What We (Don't) Know About the Optimal Non-Linear Income Tax

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Preliminary Version

Abstract

The paper studies optimal income taxation in a model with labor supply responses at the intensive and the extensive margin. In contrast to the classical result due to Mirrlees (1971), a utilitarian desire for redistribution does not pin down the signs of optimal marginal taxes and optimal participation taxes in this model. The paper also provides sufficient conditions for the optimality of tax schedules with negative marginal taxes and negative participation taxes for the working poor, complying with the main features of the US Earned Income Tax Credit. Furthermore, it uncovers a non-standard tradeoff between efficiency at the intensive margin and efficiency at the extensive margin, which provides the economic intuition behind the ambiguous sign of the optimal marginal tax.

Keywords: Optimal taxation, Utilitarian welfare maximization, Extensive margin, Intensive margin

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1 Introduction

In the last decade, the seminal paper by Saez (2002) has initiated a growing literature that aims at rationalizing the Earned Income Tax Credit (EITC), the largest tax/transfer program transferring resources towards the poor in the United States. For low-income workers, the EITC specifies a negative marginal income tax and a negative participation tax, i.e., a higher transfer than the one paid to the unemployed. Strikingly, both properties are at odds with the central result of optimal taxation theory due to Mirrlees (1971), according to which the optimal marginal income tax is strictly positive everywhere below the very top. Subsequent studies have shown the robustness of this result for all models in which, first, agents adjust their labor supply only at the intensive margin, i.e., choose how many hours or how hard to work, and second, the tax designer has a utilitarian desire for redistribution from rich (high-skill) to poor (low-skill) agents.[1]

Most prominently, two approaches have been brought forward to rationalize the EITC, each abandoning one of these basic assumption and explaining one property of the EITC. First, Saez (2002) shows that negative participation taxes might be optimal if agents adjust their labor supply only at the extensive margin, i.e., only take the binary decision whether or not to enter the labor market (see also Diamond 1980 and Choné & Laroque 2011). The basic intuition behind this result is that redistributing resources from the rich towards the working poor is less costly in efficiency terms than redistributing resources towards the unemployed. In particular, a negative participation tax for low-skill workers induces inefficient labor supply responses in this skill group only, while a rising unemployment benefit gives rise to labor supply distortions in all skill groups.

Second, Choné & Laroque (2010) show that negative marginal taxes can be rationalized in an intensive-margin model if the social planner prefers to redistribute resources from agents earning low incomes on the labor market to high-income earners. In this case, the social planner's anti-utilitarian desire to redistribute resources to high-skill agents is restricted by binding upward incentive compatibility constraints, which can only be relaxed through negative marginal taxes.[2]

These studies give rise to the questions whether an EITC-style tax scheme with negative marginal taxes and participation taxes can be optimal if, first, the social

[1] Amongst others, see Seade (1977, 1982), Diamond (1998), Hellwig (2007). Note that, under certain assumptions, the optimal marginal tax is also zero at the very bottom.

[2] In the model by Choné & Laroque (2010), agents are heterogeneous with respect to skill and, additionally, some other cost-related parameter. The authors show that an anti-utilitarian desire to redistribute from low-income to high-income workers can arise if these two type parameters exhibit a sufficiently strong correlation.

planner has a standard *utilitarian desire for redistribution* from high-skill to low-skill workers and, second, agents adjust their labor supply *at the intensive and the extensive margin*, which is arguably the most appropriate assumption from an empirical perspective.

In this case, marginal income taxes induce labor supply distortions at the intensive margin, which cannot occur in extensive-margin models by construction (Saez 2002, Choné & Laroque 2011). Relatedly, the social planner is restricted by incentive compatibility constraints as in the classical Mirrlees (1971) framework. As a consequence, it is unclear whether the simple intuition from the extensive models is still valid. If downward incentive compatibility constraints are binding in the optimal allocation, negative participation taxes for the working poor are associated with higher efficiency costs. Additional transfers to low-skill workers must then be accompanied either by stronger downward distortions at the intensive margin, or by similar transfers to workers of all higher skill types, which is at odds with the utilitarian objective. Moreover, negative marginal taxes can only be beneficial if upward incentive compatibility constraints are binding in the optimal allocation, i.e., if more resources are transferred to some group of workers than to a slightly less productive group of workers. The literature has not yet provided an explanation for why this might be in the interest of a utilitarian planner.

To some extent, this skepticism is confirmed by Jacquet et al. (2013) in a recent paper on optimal income taxation with labor supply responses at both margins. In particular, the authors show that optimal marginal taxes are positive everywhere below the very top whenever some sufficient condition is met. However, this sufficient condition is expressed in terms of endogenous variables, i.e., endogenous social weights and properties of the optimal allocation itself. Moreover, the relation between this condition and common assumptions on the economic primitives and the social planner's objective function remains unclear.

Contributions The first contribution of this paper is to show that the sign of the optimal marginal income tax is *in general ambiguous* even if the social planner holds a utilitarian desire for redistribution. For some utilitarian welfare functions, the optimal marginal tax is positive everywhere below the very top. But for other utilitarian welfare functions, the optimal marginal tax is zero throughout, or even negative at some low income levels. Complimenting these general insights, this paper is the first to provide sufficient conditions on the primitives such that an EITC-style tax scheme is indeed optimal, giving rise to upward distortions at both margins for some skill groups.

The second contribution of this paper is to *explain why* negative marginal taxes can be optimal in the model with labor supply responses at both margins. In contrast to the Mirrlees (1971) model, the sign of the optimal tax rate is not pinned down by a standard tradeoff between equity and efficiency. Instead, an additional tradeoff between intensive efficiency and extensive efficiency aspects arises, which has not been discussed in the literature so far. In Section 6, I show that both aspects of efficiency can be disentangled using an inverse elasticity rule. As will become clear below, this tradeoff between intensive efficiency and extensive efficiency drives the ambiguity of the optimal marginal tax: inducing upward distortions at the intensive margin through negative marginal taxes can be optimal if and only if this helps to reduce labor supply distortions at the extensive margin.

The final contribution of this paper is to show that the potential optimality of the EITC depends crucially on the assumed information structure. Following the related literature, I study a model in which agents are heterogeneous with respect to two type dimensions, skills and fixed costs of working. I show that an EITC-style tax scheme can be optimal in this framework if and only if agents possess private information about both type dimensions. In contrast, optimal utilitarian marginal taxes and participation taxes are always non-negative if the planner is able to observe either skills or fixed costs of working directly. Put differently, the optimal directions of labor supply distortions at both margins are ambiguous in multi-dimensional screening problems, while they are pinned down uniquely in problems of one-dimensional screening.

The paper proceeds as follows. I introduce the basic model in Section 2 and impose a set of regularity conditions in Section 3. Section 4 introduces the problem of optimal income taxation and some relevant terminology. In Section 5, I first derive the main results on the ambiguous sign of the optimal marginal taxes and participation taxes. Then, I provide sufficient conditions for the optimality of specific non-standard tax schedules, including an EITC-style tax scheme that induces upward distortions at both margins for some skill groups. Section 6 studies an auxiliary problem that helps to develop an economic intuition for this ambiguity and work out the tradeoff between intensive efficiency and extensive efficiency. Section 7 studies optimal utilitarian taxation under the alternative assumptions that either skills or fixed costs are publicly observable. Section 8 discusses the relevance of the imposed assumptions. Section 9 reviews the related literature, and Section 10 concludes. All formal proofs are relegated to the mathematical appendix.

2 Model

I study optimal Utilitarian income taxation in an economy with labor and one homogeneous good. There is a continuum of agents of mass one, each of whom is identified with a two-dimensional type (ω, δ). For reasons that will become clear below, I refer to $\omega \in \Omega$ as the skill type, and to $\delta \in \Delta$ as the fixed cost type. The skill type space Ω and the cost space Δ are compact sets, with $\underline{x}$ and $\bar{x}$ denoting the smallest and largest value of $x \in \{\omega, \delta\}$. Each agent's skill type ω and cost type δ are the realizations of two random variables $\tilde{\omega}$ and $\tilde{\delta}$ with joint probability distribution Ψ. The distribution Ψ is identical for all agents, and has full support on the type space $\Omega \times \Delta \in \mathbb{R}_+ \times \mathbb{R}$. Imposing a law of large numbers, I assume that Ψ also represents the cross-section distribution of types in the continuum of agents.[3]

The agents supply labor and consume the homogeneous good. If an agent with type (ω, δ) consumes c units and supplies labor to produce y units of this good, he receives a utility of $V(c, y, \omega, \delta)$. An allocation is given by two functions $c(\omega, \delta) \geq 0$ and $y(\omega, \delta) \geq 0$ that specify the consumption level and output level for each type in $\Omega \times \Delta$. It is feasible if and only if overall consumption does not exceed overall output, i.e.,

$$\int_{\Omega \times \Delta} c(\omega, \delta) d\Psi(\omega, \delta) \leq \int_{\Omega \times \Delta} y(\omega, \delta) d\Psi(\omega, \delta) \tag{1}$$

Each agent is privately informed about his skill ω and fixed cost δ. Thus, an allocation can only be implemented if it is incentive-compatible, i.e., if

$$V\left(c(\omega, \delta), y(\omega, \delta), \omega, \delta\right) \geq V\left(c(\omega', \delta'), y(\omega', \delta'), \omega, \delta\right) \tag{2}$$

for all types (ω, δ) and (ω', δ') in $\Omega \times \Delta$. Normative comparisons of allocations are enabled by the welfare function

$$\int_{\Omega \times \Delta} U\left[V\left(c(\omega, \delta), y(\omega, \delta), \omega, \delta\right)\right] d\Psi(\omega, \delta) \tag{3}$$

The welfare function integrates over all agents' utilities, subject to some positive-monotone transformation U. Its properties capture the planner's objective with respect to redistributive taxation, beyond the properties of the utility function V. Thus, the desirability of redistribution depends on both V and U. To guarantee existence of a solution, let $\lim_{z \to \infty} U'(z) \leq 1$.

[3] For conditions justifying this approach, see Sun (2006).

3 Assumptions

Throughout the paper, I will impose the following assumptions.

Regularity Conditions (RC): *The utility function* $V : \mathbb{R}^4 \mapsto \mathbb{R}$ *is twice continuously differentiable in* c*,* ω*,* δ *and, for* $y > 0$*, in* y*. It is strictly concave and increasing in* c*. For* $y > 0$*, it is strictly concave and decreasing in* y*, increasing in* ω *and decreasing in* δ*.*

Strict Single-Crossing (SSC): *For all* $(c, y, \omega) \in \mathbb{R}^3_{+++}$*, the utility function satisfies*

$$\frac{\partial}{\partial \omega} \left[\frac{V_c(c, y, \omega, \delta)}{V_y(c, y, \omega, \delta)} \right] < 0 \tag{4}$$

Assumptions *RC* and *SSC* are standard in the literature, and will not be discussed further.

Additive Fixed Costs (AFC): *The utility function consists of a gross utility component* $\tilde{V}$ *and an additively separable fixed cost component* δ*:*

$$V(c, y, \omega, \delta) = \tilde{V}(c, y, \omega) - 1_{y>0}\delta \tag{5}$$

Function U *is twice continuously differentiable and strictly increasing in its argument, while* $\tilde{V}$ *inherits the properties of* V *with respect to* c*,* y*, and* ω*.*

Assumption *AFC* is made for tractability, allowing to study the optimal tax problem with the random participation approach due to Rochet & Stole (2002). It has also been made in related papers on optimal taxation with labor supply responses at the extensive margin (Jacquet et al. 2013, Choné & Laroque 2011).

Quasi-Linearity in Consumption (QLC): *The gross utility component* $\tilde{V}$ *is quasi-linear in consumption:*

$$\tilde{V}(c, y, \omega) = c - h(y, \omega) \tag{6}$$

For $(y, \omega) \in \mathbb{R}^2_{++}$*, the effort cost function is strictly increasing and convex in* y*, strictly decreasing in* ω *and has a strictly negative cross derivative* $h_{y\omega}(y, \omega)$*. For* *any $\omega \in \Omega$, the effort cost function satisfies $h(0,\omega) = 0$ and the Inada conditions* $\lim_{y\to 0} h_y(y,\omega) = 0$ *and* $\lim_{y\to\infty} h_y(y,\omega) = \infty$.

Assumption *QLC* rules out income effects in labor supply, which considerably simplifies the analysis. For this reason, it has also been imposed in a number of related papers, including Diamond (1998). Moreover, it implies that the desirability of redistribution depends only on the properties of transformation U in the planner's objective function. For example, if transformation U were given by the identity function, welfare could not be increased through redistributive taxation.

Relevance of Extensive Margin (REM): *For any type (ω, δ), let*

$$y^{LF} = \arg\max_{y} V(y, y, \omega, \delta) \tag{7}$$

be the output level that an agent of type (ω, δ) would choose under laissez-faire. Heterogeneity in fixed costs is large enough to ensure that $y^{LF}(\underline{\omega}, \underline{\delta}) > 0$, and $y^{LF}(\bar{\omega}, \bar{\delta}) = 0$ hold.

By Assumption *REM*, every skill group would involve active workers and unemployed agents without redistributive income taxation. This guarantees that changes in the tax schedule will induce labor supply responses at the extensive margin by agents of all skill groups in some neighborhood of the laissez-faire allocation. The assumption is imposed to work out very clearly the differences to the standard Mirrleesian framework, where agents adjust their labor supply at the intensive margin only.

Discrete Skill Space (DSS): *The skill space Ω is given by the finite set $\{\omega_1, \omega_2, \ldots, \omega_n\}$ with $\omega_{j+1} > \omega_j$ for all natural numbers below n. The cost space Δ is given by some interval $[\underline{\delta}, \bar{\delta}]$ on the real line.*

By assumption *DSS*, the skill space is discrete, while the cost space is continuous. While this type space corresponds to the model studied by Saez (2002), it differs from Choné & Laroque (2011) and Jacquet et al. (2013) who consider models in which Ω and Δ are both given by an interval.

The next two assumptions restrict the joint type distribution Ψ, rewritten as $(F, G_1, \ldots, G_n)$. F denotes the cumulative distribution function of skills, with $f_j > 0$ representing the probability that an agent has skill type $\omega_j \in \Omega$. G_j denotes the cdf of fixed costs in the group of agents with skill type ω_j, and has a corresponding pdf g_j that is strictly positive if and only if $\delta \in \Delta$.

Log-Concave Fixed Cost Distributions (LC): *In all skill groups $\omega_j \in \Omega$, the distribution of fixed costs G_j is strictly log-concave, i.e., the inverse hazard rate $\frac{G_j(\delta)}{g_j(\delta)}$ is strictly increasing on the cost space Δ.*

This regularity assumption is satisfied for most commonly used distributions, including the uniform, normal, log-normal, exponential and Pareto distributions.

Ordered Fixed Cost Distributions (OFCD): *For each pair of skill levels ω_j and ω_{j+1} in Ω, the skill-dependent fixed cost distributions satisfy*

(i) $G_{j+1}(\delta) \geq G_j(\delta)$ *for all* $\delta \in \Delta$, *and*

(ii) $\frac{G_{j+1}(\delta)}{g_{j+1}(\delta)} \geq \frac{G_j(\delta)}{g_j(\delta)}$ *for all* $\delta \in \Delta$.

By the first part of Assumption *OFCD*, G_j weakly dominates G_{j+1} in the sense of first-order stochastic dominance. By the second part, the inverse hazard rate at any cost level δ is larger for high-skill groups than for low-skill groups. In general, both properties are closely related but not identical (for the uniform distribution, the second property is implied by first-order stochastic dominance). Note that *OFCD* covers the benchmark case of independence, in which $G_j(\delta) = G(\delta)$ for all $\delta \in \Delta$ and all $\omega_j \in \Omega$. As will become clear below, the results of this paper depend crucially on this assumption.[4]

The final assumption restricts the social objective as captured by the positive-monotone transformation U. To simplify notation, define the *endogenous social weight* α_j of workers of skill levels ω_j in allocation (c, y) by

$$\alpha_j^U(c,y) \equiv \frac{1}{\bar{\alpha}(c,y)} \mathbb{E}_\delta \left[U'\left(V\left(c(\omega_j,\delta), y(\omega_j,\delta), \omega_j, \delta\right)\right) | \delta \in \Delta : y(\omega_j,\delta) > 0 \right] \tag{8}$$

and the endogenous social weight $\alpha_0^U(c, y)$ of unemployed agents by

$$\alpha_0^U(c,y) \equiv \frac{1}{\bar{\alpha}(c,y)} \mathbb{E}_{(\omega,\delta)} \left[U'\left(V\left(c(\omega,\delta), 0, \omega, \delta\right)\right) | \omega \in \Omega, \delta \in \Delta : y(\omega,\delta) = 0 \right], \tag{9}$$

where $\bar{\alpha}(c,y) = \mathbb{E}_{(\omega,\delta)} \left[U'\left(V\left(c(\omega,\delta), y(\omega,\delta), \omega, \delta\right)\right)\right]$.

Economically, the social weight α_j measures the average welfare increase induced by a lump-sum transfer of a marginal unit to all workers with skill type ω_j, relative to the average welfare effect of a marginal lump-sum transfer to all agents in the

[4] In Section 8, I discuss the effects of Assumption *OFCD* and the robustness of the results in more detail.

economy, $\bar{\alpha}^U(c, y)$. Thus, the sequence of social weights measures the social planner's redistributive concerns.[5]

Desirability of Utilitarian Redistribution (DUR): *For every $\omega_j \in \Omega$, the following is true in every implementable allocation (c, y)*

$$0 < \alpha_{j+1}^U(c, y) < \alpha_j^U(c, y) < \alpha_0^U(c, y) \tag{10}$$

Condition *DUR* provides the rationale for optimal redistributive taxation. It implies that, if incentive considerations could be ignored, the social planner would unambiguously prefer redistributing resources from the workers within each skill group to each group of workers with lower skill type and to unemployed agents. It captures the same idea as condition *Desirability of Redistribution* in Hellwig (2007), which guarantees the optimality of positive marginal taxes in a model with labor supply responses at the intensive margin only.[6]

In the following, I distinguish between the *economy E* and the *social objective U* as two separate parts of the optimal tax problem. I refer to the economy E as the collection of the type space $\Omega \times \Delta$, the type distribution Ψ and the utility function V.

Definition 1. *Economy E is* regular *if and only if it satisfies assumptions* RC*,* SSC*,* AFC*,* QLC*,* REM*,* DSS*,* LC *and* OFCD*.*

For any regular economy, the set of feasible and incentive-compatible allocations is uniquely pinned down. In contrast, the normative ranking of the allocations in this set is enabled by the planner's objective, in particular by transformation U.

Definition 2. *For any regular economy E, the set of* utilitarian allocations $\mathcal{U}(E)$ *is given by all allocations that maximize some welfare function satisfying* DUR *over the set of feasible and incentive-compatible allocations.*

These definitions allow to rephrase the research question of this paper. In the following, I derive the properties of the income tax schedules that allow to decentralize utilitarian allocations. In particular, I shall show that some utilitarian allocations cannot be decentralized with positive marginal taxes.

[5]By construction, the average social weight over all subgroups in the population equals unity.

[6]Note, however, that *DUR* is slightly stronger as it is assumed to hold for all implementable allocations, while Hellwig (2007) assumes *Desirability of Redistribution* only for a subset of implementable allocations.

4 The optimal taxation problem

The optimal taxation problem is given by the problem of maximizing social welfare by designing an income tax schedule T that maps gross income levels into tax payments, and letting each agent choose labor supply to solve the problem of household utility maximization:

Household Problem. *Given individual type (ω, δ), maximize over $y \geq 0$ individual utility*

$$y - T(y) - h(y, \omega) - 1_{y>0}\delta \tag{11}$$

Denote by $y_T(\omega, \delta)$ the gross income solving this problem for an agent with type (ω, δ), and by Y_T the set of all income levels solving this problem for some type in $\Omega \times \Delta$. I shall be interested in two key properties of the optimal utilitarian tax schedule for all income levels $y \in Y_T$. The effects of the tax schedule on individual labor supply decision depend on two characteristics.

If the tax function T is continuously differentiable, the *marginal tax* $T'(y)$ is given by the derivative of T with respect to y. Under the imposed assumptions, every implementable allocation can indeed be decentralized through a continuously differentiable tax schedule.[7] As common in models with finite skill spaces, the set of implemented income levels Y_T in the optimal allocation will be finite. As a result, the optimal tax schedule might be increasing (or decreasing) over Y_T, even if the marginal tax is not positive (or negative) at any level y in Y_T.[8]

The *participation tax* $T^P(y) = T(y) - T(0)$ measures the increase in tax liabilities that an unemployed agent experiences if he enters the labor market and earns a gross income of y.[9] Depending on the sign of $T^P(y)$, the governmental budget constraint is constrained or relaxed if a positive mass of agents enter the labor market and earn gross income y.

[7] In the following, we thus assume that T is indeed continuously differentiable. For non-differentiable tax schedules, the implicit marginal tax $T_i'(y)$ can be defined for any consumption bundle $(y - T(y), y)$ with $y \in Y_T$. If this bundle is allocated to agents with skill type ω_j, the implicit marginal tax is given by one minus the marginal rate of substitution between output and consumption for this skill type, i.e., $T_i'(y) = 1 - h_1(y, \omega_j)$.

[8] Related to this issue, the marginal income tax is sometimes defined differently for models with discrete skill spaces. In particular, the marginal tax between two adjacent skill levels y_a and $y_b > y_a$ in Y_T can alternatively be defined as $T'(y_a, y_b) = \frac{[y_b - T(y_b)] - [y_a - T(y_a)]}{y_b - y_a}$ (see, e.g., Saez 2002). Defined this way, the marginal income tax does not convey information about the efficiency properties of implemented allocations. In contrast, the definition used here implies that the marginal tax is positive (negative) if and only if labor supply is upward distorted at the intensive margin.

[9] The term *participation tax* was first introduced by Choné & Laroque (2011). Referring to the same concept, Beaudry et al. (2009) use the term *employment tax/subsidy*.

Under the imposed assumptions, the taxation principle by Hammond (1979) and Guesnerie (1995) applies. Thus, the optimal tax problem is equivalent to the problem of maximizing the welfare function (3) subject to feasibility (1) and incentive compatibility (2). By standard arguments, the solution to this problem must be Pareto-efficient within the set of implementable, i.e., feasible and incentive-compatible, allocations.

Lemma 1. *Every implementable and Pareto-efficient allocation* $(c, y) : \Omega \times \Delta \to \mathbb{R}^2$ *can be characterized by two vectors* $(y_j)_{j=1}^n$, $(c_j)_{j=1}^n$ *and a scalar* $b \geq 0$ *such that*

- *within each skill group* $\omega_j \in \Omega$*, there is a threshold cost type* $\hat{\delta}_j \in \Delta$ *such that all*
- *all agents with types* (ω_j, δ) *such that* $\delta > \hat{\delta}_j = c_j - h(y_j, \omega_j) - b$ *provide zero output and enjoy the same consumption level* b*,*
- *all agents with skill type* ω_j *and cost type* $\delta \leq \hat{\delta}_j$ *provide the same output* y_j *and enjoy the same consumption level* c_j*.*

By Lemma 1, every implementable allocation involves pooling of all unemployed agents and of all workers of the same skill group. The social planner's problem is thus reduced to choosing a universal unemployment benefit b and a consumption-output bundle for each skill type $\omega_j \in \Omega$. This simplification directly results from the additive separability of the fixed cost δ, imposed by assumption *AFC*. In the appendix, I demonstrate that the existence of a universal unemployment benefit b and identical gross utilities $c_j - h(y_j, \omega_j)$ in each skill group follow directly from implementability. In a second step, the Pareto criterion implies that all workers of the same skill group enjoy the same consumption level c_j and provide the same output level y_j.

