Sequential Moral Hazard under Financial Limits: Sequential versus Partnership Contracts

Marco Buso[*] Luciano Greco[†]

February 2017
Preliminary and Incomplete Version

Abstract

In a moral hazard problem, the principal delegates the implementation of two sequential tasks. The principal and the agents are risk-neutral and may face financial constraints. There is no production externality between different tasks. Under sequential contracts, two agents are hired to separately implement the tasks, and the optimal second task contract is independent of first task output (i.e., it has no memory). Under partnership contract, the principal delegates both tasks to a single agent through a memory contract. When all agents face financial constraints, partnership is found to be more effective than sequential contracts to deal with the moral hazard issue. The same result holds and could be reinforced if the principal faces financial constraints. Our model provides a novel explanation of the observed correlation between financial conditions and the likelihood that infrastructure investment and operation are implemented through public-private partnerships.

Keywords: Complete Contracts, Limited liability, Budget constraint, Public-Private Partnerships

JEL classification: D86, L33, H11, H57, C61

[*]buso.marco@unipd.it, dSEA and CRIEP, University of Padua.
[†]luciano.greco@unipd.it, dSEA and CRIEP, University of Padua.

1 Introduction

The involvement of private companies in the construction and operation of public infrastructures and services is an old practice. In public works, it became quite common since the introduction of concession contracts in the legal system, as early as the beginning of XIX century. However, two distinctive features have characterized the evolution of new forms of public-private partnerships (PPPs) in the last thirty years: the greater emphasis on "value for money" for the public sector, implying a careful analysis of risk-and-task sharing agreements; the greater complexity of these new arrangements, as compared to traditional concession contracts.[1]

The long-run growing trend in the number and size of PPPs across countries, with the exception of recent years, is the main evidence of the spreading relevance of such new forms of public-private contracts (see Figure 1 for the European countries). Which factors can explain the described success of PPPs? The advantages claimed by practitioners, both from public sector and private industry, are not always in accordance with economic theory and empirical evidence. In particular, governments and private contractors have often credited PPPs with substantial benefits in the forms of efficiency gains and public finance relief. Though, the economic literature has uncovered the channels through which efficiency advantages (or disadvantages) of PPPs may materialize (Iossa and Martimort, 2015), it has not yet provided any support to the idea of their ability to relieve strained governments' budgets (Engel et al., 2013).

Yet, empirical cross-country and country-specific evidence has shown that tax burden, public debt levels, and financial conditions are significantly and positively correlated with the choice to undertake public investments in the form of PPPs (Hammami et al., 2006; Albalate et al., 2012). Political economy explanations of the correlation between government investment decisions and strained public finance – such as debt-hiding and non-compliance to fiscal rules – have been provided (Buti et al., 2007; Von Hagen and Wolff, 2006). However, by recent empirical studies (Buso et al., ress), they seem not to be valuable in explaining public preference towards

[1] An important characteristic of new forms of PPPs is the assignment of different phases of the project to a single consortium made by different firms that also act as subcontractors of the consortium itself. Such bundling agreements are implemented through different contractual arrangements, taking into account country-specific legislations (Engel et al., 2013).

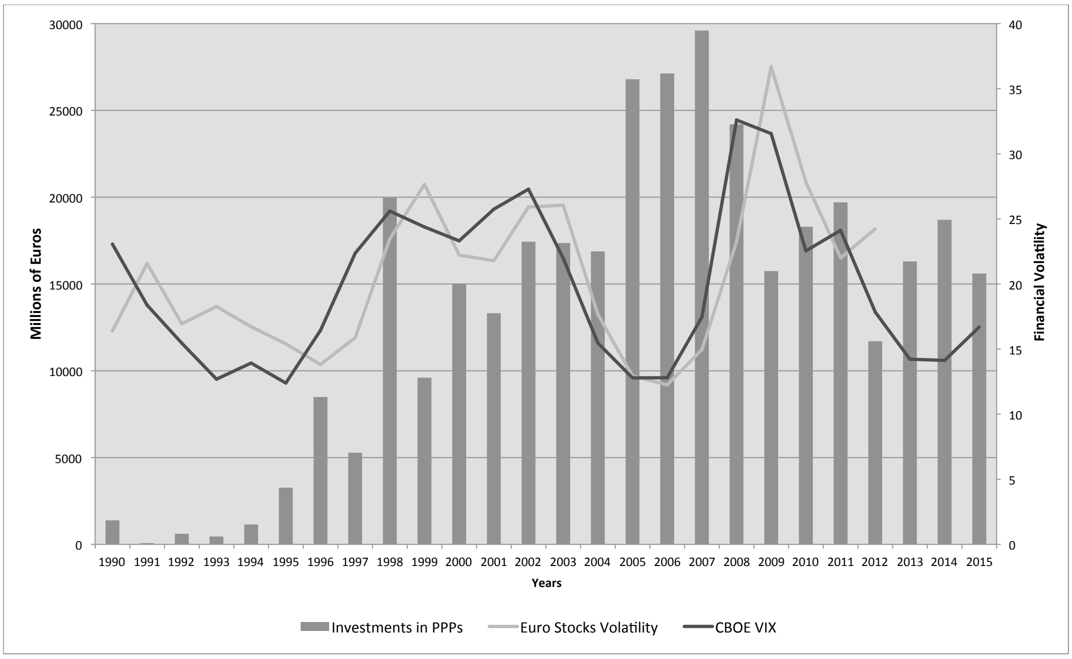


Figure 1: European Market of PPPs and Financial Volatility from 1990 to 2015

Source: Our elaboration on dataset by EPEC PPP Market Updates (http://www.eib.org/epec/), CBOE (www.cboe.com), and St. Louis Federal Reserve (https://research.stlouisfed.org/fred2/). Legend: Only contracts above ten millions of euros closed each year in one of the 28 countries of the European Union, West Balkans and Turkey are considered.

PPPs.

We contribute to this debate by providing a deeper understanding of the role of financial constraints in the optimal contract design under sequential moral hazard. The latter is the stylized representation of public procurement of infrastructure building and operation in the literature (Martimort and Pouyet, 2008; Iossa and Martimort, 2015; Engel et al., 2014). To this aim we consider a moral hazard problem in which a risk-neutral principal (i.e., the government in the procurement problem), potentially facing a financial constraint, delegates the implementation of two sequential tasks (i.e., building and operating) to risk-neutral agents, who may be financially-constrained. Each task brings to a contractible output (e.g., infrastructure quality and operational costs) that is randomly affected by the agent's task-specific effort.

To better understand the role of financial constraints in the design of the optimal contracts, and in the comparison between PPPs and traditional procurement schemes, in our model we abstract from any production (or technological) externality between the building and operating tasks.[2] As usual, the government can select between alternative contractual schemes. Under *sequential contracts*, two agents (i.e., the builder and the operator) are hired to implement the tasks. The second task optimal contract does not have memory (i.e., it is independent of first task performance). Under *partnership contract*, a single agent (i.e., a consortium of the building and operating firms) is hired to implement the two tasks, through a memory contract (i.e., the second task incentives depend on first task performance).

Contrasting the two contractual arrangements in terms of social welfare, we find that when financial (or limited liability) constraints are binding for all private firms – i.e., for the builder and the operator in the sequential contracts case and for the consortium in the partnership case – then partnership has a further advantage compared to sequential contracts in dealing with moral hazard problems. The intuition of this result is that, under partnership contract, the principal efficiently relies on a more powerful incentive mechanism, thanks to the financial externality that implicitly arises between the building and the operating phases. However, the comparison further depends on the difference between available liabilities under partnership and

[2]Such externality is a crucial determinant of the gains from PPPs in the extant contract theory literature (Iossa and Martimort, 2015; Martimort and Pouyet, 2008).

sequential contracts. Bundling the two tasks may either increase or decrease available liabilities to the private agent: *trading adjuvant* vs *insulation effect*(Farhi and Tirole, 2015). Finally, the introduction of a financial (or fiscal) constraint restricting the contracts that the principal may design could change the government's preference from sequential to partnership contracts or viceversa.

Our results show that, even if the government is perfectly benevolent, stricter financial constraints on all firms (or sectors) involved in procurement of public investment construction and operation explain why PPPs may outperform sequential contracts in terms of social welfare. However, if the impact of adverse financial conditions is differentiated across economic sectors (and firms) – e.g., if building firms are more financially constrained than operating ones – then, PPPs consortia may prove insufficient to relax limited liability constraints on private contractors. In the such a case, sequential contracts may prove more efficient than PPPs. This last finding could explain recent trend for PPPs. In particular, the model's interpretation of the negative trend of last years stays in the heterogeneous effect of the financial crisis that was more pronounced for some industries rather than other.[3] By a numerical analysis, we assess the role of harder financial conditions, both private and public, on the differential welfare that can be obtained by partnership versus sequential contracts.

The paper is structured as follows. In Section 2, we briefly review the literature that is relevant to assess our contribution. Section 3 presents the model. Section 4 analyzes sequential and partnership contracts without taking into consideration the principal's financial constraint. In Section 5, a binding budget constraint for the principal is taken into account. Then, in Section 6, we numerically run the comparative statics analysis of the impact of stricter financial conditions on the social welfare differential between partnership and sequential contracts. Section 7 concludes.

[3]The financial crisis had a more pronounced adverse effect in industries that are more dependent on external finance, and also in those industries that rely on trade credit due to underdeveloped financial intermediation (Moore and Mirzaei, 2016).

