PRELIMINARY AND INCOMPLETE

International Monetary Equilibrium with Default

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Abstract

This paper proposes a finite horizon general equilibrium model of international finance with fiat money, hetergeneous agents, multiple goods, multiple assets, multiple countries each with their own money supply, default and regulation. Nominal and real determinacy is obtained and money is non-neutral. IMED provides a coherent framework consistent with standard general equilibrium theory to study the effects of monetary, fiscal and regulatory policy in an international context in view of the current financial crisis.

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1 Introduction

Recent events in the global financial markets have posed a significant challenge to the economics profession. Historically it has been concerned with the effects of monetary aggregates, via various assumptions, on trade and output. It is now necessary to explain, in addition, the precise relationship between liquidity and default. The transmission of the crisis from a single sector of the US to the entire global economy demands that this relationship be understood in an international context and the mechanisms that propogate local effects to global ones. The orthodoxy in the international finance literature has primarily rested on studying the transmission of domestic shocks via purchasing power parity and uncovered interest rate parity, supplemented by assumptions regarding nominal or real frictions that allow money to have effects on the real economy. Clearly there is no room in this setting to study liquidity or default as there are no nominal price effects that the real sector depends on. As a consequence, discussion of the macroeconomic effects of asset accumulation, default and regulation are ad-hoc at best.

International Monetary Equilibrium with Default (IMED) provides a coherent framework to analyse the international effects of monetary policy and hence liquidity, prices and hence trade, default and hence regulation. It captures the foundations of a full-bodied monetary general equilibrium model of agent optimisation, market clearing and a positive value for money (and hence nominal determinacy). IMED extends Geanakoplos and Tsomocos (2002) to uncertainty, incomplete markets and endogeneous default and as a result combines international trade with asset pricing and finance. It can be easily adapted to allow for institutional realism by allowing for an explicit banking system, fiscal policy, government budget constraints or regional trade blocs. Incomplete markets is important, aside for the sake of realism, to allow price effects to have a meaningful effect on trade and welfare via the financial assets available, and for default to have a meaningful presence in the economy. Default is an important consideration when studying how domestic macroeconomic conditions are transmitted internationally. Default not only changes the asset span but also potentially the dimensions of the asset span. In IMED monetary policy affects interest rates, that in turn affects the cost of repayment and hence default rates, that then affect the relative attractiveness of assets. As a consequence, previously robustly traded assets may lay untraded in equilibrium. Allowing for also default results in the exchange rate becoming a non-trivial relative price. The literature has dealt with default poorly, particularly in an international setting, either ignoring it entirely or studying partial equilibrium effects which cannot be used to analyse international transmission. None have studied in an international context the effect of nominal variables on default in an economy and hence the spanning opportunities of domestic and foreign agents.

The predictions of our framework are consistent with recent financial history without any ad-hoc assumptions to supplement the model. We predict that lower short term rates transmit to lower long term yields globally, higher leverage globally, and when short term rates rise, higher default. Furthermore we make the prediction that such an exercise is welfare worsening for the home country and a gain in welfare for the rest of the world. Clearly when the model is extended to allow for production and unemployment the consequences of default may be more severe, however the predictions of terms of trade moving away from the home country, higher asset prices internationally and subsequently higher default globally are consistent with observation.

IMED is an extension of the Geanakoplos and Tsomocos (2002) model of international fiance that in turn is based on the model of Dubey and Geanakoplos (1992, 2003a,b), that proposed a model of inside and outside money general enough to encompass GE and GEI, and showed that monetary equilibrium (ME) always exists. Dubey and Geanakoplos (2006) extend their one-period model to a finite horizon with uncertainty, incomplete markets, and nominal assets and can show that determinacy still obtains for interest rates, inflation, and commodity allocations. Monetary policy is not neutral, and its effects can in principle be tracked because ME are determinate. Tsomocos(2008) shows that generic determinacy exists for international monetary equilibrium when the private monetary endowment in the economy is non zero and that this also results in money having non-neutral effects on the real economy.

International finance has traditionally been characterised as an appendage to international trade or growth models. As a consequence the role of money was seen in the context of facilitating such trade though tended to underplay the intratemporal demand and supply motives for money. Indeed it is only since the New Keynesian Open Economy (NOEM) models of Obstfeld and Rogoff (1995) that money has both real effects and crucial to the intertemporal optimisation problem. The bulk of the NOEM literature is characterised by monopolistically competitive producers doubling as consumers who set the prices of imperfectly substitutable goods. Price stickiness in the short term is modelled as arising from monopolistic competition as in the new Keynesian literature (e.g., Blanchard and Kiyotaki, 1987; Ball and Romer, 1990, 1991). This results in wages, prices, output and consumption being different from the social optimum and suggesting that the aggregate demand management policies can increase both domestic and global welfare. Although the NOEM models have attractive features such as the non-neutrality of money, intertemporal budget constraints and explicit microfoundations, it suffers from the same fundamental problem as Mundell Flemming (MF). These models rely on changes to real money balances to affect demand, output and trade, that is possible only because of the assumption of sticky prices. As a result neither has a meaningful quantity theory of money equation nor Fisher effect, features that are a natural outcome of IMED.

In MF agents calculate the marginal benefit of money invested domestically against money invested abroad while in the NOEM models agents weigh the marginal benefit of asset accumulation today and consumption tomorrow. IMED has both these channels but also studies the marginal benefit of money spent on consumption domestically compared to abroad. This allows price changes to have real effects on allocation and in consequence an meaningful quantity theory of money and Fisher effect. On balance, our model provides a significant challenge to the accepted orthodoxies in international finance of the NOEM models and see ourselves as bringing forth MF into a model with microfoundations and tradeable contingent risk while preserving the powerful Keynesian intuition of the uses and roles of money.

Our model is consistent with the asset pricing flavour of Lucas (1982) that described a two country world, each with its own currency and interest rate. A representative agent in each country would sell her whole endowment on the international market and buy it back in quantities required for domestic consumption. The requirement that agents sell their whole endowment has several major shortcomings including pre-specifying the size of the domestic economy and hence global transactions beforehand. Moreover, the model is dichotomous in that there is no significant interaction between the real and the nominal sectors of either the international or national economies. There is also no scope for interaction within economies between agents, that are inevitably of keen interest when examining financial prices in the short-term.

IMED resembles the real business cycle literature in that all markets clear all the time though our results contain many attractive features of Keyenesian analysis. An expansion in monetary policy lowers interest rates and expands output, trade in our model. The mechanism through which we have both flexible prices and non-neutrality lies in the transaction cost introduced by the (short term) interest rate rendering trade inefficient in a cash in advance world. A change in money supply will then change the spot rate and induce agents to re-asses their consumption bundle and portfolio holdings. Price flexibility then, is a consequence of these changes, rather than a direct consequence of the increased quantity of money, as in the Lucas (1982) framework.

In IMED, the presence of the short term interest rate results in a "price wedge" in that the costs agents face in buying or selling a good or asset depend on the interest rate and hence affecting the marginal utilities of agents. Following Espinoza and Tsomocos (2008), this results in the finacing cost being an addition to the correlation between aggregate consumption and real asset payoffs in determining the risk-premia in asset prices. This risk premia exists whenever the volume of trade is positive and is independent of aggregate uncertainty, unlike representative agent models. Financing costs are generated within the framework of a monetary general equilibrium model, with cash-in-advance constraints built along the lines of Dubey and Geanakoplos (1992, 2003a, 2003b, 2006), Espinoza and Tsomocos (2008), Geanakoplos and Tsomocos (2002), Goodhart et al. (2006) and Tsomocos (2008). Demand for money is endogeneous in our model and depend on the goods prices, exchange rates, yield curve and asset prices in the world economy. Given the existence of outside money, such models generate demand for liquidity that results in positive nominal interest rates (Dubey and Geanakoplos, 2006) and as a result nominal determinacy is obtained. In an international context, Tsomocos (2008) shows nominal determinacy under the presence of private liquid wealth contrary to the result of Karekan and Wallace (1981).

In this paper Section 2 describes and defines the model while in 3 we present a comparison of the IMED with GEI. In 4 we show how the liquidity trap can be obtain from within IMED . In Section 5 we show that money is non-neutral in IMED. Section 6 presents the properties of IMED with Section 6.1 presenting the term structure of interest rates and quantity theory of money, Section 6.2 presenting asset pricing in IMED while Section 6.3 presents the propositions through that international effects can be analysed.

1.1 An Aside on Currencies, Inside and Outside Money

In our model money is the stipulated medium of exchange. Trade is facilitated by the Central Bank offering loans before the commodity markets meet and are repaid afterwards. Agents are also endowed with private money, outside money, that is given to them free and clear of any obligations or liabilities, and that they may spend to purchase goods or invest intertemporally. Agents wishing to spend more money than they are endowed with may borrow from the central bank at an interest rate determined endogenously in equilibrium. Repayments are made by selling a fraction of their commodity endowments and/or rolling over the loan into the next period where they may repay it at the beginning of the period with the private money they are endowed with then or at the end of the period by again taking out a short term loan and repaying the accumulated debt by selling a fraction of their commodity endowments. From this we can see that the demand for money in our model stems from the immediate transactions need as well as for speculative motives. The ability to roll money over into the different interest rate markets means that we have a fully integrated money market and an endogenous term structure that will be determined in equilibrium. So although the profit of the central bank will always be the sum of the outside money in the system[1], different patterns of the term structure will have different consequeces on trade and consumption.

The first question that arises is whether modelling outside money is a fair representation of reality. Dubey and Geanakoplos (2006) argue that whenever the government prints money and purchases real assets, like labor, from the private sector, it creates outside money. In the US, the trasury must borrow money from the Federal reserve (that can print money) on the basis of an IOU note that may be rolled over into perpetuity, thus injecting the economy with outside money. Espinoza et. al.(2007) rationalises outside money as a nominal friction that pins down the price of money while Shubik and Wilson (1977), Shubik and Tsomocos (1992) and Espinoza et al. (2007) note that introducing default on the money market plays a similar role in ensuring the existence of a positive interest rate.

2 The Model

We extend the work of Geanakoplos and Tsomocos (2002) to uncertainty and default and provides a more natural comparison with the models of Lucas and NOEM. The monetary economy within each country of our model is as follows. There is a central bank in each period who lends money to the economy endogenously determined interest rate. All agents in our model have the opportunity to borrow from the central bank of their country of origin and indeed will do so if there are sufficient gains to trade in doing so. Households (agents who are endowed with goods and a small amount of fiat money) will, given reasonable short term interest rates, borrow from the central bank in each period. This is because householders weigh the marginal gain in utility in purchasing more of a good with the marginal loss in utility in needing to sell more of their own good to finance it, and provided interest rates

[1] See the section for the term structure of interest rates equation

are not exorbitantly high, borrowing will be sufficiently attractive.

2.1 The International Monetary Economy

We consider the canonical General Equilibrium model without production. There are H agents inhabiting C countries and consuming L goods. Each country has its own currency that needs to be used to purchase goods from that country. Exchange of currency occurs only at the foreign exchange market. Agents are endowed with a goods and/or money. There are two time periods, with the second period having S possible states of nature. Specifically, the model is given as follows :

- $t \in T = \{0, 1\}$ time horizon.
- $s \in S^* = \{0, 1, .., S\}$ states of nature.
- State 0 occurs in period 0, while in period 1 nature chooses $s \in S$ states of nature.
- We consider countries $c \in C = \{1, 2, ..., C\}$. Where we generically denote a country $\alpha \in C$ we denote another country as $\beta \in C \neq \alpha$.
- $\gamma \in C$ set of governments.
- $h \in H$ set of agents in the international monetary economy. Each agent $h \in H = \bigcup_{\alpha \in C} H^\alpha$ belongs to a country. We write h^α if agent h belongs to country α. We denote a foreign agent to be h^β.
- $l \in L = \{1, 2, ..., L\}$ perishable commodities exist in the international economy and cannot be inventoried between periods. We also associate each commodity with a single country, and we write for example $l \in L^\alpha$ [2]. That is, $l \in L = \bigcup_{\alpha \in C} L^\alpha$. The commodity space can be viewed as $R_+^{S^*L}$ whose axes are indexed by $S^* \times L$.
- $e_s^h = e_{sl}^h \in R_+^{S^*L}$ endowment vector for agent $h \in H^\alpha$ in state s.
- Each asset A^j for $j \in J = \{1, ..., J\}$ is an (A, λ, Q) triple and is an $(L + C) \times S$ dimensional vector whose sth components $(A_1^j, ..., A_L^j, ..., A_\alpha^j, ..., A_C^j)$ represents the amount A_{sl}^j of commodity $l \in L$ and the money of country $\alpha \in C$, $A_{s\alpha}^j$, due in state $s \in S$. Note that the we associate each asset with a single country so that assets from country α can be viewed as $j \in J^\alpha$. We further consider assets in $j^* \in J^* = (1, ..., J, ..., J + C(s+1) + C)$ that includes deliveries in all assets $j \in J$ as well as deliveries in the money market (μ) as well as deliveries in the intertemporal bond market $(\bar{\mu})$. The larger set of assets in each country α is given by $j^* \in J^{*\alpha}$.

