"Application bundling in system markets"

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Abstract

Motivated by recent investigations over Google's practices in the smartphone industry, we study bundling in markets for devices that allow consumers to use applications. The presence of applications on a device increases demand for it, and application developers earn revenues by interacting with consumers. A firm that controls multiple applications can offer them to device manufacturers either individually or as a bundle. We present a novel mechanism through which anti-competitive bundling can be profitable: Bundling reduces rival application developers' willingness to pay manufacturers for inclusion on their devices, and allows a multi-application developer to capture a larger share of industry profit. Bundling can also strengthen competition between manufacturers and thereby increase consumer surplus, even if it leads to foreclosure of application developers and a loss in product variety.

1 Introduction

Competition authorities in Europe and in the US have recently been investigating potentially anti-competitive practices by Google on the mobile applications market. Google, which develops the open-source mobile operating system Android as well as many mobile applications, has in particular been accused by the European Commission of abusing its dominant position by imposing restrictions on Android device manufacturers.[1] One such

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[1]The EC's statement of objections is summarized at `http://europa.eu/rapid/press-release_IP-16-1492_en.htm`. See also Edelman and Geradin, 2016 for a detailed analysis criticizing Google's practices. A Google response is available at `https://blog.google/topics/google-europe/androids-model-of-open-innovation/`.

restriction is the bundling of applications: manufacturers who want to install Google Play also have to pre-install other Google applications (notably Google Search and the Google Chrome browser). Because Google Play is by far the largest Android application store,[2] the Commission argues that it is commercially important for manufacturers to be able to offer it to their customers. On the other hand the "tied" applications (Search, Chrome and others) face stronger competition, and Google's practices prevent its competitors from being installed by manufacturers.

Beside its importance for the future of the mobile internet market, the case is interesting to economists as it touches upon an issue that has received little attention to date, namely that of anti-competitive bundling in multi-sided markets. Indeed, application developers are not mere input suppliers to smartphone manufacturers: most developers also directly benefit from their applications being installed and used by consumers, either through advertising revenues, in-app purchases, or collection of valuable data about consumers. We refer to these as "application revenues". The presence of application revenues opens up the possibility that application developers would be willing to pay manufacturers in exchange for being installed exclusively or as a default option. It also introduces externalities between the applications installed by a given manufacturer: the presence of a popular application, by attracting more consumers to a manufacturer's device, makes being installed on the device more valuable to other developers.

In this paper, we show that these features lead to a novel rationale for bundling, and we study the implications of such bundling. More specifically, we consider a market for a device that allows consumers to run applications. There are two types of applications: A and B, and the device manufacturer(s) can install at most one application of each type. There are two application developers: firm 1 offers each type of application (A_1 and B_1), whereas firm 2 only offers a B-application (B_2). Applications generate direct per-consumer revenues for their developers, and developers can offer payments to the device manufacturers in exchange for being installed.

Our first contribution is to present a novel mechanism through which bundling is profitable. Bundling by firm 1 prevents manufacturers from jointly installing A_1 and B_2. A manufacturer who installs B_2 must therefore forego application $A1$, which lowers the demand for its device. This in turn reduces firm 2's willingness to bid for inclusion on the device, as it expects to reach fewer consumers. Firm 1 can therefore offer lower payments to manufacturers in exchange for its applications being installed. When firm 2 is more efficient than firm 1 (i.e. generates higher application revenues), but not too much, bundling is profitable. The ensuing foreclosure of firm 2 reduces total welfare compared to a situation without bundling. When firm 1 is more efficient than firm 2, bundling does not affect which applications are installed. However it is always a strictly profitable strategy

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for firm 1, as it allows it to capture more of the industry profit. We discuss how the logic extends to various environments, depending on which contracts (lump sum or two-part tariffs) can be offered and whether there is competition between manufacturers.

Our second main contribution (presented in section 5) is to show that bundling can intensify competition between manufacturers and benefit consumers. With two competing manufacturers and differentiated B applications, we show that, absent bundling, manufacturers would install different B applications in equilibrium and enjoy relatively soft competition. When firm 1 sells $A1$ and $B1$ as a bundle, both manufacturers install $B1$, and are therefore perceived as less differentiated by consumers. This leads to lower prices and can increase consumer surplus.

Literature review   The study of bundling and tying as an anticompetitive practice has a rich intellectual history. First dealt a blow by the Chicago School's Single Monopoly Profit Theory (e.g., Director and Levi, 1956; Stigler, 1963), the leverage theory of bundling was reinvigorated by various scholars who showed bundling could be profitably used to deter entry (e.g., Whinston, 1990; Choi and Stefanadis, 2001; Carlton and Waldman, 2002; Nalebuff, 2004). In a setup with heterogeneous consumers, Carbajo, De Meza, and Seidmann, 1990 and Chen, 1997 showed that bundling could be used to soften competition by increasing perceived differentiation. Since bundling in these models is used as a tool to weaken competition, it is typically detrimental to consumers.

The profitability of bundling in our paper comes neither from entry deterrence nor from increased differentiation. Rather, bundling lowers the device manufacturer's outside option and allows the multi-application developer to capture more of the industry profit. Moreover, bundling can increase consumer surplus even when it does result in exclusion (by reducing differentiation between manufacturers, leading them to compete more fiercely).

A few recent papers consider bundling in platform markets. Amelio and Jullien (2012) and Choi and Jeon (2016) consider models with platforms that are unable to charge negative prices. As in many models of two-sided markets, platforms would like to subsidize one side in order to capture profit on the other, but the non-negative price constraint limits their ability to do this. A platform owner, though, can implicitly subsidize participation by tying the platform to another product and then reducing the price charged for that product. By relaxing the zero price constraint, bundling can therefore be profitable for the firm. Indeed, Choi and Jeon (2016) show that, consistent with the leverage theory, a monopolist in one market can exploit this mechanism to profitably extend its market power. Another paper that studies bundling in a two-sided context is Choi (2010). Suppose that there is some enabling good that is necessary to use a platform, and that this good is bundled with one of two competing platforms. This puts the rival platform at a disadvantage and causes it to reduce its price. Choi shows that the result can be an increase in welfare because more consumers choose to multi-home and consume exclusive content only available at the (now

cheaper) rival platform. The mechanism we study is distinct from that at work in these papers: we do not impose a non-negative price constraint,[3] and focus on environments in which buyers of the potentially bundled products are themselves manufacturers who subsequently interact with consumers (and compete).[4]

Because the "buyers" of the applications are the manufacturers and not the final users, our paper also relates to the small literature on bundling and vertical relations (e.g O'Brien and Shaffer (2005) on mergers between wholesalers and bundling, Ide and Montero (2016) on bundling and foreclosure in wholesale markets).[5]

Our paper is also reminiscent of the literature on compatibility in systems markets (Matutes and Regibeau, 1988; Kim and Choi, 2015), where firms who sell differentiated components of a system choose whether to make them compatible with their rivals' components. Gans and King (2006) provide a related analysis in the context of bundling. In these papers however, bundling (or compatibility) is over final products, meaning that consumers must choose between different bundles. In our model, bundling occurs at the upstream stage, and essentially prevents manufacturers from offering different bundles to consumers.

This last point also distinguishes our paper from the literature on bundling as an instrument to price discriminate (e.g., Adams and Yellen, 1976; Schmalensee, 1984; Armstrong and Vickers, 2010; Zhou, 2017).

2   Monopolist manufacturer and lump-sum payments

We consider the market for a device which allows consumers to use various applications. The device is produced by a single manufacturer, who sells it directly to consumers. There are two categories of applications, A and B (for instance A is an application store and B a browser), and two application developers, 1 and 2. Firm 1 produces one application in each category (A_1 and B_1), whereas firm 2 only produces an application of category B (B_2).

The manufacturer decides which applications to install on its device, but is constrained

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[3]Indeed, when applications are licensed to platforms rather than end users, negative prices are fairly common. For example, court proceedings revealed that, in 2014, Google paid Apple \$1bn for the right to be installed as the default search engine on iPhone devices. See https://www.bloomberg.com/news/articles/2016-01-22/google-paid-apple-1-billion-to-keep-search-bar-on-iphone, accessed 31 October 2016.

