Global Warming and Heterogeneous (Green and Brown) Consumers

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Abstract

This paper considers a familiar dynamic tragedy of the commons ('global warming') and investigates whether and by how much green polluters can mitigate such tragedies. Green consumers feel penalized ('pain') for any consumption in excess of the social optimum, which can arise e.g., from Kant's morale imperative. In addition the interplays among and between green and brown consumers are investigated as differential games. Green preferences, heterogeneity of consumers and the irreversibility of emissions lead to discontinuous strategies, a number non-trivial and even puzzling features.

Keywords: green versus brown preferences, stock externality, differential game, (linear) Markov strategies.

JEL #: C61, D62.

1 Introduction

This paper considers heterogenous players (consumers) in a stock externality game, for concreteness: burning fossil fuels leads through the accumulation of carbon dioxide in the atmosphere to global warming. Consumers are either brown - only concerned about their own benefit and free riding on others' efforts - or green, more precisely, accounting for what should be done, arithmetically: any individual deviation from socially optimal emissions incurs a cost (penalty). Such preferences can arise from Kant's categorical imperative: individual behavior should be such that it can be accepted as a universal rule ("act only according to that maxim whereby you can at the

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same time will that it should become a universal law" (Kant 1785, p. 421, quoted from White (2004)). Guilt and self-identity as an environmentally aware consumer are other elated forms, may be as a result of social pressure. Similar preferences are assumed in Brekke, Kvernedokk and Nyborg (2003) and in Benabou and Tirole (2006) with intrinsic motives (compare on this also Frey, e.g. Frey (1997) and its potential crowding out by external incentives with an environmental example in Frey and Oberholzer-Gee (1997)). Other and related alternatives to explain behavior deviating from the standard economic model include the 'warm glow' as advocated Andreoni (1990) and the gain for social approval, e.g. Holländer (1990). In addition social interactions in the sense of Glaeser and Scheinkman (2003) based on Cooper and John (1988) and extended to a dynamic setting in Wirl (2007a) may reenforce such preferences in particular the gain for social approval and are also part of the preferences in Holländer (1990). However, the purpose is not to add another review to this already often and extensively reviewed literature (theoretical and even more experimental), because the contribution of this paper is to extend all these static investigations into two directions: heterogeneity of consumers and dynamics due to the focus on global warming as a stock externality. These interactions among strategic players (i.e., nations or blocs, EU, OECD, BRIC, developing countries, etc. and not consumers in the ordinary sense) are modelled as a differential game; a brief subsection treats the case of individual and thus competitive consumers. The basic framework is taken from Hoel (1992), Dockner and Long (1993) with many follow ups focusing however on how nonlinear strategies can mitigate the tragedy of the commons, Rubio and Casino (2002), Rowat (2006), and Wirl (2007). A central motivation is how green consumers can mitigate environmental tragedies and to analyze the interplay of heterogenous consumers. A major finding is that the consideration of green preferences in a dynamic stock pollution game coupled with the realistic constraint of irreversible emissions results in nontrivial and partially even surprising properties of the equilibria (of the linear Nash-Markov type).

The paper is organized as follows. Section 2 introduces the framework. The different outcomes - social optimum, only brown consumers (follows actually from Hoel (1992), Dockner and Long (1993) and others), only green consumers, and heterogenous (green and brown) consumers - are derived in Section 3. Examples (Section 4) complement the theoretical analysis.

2 Model

The stock of pollution X accumulates over time the emissions $x_i \geq 0$ of the players $i \in \{1, ..., N\}$,

$$\dot{X}(t) = \sum_{i=1}^{N} x_i(t) - \delta X(t), \; X(0) = X_0 \text{ given,} \tag{1}$$

and $\delta \geq 0$ is the (constant) depreciation rate. Simplifications of this kind have a long tradition in theoretical models of global warming, e.g., Hoel (1992). The non-negativity of emissions reflects irreversibility: only depreciation can reduce once accumulated pollution but active measures are negligible (e.g., reforestation in the case of global warming, which is anyway questionable in its net effect).

2.1 Brown consumers

Following Dockner and Long (1993), each player chooses x_i such that the individual net present value (using the discount rate $r > 0$) is maximized,

$$V_i(X(0)) = \max_{\{x_i(t) \geq 0\}} \int_0^{\infty} e^{-rt} \left[u(x_i(t)) - C(X(t))\right] dt, \; i = 1, ..., N, \tag{2}$$

subject to (1). The non-negativity constraint is irrelevant for a game of brown consumers (unless they all make a very big error and overshoot not only the steady state but the stopping levels, i.e., where even brown consumers stop emitting), but crucial for the social optimum and thereby for green consumers (see below). The instantaneous utility consists of the individual (linear-quadratic and normalized) benefit

$$u(x) = x - \frac{1}{2}x^2, \tag{3}$$

minus the quadratic external costs

$$C = \frac{c}{2}X^2. \tag{4}$$

2.2 Social welfare

Accounting for symmetry, the social objective is,

$$W(X(0)) = \max_{\{x(t) \geq 0\}} \int_0^{\infty} e^{-rt} u(x(t)) - C(X(t))\, dt, \tag{5}$$

subject to the dynamic constraint (1) after setting $x_i(t) = x_j(t) = x(t)$. The result is the socially optimal emission, $x^*(t)$.

2.3 Green consumers

Green consumers feel guilt if their consumption deviates from what it should be. Arithmetically, green consumers face a penalty (from pain, lack of self-identity, etc.) for surpassing the norm and this cost is increasing and convex with respect to the amount of exceeding the social optimum x^*. External costs are identical to the brown consumers. Therefore, the objective of such green consumers is,

$$\max_{\{x_i(t)\geq 0\}} \int_0^\infty e^{-rt} \left[u(x_i(t)) - P(x_i(t), x^*(t)) - C(X(t))\right], \tag{6}$$

subject to the evolution of the stock externality (1). The penalty is assumed to be quadratic in line with the external costs and most of the related differential game literature,

$$P(x_i, x^*) = \frac{p}{2}(x_i - x^*)^2. \tag{7}$$

This modelling is very similar to Brekke, Kvernedokk and Nyborg (2003) and in line with the given motivation it excludes subjective and psychological aspects in establishing the norm itself (i.e., the parameter p does not affect the choice of x^*). However, the interpretation of these preferences to include Kantian based morale aspects is different from the one given in White (2004, p99) about perfect duties that take precedence over individual inclinations and should thus enter as a constraint, $x_i(t) \leq x^*(t)$; White (2009) argues that the Pareto criterion is therefore incompatible with Kant's categorical moral imperative. This renders however the problem trivial since green consumers will then follow strictly the socially efficient strategy and this entire lack of a trade off is psychologically not convincing. Indeed on p100 White (2004) allows the incorporation of Kant's imperative as a part of the preferences via probabilistic weighting since only God is perfect; see also the comment in Balleta and Bazin (2005).

3 Intertemporal Strategies

3.1 Social optimum

The social optimum results from solving the following Hamilton-Jacobi-Bellman equation for the value function,

$$rW = \max_{x} \left\{ u(x) - C + W' \left(Nx - \delta X \right) \right\}, \tag{8}$$

which implies the socially optimal emission strategy (identified by superscripted asterisks),

$$x^{*}\left(t\right) = \varphi\left(X\left(t\right)\right) = \begin{matrix} \alpha + \beta X\left(t\right) \\ 0 \end{matrix} \quad \text{if } X\left(t\right) \begin{matrix} \leq \\ > \end{matrix} \bar{X}^{*} := -\frac{\alpha}{\beta}, \tag{9}$$

where $\bar{X}^{*}$ is the pollution level where emissions should stop (bars are used throughout the paper to identify such thresholds) for the socially optimal emission policy, and the coefficients of the optimal policy are,

$$\beta = \frac{(r+2\delta) - \sqrt{(r+2\delta)^{2} + 4cN^{2}}}{2N}, \tag{10}$$

$$\alpha = 1 + \frac{N\beta}{(r+\delta) - N\beta}. \tag{11}$$

Irreversibility of emissions requires patching of the interior with the boundary solution at $\bar{X}^{*}$ in order to obtain a global description of the first best, which is needed as the benchmark for the green behavior. The steady state pollution level is,

$$X_{\infty}^{*} = \frac{N(r+\delta)}{(r+\delta)\delta + N^{2}c}. \tag{12}$$

The proof is similar to below and also given in the quoted papers, e.g., in Dockner and Long (1993) for $N = 2$ and for arbitrary N as the limiting case of a stochastic framework in Wirl (2008). Note that

$$\frac{\partial X_{\infty}^{*}}{\partial N} < 0 \text{ for } N > \frac{(r+\delta)\delta}{c} \text{ and } \lim_{N\to\infty} X_{\infty}^{*} = 0,$$

i.e., more players (e.g., a larger population) would be beneficial for the environment if it were optimally managed (Wirl (2008)).

The differentiation in (9) between interior and boundary ($x^{*} = 0$) is irrelevant for the social optimum, since this domain - $x = 0$ for $X > \bar{X}^{*}$= the threshold at which emissions must stop - is never reached along the social optimal emission strategy. However, it is crucial for the analysis of green consumers, because the benchmark for the green consumers must also account for the irreversibility of emissions and thus for $x^{*} \geq 0$ if they continue emitting for $X > \bar{X}^{*}$.

