Measuring Prosumer Welfare: Modelling Household Demand for Distributed Energy Resources and Residual Electricity Supply

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Abstract

Households are increasingly investing in distributed energy resources (DERs) such as rooftop photovoltaic (PV) solar panels and high-capacity batteries. Such households become "prosumers", effectively competing with traditional energy suppliers by producing some of – or even more than – their energy needs. This extends conventional "household production" by enabling households to self-produce a primary input (i.e. electricity), not just to combine third-party inputs with household investments (e.g. in electrical appliances) to produce household services. It also complicates welfare measurement, which is essential for antitrust, regulatory, climate change and distributional assessments of the likely unequal uptake of such new technologies. This paper jointly models the impact of DERs on household electricity demand and the underlying demand for DER investments from microeconomic first principles. It provides theoretically-supported representations of DER and residual electricity demand, and prosumer welfare, to support antitrust, regulatory and climate change analyses.

JEL Classifications: D13, L51, L94, Q41, Q42.

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1 Introduction

It has long-been recognised that consumers do not demand electricity as an end consumer product. Instead, electricity demand is a derived demand, conditioned by users' investments in electrical appliances (e.g. Hausman (1979), Dubin and McFadden (1984)). Having invested in appliances, electricity users combine electricity and other inputs (e.g. water, or personal labour) to produce the services they ultimately desire (e.g. Davis (2008)).[1] An important trade-off in this choice is between the capital costs of more energy-efficient appliances, and their lower operating costs.

Distributed technologies for generating and storing electricity – collectively referred to as distributed energy resources (DERs) – include both rooftop photovoltaic (PV) solar panels and home-scale batteries (including electric vehicles, EVs, which represent portable storage). The falling costs of such technologies have accompanied their rapid uptake at the household level. For example, La Nauze (2018) reports that PV uptake in the United States has grown at 50% per annum, with 5% of dwellings in California having rooftop panels (the same penetration rate as Germany). Meanwhile, 15% of Australian households had rooftop solar by 2015.

Household investments in DERs have the potential to fundamentally alter the nature of household electricity demand. They are analogous to investments in energy-efficient appliances, in that they reduce the electricity demanded when producing a given level of household services such as lighting or heating. However, they do so for all electric appliances simultaneously, rather than for just selected energy-efficient appliances. More significantly, they offer the potential to transform a household from a pure consumer into a net producer of electricity, should their investment in DER capacity be sufficient to exceed their own consumption requirements.

This means households that invest in DERs may – under certain circumstances – become "prosumers" (i.e. they are, depending on circumstances, net producers and/or net consumers of a service conventionally supplied by firms). They potentially compete with traditional generators and electricity transporters (e.g. electricity distributors) by self-supplying some of their own energy demand. Indeed, they can become net producers of electricity, selling their surplus production to others (thereby relaxing their household budget constraint, and affecting overall consumption patterns). DER invest-

[1]A simple example is combining electricity, water, soap powder, labour and a washing machine to produce clean clothes. These inputs are not inherently demanded by electricity users – indeed, using labour represents the sacrifice of leisure time, or of time that could be used to earn income for funding other consumption items. Instead, demand for these inputs is derived from the users' desire for clean clothes.

ment also means that such households might provide complementary services, such as network reinforcement during periods of peak demand and tight distribution capacity, while representing traditional load at other times. This chameleon-like quality of DERs complicates analysis of their antitrust, regulatory, climate change and distributional (i.e. as between different customer classes) implications. As stated by Castagneto-Gissey et al. (2018, p. 784), in relation to batteries:

> "There is a fundamental question about the role of storage which remains unanswered, in whether it provides an add-on service, in competition on the margin with networks and generation, or whether it instead complements networks and generation."

This paper contributes to addressing this fundamental question by modelling a household's demand for DER investments using microeconomic foundations. Conditional on such investments, a household's residual demand for electricity is also modelled. We use the random utility framework from the demand literature (e.g. Train (2009)) to translate households' discrete choices as to invest or not in DER capacity into demand for DER capacity that is continuous in key parameters. Finally, we derive expressions for measuring prosumer welfare, which are critical for any policy analysis of unequal DER uptake by different parties.

