

# Electricity Demand Response and Responsiveness Incentives

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# Agenda

- 1 Motivations
- 2 Model
- 3 Optimal contract
- 4 Numerical illustration
- 5 Perspectives
- 6 Practice

# Motivations

## Demand Response (DR)

- **Contract** between a consumer and a producer (or a retailer)
- The consumer is paid to reduce consumption a certain number of days chosen by the producer.
- The number of days of price events is determined at inception.
- The days are chosen dynamically (price event day) and the customer is informed the day before.

## Forms of DR

- **Domestic**: lower price on non-event price day (10 c/kWh vs normal tariff of 15 c/kWh) higher price during price event days (67 c/kWh). Around 30 price-event days per year.
- **Rebate**: consumer's receives money for the consumption they saved compared to a baseline. Used in industry. Potential baseline manipulation.

## Remark

- DR programs reduce consumption level on average but with a significant variance in consumers response.
- Faruqui and Sergici (2010) reports a range of response between 10% and 50% across experiments.
- Low Carbon London (LCL) pricing trial in 2013 reports a range of variation between -200 W and +200 W for consumptions of order of 1,000 W (Schofield et. al. 2014)
- Other experiment reports an average reduction of 78 kW with a standard deviation of 30 kW for a furniture store (Mathieu 2011).

DR contract fails to enhance **responsiveness**, i.e. achieving a regular consumption reduction.

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## Question

What is the optimal contract?

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- We show that **the optimal contract has a rebate form...**

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- We provide **closed-form solution of the optimal contract** in the case of linear energy value.
- We show that **the optimal contract has a rebate form... which is baseline-proofness**.
- Using LCL data, we illustrate the **potential benefits from responsiveness incentives**.

## Non-exhaustive literature on PA and continuous-time

- Laffont and Martimort, *The Theory of Incentives*, Princeton, 2002.
- Holmström and Milgrom, *Econometrica*, 1987.
- Sannikov, *Rev. Econ. Stud.*, 2008.
- Cvitanich, Possamaï & Touzi, Dynamic programming approach to Principal-Agent problems, *Finance & Stochastics*, 2018.
- Crampes & Léautier, *J. Reg. Eco.*, 2015.

## Non-exhaustive literature on DR

- Brown and Sappington, *J. Reg. Eco.*, 2016.
- Chao, *J. Reg. Eco.*, 2011.
- Chao and De Pillis, *J. Reg. Eco.*, 2013.
- Crampes & Léautier, *J. Reg. Eco.*, 2015.
- Hogan, FERC, 2009, 2010.

# Model



## The consumer (The Agent)

We focus on one single price event of duration  $T$ . Dynamics of the consumption during the price event follows:

$$X_T^{a,b} = X_0 + \int_0^T \left( - \sum_{i=1}^N a_i(s) \right) ds + \int_0^T \sum_{i=1}^N \sigma_i \sqrt{b_i(s)} dW_s^i$$

$a_i$  and  $b_i$  efforts to reduce average consumption and volatility of usage  $i$   
Consumer's criterion:

$$V^A(\xi) := \sup_{\nu := (a,b)} J_A(\xi, \nu) := \mathbb{E}^\nu \left[ U_A \left( \xi + \int_0^T (f(X_s^\nu) - c(\nu_s)) ds \right) \right]$$

with  $U_A(x) = -e^{-r x}$ ,  $f(x) = \kappa x$ , constant marginal value  $\kappa$

$$c(a, b) := \underbrace{\frac{1}{2} \sum_{i=1}^N \frac{a_i^2}{\mu_i}}_{c_1(a)} + \underbrace{\frac{1}{2} \sum_{i=1}^N \frac{\sigma_i (b_i^{-1} - 1)}{\lambda_i}}_{c_2(b)}, \quad 0 \leq a_i, \quad 0 < b_i \leq 1.$$

## The producer (The Principal)

$$J_P(\xi, \nu) := \mathbb{E}^\nu \left[ U \left( -\xi - \int_0^T g(X_s) ds - \frac{h}{2} \langle X \rangle_T \right) \right] \text{ with } U(x) = -e^{-\rho x}$$

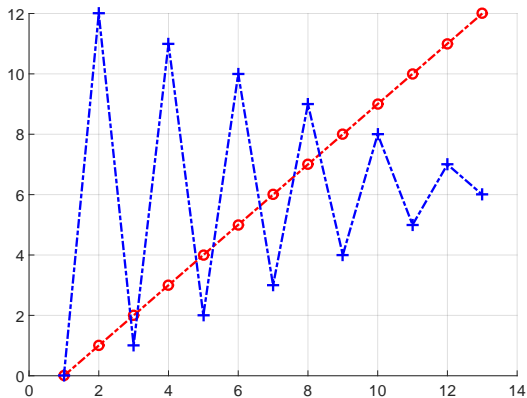
- $g(x) = \theta x$  generation cost function with constant marginal cost  $\theta$
- $h$  direct cost of volatility