[2] In the interest of simplifying notation we claim there is a single type of good in the international economy but that is endowed in both countries and hence is characterised by the country of origin. For example the good may be cars but the cars in the UK would be British Cars and would be distinct from cars from the American Cars

- The private monetary endowment from Country α in state $s \in S^*$ belonging to agent h is $m^h_{s\alpha}$.
- $u^h : R_+^{S^* \times L} \to R$ utility function of agent $h \in H$.
- $U^h(w^h) = u^h(x) - \sum_{j^* \in J^*} \sum_{s \in S^*} \lambda_s^{hj} \frac{[\phi^{hj^*} A^{j^*}_{s\alpha} - D^{hj^*}_{s\alpha}]^+}{p_s \cdot v_s}$. This is the final payoff to each agent of his optimising decisions.

2.2 Assumptions

2.2.1 Endowments

(A1). $\forall s \in S^*$, $\alpha \in C$ and $l \in L$, $\sum_{h \in H} e^h_{s^*l} > 0$ and $\sum_{h \in H} m^h_{s\alpha} > 0$. That is, every commodity is present in the economy and there is a positive amount of private money in each economy.

(A2). $\forall s \in S^*$ and $h \in H$, $e^h_{s^*l} > 0$ and/or $m^h_{s\alpha} > 0$ for some $l \in L$ and $\alpha \in C$. That is, no agent has the null endowment of both goods and private money.

(A3). Let A be the maximum amount of any commodity sl that exists and let 1 denote the unit vector in $A^{SL \times L}$. Then $\exists Q > 0 \ni u^h(0,...,Q,...0) > u^h(A1)$ for Q in an ordinary component (i.e. strict monotonicity in every component). Also, continuity and concavity are assumed.

2.2.2 Assets and Default

In an economy with default, each contract is described not only by its payoff in state s, A^j_s, but by its default penalty and quantity restriction. The default penalty, λ^{hj} is a real penalty and enforces a utility punishment on agent h for failing to repay the specified amount in the contract.

(A4). We assume that $A^j \neq 0$ and $A^j \geq 0$. Furthermore, agents have no endowments of assets and there may or may not be a limit on the sales of the assets depending on the requirements of the contract. All asset deliveries must be made in money, though the contract may stipulate delivery in multiple currencies. When the asset promises are in terms of a particular commodity, the delivery must still be in the money equivalent[3].

2.2.3 Outside Money

Money is the medium of exchange in our model as all commodities and assets can be traded for money and all asset deliveries occur in money. Money enters our economy in two ways. It may be introduced through the central bank or it may be a private endowment of agents.

[3]Note that deliveries may occur in multiple currencies.

We denote $m^h_{s\alpha}$ as the private monetary endowment of country α money of agent h in state $s \in S^*$. We assume that in each country there is a positive amount of private money in each period and state. The vector $(m^h_s)^{h\in H}_{s\in S^*}$ is called *outside money*, because it enters the system free and clear of any offsetting obligations.

2.2.4 Inside Money

In each country and state of the world there is a central bank willing to loan an exogeneously specified quantity of money at an endogeneously determined interest rate. The total quantity of money in the system, then, is the inside money plus the outside money.

Practically, the mechanism by that inside money is injected into the system can be thought of as government loans to the banking system. In return for the central bank of country $\gamma \in C$ loaning an amount of money of country $\alpha \in C$, $M^\gamma_{s\alpha}$, central bank γ receives an interest rate of $r_{s\alpha}$% (ex-ante i.e. before default). The vector $(M^\gamma_s)^{\gamma\in C}_{s\in S^*}$ is called *inside money* because it enters the system accompanied by an offsetting obligation.

2.3 Governments and Central Banks

There is a central bank in each country that has the authority to act on markets on behalf of its government. The actions of the central bank will be taken as exogenous allowing us to analyse the consequences of government activities on the dynamics of the market.

In the short term (intra period) bond market as well as the long term (inter period) bond market, the central bank will fix the amount of money lent to agents in the short term market($\{M^\gamma_{s\alpha}\}$) with the interest rate being endogenously determined ($\{r_{s\alpha}\}$).

Governments and central banks regularly borrow and lend money in the financial markets. When such government bonds are bought or sold, typically there is no default on them. This can be rationalised as the result of extremely high default penalties. In this model we assume that there exists a safe bond market in each country and hence a safe endogeneously determined inter temporal interest rate ($\{\bar{r}_\alpha\}$). The government then commits to repay a fixed amount of money and spend a fix amount of money in purchasing such safe bonds. In this way the government has the ability to affect longer term as well as shorter term interest rates.

We also allow each government to buy commodities and to buy and sell bonds and foreign currency. Government purchases of domestic commodities are considered part of fiscal policy; transactions in the bond market are regarded as open market operations for monetary policy; and transactions in foreign currencies are thought of as efforts to control exchange rates.

Finally the government is able to buy and sell (defaultable) assets, spending a fixed amount on purchasing them or comitting to repay a fixed amount for each asset.

The government actions of country $\gamma \in C$ are given by the vector:

$$(\theta^{\gamma}, \phi^{\gamma}, D^{\gamma}, b) \equiv (d^{\gamma}_{s\alpha}, \mu^{\gamma}_{s\alpha}, b^{\gamma}_{s\alpha\beta}, \bar{\mu}^{\gamma}_{s\alpha}, \theta^{\gamma}_{\alpha}, \phi^{\gamma}_{\alpha}, D^{\gamma j^*}_{s\alpha}, b^{\gamma}_{s^* l})$$

for $s \in S^*$, $\gamma, \alpha, \beta \in C$, and $l \in L$. The notation is as follows (in order): money lent to money market, number of bonds sold in money market, money spent in foreign exchange transactions, money lent in intertemporal bond market, number of bonds sold in intertemporal bond market, quantity of assets bought, quantity of assets sold, repayments on assets and intertemporal bonds and money spent on goods market. Note that currency holdings of government γ are exogeneous to the model. Practically, if the money is domestic it can be rationalised as having been created through printing, while if it were foreign currency then it can be considered to have come out of reserves. Geanakoplos and Tsomocos consider taxes within their framework, however here we abstract from them for the sake of notational parsimony as well as for the sake of focusing our attention on the financial and asset implications of our model but is a straightforward extension.

2.4 The Time Structure of Markets

In each period $t \in T = \{0, 1\}$, ($s \in S^*$ where $s = 0$ when $t = 0$ and $s \in S$ when $t = 1$), four markets meet: first the three financial markets, beginning with the short-term (intraperiod) money market, followed by the foreign exchange market and the asset and long-term (interperiod) bond market. The commodity market then meets. Finally, short-term bonds come due at the end of the period. Long-term bonds and assets are delivered before the foreign exchange market meets but after the short term market. This set up maximises the number of transactions possible and allows for agents to roll over liabilities in the short term money market to the long term one. It also allows for an explicit speculative motive for holding money. Agents have the option of investing money in the short bond market then carrying it over to the next period. The only reason they do not do this is because they believe they will get a higher return from transacting the intertemporal bond market. While preserving Keyenesian thinking on the uses of money, it also provides a rationale for an upward sloping term structure.

The first period thus has five transaction moments: short bonds, foreign exchange, assets and long bonds, commodities, short-bond deliveries while the second period has the following transaction moments: short bonds, assets and long bond deliveries, foreign exchange, commodities, short-bond deliveries. In the first period there is no delivery of assets or long bonds while in the last period there is no market for assets or long bonds.

Figure 1 indicates our time line, including the moments at that the various loans come due. We make the sequence precise when we formally describe the budget set.

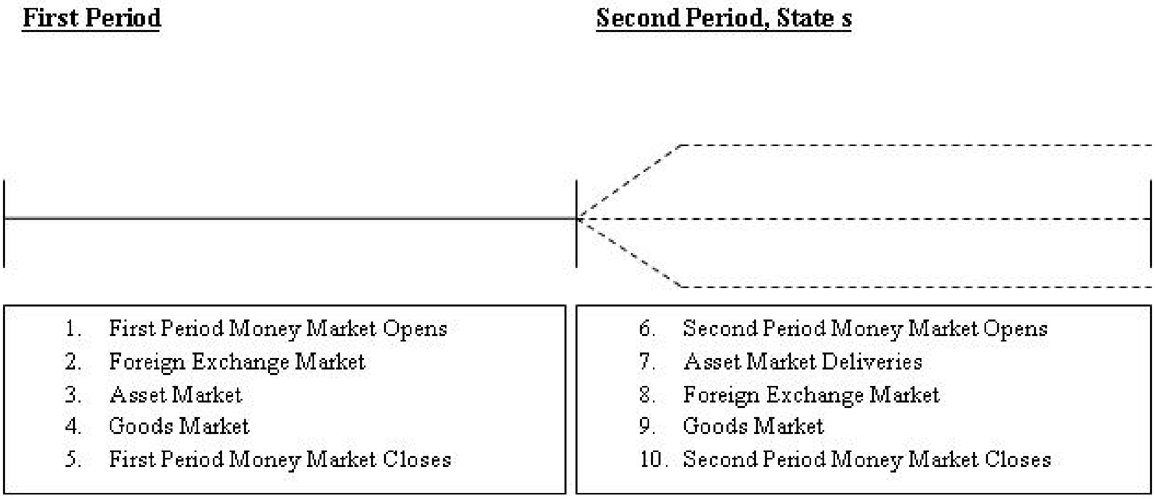

Figure 1: Time Structure

2.5 International Monetary Equilibrium with Default

2.5.1 Macro variables and individual choice variables

Denote the macro variables that are determined in equilibrium, and that every agent regards as fixed, by $\eta = (p, \pi, \psi, K)$:

- $p \in \mathbb{R}_{++}^{S^* \times L}$ = commodity prices;
- $\pi \in \mathbb{R}_{++}^{S^* \times C(C-1)/2}$ = exchange rates;
- $\psi \in \mathbb{R}_{+}^{S^* \times J}$ = asset prices. Elements of ψ will refer to the short term interest rate as well as the long term bond rate;
- $K \in [0,1]^{S^* \times J^*}$ = expected delivery rates on assets;

Denote the choices of agent h by $\sigma^h \in \Sigma^h(\eta)$ where $\sigma^h = (x^h; q^h; b^h; \theta^h; \phi^h; D^{hj})$

- $x^h \in \mathbb{R}_{+}^{L \times S^*}$ = consumption of h;
- $q^h \in \mathbb{R}_{+}^{L \times S^*}$ = sales of good l by h;
- $b^h \in \mathbb{R}_{+}^{L \times S^* + C(C-1) \times S^*}$ = amount offered to goods and foreign exchange market by h;
- $\theta^h \in \mathbb{R}_{+}^{J^*}$ = purchases of asset $j^* \in J^*$ by h. There is double counting in notation here as elements of θ will correspond to the quantities bought in the money markets (d) and bond markets ($\bar{d}$);

- $\phi^h \in \mathbb{R}_+^{J^*}$ = sales of asset $j^* \in J^*$ by h. There is double counting in notation here as elements of ϕ will correspond to the quantities sold in the money markets (μ) and bond markets ($\overline{\mu}$);
- $D^h \in \mathbb{R}_+^{(S^* \times (L+1)) \times J^*}$ = deliveries by agent h on money markets in each country, intertemporal bond markets and assets $j \in J$. Note that deliveries may occur in multiple currencies;

