

# Multi-product Supply Function Equilibria\*

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## Abstract

We solve for Nash equilibria in a procurement auction with two heterogeneous divisible goods. There are (dis)economies of scope in production and goods could be substitutes or complements for the procurer. Before demand is realized, each firm offers a vector of supply functions where supply of a good depends on the prices of both goods, which is similar to the product-mix auction and electricity markets with complex bids. We show that outcomes are not influenced by bundling of the goods. For quadratic costs and linear demand, we can use this property to transform the multi-product problem into an equivalent set of separated single-good markets, which can be analysed independently. We show that eigenvalues of Lerner and pass-through tensors are fundamental measures of multi-product market performance that do not depend on bundling. We also derive welfare for the linear equilibrium.

**Keywords:** Supply function equilibrium, multiproduct pricing, divisible-good auction, wholesale electricity market, bundling

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