Subsidising Renewables but Taxing Storage? Second-Best Policies with Imperfect Carbon Pricing*

Carsten Helm[†] *and Mathias Mier*[‡]

Abstract

We consider an economy in which competitive firms use three technologies for electricity production: pollutive fossils, intermittent renewables like wind or solar, and storage. We determine optimal subsidies for renewables and storage capacities when carbon pricing is imperfect. This policy is efficient for low market shares of intermittent renewables in the energy system, but it turns inefficient once there are sufficient renewables to partly displace fossil electricity production at times of high availability. Moreover, the subsidy scheme is substantially more complex than a first-best Pigouvian tax. The optimal renewable subsidy is always positive but tends to decrease as electricity production becomes less reliant on fossils. The optimal storage subsidy even changes its sign. It is usually negative as long as fossils contribute to filling the storage, but turns positive if fossils are used only during times of low availability of renewables. This is because more storage capacity reduces the price during times of destorage, but raises it when electricity is taken from the market to fill the storage. This has countervailing effects on firms' incentives to invest in fossil capacities, and these effects are more pronounced the higher the round-trip efficiency losses during a storage cycle.

Keywords: intermittent renewable energies, electricity storage, carbon externality, subsidies, peak-load pricing, optimal control

JEL Classification: H23, Q42, Q58, O33

1 Introduction

Dramatic cost reductions and substantial subsidies have created a worldwide boom of renewable energies (IRENA, 2017b). Subsidies for renewables are mainly motivated by the environmental costs of electricity production from fossil energies, which include climate change due to CO_2 emissions as well as local air pollution. However, the fastest-growing renewable energies, wind and solar, are characterised by a highly intermittent supply and, in particular, low reliability (dark doldrums). These characteristics are the main obstacles in transitioning to an energy system based primarily on renewable energy sources, which is widely deemed necessary in order to meet the target of the 2015 Paris agreement to keep the increase in global average temperature to well below 2°C above pre-industrial levels.

The intermittency in supply from wind and solar energies may be reduced by technological improvements like wind turbines that are able to operate at lower wind speeds, panels that are more

*We gratefully acknowledge helpful comments from participants at the 2018 Conference on Sustainable Resource Use and Economic Dynamics (SURED), at the 2018 meeting of the German Association of Environmental and Resource Economists, and at the ETH Zurich Seminar in Energy, Environmental and Resource Economics. Parts of this paper were written while Carsten Helm was visiting the Mercator Research Institute on Global Commons and Climate Change (MCC).

†University of Oldenburg, Department of Business Administration, Economics and Law, 26111 Oldenburg, Germany, carsten.helm@uni-oldenburg.de, ph +49 441 798-4113.

‡ifo Institute, Chair of Energy, Climate and Resources, 81679 Munich, Germany, mier@ifo.de, ph +49 89 9224-1365.

efficient at absorbing low levels of solar radiation, or enhanced power transmission grids that can exploit spatial differences in the availability of intermittent renewables. Moreover, it is widely perceived that electricity storage should be part of the solution. For example, according to IEA's sustainable development scenarios (SDS) generation capacity of energy storage must increase from 176.5 GW in 2017 to 266 GW in 2030 (see also IRENA, 2017a). Such storage will probably be a mix of traditional pumped hydro storage, small- (as in electric vehicles) and large-scale batteries, power-to-gas (mainly hydrogen), and compressed air storage. Since the deployment of pumped hydro storage is limited (Gimeno-Gutiérrez and Lacal-Arántegui, 2015; Sinn, 2017), much of the build-up must come from technologies that are not competitive yet, which leads to requests for additional policy support.[1]

We analyse optimal support policies for renewable and storage capacities when a Pigouvian tax to internalise the carbon externality from burning fossil fuels cannot be implemented, for example, due to political economy reasons.[2] For parsimony, we focus on a per unit subsidy for capacity investments rather than on the more widely used feed-in tariffs, market premiums, and, more recently, tenders.[3] These instruments also imply an implicit subsidy for investments in renewables and storage so that their effects are quite similar. Indeed, we later argue that they are often identical within our specific modelling framework where support policies are financed by lump-sum taxation. We build on the peak-load pricing model and consider an economy with three types of firms: those that produce with a polluting fossil energy, those that use carbon-neutral but intermittent renewable energies, and those that engage in electricity storage. Starting in a greenfield setting, firms make long-term investments in their respective capacities, taking into account the subsidies granted by a benevolent regulator. Thereafter, firms produce electricity and interact with consumers in a perfectly competitive market. Note that storage firms have a dual role. They buy electricity—that is, act like consumers—at the low prices that prevail during times of high availability of renewables, but supply electricity at the high prices that obtain during times of low availability. This exploitation of price differences and the increasing role of flexible pricing schemes motivates our assumption of dynamic pricing.[4]

For a model comprised of competitive markets, dynamic pricing, lump-sum taxation and no R&D spillovers, it is straightforward to see that a Pigouvian tax per unit of pollution would implement the first-best solution. Given that we neglect dynamic aspects of resource extraction and of the climate system, the tax would even be constant for constant marginal damage costs (see Lemoine and Rudik (2017) for dynamic taxing schemes). Subsidies for renewables reduce pollution only indirectly. First, more renewables capacities reduce the expected electricity price and, therefore, incentives to invest in fossils. Second, fossil capacities may remain unused when the availability of intermittent renewables, which have lower variable costs, is high. We show that supplementing a renewable subsidy by a (usually negative) subsidy for storage may even lead to the first-best solution. However, this is only the case when renewables are still relatively expensive, leading to a low optimal market share in the electricity system so that fossil capacities are fully used during storage. For higher market shares of renewables, the subsidy scheme is only second-best. In addition, it is substantially more complex than a Pigouvian tax because optimal subsidy levels depend on the market share of renewable energies.

[1] Costs of batteries fell by 22% from 2016 to 2017 (https://www.iea.org/tcep/energy-integration/energystorage/). Schmidt, Hawkes, Gambhir, and Staffell (2017) predict (using experience curves) that battery storage will be competitive in the next 10 (electric vehicle transportation) to 20 years (residential energy storage) (see Kittner, Lill, and Kammen (2017) for similar predictions), although other studies are less optimistic (e.g., Brouwer, van den Broek, Zappa, Turkenburg, and Faaij, 2016).

[2] The literature discusses equity issues (e.g., Polinsky, 1979), lobbying and rent seeking (e.g., Fredriksson, 1997), and distributional implications (see Goulder and Parry (2008) for a discussion and Reguant (2019) for empirical evidence).

[3] See, e.g., Eichner and Runkel (2014) for a similar approach. In 2016, 83 countries used feed-in tariffs or premiums to promote renewable energy, 58 countries used investment subsidies (capital subsidies, grants, or rebates), and 73 countries used auctions that do not exclude the use of an investment subsidy (IRENA and CPI, 2018). Moreover, most of storage subsidization is constructed as an investment subsidy (ESC, 2015).

[4] Dynamic pricing of electricity is still often restricted to larger commercial customers (e.g., Borenstein and Holland, 2005; Joskow and Wolfram, 2012), but according to Helm and Mier (2019), this may be sufficient to create appropriate price signals. Moreover, recent technological advances have dramatically lowered the costs of smart metering technologies, and many regions have set ambitious targets for their deployment (e.g., in the EU Third Energy Package). In addition, several studies have found evidence that households actually do respond to higher electricity prices by reducing usage (e.g., Faruqui and Sergici, 2010; Jessoe and Rapson, 2014).

For the example of linear demand and constant marginal capacity costs, the renewable subsidy is constant as long as fossils are always the price-setting technology—that is, even when the availability of intermittent renewables is high—but falls in the level of renewable capacities thereafter. For more general demand functions, the optimal subsidy scheme becomes even more complex.

Storage capacities even out the intermittent supply of renewables and, thereby, raise their competitiveness. This makes subsidising storage seem reasonable, but it turns out that this intuition is wrong. Storage reduces the electricity price when stored energy is supplied to the market, but it raises the price when the storage is filled. This has countervailing effects on average electricity prices and, therefore, on the incentives to invest in fossil capacities. Due to round-trip efficiency losses during a storage cycle, more electricity has to be taken from the market than can be supplied to it during times of destorage.[5] Therefore, as long as fossils contribute to electricity production during times of storage, the price increasing effect tends to dominate and storage capacities should be taxed in order to make investments in fossils less attractive. By contrast, there should be no storage subsidy with the aim to make investments in renewables more attractive. The reason is simply that a direct subsidy of renewables is more suitable for this.

Once the level of renewable capacities is large enough to fill the storage during times where they are highly available, fossils no longer benefit from the price increasing effect during storage. However, they still suffer from the additional electricity supply during destorage so that it now becomes optimal to subsidise storage. This subsidy is constant under the same conditions that lead to a decreasing renewable subsidy. Roughly speaking, as the market share of fossil energies falls, it is optimal to gradually switch from the subsidisation of renewables to subsidising storage. To summarize, the main contribution of the paper is twofold. First, it extends the peak-load pricing model by developing an analytically tractable model that integrates the optimal control problem of storage firms and accounts for rather general intermittency patterns of renewables. Second, we use this model to examine subsidies for storage and renewable technologies as an alternative to Pigouvian taxation to address the carbon externality of fossils.

Accordingly, our paper is related to several literatures. The first of these is the literature on the economics of intermittent sources of electricity production, of which Ambec and Crampes (2012, p. 321) wrote some years ago that they are 'still in their infancy'. Since then, the literature has grown substantially, but most contributions rely heavily on numerical simulations (e.g., Després, Mima, Kitous, Criqui, Hadjsaid, and Noirot, 2017) or are empirical (e.g., Abrell, Kosch, and Rausch, 2019; Cullen, 2013; Fell and Kaffine, 2018; Liski and Vehviläinen, 2017). In this paper, we build on the contribution by Helm and Mier (2019), who analyse the market diffusion of renewable energies as they become cheaper. We extend their model by introducing storage as a third technology and, additionally, analyse renewables and storage support policies. Abrell, Rausch, and Streitberger (2018), using a simulation and a simpler analytical model, analyse a larger set of renewables support policies but abstract from storage. Like Fell and Linn (2013), the authors focus on the heterogeneity of intermittent natural resources. Hence both contributions distinguish between wind and solar, but use a simple binary pattern for their availability. Andor and Voss (2016) also consider efficient subsidy schemes for renewables, but their model neither includes fossils nor a storage technology.

Another strand of literature to which this paper relates is the economics of storage. Traditional applications include balancing stochastic production disturbances in agriculture (e.g., Newbery and Stiglitz, 1979; Wright and Williams, 1984) and the combination of thermal capacity with mainly pumped hydro storage (e.g., Crampes and Moreaux, 2001). In a seminal contribution, Gravelle (1976) studies the implications of storage for peak-load pricing with variable demand. He finds that peak consumption increases less than off-peak production increases, due to round-trip losses of storage. This is similar to the effect of storage during times with high and low availability of intermittent renewables in our model. More recently, the focus has shifted toward the role of pumped storage as a natural complement to the intermittency of renewables (e.g., Crampes and Moreaux, 2010). Similar to us, Steffen and Weber (2013) determine optimal capacity investments, but only for the fossil and

[5] Round-trip efficiency is usually in the range of 65 to 90 per cent, depending on the storage technology (IRENA, 2017a).

storage technologies. They then use a load duration curve to determine the effect of intermittent renewable energies and demonstrate their results numerically by using a case study for Germany. In a related contribution, Steffen and Weber (2016) use optimal control theory to provide a more precise representation of storage dynamics. However, like Horsley and Wrobel (2002), they only consider the problem of an individual storage firm, and they focus on differences between large (unconstrained) and small (constrained) reservoirs. Ambec and Crampes (2017) analyse the social value of storage in a model with renewables, but the availability of renewables is restricted to either 0 or 1. The related model by Durmaz (2014) uses discrete time and dynamic programming to determine the optimal storage pattern. However, he does not consider policy instruments and his problem is analytically not fully tractable. Finally, Pommeret and Schubert (2019) also integrate storage into a model with electricity production from renewable and fossil technologies. However, their focus is on the optimal allocation of a fixed carbon budget over time, whereas the availability of sufficient storage capacities is taken as exogenously given.

Our paper also contributes to the more general literature on second-best policies and the ranking of policy instruments to incentivize pollution abatement. For a given abatement cost function, pollution taxes and abatement subsidies are usually seen as equivalent in the short run, whereas in the long run, subsidies lead to excessive firm entry (e.g., Kohn, 1992). In an extension of this literature that is more similar to our approach, firms can decide whether to incur the fixed cost of a new technology that reduces costs of emission abatement. In this framework, taxes on emissions and subsidies for emission abatement are usually equivalent (e.g., Milliman and Prince, 1989; Requate and Unold, 2003). Although this literature is often motivated by the problem of mitigating CO_2 emissions, specific aspects of energy markets such as the intermittency of renewables are usually neglected (see also Fischer, Preonas, and Newell, 2017). Accounting for them fundamentally changes the comparison of taxes and subsidies, as we show.

In accordance with our results, there is a broad consensus that no additional subsidies are necessary to tackle an environmental externality if perfect carbon taxation is possible (Golosov, Hassler, Krusell, and Tsyvinski, 2014; Van Der Ploeg and Withagen, 2014). Positive externalities from R&D may require renewables subsidisation (Acemoglu, Aghion, Bursztyn, and Hemous, 2012), but Parry, Pizer, and Fischer (2003) argue that the welfare effect from tackling climate change externalities is greater than the positive effect of R&D subsidisation (see also Goulder and Parry, 2008). Other reasons that have been put forward to motivate renewables subsidies are international tax competition with mobile capital (Eichner and Runkel, 2014), learning externalities and imperfect competition (Reichenbach and Requate, 2012), lumpy entry cost (Antoniou and Strausz, 2017) and imperfections in demand for energy efficiency (Fischer et al., 2017). We abstract from such market failures so as to focus on the role of intermittency of renewable energies and of storage when addressing the carbon externality.

The remainder of the paper is structured as follows. In Section 2, we introduce the model and the timing of the decisions. The game is then solved by backward induction. Section 3 considers electricity production and storage decisions, Section 4 examines capacity choices, and Section 5 determines the optimal subsidy levels. Section 6 concludes, and an Appendix contains the proofs.

2 The Model

Consider an electricity market with three technologies, indexed $j = f, r, s$. Technology f represents a dispatchable fossil technology—like conventional power plants that burn coal or gas. Dispatchability means that electricity production can be freely varied at every point in time up to the limit of its installed capacity (see Joskow, 2011). Since we abstract from uncertainty, we can ignore ramp-up times because conventional power plants are well able to adapt production some time ahead. Technology r is a renewable technology with intermittent supply—like wind turbines, solar PV, or solar thermal plants. The third technology s does not generate electricity, but is able to store it for later usage.