Denote the choices of government γ by $\sigma^\gamma \in \Sigma^\gamma(\eta)$ where $\sigma^\gamma = (b^\gamma; \theta^\gamma; \phi^\gamma; D^{\gamma j})$

- $b^\gamma \in \mathbb{R}_+^{L \times S^* + C(C-1) \times S^*}$ = amount offered to goods and foreign exchange market by γ;
- $\theta^\gamma \in \mathbb{R}_+^{J^*}$ = purchases of asset $j^* \in J^*$ by γ. There is double counting in notation here as elements of θ will correspond to the quantities bought in the money markets (d) and bond markets ($\overline{d}$);
- $\phi^\gamma \in \mathbb{R}_+^{J^*}$ = sales of asset $j^* \in J^*$ by γ. There is double counting in notation here as elements of ϕ will correspond to the quantities sold in the money markets (μ) and bond markets ($\overline{\mu}$);
- $D^\gamma \in \mathbb{R}_+^{(S^* \times (L+1)) \times J^*}$ = deliveries by γ on money markets in each country, intertemporal bond markets and assets $j \in J$;

The possibility of default means that the expected delivery rates, K, are macro variables. The asset market is an anonymous market with promises between different sellers not allowed to be distinguished even though they may deliver differently. All deliveries are pooled and buyer of the pool for each asset receive a pro rata share of the net deliveries. Each share, θ, of the pool recieves a fraction $K_s^{j^*}$ of the promised delivery $A_s^{j^*}$ for all $j^* \in J^*$. The expected delievery rate is defined as:

$$K_s^{j^*} = \frac{\sum_{h \in H} D_s^{hj^*} + \sum_{\gamma \in C} D_s^{\gamma j^*}}{\sum_{h \in H} A_s^{j^*} \phi^{hj^*}}.$$

where D is the domestic currency (of the asset) value of the asset delivery[4], $\theta = d$ and $\phi = \mu$ for the money market, $\phi = \overline{\mu}$ and $\phi = \bar{\mu}$ for the bond market, $\lambda = \infty$ for the money market, $\lambda = \infty$ for the bond market and $K = \tilde{K}$ for the money market and $K = \overline{K}$ for the bond market. The terms of the contract (A, λ, Q) are set exogeneously while the price and delivery rate are determined endogeneously in equilibrium.

[4]That is, if the asset was sold in the UK and had payoffs denominated in dollars, pounds and euro, then these values would all be converted into a pound equivalent at the prevailing exchange rate.

2.5.2 Household Budget set

The budget set $B^h(\eta) = \{\sigma^h \in \Sigma^h : \text{(1-8)below}\}$

$$d^h_{0\alpha} \leq m^h_{0\alpha} \tag{0α1}$$

$$\sum_{\beta \in C} b^h_{0\alpha\beta} \leq \Delta(0\alpha1) + \frac{\mu^h_{0\alpha}}{1+r_{0\alpha}} \tag{0α2}$$

$$\sum_{\beta \in C/\alpha} b^h_{s\alpha\beta} + \sum_{j \in J^\alpha} D^{hj}_{s\alpha} + \bar{D}_\alpha \leq \frac{\mu^h_{s\alpha}}{1+r_{s\alpha}} + \hat{m}^h_{s\alpha} \tag{sα2}$$

$$\bar{d}^h_\alpha + \sum_{j \in J^\alpha} \psi^j \theta^{hj} \leq \Delta(0\alpha2) + \sum_{\beta \in C} b^h{}_{0\beta\alpha}\pi_{0\beta\alpha} \tag{0α3}$$

$$\sum_{l \in L^\alpha} b^h_{sl} \leq \Delta(s\alpha2) + \sum_{\beta \in C} b^h{}_{s\beta\alpha}\pi_{s\beta\alpha} + \sum_{j \in J^\alpha} K^j_{s\alpha}\theta^{hj} + \bar{d}^h_\alpha(1+\bar{r}_\alpha) \tag{sα3}$$

$$\sum_{l \in L^\alpha} b^h_{0l} \leq \Delta(0\alpha3) + \sum_{j \in J^\alpha} \psi^j \phi^{hj} + \frac{\bar{\mu}^h_\alpha}{1+\bar{r}_\alpha} \tag{0α4}$$

$$q^h_{s^*l} \leq e^h_{s^*l} \tag{sα5}$$

$$x^h_{s^*l} \leq \Delta(s\alpha5) + \frac{b^h_{s^*l}}{p_{s^*l}} \tag{sα6}$$

$$D^h_{0\alpha} \leq \Delta(0\alpha4) + \sum_{l \in L^\alpha} p_{0l} q^h_{s^*l}$$

$$D^h_{s\alpha} \leq \Delta(s\alpha3) + \sum_{l \in L^\alpha} p_{sl} q^h_{sl} \tag{sα7}$$

$$\hat{m}^h_{s\alpha} \leq \Delta(0\alpha7) + (1+r_{0\alpha}) d^h_{0\alpha} \tag{0α8}$$

The final outcome to h from his choices $\sigma^h \in \Sigma^h_\eta \subset \Sigma \equiv \mathbb{R}^{L\times S^*}_+ \times \mathbb{R}^{L\times S^*}_+ \times \mathbb{R}^{L\times S^*+C(C-1)\times S^*}_+ \times \mathbb{R}^{J^*}_+ \times \mathbb{R}^{J^*}_+ \times \mathbb{R}^{(S^*\times(L+1))\times J^*}_+$ is a bundle $F^h_\eta(\sigma^h) = w^h = (x^h, \phi^h, D^h) \in (\mathbb{R}^{(S^*)\times L}_+ \times \mathbb{R}^{J^*}_+ \times \mathbb{R}^{(S^*\times(L+1))\times J^*}_+)$.

The utility to h of the outcome w^h is given by:

$$U^h(w^h) = u^h(x) - \sum_{j^* \in J^*} \sum_{s \in S^*} \lambda^{hj}_s \frac{[\phi^{hj^*} A^{j^*}_{s\alpha} - D^{hj^*}_{s\alpha}]^+}{p_s \cdot v_s}.$$

where $v_s \in \mathbb{R}^L_{++}$ is exogeneously specified and $v_s \neq 0$.

2.5.3 Government Budget set

The budget set $B^\gamma(\eta) = \{\sigma^\gamma \in \Sigma^\gamma : \text{(1-4)below}\}$

$$d^{\gamma}_{0\alpha}+\sum_{\beta\in C/\alpha} b^{\gamma}_{0\alpha\beta}+\leq M^{\gamma}_{0\alpha}+\frac{\mu^{\gamma}}{1+r_{0\alpha}} \qquad (0\alpha 1)^{\gamma}$$
$$\sum_{l\in L^{\alpha}} b^{\gamma}_{0l}+\bar{d}^{\gamma}_{\alpha}+\sum_{j\in J^{\alpha}}\psi^{j}\theta^{\gamma j}\leq \Delta(0\alpha 1)^{\gamma}+\sum_{\beta\in C} b^{\gamma}{}_{0\beta\alpha}\pi_{0\beta\alpha} \qquad (0\alpha 2)^{\gamma}$$
$$d^{\gamma}_{s\alpha}+\sum_{\beta\in C/\alpha} b^{\gamma}_{s\alpha\beta}+\sum_{j\in J^{\alpha}} D^{\gamma j}_{s\alpha}+\bar{D}_{\alpha}\leq M^{\gamma}_{s\alpha}+\frac{\mu^{\gamma}}{1+r_{s\alpha}}+\Delta(s\alpha 4)^{\gamma} \qquad (s\alpha 2)^{\gamma}$$
$$\tilde{D}^{\gamma}_{0\alpha}\leq \Delta(0\alpha 2)^{\gamma}+\frac{\bar{\mu}^{\gamma}_{\alpha}}{1+\bar{r}_{\alpha}}+\sum_{j\in J^{\alpha}}\psi^{j}\phi^{\gamma j} \qquad (0\alpha 3)^{\gamma}$$
$$\tilde{D}^{\gamma}_{s\alpha}\leq \Delta(s\alpha 2)^{\gamma}+\sum_{j\in J^{\alpha}} K^{j}_{s\alpha}\theta^{\gamma j}+\bar{d}^{\gamma}_{\alpha}(1+\bar{r}_{\alpha}) \qquad (s\alpha 3)^{\gamma}$$
$$w^{\gamma}_{0\alpha}\leq \Delta(0\alpha 3)^{\gamma}+d^{\gamma}_{\alpha}(1+r_{\alpha})$$
$$w^{\gamma}_{s\alpha}\leq \Delta(s\alpha 3)^{\gamma}+d^{\gamma}_{\alpha}(1+r_{\alpha}) \qquad (s\alpha 4)^{\gamma}$$

where w^{γ} is the money withdrawn from the system at the end of each period.

2.6 Equilibrium

We say that $(\eta,(\sigma^{h})_{h\in H})$ is an **International Monetary Equilibrium with Default** and denote it IMED for the world economy
$E=((u^{h},e^{h},m^{h})_{h\in H},M^{\gamma},\mu^{\gamma})_{h\in H,\ \gamma\in C}$ if and only if:

1. $(\sigma^{h})\in Argmax_{\sigma^{h}\in B(\eta)}\ U(x^{h})$
2. $p_{s^{*}l}\sum_{h\in H^{\alpha}} q^{h}_{s^{*}l}=\sum_{h\in H} b^{h}_{s^{*}l}+\sum_{\gamma\in C} M^{\gamma}_{s^{*}l},$
3. $\pi_{s\alpha\beta}(\sum_{h\in H} b^{h}_{s\alpha\beta}+\sum_{\gamma\in C} M^{\gamma}_{s\alpha\beta})=\sum_{h\in H} b^{h}_{s\beta\alpha}+\sum_{\gamma\in C} M^{\gamma}_{s\beta\alpha}$
4. $\dfrac{\sum_{h\in H}\mu^{h}_{s\alpha}+\sum_{\gamma\in C}\mu^{\gamma}_{s\alpha}}{(1+r_{s\alpha})}=\sum_{h\in H} d^{h}_{\alpha}+\sum_{\gamma\in C} M^{\gamma}_{s\alpha}$
5. $\dfrac{\sum_{h\in H}\bar{\mu}^{h}_{\alpha}+\sum_{\gamma\in C}\bar{\mu}^{\gamma}_{\alpha}}{(1+\bar{r}_{\alpha})}=\sum_{h\in H}\bar{d}^{h}_{\alpha}+\sum_{\gamma\in C}\bar{M}^{\gamma}_{\alpha}$
6. $K^{j^{*}}_{s}=\left\{\begin{array}{ll}\dfrac{\sum_{h\in H} D^{hj^{*}}_{s}+\sum_{\gamma\in C} D^{\gamma j^{*}}_{s}}{\sum_{h\in H} A^{j^{*}}_{s}\phi^{hj^{*}}} & \text{if } \sum_{h\in H} A^{j^{*}}_{s}\phi^{h}_{j^{*}}>0\\ \text{arbitrary} & \text{if } \sum_{h\in H} A^{j^{*}}_{s}\phi^{h}_{j^{*}}=0\end{array}\right\}$

for agents $\forall s\in S^{*}$ and $\forall\gamma\in C$, $\forall\alpha,\beta\in C$, $\forall j^{*}\in J^{*}$ and $h\in H$.

Condition 1 says that all agents optimise; 2 says that all commodity markets clear, or equivalently that price expectations are correct, 3 says that all currency markets clear, or equivalently, that currency forecasts are correct, 4 says that all short-term credit markets clear, or equivalently, that predictions of short-term interest rates are correct, 6, together with the budget set, says that each potential buyer of an asset is correct in his expectation about the fraction of promises that get delivered.

2.7 Inactive Markets Hypothesis

A difficulty associated with allowing default in a rational expectations economy arises for untraded assets. Agents decide whether to buy or sell an asset depending on the price and delivery rates associated with the asset. However if an asset is untraded unduly pessimistic expectations may mean tha the asset remains untraded. We surmount this problem by imposing the following hypothesis:

Whenever credit or asset markets are inactive (i.e., asset supply, credit extension or deposits are 0) the corresponding rates of delivery are set equal to 1.