3.2 Brown consumers

The noncooperative Nash equilibrium of all consumers being 'brown' follows from solving the following Hamilton-Jacobi-Bellman equation,

$$rV^b = \max_{x\geq 0} \left\{ u(x) - C + V^{b\prime} \left(x + (N-1) x_j - \delta X \right) \right\}, \tag{13}$$

in which

$$x^b = 1 + V^{b\prime} \tag{14}$$

is the interior Nash emission strategy. Assuming symmetry, the linear Nash equilibrium strategy is known (for $N = 2$, e.g. from Dockner and Long (1993), yet the generalization to N is straightforward) and given by[1]:

$$\begin{aligned} x^b &= 1 + v_1^b + v_2^b X, \ i = 1, \ ..., \ N, & (15) \\ v_2^b &= \frac{r + 2\delta - \sqrt{(r+2\delta)^2 + 4c(2N-1)}}{2(2N-1)} < 0, & (16) \\ v_1^b &= \frac{N v_2^b}{(r+\delta) - v_2^b(2N-1)} < 0, & (17) \end{aligned}$$

which implies the steady state,

$$X_\infty^b = \frac{N\left(1 + v_1^b\right)}{\delta - N v_2^b} > X_\infty^*. \tag{18}$$

3.3 Green Consumers

Assuming only green players, we have to solve the Hamilton-Jacobi-Bellman equation,

$$rV^g = \max_x \left\{ u(x) - \frac{p}{2} (x - x^*)^2 - C + V^{g\prime} \left(x + (N-1) x_j - \delta X \right) \right\}. \tag{19}$$

The maximization on the right hand side must take into account the irreversibility of emissions along the social optimum, i.e., if $x^* = 0$ yet $x^g > 0$; small superscript g identifies these green players. Therefore, two or better two parts of the value function, $X \leq \bar{X}^*$ and $X > \bar{X}^*$, are relevant iff $\bar{X}^* < X_\infty^g$, i.e., if it is socially optimal to stop emissions at a level of pollution that is below the steady state attained by green consumers. Substituting the socially

[1] The domain of brown consumers emitting along the boundary, i.e., $x^b = 0$, is irrelevant for the following and thus ignored.

optimal feedback φ rule for x^*, yields for the right hand side maximizing strategy

$$x^g = \begin{array}{ccc} \frac{1+\alpha p+\beta pX+V^{g\prime}}{1+p} & & X \leq \bar{X}^* \\ & \text{if} & \\ \frac{1+V^{g\prime}}{1+p} & & X > \bar{X}^* \end{array}. \tag{20}$$

As in the case of only brown consumers, the boundary solution $x^g = 0$ can be ignored as irrelevant if all consumers are green (but can play a role for heterogenous consumers in the next section). Therefore, substituting (20) into (19) yields two functional equations, one for $x^* = \alpha + \beta X$,

$$\begin{aligned} rV^g &= \frac{1+\alpha p\,(2-\alpha)}{2\,(1+p)} + \frac{\beta p\,(1-\alpha)}{1+p} X - \frac{c}{2} X^2 - \frac{\beta^2 p}{2\,(1+p)} X^2 - \delta X V^{g\prime} \\ & \frac{(1+(\alpha+\beta X)\,p)\,N}{2\,(1+p)} V^{g\prime} + \frac{2n-1}{2\,(1+p)} \left(V^{g\prime}\right)^2, \ X \leq \bar{X}^*, \end{aligned} \tag{21}$$

and one for $x^* = 0$,

$$rV^g = \frac{1}{2\,(1+p)} - \frac{c}{2} X^2 - \delta X V^{g\prime} + \frac{N}{2\,(1+p)} V^{g\prime} + \frac{2n-1}{2\,(1+p)} \left(V^{g\prime}\right)^2, \ X > \bar{X}^*. \tag{22}$$

Both functional equations are solved by guessing and then verifying that quadratic value functions

$$V^g = v_0^g + v_1^g X + \frac{v_2^g}{2} X^2, \tag{23}$$

satisfy the two Hamilton-Jacobi-Bellman equations (21) and (22). In order to economize on the notation, the coefficients in the domain $X < \bar{X}^*$ are those in (23) and identified by bars for $X > \bar{X}^*$. In fact, the values of the barred coefficients are just the special cases in the formulas below after setting $\alpha = \beta = 0$. The emission strategies are linear (from now on, the derivation is reduced to the case $x^* > 0$ for the reason just given) ,

$$x^g = \frac{1+\alpha p+\beta pX+v_1^g+v_2^g X}{1+p}, \ X \leq \bar{X}^*. \tag{24}$$

The coefficients follow from substituting the first line in (24) combined with the quadratic guess (23) into (21) and comparing coefficients (ignoring the intercept),

$$\begin{aligned} \left[(r+\delta)\,(1+p) - N\beta p - (2N-1)v_2^g\right] v_1^g &= \beta p(1-\alpha) + N\,(1+\alpha p)\,v_2^g, && (25) \\ \left[(r+2\delta)\,(1+p) - 2\beta Np\right] v_2^g &= (2N-1)\,{v_2^g}^2 - c\,(1+p) - \beta^2 p. && (26) \end{aligned}$$

The solution of these two equations (choosing the root that implies a stable strategy) is,

$$
\begin{aligned}
v_2^g &= \frac{(r+2\delta)(1+p) - 2N\beta p}{2(2N-1)} \\
&\quad - \frac{\sqrt{[(r+2\delta)(1+p) - 2N\beta p]^2 + 4\left(c(1+p) + \beta^2 p\right)(2N-1)}}{2(2N-1)} \qquad (27) \\
v_1^g &= \frac{N(1+\alpha p)\, v_2^g + \beta p(1-\alpha)}{(r+\delta)(1+p) - (2N-1)v_2^g - N\beta p}. \qquad (28)
\end{aligned}
$$

Although the above solution (as well as those below) could be given explicitly in model parameters. this is suppressed, because such a representation would be extremely cumbersome due the dependence on the expressions for (α, β).

A consequence of the two parts of the value function is that two different outcomes are possible among only green minded consumers depending on whether $\bar{X}^* < X_\infty^g$ or not. First of all these two regimes exist of course, because varying the parameter p has as its limits socially optimal $(p \to \infty)$ or brown behavior $(p \to 0)$ such that $\bar{X}^* < X_\infty^b$ (which holds at least for sufficiently large N, but usually $N = 2$ suffices). Secondly, they imply a discontinuity in the strategies. The economic reason is that the strategies must account for each player's contribution to the state dependent evolution of the benchmark x^* in the penalty for $X \leq \bar{X}^*$, while the stock of pollution has no impact on the reference level for any $X > \bar{X}^*$. The arithmetic reason is that the second part of the value function follows from setting $\alpha = \beta = 0$ in the above formulas (24) - (28) and the corresponding coefficients are given below and identified by a bar, i.e.,

$$
\begin{aligned}
\bar{v}_2^g &= \frac{(r+2\delta)(1+p)}{2(2N-1)} - \frac{\sqrt{[(r+2\delta)(1+p)]^2 + 4\left(c(1+p)\right)(2N-1)}}{2(2N-1)} \qquad (29) \\
\bar{v}_1^g &= \frac{N\bar{v}_2^g}{(r+\delta)(1+p) - (2N-1)\bar{v}_2^g}. \qquad (30)
\end{aligned}
$$

Proposition 1: *The strategy for $X \leq \bar{X}^*$ is characterized by a steeper slope $(0 > \bar{v}_2^g > v_2^g)$ and a larger intercept $(0 < \bar{v}_1^g < v_1^g)$ compared with strategy applicable to $X > \bar{X}^*$. The transition at $X = \bar{X}^*$ is (generically) discontinuous with the possible consequence that no steady state exists if $\delta X/n$ crosses at the discontinuity. If existing, then*

$$
X_\infty^* < X_\infty^g = \begin{array}{c} \frac{N\left(1+v_1^g\right)}{\delta - N v_2^g} \\ \\ \frac{N\left(1+\bar{v}_1^g\right)}{\delta - N\bar{v}_2^g} \end{array} \quad \text{if } \frac{N\left(1+v_1^g\right)}{\delta - N v_2^g} \begin{array}{c} < \\ \\ > \end{array} \bar{X}^*, \qquad (31)
$$

which is of course below the steady state of brown consumers, $X_\infty^g < X_\infty^b$*.*

The proof follows from the explicit arithmetical solutions and $\alpha > 0$, and $\beta < 0$. The economic reason for the steeper slope is that the strategy linked to the interior socially optimal strategy $x^* > 0$ must keep track of the declining benchmark x^*, and the reason for a larger intercept is that the reference point $x^*(0) = \alpha$ is positive, compared with $x^* = 0$ for $X > \bar{X}^*$. The discontinuity is a consequence of the difference in coefficients; it is only avoided in the non-generic case that the intersection between the two strategies is at $\bar{X}^*$. Fig. 1 shows an example, which suggests that the discontinuity of the emission strategies is a drop at $\bar{X}^*$ (as presumably expected), but this does not hold in general (see also the example in the following section). It shows furthermore the three possible locations[2] of the steady state: in the domain $X > \bar{X}^*$ with the indicated steady state (X_∞^g), a steady state in the domain $X < \bar{X}^*$ that requires sufficient discounting and penalties, and also the case where $\delta X/n$ crosses through the discontinuity of the green strategy. In the last case no steady state exists since applying the interior strategy results in a steady state $X_\infty^g > \bar{X}^*$ and thus outside the applicability of this part of the strategy exists and conversely using the strategy based on $x^* = 0$ produces a steady state with $X_\infty^g < \bar{X}^*$ and thus also outside its domain. A further consequence of the discontinuity of the strategy is that the dependence of steady states on model parameters leads at least to a kink (i.e., continuity but non-differentiability) or even an interval of non-existence for parameter variations that move the steady state across $\bar{X}^*$.