[4]See also Lee (2013) and Pouyet and Trégouët (2016) for papers on vertical integration in multi-sided markets, the latter with a particular focus on the smartphone industry.

[5]The literature on exclusive dealing also considers downstream competition (e.g. Fumagalli and Motta, 2006, Abito and Wright, 2008), but the logic is also quite different.

to install at most one application of each category.[6,7]

The demand for the device depends on which applications are installed. If the manufacturer sets a price p, demand is $Q(p)$ if the device supports both categories of applications (i.e. A_1 and either B_1 or B_2 are installed), and $q(p) < Q(p)$ if only one category is installed.[8] We assume that the manufacturer's marginal cost is constant and normalize it to zero. We also assume that the manufacturer's gross profit (i.e. ignoring payments to/from application developers) is quasi-concave. Denote P^* the solution to $\max_p pQ(p)$, and $\pi_{AB} \equiv P^*Q(P^*)$. Similarly define $p^* \equiv \mathrm{argmax}_p\{pq(p)\}$ and $\pi_B \equiv p^*q(p^*)$.[9] We have $\pi_{AB} > \pi_B$. A further assumption that we make is that $Q(P^*) \geq q(p^*)$: the manufacturer optimally serves more consumers when both categories of applications are installed on the device.[10] We denote these quantities as Q^* and q^* below.

Applications differ from standard components of a final product in the sense that they generate direct revenues from their interactions with consumers. These revenues may come from advertising, sale of consumer data to third parties, or "in-app purchases". We normalize application A_1's revenue to zero[11] and allow B-applications to be asymmetric. If the manufacturer installs application B_i and serves N consumers, i's revenue is $R_i(N)$. These revenues may induce application developers to offer payments to the manufacturer in exchange for being installed.

The timing is the following: At $t = 0$, firm 1 decides whether to bundle its applications. At $t = 1$, application developers offer payments to the manufacturer. At $t = 2$ the manufacturer chooses which application(s) to install. At $t = 3$ it chooses a price for its device, and payoffs are realized.

We focus on sub-game perfect equilibria that do not involve weakly dominated strategies, and simply refer to them as *equilibria* throughout the paper.

In this section we analyze the case where firms offer lump-sum payments. This setup allows us to expose our results in the simplest way. We introduce two-part tariffs in section 3.

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[6]The debate around bundling of smartphone applications has mostly focused on the manufacturer's choice of a default application (or on which application makes it onto the phone's home screen). Capacity is constrained because there can be only one default for each task and space on the home screen is limited. In Section 5 we allow consumers to change the default application configuration.

[7]Jeon and Menicucci (2012) also study bundling in a setup where the buyer has a limited capacity. The difference between their model and ours is that the capacity constraint is over the whole set of products, whereas we impose a constraint on the B-applications only. More specifically, we don't allow the manufacturer to install B_1 and B_2 only, i.e. A_1 never competes against the B applications.

[8]The implied symmetry between applications B_1 and B_2, and between categories A and B, is not essential to our arguments and merely comes to simplify the exposition.

[9]We use the notation π_B because the manufacturer never actually considers installing only A_1 in equilibrium, because both developers offer positive payments to have their B application installed.

[10]This essentially rules out situations where having both categories of applications as opposed to one increases the willingness to pay of a small number of consumers by a very large amount, whereas the other consumers see their willingness to pay increase by a relatively small amount.

[11]But our analysis easily extends to positive revenues for A_1.

For $X \in \{A, B\}$ and $i \in \{1, 2\}$, denote by T_{Xi} the payment offered by i to the manufacturer in return for installation of application X_i (with the convention that $T_{Xi} < 0$ means that the manufacturer pays i).

When payments take a lump-sum form, they do not affect the manufacturer's pricing strategy. At $t = 3$ the optimal price and quantity are thus P^* and Q^* if A_1 and a B application are installed, p^* and q^* otherwise.

We study in turn the cases where firm 1 offers its A_1 and B_1 applications independently to the manufacturer and when it only offers them as a bundle.

2.1   No bundling

Let us start with the subgame where firm 1 offers A_1 and B_1 independently.

Proposition 1. *(i) Suppose that $R_i(Q^*) > R_j(Q^*)$. In the equilibrium of the subgame without bundling, the manufacturer installs applications A_1 and B_i. Equilibrium offers are given by:*[12]

$$T_{A1} = \pi_B - \pi_{AB}, \quad T_{Bi} = R_j(Q^*), \quad T_{Bj} = R_j(Q^*)$$

(ii) If $R_1(Q^*) \geq R_2(Q^*)$, firm 1's profit is $\pi_1 = \pi_{AB} - \pi_B + R_1(Q^*) - R_2(Q^*)$. If $R_1(Q^*) < R_2(Q^*)$, $\pi_1 = \pi_{AB} - \pi_B$.

Proof. First, in equilibrium application A_1 must be installed. Indeed having A_1 increases the manufacturer's gross profit by $\pi_{AB} - \pi_B$ without cost for firm 1, so firm 1 always makes an offer that is accepted. Moreover, firm 1 finds it optimal to require a payment of $\pi_{AB} - \pi_B$ from the manufacturer, i.e. $T_{A1} = \pi_B - \pi_{AB}$.

Second, given, that A_1 is installed, firm 1 is willing to pay up to $R_1(Q^*)$ and firm 2 up to $R_2(Q^*)$ to have their B application installed. Because the applications are symmetric with respect to consumer demand, the manufacturer installs the application that offers the largest payment. Standard arguments then lead to the result that the most efficient firm (say i) bids up to $R_j(Q^*)$ and wins the auction in the unique equilibrium in undominated strategies, while the least efficient firm bids up to its value.

Firm 1's profit is then obtained straightforwardly. ∎

Firm 1's monopoly allows it to extract all of the joint profit attributable to application A. Competition for access to B-market consumers, on the other hand, means that the joint profit in this market is largely captured by the bottleneck manufacturer. Indeed, if $R_1(Q^*) = R_2(Q^*)$ then the manufacturer captures all of the B-market profit. We will see that this situation can change markedly when firm 1 bundles its two applications.

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[12]We use the tie-breaking rule that when indifferent, the manufacturer chooses the most efficient application.

2.2 Bundling

Proposition 2. *When firm 1 offers A_1 and B_1 as a bundle:*

1. *If $\pi_{AB} + R_1(Q^*) \geq \pi_B + R_2(q^*)$, equilibrium offers are $T_1 = -\pi_{AB} + \pi_B + R_2(q^*)$ and $T_{B2} = R_2(q^*)$. The manufacturer installs $A_1 - B_1$, and firm 1's profit is $\pi_1 = \pi_{AB} - \pi_B + R_1(Q^*) - R_2(q^*)$.*

2. *If $\pi_{AB} + R_1(Q^*) < \pi_B + R_2(q^*)$, equilibrium offers are $T_1 = R_1(Q^*)$ and $T_{B2} = \pi_{AB} - \pi_B + R_1(Q^*) < R_2(q^*)$. The manufacturer installs B_2, and firm 1's profit is zero.*

Proof. Given that A_1 is only available along B_1, firm 2 expects the manufacturer to only serve q^* consumers if it chooses to install B_2. Thus firm 2's willingness to pay is $R_2(q^*)$. If the manufacturer installs the $A_1 - B_1$ bundle, firm 1's revenue, and hence its willingness to pay, is $R_1(Q^*)$.

If $\pi_{AB} + R_1(Q^*) \geq \pi_B + R_2(q^*)$, firm 2 is not able to compensate the manufacturer for the loss of A_1. In equilibrium, by a reasoning similar to Proposition 1, firm 2 offers the maximum it can (i.e. $T_{B2} = R_2(q^*)$) and firm 1 offers just enough to be picked by the manufacturer ($T_1 = -\pi_{AB} + \pi_B + R_2(q^*)$

If $\pi_{AB} + R_1(Q^*) < \pi_B + R_2(q^*)$, firm 1 cannot compensate the manufacturer for the loss of the more efficient B_2, and in equilibrium $T_1 = R_1(Q^*)$ and $T_{B2} = \pi_{AB} - \pi_B + R_1(Q^*) < R_2(q^*)$. ∎

We can now state the condition for bundling to be profitable by comparing Proposition 1 and Proposition 2.