This guarantees that any agent wishing to buy or sell an asset is not discouraged from doing so because of expectations about the asset. In this way agents are encouraged to trade an asset. The existence proofs we provide are essentially for economies where every asset is traded. However it is straight forward, if computationally intensive, to consider what would occur for untraded assets as the following section will show.

2.8 Asset Market Participation and Untraded Assets

An important phenomenon of default is that it not only changes asset prices but also the decision whether or not to trade an asset at all: it changes the asset span as well as the dimensions of the asset span. Consider identical assets but that are regulated in different countries. Depending on the default penalty imposed by each government, the delivery rate will vary and hence the agents buying and selling each contract will vary. Furthermore, depending on the policy variables of the government, the relative attractiveness of trading assets changes for agents. For instance, an agent in country A may find that the price of an asset in country B too high for him to purchase. If the government of country A decides to appreciate their exchange rate (or conversely the government of country B decides to depreciate their's), the home price of that asset will fall for the home agent and he may now decide to purchase it. Similarly different sellers will enter into different asset markets depending on government policy. Government policy may also alter the equilibrium enough to render previously untraded assets liquid and thus significantly affecting the risk-sharing ability and ultimately welfare of agents both home and abroad. In this setting it is important to explicitly characterise what can occur in equilibrium by describing what is occuring out of it[5].

On the verge trading

[5]See Dubey et al (2005) for a thorough discussion on this.

In equilibrium assets will be bought by agents who offer the highest price for the asset and will be sold by those offering the lowest price. If the asset is traded then the buyers and sellers trivially agree on a price. If an asset remains untraded then the highest price offered by a buyer will be less than the lowest price offered by a seller.

Formally, we say that an agent is on the verge of buying an asset if $\psi^j = MU_j^h/\mu_0^h$ and on the verge of selling an asset if $\psi^j = MDU_j^h/\mu_0^h$ where MU is the marginal utility of buying the asset (typically $\sum_{s\in S} K_s^j \mu_s$) and MDU is the marginal disutility of selling the asset (typically $\sum_{s\in S} \min(\mu_s, \lambda_s^{hj}\theta_s)$ where θ_s is the subjective belief of the state occuring) and where μ is the marginal utility in each state $s \in S^*$.

The following two examples highlight the importance of (i) different monetary policy regimes and (ii) different regulatory regimes in determining that assets are traded and by that agents. For the first argument we use the same logic as that of the gains-to-trade hypothesis while the second example assumes no transaction cost.

Example 1: Liquidity

Consider a single exchange economy with three agents (α, β and γ) with logarithmic utility, two time periods, a single perishable good and a single bond with an infinite default penalty (i.e. default is not allowed). We will capture the effect of monetary policy by forcing agents who purchase an asset at market price ψ to pay $\psi(1+r_h)$ where r_h represents the borrowing rate for agent h. Let the endowment of agent h in period $t = (0,1)$ be given by e_t^h and assume the endowments are such that agent γ is selling the bond (with quantity sold being ϕ) and agents α and β wish to buy the bond. Finally assume that $r_\alpha < r_\beta$.

We can show that for a sufficiently high r_β, agent β will not purchase the bond and trade in the asset will be conducted solely by the other two agents. Let us assume that such an equilibrium exists. In this case the price of the asset will be given by (using the first order conditions of agents equated to price) $\psi = \frac{1-\psi\phi}{(1+r_\alpha)\phi} = \frac{\psi\phi}{1-\phi}$ that gives us $\phi = \frac{1}{2}$ and $\psi = \frac{2}{2+r_\alpha}$. Now in this equilibrium agent β has no opportunity to smooth his consumption and so will be consuming his endowment only and would be willing to pay at most $\psi = \frac{e_0^{h\beta}}{(1+r_\beta)e_1^{h\beta}}$ for this asset. That is, if $(1+r_\beta) > \frac{e_0^{h\beta}}{e_1^{h\beta}}\frac{2+r_\alpha}{2}$ then β will not be willing to trade the asset. If the government increases monetary policy and reduces r_β sufficiently then he too will be willing to trade the asset.

The following example will describe a similiar phenomenon when there are different regulatory regimes with respect to default.

Example 2: Regulation

Consider the economy in the previous example with three agents (α, β and γ) with logarithmic utility, two time periods, a single perishable good and a single bond with default penalty λ^h for agent h. Assume now that the endowments are such that α and β wish to sell the bond and γ wishes to buy the bond and $\lambda_\beta > \lambda_\alpha$.

Consider an interior equilibrium in that only α sells the bond and default partially to γ. The price at that trade in the bond occurs is given by $\lambda^\alpha[e^\alpha + \frac{e_0^\gamma}{\frac{e_1^\gamma}{e_1^\alpha - \frac{1}{\lambda}} + 2}]$. As before, β will consume his endowment in this equilibrium and would offer only a price of $\lambda^{h^\beta} e_0^{h^\beta}$ for the asset. If $\lambda^{h^\beta} > \frac{\lambda^\alpha}{e_0^{h^\beta}}[e_0^\alpha + \frac{e_0^\gamma}{\frac{e_1^\gamma}{e_1^\alpha - \frac{1}{\lambda^\alpha}} + 2}]$ then β will not be require a price higher than α and so not participate in the market.

The previous two examples have show that monetary policy and regulation will affect not only the asset span but also the *dimensionality* of the asset span. This has particularly important implications for emerging markets where an influx of capital from developed markets, that may have a lower cost of capital and potentially more suitable regulatory regimes, results in asset price appreciation. When these conditions change, particularly a liquidity shortage in the developed markets, these foreign participants are forced to re-asses the attractiveness of the investments in the emerging markets. If they decide to withdraw their money, the price of these assets must fall to accomodate the higher cost of capital in the domestic market.

2.9 Existence

This section presents the existence theorems.

2.9.1 Gains to Trade Hypothesis

If the interest rate is sufficiently large there may be no opportunities for trade to occur. We deal with this by imposing the following condition:
Definition: Let $x^h \in \mathbb{R}_+^{S^*\times L}$ $\forall h \in H$. $\forall \delta > 0$, we will say that $(x^1,...,x^H) \in \mathbb{R}_+^{S^*\times L\times H}$ permits at least δ-gains-to-trade in state s if there exists $\tau_s^1,...,\tau_s^H$ in $\mathbb{R}^{L+1}$ such that:

$$\sum_{h\in H} \tau_s^h = 0$$

and

$$x_s^h + \tau_s^h \in \mathbb{R}_+^L, \quad \forall h \in H$$
$$u^h(\bar{x}^H) > u^h(x^h), \quad \forall h \in H$$

where

$$\bar{x}^h_{tl} = \left\{ \begin{array}{ll} x^h_{tl} & t \in S^* \setminus \{s\} \\ x^h_{tl} + min\{\tau^h_{sl}, \tau^h_{sl}/(1+\delta), & \text{for } l \in L \text{ and } t = s \end{array} \right\}$$

Note that when $\delta > 0$, $\bar{x}^h_l < x^h_{tl} + \tau^h_l$, if $\tau^h_l > 0$ and $\bar{x}^h_{tl} = x^h_{tl} + \tau^h_l$ if $\tau^h_l \leq 0$. Formally, the hypothesis we impose on the economy for sufficient gains-to-trade is:

G to T: $\forall s \in S$, the initial endowment $(e^h)_{h \in H}$ permits at least δ_s -gain to trade in state s, where

$$\delta_s = \frac{\sum_{h \in H} m^h_0 + \sum_{h \in H} m^h_s}{M^\gamma}$$

Condition (G to T) needs to be valid $\forall s \in S$ but not for $s = 0$. G to T precludes the case where $L = 1$, $\forall s \in S^*$. Moreover, if the initial endowment is not Pareto optimal $\forall s \in S$, then holding other government acrtions fixed as we vary $M^\gamma \to \infty$ (G to T) is automatically satisfied[6]. The following theorem is proved in the Appendix:

Theorem 1

There always exists an IMED of $E(M, \lambda,)$ provided

1. Gains-to-trade and Inactive Market Hypothesis holds
2. $\forall s \in S^*$ and $\alpha \in C$, $\sum\limits_{h \in H} m^h_{s\alpha} > 0$ and
3. M, λ>0.

3 IMED vs GEI

Recall that $(p, \Psi(x^h, \phi)_{h \in H})$ is a GEI for the underlying economy $E = ((u^h, e^h)_{h \in H}, A)$ iff:

1. $\sum\limits_{h \in H} x^h = \sum\limits_{h \in H} e^h$
2. $\sum\limits_{h \in H} \phi^h = 0$
3. $(x^h, \phi) \in B^h(p, \pi, \Psi) = \{(x, \phi) \in \mathbb{R}^{S^*L): p(x_0 - e^h_0) + \Psi\phi \leq 0}$ and $\forall s \in S$ $p_s(x_s - e^h_s) \leq \sum\limits_{j \in J} \sum\limits_{l \in L} p_{sl} S^j_{sl} \phi_j\}$
4. $(x^h, \phi) \in B^h(p, \Psi) \to u^h(x) \leq u^h(x^h)$

Proposition 1: IMED vs GEI

Suppose that $\sum\limits_{h \in H} m^{h8}_s = 0, \forall s \in S^*$, *and* $\lambda = +\infty$. *Moreover, there exists an asset* $A^j = (1, ..., 1)$. *Then IMED and GEI coincide.*

[6] Alternatively, if $\sum\limits_{h \in H} m^h_s = 0$ and $\sum\limits_{h \in H} m^h_s = 0$ $\forall s \in S^*$ then G to T is automatically satisfied.

4 The Liquidity Trap

An extreme case of financial instability is the well-known liquidity trap. An economy manifests a liquidity trap whenever financial instability is coupled with monetary policy ineffectiveness. The Keynesian liquidity trap describes a situation in that monetary policy would not affect the nominal variables of the economy because consumers simply hold extra real money balances for speculative purposes. If that interest rates are sufficiently low and investors expect them to go up in the future, then they do not invest into assets like bonds whose value will decrease when interest rates rise. Various authors provide explanations and formalizations of the liquidity trap, e.g. Tobin (1982), Grandmont and Laroque (1973), and Hool (1976) among others, based on non-rational expectations. Dubey and Geanakoplos (2003) provide an alternative explanation based on the incompleteness of asset markets.

Proposition 2: Liquidity Trap

Suppose that the economy has a riskless asset (i.e. monetary payoffs in every state are equal to one) and $A_s^j = 0, \forall s \in S$ *and* $j \in J^\alpha$ *and* $\alpha \in C$. *Also consider the case in that the underlying economy has no GEI. Then as* $M^\gamma \to \infty$ *then* $M^\gamma / \|p_{0l}\|$ *and* $\sum_{h \in H} \phi^{hj} \to \infty$. *Moreover, there exist* $\bar{D}$ *such that* $D_s^{h^* j} > \bar{D} > 0$ *for some* $h^* \in H$, $j \in J^\alpha$ *and* $\alpha = \gamma \in C$.
Note that regulators can break this by imposing quantity constrains on the assets sold.

5 Non-Neutrality of Money

Proposition 3: No Money Illusion

A proportionate increase of all $\left(m^h\right)_{h \in H^\alpha}$ *and* M^α *will have no affect on consumption in the IME*

Increasing all money in the economy by a fixed proportion will lead to a proportionate rise in prices and will support the same allocation of consumption. This can be proved trivially by scaling all financial variables by a scalar representing the nominal value of currency.

The following proposition states that changes in the quantities of money in the international economy will have real effects on trade and consumption.

Lemma 3: Relative Prices

For agent $h \in H$ *belonging to Country* α *who borrows in the short term market of Country* α *and sells a good from Country* α *and purchases a good from country* $\beta \neq \alpha$ *who faces* *exchange rate $\pi_{s\alpha\beta}$ that is foreign currency per unit of home currency we have*[7]*:*

$$\frac{u'(c^h_{s\alpha})\,(1+r_{s\alpha})}{u(c^h_{s\beta})} = \frac{\pi_{s\alpha\beta}p_{s\alpha}}{p_{s\beta}}$$

Proposition 4: Non-Neutrality of Money

In this section we show that as long as there exists some private endowment of money within each national economy, and the IME is different from CE, government monetary policy or changes in private monetary endowments necessarily has real effects on consumption.