2 Related Literature

Our analysis provides a direct contribution to the literature on PPPs. Most of the theoretical works belonging to this strand of economic literature aim at assessing the positive and negative impact of alternative contractual schemes in terms of social welfare. This may derive by cost-reducing innovations (Hart, 2003; Bennett and Iossa, 2006) and, more generally, by production externalities among different (sequential) tasks of the public investment cycle (Martimort and Pouyet, 2008; Iossa and Martimort, 2015). However, because of necessary early commitment, PPPs do not allow for needed flexibility to face uncertain future determinants (Martimort and Straub, 2012; Iossa and Martimort, 2015). The scope for welfare-improving PPPs as compared to traditional, sequential contracting may be further restricted when there are: limitations on the governments' ability to commit to long term projects (Guasch et al., 2007; Valero, 2015); soft budget constraints and re-negotiations (de Bettignies and Ross, 2009; Engel et al., 2009); government's preference for favored groups (Maskin and Tirole, 2008); agency problems within the private consortium (Greco, 2015).

In this paper, we find that, even when production externalities are excluded, PPPs may improve on traditional procurement in terms of social welfare when financial constraints limit the capacity of the government to contract with private firms. To our knowledge, we provide a first theoretical contribution highlighting the role of financial constraints as driver of the choice between PPPs and traditional procurement in a model without political agency issues.

Our analysis is also related to the literature on moral hazard with multiple tasks and moral hazard in teams. On this topic a number of authors studied the optimal grouping of tasks when agents are risk-averse and activities are correlated with either performance or production externalities (Lockwood, 2000; Macho-Stadler and Perez-Castrillo, 1993; Choi, 1993; Itoh, 1992; Holmström and Milgrom, 1991).

Closer to our contribution is the strand of this literature that introduces dynamics in the form of repeated moral hazard. In this case, Che and Yoo (2001) show that incentives to team production can be strengthened relying on memory contracts, where past performance can be used to punish group members. Schmitz (2005) analyzes optimal contracts in repeated moral hazard under agents' limited liability and production externality (i.e., success in the first stage makes effort in the second stage more effective). Schmitz (2005) finds that assigning both tasks to a single agent (i.e., partnership, in our setting) is preferable to separation if the stakes are relatively small. But, when high effort should be exerted in the second stage regardless of the outcome of the first stage, integration is suboptimal from the principal's point of view.

Closer to our framework, Ohlendorf and Schmitz (2012) consider a repeated moral hazard problem where both the principal and the wealth-constrained agent are risk-neutral and there is no production externality. In their analysis, the optimal incentive mechanism is a memory contract (i.e., the principal induces the agent to choose a particularly high second-period effort following a first-period success and a particularly low second-period effort level following a first-period failure). Ohlendorf and Schmitz (2012) show that memory contracts can improve the capacity of the principal to motivate the agent in the first period. As we show in this paper, such a mechanism may explain why – under agents' limited liability – partnership contract can improve social welfare as compared to sequential contracts with different agents, that (optimally) prevent the principal from writing memory contracts.

In Schmitz (2013), the government has to decide whether to bundle two tasks together or contract with different private parties, each in charge of only one task. In the model, the principal is budget-constrained and the two tasks are symmetric. The latter differs with respect to our setting where tasks are sequential (i.e., one of them comes before the other and they affect in different ways the principal's objective function). Schmitz (2013)'s model is in line with the literature in that bundling dominates unbundling, when the budget constraint is not relevant. However, when the principal is constrained by the budget constraint, bundling is suboptimal since it becomes too costly for the principal to give incentives from the beginning to implement high efforts in both tasks. The difference between the latter finding by Schmitz (2013)'s and ours is driven by the intrinsic asymmetry between tasks in our setting and, related to that, by the fact that the principal cannot distinguish "which" task is likely to determine the final outcome in some states of the world.

While the strand of the literature on team production acknowledges that intrinsically different tasks may contribute to the final output, the literature on dynamic moral hazard has typically considered repeated moral hazard. In our setting, we consider a case of sequential moral hazard where the second period task does not replicate the first period one (e.g., in terms of impact on government's and agents' objective functions).

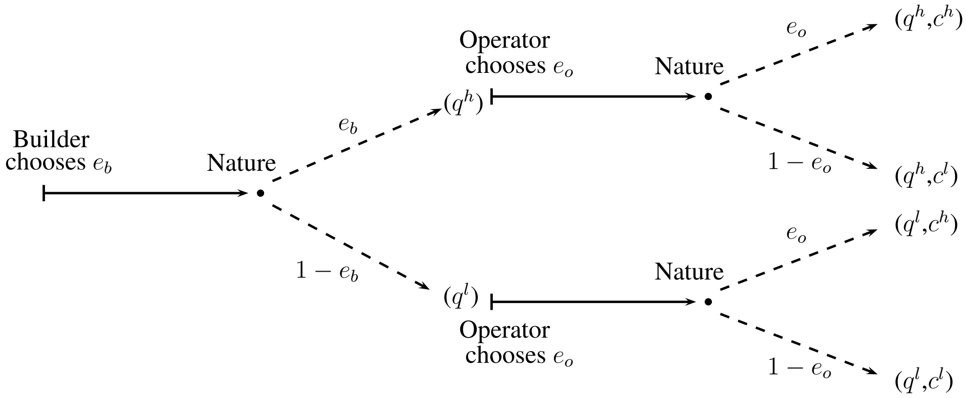


Figure 2: Sequential decisions of agents in the baseline setting

3 The model

A public infrastructure has to be built and operated. The gross social surplus generated by the public infrastructure is Sq, where q is the level of infrastructure quality, and $S > 0$ is the social marginal benefit of it. Infrastructure quality is determined in the first phase of the public infrastructure cycle (see Figure 2), as a random outcome of the builder's investment effort, $e_b \geq 0$. We assume that the quality is high, q^h, with a probability e_b, and low, q^l, with a probability $1 - e_b$. Investing in quality entails a monetary cost kq (with $k < S$) and a non-monetary (or management) cost for the builder, $\phi(e_b)$ (where: $\phi(0) = \phi'(0) = 0$, $\phi(1) = \infty$, $\phi'(.) \geq 0$, $\phi''(.) > 0$, and $\phi'''(.) \geq 0$). Thus, the state-contingent monetary profit of the building firm is $\pi_b = t_b - kq$, where t_b is the (state-contingent) payment received by the government, and the state-contingent utility of the building firm's management is $u_b = \pi_b - \phi(e_b)$.

The operational costs, C, are determined during the second, service-provision phase of the public infrastructure cycle (see Figure 2) as a random variable of the operator management's effort to cut costs, $e_o \geq 0$. Operation costs are low, c^l, with a probability e_o, and high, c^h, with a probability $1 - e_o$.[4] The non-monetary cost of management effort for the operator is $\psi(e_o)$ (where: $\psi(0) = \psi'(0) = 0$, $\psi(1) = \infty$, $\psi'(.) \geq 0$, $\psi''(.) > 0$, and $\psi'''(.) \geq 0$). Thus, the state-contingent monetary profit of the operating firm is $\pi_o = t_o - C$, where t_o is the (state-contingent) payment received by the government, and the state-contingent utility of the operating firm's management is $u_o = \pi_o - \phi(e_o)$.

In our model, firms face a financial constraint, i.e., a state-independent limited-liability constraint (LLC) such that the monetary profit of the firm cannot drop below a given threshold, that we consider as exogenous. The interpretation is that the contracting activity with the principal may lead to negative rents that however cannot be higher than the firms' own liabilities l (maximum amount of losses cannot be higher than $-l$).[5]

We assume that the government maximizes the expectation of the social value of the public infrastructure net of transfer costs, $W = Sq - T$, possibly facing a state-independent budget constraint (BC), i.e., an upper bound to total transfers paid to the private contractors, F.[6]

The government cannot directly verify the effort of its contractor(s) during the investment and operation phases. But it can *ex post* verify the level of infrastructure's quality, q, and operational cost, c. We assume that the public procurement procedures are such that the government has all the bargaining power. In our analysis, we focus on two contractual schemes that the government may choose. Under *sequential contracts*,[7] the structure of the contracting game is such that: the gov-

[4]For the sake of our argument, we abstract from possible production externalities between building and operation, which are common in the literature on PPPs. These would imply that a component of costs is determined by the quality of infrastructure.

[5]Such exogenous financial constraints may be explained by previous financial contracts that firms might have already signed.

[6]It is worth remarking that the social welfare loss associated to these transfers is not weighted by the marginal cost of public funds, as it is common in the literature. In our analysis, the marginal cost of public funds is endogenously determined as the Lagrangian multiplier associated with the public budget constraint (i.e., $F \geq T$). A more sophisticated representation of fiscal constraints may be introduced limiting the transfer in each of the two phases of the public infrastructure lifecycle. Finally, a different, more general government's objective function can be considered, $W = Sq + \alpha U - T$, where U is the sum of utilities of private contractors' managers, and α is marginal impact of such utility on social welfare. In our setting, $\alpha = 0$. The qualitative results we obtain are not significantly affected by these changes to government's objective function and financial constraints.

[7]The so-called "traditional procurement" in the literature on PPPs.

ernment proposes a take-it-or-leave-it contract to the builder, specifying a transfer $t_b(q,c)$; then it offers a contract to the operator with a transfer $t_o(q,c)$. Under the *partnership contract*, the government chooses to bundle all tasks by contracting with a single consortium of firms acting as builder and operator. The total transfer to the consortium that is specified by the bundled contract is $t(q,c)$. Finally, we assume that the government can perfectly commit to implement the contracted clauses.