Another implication of assumption *AFC* is that the value of employment is monotonically decreasing in δ within each skill group ω_j, while the outside option of unemployment has the same value for all types. Thus, there is at most one *threshold cost type* $\hat{\delta}_j \in [0, \bar{\delta}]$ for each skill group such that an agent with type (ω_j, δ) weakly prefers labor market participation if and only if $\delta \leq \hat{\delta}_j$.

Consequently, the social planner's problem can be formally defined much simpler.

Lemma 2. *The social planner's problem is equivalent to maximizing the utilitarian welfare function*

$$\sum_{j=1}^{n} f_j \left\{ \int_{\underline{\delta}}^{\hat{\delta}_j} g_j(\delta) U \left[c_j - h(y_j, \omega_j) - \delta \right] d\delta + \left[1 - G_j \left(\hat{\delta}_j \right) \right] U[b] \right\} \tag{12}$$

over $y = (y_j)_{j=1}^n$, $c = (c_j)_{j=1}^n$, *subject to the constraints*

$$b = \sum_{j=1}^{n} f_j G_j \left(\hat{\delta}_j\right) \left[y_j - c_j + b\right], \tag{13}$$

$$\hat{\delta}_j = \max\left\{\underline{\delta}, \min\left\{c_j - h\left(y_j, \omega_j\right) - b, \bar{\delta}\right\}\right\} \qquad \forall\ \omega_j \in \Omega, \tag{14}$$

$$c_{j+1} - c_j \geq h\left(y_{j+1}, \omega_{j+1}\right) - h\left(y_j, \omega_{j+1}\right) \qquad \forall\ \omega_j \in \Omega, \tag{15}$$

$$c_{j+1} - c_j \leq h\left(y_{j+1}, \omega_j\right) - h\left(y_j, \omega_j\right) \qquad \forall\ \omega_j \in \Omega \tag{16}$$

Constraint (13) represents the feasibility constraint. The incentive compatibility constraints along the fixed cost dimension are given by (14), boiled down to a set of indifference condition for the threshold cost types $\left(\omega_j, \hat{\delta}_j\right)$ in all skill groups. As argued above, the threshold worker type $\hat{\delta}_j$ and the set of active workers are uniquely determined by c_j, y_j and b for each skill level. Finally, (15) and (16) represent the set of local downward and upward incentive compatibility constraints along the skill dimension. By the single-crossing property, local incentive compatibility between all adjacent skill pairs ensures global incentive compatibility within each skill group. Note that the problem stated above does not explicitly take into account incentive-compatibility constraints between types that differ both along the skill dimension and along the fixed cost dimension. Due to the additive separability of the fixed cost component AFC, piece-wise incentive compatibility along each dimension guarantees global incentive compatibility between all types (ω, δ) and (ω', δ') in $\Omega \times \Delta$.

In the interest of readability, but with some abuse of terminology, I will refer to constraint (14) as participation constraint, and to (15) and (16) as incentive compatibility (IC) constraints. The social objective U does not appear in any of the constraints. Thus, it has no effect on the set of implementable and Pareto-efficient allocations, a subset of which is given by the set of utilitarian allocations.

The IC constraints have the same immediate implications as in the intensive model by Mirrlees (1971). First, both IC constraints can only simultaneously be satisfied if output is monotonically increasing in the skill type, $y_{j+1} \geq y_j$. Second, the single crossing property implies that the following inequality is true whenever $y_{j+1} > y_j > 0$:

$$h\left(y_{j+1}, \omega_j\right) - h\left(y_j, \omega_j\right) > h\left(y_{j+1}, \omega_{j+1}\right) - h\left(y_j, \omega_{j+1}\right) > 0$$

Thus, as long as there is no pooling across skill types with $y_{j+1} = y_j$, high-skill workers must enjoy strictly higher consumption than low-skill workers, and at most one IC constraint can be binding with respect to each pair of adjacent skill levels. In the model with labor supply responses at the intensive and extensive margin, the downward IC constraint has a third implication that does not apply in models with only one margin. The threshold cost types for high-skill groups must be strictly higher than for low-skill groups, $\hat{\delta}_{j+1} > \hat{\delta}_j$, as long as $\hat{\delta}_j$ is below the upper bound $\bar{\delta}$. This property holds because high-skill workers enjoy higher utility than low-skill workers with the same fixed cost type, whether or not there is pooling.

As argued above, Lemma 2 implies that any implementable allocation involves pooling of all active workers with the same skill type, and of all unemployed agents. With other words, the social planner can only vary the allocations and utility levels of agents in these $n+1$ (or less) sets simultaneously. The desirability of all viable changes is thus entirely captured by the sequence of endogenous social weights α^U, which varies over the set of implementable allocations. Assumption DUR requires this sequence to be strictly decreasing for any implementable allocation.

In the following, I will be interested in the efficiency properties of optimal allocations. For this purpose, it is convenient to introduce as an auxiliary function the (gross) employment surplus

$$s(y,\omega) = y - h(y,\omega). \tag{17}$$

By assumption QLC, function $s(y,\omega_j)$ has a well-defined maximizer in $\mathbb{R}_+$, which I denote as $\hat{y}_j = \arg\max_y s(y,\omega_j)$ in the following. Furthermore, denote by $\hat{s}_j = s\left(\hat{y}_j,\omega_j\right)$ the maximum level of employment surplus for an agent with skill type ω_j. The single-crossing property SSC implies that $\hat{y}_{j+1} > \hat{y}_j$ and $\hat{s}_{j+1} > \hat{s}_j$ for all $\omega_j \in \Omega$.

For any type (ω,δ), the efficient labor supply $y^*(\omega,\delta,v)$ and the efficient consumption level $c^*(\omega,\delta,v)$ are given as the pair of output and consumption that requires the lowest transfer of net resources to provide an agent of this type with utility level v, i.e., solves the problem

$$\min_{y,c}(c-y) \text{ subject to } V(c,y,\omega,\delta) \geq v$$

Lemma 3. *For any v in the domain of V, efficient labor supply is given by*

$$y^*(\omega_j,\delta) = \begin{cases} \hat{y}_j & \textit{for} \quad \delta \leq \hat{s}_j \\ 0 & \textit{for} \quad \delta > \hat{s}_j \end{cases} \tag{18}$$

By the quasi-linearity of V, the required utility level v does not affect the level of efficient labor supply. Using Lemma 1, distortions in labor supply can be defined as follows.

Definition 3. *At the intensive margin, labor supply by workers of skill group ω_j is said to be undistorted if $y_j = \hat{y}_j$, downward distorted if $y_j \in (0, \hat{y}_j)$, and upward distorted if $y_j > \hat{y}_j$.*

Definition 4. *At the extensive margin, labor supply by workers of skill group ω_j is undistorted if $\hat{\delta}_j = \hat{s}_j$, downward distorted if $\hat{\delta}_j < \hat{s}_j$, and upward distorted if $\hat{\delta}_j > \hat{s}_j$.*

5 Results

The results of this paper are provided in the two following subsections. First, Subsection 5.1 provides the main results of this paper, which hold under the regularity assumptions imposed in Section 3. The subsection mainly provides existence results, including an "anything-goes result" with respect to the sign of the optimal utilitarian marginal tax.

Second, Subsection 5.2 provides sufficient conditions for the optimality of specific tax schedules, including an EITC-style tax schedule with negative marginal tax rates and negative participation tax rates. For this purpose, I impose further assumptions that allow me to focus on a smaller class of economies.

5.1 Main results

Proposition 1. *For every regular economy, labor supply by the workers of the highest skill group ω_n is undistorted at the intensive margin, and distorted downward at the extensive margin in any utilitarian allocation.*

Proposition 1 clarifies that the famous "no distortion at the top" result, a robust property of optimal tax schedules in intensive models á la Mirrlees (1971), continues to hold. However, it only applies to the intensive margin. At the extensive margin, labor supply of the highest skill group is strictly downwards distorted in any Utilitarian allocation.

Proposition 2. *For every regular economy, there is a utilitarian allocation in which labor supply of all skill groups is undistorted at the intensive margin, and labor supply of some skill groups is distorted upward at the extensive margin.*

Proposition 3. *For some but not all regular economies, there is a utilitarian allocation in which labor supply is distorted downward at the intensive margin everywhere below the top, and distorted downward at the extensive margin everywhere.*

Proposition 4. *For some but not all regular economies, there is a utilitarian allocation in which labor supply of at least one skill group is distorted upward at both margins, and undistorted at the intensive margin for all other skill groups.*

Propositions 2 to 4 establish the indeterminacy of optimal marginal taxes in utilitarian redistribution programs. Propositions 2 and 3 cover extreme cases in which labor supply is either downward distorted at the intensive margin everywhere below the top, or undistorted throughout. Of course, there are also utilitarian allocations in which labor supply is downward distorted for some, and undistorted for other skill groups at this margin. Proposition 4 confirms the potential optimality of EITC-style tax-transfer schemes with upward distortions at both margins for some skill groups for some economies that satisfy the imposed regularity conditions. More precisely, it establishes the potential optimality of an extreme version of the EITC, in which labor supply is weakly upward distorted at the intensive margin for all skill groups. This result sharply contrasts with the unambiguous positivity of optimal marginal taxes in the intensive model (see, e.g., Mirrlees 1971 and Hellwig 2007.

The proofs of Propositions 1 to 4 are based on the analysis of a relaxed problem in which the incentive compatibility constraints between workers of different skill groups are not taken into account. In the solution to this relaxed problem, labor supply is generally undistorted at the intensive margin, because the social planner has no interest in slackening any IC constraints. In contrast to the intensive model, the solution to this relaxed problem satisfies any pair of IC constraints between skill levels ω_j and ω_{j+1} if the utilitarian welfare function is only mildly concave in the relevant range. For transformations U with sufficiently small second derivative $|U''|$, the solution to the relaxed problem actually also solves the full problem, including the complete set of IC constraints.

By Propositions 3 and 4, utilitarian allocations with downward or upward distorted labor supply at the intensive margin do not exist for all regular economies. Rather, the existence of both the "standard" case with downward distortions and of the "non-standard" case with upward distortions depend on details of the economic environment, in particular, on properties of the type set and type distribution. In the following subsection, I take a closer look at this issue by considering a class of economies with certain functional forms. Within this class of economies, I then provide sufficient conditions on the economic primitives–the type space $\Omega \times \Delta$, the joint type distribution Ψ, and the effort cost function h–under which utilitarian allocations with especially interesting properties exist.

5.2 Sufficient conditions

By Propositions 3 and 4, utilitarian allocations with labor supply distortions at the intensive margin exist for some, but not all regular economies. First, this is true for the standard constellation with downward distortions at the intensive margin everywhere below the top. Second, this also holds for extreme versions of EITC-style allocations with upward distortions at the intensive margin for some skill groups and no distortions for all other skill groups.

In this subsection, I provide sufficient conditions for the existence of utilitarian allocations with the discussed properties. For this purpose, I focus on a class of economies defined by the following assumption.

Assumption 1. *The economy satisfies the following conditions:*

(i) The effort cost function is given by $h(y,\omega) = \frac{1}{2}\,\frac{y^2}{\omega}$*,*

(ii) the skill space is given by $\Omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$ *with constant relative distances* $\frac{\omega_{j+1}}{\omega_j} = a > 1$*, and*

(iii) in each skill group $\omega_j \in \Omega$*, fixed costs are uniformly distributed on the interval* $\left[0, \bar{\delta}\right]$*, with* $\bar{\delta} > \frac{\omega_n}{2}$*.*

By assumption 1, we focus on a class of economies with simple functional forms that enable relatively simple expressions for the sufficient conditions in the remainder of this subsection. This includes the quadratic effort cost function, the constant relative distances between all adjacent skill levels, and the uniform distribution of fixed costs. The lower bound on $\bar{\delta}$ guarantees that agents with maximum skill and maximum fixed cost do not work under laissez-faire, as required by assumption *REM*. Note that assumption 1 also restricts attention to the benchmark case in which skills and fixed costs are independently distributed.

First, I provide necessary and sufficient conditions for the existence of a utilitarian allocation with standard properties, i.e., downward distortions at the intensive margin everywhere below the top.

Proposition 5. *If assumption 1 holds and* $a < 2$*, there is a utilitarian allocation in which labor supply is downward distorted at the intensive margin everywhere below the top, and downward distorted at the extensive margin everywhere.*

Proposition 6. *If assumption 1 holds,* $n = 2$ *and* $f_1 > \frac{1}{2}$*, there is a threshold* $\hat{a}(f_1) \in \left(2 + \sqrt{2}, \infty\right)$ *such that, if* $a > \hat{a}(f_1)$*, labor supply by workers of both skill groups is undistorted at the intensive margin in every utilitarian allocation.*

Note that the last result also extends to the Rawlsian (Maximin) welfare function. This is in contrast to the results by Jacquet et al. (2013) for a model with continuous skill space, according to which the Rawlsian allocation always involves downwards distortions at the intensive margin. The difference results only due to the assumed skill space with only two skill types, while all other assumption are nested in the model of Jacquet et al. (2013).[10]

Assumption 2. *The cardinality of the skill space is large enough to satisfy* $n > \inf\{z \in \mathbb{N} : z > 2 + \ln(a+1)/\ln(a)\}$. *The upper bound of the fixed cost space satisfies* $\bar{\delta} < \frac{\gamma_0 - \gamma_n}{\gamma_0 - 1} \frac{\omega_n}{2(2-\gamma_n)}$, *where* $\gamma_0 = 2 - \frac{1}{a}$ *and* $\gamma_n = 2 - \frac{a}{1+a^{2-n}(a^2-1)}$.

Assumption 3. *The share of agents with top skill level* ω_n *is high enough to satisfy*

$$f_n > \frac{\bar{\gamma}_{-n} - 1}{\bar{\gamma}_{-n} - \bar{\gamma}_n} \in [0, 1)$$

where $\bar{\gamma}_{-n} = \frac{\sum_1^{n-1} f_j \bar{\gamma}_j}{1-f_n}$ *and* $\bar{\gamma}_j = \gamma_0 - \frac{\omega_j}{2(2-\gamma_j)}(\gamma_0 - \gamma_j)$ *with* $\gamma_0 = \gamma_1 = \gamma_2 = 2 - \frac{1}{a}$ *and* $\gamma_j = 2 - \frac{a}{1+a^{2-j}(a^2-1)}$ *for* $j \geq 3$.

Assumption 2 excludes cases with particularly limited type heterogeneity. First, it requires sufficient heterogeneity in skills, depending on relative distance between two adjacent skill levels, $a = \frac{\omega_{j+1}}{\omega_j}$. Second, it imposes an upper limit on $\bar{\delta}$, so that a majority of agents with top skill ω_n participate on the labor market in the optimal allocations identified below.

Assumption 3 requires that the share of high-skill workers is sufficiently large. The exact level of the threshold share for f_n depends on the complete set of parameters, including the share f_j for all lower skill levels. However, it can be shown that this threshold is always below 1, and may even be negative. An increasing cardinality of the skill space, as measured by n, makes the assumption less demanding. Intuitively, assumption 3 seems more restrictive than Assumptions 1 and 2.

Proposition 7. *If assumptions 1, 2 and 3 hold, there is a utilitarian allocation in which labor supply by skill type* ω_2 *is upward distorted at both margins, and labor supply by all other agents is undistorted at the intensive margin.*

The economic mechanism behind this result is studied in more detail in the following section.

[10]More precisely, maximizing the Rawlsian welfare function involves undistorted labor supply at the intensive margin under even slightly weaker conditions than those imposed in 6. In particular, labor supply by all workers is undistorted at the intensive margin in the Rawlsian allocation if $a > 2$ and assumption 1 holds.

6 Intuition: The tradeoff between intensive and extensive efficiency

Propositions 2 to 4 imply that a utilitarian desire for redistribution does not pin down the direction of labor supply distortions at any margin, in contrast to the classical result in the Mirrlees (1971) model. This section aims at developing an economic intuition for the indeterminate sign of the optimal marginal tax, and its interdependence with the optimal participation tax.[11] First, I show that an elasticity rule helps to identify the optimal tax schedule. Second, I explain that, and why, labor supply responses at two margins can give rise to a tradeoff between intensive efficiency and extensive efficiency, which drives the indeterminacy of labor supply distortions. To work out this economic intuition, this section studies a simple auxiliary problem in which redistributive concerns are eliminated.

Consider an economy with only two skill levels, ω_1 and $\omega_2 > \omega_1$. The mass of low-skill agents is given by $f_1 > 0$, the mass of high-skill agents by f_2. In the social planner's objective function, let transformation U be given by the identity function. In contrast to assumption DUR, the social planner is thus interested in maximizing social surplus, i.e., the unweighted sum of individual utilities. Assume moreover that the planner is restricted by incentive compatibility and two additional constraints.[12] First, the allocation must satisfy a (positive or negative) exogenous revenue requirement A:

$$\sum_{j=1}^{2} f_j \int_{\underline{\delta}}^{\bar{\delta}} [y(\omega_j, \delta) - c(\omega_j, \delta)]\, dG_j(\delta) \geq A \tag{19}$$

Second, every unemployed agent with $y(\omega_j, \delta) = 0$ must receive an exogenously determined benefit b.[13]

As Lemma 1 applies, the social planner only has to consider allocations in which all workers with the same skill type ω_j provide identical output y_j and receive identical consumption $c_j = y_j - T_j^P + b$. Using the definition of the employment

[11] So far, the literature has only studied the potential optimality of upward distortions at the extensive margin (Diamond 1980, Saez 2002, Choné & Laroque 2011, Christiansen 2012).

[12] Except DUR, all assumptions imposed in Section 3 are taken to hold.

[13] In the following sense, the auxiliary problem can be interpreted as a part of the larger problem of optimal tax problem, rewritten as a two-step problem. In the first step, the social planner chooses (a) the amount of net resources A to be transferred from the group of high-skill agents with $\omega_j > \omega_2$ to the group of low-skill agents with skill types ω_1 and ω_2, and (b) the universal benefit to each unemployed agent b. In the second step, the planner decides how to redistribute resources within the low-skill and high-skill groups given A and b, subject to incentive-compatibility. This section focuses on the optimal amount of redistribution within the low-skill group only, and considers the benchmark case without equity concerns, i.e., $U'' \to 0$ on the relevant interval.

surplus $s(y_j, \omega_j) = y_j - h(y_j, \omega_j)$, the problem can be formally written as follows:

Auxiliary Problem. *Maximize over y_1, y_2, T_1^P and T_2^P social surplus*

$$\sum_{j=1}^{2} f_j \left[\int_0^{\hat{\delta}_j} \left(s\left(y_j, \omega_j\right) - T_j^P + b - \delta \right) dG_j(\delta) + \left[1 - G_j\left(\hat{\delta}_j\right) \right] b \right] \tag{20}$$

subject to the constraints

$$\tilde{A} = A + (f_1 + f_2)\, b \leq \sum_{j=1}^{2} f_j G_j\left(\hat{\delta}_j\right) T_j^P, \tag{21}$$

$$\hat{\delta}_j = s(y_j, \omega_j) - T_j^P \text{ for } j \in \{1, 2\}, \tag{22}$$

$$s\left(y_2, \omega_2\right) - s\left(y_1, \omega_2\right) \geq T_2^P - T_1^P, \text{ and} \tag{23}$$

$$s\left(y_2, \omega_1\right) - s\left(y_1, \omega_1\right) \leq T_2^P - T_1^P \tag{24}$$

Besides the existence of only two skill groups, there are two differences to the problem of optimal taxation defined above. First, the concave transformation U in the objective function is replaced by the identity function, which eliminates any redistributive concerns. Second, the feasibility constraint (21) contains the exogenous revenue requirement A. Participation constraints (22) and incentive compatibility constraints (23), (24) are given as before. To avoid irrelevant complications, I assume here that $\bar{\delta}$ is large enough to exceed $\hat{\delta}_j$ in every implementable allocation. Finally, recall that the unemployment benefit b is exogenously determined in the auxiliary problem.

The formal analysis of this auxiliary problem is presented in Subsection 6.1, while Subsection 6.2 illustrates the auxiliary problem and its solution graphically.

6.1 Formal analysis of the auxiliary problem

In the following, I refer to the solution of this problem, $\left(y_1^S, y_2^S, T_1^{P^S}, T_2^{P^S}\right)$, as the surplus-maximizing allocation. Lemmas 4 to 6 below imply that the level of the adjusted revenue requirement $\tilde{A} = A + (f_1 + f_2)\, b$ determines important properties of this solution, including the direction of labor supply distortions at both margins.

Lemma 4. *There are levels $A_{max} > 0$ and $A_{min} < 0$ such that*

(a) the auxiliary problem has a solution in $\mathbb{R}^4$ if and only if $\tilde{A} \leq A_{max}$, and

(b) *this solution involves threshold worker types $\hat{\delta}_j < \bar{\delta}$ for $j \in \{1, 2\}$ if and only if $\tilde{A}\,(A_{min}, A_{max}]$.*

On the one hand, the existence of unemployment as an outside option implies that the social planner can collect at most a tax revenue of A_{max}, which is realized if both skill groups are taxed at (incentive-compatible) Laffer rates. On the other hand, the auxiliary problem has a solution for any negative revenue requirement $\tilde{A} < 0$. For very negative levels of $\tilde{A}$, however, all agents of the high-skill group (or even of both groups) enter the labor market and the participation constraints are not binding anymore, i.e., labor supply becomes completely inelastic at the extensive margin. In the following, we focus on levels of the revenue requirement in the interval $[A_{min}, A_{max}]$.

Lemma 5. *For all $\tilde{A} \in [A_{min}, A_{max}]$, surplus maximization involves higher output by high-skill workers than by low-skill workers, $y_2^S > y_1^S$.*

(i) *If $\tilde{A} \in [A_{min}, 0)$, high-skill workers receive higher participation subsidies than low-skill workers, $T_2^{P^S} < T_1^{P^S} < 0$.*

(ii) *If $\tilde{A} \in (0, A_{max}]$, high-skill workers pay higher participation taxes than low-skill workers, $T_2^{P^S} > T_1^{P^S} > 0$.*

Lemma 6. *Let fixed costs in both skill groups be distributed uniformly on the interval $[0, \bar{\delta}]$, with $\bar{\delta}$ sufficiently large. There are values $A_U \in (A_{min}, 0)$ and $A_D \in (0, A_{max}]$ such that, in the surplus-maximizing allocation,*

- *high-skill labor is upward distorted at the intensive margin if $\tilde{A} \in [A_{min}, A_U)$, and*
- *low-skill labor is downward distorted at the intensive margin if and only if $\tilde{A} \in (A_D, A_{max})$.*

Thus, the relevant properties of surplus-maximizing participation taxes depend on the level of the revenue requirement $\tilde{A}$. Lemma 5 implies that optimal participation taxes for both skill groups are non-negative, inducing labor supply distortions at the extensive margin, whenever $\tilde{A}$ differs from zero. Moreover, high-skill workers always face either higher participation taxes or higher participation subsidies than low-skill workers. Lemma 6 focuses on the special case where fixed costs are distributed uniformly and identically across skill groups. For this case, the surplus-maximizing allocation also involves labor supply distortions at the intensive margin if the revenue requirement $\tilde{A}$ differs sufficiently from zero.