These non-neutrality conclusions are contrary to those derived by Lucas. The explanation is that in our model, IME is not Pareto efficient because of the distortion caused by trading via money borrowed at positive interest rates. When the government eases credit (by putting more money up at the banks) it facilitates borrowing, reduces interest rates, and increases real activity. Further, the requirement that Lucas places on agents needing to sell *all* of their endowment necessarily means that money can have no meaningful affect on trade.

A change in either monetary endowments of agents or of monetary policy by the central bank must result in a different consumption allocation in the IME. For an agent h selling good α from country α and agent h^* from Country β selling good β in states $s \in S^*$, a change in M^γ for $\gamma \in C$ must have an impact on the consumption allocation of the world endowment of goods.

Proof: Following a change in M^γ for $\gamma \in C$ we know from the term structure of interest rates that at least one interest rate must change. Assume that there is no affect on consumption. Suppose $r_{\hat{s}\alpha}$, for some $\hat{s} \in S^*$, increases without affecting the quantities of goods traded by agents. This would imply that for good β bought and good α sold by agent h, $\frac{p_{\hat{s}\beta}}{p_{\hat{s}\alpha}\pi_{\hat{s}\alpha\beta}}$ has fallen. However agent h^* is on the other side of this transaction and finds that $\frac{p_{\hat{s}\alpha}\pi_{\hat{s}\alpha\beta}}{p_{\hat{s}\beta}}$ has increased - a contradiction. Hence the allocation of consumption will change.

Assume that a change in $r_{\hat{s}\alpha}$ has resulted in a change in $\frac{\pi_{\hat{s}\alpha\beta}p_{\hat{s}\alpha}}{p_{\hat{s}\beta}}$ and no change in consumption for agent h. Now take another agent $h^* \neq h \in H$ who is not liquidity constrained. He will find his relative prices have changed and so necessarily must change his allocation of consumption.

[7] If the agent is not liquidity constrained then the proposition would be as follows:

$$\frac{u'(c^h_{s\alpha})}{u(c^h_{s\beta})} = \frac{\pi_{s\alpha\beta}p_{s\alpha}}{p_{s\beta}}$$

Assume now that a change in M^γ for $\gamma \in C$ in period 0 has resulted in no change in interest rate, but a change in delivery rate for the money market. An investor in the money market would have optimised his investment according to the following first order equation:

$$\frac{u'(c^h_{0\alpha})}{p_{0\alpha}} = (1+r_{0\alpha})\,\tilde{K}_{0\alpha}\sum_{s\in S}\theta^h_s\frac{u'(c^h_{s\alpha})}{p_{s\alpha}}$$

If following a change in M^γ this equation results in an inequality, then the agent can improve welfare by depositing an ε more or less (as in the previous example). If the result is a change in prices and no change in consumption then we can (as before) find an agent for whom relative prices have changed such that his allocation has changed. A similar argument will hold for the long bond and the asset markets.

6 Properties of the International Monetary Equilibrium

In a previous section we discussed the significance of ensuring positive interest rates in cash-in-advance models such as ours. In this section we shows that we have nominal determinacy in our International Monetary Equilibrium. Proposition 7 shows how short term interest rates are determined and in proposition 5 we show that this affects the value of trade.

Given that there exists some private liquid wealth in each state then the short term interest rates will be strictly positive. We will show this in the following proposition.

6.1 Money Demand, Interest Rates and Quantity Theory of Money

Letting $M_{s\alpha} \equiv \sum_{\gamma\in C} M^\gamma_{s\alpha}$, $\mu_{s\alpha} \equiv \sum_{\gamma\in C}\mu^\gamma_{s\alpha}$,$d_{s\alpha} \equiv \sum_{\gamma\in C} d^\gamma_{s\alpha}$, $\bar{\mu}_{s\alpha} \equiv \sum_{\gamma\in C}\bar{\mu}^\gamma_{s\alpha}$, $\bar{d}_\alpha \equiv \sum_{\gamma\in C}\bar{d}^\gamma_\alpha$, $\theta^j \equiv \sum_{\gamma\in C}\theta^{\gamma j}$, $\phi^j \equiv \sum_{\gamma\in C}\phi^{\gamma j}$, $D_\alpha \equiv \sum_{j\in J^\alpha}\sum_{\gamma\in C} D^{\gamma j}_\alpha$, $\tilde{D}_\alpha \equiv \sum_{\gamma\in C}\tilde{D}^\gamma_\alpha$, $b_{s\alpha C} \equiv \sum_{\gamma\in C}\sum_{\beta\in C/\alpha} b^\gamma_{s\alpha\beta}$, $b_{sC\alpha} \equiv \sum_{\gamma\in C}\sum_{\beta\in C/\alpha} b^\gamma_{s\beta\alpha}$,$b_{sL} \equiv \sum_{l\in L^\alpha}\sum_{\gamma\in C} b^\gamma_{sl}$, $m_{s\alpha} \equiv \sum_{h\in H} m^h_{s\alpha}$ and $\hat{m}_\alpha \equiv \sum_{h\in H}\hat{m}^h_\alpha$ we have:

Proposition 5: Quantity Theory of Money Proposition

In IMED the aggregate income of country α in each state $s \in S^*$, namely the value of all domestic commodity sales $\sum_{h\in H^\alpha}\sum_{l\in L^\alpha} p_{s^*l}q^h_{s^*l}$ is at most equal to the total stock of bank money and net government injections and remaining private money

Period 0: $\sum_{h\in H}\sum_{l\in L^\alpha} p_{0l}q^h_{0l} \le M_{0\alpha} + m_{0\alpha}$

Period 1: $\sum_{h\in H}\sum_{l\in L^{\alpha}} p_{sl}q^h_{sl} \leq M_{s\alpha}+m_{s\alpha}+\hat{m}_{s\alpha}$

Proof: In all states $s \in S^*$, any money agents have before the commodity market meets can be used to purchase goods, repay the money market or, (in period 0) if long term interest rates are not positive, to carry over to period 1. Money obtained after the commodity markets meet (i.e. through sales of commodities) can be used to repay the money market or (again, in period 0) to carry over to period 1. Clearly any money obtained *before* the commodity market need not enter the commodity market at all. Thus our velocity of money in our model will be generally less than 1. The reason for this phenomenon is unique to an international economy. Consider the following scenario. In period 0, an agent from country α borrows in the money market of country $\beta \neq \alpha$ that is then converted to country α money and spent on assets, goods or stored. The liability in country β is funded by asset sales there. In period 1 the same agent could borrow in the money market in country β using the proceeds to defray his asset liability. When the currency markets open up he would then convert country α money (obtained from the money market there or from his endowment) to country β and repay the money market loan. The (possible) remaining liability in country β could be repaid through sales of his real endowment. Thus in this simple scenario we have seen money in country α in *both* periods not used for the goods market. This situation is not possible in a closed economy and reflects the added complications when conducting monetary policy in an open economy. It also reflects how phenomenon such as inflation are more pronounced in an economy with tight capital controls as any extra increase in the money supply is more likely to result directly in an increase in the price level.

Note that in an incomplete markets economy with default it is not certain that hoarding would not take place unless an asset exists that provides a return greter than or equal to zero in every state. In this setting a default-free government bond becomes important in generating trade in assets and commodities and provides a key insight into the importance of central banks and governments in the orderly functioning of markets.

Proposition 6

At any IMED (i) $r_{s\alpha} \geq 0$ and $\bar{r}_{\alpha} \geq 0$ and (ii) $(1+\bar{r}_{\alpha}) \geq (1+r_{0\alpha})\tilde{K}_{0\alpha}$ and the following:

Proposition 7: Term Structure of Interest Rates

At any IMED, for all $s \in S^*$ and for all $\alpha \in C$,

Period 0:

$$d_{0\alpha}(1+r_{0\alpha})\tilde{K}_{0\alpha}+\frac{\mu_{0\alpha}}{1+r_{0\alpha}}+b_{0C\alpha}+\frac{\bar{\mu}_{\alpha}}{1+\bar{r}_{\alpha}}+\sum_{j\in J^{\alpha}}\psi^j\phi^j+\hat{m}_{s\alpha}$$
$$=d_{0\alpha}+\tilde{D}_{0\alpha}+b_{0\alpha C}+\bar{d}_{\alpha}+\sum_{j\in J^{\alpha}}\psi^j\theta^j+b_{0L}+m_{0\alpha}$$

Period 1:

$$d_{s\alpha}(1+r_{s\alpha})\tilde{K}_{s\alpha}+\frac{\mu_{s\alpha}}{1+r_{s\alpha}}+b_{sC\alpha}+\bar{d}_{\alpha}(1+\bar{r}_{\alpha})+\sum_{j\in J^{\alpha}}\theta^{j}K_{s}^{j}A_{s}^{j}$$
$$=d_{s\alpha}+\tilde{D}_{s\alpha}+b_{s\alpha C}+\bar{\mu}_{s\alpha}+\sum_{j\in J^{\alpha}}D_{s}^{j}+b_{sL}+m_{s\alpha}+\hat{m}_{s\alpha}$$

The RHS represents the money flowing into the system, and the LHS represents the money flowing out of the system. The interest rates must satisfy a sequence of inequalities, but only one equality per currency. If all government expenditures and transfers are zero and there is no private money in the economy then interest rates are not determined endogeneously and are zero.

Corallary: *At any IMED for all $s \in S$ and $\alpha \in C$,*

$$1+r_{0\alpha} \leq \frac{M_{0\alpha}+m_{0\alpha}}{\tilde{K}_{0\alpha}d_{0\alpha}^{\gamma}}$$

$$1+r_{s\alpha} \leq \frac{M_{s\alpha}+m_{s\alpha}+\hat{m}_{s\alpha}}{\tilde{K}_{s\alpha}d_{s\alpha}^{\gamma}}$$

Clearly if the government spends too much money without expanding the money supply, then interest rates will rise and trade will come to a complete halt. Furthermore, if default penalties on the money market are too low then the delivery rate there will approach zero and interest rates will again rise rendering borrowing inefficient. Through this we can see how fiscal policy, monetary policy and default are connected in a financial economy.

6.2 Asset Pricing with Liquidity Constraints

Positive interest rates and the liquidity based market transactions introduce a "price wedge" whose size depends on period zero interest rates. The "price wedge" manifests itself both in the commodity and the asset markets. The complication that positive interest rates introduce is the failure of the exact linear pricing rule of assets.

Proposition 8: Asset Pricing

Suppose that $A^j = \lambda_1 A^1 - \lambda_2 A^2$ and $\lambda_1, \lambda_2 \geq 0$. If asset $j \in J$ is traded at an IMED then:

$$\lambda_1\theta_1 - (1+r_{0\alpha})\lambda_2\theta_2 \leq \theta_j \leq \lambda_1\theta_1 - \frac{1}{1+r_{0\alpha}}\lambda_2\theta_2$$

$\forall \alpha \in C$.

Note that the linearity principle obtains only if all the private monetary endowments, and initial capital condition of commercial banks are zero. Moreover, the bankruptcy penalties should be such that they preclude bankruptcy altogether. Thus, interest rates will all be zero and linear pricing will obtain.

6.3 PPP, Fisher Effect, UIP and Risk Neutral Probabilities

We will use the Purchasing Power Parity theorem and the Fisher Effect hypothesis to prove the Uncovered Interest Rate Parity hypothesis in our model. subsectionInterest Rates as Central Bank Policy

Proposition 9: Purchasing Power Parity Proposition

For agent $h \in H$ who buys good α from Country α and purchases good β from country $\beta \neq \alpha$ who faces exchange rate $\pi_{s\alpha\beta}$ that is foreign currency per unit of home currency we have:

$$\frac{u'(c^h_{s\alpha})}{u(c^h_{s\beta})} = \frac{\pi_{s\alpha\beta} p_{s\alpha}}{p_{s\beta}}$$

Proof: If LHS>RHS the agent can buy ε less of good β and use the money saved to but $\varepsilon \dfrac{p_{s\beta}}{\pi_{s\alpha\beta}}$ more of the good in Country α, thereby decreasing his utility by $\varepsilon u'(c^h_{s\beta})$ and increasing his utility by $\varepsilon \dfrac{p_{s\beta}}{p_{s\alpha}\pi_{s\alpha\beta}} u'(c^h_{s\alpha})$. From our assumption, his gain in utility is greater than his loss in utility and hence the agent is better off - a contradiction that the agent has optimised.