Producer's objective:

$$V^{\text{sb}} := \sup_{\xi} J_P(\xi, \nu^*(\xi)).$$

together with the participation constraint of the consumer

$$V_A(\xi) \geq R_0 =: -e^{-rL_0}.$$



Total consumption  $X$  = Total consumption  $X$

$$\langle X \rangle = 1^2 + 1^2 + \dots + 1^2 = 12$$

$$\langle X \rangle = 12^2 + 11^2 + 10^2 \dots + 1^2 = 650$$

## Remarks

- Timing: first, the producer proposes a paying rule, knowing  $L_0$ ; then, the consumer accepts or reject the contract: if he accepts, the price event happens later; the producer measures  $X$ , and pays or charges the consumer.
- The producer does not observe the efforts  $a$  and  $b$  on usages. She only observes the consumption  $X$ .
- The problem is non-Markovian. The contract is written on the observation of the whole path of the consumption on  $[0, T]$ .
- This problem is designated as the **second-best**.

## First-best

$$V^{\text{FB}} := \sup_{\xi, \nu} \left\{ J_P(\xi, \nu) \quad : \quad J_A(\xi, \nu) \geq R_0 \right\}$$

## Consumer's Hamiltonian

$$H(z, \gamma) := H_m(z) + H_v(\gamma), \quad z, \gamma \in \mathbb{R},$$

where

$$H_m(z) := - \inf_{a \geq 0} \{a \cdot \mathbf{1}z + c_1(a)\}, \quad H_v(\gamma) := -\frac{1}{2} \inf_{b \in (0,1]} \{c_2(b) - \gamma |\sigma(b)|^2\},$$

which admits the minimizer

$$\hat{a}_j(z) := \mu_j z^-, \quad \hat{b}_j(\gamma) := 1 \wedge (\lambda_j \gamma^-)^{-\frac{1}{2}}.$$

## Optimal contract [Cvitanic, Possamaï & Touzi (2018)]

- The optimal contract is of the form:

$$Y^{Y_0, Z, \Gamma} := Y_0 + \int_0^t Z_s dX_s + \frac{1}{2} \int_0^t (\Gamma_s + r Z_s^2) d\langle X \rangle_s - \int_0^t (H(Z_s, \Gamma_s) + f(X_s)) ds,$$

where  $Z_s$  and  $\Gamma_s$  are payment rates for efforts on the average consumption and on volatility, and  $H(Z_s, \Gamma_s) + f(X_s)$  is the natural benefit the consumer gets when receiving incentives  $Z_s, \Gamma_s$ .

- Whatever the processes  $Z$  and  $\Gamma$ , one has  $V^A(Y^{Y_0, Z, \Gamma}) = U_A(Y_0)$ .
- Whatever the payment rates  $Z$  and  $\Gamma$ , the Agent will receive his required reservation utility.
- Thus, the Principal can use the payment rates to solve his own optimisation problem, using standard stochastic control methods.

# Optimal contract

## First-best

The optimal first-best contract is given by:

$$\xi_{fb} = L_0 - \kappa X_0 T + \int_0^T c(\nu_t) dt + \int_0^T \pi_{fb}^e (X_0 - X_t) dt - \frac{1}{2} \int_0^T \pi_{fb}^v d\langle X \rangle_t,$$

where

$$\pi_{fb}^e := \frac{r}{r+p} \kappa + \frac{p}{r+p} \theta, \quad \pi_{fb}^v := \frac{p}{r+p} h,$$

and the optimal efforts are:

$$a_{fb}(t) := \mu \delta^- (T - t), \quad b_{fb}(t) := 1 \wedge \left( \lambda (h + \rho \delta^2 (T - t)^2) \right)^{-\frac{1}{2}},$$

with  $\delta := \kappa - \theta$ .

- $\delta > 0 \Rightarrow$  off-peak hours;  $\delta < 0 \Rightarrow$  peak hours.
- Price of energy: constant convex combination of marginal cost and value
- Price of volatility: constant risk-sharing of the direct cost of volatility
- The contract has a rebate form.



## Second-best without responsiveness incentives

The second-best optimal contract without responsiveness incentives is given by

$\xi_{sb_m} = \xi_{sb_m}^f + \xi_{sb_m}^v$  where

$$\xi_{sb_m}^f = L_0 - \kappa TX_0 + \frac{1}{2} \int_0^T rz_{sb_m}^2(t) |\sigma|^2 dt - \int_0^T H_m(z_{sb_m}(t)) dt,$$

$$\xi_{sb_m}^v = \int_0^T \pi_{sb_m}^e (X_0 - X_t) dt,$$

where

$$\pi_{sb_m}^e := (1 - \Lambda)\kappa + \Lambda\theta,$$

and the optimal payment rate is  $z_{sb_m}(t) = \Lambda\delta(T - t)$  with

$$\Lambda := \frac{\rho|\sigma|^2 + \bar{\mu}\mathbf{1}_{\{\delta < 0\}}}{(\rho + r)|\sigma|^2 + \bar{\mu}\mathbf{1}_{\{\delta < 0\}}}.$$

- No price of volatility.
- Prices of energy is a constant convex combination of marginal cost and value of energy.
- In peak-period, the price depend on the volatilities and the costs of efforts.
- The contract has a rebate form where the initial consumption level is the baseline.
- Baseline-proofness: Whatever the initial condition (baseline), the consumer gets no more than  $L_0$ .

