A Model of Merchants

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Abstract

I propose a simple model of merchants who are specialized in buying and selling a homogenous good. Facing the same frictions as in the buyer-seller direct trades, merchants can make profits with an ability to buy and sell many units of the good. They set the price to compete in the market and provide buyers with a strong likelihood of obtaining the good. This paper establishes a turnover equilibrium where some agents choose to become merchants endogenously. An interesting multiplicity can emerge.

Keywords: Retail trades, Specialization, Turnover behaviors, Directed search equilibrium

JEL Classification Number: D4, J6, L8

1 Introduction

This article is about economic agents called merchants who are specialized in buying a good from one individual and selling it to another at a higher price, and have the ability to do deals with many agents. Rubinstein and Wolinsky [22] propose that such agents can be active in equilibrium under market frictions if they have a higher matching rate than the other agents.

I propose a new framework in which agents can choose to be merchants. The idea is simple. To become a shopkeeper, one does not require technologies to produce a good, but must be able to buy and sell many units of the good.[1] Such an ability can be acquired by investing in technologies to manage inventories on a regular basis, and to run a store, typically located in a thickly settled area, and storage facilities that allow him to stand ready to serve many buyers. These retail technologies enable merchants to facilitate buyers to obtain the good with ease and convenience under market frictions. Somewhat surprisingly, this simple but essential idea has never been formulated explicitly in the economics literature.

In contrast to Rubinstein and Wolinsky [22], where the merchants' rates of serving buyers are exogenous, I present a simple model in which those rates are determined endogenously, using a standard directed-search approach.[2] Consider an economy that has two distinct markets for a homogeneous good, e.g., potato. A group of suppliers called farmers are located in one market and each farmer is able to serve only one buyer. In the other market another group of suppliers called merchants are located and each merchant is able to serve $k_m \geq 1$ buyers. Buyers can choose which market to search for the potato. They prefer low prices and a high

[1] To take an example from a modern specialized retailer, Wal-Mart receives more than 127 million US customers per week, which is more than one-third of the US population, making 312.4 billion total annual sales, while it stocks its merchandise from 61,000 U.S. suppliers (see Wal-Mart 2006 Annual Report available at http://www.walmartstores.com). If it is in the context of modern specialized retailers like Wal-Mart, then the ability of merchants may also include the adaptation of new information technology, such as bar cords and computer tracking of inventories, that can complement its ability to stock goods frequently, as pointed out by Holmes [11]. Looking into history, as emphasized by Botticini and Eckstein [4], the acquisition of trading skills, which started from the second century CE among the Jewish farmers and later provided them with the comparative advantage to become merchants, included education and investment in religious literacy.

[2] Unlike in traditional random matching models, directed-search equilibria incorporate price competition among sellers and buyers' choice of where to search. See, for example, Accemoglu and Shimer [1], Albrecht, Gautier and Vroman [2], Burdett, Shi and Wright [5], Coles and Eeckhout [8], Faig and Jerez [9], Julien, Kennes, and King [13], McAfee [17], Moen [18], Montogomery [19], Peters [20], and Shi [25, 26].

likelihood of finding it. In a market equilibrium, buyers are indifferent between searching in the farmers' market where both the price and the likelihood of obtaining the potato are low, and in the merchants' market where both the price and the likelihood are high. The capacity advantage of merchants in such an equilibrium generates a demand stimulating effect that directs more buyers to search in the merchants' market rather than in the farmers' market. The demand effect pushes up the price of merchants. At the same time, a larger capacity of merchants implies that excess demand is less likely to occur at individual merchants. This effect, which shall be referred to as a stockout effect, puts a downward pressure on the merchants' price. The stockout effect is relatively strong when the population of buyers is sufficiently low and the capacity of merchants is relatively large, so that the likelihood of excess demand at individual merchants is relatively small.

In this economy the ability of suppliers to serve buyers depends on how many potatoes they can have ready for sale. The farmers have production technologies but are not able to sell multiple units per unit of time. In contrast, the merchants do not have production technologies but are able to buy and sell multiple units of potatoes. The merchants can buy potatoes from different farmers and transport them from the farmers' market to the merchants' market, and keep restocking the potatoes to operate the market all the time.

Within this setup, I allow for suppliers to choose which market to operate. All suppliers are born with the ability to produce. One can become a merchant if he acquires the costly technologies that enable him to buy and sell multiple units of the good. Once becoming a merchant, he specializes in buying and selling so that he cannot produce the good. When deciding to be merchants, suppliers compare the expected benefit of serving a larger number of buyers against the net technology cost. An equilibrium is established in which some suppliers choose to be farmers and others choose to be merchants.

If the population of buyers is sufficiently high, the profits of merchants are sufficiently high and so an equilibrium exists with a positive measure of merchants for any level of the retail technology costs. Otherwise, such an equilibrium exists only for a sufficiently low level of the technology costs, and it turns out to be multiple – one is stable and has many merchants, each with few units and a high price, and the other is unstable and has few merchants, each with many units and a low price. This multiplicity is driven by the stockout effect of merchants' capacity. If it is expected that the retail price in the merchants' market is high, then many suppliers choose to become a merchant. Given a finite amount of potatoes, this means that each merchant holds few units of the good. The stockout effect of the relatively low capacity justifies the initial belief of suppliers that the merchants' market has a high price. A similar logic applies to the other equilibrium with few merchants, each with many units and a low price. As the technology costs decrease, there are a larger number of merchants, each holding fewer units, in the former equilibrium. However, the opposite conclusion follows in the latter equilibrium. This is because a lower price and lower profits of merchants should accompany an increase in the unit of each merchant, due to the stockout effect, and a decrease in the number of merchants. Therefore, in the latter equilibrium a decrease in the technology costs leads to a smaller number of merchants, each holding more units.

The paper is organized as follows. Section 2 constructs a static market equilibrium, which will be extended in the following sections, taking the number and capacity of merchants as exogenously given. Section 3 extends the analysis to allow for the restocking behavior of merchants in a stationary environment. Section 4 then describes a turnover equilibrium where the decision of suppliers to become a merchant is made explicit. Section 5 discusses the related literature. Section 6 concludes. All proofs are contained in the Appendix.

2 Market equilibrium

Consider an economy where there are a continuum of buyers and suppliers. In this section I assume there is only one period. The measure of buyers is normalized to one. The suppliers-buyers ratio is denoted by $S \in (0, \infty)$. A proportion $M \in (0, 1)$ of suppliers are called merchants while the remaining proportion $1 - M$ are called farmers. Index f, m and b refer to a farmer, merchant and buyer, respectively. Each buyer wishes to purchase one unit of a homogeneous good but can visit only one supplier. Each farmer can serve only one buyer whereas each merchant can serve $k_m \geq 1$ buyers. Buyers who are served at a price p obtain utility $1 - p$ whereas those who do not purchase obtain zero utility. Given their capacity, serving a buyer or buyers requires no costs so that suppliers who serve z buyers at a unit price p obtain profits zp. Within this section, I consider an economy as illustrated in Figure 1, taking M and k_m as exogenously given.

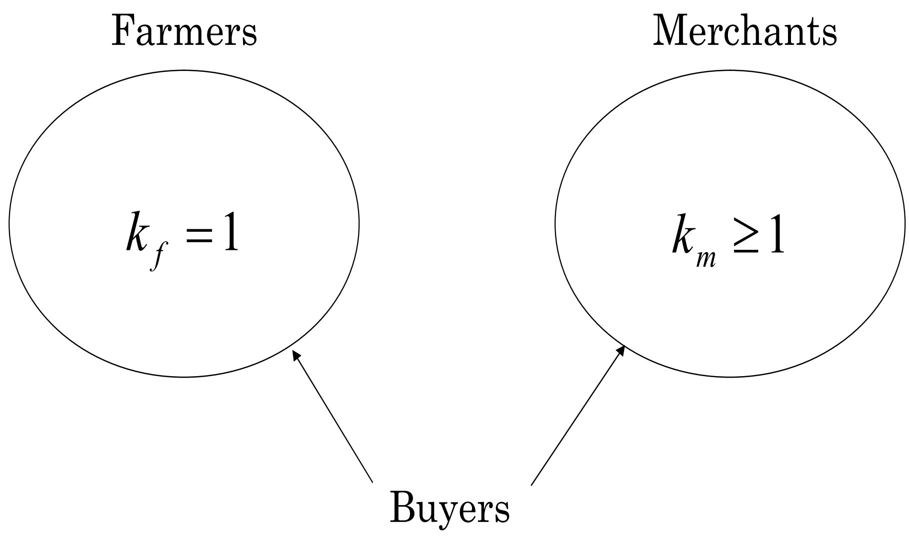

Figure 1: Search markets

The environment is the same as in standard directed search models (except that some suppliers can serve k_m buyers)[3] which can be described as a simple two-stage price posting game. In the first stage, suppliers simultaneously post a price they are willing to sell at, given the capacity. Observing the posted prices and capacities, buyers decide simultaneously which supplier to visit in the second stage. Assuming buyers cannot coordinate their actions over which suppliers to visit, I study a symmetric equilibrium in which all buyers use the identical mixing strategies. The mixing equilibrium in the second stage provides a reasonable description of a large game that captures coordination frictions among buyers, as is commonly employed in the directed search literature. Given the number of buyers who show up, the

[3] Burdett, Shi and Wright [5] and Shi [25] examine the case $k_m = 2$. The relationship to the directed search literature will be discussed in Section 5. In the Appendix, I show that the market equilibrium presented in this section is identical to the limiting solution in a finite setup counterpart as the population gets large.

farmer or merchant serves a buyer or buyers at the posted price. If there are more buyers than its capacity, then any buyer is served with equal probability.

In what follows, I construct a symmetric equilibrium where each merchant posts the identical price p_m and each farmer posts the identical price p_f. All buyers use the identical mixing strategies for any configurations of the announced prices including those where suppliers deviate from the equilibrium. Given those visiting strategies, each supplier is characterized by a queue of buyers denoted by x_i. The number of buyers visiting a supplier i is a random variable, denoted by n, which has the Poisson distribution $P(n = k) = \frac{e^{-x_i} x_i^k}{k!}$. In a symmetric equilibrium where x_i is the queue of buyers at a supplier i, each buyer visits some merchant (and some farmer) with probability SMx_m (and $S(1-M)x_f$), assigning an equal probability to each merchant (and each farmer). These visiting probabilities satisfy the adding-up restriction,

$$SMx_m + S(1-M)x_f = 1, \tag{1}$$

so that the number of buyers at all suppliers equals the total number of buyers.

Buyers' directed search: Assuming for the moment the existence of a symmetric equilibrium, the following lemma computes the probability of a buyer to get served by a supplier who has capacity k_i, denoted by $\eta(x_i, k_i)$. This probability is derived as follows. Suppose a buyer visits a supplier who has capacity k_i and n_i other buyers also visit it. Then the buyer is served with probability $\min\{\frac{k_i}{n_i+1}, 1\}$. $\eta(\cdot)$ is the sum of this probability over all $n_i = 0, 1, 2, ..., \infty$ with the Poisson density.