First Best Solution. As a benchmark, we consider the case where the government can observe the contractors' efforts, i.e., e_b and e_o. Without loss of generality, payments to contractors can be state-inpedendent (i.e., conditioned only on the optimal level of effort) and have to satisfy only the following participation constraint (PC):[8]

$$t_b - k[e_b q^h + (1-e_b)q^l] - \phi(e_b) + t_o - e_o c^l - (1-e_o)c^h - \psi(e_o) \geq 0. \tag{1}$$

The government aims at reducing transfers to contractors. Thus (1) is binding and the maximization problem of the government is:

$$\max_{e_b, e_o} (S-k)[e_b q^h + (1-e_b)q^l] - \phi(e_b) - e_o c^l - (1-e_o)c^h - \psi(e_o).$$

The first-best optimal efforts, e_b^* and e_o^*, are such that:

$$\phi'(e_b^*) = (S-k)(q^h - q^l), \tag{2}$$
$$\psi'(e_o^*) = c^h - c^l. \tag{3}$$

It is worth remarking that the first best solution can be implemented by the government even if it cannot observe the effort levels of the agents, provided that the LLCs of the agents do not bind at the (second best) optimum. In this case, the government can extract the full information rent from the firms. Therefore,

[8]It is worth to notice that the same first best optimal solution can be obtained if, instead of a single participation constraint (1), we consider two separate participation constraints for the builder and the operator:

$$t_b - k[e_b q^h + (1-e_b)q^l] - \phi(e_b) \geq 0,$$
$$t_o - e_o c^l - (1-e_o)c^h - \psi(e_o) \geq 0.$$

Proposition 1 *If limited liability constraints are not binding, then sequential and partnership contracts determine the same (first-best) effort and social welfare levels.*

4 Contracting with Limited Liability Agents

In this section we contrast partnership versus sequential contracts in a framework where agents may face financial constraints (i.e., limited liability), but the government is not financially constrained. We introduce the latter constraint in Section 5.

All agents, i.e., the builder and the operator under sequential contracts and the consortium under partnership contract, face a binding LLC, and we consider the lower bound of each firm's profit as equal respectively to $-l_b$ for the builder, $-l_o$ for the operator and $-l_c$ for the consortium of firms, with l_b, l_o and l_c greater or equal than zero.

4.1 Sequential Contracts

In this case, the government awards two contracts (one for each phase of public infrastructure cycle) to different firms: the builder and the operator.

Payments satisfying PC, ICC and LLC. For the characterization of implementable contracts, we proceed by backward induction. The optimal contract awarded by the government to the operator has to satisfy the participation (PC), the incentive-compatibility (ICC) and the limited-liability (LLC) constraints. As shown in Figure 2, the state of the world in the operation phase depends on the realized quality of the infrastructure. Thus, PC and ICC of the operator are, in general, contingent on the realization of q:

$$\max_{e_o} e_o(t_o(q, c^l) - c^l) + (1 - e_o)(t_o(q, c^h) - c^h) - \psi(e_o) \geq 0. \tag{4}$$

The LLCs can be written as:

$$\pi_o(q, c^l) = t_o(q, c^l) - c^l \geq -l_o; \tag{5}$$

$$\pi_o(q, c^l) = t_o(q, c^h) - c^h \geq -l_o. \tag{6}$$

As regards the problem (4), by the assumptions on the shape of $\psi(.)$: corner solutions (i.e., 0 or 1) cannot be optimal, the second order condition of the problem is strictly negative, hence the solution of the problem is unique. By the first order approach, the ICC can be written as:

$$(t_o(q, c^l) - c^l) - (t_o(q, c^h) - c^h) = \psi'(e_o) \geq 0; \tag{7}$$

thus, $\pi_o(q, c^l) \geq \pi_o(q, c^h)$. At the optimum, by LLC and ICC: $t_o(q, c^h) = c^h - l_o$ and $t_o(q, c^l) = c^l + \psi'(e_o) - l_o$.

It is worth to notice that the optimal transfers from the government to the operator have no memory, i.e., do not depend on the level of q. Thus, also the optimal operator's effort depends only on c^l, c^h and the shape of the non-monetary cost function, $\psi(.)$.[9]

Anticipating the effort of the operator, e_o, the optimal contract awarded by the government to the builder have to satisfy PC and ICC,

$$\begin{aligned} \max_{e_b} e_b(e_o t_b(q^h, c^l) + (1 - e_o) t_b(q^h, c^h) - kq^h) + \\ +(1 - e_b)(e_o t_b(q^l, c^l) + (1 - e_o) t_b(q^l, c^h) - kq^l) - \phi(e_b) \geq 0, \end{aligned} \tag{8}$$

as well as LLCs,

$$\pi_b(q^h, c^l) = t_b(q^h, c^l) - kq^h \geq -l_b \tag{9}$$

$$\pi_b(q^h, c^h) = t_b(q^h, c^h) - kq^h \geq -l_b \tag{10}$$

$$\pi_b(q^l, c^l) = t_b(q^l, c^l) - kq^l \geq -l_b \tag{11}$$

$$\pi_b(q^l, c^h) = t_b(q^l, c^h) - kq^l \geq -l_b. \tag{12}$$

By the first order approach, the ICC can be written as:

$$\begin{aligned} (e_o t_b(q^h, c^l) + (1 - e_o) t_b(q^h, c^h) - kq^h) + \\ -(e_o t_b(q^l, c^l) + (1 - e_o) t_b(q^l, c^h) - kq^l) = \phi'(e_b) \geq 0, \end{aligned} \tag{13}$$

[9]This result does not hold anymore if realized level of infrastructure quality affects the operational costs because of production externalities between building and operation.

Since, the government aims at reducing (expected) transfers to contractors, at the optimum, by LLCs and ICC $t_b(q^l, c^l) = t_b(q^l, c^h) = kq^l - l_b$: the optimal transfer to the builder is independent of realized operational cost when the quality of infrastructure is low. Thus, by (13), $e_o t_b(q^h, c^l) + (1 - e_o)t_b(q^h, c^h) = kq^h + \phi'(e_b) - l_b$: when the quality of the infrastructure is high, transfers to the builder may (or may not) depend on the realization of operational costs.

By the characterization of feasible transfers to the builder and the operator, we have

Lemma 2 *If the optimal sequential contracts for the builder and operator satisfy ICC and LLC, PC are satisfied if we assume that, respectively:*

$$l_b \leq e_b\phi'(e_b) - \phi(e_b); \tag{14}$$

$$l_o \leq e_o\psi'(e_o) - \psi(e_o). \tag{15}$$

Proof. Substituting the transfer functions obtained, under ICC and LLC, for the builder in (8) and for the operator in (4), PC can be written, respectively, as:

$$e_b\phi'(e_b) - \phi(e_b) \geq l_b;$$
$$e_o\psi'(e_o) - \psi(e_o) \geq l_o.$$

Both right sides of (14) and (15) are equal to zero when $e_b = 0$ and $e_o = 0$, respectively; moreover, $\frac{\delta}{\delta e}(e\phi'(e) - \phi(e)) = e\phi''(e)$ and $\frac{\delta}{\delta e}(e\psi'(e) - \psi(e)) = e\psi''(e)$, hence these are equal to zero for $e = 0$ and strictly positive for all all $e \in (0, 1)$. Thus, if firms' liabilities are sufficiently low, (14) and (15) are satisfied and expected rents are positive. On the other hand, if firms' liabilities are sufficiently high, (14) and (15) are binding and expected rents are equal to zero. In this Section we deal with the first case, while in the case of binding participation constraints we have already shown in Proposition 1 that we obtain the first best solutions. ■

Optimal Sequential Contracts. Substituting the transfer schedules that satisfy ICC and LLC of the builder and the operator in government's objective function, its maximization problem can be written as:

$$\max_{e_b,e_o} e_b((S-k)q^h - \phi'(e_b)) + (1-e_b)(S-k)q^l - e_o(c^l + \psi'(e_o)) - (1-e_o)c^h + l_b + l_o \quad (16)$$

By the first order conditions of the problem (16), the second best optimal efforts determined under sequential contracts are:

$$\phi'(e_b^s) = (S-k)(q^h - q^l) - e_b^s \phi''(e_b^s); \quad (17)$$

$$\psi'(e_o^s) = c^h - c^l - e_o^s \psi''(e_o^s). \quad (18)$$

By inspection of optimization conditions under first best (2-3) and under second best contracts (17-18), we have:

Proposition 3 *Under sequential contracts, the second best optimal efforts of the builder and the operator are strictly smaller than under first best.*

This result derives by the introduction of LLCs. As usual in moral hazard problems with risk-neutral agents, the presence of incentive constraints does not prevent the implementation of first best optimal efforts. However, the introduction of LLCs limits the scope for risk transfer from the principal to the agent, thus bringing to the distortion of second best optimal efforts.

4.2 Partnership Contract

In this case, the government awards a single (bundled) contract to a consortium carrying out both building and operation tasks.

Payments satisfying PC, ICC and LLC. In this case, constraints have to be satisfied, at the optimum, taking into account the total profit and utility of the single contractor.[10] In particular, PC and ICC are satisfied by the optimal partnership

[10]In our analysis, we abstract from possible agency problems within the consortium of builder and operator. Such problems may reduce the value for money that the government can get out of the partnership contract (Greco, 2015).

contract whenever

$$\begin{aligned}\max_{e_b,e_o^h,e_o^l} & e_b(e_o^h(t(q^h,c^l)-c^l)+(1-e_o^h)(t(q^h,c^h)-c^h)-kq^h-\psi(e_o^h))+\\ &+(1-e_b)(e_o^l(t(q^l,c^l)-c^l)+(1-e_o^l)(t(q^l,c^h)-c^h)-kq^l-\psi(e_o^l))+\\ &-\phi(e_b)\geq 0.\end{aligned} \tag{19}$$

In the same way, LLCs are satisfied, in all possible states of the world, whenever:

$$t(q^h,c^l)-kq^h-c^l\geq -l_c; \tag{20}$$

$$t(q^h,c^h)-kq^h-c^h\geq -l_c; \tag{21}$$

$$t(q^l,c^l)-kq^l-c^l\geq -l_c; \tag{22}$$

$$t(q^l,c^h)-kq^l-c^h\geq -l_c. \tag{23}$$

Also in this case we can rely on the first order approach[11]. Thus, we can substitute the ICC by the following system of optimization conditions:[12]

$$\begin{aligned}&(e_o^h(t(q^h,c^l)-c^l)+(1-e_o^h)(t(q^h,c^h)-c^h)-kq^h-\psi(e_o^h))+\\ &-(e_o^l(t(q^l,c^l)-c^l)+(1-e_o^l)(t(q^l,c^h)-c^h)-kq^l-\psi(e_o^l))=\\ &=\phi'(e_b)\geq 0;\end{aligned} \tag{24}$$

$$(t(q^h,c^l)-c^l)-(t(q^h,c^h)-c^h)=\psi'(e_o^h)\geq 0; \tag{25}$$

$$(t(q^l,c^l)-c^l)-(t(q^l,c^h)-c^h)=\psi'(e_o^l)\geq 0. \tag{26}$$

The following Lemmas simplify the set of relevant constraints.