[Insert Fig. 1 approximately here]

3.4 Green and brown consumers

Assume that there are m green and n brown consumers, $m + n = N$. This heterogeneity requires to determine two value functions, one for the green consumers,

$$rV^G = \max_x \left\{ u(x) - \frac{p}{2}(x - x^*)^2 - C + V^{G\prime}(x + (m-1)x_i + n\xi - \delta X) \right\}, \tag{32}$$

and one for the brown consumers (using Greek letters for their strategies),

$$rV^B = \max_\xi \left\{ u(\xi) - C + V^{B\prime}\left(\xi + mx + (n-1)\xi_j - \delta X\right) \right\}; \tag{33}$$

the capital superscripts indicate that these value functions of heterogeneous consumers differ from the cases of identical consumers (identified by small

[2] This sketch is stylized by varying depreciation in order to show the different possibilities. Of course, changing δ affects also the strategies.

superscripts, V^g and V^b in the previous subsections). This pair of interrelated functional equations must differentiate in addition between four domains, one interior,

$$x^* > 0, x > 0, \xi > 0, \tag{34}$$

and domains including boundary solutions, firstly, of the social optimum when green consumers still emit,

$$x^* = 0, x > 0, \xi > 0, \tag{35}$$

secondly, after the green consumers have stopped their emissions,

$$x^* = 0, x = 0, \xi > 0, \tag{36}$$

and the final domain,

$$x^* = 0, x = 0, \xi = 0, \tag{37}$$

is of course trivial. The optimal strategies are obtained by patching the interior with the neighboring boundary strategy.

In the interior domain (34), the maximizations on the right hand sides in (32) and (33) deliver the already known characterization (15) and (20) of the strategies of green and brown consumers respectively (of course replacing the small by the capital superscripts) such that the following interdependent functional equations result:

$$\begin{aligned} rV^G &= \frac{1+(2-\alpha)\,\alpha p}{2\,(1+p)} - \frac{c}{2}X^2 + \frac{(1-\alpha)\,\beta p}{1+p}X - \frac{\beta^2 p}{2}X^2 \\ &\quad -\delta X V^{G\prime} + \frac{\beta m p}{1+p}XV^{G\prime} + \left[\frac{m\,(1+\alpha p)}{1+p} + n\right]V^{G\prime} \\ &\quad + nV^{G\prime}V^{B\prime} + \frac{2m-1}{2}\left(V^{G\prime}\right)^2, \end{aligned}$$

$$\begin{aligned} rV^B &= 1 - \frac{c}{2}X^2 - \frac{\left(1+V^{B\prime}\right)^2}{2} \\ &\quad + V^{B\prime}\left(1+n+nV^{B\prime} + m\frac{1+\alpha p+\beta p X + V^{G\prime}}{1+p}\right). \end{aligned}$$

Guessing quadratic value functions,

$$\begin{aligned} V^G &= v_0^G + v_1^G X + \frac{v_2^G}{2}X^2, \\ V^B &= v_0^B + v_1^B X + \frac{v_2^B}{2}X^2, \end{aligned}$$

and comparing coefficients yields the following system of equations:

$$\begin{aligned}&\left[(1+p)(r+\delta)-m\beta p-(1+p)\,nv_2^B-(2m-1)\,v_2^G\right]v_1^G\\=\;&\beta p(1-\alpha)+\left[m(1+\alpha p)+n(1+p)\left(1+v_1^B\right)\right]v_2^G,\end{aligned}$$

$$\begin{aligned}&\left[(r+2\delta)(1+p)-2\beta mp\right]v_2^G\\=\;&-(1+p)\,c-\beta^2p+\left[2n(1+p)v_2^B+(2m-1)\,v_2^G\right]v_2^G,\end{aligned}$$

$$\begin{aligned}&\left[(r+\delta)(1+p)-\beta mp-(2n-1)(1+p)\,v_2^B-mv_2^G\right]v_1^B\\=\;&\left[n(1+p)+m\left(1+\alpha p+v_1^G\right)\right]v_2^B,\end{aligned}$$

$$\left[(r+2\delta)-2\beta mp\right]v_2^B=\left[2mv_2^G+(2n-1)(1+p)\,v_2^B\right]v_2^B.$$

This system can be solved analytically (to some surprise, since the equations for v_1^G and v_1^B are linear and substituting this solution into the quadratic equations for $\left(v_2^G,v_2^B\right)$ results in a 4^{th} order polynomial), yet the cumbersome solution is not worth reporting and the theoretical results below and the examples in Section 4 complement for this lack.

The analysis of the domain (35), i.e., socially optimal emissions are zero due to the irreversibility constraint yet $x^G>0$, does not require solving another pair of functional equations since the coefficients of the corresponding value functions, $\left(\bar{v}_1^G,\bar{v}_2^G,\bar{v}_1^B,\bar{v}_2^B\right)$ identified by upper bars, and the implied strategies follow immediately after setting $\alpha=\beta=0$ in the above expressions similar to the analysis in Section 3.3. This explicit solution implies Proposition 2.

Proposition 2: *Similar to Proposition 1, the strategies of green and brown consumers are discontinuous at $\bar{X}^*$ with the possibility that no steady state exists.*

Although the domain of no emissions by green consumers is irrelevant for all being green, it may play a role among heterogenous consumers. When only brown consumers emit, i.e., in the domain (36), the strategy is given by (15) with the only difference that the number of brown ($n<N$) replaces all consumers (N) in the formulas (16) and (17). Hence, a corresponding steady state in this domain (36) follows from the already solved case of brown consumers. As a consequence, greening already green consumers would have no impact on the stationary level of pollution and only the number of green consumers would matter. However, this case cannot arise due to the following result.

Proposition 3: *Assume that the steady state X_∞^{GB} exists. Then it is always in the domain where greens emit (i.e. when (35) holds) and is therefore* *never determined solely by the behavior of the brown players. As a consequence, the domain (36) is irrelevant for the determination of a steady state.*

Proof: See Appendix.

Further general results are difficult to obtain, because the route through the explicit analytical solution is hopeless given the very cumbersome expressions for the coefficients. The alternative, to exploit properties of the value function and the optimality conditions (in particular smooth pasting), faces the problem of multiple equilibria (Dockner and Long (1993) and the quoted follow ups), which are not an ideal condition for robust results. Since this is the route taken in the Appendix to prove Proposition 3, this claim extends actually beyond the linear strategies.

Proposition 3 is quite surprising since no matter how high the penalty and no matter how few or many brown players participate, the greens will never surrender the endgame to the browns. The economic reason why green minded consumers do not stop their emissions before the steady state is reached is that choosing very small emission renders the penalty and the marginal penalty arbitrarily small, while accrueing a high marginal benefit (1 for $x \to 0$).

3.5 Competitive Consumers

For the sake of completeness, competitive consumers under laissez faire (i.e., none of the governments exercises an environmental policy) are also briefly investigated. More precisely, the representative consumer of each strategic player $i = 1, .., N$ (nations, blocs, etc.) is the decision maker who is of infinitesimal size (aggregates, i.e., each player = nation is of the size 1 as considered so far). Common to all competitive consumers is their (rational) ignorance of the externality. Therefore, their consumption follows from a static optimization and not as the limit $N \to \infty$ in the above calculations since that would take the number of nations to infinity. This implies for brown and competitive consumers, $x^{bc} = 1$ (the additional superscript ^{c} refers to competitive agents) and thus the steady state,

$$X_{\infty}^{bc} = \frac{N}{\delta}, \tag{38}$$

if all consumers were competitive and brown.