Lemma 1 *Given $x_i \in (0, \infty)$ and $k_i \geq 1$, the buyers' probability of being served by a supplier i that has capacity k_i is given by the following closed form expression.*

$$\eta(x_i, k_i) = \frac{\Gamma(k_i, x_i)}{\Gamma(k_i)} + \frac{k_i}{x_i}\left(1 - \frac{\Gamma(k_i+1, x_i)}{\Gamma(k_i+1)}\right),$$

where $\Gamma(k) = \int_0^\infty t^{k-1}e^{-t}dt$ and $\Gamma(k,x) = \int_x^\infty t^{k-1}e^{-t}dt$. $\eta(x_i, k_i)$ is strictly decreasing in x_i and strictly increasing in k_i, and satisfies $\eta(x_i, 1) = (1 - e^{-x_i})/x_i$.

Given $\eta(\cdot)$ derived above, I now characterize the expected queue of buyers. In any equilibrium where V^b is a buyer's expected utility, should a farmer or merchant deviate by setting a price p, the expected queue length denoted by x satisfies

$$V^b = \eta(x, k_i)(1 - p). \tag{2}$$

A buyer choosing a price p that has an expected queue x gets served and obtains $1 - p$ with probability $\eta(x, k_i)$. The situation is the same for all the other buyers. As $\eta(\cdot)$ is strictly decreasing in x, (2) determines $x = x(p, k_i \mid V^b) \in (0, \infty)$ as a strictly decreasing function of price p given k_i and V^b.

Optimal pricing: Given the buyers' directed search described above, the next step is to describe the optimal price of a supplier. In any equilibrium where V^b is buyers' expected utility, the optimal price of a supplier that has capacity k_i, denoted by $p_i(V^b)$, is given by

$$p_i(V^b) = \arg\max_p \; p x(p, k_i \mid V^b)\eta(x(p, k_i \mid V^b), k_i).$$

The expected number of buyers is x for a given price p, and each buyer is served with probability $\eta(x, k_i)$. Hence, the expected number of sales is given by $x\eta(x, k_i)$. The expected profits of the supplier i are price times the expected number of sales. Using (2) to substitute out price p, yields an objective function, denoted by $\pi_i(x)$, given by:

$$\pi_i(x) = x\eta(x, k_i) - xV^b.$$

Setting $\frac{\partial \pi_i(x)}{\partial x} = 0$ and rearranging it using (2), we have

$$p_i(V^b) = -\frac{\partial \eta(x, k_i)/\partial x}{\eta(x, k_i)/x} = \frac{k_i\left(1 - \frac{\Gamma(k_i+1, x)}{\Gamma(k_i+1)}\right)}{x\eta(x, k_i)} \tag{3}$$

where $x = x(p_i(V^b), k_i \mid V^b)$ satisfies (2). Note this objective function has a unique maximum because $\pi_i(x)$ is strictly concave in $x \in (0, \infty)$,

$$\frac{\partial^2 \pi_i(x)}{\partial x^2} = 2\frac{\partial \eta(x, k_i)}{\partial x} + x\frac{\partial^2 \eta(x, k_i)}{\partial x^2} = -\frac{x^{k_i - 1} e^{-x}}{\Gamma(k_i)} < 0.$$

Existence, uniqueness and characterization of market equilibrium:

Definition 1 *Given each farmer has capacity $k_f = 1$ and each merchant has capacity $k_m \geq 1$, a market equilibrium in this economy defines a set of payoffs $\{V^j\}$ for $j = b, f, m$, and market outcomes $\{x_i, p_i\}$, for $i = f, m$, such that:*

1. *Buyers' directed search satisfies (1) and (2);*

2. *Farmers' and merchants' price p_i satisfies the first order conditions (3) for $i = f, m$;*

3. *Strategies of agents are symmetric. Agents of the same type earn the same expected utility or profits V^j for $j = b, f, m$;*

4. *Agents' expectations are rational.*

The analysis above has established the equilibrium prices $p_i(V^b)$ for $i = f, m$ given V^b. Equilibrium implies buyers must be indifferent between these prices.

$$
\begin{aligned}
V^b &= \eta(x_f, 1)(1 - p_f) && (4)\\
&= \eta(x_m, k_m)(1 - p_m). && (5)
\end{aligned}
$$

These conditions determine the equilibrium V^b where $x_i = x(p_i, k_i \mid V^b)$ is the equilibrium queue and so buyers successfully get served by the farmer or merchant with probability $\eta(x_i, k_i)$. A supplier that has capacity k_i obtains equilibrium expected profits given by

$$V^i = x_i \eta(x_i, k_i) p_i. \tag{6}$$

Identifying a market equilibrium is now reduced to a standard fixed point problem.

Theorem 1 (Market equilibrium) *For any $k_m \geq 1$, $S \in (0, \infty)$ and $M \in (0, 1)$, a market equilibrium exists and is unique.*

Figure 2 illustrates a market equilibrium. The downward sloping line represents the adding-up constraint (1), while the upward sloping curve represents the buyers' indifference conditions (4) and (5). The latter curve can be obtained by substituting out prices in (4) and (5) using (3) that yields

$$\frac{\Gamma(k_m, x_m)}{\Gamma(k_m)} = e^{-x_f}. \tag{7}$$

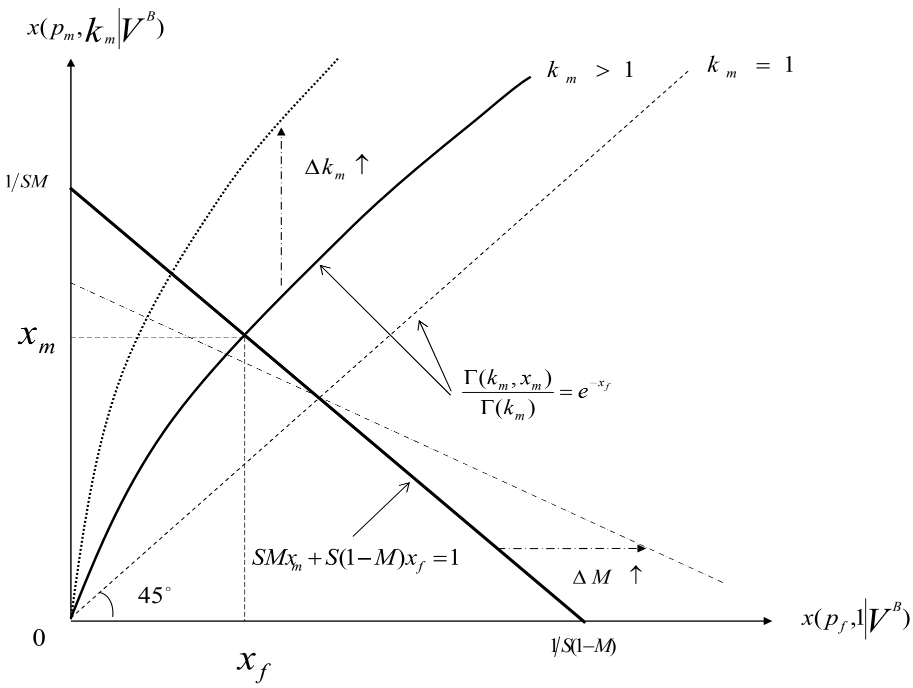

Figure 2: Market equilibrium

An intersection of these two curves identifies $x_f, x_m > 0$ that are unique. The upward sloping curve becomes a 45 degree line when $k_m = 1$ in which case all suppliers receive the identical number of buyers $x_f = x_m$; post the identical price $p_f = p_m$; and make the identical profits $V^f = V^m$. Figure 3 illustrates the equilibrium price formation. The iso-profit curve of a supplier with capacity k_i has a slope given by $\frac{dp_i}{dx_i} = -\frac{p_i}{x_i\eta}\frac{\partial x_i\eta}{\partial x_i}$, while the indifference curve has a slope given by $\frac{dp_i}{dx_i} = \frac{1-p_i}{\eta}\frac{\partial\eta}{\partial x_i}$. Higher price and/or longer queue increase profits of a supplier and decrease utility of buyers. The equilibrium price is determined by the tangency point of the iso-profit curve and the indifference curve, and is unique.

An increase in the proportion of merchants leads to lower market prices and higher utility of buyers. That is, an increase in M is represented by a flatter downward sloping line in Figure 2 which decreases x_f, x_m for $k_m > 1$, and by an inward shift of the indifference curve in Figure 3 which decreases p_f, p_m. An increase in the capacity of merchants k_m creates a demand stimulating effect that induces buyers to visit merchants more intensively and to visit farmers less intensively. In Figure 2, this effect is represented by a pivot up of the upward sloping

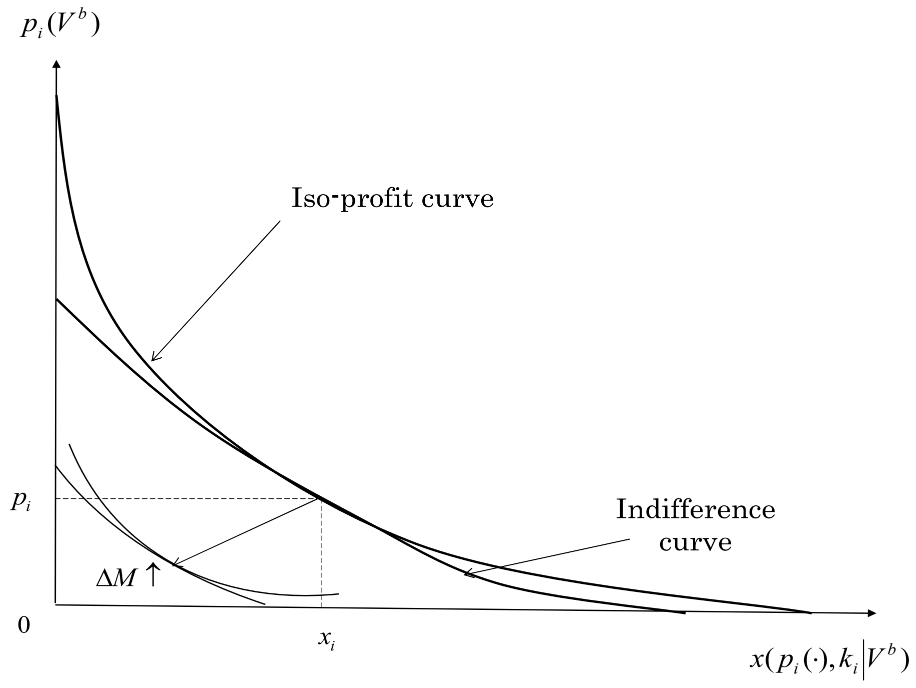

Figure 3: Equilibrium price

curve. The demand effect increases p_m, x_m and decreases p_f, x_f. At the same time, a larger k_m implies it is less likely that excess demand occurs at individual merchants. This effect, referred to as a stockout effect, generates a downward pressure on p_m. While it is difficult to illustrate it in Figure 3, it is intuitive that the stockout effect is relatively strong when the population of buyers is small, i.e. S is large, or the capacity of merchants k_m is large so that the likelihood of excess demand at individual merchants is relatively small. Interested readers can refer to Watanabe [30, 31] for more detailed analysis.