Lemma 4 *If the optimal partnership contract satisfies ICC and LLC, PC is satisfied if we assume that:*

$$l_c\leq e_b\phi'(e_b)-\phi(e_b)+e_o^l\psi'(e_o^l)-\psi(e_o^l) \tag{27}$$

[11]It is worth to notice that the Hessian matrix of second-order partial derivatives of the agent's objective function (19) is characterized by negative terms on the principal diagonal, while all cross second-order derivatives are equal to zero. Thus, the Hessian matrix is definite negative.

[12]It is worth to remark that the conditions (25-26) imply that the contract is robust also against state-contingent deviations, after q is realized. In other terms, the system of equations (24-26) implies both ex ante and ex interim ICC.

Proof. Substituting (24), (25) and (26) in the agent's objective function, PC can be written as:

$$t(q^l, c^h) - kq^l - c^h + e_b\phi'(e_b) - \phi(e_b) + e_o^l\psi'(e_o^l) - \psi(e_o^l) \geq 0. \quad (28)$$

By the proof of Lemma 2, $e_b\phi'(e_b) - \phi(e_b) \geq 0$ and $e_o^l\psi'(e_o^l) - \psi(e_o^l) \geq 0$. Thus, (23) implies (28) if $l_c \leq e_b\phi'(e_b) - \phi(e_b) + e_o^l\psi'(e_o^l) - \psi(e_o^l)$. Also in this case we consider this assumption to be satisfied, if not we are back to the first best solutions (Proposition 1). ■

Lemma 5 *Under the optimal partnership contract, the only binding LLCs are (21) and/or (23).*

Proof. By (25), if (21) is satisfied, then also (20) is satisfied. In the same way, by (26), if (23) is satisfied, also (22) is satisfied. ■

To understand which one of the LLCs is biding, we substitute (25) and (26) in (24); after some algebra we obtain:

$$t(q^h, c^h) - kq^h - c^h = t(q^l, c^h) - kq^l - c^h + A \quad (29)$$

where $A = e_o^l\psi'(e_o^l) - \psi(e_o^l) - e_o^h\psi'(e_o^h) + \psi(e_o^h) + \phi'(e_b)$. In the Appendix we prove the following

Lemma 6 *The optimal partnership contract is such that $A \geq 0$.*

Proof. See the appendix ■

Thus, the only relevant LLC is (23) – i.e., (21) is always satisfied when (23) is satisfied.[13] Moreover, since the government aims at reducing the transfer to the contractor: $t(q^l, c^h) = kq^l + c^h - l_c$.

Optimal Partnership Contract. We, now, substitute the transfer schedules that satisfy the ICC and LLC of the single private contractor in the government's objective function. After some algebra, the government's maximization problem can be

[13]Conversely, if $A < 0$, (21) is the only relevant LLC.

written as:

$$\max_{e_b,e_o^h,e_o^l} (S-k)q^l - c^h + e_o^l(c^h - c^l - \psi'(e_o^l)) + \tag{30}$$
$$+e_b[(S-k)(q^h-q^l) + (e_o^h - e_o^l)(c^h-c^l) + \psi(e_o^l) - \psi(e_o^h) - \phi'(e_b)] + l_c$$

Under the partnership contract, the optimal second best efforts in the building phase, e_b^p, and in the operation phase, e_o^{hp} –when the quality of infrastructure is high– and e_o^{lp} –when it is low, are determined by the following optimization conditions:[14]

$$\phi'(e_b^p) = (S-k)(q^h-q^l) + (e_o^{hp} - e_o^{lp})(c^h-c^l) + \psi(e_o^{lp}) - \psi(e_o^{hp}) - e_b^p\phi''(e_b^p) \tag{31}$$
$$\psi'(e_o^{hp}) = c^h - c^l \tag{32}$$
$$\psi'(e_o^{lp}) = c^h - c^l - \frac{e_o^{lp}}{1-e_b^p}\psi''(e_o^{lp}) \tag{33}$$

By these conditions, the following results characterizing the optimal partnership contract are drawn:

Proposition 7 *Under partnership contract, the optimal effort of the builder can be smaller, equal or larger than the first best optimal effort. The optimal effort of the operator is equal (or lower) than the first best one when the quality of the infrastructure is high (or low).*

Proof. *Optimal efforts of the operator.* Contrasting (3) and (32-33): $e_o^* = e_o^{hp} > e_o^{lp}$. *Optimal effort of the builder.* By (32),

$$(e_o^{hp} - e_o^{lp})(c^h - c^l) + \psi(e_o^{lp}) - \psi(e_o^{hp}) = e_o^{hp}\psi'(e_o^{hp}) - \psi(e_o^{hp}) - z(e_o^{lp}) \tag{34}$$

where $z(e) \equiv e\psi'(e_o^{hp}) - \psi(e)$ is such that: $z'(e) = \psi'(e_o^{hp}) - \psi'(e)$, that is strictly positive (or negative) for all $e < e_o^{hp}$ (or $e > e_o^{hp}$) and it is zero when $e = e_o^{hp}$; $z''(e) = -\psi''(e) < 0$; and $z(e_o^{hp}) = e_o^{hp}\psi'(e_o^{hp}) - \psi(e_o^{hp})$. Thus, $e_o^{hp}\psi'(e_o^{hp}) - \psi(e_o^{hp}) > z(e)$ for all $e \neq e_o^{hp}$, and in particular: $e_o^{hp}\psi'(e_o^{hp}) - \psi(e_o^{hp}) - z(e_o^{lp}) > 0$. Contrasting (2) and (31), e_b^p can be larger, equal or smaller than e_b^* whenever $(e_o^{hp} - e_o^{lp})(c^h - c^l) +$

[14] A sufficient condition for the second order conditions to be satisfied is that $2\psi''(0)\phi''(0) > (c^h - c^l)^2$.

$\psi(e_o^{lp}) - \psi(e_o^{hp}) > 0$ is larger, equal or smaller than $e_b^p \phi''(e_b^p) > 0$. ■

Proposition 8 *The second best optimal effort of the builder under partnership contract is strictly larger than under sequential contracts. The second best optimal effort of the operator under partnership contract, when infrastructure quality is high (or low), is strictly larger (smaller) than under sequential contracts.*

Proof. By the proof of the Proposition 7, we know that: $(e_o^{hp} - e_o^{lp})(c^h - c^l) + \psi(e_o^{lp}) - \psi(e_o^{hp}) > 0$. Contrasting (17) and (31), $e_b^p > e_b^s$. Contrasting (18) and (32-32): $e_o^{hp} > e_o^s > e_o^{lp}$. ■

The Proposition 8 highlights a result similar to Ohlendorf and Schmitz (2012), however in a different, less restrictive setting.[15] Even though no production externality exists between building and operating tasks, the optimal partnership contract exploit a memory incentive mechanism in the second phase to reward/punish the effort that the agent exerts in the first phase.

4.3 Welfare Analysis

Substituting the optimal levels of efforts in the government's objective function, we can write the value of the social welfare under partnership contract as:

$$W^p = (S-k)q^l - c^h + (e_b^p)^2 \phi''(e_b^p) + \frac{(e_o^{lp})^2}{1-e_b^p} \psi''(e_o^{lp}) + l_c; \tag{35}$$

and the value of the social welfare under sequential contracts as:

$$W^s = (S-k)q^l - c^h + (e_b^s)^2 \phi''(e_b^s) + (e_o^s)^2 \psi''(e_o^s) + l_b + l_o. \tag{36}$$

The difference between the social welfare under partnership and sequential contracts is:

[15] At first, they analyze a repeated moral hazard problem where the second period task replicates the first period one. Second, in their model the principal can decide to terminate the project after observing the first period outcome. Finally, in our paper we extends the analysis and its application by looking at the comparison between partnership and sequential contracts.

$$\Delta W = \qquad (37)$$
$$e_o^{lp}(c^h - c^l - \psi'(e_o^{lp})) - e_o^s(c^h - c^l - \psi'(e_o^s)) + (e_b^p)^2\phi''(e_b^p) - (e_b^s)^2\phi''(e_b^s) + \\ + l_c - l_b - l_o.$$

The first part of this difference comes from the presence of moral hazard together with limited liability constraints. We call this part “Moral Hazard component”. The second part of this expression reflects differences in available liabilities of the consortium in the case of the partnership contract with the ones of the builder plus the operator in the case of sequential contracts. We call this part “Available Liabilities component”. A positive or a negative value of this second component mainly depends on the presence of asymmetric information. On one hand, if firms bundle within the same consortium, the level of asymmetric information between the financier and this new borrower will increase; this effect is called “insulation effect” and implies a reduction on the level of granted liabilities (DeMarzo and Duffie, 1999; Gorton and Pennacchi, 1990). On the other hand, if two or more firms bundle, the level of available liabilities could increase for two reasons. First, the firm with more problems to obtain liabilities from external financier can reduce its level of risk perception, thus becoming more able to collect funds. Second, if we think about our model as applied to the case of PPPs, a benefit that arises is related to the involvement of outside financiers in evaluating risks that reduces the level of asymmetric information, thus increasing the potential amount of granted liabilities (Iossa and Martimort, 2015). If this last “trading adjuvant effect” (Farhi and Tirole, 2015; Whinston, 1990) encompasses the cited “insulation effect”, it means that bundling can increase the total amount of liabilities firms can receive.