For each of the three technologies there are a large number, n_j, of identical firms that interact on competitive markets. We use lower-case letters to denote choices of firms and upper-case letters for aggregate values. Accordingly, the production and capacity choices of a firm that produces with technology j are y_j and q_j, respectively, whereas overall production and capacity are $Y_j = n_j y_j$ and $Q_j = n_j q_j$. To keep the analysis focused, we consider only the most interesting case where strictly positive capacities are installed for all three technologies. Obviously, this depends on the cost parameters of the technologies.

A firm operating with technology j has capacity costs $c_j(Q_j)q_j$, where $c_j(Q_j)$ are the costs of providing one unit of capacity, which are constant from the perspective of an individual firm. Thus, we assume that each firm is too small to affect technology costs $c_j(Q_j)$, just as it takes prices as given by the assumption of competitive markets. However, technology costs depend on the overall capacity level, which allows us to account for different assumptions used in the literature in regard to renewables. In particular, $c_j'(Q_j) < 0$ would capture that economies of scale or learning reduces unit costs (as in Green and Léautier, 2017). By contrast, if one wants to emphasize that the most efficient sites for wind and solar energies are used first, then $c_j'(Q_j) > 0$ seems more appropriate (as in Abrell et al., 2018). Similarly, for storage, increasing unit costs could result from less suitable pump storage locations and the scarcity of the rare earths that are needed for batteries. For the renewable and storage technology we impose no restriction on the sign of $c_j'(Q_j)$. For the established fossil technology we adopt the standard assumption that $c_f'(Q_f) \geq 0$.

Electricity produced by the fossil and renewable technology is $y_j \geq 0$, $j = f, r$. We assume constant costs, $k_f > 0$, of producing one unit of output with the fossil technology, which are mainly variable costs for coal, oil, or gas. Moreover, fossil production leads to an environmental unit cost, $\delta > 0$, that may be internalised by a tax, τ. Hence a fossil firm's total unit costs are $b_f = k_f + \tau$, and this is equal to social costs if $\tau = \delta$. Variable costs of renewables are negligible and, therefore, ignored, as often done in other work.

Turning to the storage technology, $s(t)$ denotes a firm's level of stored electricity and $y_s(t)$ its supply of stored electricity at time t. Accordingly, $y_s(t) > 0$ characterises states in which stored electricity is fed into the electricity system, and $y_s(t) < 0$ means that electricity is stored. Storage leads to conversion losses so that the change in the level of stored electricity differs from the quantity of electricity that is fed into or taken out of the storage. Specifically, $\dot{s} := \frac{ds}{dt} = -\eta(y_s)\, y_s(t)$, where the parameter $\eta(y_s) > 0$ represents conversion losses per unit of y_s that differ for storage and destorage. In particular, we assume that $\eta(y_s) = \eta_s \in (0, 1]$ during times of storage ($y_s(t) < 0$) so that more than one unit of electricity is needed to fill the storage by one unit. Similarly, during times of destorage ($y_s(t) > 0$), we assume $\eta(y_s) = \eta_d \geq 1$ so that more than one unit of electricity has to be taken out of the storage to sell one unit on the market. Finally, for intermediate periods during which the storage capacity is not used ($y_s(t) = 0$), we assume that no electricity is lost ($\dot{s} = 0$) and $\eta(y_s) = 1$.

Intermittency of renewables is represented by an availability factor $\alpha(t) \in [0, 1]$ that is a continuous function of time. Hence the level of renewable capacities that is available at time t is $\alpha(t)\, Q_r$. To keep the analysis tractable, we assume that $\alpha(t)$ can be forecasted perfectly and follows an identical repetitive pattern, described in more detail below.[6] For example, this pattern could represent daily fluctuations of solar power or seasonal fluctuations of wind. Storage serves to balance these fluctuations so that we choose one cycle during which the storage is filled and emptied as a "representative" period. The lifetime of installed capacities (assumed to be the same for all technologies) consists of m such representative storage cycles.

The timing is as follows. In Stage 1, the government chooses subsidy levels for renewable and storage capacities. In Stage 2, competitive firms build their respective fossil, renewable, or storage capacities. In line with the literature on peak-load pricing, we assume a greenfield setting that disregards any capacity that is currently in place. Finally, in Stage 3, firms choose their production levels on the competitive electricity market and consumers demand electricity.

[6] Short-term forecasts over a day-night cycle are actually quite accurate (e.g., Iversen, Morales, Møller, and Madsen, 2016), and seasonal wind availability is, at least in the historic average, well known. Moreover, in their empirical study for southeastern Arizona, Gowrisankaran, Reynolds, and Samano (2016) find that social costs of unforecastable intermittency are small in comparison to those of intermittency overall.

3 Production and Consumption Decisions

3.1 Derivation of Optimality Conditions

The game is solved by backward induction, and we first analyse production and consumption decisions during the lifetime of installed capacities for given subsidies and taxes. The competitive market equilibrium follows from firms' profit maximisation and consumers' utility maximisation, subject to electricity prices, $p(t)$, which balance supply and demand. Due to our assumption that the availability of the renewable technology follows a repetitive pattern, the market outcome will be the same for each representative storage cycle. We denote the initial and terminal time of a storage cycle by t_0 and T, respectively, and ignore discounting within a cycle for parsimony.

First consider the production decisions of fossil and renewable firms. Capacity costs are sunk so that firms' objective is to maximise the difference between revenues, $p(t)\,y_j(t)$, and variable production costs, $b_j y_j(t)$, over the length of a representative period, where variable costs are zero for the renewable firm. Production by fossil and renewable firms is restricted by the (available) capacity, $y_f(t) \leq q_f$, $y_r(t) \leq \alpha(t)\,q_r$, and it must be non-negative, $y_f(t), y_r(t) \geq 0$. The latter constraint can be ignored because profit maximising renewable and fossil firms will never choose negative quantities in the unconstrained equilibrium. Thus, a fossil firm's profit maximisation problem for a representative cycle in Stage 3 is

$$\pi_f\left(y_f^*(q_f)\right) := \max_{y_f(t)} \int_{t_0}^{T} (p(t) - b_f)\, y_f(t)\, dt \ \text{ such that} \tag{1}$$

$$y_f(t) \leq q_f. \tag{2}$$

Using asterisks to characterise values in the competitive market solution, $\pi_f(y_f^*(q_f))$ denotes the value function of this problem, that is, the maximum profits a firm can achieve by optimising production y_f for all $t \in [t_0, T]$, given the fixed capacity parameter q_f. Differentiation of the corresponding Lagrangian yields the first-order and complementary slackness conditions ($\mu_f(t)$ is the Lagrangian multiplier) for each $t \in [t_0, T]$:

$$p(t) - b_f - \mu_f(t) \leq 0 \qquad \left[= 0, \text{ if } y_f^*(t) > 0\right], \tag{3}$$

$$q_f - y_f(t) \geq 0, \qquad \mu_f(t) \geq 0,\ \mu_f(t)\left[q_f - y_f(t)\right] = 0. \tag{4}$$

Due to the linearity of the objective function, the first-order condition is sufficient and leads to corner solutions. Specifically, if the price exceeds variable production costs, the firm produces at full capacity; i.e., $y_f(t) = q_f$ if $p(t) > b_f$. By contrast, fossil firms do not produce during times t for which $p(t) < b_f$, while any $y_f(t) \in [0, q_f]$ is optimal if $p(t) = b_f$.

Renewable firms face no variable costs, but their capacity constraint depends on the availability, $\alpha(t)$, of renewable capacities. Thus, the profit maximisation problem is

$$\pi_r\left(y_r^*(q_r)\right) := \max_{y_r(t)} \int_{t_0}^{T} p(t)\, y_r(t)\, dt \ \text{ such that} \tag{5}$$

$$y_r(t) \leq \alpha(t)\, q_r, \tag{6}$$

where $\pi_r\left(y_r^*(q_r)\right)$ denotes the value function. The first-order and complementary slackness conditions, which are again sufficient due to the linearity of the objective function, follow from differentiation of the corresponding Lagrangian ($\mu_r(t)$ is the Lagrangian multiplier) for each $t \in [t_0, T]$ as

$$p(t) - \mu_r(t) = 0, \tag{7}$$

$$\alpha(t)\, q_r - y_r \geq 0, \qquad \mu_r(t) \geq 0,\ \mu_r(t)\left[\alpha(t)\, q_r - y_r(t)\right] = 0. \tag{8}$$

Here, the binding condition (7) reflects that $y_r^*(t) > 0$ for any $\alpha(t), p(t) > 0$ because, in contrast to fossils, renewables have no variable costs.[7] Moreover, the complementary slackness condition in (8) then implies $y_r^*(t) = \alpha(t) q_r$ for all t with $p(t) > 0$, i.e., renewables are used at full capacity. However, if the level of renewable capacities and their current availability are very large, supply at full capacity may exceed demand from consumers and storage firms, leading to an equilibrium price of zero.

Next, consider storage firms. They control the level of stored electricity $s(t)$ (the state variable) so as to exploit price differences by buying and storing electricity ($y_s(t) < 0$) during times of low prices, while destoring ($y_s(t) > 0$) and selling when prices are high. Hence, a storage firm faces the following optimal control problem ($\pi_s(y_s^*(q_s))$ is the value function):

$$\begin{aligned}
\pi_s(y_s^*(q_s)) &:= \max_{y_s(t)} \int_{t_0}^{T} p(t) y_s(t) \, dt \text{ such that} && (9)\\
\dot{s}(t) &= -\eta(y_s) y_s(t), && (10)\\
s(t_0) &= s(T), && (11)\\
s(t) &\leq q_s, && (12)\\
s(t) &\geq 0. && (13)
\end{aligned}$$

The first constraint, (10), is the equation of motion for the level of stored energy, $s(t)$. Condition (11) requires that the initial and terminal storage level must be the same, which follows from our assumption of a representative storage cycle, while the overall game consists of m such cycles.[8] Finally, (12) is the capacity constraint of storage firms, and (13) is the constraint that the level of stored energy must be non-negative. The Hamiltonian is

$$\mathcal{H}_s(y_s(t)) = p(t) y_s(t) - \lambda(t) \eta(y_s) y_s(t), \tag{14}$$

where $\lambda(t)$ is the adjoint variable of $s(t)$. Conditions (12) and (13) are pure state space constraints and can be accounted for by forming the Lagrangian

$$\mathcal{L}_s(t) = \mathcal{H}_s(y_s(t)) + \varphi_s(t)(q_s - s(t)) + \varphi_d(t) s(t), \tag{15}$$

where $\varphi_s(t)$ and $\varphi_d(t)$ are the Lagrangian multipliers for the respective constraints. The Hamiltonian $\mathcal{H}_s$ is linear in $y_s(t)$ and the constraints (12) and (13) are linear in $s(t)$. Therefore, the following conditions are sufficient for optimality, *if* $\lambda(t)$ *is continuous* (see Seierstad and Sydsaeter, 1987, pp. 317-318):[9]

$$\begin{aligned}
\max_{y_s(t)} \mathcal{H}_s(y_s(t)) &= [p(t) - \lambda(t) \eta(y_s)] y_s(t), && (16)\\
\dot{s}(t) &= \frac{\partial \mathcal{L}_s(t)}{\partial \lambda(t)} = -\eta(y_s) y_s(t), && (17)\\
\dot{\lambda}(t) &= -\frac{\partial \mathcal{L}_s(t)}{\partial s(t)} = \varphi_s(t) - \varphi_d(t), && (18)
\end{aligned}$$

[7] More formally, note that the standard first-order condition is $p(t) - \mu_r(t) \leq 0\,[= 0, \text{ if } y_r^*(t) > 0]$ and suppose by contradiction that $y_r^*(t) = 0$ although $\alpha(t) > 0$ (for $\alpha(t) = 0$, we trivially have $y_r^*(t) = 0$). Given that $q_r > 0$ by assumption, the complementary slackness condition $\mu_r(t)[\alpha(t) q_r - y_r(t)] = 0$ then implies $\mu_r(t) = 0$. For any positive price, this violates the condition $p(t) - \mu_r(t) \leq 0$.

[8] The overall length of the game coincides with the lifetime of the storage capacities. Therefore, in the last period it would be optimal to completely empty the storage. For parsimony, we ignore this complication by simply requiring $s(t_0) = s(T)$, just as we ignore other (more relevant) problems such as the fact that the storage capacity of batteries gradually decreases over time.

[9] The sufficient conditions for problems with pure state constraints as stated in Seierstad and Sydsaeter (1987, pp. 317-318) are less restrictive because they allow for jumps in the state variable, $\lambda(t)$, if further conditions are satisfied. However, in the solution that we derive in Section 3.2, $\lambda(t)$ is continuous so that we can omit the related conditions.

$$\frac{\partial \mathcal{L}_s(t)}{\partial \varphi_s(t)} = q_s - s(t) \geq 0, \;\; \varphi_s(t) \geq 0, \;\; \varphi_s(t)[q_s - s(t)] = 0, \tag{19}$$

$$\frac{\partial \mathcal{L}_s(t)}{\partial \varphi_d(t)} = s(t) \geq 0, \;\; \varphi_d(t) \geq 0, \;\; \varphi_d(t)\, s(t) = 0, \tag{20}$$

$$s(t_0) = s(T). \tag{21}$$

Here, (16) is the optimality condition for the control variable $y_s(t)$, whereas (17) and (18) are the differential equations for the state and adjoint variable. Conditions (19) and (20) account for the pure state space constraints that the level of stored electricity can neither exceed the available storage capacities nor become negative.

Turning to consumers, utility maximisation leads to a demand function $x(p(t))$, for which we impose no restrictions other than $\frac{\partial x}{\partial p} < 0$. Consumption choices on the competitive electricity market maximise consumer surplus and are restricted by production. Thus, the maximisation problem of consumers is ($w(x^*)$ is the value function)

$$w(x^*) := \max_{x(t)} \int_{t_0}^{T} \left[\int_{p(t)}^{p_{max}} x(\tilde{p})\, d\tilde{p} \right] dt \;\; \text{such that} \tag{22}$$

$$x(t) \leq \sum_j Y_j(t), \tag{23}$$

where p_{max} is the maximum willingness to pay and $\sum_j Y_j(t)$ is aggregate production of all firms producing with technologies $j = f, r, s$. It is straightforward to show that in equilibrium $x(t) = \sum_j Y_j(t)$, that is, demand equals supply. In conclusion, the competitive solution follows from this market clearing condition, the inverse demand function, $p(x(t))$, and the optimality conditions of fossil firms, (3) and (4), renewable firms, (7) and (8), and storage firms, (16) to (21).

3.2 Determination of Competitive Equilibrium

Remember that storage capacities are given in Stage 3. Thus, a priori, they might always exceed the level at which profit maximising firms would want to store electricity. To exclude this unrealistic case, we assume that unit costs of storage capacities, $c_s(Q_s)$, are sufficiently large so that the level of capacities installed in Stage 2 make them a scarce resource in Stage 3; meaning that the constraint $s(t) \leq q_s$ binds for at least some t.