3 Steady state equilibrium

Consider an infinite period version of the above model. In each period buyers wish to obtain one unit of a storable good and each farmer can serve one buyer and each merchant can serve k_m buyers. The environment is exactly the same as before. I assume infinite discounting so that all agents have zero future payoffs at any given time period. As the future is fully discounted, myopic agents solve a static problem, which is identical to the one described in the previous section, each period. A market equilibrium in this modified setup is an infinite sequence of the one described in Theorem 1 where each farmer serves $x_f\eta(x_f, 1) = 1 - e^{-x_f}$ buyers and each merchant serves $x_m\eta(x_m, k_m)$ buyers in each period.

Suppose now that farmers are able to sell only one unit but are able to produce the good with zero cost each period. In contrast, merchants are able to buy and sell multiple units each period but do not have production technologies. In order to operate in the market all through the periods, merchants must keep restocking their units each period from farmers. Interpreting the capacity of suppliers as the number of goods they hold, a steady state implies each supplier holds the same unit at the beginning of all periods.

Definition 2 *Given the initial endowments, a steady state equilibrium defines an infinite sequence of the market equilibrium described in Theorem 1 in which merchants restock their units from farmers at the end of each period and each farmer holds one unit and each merchant holds k_m units at the start of each period.*

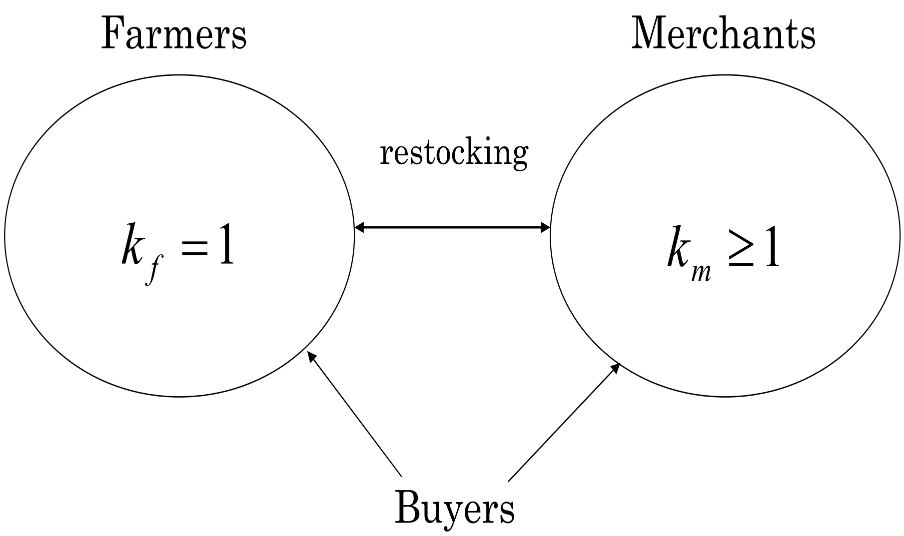

Figure 4: Steady state equilibrium

At the end of each period, there are $S(1-M)e^{-x_f}$ units in total that farmers have failed to sell. The farmers do not have strict incentives to carry over the remaining unit. On the other hand, the steady state equilibrium requires that each merchant restock $x_m\eta(x_m, k_m)$ units (to hold k_m units at the start of each period). The merchants can restock the necessary units from the remaining farmers as long as it holds that

$$Mx_m\eta(x_m, k_m) \leq (1 - M)e^{-x_f}, \tag{8}$$

where $x_m = x_m(k_m, S, M)$ and $x_f = x_f(k_m, S, M)$ are determined in Theorem 1 conditioned on values of k_m, S, M. Under the infinite discounting of agents, the market structure and storage capability of farmers are irrelevant for merchant-farmer trades and the restocking price must equal zero. That is, as the future payoffs are irrelevant merchants do not buy the units at the end of any given period unless they can be obtained at zero price. A usual tie-breaking assumption guarantees that farmers sell to a merchant. To guarantee the existence of a steady state equilibrium, a set of parameters k_m, S, M that satisfies the steady state condition (8) should be identified. Define $\underline{M} \equiv \max\{0, (S-1)/S\}$ and $\bar{M} \equiv e^{-1/S}$, and denote by $\bar{k}_m$ a solution to the equality (8). It turns out that $\bar{k}_m = k_m(M) \in [1, \infty) \subset \mathbf{R}_+$ is unique and strictly decreasing in M (see the proof of Theorem 2).

Theorem 2 (Steady state equilibrium)

1. *If $M \in (0, \underline{M}]$, then a steady state equilibrium exists and is unique for all $k_m \geq 1$.*
2. *If $M \in (\underline{M}, \bar{M}]$, then a steady state equilibrium exists and is unique for $k_m \leq \bar{k}_m$ and no steady state equilibrium exist for $k_m > \bar{k}_m$.*
3. *If $M \in (\bar{M}, 1)$, then no steady state equilibrium exist.*

In a steady state equilibrium there is a parameter restriction in terms of the scale and quantity of merchants: if the proportion of merchants M is relatively large, then the units of each merchant k_m need to be relatively small.

It is instructive to mention what would happen if the full-discounting assumption is relaxed. If agents have a non-zero discount factor then there would exist a non-zero surplus to be shared between merchants and farmers, thus the future returns of units matter to agents' decisions at the end of each period. In such a case, the market structure at the restocking stage and the storage capability of farmers need to be made explicit. This line of extension is pursued in Watanabe [31], where I consider a frictionless restocking market and the storage capability of both merchants and farmers. The former is a simplifying assumption that would not affect the essentials, whereas the latter renders farmers with a non-zero option value of not selling to a merchant. While it is true that much complication can arise once the infinite-discounting assumption is relaxed, Watanabe [31] develops a simple methodology that the results presented in Theorem 1 and 2 of the current paper can be still valid for all values of discount factor.[4]

4 Turnover equilibrium

The previous section has established a steady state equilibrium given that merchants are able to buy multiple units from different farmers and to sell them to buyers each period. This section describes the origin of merchants and turnover behavior of farmers.

Suppose now that suppliers can choose to be either a farmer or a merchant before the entire period starts. Suppliers are born with the ability to produce. I assume that one can acquire technologies to be merchants by paying fixed costs $c > 0$. Once becoming a merchant he specializes in buying and selling so that he can not produce the good. Investing in these technologies may imply establishing distribution facilities, e.g., trucks, warehouses and shops. These technologies enable one to buy multiple units from different farmers and to serve more than one buyers per unit of time.

Definition 3 *Suppose that suppliers can choose to be either a farmer or a merchant. A turnover equilibrium defines a steady state equilibrium where the proportion of merchants $M > 0$ satisfies*

$$V^m - V^f = c \tag{9}$$

where $V^i = V^i(M \mid k_m, S)$ for $i = f, m$ are determined in Theorem 1 and 2 given that $k_m \geq 1$, $S \in (0, \infty)$ and $M \in (0, 1)$ satisfy the steady state condition (8), and $c \in (0, \infty)$ represents the costs of becoming a merchant.

[4]Watanabe [31] establishes a unique steady-state equilibrium allocation that is independent of the discount factor, thus the condition for the existence of a steady state equilibrium, similar to the one described in Theorem 2, still holds for all values of discount factor.

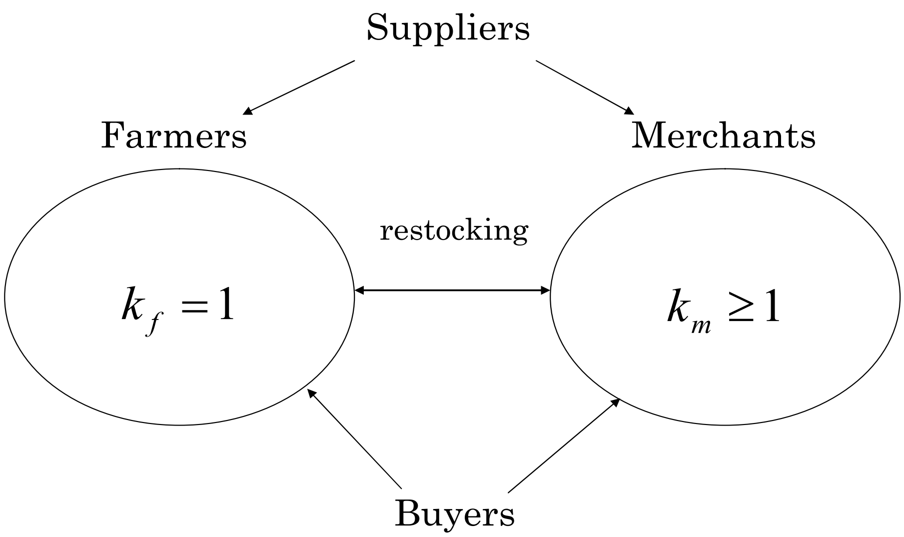

Figure 5: Turnover equilibrium (1)

In what follows, I focus on a case where the fraction of merchants' trades is large enough to make the steady state condition binding, i.e. case 2 in Theorem 2. This case is *essential* in the sense that case 1 would have some produced units never passed on to buyers, whereas case 2 has no such wastes each period.[5] In other words, I focus on a Walrasian restocking market where in aggregate supply equals demand at price equal to marginal production costs of farmers (normalized to zero). The binding steady state condition implies a negative, one to one relationship between the number of merchants and the units of each merchant.

Proposition 1 (Turnover equilibrium) *Consider a steady state equilibrium in which the steady state condition (8) is binding. If $S \in (0,1)$, then a turnover equilibrium exists for all $c \in (0,\infty)$. Otherwise, there exists some $\bar{c} < \infty$ such that a turnover equilibrium exists for $c \in (0,\bar{c}]$ but not for $c \in (\bar{c},\infty)$.*

A turnover equilibrium exists in which a positive proportion of suppliers $M > 0$ choose to become merchants instead of being farmers. In such an equilibrium, the marginal farmer is indifferent between being a farmer and choosing to be a merchant. The farmers' turnover

[5]More precisely, case 1 has some units wasted within the same period of production if farmers have no storage capability, or in the following period if produced units can be stored but for no more than two periods in the hands of the farmers. Either of these assumptions is consistent with the condition (8).

behavior reflects a tradeoff between the costs of investing in the technologies and the expected benefits from operating as a merchant where he can make a larger number of sales than it would be possible to make as a farmer. The existence of equilibrium depends on the population parameter. If the population of buyers is sufficiently large (i.e. $S < 1$), then profits of middlemen are sufficiently large and a turnover equilibrium exists for all $c \in (0, \infty)$. Otherwise, profits of merchants are not high and an equilibrium exists for $c \leq \bar{c}$ but not for $c > \bar{c}$ for some $\bar{c} \in (0, \infty)$.

