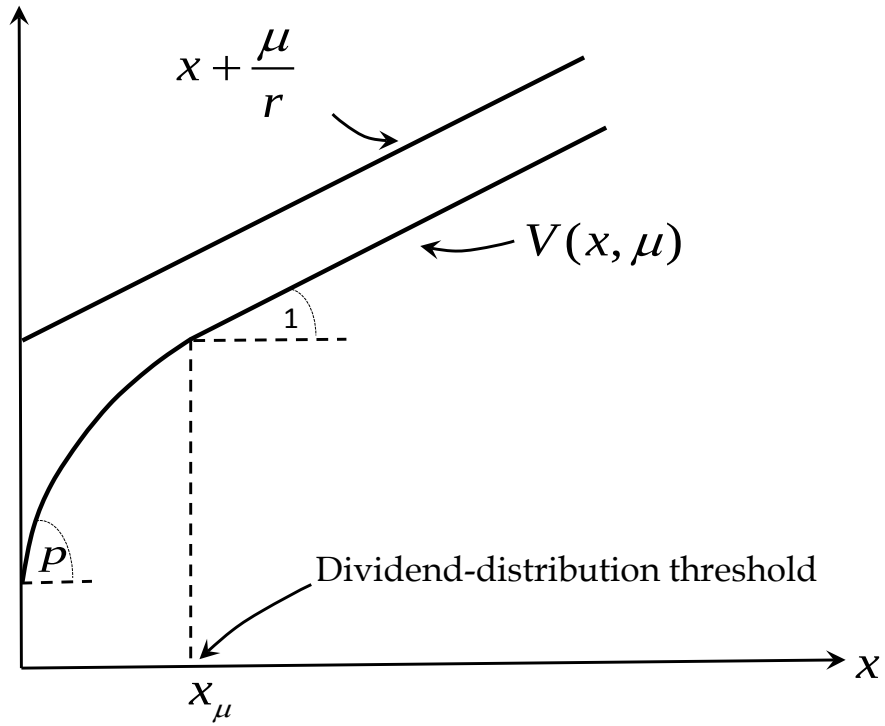
$$V_\mu(x) = \sup_{I,L \in \mathcal{A}} \mathbb{E}\left[\int_0^{\tau_0} e^{-rs}\left(dL_s - dI_s\right)\right].$$

$\tau_0 = \inf\{t \geq 0 \,:\, X_t = 0\}$.

Lokka and Zervos (2009)

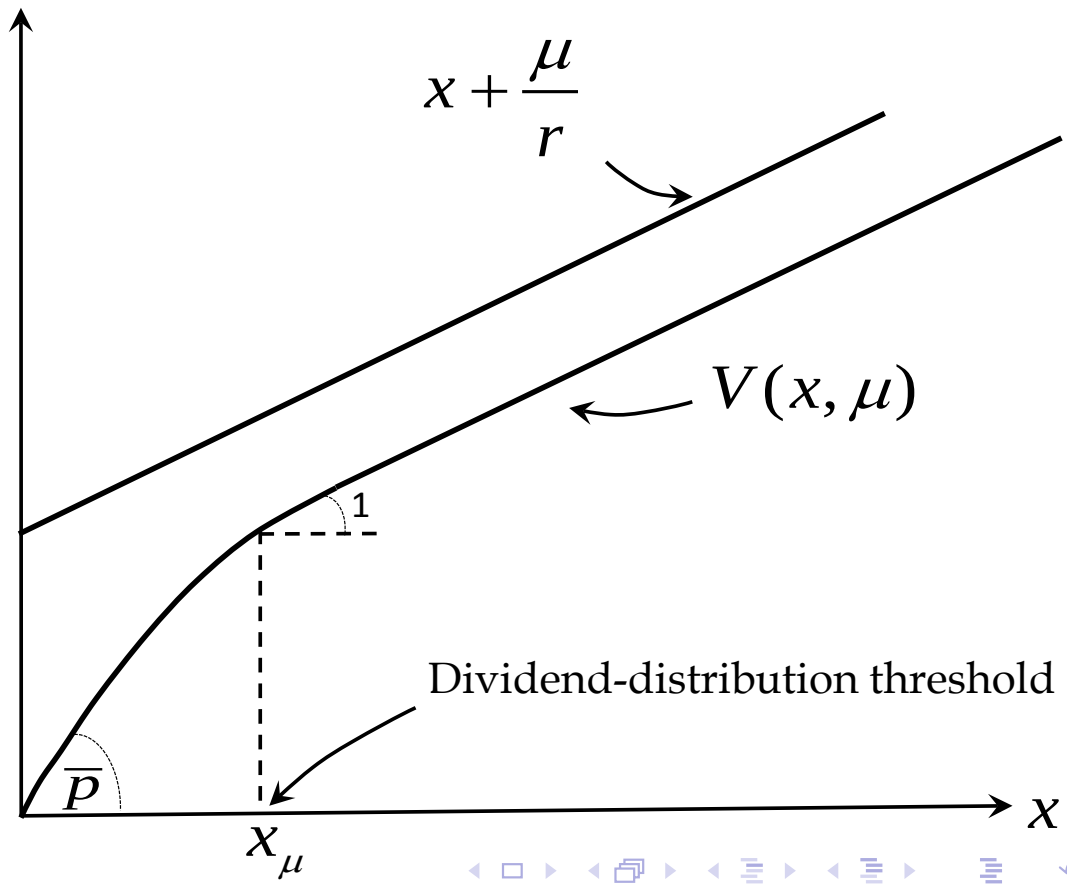
Benchmark 2: complete information ($Y = \mu$)