Related to the sign of equation 37, we first have:

Proposition 9 *When the “Available Liabilities component” is equal or greater than zero ($l_c \geq l_b + l_o$), then the partnership contract always dominates the sequential contracts in social welfare terms. This result derives from the “Moral Hazard component component” that is always higher than zero.*

Proof. Let us remark that $\Delta W = W^p - W^p(e_b^s, e_o^s) + W^p(e_b^s, e_o^s) - W^s$, where

$$W^p(e_b^s, e_o^s) = (S-k)q^l - c^h + (e_b^s)^2\phi''(e_b^s) + \frac{(e_o^s)^2}{1-e_b^s}\psi''(e_o^s) + l_c$$

is the value of the social welfare function under partnership contract if the agent implements the sequential-contracts optimal efforts. Given that the social welfare function $W^p(.,.)$ is concave and reaches its maximum when the building effort is e_b^p and the operation effort – when infrastructure quality is low – is e_o^{lp}, then $W^p - W^p(e_b^s, e_o^s) \geq 0$. Moreover, when $l_c \geq l_b + l_o$,

$$W^p(e_b^s, e_o^s) - W^s = \frac{e_b^s}{1-e_b^s}(e_o^s)^2\psi''(e_o^s) + l_c - (l_b + l_o) > 0.$$

Thus, $\Delta W > 0$. ■

The intuition of Proposition 9 is the following. The partnership contract involves less restrictive constraints on the agents objective function. In particular, because of the bundling of construction and operation tasks, the LLCs involve weaker financial limits for the agent with respect to the sequential contracts case. This allows the government to exploit the partnership contract sequentiality to relax the moral hazard constraint of the second phase when the first phase outcome is high quality of the infrastructure.

However, when $l_c < l_b + l_o$, we have that:

Corollary 10 *The partnership contract dominates the sequential contracts in social welfare terms if*

$$MHc \geq l_b + l_o - l_c \tag{38}$$

where MHc is the Moral Hazard component that has been shown to be always higher than zero in Proposition 9.

The intuition of this last Corollary 10 is the following. The partnership contract dominates the sequential contracts when the MHc is higher than the difference between between available liabilities (of the builder and the operator) under sequential contracts and available liabilities (of the consortium) under the partnership contract. If not, sequential contracts are able to maximize social welfare by granting incentives to private agents at a lower cost for taxpayers.

4.4 Discussion

Starting from Equation 38, we can generalize the analysis of financial constraints by considering the case that only some firms face limited liability at the optimum. In fact, if l_c, l_b or l_o are sufficiently high, then the limited liability constraint of respectively the consortium, the builder or the operator will become less stringent that the corresponding participation constraint. If such condition is satisfied for all agents, we already shown in Proposition 1 that both partnership and sequential contracts lead to the first best solutions. On the other hand, participation constraints may become binding only for some agents.

The case of asymmetric financial constraints may reinforce the social welfare difference between partnership and sequential contracts or dampen it. Two possible cases may emerge. If the consortium of firms does not face any binding financial constraint under the optimal partnership contract, but at least one of the firms (either the builder or the operator) is constrained by limited liability under optimal sequential contracts, then the partnership contract leads to the first best outcomes and is always preferred with respect to sequential contracts.

If the limited liability constraint is binding under the optimal partnership contract, and at least one of the firms (either the builder or the operator) is not constrained at the optimum sequential contracts by financial constraints, then the condition to have sequential contracts preferred than the partnership contract may become less stringent.

These two cases may be both relevant, as emphasized by the corporate finance literature. Despite focusing on the role of adverse selection, the paper of Farhi and Tirole (2015) shows as bundling of a safe and a risky assets: on the one hand, hurts the safe component by increasing the risk of illiquidity (Gorton and Pennacchi, 1990); on the other hand, reduces the cost of trading of the risky component. The trade-off between these two opposite effects (insulation and adjuvant effects) is solved in favour of bundling whenever the bundle is liquid. Our context is different since it deals with a problem of a moral hazard, but this argument may explain why both conditions we are considering may exist looking at real world situations.

By solving the sequential contracts maximization problems as in Section 4.1 and by considering that either the builder's or the operator's limited liability constraint may be less stringent that the corresponding participation constraint, we can compute first best solutions that are, as in Section 4.1 for the agent that has a binding limit on the level of available liabilities, and at the first best for the agent that is not constrained by any limited liability budget.

When these two cases are compared with the partnership contract with a binding consortium's limited liability constraint, the condition to have sequential dominated by the partnership contracts is respectively equal to:

$$MHc > l_b + e_o^s \psi'(e_o^s) - \psi(e_o^s) - l_c \qquad \text{Builder's Limited Liability} \tag{39}$$

$$MHc > l_o + e_b^s \phi'(e_b^s) + \psi(e_b^s) - l_c \qquad \text{Operator's Limited Liability} \tag{40}$$

These two dis-equations are comparable with the Condition 38. Differences are twofold. At first, the MHc is lower in such situations, it is respectively equal to $(e_b^p)^2\phi''(e_b^p) - (e_b^s)^2\phi''(e_b^s)$ in the case of sequential contracts with builder's limited liability constraint, and $e_o^{lp}(c^h - c^l - \psi'(e_o^{lp})) - e_o^s(c^h - c^l - \psi'(e_o^s))$ in the case of sequential contracts with operator's limited liability constraint. In both cases, when the participation constraint is binding the corresponding agent's contract is not distorted and the benefit of the partnership contract in solving the moral hazard problem will disappear. Second, in the two dis-equations l_o or l_b is replaced by the maximum level of losses (negative payoff) coming form the corresponding agent's participation constraint.

5 Constrained Public Finance

In this section, we introduce an additional constraint limiting the capacity of the government to pay its agents (BC). By Engel et al. (2013) we know that even if the government faces a binding budget constraint, partnership and sequential contracts are equivalent, when financial constraints do not affect the agents. The government can implement second best contracts taxing all agents' informational rents and therefore the public budget constraint does not add any asymmetry across sequential and partnership contracts.[16]

By the analysis of implementable sequential and partnership contracts in Sections 4.1 and 4.2, we can derive the state-contingent payments from the government to the agents, and then study transfers under partnership or sequential contracts.

Sequential Contracts. Under sequential contracts we find the optimal transfers that minimize the maximum outlays for the government:

$$\begin{aligned}
t_b(q^h, c^h) &= kq^h + \phi'(e_b) - (e_o^s)^2\psi''(e_o^s) - l_b; \\
t_b(q^h, c^l) = kq^h + \phi'(e_b) + e_o^s(1 - e_o^s)\psi''(e_o^s) - l_b &\geq t_b(q^h, c^h) : \\
t_o(q^h, c^h) = t_o(q^l, c^h) &= c^h - l_o; \\
t_o(q^h, c^l) = t_o(q^l, c^l) &= c^l + \psi'(e_o^s) - l_o. \\
t_b(q^l, c^h) = t_b(q^l, c^l) &= kq^l - l_b
\end{aligned}$$

Then, by substituting the optimal values maximizing the social welfare, we obtain the level of maximum transfers that ex-post the government should provide to private agents after observing an high infrastructure quality and low (or high) operational costs:

$$\begin{aligned}
T^s = t_b(q^h, c^h) + t_o(q^h, c^h) = S(q^h - q^l) + kq^l + c^h - e_b^s\phi''(e_b^s) - (e_o^s)^2\psi''(e_o^s) - l_b - l_o = \\
= t_b(q^h, c^l) + t_o(q^h, c^l)
\end{aligned}$$

The equalization of the transfers in the state of the world (q^h, c^h) and (q^h, c^l) is possible only if $\phi'(e_b) \geq (e_o)^2\psi''(e_o)$. If such condition is violated, the LLC in the state of the world (q^h, c^h) is binding and requires a sufficiently large transfer to the agents in such a state. Therefore, if such condition is violated the following argument – stating that the maximum state-contingent transfer is lower under participation contract – holds *a fortiori*.

Partnership Contract. By ICCs and LLCs, we characterize the transfers in the

[16] The result by Engel et al. (2013) can be proven also in our framework, assuming that LLCs are not binding.

different states of the world:

$$t(q^h, c^h) = c^h + kq^h + A - l_c;$$
$$t(q^h, c^l) = t(q^h, c^h) - c^h + c^l + \psi'(e_o^{hp}) \geq 0;$$
$$t(q^l, c^l) = t(q^l, c^h) - c^h + c^l + \psi'(e_o^{lp}) \geq 0.$$
$$t(q^l, c^h) = kq^l + c^h - l_c;$$

Knowing that $A > 0$ and $e_o^h > e_o^l$, it is straightforward to observe that:

$$t(q^h, c^h) \geq t(q^l, c^h);$$
$$t(q^h, c^l) \geq t(q^l, c^l).$$

Then, by substituting the optimal values that maximize the social welfare function, we find the maximum ex-post transfer the government could provide to the private consortium:

$$T^p = t(q^h, c^h) = S(q^h - q^l) + kq^l + c^h - \frac{(e_o^{lp})^2 \psi''(e_o^{lp})}{1 - e_b^p} - e_b^p \phi''(e_b^p) - l_c = t(q^h, c^l)$$

On the other hand, if respectively $F \leq kq^h + c^h - l_b - l_o$ or $F \leq kq^h + c^h - l_c$, then agent/s under sequential or partnership contracts will not have any incentives to participate as limited liability constraints are violated. Thus, the only interesting case for our model corresponds respectively to the situation when $F > kq^h + c^h - l_b - l_o$ under sequential contracts and $F > kq^h + c^h - l_c$ under the partnership contract. As transfers contingent to the realizations of low infrastructure quality and high (or low) operational costs are always higher than such thresholds, in the following paragraphs we will study how final results change (respectively under sequential and partnership contracts) considering the possibility that the budget constraint limits the level of private transfers contingent to the realizations of high infrastructure quality and high (or low) operational costs.