Proposition 2 (Multiplicity of turnover equilibria) *If there are a relatively small population of buyers (i.e. $S \geq 1$) then multiple turnover equilibria exist – one is stable and has many merchants, each with few units and a high price, while the other is unstable and has few merchants, each with many units and a low price.*

The multiplicity of turnover equilibria is driven by the stockout effect of merchants' capacity on their price. If it is expected that the retail price in the merchants' market is high, then many suppliers choose to become a merchant. The turnover equilibrium in this case has many merchants each holding few units of the good. The stockout effect of the relatively low k_m justifies the initial belief of suppliers that the price in the merchants' market is high. A similar logic applies to the other equilibrium with few merchants, each holding many units, and a low price in the merchants' market.

Figure 6 illustrates turnover equilibria. For $S < 1$, as the technology costs of becoming a merchant c decrease, a turnover equilibrium has a larger number of merchants, each with fewer units, as depicted by point A in the figure. In the limit as $c \to 0$, all suppliers hold the identical unit; charge the identical price; receive the identical number of buyers; and make identical profits. The same happens for $S \geq 1$ if the turnover equilibrium has a relatively large number of merchants, initially located at point B. In contrast, if the equilibrium is depicted at point C, then there are a relatively small number of merchants, each holding many units. The stockout effect, which is dominant in this case, is to lower their price and profits, accompanying an increase in the units of each merchant and a decrease in the number of merchants. Therefore, if the equilibrium is at point C initially, then a decrease in the technology costs leads to a smaller number of merchants, each holding more units.

To see the stability property of turnover equilibria, imagine a situation in which a farmer deviates to be a merchant and buyers respond accordingly. As evident from the figure, a marginal increase in M increases the profitability of being a merchant $V^m - V^f$ if the equilibrium allocation is at point C. This implies further entry increases returns above the costs, so the equilibrium at point C is unstable. In contrast, under the equilibrium allocation at point A and B, the opposite happens, and hence this equilibrium is stable.

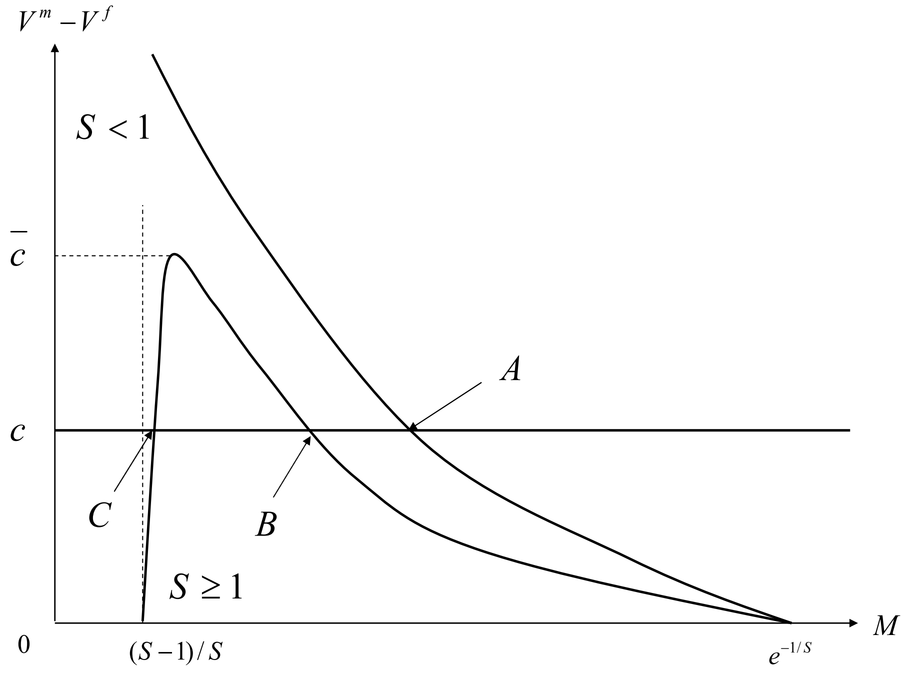

Figure 6: Turnover equilibrium (2)

The total welfare in this economy, denoted by W, is given by

$$W \equiv SMV^m + S(1-M)V^f + V^b - SMc = SV^f + V^b = S\left(1 - e^{-x_f} - x_f e^{-x_f}\right) + e^{-x_f}$$

where the second equality is by the turnover equilibrium condition (9). Observe that profits of merchants are irrelevant and the total welfare remains unchanged even if the specification of the technology costs is modified to be non-constant. As W is decreasing in x_f, the larger the total welfare, the larger the buyers' welfare. In case of multiple equilibria, the number of buyers x_f in the farmers' market is smaller when each merchant holds more units, as in point C, than when each merchant holds fewer units, as in point B. Therefore, each merchant serves a larger number of buyers, which leads to a higher welfare, in the C-type equilibrium than in the B-type equilibrium.[6] By the same reason, the welfare increases with the technology costs c if the number of merchants decreases with c, as in type A and B, whereas the welfare decreases with the technology costs c if the number of merchants increases with c, as in type C.

5 Discussion

The literature to which the present paper bares comparison would be that on middlemen.[7] The most closely related paper is Rubinstein and Wolinsky [22]. Within a bilateral random matching framework, they show that middlemen can be active in equilibrium, assuming that the middlemen have a higher meeting rate than the other agents. Deriving the middlemen's advantage in the matching technology endogenously has been at the center of the subsequent extensions. Li [14] assumes that the qualities of goods are private information and shows middlemen invest in identifying the quality of goods. Shevichenko [24] assumes ex ante heterogeneity of goods and agents' preferences and shows that middlemen can mitigate the severity of double-coincidence of wants problem. However, middlemen can provide buyers with a variety of goods only if they are able to buy and sell multiple units, hence ex ante heterogeneity of tastes/goods is not a necessary condition for the high meeting rates between middlemen and buyers. Masters (2007) assumes heterogeneity in production costs for a divisible good and shows that agents with relatively high production costs choose to become a middleman. In his

[6]If individual buyers have a downward sloping demand schedule, then an intensive margin may matter and the monopoly power of few merchants may deteriorate the welfare. However, the welfare dominance of C-type equilibrium would still survive as long as the price elasticity of individual demand is not so high that the extensive margin of large-scaled merchants outweighs the intensive margin of few merchants.

[7]See also Biglaiser [3], Caillaud and Jullien [6], Johri and Leach [12], and Smith [27].

model, agents trade off the opportunity cost of waiting consumption, which is heterogeneous across agents, against a better terms of trade that can be obtained as a middlemen. However, the meeting rates of agents in his model depend exogenously on the aggregate quantity (like in Pissarides [21]), and cannot influence the incentives of individuals to become a middleman. In contrast, the meeting rates of individuals in my model have an endogenous link to the benefit of becoming a middleman, selling to a larger number of buyers at a higher price. It is exactly this mechanism that the size and the number of middlemen generate differential implications on the overall economy in my model. Another feature in this strand is that the terms of trades are determined by bilateral bargaining, hence price competition among middlemen is absent in these models.

Price competition among middlemen is considered by Spulber [28] who also uses a random meeting model. Because middlemen are assumed to be the only medium of exchange in his model, recent works in this strand incorporate an additional avenue of exchange – a monopolist market maker in Rust and Hall [23] and buyer-seller direct trades in Hendershott and Zhang [10]: recently, Loertscher [15] has allowed for horizontal product differentiation. However, since the matching technology in this approach is exogenous, middlemen's advantage in the matching rate cannot be addressed in these models. Further, while the importance of the role of middlemen's inventory to mitigate market frictions is well recognized,[8] there has been no attempt to incorporate such a role of middlemen into the framework in this strand.

In the present paper, I have proposed a simple model using a standard directed-search approach that integrates the key roles of merchants mentioned above – the high meeting rate and price competition. In particular, it has focussed on the issue of turnover decision to become a merchant under the simplifying assumption of myopic agents. Watanabe [31] provides a general model in which suppliers are allowed to be forward-looking. This allows merchants

[8]Rust and Hall put "An important function of intermediaries is to hold inventory to provide a buffer stock that offers their customers liquidity at times when there is an imbalance between supply and demand. In the securities business, liquidity means being able to buy or sell a reasonable quantity of shares on short notice. In the steel market, liquidity is also associated with a demand for immediacy so that a customer can be guaranteed of receiving shipment of an order within a few days of placement. Lacking inventories and stockouts, this model cannot be used to analyze the important role of intermediaries in providing liquidity." (page 401).

and farmers to share surplus in the restocking market, thus the equilibrium restocking price can depend on the size and number of merchants. The model presented in the current paper is a special case of the one developed in Watanabe [31]. Assuming away the turnover decision of farmers, that paper studies the behavior of the bid-ask spread of merchants.

In relation to the directed-search literature, the closest paper to mine is Burdett, Shi, and Wright [5]. Indeed, the market equilibrium described in Section 2 corresponds to theirs, if $k_m = 2$. On page 1076, they wrote "It might be interesting to endogenize capacity along these lines in the general case of n buyers and m sellers." The turnover equilibrium constructed in Section 4 can be taken as one attempt in line with their suggestion. As for the welfare implication, it turns out that an economy with few sellers each holding many units can achieve a higher welfare than another economy with many sellers each holding few units, given fixed total supply. Coles and Eeckhout [8] study a setup in which sellers can post a more general trading mechanism for a finite number of agents. They show that a continuum of equilibria exist including an equilibrium with a simple form of price posting, i.e., the one studied in Burdett, Shi and Wright, while sellers prefer an equilibrium with auction. With a continuum of agents, auction and price posting are practically equivalent, with sellers achieving the same revenue and guaranteeing buyers the same utility. A usual argument applies: relatively high transaction costs associated with establishing and implementing auction can make sellers prefer price posting. This makes sense in particular for the economy considered here where retail technologies are made explicit and play an important economic role for merchants' profits.

6 Conclusion

This paper has presented a simple theory of merchants. The idea proposed in this paper that some agents can choose to become merchants specialized in buying and selling many units is truly simply and intuitively appealing. Clower and Leijonhufvud [7] use an inventory-based competitive 'supermarket' story and observe that the presence of frictions and the absence of coordinations in market exchanges may lead to the rise of merchant traders and organized markets. I have used a simple model that has coordination frictions and demonstrated that it is essential that merchants are able to buy and sell many units of the good in the first place. An interesting extension is to incorporate ex-ante heterogeneity of goods, buyers' preferences or traveling costs with imperfect information. These additional ingredients would magnify the matching effectiveness and profits of merchants. It will also be interesting to insert the model into the standard neoclassical growth framework and explore macroeconomic implications of labor reallocation from the manufacturing sector to the service sector.