Notably, income effects are found to play a key role in affecting DER demand. DER demand might be predicted to be decreasing in DER "price" (i.e. per-period rental cost), and increasing in each of electricity price, DER productivity and DER capacity. However, these results are found to be sensitive to the assumed form of utility. Specifically, with quasi-linear utility in which income effects are suppressed, DER demand responds to each of these variables as predicted. However, with Cobb-Douglas utility, in which income and substitution effects are offsetting, DER demand responds to each of these variables with the opposite signs.

More generally, DER investment is shown to "contract" electricity demand by that investment's output (i.e. its productivity times its capacity). DER investment also reduces electricity demand through effectively reducing household income by DER capital cost. Finally, we derive both social welfare, and the profit function of a monopolist DER supplier based on our modelled DER demand.

These tools pave the way for strategic and regulatory analyses of DER investments. For example, they enable an analysis of the impacts on consumer welfare of different parties making DER investments (e.g. households, generators or distributors). This is not just by highlighting how consumer surplus is affected by DER investment, but also how DER investment choices will differ between different parties. Significantly, generators and distributors are likely to face differing strategic incentives to invest in DERs, depending on the extent to which those technologies are net substitutes or net complements for their existing activities.

Furthermore, households will have differing capacity to invest in DERs (e.g. due to differences in home tenure), and face different incentives for DER investments depending on strategic choices by generators and distributors, or by regulators (e.g. of distributors). Finally, how households change their energy usage and net demand/supply in response to DER uptake and market prices will have important implications for policies intended to reduce greenhouse gas emissions. Accounting for such changes will increasingly become necessary in formulating and assessing climate change policies. Analyses of these types are left to future work.

Existing studies of the strategic implications of DERs are limited, and use only simple characterisations of household demand. For example, Sioshansi (2014) assumes linear demand for electricity when modelling the strategic effects of storage. Conversely, Munoz-Alvarez et al. (2017) posit a general surplus function for consumers when modelling the strategic impacts of DERs for different types of DER owner, without relating that surplus to microeconomic foundations. This paper contributes to this emerging literature by providing such foundations.

Closest in spirit to this paper is De Groote and Verboven (2018), who model households' choices over DER (specifically PV) adoption in terms of the present value of expected net cost savings (i.e. upfront capital outlays less the present values of subsidies and savings from reduced energy purchases). However, unlike this paper, they do not jointly model households' choices over both DER adoption and energy demand contingent on such adoption. To our knowledge we are the first to model this joint decision using microeconomic foundations.[2]

This paper is structured as follows. Section 2 models household's utility maximisation problem when combining the consumption of market goods and self-produced household services that consume electricity. Section 3 uses this framework to derive a household's optimal residual derived demand for supplied electricity, conditional on its investments in DER capacity and electric appliances. It does so in the general case, and for simpler specific cases, and then derives a household's demand for DER capacity anticipating how that capacity affects the household production problem. Section 4 then dis-

[2]La Nauze (2018) presents evidence that Australian households with PV investments respond to income changes created from net energy sales differently to changes in income from other sources. In a different spirit to this paper, she offers behavioural explanations for this phenomenon (i.e. mental accounting and category budgeting).

cusses some illustrative applications of these demand derivations, including monopoly DER supply. Section 5 concludes, including a discussion of limitations of this study, and likely useful extensions.

2 Model

We model the following sequence of household choices:

1. Conditional on existing household appliance investments, households choose their preferred level of investment in DER capacity; and
2. Conditional on both appliance and DER investments, households then choose their utility-maximising mix of electricity-consuming household services and other consumption goods and services.

It is from these choices that household electricity demand can be determined as a derived demand. For notational convenience we suppress household index i, but introduce it in Section 3.2 where doing so is more necessary.

