

# Concordance among Holdouts\*

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## Abstract

A *holdout* problem arises when a disparately-owned good is desired by a prospective buyer only in its entirety, as in land assembly. Market design is then crucial, as fully respecting property rights of many sellers eventually eliminates all trade. We propose a *Concordance principle*, inspired by the Cournot (1838) theory of *concours de producteurs*: divide profits by exogenously determined shares and impose a Pigouvian tax. This protects collective and approximate individual property rights while solving the holdout problem and, asymptotically, achieving full efficiency. Melding Concordance with most standard auction procedures encourages truth-telling. Extensions of our approach yield mechanisms for collaboration and public goods problems.

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*Very few commodities are consumed in just the form in which they are left in the hands of the first producer. . . [S]everal raw materials are generally brought together in the manufacture of each of these products. . . [T]he more there are of articles thus related, the higher the price determined by the division of monopolies will be, than that which would result from the fusion or association of monopolists.*  
 –Antoine-Augustin Cournot, *Recherche sur les Principes Mathématiques de la Théorie des Richesses* (1838)

Cournot (1838) established perhaps the most celebrated theorem in the theory of industrial organization: sufficient competition (*concurrence*) between the producers of substitutable goods leads to efficient price-taking behavior. As the number of competing sellers grows large none has much influence on price and therefore each eventually acts as a price-taker. While Cournot’s theorem is now more than one hundred seventy years old and originally treated only a very special model of competition, his fundamental insight has (Section I) proved robust and has had a profound impact on economics. It is the basis of much of modern antitrust policy (U.S. Department of Justice and the Federal Trade Commission, 1992) and is one of the most fundamental principles of market design. Extension of Cournot’s reasoning shows that a wide variety of simple market institutions, such as double auctions, are highly efficient (Satterthwaite and Williams, 1989; Rustichini et al., 1994) as long as the number of participants is large, because in that case no participant has much influence on price. Therefore in eliminating distortions from market power, clever market design is often less important, in markets where buyers (or sellers) are substitutes for each other, than is ensuring vigorous competition (Bulow and Klemperer, 1996; Klemperer, 2002).

Less widely known than but nearly as important as Cournot’s theory of competition was his treatment of *concours*, collaboration<sup>1</sup> among the producers each of perfectly complementary pieces of a good described in the quote above. As Sonnenschein (1968) argued, *Cournot collaboration* is the dual of Cournot competition: just as an increase in the number of competitors drives down mark-ups and increases production, production of a good being subdivided into more monopolized components drives down production, raising prices. As the number of *collaborators* grows large no individual has much impact on the quantity of the good purchased, as each individual only constitutes a small part of the price. Each

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<sup>1</sup>In modern French, *concours* refers to a competitive examination. But its more classical meaning is either a meeting of many individuals or a collaboration (Le Grand Robert de la Langue Française, 2009). We assume that it is this more classical sense that Cournot intended. “Collaboration” is the word most commonly given as the best translation of *concours* in this sense, and thus we have adopted it. In contrast, Irving Fisher and his translation assistant Nathaniel T. Bacon (Cournot, 1897) instead used the phrase “mutual relations” which we find cumbersome, a judgement that we believe is born out by the fact that this phrase virtually never appears in subsequent literature to describe perfectly complementary monopoly.

collaborator therefore eventually takes quantity rather than price as given, demanding an arbitrarily high price and eliminating all gains from trade.

This pessimistic dual of Cournot’s theorem is at least as significant for market design as his more famous result. Suppose that each piece of a good that a buyer desires in its entirety is owned by a different self-interested seller. Then as the number of such sellers grows, all possibility of beneficial trade in the absence of expropriation or subsidies disappears (Mailath and Postelwaite, 1990). Simply eliminating strategic incentives for sellers to misreport their values by having a buyer make a take-it-or-leave-it offer is insufficient, as such a buyer must be willing to offer each seller her highest possible value.

This basic *holdout* problem, that no individual is likely to scuttle a deal and therefore each can demand all gains from trade, arises in the many practical settings discussed in Section IV and more extensively in a separate appendix accompanying this paper. These include land assembly, an example we return to often throughout the paper, corporate acquisitions, patent pools, and spectrum management. Furthermore unlike classical auctions, there is no way around the fundamental market design problem holdout poses. The more sellers there are, the worse matters get; thus, if any trade is to be possible, it will rely on market engineering. Relatively conservative back-of-the-envelope calculations in Subsection IV.E suggest that deadweight loss on the order of many trillions of dollars, net present value, could potentially be avoided by a mechanism overcoming such inefficiencies.

Unfortunately, as described in Subsection V.C, most practical mechanisms used or proposed to solve the holdout problem involve some form of weighted voting to determine whether a sale takes place. These typically either poorly protect property rights or are highly inefficient (sometimes failing to solve the holdout problem at all).

This sharply contrasts with the problem of auction design. While there is strong reason to believe that the success of an auction will depend more on the degree of competition than its design, the success of corporate acquisitions is impossible without an appropriate mechanism. While “optimal” auctions (Myerson, 1981) are typically variants of those used since time immemorial, the mechanisms currently used for land assembly seem far from what a thoughtful analyst would prescribe. Yet while a substantial, burgeoning literature has studied the auction problem (Klemperer, 2000; Milgrom, 2004), we not aware of a single formal market design paper proposing a novel solution to the holdout problem. This article begins to fill that gap, after providing a general model of the holdout problem in Section II.

Section III presents our primary contribution: inspired by Cournot, we propose a principle underlying our approach to solving holdout. Cournot argued that both public and private interests would be served by merging collaborating producers into single firm. In merged firms, divisions typically share revenue according to predetermined formulae and internalize

externalities they cause. Therefore the natural extension of Cournot’s insight to the holdout problem is that each seller should have an option, if she chooses not to influence the sale decision, to receive at worst a pre-determined share of the buyer’s offer whenever a sale takes place. If instead she chooses to influence the decision of the sale, she pays a Pigouvian tax that forces her to bear the externalities she imposes on other sellers.

In honor of Cournot’s theory of *concours*, we refer to this principle by the only word<sup>2</sup> in English that, as far as we know, is directly derived from *concours*: Concordance. While no mechanism can simultaneously protect individual property rights and achieve efficiency, we show in Section III that any mechanism based on the Concordance principle strikes an attractive balance between these goals. Concordance mechanisms are always as efficient as bilateral bargaining and become perfectly efficient as the number of sellers grows large, while protecting the property rights of the community of sellers as a whole and an approximation to individual property rights based on information available to the government and other sellers. Unlike eminent domain, implementing Concordance does not require that the government know sellers’ subjective valuations; rather, it only needs to have an approximation of their share of the total land value. The quality of this share approximation determines the degree to which property rights are upheld.

Specific Concordance mechanisms (Subsections V.A-B) use standard concepts from auction theory to calculate the Pigouvian taxes sellers owe. The simplest enforcement strategy is that suggested by Vickrey (1961), Clarke (1971) and Groves (1973): each seller to pay a tax for any negative externalities their reported valuation causes others, based on others’ (truthfully) reported valuations. Extending ideas of Green and Laffont (1977) shows that this is the only Concordance approach to enforcement that is straightforward for sellers, in the sense that sellers need not engage in strategic thinking to determine their optimal action.

This *Straightforward Concordance* (SC) mechanism (Subsection V.A) has the typical disadvantage of VCG mechanisms: taxes are lost to a central authority. We propose a tax refund system that mitigates this problem; adapting the suggestion of Cavallo (2006) to our context, each seller receives her share of the minimum taxes she could cause other sellers to pay. Nonetheless potential for manipulation and budget surpluses persist.

Many of SC’s defects can be overcome if externalities are assessed on some basis other than other sellers’ reports (Arrow, 1979; d’Aspremont and Gérard-Varet, 1979). We refer to the mechanism in which each seller pays her expected externality, rather than her actual externality, as the *Bayes-Nash Concordance* (BNC) mechanism as described in Subsubsection V.B.1. While not entirely straightforward, truthfulness is in each seller’s interest as long as

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<sup>2</sup>Of course the root of “concordance,” “concord,” also derives from *concours*, but we thought “concordance” better captured the active role the mechanism plays in achieving an efficient outcome.

other sellers are truthful. Furthermore tax refunds can fully balance the budget, addressing some of the incentives for collusion in SC. Thus, in theory, BNC may be superior to SC.

Unfortunately, BNC requires calculation of the expected externalities associated with any reported valuation. Without a specific, known joint distribution of valuations, which may be hard to come by in practice, there is no reliable way to translate valuations into taxes, or vice-versa. Thus perhaps the best practical approximation to BNC is simply to have each seller pay her reported surplus, as an approximation of her expected externality. That mechanism is equivalent to asking each seller to put down an amount of cash in the cause of avoiding or promoting a sale at an announced offer price. Whichever pile is larger in aggregate wins, roughly analogous to the standard all-pay auction.

This *All-pay Concordance* (APC) mechanism (Subsubsection V.B.2) has the corresponding disadvantage of the all-pay auction: sellers must engage in complex calculations to determine their optimal strategy. It is not even clear what the equilibrium of APC, or the analogous *First-price Concordance* (FPC) mechanism (Subsubsection V.B.3), is. This leads us to prefer SC for now, despite the potential advantages of APC and FPC. Regardless, we argue in Section V.D that in many high-stakes settings Concordance mechanisms are superior to those based on voting by sellers.

The holdout problem is closely related to the large literature on mechanism design for the provision of public goods. In fact (Subsection VI.A), it is nearly equivalent to a binary public goods game. The reason we believe the public goods literature missed the approximation to Lindahl equilibrium, which our mechanism proposes, was the focus on more general continuous public goods environments with income effects for which our mechanism has no obvious analog. However, our mechanism does naturally extend to the (constant marginal cost version of) Cournot’s original collaboration problem (Subsection VI.B) and to an analogous continuous public goods setting (Subsection VI.C).

Our paper is a first pass at practical solution of the holdout problem. However, given the paucity of work on this important question, we suspect much of our contribution will be in raising rather than finally answering it. We therefore hope that future research will design superior mechanisms for solving the holdout problem. In Section VI we conclude by discussing some directions such work might follow. Most proofs are collected in an appendix following the main text of the paper. A more extensive appendix that discusses applications and the connections among auctions, holdout problems and Cournot’s theory, as well as software designed by William Weingarten for implementing and simulating all mechanisms described in this paper, appear separately at the authors’ websites.

# I Cournot and the Holdout Problem

Mathematics formalizing the results described here are summarized in the online appendix.

## I.A Cournot’s theory and its critics

In his 1838 classic *Recherche sur les Principes Mathématiques de la Théorie des Richesses* (Research into the Mathematical Principles of the Theory of Wealth), Antoine-Augustin Cournot famously argued that a firm monopolizing the production of a good will produce less than is socially optimal. The firm produces where marginal revenue equals marginal cost and because selling an extra unit requires lowering the price, marginal revenue is less than price. However, if production is divided among sufficiently many firms, each firm, feeling the impact of lower prices on a smaller number of units, would have little motive to reduce production. His celebrated theorem, a general modern presentation of which is given by Frank (1965), shows that sufficient competition leads to efficient marginal cost pricing.

Less well-known but equally important was Cournot’s theory of the collaboration which must occur when many firms monopolize the production of components (like left and right shoes) of a composite good. In this case, rather than production being divided among many competitors, profits are divided among many collaborators. As Sonnenschein (1968) observed, this problem is dual to competition. Rather than mark-ups falling (and production rising), collaboration causes production to dwindle as the multiple mark-ups (Spengler, 1950) of the collaborators pile up.<sup>3</sup> Bergstrom (1978) showed that this eventually leads to a pessimistic dual of Cournot’s theorem: all production is eventually eliminated as each firm grabs its piece of a diminishing pie. This is precisely the holdout problem as it is commonly understood: as each owner tries to extract as much as she can from the buyer, she fails to take into account the harm she risks causing the rest of the community by scuppering the deal. The most famous application<sup>4</sup> of this reasoning in economics is the Samuelson (1954) “free-rider” problem: voluntary contributions to the provision of a public good fail because of the “tragedy of the commons” (Hardin, 1968).

Cournot’s results quickly came under fire. Dupuit (1844) argued that the distortion of monopoly can be avoided if the firm is legally obliged to price at cost.<sup>5</sup> Along the lines of Dupuit, Lindahl (1919) argued that public goods can be efficiently funded by voluntary contributions if the state commits to magnify the impact of individual contributions with

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<sup>3</sup>Thus the common terminology of the “double marginalization” problem.

<sup>4</sup>This symmetry (Buchanan and Yoon, 2000) has spawned the name “anti-commons” (Michelman, 1967) for the holdout problem. Another common name is the “paradox of collective action” (Olson, 1965).

<sup>5</sup>Bertrand (1883) showed such regulation was unnecessary if two firms could commit to prices.

the contributions of others. Thus the holdout problem could be overcome simply by making appropriate take-it-or-leave-it offers to all property owners (Shapiro and Pincus, 2007).

Cournot's models were thus insufficient to support his arguments. Yet the central arguments were not so easily dismissed. Samuelson (1954) criticized Lindahl's theory as impractical, requiring detailed knowledge by the government of individual preferences<sup>6</sup> which those individuals could never be persuaded to divulge. Baron and Myerson (1982) went further, observing that regulating price to cost is only feasible—even with monopoly—if costs are known by a government that could never persuade the monopolist to reveal this information.

## **I.B Myerson-Satterthwaite and the redemption of Cournot**

The resulting confusion was resolved by Myerson and Satterthwaite (1981), who noted that the reason for monopoly distortion is the monopolist's ignorance of her consumers' willingness to pay for her product which prevents perfect price discrimination. The same problem confronts a regulator attempting to force that monopolist to price at cost. Any time it is uncertain whether a transaction should take place, at least one party must play a role in determining a mutually beneficial price. Whichever party is given that responsibility and protected from coercion or expropriation will have an incentive to reduce the probability of the sale. Thus Cournot's monopoly distortion cannot be eliminated without intervention, such as coercion or the injection of outside funds to grease the wheels of trade.

However it may be mitigated through competition reducing the probability/quantity of sales<sup>7</sup> any individual participates in and thus her incentive to manipulate prices. Satterthwaite and Williams (1989), Rustichini et al. (1994) and Cripps and Swinkels (2006) show that (double auction) competition eventually leads to efficiency. Mailath and Postlewaite (1990) vindicated Cournot's collaboration theory: any system of procuring property from disparate owners without coercion must pay each close to the maximal value any could have. Thus as long as it is not ex-ante certain a transaction should occur or a public work be created, the chance it can occur must dwindle as the number of owners grows.<sup>8</sup> Just as removing pricing power from firm's hands does not eliminate the monopoly distortion, eliminating strategic incentives for sellers to overstate values is insufficient to solve the holdout problem.

Cournot's basic contentions have thus been redeemed, even if the models first used to

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<sup>6</sup>Vickrey (1961) showed Bertrand competition only gradually drives down prices when competitors have heterogeneous costs.

<sup>7</sup>This can also be done by partially abrogating property rights (Segal and Whinston, Forthcoming).

<sup>8</sup>This indicates why it is so difficult for a coalition of small buyers in a package auction to beat a bidder for a collected lot and calls into question claims that auctions for such settings can possibly lead efficiency with even approximate incentive compatibility. To the extent that such auctions resemble the holdout problem it seems unlikely that any non-coercive mechanism can avoid serious inefficiencies.

formalize them have been displaced. Property rights (monopoly) impede efficient allocations. Competition alleviates this problem gradually. But, most importantly for our purpose, dispersed property rights over a single good make that good's sale impossible.

## I.C Market design and our contribution

A large literature, beginning with Myerson (1981), studies how the benefits of competition can be maximized; recent work in that tradition has largely vindicated Cournot, concluding that the benefits of competition itself are greater than those of auction design (Bulow and Klemperer, 1996) and thus design should focus on encouraging competition (Klemperer, 2002). Yet even more clearly, Cournot's results imply that market design should focus on solving the holdout problem, where without proper design outcomes are disastrous.

Despite this, research on the design of practical institutions for handling holdout problems has been severely limited. As far as we know every mechanism thus far proposed suffers from at least one of three serious flaws. Many mechanisms Nash-implement Lindahl allocations (Groves and Ledyard, 1977; Hurwicz, 1977; Walker, 1981; Bagnoli and Lipman, 1989; Tian, 1989) but have impractically large multiplicities of equilibria or assume complete information and thus effectively rely on an implausible common knowledge of values to achieve efficiency (Bailey, 1994). Property-rights-preserving mechanisms<sup>9</sup> that merely eliminate strategic misrepresentation of valuations (Bagnoli and Lipman, 1988; Shapiro and Pincus, 2007) do not solve the more fundamental holdout problem.<sup>10</sup> Classic market value-based takings and more efficient proposals for simple Vickrey-Clarke-Groves procedures (Plassmann and Tideman, 2009) abrogate all semblance of property rights.

Thus we view the problem of designing a practical mechanism<sup>11</sup> for the holdout problem

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<sup>9</sup>Another class of such mechanisms that is just beginning to be studied are so-called package exchanges (Milgrom, 2007). Because in these mechanisms truthfulness is not incentive compatible it is not clear how to square their suggestions of efficiency with Mailath and Postlewaite's theorem. However we strongly suspect that incentives for deceit in those mechanisms will be most acute in holdout setting and thus, in practice, they will perform in the long-run roughly like other property right preserving mechanisms.

<sup>10</sup>Another potential definition of holdout that we do not consider essential, namely delays in the bargaining process, has been suggested by Menezes and Pitchford (2004) and Miceli and Segerson (2007) among others.

<sup>11</sup>Various authors have proposed systems for solving the holdout problem without a centralized mechanism. However, these systems present problems of their own. For example, Kelly (2006) argues that the ability of developers to secretly purchase land for assembly is sufficient to address the problem. While secret purchases will help reduce the holdout problem for any particular assembler, they increase the difficulty in the real estate market generally, as each seller will come to suspect that her land may be part of an assembly. Thus, the loss from holdout is just spread out over many transactions as an increased Myerson-Satterthwaite distortion. Bell and Parchomovsky (2007) and Plassmann and Tideman (2009) have proposed using property self-assessment to elicit truthful revelation of values. However, this mechanism is decentralized and by the extension of the Green and Laffont (1977) theorem we prove in the Appendix, any such mechanism, if incentive compatible, must be payoff-equivalent to a Groves mechanism. Shoup (2008) proposes regulatory subsidies for land assembly, an approach with limited applicability (and potential incentives for misuse)



that strikes a reasonable compromise between the competing goals of efficiency and fairness (property rights) to be almost entirely open. The only mechanisms we are aware of that make an attempt at this are those based on voting within the community over the sale, perhaps the oldest mechanism for collective decision-making. We refer to such mechanisms as  $X$ -plurality mechanisms and in Subsection V.D show how, despite some merits, they often have an unattractive mix of inattention to property rights and inefficiency.

Our paper therefore attempts for the first time to bring the insights of modern market design to bear on the holdout problem, exploiting the special structure of quasi-linearity and binary choice it provides. Our starting point is Cournot's deep observation that collaboration externalities can be internalized through a merger. While the approach we develop applies to Cournot's original collaboration problem and to public goods (see Section VI), we focus on holdout both because of its wide range of practical applicability (see Section IV), the typically greater emphasis on property rights in holdout settings, and holdout's natural incorporation of the special assumptions (quasi-linear preferences and binary decisions) which we exploit.