We can distinguish three outcomes. *Storage periods* in which the storage is filled ($y_s(t) < 0$), *destorage periods* in which the storage is emptied ($y_s(t) > 0$), and *intermediate periods* with neither storage nor destorage ($y_s(t) = 0$). First, consider the two outcomes with $y_s(t) \neq 0$. If we had $p(t) \neq \lambda(t)\eta(y_s)$, it would be impossible for any $y_s^*(t)$ to maximize (16). Therefore, we must have $p(t) = \lambda(t)\eta(y_s)$ during storage and destorage. Intuitively, note that the adjoint variable $\lambda(t)$ is usually interpreted as the change in the value function due to a unit increase in the state variable, $s(t)$. Thus, $\lambda(t)$ is the value of stored electricity which, after being weighted by conversion losses, must equal the price of electricity. Moreover, during storage and destorage periods the storage can neither be full nor empty (except at the boundaries), i.e., $s(t) < q_s$ and $s(t) > 0$. Thus, from the complementary slackness conditions in (19) and (20) we have $\varphi_s(t) = \varphi_d(t) = 0$ so that $\dot{\lambda}(t) = \varphi_s(t) - \varphi_d(t) = 0$ from (18). Finally, by assumption the round-trip efficiency loss parameter is constant at $\eta(y_s) = \eta_s$ during storage and at $\eta(y_s) = \eta_d$ during destorage. Using $p(t) = \lambda(t)\eta(y_s)$ it follows that not only $\lambda(t)$, but also prices are constant during each individual storage and destorage period. Note that we have imposed no technical restrictions on the speed of filling and depleting the storage. Thus, if prices varied during an individual storage or destorage period, firms would have arbitrage opportunities and use their storage to buy electricity at low prices and sell it at high prices.

To this point, we have not restricted the evolution of $\alpha(t)$ over time that drives the storage/destorage pattern. To keep the model tractable, we need to impose more structure as otherwise nearly any sequence of storage, destorage, and intermediate periods is conceivable. Therefore, the remainder of the paper is based on the following additional assumption.

Assumption 1. *For each representative cycle $t \in [t_0, T]$, the availability of renewables energies, $\alpha(t)$, is given by the same single-peaked function with $\alpha(t_0) = \alpha(T) = 0$ and $\max\{\alpha(t)\} = 1$.*

The assumption $\alpha(t_0) = \alpha(T)$ captures that, by continuity of $\alpha(t)$, the availability at the end of the current and at the beginning of the next representative cycle must be the same. The choice of a time with zero availability as the starting point of a representative period, i.e., $\alpha(t_0) = 0$, is somewhat arbitrary but facilitates the exposition. The black solid curve in Figure 1 is an example for a distribution that satisfies Assumption 1. It depicts solar PV production in Germany on 30 June 2018, where maximum production has been normalised to 1 and the length of a representative cycle from t_0 to T is 24 hours (downloaded from https://transparency.entsoe.eu on 2 July 2018). The transparent segments to the left and to the right illustrate our simplifying assumption that the availability of renewables is the same in periods prior and subsequent to the depicted one.

Obviously, firms should destore electricity when the availability of renewables is low (lower bold parts of the curve), and store electricity when the availability is high (upper bold part of the curve), leading to the following sequence of periods: destorage, intermediate, storage, intermediate, destorage, ... Together with our assumption that we consider a representative cycle, a repeated pattern of identical destorage and storage periods obtains. Therefore, the storage should be completely emptied at the end of each destorage period, and completely filled at the end of each storage period. Otherwise, some stored electricity and/or some storage capacity would never be used, which cannot be optimal. This property keeps the analysis tractable and is the reason for Assumption 1. In the remainder of this subsection, we derive an intuitive solution for the competitive equilibrium and show in Appendix A that it satisfies all optimality conditions from Subsection 3.1. For later reference, we state the solution in terms of aggregate values, $Y_j = n_j y_j$, $Q_j = n_j q_j$, and $S = n_s s$.

Dispatchable electricity from storage and fossils is most valuable when the availability of renewables is minimal, i.e., at t_0 for which $\alpha(t_0) = 0$. Therefore, fossils must be fully used at t_0 as, otherwise, some capacities would always lie idle. Given that renewables have lower variable costs, they obviously must be fully used too. Moreover, we have already shown that the electricity price is constant at $p_d = \lambda(t_0)\eta_d$ during the destorage period that starts at t_0. Hence, fossils and renewables continue to be fully used during this destorage period. It follows immediately that the level of destorage must balance the fluctuation of renewables in order to keep electricity supply and, thus, the price constant, i.e.,

$$Y_s(t) \quad = \quad (\alpha(t_d) - \alpha(t)) Q_r \;\text{ for all } t \in [0, t_d], \tag{24}$$

where t_d denotes the end of the first destorage period.[10]

Now turn to the second destorage period that starts at $t = t'_d$. In Figure 1, this is the bold segment to the left of T. Noting that we consider a representative cycle, this period is identical to the one that precedes t_0. Accordingly, the two destorage periods can be viewed as being connected and, therefore, must have the same constant electricity price. It follows that production is also the same, i.e., fossils and renewables produce at full capacity and destorage is as given by (24). Moreover, $\alpha(t_d) = \alpha(t'_d) =: \alpha_d$ as, otherwise, destorage should be shifted to periods with lower availability of renewables.

[10] More formally, our assumptions about the availability of renewable energies, $\alpha(t)$, imply that the destorage period continues until the storage runs empty at $t = t_d$, i.e., $S(t_d) = 0$ (see Figure 1). Thus, t_d is characterised by $Y_f(t_d) = Q_f$, $Y_r(t_d) = \alpha(t_d) Q_r$, and $Y_s(t_d) = 0$. By continuity of $\alpha(t)$ and, thus, of available production capacities, we must have $p(t_d) = p_d$. This implies $x(t_d) = x_d$, where demand x_d is constant during destorage. Solving the market clearing condition for $Y_s(t) = x_d - Y_f(t) - Y_r(t)$ and using $x_d = x(t_d) = Q_f + \alpha(t_d) Q_r$ yields (24).

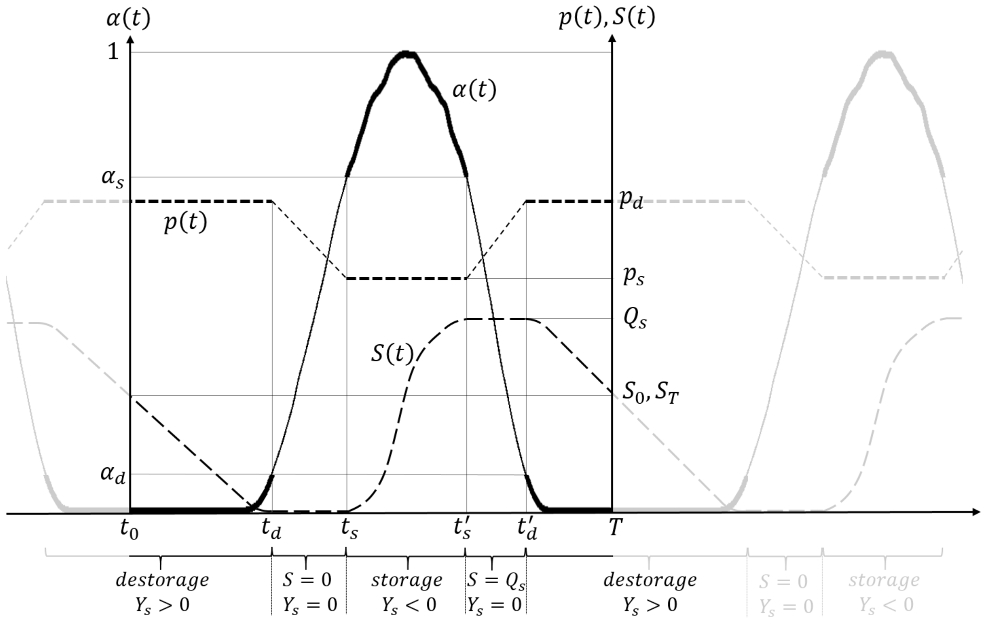

Fig. 1: Availability of renewables and competitive equilibrium

Finally, the "connectedness" of the two destorage periods and the repeated cycle of identical destorage and storage periods imply that the storage must be full at $t = t'_d$ and run empty at $t = t_d$ (see Figure 1). Hence, integration of the equation of motion (17) over the destorage periods must satisfy $Q_s = \eta_d \int_{t_0}^{t_d} Y_s(t)\,dt + \eta_d \int_{t'_d}^{T} Y_s(t)\,dt$. Substitution from (24) gives

$$Q_s \quad = \quad \eta_d \int_d (\alpha_d - \alpha(t))\, Q_r dt, \tag{25}$$

where $\int_d dt := \int_{t_0}^{t_d} dt + \int_{t'_d}^{T} dt$ denotes the combined duration of the two destorage periods. Condition (25) implicitly determines the critical availability α_d where destorage ends. Intuitively, the destorage period is shorter when the storage capacity Q_s is low, when conversion losses η_d are large, and when the level of renewable capacities that has to be substituted by destorage, $(\alpha_d - \alpha(t))\, Q_r$, is large. The first line in Table 1 summarises production in the destorage period.

Tab. 1: Solution of production stage for fossils, renewables, and storage

period	availability of renewables	$Y_r(t)$	$Y_f(t)$	$Y_s(t)$
d	$0 \le \alpha(t) \le \alpha_d$	$\alpha(t)\, Q_r$	Q_f	$(\alpha_d - \alpha(t))\, Q_r$
case 1	$\alpha_d < \alpha(t) \le \min\{\alpha_1, \alpha_s\}$	$\alpha(t)\, Q_r$	Q_f	0
case 2	$\min\{\alpha_1, \alpha_s\} < \alpha(t) \le \min\{\alpha_2, \alpha_s\}$	$\alpha(t)\, Q_r$	$x(b_f) - \alpha(t)\, Q_r$	0
case 3	$\min\{\alpha_2, \alpha_s\} < \alpha(t) \le \min\{\alpha_3, \alpha_s\}$	$\alpha(t)\, Q_r$	0	0
case 4	$\min\{\alpha_3, \alpha_s\} < \alpha(t) \le \alpha_s$	$x(0)$	0	0
s	$\alpha_s < \alpha(t) \le 1$	$\min\{\alpha(t)\, Q_r,\, x(0) - Y_s(t)\}$	$Y_f(\alpha_s)$	$(\alpha_s - \alpha(t))\, Q_r$
α_d implicitly solves (25), $\alpha_1 = \frac{x(b_f) - Q_f}{Q_r}$, $\alpha_2 = \frac{x(b_f)}{Q_r}$, $\alpha_3 = \frac{x(0)}{Q_r}$, and α_s implicitly solves (27)				

As $\alpha(t)$ starts to exceed α_d, we enter the first intermediate period where neither storage nor destorage occurs. Hence $y_s(t) = 0$ and the solution follows from the first-order condition (3) for fossils and (7) for renewables, as well as the related complementary slackness conditions (4) and

(8). Initially, fossils and renewables continue to be fully used. This is case 1 in Table 1. As the availability of renewables, $\alpha(t)$, rises, supply increases and the equilibrium price falls. Case 1 obtains until the price has fallen to the variable costs of fossils, b_f. Thereafter, case 2 obtains, for which the maximum feasible electricity output from renewables and fossils exceeds demand at $p = b_f$. Hence only renewable capacities are fully used (due to lower variable costs), and fossils serve the remaining demand, $x(b_f) - \alpha(t) Q_r$. For even larger values of $\alpha(t)$, the equilibrium price falls below b_f so that only renewables are used, but still at full capacity (case 3). Finally, for very high levels of $\alpha(t)$, case 4 may obtain, in which the available capacity of renewables, $\alpha(t) Q_r$, exceeds demand at a price of zero, $x(0)$, leading to excess capacities of renewables.

Let $\alpha_1 \leq \alpha_2 \leq \alpha_3$ denote the availabilities where the respective cases end, and $t_1 \leq t_2 \leq t_3$ the associated times. Hence $\alpha_i = \alpha(t_i)$ for $i = 1, 2, 3$. Depending on the size of storage and renewable capacities, not all cases need obtain. Ceteris paribus, a larger storage capacity takes longer to fill so that storage starts earlier, that is, already during case 1, 2, or 3 of the intermediate period. Conversely, larger renewable capacities imply that a given storage can be filled faster, hence storage starts later and more cases obtain. In Table 1, this is represented by the minimum operator in the column for availability, where $\alpha_s := \alpha(t_s) = \alpha(t'_s)$ is the availability when the intermediate period ends and storage starts (t'_s denotes the end of the storage period; see Figure 1).

Accordingly, any one of cases 1 to 4 can prevail at the start of the storage period, during which the price remains constant (see above). By continuity of the available production capacities, this price, p_s, must be the same as that at the end of the intermediate period, i.e., $p_s = p(t_s)$. This results in a constant demand, $x_s = x(t_s)$, a constant supply from fossils, $Y_f = Y_f(t_s)$, and additional renewable capacities that become available as $\alpha(t)$ rise above α_s are used to refill the storage, i.e.,[11]

$$Y_s(t) = (\alpha_s - \alpha(t)) Q_r \text{ for all } t \in \left[t_s, t_s^{'}\right]. \tag{26}$$

The last line in Table 1 summarizes production during the storage period.[12] The empty storage is completely filled during the storage period. Hence integration of the equation of motion (17) yields $Q_s = -\eta_s \int_{t_s}^{t'_s} Y_s(t)\, dt$. Upon substitution from (26),

$$Q_s = -\eta_s \int_s (\alpha_s - \alpha(t)) Q_r dt, \tag{27}$$

where $\int_s dt := \int_{t_s}^{t'_s} dt$ denotes the duration of the storage period. This condition implicitly defines the critical availability, α_s, when the storage period starts. Intuitively, the storage period lasts longer when the storage capacity Q_s is large, when the storage efficiency η_s is small, and when the level of renewable capacities that is available for storage, $(\alpha_s - \alpha(t)) Q_r$, is small.

Finally, for the second intermediate period from t'_s to t'_d the solution follows from the same equilibrium conditions as for the first intermediate period. Thus, for each $\alpha(t)$, the solution is the same as already summarised by cases 1 to 4 in Table 1. Note that $\alpha(t)$ is (weakly) decreasing in t so that the four cases obtain in reverse order. Hence, in the second intermediate period $t'_3 \leq t'_2 \leq t'_1$, where t'_i is the time at which case i ends, and $\alpha(t'_i) = \alpha_i$, $i = 1, 2, 3$. Lemma 1 summarises these results.