5.1 Sequential Contracts

The government budget constraint is binding under the sequential contract whenever $T^s \geq F$. Thus, we add to the government's program the following budget constraints:[17]

$$t_b(q^h, c^l) + t_o(q^h, c^l) \leq F, \tag{41}$$

$$t_b(q^h, c^h) + t_o(q^h, c^h) \leq F. \tag{42}$$

Optimal Sequential Contracts. Substituting the transfer schedules that satisfy ICC, and LLC of the builder and the operator in government's objective function, and considering the budget constraints, its maximization problem can be written as:

$$\begin{aligned}
\max_{e_b, e_o^h, e_o^l, t_b(q^h, c^h)} \; & e_b(Sq^h) + (1 - e_b)(Sq^l) + \\
& -e_b[kq^h + \phi'(e_b) - l_b - (1 - e_o^h)t_b(q^h, c^h)] \\
-e_b(1 - e_o^h)t_b(q^h, c^h) - e_b e_o^h(\psi'(e_o^h) + c^l - l_o) - e_b(1 - e_o^h)(c^h - l_o) & \\
-(1 - e_b)e_o^l(kq^l - l_b + c^l + \psi'(e_o^l) - l_o) - (1 - e_b)(1 - e_o^l)(kq^l - l_b + c^h - l_o) & \\
s.t. : t_b(q^h, c^h) + c^h - l_o - F \leq 0 \quad (\lambda_s) & \\
kq^h + \phi'(e_b) - l_b - (1 - e_o^h)t_b(q^h, c^h) + e_o^h c^l + e_o^h \psi'(e_o^h) - e_o^h l_o - e_o^h F \leq 0 \quad (\mu_s) & \\
-t_b(q^h, c^h) + kq^h - l_b \leq 0 \quad (\alpha_s) &
\end{aligned}$$

[17] If the government budget limit becomes very stringent such that the builder's limited liability constraint in the case of high infrastructure quality is not satisfied, then the project is not realized anymore.

First order conditions:

$$e_b : S(q^h - q^l) - [kq^h + \phi'(e_b) - l_b + e_b\phi''(e_b)] +$$
$$-e_o^h(\psi'(e_o^h) + c^l - c^h) + e_o^l(c^l - c^h + \psi'(e_o^l)) + kq^l - l_b - \mu_s\phi''(e_b) = 0$$
$$e_o^h : -e_b(\psi'(e_o^h) + c^l - c^h + e_o^h\psi''(e_o^h)) +$$
$$-\mu_s(t_b(q^h, c^h) + c^l + \psi'(e_o^h) + e_o^h\psi''(e_o^h) - l_o - F) = 0$$
$$e_o^l : -(1 - e_b)(c^l - c^h + \psi'(e_o^l) + e_o^l\phi''(e_o^l)) = 0$$
$$t_b(q^h, c^h) : -\lambda_s + (1 - e_o^h)\mu_s = 0$$
$$+[t_b(q^h, c^h) + c^h - l_o - F]\lambda_s = 0$$
$$+[kq^h + \phi'(e_b) - l_b - (1 - e_o^h)t_b(q^h, c^h) + e_o^h c^l + e_o^h\psi'(e_o^h) - e_o^h l_o - e_o^h F]\mu_s = 0$$

where λ_s and μ_s are the Lagrangian multipliers associated with the BCs. If $\lambda_s = \mu_s = 0$, the model replicates the one analyzed in Section 4.1. Moreover, if $\lambda_s > 0$ then also $\mu_s > 0$ (and the reverse); thus, both budget constraints are binding. By the first order conditions, we obtain the characterization of the second best optimal efforts:

$$\phi'(e_b) = (S - k)(q^h - q^l) - (e_b + \mu_s)\phi''(e_b)$$
$$\psi'(e_o^h) = c^h - c^l - e_o^h\psi''(e_o^h)$$
$$\psi'(e_o^l) = c^h - c^l - e_o^l\psi''(e_o^l)$$
$$\lambda_s = (1 - e_o^h)\mu_s$$
$$t_b(q^h, c^h) = F - c^h + l_o$$
$$kq^h + \phi'(e_b) - l_b - (1 - e_o^h)t_b(q^h, c^h) + e_o^h c^l + e_o^h\psi'(e_o^h) - e_o^h l_o - e_o^h F = 0$$

We come to some interesting conclusions. First, the operation contract does not have memory, and the operator's effort is equal to the case where the government faces no financial constraint (i.e.,$e_o^h = e_o^l = e_o^s$). Second, the builder's effort is lower than when the government is financially unrestricted: $e_b^{s,BC} < e_b^s$.

5.2 Partnership Contract

In the partnership contract the budget constraint is binding when $T^p \geq F$. Implementability conditions deriving by agent's PC, LLC and ICC are as in Section 4.2, however the government's program has to satisfy also the budget constraints:[18]

$$t(q^h, c^l) \leq F, \tag{43}$$

$$t(q^h, c^h) \leq F. \tag{44}$$

Optimal Partnership Contract.

Substituting the transfer schedules that satisfy the ICC and LLC of the single private contractor in the government's objective function, after some algebra, the constrained maximization problem can be written as:

$$\begin{aligned}
\max_{e_b, e_o^h, e_o^l} \; & e_b S(q^h - q^l) + Sq^l + \\
-e_b e_o^h [c^l + (1 - e_o^h)\psi'(e_o^h) + kq^h + \psi(e_o^h) + (e_o^l \psi'(e_o^l) - \psi(e_o^l)) + \phi'(e_b) - l_c] & + \\
-e_b (1 - e_o^h)[c^h - e_o^h \psi'(e_o^h) + kq^h + \psi(e_o^h) + (e_o^l \psi'(e_o^l) - \psi(e_o^l)) + \phi'(e_b) - l_c] & + \\
-(1 - e_b) e_o^l (\psi'(e_o^l) + kq^l + c^l - l_c) - (1 - e_b)(1 - e_o^l)(kq^l + c^h - l_c) & \\
s.t. : c^h - e_o^h \psi'(e_o^h) + kq^h + \psi(e_o^h) + (e_o^l \psi'(e_o^l) - \psi(e_o^l)) + \phi'(e_b) - l_c - F \leq 0 & \quad (\lambda_p) \\
c^l + (1 - e_o^h)\psi'(e_o^h) + kq^h + \psi(e_o^h) + (e_o^l \psi'(e_o^l) - \psi(e_o^l)) + \phi'(e_b) - l_c - F \leq 0 & \quad (\mu_p)
\end{aligned}$$

where λ_p and μ_p are the Lagrangian multipliers associated with the BCs. First order conditions are as follows:

$$\begin{aligned}
e_b : S(q^h - q^l) + (e_o^h - e_o^l)(c^h - c^l) - k(q^h - q^l) & + \\
-\psi(e_o^h) + \psi(e_o^l) - \phi'(e_b) - e_b \phi''(e_b) - (\lambda_p + \mu_p)\phi''(e_b) & = 0 \\
e_o^h : e_b(c^h - c^l - \psi'(e_o^h)) - \alpha_p e_o^h \psi''(e_o^h) - \mu_p (1 - e_o^h)\psi''(e_o^h) & = 0 \\
e_o^l : -(1 - e_b)(\psi'(e_o^l) + c^l - c^h + e_o^l \psi''(e_o^l)) - (\lambda_p + \mu_p) e_o^l \psi''(e_o^l) & = 0 \\
\lambda_p [c^h - e_o^h \psi'(e_o^h) + kq^h + \psi(e_o^h) + (e_o^l \psi'(e_o^l) - \psi(e_o^l)) + \phi'(e_b) - l_c - F] & = 0 \\
\mu_p [c^l + (1 - e_o^h)\psi'(e_o^h) + kq^h + \psi(e_o^h) + (e_o^l \psi'(e_o^l) - \psi(e_o^l)) + \phi'(e_b) - l_c - F] & = 0
\end{aligned}$$

[18] Government budget constraints in the state of the world lh or ll are never binding as in such cases the limited liability constraint of the consortium in the state of the world hh is not respected and the project is not realized.

Lemma 11 $\lambda_p > 0$ *and* $\mu_p > 0$.