7 Appendix

7.1 Proof of Lemma 1

I simplify the notation to set $x = x_i$ and $k = k_i$. Because the number of buyers arriving at a supplier n follows the Poisson distribution, if a buyer chooses a supplier that can serve k buyers, then the probability of being served by this supplier is given by:

$$\begin{aligned}\eta(x,k) &= \Pr(n \leq k-1)\Pr(\text{served} \mid n \leq k-1) + \Pr(n \geq k)\Pr(\text{served} \mid n \geq k) \\ &= \sum_{j=0}^{k-1} \frac{x^j e^{-x}}{j!} + \sum_{j=k}^{\infty} \frac{x^j e^{-x}}{j!} \frac{k}{j+1}\end{aligned}$$

where x is the expected number of buyers at this supplier. If n is less than the given supplier's capacity, a buyer choosing the given supplier gets served with probability one (the first term above). Otherwise, there is a possibility of being rationed and she/he gets served with probability $\frac{k}{j+1}$, where $j \in \{k, k+1, ..., \infty\}$ counts the realized number of buyers in these cases (the second term above).

A rearrangement yields:

$$\begin{aligned}\eta(x,k) &= \sum_{j=0}^{k-1} \frac{x^j e^{-x}}{j!} + k\sum_{j=0}^{\infty} \frac{x^j e^{-x}}{(j+1)!} - k\sum_{j=0}^{k-1} \frac{x^j e^{-x}}{(j+1)!} \\ &= \frac{\Gamma(k,x)}{\Gamma(k)} + \frac{k}{x}\left(1-e^{-x}\right) - \frac{k}{x}\left(\frac{\Gamma(k+1,x)}{\Gamma(k+1)} - e^{-x}\right) \\ &= \frac{\Gamma(k,x)}{\Gamma(k)} + \frac{k}{x}\left(1 - \frac{\Gamma(k+1,x)}{\Gamma(k+1)}\right).\end{aligned}$$

To reach the second equality, the following manipulations are performed. The first term follows from $\sum_{j=0}^{k-1} \frac{x^j e^{-x}}{j!} = \frac{\Gamma(k,x)}{\Gamma(k)}$ (i.e., the series definition of the cumulative gamma function) where $\Gamma(k) = \int_0^\infty t^{k-1}e^{-t}dt$ and $\Gamma(k,x) = \int_x^\infty t^{k-1}e^{-t}dt$. To obtain the second term, set $h = j+1$ and go as follows:

$$\sum_{j=0}^{\infty} \frac{x^j e^{-x}}{(j+1)!} = \sum_{h=1}^{\infty} \frac{x^{h-1}e^{-x}}{h!} = \frac{1}{x}\sum_{h=1}^{\infty} \frac{x^h e^{-x}}{h!} = \frac{1}{x}\left(\sum_{h=0}^{\infty} \frac{x^h e^{-x}}{h!} - e^{-x}\right) = \frac{1}{x}\left(1-e^{-x}\right).$$

The third term is obtained by combining the two manipulations described above. It is immediate that $\eta(x,k)$ is strictly decreasing in x and strictly increasing in k, and satisfies $\eta(x,1) = (1-e^{-x})/x$. ■

7.2 Proof of Theorem 1

The proof takes 3 steps. Step 1 establishes that equilibrium requires $V^b \in [0,1]$. For any $V^b \in [0,1]$, Step 2 establishes that (3),(4),(5) imply a unique solution x_i for $i = f, m$. With a slight abuse of notation, let $x_i(V^b)$ denote this solution. An equilibrium is then identified by noting (1) requires V^b satisfies the fixed point condition

$$SMx_m(V^b) + S(1-M)x_f(V^b) = 1 \tag{10}$$

where S, M is a positive constant. Using Steps 2, Step 3 establishes that there exists a unique $V^b \in (0,1)$ satisfying this condition. Hence Step 3 establishes an equilibrium exists and is unique: given V^b satisfying the fixed point condition (10), $x_f, x_m \in (0,\infty)$ are uniquely determined in Step 2, $p_f, p_m \in (0,1)$ are uniquely determined by (4), (5), $V^i \in (0,k_i)$ for $i = f, m$ is uniquely determined by (6), respectively. By construction, this solution then satisfies the equilibrium requirements (1), (3)-(6) for $i = f, m$ and so describes equilibrium.

Step 1 Equilibrium implies $V^b \in [0,1]$.

Proof of Step 1. (3) and (4) for $i = f$ imply $V^b = e^{-x_f}$. As equilibrium implies $x_f \geq 0$, it follows that $V^b \in [0,1]$. This completes the proof of Step 1.

Step 2 For any $V^b \in [0,1]$, a solution x_i for $i = f, m$ defined by (3),(4),(5) exists, is unique and implies: $x_i(V^b)$ is continuous and strictly decreasing in V^b and satisfies $x_i(V^b) \to \infty$ as $V^b \to 0$ and $x_i(1) = 0$.

Proof of Step 2. The claim for $i = f$ is immediate by Step 1. To prove the claim for $i = m$, substituting out p_m from (5) using (3) yields

$$\frac{\Gamma(k_m, x_m)}{\Gamma(k_m)} = e^{-x_f(V^b)} \ .$$

The R.H.S. of this equation is strictly increasing in V^b and satisfies $e^{-x_f(0)} = 0$ and $e^{-x_f(1)} = 1$ while L.H.S. is strictly decreasing in x_m and satisfies $\frac{\Gamma(k_m,0)}{\Gamma(k_m)} = 1$ and $\frac{\Gamma(k_m,x_m)}{\Gamma(k_m)} \to 0$ as $x_m \to \infty$. Therefore, there exists a unique solution $x_m(V^b)$ that is continuous and strictly decreasing in V^b that satisfies $x_m(V^b) \to \infty$ as $V^b \to 0$ and $x_m(1) = 0$. This completes the proof of Step 2.

Step 3 There exists a unique $V^b \in (0,1)$ that satisfies the fixed point condition (10).

Proof of Step 3. By Step 2, the L.H.S. of (10), denoted by $L_m(V^b)$, is continuous and strictly decreasing in V^b and satisfies $L_m(V^b) \to \infty$ as $V^b \to 0$ and $L_m(1) = 0$. Because the R.H.S. of (10) is a positive constant, there exists a unique $V^b \in (0,1)$. This completes the proof of Step 3. ■

7.3 Proof of Theorem 2

The proof takes two steps. Step 1 establishes that (1),(3),(4),(5) imply a unique solution x_i for all $k_m \geq 1$, $S \in (0,\infty)$, $M \in (0,1)$ and $i = f, m$. With a slight abuse of notation, let $x_i(k_m, S, M)$ denote this solution. Step 2 then identifies a set of parameters k_m, S, M that satisfies the steady state condition (8),

$$Mx_m(k_m, S, M)\eta(x_m(k_m, S, M), k_m) \leq (1-M)e^{-x_f(k_m,S,M)}. \tag{11}$$

Given k_m, S, M satisfying (11), Step 1 pins down a unique $x_i \in (0,\infty)$ for $i = f, m$; the other equilibrium values are identified by the same procedure as in Theorem 1 using (4),(5),(6) for $i = f, m$. This solution then satisfies the equilibrium requirements (1), (3)-(6),(8) and so describes a unique steady state equilibrium.

<u>**Step 1**</u> For any $k_m \geq 1$, $S \in (0,\infty)$ and $M \in (0,1)$, a solution $x_i = x_i(k_m, S, M)$ to (1),(3),(4),(5) exists and is unique for $i = f, m$ that is: continuous in $S, M, k_m \in \mathbf{R}_+$; strictly decreasing in $S \in (0,\infty)$ for all $k_m \geq 1$ and $M \in (0,1)$; strictly decreasing in $M \in (0,1)$ for $k_m > 1$ and $S \in (0,\infty)$; strictly increasing (decreasing) in k_m for $S \in (0,\infty)$ and $M \in (0,1)$ if $i = m$ $(i = f)$ satisfying $x_f(1,\cdot) = x_m(1,\cdot) = 1/S$, $x_f(k_m,\cdot) \to 0$ and $x_m(k_m,\cdot) \to 1/SM$ as $k_m \to \infty$.

Proof of Step 1. Note (3),(4),(5) are reduced to (7). Substituting out x_m in (7) by using (1),

$$\frac{\Gamma(k_m, \frac{1-S(1-M)x_f}{SM})}{\Gamma(k_m)} = e^{-x_f}. \tag{12}$$

The L.H.S. of this equation, denoted by $\Phi(x_f, k_m, S, M)$, is continuous and strictly increasing in x_f and $k_m \in \mathbf{R}_+$, satisfying for any $S \in (0,\infty)$ and $M \in (0,1)$:

$$\Phi(x_f,\cdot) \to \frac{\Gamma(k_m, 1/SM)}{\Gamma(k_m)} < 1 \text{ as } x_f \to 0; \quad \Phi(1/S,\cdot) = \frac{\Gamma(k_m, 1/S)}{\Gamma(k_m)}$$

which is equal to (greater than) $e^{-1/S}$ when $k_m = 1$ $(k_m > 1)$;

$$\Phi(x_f, 1, \cdot) = e^{-(1-S(1-M)x_f)/SM}; \quad \Phi(x_f, k_m, \cdot) \to 1 \text{ as } k_m \to \infty.$$

Similarly, $\Phi(\cdot)$ is continuous and strictly increasing in S and M for any $x_f \in (0, 1/S)$ and $k_m \geq 1$. It follows therefore that a unique solution $x_f = x_f(k_m, S, M) \in (0, 1/S]$ exists that is: continuous and strictly decreasing in $k_m \in [1,\infty) \subset \mathbf{R}_+$ satisfying $x_f(1,\cdot) = 1/S$ and $x_f(k_m,\cdot) \to 0$ as $k_m \to \infty$ for any S, M; continuous in S and M; strictly decreasing in S for all $k_m \geq 1$; strictly decreasing in M for $k_m > 1$.

Applying this solution to (1), it is immediate that a unique solution $x_m = x_m(k_m, S, M) \in [1/S, 1/SM)$ exists that is: continuous in $S, M, k_m \in \mathbf{R}_+$; strictly decreasing in S for all $k_m \geq 1$; strictly decreasing in M for $k_m > 1$; strictly increasing in k_m satisfying $x_m(1,\cdot) = 1/S$ and $x_m(k_m,\cdot) \to 1/SM$ as $k_m \to \infty$. This completes the proof of Step 1.