Similarly, if LHS<RHS the agent can spend $\varepsilon p_{s\alpha}$ less on good α and buy $\varepsilon \dfrac{p_{s\alpha}\pi_{s\alpha\beta}}{p_{s\beta}}$ more of good β and ended up better off - again a contradiction that the agent has optimised.

In our model agents weigh the marginal benefit of domestic and foreign consumption. Typical analysis of the purchasing power parity conditions compares the prices of identical goods in different places however, agents in our model, and indeed in reality, have no interest in purchasing identical goods. What is a more pertinent measure of relative prices is comparing the marginal benefit of consumption of two goods. If the two goods are identical, then the prices will still not be identical as it is the *cost* of purchasing a foreign good that will be equated to the price of the domestic good - the purchasing power parity condition based on the law of one price and adjusted for the borrowing cost. If however, the goods are not identical, then the relative marginal benefit of consumption of the two goods must equal their relative costs.

Proposition 10: Fisher Effect

Suppose that some agent h in Country α *who sells a bond domestically and in Country* α *and chooses* $b^h_{0\alpha} > 0$ *and* $b^h_{s\alpha} > 0$ *for state both state 1 and state 2 and has some Country* α *money left over when long loans come due in period 1, then at IME we must have:*

$$\frac{\frac{u'(c^h_{0\alpha})}{p_{0\alpha}}}{\sum_{s\in S}\theta^h_s \frac{u'(c^h_{s\alpha})}{p_{s\alpha}}} = 1 + \bar{r}_\alpha$$

where $\alpha \in C$ and θ^h_s is the subjective belief of agent h that state s will occur.

Rearranging the above and taking logarithms allows us to interpret the above as the nominal rate of interest being equal to the real rate of interest plus (expected) inflation plus risk premium term.

Proof: From the first order equations of agent h we have:

$$\frac{u'(c^h_{0\alpha})}{p_{0\alpha}}\frac{1}{1+\bar{r}_\alpha} = \sum_{s\in S}\theta^h_s \frac{u'(c^h_{s\alpha})}{p_{s\alpha}}$$

If in the above the LHS>RHS the agent can deposit $\varepsilon p_{0\alpha}$ less in the long bond market and consume ε more of good α in period 0 increasing his utility by $\varepsilon u'(c^h_{0\alpha})$. In the next period he will consume $(1+\bar{r}_\alpha)\varepsilon\frac{p_{0\alpha}}{p_{s\alpha}}$ less for each $s \in S$ thus his expected utility will fall by

$$p_{0\alpha}(1+\bar{r}_\alpha)\varepsilon\left[\sum_{s\in S}\theta^h_s \frac{u'(c^h_{s\alpha})}{p_{s\alpha}}\right]$$

- less than his gain in utility and a contradiction that he has optimised.

Conversely if LHS<RHS the agent can deposit $\varepsilon p_{0\alpha}$ more in the long bond market and consume ε less of good α in period 0. In the next period he will consume $(1+\bar{r}_\alpha)\varepsilon\frac{p_{0\alpha}}{p_{s\alpha}}$ more for each $s \in S$ and be better off.

Now rearranging these equations we obtain:

$$\frac{\frac{u'(c^h_{0\alpha})}{p_{0\alpha}}}{\sum_{s\in S}\theta^h_s \frac{u'(c^h_{s\alpha})}{p_{s\alpha}}} = 1 + \bar{r}_\alpha$$

and finally

$$log(\frac{u'(c^h_{0\alpha})}{u'(c^h_{1\alpha})}) + log(\frac{p_{1\alpha}}{p_{0\alpha}}) + \left[log(\frac{\theta^h_1 \frac{u'(c^h_{1\alpha})}{p_{1\alpha}}}{\sum_{s\in S} \theta^h_s \frac{u'(c^h_{s\alpha})}{p_{s\alpha}}}) - log(\theta^h_1) \right] \approx \bar{r}_\alpha$$

From here we can see that the nominal interest rate is approximately the real interest rate plus the rate of inflation plus a risk premium term[8].

Proposition 11: Uncovered Interest Rate Parity

Suppose in period 0 agent h has one unit of country α money. He can either deposit the money domestically and convert it to country β money in the future or he can convert it to country β money immediately and invest the money there. These two strategies will have the same value in expectation. That is, we must have[9]:

$$\begin{aligned} \frac{\pi_{0\alpha\beta}\,(1+\bar{r}_\beta)}{1+\bar{r}_\alpha} &= \sum_{s\in S} \pi_{1\alpha\beta} \times \tilde{\theta}_{\beta s} \\ &= \mathbb{E}_{\tilde{\theta}_\beta}[\pi^1_{\alpha\beta}] \end{aligned}$$

That is, the UIP proposition gives the future exchange rate (the exchange rate in period 1 is given by $\pi^1_{\alpha\beta}$) under the risk neutral measure ($\tilde{\theta}_\beta$) [10].

Proof: We obtain the UIP hypothesis when we combine the Fisher Effect result with the Purchasing Power Parity result:

$$\begin{aligned} \pi_{0\alpha\beta} \frac{u'(c^h_{0\beta})}{p_{0\beta}} \frac{1}{1+\bar{r}_\alpha} &= \sum_{s\in S} \pi_{s\alpha\beta} \theta^h_s \frac{u'(c^h_{s\beta})}{p_{s\beta}} \\ \frac{u'(c^h_{0\beta})}{p_{0\beta}} \frac{1}{1+\bar{r}_\beta} &= \sum_{s\in S} \theta^h_s \frac{u'(c^h_{s\beta})}{p_{s\beta}} \end{aligned}$$

[8] If the agent is liquidity constrained then this would become

$$log(\frac{u'(c^h_{0\alpha})}{u'(c^h_{1\alpha})}) + log(\frac{p_{1\alpha}}{p_{0\alpha}}) + \left[log(\frac{\theta^h_1 (1+r_{1\alpha}) \frac{u'(c^h_{1\alpha})}{p_{1\alpha}}}{\sum_{s\in S} \theta^h_s (1+r_{s\alpha}) \frac{u'(c^h_{s\alpha})}{p_{s\alpha}}}) - log(\theta^h_1) \right] \approx \bar{r}_\alpha + r_{1\alpha}$$

[9] We define the exchange rate here as the ratio of Country α money over Country β money, The risk neutral probabilities are defined for Country β

[10] If an agent is borrowing in the home market and depositing in the long market, the foreign interest rate should be deflated by the short term interest rate in his home country. If the agent is borrowing abroad and depositing at home, the home long rate should be deflated by the home short rate

Now, combining the two equations

$$\pi_{0\alpha\beta}\frac{(1+\bar{r}_{\beta})}{(1+\bar{r}_{\alpha})} = \sum_{s\in S}\pi_{s\alpha\beta}\left[\frac{\theta_s^h\frac{u'(c^h_{s\beta})}{p_{s\beta}}}{\sum_{s\in S}\theta_s^h\frac{u'(c^h_{s\beta})}{p_{s\beta}}}\right]$$

that gives the desired result.

In models of no uncertainty, agents weigh the marginal benefit of consumption in the future over consumption today. They achieve this by weighing the marginal benefit of investing in one country over the other and the resulting arbitrage relationship determines the UIP condition. When uncertainty is introduced into the model the process is not so mechanical. Agents now need to weigh the marginal benefit of consumption in one state over the other and must achieve this with available assets. This means that the UIP condition is not a straightforward arbitrage pricing relationship.

In developing the UIP hypothesis as a product of the Purchasing Power Parity hypothesis and the Fisher Effect Hypothesis we argue that any test of UIP must be a joint test of PPP and Fisher Effect. This becomes more crucial when you do not make the assumption of incomplete markets where you need to find the same risk neutral measure for the Fisher Effect as well as UIP.

Our derivation of the UIP condition above briefly noted the effect of the period 0 transaction cost. In the following section we show how the transaction cost explicitly affects the valuation of risk by agents.

We can also see how exchange rates are intimately related to marginal utilities. Consider an appreciation of the exchange rate of country α in some state $\hat{s}\in S$. Now from PPP we can see that for agent h who belongs to any country $c\in C$: $\frac{u'(c^h_{s\alpha})}{u(c^h_{s\beta})} < \frac{\pi_{s\alpha\beta}p_{s\alpha}}{p_{s\beta}}$. Hence an agent will optimise by reducing his consumption in the country α good and increasing his consumption in country β good. A reduction in his consumption of the country α good will have a downward effect on the price of that good and vice versa for the country β good. Hence we find that the marginal rate of substitution for state α will rise following an appreciation of the exchange rate.

Proposition 12: National Accounts

National Income/GDP

National Income is defined as value of domestic goods and will depend on the amount of money in the economy. The National Income for country α is:

$$GDP(s,\alpha) = \sum_{h\in H^\alpha}\sum_{l\in L^\alpha} p_{sl}e^h_{sl}$$

Agents in our model can exchange their α-money for β-money in any state. They can also trade these monies for assets or bonds in the β country and hence we may say that there is perfect capital mobility. Explicitly modelling uncertainty and the opportunity to undertake international financial investments makes the balance of trade interesting.

Trade Balance

We define the balance of trade surplus for country α with respect to country β in state $s \in S^*$ by

$$B(s,\alpha,\beta) = \sum_{h\in H^\beta}\sum_{l\in l^\alpha} b^h_{sl} - \pi_{s\beta\alpha}\sum_{h\in H^\alpha}\sum_{l\in L^\beta} b^h_{sl} + \sum_{l\in l^\alpha} b^\beta_{sl} - \pi_{s\beta\alpha}\sum_{l\in l^\beta} b^\alpha_{sl}$$

That is, the net exports of residents plus the net exports of the governments of countries α and β. The α-balance of trade surplus is given by:

$$B(s,\alpha) = \sum_{\beta\in C/\{\alpha\}} B(s,\alpha,\beta)$$

where money offered to the goods market is given by b.

With floating exchange rates such as we have, the only way this number could be nonzero is if an a agent exchanged α-money for β-money, and instead of spending it on commodities, deposited in a β bank, or vice versa. Given that financial flows and related heding activities are crucial in this economy, we should expect the trade balance to be non-zero for all countries.

Capital Account

$$\begin{aligned} CAP(0,\alpha) = &\sum_{h\in H^\alpha}\sum_{\beta\in C/\alpha}\sum_{j\in J^\beta} \psi^j\{\theta^{hj} - \phi^{hj}\}\pi_{0\beta\alpha} - \sum_{\beta\in C/\alpha}\sum_{h\in H^\beta}\sum_{j\in J^\alpha}\{\theta^{hj} - \phi^{hj}\} \\ &+ \sum_{\beta\in C/\alpha}\sum_{j\in J^\beta} \psi^j\{\theta^{\alpha j} - \phi^{\alpha j}\}\pi_{0\beta\alpha} - \sum_{\beta\in C/\alpha}\sum_{j\in J^\alpha}\{\theta^{\beta j} - \phi^{\beta j}\} \end{aligned}$$

That is, the net holding of assets by individuals and government of country α less net claims on country α assets by foreign individuals and governments. The capital account in state $s \in S$ will be the negative of that in period 0 as all capital must return home.

Net Interest Payments

The interest income is defined as the net value of assets in state $s \in S$ less their amount in period 0

$$NIP(s,\alpha) = \sum_{h\in H^\alpha}\sum_{\beta\in C/\alpha}\sum_{j\in J^\beta}\{K^jA^j\theta^{hj} - D^{hj}\}\pi_{s\beta\alpha} - \sum_{\beta\in C/\alpha}\sum_{h\in H^\beta}\sum_{j\in J^\alpha}\{K^jA^j\theta^{hj} - D^{hj}\}$$
$$+ \sum_{\beta\in C/\alpha}\sum_{j\in J^\beta}\{K^jA^j\theta^{\alpha j} - D^{\alpha j}\}\pi_{s\beta\alpha} - \sum_{\beta\in C/\alpha}\sum_{j\in J^\alpha}\{K^jA^j\theta^{\beta j} - D^{\beta j}\} - CAP(0,\alpha)$$

Current Account
The current account for country α is defined as the trade balance plus net income from abroad. In period 0 this is simply the trade balance but in state $s \in S$ the net income from abroad becomes an important component of the current account. We define the current account as:

$$CA(s,\alpha) = \sum_{\beta\in C/\{\alpha\}} B(s,\alpha,\beta) + NIP(s,\alpha)$$

Proposition 13: Intertemporal Balance of Payments

The Current Account and Capital Account are zero intertemporally:

The Current Account plus the Capital Account must be zero in every period and state by definition. Intertemporally the Capital Account in state $s \in S$ must reverse that in period 0 and hence the Current Account must be 0 intertemporally also. Note that government holdings of foreign currency will either be invested in assets or used to purchase foreign goods with the result that the budget constraint forcing the government to default on liabilities.