Proof. Assume that $\lambda_p > 0$. Then, by the first order conditions, to have $c^l + \psi'(e_o^h) - c^h = 0$, it is necessary that: $\lambda_p e_o^h \psi''(e_o^h) - \mu_p(1 - e_o^h)\psi''(e_o^h) = 0$. This condition is satisfied if $\lambda_p = \frac{\mu_p(1-e_o^h)}{e_o^h}$. If $\mu = 0$, then $c^l + \psi'(e_o^h) - c^h < 0$. This is not possible since $\lambda_p e_o^h \psi''(e_o^h) > 0$. Assuming now that $\mu_p > 0$. Then, by the first order conditions, if $\lambda_p = 0$ we need to have $c^h - \psi'(e_o^h) - c^l < 0$, but this is not verified since $\mu_p(1 - e_o^h)\psi''(e_o^h)$ is higher than 0. Then, if $\mu_p > 0$, it means that $\lambda_p > 0$ ■

By the first order conditions, we find the following second best optimization conditions:

$$\phi'(e_b) = (S - k)(q^h - q^l) + (e_o^h - e_o^l)(c^h - c^l) - \psi(e_o^h) + \psi(e_o^l) - (e_b + \frac{\mu_p}{e_o^h})\phi''(e_b)$$
$$\psi'(e_o^h) = c^h - c^l$$
$$\psi'(e_o^l) = c^h - c^l - \frac{\mu_p + e_o^h}{e_o^h(1 - e_b)} e_o^l \psi''(e_o^l)$$
$$c^h - e_o^h \psi'(e_o^h) + kq^h + \psi(e_o^h) + (e_o^l \psi'(e_o^l) - \psi(e_o^l)) + \phi'(e_b) - l_c - F = 0$$
$$c^l + (1 - e_o^h)\psi'(e_o^h) + kq^h + \psi(e_o^h) + (e_o^l \psi'(e_o^l) - \psi(e_o^l)) + \phi'(e_b) - l_c - F = 0$$

By these conditions, we derive two interesting results. First, the contract has memory, precisely the operator effort differs depending on the quality of the infrastructure ($e_o^h = e_o^* > e_o^{l,BC}$). It is worth mentioning that $e_o^{l,BC}$ is lower with respect to the result of Section 4.2 (Unconstrained Public Finance). Second, even the builder effort is lower with respect to the situation with unconstrained public finance, while, as before, it can be either higher or lower than the first best.

It is important to emphasize that if private limited liabilities constraints under the states of the world *hh* or *hl* are not respected because of the budget constraint, then the project is never realized. Moreover, results suggest as it is always optimal in the case of partnership contract to implement a memory contract.

5.3 Welfare Comparison between Partnership and Sequential Contracts

In this paragraph, we will first study whether the budget constraint is more easily binding under sequential or partnership contracts. Second, we will compare sequential with partnership contracts considering that both contracts are constrained by the presence of a budget limit.

Maximum transfer. The difference between the largest financial transfers between sequential (T^s) and partnership (T^p) contracts under unconstrained public finance is equal to:

$$T^s - T^p = -[e_o^s(c^h - c^l - \psi'(e_o^s)) - e_o^{lp}(c^h - c^l - \psi'(e_o^{lp}))] - [e_b^s\phi''(e_b^s) - e_b^p\phi''(e_b^p))] + l_c - l_b - l_o. \quad (45)$$

If we compare $T^s - T^p$ with $W^p - W^s$ under unconstrained public finance, we find that:

$$T^s - T^p = [W^p - W^s] + (e_b^p)\phi''(e_b^p)(1 - e_b^p) - (e_b^s)\phi''(e_b^s)(1 - e_b^s) \quad (46)$$

Main implication of Equation 46 is summarized by the following proposition:

Proposition 12 *The presence of a budget constraint may change government's preference either from sequential to partnership contracts orviceversa.*

Proof. Looking at Equation 46, we can distinguish two cases. The first is when $(e_b^p)\phi''(e_b^p)(1 - e_b^p) > (e_b^s)\phi''(e_b^s)(1 - e_b^s)$. Under such condition, if initially $W^p > W^s$, then $T^s > T^p$, meaning that the budget constraint will be more easily binding under sequential rather than partnership contracts. Otherwise, if initially $W^p > W^s$, then T^s can be either higher or lower that T^p. Under the first scenario, if $T^p < T^s$, then the budget constraint will be more easily binding under sequential rather than partnership contracts, as a consequence the government preference could change from sequential to partnership contracts. The second case is when $(e_b^p)\phi''(e_b^p)(1 - e_b^p) < (e_b^s)\phi''(e_b^s)(1 - e_b^s)$. Under such condition, the reasoning is similar. Precisely, if initially $W^p > W^s$, and finally $T^s < T^p$, the the government preference could change from partnership to sequential contracts ■

Welfare analysis

Substituting the optimal levels of efforts considering a binding budget constraint in the government's objective function, we can write the value of the social welfare under partnership contract as:

$$W^p = e_b^{pc}[S(q^h - q^l) + kq^l + c^h - l_c - F] + (S-k)q^l - c^h +$$
$$+(1-e_b^{pc})(e_o^{lpc})^2 \frac{\mu_p + e_o^{hpc}}{e_o^{hpc}(1-e_b^{pc})}\psi''(e_o^{lpc}) + l_c$$
$$W^p = (S-k)q^l - c^h + l_c + e_b(e_b + \frac{\mu_p}{e_o^h})\phi''(e_b) + (e_o^{lpc})^2 \frac{\mu_p + e_o^{hpc}}{e_o^{hpc}(1-e_b^{pc})}\psi''(e_o^{lpc})$$

and under sequential contracts as:

$$W^s = e_b^{sc}[S(q^h - q^l) + kq^l + c^h - F - l_b - l_o] + (S-k)q^l - c^h + (1-e_b^{sc})(e_o^{sc})^2\psi''(e_o^{sc}) + l_b + l_o$$
$$W^s = (S-k)q^l - c^h + e_b^{sc}(e_b^{sc} + \mu_s)\phi''(e_b^{sc}) + (e_o^{sc})^2\psi''(e_o^{sc}) + l_b + l_o$$

If we compute the difference in welfare function between partnership and sequential contracts, we will find:

$$\begin{aligned} \Delta W = W^{pc} - W^{sc} \\ +(e_b^{pc} - e_b^{sc})[S(q^h - q^l) + kq^l + c^h - F] \\ +(1-e_b^{pc})(e_o^{lpc})^2 \frac{\mu_p + e_o^{hpc}}{e_o^{hpc}(1-e_b^{pc})}\psi''(e_o^{lpc}) - (1-e_b^{sc})(e_o^{sc})^2\psi''(e_o^{sc}) \\ +l_c(1-e_b^{pc}) - (l_b + l_o)(1-e_b^{sc}). \end{aligned} \tag{47}$$

A first result is:

Proposition 13 *When the "Available Liabilities component" is equal or greater than zero ($l_c \geq l_b + l_o$), then the partnership contract always dominates the sequential contracts in social welfare terms also in the presence of constrained public finance.*

Proof. Knowing that FOCs of the partnership problems are global max under our restricted range of values (e_o such that $e_o < e_o^*$), we can substitute on the partnership welfare function the FOCs of the sequential problem and we should have that:

$$\begin{aligned} W^p &= e_b^{pc}[S(q^h - q^l) + kq^l + c^h - F] + (S-k)q^l - c^h + \\ &+(1-e_b^{pc})(\frac{\mu_p + e_o^{hpc}}{e_o^{hpc}(1-e_b^{pc})})(e_o^{lpc})^2\psi''(e_o^{lpc}) + l_c(1-e_b^{pc}) > \\ W^p(e_b^{sc}, e_o^{sc}) &= e_b^{sc}[S(q^h - q^l) + kq^l + c^h - F] + (S-k)q^l - c^h + \\ &+(1-e_b^{sc})(\frac{\mu_s + e_o^{sc}}{e_o^{sc}(1-e_b^{sc})})(e_o^{sc})^2\psi''(e_o^{sc}) + l_c(1-e_b^{pc}) \end{aligned}$$

Moreover we can easily verify that:

$$\begin{aligned} W^p(e_b^{sc}, e_o^{sc}) - W^s(e_b^{sc}, e_o^{sc}) = \\ (1-e_b^{sc})[(e_o^{sc})^2\psi''(e_o^{sc})(\frac{\mu_s + e_o^{sc}e_b^{sc}}{e_o^{sc}(1-e_b^{sc})}) + (l_c - l_b - l_o)] \end{aligned}$$

that is higher than zero if $lc \geq l_b + l_o$. By putting together the two equations, we conclude that, if $lc \geq l_b + l_o$:

$$W^p > W^p(e_b^s, e_o^s) > W^s$$

■

Secondly, if we want to study how this difference changes with F, we can compute the derivative and using the envelope theorem, we will find:

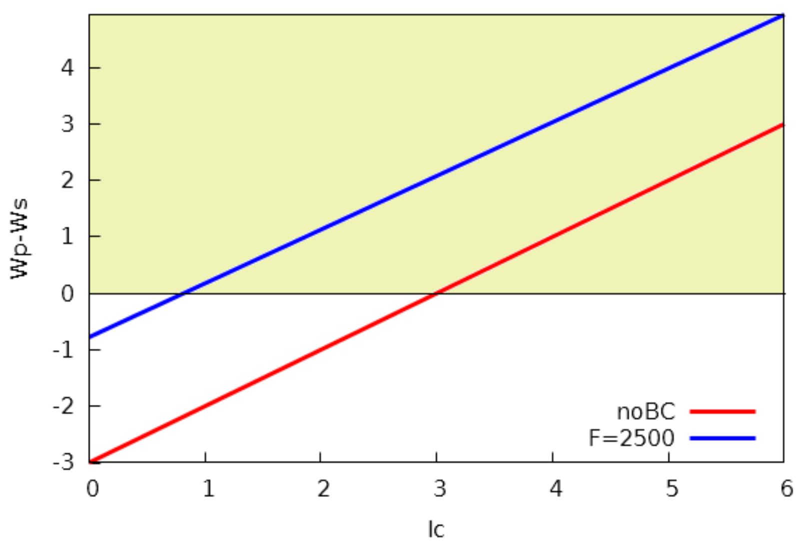
$$\frac{d(W^{pc} - W^{sc})}{dF} = -(e_b^{pc} - e_b^{sc}) \tag{48}$$

Depending on the parameters, e_b^{pc} can be either higher or lower that e_b^{sc}. Thus, the presence of a binding budget constraint can either increase or decrease the government's preference towards the partnership contract. In the following section we will perform a numerical simulation to study how the government's choice can be affected with the change in the parameters: F, l_c, l_b and l_o.