Case: $1 < p < \bar{p}$



Benchmark 2: complete information ($Y = \mu$)

Case: $p > \bar{p}$



Financing frictions and imperfect information

$$X_t = \phi(Y_t) - \phi(y) + x + \frac{I_t}{p} - L_t$$

$$Y_t = y + \frac{1}{\sigma}\int_0^t (\mu^2 - Y_s^2)\, dW_s$$

$$V(x,y) = \sup_{I,L\in\mathcal{A}} \mathbb{E}\left[\int_0^{\tau_0} e^{-rs}\,(dL_s - dI_s)\right].$$

$\tau_0 = \inf\{t \geq 0 \,:\, X_t = 0\}.$

Heuristics

- Dividend policy
  - Corporate cash policy, in terms of cash target levels, changes as the firm learns about its profitability.
  - The cash target level is not a constant, but a function of the profitability prospects ($V_x(b(y), y) = 1$)
- Issuance policy
  - If $p \geq \bar{p}$ firm liquidated when cash is depleted
  - If $1 < p < \bar{p}$ Issuance should require sufficiently high level of profitability prospects: $V(0, y) = 0$ for $y \leq y_i$ and $V_x(0, y) = p$ for $y \geq y_i$

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Heuristics ($1 < p < \bar{p}$).

Find a function V, a function b, and a constant y_i that solve variational system

$$\frac{1}{2\sigma^2}(\mu^2 - y^2)^2 V_{yy} + \frac{1}{2}\sigma^2 V_{xx} + (\mu^2 - y^2)V_{xy} + yV_x - rV = 0 \quad (1)$$

$$\text{on the domain } \{(x, y),\ 0 < x < b(y),\ -\mu < y < \mu\},$$

$$V(0, y) = 0 \quad \forall y \in (-\mu, y_i], \quad (2)$$

$$V_x(0, y) = p \quad \forall y \in [y_i, \mu), \quad (3)$$

$$V_x(x, y) = 1, \text{ for } x \geq b(y), \quad (4)$$

$$V_{xy}(b(y), y) = 0, \quad (5)$$

$$\lim_{y \longrightarrow \mu} V(x, y) = V_\mu(x). \quad (6)$$

A change of variable

$$X_t = \phi(Y_t) - \phi(y) + x + \frac{I_t}{p} - L_t$$

$$Z_t = \phi(Y_t) - X_t = L_t - \frac{I_t}{p} + (\phi(y) - x)$$

- The process Z increases whenever the firm reaches a cash target level.
- The process Z decreases whenever the firm issues new shares.
- Between issuance and payment dates, the process Z remains constant.

We solve the shareholders' problem in the (z, y)-space

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The variational system in the (z,y) space.