It will become clear below that these distortions at the intensive margin are optimal due to a tradeoff between two aspects of efficiency, in the following labeled *intensive efficiency* and *extensive efficiency*. I will then show that this tradeoff between *intensive efficiency* and *extensive efficiency* is the basis of the indeterminate sign of the optimal marginal tax established in Propositions 2, 3 and 4. In the remainder of this section, I focus on the case of a negative requirement $\tilde{A}$, for which surplus can be maximized through an EITC-style tax schedule inducing upward distortions in labor supply at both margins.[14]

The intuition behind both lemmas can be explained using an adapted version of the inverse elasticity (Ramsey) rule for optimal commodity taxation. Consider first a relaxed version of the auxiliary problem in which both IC constraints are ignored. For clarity, we denote by $\left(\tilde{y}_1, \tilde{T}_1^P, \tilde{y}_2, \tilde{T}_2^P\right)$ the relaxed problem's solution in the following. Without IC constraints, the social planner then treats high-skill and low-skill labor just as two separate varieties of labor, or two distinct tax bases. As there is no need to slacken an incentive constraint, optimal labor supply is undistorted at the intensive margin, $\tilde{y}_j = \hat{y}_j = \arg\max_y s(y, \omega_j)$, and the employment surplus equals its efficient level $\hat{s}_j = \max_y s(y, \omega_j)$. In the relaxed problem, *intensive efficiency* is consequently ensured. Thus, the social planner only needs to care about maximizing *extensive efficiency*, i.e., minimizing labor supply distortions at the extensive margin.

The mathematical structure of the relaxed auxiliary problem coincides with the classical Ramsey problem.[15] Consequently, the optimal pattern of taxes follows the familiar elasticity logic, according to which higher taxes or subsidies should be set for less elastic tax bases and vice versa. Formally, optimal participation taxes for both skill groups are characterized by the following version of the inverse elasticity rule

$$\tilde{T}_j^P = \frac{\lambda - 1}{\lambda} \frac{G_j(\hat{s}_j - T_j^P)}{g_j(\hat{s}_j - T_j^P)} \text{ for } j \in \{1, 2\} \tag{25}$$

This condition relates the optimal participation tax liability for each skill group to

[14]The case of a negative $\tilde{A}$ is plausible, e.g., if the economy is additionally populated by workers with higher skill types $\omega_j > \omega_2$, from which the utilitarian planner redistributed resources towards workers with the lowest skill levels ω_1 and ω_2. More precisely, $\tilde{A}$ is negative if and only if the social planner prefers negative participation taxes, i.e., higher transfers to be paid to the working poor than to the unemployed. As Proposition 2 implies, there is always a well-behaved utilitarian welfare function such that negative participation taxes to the lowest skill levels are indeed optimal.

[15]A minor difference between the commodity tax and the labor tax setting is given by the elasticities of demand and supply functions. The assumptions on primitives taken here imply that labor demand is completely elastic, while labor supply is upward sloping for $r_j \in \left[0, \bar{\delta}\right]$ and completely inelastic otherwise.

the semi-elasticity of its labor market participation, $\frac{g_j(\hat{\delta}_j)}{G_j(\hat{\delta}_j)} = \frac{\partial G_j(\hat{\delta}_j)/\partial\left(y_j - T_j^P\right)}{G_j(\hat{\delta}_j)}$.[16]

The intuition behind this rule rests on the social planner's desire to reduce distortions in labor supply as much as possible. Participation taxes differing from zero induce extensive margin responses, giving rise to labor supply distortions as agents enter (leave) the labor market which would stay unemployed (employed) in the first-best allocation. Thus, the surplus-maximizing social planner seeks to keep both participation rates T_1^P and T_2^P as close as possible to zero. This requires the optimal participation taxes for both skill groups to have the same sign, positive for $\tilde{A} > 0$ and negative for $\tilde{A} < 0$.[17] Otherwise, both participation taxes could be decreased in absolute terms, thereby also reducing labor supply distortions.

Moreover, the higher the semi-elasticity of participation for skill group ω_j, the larger is the extensive margin response induced by distributing an additional unit of resources to workers of this skill group. Consequently, it is optimal to set higher participation taxes (or higher subsidies) for the less responsive skill group.

Crucially, the relative sizes of these semi-elasticities are unambiguously pinned down by the imposed assumptions *LC* and *OFCD* on the type distribution. Recall that, in skill group ω_j, only workers with fixed cost types below some threshold $\hat{\delta}_j$ enter the labor market. As usually, incentive-compatibility implies that a high-skill worker enjoys a higher utility than a low-skill worker with the same fixed cost type δ in every implementable allocation. In contrast, the outside option of unemployment has the same value for all agents. Thus, agents in the high-skill group enter the labor market even with higher fixed costs than agents of the low-skill group, implying a higher cost threshold $\hat{\delta}_2 > \hat{\delta}_1$. The assumption of log-concavity implies that an increase in the threshold cost type $\hat{\delta}_j$ decreases the semi-elasticity $\frac{g_j(\hat{\delta}_j)}{G_j(\hat{\delta}_j)}$. Thus, the semi-elasticity of high-skill labor is smaller than the semi-elasticity of low-skill labor if skills and fixed cost are independently distributed, $G_1 = G_2$. Assumption *OFCD* also allows for some correlation, as long as the hazard rate for high-skill workers is larger, $\frac{g_2(\delta)}{G_2(\delta)} < \frac{g_1(\delta)}{G_1(\delta)}$ for all $\delta \in \Delta$. In this case, the difference between the semi-elasticities of high-skill participation and low-skill participation is even larger than in the case of independence.

By the inverse elasticity rule, the optimal ratio of high-skill to low-skill participation taxes thus exceeds unity in the solution to the relaxed problem whenever the

[16]The semi-elasticity of participation measures the percentage increase in the participation rate that results if the net-of-tax labor income increases by one unit (instead of one percent as with the standard elasticity). In the framework of optimal commodity taxation, the inverse elasticity rule is usually expressed in terms of the standard elastity $\epsilon_j^P = \frac{y_j - T_j^P}{G_j(\hat{\delta}_j)} \frac{\partial G_j(\hat{\delta}_j)}{\partial\left(y_j - t_j^P\right)}$.

[17]Note that, for positive (negative) $\tilde{A}$, the Lagrange multiplier λ attains a level above (below) unity.

revenue requirement differs from zero:

$$\frac{\tilde{T}_2^P}{\tilde{T}_1^P} = \frac{G_2(\hat{s}_2 - T_2^P)/g_2(\hat{s}_2 - T_2^P)}{G_1(\hat{s}_1 - T_1^P)/g_1(\hat{s}_1 - T_1^P)} > 1 \text{ for all } \tilde{A} \neq 0 \tag{26}$$

For any negative revenue $\tilde{A}$, equation 26 implies that *extensive efficiency* is maximized by setting a strictly higher participation subsidy for high-skill workers than for low-skill workers: $\tilde{T}_2^P < \tilde{T}_1^P < 0$.

This implies that the solution to the relaxed problem satisfies the downward IC constraint (24). Whether it also satisfies the upward IC constraint (23), however, is in general unclear. If this is indeed true, the relaxed problem's solution represents the surplus-maximizing allocation. Thus, it is possible to maximize extensive efficiency and intensive efficiency at the same time, and surplus-maximization does not give rise to labor supply distortions at the intensive margin.

If the relaxed problem's solution instead violates the upward IC constraint, a tradeoff between intensive efficiency and extensive efficiency arises. To maximize extensive efficiency according to the inverse elasticity rule, the social planner would like to redistribute more resources to the high-skill workers than compatible with the upward IC constraint. The upward IC constraint will consequently be binding. Moreover, the social planner can only increase extensive efficiency if he slackens this constraint by distorting labor supply of high-skill workers upwards. As this initially involves only negligible losses in intensive efficiency, surplus maximization gives rise to strict upward distortions in high-skill labor supply at the intensive margin, and strictly negative marginal taxes.[18]

The sign of the surplus-maximizing marginal tax thus depends on whether the relaxed problem's solution satisfies or violates the upward IC constraint. Without further assumptions on the revenue requirement $\tilde{A}$ and the properties of the fixed cost distributions G_1 and G_2, this can not be determined though.

Lemma 6 considers the simple case of identical uniform distributions of fixed

[18] The surplus-maximizing allocation then exactly balances marginal gains in extensive efficiency and marginal losses in intensive efficiency. Formally, this intuition can be captured by a generalized version of the inverse elasticity rule. For any $\tilde{A} \in [A_{min}, A_{max}]$, the surplus-maximizing participation tax $T_j^{P^S}$ for $j \in \{1, 2\}$ is characterized by

$$T_j^{P^S} = \left[\frac{\lambda - 1}{\lambda} + \frac{s_1(y_j, \omega_j)}{s_1(y_j, \omega_k) - s_1(y_j, \omega_j)}\right] \frac{G_j\left(\hat{\delta}_j\right)}{g_j\left(\hat{\delta}_j\right)} - \frac{f_k}{f_j} \frac{s_1(y_k, \omega_k)}{s_1(y_j, \omega_j) - s_1(y_k, \omega_k)} \frac{G_k\left(\hat{\delta}_k\right)}{g_j\left(\hat{\delta}_j\right)},$$

where $k \neq j$ refers to the other skill group, $\hat{\delta}_j = s(y_j, \omega_j) - T_j^P$ denotes the threshold worker type in skill group ω_j, and λ denotes the Lagrange multiplier associated with the planner's budget constraint (21). Note that $s_1(y_j, \omega_j) = 0$ if and only if $y_j = \hat{y}_j$, i.e., labor supply by workers of skill group ω_j is undistorted at the intensive margin.

costs in both skill groups. For this case, the solution to the relaxed problem violates upward incentive compatibility if the revenue requirement $\tilde{A}$ is below some threshold $A_U < 0$.[19] As argued above, the surplus-maximizing allocation consequently involves a binding upward IC and upward distorted labor supply at the intensive margin, $y_2 > \hat{y}_2$, in this case.

6.2 Graphical illustration of the auxiliary problem

Figures 6.2 and 6.2 on the following pages illustrate the tradeoff between intensive efficiency and extensive efficiency graphically for some negative revenue requirement $\tilde{A}$.

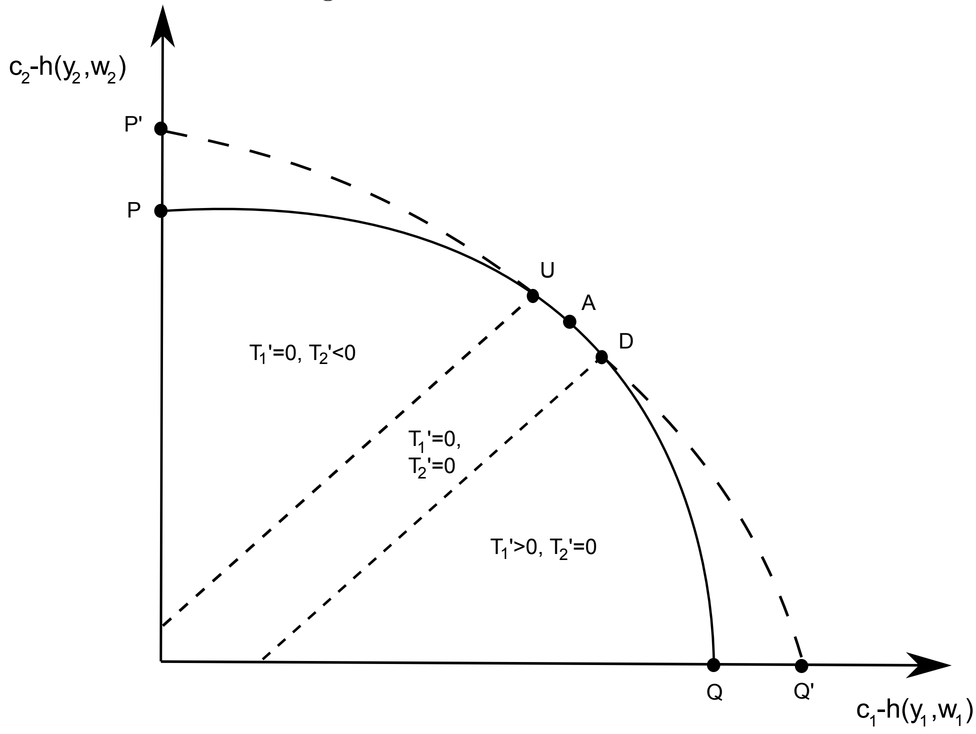
Figure 6.2 depicts the Pareto frontiers for the relaxed and the non-relaxed versions of the auxiliary problem. More precisely, it plots the gross utility levels $\tilde{V}_j \equiv \tilde{V}(c_j, y_j, \omega_j) = c_j - h(\hat{y}_1, \omega_1)$ of low-skill workers and high-skill workers corresponding to all (second-best) Pareto efficient allocations $\left(y_1, y_2, T_1^P, T_2^P\right)$. Recall that the utility level of a worker with type (ω_j, δ) is given by $V(c_j, y_j, \omega_j, \delta) = \tilde{V}_j - \delta$, so that $\tilde{V}_j$ represents the common element for all workers with the same skill type. In Figure 6.2, the gross utility $\tilde{V}_1$ of low-skill workers is depicted on the horizontal axis, while the gross utility $\tilde{V}_2$ of high-skill workers is on the vertical axis.

The dashed line $P'Q'$ represents the Pareto-frontier for the relaxed auxiliary problem, in which the social planner is not restricted by IC constraints. Moving this line down and to the right corresponds to reductions in the low-skill participation tax T_1^P, financed by an increasing level of the high-skill participation tax T_2^P. In the relaxed problem, these tax changes induce labor supply responses at the extensive margin, pulling some unemployed low-skill agents into employment and forcing high-skill workers out of the labor market. As the IC constraints can be ignored, labor supply by both skill groups is undistorted at the intensive margin in the allocation corresponding to all points on the dashed line $P'Q'$. Nevertheless, the Pareto frontier for the relaxed problem is strictly concave due to the extensive margin responses.

The solid line PQ represents the Pareto frontier for the non-relaxed auxiliary problem, which encloses the set of implementable allocations. Between points U and D, it coincides with the relaxed problem's Pareto frontier $P'Q'$. In the allocations corresponding to this interval, the participation taxes T_1^P and T_2^P are sufficiently close to each other to satisfy both IC constraints (23) and (24) even without distortions at the intensive margin. This necessarily includes point A, in

[19]The same result holds if G_2 first-order stochastically dominates G_1, i.e., if high-skill agents have overall lower fixed costs than low-skill agents. In this case, we find threshold values A'_U and A'_D that are even closer to zero, implying a larger propensity of intensive margin distortions in both directions.

Figure 1: The Pareto frontier



The figure shows the Pareto frontier for the auxiliary problem (solid line PQ) and the relaxed auxiliary problem (dashed line $P'Q'$). Horizontal axis: gross utility of low-skill workers, $c_1 - h(y_1, \omega_1)$. Vertical axis: gross utility of high-skill workers, $c_2 - h(y_2, \omega_2)$.

which both participation taxes are identical $T_1^P = T_2^P$. The social planner can implement the allocations in this interval without distorting labor supply at the intensive margin. More generally, all combinations of $\tilde{V}_1$ and $\tilde{V}_2$ between both dashed lines can be implemented without intensive margin distortions, i.e., with marginal taxes $T_1' = T_2' = 0$.

To the left of point U, the solid Pareto frontier PQ for the non-relaxed problem is below the dashed line $P'Q'$. In this region, the gross utility $\tilde{V}_2$ of high-skill workers is so much higher than $\tilde{V}_1$ that the upward IC constraint would be violated without intensive margin distortions. Thus, all points on the solid Pareto frontier left of U correspond to allocations with a binding upward IC constraint, and upwards distorted high-skill labor y_2 at the intensive margin. These allocations can only be implemented with EITC-style tax schemes, involving negative marginal taxes T_2' for high-skill workers. Note also that movements along the Pareto frontier PQ thus involve labor supply responses at the intensive and the extensive margin. Moving up from point U, the upward IC constraint is tightened more and more, and can only be restored through stronger upwards distortions in y_2.

Symmetrically, the solid Pareto frontier PQ is below the dashed line $P'Q'$ to the right of point D, where the downward IC becomes binding. In the allocations below D and the lower dashed line, positive marginal taxes $T_1' > 0$ for low-skill worker induce downwards distortions in y_1, which are necessary to satisfy the downward IC constraint. Altogether, Figure 6.2 allows to distinguish three parts of the Pareto frontier with respect to the marginal effects on intensive efficiency. If agents would adjust their labor supply only at the intensive margin, surplus would be maximized in every point between U and D. But this is only one aspect of efficiency if labor supply also respond at the extensive margin.

Figure 6.2 allows to take into account extensive efficiency aspects as well. The dotted line EF depicts the locus of allocations maximizing extensive efficiency, i.e., satisfying the inverse elasticity condition[20]

$$\frac{T_2^P}{T_1^P} = \frac{G_2\left(s\left(y_2,\omega_2\right) - T_2^P\right)/g_2\left(s\left(y_2,\omega_2\right) - T_2^P\right)}{G_1\left(s\left(y_1,\omega_1\right) - T_1^P\right)/g_1\left(s\left(y_1,\omega_1\right) - T_1^P\right)} > 1 \text{ for all } \tilde{A} \neq 0.$$

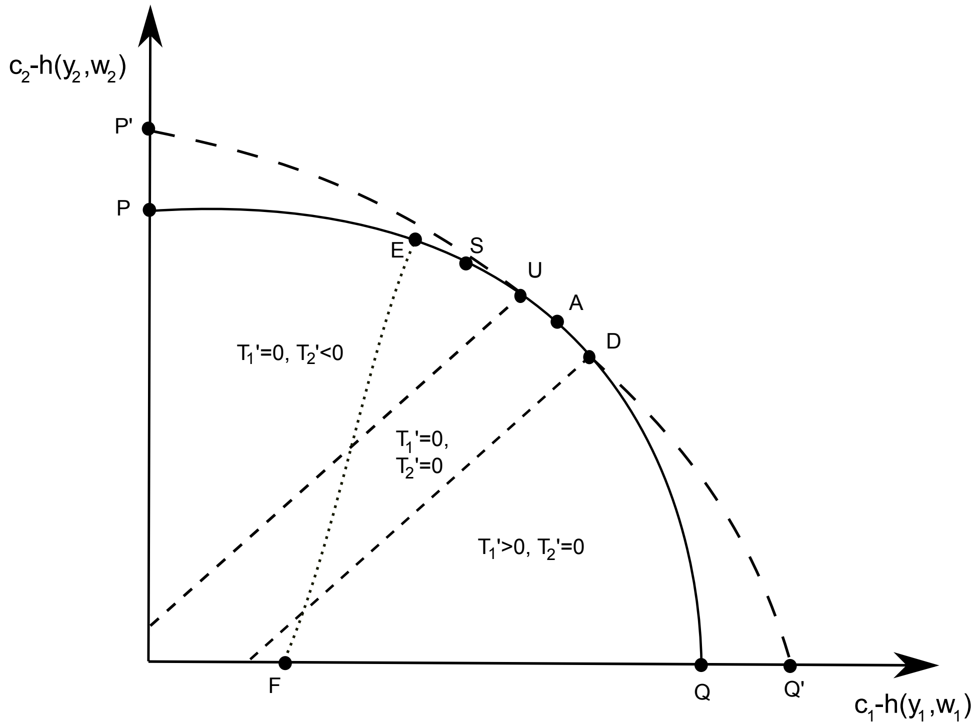
The intersection of this line with the Pareto frontier PQ is given by point E, which would be optimal if movements along the Pareto frontier would only induce labor supply responses at the extensive margin, but not at the intensive margin. Thus, the dotted line EF allows to distinguish two parts of the Pareto frontier with respect

[20]Note that, for the non-relaxed problem, this condition does not necessarily involve the first-best workload levels $\hat{y}_1$ and $\hat{y}_2$. In contrast, it involves the levels of $y_1 \leq \hat{y}_1$ and $\hat{y}_2 \geq y_2$ that are required to ensure incentive-compatibility for the corresponding allocations.

to the marginal effects on extensive margin. In particular, extensive efficiency is increased by movements down the Pareto frontier in the region left of point E, and decreased in the region right of E.

The properties of the surplus-maximizing allocations thus depend on the location of point E, the intersection of this line with the Pareto frontier PQ. By Lemma 5, E must be located above the uniform-taxation point A for any negative revenue requirement $\tilde{A}$. Depending on the exact level of $\tilde{A}$ and the properties of G_1 and G_2, it may either lie to the left or to the right of point U, where the upward IC constraint becomes binding.

Figure 2: The surplus-maximizing allocation



The figure shows the allocations maximizing social surplus (S), extensive efficiency (E) and intensive efficiency (between U and D). Horizontal axis: gross utility of low-skill workers, $c_1 - h(y_1, \omega_1)$. Vertical axis: gross utility of high-skill workers, $c_2 - h(y_2, \omega_2)$.

Figure 6.2 illustrates the case in which this intersection is located to the left of U. Lemma 6 implies that this case indeed occurs under reasonable assumptions. For this case, a tradeoff between *intensive efficiency* and *extensive efficiency* arises between points E and U on the Pareto frontier. E maximizes extensive efficiency, but requires upward distortions in high-skill labor supply at the intensive margin. In contrast, intensive efficiency is maximized at point U, which does not satisfy the inverse elasticity condition.

Starting from U and moving the Pareto frontier up towards E initially induces first-order gains in extensive efficiency, but only second-order losses in intensive efficiency. Starting instead from E and moving the Pareto frontier down towards U initially induces first-order gains in intensive efficiency, but only second-order losses in extensive efficiency. Consequently, the surplus-maximizing allocation must be located at some point S in the interior of this region, balancing marginal gains in intensive efficiency and marginal losses in extensive efficiency (see Figure 6.2).

Finally, the set of utilitarian allocations is given by the collection of all points on the Pareto frontier between points S and Q in Figure 6.2. By assumption DUR, the social planner would prefer to redistribute resources from high-skill workers to low-skill workers if he were not restricted by incentive considerations. Thus, any movement down the Pareto frontier induces a strict equity gain. At any point to the right of the surplus-maximizing allocation S, however, it also involves a loss in overall efficiency (combining intensive and extensive aspects). Thus, each point on the Pareto frontier below S corresponds to a utilitarian allocation. With respect to the intensive margin, this set contains allocations with upwards distortions in y_2 (between S and U), without distortions (between U and D) and with downward distortions in y_1 (between D and Q).

This clarifies that, and why, the *existence* of a utilitarian desire for redistribution does not pin down the direction of intensive margin distortions, nor the sign of the optimal marginal income tax. In cases as the one illustrated in Figure 6.2, this optimal sign instead depends on the *intensity* of the planner's local redistributive concerns. With a strong desire for redistribution between adjacent skill groups, he will typically prefer tax schedules with positive marginal taxes, implementing allocations in the lower right corner (between D and Q). If he instead values additional resources in the hands of workers of both skill groups almost equally, in contrast, an EITC-style tax scheme with negative marginal taxes is optimal, implementing an allocation between (S and U).[21]

In the latter case, the optimal upward distortion in labor supply cannot be understood in terms of the classical tradeoff between equity and intensive efficiency. Above U, moving down the Pareto frontier instead induces gains both in equity and intensive efficiency, which are counteracted by losses in extensive efficiency. Thus, the potential optimality of upward distortions at the intensive margin is not driven,

[21]Proposition 6 however implies that for some regular economies, labor supply is undistorted in all utilitarian allocations. In these cases, the distance between points U and D is so large, that they enclose all points on the Pareto frontier. Put differently, the Pareto frontier for the non-relaxed problem coincides with the one for the relaxed problem.

but rather reduced by local equity concerns, and can only be understood in terms of the efficiency-efficiency tradeoff studied in this section.