Step 2 (i) For $M \in (0, 1 - \frac{1}{S}]$, (11) holds for all $k_m \geq 1$. (ii) For $M \in (\max\{0, 1 - \frac{1}{S}\}, e^{-\frac{1}{S}}]$, (11) holds for $k_m \leq \bar{k}_m$ and (11) does not hold for $k_m > \bar{k}_m$ where $\bar{k}_m = k_m(M) \in [1, \infty) \subset \mathbf{R}_+$ is strictly decreasing in M. (iii) For $M \in (e^{-\frac{1}{S}}, 1)$, there is no $k_m \geq 1$ that satisfies (11).

Proof of Step 2. For $S \in (0, \infty)$, $M \in (0, 1)$ and $k_m \in [1, \infty) \subset \mathbf{R}_+$, define

$$\Psi(k_m, S, M) \equiv M x_m(k_m, S, M)\eta(x_m(k_m, S, M), k_m) - (1 - M)e^{-x_f(k_m, S, M)}$$

where $x_i(k_m, S, M)$ satisfies the properties obtained in Step 1 for $i = f, m$. (11) requires $\Psi(\cdot) \leq 0$. Observe that $\Psi(\cdot)$ is continuous and strictly increasing in both M and k_m for any $S \in (0, \infty)$, and satisfies: $\Psi(1, S, M) = M - e^{-1/S}$; $\Psi(k_m, S, M) \to 1/S - (1 - M)$ as $k_m \to \infty$. Therefore, it holds that for all $k_m \geq 1 \in \mathbf{Z}_+$: $\Psi(\cdot) \leq 0$ if $M \leq 1 - \frac{1}{S}$; $\Psi(\cdot) > 0$ if $M > e^{-\frac{1}{S}}$. Hence, the claims (i) and (iii) follow. If $M \in (\max\{0, 1 - \frac{1}{S}\}, e^{-\frac{1}{S}}]$ then $\Psi(\cdot) \leq 0$ for $k_m \leq \bar{k}_m$ and $\Psi(\cdot) > 0$ for $k_m > \bar{k}_m$, where $\bar{k}_m = k_m(M) \in [1, \infty) \subset \mathbf{R}_+$ is a unique solution to $\Psi(\bar{k}_m, \cdot) = 0$, which is strictly decreasing in M. Hence, the claim (ii) follows. This completes the proof of Step 2. ■

7.4 Proof of Proposition 1

The proof here is to identify the value of $M \in (\underline{M}, \bar{M}]$, where $\underline{M} \equiv \max\{0, 1 - \frac{1}{S}\}$ and $\bar{M} \equiv e^{-\frac{1}{S}}$, that satisfies the binding steady state condition (8) and the free-entry condition (9). Remember that Step 1 in the proof of Theorem 2 has derived a solution $x_i(\cdot)$ to (1),(3),(4),(5), for $i = f, m$. Remember also that Step 2 in the proof of Theorem 2 has derived a solution to the equation (8), denoted by $k_m(M) \in [1, \infty) \subset \mathbf{R}_+$. Using this solution, the proof here proceeds as follows. Step 1 establishes a solution, denoted by $x_i = x_i(k_m(M), M)$, for $M \in (\underline{M}, \bar{M}]$ and $i = f, m$, taking $S \in (0, \infty)$ as given. Step 2 then identifies an equilibrium by noting (9) requires M satisfy the fixed-point condition, $V^m - V^f = c$ or

$$\Omega(M) \equiv -1 + \frac{e^{-x_f(k_m(M), M)}}{M}\left(x_f(k_m(M), M) + 1 - \frac{1}{S}\right) = c \tag{13}$$

for all $c \in (0, \infty)$ if $S < 1$, and for $c \in (0, \bar{c}] < \infty$ at some $\bar{c} < \infty$ if $S \geq 1$. How to reach the fixed point condition (13) will be detailed in Step 2. Hence using Step 1, Step 2 establishes that $M \in (\underline{M}, \bar{M})$ exists, satisfying this condition. Given the M satisfying (13), Theorem 2 pins down $\bar{k}_m = k_m(M) \geq 1$ and $x_i \in (0, \infty)$, $i = f, m$; the other equilibrium values are identified by the same procedure as in Theorem 1 using (4),(5),(6), $i = f, m$. This solution then satisfies the equilibrium requirements (1), (3)-(6),(8),(9) and so describes a turnover equilibrium.

Step 1 Given $k_m = k_m(M) \in [1, \infty) \subset \mathbf{R}_+$ satisfying the equation (8), a solution $x_i = x_i(k_m(M), M)$ to (1) and (7) exists for $i = f, m$ that is continuous in $M \in (\underline{M}, \bar{M})$ and satisfies: $x_f(1, \bar{M}) = x_m(1, \bar{M}) = 1/S$; $x_s \to 0$ and $x_m \to 1/(S - 1)$ as $M \to \underline{M} = (S - 1)/S$ if and only if $S \in [1, \infty)$; $x_f \in (0, 1/S)$ and $x_m \to \infty$ as $M \to \underline{M} = 0$ if and only if $S \in (0, 1)$.

Proof of Step 1. From the equation (8), it follows that: $k_m(M)$ is strictly decreasing in $M \in (\underline{M}, \bar{M})$; $k_m(\cdot) \to \infty$ as $M \to \underline{M}$; $k_m(\bar{M}) = 1$. Applying $k_m = k_m(M) \in [1, \infty) \subset \mathbf{R}_+$, one obtains a solution to (12), denoted by $x_f = x_f(k_m(M), M)$; applying this solution, one obtains a solution to (1), denoted by $x_m = x_m(k_m(M), M)$. Both solutions are continuous in $M \in (0, \bar{M}]$. Now I prove the last statement. $x_f(1, \bar{M}) = x_m(1, \bar{M}) = 1/S$ follows immediately from $k_m(\bar{M}) = 1$. The limit as $M \to \underline{M} \equiv \max(0, (S-1)/S)$ can be examined by using the following property (see Temme [29] p.285):

$$\frac{\Gamma(k_m, x_m)}{\Gamma(k_m)} \to D \quad \text{as} \quad k_m \to \infty \tag{14}$$

where $D \in [0, 1]$ satisfies: $D = 1$ if and only if $x_m < k_m$; $D = 0$ if and only if $x_m > k_m$.

There are three possibilities. Suppose first $x_f \to 1/S$ as $M \to \underline{M}$. Then, (7) and (14) imply $x_m = k_m$ as $M \to \underline{M}$ while (1) implies $x_m \to 1/S$ as $M \to \underline{M}$, reaching a contradiction: $k_m \to \infty > 1/S$ as $M \to \underline{M}$ when (8) holds with equality.

Suppose next that $x_f \to 0$ as $M \to \underline{M}$. Then, (7) and (14) imply $x_m < k_m$ as $M \to \underline{M}$ while (1) implies $x_m \to 1/S\underline{M}$ as $M \to \underline{M}$. Hereafter, note that $\eta(x_m(k_m), k_m) \to 1$ as $k_m \to \infty$ for $x_m \leq k_m$. Applying these limiting values to (8), $Mx_m\eta(\cdot) \to 1/S$ and $(1-M)e^{-x_f} \to 1-\underline{M}$ as $M \to \underline{M}$, which implies (8) holds with equality if and only if $S \geq 1$, leading to $\underline{M} = (S-1)/S$. Hence, $x_f \to 0$ and $x_m \to 1/(S-1)$ as $M \to (S-1)/S$ if and only if $S \geq 1$.

Finally, suppose $x_f \in (0, 1/S)$ as $M \to \underline{M}$. Then, (7) and (14) imply $x_m = k_m$ as $M \to \underline{M}$ while (1) implies $x_m \to (1 - S(1-\underline{M})x_f)/S\underline{M}$ as $M \to \underline{M}$. Applying these limiting values it turns out that the equation (8) has a solution $x_f \in (0, 1/S)$ if and only if $S < 1$, leading to $\underline{M} = 0$. The solution is unique. Hence, $x_f \in (0, 1/S)$ and $x_m \to \infty$ as $M \to 0$ if and only if $S < 1$. This completes the proof of Step 1.

<u>**Step 2**</u> There exists $M \in (\underline{M}, \bar{M})$ that satisfies the fixed-point condition (13) for all $c \in (0, \infty)$ if $S \in (0, 1)$ and for $c \in (0, \bar{c}]$ at some $\bar{c} < \infty$ if $S \in [1, \infty)$.

Proof of Step 2. To reach the fixed point condition (13), observe first that

$$V^m - V^f = k_m \left(1 - \frac{\Gamma(k_m+1, x_m)}{\Gamma(k_m+1)}\right) - \left(1 - e^{-x_f} - x_f e^{-x_f}\right).$$

On the other hand, the binding steady-state condition (8) implies that

$$\begin{aligned} k_m \left(1 - \frac{\Gamma(k_m+1, x_m)}{\Gamma(k_m+1)}\right) &= -x_m \frac{\Gamma(k_m, x_m)}{\Gamma(k_m)} + \frac{1-M}{M} e^{-x_f} \\ &= \frac{e^{-x_f}}{M} \left(-Mx_m + 1 - M\right) \\ &= \frac{e^{-x_f}}{M} \left(-\frac{1}{S} + (1-M)(x_f + 1)\right) \end{aligned}$$

where the second equality is by the indifference condition (7) and the third equality is by the adding-up restriction (1). Applying this expression to $V^m - V^f$ leads to (13).

Given $k_m(M) \in [1, \infty) \subset \mathbf{R}_+$ and $x_f(k_m(M), M)$ established in Step 1, we now find a solution $M \in (\underline{M}, \bar{M}]$ to the fixed point condition (13), $\Omega(M) = c$. If $S < 1$ then it holds that $\Omega(\bar{M}) = 0 < c$; $\Omega(M) \to \infty > c$ as $M \to \underline{M} = 0$. As $\Omega(M)$ is continuous in $M \in (\underline{M}, \bar{M})$, this proves the existence of a solution $M \in (\underline{M}, \bar{M})$ to (13) for all $c \in (0, \infty)$. If $S \geq 1$ then it holds that $\Omega(\bar{M}) = 0 < c$; $\Omega(M) \to 0 < c$ as $M \to \underline{M} = 1 - \frac{1}{S} \geq 0$. Observe also that $\Omega(\cdot) = \frac{e^{-x_f(\cdot)}}{M}\Lambda(M)$ where

$$\Lambda(M) \equiv -Me^{x_f(k_m(M),M)} + x_f(k_m(M), M) + 1 - \frac{1}{S},$$

satisfying $\Lambda(\bar{M}) = 0$ and

$$\begin{aligned} \frac{d\Lambda(\bar{M})}{dM} &= -e^{x_f(k_m(\bar{M}),\bar{M})} - \left(\bar{M}e^{x_f(k_m(\bar{M}),\bar{M})} - 1\right)\left(\frac{\partial x_f(k_m, \bar{M})}{\partial M} + \frac{\partial x_f(k_m, \bar{M})}{\partial k_m}\frac{dk_m(M)}{dM}\right) \\ &= -e^{1/S} < 0 \end{aligned}$$

where the second equality is by $x_f(k_m(\bar{M}), \bar{M}) = x_f(1, \bar{M}) = 1/S$, as shown in Step 1, and hence by $\bar{M}e^{x_f(k_m(\bar{M}),\bar{M})} = e^{-1/S}e^{1/S} = 1$. Therefore, there exists some $M' \in (\underline{M}, \bar{M})$ such that $\Lambda(M') > 0$, leading to $\Omega(M') = \frac{e^{-x_f(k_m(M'),M')}}{M'}\Lambda(M') > 0$. As $\Omega(M) \geq 0$ is continuous and bounded for all $M \in (\underline{M}, \bar{M}]$, this implies there exists some $\bar{c} \in (0, \infty)$ such that a solution $M \in (\underline{M}, \bar{M})$ exists for $c \leq \bar{c}$ but not for $c > \bar{c}$. In any case, select a nearest value of $M \in (\underline{M}, \bar{M})$ to the solution that leads to $k_m > 1 \subset \mathbf{Z}_+$. This completes the proof of Step 2. ■

7.5 Proof of Proposition 2

The analysis provided in Step 2 in the proof of Proposition 1 implies that there exist more than one solution to the fixed point condition (13) if $S \geq 1$. In what follows, I prove the latter part of the proposition.