Find a function U, a function k, and a constant z_i that solve variational system

$$\frac{1}{2\sigma^2}(\mu^2 - y^2)^2 U_{yy}(z,y) - rU(z,y) = 0$$

$$\text{on the domain } \{(z,y),\ z \in \mathbb{R},\ \phi^{-1}(z) < y < k(z)\},$$

$$U(z, \phi^{-1}(z)) = 0 \quad \forall z \leq z_i,$$

$$U_z(z, \phi^{-1}(z)) = -p \quad \forall z \geq z_i,$$

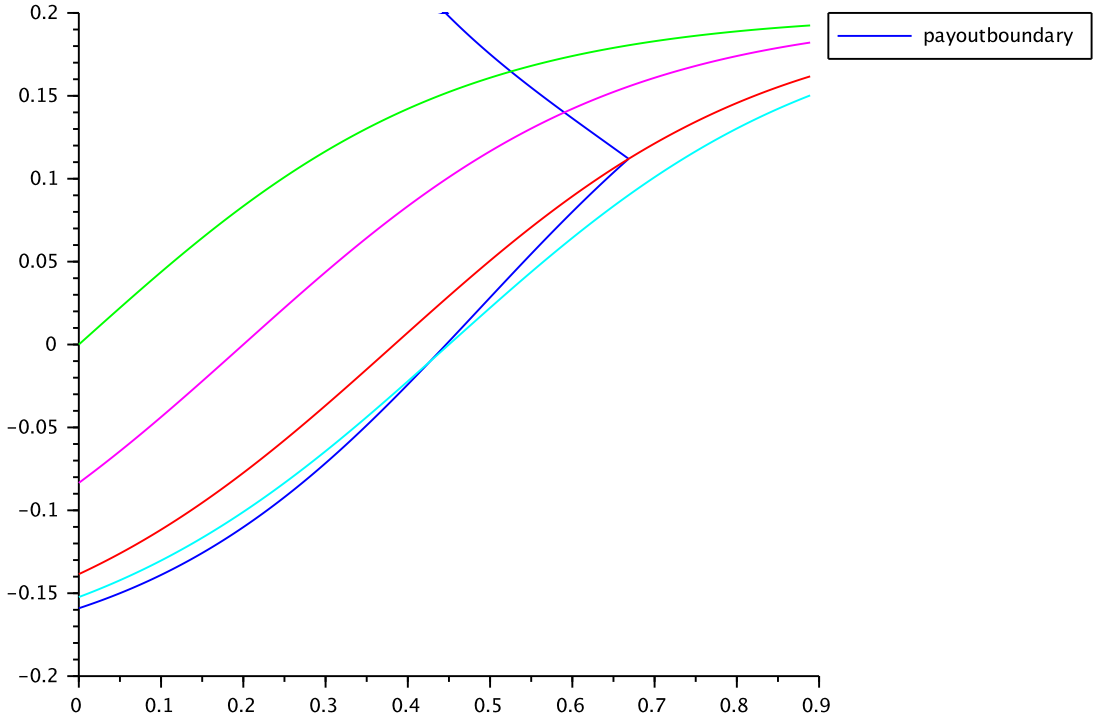
$$U_z(z,y) = -1, \text{ for } k(z) \leq y,$$

$$U_{xy}(z, k(z)) = 0,$$

$$\lim_{z \longrightarrow \infty} U(z, \phi^{-1}(x+z)) = V_\mu(x),$$

$$U(z,y) = V(\phi(y) - z, y),\ \ k(z) = (\phi - b)^{-1}(z),\ \ z_i = \phi(y_i).$$

The payout boundary function $y \longrightarrow b(y)$



Parameters: $r = 0.1$, $\sigma = 0.3$, $\mu = 0.2$, $p = 1.5$.
Performance indexes $z = \phi(y) - x$ for $z = -0.48$, $z = -0.19$, $z = 0$,
$z_i = \phi(y_i) = -0.38$, $y_i = -0.14$.
$\max_y b(y) = 0.67$, $x_\mu = 0.44$.