Along the same lines, it can be explained why low-skill labor y_1^S is downwards distorted in the surplus-maximizing allocation if and only if the revenue requirement A is large enough (above some threshold A_D), but still below the maximal tax revenue A_{max}. In this case, the surplus-maximizing allocation is located to the right of point D. The same is true for the complete set of utilitarian allocations, which implies that optimal marginal taxes are unambiguously positive in this case. One can conclude that negative participation taxes, which only arise for negative revenue requirements $\tilde{A}$, represent a necessary but not sufficient condition for the optimality of negative marginal taxes.

Summing up, I have shown that the problem of constrained surplus maximization gives rise to a tension between labor supply distortions at the intensive margin and labor supply distortions at the extensive margin, which has not been discussed in the literature so far. To minimize efficiency losses due to labor supply responses at the extensive margin, the social planner would prefer implementing an allocation that potentially violates upward incentive compatibility. Surplus-maximization then gives rise to a tradeoff between *intensive efficiency* and *extensive efficiency*, while welfare maximization involves a threeway-tradeoff between *equity*, *intensive efficiency* and *extensive efficiency.*[22]

7 One-dimensional private information

In the Mirrlees (1971) framework, agents differ in and are privately informed about their skill types only. In accordance with the recent literature on labor supply responses at the extensive margin, or at both margins, I have studied a model in which agents are heterogeneous with respect to skills and fixed costs of working (Saez 2002, Choné & Laroque 2011, Jacquet et al. 2013). In the previous sections of this paper as in all previous studies, it is moreover assumed that agents are privately informed about both dimensions of heterogeneity, so that the social planner can exclusively observe the gross income an agent earns on the labor market.

This gives rise to the question whether the derived results, in particular the potential optimality of the EITC, are driven by multi-dimensional heterogeneity or by multi-dimensional private information. From a theoretical perspective, this is important to understand the economic mechanism behind this result. From an applied perspective, one might argue that governments actually possess at least some

22

information about these individual characteristics. Notice for example that the US earned income tax credit (EITC) conditions tax liabilities on individual characteristics and life circumstances such as family status and the number of dependent children, which are commonly brought forward in the literature to motivate the assumption of heterogeneity in fixed costs of working. Similarly, tax authorities in many countries make use of tagging with respect to, e.g., disabilities or spatial distance between the place of residence and the workplace of tax payers.

This section aims at clarifying the importance of the imposed information structure. For this purpose, I study optimal taxation under the alternative assumptions that the social planner is able to observe one individual parameter directly.

7.1 Observable fixed costs

The first alternative to the information structure considered so far is to abandon the assumption of private information on fixed cost types. Instead, assume that the social planner is able to observe individual fixed cost types, while agents remain privately informed about their skill types. The information structure is thus similar to the one in the classical Mirrlees (1971) framework. In contrast to the latter, however, there is observable heterogeneity with respect to fixed costs, which can be used for tagging, i.e., to condition tax payments on individual fixed cost types. Nevertheless, changes in the tax schedule can give rise to labor supply responses at both margins.

With observable fixed costs, the planner only needs to take into account a limited set of incentive-compatibility constraints. In particular, only the incentive-compatibility constraints between agents with alternative skills, but the same cost parameter δ need to be satisfied:

$$c(\omega, \delta) - h\left(y(\omega, \delta), \omega\right) - 1_{y>0}\delta \geq c(\omega', \delta) - h\left(y(\omega', \delta), \omega'\right) - 1_{y>0}\delta \quad \text{for all } \omega, \omega' \in \Omega \text{ and } \delta \in \Delta \tag{27}$$

With observable fixed cost types, the social planner's problem is to maximize social welfare (3), subject to feasibility (1) and the reduced set of incentive compatibility constraints (27). However, this problem can be rewritten as a two-step problem. In the first step, the social planner maximizes overall welfare by redistributing resources between all fixed cost groups, without being constrained by any IC constraints. In the second step, the planner maximizes the group-specific welfare in each fixed cost group, subject to the group-specific IC constraints (27) and a group-specific feasibility constraint. Thus, he essentially solves separate optimal tax problems for each groups of agents with each fixed cost type $\delta \in \Delta$.

As Assumption *DUR* does not pin down the redistributive concerns of the social planner within the group of agents with fixed cost type δ, we need to replace it with the following assumption.

Desirability of Utilitarian Redistribution with Observable Costs (DR δ): *For each fixed cost level $\delta \in \Delta$, the following is true in every implementable allocation (c, y):*

$$0 < \alpha'_{j+1}(c, y, \delta) < \alpha'_j(c, y, \delta) < \alpha'_0(c, y, \delta), \tag{28}$$

where

$$\begin{aligned} \alpha'_j(c, y, \delta) &= U'\left[c(\omega_{j+1}, \delta) - h\left[y(\omega_{j+1}, \delta), \delta\right] - \delta\right] \quad \textit{and} \\ \alpha'_0(c, y, \delta) &= \mathbb{E}_{\omega_k}\left[U'\left[c(\omega_k, \delta)\right] | y(\omega_k, \delta) = 0\right] \end{aligned}$$

denote the endogenous weights associated to working agents with type (ω_j, δ) and to unemployed agents, respectively.

This assumption is clearly satisfied if function U is strictly concave on $\mathbb{R}$. Defining the set of utilitarian allocation based on Assumption *DUR δ* instead of *DUR*, the optimal structure of income tax schedule has similar effects on labor supply distortions as in the Mirrlees (1971) model.

Proposition 8. *With observable fixed cost types, labor supply in any utilitarian allocation is*

(i) undistorted at the intensive margin at the top skill, i.e., for all agents with skill type ω_n,

(ii) strictly downwards distorted at the intensive margin for all agents with lower skill types, and

(iii) weakly downwards distorted at the extensive margin for all types (ω, δ)

in all fixed cost groups for any regular economy.

Proposition 8 is closely related to the main results by Mirrlees (1971) and subsequent papers. In particular, parts (i) and (ii) correspond to the traditional results on optimal distortions at the intensive margin. These papers do not provide insights on optimal distortions at the extensive margin, though.[23] Nevertheless, similar arguments can be applied to show that all downward IC constraints in each fixed cost group must be binding in the optimal allocation. Distorting labor supply downwards at the intensive margin then helps to slacken these downward IC constraints, and to achieve further equity gains.

The crucial difference to the model with two-dimensional difference is directly related to the different information structure. With two-dimensional private information, there are agents in all skill groups that are indifferent between employment and unemployment. Thus, a small increase in the unemployment benefit induces unintended labor supply responses at the extensive margin in all skill groups. With observable fixed costs, only the least productive workers are indifferent between employment and unemployment, while all workers with higher skill types strictly prefer working. Consequently, a small increase in the benefit induces only extensive margin responses among the least productive workers, but does not drive high-skill workers out of the labor market.

7.2 Observable skill types

The second alternative to the information structure in the main part of this paper involves observable skill types. In contrast, let the agents be privately informed about their fixed cost types. Thus, the social planner again faces a one-dimensional screening problem. Given this information structure, an allocation is incentive-compatible if and only if

$$c(\omega,\delta) - h\left(y(\omega,\delta),\omega\right) - 1_{y>0}\delta \geq c(\omega,\delta') - h\left(y(\omega,\delta'),\omega\right) - 1_{y>0}\delta' \quad \text{for all } \omega \in \Omega \text{ and } \delta,\delta' \in \Delta \tag{29}$$

With observable fixed costs, the optimal tax problem is to maximize social welfare (3), subject to feasibility (1) and the new set of incentive compatibility constraints (29).

Again, Assumption *DUR* needs to be replaced with an assumption on the planner's redistributive concerns within the group of agents with each skill type ω_j.

Desirability of Utilitarian Redistribution with Observable Skills (DUR ω): *For every skill level $\omega_j \in \Omega$, the following is true in every implementable allocation*

[23] Typically, Inada conditions ensure that all agents provide strictly positive output in the optimal allocation, thereby ruling out extensive margin distortions.

(c, y)

$$0 < \alpha_w^U(c, y, \omega_j) < \alpha_0^U(c, y, \omega_j), \tag{30}$$

where

$$\begin{aligned} \alpha_w^U(c, y, \omega_j) &= \mathbb{E}_\delta \left[U' \left(c(\omega_j, \delta) - h\left[y(\omega_j, \delta), \omega_j\right] - \delta\right) | y(\omega_j, \delta) > 0\right] \quad \textit{and} \\ \alpha_0^U(c, y, \omega_j) &= \mathbb{E}_\delta \left[U' \left[c(\omega_j, \delta]\right) | y(\omega_j, \delta) > 0\right] \end{aligned}$$

denote the average weights associated to working agents and to unemployed agents, respectively.

Again, this assumption is satisfied whenever function U is strictly concave on $\mathbb{R}$. The following proposition clarifies that optimal utilitarian income taxation cannot give rise to upward distortions in labor supply at any margin, as long as the social planner faces a one-dimensional screening problem.

Proposition 9. *With observable skill types, labor supply in any utilitarian allocation is*

- *undistorted at the intensive margin everywhere, and*
- *distorted downward at the extensive margin*

in all skill groups $\omega_j \in \Omega$ for any regular economy.

This insight and the logic behind it differ more strongly from the results by Mirrlees (1971) as well as Saez (2002), Choné & Laroque (2011). Given this information structure, the social planner only needs to consider incentive compatibility constraints between agents with identical skill types, but different cost types. As there is no single-crossing condition imposed with respect to the fixed cost type δ, labor supply distortions at the intensive margin cannot help to slacken IC constraints and are thus never optimal. Because Assumption *AFC* imposes additive separability of the fixed cost component δ, every implementable allocation involves pooling by all workers with the same skill, and by all unemployed agents with the same skill. Thus, the social planner's problem is basically reduced to choosing a benefit level b_j for unemployed agents and a consumption-output bundle (c_j, y_j) for workers of each skill group $\omega_j \in \Omega$.

In any skill group, redistributing additional resources from workers to unemployed agents induces an equity gain, but also forces some previously indifferent workers out of the labor market. As long as labor supply is not downwards distorted at the extensive margin, this also implies an efficiency gain and, consequently, a strict increase in social welfare. Thus, the optimal allocation must involve a strict downward distortion at the extensive margin in each skill group.

To summarize, this section has clarified that neither two-dimensional heterogeneity nor the existence of labor supply responses at the intensive and the extensive margin per se alter the main insights of Mirrlees (1971). As long as the utilitarian planner is able to observe one dimension of heterogeneity, and needs to solve a one-dimensional screening problem, the optimal allocation will never involve upward distortions in labor supply. If agents are instead privately informed about skill types as well as fixed cost types, a utilitarian desire for redistribution does not pin down the optimal direction of labor supply distortions as implied by Propositions 2 to 4.

8 Discussion of assumptions

This paper studies optimal utilitarian income taxation under a number of regularity assumptions imposed in Section 3. In the following, I discuss the implications of these assumptions for the results of this paper, in particular for the ambiguous sign of the optimal marginal income tax.

Assumption AFC and QLC restrict individual preferences. Assumption AFC follows Jacquet et al. (2013), the most prominent previous paper on optimal income taxation with labor supply responses at two margins. It imposes additive separability of the fixed cost component δ, which is required for reasons of tractability, as it allows to study the model using the random participation approach due to Rochet & Stole (2002). Under AFC, the fixed cost type δ only affects an agent's decision whether or not to enter the labor market. Conditional on entering the labor market, in contrast, the individually optimal level of workload y only depends on the skill type ω for any given tax schedule T. Thus, all workers with the same skill type react identically to changes in T. In mechanism perspective, assumption AFC implies that an allocation is implementable whenever it satisfies dimension-wise incentive compatibility, i.e., if no agent with type (ω, δ) prefers the allocations of types that differ in only one type parameter.

Assumption QLC follows the seminal paper by Diamond (1998) and has two implications. First, the imposed quasi-linearity in consumption considerably simplifies the optimal tax problem by eliminating income effects in labor supply. In particular, assumption QLC implies that individually optimal choices of workload y only depend on marginal income taxes, but are unaffected by lump-sum taxes. Thus, it simplifies the definition and analysis of labor supply distortions at the intensive margin.[24]

Second, the assumed quasi-linearity implies that the social planner's desire for redistribution only depends on the properties of the social objective function U (and the joint type distribution Ψ). This simplifies the analysis of sufficient conditions for condition DUR to be satisfied. In particular, the limit-case of a social planner without redistribute concerns is attained for U equaling the identity function.

By assumption REM, there would be unemployed as well as employed agents with each skill type under laissez-faire. This guarantees that variations in tax liabilities induce labor supply responses at the extensive margin in all skill groups, as long as the highest skill group faces a positive participation tax. By Proposition 1, this is always true for the optimal tax schedule. From a theoretical perspective, this assumption simplifies the comparison between the model studied here and the Mirrlees (1971) model, where no extensive margin responses occur.

The main results of this paper survive, however, under the weaker condition that extensive margin responses occur for more than two skill groups. Consider an intermediate model in which extensive margin responses in labor supply only occur up to some threshold skill level $\omega_k < \omega_n$. Then, labor supply by all agents with skill types $\omega_j \in [\omega_k, \omega_{n-1}]$ is strictly downward distorted at the intensive margin in every utilitarian allocation, just as in the intensive model á la Mirrlees (1971). In contrast, the direction of optimal distortions at the intensive margin is ambiguous for all skill groups below ω_k, as in the model studied here.

Finally, assumption LC requires that the fixed cost distribution G_j for each skill group is strictly log-concave, i.e., has a strictly increasing reverse hazard rate, which is true for most commonly used distribution functions, including the uniform, normal, log-normal, Pareto and exponential distributions. Assumption $OFCD$ imposes two conditions on the joint type distribution. By part (i), fixed costs must be larger among low-skill workers than among high-skill workers in the sense of first-order stochastic dominance. By part (ii), the hazard rate $G_j(\delta)/g_j(\delta)$ must be weakly lower for low-skill groups than for high-skill groups. Clearly, both properties are closely related, although not equivalent in general. However, they have two separate, crucial implications.

First, LC and Assumption $OFCD$ (ii) jointly imply that the semi-elasticity of the participation rate is strictly lower for low-skill types than for high-skill types in every implementable allocation. As the analysis of the auxiliary problem in Section

[24] In a slightly weaker version of assumption QLC, income effects could also be ruled out by assuming that the utility function is given by $V(c, y, \omega, \delta) = \Phi\left[c - h(y, \omega) - 1_{y>0}\delta\right]$, where Φ is some strictly increasing function.

6 has revealed, this is a necessary condition for the ambiguous sign of the optimal marginal tax for the working poor, who receive employment subsidies. Crucially, strong empirical evidence confirms that low-skill workers indeed react more responsively on the extensive margin (see, e.g., Juhn et al. 1991, Immervoll et al. 2007, Meghir & Phillips 2010).[25] Thus, assumptions *LC* and *OFCD* guarantee the empirical relevance of the derived results.[26]

Second, *LC* and *OFCD* (i) jointly ensure that condition *DUR* does not restrict the analysis to the empty set. Although the planner's desire for redistribution from high-skill to low-skill workers is imposed directly through *DUR*, it actually represents a joint assumption on properties of the joint type distribution and the social objective U. It can be shown that the sequence of social weights is strictly decreasing whenever the social objective U is strictly concave and *LC* and *OFCD* (ii) hold. In contrast, concavity of U would neither be sufficient nor necessary if *LC* or *OFCD* (ii) would be violated.[27]

Intuitively, concavity of U implies that the planner prefers to redistribute from skill groups with high average utility to skill groups with lower average utility. With a strong positive correlation between skills and fixed costs, however, high-skill workers might be on average worse off than low-skill workers. Thus, the social planner might hold an anti-utilitarian desire to redistribute from low-skill to high-skill workers. More generally, there might exist joint type distributions Ψ such that Assumption *DUR* would not be satisfied for *any* strictly increasing function U.[28]

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[25] More precisely, these studies find that the elasticity of participation, $[y_j - T(y_j)] \frac{g_j[y_j - T(y_j)]}{g_j[y_j - T(y_j)]}$ is decreasing along the skill dimension. The same must be true for the semi-elasticities of participation, however, as all estimated elasticities are positive and $y_j - T(y_j)$ is strictly increasing in the skill type. Note, however, that these empirical studies only reveal relative semi-elasticities under the current tax schedules, which will typically differ from the optimal tax schedule.

[26] It is nevertheless interesting to note that the sign of the optimal marginal tax would even be ambiguous in the opposite case, in which high-skill groups would react more strongly at the extensive margin. Then, however, optimal marginal taxes would be strictly positive for low-skill workers and potentially negative for high-skill workers. I am not aware of real-world tax schedules with this property, however.

[27] See Propositions 2 and 3 by Choné & Laroque (2011) for the same result in a model with labor supply responses at the extensive margin only. In the Mirrlees (1971) framework with one-dimensional heterogeneity, in contrast, concavity of U is a sufficient condition for a standard utilitarian desire for redistribution, irrespective of the properties of the type distribution.

[28] Choné & Laroque (2010) study the roots and effects of increasing social weight functions in a model with labor supply responses at the intensive margin only, also referring to settings with two-dimensional heterogeneity and strong correlation between both private parameters. In particular, they use this logic to rationalize an EITC-style income tax schedule with negative marginal taxes.

9 Related Literature

The paper studies the implications of optimal utilitarian income taxation in a model with labor supply responses at two margins. Thus, it builds on the rich literature on optimal taxation with labor supply responses at the intensive margin only, starting with the seminal paper by Mirrlees (1971). Further important studies include Seade (1977, 1982) and Hellwig (2007). In their models, a utilitarian desire for redistribution leads to the optimality of strictly positive marginal taxes everywhere below the very top. In contrast, the optimal sign of marginal taxes is ambiguous in my paper, which is a joint result of, first, the existence of two margins of labor supply responses, and second, individual heterogeneity in two dimensions that are both associated with private information.

Regarding the theoretical model, this paper is more closely related to the literature on optimal taxation with labor supply responses at the extensive margin. This strand of the literature was initiated by Saez (2002), building on previous work by Diamond (1980). A rigorous theoretical treatment of the extensive model is provided by Choné & Laroque (2011). In these papers, agents differ in two individual parameter, interpreted as skills and fixed costs or opportunity costs of employment. Thus, the social planner faces a multi-dimensional screening problem. In contrast to this paper, however, they focus on models in which the agents only face fixed costs of working, but no continuous cost of increasing their workload as in Mirrlees (1971).[29] Thus, agents only choose whether or not to work at all; if an agent enters the labor market, he always produces at full capacity. Consequently, distortions in labor supply can only occur at the extensive margin.

The main finding of these models is that negative participation taxes for low-skill workers are optimal if and only if the utilitarian planner associates to them a social weight above the population average. The intuition for this result rests on an efficiency argument, comparing the efficiency costs of two changes in the allocation: redistributing resources towards the working poor induces some upwards distortions in the labor supply of these groups, but redistributing resources towards the unemployed leads to adverse labor supply responses by workers of *all* skill groups.[30]

In the extensive models by Diamond (1980), Saez (2002), Choné & Laroque (2011), the economic role of marginal income taxes differs strongly from the one in the Mirrlees (1971) model and in my model. First, non-zero marginal taxes do

[29]While focusing on models with one margin only, Saez (2002) also discusses the general model with labor supply responses at both margins. He simulates the optimal tax schedule for this general model, but does not study the properties of optimal tax schedule analytically.

[30]Christiansen (2012) studies in detail the economic mechanism giving rise to the optimality of negative participation taxes.

not induce labor supply distortions at the intensive margin. Second, labor supply distortions do not help to relax incentive compatibility contraints. In their models, there are no upward incentive compatibility constraints, and only degenerate downward incentive compatibility constraints.[31] In my model, negative (or positive) marginal taxes can in contrast only be optimal because they induce intensive margin distortions that help to relax incentive compatibility constraints. While Diamond (1980) and Choné & Laroque (2011) also provide examples under which negative marginal taxes for the working poor are optimal, the mathematical and theoretical arguments explaining these phenomena consequently differ from those provided above.[32]

More generally, the social planner in my model needs to take into account labor supply responses at the intensive and the extensive margin responses. As shown above, the maximization of a utilitarian welfare function can give rise to a tradeoff between intensive efficiency and extensive efficiency, which is key to understand the ambiguous sign of optimal marginal taxes. This tradeoff is absent in the extensive models as well as the intensive models discussed above.

Most closely related to this paper is the analysis by Jacquet et al. (2013), who also study optimal income taxation with labor supply responses at both margins. As in my model, agents face fixed costs of employment (as in the extensive model) as well as variable costs of providing effort in the job (as in the intensive model). The research questions of both papers differ strongly. This paper contributes to the literature by showing that, and why, the optimal signs of marginal income taxes and participation taxes are ambiguous even if the social planner has a desire for utilitarian redistribution. Jacquet et al. (2013) focus on identifying conditions under which optimal marginal taxes are unambiguously positive. In particular, they provide a sufficient condition under which marginal taxes are throughout positive, expressed in terms of endogenous social weights and of the optimal allocation itself. They argue that this sufficient condition does not seem very restrictive, and provide some examples under which is is certainly satisfied. In contrast, I show that the optimal sign of marginal income taxes and participation taxes is in general ambiguous, and provide a sufficient condition for the optimality of negative marginal taxes, which is expressed in terms of the primitives, i.e., the type set, the type distribution and utility functions. One interpretation of my results is that, for a large class of

[31] In the paper by Choné & Laroque (2011), the optimal allocation always involves slack downward incentive compatibility constraints for all skill levels with relevant extensive margin, i.e., with positive shares of unemployed agents.

[32] The example provided in Choné & Laroque (2011) is based on the assumptions that first, the skill space is continuous and second, no agent of the lowest skill type would work under laissez-faire. Both assumptions do not hold in my paper.

economies, it mainly depends on the intensity of the planner's redistributive concerns whether or not the condition identified by Jacquet et al. (2013) is satisfied. As Jacquet et al. (2013) concentrate on cases in which the optimal marginal tax can be signed unambiguously, they are not concerned with working out the economic mechanism underlying the indeterminacy of this sign. Correspondingly, they do not discuss the tradeoff between intensive efficiency and extensive efficiency, which is identified as the source of ambiguity in my model.

There are two minor differences between this paper and Jacquet et al. (2013). First, their model is more general as they allow for income effects in labor supply which are assumed away in this paper. Second, the skill space in their model is given by an interval, while I study a finite set of skill types. Reflecting this difference, the mathematical proofs applied in both papers differ considerably.[33]

Two further papers aim at rationalizing negative marginal income taxes, both based on a desire to redistribute resources locally upwards. Choné & Laroque (2010) study a model á la Mirrlees (1971) with labor supply reponses at the intensive margin only, but with two-dimensional heterogeneity in individual characteristics. They argue that, if there is a specific correlation between both dimensions of heterogeneity, the social planner might want to redistribute resources locally upwards to the group of more skilled, but more disadvantaged (in the second dimension) agents. In this case, the anti-utilitarian desire to redistribute resources from low-skill to high-skill agents gives rise to a reversed equity-efficiency tradeoff, and to optimal upward distortions in labor supply by high-skill workers. In contrast, I assume the social planner to be a utilitarian who would strictly prefer to transfer resources from high-skill to low-skill workers, if he could ignore incentive considerations. In my framework, optimal upward distortions can thus result for efficiency reasons only, more precisely due to the tradeoff between intensive efficiency and extensive efficiency.