6 Comparative Statics on Financial Limits

Main drivers of the theoretical model are the financial parameters: F for the public regulator, and respectively lc, lb and lo for the private consortium, builder and operator. In the subsection 4.4 we discussed the situation where: under the partnership contract, the limited liability constraint of the consortium is binding; while under the sequential contract either one between the builder or the operator's limited liability constraint is binding. In such a case, we derived sequential contracts dominate the partnership contract when lc is sufficiently low with respect to lb or lo, depending on whether the builder or the operator's limited liability constraint is relevant. The next graph summarizes the statement of these propositions. Comparative statics are based on numerical simulations; assumptions on functional forms and model's parameters are reported in the appendix.

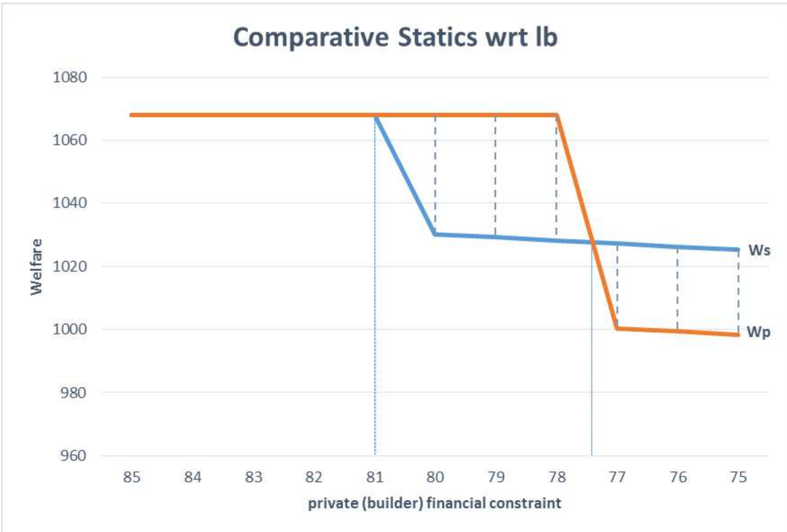
Figure 3: Comparative Static on private financial limits



The graph considers the situation where the builder's and the consortium limited liability constraints are binding, while the one of the operator is slack. In the figure, the x axis reports the magnitude of lc; the higher the value, the lower the consortium's (with respect to the builder's) financial limit, thus the less strict the partnership financial constraint. The y axis reports the difference between the partnership and the sequential value function of the government. Sequential contract dominates partnership contract when lc is sufficiently low, otherwise under the yellow region partnership is welfare improving with respect to the sequential contract. In the graph, we interrelates private with public financial limits, and we shows as considering a binding government's budget constraint (blue line) reduces the region where sequential dominates partnership contract.

In order to better disentangle results of our analysis, we propose a further comparative static. In the next graph we consider the specific case when $lc = lb + lo$, thus we can report the comparative static of $Wp - Ws$ with respect to lb.

Figure 4: Comparative Static of Wp and Ws with respect to lb



In the graph the builder's financial limit (x axis) is reported in a descending order, while the y axis represents the level of the government's value function. When lb is sufficiently high, both the builder's and the consortium's limited liability constraints are slack, then sequential and partnership contracts are equivalent to the first best.[19] In the second part of the graph only the builder's limited liability constraint is binding, while under the partnership contract, the bundling of agents increases the

[19] As in the previous analysis, in this case we consider the operator to have a slack limited liability constraint.

contract flexibility and allows the government to achieve the first best. Finally, in the third part of the graph lb is extremely low, thus the consortium's limited liability constraint becomes binding. In this case the sequential contract is preferred since, under the partnership contract, the builder's financial limit distorts downward both the builder and the operator level of effort. On the other hand, under the sequential contract, tasks are unbundled, thus the operational contract is not distorted and the operator can apply the first best level of effort.

7 Conclusions

This theoretical paper analyses the involvement of private companies to provide citizens private goods and services. Precisely, we focus on projects characterized by sequential activities, as the building of an infrastructure and the operation of the realized asset. In such a context the literature on PPPs emphasized advantages of bundling these sequential tasks within a single contract between the government and a private agent. A relevant benefit is explained by the presence of positive production externalities within activities (Iossa and Martimort, 2015; Martimort and Pouyet, 2008; Hart, 2003) that can provide incentives for innovations during the building stage able to reduce operational costs. A different aspect studied by the literature on PPPs is the role of public budget constraints on the government's choice between sequential and partnership contracts. The paper of Engel et al. (2013) stated the " irrelevance results" saying that the presence of a budget constraint is not able to explain why governments should prefer to allocate sequential unrelated activities through a single long-term contract rather than through independent sequential contracts.

In this analysis we introduced a model without production externalities and characterized by moral hazard with the presence of both private limited liability and budget constraints. With this choice we aimed at studying the government's aptitude towards partnership or sequential contracts considering the presence of financial private and public restraints. Moral hazard is introduced in the model as the builder's and the operator's efforts cannot be observed by the principal that however can set contingent transfers on the base of observable ex-post outcomes that are directly linked to the levels of efforts. Precisely, the infrastructure quality as well as the operational cost can be high or low.

Differences between partnership and sequential contracts in our framework are the following. In the case of sequential contracts the government awards the two phases (building and operation) separately through sequential contracts to a builder and an operator. Agents' payoff are made of government's transfers minus monetary and non-monetary costs. Contracts should precise the optimal levels of ex-post transfers able to satisfy private participation, incentive-compatibility and limited liability constraints. Each set of constraints is separately defined for each agent and the problem is solved backwards. In the case of partnership contract the government awards the two sequential tasks through a single contract to a consortium of agents. Under this scenario, constraints are bundled and the consortium maximizes the total payoff coming from the building and operation activities. By solving the problem and by studying as welfare is different between sequential and partnership contracts we obtain two relevant results.

At first, without considering the presence of a budget constraint, we show as private financial restraints may affect the government's preference between partnership and sequential contracts even in the absence of production externalities. In fact, optimality under the partnership contract implies the settlement by the government of a memory contract where second order optimal depend not only on the level of operation costs, but also on the level of the building's quality. Such a mechanism creates an implicit and costly incentive for the private agent to increase its level of effort during the building task. This endogenous memory contract derives from the presence of limited liability constraints together with moral hazard and it emphasizes a benefit of the partnership contract that is independent by the presence of production externalities (Iossa and Martimort, 2015; Martimort and Pouyet, 2008). Then, the choice between partnership and sequential contracts further depend on the difference between available liabilities of private agents under partnership and sequential contracts.

Second, we show as the presence of a budget constraint does not cancel out the benefit of a memory partnership contract. Moreover, despite confirming the result of Engel et al. (2013) in the absence of private financial restraints, we find that considering limited liability constraints the government's choice between sequential and partnership contract may be affected by the presence of a budget constraint.

Results of this paper are relevant essentially for the literature on PPPs, as we are able to provide a theoretical explanations of empirical data showing as governments' propensity to adopt PPPs moves together with the strictness of the financial environment. Precisely, with respect to the related literature, we first explain why government's could have an incentive to adopt PPPs even if externalities within stages are negative or not relevant, second we provide a channel able to explained the observed link between governments' budget constraints and PPPs' adoption (Buso et al., ress).

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Appendix

Proof of Lemma 6. When $A < 0$, substituting the relevant constraints (21, 24, 25 and 26), the government's optimization program can be written as:

$$\max_{e_b, e_o^h, e_o^l} (S-k)q^l - c^h - e_o^l(c^l - c^h) - e_o^h \psi'(e_o^h) - \psi(e_o^l) + \psi(e_o^h) + \phi'(e_b) + \\ + e_b[(S-k)(q^h - q^l) + (e_o^h - e_o^l)(c^h - c^l) + \psi(e_o^l) - \psi(e_o^h) - \phi'(e_b)] + l_c$$

By the first order conditions, we find that:

$$\phi'(e_b^p) = (S-k)(q^h - q^l) + (e_o^{hp} - e_o^{lp})(c^h - c^l) + \psi(e_o^{lp}) - \psi(e_o^{hp}) + (1 - e_b^p)\phi''(e_b^p)$$
$$\psi'(e_o^{hp}) = c^h - c^l - \frac{e_o^{hp}}{e_b^p}\psi''(e_o^{hp})$$
$$\psi'(e_o^{lp}) = c^h - c^l$$

By the properties of the ψ function we know that $e_o^{lp} \geq e_o^{hp}$, but then the initial condition to have $A < 0$ is not satisfied.[20] ■

7.1 Numerical Simulations

To perform the comparative statics with respect to the financial parameters, we assumed some specific forms for the effort functions and we assigned some numerical values to the model parameters. Related to the effort functions, we assumed the following quadratic forms that are in line with our initial assumptions:

[20] By the proof of Lemma 2, $e_o\psi'(e_o) - \psi(e_o) > 0$. Thus, $A < 0$ only if $e_o^{lp} \leq e_o^{hp}$

$$\phi(e_b) = p * (e_b^2)/2\psi(e_o) = q * (e_o^2)/2 \tag{49}$$

where respectively $p = 1000$ and $q = 100$. Related to the model parameters, we assigned the following values: $S = 100$; $q^h = 40$; $q^l = 20$; $c^h = 60$; $c^l = 20$; $k = 60$. Values are set such that there are incentives for the private agents to increase the level of effort, and there is the interest for the public principal to realize the investment.

Finally, final limits are equal to $lb = 0$ for the private builder, and $F = 2500$ for the public principal. For the private builder, we considered the standard limited liability constraint we used in our benchmark case. However, the budget financial limit is set such that the budget constraint of the principal becomes binding.