We modify Davis (2008), who applies the original household production problem introduced by Becker (1965) and Lancaster (1966) to the problem of appliance choice and electricity demand. Electricity demand is denoted by x. The consumption of electricity-consuming household services is z_1, while the consumption of a composite other good is denoted z_2. Given a household's existing investment Φ in a stock of electricity-consuming appliances (with Φ assumed exogenous), the household's problem is to choose the level of DER investment yielding maximal utility:

$$\max_{j \in 1,\dots J} \{V(K_1;\Phi),\dots,V(K_J;\Phi)\} \tag{1}$$

where $V(K_j;\Phi)$ is an indirect utility function conditional on Φ and level of DER capacity K_j. In turn, $V(.)$ results from the household's utility maximisation problem:

$$V(K_j;\Phi) = \max_{\{x, z_2\}} U(z_1, z_2) \tag{2}$$

subject to the constraints:

$$z_1 = f(x;\Phi) \tag{3}$$

$$p(x - \gamma K_j) + z_2 = y - rK_j \tag{4}$$

A household chooses electricity and composite good consumption so as to maximise its utility from consuming the composite good and electricity-consuming household services. Constraint (3) represents how the household's given stock of electrical appliances can be combined with electricity to produce those services. Conversely, constraint (4) represents the household's budget constraint, given exogenous income y, and with p being the retail price of purchased electricity, which in turn is $x - \gamma K_j$. Without loss of generality, we assume that the price of the composite good is normalised to one.[3]

Net electricity purchases at price p are represented by electricity consumption x less self generation γK_j, where γ is a technical parameter reflecting the productivity of DER capacity K_j (i.e. the rate at which K_j units of DER capacity produce electricity). Since households will have differing roof areas and orientations, and different locales will have different sunshine patterns, it should be expected that γ will vary by household. The marginal cost of self-generation is assumed to be zero. We do not constrain x to exceed own-production capacity γK_j, but instead simply assume that any generation in excess of own-consumption earns the price p, as often the case with "net metering".[4] This assumption is further motivated by the emergence of peer-to-peer trading of surplus household energy production, since this enables households with surplus production to sell it at the prevailing retail price (even if, by regulation or otherwise, electricity retailers offer some lower price when buying excess household generation).

The right-hand side of the budget constraint deducts an assumed per-period capital charge rK_j, representing the cost of owning DER capacity K_j.[5] Hence, DER investment K_j respectively reduces both net electricity purchases and effective household income.[6] We assume rental rate r and electricity price p are common to all households.

[3] For simplicity, we suppress the household's time allocation problem, assuming instead that electricity-consuming household services do not require labour inputs, and therefore create no work-leisure trade-offs in the household's budget constraint.

[4] We leave it to an extension to model the situation in which excess self-generation earns some price other than p – e.g. some subsidised higher price, or some lower price such as the wholesale electricity price. Time-of-use and household-differentiated pricing are other useful extensions left to future work.

[5] For example, this could represent the per-period cost of leasing K_j.

[6] I.e. committing to purchase DER capacity K_j requires the household to make a per-period commitment to expend rK_j, leaving only $y - rK_j$ to spend on x and z_2.

3 Solution

3.1 Conditional Derived Demand for Electricity and Associated Conditional Welfare

3.1.1 General Case

Constraint (3) can be directly substituted into (2) for z_1, while constraint (4) can be solved for z_2 before substitution in (2). Doing so simplifies utility maximisation problem (2) subject to constraints (3) and (4) into the following unconstrained and univariate maximisation, conditional on appliance choice Φ and DER capacity choice K_j:

$$V\left(K_j;\Phi\right) = \max_{x} U\left(f\left(x;\Phi\right), y - rK_j - p\left(x - \gamma K_j\right)\right) \tag{5}$$

Taking the household's first order condition with respect to x, electricity demand – conditional on K_j and Φ – is $x^*\left(p, r; K_j, \Phi, y, \gamma\right)$ defined implicitly by:

$$U_1'\left(x; p, r; K_j, \Phi, y, \gamma\right) f'\left(x;\Phi\right) - U_2'\left(x; p, r; K_j, \Phi, y, \gamma\right) p = 0 \tag{6}$$