[11] More formally, solving the market clearing condition for $Y_s(t) = x_s - Y_f(t) - Y_r(t)$ and noting that there is no storage at t_s so that $x_s = Y_r(t_s) + Y_f(t_s)$, we obtain $Y_s(t) = Y_r(t_s) + Y_f(t_s) - Y_f(t) - Y_r(t)$ during storage. From the constant price it follows that constant production of fossils, $Y_f = Y_f(t_s)$, constant supply of renewables for consumption, $Y_r = Y_r(t_s) = \alpha_s Q_r$, and using the remaining renewable capacities, $(\alpha(t) - \alpha_s) Q_r$, to fill the storage is a profit maximizing schedule. However, if storage starts in case 2 or case 4, this is not the only solution. In particular, case 2 and a storage period that starts during case 2 are characterised by excess capacities of fossils and a price that equals variable production costs. These idle fossil capacities could be used to reschedule some of the storage to $\alpha \in [\alpha_1, \alpha_s]$ without affecting profits. An equivalent argument applies to excess renewable capacities if the storage period starts during case 4.

[12] The term $Y_r(t) = \min\{\alpha(t) Q_r, x(0) - Y_s(t)\}$ accounts for the fact that storage may start in case 4 so that available renewable capacities, $\alpha(t) Q_r$, exceed the sum of consumer demand at price 0 and storage, $x(0) - Y_s(t)$.

Lemma 1. *Equilibrium levels for production and storage are as given in Table 1. Demand and prices follow straightforwardly from the market clearing condition,* $x(t) = \sum_j Y_j(t)$*, and the inverse demand function* $p(x(t))$*. For low availabilities of renewable capacities, there is destorage, during which prices are maximal and constant. Conversely, for high availabilities there is storage, during which prices are minimal and constant. For intermediate availabilities, the electricity price and supply from fossil capacities are weakly decreasing in the availability of renewables.*

For the subsequent analysis of optimal subsidies, we need to know how the triggered changes in capacities affect production and demand. In principle, this follows from Lemma 1, but for the storage and destorage periods it depends in a nontrivial way on effects via the boundaries of these periods, α_d, α_s. Lemma 2 summarises the relevant comparative statics.

Lemma 2. *Marginal changes in capacities* Q_f, Q_r, Q_s *have the following comparative static effects on demand,* $x(t)$*, and on the availability levels of renewables that determine the length of the storage and destorage periods,* α_s, α_d*.*

(a) Fossil capacities: $\frac{\partial \alpha_s}{\partial Q_f} = \frac{\partial \alpha_d}{\partial Q_f} = 0$ *and* $\frac{\partial x(t)}{\partial Q_f} = \frac{\partial}{\partial Q_f} \sum_j Y_j(t)$*, where* $Y_j(t)$ *follows straightforwardly from Lemma 1.*

(b) Renewable and storage capacities: For the intermediate period, $\frac{\partial x(t)}{\partial Q_r} = \frac{\partial}{\partial Q_r} \sum_j Y_j(t)$ *and* $\frac{\partial x(t)}{\partial Q_s} = 0$*. For the storage and destorage period, the derivatives are as given in the table below, with the following exception: if cases 2 or 4 obtain at the beginning of the storage period, then* $\frac{\partial x_s}{\partial Q_r} = \frac{\partial x_s}{\partial Q_s} = 0$*.*

	$\partial \alpha_d / \partial$	$\partial x_d / \partial$	$\partial \alpha_s / \partial$	$\partial x_s / \partial$
Q_r	$-\frac{\int_d (\alpha_d - \alpha(t)) dt}{Q_r \int_d dt} < 0$	$\frac{\int_d \alpha(t) dt}{\int_d dt} > 0$	$-\frac{\int_s (\alpha_s - \alpha(t)) dt}{Q_r \int_s dt} > 0$	$\frac{\int_s \alpha(t) dt}{\int_s dt} > 0$
Q_s	$\frac{1}{\eta_d Q_r \int_d dt} > 0$	$\frac{1}{\eta_d \int_d dt} > 0$	$-\frac{1}{\eta_s Q_r \int_s dt} < 0$	$-\frac{1}{\eta_s \int_s dt} < 0$

Accordingly, only changes in renewable and storage capacities have a direct effect on the length of the storage and destorage period. In particular, an increase in Q_r (weakly) raises renewable production for all t (due to lower variable costs than fossils). Therefore, a given storage capacity is filled faster and storage starts later, i.e., $\frac{\partial \alpha_s}{\partial Q_r} > 0$. The magnitude of this effect is given by the additional production of a marginal renewable capacity unit over the storage cycle, $\int_s (\alpha_s - \alpha(t)) dt$, weighted by the overall capacity, Q_r, and the length of the storage period, $\int_s dt$. Similarly, the destorage period lasts shorter ($\frac{\partial \alpha_d}{\partial Q_r} < 0$) because a given level of stored electricity, Q_s, has to substitute for a larger amount of renewables over the destorage period. The corresponding marginal changes in demand during destorage and storage, $\frac{\partial x_d}{\partial Q_r}, \frac{\partial x_s}{\partial Q_r}$, are simply additional renewable production over the destorage and storage period, weighted by the length of the respective period; with the following exception: if storage starts during case 2 or 4, then demand is constant at $x_s = x(b_f)$ and $x_s = x(0)$, respectively.

Next, consider an increase in the storage capacity, Q_s. Ceteris paribus, this trivially leads to longer storage and destorage periods, i.e., $\frac{\partial \alpha_s}{\partial Q_s} < 0$ and $\frac{\partial \alpha_d}{\partial Q_s} > 0$. The size of this effect is smaller if the level of intermittent renewables, Q_r, that are balanced by storage and destorage is larger, and if the respective periods last longer. In addition, if conversion losses of storage are large (small η_s), then it takes longer to fill an additional unit of storage so that α_s falls more strongly in Q_s. Similarly, if conversion losses of destorage are large (high η_d), then the electricity that can be stored with an additional unit of Q_s is depleted more quickly so that α_d and the length of the destorage period increase less.

Finally, consider again effects on demand in the table. When an additional unit of Q_s must be filled during storage, prices increase and demand, x_s, is reduced. The effect is stronger when more electricity has to be taken from the market because only little arrives in the storage due to conversion losses, that is, if $\eta_s \int_s dt$ is small. During the destorage period, an additional unit of Q_s reduces the price and raises demand, x_d. The effect is smaller when more of this additional electricity is lost over the destorage period due to conversion losses, that is, if $\eta_d \int_d dt$ is large.

4 Capacity Choices of Competitive Firms

We now turn to Stage 2, in which fossil, renewable, and storage firms choose their respective capacities, thereby anticipating the outcome of production decisions in Stage 3. Remember that the value functions $\pi_j\left(y_j^*\left(q_j\right)\right)$, $j = f, r, s$, as given by (1), (5), and (9), represent, respectively, the maximum profits that fossil, renewable, and storage firms can achieve for given capacities, q_j, during one representative cycle, $t \in [t_0, T]$. Note that, by construction, production choices in one cycle have no effect on other cycles. Therefore, the net present value of profits over the lifetime of capacities—which is m representative cycles—is simply $\sum_{z=1}^{m} \frac{1}{(1+r)^z} \pi_j\left(y_j^*\left(q_j\right)\right) = \rho \pi_j\left(y_j^*\left(q_j\right)\right)$, where $\rho := \frac{1}{r} - \frac{1}{r(1+r)^m}$ and r is the discount factor. Accounting for capacity costs, $c_j\left(Q_j\right) q_j$, and for a given policy vector of subsidies and taxes, $\theta = (\sigma_r, \sigma_s, \tau)$, the profits that the respective firms maximise in Stage 2 are

$$\pi_f\left(q_f^*(\theta), \theta\right) := \max_{q_f} \rho \int_{t_0}^{T} \left(p\left(t\right) - k_f - \tau\right) y_f^*(t, q_f) dt - c_f\left(Q_f\right) q_f, \tag{28}$$

$$\pi_r\left(q_r^*(\theta), \theta\right) := \max_{q_r} \rho \int_{t_0}^{T} p\left(t\right) y_r^*(t, q_r) dt - \left(c_r(Q_r) - \sigma_r\right) q_r, \tag{29}$$

$$\pi_s\left(q_s^*(\theta), \theta\right) := \max_{q_s} \rho \int_{t_0}^{T} p\left(t\right) y_s^*(t, q_s) dt - \left(c_s(Q_s) - \sigma_s\right) q_s, \tag{30}$$

where we have substituted for the value functions $\pi_j\left(y_j^*\left(q_j\right)\right)$ from (1), (5), and (9).

When choosing capacity levels, competitive firms take as given the capacity choices of other firms, unit capacity costs, $c_j(Q_j)$, the equilibrium electricity demand and price, $x\left(t\right), p\left(t\right)$, as well as the occurrence of cases and the t where they start (columns 1 and 2 of Table 1). Using this, differentiation of the objective functions in (28) to (30) with respect to the respective capacities yields the following first-order conditions for the representative fossil, renewable, and storage firm ($\pi_{jj} := d\pi_j\left(q_j^*(\theta), \theta\right)/dq_j$ for $j = f, r, s$):

$$\pi_{ff} = \rho \int_{t_0}^{T} \left(p\left(t\right) - k_f - \tau\right) \frac{dy_f^*\left(t, q_f\right)}{dq_f} dt - c_f\left(Q_f\right) = 0, \tag{31}$$

$$\pi_{rr} = \rho \int_{t_0}^{T} p\left(t\right) \frac{dy_r^*\left(t, q_r\right)}{dq_r} dt - c_r\left(Q_r\right) + \sigma_r = 0, \tag{32}$$

$$\pi_{ss} = \rho \int_{t_0}^{T} p\left(t\right) \frac{dy_s^*\left(t, q_s\right)}{dq_s} dt - c_s\left(Q_s\right) + \sigma_s = 0, \tag{33}$$

where the derivatives $dy_j^*\left(t, q_j\right)/dq_j$ follow straightforwardly from Table 1 for the respective cases. Intuitively, firms equalise the net present value of additional production from a marginal capacity unit—the integral terms—and its costs, $c_j\left(Q_j\right)$, thereby accounting for subsidies, and a tax on fossils if implemented. A priori, corner solutions might obtain. However, our focus on situations where optimal capacity levels are positive for all three technologies excludes the case that $\pi_{jj} < 0$. Conversely, $\pi_{jj} > 0$ and, thus, positive marginal (and total) profits would lead to entry until the conditions bind and profits are zero again. Indeed, by substituting $c_j\left(Q_j\right), j = f, r, s$ and, thus, the equilibrium capacity levels from (31) to (33) into the profit functions (28) to (30), it is straightforward to show that all firms make zero profits in equilibrium.

5 Optimal Subsidy and Tax Levels

Now consider the regulator's choice of a tax and, alternatively, of subsidies for renewable and storage capacities in Stage 1. Remember from Section 3.1 that $w(x^*)$ is the value function of the consumer surplus maximisation problem in Stage 3, that is, for one representative cycle. Based on the arguments presented in Section 4, the net present value over the lifetime of capacities is $\rho w(x^*)$. Similarly, the net present value of environmental damages from fossil production is $\rho\delta \int_{t_0}^{T} Y_f(t, \mathbf{Q})\, dt$, where the integral term is aggregate fossil production during one representative period, δ the associated environmental unit cost, and $\mathbf{Q} = (Q_f, Q_r, Q_s)$ the vector of overall capacities. Finally, a direct tax τ on fossil production leads to revenues for the government with net present value $\rho\tau \int_{t_0}^{T} Y_f(t, \mathbf{Q})\, dt$.

Optimal subsidies and taxes maximise the discounted stream of consumer and producer surplus minus subsidy and environmental costs plus tax income:

$$\max_{\sigma_r, \sigma_s, \tau} W := \rho w(x^*) + \sum_j n_j \pi_j \left(q_j^*(\theta), \theta\right) - \sigma_r Q_r - \sigma_s Q_s - \rho(\delta - \tau) \int_{t_0}^{T} Y_f(t, \mathbf{Q})\, dt, \tag{34}$$

where $\sum_j n_j \pi_j \left(q_j^*(\theta), \theta\right)$ is the present value of overall producer surplus, and $\delta - \tau$ can be viewed as that part of environmental damages that is not internalised by a tax on fossils. Note that all production and capacity levels are equilibrium values from Stages 2 and 3, but for the sake of parsimony we drop the asterisk in the remainder, except in the value functions.

First, consider the regulator's choice of subsidies, σ_r, σ_s. These do not show up directly in the consumer surplus maximisation problem (22) and (23). Hence the constrained envelope theorem implies $\frac{dw(x^*)}{d\sigma_j} = 0$ for $j = r, s$. Similarly, profits, $\pi_j \left(q_j^*(\theta), \theta\right)$, are also value functions so that the envelope theorem again implies that only direct effects of subsidies need to be taken into account when differentiating. Hence, for $j = f, r, s$ and $k = r, s$ we have

$$\frac{d\pi_j \left(q_j^*(\theta), \theta\right)}{d\sigma_k} = \frac{\partial \pi_j \left(q_j^*(\theta), \theta\right)}{\partial \sigma_k} = \begin{cases} 0 & \text{for } j \neq k, \\ q_r & \text{for } j = k = r, \\ q_s & \text{for } j = k = s. \end{cases} \tag{35}$$

Using this and collecting terms, the first-order conditions for optimal subsidies are[13]

$$-\sigma_r \frac{dQ_r}{d\sigma_j} - \sigma_s \frac{dQ_s}{d\sigma_j} - \rho(\delta - \tau) \int_{t_0}^{T} \frac{dY_f(t, \mathbf{Q})}{d\sigma_j} dt = 0, \quad j = r, s. \tag{36}$$

Subsidies constitute revenues for firms but costs of the same size for the government. These direct effects cancel in the aggregate if subsidies are financed by lump-sum taxation. Therefore, optimal subsidies follow from their indirect effects via affecting capacity choices. From (32) and (33), σ_r and σ_s represent the amount by which the costs of a marginal capacity unit exceed the net present value of the resulting additional production. Hence, $\sigma_r \frac{dQ_r}{d\sigma_j}$ and $\sigma_s \frac{dQ_s}{d\sigma_j}$ can be interpreted as marginal costs of distorting renewable and storage capacities. At the optimal subsidy level, these must be equal to the present value of non-internalised avoided marginal damages.

The derivation of the first-order condition for taxes is more complex because during case 2 of the intermediate period and during storage that starts with case 2, the price is $p(t) = b_f = k_f + \tau$. Thus, compared to subsidies, a tax can also have a direct effect on the electricity price that must be taken into account. However, the additional expenses of consumers that result from a tax-induced higher price are additional revenues for firms so that these effects cancel out in the aggregate (see Appendix C). Hence the first-order condition of the tax has a very simple form, too:

$$-\sigma_r \frac{dQ_r}{d\tau} - \sigma_s \frac{dQ_s}{d\tau} - \rho(\delta - \tau) \int_{t_0}^{T} \frac{dY_f(t, \mathbf{Q})}{d\tau} dt = 0. \tag{37}$$

[13] It is straightforward to show that all terms under the integral as well as their derivatives are continuous so that one can apply the Leibniz rule and differentiate under the integral sign (see Sydsaeter, Hammond, Seierstad, and Strom 2005, p. 156).