Proposition 3. *(i) If $R_2(Q^*) > R_1(Q^*)$, bundling is strictly profitable if and only if $R_1(Q^*) > R_2(q^*)$. Bundling is welfare decreasing.*
(ii) If $R_1(Q^*) \geq R_2(Q^*)$, bundling is strictly profitable and welfare neutral.

When application B_2 generates more revenue than B_1, a necessary and sufficient condition for bundling to be profitable is that application A_1 significantly boost demand for the device (i.e. $Q^* >> q^*$). Indeed in that case bundling considerably softens firm 2's incentive to bid for inclusion on the device, resulting in inefficient exclusion of firm 2, whose application would be installed absent bundling.

When $R_1(Q^*) \geq R_2(Q^*)$, bundling does not affect which applications are installed (A_1 and B_1). However, because it always reduces firm 2's bid, it is strictly profitable for firm 1.

Bundling with cost complementarity   At this point it may be useful to cast our results in a framework that makes the comparison with the established literature on bundling more transparent. We focus on the symmetric setup. Suppose that there are

two products, A and B, that bring utility v_A and v_B to buyers. Firm 1 produces both A and B, while firm 2 only produces B. Products A and B are *cost-complements* in the following sense: providing a consumer with product B is less costly if that consumer has already installed A. To make things simple, suppose that the cost is c_B if the consumer does not buy A, and 0 otherwise. The timing is the following: At $t = 1$ firm 1 decides whether to bundle A and B. At $t = 2$ firms make simultaneous price offers to consumers. At $t = 3$ consumers decide which products to buy.

Suppose first that products are not bundled. In equilibrium firm 1 must charge $p_A^* = v_A$ for product A. Then, anticipating that consumers will buy product A, firms compete à la Bertrand and offer $p_B^* = 0$ to consumers. Firm 1's profit is $v_A - c_A$.

With bundling, the lowest price that firm 2 can offer is $p_{B2} = c_B > 0$. Consumers buy the bundle at price P if and only if $v_A + v_B - P \geq v_B - c_B$, and therefore the equilibrium price for the bundle is $v_A + c_B$. Firm 1's profit is then $v_A + c_B - c_A$, i.e. bundling is profitable.

Note that cost complementarity is different from standard consumption complementarity (e.g. a Δ increase in v_B if the consumer also buys A). Indeed in the latter case firm A could charge $p_A = v_A + \Delta$ and $p_B = v_B$ without bundling, and make the same profit as under bundling.

Our model is an instance of cost complementarity, where the cost of installing the B application on a manufacturer's devices is negative (it is minus the application revenues) and goes up when application A is not installed (from $-R(Q^*)$ to $-R(q^*)$).

While our focus on lump-sum payments allows us to expose our argument in a clean way, one may wonder if it would hold if payments could be conditioned on quantities sold. In the next section we allow application developers to offer two-part tariffs to the manufacturer and show that bundling can still be profitable.

3 Two-part tariffs

Suppose that application developers can offer two-part tariffs to the manufacturer. If the manufacturer installs application i and sells N devices, the payment it receives from developer i is $f_i + w_i N$. We focus on the case with symmetric B-applications: $R_1(N) = R_2(N) = R(N)$. The revenue generated by application A_1 is still assumed to be zero for simplicity.

3.1 No bundling

First, it is easy to see that in equilibrium the manufacturer installs application A_1. Indeed installing A_1 increases demand, and therefore there is a positive amount that the manufacturer is willing to pay to firm 1.

Let p^* be the price that maximizes the industry profit given that A_1 is installed, i.e.

$$p^* = \arg \max_p \{pQ(p) + R(Q(p))\}$$

Let $\pi^* \equiv p^*Q(p^*) + R(Q(p^*))$.

The first-order condition for p^* is

$$(p^* + R'(Q(p^*)))\, Q'(p^*) + Q(p^*) = 0 \tag{1}$$

If the manufacturer installs a B application with a per-unit payment of w_B alongside A_1, it chooses p so as to maximize $(p + w_B)Q(p)$. If the application developers offers $w_B^* \equiv R'(Q(p^*))$, then it induces the manufacturer to choose the price that maximizes their joint surplus.

Lemma 1. *In the equilibrium of the subgame without bundling, the B-application chosen by the manufacturer offers the two-part tariff: $T_B^*(q) = w_B^* q + f_B^*$, with $f_B^* = R(Q(p^*)) - w_B^* Q(p^*))$. The developer of the chosen B application makes zero profit.*

Proof. If B_i is installed in equilibrium, B_j makes zero profit. Therefore (w_B^*, f_B^*) cannot be such that B_i makes a positive profit, otherwise B_j could offer $f' = f_B^* + \epsilon$ and be chosen. Suppose now that B_i offers a contract $(w', f') \neq (w_B^*, f_B^*)$ that allows it to break even. Because w_B^* maximizes the joint surplus of the manufacturer and the B-developer, B_j could deviate and offer w_B^* along with a fixed fee that makes the manufacturer better-off than under (w', f'). ∎

A similar line of reasoning reveals that firm 1 will only offer a (negative) lump-sum payment for A_1 to be installed. Indeed, offering a per-unit fee would not create any further alignment of incentives, and would thus be superfluous.

Suppose that the manufacturer deviates and does not install A_1, but accepts the contract (w_B^*, f_B^*) offered by B_i. Its profit is then $(p + w_B^*)q(p) + f_B^*$. Let $\hat{p}$ be the price that maximizes this expression, and $\hat{\pi}$ the associated profit.

The payment f_{A1}^* must be such that $\pi^* + f_{A1}^* \geq \hat{\pi}$. Suppose that the constraint is satisfied with equality. Can B_j offer a contract that allows it to make a positive profit? We have seen that it cannot induce the manufacturer to keep A_1 and switch to B_j, but it could potentially offer a contract that induces the manufacturer to drop A_1. The most aggressive such contract is $(\tilde{w}_B, \tilde{f}_B)$ such that $\tilde{w}_B = R'(q(\tilde{p}))$ (where $\tilde{p}$ maximizes $pq(p) + R(q(p))$) and $\tilde{f}_B$ allows B_j to break-even ($\tilde{f}_B = R(q(\tilde{p})) - \tilde{w}_B q(\tilde{p})$). Let $\tilde{\pi}$ be the manufacturer's profit if it accepts $(\tilde{w}_B, \tilde{f}_B)$ and rejects A_1's offer.

The next lemma finishes the characterization of the equilibrium:

Lemma 2. *In the equilibrium of the subgame without bundling, $f_{A1}^* = \max\{\hat{\pi}, \tilde{\pi}\} - \pi^* < 0$. If $\hat{\pi} > \tilde{\pi}$, B_j offers the contract (w_B^*, f_B^*) and gets rejected. If $\tilde{\pi} > \hat{\pi}$, B_j offers the contract*

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$(\tilde{w}_B, \tilde{f}_B)$ *and gets rejected. Firm 1's profit is* $-f^*_{A1}$.

Proof. The two constraints for A_1 are $f^*_{A1} \geq \hat{\pi} - \pi^*$ (otherwise the manufacturer deviates and installs only B_i) and $f^*_{A1} \geq \tilde{\pi} - \pi^*$ (otherwise B_j can offer $(\tilde{w}_B, \tilde{f}_B)$ and the manufacturer will choose to only install B_j). B_j's equilibrium offer is chosen so as to make f^*_{A1} optimal given B_j's offer. Finally, given that no B application generates a positive profit, firm 1's profit solely come from the licensing of A_1. ∎

3.2 Bundling

Under bundling, firm 1 offers a contract (w^*_{AB}, f^*_{AB}) where $w^*_{AB} = w^*_B$ from the no-bundling case. Indeed such a unit fee induces the manufacturer to choose the price that maximizes the joint profit, and the fixed fee is then used to share that profit.