Green and competitive consumers also know that individuals (one among six billion polluters in the case of global warming) have no influence on the external costs. Therefore, they ignore external costs in their objective (6) too but still account for the social norm. This implies that their optimal consumption (x^{gc}),

$$x^{gc}(t) = \frac{1 + px^*(t)}{1+p} = \begin{array}{c} \frac{1+p(\alpha+\beta X(t))}{1+p} \\ \\ \frac{1}{1+p} \end{array} \quad \text{if } X(t) \begin{array}{c} \leq \\ \\ > \end{array} \bar{X}^*, \tag{39}$$

is time dependent due to its link to what should be done, namely $x^*(t)$. The associated steady state depends on whether it is in the domain of still positive emissions in the first best (and choosing p sufficiently large ensures the existence of this case since $x^{gc} \to x^*$ for $p \to \infty$),

$$\bar{X}^* > X_\infty^{gc} = \frac{1 + p\alpha}{(1+p)\delta - p\beta}, \text{ with } \alpha \text{ and } \beta \text{ from (11) and (10)}, \tag{40}$$

or is independent of the parameters of the socially optimal policy (highly probable unless p is very large),

$$X_{c\infty}^g = \frac{1}{(1+p)\delta} > \bar{X}^*, \tag{41}$$

since the socially optimal emissions cannot turn negative due to assumption of irreversible emissions.

Obviously, brown competitive consumers never stop polluting, but neither do green consumers stop, no matter how high the level of pollution is, because irreversibility implies that the reference emission cannot turn negative. Hence, the steady state of brown and green competitive consumers is,

$$X_\infty^{GBc} = \frac{m}{(1+p)\delta} + \frac{n}{\delta}. \tag{42}$$

This expression assumes (realistically) that the corresponding steady state exceeds the stopping rule of the social optimum, $X_\infty^{GBc} > \bar{X}^*$, otherwise (40) must be substituted.

4 Examples

Given the cumbersome analytical expressions, numerical examples complement the theoretical results. Assuming,

$$N = 4, r = 0.05, c = 0.01, p = 1, \delta = 0.01, m = 2, n = 2, \tag{43}$$

we obtain the steady states,

$$X_\infty^* = 1.49 < X_\infty^g = 11.90 < X_\infty^{GB} = 13.46 < X_\infty^b = 15.33. \tag{44}$$

For completeness, the competitive cases imply even much much larger steady states:

$$X_\infty^{bc} = 400,\ X_\infty^{gc} = 200,\ X_\infty^{GBc} = 300.$$

Returning to strategic players (44), a substantial factor (of above 10!) characterizes the difference between efficient and only brown consumers. However, the steady state associated with green consumers is not dramatically lower due to the modest penalty; in fact, the 'qualitative' picture in Fig. 1 is actually based on the above example. Having a 50-50 distribution between green and brown consumers, the green consumers stop emissions at $X = 17.2$ and the steady state $X_\infty^{GB} = 13.46$ is in the domain where greens and browns emit as claimed. Fig. 2 shows the strategies and surprises with the large discontinuities when the social optimal emissions stop (at $X = \bar{X}^* = 1.535...$) and that the emissions of the green consumers jump upwards (!) while the brown consumers reduce their emissions dramatically. An economic explanation of this upward jump (of course, that does not occur in general) is that green consumers do not have to take into account anymore the decline in the socially optimal emissions for $X > \bar{X}^*$. Furthermore note how the presence of green consumers leads to an increase of the pollution of brown consumers far above the level if all consumers ($N = n$) were brown in the domain $X \leq \bar{X}^*$! The economic reasons for this robust result (i.e., across the many parameter constellations performed) that this aggressive strategy enhances the decline of the green emissions due to their dependence on the social optimum, and this decline encourages free riding of brown consumers above the usual level. In contrast, green consumers lower their consumption in response to the existence of brown consumers, but only for $X \leq \bar{X}^*$, which in turn encourages higher emissions of brown players. However, at the steady state, a reversion and surprising relation between green and brown consumers emerge: the greens emit more than the browns (partially due to the low penalty and of course this does not hold generally, in particular not for higher penalties)!

[Insert Fig. 2 approximately here]

Fig. 3 top shows the sensitivity of steady states with respect to the crucial penalty parameter p but for a higher depreciation rate, $\delta = 0.2$ (all other parameters are as in (43) and in Fig. 2), in order to highlight the possibility of the non-existence of a steady state, here for heterogenous consumers but not for only green consumers. This non-existence holds over a substantial domain of parameter values and hence the chart at the bottom of Fig.3 shows the aggregate emission strategies for a particular value of p and how δX intersects at the discontinuity.

[Insert Fig. 3 approximately here]

Finally, Fig. 4 shows the impact of the share of green (or respectively brown) consumers for $N = 10$ in order to allow for some variability. This chart is less surprising and just reveals that turning brown into green consumers fosters less (stationary) pollution.

[Insert Fig. 4 approximately here]

5 Summary

This short paper considered how green preferences can mitigate tragedies of the commons involving stock pollutions such as global warming. The major findings are that strategic green consumers introduce non-trivial and discontinuous strategies with many surprising features (e.g., possible lack of a steady state). Intuition suggests that green consumers may stop their emissions before a steady state associated with heterogenous consumers (i.e., with brown consumers present) is reached, in particular for high penalties and high external costs such that the domain of no emissions in the first best is large. However, green consumers will never leave the endgame to the browns since choosing small emissions allows green consumers to minimize the penalty yet to obtain the high marginal benefit from emissions further enhanced by the fact that these emissions reduce the emissions of others in particular of the brown consumers.

This simple dynamic game of heterogenous and strategic consumers is admittedly only a first step that can be extended into different directions. For example, one may include social pressure, i.e., the more consumers turn green the higher is the pressure on browns to turn also green but on the other hand enhances free riding of those remaining brown. Including uncertainty about the damage is another natural extension, or the consideration of private information about the types (green, brown or continuous shades between green and brown), the possibility of negotiations, contracts and incentives (as a part e.g., in the Montreal agreement about CFCs).

6 Appendix: Proof of Proposition 3

A few lemmas precede the proof.

Lemma 1: Consider the domain (37), $x^G = x^B = 0$, then the value functions are

$$V^G = A_G X^{-r/\delta} - \frac{c}{2}\frac{X^2}{r+2\delta}, \quad (45)$$
$$V^B = A_B X^{-r/\delta} - \frac{c}{2}\frac{X^2}{r+2\delta}, \quad (46)$$

where A_j $j = G, B$ are corresponding integration constants.

Proof: Setting, $x^* = x^G = x^B = 0$ in the Hamilton-Jacobi-Bellman equations (33) and (32) yields,

$$rV^G = -C - \delta X V^{G\prime},$$
$$rV^B = -C - \delta X V^{B\prime},$$

and these two independent differential equations have the claimed solutions. QED.

Lemma 2: Let $\bar{X}^G$ and $\bar{X}^B$ denote the levels where the emission strategies hit the abscissa (i.e., the stopping levels, or more precisely, the levels where emissions start if the initial condition X_0 is to the right of these levels). Then,

$$V^G\left(\bar{X}^G\right) = \frac{\delta}{r}\bar{X}^G - \frac{c}{2r}\left(\bar{X}^G\right)^2,$$
$$V^B\left(\bar{X}^B\right) = \frac{\delta}{r}\bar{X}^B - \frac{c}{2r}\left(\bar{X}^B\right)^2.$$

Proof: The optimal starting of emissions, must satisfy the smooth pasting condition,

$$V^{G\prime} = -\frac{r}{\delta}A_G X^{-r/\delta-1} - \frac{cX}{r+2\delta} = -1, \quad (47)$$
$$V^{B\prime} = -\frac{r}{\delta}A_B X^{-r/\delta-1} - \frac{cX}{r+2\delta} = -1. \quad (48)$$

Solving these equations for the coefficients (A_G, A_B) and substitution into (45) and (46) yields the claim. QED.

Corollary 1: Assume $X \geq \max\left(\bar{X}^G, \bar{X}^B\right)$ and thus $x^G = x^B = 0$, then $V^G(X) < V^B(X) \Longleftrightarrow A_G < A_B$.

Corollary 2: $\bar{X}^G < \bar{X}^B$ iff $A_G > A_B$.

Proof: Emissions start when the derivative of the value function reaches the critical level of -1 coming from the right and below. Therefore, $\bar{X}^B > \bar{X}^G$ implies

$$-1 = V^{B\prime}\left(\bar{X}^B\right) > V^{G\prime}\left(\bar{X}^B\right),$$

or in detail,

$$-\frac{r}{\delta}A_B\left(\bar{X}^B\right)^{-r/\delta-1} - \frac{c\bar{X}^B}{r+2\delta} > -\frac{r}{\delta}A_G\left(\bar{X}^B\right)^{-r/\delta-1} - \frac{c\bar{X}^B}{r+2\delta},$$

which holds iff $A_G > A_B$. QED.

Hence, combining **Corollary 1** and **2**, a higher value (of course in this stopping domain, $X \geq \max\left(\bar{X}^G, \bar{X}^B\right)$) is associated with a later start (i.e., at a lower level of pollution) of emissions.