7 A Numerical Example

This section presents a numerical simulation of an IMED. The world economy is paramaterised with a suitable initial equilibrium with that a comparative statics exercise is performed by iterating the data of the economy followed by an analysis of the results.

The IMED under consideration is characterised by two countries and a single representative agent in each country. There are two time periods with two possible states of the world in the second time period. We call Country α the UK and Country β America. Agent h^α is from the UK while Agent h^β is from America. Agents h^α and h^β act intertemporally and maximise their utility by spending their endowments of money, purchasing goods available in the market, selling their endowment of goods and borrowing in the short term money market. We call them households as they are endowed with goods to sell.

7.1 The International Monetary Economy

The IMED is given as follows :

- $t \in T = \{0,1\}$ time horizon.
- $s \in S^* = \{0,1,2\}$ states of nature.
- State 0 occurs in period 0, while in period 1 nature chooses $s \in S = \{1,2\}$ states of nature.
- We consider countries $\alpha, \beta \in C$.
- $h \in H = \{h^\alpha, h^\beta\}$ set of agents in the international monetary economy.
- $l \in L = \{1\}$ perishable commodities and cannot be inventoried between periods. We also associate each commodity with a single country, and we write for example $l \in L^\alpha$ [11]. The commodity space can be viewed as $R_+^{S^*L}$ whose axes are indexed by $\{0,1,2\} \times \{1\}$. Goods are allocated to particular countries with a good belonging to Country α being denoted by 1 and goods belonging to country β being denoted by 2.
- $e_s^h = \left(e_{sl}^h\right) \in R_+^{S^*L}$ endowment for agent $h \in H$ in state s.
- The private monetary endowment from Country α in state s belonging to agent h is $m_{s\alpha}^h$.
- There is a single non-defaultable bond (i.e. $\lambda = \infty$) in each country.
- $u^h : R_+^{S^* \times L} \to R$ utility function of agent $h \in H$.

Agents in this model have CRRA utility functions.

7.2 Parameterisation

Table 7.2 presents the underlying data of the world economy. The total monetary endowment in the UK is similar to that in the US as is the money supplies provided by the central bank and endowment of goods. We will see in the following sections the effect of these characteristics on the nature of the equilibrium in the model.

[11] In the interest of simplifying notation we claim there is a single type of good in the international economy but that is endowed in both countries and hence is characterised by the country of origin. For example the good may be cars but the cars in the UK would be British Cars and would be distinct from cars from the American Cars

	Agent h^{α}	**Agent h^{β}**		**Agent h^{α}**	**Agent h^{β}**
Outside Money in State 0,1,2			Endowments of Goods 1,2 in State 0,1,2		
$m^{h^{\alpha}}_{0\alpha}, m^{h^{\beta}}_{0\beta}$	10.000	10.000	e_{01}, e_{02}	100.000	100.000
$m^{h^{\alpha}}_{1\alpha}, m^{h^{\beta}}_{1\beta}$	10.000	10.000	e_{11}, e_{12}	100.000	100.000
$m^{h^{\alpha}}_{1\alpha}, m^{h^{\beta}}_{2\beta}$	10.000	10.000	e_{21}, e_{22}	100.000	100.000
Preferences for home good			Subjective Probabilities for State 1		
h^{α}, h^{β}	0.500	0.500	$\theta_1^{h\alpha}, \theta_1^{h\beta}$	0.500	0.500
Money Supply in States 0,1,2			CRRA Risk Aversion		
	UK	**US**	$\rho_{h^{\alpha}}, \rho_{h^{\beta}}$	.500	.500
M_{01}, M_{02}	120.000	120.000	Default Penalty		
M_{11}, M_{12}	75.000	75.000		**UK**	**US**
M_{21}, M_{22}	100.000	100.000	$\lambda^{\alpha}, \lambda^{\beta}$	0.077	0.077

Table 1: Parameters of Initial Equilibrium

7.3 Initial Equilibrium

We present the endogenous variables of the initial equilibrium here. Table 2 presents the microeconomic variables of the economy or the individual choice variables while Table 3 shows the macroeconomic variables in each country. Money enters the economy through the money markets in IMED and, in equilibrium, the demand for money will meet the money supply at a price of the short term interest rates. The lower the interest rate, the more efficient trade becomes so trade increases inversely with interest rates (see Espinoza and Tsomocos, 2008, and Espinoza et al, 2009). Table 2 shows a bias toward home consumption for each agent due to the financing cost creating an inefficiency in purchasing foreign goods.

Table 3 shows that the price level in each country depends on the quantity of money offered to the goods market that in turn depends on the money supply. In period 0 only a fraction of the money available is used in the goods market while in period 1 all the money available will be used in the goods market and captures the primary result of the quantity theory of money. Similarly the liquidity in the currency markets are dependent on the quantity of money available as can be seen by the money offered to the foreign exchange market.

In period 1 agents weigh the marginal rate of substitution (MRS) of repayment of the loan with that of the default punishment. As there is a real default penality, the price level does not enter into the decision. Rather it is the interest rate. The higher the interest rate, the higher will be MRS of the agent with respect to the penalty and hence higher default. The higher spot rate of %13.3 in state 1 means that agents will find it optimal to default, and given the default penalty imposed there is only default in state 1.

	Agent h^{α}	**Agent h^{β}**		**Agent h^{α}**	**Agent h^{β}**
Consumption of Good 1 in State 0,1,2			*Consumption of Good 2 in State 0,1,2*		
$c_{01}^{h^{\alpha}}, c_{01}^{h^{\beta}}$	53.977	46.023	$c_{02}^{h^{\alpha}}, c_{02}^{h^{\beta}}$	46.023	53.977
$c_{11}^{h^{\alpha}}, c_{11}^{h^{\beta}}$	56.455	43.545	$c_{12}^{h^{\alpha}}, c_{12}^{h^{\beta}}$	43.545	56.455
$c_{21}^{h^{\alpha}}, c_{21}^{h^{\beta}}$	54.758	45.242	$c_{22}^{h^{\alpha}}, c_{22}^{h^{\beta}}$	45.242	54.758
Money Offered for Goods 1, 2 in State 0,1,2			*Money Offered to Foreign Exchange Market in State 0,1,2*		
$b_{02}^{h^{\alpha}}, b_{01}^{h^{\beta}}$	101.767	101.767	$b_{012}^{h^{\alpha}}, b_{021}^{h^{\beta}}$	130.000	130.000
$b_{12}^{h^{\alpha}}, b_{11}^{h^{\beta}}$	85.000	85.000	$b_{112}^{h^{\alpha}}, b_{121}^{h^{\beta}}$	61.597	61.597
$b_{22}^{h^{\alpha}}, b_{21}^{h^{\beta}}$	110.00	110.000	$b_{212}^{h^{\alpha}}, b_{221}^{h^{\beta}}$	78.346	78.346
Amount Repaid to Short-Term Money Market in State 0,1,2			*Lending/Borrowing in Country α Long Term Bond Market*		
$\mu_{0\alpha}^{h^{\alpha}}, \mu_{0\beta}^{h^{\beta}}$	130.000	130.000	$\overline{\mu}_{1}^{\alpha}, \overline{d}_{1}^{h^{\beta}}$	31.654	28.233
$\mu_{1\alpha}^{h^{\alpha}}, \mu_{1\beta}^{h^{\beta}}$	85.000	85.000	*Lending/Borrowing in Country β Long Term Bond Market*		
$\mu_{2\alpha}^{h^{\alpha}}, \mu_{2\beta}^{h^{\beta}}$	110.00	110.000			
Utility of Agents			$\overline{d}_{2}^{h^{\alpha}}, \overline{\mu}_{2}^{h^{\beta}}$	28.233	31.654
$U^{h^{\alpha}}, U^{h^{\beta}}$	28.088	28.088			

Table 2: Microeconomic Variables of Economy

	UK	**US**		**UK**	**US**
Price of Goods 1,2 in States 0,1,2			*Short Term interest Rates in States 0,1,2*		
p_{01}, p_{02}	2.211	2.211	$r_{0\alpha}, r_{0\beta}$	0.083	0.083
p_{11}, p_{12}	1.952	1.952	$r_{1\alpha}, r_{1\beta}$	0.133	0.133
p_{21}, p_{22}	2.431	2.431	$r_{2\alpha}, r_{2\beta}$	0.1	0.1
Sales of Goods 1,2 in States 0,1,2			*Long Term interest Rate*		
q_{01}, q_{02}	46.023	46.023	R_{α}, R_{β}	1.121	1.121
q_{11}, q_{12}	43.545	43.545	*Delivery Rate*		
q_{21}, q_{22}	45.242	45.242	K_{11}, K_{12}	0.739	0.739

UK Nominal Trade Balance in States 0,1,2			*Exchange Rates in States 0,1,2*		
$TB0\alpha$	$TB1\alpha$	$TB2\alpha$	$\pi_{0\alpha\beta}$	$\pi_{1\alpha\beta}$	$\pi_{2\alpha\beta}$
0	0	0	1	1	1

Table 3: Macroeconomic Variables of Economy

7.4 Comparative Statics

In this section we iterate our initial parameterisation in order to determine the effect on all the endogenous variables of the economy of either an anticipated or unanticipated change in a policy parameter or the physical data of the international economy. The comparative statics exercise is interpreted through the propositions derived in earlier sections and we do not need to resort to a stationary state, a representative agent, nor ignore equilibrium conditions.

The unique insight given by cash-in-advance models is through explicitly modeling liquidity and the effects on real variables of the economy by nominal variables. In the following section we examine the effect of a change in monetary policy by the central banks followed by what occurs when there is an increase in the endowment of real goods in the international economy.

7.4.1 Tables

Here we present the comparative statics results. We iterate the initial equilibrium by each parameter, in each case increasing the parameter by 1% 5 times. Cases where there is a strictly monotonic increase we denote a $+$ (when there is a monotonic increase $+/=$), when there is a strictly montonic decrease we donote $-$ (when there is a monotonic decrease $-/=$), when there is an increase then a decrease $+/-$, when there is a decrease then an increase $-/+$ and finally when there is no change we denote 0.

7.4.2 Monetary Policy

Non-Coordinated Expansion in Money Supply in Period 0

The two-country Mundell Flemming Model posits that an (unanticipated) expansionary monetary policy will depreciate the exchange rate, reduce global interest rates and have a beggar-thy-neighbor effect on output. The Obstfeld Rogoff model claims that such an expansion will reduce world interest rates, lead to a current account surplus at home and an increase in world demand leading to welfare gains both at home and abroad. IMED captures the channels in these models as well as the effects of prices on the final allocation and welfare. As a consequence of the broader analysis we find vastly different results.

In IMED, from the term structure of interest rates (TS), an expansion in domestic (UK) money supply lowers the money market interest rate there. A lower short term interest rate results in a lower transaction cost of imports. From purchasing power parity (PPP) UK households then optimise by exporting more British goods and offering more money to the currency market in order to purchase more US goods, that in turn depreciates the Pound. The stronger US Dollar means that Americans will now wish to purchase more UK goods and reduce consumption of American goods. In this way we see an increase in the volume of global trade and a slight deterioration in the UK nominal trade balance.