Unlike the case of no-bundling, firm 2 knows that if the manufacturer installs B_2 it will not have A_1. The best offer it can make is thus $(\tilde{w}_B, \tilde{f}_B)$. Therefore, the manufacturer will choose the bundle if and only if $\pi^* + f_{AB} \geq \tilde{\pi}$.

Lemma 3. *In the equilibrium of the subgame with bundling, firm 1 offers* (w^*, f^*_{AB}) *with* $f^*_{AB} = \tilde{\pi} - \pi^* < 0$. *Firm 2 offers* $(\tilde{w}_B, \tilde{f}_B)$. *The manufacturer installs the bundle, and firm 1's profit is* $-f^*_{AB}$.

We can now compare the two strategies. Because B-applications are symmetric, bundling is always weakly profitable. More interesting are the instances where it is strictly profitable.

Proposition 4. *With symmetric B-applications and two-part tariffs, bundling is strictly profitable if and only if* $\hat{\pi} > \tilde{\pi}$.

To get a better intuition, let us discuss a few special cases. First, when revenues are linear $(R(N) = rN)$, bundling is not strictly profitable. Indeed in this case $w^*_B = \tilde{w}_B = r$, and therefore $\hat{\pi} = \tilde{\pi}$. With linear revenues, B_2 offers a unit fee equal to r irrespective of the bundling decision, and therefore bundling does not succeed in making it less aggressive.[13]

Let us now focus on linear demands: $q(p) = \max\{1 - p, 0\}$ and $Q(p) = \max\{A - p, 0\}$ with $A > 1$. We look at two possible scenarios. First, suppose that the revenue function is concave and takes the form $R(N) = rN - \sigma N^2$, with $\sigma \in (0, r/2A)$ (so that $R' > 0$). This corresponds to situations where the marginal consumer brings lower revenues to application developers, for instance because his income is lower than infra-marginal consumers. In this example, we have $\hat{\pi} - \tilde{\pi} = \frac{(A-1)^2 \sigma}{4(1+\sigma)} > 0$, so that bundling is strictly profitable.

The second scenario is one where $R(N)$ is convex and takes the form $R(N) = N(r + \lambda N - \delta N^2)$. With this example the per-user revenue is increasing but concave. A

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[13]This point is more general: if B-developers could offer fully general contracts of the form $T_B(q)$, B_2 would offer $T_B(q) = R(q)$ and bundling would never be profitable.
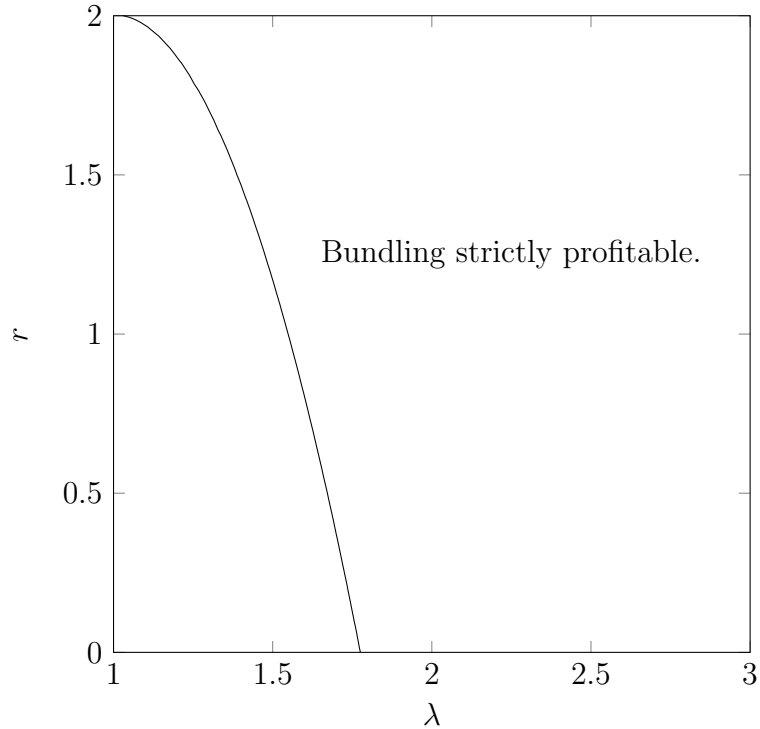
Figure 1: Profitability of bundling under two-part tariffs, with $Q(p) = A - p$, $q(p) = 1 - p$, $R(N) = N\left(r + \lambda N - \delta N^2\right)$. Parameter values: $A = 1.5$, $\delta = 0.1$.

possible justification for this could be the existence of network effects at the application level, which allow the developer to generate higher per-user revenues as the size of the network increases, although at a decreasing rate. λ and δ then capture the magnitude and the rate of decay of network effects. As Figure 1 illustrates, bundling can be strictly profitable if r and λ are large enough.

4 Profitability of bundling with competing manufacturers

The mechanisms that can make bundling profitable with a monopoly manufacturer continue to operate when we introduce competition. Competition also introduces new considerations: applications become a potential source of differentiation between manufacturers, and bundling can therefore affect equilibrium device prices and consumer surplus.

In this section we use a reduced-form approach to model competition between manufacturers, and give conditions for bundling to be profitable for firm 1. We introduce an additional manufacturer to the market, and denote the two manufacturers by L and R. Manufacturers' gross profit depends on which applications they choose to install. To keep the analysis concise we make the following set of assumptions: [14]

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[14]These assumptions, in particular the essentiality of A_1, are not critical to the core argument but

1. B-application revenues are linear and symmetric: if N_i is the number of consumer who use application B_i, $R_i(N_i) = rN_i$ for $i \in \{1,2\}$.

2. Application A_1 is essential: without it a manufacturer cannot sell any device.

3. If the two manufacturers install A_1 and the same B application, their gross profit is π_S. If they install different B applications, their profit is π_D. In both cases the number of consumers served by each manufacturer in equilibrium is Q.

The timing is the following: At $t = 0$, firm 1 decides whether to bundle A_1 and B_1. At $t = 1$ firms 1 and 2 make simultaneous secret offers to the manufacturers. We also restrict ourselves to lump-sum offers for simplicity. At $t = 2$, manufacturers choose which applications to install, and payments are made. At $t = 3$ manufacturers compete on the market and profits are realized.

Lemma 4. *In equilibrium without bundling: (i) both manufacturers install A_1; (ii) They install different B applications if $\pi_D > \pi_S$, and the same B application if $\pi_D < \pi_S$.*

Proof. Suppose that one manufacturer, say L, does not install A_1 in equilibrium. Then, because offers are secret, firm 1 could increase its profit by requiring a small payment from L in exchange for installing A_1. This offer would be accepted by L.

Suppose now that L expects R to choose A_1 and B_i. If firms 1 and 2 expect L to install A_1, they are willing to offer L up to rQ to be installed on L's device. If $\pi_D > \pi_S$, firm j can convince L to install B_j even when i offers $T^L_{Bi} = rQ$ by offering $T^L_{Bj} = rQ + \pi_S - \pi_D + \epsilon < rQ$. A symmetric reasoning applies when $\pi_D < \pi_S$. ∎

Under no-bundling, there is a multiplicity of offers that are compatible with the equilibrium allocation described by Lemma 4. Because our point is to show that bundling can be profitable for firm 1, we focus on the best equilibrium for firm 1 under no-bundling thereafter.

4.1 Efficient differentiation: $\pi_D > \pi_S$

One possible scenario is that installing different B-applications allows manufacturers to differentiate from one another and relax competition. This happens in the following example.

Example Suppose that manufacturers are horizontally differentiated à la Hotelling, with a transportation parameter τ_m, and that applications B_1 and B_2 are also horizontally differentiated, with a transportation cost τ_b, that $\tau_b > \tau_m$, and that τ_b is small enough so that the market is covered. As we show in section 5, this implies that $\pi_D - \pi_S = \frac{\tau_b - \tau_m}{2} > 0$.

greatly simplify the exposition. A more general analysis is available from the authors upon request.

Proposition 5. *When $\pi_D > \pi_S$, bundling is profitable if $rQ \geq \pi_D - \pi_S$.*

Proof. Let us start with the case of no bundling. We know from Lemma 4 that the manufacturers install different B applications in equilibrium. Suppose that manufacturer L installs A_1 and B_1 whereas R installs A_1 and B_2.