Given $S \geq 1$, denote by $M^l, M^h \in (\underline{M}, \bar{M})$ the solution to (13). Without loss of generality, assume $M^l < M^h$. Below, I shall refer to the corresponding equilibria using the superscript l, h. The binding steady-state condition (8) implies $k_m^l = k_m(M^l) > k_m^h = k_m(M^h)$. This further implies that $x_m^l = x_m(k_m(M^l), M^l) > x_m^h = x_m(k_m(M^h), M^h)$, because $x_m(k_m(M), M)$ is increasing in k_m and decreasing in M as shown in Step 1 of the proof of Theorem 2. Now, observe that $\Omega(M) = -1 + \frac{\Theta(x_f(\cdot))}{M}$ where

$$\Theta(x_f(\cdot)) \equiv e^{-x_f(\cdot)}\left(x_f(\cdot) + 1 - \frac{1}{S}\right)$$

is strictly increasing in $x_f(\cdot) \in (0, 1/S)$. As the fixed point condition requires $\Omega(M) = c$, it has to hold that $x_f^h = x_f(k_m(M^h), M^h) > x_f^l = x_f(k_m(M^l), M^l)$.

Finally, I examine the equilibrium price of merchants. Notice that: $x_f^h > x_f^l$ implies $V^{f,h} > V^{f,l}$; $x_m^h < x_m^l$ and $k_m^h < k_m^l$ imply $x_m^h\eta(x_m^h, k_m^h) < x_m^l\eta(x_m^l, k_m^l)$. As the fixed point condition requires $V^{m,\iota} - V^{f,\iota} = c$ for $\iota = l, h$, it then follows that $V^{m,h} > V^{m,l}$ which leads to $p_m^h x_m^h\eta(x_m^h, k_m^h) > p_m^l x_m^l\eta(x_m^l, k_m^l)$ and $p_m^h > p_m^l$. ■

7.6 A large economy

The objective of this subsection is to show that a finite version of the economy obtains the same solution as in the main text (described in Section 2) when the population gets large. Suppose there are n_b buyers and n_s sellers. Denote by m the number of k_m-sellers, each with capacity $k_m \geq 1$, and by $f = n_s - m$ the number of k_f-sellers, each with capacity $k_f = 1$. Assume $n_b > k_m$. A directed search equilibrium is where all sellers with capacity k_i set price p_i and all buyers visit each k_i-seller with probability α_i for $i = f, m$, satisfying $m\alpha_m + f\alpha_f = 1$. To characterize the equilibrium conditions, the first step is to calculate the probability that a buyer will get served, conditional on arriving at a k_i-seller, denoted by η^i.

Lemma 2 *The probability that a buyer will get served, conditional on arriving at a k_i-seller, is given by*

$$\eta^i = I_{1-\alpha_i}(n_b - k_i, k_i) + \frac{k_i}{n_b\alpha_i}\left(1 - I_{1-\alpha_i}(n_b - k_i, k_i + 1)\right)$$

for $i = f, m$, where

$$I_{1-\alpha}(n-k,k) \equiv \frac{\Gamma(n)}{\Gamma(n-k)\Gamma(k)} \int_0^{1-\alpha} t^{n-k-1}(1-t)^{k-1}dt$$

is the regularized incomplete beta function.

Proof of Lemma 2. If a buyer visits a given seller with capacity k_i, he will get served with the following probability:

$$\eta^i = \sum_{j=0}^{k_i-1} C^j_{n_b-1}\alpha_i^j (1-\alpha_i)^{n_b-1-j} + \sum_{j=k_i}^{n_b-1} \frac{k_i}{j+1} C^j_{n_b-1}\alpha_i^j (1-\alpha_i)^{n_b-1-j},$$

where $C^j_{n_b-1} = \frac{(n_b-1)!}{j!(n_b-1-j)!}$. This equation reads as follows. If the number of the other buyers visiting then given k_i-seller is less than k_i, then a given buyer gets served with probability one (the first term). Otherwise, the given buyer gets served with probability $\frac{k_i}{j+1}$ (the second term). In what follows, I simplify step by step the terms in η^i. To simplify the first summation, observe that

$$\begin{aligned}
\sum_{j=0}^{k_i-1} C^j_{n_b-1}\alpha_i^j(1-\alpha_i)^{n_b-1-j} &= (1-\alpha_i)^{n_b-1} + (n_b-1)\alpha_i(1-\alpha_i)^{n_b-2} + ... \\
&\quad ... + \frac{(n_b-1)(n_b-2)\cdots(n_b-h)}{h!}\alpha_i^h(1-\alpha_i)^{n_b-h-1} + ... \\
&\quad ... + \frac{(n_b-1)(n_b-2)\cdots(n_b-k_i+1)}{(k_i-1)!}\alpha_i^{k_i-1}(1-\alpha_i)^{n_b-k_i} \\
&= \frac{(n_b-1)!}{(n_b-k_i-1)!(k_i-1)!}\int_0^{1-\alpha_i} t^{n_b-k_i-1}(1-t)^{k_i-1}dt \\
&\equiv I_{1-\alpha_i}(n_b-k_i,k_i),
\end{aligned}$$

where the second equality follows from integration by parts k times. To simplify the second summation in η^i, observe that

$$\begin{aligned}
\sum_{j=0}^{n_b-1} \frac{k_i}{j+1} C_{n_b-1}^{j} \alpha_i^j (1-\alpha_i)^{n_b-1-j} &= k_i \sum_{j=0}^{n_b-1} \frac{(n_b-1)!}{(j+1)!(n_b-1-j)!} \alpha_i^j (1-\alpha_i)^{n_b-1-j} \\
&= \frac{k_i}{n_b\alpha_i} \sum_{h=1}^{n_b} \frac{n_b!}{h!(n_b-h)!} \alpha_i^h (1-\alpha_i)^{n_b-h} \\
&= \frac{k_i}{n_b\alpha_i} \left(1-(1-\alpha_i)^{n_b}\right),
\end{aligned}$$

where I set $h=j+1$ and

$$\begin{aligned}
\sum_{j=0}^{k_i-1} \frac{k_i}{j+1} C_{n_b-1}^{j} \alpha_i^j (1-\alpha_i)^{n_b-1-j} &= k_i \sum_{j=0}^{k_i-1} \frac{(n_b-1)!}{(j+1)!(n_b-1-j)!} \alpha_i^j (1-\alpha_i)^{n_b-1-j} \\
&= \frac{k_i}{n_b\alpha_i} \sum_{h=1}^{k_i} \frac{n_b!}{h!(n_b-h)!} \alpha_i^h (1-\alpha_i)^{n_b-h} \\
&= \frac{k_i}{n_b\alpha_i} \left(\sum_{h=0}^{k_i} \frac{n_b!}{h!(n_b-h)!} \alpha_i^h (1-\alpha_i)^{n_b-h} - (1-\alpha_i)^{n_b} \right) \\
&= \frac{k_i}{n_b\alpha_i} \left(I_{1-\alpha_i}(n_b-k_i, k_i+1) - (1-\alpha_i)^{n_b} \right).
\end{aligned}$$

In the very last equality, I have applied the same procedure as in the simplification made in the first summation, i.e., integration by parts $k+1$ times that leads to

$$\begin{aligned}
\sum_{h=0}^{k_i} \frac{n_b!}{h!(n_b-h)!} \alpha_i^h (1-\alpha_i)^{n_b-h} &= \frac{n_b!}{(n_b-k_i-1)!k_i!} \int_0^{1-\alpha_i} t^{n_b-k_i-1}(1-t)^{k_i} dt \\
&\equiv I_{1-\alpha_i}(n_b-k_i, k_i+1).
\end{aligned}$$

Combining all these expressions leads to the one stated in the lemma. This complete the proof of Lemma 2. ■

Given the η^i derived above, the next step is to describe the buyers' directed search and the equilibrium price. As buyers must be indifferent between any sellers, we have

$$(1-p_f)\eta^f = (1-p_m)\eta^m. \tag{15}$$

Suppose now that a seller i' deviates to a price $p_{i'} \neq p_i$. Denoting by $\alpha_{i'}$ the probability that a buyer visits the deviant, the adding-up restriction implies

$$\alpha_{m'} + (m-1)\alpha_m + f\alpha_f = 1 \tag{16}$$

if $i' = m$ and

$$m\alpha_m + \alpha_{f'} + (f-1)\alpha_f = 1, \tag{17}$$

if $i' = f$, and the indifferent condition implies

$$(1-p_{i'})\eta^{i'} = (1-p_i)\eta^i, \tag{18}$$

$i = f, m$, where

$$\eta^{i'} = I_{1-\alpha_{i'}}(n_b - k_i, k_i) + \frac{k_i}{n_b\alpha_{i'}}\left(1 - I_{1-\alpha_{i'}}(n_b - k_i, k_i + 1)\right)$$

represents the buyer's probability of being served by visiting the deviating seller i'. The optimality condition then requires that

$$p_{i'} = \mathrm{argmax}_{p\in[0,1]} p n_b \alpha_{i'} \eta^{i'} \tag{19}$$

where the probability function $\alpha_{i'} = \alpha_{i'}(p, p_f, p_m)$, with $p = p_{i'}$, is determined by conditions (15), (16) if $i' = m$ or (17) if $i' = f$, (18), given p_i, $i = f, m$. Notice that $\eta^{i'}$ and the expected number of buyers, given by $n_b\alpha_{i'}\eta^{i'}$, depend on this probability.