It is straightforward to see that $\sigma_r, \sigma_s = 0$ and a tax $\tau = \delta$ satisfies all three first-order conditions. Moreover, in the proof of Proposition 2 below we show that $\tau = \delta$ is not only a critical point but a global maximum. Hence we have the following result.

Proposition 1. *The first-best solution obtains with a Pigouvian tax on fossils, $\tau^* = \delta$, and no subsidies for renewable and storage capacities.*

Proposition 1 confirms the expectation that Pigouvian taxation also works in a model that accounts for intermittency of renewables and storage. However, it neither excludes the possibility that solutions with $\sigma_r, \sigma_s \neq 0$ are also efficient, nor does it provide guidance for second-best subsidy levels when Pigouvian taxes are not feasible—for example, due to political economy reasons. To examine this, we need a closer evaluation of the derivatives $\frac{dQ_r}{d\theta}, \frac{dQ_s}{d\theta}, \frac{dY_f}{d\theta}$, $\theta = \sigma_r, \sigma_s, \tau$ that capture the interaction in Stages 2 and 3. As discussed in Subsection 3.2, the outcome of Stage 3 depends on which of the intermediate cases 1 to 4 obtain in equilibrium. If storage starts during case 1 (case 2), fossil capacities are fully (partially) used during storage. By contrast, if it starts in case 3 or 4, no fossils are used during storage. We now examine these cases in turn.

5.1 Fossil Capacities Fully Used During Storage

First, suppose that $\alpha_s < \alpha_1$ so that only case 1 obtains in the intermediate period. From Lemma 1, in this case fossil capacities are always fully used, i.e., $y_f(t, \mathbf{Q}) = q_f$ so that $\frac{dy_f(t,\mathbf{Q})}{dq_f} = \frac{dY_f(t,\mathbf{Q})}{dQ_f} = 1$. Moreover, for each t the equilibrium electricity price that obtains in Stage 3 does not directly depend on σ_j, but is a function of capacities that are given at this stage, i.e., $p(t) = p(t, \mathbf{Q})$. Thus, total differentiation of the first-order condition (31) for fossil capacities yields

$$d\pi_{ff} = \left(\rho \int_{t_0}^{T} \frac{\partial p(t)}{\partial Q_f} dt - c_f'(Q_f)\right) dQ_f + \left(\rho \int_{t_0}^{T} \frac{\partial p(t)}{\partial Q_r} dt\right) dQ_r + \left(\rho \int_{t_0}^{T} \frac{\partial p(t)}{\partial Q_s} dt\right) dQ_s. \quad (38)$$

Using the compact notation $\int_s dt$ and $\int_d dt$ for the destorage and storage periods, the last integral term in (38) becomes $\int_{t_0}^{T} \frac{\partial p(t)}{\partial Q_s} dt = \frac{\partial p_s}{\partial Q_s} \int_s dt + \frac{\partial p_d}{\partial Q_s} \int_d dt$, where p_s and p_d are the constant prices during storage and destorage. This takes into account that the price in the intermediate period does not (directly) depend on Q_s, simply because storage capacities are not used in this period (see Lemma 1). Setting $d\pi_{ff} = 0$, dividing by $d\sigma_j$, and rearranging yields

$$\frac{dQ_f}{d\sigma_j} = \frac{\int_{t_0}^{T} \frac{\partial p(t)}{\partial Q_r} dt}{\frac{c_f'(Q_f)}{\rho} - \int_{t_0}^{T} \frac{\partial p(t)}{\partial Q_f} dt} \frac{dQ_r}{d\sigma_j} + \frac{\frac{\partial p_s}{\partial Q_s} \int_s dt + \frac{\partial p_d}{\partial Q_s} \int_d dt}{\frac{c_f'(Q_f)}{\rho} - \int_{t_0}^{T} \frac{\partial p(t)}{\partial Q_f} dt} \frac{dQ_s}{d\sigma_j}, \quad (39)$$

where the two fractions are simply the partial derivatives $\frac{\partial Q_f}{\partial Q_r}$ and $\frac{\partial Q_f}{\partial Q_s}$, respectively. We can apply the chain rule when partially differentiating prices $p(t) = p(x(t))$ with respect to Q_j. Using Lemma 2, this yields $\frac{\partial p_s}{\partial Q_s} = \frac{\partial p_s}{\partial x_s} \frac{\partial x_s}{\partial Q_s} = -\frac{\partial p_s}{\partial x_s} \frac{1}{\eta_s \int_s dt}$ and $\frac{\partial p_d}{\partial Q_s} = \frac{\partial p_d}{\partial x_d} \frac{1}{\eta_d \int_d dt}$. Similarly, $\int_{t_0}^{T} \frac{\partial p(x(t))}{\partial Q_r} dt = \int_{t_0}^{T} \frac{\partial p(t)}{\partial x(t)} \alpha(t)\, dt$ and $\int_{t_0}^{T} \frac{\partial p(x(t))}{\partial Q_f} dt = \int_{t_0}^{T} \frac{\partial p(t)}{\partial x(t)} dt$ from Lemma 1 and 2 for the current case where fossil capacities are fully used for all t. Moreover, for this case $\int_{t_0}^{T} \frac{dY_f(t,\mathbf{Q})}{d\sigma_j} dt = \int_{t_0}^{T} dt \frac{dQ_f}{d\sigma_j}$, which allows us to substitute (39) into the condition for optimal subsidies (36). Using the compact notation $\frac{\partial Q_f}{\partial Q_r}$ and $\frac{\partial Q_f}{\partial Q_s}$ for the two fractions in (39) and rearranging yields the two first-order conditions for subsidies,

$$\left(\sigma_r + (\delta - \tau)\rho \int_{t_0}^{T} dt \frac{\partial Q_f}{\partial Q_r}\right) \frac{dQ_r}{d\sigma_j} + \left(\sigma_s + (\delta - \tau)\rho \int_{t_0}^{T} dt \frac{\partial Q_f}{\partial Q_s}\right) \frac{dQ_s}{d\sigma_j} = 0, \quad j = r, s. \quad (40)$$

Note that the two terms in brackets have the same value for $j = r, s$. For any given τ, it follows that subsidies for which these two terms are both equal to zero satisfy the first-order condition. We obtain the following result.[14]

Proposition 2. *Suppose that $\alpha_s < \alpha_1$ so that the fossil technology always produces at full capacity. Then the optimal subsidies of renewable and storage capacities are*

$$\sigma_r^* = \rho \int_{t_0}^{T} dt \frac{\int_{t_0}^{T} \frac{\partial p(t)}{\partial x(t)} \alpha(t)\, dt}{\int_{t_0}^{T} \frac{\partial p(t)}{\partial x(t)} dt - \frac{c_f'(Q_f)}{\rho}} (\delta - \tau), \tag{41}$$

$$\sigma_s^* = \rho \int_{t_0}^{T} dt \frac{-\frac{1}{\eta_s}\frac{\partial p_s}{\partial x_s} + \frac{1}{\eta_d}\frac{\partial p_d}{\partial x_d}}{\int_{t_0}^{T} \frac{\partial p(t)}{\partial x(t)} dt - \frac{c_f'(Q_f)}{\rho}} (\delta - \tau). \tag{42}$$

These subsidies implement the first-best solution. Moreover, for any $\tau < \delta$, the renewable subsidy is strictly positive, whereas the storage subsidy is negative if and only if $\frac{1}{\eta_d}\left|\frac{\partial p_d}{\partial x_d}\right| < \frac{1}{\eta_s}\left|\frac{\partial p_s}{\partial x_s}\right|$.

Accordingly, a combination of subsidising renewables and—usually (see below)—a tax on storage can implement the first-best solution not only for $\tau = 0$, but for any imperfect taxation of the carbon externality.

To examine the optimal values for σ_r and σ_s, note that we switched the terms in the denominator so that the fraction in (41) now stands for the term $-\frac{\partial Q_f}{\partial Q_r} = -\frac{\partial Y_f}{\partial Q_r}$. Accordingly, the optimal subsidy for renewables equals the net present value of non-internalised avoided damages, $\delta - \tau$, from the effect that renewable capacities displace fossils. Moreover, note that $\rho = \sum_{z=1}^{m} \frac{1}{(1+r)^m}$ is decreasing in the discount factor r, increasing in the number of representative cycles m, and $\int_{t_0}^{T} dt$ is the length of such a representative cycle. Thus, σ_r rises if renewable capacities are used longer, and if firms discount future profits less. Finally, σ_r is larger when the resulting higher renewable capacities are more effective in displacing fossil capacities, i.e., when $-\frac{\partial Q_f}{\partial Q_r}$ is large. This is the case when $\int_{t_0}^{T} \left|\frac{\partial p(t)}{\partial x(t)}\right| \alpha(t)\, dt = \int_{t_0}^{T} \left|\frac{\partial p(t)}{\partial Q_r}\right| dt$ is large so that renewable capacities greatly reduce the electricity price (making fossils less attractive); when $\int_{t_0}^{T} \left|\frac{\partial p(t)}{\partial x(t)}\right| dt = \int_{t_0}^{T} \left|\frac{\partial p(t)}{\partial Q_f}\right| dt$ is low so that lower fossil capacities raise the electricity price only little; and when lower fossil capacities lead only to a small reduction of their unit costs $c_f(Q_f)$.

Now consider expression (42) for the optimal storage subsidy. The terms $(\delta - \tau)\rho \int_{t_0}^{T} dt$ are the same as in (41), and the fraction now stands for $-\frac{\partial Q_f}{\partial Q_s}$. Accordingly, the optimal storage subsidy is equal to the net present value of the change in non-internalised damages from the use of fossil capacities that results from a marginal change in storage capacities. However, the sign of $\frac{\partial Q_f}{\partial Q_s}$, i.e., whether more storage capacities reduce or raise the equilibrium level of fossil capacities, is ambiguous and depends on the numerator in (42). In particular, subsidies σ_s raise the storage capacity, Q_s, and, thereby, reduce the price of the destorage period ($\int_d \frac{\partial p_d}{\partial Q_s} dt = \frac{1}{\eta_d}\frac{\partial p_d}{\partial x_d} < 0$). This makes investment in fossils less attractive and reduces associated pollution. The effect is smaller if efficiency losses are large so that only a small share of electricity from the storage arrives in the market. Thus, a high η_d reduces the reason for subsidising Q_s. Conversely, if storage subsidies induce a higher Q_s, then the price of the storage period rises ($\int_s \frac{\partial p_s}{\partial Q_s} dt = -\frac{1}{\eta_s}\frac{\partial p_s}{\partial x_s} > 0$) and, thus, the profitability of investment in fossils. This effect is larger when efficiency losses are large so that more electricity has to be taken from the market to fill the storage. Therefore, a low η_s motivates taxing Q_s. In conclusion, high efficiency losses during destorage (high η_d) and storage (low η_s) both strengthen the case for taxing storage capacities. Moreover, one would expect that demand is more price responsive when prices are high, that is, during destorage, than at the low prices during storage (see Faruqui and Sergici, 2010). This

[14] For any given τ, the optimal subsidies in Proposition 2 are the only solution to (40) if one abstracts from pathological cases such as $\frac{dQ_r}{d\sigma_s}\frac{dQ_s}{d\sigma_r} - \frac{dQ_s}{d\sigma_s}\frac{dQ_r}{d\sigma_r} = 0$.

implies that $|\frac{\partial x_d}{\partial p_d}| > |\frac{\partial x_s}{\partial p_s}| \Longleftrightarrow |\frac{\partial p_d}{\partial x_d}| < |\frac{\partial p_s}{\partial x_s}|$, further supporting the argument that storage capacities should be taxed rather than subsidised.

Given the relevance of this result, it is worth recapitulating the main intuition. If fossils always produce at full capacity, storage can reduce pollution only by making investment in fossils less attractive. This happens when the price reducing effect of storage capacities in the destorage period dominates the price increasing effect during the storage period. Importantly, this is the case although, ceteris paribus, the positive effects on investment incentives for renewables are even larger because they sell most during storage. However, the second policy instrument—the subsidy for renewables—is used to compensate this effect. Therefore, the optimal subsidy σ_s only depends on its indirect effects on fossil capacities and the resulting pollution damage. With more specific assumptions about the functions for demand and capacity costs, the optimal subsidies obtain a very simple structure.

Corollary 1. *Consider the case where the fossil technology always produces at full capacity (i.e., $\alpha_s < \alpha_1$), and assume that $\frac{\partial^2 p}{\partial x^2} = 0$. For any $\tau < \delta$, it is optimal to tax storage capacities if there are conversion losses of storage. If the fossil technology has constant unit costs—i.e., $c_f'(Q_f) = 0$—then optimal subsidies are $\sigma_r^* = \rho(\delta - \tau) \int_{t_0}^{T} \alpha(t)\, dt$ and $\sigma_s^* = -\rho(\delta - \tau)\left(\frac{1}{\eta_s} - \frac{1}{\eta_d}\right)$.*

Renewable capacities are subsidised because they displace pollutive fossils. For linear demand and constant unit costs, this effect is very simple. An additional unit of renewable capacities reduces fossil capacities and production by $\int_{t_0}^{T} \alpha(t)\, dt$. The optimal renewable subsidy equals the net present value of the non-internalised damage avoided.

The storage subsidy affects investments in fossil capacities by changing prices during destorage and storage in opposite directions. For linear demand and no round-trip efficiency losses, these countervailing effects cancel each other, and the optimal storage subsidy would be zero. If we account for conversion losses (low η_s and high η_d), a tax is optimal because lower storage capacities reduce the price during storage by more than they raise the price during destorage. According to Corollary 1, the optimal tax equals the net present value of the non-internalised damage from round-trip losses during a storage cycle, $\frac{1}{\eta_s} - \frac{1}{\eta_d}$.