First, we know that the B application that is not chosen by a manufacturer must offer a payment of rQ to this manufacturer, because of our focus on non-dominated strategies.

Second, we look at the conditions for manufacturer L to choose $\{A_1, B_1\}$ given offers T_{A1}^L, T_{B1}^L and $T_{B2}^L = rQ$, and given that R installs A_1 and B_2. L must prefer $\{A_1, B_1\}$ to $\{A_1, B_2\}$, i.e.

$$T_{B1}^L \geq rQ + \pi_S - \pi_D \tag{2}$$

It must also prefer $\{A_1, B_1\}$ to $\{\emptyset, B_1\}$, which implies

$$T_{A_1}^L \geq -\pi_D \tag{3}$$

Last, it must prefer $\{A_1, B_1\}$ to $\{\emptyset, B_2\}$, i.e.

$$T_{A_1}^L + T_{B_1}^L \geq rQ - \pi_D \tag{4}$$

Condition (4) is actually binding, and therefore the profit that firm 1 obtains from its interaction with L is $rQ - (T_{A_1}^L + T_{B_1}^L) = \pi_D$: all the profit from selling the device to consumers is captured by firm 1, but the manufacturer still enjoys a rent of rQ due to the competing offer by firm 2.

We now turn our attention to manufacturer R. There are multiple equilibria here, but for our purpose (finding sufficient conditions for bundling to be profitable), we can focus on the best equilibrium for firm 1. Firm 1 cannot charge more than π_D for installing A_1, but there is an equilibrium in which it charges exactly this: $T_{A1}^R = -\pi_D$, $T_{B1}^R = rQ$ and $T_{B2}^R = rQ$. With such offers, R chooses A_1 and B_2 and gets a profit equal to rQ. Firm 2 gets a profit of 0 but cannot offer less to R, as otherwise R would simply install B_1 alone and get rQ.

Putting the two previous paragraphs together, firm 1's total profit is $2\pi_D$.

When firm 1 bundles A_1 and B_1, firm 2 cannot offer manufacturers any payment in exchange for installing B_2, because A_1 is essential. Firm 1 can therefore offer $T_1^L = T_1^R = -\pi_S$ and generates a profit of $2(\pi_S + rQ)$. Comparing this profit to the maximal profit without bundling ($2\pi_D$) gives the result. ∎

By a logic similar to the case with one manufacturer, bundling allows firm 1 to capture more of the application revenues. However firm 1 cannot extract as much of manufacturers' profit, because the lack of differentiation intensifies competition. Bundling is profitable when the latter effect is relatively small.

Note that bundling is inefficient from the industry standpoint, because it prevents differentiation. We cannot say more that this without putting more structure on consumers' preferences, which is the purpose of Section 5.

Before doing so, we look at the case where differentiation is not efficient, $\pi_S > \pi_D$.

4.2 Inefficient differentiation: $\pi_S > \pi_D$

A scenario compatible with $\pi_S > \pi_D$ is one where the B applications exhibit network effects.

Example Suppose now that while manufacturers are still horizontally differentiated with a transportation cost τ_m, B applications are no longer horizontally differentiated but instead exhibit network externalities: if a mass n_i of consumers use application i (irrespective of which manufacturer they use), the utility from using i is γn_i, with $\gamma \in (0, \tau_m)$. If both manufacturers install the same B application, and if τ_m is small enough that the market is covered, the network externalities cancel out (consumers get the same network benefit on either manufacturer), and equilibrium (gross) profit is $\pi_S = \tau_m/2$. If, on the other hand, they install a different B application, network effects intensify competition and lead to a gross profit of $\pi_D = \frac{\tau_m - \gamma}{2} < \pi_S$.

Proposition 6. *Suppose that $\pi_S > \pi_D$. Then bundling is strictly profitable for firm 1.*

Proof. Under no bundling, we know that both manufacturers install the same B application, and that the losing B application must offer rQ to both manufacturers.

If manufacturers install B_2, the best equilibrium for firm 1 is such that $T_{A1}^L = T_{A1}^R = -\pi_S$, $T_{B1}^L = T_{B1}^R = rQ$, and $T_{B2}^L = T_{B2}^R = rQ$ (firm 2 cannot offer less because the manufacturers would deviate by accepting B_1 and not installing A_1). Firm 1's profit is $2\pi_S$.

If manufacturers install B_1 instead, the best equilibrium for firm 1 is such that $T_{A1}^L = T_{A1}^R = -\pi_S$, $T_{B1}^L = T_{B1}^R = rQ$, and $T_{B2}^L = T_{B2}^R = rQ$. Firm 1's profit is $2\pi_S$.

In both cases firm 1 extracts the whole profit generated by device sales to consumers ($2\pi_S$), but relinquishes a rent equal to $2rQ$ to manufacturers.

Under bundling, firm 2 makes no offer and firm 1 offers $T_1^L = T_1^R = -\pi_S$, for a profit of $2(\pi_S + rQ)$. ∎

5 Bundling and competing manufacturers: a Hotelling model

We have established that bundling can be a strictly profitable strategy. To examine the effect of bundling on prices and consumer surplus, we must go beyond a reduced-form

specification of profits and add additional structure to the model. To this end, we now further develop the Hotelling model of differentiation introduced in Section 4.1.

To be more precise, suppose that each consumer has a type $\mathbf{x} = (x_m, x_b)$, uniformly distributed in $[0, 1]^2$. Manufacturers L and R are respectively located at distance $d_L = x_m$ and $d_R = 1 - x_m$ from the consumer's ideal. Similarly, the distance to applications B_1 and B_2 is x_b and $1 - x_b$ respectively. Let d_{ib} denote the distance to the B-market application installed by manufacturer i. A consumer of manufacturer i obtains, by default, utility

$$V - \tau_m d_i - \tau_b d_{ib} - P_i,$$

where V is the standalone value of the consumer's ideal manufacturer, τ_m and τ_b are transport costs, and P_i is the price charged by manufacturer i.

We allow consumers to incur a cost, Δ, to install a different B-market application on their chosen hardware. If the consumer chooses to do this then his utility becomes

$$V - \tau_m d_i - \tau_b (1 - d_{ib}) - P_i - \Delta.$$

We assume that V is large enough to guarantee the market is covered and focus on the case in which application A_1 is essential and is installed by both manufacturers.

We start by assuming that there is no bundling, and compute the sub-game perfect equilibrium.

5.1 Subgame without application differentiation

Suppose that both manufacturers have identical application configurations (both $\{A_1, B_1\}$ or both $\{A_1, B_2\}$). Any consumer who finds it optimal to install a custom application on manufacturer L would also do so on R. Thus, for every consumer, the relevant comparison is between two manufacturers with identical applications and the game collapses to standard one-dimensional Hotelling competition between differentiated manufacturers.

Lemma 5. *If both manufacturers install the same applications then prices in a symmetric equilibrium are $P_L = P_R = \tau_m$. Manufacturer gross profit is $\pi_L = \pi_R = \tau_m/2$. Consumer surplus is*

$$CS = \begin{cases} V - \frac{1}{4}\left(\tau_b + 2\Delta - \frac{\Delta^2}{\tau_b} + 5\tau_m\right) & \text{if } \Delta < \tau_b \\[2mm] V - \frac{1}{4}\left(2\tau_b + 5\tau_m\right) & \text{if } \tau_b \leq \Delta. \end{cases} \tag{5}$$

Proofs for this section are in the Appendix.