In the model by Beaudry et al. (2009), agents differ in and are privately informed about their productivities in the formal sector as well as in the informal sector. Within each group of workers with identical productivity in the formal sector, the ones with highest informal productivity choose to stay officially unemployed in order to maximize their income. Thus, the social planner assigns lower social weights to the unemployed than to the employed within the same skill group, which again conflicts with the assumed desire for utilitarian redistribution in this paper. In

[33]In a related paper, Lorenz & Sachs (2011) study optimal income taxation with two margins of labor supply responses, where the extensive margin results from an minimum-hours constraint. While they do not study the sign of optimal marginal taxes, they provide a sufficient condition for the positivity of optimal participation taxes.

Beaudry et al. (2009), the non-monotonic weight sequence implies that employment subsidies up to some threshold skill level are optimal. Their model differs in two further aspects from the classical Mirrlees (1971) setting. First, effort costs are linear so that all agents choose either to work at full capacity in the formal sector or to move towards the informal sector (except one threshold skill type). As in the extensive models discussed above, the optimal allocation cannot involve upward distortions at the intensive margin by construction. Second, they assume that the social planner can observe hours worked in the formal sector, thereby deviating from the conventional information structure.

10 Conclusion

The largest US program transferring resources towards the poor, the Earned Income Tax Credit (EITC), involves negative marginal taxes and negative participation taxes for the working poor. Given a utilitarian desire for redistribution, this cannot be rationalized in a model in which agents adjust their labor supply only at the intensive margin as in the classical Mirrlees (1971) framework; the optimal *marginal tax* is then positive everywhere below the very top. In contrast, recent research finds that optimal *participation taxes* can be negative if agents adjust their labor supply at the extensive instead of the intensive margin (Saez 2002, Choné & Laroque 2011). This paper is the first to show that, and explain why, EITC-style tax schemes with *negative marginal taxes and negative participation taxes* can indeed be optimal if labor supply responses take place at the intensive and the extensive margin, which is arguably the most appropriate assumption from an empirical perspective.

More generally, I show that the existence of a utilitarian desire to redistribute resources from high-skill to low-skill workers does neither pin down the optimal signs of marginal and participation taxes nor the optimal directions of labor supply distortions at both margins. Instead, the properties of the optimal tax scheme depend on the intensities of the social planner's concerns for redistribution, first, from the very rich to the very poor, and second, within the group of the working poor. The paper works out the economic intuition behind this ambiguity, which is driven by an inherent, but yet undiscussed, tradeoff between intensive efficiency and extensive efficiency aspects. Negative marginal taxes create efficiency losses at the intensive margin; in certain situation, they can however help to increase extensive efficiency by slackening upward incentive compatibility constraints.

A number of questions remain unresolved. First, the theoretical analysis clarifies that the properties of the optimal tax scheme depend strongly on the relative (semi-

)elasticities of labor market participation shares in different skill groups. While there is already some empirical evidence on this issue, future research should focus more strongly on the heterogeneity of labor supply responses, instead of mainly estimating average elasticities. Second, the analysis has been simplified considerably by a number of assumptions. In my view, the most restrictive of these assumptions are given by the quasi-linearity of preferences in consumption, which rules out any income effects in labor supply, and the discreteness of the skill type space. Although I conjecture that the basic insights would remain valid, relaxing these assumptions could improve the economic understanding of the mechanisms at work and complete the picture.

Appendix

A Proofs for Sections 4 to 6

Proof of Lemma 1 An allocation (c, y) is incentive compatible if it satisfies the following inequality for all pairs of (ω, δ) and (ω', δ') in $\Omega \times \Delta$:

$$c(\omega, \delta) - h\left[y(\omega, \delta), \omega\right] - 1_{y(\omega,\delta)>0}\delta \geq c(\omega', \delta') - h\left[y(\omega', \delta'), \omega\right] - 1_{y(\omega',\delta')>0}\delta$$

The proof of Lemma 1 requires to distinguish between several cases. First, consider two agents of types (ω, δ) and (ω', δ') such that both provide zero output. Incentive compatibility requires identical consumption $c(\omega, \delta) = c(\omega', \delta') \equiv b$.

Second, consider two agents with identical skill type ω_j and different cost types $\delta \neq \delta'$ such that both provide positive effort. As both IC constraints need to be satisfied, both agents need to receive the same gross (of fixed costs) utility level $c(\omega_j, \delta) - h\left[y(\omega_j, \delta), \omega_j\right] = c(\omega_j, \delta') - h\left[y(\omega_j, \delta'), \omega_j\right] = z_j$. In general, incentive compatibility does not imply $c(\omega_j, \delta) = c(\omega_j, \delta')$ and $y(\omega_j, \delta) = y(\omega_j, \delta')$, because different consumption bundles provide the same gross utility level z_j to workers with identical skill types. Incentive compatibility only requires that non of the bundles meant for some worker with skill ω_j is preferred by some worker with skill ω_k, i.e., that $c(\omega_j, \delta) - h\left[y(\omega_j, \delta), \omega_k\right] \leq z_k$ holds.

Second-best Pareto efficiency, however, requires identical bundles (c_j, y_j) for all workers of skill level ω_j. By the properties of effort cost function h, there is always a unique bundle (c_j, y_j) that minimizes the net transfer $c - y$ subject to incentive compatibility, i.e., subject to $c - h(y, \omega_j) = z_j$ and $c - h(y, \omega_k) < z_k$. This may either involve the efficient level $\hat{y}_j$ or the closest level to $\hat{y}_j$ that is still consistent with all IC constraints. If some agent with type (ω_j, δ) receives bundle $(c', y') \neq (c_j, y_j)$ with positive output $y' \neq y_j$, then net resources can be saved by changing his allocation to (c_j, y_j) without changing his utility level. But then, redistributing these resources lump-sum to all agents in the economy leads to a strict (incentive-compatible) Pareto improvement.

Finally, consider two agents with the same skill type ω_j and different cost types δ, δ' such that $y(\omega_j, \delta) > 0$ and $y(\omega_j, \delta') = 0$. By incentive compatibility, $\delta \leq c_j - h\left(y_j, \omega_j\right) - b \equiv \hat{\delta}_j$ and $\delta' \geq \hat{\delta}_j$.

Proof of Lemma 2 Assumption *DUR* ensures that the social planner associates positive weight to all skill groups. By standard arguments, any utilitarian allocation must then be Pareto-efficient, which implies identical bundles (c_j, y_j) for all workers of each skill group ω_j, and all unemployed agents. The welfare function (12) and the feasibility constraint (13) directly follow from inserting the skill-conditional levels c_j and y_j and the universal benefit b.

Incentive compatibility along the fixed cost dimension, i.e., between types with identical skills ω_j and different cost types δ, δ' is given if and only if the participation constraint (14) is satisfied. It takes into account the possibility of corner solutions, in which all agents of some skill groups are either unemployed, $\hat{\delta}_j = \bar{\delta}$, or employed, $\hat{\delta}_j = \underline{\delta}$. As all unemployed agents receive the same benefit, constraint (14) also ensures that no worker of skill group ω_j wants to mimic an unemployed agent of some other skill group.

Incentive compatibility between two workers with adjacent skill types ω_j, ω_{j+1} and arbitrary fixed cost types $\delta \leq \hat{\delta}_j$, $\delta' \leq \hat{\delta}_{j+1}$ is satisfied if and only if

$$\tilde{V}(c_{j+1}, y_{j+1}, \omega_{j+1}) - \delta' \geq \tilde{V}(c_j, y_j, \omega_{j+1}) - \delta' \text{ and}$$
$$\tilde{V}(c_j, y_j, \omega_j) - \delta \geq \tilde{V}(c_{j+1}, y_{+1}, \omega_j) - \delta \ ,$$

which is equivalent to constraints (15) and (16). By the single-crossing property, they also ensures incentive compatibility between non-adjacent skill types. Finally, (14), (15) and (16) jointly guarantee that no unemployed agent of skill type ω_j wants to mimic some worker with some other skill type ω_k, because $b > \tilde{V}(c_j, y_j, \omega_j) - \delta \geq \tilde{V}(c_k, y_k, \omega_j) - \delta$ for all unemployed agents with $\delta > \hat{\delta}_j$ and any $k \neq j$.

Proof of Lemma 3

Proof. For any type (ω_j, δ), efficient labor supply is given by the minimizer of the net transfer of resources $(c - y)$ subject to the constraint $V(c, y, \omega_j, \delta) \geq v$. This problem is equivalent to maximizing the following Lagrangian

$$\mathcal{L}(c, y) = y - c + \lambda \left[c - h(y, \omega_j) - 1_{j>0}\delta - v\right]$$

The discontinuity at $y = 0$ requires a case distinction. For the corner solution $y = 0$, the required net transfer trivially follows as $c_0(v) = v$.

For the interior solution $y > 0$, monotonicity and convexity of h ensure a unique solution, given by $y = \hat{y}_j$ and $c = \hat{c}_j(v) = h\left(\hat{y}_j, \omega_j\right) + \delta + v$, where $\hat{y}_j$ is implicitly defined by the first-order condition $1 - h_1\left(\hat{y}_j, \omega_j\right) = 0$. The net transfer is given by $\hat{c}_j(v) - \hat{y}_j = v - \hat{s}_j + \delta$. If and only if $\delta \leq \hat{s}_j$, the interior solution dominates the corner solution, so that $y^*(\omega_j, \delta) = \hat{y}_j$. □

Proofs of Propositions 1-2

Proposition 1 implies that the famous no-distortion-at-the-top result still holds with labor supply responses at the intensive margin, although only with respect to the intensive margin. At the extensive margin, labor supply is instead strictly downward distorted at the top. Proposition 2 derives the existence of utilitarian allocations without distortions at the intensive margin for any regular economy.

Both propositions are proven through a series of lemmas. To simplify notation in the following, I find it convenient to define the employment rent $r_j = c_j - h(y_j, \omega_j) - b$ as an auxiliary function. It measures the utility gain that a worker of skill level ω_j receives if he provides output $y_j > 0$ instead of staying unemployed, conditional on the mechanism (c, y, b) and gross of fixed costs.

First, consider a relaxed problem in which the incentive compatibility constraints (15 and (16) between active workers of different skill types are not taken into account. However, we still include the constraint that unemployed agents of all skill types must receive the same benefit b. Moreover, the planner is still restricted by the set of participation constraints (14), i.e., needs to take into account labor supply responses at the extensive margin. Note that this relaxed problem corresponds to the first-and-half problem in Jacquet et al. (2013), which is however studied under the assumption of a continuous skill space. Given the definition of the employment rent, the social planner's problem can be defined as follows

Relaxed Problem. *Maximize over* $y = (y_j)_{j=1}^n$, $r = (r_j)_{j=1}^n$, *and* b *the welfare function*

$$\sum_{j=1}^{n} f_j \left[\int_{\underline{\delta}}^{\hat{\delta}_j} U(r_j + b - \delta)\, dG_j(\delta) + \left[1 - G_j\left(\hat{\delta}_j\right)\right] U(b) \right]$$

subject to the constraints

$$b = \sum_{j=1}^{n} f_j G_j\left(\hat{\delta}_j\right) \left[s(y_j, \omega_j) - r_j \right],$$

$$\hat{\delta}_j = \max\left\{\underline{\delta}, \min\left\{r_j, \bar{\delta}\right\}\right\} \text{ for all } \omega_j \in \Omega$$

In this model, the planner's objective is not necessarily globally concave in all choice variables. The same problem arises in the model with labor supply responses at the extensive margin only, Choné & Laroque (2011) show that the Lagrangian can become convex in consumption levels if social weights are particularly high. The following assumption assumes away this irregularity in order to concentrate on the economic problem.

Assumption 4. *For any skill level ω_j, the social weight $\alpha_j^U(c,y)$ associated to workers with this skill type satisfies*

$$\alpha_j^U(c,y) < \chi_j(\delta) = \left(2 - \frac{G_j(\delta)g_j'(\delta)}{g_j(\delta)^2}\right) / \left(1 - \frac{G_j(\delta)g_j'(\delta)}{g_j(\delta)^2}\right)$$

for all $\delta \in \Delta$. Moreover, $\alpha_j^U(c,y)$ is weakly decreasing in c_j.

The log-concavity of G_j imposed by assumption LC ensures that $g_j(\delta)^2 > G_j(\delta)g_j'(\delta)$. Thus, the upper bound $\chi_j(\delta)$ exceeds unity for $\omega_j \in \Omega$ and $\delta \in \Delta$. For uniformly distributed fixed costs, $\chi_j(\delta) = 2$ for all δ and ω_j. All results derived in this paper follow for utilitarian welfare functions that satisfy this assumption.

Lemma 7. *Let the relative social weight $\alpha_j^U(c^R, y^R)$ be defined as in equation 8 on page 8. In the solution to the relaxed problem, (r^R, y^R, b^R), all workers of skill type ω_j*

- *provide the efficient output level $y_j^R = \hat{y}_j$, and*
- *receive an employment rent that is implicitly defined by*

$$g(r_j^R)\left[r_j^R - \hat{s}_j\right] = G_j(r_j^R)\left(\alpha_j^U(c^R, y^R) - 1\right) .$$

The unemployment benefit is given by

$$b^R = \sum_{j=1}^{n} f_j G_j\left(r_j^R\right)\left(\hat{s}_j - r_j^R\right)$$

Proof. Assume that $r_j \in \left(\underline{\delta}, \bar{\delta}\right)$ for all skill levels $\omega_j \in \Omega$. Then, the Lagrangian of the relaxed problem is given by

$$\begin{aligned}\mathcal{L} = &\sum_{j=1}^{n} f_j \left[\int_{\underline{\delta}}^{r_j} g_j(\delta) U\left(r_j + b - \delta\right) d\delta + \left(1 - G_j(r_j)\right) U(b)\right] \\ &+ \lambda \left[\sum_{j=1}^{n} f_j G_j(r_j)\left(s\left(y_j, \omega_j\right) - r_j\right) - b\right]\end{aligned}$$

The first-order conditions with respect to r_j, y_j and b are given by

$$\mathcal{L}_{r_j} = f_j \left[\int_{\underline{\delta}}^{r_j^R} g_j(\delta) U'\left(r_j^R + b^R - \delta\right) d\delta - \lambda G_j(r_j^R)\right.$$

$$
\begin{aligned}
&+\lambda g_j(r_j^R)\left(s\left(y_j^R,\omega_j\right)-r_j^R\right)\Big] &&=0\\
\mathcal{L}_{y_j} =&\lambda f_j G_j(r_j^R) s_1(y_j^R,\omega_j) &&=0\\
\mathcal{L}_b =&\underbrace{\sum_{j=1}^n f_j\left[\int_{\underline{\delta}}^{r_j^R} g_j(\delta)U'\left(r_j^R+b^R-\delta\right)d\delta+\left(1-G_j(r_j^R)\right)U'(b^R)\right]}_{=\bar{\alpha}(c^R,y^R)}-\lambda &&=0
\end{aligned}
$$

By the last FOC, the value of multiplier λ equals the average marginal utility $\bar{\alpha}(c^R, y^R)$ in the optimal allocation. The same will be true for the full problem. By the FOC with respect to y_j, $s_1(y_j^R, \omega_j)$ must be zero in the solution to the relaxed problem. Thus, workers of all skill levels provide efficient output $y_j = \hat{y}_j$. Rearranging the FOC with respect to r_j and substituting in $\alpha_j^U(c^R, y^R) = \left[\int_{\underline{\delta}}^{r_j^R} g_j(\delta)U'\left(r_j^R + b^R - \delta\right) d\delta\right] / \bar{\alpha}(c^R, y^R)$ gives the expression in Lemma 7. By assumption *REM*, the first derivative is strictly positive for $r_j \to \underline{\delta}$. For $r_j \to \infty$, it is strictly negative by $\lim_{z\to\infty} U'(z) < 1$. By the continuity of the first-order condition in r_j, it must have at least one root. Assumption 4 guarantees concavity of the Lagrangian in r_j is for all $r_j \geq \underline{\delta}$. Thus, the first-order condition with respect to r_j has a unique root, which involves $r_j^R > \underline{\delta}$. □

The conditions defining the relaxed problem's solution have the same structure as those defining the optimal allocations in the extensive models by Saez (2002) and Choné & Laroque (2011), and the solution to the first-and-half problem in Jacquet et al. (2013). Due to the lack of IC constraints, labor supply is generally undistorted in the solution to the relaxed problem. The optimal vector of employment rents is determined by the sequence of endogenous social weights α^U. For $\alpha_j > 1$, workers of skill type ω_j receive an employment rent that exceeds the efficient surplus $\hat{s}_j = \max_y [y - h(y, \omega_j)]$. For $\alpha_j < 1$, workers of skill type ω_j receive an employment rent below $\hat{s}_j$. By assumption *REM*, this implies an interior solution $r_j < \hat{s}_j \leq \hat{s}_n < \bar{\delta}$. Note that $\alpha_n < 1$ is ensures for all utilitarian allocations.

Next, we identify conditions on the pair of social weights α_j and α_{j+1} such that the solution to the relaxed problem satisfies both IC constraints. For this purpose, I ignore the endogeneity of the weight sequence α^U for a while. In particular, assume that α^U equals some exogenous sequence $\beta = (\beta_0, \beta_1, \ldots, \beta_n)$, which determines the optimal employment rent $\tilde{r}_j(\beta_j)$ in the relaxed problem.

Lemma 8. *For any skill level ω_j, there is a threshold $\gamma_j^E > 1$ such that the function $Z_j(\beta_j, r) = g(r)[r - \hat{s}_j] - G_j(r)(\beta_j - 1)$ has a unique root $\tilde{r}_j(\beta_j) \in \left(\underline{\delta}, \bar{\delta}\right)$ in r if and only if $\beta_j < \gamma_j^E$. Moreover, $\tilde{r}_j(\beta_j)$ is strictly increasing in its argument for all $\beta_j < \gamma_j^E$.*

Proof. First, note that $\lim_{r \to \underline{\delta}} Z_j(r) < 0$ for all β_j. Second, for $r \in (\underline{\delta}, \bar{\delta})$, the derivative of Z_j with respect to r is given by

$$\frac{\partial Z_j(r, \beta_j)}{\partial r} = g_j(r)(2 - \beta_j) + g'(r_j)(r - \hat{s}_j)$$

Assumption 4 ensures that this derivative is strictly positive at any root of Z in r. By continuity, there is consequently at most one root in the interval $(\underline{\delta}, \bar{\delta})$. For $r \to \bar{\delta}$, Z approaches $1 + g_j(\bar{\delta})(\bar{\delta} - \hat{s}_j) - \beta_j$. Thus, a unique root in r exists if β_j is smaller than the minimum of $1 + g_j(\bar{\delta})(\bar{\delta} - \hat{s}_j) > 1$ and $\chi_j(\bar{\delta}) > 1$ as defined in Assumption 4. The derivative of $\tilde{r}_j$ with respect to β_j is given by

$$\frac{d\tilde{r}_j}{d\beta_j} = \frac{G_j(\tilde{r}_j)}{(\partial Z_j(\tilde{r}_j, \beta_j))/(\partial r)} > 0.$$

The numerator is positive for all $\beta_j < \gamma_j^E$, where $\tilde{r}_j < \bar{\delta}$. As argued above, Assumption 4 ensures the same for the denominator. □

Lemma 9. *Consider any two adjacent skill groups ω_j and ω_{j+1} in Ω with weights β_j and β_{j+1}. There are a value $\gamma_j^D \in (1, \gamma_j^E)$ and a strictly increasing function $\beta_j^D : (-\infty, \gamma_j^D) \to (-\infty, \gamma_{j+1}^E)$ such that the solution to the relaxed problem satisfies the downward IC constraint if and only if $\beta_j < \gamma_j^D$ and $\beta_{j+1} \in [\beta_j^D(\beta_j), \gamma_{j+1}^E)$. There is a threshold level $\underline{\beta}_j < 1$ such that $\beta_j^D(x) < x$ for all $x \in (\underline{\beta}_j, \gamma_j^D)$.*

Proof. Using the definition of the employment rent, the downward IC constraint (15) reads $\tilde{r}_{j+1}(\beta_{j+1}) - \tilde{r}_j(\beta_j) \geq h(\hat{y}_j, \omega_j) - h(\hat{y}_j, \omega_{j+1})$. Note that the right-hand side does not depend on the weights β_j, β_{j+1}. By Lemma 8, $\tilde{r}_j$ and $\tilde{r}_{j+1}$ are defined and below $\bar{\delta}$ if and only if $\beta_j < \gamma_j^E$ and $\beta_{j+1} < \gamma_{j+1}^E$.

First, define γ_j^D implicitly by $\tilde{r}_j(\gamma_j^D) = \bar{\delta} - h(\hat{y}_j, \omega_j) + h(\hat{y}_j, \omega_{j+1}) < \bar{\delta}$. We have $\gamma_j^D > 1$ due to $\tilde{r}_j(1) < \hat{s}_{j+1} - h(\hat{y}_j, \omega_j) + h(\hat{y}_j, \omega_{j+1})$ and $\hat{s}_{j+1} < \bar{\delta}$. By the monotonicity of $\tilde{r}_k$ in β_k for $k \in \{j, j+1\}$, the downward IC can only be satisfied for $\beta_{j+1} \to \gamma_{j+1}^E$ if $\beta_j < \gamma_j^D$. For any $\beta_j < \gamma_j^D$, there is moreover a unique level $\beta_j^D(\beta_j) < \gamma_{j+1}^E$ such that $\tilde{r}_{j+1}(x) - \tilde{r}_j(\beta_j) \geq h(\hat{y}_j, \omega_j) - h(\hat{y}_j, \omega_{j+1})$ is satisfied with equality if and only if $x = \beta_j^D(\beta_j)$, and with strict inequality if and only if $x \in [\beta_j^D(\beta_j), \gamma_{j+1}^E)$. Moreover, β_j^D is strictly increasing in β_j due to the monotonicity of $\tilde{r}_k$ in β_k.