Lemma 3: *Assume* $X_\infty^{GB} > \bar{X}^G$, *then*

$$V^B\left(X_\infty^{GB}\right) - V^G\left(X_\infty^{GB}\right) = \frac{1}{r}u\left(\frac{\delta X_\infty^{GB}}{n}\right) > 0.$$

Proof: Substituting verifies the claim,

$$V^G\left(X_\infty^{GB}\right) = -\frac{C\left(X_\infty^{GB}\right)}{r} < V^B\left(X_\infty^{GB}\right) = \frac{u\left(\frac{\delta X_\infty^{GB}}{n}\right) - C\left(X_\infty^{GB}\right)}{r}. \text{ QED.}$$

Now assume indirectly,

$$\bar{X}^B > X_\infty^{GB} > \bar{X}^G. \tag{49}$$

Corollary 2 coupled with **Corollary 1** suggests a higher value for the value functions of the greens, while **Lemma 3** implies a higher value of the value functions of the browns at least at the steady state. Therefore by continuity, these value functions must intersect in the open interval $\left(\bar{X}^G, \bar{X}^B\right)$, i.e., $rV^G = rV^B$ for an $\hat{X} \in \left(\bar{X}^G, \bar{X}^B\right)$. Moreover, V^G must cut V^B from below, thus

$$V^{G\prime} > V^{B\prime} \text{ at } X = \hat{X} \in \left(\bar{X}^G, \bar{X}^B\right). \tag{50}$$

This is however, impossible because the domain (36) requires $x = 0 \Longrightarrow V^{G\prime} < -1$ and $\xi > 0 \Longrightarrow V^{B\prime} > -1$. Contradiction. QED.

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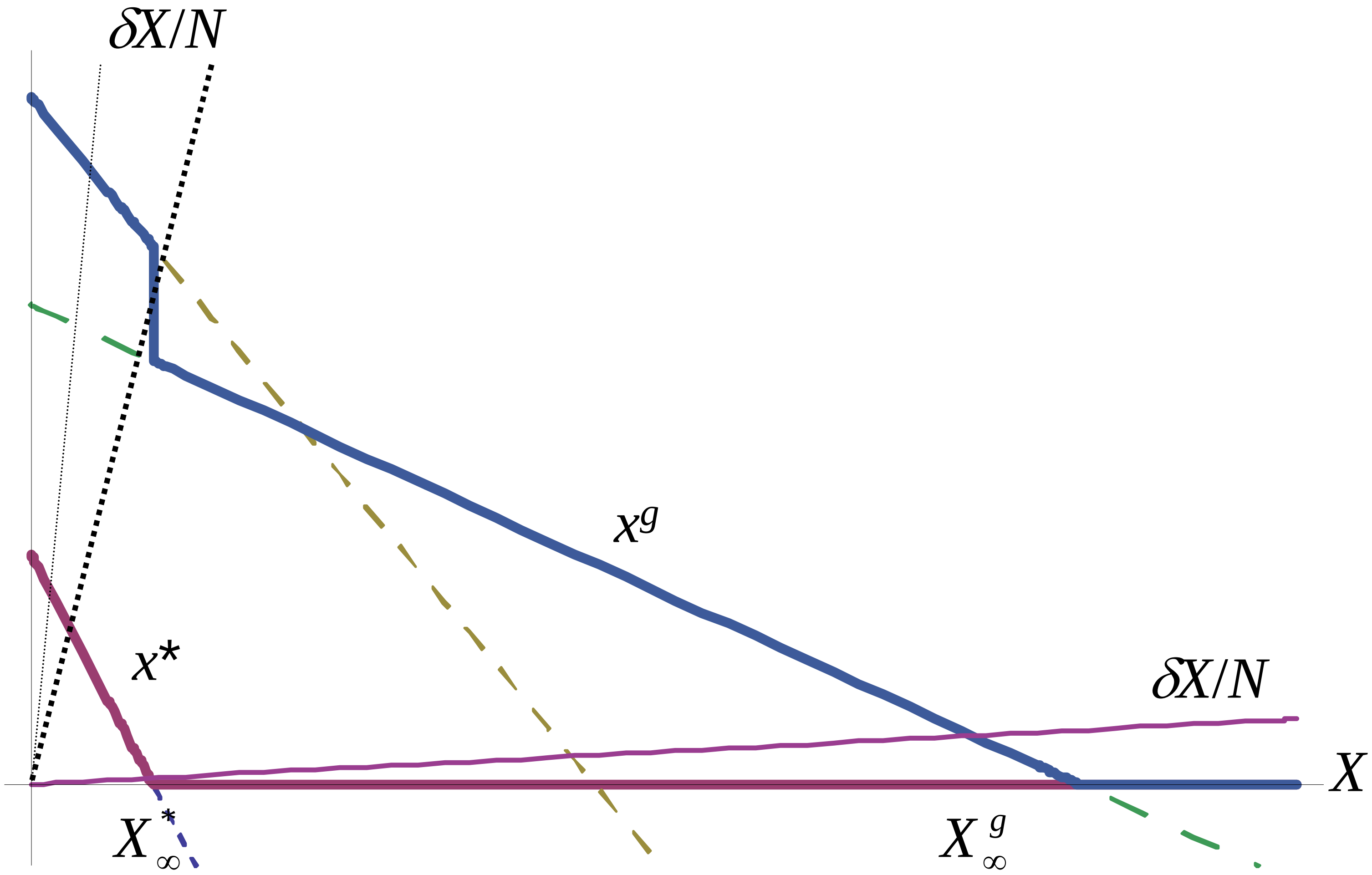


Fig. 1: Emission strategies of green consumers
dashed lines refer to the solution ignoring irreversibility

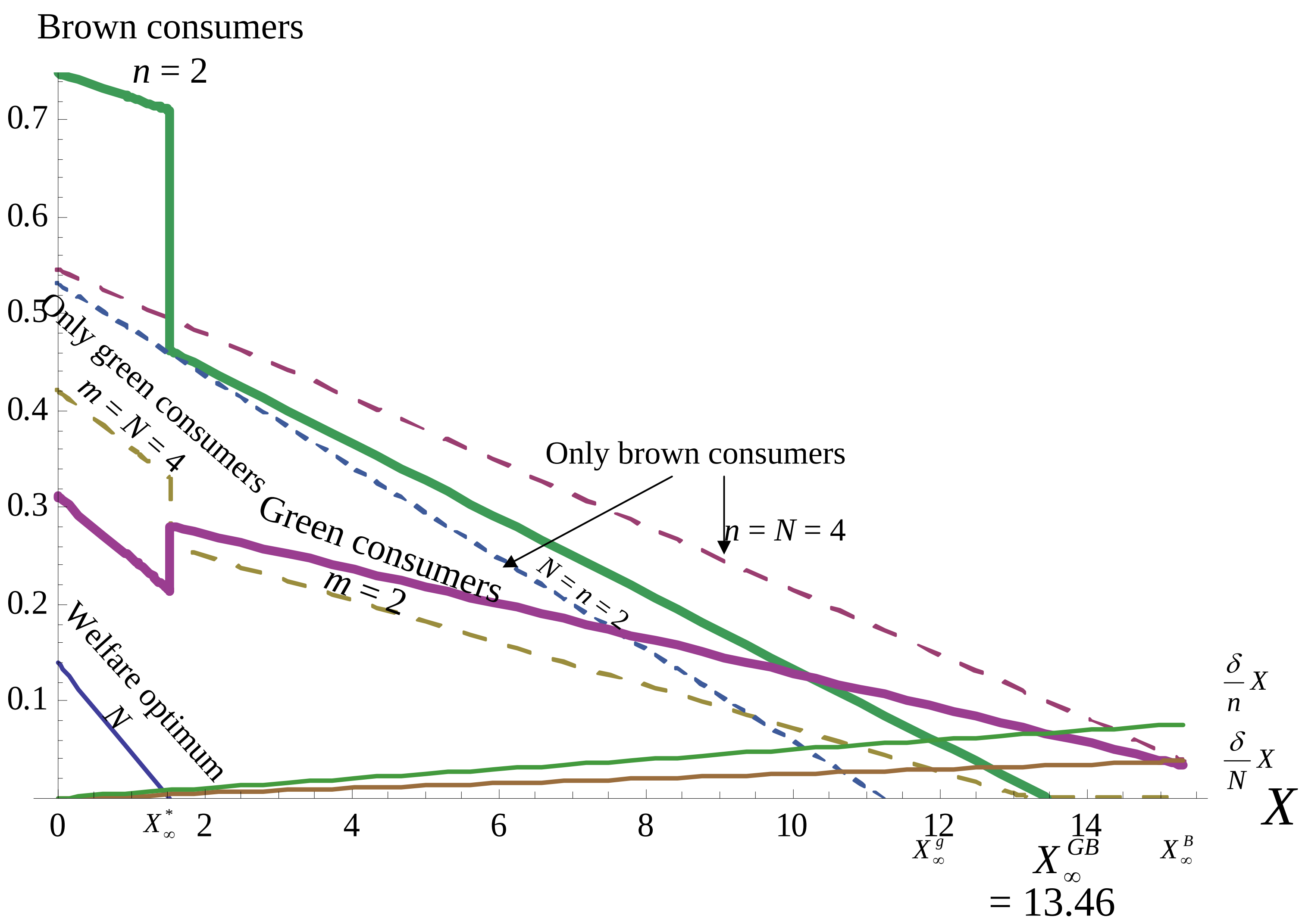
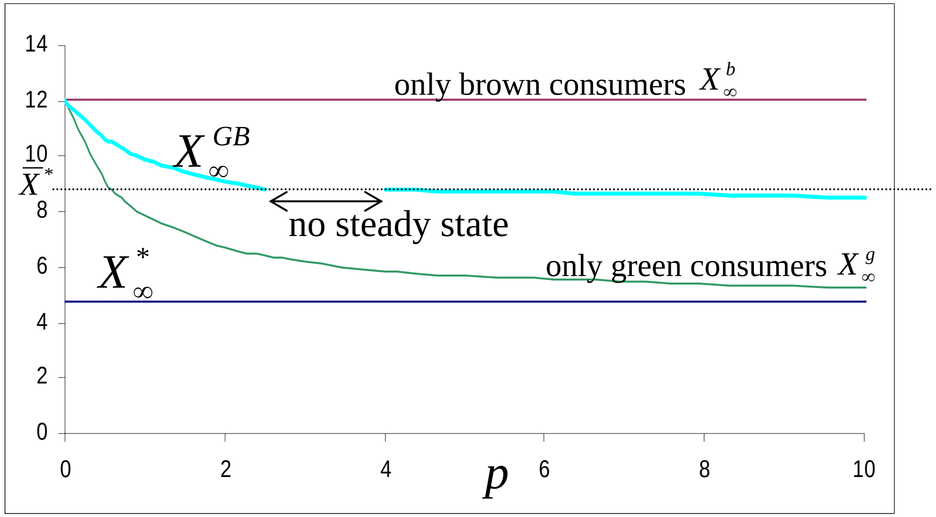