5.2 Subgame with application differentiation

Now suppose that the manufacturers differentiate in their choice of B market applications: manufacturer L installs $\{A_1, B_1\}$ and R installs $\{A_1, B_2\}$. We obtain demand for the two
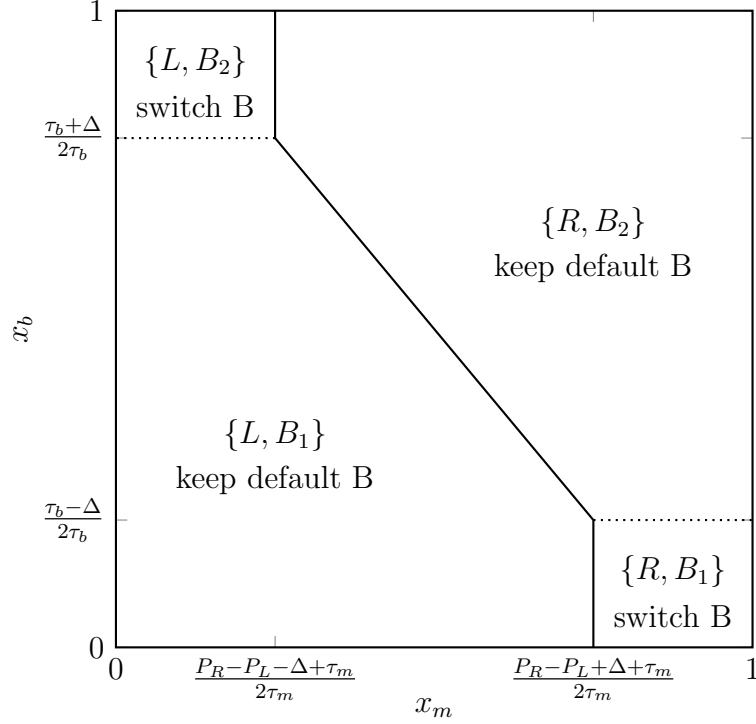
Figure 2: Demand when L installs B_1 by default and R installs B_2.

manufacturers (illustrated in Figure 2), equilibrium prices, profits, and consumer surplus. Lemma 6 summarizes.

Lemma 6. *If manufacturer L installs $\{A_1, B_1\}$ and R installs $\{A_1, B_2\}$ then prices in a symmetric equilibrium are*

$$P_L = P_R = \begin{cases} \tau_b \ \text{if } \tau_m \leq \min\{\tau_b, \Delta\} \\ \tau_m \ \text{otherwise.} \end{cases}$$

Manufacturer gross profit is $\pi_L = \pi_R = P/2$. Consumer surplus is

$$CS = \begin{cases} V - \dfrac{1}{12}\left(6\tau_b - \dfrac{\tau_b^2}{\tau_m} + 15\tau_m\right) \ \text{if } \tau_b < \min\{\tau_m, \Delta\} \\[2ex] V - \dfrac{1}{12}\left(6\tau_m - \dfrac{\tau_m^2}{\tau_b} + 15\tau_b\right) \ \text{if } \tau_m \leq \min\{\tau_b, \Delta\} \\[2ex] V - \dfrac{1}{12}\left(3\tau_b - 3\dfrac{\Delta(\Delta - 2\tau_m) - t\tau_m^2}{\tau_m} - \Delta^2\dfrac{3\tau_m - 2\Delta}{\tau_b\tau_m}\right) \ \text{if } \Delta < \min\{\tau_m, \tau_b\}. \end{cases}$$

5.3   Overall equilibrium and welfare effects of bundling

In both of Lemmas 5 and 6, equilibrium prices are such that each manufacturer serves half of the market. Each developer is therefore willing to pay up to $r/2$ for its B-application to be installed by a given manufacturer. This implies that, absent bundling,

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manufacturers' revenue from application developers is independent of their choice of application configuration and this choice is made to maximize profits from downstream hardware sales.

If $\tau_m < \min\{\tau_b, \Delta\}$ then manufacturers earn $\tau_b/2$ when B-applications are differentiated and $\tau_m/2$ when they are not. Any equilibrium must therefore have application differentiation. If $\tau_m \geq \min\{\tau_b, \Delta\}$ then, absent bundling, a manufacturer earns profit $\tau_m/2$ irrespective of whether it has the same B-application as its rival or a different one; there are therefore two equilibria. However, the equilibrium with application differentiation is a knife-edge case that only works for $r_1 = r_2$. Any perturbation $r_1 + \epsilon$ would lead both manufacturers to install B_1. We therefore select equilibria where manufacturers install the same B-application if $\tau_m \geq \min\{\tau_b, \Delta\}$.

Corollary 1. *Suppose there is no bundling. Manufacturers install different B-applications in equilibrium if $\tau_m < \min\{\tau_b, \Delta\}$, and install the same B-application otherwise.*

Bundling of A_1 and B_1 prevents manufacturers from differentiating through applications. This does not change surplus or welfare when manufacturers would choose the same B-application anyway (i.e. when $\tau_m \geq \min\{\tau_b, \Delta\}$). When $\tau_m < \min\{\tau_b, \Delta\}$, the welfare implications of bundling can be found by comparing Lemmas 5 and 6. The following result summarizes the effect of bundling on consumer surplus.

Proposition 7. *Bundling causes consumer surplus to*

1. *increase if*

$$\tau_m \leq \min\left\{\widetilde{\Delta}, \frac{1}{2}\left(\sqrt{3}\sqrt{43\tau_b^2 + 4\widetilde{\Delta}^2 - 8\widetilde{\Delta}\tau_b} - 9\tau_b\right)\right\} \tag{6}$$

   (where $\widetilde{\Delta} = \min\{\tau_b, \Delta\}$),

2. *decrease if*

$$\frac{1}{2}\left(\sqrt{3}\sqrt{43\tau_b^2 + 4\widetilde{\Delta}^2 - 8\widetilde{\Delta}\tau_b} - 9\tau_b\right) < \tau_m < \widetilde{\Delta}, \tag{7}$$

3. *remain unchanged if $\widetilde{\Delta} \leq \tau_m$.*

Figure 3 illustrates Proposition 7. If installing applications is not feasible for consumers ($\Delta > \tau_b$) then (6) simplifies to $\tau_m < 0.91\tau_b$.

Bundling can benefit consumers. This makes for a striking contrast with most extant theories, where bundling is used as a tool to either foreclose or soften competition—typically resulting in higher prices and lower consumer surplus. Here, bundling has the opposite effect: installing different applications is a way for manufacturers to differentiate and thus make their residual demand less elastic. But bundling prevents such differentiation. While this harms consumers through a loss of product variety, the net effect can be beneficial because consumers pay lower prices for access to the manufacturer. This effect
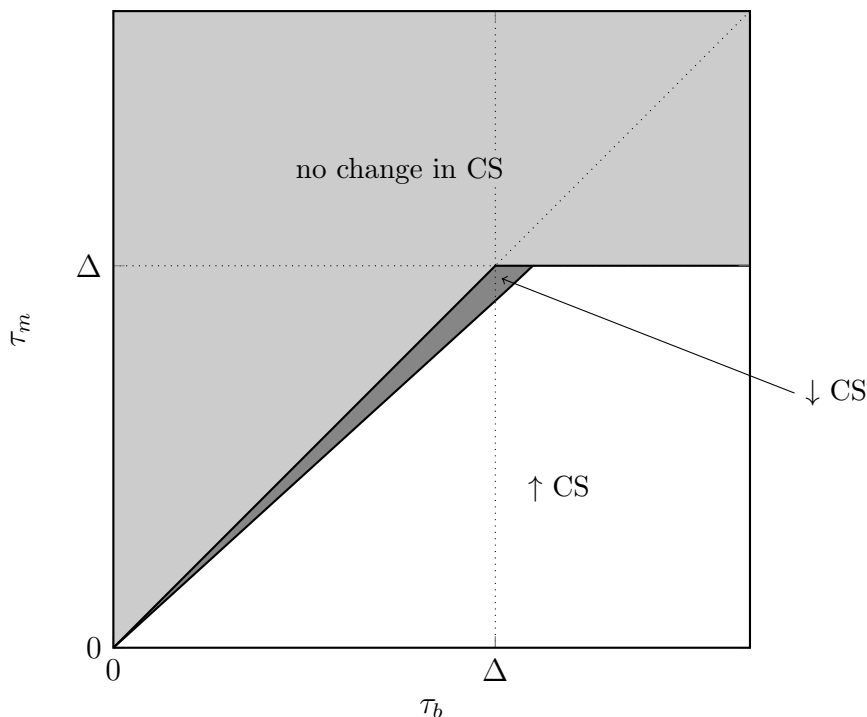
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Figure 3: Effect of bundling on consumer surplus. Consumer surplus increases in the unshaded region, decreases in the dark gray region, and remains unchanged in the light gray region.

is strongest when manufacturers are not intrinsically very differentiated because this is when the inability to differentiate through applications forces manufacturers into fierce price competition.