For the threshold $\underline{\beta}_j$, consider the case where $\beta_j = \beta_{j+1} = x$. As the derivative of $\tilde{r}_j(x)$ with respect to ω_j is strictly positive, we have $\tilde{r}_{j+1}(x) > \tilde{r}_j(x)$. Combining both first-order conditions, we have

$$\tilde{r}_{j+1}(x) - \tilde{r}_j(x) = \hat{s}_{j+1} - \hat{s}_j + (x - 1)\left[\frac{G_{j+1}(\tilde{r}_{j+1}(x))}{g_{j+1}(\tilde{r}_{j+1}(x))} - \frac{G_j(\tilde{r}_j(x))}{g_j(\tilde{r}_j(x))}\right]$$

By assumptions *LC* and *OFCD*, the inequality $\tilde{r}_{j+1} > \tilde{r}_j$ implies that the last term in brackets is strictly positive. Moreover, note that

$$\hat{s}_{j+1} - \hat{s}_j > s\left(\hat{y}_j, \omega_{j+1}\right) - s\left(\hat{y}_j, \omega_j\right) = h(\hat{y}_j, \omega_j) - h(\hat{y}_j, \omega_{j+1}).$$

Thus, the downward IC constraint is satisfied with strict inequality for all $x \geq 1$. On the other hand, $\lim_{x \to -\infty} \tilde{r}_{j+1}(x) = \lim_{x \to -\infty} \tilde{r}_j(x) = \underline{\delta}$. Thus, the downward IC constraint is violated for sufficiently small $x \to -\infty$. By continuity, there must be at least one value $x < 1$ such that the downward IC is satisfied with equality for $\beta_{j+1} = \beta_j = x$. The threshold $\underline{\beta}_j$ is given by the highest value with this property. Thus, the downward IC constraint is satisfied with strict inequality for all $\beta_j = \beta_{j+1} > \underline{\beta}_j$. By the continuity of $\tilde{r}_{j+1}$ in β_{j+1}, $\beta_j^D(\beta_j) < \beta_j$ must be satisfied for all $\beta_j \in \left(\underline{\beta}_j, \gamma_j^D\right)$. □

Lemma 10. *Consider any two adjacent skill groups ω_j and ω_{j+1} in Ω with weights β_j and β_{j+1}. There are a value $\gamma_j^U < \gamma_j^E$ and a strictly increasing function β_j^U : $\left(-\infty, \gamma_j^U\right) \to \left(-\infty, \gamma_{j+1}^E\right)$ such that the solution to the relaxed problem violates the upward incentive compatibility constraint between groups ω_j and ω_{j+1} if and only if $\beta_j < \gamma_j^U$ and $\beta_{j+1} \in \left[\beta_j^U\left(\beta_j\right), \gamma_{j+1}^E\right)$. There is a threshold level $\bar{\beta}_j \in \left(1, \gamma_j^D\right]$ such that $\beta_j^U(x) > x$ for all $x < \bar{\beta}_j$.*

Proof. The upward IC constraint (16) can be rewritten $\tilde{r}_{j+1}\left(\beta_{j+1}\right) - \tilde{r}_j\left(\beta_j\right) \leq h(\hat{y}_{j+1}, \omega_j) - h(\hat{y}_{j+1}, \omega_{j+1})$. By the monotonicity of $\tilde{r}_j$ in β_j, there is a unique $\gamma_j^U < \gamma_j^E$ such that $\tilde{r}_j\left(\gamma_j^D\right) = \bar{\delta} - h(\hat{y}_{j+1}, \omega_j) + h(\hat{y}_{j+1}, \omega_{j+1}) < \bar{\delta}$. The upward IC is satisfied for all $\beta_j \geq \gamma_j^U$ and $\beta_{j+1} < \gamma_{j+1}^E$. For all $\beta_j < \gamma_j^U$, there is in contrast a unique level β_j^U such that $\tilde{r}_{j+1}\left(x\right) - \tilde{r}_j\left(\beta_j\right) \leq h(\hat{y}_j, \omega_j) - h(\hat{y}_j, \omega_{j+1})$ is satisfied with equality if and only if $x = \beta_j^U(\beta_j)$, and violated if and only if $x \in \left[\beta_j^U(\beta_j), \gamma_{j+1}^E\right)$. Moreover, β_j^U is strictly increasing in β_j due to the monotonicity of $\tilde{r}_j$ and $\tilde{r}_{j+1}$.

For the threshold $\bar{\beta}_j$, consider the case where $\beta_j = \beta_{j+1} = x$. Combining both first-order conditions, we have

$$\tilde{r}_{j+1}(x) - \tilde{r}_j(x) = \hat{s}_{j+1} - \hat{s}_j + (x-1)\left[\frac{G_{j+1}(\tilde{r}_{j+1}(x))}{g_{j+1}(\tilde{r}_{j+1}(x))} - \frac{G_j(\tilde{r}_j(x))}{g_j(\tilde{r}_j(x))}\right]$$

Recall that the last term in brackets is strictly positive. Moreover, note that

$$\hat{s}_{j+1} - \hat{s}_j < s\left(\hat{y}_{j+1}, \omega_{j+1}\right) - s\left(\hat{y}_{j+1}, \omega_j\right) = h(\hat{y}_j, \omega_j) - h(\hat{y}_j, \omega_{j+1}).$$

Thus, the upward IC constraint is satisfied with strict inequality for all $x \leq 1$. Depending on parameters and the properties of G_j and G_{j+1}, it is possible that either the upward IC is satisfied for all levels of $x < \gamma_j^U$ so that $\bar{\beta}_j = \gamma_j^U$, or the upward IC is violated for some $x \in \left(1, \gamma_j^U\right)$. In the latter case, $\bar{\beta}_j < 1$ is given by the lowest level $x < \gamma_j^U$ such that the upward is satisfied with equality for $\beta_j = \beta_{j+1} = x$ and violated for $\beta_j = \beta_{j+1} = x + \varepsilon$ with ε approaching 0 from above. By continuity, $\beta_j^U(\beta_j) > \beta_j$ for all $\beta_j < \bar{\beta}_j$. □

Proof of Proposition 1

Proof. The sequence of endogenous social weights must be strictly decreasing in every implementable allocation for every Utilitarian objective as defined in DUR. Thus, $\alpha_n^U(c, y) < 1$ must be true in every utilitarian allocation. By Lemma 7, this implies $r_n < \hat{s}_n$ in the solution to the relaxed problem.

First, consider the intensive margin. By assumption REM, $\hat{s}_n < \bar{\delta}$ so that the extensive margin is relevant in each utilitarian allocation. By Lemma 10, $\alpha_n^U < 1$ then implies that the relaxed problem's solution cannot violate the upward IC constraint. The same is true for the solution to a semi-relaxed problem in which all IC constraints below skill level ω_{n-1} are taken into account. By standard arguments, labor supply y_n is undistorted at the intensive margin, whether or not the downward IC constraint between skill types ω_{n-1} and ω_n is binding.

Second, consider the extensive margin. Labor supply by workers with skill ω_n is downward distorted at the extensive margin if and only if $r_n < \hat{s}_n$. Because $y_n = \hat{y}_n$ as argued above, this is equivalent to $T(y_n) > -b$, where $b \geq 0$. If the downward IC between skill types ω_n and ω_{n-1} is not binding, $\alpha_n < 1$ and $\hat{s}_n < \bar{\delta}$ jointly imply that $\hat{\delta}_j = r_j < \hat{s}_j$.

Assume instead that the downward ICs between ω_n and some skill type ω_k with $k \in [1, n-1]$ are binding, while the downward IC between ω_k and ω_{k-1} is not binding. By standard arguments, this implies that $r_k < \tilde{r}_k = \hat{s}_k + \frac{G_k(\tilde{r}_k)}{g_k(\tilde{r}_k)}(\beta_k - 1)$. If $r_k < \hat{s}_k$, then this implies that $T(y_k) > -b$. The binding downward ICs imply that $T(y_n) > T(y_{n-1}) > \cdots > T(y_k)$. Thus, labor supply is downwards distorted at skill level ω_n.

If instead $r_k > \hat{s}_k$, this requires that $\beta_j > 1$ for all $j \leq k$. Then, workers of all skill levels $\omega_j < \omega_k$ upward distorted labor supply at the extensive margin, and $T(y_j) < -b < 0$. This would directly be true for all skill levels for which either no IC is binding, and for which the downward IC is binding. Assume finally that there are skill types ω_a and ω_b between all upward IC constraints are binding. Then, r_a must exceed $\tilde{r}_a = \hat{s}_a + \frac{G_a(\tilde{r}_a)}{g_a(\tilde{r}_a)}(\beta_a - 1) > \hat{s}_a$, so that $T(y_a) < -b$. Moreover, the binding upward ICs imply that $T(y_b) < T(y_{b-1} < \cdots < T(y_a) < -b$. Altogether, this implies that $T(y_n)$ can only be below $-b$ if $T(y_j) < -b$ is also true for all other skill levels. But this is clearly not consistent with the feasibility constraint. More concretely, budget balance requires that $T(y_n) > 0$ so that labor supply is strictly downward distorted at the extensive margin. $\square$

Lemma 11. *There is a strictly decreasing sequence $\beta = (\beta_0, \beta_1, \ldots, \beta_n)$ such that the following conditions are satisfied*

1. $\sum_{j=1}^{n} f_j \left[G_j(x_j)\beta_j + (1 - G_j(x_j))\beta_0\right] = 1$ *with x_j implicitly defined by* $x_j - \hat{s}_j = \frac{G_j(x_j)}{g_j(x_j)}(\beta_j - 1)$ *for all $\omega_j \in \Omega$,*
2. $\beta_{j+1} \in \left[\beta_j^D(\beta_j), \beta_j^U(\beta_j)\right]$ *for all $\omega_j \in \Omega$, and*
3. $\beta_0 > \beta_1 > 1$.

Proof. Consider the following family of sequences: Let $\tilde{\beta}_1(\varepsilon, \phi) = 1 + \phi$, while $\tilde{\beta}_j(\varepsilon, \phi) = 1 - (j-1)\varepsilon$ for all $j \in [2, n]$, and

$$\tilde{\beta}_0(\varepsilon, \phi) = \frac{1 - \sum_{j=1}^{n} f_j G_j(x_j)\tilde{\beta}_j(\varepsilon, \phi)}{\sum_{j=1}^{n} f_j \left[1 - G(x_j)\right]}$$

For any ε and ϕ, the sequence has average 1. For any $\varepsilon > 0$ and $\phi > 0$, the sequence is strictly decreasing from α_1 on, and $\alpha_1 > 1$. If ϕ is small enough compared to ε, the sequence satisfies $\alpha_0 > \alpha_1$. Lemmas 9 and 10 imply that $\beta_j^U(x) > x > \beta_j^D(x)$ for all x close enough to 1 and all $\omega_j \in \Omega$. This implies that there is some threshold $\varepsilon_1 > 0$ such that $\tilde{\beta}_{j+1} \in \left[\beta_j^D(\tilde{\beta}_j), \beta_j^U(\tilde{\beta}_j\right]$ for all $j \in [2, n-1]$ for any $\varepsilon \in (0, \varepsilon_1]$. If $\varepsilon > 0$ is small enough, there is moreover a threshold $\phi_1 > 0$ such that $\tilde{\beta}_2 \in \left[\beta_1^D(\tilde{\beta}_1), \beta_1^U(\tilde{\beta}_1\right]$ for all $\phi \in (0, \phi_1]$. If ϕ is small enough compared to ε, the sequence finally satisfies $\tilde{\beta}_0 > \tilde{\beta}_1$. $\square$

Proof of Proposition 2

Proof. By Lemma 11, there exists a strictly decreasing sequence β such that (a) the solution to the relaxed problem also solves the full problem because $\beta_j \in \left[\alpha_j^D(\beta_{j-1}), \alpha_j^U(\beta_{j-1})\right]$ for all $\omega_j \in \Omega$, and (b) the social weight associated to workers of skill level ω_1 is above the population average of 1. By Lemma 7, labor supply is undistorted at the intensive margin for all workers in the relaxed problem's solution. Moreover, $\alpha_j^U > 1$ implies that $\hat{\delta}_1 = r_1 > \hat{s}_1$. Thus, labor supply by workers of skill group 1 is strictly upward distorted at the extensive margin. By construction, any strictly decreasing weight sequence satisfies Assumption *DUR*.

For an example with endogenous social weights, assume that the social objective is given by a member of some family of functions K such that $U(x) = K(a, x)$, where K is twice continuously differentiable in both arguments and satisfies for all $x \in \mathbb{R}$ the following properties: a) $K'(a,x) > 0$ for all $a \geq 0$ and $x \in \mathbb{R}$, and b) $K''(a,x) < 0$ for all $a > 0$ and $\lim_{a \to 0} K''(a,x) = 0$. If assumptions *LC* and *OFCD* hold and $G_{j+1}(\delta) \geq G_j(\delta)$ for all skill levels, the endogenous weight sequence α^U is strictly decreasing for all $a > 0$ (see Proposition 3 in Choné & Laroque 2011). Moreover, there exists again some $\bar{a} > 0$ such that the optimal utilitarian allocation involves no distortions at the intensive margin at any skill level for all $a \in (0, \bar{a})$. If the curvature of $K(a,x)$ is sufficiently small on the interval $x \in [0, \hat{s}_1]$ relative to the interval $[\hat{s}_1, \hat{s}_n]$, then the resulting social weight α_1^U will certainly be below unity, giving again rise to upward distortions at the extensive margin. □

Propositions 3 and 4 are proven by example in Section 5.2.

Proofs of Propositions 5, 6 and 7 Proposition 5 is proven by a series of lemmas. In particular, a redistributive weight sequence is constructed for which the upward incentive compatibility constraint between skill groups 1 and 2 is binding and y_2 is upwards distorted in the optimal second-best allocation, while labor supply by all other skill groups is undistorted at the intensive margin. The strategy taken is, first, to solve a relaxed problem in which all incentive compatibility constraints are ignored, and second, to construct a sequence of decreasing exogenous weights such that the solution to the relaxed problem also solves the full problem in which the local incentive compatibility constraints are taken into account if and only if Assumptions 1 and 2 are met. However, the average weight implied by this weight sequence will generally differ from unity. In the third step, we prove that a redistributive weight sequence (with unity average) with the same properties exists, if additionally f_n exceeds some threshold level $\hat{f}_n$. The final step is then to construct a redistributive weight sequence for which y_2 is upwards distorted in the second-best allocation.

Lemma 12. *Under assumption 1, the solution to the relaxed problem involves*

- *an employment rent of* $r_j^R = \frac{\omega_j/2}{2 - \alpha_j(r_j, b)}$ *if* $\bar{\delta} > r_j^R$*, and*
- *the efficient output level* $y_j^R = \omega_j$ *for all skill levels* $\omega_j \in \Omega$*.*

Proof. For the quadratic effort cost function, the efficient levels of output and employment surplus are given by $\hat{y}_j = \omega_j$ and $\hat{s}_j = \hat{y}_j - \frac{\hat{y}_j^2}{2\omega_j} = \frac{\omega_j}{2}$. For the uniform distribution on some interval $\left[0, \bar{\delta}\right]$, we have $\frac{G_j(r_j)}{g_j(r_j)} = r_j$ for any $r_j < \bar{\delta}$. For all $r_j^R < \bar{\delta}$, the first-order condition with respect to r_j can thus be rearranged to have $r_j^R - \hat{s}_j = r_j^R \left(\alpha_j^U - 1\right)$. Solving for r_j^R then gives the equation in Lemma 12. □

Lemma 13. *Under assumptions 1, the thresholds introduced in Lemmas 9 and 10 are given by* $\underline{\beta}_j = 2 - a < 1$ *and* $\bar{\beta}_j = 2 - \frac{1}{a} > 1$ *for all* $\omega_j \in \Omega$*. Furthermore,* $\beta_j^D(\beta) = 2 - \frac{a}{\frac{1}{2-\beta}+1-\frac{1}{a}}$ *and* $\beta_j^U(\beta) = 2 - \frac{a}{\frac{1}{2-\beta}+a(a-1)}$ *for all skill levels in* Ω*. This implies that* $\beta_j^D(x) > x$ *for all* $x < \underline{\beta}_j$ *and* $\beta_j^U(x) < x$ *for all* $x > \bar{\beta}_j$*.*

Proof. First, note that under Assumption 1, $\frac{\hat{s}_{j+1}}{\hat{s}_j} = \frac{\hat{y}_{j+1}}{\hat{y}_j} = \frac{\omega_{j+1}}{\omega_j} = a > 1$. Thus, the IC constraints are given by

$$r_{j+1} - r_j \geq \frac{y_j^2}{2}\left(\frac{1}{\omega_j} - \frac{1}{\omega_{j+1}}\right) \text{ , and}$$
$$r_{j+1} - r_j \leq \frac{y_{j+1}^2}{2}\left(\frac{1}{\omega_j} - \frac{1}{\omega_{j+1}}\right)$$

Plugging in the solution to the relaxed problem gives

$$\frac{\omega_j}{2}\left(\frac{a}{2-\beta_{j+1}} - \frac{1}{2-\beta_j}\right) \geq \frac{\omega_j}{2}\frac{a-1}{a} \text{ , and}$$
$$\frac{\omega_j}{2}\left(\frac{a}{2-\beta_{j+1}} - \frac{1}{2-\beta_j}\right) \leq \frac{a\omega_j}{2}(a-1)$$

Solving for β_{j+1} in both constraints gives the functions β_j^D and β_j^U. Setting $\beta_{j+1} = \beta_j = \beta$, the downward IC is satisfied if $\beta \in \left[2-a, \gamma_D^j\right]$, and violated for all $\beta < 2 - a = \underline{\beta}_j$. The upward IC is satisfied for all $\beta \leq \min\left\{2 - \frac{1}{a}, \gamma_j^U\right\}$, and violated for all $\beta \in \left(2 - \frac{1}{a}, \gamma_j^U\right)$, if the latter interval is non-empty. □

Proof of Proposition 5

Proof. If assumption 1 holds, the downward IC constraint is binding whenever $\beta_{j+1} < \beta_j \leq 2 - a$. As $a \in (1, 2)$, this is compatible with strictly positive weights for all skill types. Consider for example weight function β' with $\beta'_1 = 2 - a \in (0, 1)$ and $\beta'_{j+1} = \beta_j - \frac{2-a}{n}$ for all $j \in \{2, \dots, n\}$. Given these weight function, there is a unique weight $\beta'_0 > 1$ associated to the unemployed such that average social weight is one. By construction, the social objective corresponding to weight function β' satisfies assumption *DUR*. The social weight of all worker groups is below unity, thus giving rise to downward distortions at the extensive margin. In particular, the optimal level of r_1 will be below $r_1^R < \hat{s}_1$ because the downward IC between skill levels ω_1 and ω_2 is binding. Thus, workers of skill level ω_1 will pay positive participation taxes and have downward distorted labor supply at the extensive margin. As all downward IC constraints are binding, workers of all higher skill levels pay even higher participation taxes and have downward distortions at the extensive margin, too. Thus, there is a utilitarian allocation in which labor supply is distorted downward at the intensive margin everywhere below the very top, and at the extensive margin everywhere. Note that, with $a < 2$, the same pattern of distortions arises in the Rawlsian allocation, which results for social weights $\alpha_j = 0$ for all worker types and $\alpha_0 > 1$ for unemployed agents. $\square$

Proof of Proposition 6

Proof. In the following, I proof that there is a threshold $\hat{a}(f_1)$ such that, if $a > \hat{a}(f_1)$ both IC constraints are slack for all welfare functions satisfying Assumption DUR. First, note that $\alpha_2^U < 1 < \bar{\beta}_2 = 2 - \frac{1}{a}$ for all $a > 1$ and all utilitarian welfare functions. Thus, the upward IC cannot be violated by the relaxed problem's solution (this is a corollary of Proposition 1).

Second, the downward IC is slack for all utilitarian welfare function. By the monotonicity of $\tilde{r}_j$ in β_j, it suffices to show that the downward IC is still slack if α_1^U is at the highest possible level and α_2^U is at the lowest possible level. The lower bound of α_2^U is clearly given by 0. For the upper bound of α_1^U, $\alpha_0^U > \alpha_1^U$ implies that

$$\begin{aligned} f_1\left[G_1(r_1)\alpha_1^U + [1 - G_1(r_1)]\,\alpha_0^U\right] + f_2\left[G_2(r_2)\alpha_2^U + [1 - G_2(r_2)]\,\alpha_0^U\right] = 1 \\ \Leftrightarrow \alpha_1^U < \frac{1}{f_1 + f_2\,[1 - G_2(r_2)]} \leq \frac{1}{f_1}. \end{aligned}$$

Using function $\beta_j^D(\beta)$ as given in Lemma 13, the downward IC constraint is satisfied for all combinations $\left(\alpha_1^U, \alpha_2^U\right)$ compatible with DUR if

$$\begin{aligned} \beta_j^D(\zeta) = 2 - \frac{a}{\frac{1}{2-1/f_1} + 1 - \frac{1}{a}} &< 0 \\ \Leftrightarrow a^2 - 2\frac{3f_1 - 1}{2f_1 - 1}a + 2 &> 0 \\ \Rightarrow a > \frac{3f_1 - 1}{2f_1 - 1} + \sqrt{\left(\frac{3f_1 - 1}{2f_1 - 1}\right)^2 - 2} = \hat{a}\left(f_1\right) & \end{aligned}$$

Note that the lower root of this quadratic function is below 1, and thus irrelevant due to $a > 1$. Finally, note that $\hat{a}\left(f_1\right)$ goes to ∞ for f_1 approaching $1/2$ (from above) and to $2 + \sqrt{2}$ for f_1 approaching 1. $\square$

Lemma 14. *Under assumptions 1 and 2, and with the social weight associated to unemployed agents and workers of the highest skill type ω_n given by $\gamma_0 = 2 - \frac{1}{a}$ and $\gamma_n = 2 - \frac{a}{1+a^{2-n}(a^2-1)}$, respectively, the average weight $\bar{\gamma}_n = G_n\left(\tilde{r}_n\right)\gamma_n + \left[1 - G_n\left(\tilde{r}_n\right)\right]\gamma_0$ associated to agents of skill type ω_n is below unity.*

Proof. First, the average $G_n(\tilde{r}_n)\gamma_n + [1 - G_n(\tilde{r}_n)]\gamma_0$ can only be below 1 if γ_n is below 1. Given the definition of n, this is true if and only if

$$\begin{aligned}\gamma_n = 2 - \frac{a}{1 + a^{2-n}(a^2-1)} < 1 \\ \Leftrightarrow a^{2-n}\left(a^2-1\right) < a - 1 \\ \Leftrightarrow (n-2)\ln(a) > \ln(a+1) \\ \Leftrightarrow n > 2 + \frac{\ln(a+1)}{\ln(a)},\end{aligned}$$

which is identical to the lower bound imposed on n. Then, the average is negative if the share of workers $G_n(\tilde{r}_n)$ is above $\frac{\gamma_0 - 1}{\gamma_0 - \gamma_n} < 1$. By Lemma 12, $G_n(r_n) = \frac{1}{\bar{\delta}}\frac{\omega_n}{2(2-\gamma_n)}$ for $\alpha_n^U = \gamma_n$. Solving for $\bar{\delta}$ gives the upper bound imposed on the length of the fixed cost space $\left[0, \bar{\delta}\right]$. □

Lemma 15. *Under assumptions 1 and 2, and with the social weight sequence β equaling sequence γ as defined in Assumption 3, the upward IC constraint between skill types ω_1 and ω_2, and the downward IC constraints between all other skill types are satisfied with equality. Moreover, $\gamma_j > \underline{\beta}_j = 2 - a$ for all $\omega_j \in \Omega$.*

Proof. The elements of sequence γ are defined as $\gamma_0 = \gamma_1 = \gamma_2 = \bar{\beta}_j = 2 - \frac{1}{a}$ and $\gamma_j = 2 - \frac{a}{1+a^{2-j}(a^2-1)}$ for all $j \geq 3$. This sequence is designed in such a way that, by functions β_j^D and β_j^U defined in Lemma 13, $\gamma_2 = \beta_1^U(\gamma_1)$ and $\gamma_j = \beta_j^D(\gamma_{j-1})$ for all $j \geq 3$. As long as $\tilde{r}_j(\gamma_j) < \bar{\delta}$ for all skill types, this implies that the relaxed problem's solution satisfies with equality one of the IC constraints for each pair ω_j, ω_{j+1} in Ω. By the construction of sequence γ, $\tilde{r}_n(\gamma_n) > \tilde{r}_j(\gamma_j)$ for all $j < n$ (otherwise, the downward ICs could not be satisfied). As $\gamma_n < 1$ by Lemma 14, $\tilde{r}_n(\gamma_n) < \hat{s}_n < \bar{\delta}$, where the last inequality follows from assumption *REM*.

Thus, if the weight sequence α^U would be identical to γ, then the upward IC constraint between skill types ω_1 and ω_2 would be satisfied with equality. Moreover, it would be violated for any $\alpha_1^U = \alpha_2^U > \gamma_1$. Furthermore, if $\alpha^U = \gamma$, the downward IC constraints between all pairs ω_j and ω_{j+1} in Ω with $j \geq 2$ are satisfied with equality.