Non monotonic relationship between cash target levels and profitability prospects

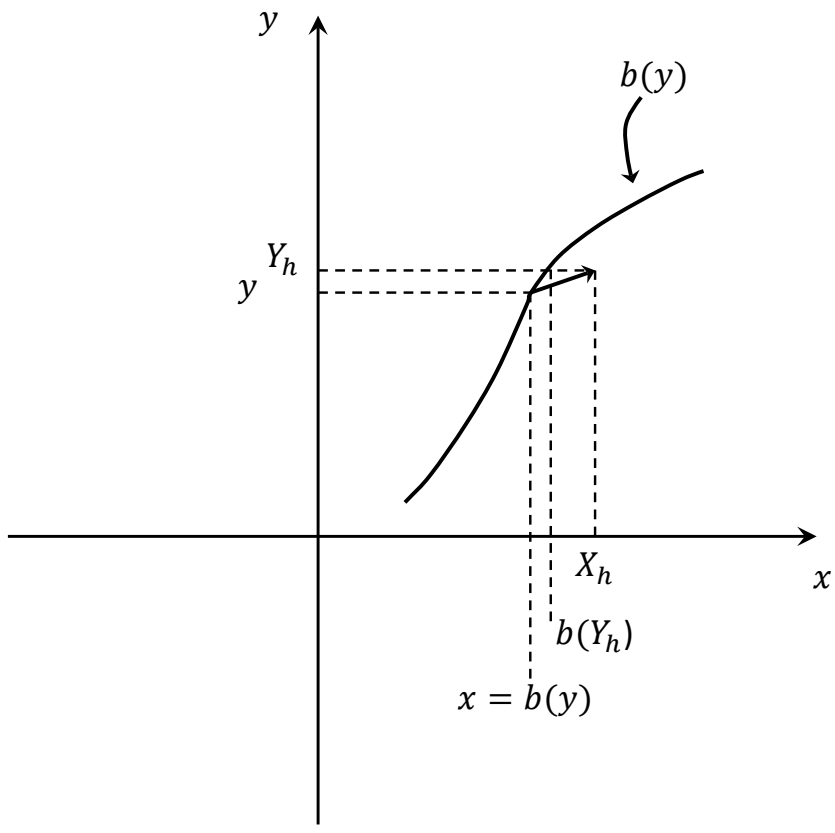
Two effects:

- Positive shocks on earnings increase profitability prospects and can induce firm's management to lower cash target levels because accumulating cash is costly.
- A firm has more to lose from liquidity constraints when profitability prospects increase. This can induce firm's management to increase cash target levels.

Payout ratio

- What a firm does with a marginal \$1 when it is exactly at its target level of cash?
- In standard model with complete information about profitability, the firm pays out \$1 as a dividend
- Not here! The firm can save a fraction of the marginal \$1 and pays out the remainder. The fraction paid out depends on the profitability prospects and defines the payout ratio of the firm.