Consumers are more likely to benefit from bundling when there are sufficient barriers to end-user installation of applications (i.e., when Δ is not too small). If consumers can easily undo differentiation by installing alternative applications then manufacturers compete *as if* there were no application differentiation at all. Thus, prices do not fall further when differentiation is actually eliminated.

The power of bundling to commodify a hardware market also suggests an additional mechanism through which bundling can be profitable. Suppose that manufacturers L and R produce telephones within the Android ecosystem, while a vertically-integrated third manufacturer ("Apple") produces telephones that are pre-installed with its own applications. By bundling, firm 1 can induce L and R to lower the price of an Android phone. This will typically cause users to switch from Apple to Android, further increasing firm 1's application revenues.

6 Conclusion

We have developed a model of bundling in which hardware manufacturers license (bundles of) applications from developers, and those developers receive revenues direct from consumers via advertising or in-app purchases. Our first contribution was to show that these features jointly imply a novel channel through which bundling is profitable: it softens competitive pressure from rival application developers during licensing negotiations. If the bundling firm is the least efficient then bundling can profitably result in inefficient exclusion provided the efficiency disadvantage is not too great. If the bundling firm is the most efficient then bundling is strictly profitable even though it does not change the applications that are installed in equilibrium. This mechanism is robust to environments with two-part tariffs and to the introduction of manufacturer competition.

Building on these insights, we also analysed the implications of bundling for consumer surplus when applications and hardware are differentiated. We showed that bundling reduces effective product differentiation by causing all device manufacturers to install the same applications. The resulting commoditization of hardware causes device prices to fall, and bundling can therefore make consumers better-off.

A Omitted Proofs

A.1 Proof of Lemma 5

The indifferent consumer has

$$\tau_m x_m^* + P_L = \tau_m(1 - x_m^*) + P_R \implies x_m^* = \frac{P_R - P_L + \tau_m}{2\tau_m}.$$

manufacturer L chooses P_L to maximize $P_L x_m^*$. This gives rise to the symmetric equilibrium price $P_L = P_R = \tau_m$, with corresponding equilibrium profit $\pi_L = \pi_R = \tau_m/2$. This is just the standard symmetric Hotelling outcome when firms have a single dimension of differentiation.

Suppose both manufacturers install B_1 by default (the case where both install B_2 is symmetric). A consumer finds it worthwhile to switch to application B_2 if

$$x_b \tau_b > (1 - x_b)\tau_b + \Delta \iff x_b > \frac{\tau_b + \Delta}{2\tau_b}. \tag{8}$$

If $\Delta < \tau_b$ (which is a necessary condition for some consumers to switch), consumer surplus is therefore

$$V - 2 \int_0^{1/2} \left(\int_0^{\frac{\tau_b + \Delta}{2\tau_b}} (\tau_m x_m + \tau_b x_b + P_L)\, dx_b + \int_{\frac{\tau_b + \Delta}{2\tau_b}}^1 (\tau_m x_m + \tau_b(1 - x_b) + P_L + \Delta)\, dx_b \right) dx_m.$$

Substituting $P_L = P_R = \tau_m$ yields the equilibrium consumer surplus:

$$V - \frac{1}{4}\left(\tau_b + 2\Delta - \frac{\Delta^2}{\tau_b} + 5\tau_m\right).$$

If $\tau_b \leq \Delta$ then installing a different B-application from the default one is prohibitively costly. Consumers then stick with the default and obtain surplus

$$V - 2\left(\int_0^{1/2}\int_0^1 (\tau_m x_m + \tau_b x_b + P_L)\,dx_b\,dx_m\right) = V - \frac{1}{4}(2\tau_b + 5\tau_m).$$

A.2 Proof of Lemma 6

A.2.1 Step 1: checking who consumers what

By analogy to (8), we can determine when consumers find it worthwhile to change the default applications on their chosen manufacturer and identify three types of consumer:

1. Consumers with $x_b < \frac{\tau_b - \Delta}{2\tau_b}$ who use manufacturer R install application B_1. Thus, they are indifferent between L and R if

$$\tau_m x_m + \tau_b x_b + P_L = \tau_m(1 - x_m) + \tau_b x_b + P_R + \Delta. \tag{9}$$

2. Consumers with $x_b \in [\frac{\tau_b - \Delta}{2\tau_b}, \frac{\tau_b + \Delta}{2\tau_b}]$ never choose to change the default application. Thus, they are indifferent between L and R if

$$\tau_m x_m + \tau_b x_b + P_L = \tau_m(1 - x_m) + \tau_b(1 - x_b) + P_R. \tag{10}$$

3. Consumers with $x_b > \frac{\tau_b + \Delta}{2\tau_b}$ who use manufacturer L install application B_2. Thus, they are indifferent between L and R if

$$\tau_m x_m + \tau_b(1 - x_b) + P_L + \Delta = \tau_m(1 - x_m) + \tau_b(1 - x_b) + P_R. \tag{11}$$

Let $\widetilde{\Delta} = \min\{\Delta, \tau_b\}$. Solving the three indifference conditions in (9)–(11) yields a function for the indifferent consumer's x_p:

$$x_p^*(x_b) = \begin{cases} \dfrac{P_R - P_L + \Delta + \tau_m}{2\tau_m} & \text{if } x_b < \dfrac{\tau_b - \widetilde{\Delta}}{2\tau_b} \\[2ex] \dfrac{P_R - P_L + \tau_b - 2x_b\tau_b + \tau_m}{2\tau_m} & \text{if } x_b \in \left[\dfrac{\tau_b - \widetilde{\Delta}}{2\tau_b}, \dfrac{\tau_b + \widetilde{\Delta}}{2\tau_b}\right] \\[2ex] \dfrac{P_R - P_L - \Delta + \tau_m}{2\tau_m} & \text{if } x_b > \dfrac{\tau_b + \widetilde{\Delta}}{2\tau_b}. \end{cases} \tag{12}$$

The smallest x_b for which some consumers buy R is the $\underline{x}_b$ solving $\underline{x}_b = \min\{x_b \geq 0 : x_p^*(\underline{x}_b) \leq 1\}$. Similarly, define $\overline{x}_b$ as the largest x_b for which some consumers buy L: $\overline{x}_b = \max\{x_b \leq 1 : x_p^*(\underline{x}_b) \geq 0\}$.

A.2.2 Step 2: solving for equilibrium prices

Firm L's profit is

$$\pi_L(P_L, P_R) = P_L \left(\underline{x}_b + \int_{\underline{x}_b}^{\overline{x}_b} x_p^*(x_b) \, dx_b \right). \tag{13}$$

At a symmetric equilibrium we face two possibilities: Firstly, if $\Delta < \tau_b$ then $\underline{x}_b, \overline{x}_b \in (0,1)$ if and only if $\tau_m < \Delta$ (this can be seen by looking at the first and last lines of (12)). The second possibility is $\Delta \geq \tau_b$, in which case $\underline{x}_b, \overline{x}_b \in (0,1)$ if and only if $\tau_m < \tau_b$ (this can be seen by looking at the middle line of (12) and noting that the first and last lines of (12) are irrelevant when $\Delta \geq \tau_b$).

Evaluating (13) accounting for the possibility of corner solutions, the profit around a symmetric equilibrium is therefore

$$\pi_L(P_L, P_R) = \begin{cases} \dfrac{P_R - P_L + \tau_b}{2\tau_b} P_L \text{ if } \tau_m \leq \widetilde{\Delta} \\[4mm] \dfrac{P_R - P_L + \tau_m}{2\tau_m} P_L \text{ if } \tau_m > \widetilde{\Delta}. \end{cases} \tag{14}$$

We now proceed as follows: Firstly, we identify a putative symmetric equilibrium such that a small deviation is not profitable—where "small" means that we remain within the same piecewise case in (14). Secondly, we verify that this is indeed an equilibrium be checking whether a firm could profit from a larger deviation that causes us to switch between the two cases in (14).