Finally, $\beta_j^D(\beta) \in (2 - a, \beta)$ holds if and only if $\beta > 2 - a$. Thus, $\gamma_j > \underline{\beta}_j = 2 - a$ for all j and the sequence is strictly decreasing with $\gamma_{j+1} < \gamma_j$ for all $j \geq 2$. □

Proof of Proposition 7

Proof. Under Assumption 3, the population average over sequence γ in the relaxed problem's solution is below unity:

$$\begin{aligned}\bar{\gamma} &= \sum_{j=1}^{n} f_j \left[G_j(\tilde{r}_j)\gamma_j + [1 - G_j(\tilde{r}_j)]\,\gamma_0\right] = \sum_{j=1}^{n} f_j \bar{\gamma}_j \\ &= (1 - f_n)\,\bar{\gamma}_{-n} + f_n \bar{\gamma}_n < 1\end{aligned}$$

Thus, the sequence γ as defined above cannot be a sequence of social weights. It is however possible to construct a similar sequence $\tilde{\gamma}$ with $\tilde{\gamma}_0 > \tilde{\gamma}_1 > \gamma_1$, $\tilde{\gamma}_2 \in \left(\beta_1^U(\tilde{\gamma}_1), \tilde{\gamma}_1\right)$, $\tilde{\gamma}_3 > \beta_2^D(\tilde{\gamma}_2)$ and $\tilde{\gamma}_{j+1} = \beta_j^D(\gamma_j)$ for all $j \geq 3$ such that the average weight is given by 1. Recall that $\beta_1^U(x) < x$ for all $x > \gamma_1 = 2 - \frac{1}{a}$ by Lemma 13.

By construction, the sequence $\tilde{\gamma}$ is strictly decreasing throughout and thus satisfies assumption DUR. Furthermore, the relaxed problem's solution satisfies all downward ICs between skill types ω_2 and ω_n, but violates the upward IC constraint between skill types ω_1 and ω_2. The solution to the optimal tax problem thus involves an upward distortion in y_2 at the intensive margin (the proof of Lemma 6 below shows in more detail that a binding upward IC constraint gives rise to an upward distortions at the intensive margin). Labor supply by all other skill groups is undistorted at the intensive margin. In particular, as $\tilde{\gamma}$ is designed so that the downward IC between skill types ω_2 and ω_3 is slack, it clearly is still slack with a small upward distortion in y_2.

Furthermore, labor supply by skill types ω_1 and ω_2 is upward distorted at the extensive margin, as $\tilde{r}_1 > \hat{s}_1$ by $\alpha_1 > 1$ and $r_1 > \tilde{r}_1$ due to the binding upward IC, while $r_2 = r_1 + h(y_2, \omega_1) - h(y_2, \omega_2) > \tilde{r}_1 + h(\hat{y}_2, \omega_1) - h(\hat{y}_2, \omega_2) > \hat{s}_2$. □

Under Assumption 1, a social objective U giving rise to social weight sequence $\alpha^U = \tilde{\gamma}$ can be derived explicitly. Given the uniform distribution of fixed costs, the social weights are given by $\alpha_j^U(c, y) = \frac{1}{r_j}\left[U(r_j + b) - U(0)\right]$ for all $j \in [1, n]$, while $\alpha_0^U(c, y) = U'(b)$. Thus, $\alpha^U = \tilde{\gamma}$ if and only if $U(r_j + b) = U(b) + r_j \tilde{\gamma}_j$ and $U'(b) = \tilde{\gamma}_0$, where $(r_j)_{j=1}^n$ and b solve the set of first-order conditions of the optimal tax problem setting weights according to sequence $\tilde{\gamma}$.

Proof of Lemma 4 For $\tilde{A} = 0$, the auxiliary problem is solved by setting $T_1^P = T_2^P = 0$. Consider a relaxed problem in which both IC constraints (23) and (23) are ignored. Then, $y_j = \hat{y}_j$ for $j \in \{1, 2\}$. For $\hat{\delta}_j < \bar{\delta}$, the first-order condition with respect to T_j^P is given by

$$\mathcal{L}_{T_j^P} = f_j \left[G_j(\hat{\delta}_j)\,(-1 + \lambda) - \lambda g_j(\hat{\delta}_j) T_j^P\right] = 0,$$

where λ represents the Lagrange multiplier associated with the feasibility constraint. Combining the FOCs with respect to T_1^P and T_2^P, both need to have the same sign. Thus, the feasibility constraint (21) can only be satisfied if $T_1^P = T_2^P = 0$, which also satisfies the second-order condition. This solution satisfies both IC constraints (23) and (23), and involves $\hat{\delta}_1 < \hat{\delta}_2 < \bar{\delta}$ by Assumption *REM*. Thus, the solution to the relaxed problem is given by $T_j^P = 0$ and $y_j = \hat{y}_j$ for both skill groups.

With respect to the upper bound A_{max}, consider first the problem of maximizing revenue from participation taxes, $\sum_{j=1}^{2} f_j G_j \left[s\left((y_j, \omega_j) - T_j^P\right] T_j^P\right.$, if the social planner is not restricted by IC constraints. The first-order conditions with respect to T_j^P is given by

$$\begin{aligned} f_j \left[G_j\left(s\left(y_j, \omega_j\right) - T_j^P\right) - g_j\left(s\left(y_j, \omega_j\right) - T_j^P\right) T_j^P\right] = & \quad 0 \\ \Leftrightarrow T_j^P = \frac{G_j\left(s\left(y_j, \omega_j\right) - T_j^P\right)}{g_j\left(s\left(y_j, \omega_j\right) - T_j^P\right)} \end{aligned}$$

While the left-hand side is increasing in T_j^P, the right-hand side is strictly decreasing by the log-concavity of G_j. As the left-hand side is smaller than the right-hand side for $T_j^P = 0$ and larger for $T_j^P \leq s(y_j, \omega_j) - \underline{\delta}$, the tax maximization problem has a unique maximizer $\left(T_1^{P^*}, T_2^{P^*}\right)$ and a unique maximum $\bar{A} < \sum_{j=1}^{2} f_j \left(s(y_j, \omega_j) - \underline{\delta}\right)$. Taking the IC constraints into account, this maximum is weakly lower, given by some level $A_{max} \leq \bar{A}$. By the construction of A_{max}, the auxiliary problem has no solution in reals for revenue requirements $\tilde{A} > A_{max}$.

With respect to the lower bound A_{min}, note first that, for any level $\tilde{A}$, one Pareto efficient allocation involves uniform taxation $T_1^P = T_2^P = T_E$. In this point, no IC is binding, so that $y_j = \hat{y}_j$ for both groups of workers. If $T_E < \hat{s}_1 - \bar{\delta} < \hat{s}_2 - \bar{\delta}$, then all workers of both skill levels would work under uniform taxation, i.e., $\hat{\delta}_j = \bar{\delta}$ for $j \in \{1, 2\}$. Then, the negative revenue created by tax level T_E is given by $[f_1 + f_2]\, T_E$. In every other allocation on the Pareto frontier, workers of one skill group must be better off. Thus, $s(y_j, \omega_j) - T_j^P > \hat{\delta}_j = \bar{\delta}$ for at least one skill group and any $\tilde{A} < [f_1 + f_2]\left(\hat{s}_1 - \bar{\delta}\right) \equiv \underline{A}$. One can conclude that there must be some $A_{min} \in (\underline{A}, 0)$ such that $\hat{\delta}_j < \bar{\delta}$ for both skill levels is true in the surplus-maximizing allocation only if $\tilde{A} < A_{min}$.

It remains to show the *if* part, i.e., uniqueness of the threshold A_{min} satisfying $s(y_j, \omega_j) - T_j^P = \hat{\delta}_j = \bar{\delta}$ for one group and $\hat{\delta}_k < \bar{\delta}$ for the other group. By the downward IC constraint, $\hat{\delta}_2 > \hat{\delta}_1$ as long as both are below $\bar{\delta}$. Reducing $\tilde{A}$ further requires either reducing T_1^P or T_2^P. While the former induces further distortions at the extensive margin, the latter has no effect on labor market participation. Thus, the social planner will choose $\hat{s}_2 - T_j^2$ strictly above $\bar{\delta}$ for any $\tilde{A} < A_{min}$. This implies that $\hat{\delta}_2 = \bar{\delta}$ for all $\tilde{A} < A_{min}$, which is consequently unique.

Proof of Lemma 5

Proof. Again, I first solve the auxiliary problem in terms of employment rents (r_1, r_2) and workloads (y_1, y_2). Then, I substitute in the participation tax levels $T_j^P = s(y_j, \omega_j) - r_j$. By Lemma 4, $r_j = \hat{\delta}_j = s(y_j, \omega_j) - T_j^P < \bar{\delta}$ for all $\tilde{A} \in (A_{min}, A_{max})$. Thus, the Lagrangian of the auxiliary problem can be written

$$
\begin{aligned}
\mathcal{L} = \quad & \sum_{j=1}^{2} f_j \left[\int_{\underline{\delta}}^{r_j} g_j(\delta) \left(r_j + b - \delta \right) d\delta + \left(1 - G_j(r_j) \right) b \right] \\
& + \lambda \left[\sum_{j=1}^{2} f_j G_j(r_j) \left[s\left(y_j, \omega_j \right) - r_j \right] - A - \left(f_1 + f_2 \right) b \right] \\
& + \mu_D \left[r_2 - r_1 - h\left(y_1, \omega_1 \right) + h\left(y_1, \omega_2 \right) \right] \\
& + \mu_U \left[r_1 - r_2 + h\left(y_2, \omega_1 \right) - h\left(y_2, \omega_2 \right) \right],
\end{aligned}
$$

where $\mu_D > 0$ ($\mu_D = 0$) if the downward IC is binding (not binding), and $\mu_U > 0$ ($\mu_U = 0$) if the upward IC is binding (not binding). The first-order conditions with respect to T_1^P, T_2^P, y_1, y_2 are given as

$$
\begin{aligned}
\mathcal{L}_{r_1} &= f_1 \left[G_1(r_1) \left(1 - \lambda \right) - \lambda g_1(r_1) \left[s\left(y_1, \omega_1 \right) - r_1 \right] \right] - \mu_D + \mu_U &&= 0 \\
\mathcal{L}_{r_2} &= f_2 \left[G_2(r_2) \left(1 - \lambda \right) - \lambda g_2(r_2) \left[s\left(y_2, \omega_2 \right) - r_2 \right] \right] + \mu_D - \mu_U &&= 0 \\
\mathcal{L}_{y_1} &= \lambda f_1 G_1(r_1) s_y\left(y_1, \omega_1 \right) - \mu_D \left[h_y(y_1, \omega_1) - h_y(y_1, \omega_2) \right] &&= 0 \\
\mathcal{L}_{y_2} &= \lambda f_2 G_2(r_2) s_y\left(y_2, \omega_2 \right) + \mu_U \left[h_y(y_2, \omega_1) - h_y(y_2, \omega_2) \right] &&= 0
\end{aligned}
$$

By the Lagrange theorem, the multipliers μ_1 and μ_2 are positive if the corresponding IC constraint is binding. If the downward IC constraint is not binding, $\mu_D = 0$, then the first-order condition with respect to Y_1 implies that $s_y\left(y_1, \omega_1 \right) = 0$, i.e., labor supply by low-skill workers is undistorted at the intensive margin with $y_1 = \hat{y}_1$. If the downward IC constraint is instead binding, $\mu_D > 0$, then the single-crossing condition implies that $s_y\left(y_1, \omega_1 \right) > 0$ must be true, i.e., y_1 is strictly downward distorted. By the corresponding arguments, high-skill labor supply is undistorted if the upward IC is not binding, $\mu_U = 0$, and strictly upward distorted if it is binding.

Thus, we have $y_2 \geq \hat{y}_2 > \hat{y}_1 \geq y_1$ in every solution to this problem, implying that there cannot be pooling of high-skill workers and low-skill workers. The single-crossing condition then implies that $h(y_2, \omega_1) - h(y_2, \omega_2) > h(y_1, \omega_1) - h(y_1, \omega_2)$ holds. Consequently, at most one IC constraint is binding in any implementable allocation.

The main question then is which, if any, of the IC constraints is actually binding in the surplus-maximizing allocation. I first study a relaxed problem in which both incentive compatibility constraints are ignored, and then check explicitly whether the solution to this relaxed problem violates one of the ignored constraints. For clarity, we denote the relaxed problem's solution for variable x by $\tilde{x}$.

The FOCs of this relaxed problem equal the ones of the auxiliary problem, setting $\mu_1 = \mu_2 = 0$. As argued above, the FOC with respect to y_j then requires $s_y(y_j, \omega_j) = 0$. Thus, labor supply is undistorted at the intensive margin, with $y_j = \hat{y}_j$ and $s(y_j, \omega_j) = \hat{s}_j$ for both skill groups. Second, rearranging the first-order conditions with respect to r_j gives

$$\hat{s}_j - \tilde{r}_j = \frac{\lambda - 1}{\lambda} \frac{G_j(\tilde{r}_j)}{g_j(\tilde{r}_j)}$$

Recall that $r_j = c_j - h(y_j, \omega_j) - b$, so that $\frac{\partial G_j(r_j)}{\partial c_j} = g_j(r_j)$. Thus, the semi-elasticity of the participation share of type ω_j workers with respect to the net labor income c_j is given by the fraction $\frac{g_j(r_j)}{G_j(r_j)}$. Replacing r_j by $s(y_j, \omega_j) - T_j^P$ gives the inverse elasticity rule (25).

The inverse elasticity rule has the following two implications for the relaxed auxiliary problem. First, both participation taxes have the same sign as the semi-elasticities of both skill groups are strictly positive for any $\tilde{A} \in [A_{min}, A_{max}]$. To satisfy the feasibility constraint (21), they have to be positive (negative) if $\tilde{A} = A + (f_1 + f_2)b$ is positive (negative). For $\tilde{A}$, both participation taxes have to equal zero.

Second, the higher the semi-elasticity of participation, the lower is the absolute value of the surplus-maximizing participation tax $\tilde{T}_j^P$. Thus, the surplus-maximizing taxes satisfy

$$\frac{\tilde{T}_2^P}{\tilde{T}_1^P} = \frac{G_2(\tilde{r}_2)}{g_2(\tilde{r}_2)} \frac{g_1(\tilde{r}_1)}{G_1(\tilde{r}_1)}$$

Thus, the optimal participation taxes depend crucially on the relative sizes of both semi-elasticities. For any allocation with $r_2 > r_1$, Assumptions *LC* and *OFCD* imply that the semi-elasticity for low-skill workers must be larger than the one for high-skill workers. More precisely, Assumption *LC* ensures that $\frac{G_2(\tilde{r}_2)}{g_2(\tilde{r}_2)} > \frac{G_2(\tilde{r}_1)}{g_2(\tilde{r}_1)}$ if $r_2 > r_1$. Assumption *OFCD* implies that $\frac{G_2(\tilde{r}_1)}{g_2(\tilde{r}_1)} \geq \frac{G_1(\tilde{r}_1)}{g_1(\tilde{r}_1)}$.

For the non-relaxed auxiliary problem, the inequality $r_2 > r_1$ is ensured for all levels of $\tilde{A}$ by the downward IC constraint. For the relaxed problem, this is immediately clear only for $\tilde{A} = 0$, where $T_2^P = T_1^P = 0$ ensures $r_2 = \hat{s}_2 > r_1 = \hat{s}_1$. It can be shown, however, that there is no level $\tilde{A}$ for which $\frac{G_2(\tilde{r}_2)}{g_2(\tilde{r}_2)} = \frac{G_1(\tilde{r}_1)}{g_1(\tilde{r}_1)}$. By the inverse elasticity rule, it would then be optimal to set identical taxes, $T_2^P = T_1^P$. But then, we would again have $r_2 = \hat{s}_2 - T_2^P > r_1 = \hat{s}_1 - T_1^P$, which implies $\frac{G_2(\tilde{r}_2)}{g_2(\tilde{r}_2)} > \frac{G_1(\tilde{r}_1)}{g_1(\tilde{r}_1)}$. Because the solution $(\tilde{r}_2, \tilde{r}_1)$ is continuous in $\tilde{A}$, this also rules out $\frac{G_2(\tilde{r}_2)}{g_2(\tilde{r}_2)} < \frac{G_1(\tilde{r}_1)}{g_1(\tilde{r}_1)}$ for any levels of $\tilde{A}$. We can conclude that the semi-elasticity of low-skill workers is larger than the one of high-skill workers in every solution to the relaxed auxiliary problem as well.

Thus, the optimal ratio of participation taxes always satisfies $\frac{\tilde{T}_2^P}{\tilde{T}_1^P} > 1$. For all $\tilde{A} < 0$, this implies $\tilde{T}_2^P < \tilde{T}_1^P < 0$. Thus, the relaxed problem's solution satisfies the downward IC $T_2^P - T_1^P \leq s(y_2, \omega_2) - s(y_1, \omega_2)$, where the right-hand side is strictly positive. Without further assumptions on the properties of G_1 and G_2, it cannot be determined whether relaxed problem satisfies the upward IC constraint. If it does, the relaxed problem's solution $(\tilde{T}_1^P, \tilde{T}_2^P, \hat{y}_1, \hat{y}_2)$ also solves the non-relaxed problem. Then, no IC constraint is binding in the surplus-maximizing allocation, which furthermore involves $T_2^{P^S} < T_1^{P^S} < 0$ and $r_j > \hat{s}_j$ for both skill levels, as claimed in Lemma 5. This will certainly be true in some neighborhood of $\tilde{A} = 0$, where $T_2^P \approx T_1^P$.

If the relaxed problem's solution instead violates the upward IC constraint, this constraint will be binding, and its Lagrange multiplier μ_U will be strictly positive in the surplus-maximizing allocation. Then, the first-order condition with respect to y_2 implies

$$s_y(y_2, \omega_2) = -\frac{\mu_U}{\lambda f_2 G_2(r_2)} \left[h_y(y_2, \omega_1) - h_y(y_2, \omega_2)\right] < 0 \ ,$$

where the term $[h_y(y_2, \omega_1) - h_y(y_2, \omega_2)]$ is strictly positive by the single-crossing property. By the strict concavity of s in y, we have $y_2 > \hat{y}_2 > \hat{y}_1$ and, by standard arguments, $T_1^{P^S} < \tilde{T}_1^P < 0$. Jointly, this implies $T_2^{P^S} = T_1^{P^S} + s(y_2, \omega_1) - s(\hat{y}_1, \omega_1) < T_1^{P^S} < 0$.

In the second case, $\tilde{A} > 0$, a similar argument implies that either the relaxed problem's solution also solves the non-relaxed problem, so that $T_2^{P^S} = \tilde{T}_2^P > T_1^{P^S} = \tilde{T}_1^P > 0$, or the downward IC is binding, $y_1 < \hat{y}_1 < \hat{y}_2$. Then, we have $T_1^{P^S} > \tilde{T}_1^P > 0$, and $T_2^{P^S} = T_1^{P^S} + s(\hat{y}_2, \omega_2) - s(y_1, \omega_2) > T_1^{P^S} > 0$. □

Proof of Lemma 6

Proof. First, consider again the relaxed problem. With the assumed uniform distribution on $\left[0, \bar{\delta}\right]$, we have $\frac{G_j(r_j)}{g_j(r_j)} = r_j = \hat{s}_j - T_j^P$. Inserting this into the inverse elasticity formulas for optimal tax rates (25), the optimal ratio of participation tax rates is given by

$$\frac{\tilde{T}_2^P}{\tilde{T}_1^P} = \frac{\hat{s}_2 - T_2^P}{\hat{s}_1 - T_1^P} = \frac{\hat{s}_2}{\hat{s}_1} > 1$$

This implies that both participation tax levels are strictly increasing in $\tilde{A}$, and that $\frac{dT_2^P}{d\tilde{A}} > \frac{dT_1^P}{d\tilde{A}}$ on the interval $[A_{min}, A_{max}]$. Thus, if there is some level of $\tilde{A}$ at which the upward (downward) IC is violated by the relaxed problem's solution, then the same is also true for all lower (higher) levels.

For ease of notation, define the auxiliary parameter $q \equiv \frac{\hat{s}_1}{\hat{s}_2} < 1$. Thus, the difference in participation taxes is given by $\tilde{T}_2^P - \tilde{T}_1^P = (1-q)\tilde{T}_2^P$. For any level $\tilde{A} < 0$, this difference is negative by Lemma 5. The relaxed problem's solution violates the upward IC if

$$\begin{aligned} \tilde{T}_2^P - \tilde{T}_1^P = (1-q)\tilde{T}_2^P &< s\left(y_2, \omega_1\right) - s\left(y_1, \omega_1\right) \\ \Leftrightarrow \tilde{T}_2^P &< \frac{s\left(\hat{y}_2, \omega_1\right) - \hat{s}_1}{1-q} \equiv z_U \end{aligned}$$

Note that term z on the right-hand side of this inequality only depends on exogenous parameters, while the left-hand side is strictly increasing in $\tilde{A}$. On the Pareto-frontier, the feasibility condition holds with equality. Substituting in the optimal ratio of participation tax levels then gives

$$\tilde{A} = f_1 G_1\left(\hat{\delta}_1\right) T_1^P + f_1 G_2\left(\hat{\delta}_2\right) T_2^P = \left(f_1 q^2 + f_2\right) \frac{\hat{s}_2 - T_2^P}{\bar{\delta}} T_2^P .$$

By Lemma 6, if the highest fixed cost type $\bar{\delta}$ is sufficiently large, there is a threshold $A_U \in (A_{min}, 0)$ such that the surplus-maximizing allocation involves upward distortions in y_2 for all $\tilde{A} \in (A_{min}, A_U)$. In particular, $A_U > A_{min}$ holds if and only if $\bar{\delta} > \hat{s}_2 - z_U$ is true.

First, the solution to the relaxed problem involves $\hat{\delta}_2 < \bar{\delta}$ if and only if $\tilde{T}_2^P > \hat{s}_2 - \bar{\delta}$. Second, it violates the upward IC constraint if and only if $\tilde{T}_2^P < z_U$. If $\bar{\delta} < \hat{s}_2 - z$, both conditions cannot hold at the same time. Then, the lower bound of $\tilde{A}$ for interior solutions is given by $A_{min} = \left(f_1 q^2 + f_2\right)\left[\hat{s}_2 - \bar{\delta}\right]$, and the upward IC constraint is satisfied for all $\tilde{A} > A_{min}$.

If instead $\bar{\delta} > \hat{s}_2 - z_U$, then both conditions can hold simultaneously. In this case, the upward IC constrained is satisfied by the relaxed problem's solution, and is slack in the surplus-maximizing allocation if and only if $\tilde{A} \geq A_U = \left(f_1 q^2 + f_2\right) \frac{\hat{s}_2 - z}{\bar{\delta}} z$. If $\tilde{A}$ is between A_U and $\left(f_1 q^2 + f_2\right)\left[\hat{s}_2 - \bar{\delta}\right]$, the relaxed problem has an interior solution with $\hat{\delta} = q\hat{\delta}_2 < \hat{\delta}_2 < \bar{\delta}$ and violated the upward IC constraint.

In the non-relaxed problem, the upward IC is thus binding and high-skill labor supply is upwards distorted at the intensive margin, $y_2 > \hat{y}_2$. Moreover, $T_2^P > \tilde{T}_2^P$ because further reductions in T_2^P would require even stronger upward distortions in y_2. Thus, the threshold A_{min} for an interior solution with $\hat{\delta}_2 < \bar{\delta}$ is given by some level $A_{min} < (f_1 q^2 + f_2) \left[\hat{s}_2 - \bar{\delta}\right] < A_U$.