## II The Model

For concreteness, we will discuss our model in terms of a land assembly example: there are  $N$  (potential) sellers  $i$ , each of whom owns a piece of a plot of land.

**Assumption 1** (Single-minded, Quasi-Linear Sellers). *Seller  $i$ 's utility is  $v_i 1_{no\ sale} + t_i$  where  $v_i$  is her private-information land valuation and  $t_i$  is any net transfer to her. That is the seller values only her own plot of land and (quasi-linearly) money.*

There is a single (potential) buyer  $\beta$ .<sup>12</sup>

**Assumption 2** (Perfect Complements). *The buyer is only interested in buying the entire plot. Her utility is  $b_i 1_{sale} - p_i$  where  $b_i$  is her private-information land valuation and  $p_i$  is any net transfer from the buyer. Thus the buyer views parts of the plot as perfect complements.*

A social planner (such as the government) has some beliefs about each  $v_i$ , believing that  $\mathbf{v} \equiv \langle v_1, \dots, v_N \rangle$  is drawn from  $[\underline{v}, \bar{v}]^N$  according to a smooth, full support joint probability density function  $g$ . Let  $V \equiv \sum_i v_i$  and let seller  $i$ 's share  $s_i(\tilde{V}) \equiv E \left[ \frac{v_i}{V} | V = \tilde{V} \right]$ .

**Assumption 3** (Shares).  *$s_i(\tilde{V}) \equiv s_i$ : the social planner believes that, regardless of the total value of the plot, the expected share of any given seller is fixed at  $s_i$ .  $\mathbf{s} \equiv \langle s_1, \dots, s_N \rangle$ .*

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because it is not self-financing.

<sup>12</sup>We will typically suppress the notation  $\beta$  except in mathematical expressions.

Section IV discusses how shares can be ascertained in practice for particular applications. Transactions are structured by a *mechanism* (Hurwicz, 1973) consisting of a collection of *offers*  $\mathcal{O} \subseteq \mathbb{R}$  specifying actions that buyers can take, *reserve values*  $\mathcal{R} \subseteq \mathbb{R}$  specifying actions that sellers can take, a *purchase rule*  $P : \mathcal{B} \times \mathcal{R}^N \rightarrow \{0, 1\}$  specifying the actions of buyers and sellers following which a transfer of the plot takes place, and a *transfer rule*  $\mathbf{T} : \mathcal{B} \times \mathcal{R}^N \rightarrow \mathbb{R}^{N+1}$  specifying transfers to or from buyers and sellers that follow actions.<sup>13</sup>

The mechanism may also have a pair of *suggested strategies*  $\langle o^*, r^* \rangle$  that, while not technically part of the mechanism *per se*, help in structuring thought about the mechanism. Here,  $r^* : [\underline{v}, \bar{v}] \rightarrow \mathcal{R}$  is a *suggested seller reserve function*<sup>14</sup>, and  $o^*$  is a *suggested buyer offer*.

It is useful to consider an example: the  $\frac{1}{2}$ -plurality mechanism in which the buyer makes a take-it-or-leave-it offer, which is revealed to the sellers who then vote on whether to accept it. The share-majority rules and all ties break in favor of a sale. If no sale is made, no money changes hands; if a sale is made, each seller receives her share of the buyer's offer. In this case  $\mathcal{O} = \mathbb{R}_+$  and represents the buyer's offer.  $\mathcal{R} = \mathbb{R}_+$  and represents the seller's valuation of her land. The purchase rule  $P(o, \mathbf{r}) = 1_{\sum_i s_i 1_{r_i \leq s_i o} \geq \sum_i s_i 1_{r_i > s_i o}}$  where  $1_x$  is 1 is the indicator function for  $x$ . The transfer rule  $\mathbf{T}(o, \mathbf{r}) = \langle -1, \mathbf{s} \rangle P(o, \mathbf{r})o$ : conditional on a sale, the buyer pays her bid  $b$  to the sellers in proportion to their shares and in the event of no sale, no money changes hands.  $r^*(v) \equiv v$ : truthful revelation. Finally  $o^*(b)$  is the monopsonist optimal price facing supply  $\tilde{G}^{\frac{1}{2}}$ , the cdf of the minimum successful offer.

There are a number of properties that one might want a mechanism to satisfy. A basic one is that the mechanism not require external subsidies.

**Definition 1.**  $\mathcal{M}$  is self-financing if  $\sum_i T_i \leq 0$ , where  $T_i$  are the entries of  $\mathbf{T}$ .

Mechanisms which are not self-financing may be open to fraudulent exploitation and are therefore virtually never used in market design (Myerson and Satterthwaite, 1981) or industrial organization (Tirole, 1988). We may also want to require that no money be paid to the planner.

**Definition 2.** A self-financing mechanism  $\mathcal{M}$  is budget-balanced if  $\sum_i T_i = 0$ .

Sellers in the applications we describe in Section IV are often inexperienced, under-resourced or otherwise ill-equipped to make complex calculations. It is therefore useful for them to not have to think overly carefully about how to participate in the mechanism. One way to achieve this is for the strategy suggested to the sellers to always be in their interests.

<sup>13</sup>We write  $T_\beta(o, \mathbf{r})$  for the transfer to the buyer and write  $T_i(o, \mathbf{r})$  for the transfer to the seller  $i$ .

<sup>14</sup>In principle some mechanisms might recommend different reserves to different sellers depending on their identity (or share) and not merely based on their valuation. However, any such mechanism may easily be transformed, by the revelation principle (Gibbard, 1973), into one where the suggested strategy is truthful revelation. Thus there is no loss of generality in our approach of economizing on notation.

**Definition 3.** A mechanism  $\mathcal{M}$  is straightforward for sellers<sup>15</sup> if

$$T_i(o, r^*[v_i], \mathbf{r}_{-i}) - v_i P(o, r^*[v_i], \mathbf{r}_{-i}) \geq T_i(o, r', \mathbf{r}_{-i}) - v_i P(o, r', \mathbf{r}_{-i})$$

for each  $i \in [1, N] \cap \mathbb{Z}$ ,  $v_i \in \mathbb{R}_+$ ,  $\mathbf{r}_{-i} \in \mathcal{R}^{N-1}$ ,  $o \in \mathcal{O}$ ,  $r' \in \mathcal{R}$  where  $\mathbf{r}_{-i}$  is the  $(N-1)$ -st dimensional vector  $\mathbf{r}$  excluding  $r_i$ . That is the recommended seller strategy  $r^*$  is dominant.

However Green and Laffont (1977) show that the set of efficient straightforward mechanisms is highly restricted. A weaker requirement is (*Bayes-Nash*) *implementability*: if all other participants behave as they are told to, all others find it in their interest to do so.<sup>16</sup>

**Definition 4.** A mechanism is (Bayes-Nash) implementable if

1. For each  $b, o'$ ,

$$E[bP(o^*[b], \mathbf{r}^*[\mathbf{v}]) + T_\beta(o^*[b], \mathbf{r}^*[\mathbf{v}]) | b] \geq E[bP(o', \mathbf{r}^*[\mathbf{v}]) + T_\beta(o', \mathbf{r}^*[\mathbf{v}]) | b].$$

2. For each  $i = 1, \dots, N, v_i, r'$ ,

$$\begin{aligned} E[T_i(o^*[b], \mathbf{r}^*[\mathbf{v}]) - v_i P(o^*[b], \mathbf{r}^*[v_i], \mathbf{r}^*[\mathbf{v}_{-i}]) | v_i] \\ \geq E[T_i(o^*[b], r', \mathbf{r}^*[\mathbf{v}_{-i}]) - v_i P(o^*[b], r', \mathbf{r}^*[\mathbf{v}_{-i}]) | v_i], \end{aligned}$$

where  $\mathbf{r}^*$  takes each  $v_j$  to  $r^*(v_j)$ .

A final notion of incentive compatibility is not worth formally defining as it is used only informally below, but is nonetheless of interest. In land assembly shares are often determined by the buyer or someone with the buyer's interest primarily in mind, such as a local government. It would be desirable if that authority would have no incentive to distort the shares. We will say that a mechanism is *share incentive compatible* if a buyer could not gain by misreporting shares, given recommended strategies.

A natural goal is allocative efficiency. A fully efficient mechanism achieves this goal perfectly when it implements beneficial trades and only these.

**Definition 5.** A mechanism is fully efficient if, for all  $\langle b, \mathbf{v} \rangle \in \mathcal{B} \times \mathcal{R}^N$ , we have

$$P(o^*[b], \mathbf{r}^*[\mathbf{v}]) = 1_{b \geq \sum_i v_i}.$$

<sup>15</sup>One could similarly define straightforward for buyers, but this is not a concept we will use often.

<sup>16</sup>Implementability requires participants to anticipate others' actions, so it will typically depend on participants' beliefs. We follow the Harsanyi doctrine in modeling participants' beliefs as derived from the common prior belief held by the social planner, and updated according to whatever information they hold.

As discussed above protecting property rights (nearly) always involves sacrificing some efficiency, a trade-off one may be willing to make. A simple alternative to full efficiency is to guarantee that things get no worse than they are under bilateral trade.

**Definition 6.** *The efficiency of a mechanism  $e(\mathcal{M}) \equiv \frac{E[(b-V)P(o^*[b], r^*[\mathbf{v}])]}{E[(b-V)1_{b>V}]}$ , the fraction of total possible gains from trade realized. A mechanism  $\mathcal{M}$  is bilateral efficient relative to another mechanism  $\mathcal{M}'$  with  $N = 1$  if the efficiency of  $e(\mathcal{M}) \geq e(\mathcal{M}')$  when the ( $b$ -conditional) distribution of the single seller valuation under  $\mathcal{M}'$  is the same as that of  $V$  under  $\mathcal{M}$ .*

Most mechanisms we discuss have natural analogs for collections of sellers of any size. We can therefore think of these mechanisms as forming a series  $\{\mathcal{M}^i\}_{i=1}^\infty$  and analyze whether a mechanism becomes more or less efficient as the number of seller grows large.

**Definition 7.** *A series of mechanisms  $\{\mathcal{M}^i\}_{i=1}^\infty$  is asymptotically efficient relative to a series of joint probability density functions if  $\lim_{i \rightarrow \infty} e(\mathcal{M}^i) = 1$  under the induced measures.*

Another goal is preservation of property rights, that no seller should be forced to sell below her value. A stricter standard is that her recommended action have this property.

**Definition 8.** *A mechanism preserves individual property rights if, for all  $v_i \in [\underline{v}, \bar{v}]$ , there exists  $r \in \mathcal{R}$  such that*

$$T_i(o^*[b], r, \mathbf{r}^*[\mathbf{v}_{-i}]) \geq v_i P(o^*[b], r, \mathbf{r}^*[\mathbf{v}_{-i}])$$

*for each  $i = 1, \dots, N$  and  $\langle b, \mathbf{v}_{-i} \rangle \in \mathcal{B} \times \mathcal{R}^{N-1}$ . A mechanism strictly preserves individual property rights if  $r^*$  has this property.*

The many philosophical, legal and economic reasons why protecting property rights may be attractive are discussed more extensively in Section IV and especially our separate appendix. However the results surveyed in Section I show that preserving individual property rights is deeply inconsistent with social efficiency. This is why many practical mechanisms for solving the holdout problem must abrogate some property rights. Nonetheless a primary goal of our work is to do this “as little as possible” by defining alternative to individual property rights which approximate some of their spirit, but allow more efficiency. One alternative is to protect the property rights of the community as a whole; that is, the community should never be forced to sell the land for less than it is worth to it as a whole.

**Definition 9.** *A mechanism preserves collective property rights if for all  $\mathbf{v} \in [\underline{v}, \bar{v}]^N$ , there exists  $\mathbf{r} \in \mathcal{R}^N$  such that  $\sum_i T_i(o, \mathbf{r}) \geq V \cdot P(o, \mathbf{r})$  for each  $\langle b, \mathbf{v} \rangle \in \mathcal{B} \times \mathcal{R}^N$ . This condition holds strictly if  $r^*$  satisfies this property.*

Section IV and the separate appendix discuss some applications in which collective property rights are appealing, but they are obviously may be cold comfort to many individuals within the community, who may be drastically expropriated. While precisely preserving their rights may be impossible, a more modest goal may be that they receive a “fair” share of a collectively acceptable compensation for their property.

**Definition 10.** *A mechanism preserves approximate individual property rights if, for all  $i = 1, \dots, N$ , and  $\langle b, \mathbf{v} \rangle \in \mathcal{B} \times [\underline{v}, \bar{v}]^N$ , there exists  $r \in \mathcal{R}$  such that*

$$T_i(o^*[b], r, \mathbf{r}^*[\mathbf{v}_{-i}]) \geq \frac{s_i \sum_{j \neq i} v_j}{1 - s_i} P(o^*[b], r, \mathbf{r}^*[\mathbf{v}_{-i}]).$$

### III The Concordance Principle

Our basic approach, of which all of the mechanisms we propose below can be seen as applications, is inspired by Cournot’s solution to the collaboration problem. Cournot argued that the collaborating firms should merge<sup>17</sup> so as to fairly share in—and hence internalize—each others’ profits. We see this suggestion, applied to holdout, as consisting of two parts:

1. Sellers should divide profits from a sale according to a pre-specified formula, just as a merger divides stock in the conglomerate among the shareholders of the merging firms.
2. Sellers should be incentivized to share information by paying for externalities caused by moving the group decision towards her preference, just as divisions of a firm (Groves and Loeb, 1979) are incentivized to communicate with headquarters.

The *Concordance principle*, so-named in honor of Cournot’s theory, implements these ideas. First, decisions are made to maximize the community’s aggregate welfare and the offer is assigned according to exogenous shares. Second, any land-owner who changes the decision about a sale, relative to what would happen if that individual’s share were evenly distributed over the rest of the community, is required to pay for her impact. This procedure is detailed informally in the box below and formally in the following definition.

**Definition 11.** *A mechanism  $\mathcal{M}$  satisfies the Concordance principle if  $P(o, \mathbf{r}) = 1_{o \geq R}$ ,*

$$[r_i = s_i o] \vee [r_j = s_j o \forall j \neq i] \implies T_i(o, \mathbf{r}) \geq s_i o P(o, \mathbf{r}),$$

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<sup>17</sup>The idea that some form of “corporate merger” would be useful in solving holdout problems was also recognized by Lehari and Licht (2007). However, they only saw a role for such a structure in determining the compensation, not deciding on the taking, parts which we see as inseparable. Furthermore they provided no explicit procedure for decision making within the seller “cartel,” which we view as the heart of what distinguishes various mechanisms (even multilateral bargaining).

## The Concordance Principle

1. Sellers are asked to report their values truthfully and buyers are asked to make the monopsonist-optimal offer to the aggregate seller.
2. The buyer's offer is accepted when it exceeds the total reported reserve.
3. Each seller has the option to *exert no influence* in which case the share-scaled-up reserve of all other sellers determines whether a sale occurs. If no seller exerts influence the sale proceeds.
4. Sellers exerting no influence, and lone sellers exerting influence, receive at least their share of the offer if a sale occurs and never pay anything.
5. In order to encourage truthful reporting, sellers exerting influence may be required to pay a Pigouvian tax.

$$r^*(v) \equiv v \text{ and } o^*(b) \equiv \operatorname{argmax}_o (b - o) \overline{G}(o).$$

We henceforth call any mechanism satisfying the Concordance principle a *Concordance mechanism* and refer to the set of all such mechanism by  $\mathfrak{C}$ . We are not aware of any such mechanism prior to this paper. The most distinctive attractive properties of these mechanisms (bilateral and asymptotic efficiency, collective and approximate individual property rights) follow directly from the Concordance principle, as we show in Theorems 1 and 2.

**Theorem 1.** *Every  $\mathcal{M} \in \mathfrak{C}$  is bilaterally efficient relative to its own bilateral form (a take-it-or-leave-it offer to a single seller). Furthermore, a sequence of mechanisms  $\{\mathcal{M}^n\}_{n=1}^\infty$  with  $\mathcal{M}^n \in \mathfrak{C}$  for all  $n$  is asymptotically efficient given share vectors  $\{\mathbf{s}^n\}_{n=1}^\infty$  if*

1. *There exists an  $M > 0$  such that for all  $i, n$   $ns_i^n < M$ .*
2.  *$\left\{ \frac{v_i^n}{s_i^n} \right\}_{i=1}^n$  are drawn independently and identically across  $n$  and  $i$  from some distribution with finite support and  $b$  is drawn independently, from identical distributions across  $n$ .*

Thus Concordance mechanisms solve the holdout problem: their efficiency does not deteriorate below that achieved in bilateral trade as the number of sellers grows large. In fact, these mechanisms transform pessimistic scenario of collaboration into the optimistic one of competition—provided seller values are somewhat independent, as the number of sellers grows large the mechanism eventually becomes fully efficient.

Concordance mechanisms use the aggregate seller value as an offer threshold. This is the same rule as if the community were a single seller, implying bilateral efficiency. Furthermore if

the ratio of seller values to shares are drawn i.i.d. and shares are not too concentrated then as the number of sellers increases, by the law of large numbers, asymmetric information about aggregate valuation dwindles and efficiency results as the Cournot-Myerson-Satterthwaite distortion vanishes. As with bundling of sufficiently many independently valued goods (Armstrong, 1999), the buyer eventually appropriates all (potential) gains from trade. If aggregate uncertainty about seller values persists in the limit,<sup>18</sup> so will distortions.

Concordance mechanisms also satisfy both of our second best property guarantees. They use the same decision rule for accepting offers as the community as a whole would, so they preserve collective property rights as long as the community can avoid tax payments.<sup>19</sup> Each individual, if she exerts no influence, receives her share of the offer, which is—if accepted—at least  $s_i \frac{\sum_{j \neq i} r_j}{1 - s_i}$ ; because she has the option to do this, her approximate property rights are never violated when other agents play the recommended strategy.

**Theorem 2.** *All  $\mathcal{M} \in \mathfrak{C}$  preserve collective and approximate individual property rights.*

All the additional details distinguishing Concordance mechanisms relate to how they use taxes to encourage truthfulness. To see why these are necessary, consider a seller with a value of \$100 and a share of one half facing a buyer offer of \$1000 but anticipating that the other seller with the same share has a value of \$950. This seller would have an incentive to report a reserve of \$0 rather than \$100 if she expects her partner to report truthfully as this would induce a sale earning her a profit of \$500 while truthfully reporting her valuation would lead to no sale. Thus she must be taxed to deter her from lying.

The various techniques (VCG, expected externality, first-price, all-pay) we use to encourage truthful revelation have trade-offs familiar to auction theorists: straightforwardness, budget-balance and attention to the “Wilson (1987) doctrine” (avoidance of strong dependence on beliefs). The Concordance mechanisms we propose are, with some small tweaks, simple combinations of the Concordance principle with these approaches to encouraging truthfulness. Thus the Concordance principle is *not* a substitute for standard auction approaches. Rather it is a complement, providing a means, by setting appropriate endogenous status quo property right allocations (effectively shared among the group of sellers as a whole), of making standard auction theory relevant and appealing for the holdout problem.