Fig. 2: Different emission strategies and their implied steady states,
$r = 0.05$, $\delta = 0.01$, $c = 0.01$, $p = 1$, $N = 4$, $m = n = 2$.

Steady states



Lack of a steady state
e.g., for $p = 3$.

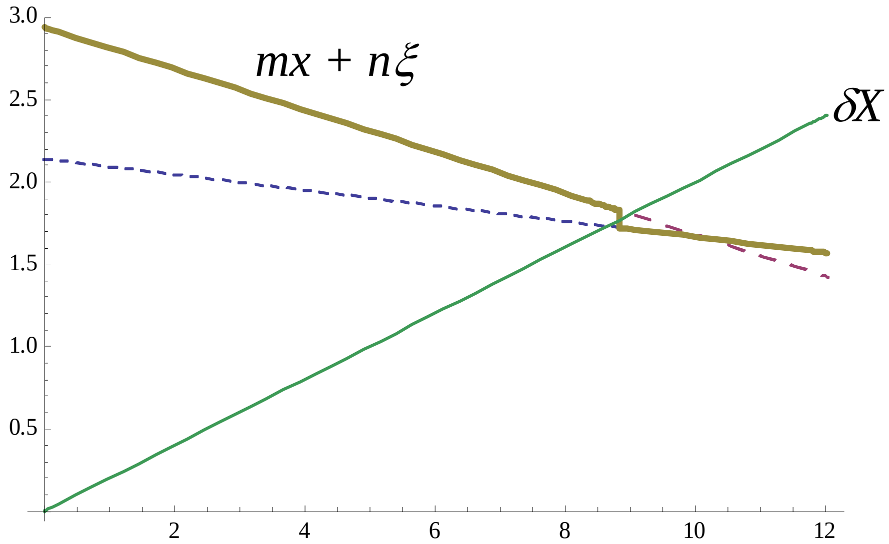
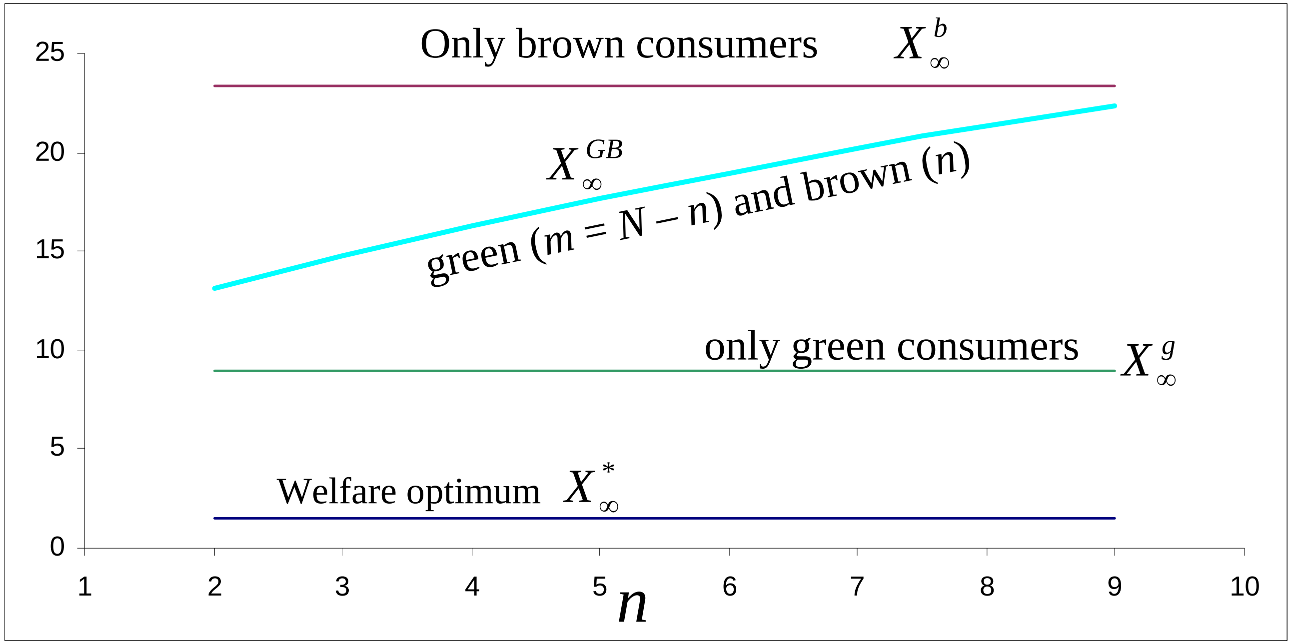


Fig. 3: Example: $r = 0.05$, $\delta = 0.2$, $c = 0.01$, $N = 4$, $m = n = 2$.



number of brown consumers

Fig. 4: Steady states versus distribution brown/green,
$r = 0.05,\ \delta = 0.1,\ c = 0.01,\ p = 5,\ N = 10$.

8 Appendix for Referees: Additional Examples

8.1 Green consumers

Fig. R1 shows the numerical examples that was used for the qualitative Fig. 1 in the paper and Fig. 2 shows the kink in the steady state relation with respect to p, i.e., in this example steady states exist globally. The critical penalty at which the steady state relation has a kink is at $p = p^g_{crit} = 194.257....$ Hence, greening already very green consumers (i.e., $p > p^g_{crit}$) has little effect, but helps much more for less green consumers. Fig. 3 shows an example where the green strategies jump upwards at $\bar{X}^*$, however this jump is irrelevant for the determination of the steady state.

[Insert Fig. R1 approximately here]
[Insert Fig. R2 approximately here]
[Insert Fig. R3 approximately here]

8.2 Heterogenous consumers

Figs. R4 and R5 show heterogenous consumers but for twice the number of players and different penalties. As expected, the higher penalty leads to low emissions of greens and now below the browns' emissions.

[Insert Fig. R4 approximately here]
[Insert Fig. R5 approximately here]