For the first step, taking a first-order condition from (14) and imposing symmetry yields

$$P_L = P_R = \begin{cases} \tau_b \text{ if } \tau_m \leq \widetilde{\Delta} \\[3mm] \tau_m \text{ if } \tau_m > \widetilde{\Delta}, \end{cases} \qquad \pi_L = \pi_R = \begin{cases} \tau_b/2 \text{ if } \tau_m \leq \widetilde{\Delta} \\[3mm] \tau_m/2 \text{ if } \tau_m > \widetilde{\Delta}. \end{cases}$$

We now need to check whether this putative equilibrium is robust to larger deviations. There are four cases: $\Delta > \tau_b, \tau_m > \tau_b$; $\Delta > \tau_b \geq \tau_m$; $\tau_b \geq \Delta \geq \tau_m$; and $\tau_b \geq \Delta, \tau_m > \Delta$. In each case, there are two large deviations to consider: an increase and a decrease in P. Here we show how to rule-out these two deviations for the first of the four cases; the other three cases follow a similar logic, with details available on request.

Suppose $\Delta > \tau_b, \tau_m > \tau_b$. This implies that

$$x_p^*(x_b) = \frac{P_R - P_L + \tau_b - 2x_b\tau_b + \tau_m}{2\tau_m}$$

for every $x_b \in [0,1]$ and the putative equilibrium price identified above is $P_L = P_R = \tau_m$.

If L cuts P_L such that $P_L < \tau_b$ then $x_p^*(x_b) = 1$ for $x_b < (\tau_b - P_L)/2\tau_b$. L's profit is therefore

$$\left[\frac{\tau_b - P_L}{2\tau_b} + \int_{\frac{\tau_b - P_L}{2\tau_b}}^{1} \frac{P_R - P_L + \tau_b - 2x_b\tau_b + \tau_m}{2\tau_m} \, dx_b \right] P_L = \frac{P_L \left[(8\tau_m - \tau_b)\tau_b - 2P_L\tau_b - P_L^2 \right]}{8\tau_b\tau_m}.$$

One can check that this is increasing for every $P_L < \tau_b$ so the best such deviation is to $P_L = \tau_b$, which yields lower than the putative equilibrium profits.

If P_L is increased such that $P_L > 2\tau_m - \tau_b$ then $x_p^*(x_b) = 0$ for $x_b > (2\tau_m + \tau_b - P_L)/2\tau_b$. This implies L's profits are

$$P_L \int_0^{\frac{2\tau_m + \tau_b - P_L}{2\tau_b}} \frac{P_R - P_L + \tau_b - 2x_b\tau_b + \tau_m}{2\tau_m} \, dx_b = \frac{P_L(2\tau_m + \tau_b - P_L)}{8\tau_b\tau_m}.$$

One can check that this is decreasing for every $P_L > 2\tau_m - \tau_b$ so the best such deviation is to $P_L = 2\tau_m - \tau_b$, which yields lower than the putative equilibrium profits.

A.2.3 Step 3: computing consumer surplus

If we let $\widetilde{x}_p(x_b) = \min\{1, \max\{0, x_p^*(x_b)\}\}$, consumer surplus can be written as

$$2 \left[\int_0^1 \int_0^{\widetilde{x}_p(x_b)} \left(V - \tau_m x_p - \tau_b x_b - P_L \right) dx_p \, dx_b - \int_{\frac{\tau_b + \widetilde{\Delta}}{2\tau_b}}^1 \int_0^{\widetilde{x}_p(x_b)} \Delta \, dx_p \, dx_b \right]. \tag{15}$$

We have four cases depending on whether $\tau_m \leq \widetilde{\Delta}$ and whether $\tau_b \leq \Delta$. Case 1: if $\tau_m \leq \widetilde{\Delta}$ and $\tau_b \leq \Delta$ then (15) becomes

$$2 \int_0^{\frac{\tau_b - \tau_m}{2\tau_b}} \int_0^1 \left(V - \tau_m x_p - \tau_b x_b - \tau_b \right) dx_p dx_b +$$

$$2 \int_{\frac{\tau_b - \tau_m}{2\tau_b}}^{\frac{\tau_b + \tau_m}{2\tau_b}} \int_0^{\frac{\tau_b - 2x_b\tau_b + \tau_m}{2\tau_m}} \left(V - \tau_m x_p - \tau_b x_b - \tau_b \right) dx_p dx_b = V - \frac{1}{12} \left(6\tau_m - \frac{\tau_m^2}{\tau_b} + 15\tau_b \right). \tag{16}$$

Case 2: if $\tau_m \leq \widetilde{\Delta}$ and $\tau_b > \Delta$ then (15) is again given by (16).

Case 3: $\tau_m > \widetilde{\Delta}$ and $\tau_b \leq \Delta$ then (15) becomes

$$\int_0^1 \int_0^{\frac{\tau_b - 2x_b\tau_b + \tau_m}{2\tau_m}} \left(V - \tau_m x_p - \tau_b x_b - \tau_m \right) dx_p dx_b = V - \frac{1}{12} \left(6\tau_b - \frac{\tau_b^2}{\tau_m} + 15\tau_m \right).$$

Case 4: $\tau_m > \widetilde{\Delta}$ and $\tau_b > \Delta$ then (15) becomes

$$2\int_0^{\frac{\tau_b-\Delta}{2\tau_b}}\int_0^{\frac{\tau_m+\Delta}{2\tau_m}}\left(V - \tau_m x_p - \tau_b x_b - \tau_m\right)dx_p dx_b +$$

$$2\int_{\frac{\tau_b-\Delta}{2\tau_b}}^{\frac{\tau_b+\Delta}{2\tau_b}}\int_0^{\frac{\tau_b-2x_b\tau_b+\tau_m}{2\tau_m}}\left(V - \tau_m x_p - \tau_b x_b - \tau_m\right)dx_p dx_b +$$

$$2\int_{\frac{\tau_b+\Delta}{2\tau_b}}^{1}\int_0^{\frac{\tau_m-\Delta}{2\tau_m}}\left(V - \tau_m x_p - \tau_b(1 - x_b) - \tau_m\right)dx_p dx_b$$

$$= \frac{\Delta^2(3\tau_m - 2\Delta) + 3\tau_b\left(\Delta^2 - 2\Delta\tau_m + (4V - 5\tau_m)\tau_m\right) - 3\tau_b^2\tau_m}{12\tau_b\tau_m}.$$

Combining cases 1–4 yields the expression for consumer surplus in the statement of the lemma.

A.3 Proof of Proposition 7

We can rewrite (5) as

$$CS = V - \frac{1}{4}\left(\tau_b + 2\widetilde{\Delta} - \frac{\widetilde{\Delta}^2}{\tau_b} + 5\tau_m\right), \tag{17}$$

which is the consumer surplus under bundling (i.e., absent differentiation).

If $\tau_m > \min\{\tau_b, \Delta\}$ then the equilibrium involves no application differentiation, regardless of whether there is bundling or not. Thus, the price (and hence consumer surplus) are unchanged.

If $\tau_m \leq \min\{\tau_b, \Delta\}$ then, by Corollary 1, there will be application differentiation in the absence of bundling. Thus, (17) should be compared with the expression for consumer surplus found in Lemma 6. The change in consumer surplus from bundling is therefore

$$\left[V - \frac{1}{4}\left(\tau_b + 2\widetilde{\Delta} - \frac{\widetilde{\Delta}^2}{\tau_b} + 5\tau_m\right)\right] - \left[V - \frac{1}{12}\left(6\tau_m - \frac{\tau_m^2}{\tau_b} + 15\tau_b\right)\right],$$

which is positive if (in addition to $\tau_m < \widetilde{\Delta}$) we have

$$\tau_m < \frac{1}{2}\left(\sqrt{3}\sqrt{43\tau_b^2 + 4\widetilde{\Delta}^2 - 8\widetilde{\Delta}\tau_b} - 9\tau_b\right).$$

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