## Second-best with responsiveness incentives

The second-best optimal contract is given by  $\xi_{sb} = \xi_{sb}^f + \xi_{sb}^v$  where

$$\xi_{sb}^f := L_0 - \kappa TX_0 - \int_0^T H(z_{sb}, \gamma_{sb})(t) dt$$

$$\xi_{sb}^v := \int_0^T \pi_{sb}^e(t)(X_0 - X_t) dt - \frac{1}{2} \int_0^T \pi_{sb}^v(t) d\langle X \rangle_t,$$

and

$$\pi_{sb}^e(t) := \kappa + z'_{sb}(t), \quad \pi_{sb}^v(t) := h + p(z_{sb}(t) - \delta(T - t))^2,$$

where  $z_{sb}$  is a deterministic function of time, solution of a scalar optimisation problem for each time. The optimal efforts are

$$a_{sb}(t) := \mu z_{sb}(t)^-, \quad b_{sb}(t) := 1 \wedge \left( \lambda \gamma_{sb}(t)^- \right)^{-\frac{1}{2}},$$

$$\gamma_{sb}(t) := -h - rz_{sb}(t)^2 - p(z_{sb}(t) - \delta(T - t))^2.$$

- Prices of energy and volatility are now non-constant deterministic function of time.
- Price of volatility is always greater than the first-best price.
- In peak-hours, the price of energy is also greater than the first-best.
- The contract has a rebate form where the initial consumption level is the baseline.
- Baseline-proofness: Whatever the initial condition (baseline), the consumer gets no more than  $L_0$ .

# Numerical illustration

## Calibration

- Make extensive use of the Low Carbon London 2013 pricing trial.
- We interpret the LCL pricing trial as the implementation of the optimal contract with uncontrolled responsiveness and linear energy value.

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## Parameters shopping list

$T$   $\sigma$   $\kappa$   $\theta$   $h$   $p$   $r$   $\mu_i$   $\lambda_i$

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$T$	$h$	$\kappa$	$\theta$	$\rho$	$r$	$\sigma$	$\mu$	$\lambda$
5.5	$4.0 \cdot 10^{-3}$	11.76	67.2	$0.6 \cdot 10^{-2}$	$0.57 \cdot 10^{-2}$	85	$9.3 \cdot 10^{-5}$	$2.8 \cdot 10^{-2}$

Table: Nominal values for model parameters.



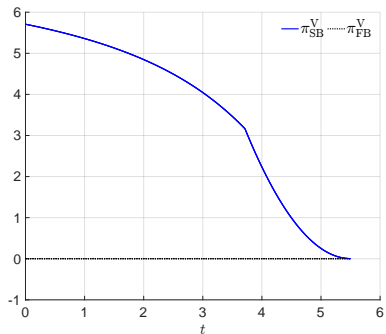
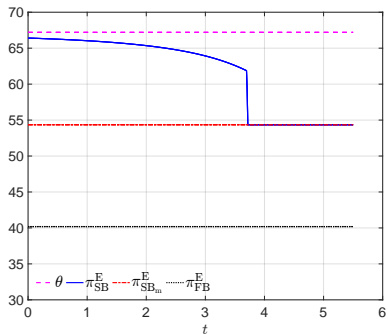


Figure: Prices for energy ( $p/kWh$ ) and volatility ( $p/kW^2$ ).

## Conservative estimate of the benefit from responsiveness incentives

	First-best	Second-best with responsiveness	Second-best without responsiveness
Cost of effort $c_1$	5.97	5.97	4.68
Cost of effort $c_2$	0.40	0.59	0
Total cost of effort	6.37	6.56	4.68
Producer's benefit	6.76	6.21	5.40
Average consumption reduction	52.15	45.17	40.00
Standard deviation consumption	46.49	39.61	85.06

**Table:** Costs in pence, consumption and standard deviation in Watt.

## Perspectives

- Limited liability (no negative payments).
- Group of consumers with different energy valuation (adverse selection)
- Making a pricing trial with responsiveness incentives.

# Practice

## Index the contract on the quadratic variation

Use the approximation of the quadratic variation  $\langle X \rangle_T$

$$\langle X \rangle_T \approx \sum_{i=0}^{n-1} (X_{t_{i+1}} - X_{t_i})^2.$$

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## Index on the event-per-event variations of the consumption

On event  $k$ , the producer measure the quantity

$$\bar{X}_k := \int_0^T X_t dt$$

and charges the consumer a cost proportional to the quantity

$$\mathbb{V}_k := |\bar{X}_k - X_c|$$

where  $X_c$  is a contractualised targeted consumption in the spirit of Chao (2011).

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