As noted earlier, p and r are assumed common to all households, while all other terms (K_j, Φ, y, and γ) are household-specific. Since this is *total* household-level conditional demand for electricity including self-generation, the household's *net* conditional electricity demand X^* from external supply (e.g. traditional electricity retailers) – whether positive or negative – is:

$$X^*\left(p, r; K_j, \Phi, y, \gamma\right) = x^*\left(p, r; K_j, \Phi, y, \gamma\right) - \gamma K_j \quad \lesseqgtr 0 \tag{7}$$

Assume a mass M of consumers, proportion θ of whom cannot install DERs (e.g. because they do not own their home, or have no roof space for PV panels), and denote distribution functions as $F\left(.\right)$.[7] The *market-level* conditional demand for *supplied* electricity $\tilde{X}^*$, as faced by other suppliers, is then:

$$\begin{aligned}\tilde{X}^*\left(p, r; M, \theta\right) =& M\theta \int x^*\left(p; y, \Phi\right) dF_y\left(y\right) dF_\Phi\left(\Phi\right) \\ &+ M(1-\theta) \int X^*\left(p, r; K_j, \Phi, y, \gamma\right) dF_K\left(K\right) dF_\Phi\left(\Phi\right) dF_y\left(y\right) dF_\gamma\left(\gamma\right)\end{aligned} \tag{8}$$

[7]We assume incomes, appliance choices and DER investments are independent for expositional convenience only. In practice they are likely to be highly correlated and hence jointly distributed.

noting that K_j is treated as being given when deriving conditional electricity demand.[8]

Noting that $K_j \equiv 0$ for those households who cannot install DERs, and denoting $x^*(p, r; K_j, \Phi, y, \gamma)$ as $x^*(.)$, a household's indirect utility conditional on K_j and Φ (and remaining parameters) is then given by:

$$V(p, r; K_j, \Phi, y, \gamma) = U(f(x^*(.); \Phi), y - rK_j - p(x^*(.) - \gamma K_j)) \quad (9)$$

Finally, assuming a standard utilitarian framework, social welfare conditional on household DER investment can be defined in terms of the weighted average utility of each household:

$$\begin{aligned} W(p, r; M, \theta) = & M\theta \int U^*(.) \, dF_y(y) \, dF_\Phi(\Phi) \\ & + M(1-\theta) \int U^*(.) \, dF_K(K) \, dF_\Phi(\Phi) \, dF_y(y) \, dF_\gamma(\gamma) \quad (10) \end{aligned}$$

$$U^*(.) \equiv U(f(x^*(.); \Phi), y - rK_j - p(x^*(.) - \gamma K_j)) \quad (11)$$

Standard measures of consumer surplus are not meaningful for derived demands such as that for electricity, since it is the total household utility from electricity-consuming services that is relevant. Consumer surplus for net electricity demand is also of limited interest, since that ignores the utility a household derives from self-generation. Hence, the approach here is to measure social welfare directly from household utility functions. This more adequately measures the welfare produced by electricity when consumed as an input to the production of other, inherently-demanded household services. It also captures the welfare gains of both self-generated and purchased electricity.

3.1.2 Cobb-Douglas Case

One particular case is that in which both production technology (3) and the unconstrained utility function take constant returns to scale Cobb-Douglas form (assuming $\alpha, \beta \in [0, 1]$):

$$z_1(x; \Phi) = \Phi^\alpha x^{1-\alpha} \quad (12)$$

$$\begin{aligned} U(z_1(x; \Phi), z_2(x; K_j)) = & \beta ln\left(\Phi^\alpha x^{1-\alpha}\right) \\ & + (1-\beta) \, ln\left((y - rK_j) - p(x - \gamma K_j)\right) \quad (13) \end{aligned}$$

[8] Equation (8) can be written unconditionally using K^* derived from the household's DER investment problem solved in Section 3.2.

Taking the first order condition with respect to x in (13) and then solving for x yields the following form of conditional derived demand for electricity:

$$x^* (p, r; K_j, \Phi, y, \gamma) = \frac{\beta (1-\alpha)}{1-\alpha\beta} \left[\gamma K_j + \frac{(y - rK_j)}{p} \right] \quad (14)$$

Thus DER capacity K_j plays offsetting roles in a household's utility-maximising conditional derived demand for electricity. On the one hand it reduces the household's effective purchasing power due to the DER rental charge rK_j. Offsetting this effect, however, is the household's demand contraction at all prices, γK_j, due to being able to self-generate that amount at zero marginal cost. It is easily verified that conditional derived electricity demand (14) is increasing in K_j, decreasing in DER rental rate r, but only decreasing in retail electricity price p if $y > rK_j$.