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<sup>18</sup>This seems unlikely, because even if the buyer did not know the distribution of seller valuations ex-ante, she would only need to persuade a small subsample to reveal their valuations truthfully in order to estimate the average aggregate valuation well as long as all values are drawn from the same distribution. This is an easy incentive problem (Kurz, 1974): two groups can be sampled and each asked incentivized to predict the others’ average reported preference, for example, in the spirit of Maskin (1999).

<sup>19</sup>Note that all tax payments can be avoided in this case if, for example, one individual submits her share of the community valuation and every other individual chooses not to exert influence.

## IV Applications

The abbreviated discussion of applications presented here is extended our Online Appendix.

### IV.A Land assembly

Large plots of contiguous land valued greatly in their entirety are often divided into pieces owned by disparate groups of individuals. To help alleviate the holdout problem that (especially public) developers face in assembling land (Merrill, 1986; Posner, 2005), the policy of “eminent domain” in the Fifth Amendment to the U.S. Constitution allows the government to take “private property ... for public use,” but only after “just compensation” has been paid. This procedure typically involves the (local) government asking for an assessment of land values by a real estate expert. Given limited legal recourse for takees following recent Supreme Court decisions (*Kelo v. City of New London, Connecticut*, 2005), this compensation is far below the minimum price at which residents would sell. States have reacted by severely curtailing the use of eminent domain (Castle Coalition, 2009; Morton, 2006; Wolf, 2008), restoring the holdout problem. Inability to resolve holdout problems is common in developing countries, but some have taken a middle path, requiring the consent of (only) some fraction of sellers as recently proposed for the United States by Heller and Hills (2008).

Land assembly has played important roles in many societies. Hoffman (1988) argues that Britain beat France to industrialization largely because, unlike France, it required only four-fifths of owners to consent to assemblies. The Mexican *ejido* system of collective land ownership requires the consent of all community members for any land to be sold. Some believe (Schmidt and Gruben, 1992) this leads to a “national agricultural system dominated by uneconomically small...farms.” In the immediate post-war period a Japanese system of partial-consent land assembly speeded development (Minerbi, 1986); its decline since then is blamed for much sprawl (Sorensen, 1999) and the eventual Narita airport debacle (Shimizu, 2005). The land assembly necessary to create a crime-free neighborhood is widely cited as one of the important reasons why extensive low-value slums continue to occupy some of the world’s most beautiful urban real estate on the hills overlooking Rio de Janeiro. Massive land assembly problems (FERC, 2005) confront builders of the energy infrastructure (pipelines and transmission lines) that will be required in coming years (Chupka et al., 2008). Efficient collective decisions about the use of land are widely thought to be reason for the dominance of the condominium form of property management (West and Morris, 2003; Heller, 2008).

Land assembly fits concepts we introduced above. Individuals’ share of the total assessed value of to-be-assembled land can be used for shares without having to base compensation directly on assessments. Collective property rights have a natural place in systems (México)



where land is legally owned jointly by a community. Approximate individual property rights are a natural formalization of the “community consensus on the severity of the harm inflicted” some legal scholars (Ellickson, 1973) believe is the appropriate foundation for compensation.

## IV.B Corporate acquisitions

When one individual or corporation seeks a controlling share in a public firm, most countries require that it makes a bid for all shares (Kirchmaier et al., 2009). These regulations are designed to protect minority shareholders’ interests in the case of take-overs by other firms whose interests do not concord with strict divisional profit maximization and to help ameliorate free-riding on corporate efficiency improvements (Grossman and Hart, 1980) by corporate “raiders.” Because individuals have heterogeneous risk-aversion and belief-driven infra-marginal utility from investing in the to-be acquired firm (Shleifer, 1985), it would be nearly impossible for a prospective buyer to purchase voluntarily all shares. Thus to allow acquisitions to take place, nearly every jurisdiction allows consent by some super-majority of share-holders to squeeze-out (Croft and Donker, 2006), as in Europe, or over-rule (Armour and Skeel, 2007), as in the U.S., the remaining holdouts.

Many of the criteria we defined seem compelling here. Collective property rights are appropriate given that it is hard to imagine shareholders making substantial *individual* investments in the value of the firm, so protecting collective, rather than individual, incentives is most important. Guaranteeing each individual a share-weighted piece of this collective settlement seems natural, as individuals are typically paid their share of the acquisition price.

## IV.C Other examples

Other examples abound. Rules in most countries (La Porta et al., 1998) require the consent of a supermajority of creditors to a debt renegotiation outside of bankruptcy, with thresholds differing across countries: high in the United States, lower in Europe and East Asia (Lee, 2007). Following Federal Communications Commission (FCC) auctions, radio spectrum has become fragmented, inhibiting efficient high-speed wireless internet (Hazlett, 2005). Re-assembling blocks of spectrum is a top priority of the FCC; their openness to sophisticated mechanisms (such as the spectrum auctions themselves) offers a promising opportunity for new solutions to holdout. To avoid collaboration problems, investors commonly assemble pools of complementary patented innovations and license them jointly (Lerner and Tirole, 2004). Difficulty forming pools can be a drag on innovation (Heller and Eisenberg, 1998). A collector trying to assemble the pieces of a fragmented triptych or a closely connected group of artworks faces holdouts among the current owners of individual pieces. Class action legal

settlements are often plagued by holdouts (Rob, 1989), resolved through voting procedures. Heller (2008) surveys a variety of other examples, from post-Communist property transitions in eastern Europe to share-cropping relations in the post-Bellum South.

#### IV.D How big is the holdout problem?

In the United States alone there are nearly active 6000 takings (Berliner, 2006) per year; these likely represent only a small fraction of all land assembly in the United States. Supposing an average stakes of \$10 million these represent \$60 billion annually; similar guesses for México and the Brazil indicate together at least \$20 billion dollars annually. Thus global land assembly activity is likely on the order of hundreds of billions of dollars each year. According to Dealogic, corporate acquisitions amounted to \$972 billion or 5.5% of global GDP in the first quarter of 2008 (Twaronite, 2009) alone. When the economy is weak reduced acquisitions are compensated by debt settlements; in 2008, according to BankruptcyData.com, the assets of U.S. firms filing for bankruptcy amounted to more than a trillion dollars. The other activities mentioned above are harder to quantify without more detailed investigation but probably amount to hundreds of billions of dollars. Aggregating these gives a ballpark estimate of many trillions of dollars for the annual volume of transactions subject to holdout problems.

Supposing an average of 20% potential gains from trade (consistent with a modest 10% monopoly mark-up under linear demand), and assuming that a very modest<sup>20</sup> one quarter of these are lost to deadweight from holdout, this amounts to 5% of transaction volume. A high 5% real interest rate would roughly indicate that the discounted NPV of social gains from an efficient mechanism for holdout is on the order of many trillions of dollars or double digit percents of global annual GDP. This is about an order of magnitude larger than the Lucas (2003) estimate of approximately 1% of global GDP for the NPV of eliminating all cyclic macroeconomic fluctuations. Many (Chauvin et al., 2009) would argue that Lucas's estimate is low and it would be generous to call the calculation here casual. Nonetheless, we believe they gesture to holdout being not just an intellectual problem where theory can shine, but rather a pressing social challenge of at least as great practical importance as the more-studied (including by ourselves) problems of auctions, matching (Roth, 2002), assignment (Abdulkadiroğlu and Sönmez, 1998) and combinatorial assignment (Budish, 2008).

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<sup>20</sup>This is linear demand monopoly deadweight loss which is much smaller than for other demand functions. Furthermore monopoly deadweight loss is very modest compared to what one would expect from holdout.

### Straightforward Concordance

1. SC is a Concordance mechanism with taxes on sellers who are pivotal in the sense that  $R$  and  $R_i$  are on different sides of  $o$ .
2. In this case she pays a tax equal to the harm she caused:  $(1 - s_i)|o - R_i|$ .
3. A refund is paid to each seller  $i$  of her share of the minimal VCG surplus independent of the reservation seller  $i$  announces:

$$s_i \min_{\hat{r}_i} \sum_{j=1}^N \left( 1_{(\hat{R}_j - o)(\hat{R} - o)} (1 - s_j) |o - \hat{R}_j| \right),$$

where  $\hat{R}$  and  $\hat{R}_j$  are computed as  $R$  and  $R_j$  with  $\hat{r}_i$  substituted for  $r_i$ .

## IV.E Straightforward Concordance (SC)

The simplest implementation of the Concordance principle uses the mechanism of Vickrey, Clarke and Groves to enforce truthful revelation of values. This mechanism incentivizes truthful revelation through Pigouvian externality taxes assessed on the base of other sellers' valuation reports.

Straightforward Concordance, outlined informally in the box above, is formally defined by  $\mathcal{B} = \mathcal{R} = \mathbb{R}_{++}$ ,  $P(o, \mathbf{r}) = 1_{o \geq R}$ ,  $T_\beta(o, \mathbf{r}) = -oP(o, \mathbf{r})$ ,

$$T_i(o, \mathbf{r}) = \underbrace{s_i o P(o, \mathbf{r})}_{\text{transfer value}} - \underbrace{1_{(R_i - o)(R - o) < 0} (1 - s_i) |o - R_i|}_{\text{VCG tax}} + \underbrace{s_i \min_{\hat{r}_i} \sum_{j=1}^N \left( 1_{(\hat{R}_j - o)(\hat{R} - o)} (1 - s_j) |o - \hat{R}_j| \right)}_{\text{tax refund}}.$$

Suggested strategies are monopsonist-optimal and truthful for the buyer and sellers respectively, as with any Concordance mechanism. The crucial advantage of SC over other implementations is that it is straightforward for sellers, thus its name. This implies that stated benefits associated with the Concordance principle apply not when sellers blindly “do what they are told” to by the mechanism, but also if they act rationally in their own interests.

**Proposition 1.**  $\mathcal{M}^*$  is self-financing, straightforward for sellers and implementable.

In the Appendix, we develop the class of “Groves Holdout” mechanisms, the natural extension of SC to accommodate alternative Groves payment rules. An extension of the main result of Green and Laffont (1977) to the (binary) case of holdout shows that only these

mechanisms are simultaneously straightforward for sellers, Pareto-optimal among sellers, and bilaterally efficient.<sup>21</sup> This characterization of the Groves Holdout class, combined with bounds derived from the Concordance principle pin down SC uniquely<sup>22</sup>: any straightforward Concordance mechanism is SC up to changes in the strictly positive refund.

**Proposition 2.** *Any straightforward-for-sellers Concordance mechanism  $\mathcal{M}$  is isomorphic to a mechanism of the form  $\mathcal{B} = \mathcal{R} = \mathbb{R}_{++}$ ,  $P(o, \mathbf{r}) = 1_{o \geq R}$ ,  $T_\beta(o, \mathbf{r}) = -oP(o, \mathbf{r})$ ,*

$$T_i(o, \mathbf{r}) = s_i o P(o, \mathbf{r}) - \underbrace{1_{(R_i - o)(R - o) < 0} (1 - s_i) |o - R_i|}_{VCG \text{ tax}} + h_i^+(o, \mathbf{r}_{-i}),$$

with  $h_i^+(o, \mathbf{r}_{-i}) \geq 0$  for all  $\langle o, \mathbf{r}_{-i} \rangle \in \mathcal{O} \times \mathcal{R}^{N-1}$ .

Furthermore, as shown in Proposition 8 in Subappendix B.3, our refund system uses the approach of Cavallo (2006) to return the maximal revenues to sellers while maintaining self-financing and without favoring any seller ex-ante.<sup>23</sup> Thus, SC is the maximally refunding, self-financing, non-discriminatory straightforward Concordance mechanism. However, in practice SC may return very little revenue; when all sellers shares are identical it never returns any.<sup>24</sup> The mechanism is also highly vulnerable to collusion or other manipulation.

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<sup>21</sup>A few innovations are required in this argument, since the argument of Green and Laffont (1977) relies upon the existence of at least three outcomes, while our setting has only two.

<sup>22</sup>In fact, we can prove a variant on this result (Proposition 9) which asymptotically pins down the pre-refund structure of any mechanism  $\mathcal{M}$  for the holdout problem which

1. is straightforward for sellers,
2. induces a Pareto-optimal outcome amongst sellers,
3. is bilaterally efficient relative to a take-it-or-leave-it offer, and
4. preserves approximate individual property rights.

is isomorphic to a mechanism of the form  $\mathcal{B} = \mathcal{R} = \mathbb{R}_{++}$ ,  $P(o, \mathbf{r}) = 1_{o \geq R}$ ,  $T_\beta(o, \mathbf{r}) = -oP(o, \mathbf{r})$ ,

$$T_i(o, \mathbf{r}) = s_i o P(o, \mathbf{r}) - \underbrace{1_{(R_i - o)(R - o) < 0} (1 - s_i) |o - R_i|}_{VCG \text{ tax}} + \hat{h}_i(o, \mathbf{r}_{-i}),$$

with  $\hat{h}_i(o, \mathbf{r}_{-i})$  bounded below by

$$\hat{h}_i(o, \mathbf{r}_{-i}) \geq \begin{cases} -s_i(o - R_i) & o \geq R_i, \\ 0 & \text{otherwise.} \end{cases}$$

<sup>23</sup>One deficiency of this approach is that no refunding occurs if seller reports are allowed to be unbounded. Seller values are bounded in practice, however, so this does not seem particularly restrictive. The optimal tightness of these bounds would trade off expressiveness of the mechanism against budget-balance and reduction of the potential gains from manipulation.

<sup>24</sup>To see this note that if any seller were to submit a reservation value maximally in the direction of the outcome that would occur if she exerted no influence, then no seller (if all had equal shares) would be influential. Thus the minimal tax for any seller is 0 and thus none can safely be returned any revenue.

In one form of collusion effective against SC, groups can avoid tax payments by share-weighted averaging their values and each reporting their share of this average. This leads to exactly the same sales rule as when members of the community act non-cooperatively, and additionally community property rights are strictly preserved. Since we are concerned with achieving efficiency and protecting property rights, rather than raising revenue, collusion in this fashion actually *improves*<sup>25</sup> outcomes. Of course, collusion among sub-groups of sellers, or imperfect collusion among all sellers, can be harmful to efficiency and property rights. The problem is well-known to be particularly severe (Ausubel and Milgrom, 2005) if, as seems likely in corporate acquisitions for example, there is a very large number of sellers and sellers can easily “de-merge,” splitting one individual into two each with half the share who can then express the same extreme preference, obtaining their desired outcome at no cost. Concerns about the possibility of such manipulation, as well as imperfect budget balance, are the primary motivation behind our other Concordance mechanisms.

## IV.F Other Concordance Mechanisms

We now discuss three alternatives to SC which, by using other standard auction concepts, sacrifice straightforwardness to improve budget balance and, potentially, reduce collusion.

### IV.F.1 Bayes-Nash Concordance (BNC)

Incentives for truthful reporting of valuations in the SC are maintained by forcing sellers to pay for any external harms caused by their influence. However, since a seller’s tax payments are assessed with respect to the value disclosures of other sellers, even with refunds SC is vulnerable to collusion amongst multiple sellers. This form of collusion may be sidestepped by using only external information in assessing a sellers’ taxes. If the expected externality a seller exerts based on having a particular value is known, she can simply be made to pay this as suggested by Arrow (1979) and d’Aspremont and Gérard-Varet (1979). Implementing such an “expected externality mechanism” of course requires knowledge by the mechanism administrator<sup>26</sup> of the distribution of other seller valuations conditional on a particular seller valuation. Such knowledge would be difficult to come by in practice, likely requiring a large data set of past similar situations to estimate the distribution of valuations as in standard

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<sup>25</sup>In fact, this is foreshadowed by Cournot’s argument: while the goal of an antitrust authority is to prevent anticompetitive mergers and collusion among horizontally related firms, it actually wishes to encourage and aid mergers and cooperation among vertically related firms. Concordance can be seen as providing sellers with a framework for overcoming the pitfalls of their self-interest and individual property rights.

<sup>26</sup>Cremer and Riordan (1985) argue that this can also be delegated to the buyer or even one of the sellers.

## Bayes-Nash Concordance

1. BNC is the Concordance mechanism with taxes equal to expected externalities, conditional on the reported valuation and the offer,

$$(1 - s_i)E_{\mathbf{v}_{-i}} [|V_i - o| 1_{(V_i - o)(V - o) < 0} \mid v_i = r_i]$$

2. Sellers receive refunds equal to her share of others' expected externalities,

$$s_i \sum_{j \neq i} E_{\mathbf{v}_{-j}} [|V_j - o| 1_{(V_j - o)(V - o) < 0} \mid v_j = r_j] .$$

structural estimation of auctions. This violates the Wilson (1987) doctrine<sup>27</sup> that mechanism should not depend too heavily for their performance on the beliefs of agents. Nonetheless the mechanism is of some intellectual interest and helps frame the connection between SC and the other, perhaps more practical, mechanisms we describe below.

We therefore assume for the purposes of this mechanism that

1. the social planner knows the distribution of  $\mathbf{v}$  precisely, and
2. the valuations  $v_i$  are independent of one another and of  $b$ , so that the distribution of other valuations does not depend on the value of any particular seller's valuation.

In this case, we can calculate for any reported  $v_i$  and offer  $o$  the expected Pigouvian tax the seller would have to pay under SC. This is just

$$(1 - s_i)E_{\mathbf{v}_{-i}} [|V_i - o| 1_{(V_i - o)(V - o) < 0} \mid v_i = r_i] .$$

It is well-known that a mechanism in which sellers pay their expected externalities will be Bayes-Nash incentive compatible. That is, if each seller is made to pay her expected externality then, if she expects all other sellers to reveal their valuations truthfully and she is risk neutral, she will have an incentive<sup>28</sup> to reveal her valuation truthfully as well.

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<sup>27</sup>Furthermore the incentive properties of the mechanism rely heavily on sellers being risk-neutral, perhaps reasonable over the relevant range of values, but perhaps not given that bidders in auctions for small stakes typically exhibit strong risk aversion (Bajari and Hortagsu, 2005). On the other hand, by helping to smooth payments over states, BNC would also be preferred by risk-averse sellers.

<sup>28</sup>In fact Williams (1999) show that any efficient Bayesian incentive compatible mechanism must have this format, a result we plan to extend in a subsequent paper to show that BNC has payoffs equivalent to those of any implementable Concordance mechanism with risk-neutral agents.