Similar arguments can be made with respect to the threshold A_D above which the downward IC becomes binding. With uniformly distributed taxes, the downward IC constraint is given by

$$T_2^P - T_1^P \leq s\left(y_2, \omega_2\right) - s\left(y_1, \omega_2\right)$$

For the relaxed problem, the Laffer rates are given by $\tilde{T}_2 = \frac{\hat{s}_2}{2}$ and $\tilde{T}_1 = \frac{\hat{s}_1}{2} = q\tilde{T}_2 < \tilde{T}_2$. Inserting the optimal ratio of taxes, the downward IC constraint then follows as

$$\begin{aligned} (1-q)\frac{\hat{s}_2}{2} &\leq \hat{s}_2 - s\left(\hat{y}_1, \omega_2\right) \\ \Leftrightarrow (1+q)\frac{\hat{s}_2}{2} &\geq s\left(\hat{y}_1, \omega_2\right) \end{aligned}$$

Both sides of this inequality contain only exogenous variables. Whether the downward IC is satisfied or violated for Laffer rates in the relaxed problem thus only depends on properties of the variable cost function h and the difference between skill levels ω_1 and ω_2. If the inequality above is satisfied, then the downward IC is slack in the surplus-maximizing allocation for all levels $\tilde{A}$ in the interval (A_{min}, A_{max}). If is is instead violated, then there is a threshold $A_D \in (0, A_{max})$ such that the downward IC is binding, and y_1 is downward distorted in the surplus-maximizing allocation for all levels of $\tilde{A} \in (A_D, A_{max})$.

This result seems to contrast with the result for threshold A_U, which is above A_{min} if and only if $\bar{\delta}$ is sufficiently large. Allowing for $\underline{\delta} \neq 0$, however, one can also show that A_D is below A_{max} if and only if $\underline{\delta}$ is sufficiently small. □

B Proofs for Section 7

Proof of Proposition 8

In the following, I assume that the social planner observes fixed cost types, while the agents are privately informed about their skill types only. Proposition 8 studies optimal utilitarian income taxation given this information structure. Then, observable fixed costs types can be used for tagging, i.e., the social planner is able to design specific tax schedules for each fixed cost group. For example, he might choose different benefit payments for unemployed agents with different fixed costs types.

For readability, I denote in the following the consumption-output bundle allocated to agents of type (ω_j, δ) by $c_j(\delta) = c(\omega_j, \delta)$, and $y_j(\delta) = y(\omega_j, \delta)$. Furthermore, I rewrite the joint type distribution Ψ using the functions $G(\delta)$ and $F(\delta)$. $G(\delta)$ denotes the unconditional cdf of fixed costs, with pdf $g(\delta) > 0$ if and only if $\delta \in \Delta$. $F(\delta)$ represents the cdf of skill types ω in the group of agents with fixed cost type δ, while the share of agents with skill type ω_j is denoted by $f_j(\delta)$.

Lemma 16. *With observable fixed cost types, an allocation is incentive compatible if and only if, in each group of agents with fixed cost type $\delta \in \Delta$,*

(i) there is a unique threshold type $k(\delta) \in \mathbb{N}$ such that all agents with skill type $\omega_j < \omega_{k(\delta)}$ are unemployed and receive the same cost-specific benefit $b(\delta) \in \mathbb{R}$, while all agents with skill type $\omega_j \geq \omega_{k(\delta)}$ provide positive output $y_j(\delta) > 0$,

(ii) if $\omega_{k(\delta)} > \omega_1$, the allocation of the threshold worker type $(\omega_{k(\delta)}, \delta)$ satisfies

$$c_{k(\delta)}(\delta) - h\left(y_{k(\delta)}, \omega_{k(\delta)}\right) \geq b(\delta) + \delta \geq c_{k(\delta)}(\delta) - h\left(y_{k(\delta)}, \omega_{k(\delta)-1}\right), \text{ and}$$

(iii) if $\omega_{k(\delta)} < \omega_n$, the allocations of all workers with skill types $\omega_j \geq \omega_{k(\delta)}$ satisfy

$$h\left(y_{j+1}(\delta), \omega_j\right) - h\left(y_j(\delta), \omega_j\right) \geq c_{j+1}(\delta) - c_j(\delta) \geq \\ h\left(y_{j+1}(\delta), \omega_{j+1}\right) - h\left(y_j(\delta), \omega_{j+1}\right).$$

Proof. For part (i), consider first two types (ω_i, δ) and (ω_j, δ) such that $y_i(\delta) = y_j(\delta) = 0$. Incentive compatibility requires that $c_i(\delta) = c_j(\delta) = b(\delta)$, which is the benefit receives by all unemployed agents with fixed cost type δ. Second, consider some employed type (ω_j, δ) with $y_j(\delta) > 0$. Incentive compatibility requires $c_j(\delta) - h\left(y_j(\delta), \omega_j\right) - \delta \geq b(\delta)$. By single-crossing, all agents with higher skill type prefer bundle $(c_j(\delta), y_j(\delta))$ strictly to bundle $(b(\delta), 0)$, and must thus provide positive output in any incentive-compatible allocation. Symmetrically, if there is some type (ω_i, δ) that weakly prefers unemployment, then all agents with lower skill type will strictly prefer unemployment. Thus, there is a unique threshold $\omega_{k(\delta)} \in [\omega_1, \omega_n]$ for each fixed cost level.

For parts (ii) and (iii), note that we only need to consider incentive compatibility constraints between agents with identical fixed cost δ. The inequalities given in part (ii) guarantee that $\omega_{k(\delta)}$ is indeed the threshold skill level. The inequalities in part (iii) represent standard IC constraints between adjacent skill types. As usual, the single-crossing property implies that global incentive-compatibility holds if and only if all local IC constraints are satisfied. □

Lemma 17. *At any utilitarian allocation, the downward IC constraint for the threshold worker type $\omega_{k(\delta)}$ is binding in each group of agents with fixed cost type $\delta \in \Delta$, i.e., $c_{k(\delta)}(\delta) - h\left(y_{k(\delta)}, \omega_{k(\delta)}\right) = b(\delta) + \delta$ holds.*

Proof. Given Lemma 16, the planner's objective can be written

$$W(c, y) = \int_{\underline{\delta}}^{\bar{\delta}} W_\delta(c(\delta), y(\delta)) dG(\delta),$$

where the cost-group welfare level $W_\delta(c(\delta), y(\delta))$ for each $\delta \in \Delta$ is given by

$$W_\delta(c(\delta), y(\delta)) = F_{k(\delta)-1}(\delta) U\left[b(\delta)\right] + \sum_{j=k(\delta)}^{n} f_j(\delta) U\left[c_j(\delta) - h\left(y_j(\delta), \omega_j\right) - \delta\right].$$

The feasibility constraint can be divided into a global constraint $\int_{\underline{\delta}}^{\bar{\delta}} A(\delta) dG(\delta) \geq 0$ and a set of cost-dependent constraints $\sum_{j=k(\delta)}^{n} f_j(\delta)\left[y_j(\delta) - c_j(\delta) + b(\delta)\right] \geq b(\delta) + A(\delta)$. The set of incentive-compatibility constraints is given as in parts (ii) and (iii) of Lemma 16.

By standard arguments, any utilitarian allocation satisfies the feasibility constraints with equality. The function of cost-specific revenues $A(\delta)$ is chosen to equate average marginal utilities (and average endogenous weights) in all fixed cost groups, which typically implies redistribution from low-cost groups to high-skill groups. Within each fixed cost group, the functions $c(\delta)$, $y(\delta)$ and the benefit $b(\delta)$ are chosen to maximize cost-specific welfare $W_\delta(c(\delta), y(\delta))$ subject to the cost-specific revenue requirement $A(\delta)$ and the cost-specific IC constraints.

A proof by contradiction demonstrates that the threshold worker type $\left(\omega_{k(\delta)}, \delta\right)$ must be indifferent between employment and unemployment, i.e., the downward IC between types $\left(\omega_{k(\delta)}, \delta\right)$ and $\left(\omega_{k(\delta)-1}, \delta\right)$ must be binding in any utilitarian allocation. Assume this were not the case, i.e., there is an incentive compatible and feasible allocation that maximizes welfare and involves $c_{k(\delta)}(\delta) - h\left(y_{k(\delta)}, \omega_{k(\delta)}\right) > b(\delta) + \delta$.

Then, leaving $y(\delta)$ unchanged, reducing $c_j(\delta)$ uniformly by a small amount $\varepsilon > 0$ for all workers with $\omega_j \geq \omega_{k(\delta)}$ and increasing the unemployment benefit $b(\delta)$ by $\varepsilon \left[1 - F_{k(\delta)-1}(\delta)\right] / F_{k(\delta)-1}(\delta)$ would be possible without violating feasibility or incentive-compatibility. The marginal welfare effect of this variation is given by

$$\frac{dW_\delta}{d\varepsilon} = \left[1 - F_{k(\delta)-1}(\delta)\right] \alpha_0(\delta) - \sum_{j=k(\delta)}^{n} f_j(\delta)\alpha_j(\delta) > 0$$

This is positive as Assumption DUR δ implies $\alpha_0'(c, y, \delta) > \alpha_j'(c, y, \delta)$ for all $j \geq k(\delta)$. Thus, the original allocation cannot be a utilitarian allocation.

Note that, with observable fixed costs, increasing $b(\delta)$ induces extensive margin responses if and only if it conflicts with the IC constraint for type $(\omega_{k(\delta)-1}, \delta)$. Thus, an equity-efficiency tradeoff can arise if and only if the downward IC of type $\left(\omega_{k(\delta)}, \delta\right)$ is binding. □

Lemma 18. *At any utilitarian allocation, all downward IC constraints between active workers with $\omega_j \geq \omega_{k(\delta)}$ are binding in each group of agents with fixed cost type $\delta \in \Delta$:*

$$\begin{aligned} c_{j+1}(\delta) - h\left(y_{j+1}(\delta), \omega_{j+1}\right) &= c_j(\delta) - h\left(y_j(\delta), \omega_{j+1}\right) \\ &= b(\delta) + \delta + \sum_{l=k(\delta)}^{j} \left[h\left(y_l(\delta), \omega_l\right) - h\left(y_l(\delta), \omega_{l+1}\right)\right] \quad . \end{aligned}$$

Proof. I only provide a sketch of the proof, because it is based on standard arguments that are familiar from the literature on optimal income taxation with labor supply responses at the intensive margin only (see, e.g., Mirrlees 1971). Consider some feasible and incentive-compatible allocation in which the downward IC constraint between types (ω_j, δ) and (ω_{j+1}, δ) is not binding, where $\omega_j \geq \omega_{k(\delta)}$. Then, it is possible to reduce consumption uniformly for all agents with skill type $\omega_i \geq \omega_{j+1}$, and using these resources for uniform transfers towards all agents with skill types $\omega_l \leq \omega_j$, until the downward IC constraint between agents with skill types ω_j and ω_{j+1} becomes binding. This is consistent with incentive-compatibility and feasibility, and yields a marginal welfare increase of

$$\begin{aligned} \frac{dW_\delta}{d\varepsilon} = \frac{1 - F_j(\delta)}{F_j(\delta)} &\left[F_{k(\delta)-1}(\delta)\alpha_0(\delta) + \sum_{l=k(\delta)}^{j} f_l(\delta)\alpha_l(\delta)\right] \\ &- \sum_{l=j+1}^{n} f_l(\delta)\alpha_l(\delta) > 0 \end{aligned}$$

As social weights are strictly decreasing in ω by Assumption *DUR* δ, this induces a strict welfare gain.

Thus, the downward IC must be binding between all pairs of skill types above $\omega_{k(\delta)}$, as well as for the threshold skill type $\omega_{k(\delta)}$. Consequently, $c_j(\delta)$ follows as a function of δ, $b(\delta)$ and the output levels $y_i(\delta)$ of all skill types $\omega_i \leq \omega_j$. □

Lemma 19. *At the intensive margin, labor supply is undistorted at the top skill level ω_n and strictly downwards distorted everywhere below the top for all workers in each group of agents with fixed cost type $\delta \in \Delta$.*

Proof. In the following, we write $x_j^\delta = x_j(\delta)$ for $x \in \{y, b, f, \lambda, A\}$ for reasons of readability. By Lemmas 17 and 18, the group-specific Lagrangian can be written

$$\begin{aligned}\mathcal{L}^\delta =& F^\delta_{k(\delta)-1} U\left[b^\delta\right] + \sum_{j=k(\delta)}^{n} f_j^\delta U\left[b^\delta + \sum_{l=k(\delta)}^{j-1}\left[h\left(y_l^\delta, \omega_l\right) - h\left(y_l^\delta, \omega_{l+1}\right)\right]\right] \\ &+ \lambda^\delta\left\{\sum_{j=k(\delta)}^{n} f_j^\delta\left[y_j^\delta - h\left(y_j^\delta, \omega_j\right) - \delta - \sum_{l=k(\delta)}^{j-1}\left[h\left(y_l^\delta, \omega_l\right) - h\left(y_l^\delta, \omega_{l+1}\right)\right]\right]\right. \\ &\left. - b^\delta - A^\delta\right\}\end{aligned}$$

Taking the derivative with respect to $b(\delta)$ implies that $\lambda(\delta)$ equals the cost-specific average weight $\bar{\alpha}(\delta)$. The derivative with respect to $y_j(\delta)$ is given by

$$\begin{aligned}\mathcal{L}_{y_j} =& \underbrace{\left[h_1\left(y_j(\delta), \omega_j\right) - h_1\left(y_j(\delta), \omega_{j+1}\right)\right]}_{>0} \underbrace{\sum_{l=j+1}^{n} f_l(\delta)\left[\alpha_l(\delta) - \lambda(\delta)\right]}_{<0} \\ &+ \lambda(\delta) f_j(\delta) \underbrace{\left[1 - h_1(y_j(\delta), \omega_j\right]}_{>0} = 0.\end{aligned}$$

By the single-crossing property, the term in the first bracket is strictly positive. As the social weights are decreasing with ω, the second term is strictly negative. Thus, the first-order condition can only be satisfied if $h_1\left(y_j(\delta), \omega_j\right) < 1$. In other words, labor supply is strictly downward distorted for all worker types below ω_n, $y_j(\delta) < \hat{y}_j$, in any utilitarian allocation. For the top skill level, the familiar "no-distortion-at-the-top" result prevails. Intuitively, the downward distortion in $y_j(\delta)$ slackens the downward IC constraint between types (ω_{j+1}, δ) and (ω_j, δ), allowing to redistribute more resources to lower skill types. Starting from $y_j(\delta) = \hat{y}_j$, this has negligible efficiency costs, but allows to achieve first-order equity gains. Again, the crucial difference to the model with two-dimensional private information is that changes in y_j do not involve labor supply responses at the extensive margin. □

Lemma 20. *At the extensive margin, labor supply is weakly downward distorted in each group of agents with fixed cost type $\delta \in \Delta$, and strictly downward distorted for some fixed cost levels $\delta \in \Delta$.*

Proof. Again, the Lemma can be proven by contradiction. Assume that a utilitarian allocation involves, for workers with skill type ω_j, some output requirements $(y_j(\delta))_{j=1}^n$ and $\hat{s}_{k(\delta)} < \delta$, i.e., upward distortions in labor supply at the extensive margin. By Lemmas 17 and 18, all downward IC constraints must be binding in any utilitarian allocation. Thus, an agent with threshold skill type $\omega_{k(\delta)}$ must be indifferent between employment and unemployment. In this allocation, the level of the unemployment benefit $b(\delta)$ is pinned down by the feasibility constraint:

$$\begin{aligned} b(\delta) = & \sum_{j=k(\delta)}^{n} f_j(\delta) \left[y_j(\delta) - h(y_j(\delta), \omega_j \right] - \delta - A(\delta) \\ & - \sum_{l=k(\delta)}^{j-1} \left[h(y_l(\delta), \omega_l) - h(y_l(\delta), \omega_{l+1}) \right] \end{aligned}$$

If $\hat{s}_{k(\delta)} \leq \delta$, welfare can be increased by removing agents of type $(\omega_{k(\delta)}, \delta)$ from the labor market by setting $y_{k(\delta)}(\delta) = 0$, while keeping the workloads and consumption levels of all agents with $\omega_j > \omega_{k(\delta)}$ constant. Because the former agents were indifferent between working and staying unemployed before, this is possible without violating any IC constraint. All else equal, the feasibility constraint is relaxed by

$$-f_{k(\delta)}(\delta) \left[y_{k(\delta)}(\delta) - h\left(y_{k(\delta)}(\delta), \omega_j \right) - \delta \right] > -f_{k(\delta)}(\delta) \left[\hat{s}_{k(\delta)} - \delta \right] \geq 0.$$

The first inequality follows due to the downward distortion in $y_{k(\delta)}(\delta)$ at the intensive margin (see Lemma 19), the second one by assumption. As the feasibility constraint is slack after this deviation, the consumption levels of all agents in the skill group can be increased uniformly, inducing a Pareto improvement. Consequently, the initial allocation with upward distortions at the extensive margin cannot represent a utilitarian optimum.

By the same argument, labor supply is strictly downward distorted at the intensive margin in all fixed costs groups such that $\delta = \hat{s}_j$ for some $\omega_j \in \Omega$. For skill groups with $\delta \in (\hat{s}_j, \hat{s}_{j+1})$, in contrast, labor supply is strictly downward distorted if and only if the social planner has a sufficiently strong desire for redistribution. □

Proof of Proposition 9

In the following, I assume that the social planner observes skill types, while the agents are privately informed about their fixed cost types only. Proposition 9 studies optimal utilitarian income taxation given this information structure. Then, the social planner can use skill types for tagging, i.e., can condition unemployment benefits as well as tax payments directly on an agent's skill type. Proposition 9 is proven by a series of lemmas.

Lemma 21. *In every implementable allocation, there is a unique fixed cost threshold type $\tilde{\delta}_j \in \Delta$ for each skill level $\omega_j \in \Omega$ such that each agent with skill type ω_j and*

(i) fixed cost type $\delta > \tilde{\delta}_j$ is unemployed and consumes a skill-specific benefit $b_j \in \mathbb{R}$,

(ii) fixed cost type $\delta \leq \tilde{\delta}_j$ provides positive output $y(\omega_j, \delta) > 0$ and enjoys a gross (of the fixed cost) utility $c(\omega_j, \delta) - h\left[y(\omega_j, \delta), \omega_j\right] = z_j = b_j + \tilde{\delta}_j$.

Proof. For part (i), consider agents with two fixed cost types δ and $\delta' \neq \delta$ such that $y(\omega_j, \delta) = y(\omega_j, \delta') = 0$. Incentive compatibility requires that $c(\omega_j, \delta) = c(\omega_j, \delta') = b_j$, which represents the unemployment benefit. For part (ii), consider agents with two fixed cost types δ and $\delta' \neq \delta$ such that $y(\omega_j, \delta) > 0$ and $y(\omega_j, \delta') > 0$. Incentive compatibility requires that $c(\omega_j, \delta) - h\left[y(\omega_j, \delta), \omega_j\right] = c(\omega_j, \delta') - h\left[y(\omega_j, \delta'), \omega_j\right] = z_j$. Note that incentive compatibility does not imply pooling of all workers with skill type ω_j. For the threshold type $\tilde{\delta}_j$, a worker with type (ω_j, δ) prefers his bundle to $(b_j, 0)$ if and only if $c(\omega_j, \delta) - h\left[y(\omega_j, \delta), \omega_j\right] - \delta = z_j - \delta \geq b_j$, i.e., if $\delta \leq z_j - b_j = \tilde{\delta}_j$. Symmetrically, unemployed agents prefer bundle $(b_j, 0)$ to the bundle of any worker if and only if $\delta \geq z_j - b_j = \tilde{\delta}_j$. □

Lemma 22. *An allocation is Pareto efficient in the set of implementable allocations if and only if, for each skill type $\omega_j \in \Omega$, all workers are allocated the same bundle $(c_j, \hat{y}_j)$ with undistorted labor supply at the intensive margin.*

Proof. By Lemma 21, each worker with type (ω_j, δ) is indifferent between his bundle $(c(\omega_j, \delta), y(\omega_j, \delta))$ and the bundles of all other types (ω_j, δ) such that $\delta \leq \tilde{\delta}_j$. With observable skills, the social planner does not have to satisfy incentive compatibility constraints between agents with different skill types. Thus, the social planner can allocate to all workers with skill type ω_j the bundle (c, y) which minimizes $(c - y)$ subject to $c - h(y, \omega_j) \geq z_j$. By Lemma 3, the solution to this problem is given by $\hat{y}_j$, i.e., undistorted labor supply at the intensive margin. The consumption level c_j follows as $c_j = z_j + h\left(\hat{y}_j, \omega_j\right)$. If a positive measure of agents would provide some positive output $y \neq \hat{y}_j$, then giving them instead bundle $(c_j, \hat{y}_j)$ and redistributing the saved resources lump-sum to all agents without violating any IC constraint would lead to a Pareto improvement. □

Lemma 23. *In any utilitarian allocation, labor supply is strictly downward distorted at the extensive margin with* $\tilde{\delta}_j \in (\underline{\delta}, \hat{s}_j)$ *in all skill groups.*

Proof. By Lemmas 21 and 22, the Lagrangian for the problem of optimally redistributing resources within skill group ω_j can be written as

$$\mathcal{L}_j = \int_{\underline{\delta}}^{\tilde{\delta}_j} g_j(\delta) U \left[c_j - h\left(\hat{y}_j, \omega_j\right) - \delta\right] d\delta + \left[1 - G_j(\tilde{\delta}_j)\right] U\left[b_j\right] \\ + \lambda_j \left[G_j(\tilde{\delta}_j)\left(y_j - c_j + b_j\right) - b_j - A - j\right],$$

with $\tilde{\delta}_j = c_j - h\left(\hat{y}_j, \omega_j\right) - b_j$ if $\tilde{\delta}_j \in \left(\underline{\delta}, \bar{\delta}\right)$. Assume for the moment that the latter is true. Combining the first-order conditions with respect to b_j and c_j, the Lagrange multiplier associated with the feasibility constraint equals the average social weight in skill group ω_j, given by

$$\lambda_j = \int_{\underline{\delta}}^{\tilde{\delta}_j} g_j(\delta) U' \left[c_j - h\left(\hat{y}_j, \omega_j\right) - \delta\right] d\delta + \left[1 - G_j(\tilde{\delta}_j)\right] U'\left[b_j\right].$$

The first-order condition with respect to b_j reads

$$\frac{\partial \mathcal{L}_j}{\partial b_j} = \left[1 - G - j(\tilde{\delta}_j)\right]\left[U'\left(b_j\right) - \lambda_j\right] - \lambda_j g_j\left(\tilde{\delta}_j\right)\left[\hat{y}_j - c_j + b_j\right] = 0.$$

For $\tilde{\delta}_j \in \left(\underline{\delta}, \bar{\delta}\right)$, the second bracket in this equation is positive by Assumption *DUR* ω. The same is true for the second bracket. Thus, the optimal level of c_j must be smaller than $\hat{y}_j + b_j$ to satisfy the first-order condition. For the threshold cost type, this implies $\tilde{\delta}_j = c_j - h\left(\hat{y}_j, \omega_j\right) - b_j < \hat{y}_j - h\left(\hat{y}_j, \omega_j\right) = \hat{s}_j$.

For $\tilde{\delta}_j = \underline{\delta}$, the first-order condition with respect to b_j cannot be satisfied. In this case, all agents in this skill group would be unemployed so that $\lambda_j = U'(b_j)$. Then, $y_j - c_j + b_j = 0$ would have to be true, implying $\tilde{\delta}_j = \hat{s}_j$. By Assumption *REM*, this is however inconsistent with $\tilde{\delta}_j = \underline{\delta}$. Similarly, the FOC with respect to c_j cannot be satisfied for the corner solution $\tilde{\delta}_j = \bar{\delta}$. Thus, labor supply is strictly downward distorted with $\tilde{\delta}_j \in (\underline{\delta}, \hat{s}_j)$ in all skill groups. □

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