Indirect utility $V (p, r; K_j, \Phi, y, \gamma)$ is derived as usual by substituting (14) in (13). After some algebra it can be shown that this takes the following convenient form, where A does not depend on K_j:[9]

$$V (p, r; K_j, \Phi, y, \gamma) = A - (\alpha\beta - 1)\, ln\, ((\gamma p - r) K_j + y) \quad (15)$$

3.1.3 Quasi-Linear Case

An even simpler specific case is that in which z_1 is proportional in x and Φ, and utility is quasi-linear in z_2:

$$z_1 = \Phi x \quad (16)$$

$$U (z_1, z_2) = z_2 + ln\,(z_1) \quad (17)$$

Substituting (16) in (17), and concentrating out z_2 using (4) as before, unconstrained utility writes as:

$$U (x; p, r; K_j, \Phi, y, \gamma) = (y - rK_j) - p (x - \gamma K_j) + ln\,(\Phi x) \quad (18)$$

In this case household-level conditional total electricity demand takes the trivial, but highly-tractable, unit iso-elastic form (devoid of both the income effect of DER capacity, rK_j, and its demand-contracting effect, γK_j):

$$x^* (p, r; K_j, \Phi, y, \gamma) = \frac{1}{p} \quad (19)$$

Indirect utility therefore takes the convenient form:

$$V (p, r; K_j, \Phi, y, \gamma) = K_j (\gamma p - r) + ln \left(\frac{\Phi}{p} \right) + y - 1 \quad (20)$$

[9]This proves useful later, when we derive households' choice probabilities for DER investments K_j. This is because terms such as A which do not depend on K_j are eliminated when a given household compares indirect utilities from different K_j choices.

3.2 Continuous Demand for Households' Discrete DER Capacity Choices

Modelling household residual electricity demand, conditional on DER capacity and electrical appliance choices, provides a clearer conceptual foundation for any antitrust, regulatory or distributional analyses where uneven household uptake of DERs is of interest. However, the important question remains as to how households make DER capacity choices, anticipating how those choices translate into optimal household service production choices, and hence household electricity demand. Knowledge of both types of household choice is therefore a necessary precondition for any antitrust, regulatory or distributional analyses based on solid micro-foundations.

Any such analyses would benefit from convenient functional forms for DER demand. In particular, each household's discrete choice regarding a particular DER capacity investment (including non-investment) would usefully be aggregated into a functional form continuous in DER cost. We illustrate how to do so here using the random utility approach from the discrete choice literature (e.g. Train (2009)). We begin with the Cobb-Douglas case analysed above, and then also the simpler, quasi-linear case.

3.2.1 Cobb-Douglas Case

To begin, we assume that household i's indirect utility function, conditional on its DER capacity choice K_{ij} and appliance choice Φ_i, is an extended version of (15):

$$V_i\left(p, r; K_{ij}, \Phi_i, y_i, \gamma_i\right) = A_i - \left(\alpha\beta - 1\right) ln\left(\left(\gamma_i p - r\right) K_j + y_i\right) + \epsilon_{ij} \tag{21}$$

In this specification, we assume that household i's utility from discrete DER capacity choice K_j includes the random utility component ϵ_{ij} which is iid Type I Extreme Value. While this formulation conveniently yields a continuous demand for $j = 1, \dots, J$ discrete levels of DER capacity, here we show this for just two capacity levels: $K_1 = 0$ and $K_1 = K$. Thus, for illustrative purposes, household i is assumed to choose between having fixed DER capacity K, or no DER capacity at all:

$$V_{i1} \equiv V_i\left(p, r; K_{i1} = 0, \Phi_i, y_i, \gamma_i\right) = A_i - \left(\alpha\beta - 1\right) ln\left(y_i\right) + \epsilon_{i1} \tag{22}$$

$$V_{i2} \equiv V_i\left(p, r; K_{i2} = K, \Phi_i, y_i, \gamma_i\right) = A_i - \left(\alpha\beta - 1\right) ln\left(\left(\gamma_i p - r\right) K + y_i\right) + \epsilon_{i2} \tag{23}$$