A directed search equilibrium in the finite economy is an interior solution $p_i, \alpha_i \in (0,1)$, $i = f, m$ to the optimality conditions (19) and the buyers' indifference condition (15) that satisfies the adding-up restriction $m\alpha_m + f\alpha_f = 1$ and $p_{i'} = p_i$, $\alpha_{i'} = \alpha_i$, $i = f, m$. Below, I show that the finite equilibrium achieves the same solution as in the continuum version (presented in the main text) when the population of buyers n_b gets large, while keeping the population ratios $S \equiv \frac{n_s}{n_b}$ and $M \equiv \frac{m}{n_s}$ constant.

Theorem 3 *In the limit as $n_b \to \infty$, keeping $S \equiv \frac{n_s}{n_b}$ and $M \equiv \frac{m}{n_s}$ constant, it holds that*

$$p_i \to \frac{k_i\left(1 - \frac{\Gamma(k_i+1, x_i)}{\Gamma(k_i+1)}\right)}{x_i \eta(x_i, k_i)}$$

where $x_i \equiv \lim_{n_b\to\infty} n_b\alpha_i$ for $i = f, m$.

Proof of Theorem 3. In what follows, I only present the proof of the k_m-sellers' price. The price of k_f-sellers can be obtained by a similar procedure.

An interior solution for the optimum satisfies the first order condition given by

$$n_b\alpha_{m'}\eta^{m'} + p_{m'}\left(n_b\eta^{m'} + n_b\alpha_{m'}\frac{\partial\eta^{m'}}{\partial\alpha_{m'}}\right)\frac{\partial\alpha^{m'}}{\partial p_{m'}} = 0$$

The dependence of the probability function $\alpha_{m'} = \alpha_{m'}(p_{m'}, p_m, p_f)$ on the price $p_{m'}$ can be made explicit by writing as $\eta^i = \eta^i(\alpha_i)$, $i = m', m, f$, and can be identified by constructing an implicit function

$$\Upsilon(\alpha_{m'}, p_{m'}) \equiv (1-p_{m'})\eta^{m'}(\alpha_{m'}) - (1-p_f)\eta^f(\alpha_f(\alpha_m(\alpha_{m'}, p_{m'}, p_m), \alpha_{m'})) = 0$$

using (15), (16), (18), where $\alpha_f = \alpha_f(\alpha_m, \alpha_{m'})$ is determined by (16) or

$$\alpha_f = \frac{1}{f}\left(1 - (m-1)\alpha_m - \alpha_{m'}\right)$$

and $\alpha_m = \alpha_m(\alpha_{m'}, p_{m'}, p_m)$ implicitly by (18) or

$$\eta^{m'}(\alpha_{m'}) = \frac{1-p_m}{1-p_{m'}}\eta^m(\alpha_m).$$

From $\Upsilon(\cdot) = 0$, it follows that

$$\frac{\partial \alpha_{m'}}{\partial p_{m'}} = -\frac{\partial \Upsilon / \partial p_{m'}}{\partial \Upsilon / \partial \alpha_{m'}} = -\frac{\left(\frac{m-1}{f}\frac{\partial \eta^f}{\partial \alpha_f}\frac{\partial \alpha_m}{\partial p_{m'}} - \eta^f \frac{1}{1-p_{m'}}\right)\frac{\eta^{m'}}{\eta^f}}{\frac{\partial \eta^{m'}}{\partial \alpha_{m'}} + \left(\frac{m-1}{f}\frac{\partial \eta^f}{\partial \alpha_f}\frac{\partial \alpha_m}{\partial \alpha_{m'}} + \frac{1}{f}\frac{\partial \eta^f}{\partial \alpha_f}\right)\frac{\eta^{m'}}{\eta^f}}$$

where

$$\frac{\partial \alpha_m}{\partial \alpha_{m'}} = \frac{\partial \eta^{m'}/\partial \alpha_{m'}}{\partial \eta^m / \partial \alpha_m}\frac{1-p_{m'}}{1-p_m}, \quad \frac{\partial \alpha_m}{\partial p_{m'}} = -\frac{\eta^m}{\partial \eta^m/\partial \alpha_m}\frac{1}{1-p_{m'}}.$$

Evaluated at $p_{m'} = p_m$, $\alpha_{m'} = \alpha_m$, the first order condition can be written as

$$\frac{p_m}{1-p_m} = -\frac{\alpha_m\left(\frac{\partial \eta^m}{\partial \alpha_m} + \frac{m}{f}\frac{\partial \eta^f}{\partial \alpha_f}\frac{\eta^m}{\eta^f}\right)}{\left(\frac{\eta^m}{\partial \eta^m/\partial \alpha_m} + \alpha_m\right)\left(\frac{\partial \eta^m}{\partial \alpha_m} + \frac{m-1}{f}\frac{\partial \eta^f}{\partial \alpha_f}\frac{\eta^m}{\eta^f}\right)} \tag{20}$$

where

$$\frac{\partial \eta^i}{\partial \alpha_i} = -\frac{k_i}{n_b \alpha_i^2}\left(1 - I_{(1-\alpha_i)}(n_b - k_i, k_i + 1)\right)$$

for $i = f, m$.

I now derive the limit of the equilibrium price. The following lemma calculates the limit of η^i.

Lemma 3 *In the limit as $n_b \to \infty$, keeping $S \equiv \frac{n_s}{n_b}$ and $M \equiv \frac{m}{n_s}$ constant, it holds that*

$$\eta^i \to \frac{\Gamma(k_i, x_i)}{\Gamma(k_i)} + \frac{k_i}{x_i}\left(1 - \frac{\Gamma(k_i+1, x_i)}{\Gamma(k_i+1)}\right)$$

where $x_i \equiv \lim_{n_b \to \infty} n_b \alpha_i$ for $i = f, m$.

Proof of Lemma 3. The proof is built on the following observations:

$$\lim (1-\alpha_i)^{n_b - h} = e^{-x_i}$$

$$\lim \frac{(n_b-1)(n_b-2)\cdots(n_b-h)}{h!}\alpha_i^h = \lim \frac{(1-\frac{1}{n_b})(1-\frac{2}{n_b})\cdots(1-\frac{h}{n_b})}{h!}n_b^h \alpha_i^h = \frac{x_i^h}{h!}$$

for $h < \infty$. Applying the former observation,

$$\lim \sum_{j=0}^{n_b-1} \frac{k_i}{j+1} C_{n_b-1}^{j} \alpha_i^j (1-\alpha_i)^{n_b-1-j} = \lim \frac{k_i}{n_b \alpha_i} (1-(1-\alpha_i)^{n_b}) = \frac{k_i}{x_i}(1-e^{-x_i})$$

Combining the former and latter observations,

$$\begin{aligned} \lim I_{1-\alpha_i}(n_b-k_i,k_i) &= \lim \sum_{j=0}^{k_i-1} C_{n_b-1}^{j} \alpha_i^j (1-\alpha_i)^{n_b-1-j} = \sum_{j=0}^{k_i-1} \frac{x_j e^{-x_j}}{j!} = \frac{\Gamma(k_i,x_i)}{\Gamma(k_i)} \\ \lim I_{1-\alpha_i}(n_b-k_i,k_i+1) &= \lim \sum_{j=0}^{k_i} C_{n_b}^{j} \alpha_i^j (1-\alpha_i)^{n_b-j} = \sum_{j=0}^{k_i} \frac{x_j e^{-x_j}}{j!} = \frac{\Gamma(k_i+1,x_i)}{\Gamma(k_i+1)}. \end{aligned}$$

Summing up all these limiting terms, it holds that

$$\begin{aligned} \eta^i &\to \frac{\Gamma(k_i,x_i)}{\Gamma(k_i)} + \frac{k_i}{x_i}(1-e^{-x_i}) - \frac{k_i}{x_i}\left(\frac{\Gamma(k_i+1,x_i)}{\Gamma(k_i+1)} - e^{-x_i}\right) \\ &= \frac{\Gamma(k_i,x_i)}{\Gamma(k_i)} + \frac{k_i}{x_i}\left(1 - \frac{\Gamma(k_i+1,x_i)}{\Gamma(k_i+1)}\right) \end{aligned}$$

in the limit as $n_b \to \infty$, keeping $S \equiv \frac{n_s}{n_b}$ and $M \equiv \frac{m}{n_s}$ constant. This completes the proof of Lemma 3. ■

Given Lemma 3, it follows that

$$\begin{aligned} \lim \frac{p_m}{1-p_m} &= \lim - \frac{n_b \alpha_m \left(\frac{\partial \eta^m}{\partial \alpha_m} + \frac{M}{1-M}\frac{\partial \eta^f}{\partial \alpha_f}\frac{\eta^m}{\eta^f}\right)}{\left(\frac{\eta^m}{\frac{\partial \eta^m}{\partial \alpha_m}\frac{1}{n_b}} + n_b \alpha_m\right)\left(\frac{\partial \eta^m}{\partial \alpha_m} + \frac{M}{1-M}\frac{\partial \eta^f}{\partial \alpha_f}\frac{\eta^m}{\eta^f}\right)} \\ &= \frac{x_m \lim \left(-\frac{\partial \eta^m}{\partial \alpha_m}\frac{1}{n_b}\right)}{\lim \eta^m - x_m \lim \left(-\frac{\partial \eta^m}{\partial \alpha_m}\frac{1}{n_b}\right)} \\ &= \frac{\frac{k_m}{x_m}\left(1 - \frac{\Gamma(k_m+1,x_m)}{\Gamma(k_m+1)}\right)}{\frac{\Gamma(k_m,x_m)}{\Gamma(k_m)}}, \end{aligned}$$

where the last equality follows from

$$\lim - \frac{\partial \eta^m}{\partial \alpha_m}\frac{1}{n_b} = \lim \frac{k_m}{n_b^2 \alpha_m^2}\left(1 - I_{(1-\alpha_m)}(n_b-k_m,k_m+1)\right) = \frac{k_m}{x_m^2}\left(1 - \frac{\Gamma(k_m+1,x_m)}{\Gamma(k_m+1)}\right).$$

This leads to the expression given in the theorem. This completes the proof of Theorem 3. ■

To sum up,

Proposition 3 *A finite setup obtains the same solution as in the continuum economy, described in Theorem 1 and illustrated in Figure 2, when the population gets large.*

Proof of Proposition 3. Applying the limit of the equilibrium price p_i (derived in Theorem 3) and the limit of η^i (derived in Lemma 3) for $i = f, m$ to the indifference condition of buyers (15), one obtains

$$\frac{\Gamma(k_m, x_m)}{\Gamma(k_m)} = e^{-x_f},$$

which is identical to (7) presented in the main text. Also, the adding-up restriction in the limit becomes

$$SMx_m + S(1 - M)x_f = 1,$$

which is identical to the one for the continuum version (1). Therefore, a finite setup obtains an equilibrium allocation, x_i, $i = f, m$, identical to the continuum setup counterpart, described in Theorem 1, when the population gets large. This completes the proof of Proposition 3. ■

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