This intuition leads to the *Bayes-Nash Concordance (BNC)* mechanism described informally in the box above and formally by  $\mathcal{B} = \mathcal{R} = \mathbb{R}_{++}$ ,  $P(o, \mathbf{r}) = 1_{o \geq R}$ ,

$$T_i(o, \mathbf{r}) = \underbrace{s_i o P(o, \mathbf{r})}_{\text{transfer value}} - \underbrace{(1 - s_i) E_{\mathbf{v}_{-i}} [ |V_i - o| 1_{(V_i - o)(V - o) < 0} \mid v_i = r_i ]}_{\text{Pigouvian tax}} + \underbrace{s_i \sum_{j \neq i} E_{\mathbf{v}_{-j}} [ |V_j - o| 1_{(V_j - o)(V - o) < 0} \mid v_j = r_j ]}_{\text{tax refund}}.$$

The mechanism is budget-balanced, as the total of taxes paid equal total refunds:

$$\begin{aligned} & \sum_j (1 - s_j) E_{\mathbf{v}_{-j}} [ |V_j - o| 1_{(V_j - o)(V - o) < 0} \mid v_j = r_j ] \\ &= \sum_j \left( \sum_{i \neq j} s_i \right) E_{\mathbf{v}_{-j}} [ |V_j - o| 1_{(V_j - o)(V - o) < 0} \mid v_j = r_j ] \\ &= \sum_i s_i \sum_{j \neq i} E_{\mathbf{v}_{-j}} [ |V_j - o| 1_{(V_j - o)(V - o) < 0} \mid v_j = r_j ]. \end{aligned}$$

Again suggested strategies follow the Concordance principle.

The fact that BNC is actually a Concordance mechanism is immediate from the mechanism's definition once we observe that the expected externality

$$(1 - s_i) E_{\mathbf{v}_{-i}} [ |V_i - o| 1_{(V_i - o)(V - o) < 0} \mid v_i = r_i ]$$

of any seller  $i$  choosing to exert no influence vanishes as

$$\text{sign}(V - o) = \text{sign}(s_i o + [1 - s_i] V_i - o) = \text{sign}([1 - s_i][V_i - o]) = \text{sign}(V_i - o).$$

While BNC is not straightforward for sellers, it is implementable: so long as other sellers report truthfully, each is incentivized to do so. But for its practical difficulties, BNC might be more attractive than SC given its budget balance and potentially greater resilience to collusion.

**Proposition 3.** *BNC is budget-balanced and implementable. Moreover BNC strictly preserves collective property rights.*

#### IV.F.2 All-pay Concordance (APC)

BNC is difficult to implement because it requires the social planner to determine the expected externality payments appropriate for each valuation. These payments can be described by

## All-pay Concordance

1. APC is the Concordance mechanism with taxes equal to the surplus the seller would obtain given her announced reserve from her desired outcome.
2. The sellers' taxes are redistributed, with each seller  $i$  receiving

$$s_i \sum_{j \neq i} \frac{|s_j o - r_j|}{1 - s_j}.$$

a function  $f(v_i - s_i o)$  (possibly idiosyncratic across sellers) with  $f(0) = 0$ ,  $f'(x)x > 0$  for all  $x$  because sellers with larger announced surplus are pivotal more often (and by larger amounts). Thus the implementability problem of BNC can be seen as arising from the fact that the appropriate functional form of  $f$  is unknown. A natural way to address this problem is to assume a simple functional form for  $f$  satisfying these properties and hope that this roughly approximates the correct form. One natural candidate is  $f(x) = |x|$ .

This suggests the *All-pay Concordance (APC)* mechanism. Because in APC each seller pays her full announced surplus and the option creating greater announced net surplus wins, this procedure is equivalent to one in which each seller announces a preferred decision and an amount she is willing to pay to obtain this decision and whichever option has greater monetary support is chosen. This latter procedure is a simple adaptation of the standard all-pay auction to the Concordance context. Formally, the APC mechanism is given by  $\mathcal{B} = \mathcal{R} = \mathbb{R}_{++}$ ,  $P(o, \mathbf{r}) = 1_{o \geq R}$ ,

$$T_i(o, \mathbf{r}) = \underbrace{s_i o P(o, \mathbf{r})}_{\text{transfer value}} - \underbrace{|s_j o - r_j|}_{\text{tax}} + \underbrace{s_i \sum_{j \neq i} \frac{|s_j o - r_j|}{1 - s_j}}_{\text{budget-balancing transfer}}$$

with Concordance recommended strategies. APC is perfectly budget-balanced, since

$$\sum_i s_i \sum_{j \neq i} \frac{|s_j o - r_j|}{1 - s_j} = \sum_j (1 - s_j) \frac{|s_j o - r_j|}{1 - s_j} = \sum_j |s_j o - r_j|.$$

This, in addition to the independence of taxes from other sellers' behavior that APC shares with BNC, seems to indicate that APC may also be more resilient to collusion than SC.



### First-price Concordance

1. FPC is the Concordance mechanism where all positive surplus gained by sellers relative to their disfavored outcome is taxed away.
2. These taxes are then redistributed, with each seller  $i$  receiving

$$s_i \sum_{j \neq i} \frac{\max([s_j o - r_j] 1_{\text{sale}}, [r_j - s_j o] 1_{\text{no sale}})}{1 - s_j}.$$

In a future draft we hope to investigate this formally by analyzing the susceptibility of APC to mergers and de-mergers. Just as in the BNC mechanism, APC strictly preserves community property rights. Furthermore APC is a Concordance mechanism by construction, hence the results of Theorems 1 and 2 apply *if buyers and sellers follow recommended strategies*.

However, APC is not implementable: recommended strategies for APC cannot form an equilibrium, as this would give seller's negative surplus with near certainty. Thus, it is not clear how relevant Theorems 1 and 2 are to APC. Nonetheless, some traditional intuitions about the revenue equivalence theorem suggest that the equilibrium outcomes of APC may be related to those of SC, but it is not clear whether these intuitions are valid in this setting. Thus further theoretical and experimental research is needed to determine when APC is as efficient and protective of property rights as Concordance mechanisms purport to be.

#### IV.F.3 First-price Concordance (FPC)

For completeness we now show how the final standard auction enforcement mechanism, first-price payments, can be translated to the Concordance setting. The standard first-price auction has bidders declare a value for the object being sold with winners forced to pay their stated values. This gives (well-known) incentives for bidders to understate their valuations. The natural Concordance analog of this approach is to have sellers announce (falsely) a value and then have them pay the associated surplus as a result of obtaining the outcome (sale or no sale) they desire, if this outcome indeed obtains. This mechanism is described in the

above box, and given formally by  $\mathcal{B} = \mathcal{R} = \mathbb{R}_{++}$ ,  $P(o, \mathbf{r}) = 1_{o \geq R}$ ,

$$T_i(o, \mathbf{r}) = \underbrace{s_i o P(o, \mathbf{r})}_{\text{transfer value}} - \underbrace{\max(0, [s_i o - r_i] 1_{\text{sale}}, [r_i - s_i o] 1_{\text{no sale}})}_{\text{tax on desired outcome}} + \underbrace{s_i \sum_{j \neq i} \frac{\max(0, [s_j o - r_j] 1_{\text{sale}}, [r_j - s_j o] 1_{\text{no sale}})}{1 - s_j}}_{\text{budget-balancing transfer}},$$

with Concordance recommended strategies. FPC mimics all APC properties: it is budget-balanced and strictly preserves community property rights, but is not implementable.

#### IV.F.4 Alternative Enforcement Procedures

Of course the VCG, Bayes-Nash, all-pay and first-price enforcement approaches far from exhaust the set of all proposed payment procedures used in auctions. For example, number of “approximately incentive compatible” procedures have been proposed in recent years for dealing with complicated package auctions, many using payment schemes that ensure allocations are in, or as near as possible to, the core (Day and Milgrom, 2008; Erdil and Klemperer, Forthcoming). Given the similarities between package auctions with complementarities and the holdout problem these, it seems natural to adapt these payment schemes, using surplus from sale  $s_i o - v_i$  as the basis. However, the core in the holdout problem is nearly always empty: any coalition excluding a seller who earns a negative surplus when she exerts no influence earns greater surplus than it would were she included. Thus one would have to consider payments that come closest to the core. We do not consider such a core-closest Concordance mechanism<sup>29</sup> here because we believe that in most of the the settings where Concordance mechanisms are likely to be applied, the complexity of the payments involved is likely to outweigh the potential benefits such mechanisms offer. However, given that at least some applications (such as to spectrum reassembly) involve sophisticated participants, such a mechanism might still prove useful. More broadly, it should be clear how the Concordance principle can easily be combined with most auction enforcement mechanisms.

### IV.G X-Plurality

A contrast to Concordance mechanisms are a class of mechanism we call *X-plurality*. These generalize all those holdout mechanisms we know of that have actually been used in the past.

As described in the box below, all these share with Concordance mechanisms the fact that

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<sup>29</sup>We thank Jeremy Bulow for suggesting this idea to us.

## X-Plurality

1. The buyer is asked to submit the monopsonist-optimal offer against the distribution of minimum offers needed to persuade  $X$  percent of the shares to consent and sellers are asked to truthfully report their values.
2. If  $\sum_i s_i 1_{s_i o \geq r_i} \geq X$ , where  $X$  is a pre-specified value, and  $o \geq \underline{V}$  where  $\underline{V}$  is the lowest possible total community valuation, then the plot is sold. In this case, the buyer pays  $o$  and each seller receives  $s_i o$ . Otherwise no transaction takes place and no money changes hands.

sellers are recommended to report truthfully and that the offer conditional on sale is divided according to shares; however, no taxes are paid. Moreover, rather than the buyer's offer being accepted if it exceeds the sum of seller valuations, it is accepted if at least a fraction of shares  $X \in [0, 1]$  would “vote for” a sale. That is, sale occurs if there is a collection of sellers constituting at least a fraction  $X$  having value-to-share ratios  $\frac{v_i}{s_i}$  below the offer.

Standard mechanisms for the holdout problem can be seen as special cases of this rule. As discussed in Section IV, the rules for corporate acquisitions used in virtually all countries and the related rules used for land assembly in various countries at various times have been voting-based,  $X$ -plurality rules with various thresholds  $X$ . The standard eminent domain system can be seen as an  $X$ -plurality system with  $X = 0$ : sellers are paid the minimum valuation they could have by revealed preference, the market valuation of their land. The systems of Bagnoli and Lipman (1988), Shavell (2007), and Shapiro and Pincus (2007), which require universal consent, arise as the  $X$ -plurality mechanism with  $X = 1$ .

The general *X-plurality mechanism* is formally, implemented as  $\mathcal{B} = \mathcal{R} = \mathbb{R}_{++}$ ,

$$P(o, \mathbf{r}) = 1_{X \leq \sum_i s_i 1_{s_i o \geq r_i}},$$

$T_i(o, \mathbf{r}) = s_i o P(o, \mathbf{r})$ ,  $r^*(v) = v$  and  $o^* \equiv \arg\max_o (b - o) \tilde{G}^X(o)$ . Here  $\tilde{G}^X$  is the cumulative distribution function of the  $\underline{o}_{N,X} \equiv \arg\min_o 1_{X \leq \sum_i s_i 1_{s_i o \geq r_i}}$ . In the case of equal shares  $\underline{o}_{N,X}$  is just the  $[N(1 - X)]$ -th order statistic of the distribution of  $\frac{v_i}{s_i}$ . The case  $X = 1$  encompasses the mechanisms of Shavell and Shapiro and Pincus; the case  $X = \frac{1}{2}$  is equivalent to majority share rule; and the case  $X = 0$  corresponds to the typical application of eminent domain.<sup>30</sup>

<sup>30</sup>Here, we have assumed that the minimal value  $\underline{V}$  is that which is assessed as compensation in a taking. This appears to be reasonable, as in practice takings are often compensated at or below market value—and therefore at the lower bound of possible subjective property valuations (Radin, 1982; Fennell, 2004).

The strength of  $X$ -plurality, and the reason it has likely be used so widely, is that it combines the straightforwardness of SC with the budget balance of the other Concordance mechanisms, while being eminently practical and simple. Budget balance is immediate as all revenues generated by the offer are disbursed to the sellers and no taxes on sellers are assessed. Straightforwardness follows from the fact that the mechanism has sellers “voting” in favor of the (generically) unique outcome that yields them weakly positive surplus and voting thus can only increase the probability of this outcome realizing. Implementability follows from the fact that the minimum successful offer that is just sufficient to achieve a sale is that equal to that which is just sufficient to have a fraction  $X$  of shares consent. The seller finds it optimal to make the monopsonist’s optimal offer against this “supply curve.”

**Proposition 4.** *For all  $X$ , the  $X$ -plurality mechanism is budget-balanced, straightforward for sellers and implementable.*

However, the  $X$ -plurality class of mechanisms suffers from two pervasive deficiencies: its inefficiency, potentially both encouraging inefficient and discouraging efficient sales, and its complicated and often unappealing relationship to property rights. We discuss these in turn.

On the efficiency front it is easy to see that, in general, the  $X$ -plurality mechanism leads to efficient community decision-making given a buyers’ offer when the  $X$ -th percentile of the share-weighted empirical distribution of value-to-share ratios coincides with the share-weighted average of that distribution. Bergstrom (1979a,b) extensively developed in the context of public goods games (effectively) the theory of efficiency (among sellers) of the response to a buyer’s offer in the case of equal (rather than share-based) voting weights and  $X = \frac{1}{2}$ . In that case in large communities efficiency results if and only if the median of the distribution of value-to-share ratios coincides with its mean. Efficiency would require the social planner setting  $X$  at the quantile of the distribution corresponding to its mean.

Of course a number of additional complexities arise in our setting. First, share-weighted voting is not strictly equivalent to equal-weights voting and thus it is the empirical quantile of weighted distribution that is relevant. Second, because communities are not infinitely large it is the *empirical* rather than *population* quantiles that are relevant. Third, in small populations where the empirical quantile is uncertain, because of the Cournot-Myerson-Satterthwaite distortion that exists in small populations it might actually be optimal from an efficiency perspective to set  $X$  somewhat below the quantile of the transformed distribution corresponding to mean of the distribution so as to raise the probability of a sale back towards efficiency. Furthermore, for very small communities the empirical distribution may diverge sufficiently from the population distribution that the efficiency properties may depend on different features of the distribution than simply its quantiles. More careful analysis of these issues is beyond the scope of our paper but an interesting direction for future research.

Our main point is two-fold. First, any efficiency guarantee for  $X$ -plurality mechanism would rely on (the social planner) having a clear sense of the distribution of valuations. Second, anytime such information is not available the  $X$ -plurality mechanisms can be highly inefficient—in either direction. If the true properly weighted distribution is such that the mean is consistently above (below) the  $X$ -th quantile and the buyer’s value lies between these, most sales (failures to make a sale) will be inefficient except in very small communities of sellers. Thus  $X$ -plurality seems likely to be inefficient, especially in large communities.

By construction  $X$ -plurality preserves the property rights of sellers constituting a fraction  $X$  of all shares. It also preserves approximate individual<sup>31</sup> and collective property rights *to the extent that the  $X$ -th empirical quantile is weakly above the mean*. To the extent it is below collective and approximate individual property rights are violated. In practice this suggests that when  $X$  is large,  $X$ -plurality mechanisms will tend to preserve property rights at least as well as Concordance mechanisms do, if not better, though precise guarantees will depend on assumptions about distributions. However raising  $X$  also inefficiently reduces the number of sales. Thus the class of  $X$ -plurality mechanisms, in practice, seems to embody an unattractive tradeoff between inefficiency and violation of property rights. Furthermore, there is much scope for vote-buying manipulation, except when  $X$  is very small.

## IV.H Comparison of Holdout Mechanisms

Table 1 presents a comparison between the holdout mechanisms. The comparison also addresses practical factors, such as individuals’ levels of exposure to risk.<sup>32</sup>

The table contains one dimension not discussed above: share incentive compatibility. We omitted this above because for almost all mechanisms misreporting shares does not benefit the buyer in any way because the seller decision is independent of shares. In fact the buyer may generally have an interest in reporting shares truthfully as this tends to reduce property rights violations (for Concordance mechanisms), thereby economizing on potential legal costs, or bringing down minimum sale prices (for high  $X$ -level  $X$ -plurality mechanisms). However for moderate  $X$ -level  $X$ -plurality mechanisms, the seller may gain by allocating all shares to buyers with low valuations. Furthermore, though this is not reported in the table, under eminent domain (0-plurality) the buyer may have an incentive to distort down the “minimum seller valuation” so as to pay compensation below market prices. This has been a common concern in much of the controversy over eminent domain (Castle Coalition, 2009).

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<sup>31</sup>Except that the share is a share of the actual  $V$  not  $V_i$ .

<sup>32</sup>In practical implementations of the Concordance mechanisms, it will be necessary for sellers to finance their tax transfers without overflowing their ex-ante budgets. This problematic for mechanisms where an individual’s net taxes can be unpredictably negative, as with SC. This issue raises some implementation questions which we leave for future work; here we just compare on the basis of uncertainty about tax bills.

	Finances	Simplicity	Efficiency	Property Rights	Risk and Budgets	Share incentive	Collusion	Practical Issues
SC	Self-financing, asymptotically balanced	Straight-forward for sellers, implementable	Bilateral, asymptotic	Collective, asymp. strict collective, approx. individual	High	Yes	Moderate?	
BNC	Balanced budget	Implementable	Bilateral, asymptotic	Strict collective, approximate individual	Low	Yes	Low?	Requires detailed knowledge of valuations
APC	Balanced budget	Complex, possibly unimplementable	Bilateral, asymptotic	Same as BNC	Low	Yes	None?	
FPC	Balanced budget	Very complex, likely unimplementable	Bilateral, asymptotic	Same as BNC	Moderate	Yes	Very low?	
$X$ -plurality (low $X$ )	Budget balanced	Like SC	Too many sales	None	None	Yes	None	
$X$ -plurality (mid $X$ )	Budget balanced	Like SC	If percentile matches mean	$X$ of shares, approximate individual if efficient	None	No	High?	
$X$ -plurality (high $X$ )	Budget balanced	Like SC	Holdout: no asymp. gains	Near-perfect individual	None	Yes	Very high?	

Table 1: Comparison of holdout mechanisms

Two comparisons are clear. BNC nearly dominates SC, were it practically implementable which sadly seems unlikely. Moderate  $X$ -plurality mechanisms dominate those with low  $X$ , though what exactly constitute “low” and “moderate” is ambiguous.

Narrowing our focus to undominated mechanisms, we have a number of interesting but difficult-to-quantify trade-offs. BNC seems the best of the Concordance mechanisms when it is feasible, while the tradeoffs between the straightforwardness of SC and the other benefits APC and FPC are subtle. APC shows more promise than FPC, but without better theoretical knowledge of their equilibria, as well as the ability of communities to reach equilibrium without infeasible training, SC seems more attractive alternative at present. However we suspect that in the long-term some variation on APC may be superior.

Among values for the  $X$ -plurality mechanism above the best guess of that optimal for efficiency, a simple trade-off between efficiency and property rights appears. Opinions about the appropriate stand on this trade-off are likely to differ widely across honest people. Given that our main goal here is economics, not philosophy, we will not opine on it further except to say that we would guess that to the extent that the current legal environment in the United States reveals policy preferences, allowing the holdout problem to persist through a very high  $X$  seems unlikely to be a widely accepted policy.

Comparing  $X$ -plurality to the Concordance mechanisms is hardest. An  $X$ -plurality mechanism is extremely easy to explain and does not require any tax payments which might conflict with the budget constraints of sellers. Furthermore the  $X$ -fraction-of-shares property rights protection may be attractive to some, though clearly Concordance mechanisms will, under appropriate comparisons of quantiles to means, also satisfy similar properties. However, if one is willing to put faith in complicated institutions, Concordance mechanisms offer a much more attractive set of guarantees about combinations of efficiency, property rights and incentives than any practical  $X$ -plurality mechanism could.