5.2 Fossil Capacities Partly Used During Storage

Next, consider the case where fossil capacities are only partly used during the storage period, which is characterised by $\alpha_1 < \alpha_s \leq \alpha_2$. Thus, storage starts during case 2, and it follows from Lemma 1 that the price in case 2 of the intermediate period and during storage equals the marginal production costs of fossil firms plus taxes, $b_f = k_f + \tau$. Hence during these periods, their marginal profits are zero, and marginal changes in capacities Q_f, Q_r, Q_s have no effect on the price. Using this and otherwise applying the same manipulations to (38) that in Subsection 5.1 led to condition (39), now yields

$$\frac{dQ_f}{d\sigma_j} = \frac{\int_{d,1} \frac{\partial p(t)}{\partial Q_r} dt}{\frac{c_f'(Q_f)}{\rho} - \int_{d,1} \frac{\partial p(t)}{\partial Q_f} dt} \frac{dQ_r}{d\sigma_j} + \frac{\int_{d} \frac{\partial p_d}{\partial Q_s} dt}{\frac{c_f'(Q_f)}{\rho} - \int_{d,1} \frac{\partial p(t)}{\partial Q_f} dt} \frac{dQ_s}{d\sigma_j}. \tag{43}$$

where subscripts to the integral sign denote the periods over which the integration applies, i.e., $\int_{d,1} dt := \int_{t_0}^{t_1} dt + \int_{t_1'}^{T} dt$ denotes the joint duration of destorage and case 1. The two fractions represent again the partial derivatives $\frac{\partial Q_f}{\partial Q_r}$ and $\frac{\partial Q_f}{\partial Q_s}$, respectively. In line with the preceding discussion, (43) is the same expression as (39) except that all derivatives $\frac{\partial p(t)}{\partial Q_j}, j = r, f, s$, are zero during case 2 and storage. To determine optimal subsidies for a given τ, we need to differentiate expected production of fossils, which yields (using Lemma 1)

$$\int_{t_0}^{T} \frac{dY_f(t, \mathbf{Q})}{d\sigma_j} dt = \int_{d,1} dt \frac{dQ_f}{d\sigma_j} - \int_{2} \alpha(t)\, dt \frac{dQ_r}{d\sigma_j} - \int_{s} dt \frac{d(\alpha_s Q_r)}{d\sigma_j}, \tag{44}$$

The expression reflects that fossil capacities are only partly used during case 2 of the intermediate period and during storage. Substitution of (44) and (43) into the first-order condition (36) for optimal subsidies yields, after some further transformations (see Appendix E), the following result.

Proposition 3. *Suppose that $\alpha_1 < \alpha_s \leq \alpha_2$ so that fossil capacities are only partly used during the storage period. Then, optimal subsidies of renewable and storage capacities follow from*

$$\sigma_r^* = \rho \left(\int_{d,1} dt \frac{\int_{d,1} \frac{\partial p(t)}{\partial x(t)} \alpha(t)\, dt}{\int_{d,1} \frac{\partial p(t)}{\partial x(t)} dt - \frac{c_f'(Q_f)}{\rho}} + \int_{2,s} \alpha(t)\, dt \right) (\delta - \tau), \tag{45}$$

$$\sigma_s^* = \rho \left(\int_{d,1} dt \frac{\frac{1}{\eta_d} \frac{\partial p_d}{\partial x_d}}{\int_{d,1} \frac{\partial p(t)}{\partial x(t)} dt - \frac{c_f'(Q_f)}{\rho}} - \frac{1}{\eta_s} \right) (\delta - \tau). \tag{46}$$

For any $\tau \neq \delta$, these subsidies do not implement the first-best solution, and for $\tau < \delta$ raising the tax on fossil pollution would increase welfare. Moreover, for any $\tau < \delta$, the renewable subsidy is strictly positive, whereas the storage subsidy is negative if and only if $\frac{1}{\eta_d} \frac{\partial p_d}{\partial x_d} \frac{\int_{d,1} dt}{\int_{d,1} \frac{\partial p(t)}{\partial x(t)} dt - \frac{c_f'(Q_f)}{\rho}} < \frac{1}{\eta_s}$.

Accordingly, subsidies are no longer efficient if the market penetration of renewables is large enough so that fossil capacities lie partly idle during times of high availability. The reason is that a tax would raise the market price $b_f = k_f + \tau$ during case 2 and storage, but subsidies not. Therefore, the latter lead to higher electricity consumption that is met with pollutive fossils during these periods. In line with this, we show in Appendix E that, for any $\tau < \delta$, at the optimal subsidy levels welfare increases in the tax by $\frac{dW}{d\tau} = -\rho(\delta - \tau) \int_{2,s} \frac{dx(b_f)}{d\tau} dt > 0$.

Similar to Proposition 2, the brackets in (45) and (46) represent the terms $-\int_{t_0}^{T} \frac{\partial Y_f(t)}{\partial Q_r} dt$ and $-\int_{t_0}^{T} \frac{\partial Y_f(t)}{\partial Q_s} dt$. Accordingly, optimal subsidies are again equal to the net present value of the change in non-internalised pollution damages induced by a marginal unit of renewable and storage capacities, respectively. During destorage and case 1, fossils are fully used so that over this range the effects are essentially the same as in Proposition 2. This is represented by the respective first fraction in the brackets. During case 2 of the intermediate periods and during storage—the other term in the brackets—fossil capacities are only partly used. Moreover, the price is constant at ($p = b_f$) so that there are no price effects, which simplifies the interaction. Specifically, a marginal increase in renewable capacities raises renewable production by $\alpha(t)$ and replaces fossil production by this amount, thus mitigating pollution by $(\delta - \tau) \int_{2,s} \alpha(t)\, dt$ (see (45)). In principle, the same intuition applies to the second term in the bracket of condition (46). However, storage capacities have no effect during the intermediate period, and during storage the change in fossil production induced by a marginal unit of storage capacities is $\int_s \frac{\partial Y_f}{\partial Q_s} dt = \int_s \frac{\partial \alpha_s}{\partial Q_s} Q_r dt = -\frac{1}{\eta_s}$ (from (44) and Lemma 2).

The renewable subsidy, σ_r, is strictly positive. By contrast, the sign of the storage subsidy, σ_s, depends again on the relative strength of its countervailing effects during storage and destorage. As in Proposition 2, higher conversion losses during destorage (high η_d) reduce the positive first term in the bracket of (46), and higher conversion losses during storage (low η_s) increase the negative second term in the bracket. In conclusion, a less efficient storage technology supports the taxation of storage.

Note that the difference between the optimal subsidies in Propositions 2 and 3 result primarily from the absence of price effects during case 2 and storage when fossils are only partly used in these periods. In line with this, the following corollary shows that the optimal subsidies are the same if these price effects are suppressed by assuming linear demand.

Corollary 2. *Consider the case where fossil capacities are only partly used during the storage period (i.e., $\alpha_1 < \alpha_s \leq \alpha_2$), and assume that $\frac{\partial^2 p}{\partial x^2} = 0$. For any $\tau < \delta$, it is optimal to tax storage capacities when $c_f'(Q_f) > 0$ and/or when there are round-trip efficiency losses of storage. If the fossil technology* *has constant unit costs—i.e., $c_f'(Q_f) = 0$—then optimal subsidies are $\sigma_r^* = \rho(\delta - \tau)\int_{t_0}^{T} \alpha(t)\,dt$ and $\sigma_s^* = -\rho(\delta - \tau)\left(\frac{1}{\eta_s} - \frac{1}{\eta_d}\right)$.*

The only difference from Corollary 1 is that the result regarding the optimality of a tax on storage capacities is even stronger. The difference obtains because changes in technology costs, i.e., $c_f'(Q_f) > 0$, have an effect only when fossil capacities are fully used. This is now the case during destorage, but not during storage.

5.3 Only Renewable Capacities Used During Storage

We now consider the situation where the intermediate period extends to case 3, and possibly also to case 4. Formally, this situation is characterised by $\alpha_2 < \alpha_s$. From Lemma 1, fossils do not produce in these cases, nor in the storage period that follows them. Therefore, subsidies have no effects on fossil production during these periods and expression (44) simplifies to

$$\int_{t_0}^{T} \frac{dY_f(t, \mathbf{Q})}{d\sigma_j}dt = \int_{d,1} dt\frac{dQ_f}{d\sigma_j} - \int_{2} \alpha(t)\,dt\frac{dQ_r}{d\sigma_j}. \tag{47}$$

As in Subsection 5.2, fossils produce at full capacity during destorage and case 1 so that (43) still applies. Substituting this and (47) into the two first-order conditions (36) for optimal subsidies yields the next result.

Proposition 4. *Suppose that $\alpha_2 < \alpha_s$ so that no fossil capacities are used during the storage period. Then, optimal subsidies of renewable and storage capacities are*

$$\sigma_r^* = \rho\left(\int_{d,1} dt\frac{\int_{d,1}\frac{\partial p(t)}{\partial x(t)}\alpha(t)\,dt}{\int_{d,1}\frac{\partial p(t)}{\partial x(t)}dt - \frac{c_f'(Q_f)}{\rho}} + \int_{2}\alpha(t)\,dt\right)(\delta - \tau), \tag{48}$$

$$\sigma_s^* = \rho\left(\int_{d,1} dt\frac{\frac{1}{\eta_d}\frac{\partial p_d}{\partial x_d}}{\int_{d,1}\frac{\partial p(t)}{\partial x(t)}dt - \frac{c_f'(Q_f)}{\rho}}\right)(\delta - \tau). \tag{49}$$

For any $\tau \neq \delta$, these subsidies do not implement the first-best solution. Moreover, for any $\tau < \delta$, both subsidies are strictly positive, and raising the tax on fossil pollution would increase welfare.

Conditions (48) and (49) are the same expressions as in Proposition 3, except that the terms for the storage period are dropped because fossils do not produce during storage. Therefore, renewables no longer replace fossil production and mitigate the associated pollution during times of storage, which reduces the reason for subsidising them.

Also, the storage subsidy, σ_s, is now unambiguously positive. This is because storage capacities now affect fossil production only during the destorage period, where they lower the price and, thus, make investment in fossil capacities less attractive. This effect is larger if conversion losses of destorage are small (η_d close to 1) so that a lot of electricity from the storage arrives on the market.

Corollary 3. *Consider the case where fossil capacities are not used during the storage period (i.e., $\alpha_2 < \alpha_s$), and assume that $\frac{\partial^2 p}{\partial x^2} = 0$ and $c_f'(Q_f) = 0$. Then, the optimal subsidy for renewable capacities is $\sigma_r^* = \rho(\delta - \tau)\int_{d,1,2}\alpha(t)\,dt$ and, thus, decreasing in the level of renewable capacities. The optimal subsidy for storage capacities is constant at $\sigma_s^* = \rho(\delta - \tau)\frac{1}{\eta_d}$.*

To see why the renewable subsidy is decreasing in Q_r, note that fossils are now used only during destorage and cases 1 and 2, as reflected by the integral $\int_{d,1,2} dt$. This period is shorter for higher Q_r because the availability of renewables for which case 2 ends, $\alpha_2 = x(b_f)/Q_r$, is decreasing in Q_r.

Accordingly, for higher Q_r, fossil capacities are used less often so that there is less reason to subsidise their replacement by renewable capacities. Figure 2 summarizes the course of the optimal subsidies for rising levels of renewable capacities—and, accordingly, falling levels of fossils—under the simplifying assumptions of the corollaries.

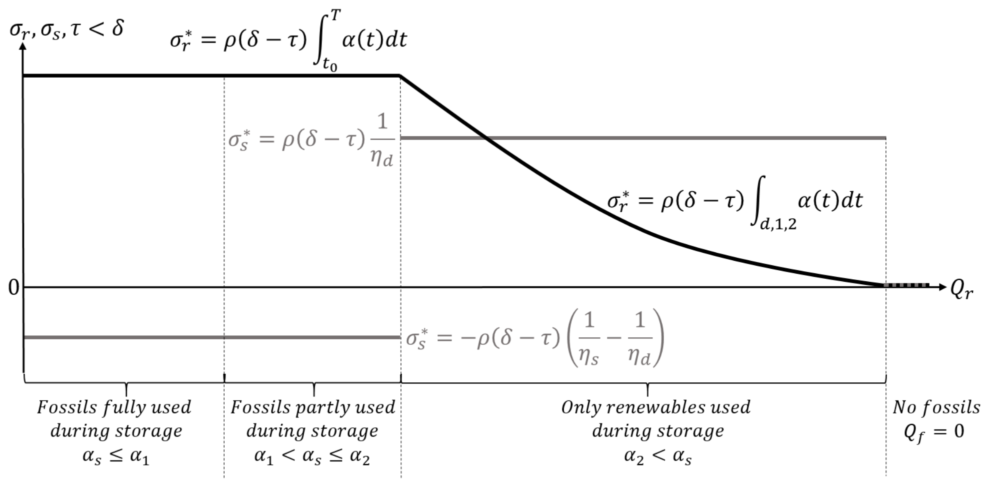

Fig. 2: Optimal subsidies for $\frac{\partial^2 p}{\partial x^2} = 0$ and $c'_f(Q_f) = 0$

6 Concluding Remarks

In this article, we analysed optimal subsidies for installing capacities of intermittent renewable energies and storage when there is imperfect carbon pricing. Renewables reduce the profitability of fossil investments by lowering expected prices and by displacing fossil production at times of high availability. Therefore, the optimal renewable subsidy is always positive. Storage capacities raise the electricity price when the storage is filled and lower it during destorage. This has countervailing effects on the profitability of fossil, and the storage subsidy is chosen on the basis of the relative strength of these effects. As long as the energy system chiefly relies on fossils so that they are always the price-setting technology, storage capacities should be taxed.

For a low market penetration of renewables, this combination of a subsidy for renewables and a tax on storage even leads to the first-best solution. However, as soon as renewable capacities are sufficient to (partly) displace fossil production at times of high availability, the subsidy scheme is only second-best. In addition, optimal subsidies are more complex than a first-best Pigouvian tax. They constitute a moving target because they vary, even in their sign, depending on the relative shares of the three technologies in the electricity system. Optimal subsidies also require substantially more knowledge about the electricity market—such as demand sensitivity—than does a Pigouvian tax, which, in our model, simply equals the environmental unit costs of fossil production, δ. Hence one should read the paper not so much as a call for taxing storage, but as a lesson in the complexity of second-best policies that strengthens the case for directly addressing the externality with a price on carbon.

Our results show that accounting for intermittency of renewables—an aspect that is still often neglected in the analytical literature—has substantial implications for the design of policy instruments. Moreover, given the substantial public funds that currently subsidise renewables, and, increasingly, storage, this paper is of high policy relevance. At least under the simplifying assumptions in the corollaries, it provides an argument for gradually reducing the subsidy for renewables as their market penetration rises, and raising the subsidy for storage instead. However, the latter is not targeted at supporting the rising share of renewables, because the market provides sufficient incentives to build storage capacities if there is more fluctuating electricity from renewables. Rather, storage of electricity is subsidised because it substitutes fossil production when the availability of renewables is low. Our analysis also has implications for the subsidization of electric vehicles, especially when their batteries are not only used for driving, but also as buffers to balance the intermittent supply of renewables. In countries where the share of renewables is still relatively low, our results weaken the case for their subsidization; although other reasons such as reducing local air pollution may still justify subsidies (see Holland, Mansur, Muller, and Yates, 2016).

This points to the limitations of our analysis that need be taken into account when drawing policy recommendations, in addition to the usual caveat that the analysis is based on a stylized model that abstracts from many important aspects of real-world electricity markets. First, we only considered the carbon externality, but there are other relevant market failures, like positive externalities from R&D (Acemoglu et al., 2012). Given the current technological progress in storage technologies such as batteries and power-to-gas, these externalities may well provide an overriding argument for subsidising storage. Moreover, we only considered subsidies for capacities, whereas dominating instruments for renewables have been feed-in-tariffs and market premiums—that is, a subsidisation of electricity output. However, for most of the cases that we analysed, renewable capacities are fully used so that a subsidy per unit of output is equivalent to a subsidy per unit of capacity that is available on average. From this perspective, a subsidy for renewable capacities is quite similar to the feed-in-tariff in that both are paid independently of the price that obtains on the market for electricity.