Using (22) and (23), and noting that terms unrelated to choice j cancel, the probability that household i chooses to install DER capacity K is therefore:

$$\begin{aligned} P_{i2} \equiv & P\left(V_{i1} < V_{i2}\right) \\ = & P\left(\epsilon_{i1} - \epsilon_{i2} < \left(\alpha\beta - 1\right) ln\left(\left(\gamma_i p - r\right) K + y_i\right) - \left(\alpha\beta - 1\right) ln\left(y_i\right)\right) \end{aligned} \tag{24}$$

Since the ϵ_{ij} are iid Type I Extreme Value, their difference is distributed as logistic. Hence the probability that household i chooses DER capacity K is thus (following Train (2009), pp 38-40 and pp 74-75):

$$P_{i2} = \frac{1}{1 + e^{\alpha\beta - 1}\left(1 + \frac{(\gamma_i p - r)K}{y_i}\right)} \tag{25}$$

Total DER demand of all households in this case is thus (recalling that only proportion $(1-\theta)$ of mass M of households can install DERs):[10]

$$K^*\left(r; M, \theta\right) = \int \frac{M\left(1-\theta\right)}{1 + e^{\alpha\beta - 1}\left(1 + \frac{(\gamma_i p - r)K}{y_i}\right)} dF_y\left(y\right) dF_\gamma\left(\gamma\right) \tag{26}$$

Note that y_i and γ_i survive differencing in (24), due to being interacted with K_{ij} in $V_{ij}(.)$. This means these parameters survive in the resulting choice probabilities – as does Φ_i implicitly – and hence the need for integrating in (26).

As desired for antitrust, regulatory and distributional analyses, DER demand is a continuous function of rental cost r. This is despite the underlying household choices being discrete – i.e. between installing DER capacity K, or no DER capacity at all.

However, in contrast to the quasi-linear case presented below in which income effects are absent, the derivatives of K^* with respect to rental rate r and retail electricity price p in this Cobb-Douglas preferences case are *positive* and *negative* respectively. More precisely:

$$\frac{\partial Intergand\left(K^*\left(.\right)\right)}{\partial r} = \frac{M\left(1-\theta\right) e^{\alpha\beta-1} K y_i}{\left(e^{\alpha\beta-1}\left(K\left(\gamma_i p - r\right) + y_i\right) + y_i\right)^2} > 0 \tag{27}$$

$$\frac{\partial Intergand\left(K^*\left(.\right)\right)}{\partial p} = -\frac{M\left(1-\theta\right) e^{\alpha\beta-1} K \gamma_i}{\left(1 + e^{\alpha\beta-1}\left(1 + \frac{(\gamma_i p - r)K}{y_i}\right)\right)^2 y_i} < 0 \tag{28}$$

[10] Clearly some of the proportion θ of households who cannot install DER capacity for whatever reason may in fact have high DER productivity (if only they could install DERs). Conversely, some of the proportion $(1-\theta)$ of households that can install DERs may have low DER productivity. Here we analyse the DER investment decisions of only those households that can install DERs.

Likewise in contrast to the quasi-linear case below, it can be easily verified that the integrand of (26) is *decreasing* in DER productivity rate γ, and also *decreasing* in DER capacity K when DER savings exceed the DER rental rate (i.e. when $\gamma p > r$).

Using (26), conditional market-level demand for supplied electricity (8), and conditional social welfare function ((10) and (11)), can each be written in unconditional form, allowing $dF_K(.)$ to be dispensed with.