To summarize, we believe potential implementers’ preferences likely fall into three camps:

1. Those strongly protective of property rights, who will favor  $X$ -plurality with a high  $X$ .
2. Those primarily interested in efficiency and simplicity, likely working in contexts where budgets and stakes are very small, who would tend to favor an  $X$ -plurality mechanism with an  $X$  chosen to roughly maximize efficiency.
3. Those interested in a mix of efficiency and property rights in high-stakes environments where they are willing to expend and require of sellers the resources (material and intellectual) necessary to implement the sophisticated Concordance mechanisms. It is in such contexts that our paper will be most valuable and we suspect such planners

will begin with SC and only consider APC, FPC and BNC once more careful research has been done on their effectiveness and practicality.

## V Public Goods and Collaboration

Despite our focus above on holdout, there is an intimate connection between this and the problems of collaboration and public goods. Modifications of our mechanisms above can thus be applied to those problems.

### V.A Binary public goods

Binary public goods<sup>33</sup>, non-rivalrous and non-excludable all-or-nothing public projects, with quasi-linear preferences are essentially equivalent to land assembly, except that the cost of building them is typically assumed to be known publicly while the buyer's valuation above is the private information of the buyer. It is easy to see that from the perspective of the mechanisms, except in efficiency calculations, this is inessential and in fact many public projects are privately supplied at privately-known cost. In all other respects the correspondence is exact. Property rights become voluntary participation, perfect complementarity non-excludability and single-mindedness non-rivalry. True shares are Lindahl prices and actual (approximate) shares are closely connected to the *pseudo-Lindahl* prices of Bergstrom (1979b), a public authority's closest approximation to Lindahl based on public information. Thus our mechanism's guarantee of approximate property rights implies a corresponding binary public goods mechanism that implements generally Bergstrom's (efficient) pseudo-Lindahl equilibrium. As far as we know ours is the first mechanism that does so in general.

While strong moral and some economic intuitions point towards approximate property rights as an appealing concept for the holdout problem, the justification of approximate non-coercion for public goods games is less obvious. However a long tradition dating back to Lindahl has emphasized the fairness of this approach and thus we consider our mechanism something of a contribution to the theory of public goods. We suspect the reason we were able to make this contribution is that we took as our starting point the less general problem of binary, quasi-linear public goods games and exploited the special structure they imply.

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<sup>33</sup>One interesting problem of recent interest with the public goods problem arises in the context of auctions where a collection of lots is valuable to one bidder only in its entirety and to a number of sellers each only for one piece of the lot. Solving this public goods problem, as with the Concordance mechanism, through coercion seems difficult given the difficulty of identifying who should be coerced to participate. Nonetheless the possibility of adding some degree of coercion (payments by losing bidders) to package auctions to solve complementarities is an interesting potential direction for future research.



### Straightforward Collaboration

1. Collaborators agree on a demand  $Q$  and are assigned shares  $\{s_i\}_{i=1}^N$ .
2. Each seller  $i$  submits a cost  $c_i$ .
3.  $P^*(C)$  is charged to consumers and sellers receive their share of revenue.
4. Each seller pays the Pigouvian tax

$$(1 - s_i) ([P^*(C_i) - C_i] Q[P^*(C_i)] - [P^*(C) - C_i] Q[P^*(C)])$$

5. Each receives a refund analogous to that in SC above.

As we will see some of the insights gained from this context can be directly generalized, but that the connection to the fully general setting is not immediately apparent.

## V.B Collaboration

Collaboration inherently involves sophisticated sellers and diffuse consumers, so it calls either for a regulator or a mechanisms that forces buyers to be price taker. This requires retooling the concordance mechanisms to put sellers in charge of bargaining. Thus we need the market demand function  $Q$  to be common knowledge among sellers and the social planner.<sup>34</sup>

Consider Cournot's model with unknown, idiosyncratic (constant marginal) costs for each firm. Firms charge or are assigned a price  $p_i$  and may also pay a tax to the market organizer. Their payoff is  $(p_i - c_i) Q\left(\sum_{j=1}^N p_j\right) - t_i$  where  $t_i$  is any tax they pay.

Suppose that, rather than firms choosing their price, they are assigned a price that is their share  $s_i$  of a total collective price  $P$ . Each collaborator  $i$ 's payoff is then  $(s_i P - c_i) Q(P)$ . If we let  $C \equiv \sum_{j=1}^N c_j$ , then clearly a cartel planner trying to maximize industry-wide profits *who know all costs* would set  $P$  like a monopolist with cost  $C$  facing demand  $Q$  according to the classic equation  $MR = C$ ; we assume marginal revenue is strictly declining in quantity so this equation implies a unique price  $P^*$ . However each firm's profits are proportional to

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<sup>34</sup>While this is a common assumption in industrial organization, it poses a substantial problem for the practical implementation of the mechanism. The mechanism explicitly uses this commonly known demand function to construct the payoffs of each agent; thus if the demand is not explicitly commonly known it would be hard to implement the mechanism. Of course this practical difficulty could be overcome by randomizing prices, or using structural econometrics (Baker and Bresnahan, 1988; Berry et al., 1995), to measure industry demand to all parties' satisfaction before implementing the mechanism.

those of a linear cost monopolist with cost  $\frac{c_i}{s_i}$ . Thus if  $s_i$  is larger (smaller) than  $\frac{c_i}{C}$ , firm  $i$  will prefer a higher (lower) price  $P$  than is in the interests of the cartel, just as a seller assigned too small of a share in the standard holdout problem above will tend to want a higher reserve price than is in the community's interest. Thus if the cartel must assess a tax on the individual collaborator for any harm she causes other members of the cartel. This suggests the mechanism shown in the box above, with  $C_i \equiv \frac{C-c_i}{1-s_i}$ .

As shown in Appendix E, the *Straightforward Collaboration* mechanism has properties analogous to the attractive features of SC except that it *does not* have the attractive asymptotic properties. The collaboration mechanism *because it gives bargaining power to the sellers* eliminates the benefits that arise from the asymptotic elimination of aggregate uncertainty about  $C$ ; the monopoly allocation is instead implemented for every  $N$ . A regulatory mechanism would be needed for such asymptotic efficiency.

Other mechanisms described above can be adapted to this context in a manner described in Appendix E. The  $X$ -plurality rules become like those of Bergstrom (1979b), where the  $X$ -th empirical quantile of the reported, share-weighted  $\frac{c_i}{s_i}$  distribution is used to determine  $C$ . BNC mirrors the extension of SC. APC and FPC respectively use the taxes

$$\left( s_i P^* \left[ \frac{c_i}{s_i} \right] - c_i \right) Q \left( P^* \left[ \frac{c_i}{s_i} \right] \right) \text{ and} \\ [s_i P^* (C) - c_i] Q [P^* (C)] - [s_i P^* (C_i) - c_i] Q [P^* (C_i)].$$

## V.C General public goods

Applying collaboration mechanisms to continuous public goods problems is simple if taxpayer preferences are quasi-linear and known up to a term affine shift  $v_i f(E)$  where  $v_i$  is the idiosyncratic valuation,  $f$  is a smooth concave function and  $E$  is the total expenditure on the public good. Of course this is still a very specific context for public goods as it disallows income effects and heterogeneity in the curvature of valuation of the good.

A more general direction for extending the concordance principle to quasi-linear but general valuation functions would be to define no influence as submitting  $v(e) = s_i e$ . Influence potentially subject to taxation would then be defined relative to that benchmark. This is an interesting direction for future research and would allow the extension of collaboration mechanisms to, for example, general cost functions. However we do not emphasize such a class of mechanisms further as the relationship between the holdout problem and such general continuous public goods games is far less direct than to the constant marginal cost collaboration setting and its public goods analog.

## VI Conclusion

This paper makes two contributions. First, we bring the holdout problem to the attention of the market design community both by emphasizing its importance and the lack of a satisfying solution thus far, and by bringing together disparate strands of literature and systematizing our knowledge of the holdout problem. Second, we introduce a potential framework for solving the holdout problem, which maintains a semblance of property rights while achieving substantial efficiency and can be implemented in a number of ways corresponding to classical auction solutions. To our knowledge, our paper is the first to propose a novel market design for the holdout problem. Hence, we expect—and in fact hope—that our work will not provide the final answer to the problem nor the most definitive comparison among the implementations we propose. This provides many interesting directions for future research.

Our analysis could naturally be extended in a number of ways. More analytical, computational and experimental work is needed to understand the behavior of the FPC and APC mechanisms, as well as the incentives for and impact of collusion in various mechanisms discussed above. More thought should be given to precisely implementing BNC. It would be useful to understand better the efficiency-optimal choice of  $X$  for  $X$ -plurality mechanisms, particularly how this varies with community size and distributions. Understanding more fully the reasons, philosophical, legal and economic (protecting investments), why property rights protections are desirable properties of a holdout mechanism would help clarify what compromises on these rights are reasonable. Extensions of the mechanisms and related property rights guarantees to broader public goods games is an interesting theoretical design problem. Measuring more carefully than our embarrassingly casual back-of-the-envelope guess the real-world losses of social value due to holdout problems is an important empirical question; we are aware of only two papers (Colwell and Munneke, 1999; Murray and Stern, 2007) that attempt to quantify these losses.<sup>35</sup>

Fine-tuning the Concordance mechanisms and their explanation to sellers will require experimental research. A field implementation of the system will be a crucial test of concept.

Many relatively minor extensions of our mechanism could expand their range of applicability. Concordance mechanisms place full property rights into community hands, but it would be simple, and natural in many eminent domain contexts, to place property rights partially into the buyer’s hands; it is well-known (Segal and Whinston, Forthcoming) that this helps mitigate the residual Cournot-Myerson-Satterthwaite distortion. Our Concordance mechanisms all require sellers to make tax payments, which are partially refunded, for enforcing on the community their preferences about sales. In the real world, sellers often

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<sup>35</sup>Unfortunately, both are limited to small and specific applications.

face budget constraints that may make this feature unattractive. Designs (similar to those of Pai and Vohra (2009) for auctions) that come close to preserving the attractive properties of Concordance mechanisms while accommodating bidders with privately known budget constraints would be a challenging but practically important extension of our work.

Three broader and more-ambitious extensions suggest themselves. First, other auction enforcement procedures than Bayes-Nash, All-pay, and First-price might be combined with the Concordance principle. Second, other mechanisms we have envisioned, ones violating the Concordance principle, might still have some attractive property rights properties and least in certain cases, as the  $X$ -plurality class suggests. Finally, and likely most interesting: we restricted our attention to a case of perfect complements and assumed no competition between aggregate land plots. In many practical settings a contiguous block of land must be assembled, but several such blocks may compete to host such a project; some of the competing collections may even overlap. This problem collaboration nested within in competition raises a number of interesting questions. Both in Cournot’s collaboration-competition model and in mechanism design, how fast must competition grow relative to collaboration for efficiency to improve (or worsen) with size? What are natural mechanisms for a setting of holdout combined with competitive procurement? More generally: how does the Concordance principle extend to imperfect complements?

## References

- Abdulkadiroğlu, Atila and Tayfun Sönmez**, “Random Serial Dictatorship and the Core from Random Endowments in House Allocation Problems,” *Econometrica*, 1998, 66 (3), 689–701.
- Armour, John and David Skeel**, “Who Writes the Rules for Hostile Takeovers, and Why? The Particular Divergence of U.S. and U.K. Takeover Regulation,” *Georgetown Law Journal*, 2007, 95 (6), 1727–1794.
- Armstrong, Mark**, “Price Discrimination by a Many-Product Firm,” *Review of Economic Studies*, 1999, 66 (1), 151–168.
- Arrow, Kenneth J.**, “The Property Rights Doctrine and Demand Revelation under Incomplete Information,” in Michael Boskin, ed., *Economics and Human Welfare*, New York: Academic Press, 1979, pp. 23–39.

- Ausubel, Lawrence M. and Paul Milgrom**, “The Lovely but Lonely Vickery Auction,” in Peter Cramton, Richard Steinberg, and Yoav Shoham, eds., *Combinatorial Auctions*, Cambridge, MA: MIT Press, 2005.
- Bagnoli, Mark and Barton L. Lipman**, “Successful Takeovers without Exclusion,” *Review of Financial Studies*, 1988, *1* (1), 89–110.
- and —, “Provision of Public Goods: Fully Implementing the Core through Private Contributions,” *Review of Economic Studies*, 1989, *56* (4), 583–601.
- Bailey, Martin J.**, “Lindahl Mechanisms and Free Riders,” *Public Choice*, 1994, *80* (1/2), 35–39.
- Bajari, Patrick and Ali Hortaçsu**, “Are Structural Estimates of Auction Models Reasonable? Evidence from Experimental Data,” *Journal of Political Economy*, 2005, *113* (4), 703–741.
- Baker, Jonathan B. and Timothy F. Bresnahan**, “Estimating the Residual Demand Curve Facing a Single Firm,” *International Journal of Industrial Organization*, 1988, *6* (3), 283–300.
- Baron, David P. and Roger B. Myerson**, “Regulating a Monopolist with Unknown Costs,” *Econometrica*, 1982, *50* (4), 911–930.
- Bell, Abraham and Gideon Parchomovsky**, “Taking Compensation Private,” *Stanford Law Review*, 2007, *59*, 871–906.
- Bergstrom, Ted**, “Cournot Equilibrium in Factor Markets,” 1978. [http://works.bepress.com/ted.bergstrom/subject\\_areas.html#Economic Theory](http://works.bepress.com/ted.bergstrom/subject_areas.html#Economic Theory).
- , “Do Governments Spend Too Much,” *National Tax Journal*, 1979, *32* (2), 81–86.
- , “When Does Majority Rule Supply Public Goods Efficiently?,” *Scandinavian Journal of Economics*, 1979, *81* (2), 216–226.
- Berliner, Dana**, *Opening the Floodgates: Eminent Domain Abuse in the Post-Kelo World*, Castle Coalition, 2006.
- Berry, Steven T., James Levinsohn, and Ariel Pakes**, “Automobile Prices in Market Equilibrium,” *Econometrica*, 1995, *63* (4), 841–890.

- Bertrand, Joseph Louis François**, “Review of “Théorie Mathématique de la Richesse Sociale,” and “Recherches sur les Principes Mathématiques de la Théorie des Richesses,”” *Journal des Savants*, 1883, 48, 499–508.
- Buchanan, James M. and Yong J. Yoon**, “Symmetric Tragedies: Commons and Anti-commons,” *Journal of Law and Economics*, 2000, 43 (1), 1–13.
- Budish, Eric**, “The Combinatorial Assignment Problem: Approximate Competitive Equilibrium from Equal Incomes,” 2008. <http://faculty.chicagobooth.edu/eric.budish/index.html>.
- Bulow, Jeremy I. and Paul D. Klemperer**, “Auctions Versus Negotiations,” *American Economic Review*, 1996, 86 (1), 180–194.
- Castle Coalition**, “Legislative Center,” 2009.
- Cavallo, Ruggiero**, “Optimal Decision-Making with Minimal Waste: Strategyproof Redistribution of VCG Payments,” in “Proceedings of the fifth international joint conference on Autonomous agents and multiagent systems,” Vol. 5 2006, pp. 882–889.
- Chauvin, Kyle, David Laibson, and Johanna Mollerstrom**, “Asset Bubbles and the Cost of Economic Fluctuations,” 2009. [http://www.szgerzensee.ch/uploads/media/jmcb\\_laibson.pdf](http://www.szgerzensee.ch/uploads/media/jmcb_laibson.pdf).
- Chupka, Marc W., Robert Earle, Peter Fox-Penner, and Ryan Hledik**, “Transforming America’s Power Industry: The Investment Challenge 2010-2030,” Technical Report, Edison Foundation, [http://www.eei.org/ourissues/finance/Documents/Transforming\\_Americas\\_Power\\_Industry.pdf](http://www.eei.org/ourissues/finance/Documents/Transforming_Americas_Power_Industry.pdf) 2008.
- Clarke, Edward H.**, “Multipart Pricing of Public Goods,” *Public Choice*, 1971, 11 (1), 17–33.
- Colwell, Peter F. and Henry J. Munneke**, “Land Prices and Land Assembly in the CBD,” *Journal of Real Estate Finance and Economics*, 1999, 18 (2), 163–180.
- Cournot, A. Augustin**, *Recherches sur les Principes Mathématiques de la Théorie des Richesses*, Paris, 1838.
- , in Nathaniel T. Bacon and Irving Fisher, eds., *Researches into the Mathematical Principles of the Theory of Wealth*, New York, NY: MacMillan, 1897.

- Cremer, Jacques and Michael H. Riordan**, “A Sequential Solution to the Public Goods Problem,” *Econometrica*, 1985, *53* (1), 77–84.
- Cripps, Martin W. and Jeroen M. Swinkels**, “Efficiency of Large Double Auctions,” *Econometrica*, 2006, *74* (1), 47–92.
- Croft, Elizabeth and Hank Donker**, “Why Bidders Like Creeping Takeovers,” *Journal of Corporate Accounting and Finance*, 2006, *17* (2), 45–51.
- d’Aspremont, Claude and Louis-André Gérard-Varet**, “Incentives and Incomplete Information,” *Journal of Public Economics*, 1979, *11* (1), 25–45.
- Day, Robert and Paul Milgrom**, “Core-Selecting Package Auctions,” *International Journal of Game Theory*, 2008, *36* (3–4), 393–407.
- Dupuit, Arsène Jules Étienne Juvénal**, *De la Mesure de L’utilité des Travaux Publics*, Paris, 1844.
- Ellickson, Robert C.**, “Alternatives to Zoning: Covenants, Nuisance Rules, and Fines as Land Use Controls,” *University of Chicago Law Review*, 1973, *40* (4), 683–781.
- Erdil, Aytek and Paul Klemperer**, “A New Payment Rule for Core-Selecting Package Auctions,” *Journal of European Economic Association*, Forthcoming.
- Fennell, Lee Anne**, “Taking Eminent Domain Apart,” *Michigan State Law Review*, 2004, *957–1004* (4).
- FERC**, “A Guide to the FERC Electric Transmission Facilities Permit Process,” Technical Report, Federal Energy Regulator Comission, <http://www.ferc.gov/for-citizens/citizen-guides/electric/guide-transmission-8-08.pdf> 2005.
- Frank, Charles R.**, “Entry in a Cournot Market,” *Review of Economic Studies*, 1965, *32* (3), 245–250.
- Gibbard, Alan**, “Manipulation of Voting Schemes: A General Result,” *Econometrica*, 1973, *41* (4), 587–602.
- Green, Jerry and Jean-Jacques Laffont**, “Characterization of Satisfactory Mechanisms for the Revelation of Preferences for Public Goods,” *Econometrica*, 1977, *45* (2), 427–438.
- Grossman, Sanford J. and Oliver D. Hart**, “Takeover Bids, the Free-Rider Problem, and the Theory of the Corporation,” *Bell Journal of Economics*, 1980, *11* (1), 42–64.