Payout ratio



$$\mathrm{p}(y) = \frac{X_h - b(Y_h)}{X_h - x}$$

Payout ratio and Saving function

$$p(y) = 1 - b'(y)\frac{\mu^2 - y^2}{\sigma}$$

$$s(y) = b'(y)\frac{\mu^2 - y^2}{\sigma}$$

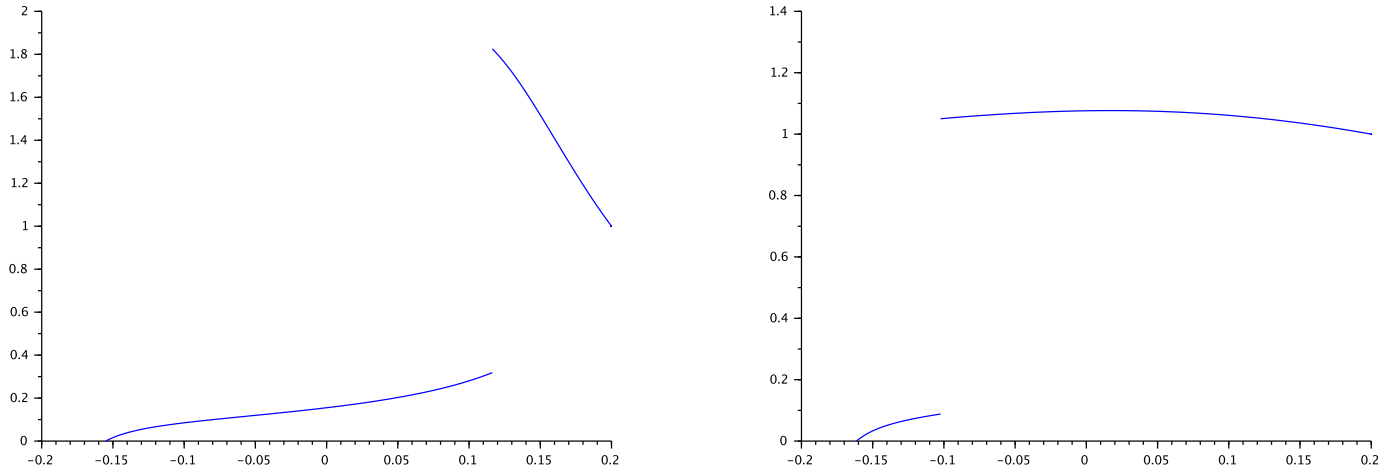


Figure: The payout ratio as a function of profitability prospects. (Left) The parameters are $r = 0.1$, $\sigma = 0.3$, $\mu = 0.2$ and $p = 1.5$. (Right) The parameters are $r = 0.1$, $\sigma = 0.3$, $\mu = 0.2$ and $p = 1.05$

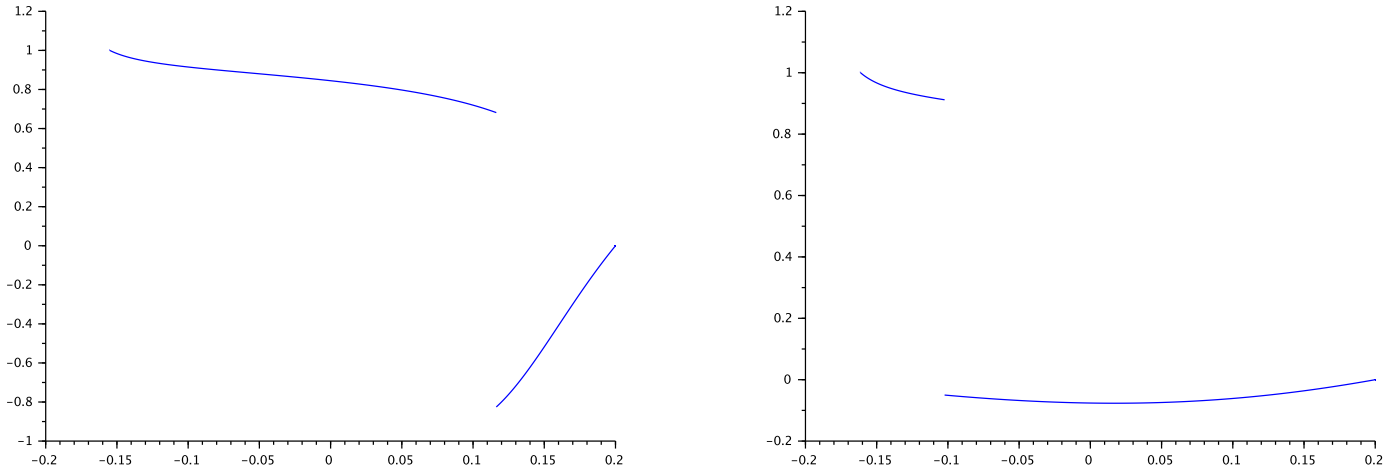


Figure: The saving function. (Left) The parameters are $r = 0.1$, $\sigma = 0.3$, $\mu = 0.2$ and $p = 1.5$. (Right) The parameters are $r = 0.1$, $\sigma = 0.3$, $\mu = 0.2$ and $p = 1.05$

Conclusion

- Simple theoretical model where a corporate life-cycle dynamics of cash holdings stems from the optimal equity issuance and payout policies.
- Dynamics of cash holdings, dynamics of firm value are drastically different in the different stages of the corporate life-cycle.