3.2.2 Quasi-Linear Case

For an even simpler specific case, we now assume that household i's indirect utility function, conditional on its DER capacity choice K_{ij} and appliance choice Φ_i, is an extended version of (20):

$$V_i\left(p, r; K_{ij}, \Phi_i, y_i, \gamma_i\right) = K_{ij}\left(\gamma_i p - r\right) + ln\left(\frac{\Phi_i}{p}\right) + y_i - 1 + \epsilon_{ij} \tag{29}$$

As above, for illustrative purposes we assume that household i chooses between having fixed DER capacity K, or no DER capacity at all:

$$V_{i1} \equiv V_i\left(p, r; K_{i1} = 0, \Phi_i, y_i, \gamma_i\right) = ln\left(\frac{\Phi_i}{p}\right) + y_i - 1 + \epsilon_{i1} \tag{30}$$

$$V_{i2} \equiv V_i\left(p, r; K_{i2} = K, \Phi_i, y_i, \gamma_i\right) = K\left(\gamma_i p - r\right) + ln\left(\frac{\Phi_i}{p}\right) + y_i - 1 + \epsilon_{i2} \tag{31}$$

Using (30) and (31), and noting again that terms unrelated to choice j cancel, the probability that household i chooses to install DER capacity K is therefore:

$$P_{i2} \equiv P\left(V_{i1} < V_{i2}\right) = P\left(\epsilon_{i1} - \epsilon_{i2} < \left(\gamma_i p - r\right) K\right) \tag{32}$$

Again assuming that the ϵ_{ij} are iid Type I Extreme Value, the probability that household i chooses DER capacity K is thus:

$$P_{i2} = \frac{1}{1 + e^{-(\gamma_i p - r)K}} \tag{33}$$

Hence, provided the unit savings from DER investment exceed the investment's rental cost (i.e. $\gamma p > r$), the probability of household i installing DER capacity K is increasing in their difference, $\gamma p - r$.

Finally, total DER demand is thus:

$$K^*(r; M, \theta) = \int \frac{M\left(1 - \theta\right)}{1 + e^{-(\gamma_i p - r)K}} dF_\gamma\left(\gamma\right) \tag{34}$$

As for (26), γ_i and (implicitly) Φ_i survive differencing in (32) when computing choice probabilities. However, y_i terms vanish as expected when assuming quasi-linear utility, so only integration with respect to γ and Φ is required.

Once again, as desired for applications, DER demand is a continuous function of rental cost r, despite the underlying household choices being discrete. However, in contrast to the Cobb-Douglas case presented above where income effects arise, here the derivatives of K^* with respect to rental rate r and retail electricity price p are *negative* and *positive* respectively. More specifically:

$$\frac{\partial Intergand\left(K^*\left(.\right)\right)}{\partial r} = -\frac{M\theta e^{-(\gamma_i p-r)K}K}{\left(1+e^{-(\gamma_i p-r)K}\right)^2} < 0 \tag{35}$$

$$\frac{\partial Intergand\left(K^*\left(.\right)\right)}{\partial p} = \frac{M\theta e^{-(\gamma_i p-r)K}K\gamma}{\left(1+e^{-(\gamma_i p-r)K}\right)^2} > 0 \tag{36}$$

It is easily verified that the integrand of (34) is increasing in DER productivity rate γ, and also increasing in DER capacity K if DER savings exceed the DER rental cost (i.e. when$\gamma p > r$).

Substituting (34) for K in (8) and (10) respectively enables market-level demand for supplied electricity, and social welfare, to be calculated unconditionally.

4 Applications

4.1 Profit Function of Monopoly Supplying DER Capacity

Section 3 used microeconomic foundations to produce utility-maximising derived electricity demand, conditional on DER capacity and electrical appliance choice, as a continuous function of electricity price p. That demand can be considered "residual" in the sense that it is a household's demand for supplied electricity after allowing for self-generation using DER capacity. Section 3 also used the random utility approach to produce households' demand for DER capacity – anticipating optimal conditional derived electricity demand – with DER demand a declining and continuous function of DER rental cost.

To show how these derivations can be applied for antitrust, regulatory or distributional analysis, this section present the profit function of a monopolist

DER supplier. Doing so highlights how any micro-founded analysis of DER impacts needs to account for:

1. The price of DER capacity – i.e. its rental cost; and
2. The productivity of DER capacity (γ) interacted with the price of supplied electricity (p).