- Groves, Theodore**, “Incentives in Teams,” *Econometrica*, 1973, *41* (4), 617–631.
- **and John Ledyard**, “Optimal Allocation of Public Goods: A Solution to the “Free Rider” Problem,” *Econometrica*, 1977, *45* (4), 783–809.
- **and Martin Loeb**, “Incentives in a Divisionalized Firm,” *Management Science*, 1979, *25* (3), 221–230.
- Hardin, Garrett**, “The Tragedy of the Commons,” *Science*, 1968, *162* (3859), 1243–1248.
- Hazlett, Thomas W.**, “Spectrum Tragedies,” *Yale Journal on Regulation*, 2005, *22* (2), 242–274.
- Heller, Michael**, *The Gridlock Economy: How Too Much Ownership Wrecks Markets, Stops Innovation and Cost Lives*, Philadelphia, PA: Basic Books, 2008.
- Heller, Michael A. and Rebecca S. Eisenberg**, “Can Patents Deter Innovation? The Anticommons in Biomedical Research,” *Science*, 1998, *280* (5394), 698–701.
- **and Rick Hills**, “Land Assembly Districts,” *Harvard Law Review*, 2008, *121* (6), 1467–1527.
- Hoffman, Philip T.**, “Institutions and Agriculture in Old Regime France,” *Politics and Society*, 1988, *16* (2–3), 241–264.
- Hurwicz, Leonid**, “The Design of Mechanisms for Resource Allocation,” *American Economic Association Papers & Proceedings*, 1973, *63* (2), 1–30.
- , “On Informationally Decentralized Systems,” in Kenneth J. Arrow and Leonid Hurwicz, eds., *Studies in Resource Allocation Processes*, Cambridge, UK: Cambridge University Press, 1977, pp. 425–459.
- Kelly, Daniel B.**, “The Public Use Requirement in Eminent Domain Law: A Rationale Based on Secret Purchases and Private Influence,” *Cornell Law Review*, 2006, *92* (1), 1–65.
- Kelo v. City of New London, Connecticut*
- Kelo v. City of New London, Connecticut***, (04–108), 2005, 545 U.S. (469).
- Kirchmaier, Tom, Jeremy Grant, and Jodie Krishner**, “Financial Tunnelling and the Mandatory Bid Rule,” 2009. [http://papers.ssrn.com/sol3/papers.cfm?abstract\\_id=613945](http://papers.ssrn.com/sol3/papers.cfm?abstract_id=613945).



- Klemperer, Paul D.**, “*Auction Theory: a Guide to the Literature*,” in Paul D. Klemperer, ed., *The Economic Theory of Auctions*, Cheltenham, UK: Edward Elgar, 2000, chapter Introduction.
- , “What Really Matters in Auction Design,” *Journal of Economic Perspectives*, 2002, 16 (1), 169–189.
- Kurz, Mordecai**, “*Experimental Approach to the Determination of the Demand for Public Goods*,” *Journal of Public Economics*, 1974, 3 (4), 329–348.
- La Porta, Rafael, Florencio Lopez de Silanes, Andrei Shleifer, and Robert W. Vishny**, “*Law and Finance*,” *Journal of Political Economy*, 1998, 106 (6), 1113–1155.
- Le Grand Robert de la Langue Française**, “*Concours*,” 2009. <http://gr.bvdep.com/>.
- Lee, Hyun Chul**, “*Efficient and Inefficient Debt Restructuring: A Comparative Analysis of Voting Rules in Workouts*,” *Cornell International Law Journal*, 2007, 40 (3), 661–690.
- Lehavi, Amnon and Amir N. Licht**, “*Eminent Domain, Inc.*,” *Columbia Law Review*, 2007, 107 (7), 1704–1748.
- Lerner, Josh and Jean Tirole**, “*Efficient Patent Pools*,” *American Economic Review*, 2004, 94 (3), 691–711.
- Lindahl, Erik**, “*Positive Lösung*,” in “*Die gerechtigkeit der Besteuerung*” number I, Lund, 1919, chapter 4, pp. 85–98.
- Lucas, Robert E.**, “*Macroeconomic Priorities*,” *American Economic Review*, 2003, 93 (1), 1–14.
- Mailath, George J. and Andrew Postelwaite**, “*Asymmetric Information Bargaining Problems with Many Agents*,” *Review of Economic Studies*, 1990, 57 (3), 351–367.
- Maskin, Eric**, “*Nash Equilibrium and Welfare Optimality*,” *Review of Economic Studies*, 1999, 66 (1), 23–38.
- Menezes, Flavio and Rohan Pitchford**, “*A Model of Seller Holdout*,” *Economic Theory*, 2004, 24 (2), 231–253.
- Merrill, Thomas W.**, “*The Economics of Public Use*,” *Cornell Law Review*, 1986, 72 (1), 61–116.

- Miceli, Thomas J. and Kathleen Segerson**, “A Bargaining Model of Holdouts and Takings,” *American Law and Economics Review*, 2007, 9 (1), 1–30.
- Michelman, Frank I.**, “Property, Utility, and Fairness: Comments on the Ethical Foundations of Just Compensation Law,” *Harvard Law Review*, 1967, 80 (6), 1165–1258.
- Milgrom, Paul**, *Putting Auction Theory to Work*, Cambridge, UK: Cambridge University Press, 2004.
- , “Package Auctions and Exchanges,” *Econometrica*, 2007, 75 (4), 935–965.
- Minerbi, Luciano**, *Land Readjustment, the Japanese System*, Cambridge, MA: Oelgeschlager, Gunn & Hain in association with the Lincoln Institute of Land Policy, 1986.
- Morton, Justin R.**, “Post-Kelo Legislation on Eminent Domain Impacts Valuations,” *Appraisal Journal*, 2006, 76 (4), 383–385.
- Murray, Fiona and Scott Stern**, “Do Formal Intellectual Property Rights Hinder the Free Flow of Scientific Knowledge? An Empirical Test of the Anti-Commons Hypothesis,” *Journal of Economic Behavior & Organization*, 2007, 63 (4), 648–687.
- Myerson, Roger B.**, “Optimal Auction Design,” *Mathematics of Operations Research*, 1981, 6 (1), 58–73.
- and **Mark A. Satterthwaite**, “Efficient Mechanisms for Bilateral Trading,” *Journal of Economic Theory*, 1981, 29 (2), 265–281.
- Olson, Mancur**, *The Logic of Collective Action*, Cambridge, MA: Harvard University Press, 1965.
- Pai, Mallesh M. and Rakesh Vohra**, “Optimal Auctions with Financially Constrained Bidders,” 2009. <http://www.kellogg.northwestern.edu/faculty/pai/research.htm>.
- Plassmann, Florenz and T. Nicolaus Tideman**, “Efficient and Fair Land Assembly,” 2009. <http://bingweb.binghamton.edu/~fplass/papers/LandAssembly.pdf>.
- Posner, Richard A.**, “Foreword: A Political Court,” *Harvard Law Review*, 2005, 119 (1), 31–102.
- Radin, Margaret Jane**, “Property and Personhood,” *Stanford Law Review*, 1982, 34 (5), 957–1015.

- Rob, Rafael**, “Pollution Claim Settlements under Private Information,” *Journal of Economic Theory*, 1989, 47 (2), 307–333.
- Roth, Alvin E.**, “The Economist as Engineer: Game Theory, Experimentation, and Computation as Tools for Design Economics,” *Econometrica*, 2002, 70 (4), 1341–1378.
- Rustichini, Aldo, Mark A. Satterthwaite, and Steven R. Williams**, “Convergence to Efficiency in a Simple Market with Incomplete Information,” *Econometrica*, 1994, 62 (5), 1041–1063.
- Samuelson, Paul A.**, “The Pure Theory of Public Expenditure,” *Review of Economics and Statistics*, 1954, 36 (4), 387–389.
- Satterthwaite, Mark A. and Steven R. Williams**, “The Rate of Convergence to Efficiency in the Buyer’s Bid Double Auction as the Market Becomes Large,” *Review of Economic Studies*, 1989, 56 (4), 477–498.
- Schmidt, Ronald H. and William C. Gruben**, “Ejido Reform and the NAFTA,” *FRBSF Weekly Letter*, 1992, 92 (34).
- Segal, Ilya and Michael Whinston**, “An Expected-Efficient Status Quo Allows Efficient Bargaining,” *Theoretical Economics*, Forthcoming.
- Shapiro, Perry and Jonathan Pincus**, “The L2H2 Auction: Efficiency and Equity in the Assemblage of Land for Public Use,” 2007. [http://www.economics.adelaide.edu.au/research/papers/doc/Efficiency\\_Equity\\_Rev5b.pdf](http://www.economics.adelaide.edu.au/research/papers/doc/Efficiency_Equity_Rev5b.pdf).
- Shavell, Steven**, “Eminent Domain versus Government Purchase of Land Given Imperfect Information about Owner’s Valuations,” 2007. [http://lsr.nellco.org/cgi/viewcontent.cgi?article=1386&context=harvard\\_olin](http://lsr.nellco.org/cgi/viewcontent.cgi?article=1386&context=harvard_olin).
- Shimizu, Kaho**, “Narita to Extend Runway, but Which Way?,” *Japan Times*, 2005, April 20.
- Shleifer, Andrei**, “Do Demand Curves for Stocks Slope Down,” *Journal of Finance Papers and Proceedings*, 1985, 41 (3), 579–590.
- Shoup, Donald**, “Graduated Density Zoning,” *Journal of Planning Education and Research*, 2008, 28 (2), 161–179.
- Sonnenschein, Hugo**, “The Dual of Duopoly is Complimentary Monopoly: or, Two of Cournot’s Theories are One,” *Journal of Political Economy*, 1968, 76 (2), 316–318.

- Sorensen, Andre**, “Land Readjustment, Urban Planning and Urban Sprawl in the Tokyo Metropolitan Area,” *Urban Studies*, 1999, 36 (13), 2333–2360.
- Spengler, Joseph J.**, “Vertical Integration and Antitrust Policy,” *Journal of Political Economy*, 1950, 50 (4), 347–352.
- Tian, Guoqiang**, “Implementation of the Lindahl Correspondence by a Single-Valued, Feasible, and Continuous Mechanism,” *Review of Economic Studies*, 1989, 56 (4), 613–621.
- Tirole, Jean**, *Theory of Industrial Organization*, Cambridge, MA: MIT Press, 1988.
- Twaronite, Lisa**, “Corporate Investment-Grade Bond Volume Surges,” *MarketWatch*, 2009, <http://www.marketwatch.com/story/corporate-investment-grade-bond-volume-surges?pagenumber=2>.
- U.S. Department of Justice and the Federal Trade Comission**, “Horizontal Merger Guidelines,” [http://www.usdoj.gov/atr/public/guidelines/horiz\\_book/hmg1.html](http://www.usdoj.gov/atr/public/guidelines/horiz_book/hmg1.html) 1992.
- Vickrey, William**, “Counterspeculation, Auctions and Competitive Sealed Tenders,” *Journal of Finance*, 1961, 16 (1), 8–37.
- Walker, Mark**, “A Simple Incentive Compatible Scheme for Attaining Lindahl Allocations,” *Econometrica*, 1981, 49 (1), 65–71.
- West, Mark D. and Emily M. Morris**, “The Tragedy of the Condominiums: Legal Responses to Collective Action Problems After the Kobe Earthquake,” *American Journal of Comparative Law*, 2003, 51 (4), 903–940.
- Williams, Steven R.**, “A Characterization of Efficient, Bayesian Incentive Compatible Mechanisms,” *Economic Theory*, 1999, 14 (1), 155–180.
- Wilson, Robert**, “Game Theoretic Analysis of Trading Processes,” in Truman F. Bewley, ed., *Advances in Economic Theory*, Cambridge, UK: Cabmridge University Press, 1987.
- Wolf, Michael Allan**, “Private Property, Community Development, and Eminent Domain,” in Robin Paul Malloy, ed., *Private Property, Community Development, and Eminent Domain*, Burlington, VT: Ashgate, 2008.

# Appendix

## A The Concordance Principle

In this section, we prove the two parts of Theorem 1.

### 1 Bilateral Efficiency

The outcome of any Concordance mechanism  $\mathcal{M} \in \mathfrak{C}$  is exactly identical to that of its bilateral form  $\mathcal{M}'$  with the distribution of the seller's value being that of  $V$ . This follows directly from the fact that the truthfulness is suggested for all sellers in  $\mathcal{M}$  and for the single seller in  $\mathcal{M}'$ , while in both  $\mathcal{M}$  and  $\mathcal{M}'$  the buyer's suggested offer is the monopsonist optimal price facing the distribution of  $V$ , proving the first half of the theorem.

### 2 Asymptotic Efficiency

We let  $\{x_i^n\}$  represent the i.i.d. process in the theorem statement, and write  $\mu$  and  $\sigma^2$  for the (strictly positive) mean and (strictly positive) variance of this process, respectively. As

$$V^n = \sum_{i=1}^n v_i^n = \sum_{i=1}^n x_i^n s_i^n$$

and  $\sum_{i=1}^n s_i^n = 1$ , we have

$$E[V^n] = \mu, \quad \text{Var}[V^n] < \frac{M^2 \sigma^2}{n}$$

by the share bound and i.i.d. hypotheses.

To show the result, it suffices to demonstrate that the total inefficiency of  $\mathcal{M}^n$  vanishes as  $n \rightarrow \infty$ .<sup>36</sup>

The buyer is instructed to make the monopsonist's optimal offer against supply function

$$\overline{G}_n(o) \equiv \text{Prob}[o \geq V^n],$$

or equivalently against inverse supply  $S_n(q) \equiv \overline{G}_n^{-1}(q)$ . That is, the buyer maximizes

$$q(b - S_n(q)) \tag{1}$$

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<sup>36</sup>This is sufficient because the gains from trade are bounded away from zero.

over  $q$ . We let  $\tilde{q}_n(b)$  be the optimal choice of  $q$  in (1), for a buyer with value  $b$ .

We first examine the case in which  $b > \mu$ .<sup>37</sup> We show that as  $n$  becomes large, a buyer with value  $b$  chooses a probability of sale approaching 1.

**Lemma 1.** *For any fixed  $b > \mu$ , we have  $\tilde{q}_n(b) \rightarrow 1$  as  $n \rightarrow \infty$ .*

*Proof.* By the one-sided Chebyshev inequality, we have for any  $\alpha > 0$

$$\text{Prob}[V^n - \mu \geq \alpha] \leq \frac{M^2 \sigma^2}{M^2 \sigma^2 + n \alpha^2},$$

which vanishes as  $n \rightarrow \infty$ . It follows immediately that  $S_n(q) \rightarrow \mu$  as  $n \rightarrow \infty$  (pointwise) for any  $q$ . Thus, for any fixed  $\epsilon > 0$ , we have

$$(q + \epsilon)(b - S_n(q + \epsilon)) > q(b - S_n(q))$$

for  $n$  sufficiently large (since  $(S_n(q + \epsilon) - S_n(q)) \rightarrow 0$ ), hence we have  $\tilde{q}_n(b) \rightarrow 1$  as  $n \rightarrow \infty$ , for any given  $b$ .

Note that because  $b \geq o$  and a sale takes place only when  $o \geq V$ , inefficient sales will never take place. Thus, the total inefficiency of  $\mathcal{M}^n$  in cases when  $b > \mu$  is bounded above by the potential surplus created by failed efficient sales:

$$\int_{\mu}^{\infty} (1 - \tilde{q}_n(b))(b - \mu)h(b) db, \quad (2)$$

where  $h(b)$  is the probability density function of  $b$ . In order for the efficiency of  $\mathcal{M}^n$  to be well-defined (which we assume),  $b$  must have a finite upper-tail integral. We therefore have

$$\int_{\bar{b}}^{\infty} (1 - \tilde{q}_n(b))(b - \mu)h(b) db \leq \int_{\bar{b}}^{\infty} (b - \mu)h(b) db < \infty$$

for any  $\bar{b} > \mu$ . Thus, to bound (2), it suffices to show that

$$\int_{\mu}^{\bar{b}} (1 - \tilde{q}_n(b))(b - \mu)h(b) db \leq (\bar{b} - \mu) \int_{\mu}^{\bar{b}} (1 - \tilde{q}_n(b))h(b) db \quad (3)$$

vanishes as  $n \rightarrow \infty$ , for any fixed  $\bar{b} > \mu$ .<sup>38</sup> We now show this fact.

<sup>37</sup>The case in which  $b < \mu$  is simpler, as we discuss below.

<sup>38</sup>If (3) vanishes for any  $\bar{b} > \mu$ , then we must have

$$\int_{\mu}^{\infty} (1 - \tilde{q}_n(b))(b - \mu)h(b) db = \int_{\mu}^{\bar{b}} (1 - \tilde{q}_n(b))(b - \mu)h(b) db + \int_{\bar{b}}^{\infty} (1 - \tilde{q}_n(b))(b - \mu)h(b) db \rightarrow 0$$

**Lemma 2.** *The integral*

$$\int_{\mu}^{\bar{b}} (1 - \tilde{q}_n(b))h(b) db \quad (4)$$

*vanishes as  $n \rightarrow \infty$ .*

*Proof.* As the integrand of (3) lies in  $[0, 1]$  for all  $n$  and  $b$ , we may take the limit of (4) as  $n \rightarrow \infty$  and apply the dominated convergence theorem. The claim then follows since  $\tilde{q}_n(b) \rightarrow 1$  (pointwise) as  $n \rightarrow \infty$  by Lemma 1.

Lemma 2 shows that the right side of (3) vanishes as  $n \rightarrow \infty$ . We therefore see that, as  $n \rightarrow \infty$ ,  $\mathcal{M}^n$  is fully efficient for buyers with values  $b > \mu$ . As the buyer never offers more than  $b$ , it is quick to show that inefficiency of  $\mathcal{M}^n$  vanishes (as  $n \rightarrow \infty$ ) when  $b < \mu$ .<sup>39</sup> Thus, we have the desired result.

## B Groves Holdout Mechanisms

We now derive our main results for the SC mechanism, as special cases of more general results for the full class of Groves mechanisms for the holdout problem.

**Definition 12.** *A mechanism  $\mathcal{M} = \{\mathcal{B}, \mathcal{R}, P, \mathbf{T}\}$  with  $\mathcal{B} = \mathcal{R} = \mathbb{R}_{++}$  is a Groves Holdout (GH) mechanism if:*

- $P(o, \mathbf{r}) = 1_{o \geq R}$ ;
- $T_{\beta}(o, \mathbf{r}) = -oP(o, \mathbf{r})$ ;
- $T_i(o, \mathbf{r}) = s_i o P(o, \mathbf{r}) + \sum_{j \neq i} (s_j o P(o, \mathbf{r}) + r_j (1 - P(o, \mathbf{r}))) + h_i(o, \mathbf{r}_{-i})$ , where,  $h_i(o, \mathbf{r}_{-i})$  is a function independent of the reserve reported by seller  $i$ ;

The first condition ensures that transfer occurs if and only if the buyer's offer exceeds the collective reserve, while the second simply requires that the buyer pay her offer in the case of transfer. The third condition holds most of the content of the definition; it means that for all sellers  $i$ , the *Groves tax* rule  $\tau_i(o, \mathbf{r}) \equiv T_i(o, \mathbf{r}) - s_i o P(o, \mathbf{r})$  takes the form of the transfer rule of a classical Groves (1973) mechanism (citation year suppressed in the

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as  $\bar{b} \rightarrow \infty$ , whence we see that (2) vanishes as  $n \rightarrow \infty$ .