There are several ways to broaden the analysis. Among them is the extension of the model to several renewable (e.g., wind and solar) and storage technologies (e.g., batteries, pumped hydro storage, and power-to-gas). This would allow a more accurate distinction between the different storage needs that result from daily and seasonal seasonal variations in the availability of intermittent renewables (see Sinn (2017) and Zerrahn, Schill, and Kemfert (2018) for a discussion). The model could also be extended by including other market failures—e.g., R&D spillovers and imperfect competition—or further aspects of electricity markets, such as variable demand, trade and the transmission grid. However, the first extension would make it more difficult to isolate the effects that were the focus of this contribution, and the second would probably come at the cost of greater reliance on numerical simulations.

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Appendix

A Proof of Lemma 1

As shown in the main text, the sufficient conditions for optimality are equations (3) and (4) for fossil firms, (7) and (8) for renewable firms, and (16) to (21) for storage firms. We now prove that the equilibrium values in Lemma 1 satisfy these conditions. Starting with the destorage period, it is straightforward to see that the solution $y_r(t) = \alpha(t) q_r$ and $y_f(t) = q_f$ satisfies (3), (4), (7), and (8) for $\mu_r, \mu_f > 0$, and results in a price above the marginal costs of fossils, b_f.[15] Turning to storage firms, conditions (16) and (18) to (20) have already been used at the beginning of Section 3.2 to derive the result of a constant price during destorage. The remaining condition (17) describes how storage is depleting ($\dot{s}(t) < 0$), where the length of the destorage period that follows from (25) ensures that destorage levels are consistent with the quantity of stored electricity.

Once the storage has run empty, we have $s(t) = 0$ so that $\varphi_d(t)$ turns (weakly) positive (see (20)). This initiates the first intermediate period, $t \in (t_d, t_s)$, during which $\dot{s}(t) = y_s(t) = 0$ (eqs. (17)). For renewables, (7) and (8) imply that $y_r(t) = \alpha(t) q_r$ for any $p(t) > 0$ (cases 1 to 3), whereas for $p(t) = 0$ any output $y_r(t)$ is profit maximizing due to marginal costs of zero. Moreover, the fact that $\alpha(t)$ is increasing during the first intermediate period implies that, ceteris paribus, $p(t)$ is decreasing. As long as $p(t) > b_f$, we have $\mu_f(t) > 0$ from (3) so that $y_f(t) = q_f$ from (4). This case 1 continues until full usage of fossil and available renewable capacities have lowered the price to the variable costs of fossils, $p(t) = b_f$. Accordingly, case 2 starts when $x(b_f) = \alpha Q_r + Q_f$ which yields $\alpha_1 = \frac{x(b_f) - Q_f}{Q_r}$. During case 2, fossils continue to be used so that (3) still binds. Hence, $p(t) = b_f$ implies $\mu_f(t) = 0$ from (3) and so that $0 < y_f(t) < q_f$ by the complementary slackness conditions in (4). Once available renewable capacities, $\alpha(t) Q_r$, are large enough to satisfy demand at $p(t) = b_f$, we enter case 3. Accordingly, case 3 starts when $x(b_f) = \alpha Q_r$ so that $\alpha_2 = \frac{x(b_f)}{Q_r}$. For $\alpha(t) > \alpha_2$, we have $p(t) - b_f < 0$ so that $y_f(t) = 0$ from (3). Finally, once $\alpha(t) Q_r$ is large enough to satisfy demand at $p = 0$, i.e., at $\alpha_3 = \frac{x(0)}{Q_r}$, we enter case 4. It is characterised by $\mu_r(t) = \mu_f(t) = 0$ and excess capacities of renewables, $q_r(t) > y_r(t) > 0$.

Turning to storage firms, condition (19) is met for $\varphi_s(t) = 0$ because $q_s > s(t)$ during the intermediate period, which implies $\dot{\lambda}(t) = -\varphi_d(t) \leq 0$ (from (18)). At the beginning of the preceding paragraph, we have already addressed conditions (20) and (17); hence it remains to show that $y_s(t) = 0$ maximizes $[p(t) - \lambda(t) \eta(y_s)] y_s(t)$ (eqs. (16)). Using $\dot{\lambda}(t) \leq 0$, $\eta_d \geq \eta_s$, and noting that continuity of $\lambda(t)$ was a precondition for (16) to (21) being sufficient, we obtain a situation as depicted by the bold lines in Figure 3. Moreover, during $t \in [t_d, t_s]$ the price $p(t)$ is monotonically decreasing from p_d to p_s, as represented by the dashed line in Figure 2. It is straightforward to see that the values of the multiplier $\varphi_d(t)$ which determines the course of $\lambda(t)$ can be chosen such that $\lambda(t)\eta_d > p(t) > \lambda(t)\eta_s$. Using this, it follows immediately that $y_s(t) > 0$ would lead to $[p(t) - \lambda(t) \eta_d] y_s(t) < 0$, and $y_s(t) < 0$ to $[p(t) - \lambda(t) \eta_s] y_s(t) < 0$. Therefore, $y_s(t) = 0$ must be optimal.

The storage period ($y_s(t) < 0$) can start during either of the cases 1 to 4. As shown in the main text, it has a price that equals the one at the end of the intermediate period, i.e., $p_s = p(t_s)$. Hence, $Y_f = Y_f(t_s)$ from (3) and (4). Moreover, for $p_s > 0$ conditions (7) and (8) imply $Y_r = \alpha(t) Q_r$, whereas for $p_s = 0$ any Y_r is profit maximizing due to the assumption of no variable costs. Turning to storage firms, the argument parallels that during the destorage period: conditions (16) and (18) to (20) have already been addressed at the beginning of Section 3.2, (17) now implies that $\dot{s}(t) > 0$, and (27) ensures that storage levels are consistent with the available storage capacity. Once the storage is completely filled, $s(t) = q_s$ and $\varphi_s(t)$ turns (weakly) positive (see (19)). Thereafter, the second intermediate and destorage periods follow. Their solution and the critical availabilities that distinguish these solutions are the same as discussed above for each respective $\alpha(t)$.

[15] Note that firms are identical so that $y_r(t) = \alpha(t) q_r$ implies $Y_r(t) = n_r y_r(t) = n_r \alpha(t) q_r = \alpha(t) Q_r$, where n_r is the number of renewable firms. An equivalent argument applies to the other quantities in the proof.

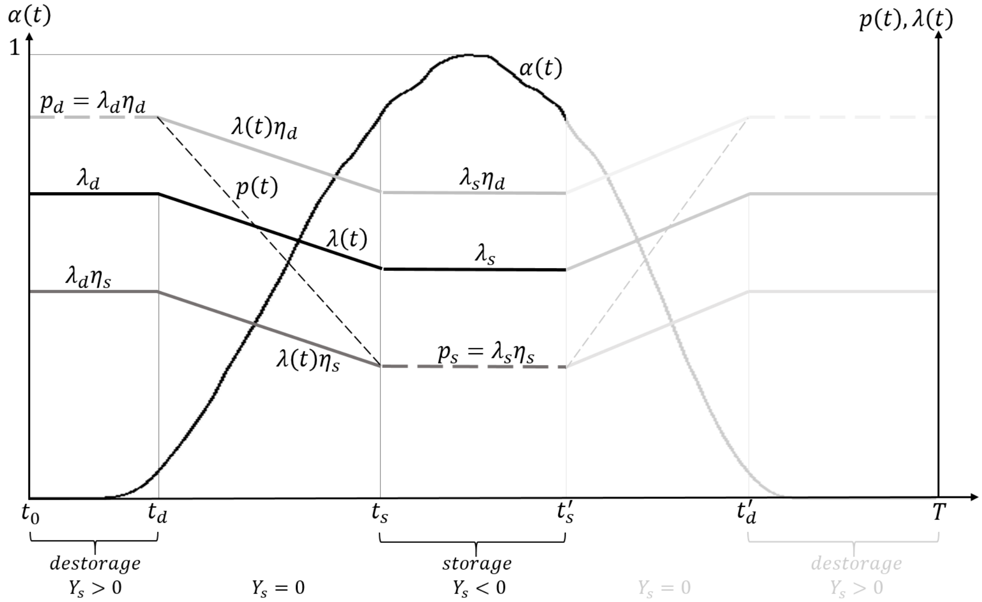

Fig. 3: Availability of renewables and resulting values of the adjoint variable

B Proof of Lemma 2

Conditions (25) and (27), which implicitly determine α_d and α_s, can be written as

$$f_d \;:=\; \eta_d \int_{t_0}^{t_d} \left(\alpha_d - \alpha\left(t\right)\right) Q_r dt + \eta_d \int_{t_d'}^{T} \left(\alpha_d - \alpha\left(t\right)\right) Q_r dt - Q_s = 0, \tag{50}$$

$$f_s \;:=\; -\eta_s \int_{t_s}^{t_s'} \left(\alpha_s - \alpha\left(t\right)\right) Q_r dt - Q_s = 0. \tag{51}$$

The comparative static effects of a change in Q_f, Q_r, or Q_s, thereby taking the other capacities as given, follow from applying the implicit function theorem, i.e., $\frac{\partial \alpha_u}{\partial Q_j} = -\frac{\partial f_u}{\partial Q_j} / \frac{\partial f_u}{\partial \alpha_u}$ for $u = d, s$ and $j = f, r, s$. It follows that $\frac{\partial \alpha_s}{\partial Q_f} = \frac{\partial \alpha_d}{\partial Q_f} = 0$. Next, note that $\alpha_d = \alpha\left(t_d\right) = \alpha\left(t_d'\right)$ and $\alpha_s = \alpha\left(t_s\right) = \alpha\left(t_s'\right)$. This implies that the integral terms in (50) and (51) are zero if evaluated at the boundaries of the integral, t_d, t_d' and t_s, t_s', respectively. Using this when applying the implicit function theorem yields the comparative statics $\frac{\partial \alpha_d}{\partial Q_r}, \frac{\partial \alpha_d}{\partial Q_s}, \frac{\partial \alpha_s}{\partial Q_r}$, and $\frac{\partial \alpha_s}{\partial Q_s}$ in Lemma 2.

Demand during destorage, $x_d = \sum_j Y_j\left(t\right) = Q_f + \alpha_d Q_r$, follows straightforwardly from Lemma 1 and Table 1. Differentiation yields $\frac{\partial x_d}{\partial Q_r} = \alpha_d + \frac{\partial \alpha_d}{\partial Q_r} Q_r$ and $\frac{\partial x_d}{\partial Q_s} = \frac{\partial \alpha_d}{\partial Q_s} Q_r$. Substitution of $\frac{\partial \alpha_d}{\partial Q_r}, \frac{\partial \alpha_d}{\partial Q_s}$ yields the values in Lemma 2, where we have used

$$\frac{\partial x_d}{\partial Q_r} \;=\; \alpha_d - \frac{\int_d \left(\alpha_d - \alpha\left(t\right)\right) dt}{\int_d dt} = \alpha_d - \alpha_d \frac{\int_d dt}{\int_d dt} + \frac{\int_d \alpha\left(t\right) dt}{\int_d dt}. \tag{52}$$

For storage, demand depends on the case that obtains at the beginning of the storage period. If it starts during case 1, 2, or 3 of the intermediate period, then $\min\left\{\alpha\left(t\right) Q_r, x\left(0\right) - Y_s\left(t\right)\right\} = \alpha\left(t\right) Q_r$ so that $x_s = \sum_j Y_j\left(t\right) = Y_f\left(\alpha_s\right) + \alpha_s Q_r$ from Table 1. If storage starts during cases 1 or 3, we obtain $x_s = Q_f + \alpha_s Q_r$ or $x_s = \alpha_s Q_r$, respectively. In both situations, $\frac{\partial x_s}{\partial Q_r} = \alpha_s + \frac{\partial \alpha_s}{\partial Q_r} Q_r$ and $\frac{\partial x_s}{\partial Q_s} = \frac{\partial \alpha_s}{\partial Q_s}$.

The values in Lemma 2 follow again after substituting for $\frac{\partial \alpha_s}{\partial Q_r}, \frac{\partial \alpha_s}{\partial Q_s}$, where we have used

$$\frac{\partial x_s}{\partial Q_r} = \alpha_s - \frac{\int_s (\alpha_s - \alpha(t))\, dt}{\int_s dt} = \alpha_s - \alpha_s \frac{\int_s dt}{\int_s dt} - \frac{\int_s \alpha(t)\, dt}{\int_s dt}. \tag{53}$$

If storage starts during case 2, then $x_s = \sum_j Y_j(t) = x(b_f)$. Similarly, if it starts during case 4, then $\min\{\alpha(t) Q_r, x(0) - Y_s(t)\} = x(0) - Y_s(t)$ so that $x_s = \sum_j Y_j(t) = x(0)$.