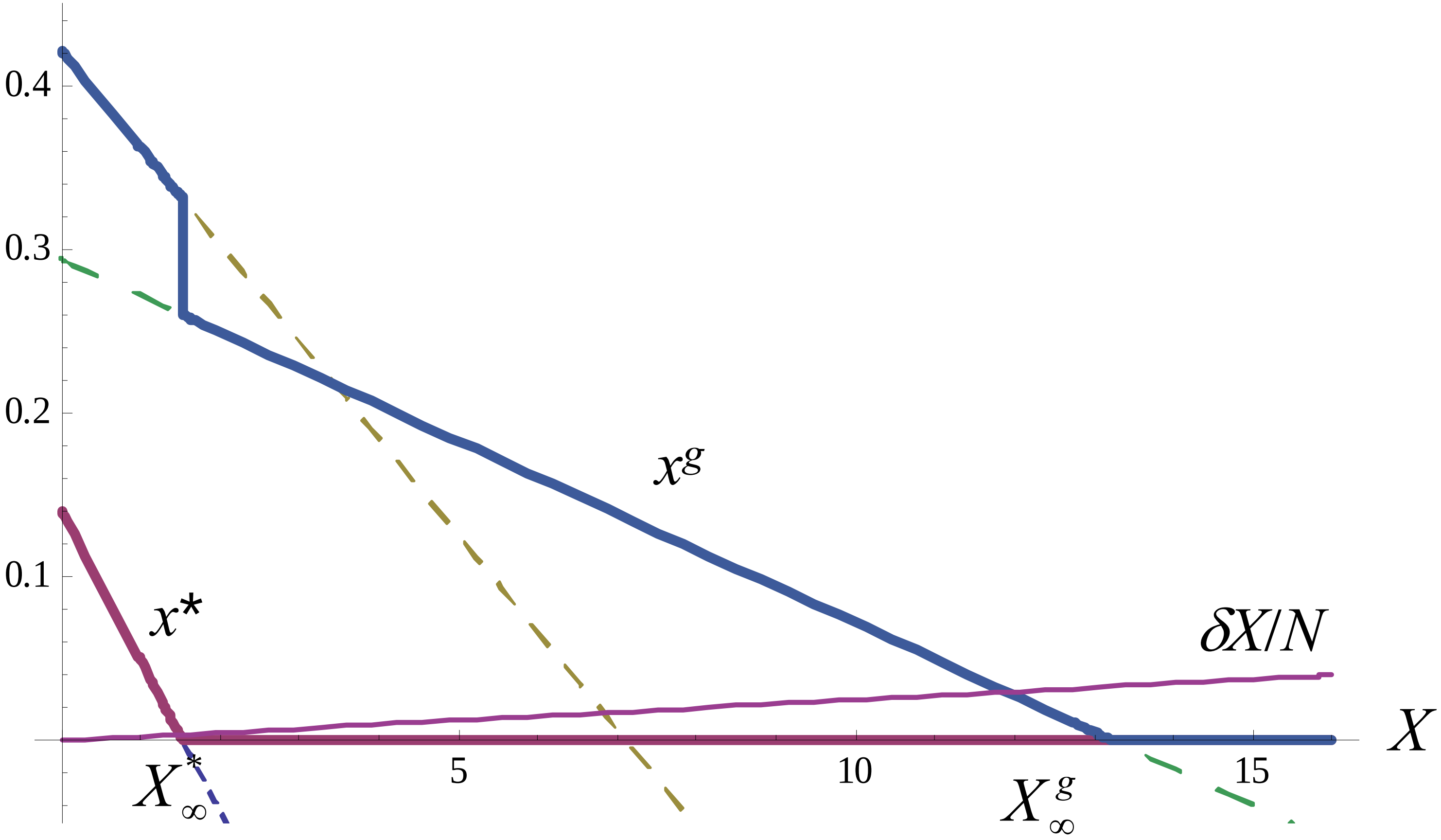


Fig. R1: Emission strategies of green consumers

r = .05, c = .01, δ = .01, p = 1, N = 4

dashed lines refer to the solution ignoring irreversibility

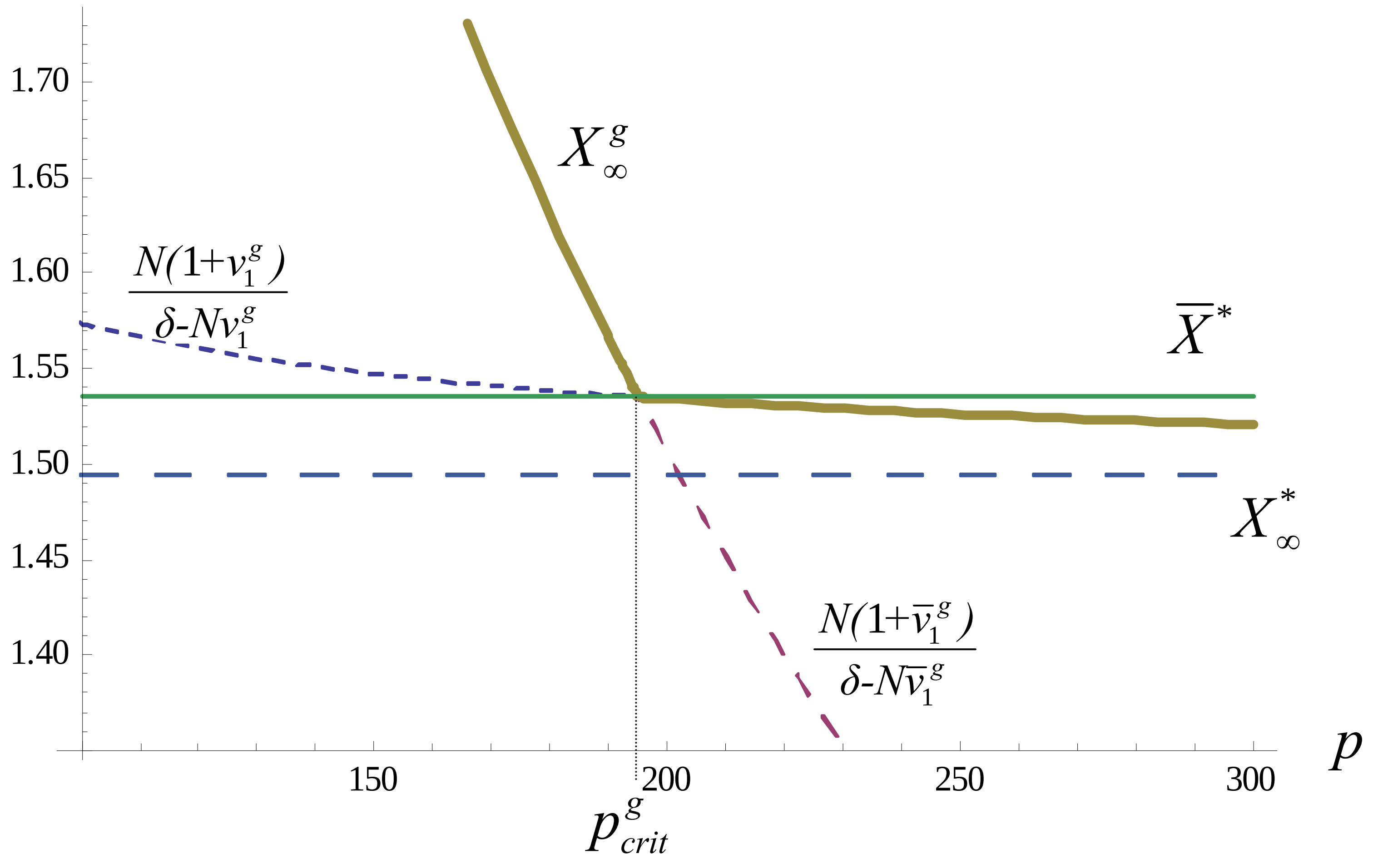


Fig. R2: Steady states versus green awareness (p)

r = .05, c = .01, δ = .01, N = 4.

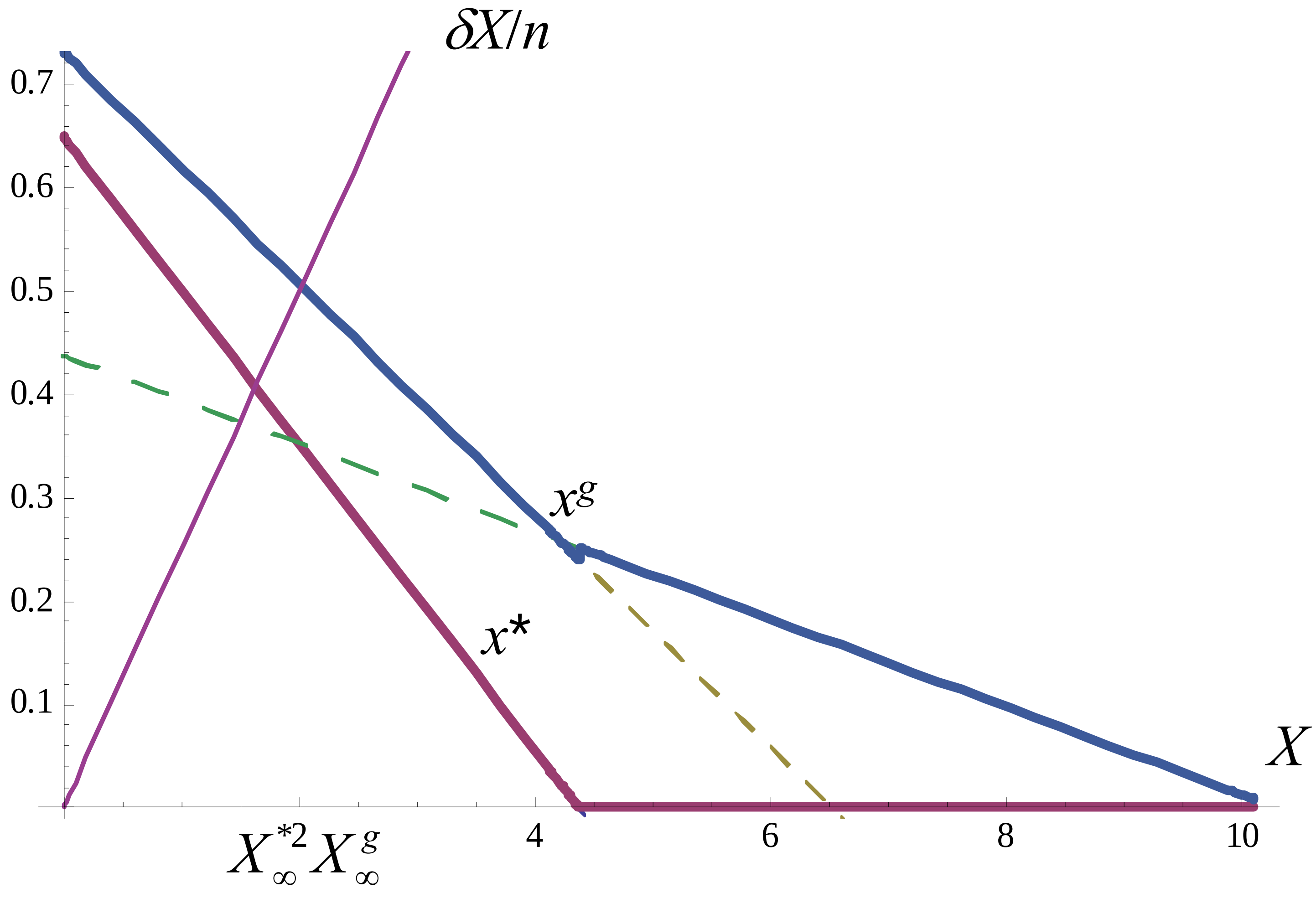


Fig. R3: Emission strategies of green consumers strategy can jump,
r = .05, c = .1, δ = .5, p = 1,
but in the domain irelevant for a steady state.