<sup>39</sup>As trade is always inefficient when  $b < \underline{V}$ , the inefficiency of  $\mathcal{M}^n$  in the case  $b < \mu$  is bounded by an integral with finite support  $[\underline{V}, \mu]$ . An argument directly analogous to that for the case  $b > \mu$  shows that the integrand (i.e. the inefficiency for fixed  $b < \mu$ ) vanishes pointwise, hence the inefficiency of  $\mathcal{M}^n$  vanishes globally by the dominated convergence theorem.

sequel).<sup>40</sup> Since a GH mechanism is completely specified by either  $h_i(o, \mathbf{r}_{-i})$  or  $\tau_i(o, \mathbf{r})$ , we use these two functions interchangeably according to convenience.

The suggested strategies for a GH mechanism are  $r^*(v) \equiv v$  and  $o^*(b) = \operatorname{argmax}_o (b - o)\overline{G}(o)$ .<sup>41</sup>

## 1 Basic Properties

It follows immediately from standard results on Groves mechanisms that  $r^*(v)$  is optimal, i.e. any GH mechanism is straightforward for sellers.<sup>42</sup> Indeed, it is quick to compute that in such a mechanism the *ex post* utility of seller  $i$  is given by

$$T_i(o, \mathbf{r}) + v_i(1 - P(o, \mathbf{r})) = oP(o, \mathbf{r}) + \left( v_i + \sum_{j \neq i} r_j \right) (1 - P(o, \mathbf{r})) + h_i(o, \mathbf{r}_{-i}).$$

It follows immediately that the seller  $i$  wants sale to happen if and only if  $o \geq v_i + \sum_{j \neq i} r_j$ , which is exactly the outcome implemented if seller  $i$  reports  $r_i = v_i$ .

Moreover, the suggested strategy of a GH mechanism is optimal for the buyer. If the sellers follow the strategy  $r^*$ , sellers will accept the buyer's offer whenever  $o \geq V$ . Thus the buyer's expected payoff in this case is  $(b - o)\overline{G}(o)$ , hence  $o^*$  is her best offer by construction.

The preceding observations are summarized in the following proposition.

**Proposition 5.** *Any GH mechanism is straightforward for sellers and implementable.*

## 2 Uniqueness of the Groves Holdout Class

We now prove that the Groves Holdout class is unique: any mechanism for the holdout problem which is straightforward for sellers and bilaterally efficient is isomorphic to a GH mechanism. We begin with a lemma which extends Theorem 3 of Green and Laffont (1977) to our setting.<sup>43</sup>

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<sup>40</sup>Note that when refunds are present, this “tax” rule incorporates both the tax and refund components of the mechanism, as we detail further below.

<sup>41</sup>Here, as in the main text,  $\overline{G}$  is the cumulative distribution function of  $V$ ; that is  $o^*(b)$  is the monoposonist optimal price facing an supply function  $\overline{G}(o)$ .

<sup>42</sup>Note that our normalization of seller utility differs slightly from that which would be expected in a standard Groves framework. We take the utility in case of no sale to be  $v_i$ . It would be more in line with the mechanism design literature—but substantially less intuitive—to treat the case of no sale as a baseline giving utility 0. Then, the utility in case of sale would be given by  $s_i o - v_i$ . To convert to this alternative normalization, it suffices to (additively) renormalize each function  $h_i(o, \mathbf{r}_{-i})$  by  $(1 - s_i)R_i$ .

<sup>43</sup>Even with the renormalization described in Footnote 42, Theorem 3 of Green and Laffont (1977) does not directly apply to our setting. The approach of Green and Laffont (1977) requires at least three outcomes, and in our setting there are only two: sale and no sale. Nonetheless, the basic approach of Green and Laffont (1977) generalizes directly to the argument we give in the proof of Lemma 3.



**Lemma 3.** *Any (direct revelation) mechanism for the holdout problem which*

- *induces a Pareto-optimal outcome amongst sellers (conditional upon share division and the buyer's offer  $o$ )<sup>44</sup> and*
- *has truthful reporting as a dominant strategy for each seller*

*is a Groves Holdout mechanism.*

*Proof.* It suffices to show that any mechanism satisfying the conditions of the lemma has the property that

$$\tau_i(o, r_i, \mathbf{r}_{-i}) - \tau_i(o, \hat{r}_i, \mathbf{r}_{-i}) = (P(o, r_i, \mathbf{r}_{-i}) - P(o, \hat{r}_i, \mathbf{r}_{-i})) \sum_{j \neq i} (s_j o - r_j). \quad (5)$$

If this property fails, then there exist  $r_i$ ,  $\hat{r}_i$ , and  $\mathbf{r}_{-i}$  such that

$$\tau_i(o, r_i, \mathbf{r}_{-i}) - \tau_i(o, \hat{r}_i, \mathbf{r}_{-i}) = (P(o, r_i, \mathbf{r}_{-i}) - P(o, \hat{r}_i, \mathbf{r}_{-i})) \sum_{j \neq i} (s_j o - r_j) + \epsilon$$

for some  $\epsilon \neq 0$ . Without loss of generality, we assume that  $\epsilon > 0$ . If  $P(o, r_i, \mathbf{r}_{-i}) = P(o, \hat{r}_i, \mathbf{r}_{-i})$ , then clearly truthful play is not a dominant strategy for seller  $i$  anticipating other sellers reporting truthfully, as  $i$  should announce reservation value  $r_i$  when  $v_i = \hat{r}_i$ . Thus, we may assume that  $P(o, r_i, \mathbf{r}_{-i}) \neq P(o, \hat{r}_i, \mathbf{r}_{-i})$ . By Pareto optimality, we have  $P(o, r_i, \mathbf{r}_{-i}), P(o, \hat{r}_i, \mathbf{r}_{-i}) \in \{0, 1\}$ . We assume that  $P(o, r_i, \mathbf{r}_{-i}) = 1$ ; the case  $P(o, r_i, \mathbf{r}_{-i}) = 0$  follows analogously.

Let  $\hat{r}'_i = s_i o - \sum_{j \neq i} (s_j o P(o, \hat{r}_i, \mathbf{r}_{-i}) + r_j (1 - P(o, \hat{r}_i, \mathbf{r}_{-i}))) - \delta$  for some small  $\delta > 0$ .<sup>45</sup> Then by Pareto optimality  $P(o, \hat{r}'_i, \mathbf{r}_{-i}) = 0 = P(o, \hat{r}_i, \mathbf{r}_{-i})$ ; it follows that  $\tau_i(o, \hat{r}'_i, \mathbf{r}_{-i}) =$

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<sup>44</sup>By a *Pareto-optimal outcome (conditional upon share division and the buyer's offer  $o$ )*, we mean a choice of  $P(o, \mathbf{r})$  such that  $\sum_{i=1}^N (s_i o P(o, \mathbf{r}) + r_j (1 - P(o, \mathbf{r})))$  is maximized.

<sup>45</sup>This  $\delta$  is only needed to prevent indifference in the sale decision.

$\tau_i(o, \hat{r}_i, \mathbf{r}_{-i})$ .<sup>46</sup> But this implies that

$$\begin{aligned}
& \tau_i(o, r_i, \mathbf{r}_{-i}) - \tau_i(o, \hat{r}'_i, \mathbf{r}_{-i}) = \tau_i(o, r_i, \mathbf{r}_{-i}) - \tau_i(o, \hat{r}_i, \mathbf{r}_{-i}) \\
& = (P(o, r_i, \mathbf{r}_{-i}) - P(o, \hat{r}_i, \mathbf{r}_{-i})) \sum_{j \neq i} (s_j o - r_j) + \epsilon \\
& = \left( \sum_{j \neq i} (s_j o P(o, r_i, \mathbf{r}_{-i}) + r_j (1 - P(o, r_i, \mathbf{r}_{-i}))) \right) - \left( \sum_{j \neq i} (s_j o P(o, \hat{r}_i, \mathbf{r}_{-i}) + r_j (1 - P(o, \hat{r}_i, \mathbf{r}_{-i}))) \right) + \epsilon \\
& = (1 - s_i) o + (\hat{r}'_i - s_i o + \delta) + \epsilon \\
& \geq \hat{r}'_i - s_i o + \epsilon.
\end{aligned}$$

But then, we have

$$\tau_i(o, r_i, \mathbf{r}_{-i}) + s_i o > \tau_i(o, \hat{r}'_i, \mathbf{r}_{-i}) + \hat{r}'_i,$$

from which it follows that truthful reporting is not a dominant strategy for  $i$  when  $i$  anticipates truthful reports from other sellers and  $v_i = \hat{r}'_i$ .<sup>47</sup>

Now, we can prove the uniqueness of the Groves Holdout class.

**Proposition 6.** *Let  $\mathcal{M}$  be a (direct revelation) mechanism for the holdout problem which*

1. *is straightforward for sellers,*
2. *induces a Pareto-optimal outcome amongst sellers (conditional upon share division and the buyer's offer  $o$ ),<sup>48</sup> and*
3. *is bilaterally efficient relative to a take-it-or-leave-it offer.*

*Then,  $\mathcal{M}$  is isomorphic to a GH mechanism  $\mathcal{M}'$ .*

*Proof.* We denote by  $r^*(v)$  the strategy recommended in mechanism  $\mathcal{M} = \{\mathbb{R}_{++}, \mathbb{R}_{++}, P, \mathbf{T}\}$  for a seller with value  $v$ . Now, we take  $\mathcal{M}'$  to be the mechanism

$$\mathcal{M}' = \{\mathbb{R}_{++}, \mathbb{R}_{++}, P(o, r^*(\mathbf{r})), \mathbf{T}(o, r^*(\mathbf{r}))\}.$$

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<sup>46</sup>If  $\tau_i(o, \hat{r}'_i, \mathbf{r}_{-i}) \neq \tau_i(o, \hat{r}_i, \mathbf{r}_{-i})$ , then we have found  $\hat{r}'_i$ ,  $\hat{r}_i$ , and  $\mathbf{r}_{-i}$  such that  $P(o, \hat{r}'_i, \mathbf{r}_{-i}) = P(o, \hat{r}_i, \mathbf{r}_{-i})$  and either  $\tau_i(o, \hat{r}'_i, \mathbf{r}_{-i}) > \tau_i(o, \hat{r}_i, \mathbf{r}_{-i})$  or  $\tau_i(o, \hat{r}'_i, \mathbf{r}_{-i}) < \tau_i(o, \hat{r}_i, \mathbf{r}_{-i})$ . But in either of these cases, truthful reporting is not always dominant. Indeed, in the first case,  $i$  should report  $\hat{r}'_i$  when  $v_i = \hat{r}_i$ ; in the second case  $i$  should report  $\hat{r}_i$  when  $v_i = \hat{r}'_i$ .

<sup>47</sup>When  $v_i = \hat{r}'_i$ , seller  $i$  should announce the reservation value  $r_i$  instead.

<sup>48</sup>By a Pareto-optimal outcome (conditional upon share division and the buyer's offer  $o$ ), we mean a choice of  $P(o, \mathbf{r})$  such that  $\sum_{i=1}^N (s_i o P(o, \mathbf{r}) + r_j (1 - P(o, \mathbf{r})))$  is maximized.

It follows quickly from the straightforwardness of  $\mathcal{M}$  that  $\mathcal{M}'$  has truthful reporting as a dominant strategy for each seller.<sup>49</sup> Additionally, since  $\mathcal{M}$  induces a Pareto-optimal outcome amongst sellers under the recommended strategies,  $\mathcal{M}'$  induces a Pareto-optimal outcome amongst sellers under truthful reporting. Then, it follows from Lemma 3 that  $\mathcal{M}'$  is a GH mechanism.

Finally, we observe that  $r^*$  must be univalent on the domain of values  $[\underline{v}, \bar{v}]$ . To see this, we suppose the contrary, i.e. that there are two valuation profiles  $\mathbf{v} \neq \mathbf{v}'$  such that  $r^*(\mathbf{v}) = r^*(\mathbf{v}')$ . Now, we fix the buyer's value at  $b$ , and observe that the (optimal) buyer offer  $o$  in  $\mathcal{M}$  submitted when seller values are given by  $\mathbf{v}$  must be the same as that submitted when seller values are  $\mathbf{v}'$ . But this contradicts the bilateral efficiency of  $\mathcal{M}$ , since  $P(o, r^*(\mathbf{v})) = P(o, r^*(\mathbf{v}'))$  but  $\mathbf{v} \neq \mathbf{v}'$ . Thus,  $r^*$  must be univalent.

From the univalence of  $r^*$ , we construct the inverse  $(r^*)^{-1}$ . Composing this function with the reserves  $\mathbf{r}$  submitted to  $\mathcal{M}$  returns the seller values  $\mathbf{v} = (r^*)^{-1}(\mathbf{r})$ . As  $\mathcal{M}$  is equivalent to  $\mathcal{M}'$  run on  $\mathbf{v} = (r^*)^{-1}(\mathbf{r})$ , we have demonstrated the desired isomorphism.

### 3 Optimal Tax Refunds

Closely following the approach of Cavallo (2006), we now present a method for redistribution of the taxes collected by a Groves Holdout mechanism  $\mathcal{M}$  with a (weakly) negative tax burden on sellers  $\tau_i(o, \mathbf{r}) \leq 0$ , and show that this redistribution method is optimal in a certain natural sense. We let

$$\sigma_i(o, \mathbf{r}_{-i}) \equiv \min_{\hat{r}_i} \sum_{j=1}^N \tau_j(o, \hat{r}_i, \mathbf{r}_{-i})$$

be the lower bound on tax surplus computed across all possible reports  $\hat{r}_i$  of seller  $i$ . We then set

$$\tau'_i(o, \mathbf{r}) \equiv \tau_i(o, \mathbf{r}) - s_i \sigma_i(o, \mathbf{r}_{-i}).$$

**Proposition 7.** *The GH mechanism  $\mathcal{M}'$  with Groves tax equal to  $\tau'_i(o, \mathbf{r})$  is self-financing.*

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<sup>49</sup>The proof of this statement exactly follows the first part of the proof of Theorem 5 of Green and Laffont (1977), hence we omit it.

*Proof.* This is immediate, as we have

$$\begin{aligned}
\sum_{i=1}^N \tau'_i(o, \mathbf{r}) &= \sum_{i=1}^N (\tau_i(o, \mathbf{r}) - s_i \sigma_i(o, \mathbf{r}_{-i})) \\
&= \sum_{i=1}^N \left( \tau_i(o, \mathbf{r}) - s_i \min_{\hat{r}_i} \sum_{j=1}^N \tau_j(o, \hat{r}_i, \mathbf{r}_{-i}) \right) \\
&= \sum_{i=1}^N \left( \sum_{j=1}^N s_j \right) \tau_i(o, \mathbf{r}) - \sum_{i=1}^N s_i \min_{\hat{r}_i} \sum_{j=1}^N \tau_j(o, \hat{r}_i, \mathbf{r}_{-i}) \\
&= \sum_{i=1}^N s_i \sum_{j=1}^N \tau_j(o, \mathbf{r}) - \sum_{i=1}^N s_i \min_{\hat{r}_i} \sum_{j=1}^N \tau_j(o, \hat{r}_i, \mathbf{r}_{-i}) \\
&= \sum_{i=1}^N s_i \sum_{j=1}^N \left( \tau_j(o, \mathbf{r}) - \min_{\hat{r}_i} \sum_{j=1}^N \tau_j(o, \hat{r}_i, \mathbf{r}_{-i}) \right) < 0.
\end{aligned}$$

A mechanism is said to be  $\mathcal{M}$ -surplus-anonymous if it deterministically maps surplus lower bounds  $(\sigma_i(o, \mathbf{r}_{-i}))$  to refund payments (to each seller  $i$ ) according to a function invariant to domain information that does not apply identically to every seller. Since the space of seller values will always be compact and simply connected, a direct adaptation of the proof of Theorem 6 of Cavallo (2006) shows the following proposition.

**Proposition 8.** *The GH mechanism  $\mathcal{M}'$  runs a minimal budget imbalance among all ex post individually rational, self-financing, bilaterally efficient,  $\mathcal{M}$ -surplus-anonymous mechanisms.*

## 4 Constructing Straightforward Concordance

We denote by  $P_{-i}(o, \mathbf{r}_{-i}) \equiv 1_{o \geq R_i}$  the sale decision which the community would make absent  $i$ , and consider the Groves Holdout mechanism  $\mathcal{M}^{\text{VCG}}$  with  $h_i(o, \mathbf{r}_{-i})$  given by

$$h_i^{\text{VCG}}(o, \mathbf{r}_{-i}) \equiv -(1 - s_i) (R_i + (o - R_i) P_{-i}(o, \mathbf{r}_{-i})).$$

The function  $h_i^{\text{VCG}}(o, \mathbf{r}_{-i})$  has associated tax function

$$\begin{aligned}
\tau_i^{\text{VCG}}(o, \mathbf{r}) &\equiv \sum_{j \neq i} (s_j o P(o, \mathbf{r}) + r_j (1 - P(o, \mathbf{r}))) + h_i^{\text{VCG}}(o, \mathbf{r}_{-i}) \\
&= \sum_{j \neq i} (s_j o P(o, \mathbf{r}) + r_j (1 - P(o, \mathbf{r}))) - (1 - s_i) (R_i + (o - R_i) P_{-i}(o, \mathbf{r}_{-i})) \\
&= \sum_{j \neq i} ((s_j o - r_j) P(o, \mathbf{r})) - (1 - s_i) (o - R_i) P_{-i}(o, \mathbf{r}_{-i}) \\
&= (1 - s_i) (o - R_i) P(o, \mathbf{r}) - (1 - s_i) (o - R_i) P_{-i}(o, \mathbf{r}_{-i}) \\
&= \begin{cases} -(1 - s_i) |o - R_i| & P(o, \mathbf{r}) = 1 - P_{-i}(o, \mathbf{r}_{-i}) \\ 0 & P(o, \mathbf{r}) = P_{-i}(o, \mathbf{r}_{-i}). \end{cases}
\end{aligned}$$

We therefore see that  $\mathcal{M}^{\text{VCG}}$  implements the tax rule of the SC mechanism  $\mathcal{M}^*$  (without a refund).

#### 4.a Uniqueness of the VCG Tax

We now prove that the SC tax is the unique enforcement mechanism which can be used in a straightforward Concordance mechanism. Specifically, we use Proposition 6 to show that every straightforward Concordance mechanism is a GH mechanism with  $h_i^+(o, \mathbf{r}_{-i})$  term of the form  $h_i^{\text{VCG}}(o, \mathbf{r}_{-i}) + h_i^+(o, \mathbf{r}_{-i})$  with  $h_i^+(o, \mathbf{r}_{-i})$  everywhere nonnegative. This pins down the SC enforcement uniquely, although the refund mechanism may still be modified while preserving straightforwardness and Concordance.