C Proof of the first-order condition (37) for taxes

As mentioned in the main text, we need to account for the direct effect of the tax on the electricity price whenever $p(t) = k_f + \tau$. First, if $\alpha_1 \geq \alpha_s$, then case 2 never obtains and $p(t) > k_f + \tau$ for all t. This is the situation where the fossil technology always produces at full capacity. Second, if $\alpha_1 < \alpha_s \leq \alpha_2$, then $p(t) = k_f + \tau$ during case 2 and during storage, for which fossil capacities are only partly used. Finally, if $\alpha_2 < \alpha_s$, then $p(t) = k_f + \tau$ obtains only during case 2, and fossils do not produce during higher cases and storage. Using this, we obtain the following derivatives of optimised profits:[16]

$$\begin{aligned}
\text{for } \alpha_1 \geq \alpha_s: \quad & \frac{\partial \pi_j\left(q_j^*(\theta), \theta\right)}{\partial \tau} = \begin{cases} -\rho \int_{t_0}^{T} y_f^*(t, q_f) dt & \text{for } j = f, \\ 0 & \text{for } j = r, s, \end{cases} \\
\text{for } \alpha_1 < \alpha_s \leq \alpha_2: \quad & \frac{\partial \pi_j\left(q_j^*(\theta), \theta\right)}{\partial \tau} = \begin{cases} -\rho \int_{d,1} y_f^*(t, q_f)\, dt & \text{for } j = f, \\ \rho \int_{2,s} y_r^*(t, q_r)\, dt & \text{for } j = r, \\ \rho \int_s y_s^*(t, q_s)\, dt & \text{for } j = s, \end{cases} \\
\text{for } \alpha_2 < \alpha_s: \quad & \frac{\partial \pi_j\left(q_j^*(\theta), \theta\right)}{\partial \tau} = \begin{cases} -\rho \int_{d,1} y_f^*(t, q_f)\, dt & \text{for } j = f, \\ \rho \int_2 y_r^*(t, q_r)\, dt & \text{for } j = r, \\ 0 & \text{for } j = s, \end{cases}
\end{aligned} \tag{54}$$

where, as usual, the subscripts to the integral sign denote the periods over which the integration applies. Intuitively, a higher tax on pollution reduces the profits of fossil firms, but only when they produce and make positive marginal profits. By contrast, renewable firms benefit if there is a tax-induced higher price during case 2 and storage. Finally, storage firms are unaffected during case 2 (they do not produce), but suffer if there is a tax-induced higher price during storage ($y_s^*(t, q_s) < 0$ during storage). An equivalent case distinction applies for the value function of consumers. The Lagrangian of the consumer surplus maximization problem (22) and (23) is ($\nu(t)$ is the Lagrangian multiplier)

$$\mathcal{L}_x(t) = \int_{p(t)}^{p_{max}} x(\tilde{p})\, d\tilde{p} + \nu(t) \left(\sum_j Y_j(t) - x(t) \right), \tag{55}$$

so that $\frac{dw(x^*)}{d\tau} = \int_{t_0}^{T} \frac{\partial \mathcal{L}_x(t)}{\partial \tau} dt = 0$ if $\alpha_1 \geq \alpha_s$, but

$$\frac{dw(x^*)}{d\tau} = \int_{t_0}^{T} \frac{\partial \mathcal{L}_x(t)}{\partial \tau} dt = \begin{cases} -\int_{2,s} x(b_f)\, dt & \text{for } \alpha_1 < \alpha_s \leq \alpha_2, \\ -\int_2 x(b_f)\, dt & \text{for } \alpha_2 < \alpha_s. \end{cases} \tag{56}$$

This reflects that consumers suffer from tax-induced higher prices. The general first-order condition for optimal taxes follows from differentiation of (34) as

$$\frac{dW}{d\tau} = \rho \frac{dw(x^*)}{d\tau} + \sum_j n_j \frac{d\pi_j\left(q_j^*(\theta), \theta\right)}{d\tau} - \sigma_r \frac{dQ_r}{d\tau} - \sigma_s \frac{dQ_s}{d\tau}$$

[16] The conditions for $\alpha_1 < \alpha_s$ follow straightforwardly after isolating case 2 and storage in the integrals in the profit functions (28) to (30) and noting that $p(t) = k_f + \tau$ during case 2.

$$+\rho\int_{t_0}^{T} Y_f\left(t,\mathbf{Q}\right) - \rho(\delta-\tau)\int_{t_0}^{T}\frac{dY_f\left(t,\mathbf{Q}\right)}{d\tau}dt. \tag{57}$$

Note that $n_j y_j^*\left(t,q_j\right) = Y_j\left(t,\mathbf{Q}\right)$, and for $\alpha_1 < \alpha_s \leq \alpha_2$ we have

$$-\int_{d,1} Y_f(t,\mathbf{Q})dt + \int_{2,s} Y_r(t,\mathbf{Q})dt + \int_{s} Y_s(t,\mathbf{Q})dt + \int_{t_0}^{T} Y_f\left(t,\mathbf{Q}\right) = \int_{2,s} x\left(b_f\right)dt \tag{58}$$

as supply must equal demand, and $\int_2 Y_s(t,\mathbf{Q})dt = 0$. Similarly, for $\alpha_2 < \alpha_s$, we have

$$-\int_{d,1} Y_f(t,\mathbf{Q})dt + \int_{2} Y_r(t,\mathbf{Q})dt + \int_{t_0}^{T} Y_f\left(t,\mathbf{Q}\right) = \int_{2} x\left(b_f\right)dt. \tag{59}$$

Using this when substituting from (54) and (56) into (57) yields the same condition (37) for all three cases.

D Proof of Proposition 2

It remains to prove that the subsidy levels $\sigma_r^*(\tau)$ and $\sigma_s^*(\tau)$ in Proposition 2 are indeed first-best for the case $\alpha_s < \alpha_1$, where the notation clarifies that they are functions of τ. These subsidies are the solution of the first-order conditions in (36), but we need to show that the remaining first-order condition for taxes (37) is also satisfied at $\sigma_r^*(\tau), \sigma_s^*(\tau)$. Dividing (38) by $d\tau$, setting $d\pi_{ff} = 0$, and rearranging yields

$$\frac{dQ_f}{d\tau} = \frac{\int_{t_0}^{T}\frac{\partial p(t)}{\partial Q_r}dt}{\frac{c_f'(Q_f)}{\rho} - \int_{t_0}^{T}\frac{\partial p(t)}{\partial Q_f}dt}\frac{dQ_r}{d\tau} + \frac{\frac{\partial p_s}{\partial Q_s}\int_s dt + \frac{\partial p_d}{\partial Q_s}\int_d dt}{\frac{c_f'(Q_f)}{\rho} - \int_{t_0}^{T}\frac{\partial p(t)}{\partial Q_f}dt}\frac{dQ_s}{d\tau}. \tag{60}$$

Moreover, for the case of Proposition 2, fossils always produce at full capacity so that $\int_{t_0}^{T}\frac{dY_f(t,\mathbf{Q})}{d\tau}dt = \int_{t_0}^{T}dt\frac{dQ_f}{d\tau}$. Multiplying this with $\rho(\delta-\tau)$, substituting for $\frac{dQ_f}{d\tau}$ from (60), and noting that the two fractions in (60) are the same as those in expressions (41) and (42) for optimal subsidies with reverse sign, yields

$$\rho(\delta-\tau)\int_{t_0}^{T}\frac{dY_f\left(t,\mathbf{Q}\right)}{d\tau}dt = -\sigma_r^*(\tau)\frac{dQ_r}{d\tau} - \sigma_s^*(\tau)\frac{dQ_s}{d\tau}. \tag{61}$$

Accordingly, any subsidy scheme that satisfies the first-order conditions (36) for subsidies—one for each τ—satisfies the first-order condition for taxes (37), too. Noting that the first-order conditions are necessary but not sufficient, this gives us all candidates for the optimal solution, but some of them may only be local maxima (or saddle points). We now show that all these solution candidates lead to the same welfare level and, therefore, constitute not only local but global maxima.[17]

In particular, substituting $\sigma_r^*(\tau)$ and $\sigma_s^*(\tau)$ into (34) gives us welfare as a function of τ *when subsidies are adjusted optimally* and, thus, welfare for all solution candidates (using $Y_f\left(t,\mathbf{Q}\right) = Q_f$):

$$W(\tau) = \rho w\left(x^*\right) + \sum_j n_j\pi_j\left(q_j^*\left(\theta\right),\theta\right) + \rho\left(\delta-\tau\right)\int_{t_0}^{T}dt\left(\frac{\partial Q_f}{\partial Q_r}Q_r + \frac{\partial Q_f}{\partial Q_s}Q_s - Q_f\right), \tag{62}$$

where we have used the compact notation $\frac{\partial Q_f}{\partial Q_r}, \frac{\partial Q_f}{\partial Q_s}$ for the fractions in (41) and (42). Note that the producer surplus of renewable and storage firms (but not for fossil firms) depends directly on subsidies, i.e., we have after substituting for the optimal subsidy scheme

$$\pi_j\left(q_j^*(\theta),\theta\right) = \rho\int_{t_0}^{T}p\left(t\right)y_j^*(t,q_j)dt - \left(c_j(Q_j) + \rho\left(\delta-\tau\right)\int_{t_0}^{T}dt\frac{\partial Q_f}{\partial Q_j}\right)q_j, \quad j = r,s. \tag{63}$$

[17] The standard approach of using the second-order conditions does not work because the derivatives in the Hessian are too complex to be interpretable.

Using this and the envelope theorem when differentiating (62) with respect to τ, cancelling common terms and rearranging yields

$$\frac{dW(\tau)}{d\tau} = \rho(\delta-\tau)\int_{t_0}^{T} dt\left[\frac{\partial Q_f}{\partial Q_r}\frac{dQ_r}{d\tau}+\frac{\partial Q_f}{\partial Q_s}\frac{dQ_s}{d\tau}-\frac{dQ_f}{d\tau}\right]. \tag{64}$$

Remember that the terms $\frac{\partial Q_f}{\partial Q_r}$ and $\frac{\partial Q_f}{\partial Q_r}$ are simply the compact notation for the two fractions in (60) so that the square bracket in (64) is equal to 0. Therefore, $\frac{dW(\tau)}{d\tau}=0$ not only at the Pigouvian tax $\tau^*=\delta$, but for any τ *provided that subsidies are adjusted optimally.* Given that $W(\tau)$ describes all local maxima, they must all have the same welfare and, therefore, constitute global maxima. We conclude that for the case $\alpha_s<\alpha_1$ the subsidy scheme $\sigma_r^*(\tau),\sigma_s^*(\tau)$ is indeed first-best.

E Proof of Proposition 3

From Lemma 2, $\frac{\partial \alpha_s}{\partial Q_f}=0$ so that total differentiation of $\alpha_s Q_r$ gives

$$d(\alpha_s Q_r) = \left(\frac{\partial \alpha_s}{\partial Q_r}Q_r+\alpha_s\right)dQ_r+\frac{\partial \alpha_s}{\partial Q_s}Q_r dQ_s. \tag{65}$$

Substitution of this and (43) into (44) and collecting terms yields

$$\begin{aligned}\int_{t_0}^{T}\frac{dY_f(t,\mathbf{Q})}{d\sigma_j}dt &= \left[\int_{d,1}dt\frac{\partial Q_f}{\partial Q_r}-\int_2\alpha(t)\,dt-\int_s dt\left(\frac{\partial \alpha_s}{\partial Q_r}Q_r+\alpha_s\right)\right]\frac{dQ_r}{d\sigma_j}\\ &\quad+\left[\int_{d,1}dt\frac{\partial Q_f}{\partial Q_s}-\int_s dt\frac{\partial \alpha_s}{\partial Q_s}Q_r\right]\frac{dQ_s}{d\sigma_j},\end{aligned} \tag{66}$$

where $\frac{\partial Q_f}{\partial Q_r}$ and $\frac{\partial Q_f}{\partial Q_s}$ represent the two fractions in (43). Substituting for $\frac{\partial \alpha_s}{\partial Q_r}$ from Lemma 2, we obtain $\left(\frac{\partial \alpha_s}{\partial Q_r}Q_r+\alpha_s\right)\int_s dt=-\int_s(\alpha_s-\alpha(t))\,dt+\alpha_s\int_s dt=\int_s\alpha(t)\,dt$. Moreover, again from Lemma 2, $\frac{\partial p_d}{\partial Q_s}\int_d dt=\frac{\partial p_d}{\partial x_d}\frac{1}{\eta_d}$ and $\frac{\partial \alpha_s}{\partial Q_s}Q_r\int_s dt=-\frac{1}{\eta_s}$. Using this when substituting (66) into the first-order conditions (36) for optimal subsidies, and collecting terms with $\frac{dQ_r}{d\sigma_j}$ and $\frac{dQ_s}{d\sigma_j}$, yields the subsidy levels in Proposition 3.

It remains to prove the statement regarding the inefficiency of σ_r,σ_s, and we proceed similarly as in the proof of Proposition 2. From Table 1,

$$\int_{t_0}^{T}Y_f(t,\mathbf{Q})\,dt=\int_{d,1}Q_f dt+\int_2(x(b_f)-\alpha(t)\,Q_r)\,dt+\int_s(x(b_f)-\alpha_s Q_r)\,dt, \tag{67}$$

where $b_f=k_f+\tau$. Differentiation yields

$$\int_{t_0}^{T}\frac{dY_f(t,\mathbf{Q})}{d\tau}dt=\int_{d,1}dt\frac{dQ_f}{d\tau}+\int_2\left(\frac{dx(b_f)}{d\tau}-\alpha(t)\frac{dQ_r}{d\tau}\right)dt+\int_s\left(\frac{dx(b_f)}{d\tau}-\frac{d(\alpha_s Q_r)}{d\tau}\right)dt. \tag{68}$$

Multiplying (43) by $\frac{d\sigma_j}{d\tau}$ yields

$$\frac{dQ_f}{d\tau} = \frac{\int_{d,1}\frac{\partial p(t)}{\partial Q_r}dt}{\frac{c_f'(Q_f)}{\rho}-\int_{d,1}\frac{\partial p(t)}{\partial Q_f}dt}\frac{dQ_r}{d\tau}+\frac{\int_d\frac{\partial p_d}{\partial Q_s}dt}{\frac{c_f'(Q_f)}{\rho}-\int_{d,1}\frac{\partial p(t)}{\partial Q_f}dt}\frac{dQ_s}{d\tau}, \tag{69}$$

where the two fractions are the same terms that show up in the optimal subsidies (45) and (46), with reverse sign. Using this when substituting (69) and $d(\alpha_s Q_r)\int_s dt = \int_s \alpha(t)\, dt dQ_r - \frac{1}{\eta_s} dQ_s$ (see above) into (68) gives

$$\rho(\delta-\tau)\int_{t_0}^{T}\frac{dY_f(t,\mathbf{Q})}{d\tau}dt = -\sigma_r\frac{dQ_r}{d\tau} - \sigma_s\frac{dQ_s}{d\tau} + \rho(\delta-\tau)\int_{2,s}\frac{dx(b_f)}{d\tau}dt. \tag{70}$$

The left-hand side of the first-order condition for taxes (37) represents the marginal effect of taxes on welfare, $\frac{dW}{d\tau}$. Accordingly, substitution of (70) yields

$$\frac{dW}{d\tau} = -\rho(\delta-\tau)\int_{2,s}\frac{dx(b_f)}{d\tau}dt. \tag{71}$$

This term is strictly positive for any $\tau < \delta$ because $\frac{dx(b_f)}{d\tau} < 0$ (remember that $p(t) = b_f = k_f + \tau$ during case 2 and storage).

F Proof of Proposition 4

It remains to prove the statement regarding the inefficiency of subsidies. The proof is the same as for Proposition 3, if one adjusts it for the fact that fossils no longer produce during storage. Therefore, (67) and (68) still apply if one drops the terms for the storage period $\int_s(\cdot)\,dt$. Moreover, as argued in the main text just before Proposition 4, condition (43) continues to holds so that multiplying by $\int_{d,1} dt\frac{d\sigma_j}{d\tau}$ yields (69). Substituting this into the adjusted ($\int_s(\cdot)\,dt$ is dropped) expression (68) gives (70) if the storage period is dropped in the last integral (i.e., $\int_{2,s}(\cdot)\,dt$ is replaced by $\int_2(\cdot)\,dt$). Accordingly, at the optimal subsidy levels

$$\frac{dW}{d\tau} = -\rho(\delta-\tau)\int_{2}\frac{dx(b_f)}{d\tau}dt, \tag{72}$$

which is strictly positive for any $\tau < \delta$.