In other words, any analysis of DER supply that simply supposes DER demand is a declining function of rental cost neglects how DER productivity interacts with electricity price to also influence that demand. The above formulations for electricity and DER demand provide an internally-consistent framework for modelling decisions by DER suppliers.

Assuming a monopolist DER supplier charging DER rental r faces unit marginal cost of production c and fixed cost F, then its profit function writes as:

$$\Pi_{DER}^{M}(r) = K(r)(r-c) - F \tag{37}$$

Using (34), this writes as:

$$\Pi_{DER}^{M}(r) = \int \frac{M(1-\theta)(r-c)}{1+e^{-(\gamma_i p - r)K}} dF_{\Phi}(\Phi)\, dF_{\gamma}(\gamma) - F \tag{38}$$

All other things being equal, such a monopolist's profit is decreasing in unit marginal cost c, but increasing in effective customer mass $M(1-\theta)$, DER capacity K and productivity γ_i, and price of supplied electricity p.

Based on profit function (38) and unconditional social welfare ((10) and (11)) using (34), it is possible to compare monopoly and first best levels of DER capacity supply. Relevant applications include regulatory analysis of the impacts of monopoly DER supply, the impact of monopoly DER supply on welfare outcomes of households that can or cannot install DERs, among many others. Deriving the comparable profit function under supply by oligopolistic, competitive or customer-owned firms (just to name a few such options) is left to future work.

4.2 Profit Function of Monopoly Supplying DER Capacity and Electricity Services

If the monopolist was involved in activities over and above DER supply – e.g. electricity generation or distribution – then profit function (38) can be modified to reflect those additional activities. For example, additional revenues would be included, incorporating residual electricity demand such as in (19), for selling either electricity or distribution services. Associated production costs would also need to be included, as would recognition of how DERs might provide services that substitute for and/or complement firms' other activities.

Doing so would highlight how different suppliers of DER capacity – e.g. generators or distributors – if they are also involved in supplying electricity services, create different strategic trade-offs. It would also provide a coherent framework for assessing how regulation of activities like distribution affects the relative strategic incentives of generators and distributors to supply DERs to households – and highlight how these incentives differ from those of a "pure play" DER supplier. Such a framework is essential for assessing the strategic, competitive and regulatory implications of DERs, and is left to future work.

5 Conclusion

This paper provides tractable formulations of residual household electricity demand, conditional on DER and appliance investments – and the demand for DER capacity itself – based on microeconomic foundations. It uses these jointly-determined formulations to enable specification of both conditional and unconditional measures of "prosumer" welfare. These formulations highlight interactions between household choices of DER capacity, appliances, and the production of electricity-consuming household services (e.g. lighting). Neglecting these interactions, and arbitrarily positing that either residual electricity or DER demand are simply functions of own price, could bias analyses of the antitrust, regulatory, climate change and distributional implications of DER investments being made by various parties (i.e. households, generators, distributors, etc).

Limitations of this analysis include assuming that household electricity production in excess of own demand requirements produces revenues based on the price of supplied electricity. They also include neglecting uncertainty (e.g. intermittent DER supply), and imposing particular functional forms such as Cobb-Douglas or quasi-linear preferences in household production choices. We leave it to future work to allow for households facing different buy and sell prices, and for DER capacity demand derivations based on more general preference specifications. We also leave it to future work to apply these formulations in antitrust, regulatory, climate change and distributional assessments of (e.g.) how different forms of DER ownership affect their strategic use and resulting social welfare.

These are important extensions that would better inform analyses of the welfare implications of DERs. However, this paper takes a first step to providing rigorous microeconomic foundations for these other analyses.

Disclosure

A preliminary version of this research was conducted in the course of preparing a policy discussion document for the Electricity Retailers' Association of New Zealand (ERANZ) on the implications of disruptive technologies, business models and players for electricity regulation. The author gratefully acknowledges the generous financial support of ERANZ for that earlier work. Any views expressed in this paper are the author's, and do not purport to represent those of ERANZ or its members.

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