*Proof of Proposition 2.* We suppose that  $\mathcal{M}$  is a straightforward Concordance mechanism with suggested strategy  $r^*(v) = v$ . Then,  $\mathcal{M}$  is bilaterally efficient relative to a take-it-or-leave-it offer (by Theorem 1), and induces a Pareto-optimal outcome among sellers (since the decision rule of  $\mathcal{M}$  is given by  $P(o, \mathbf{r}) = 1_{o \geq R}$ ). Then, by Proposition 6, there is a GH mechanism  $\mathcal{M}'$  isomorphic to  $\mathcal{M}$ . Moreover, by the proof of Proposition 6, that isomorphism is given by the map  $(r^*)^{-1} = 1$ , hence  $\mathcal{M} = \mathcal{M}'$  is a GH mechanism.

We decompose the function  $h_i^+(o, \mathbf{r}_{-i})$  in the transfer function  $T_i(o, \mathbf{r})$  of  $\mathcal{M}$  in the form

$$h_i^+(o, \mathbf{r}_{-i}) = h_i^{\text{VCG}}(o, \mathbf{r}_{-i}) + \hat{h}_i(o, \mathbf{r}_{-i});$$

it suffices to show that  $\hat{h}_i(o, \mathbf{r}_{-i}) \geq 0$  for all  $\langle o, \mathbf{r}_{-i} \rangle \in \mathcal{O} \times \mathcal{R}^{N-1}$ . But this is immediate since by the Concordance principle we must have

$$s_i o P(o, \mathbf{r}) + \hat{h}_i(o, \mathbf{r}_{-i}) \geq s_i o P(o, \mathbf{r}) + \tau_i^{\text{VCG}}(o, \mathbf{r}) + \hat{h}_i(o, \mathbf{r}_{-i}) = T_i(o, \mathbf{r}) \geq s_i o P(o, \mathbf{r})$$

when valuation  $s_i o$  is submitted by seller  $i$ .

Additionally, we can now prove a result which combines with Proposition 6 to prove the remark stated in Footnote 22.

**Proposition 9.** *Suppose that  $\mathcal{M}$  is a GH mechanism which preserves approximate individual property rights. Then, the function  $h_i(o, \mathbf{r}_{-i})$  in the transfer function  $T_i(o, \mathbf{r})$  of  $\mathcal{M}$  can be decomposed in the form*

$$h_i(o, \mathbf{r}_{-i}) = h_i^{\text{VCG}}(o, \mathbf{r}_{-i}) + \hat{h}_i(o, \mathbf{r}_{-i})$$

with  $\hat{h}_i(o, \mathbf{r}_{-i})$  bounded below by<sup>50</sup>

$$\hat{h}_i(o, \mathbf{r}_{-i}) \geq \begin{cases} -s_i(o - R_i) & P_{-i}(o, \mathbf{r}_{-i}) = 1 \\ 0 & \text{otherwise.} \end{cases}$$

*Proof.* We fix a seller  $i$  and suppose that the buyer's value is  $b$  and that the values of the sellers  $j \neq i$  are given by  $\mathbf{v}_{-i}$ . We set  $o = o^*(b)$  and  $\mathbf{r}_{-i} = r^*(\mathbf{v}_{-i})$ .

Since  $\mathcal{M}$  preserves approximate individual property rights, there must be some  $r_i \in \mathcal{R}$  such that

$$\begin{aligned} s_i o P(o, r_i, \mathbf{r}_{-i}) + \tau_i^{\text{VCG}}(o, \mathbf{r}) + \hat{h}_i(o, \mathbf{r}_{-i}) &= T_i(o^*(b), r_i, r^*(\mathbf{v}_{-i})) \geq \frac{s_i \sum_{j \neq i} v_j}{1 - s_i} P(o^*(b), r_i, r^*(\mathbf{v}_{-i})) \\ &= \frac{s_i \sum_{j \neq i} r_j}{1 - s_i} P(o, r_i, \mathbf{r}_{-i}) = s_i R_i P(o, r_i, \mathbf{r}_{-i}). \end{aligned}$$

Reorganizing this expression gives

$$\begin{aligned} s_i(o - R_i)P(o, r_i, \mathbf{r}_{-i}) &\geq -(\tau_i^{\text{VCG}}(o, \mathbf{r}) + \hat{h}_i(o, \mathbf{r}_{-i})) \\ &= \begin{cases} (1 - s_i)|o - R_i| - \hat{h}_i(o, \mathbf{r}_{-i}) & P(o, r_i, \mathbf{r}_{-i}) = 1 - P_{-i}(o, \mathbf{r}_{-i}) \\ -\hat{h}_i(o, \mathbf{r}_{-i}) & P(o, r_i, \mathbf{r}_{-i}) = P_{-i}(o, \mathbf{r}_{-i}). \end{cases} \quad (6) \end{aligned}$$

If  $P_{-i}(o, \mathbf{r}_{-i}) = 0$ , then (6) cannot hold if  $\hat{h}_i(o, \mathbf{r}_{-i}) < 0$ .<sup>51</sup> If instead  $P_{-i}(o, \mathbf{r}_{-i}) = 1$ , then we obtain exactly the desired bound on  $\hat{h}_i(o, \mathbf{r}_{-i})$ .

<sup>50</sup>This bound is only guaranteed on the domain of possible offers and values, which may be smaller than the set  $\mathcal{O} \times \mathcal{R}^N$  because the domains of buyer and seller values are bounded. As the behavior of the mechanism  $\mathcal{M}$  outside of the possible input space seems wholly irrelevant, we suppress this technical concern in the sequel.

<sup>51</sup>In the first case, the left side of (6) vanishes, and in the second case the left side of (6) is weakly negative. Meanwhile, if  $\hat{h}_i(o, \mathbf{r}_{-i}) < 0$ , then the right side of (6) is strictly positive.

## 4.b The SC Refund

Next, we obtain the full form of SC by implementing the optimal redistribution rule obtained in Section 3.

Computing the lower bound on  $\tau_i^{\text{VCG}}(o, \mathbf{r})$  gives

$$\sigma_i^{\text{VCG}}(o, \mathbf{r}_{-i}) \equiv \min_{\hat{r}_i} \sum_{j=1}^N \tau_j^{\text{VCG}}(o, \hat{r}_i, \mathbf{r}_{-i}) = - \min_{\hat{r}_i} \sum_{j=1}^N \left( 1_{(\hat{R}_j - o)(\hat{R} - o)} (1 - s_j) |o - \hat{R}_j| \right),$$

where  $\hat{R}$  and  $\hat{R}_j$  are computed as  $R$  and  $R_j$  with  $\hat{r}_i$  substituted in place of  $r_i$ . Applying the optimal refund of  $\tau_i^{\text{VCG}}(o, \mathbf{r})$ , we then obtain exactly the Groves tax used in the SC mechanism,

$$\tau_i^*(o, \mathbf{r}) \equiv \tau_i(o, \mathbf{r}) - s_i \sigma_i^{\text{VCG}}(o, \mathbf{r}_{-i}).$$

Following this construction, we see that Proposition 1 follows from Propositions 5 and 7. Since SC is also a Concordance mechanism, Theorem 1 applies in dominant-strategy equilibrium.

## C Alternative Mechanisms

### 1 Bayes-Nash Concordance (BNC)

*Proof of Proposition 3.* Budget balance and implementability are shown in Section IV.F.1. The fact that BNC preserves collective property rights is immediate because

- each seller  $i$  announces  $r_i = v_i$  in (Bayes-Nash) equilibrium,
- transfer occurs in BNC if and only if  $o \geq R = \sum_i r_i = \sum_i v_i = V$ , and
- all tax revenues collected in BNC are shared amongst the sellers.

The approximate property rights and efficiency claims follow from Theorems 1 and 2 since truthful revelation is (Bayes-Nash) equilibrium behavior for sellers.

## 2 First-price Concordance (FPC)

To see that FPC is budget-balanced, it suffices to observe that

$$\begin{aligned}
& \sum_j \max(0, [s_j o - r_j] 1_{\text{sale}}, [r_j - s_j o] 1_{\text{no sale}}) \\
&= \sum_j (1 - s_j) \frac{\max(0, [s_j o - r_j] 1_{\text{sale}}, [r_j - s_j o] 1_{\text{no sale}})}{1 - s_j} \\
&= \sum_j \left( \sum_{i \neq j} s_i \right) \frac{\max(0, [s_j o - r_j] 1_{\text{sale}}, [r_j - s_j o] 1_{\text{no sale}})}{1 - s_j} \\
&= \sum_i s_i \sum_{j \neq i} \frac{\max(0, [s_j o - r_j] 1_{\text{sale}}, [r_j - s_j o] 1_{\text{no sale}})}{1 - s_j}.
\end{aligned}$$

## D Public Goods and Collaboration

This appendix first establishes certain relationships and conditional equivalences among hold-out, collaboration and public goods problems. It then briefly analyzes the Straightforward Collaboration mechanism outlined in the text. Finally it describes in just a bit more detail the other mechanisms for the collaboration problem mentioned in the text.

### 1 Translation

#### 1.a Quasi-linear, binary public goods

$N$  members of a community have values  $v_i$  for a public good and their utility is quasi-linear in a private numeraire good (money). The good is non-excludable and non-rival in the sense that if it is created all members of the community gain their value and none gain any value if it is not (fully) created; this is analogous to perfect complements. The cost of creating the public good  $C$  is commonly known but each community member's valuation is her private information.

It is simple to see how all the notions we invoke in the text can be translated here. Property rights become voluntary participation: that individuals (have the option) not be forced to pay more than  $v_i$ . Efficiency requires that the good be created if and only if  $V \equiv \sum_i v_i > C$ . Essentially every term receives a negative sign and otherwise everything is left unchanged. The only substantial changes in the claims about the mechanisms are that the internal-to-the-community efficiency guarantees, which translate into bilateral and asymptotic efficiency, here guarantee full efficiency.



## 1.b Collaboration

With a slight reformulation and a few natural assumptions, our model also applies to Cournot's original problem of collaboration among firms.

There are a few main differences, all minor, between collaboration and the holdout problem:

1. In the holdout problem, the quantity of sale in Cournot's problem becomes a probability of sale. Formally let  $F$  be the cumulative distribution function of  $b$  and let  $Q \equiv 1 - F$ . Thus the two problems can only be compared if agents are risk-neutral.
2. Cournot's assumption that demand is commonly known to all collaborators requires that, in the holdout problem, conditional on their valuation, all sellers have the same beliefs about the distribution of buyer valuations  $Q$ .
3. In Cournot's model, all collaborators know all others' opportunity cost of sales  $c_i$ . This trivializes the mechanism design problem, as a smart group of firms will simply agree all to price at cost in exchange for a share of total profits (Spengler, 1950). What complicates this, in the holdout problem, is that values  $v_i$  (opportunity costs of sales) are private information.
4. In collaboration, all payments by buyers,  $Q(P)P$ , where  $P \equiv \sum_i p_i$ , end up in the hands of sellers. In the holdout problem, mechanisms may involve some of these revenues being lost to a third party. This means there may be some separate transfer  $t_i$  from (or, if negative, to) the collaborators. Then the expected payment of each buyer is  $Q(P)P$ , while only  $Q(P)P - \sum_i t_i$  is captured by the sellers.
5. Finally, and most importantly, collaboration accords all bargaining power to the collaborators by making them price setters, while in the holdout problem buyers need not receive a take-it-or-leave-it offer.

Otherwise the two problems are identical.

**Proposition 10.** *Cournot's collaboration model where demand never exceeds  $\bar{Q}$  at weakly positive prices is a special case of the holdout model of Section III where*

1. *All sellers are risk-neutral.*
2. *Sellers have common priors over buyer valuations and seller valuations are not informative about the distribution of seller valuations.*

3. *One only considers mechanisms where the buyer is a price-taker (the mechanism is straightforward for buyers).*

Note that this provides an alternative proof of the Bergstrom's Corollary (see the Online Appendix). Cournot's industrial organization is just another mechanism for the holdout problem: each seller announces a reservation price and the buyer either agrees to pay the sum of all such announced prices or to forgo the purchase. Because the case of known costs can be thought of as a limit of normal distributions with increasingly small variance, we know by Mailath and Postelwaite's Corollary (see the Online Appendix) that as the number of collaborators grows large the number of sales must dwindle to 0 under *any* mechanism, including Cournot's.

*Proof.* Collaborators payoffs are by definition  $Q(P)(p_i - c_i) - t_i$ . Consider any holdout mechanism in which the seller is made a take-it-or-leave-it offer of  $P$ . The probability she accepts is  $1 - F(P)$  and, in the case of acceptance each seller loses  $v_i$ . However each seller also receives an expected transfer  $T_i$ . Thus the expected payoff of seller  $i$  is  $-(1 - F[P])v_i - T_i$ . If we define  $v_i \equiv \bar{Q}c_i$ ,  $F(P) \equiv 1 - \frac{Q(P)}{\bar{Q}}$  and  $T_i \equiv Q(P)p_i - t_i$  it is clear that the payoffs of the two games are equivalent under our the proposition hypotheses.

Thus any holdout mechanism in which the buyer is a price taker immediately yields an alternative mechanism for managing collaboration. Just as with the other applications of the holdout problem, collaboration is a pervasive problem in the economy. Being one of the first mathematical economists, Cournot was not yet at the point of having to consider problems of second-order social importance! Collaboration problems arise when many firms have patents on various components of a joint product (Lerner and Tirole, 2004) and whenever one firm with substantial market power sells to another with similar power (Spengler, 1950), such as those between a union and a product market powerful employer as well as any other vertically related industry.

The extensions of the definitions of efficiency, straightforwardness and implementability to this context are obvious so we do not state them formally. The extension of property rights is *individual bankruptcy-proofness*. This is the requirement, which we abstain from defining overly formally here, that each seller can report a cost which ensures that she pays no tax and receives a price weakly above her cost. A natural weakening of this is *approximate individual bankruptcy-proofness*, that each seller can report a cost which ensure she pays no tax and receives a price weakly above  $s_i \frac{\sum_{j \neq i} c_j}{1 - s_i}$ . *Collective bankruptcy proofness* is that group of all collaborators can collude so that  $P > C$  and no taxes are paid.

Of course, the justification for various design principles may be weaker in Cournot's setting than in others. It is probably less important that the mechanism be straightforward

for producers; this is helpful as most mechanisms which make a take-it-or-leave-it offer to buyers are not straightforward for sellers. Furthermore the justifications of protecting property rights are different: here protecting property rights means ensuring no firm makes a loss. This constraint is likely only important on average (collectively) to avoid the necessity of state subsidies or individually for those firms that cannot be coerced to produce. Nonetheless these differences are fairly minor and for the most part the holdout problem can reasonably be seen to encompass mechanisms for addressing Cournot's problem of collaboration.

### 1.c More general public goods

Consider a public goods environment where consumers' values for the continuous good are quasi-linear in money. Let  $e$  be the total expenditure on the good. Suppose that each consumer has a value  $v_i$  and the utility she gains from the public good is  $v_i f(E)$ , where  $f$  is smooth, increasing, concave and has  $f(0) = 0$ . We can then write aggregate utility for expenditure level  $e$  as

$$Vf(E) - E = f(E) \left( V - \frac{E}{f(E)} \right)$$

Note that under my assumptions  $\frac{E}{f(E)}$  is monotonically increasing in  $E$  as  $f(E) - f'(E)E > 0$  for all  $E$ . Thus the function  $P(E) = \frac{E}{f(E)}$  is invertible and we can rewrite the social payoff as  $f(P^{-1}[P])(V - P)$ . Suppose each consumer  $i$  is asked to furnish an expenditure  $e_i$  of the total expenditure. Then clearly her payoff is  $f(P^{-1}[P])(v_i - p_i)$  where  $p_i \equiv \frac{e_i}{f(E)}$  where clearly  $P = \sum_i p_i$ . It should be now clear the equivalence between this problem and collaboration, except for switches of signs, as we can define  $Q \equiv f \circ P^{-1}$  and the problem is exactly equivalent except that payoffs have a negative sign and  $Q$  is increasing rather than decreasing. Thus just as any holdout mechanism applies to binary public goods games, any collaboration mechanism applies to the environment described here.

## 2 Straightforward Collaboration

The Straightforward Collaboration mechanism is as described in the text except that a refund is given to each seller  $i$  in the amount of

$$s_i \min_{\hat{c}_i} \sum_{j=1}^N (1 - s_j) \left( \left[ P^* \left( \hat{C}_j \right) - \hat{C}_j \right] Q \left[ P^* \left( \hat{C}_j \right) \right] - \left[ P^* \left( \hat{C} \right) - \hat{C}_j \right] Q \left[ P^* \left( \hat{C} \right) \right] \right)$$

where  $\hat{C}$  and  $\hat{C}_j$  are calculated as before, but with  $c_i$  replaced with  $\hat{c}_i$ . Arguments for the various properties of the VCG Collaboration mechanism are sufficiently simple as not to merit

formal proof, given the development of SC in the text. Approximate individual bankruptcy-proofness follows from the fact that a collaborator submitting  $\hat{c}_i = s_i \frac{\sum_{j \neq i} c_j}{1-s_i}$  will never pay taxes and receives a price  $P^*(C_i) \geq C_i$  as optimal monopoly prices are always weakly above marginal cost. Collective bankruptcy-proofness follows from the fact that if all collaborators submit their share of  $C$  none pays any taxes and  $P^*(C) \geq C$ . Profit maximization follows from the decision rule, but the monopoly distortion obviously persists. Straightforwardness is by the Groves construction of the mechanism.

### 3 Other collaboration mechanisms

We conclude this appendix by stating in just a bit more detail the workings of the other collaboration mechanisms described in the text.

1. APC: Each collaborator pays her full maximized profits in taxes  $\left(s_i P^* \left[ \frac{c_i}{s_i} \right] - c_i\right) Q \left( P^* \left[ \frac{c_i}{s_i} \right] \right)$ . Each collaborator receives a refund of her share of the reported maximal profits of all other collaborators  $s_i \sum_{j \neq i} \frac{\left(s_j P^* \left[ \frac{c_j}{s_j} \right] - c_j\right) Q \left( P^* \left[ \frac{c_j}{s_j} \right] \right)}{1-s_j}$ .
2. FPC: Each collaborator pays in tax the surplus she gains by the impact she has on the price  $[s_i P^*(C) - c_i] Q[P^*(C)] - [s_i P^*(C_{-i}) - c_i] Q[P^*(C_{-i})]$  and is refunded her share of all other seller's surplus  $s_i \sum_{j \neq i} \frac{[s_j P^*(C) - c_j] Q[P^*(C)] - [s_j P^*(C_j) - c_j] Q[P^*(C_j)]}{1-s_j}$ .
3.  $X$ -plurality: Each seller announces a cost  $c_i$  and this is transformed into a collective cost  $\frac{c_i}{s_i}$ . The  $X$ -th quantile collective cost is determined by ranking all  $\frac{c_i}{s_i}$  and choosing those lowest value so that the sum over all  $i$  below this of  $s_i$  is at least  $X$ . We refer to this quantity as  $C^X(\mathbf{c}, \mathbf{s})$ . A price  $P^*(C^X[\mathbf{c}, \mathbf{s}])$  is charged to consumers and each seller is given a share  $s_i$  of the revenue. No other money changes hands. This is equivalent (except over regions of increasing marginal revenue which all sellers will be unanimous in wanting to bypass) to trying out many different price levels and at each asking each for each collaborator to vote in favor of raising or lowering the price, then charging the minimum price at which at least  $X$  of the collaborators support a lower price.