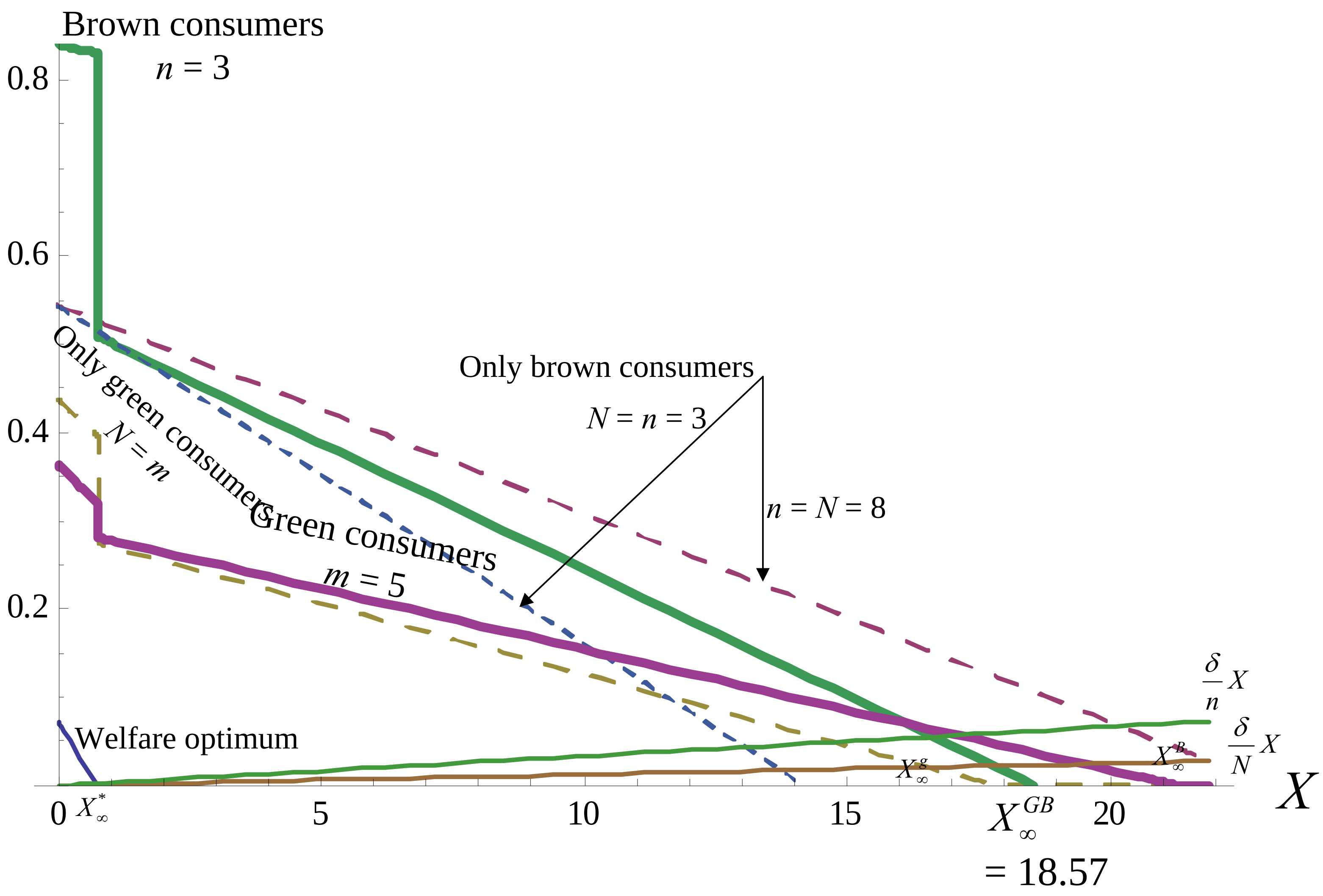


Fig. R4: Different emission strategies and implied steady states
$r = 0.05$, $\delta = 0.01$, $c = 0.01$, $p = 1$, $N = 8$, $m = 5$, $n = 3$.

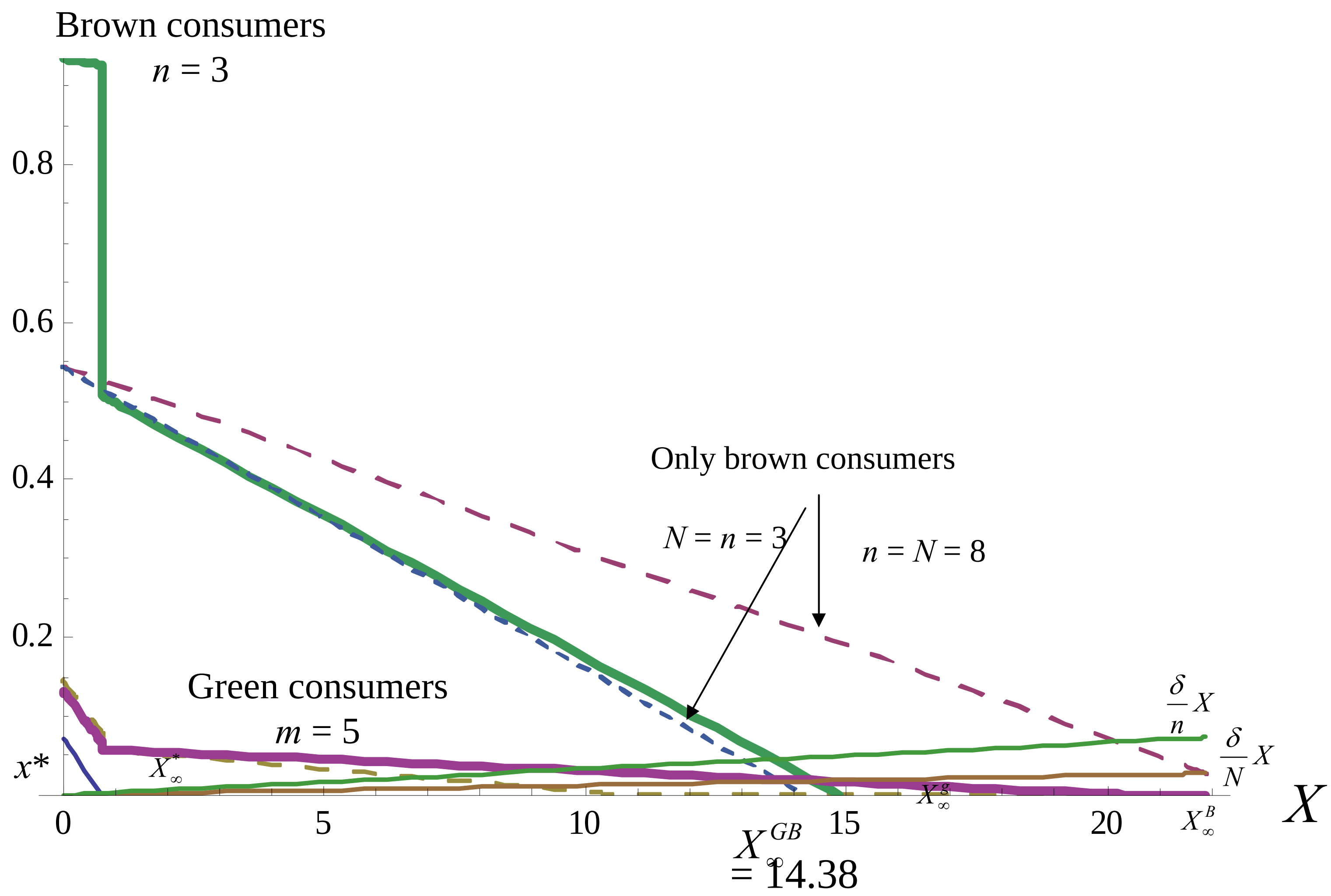


Fig. R5: Different emission strategies and implied steady states

$r = 0.05$, $\delta = 0.01$, $c = 0.01$, $\boldsymbol{p = 10}$, $N = 8$, $m = 5$, $n = 3$.