	$M_{0\alpha}$	$M_{1\alpha}$	$M_{2\alpha}$	Perm Unco	M_0	M_1	M_2	Perm Co
$c_{01}^{h^\alpha}$	-	+	-	-	-	=	=	-
$c_{11}^{h^\alpha}$	=	-	=	-	-	-	=	-
$c_{21}^{h^\alpha}$	=/-	-	+	=/+	=/-	=	-	-
$c_{02}^{h^\alpha}$	+	+	-	=/+	+	=	=	+
$c_{12}^{h^\alpha}$	=	=/-	=	=/-	=	+	=	+
$c_{22}^{h^\alpha}$	=/-	-	+	+	=	=	+	+
$c_{01}^{h^\beta}$	+	-	+	+	+	=	=	+
$c_{11}^{h^\beta}$	=	+	=	+	=	+	=	+
$c_{21}^{h^\beta}$	=/+	+	-	=/-	=	=	+	+
$c_{02}^{h^\beta}$	-	-	+	=/-	-	=	=	-
$c_{12}^{h^\beta}$	=	=/+	=	=/+	=	-	=	-
$c_{22}^{h^\beta}$	=/+	+	-	-	=	=	-	-
Utility α	-	=/-	=/+	=/-	-	=/-	=/+	=/-
Utility β	+	=/+	=/-	=/+	-	=/-	=/+	=/-
μ_{01}^{α}	+	=	=	+	+	=	=	+
μ_{11}^{α}	=	+	=	+	=	+	=	+
μ_{21}^{α}	=	=	+	+	=	=	+	+
μ_{02}^{β}	=	=	=	=	+	=	=	+
μ_{12}^{β}	=	=	=	=	=	+	=	+
μ_{22}^{β}	=	=	=	=	=	=	+	+
p_{01}	+	=/-	-	+	+	=/-	-	+
p_{11}	=	+	=	+	=	+	=	+
p_{21}	=/-	-	+	+	=	=	+	+
p_{02}	-	=/-	-	-	+	=/-	-	+
p_{12}	=	=	=	=	=	+	=	+
p_{22}	=/+	+	-	-	=	=	+	+
$b_{02}^{h^\alpha}$	-	+	-	-	+	=/-	-	+
$b_{12}^{h^\alpha}$	=	=	=	=	=	+	=	+
$b_{22}^{h^\alpha}$	=	=	=	=	=	=	+	+
$b_{01}^{h^\beta}$	+	-	=/+	+	+	=/-	-	+
$b_{11}^{h^\beta}$	=	+	=	+	=	+	=	+
$b_{21}^{h^\beta}$	=	=	+	+	=	=	+	+

	$M_{0\alpha}$	$M_{1\alpha}$	$M_{2\alpha}$	Perm Unco	M_0	M_1	M_2	Perm Co
r_{01}	-	=	=	-	-	=	=	-
r_{11}	=	-	=	-	=	-	=	-
r_{21}	=	=	-	-	=	=	-	-
r_{02}	=	=	=	=	-	=	=	-
r_{12}	=	=	=	=	=	-	=	-
r_{22}	=	=	=	=	=	=	-	-
$\bar{r}_\alpha$	-	+	+	+	-	+	+	+
$\bar{r}_\beta$	+	+	-	+	-	+	+	+
$\pi_{0\alpha\beta}$	-	=	=	-	=	=	=	=
$\pi_{1\alpha\beta}$	=	-	=	-	=	=	=	=
$\pi_{2\alpha\beta}$	=/+	+	-	-	=	=	=	=
$TB(01)$	=/-	-	+	+	=	=	=	=
$TB(11)$	=	+	=	+	=	=	=	=
$TB(21)$	+	+	-	-	=	=	=	=
d_2^{α}	+	-	+	+	+	+	+	+
μ_1^{α}	+	+	+	+	-/+	+	+	+
d_1^{β}	+	+	-	+	+	+	+	+
μ_2^{β}	+	-	+	+	-/+	+	+	+
K_{11}	-	+	+	+	-	-	+	+
K_{12}	=/-	-	+	=/+	-	-	+	+
q_{01}	+	-	+	+	+	=	=	+
q_{11}	=	+	=	+	=	+	=	+
q_{21}	=/+	+	-	=/-	=/+	=	+	+
q_{02}	+	+	-	=/+	+	=	=	+
q_{12}	=	=	=	=	=	+	=	+
q_{22}	=/-	-	+	+	=	=	+	+
$b_{012}^{h^\alpha}$	+	=	=	+	+	=	=	+
$b_{112}^{h^\alpha}$	-	+	-	+	-	+	-	+
$b_{212}^{h^\alpha}$	-	-	+	+	+/-	-	+	+
$b_{021}^{h^\beta}$	=	=	=	=	+	=	=	+
$b_{121}^{h^\beta}$	-	+	-	-	-	+	-	+
$b_{221}^{h^\beta}$	-	+	-	-	+/-	-	+	+

Table 4: Comparative Statics For Nominal Shocks

Columns 1-3 display the effects a marginal change in the money supply in the UK for each state while Column 4 a change in money supply in every state. The remaining columns perform the same exercise but with the US increasing money supply conjointly.

From the quantity theory of money (QTM), a higher domestic money supply has an inflationary effect in the UK and results in expected deflation there. The higher imports by American agents means that Americans have a lower real interest rate in terms of British goods and so they increase deposits there. Lower real interest rates and expected deflation combine to lower the nominal yield in the UK (Fisher Effect (FE)). Greater exports from the US result in deflation there (QTM). This, together with a fall in domestic consumption by Americans results in a rise in the real interest rate for Americans in terms of American goods. A higher real interest rate and and a rise in expected inflation results in a rise in the nominal yield in the US.

The lower nominal yield in the UK induces the British to accumulate more debt in the UK while the higher yields in the US induce them to accumulate assets in the US. In short, the lower interest rates in the UK result in the the global economy becoming more leveraged. Higher debt levels in the future need to be repaid in the presence of liquidity constraints. If the government does not commit to expanding money supply in the future, then from the *on-the-verge* (OTV) condition, the marginal rate of substitution will increase above the default penalty. This encourages both agents to default more as repayment is more costly than default. The result of this is that the delivery rates in both countries fall. In the alternate state of the world where default is not expected, agents also have a higher marginal rate of substitution domestically compared to abroad where they have asset payoffs. From the PPP we know that agents will then optimise by spending less money abroad and we see a fall in the liquidity in the domestic currency market. The trade deficit in period 0 for the UK is offset in expectation by a trade surplus in state 2.

Overall we see a fall in the welfare of the UK while Americans are better off. In a setting with default we need to analyse both the allocation and the deadweight loss associated with the penalty imposed on default. Allocationally the British increase imports in period 0 while in every other state Americans improve their consumption. This result is straightforward from the fact that the first order effect in the model is the change in the short term interest rate in the UK. This prompts a large expenditure switching in the UK from home to abroad and from the future to the present. As Americans respond to price effects, they are not able to accomodate British consumption and trade changes in a mutually beneficial way as would be possible in a closed economy setting. Rather they benefit from the inefficient intertemporal re-allocation of the British.

Non-Coordinated Expansion in Money Supply in Period 1

An expansion in money supply in the UK in state 1 results in a fall in the short term interest rate there from the term structure of interest rates. From PPP we know that the British will optimise by increasing exports and importing more American goods and depreciating the Pound. This induces Americans to increase imports. From the QTM we see inflation in the UK caused by an increase in money supply. From the FE, a higher real interest rate in terms of British goods for the British and expected inflation results in higher debt levels and a higher nominal yield in the UK. The higher yield also attracts greater investment from the US. From the OTV condition a lower short-term interest rate lowers the marginal rate of substitution below the default penalty and hence encourages repayment and we see an increase in the repayment rate in the UK. In the US, the incentive to reduce domestic consumption causes the marginal rate of substitution to increase above the default penalty that results in greater default there. A lower repayment rate will be compensated with by a higher expected return anad so yields rise there. In both countries prices and quantities have limited response in state 1 due to equating marginal rates of substitution with the default penalty. As a result, the main welfare implications are derived intertemporally and so we see an improvement in welfare for American while a worsening for the British agent.

A monetary expansion in the UK in state 2 lowers the interest rate there and creates expected inflation (from TS and QTM). The two effects work in the same direction of increasing the yield in the UK (expected inflation and lower transaction cost) and we see greater borrowing by the British and less deposits by Americans. From PPP, the lower UK spot rate encourages imports and depreciates the exchange rate. The stronger US dollar also encourages Americans to spend more on UK goods and export more that, from QTM, causes a fall in the price level there. The expected deflation in the US results in a lower yield supported by greater capital flows from the UK. The lower yield also stimulates greater debt levels in the US. The greater deposits by the British in the US result in a fall in the marginal rate of substitution in state 1 in the US for him and via the exchange rate to his marginal rate of substitution in the UK. From the OTV this means that the incremental debt undertaken is repaid fully and we see higher delivery rates in the UK. Expecting a higher delivery rate in the UK, Americans increase delivery at home resulting in higher delivery rates globally though also lower liquidity in the currency markets. The increase in foreign assets by the British compared to the Americans is supported by a trade surplus in period 0. These assets are then used to finance a trade deficit in state 2 and so we see a rise in imports and a fall in exports by the UK. In this scenario prices and quantities are able to respond to increases in liquidity so the main welfare come from increases in consumption in state 2 - there is an improvement in welfare for the UK and worsening for the US.

Extra liquidity in the crisis state lowers the rate of default in the UK but transmits this via the exchange rate channel to greater default in the US. Extra liquidity in the good state results in higher delivery rates globally via higher yields and lower marginal rates of substitution in the UK. However, as in the case of an expansion in period 0, the increase in trade provided by the British agent is not reciprocated by the American, so we see the allocation increasingly favouring the US even though the deadweight loss is changing in an inverse manner. This simple example demonstrates the distinction between default and welafare and the need to consider the importance of cross border flows and prices when conducting monetary policy.

Non-Coordinated Permanent Expansion in Money Supply

The permanently lower interest rates in the UK encourage imports and depreciates the Pound. From OTV we see an increase in the delivery rate in the UK and via the greater asset purchases in the UK by Americans, higher delivery in the US. The higher exports in period 0 results in a trade surplus that is offset in state 2 by a trade deficit and we see a marginal fall in exports by the UK. In this exercise there is a fall in welfare for the UK and rise for the US caused by a larger increase in exports by the UK complimented by only marginal increases in exports by the US.

Coordinated Temporary Increase in Money Supply in Period 0

The lower short term interest rate in period 0 globally results in more exports from both countries. The higher income is then used to increase the net asset position. This together with the higher inflation (and expected deflation) results in lower yields globally. In this setting the final allocation has unequivocally improved. However, even though the delivery rate has improved, the greater debt undertaken globally results in a greater deadweight loss caused by default and outweighing the welfare gains from the superior allocation. Such an effect is unique to IMED and requires a formal general equilibrium analysis incorporating trade, money and default.

Coordinated Temporary Increase in Money Supply in Period 1

A global expansion in state 1 results in an increase in exports globally and a rise in expected inflation. It also encourages greater leverage. The increased debt levels offset the lower cost of financing debt resulting in lower repayment rates. As in the case of a period 0 expansion we see an improvement in the allocation of consumption however the higher disutility of default causes welfare to fall globally.

A similar expansion in state 2 results in an increase in delivery rates as the higher global yields encourage asset accumulation and offset the higher repayment of debt. Consequently the welfare loss from default is now less than the welfare gain from the superior allocation and we find a pareto improvement.

Temporary Increase in GDP in Period 0

A higher GDP in the UK in period 0 results in a lower real interest rate and deflation. The lower real interest rate causes lower levels of debt in the UK though the expected inflation causes the yield to rise. From PPP there is an increase in exports and imports (via the UK price channel) raising the real interest rate in the US in terms of American goods. This results in higher debt levels in the US. The trade surplus in period 0 is then offset in expectation by a trade deficit in state 2 where the British import more and export less. The lower debt levels in the UK result in greater delivery rates there while in the US the opposite occurs. Ultimately the higher world product results in an increase in welfare though the gains are more favourable to the UK.

Temporary Increase in GDP in Period 1

A higher GDP in state 1 results in similar effects as in period 0 though lower yield. A higher GDP in state 2 results in a lower yield and greater debt levels as the real interest rate has risen. This causes an increase in the rate of default in state 1 though has a neutral effect

	$e_{0\alpha}$	$e_{1\alpha}$	$e_{2\alpha}$	$\rho\alpha$	$\rho\alpha\rho_\beta$	$\lambda\alpha$	Global		$e_{0\alpha}$	$e_{1\alpha}$	$e_{2\alpha}$	$\rho\alpha$	$\rho\alpha\rho_\beta$	$\lambda\alpha$	Global
$c_{01}^{h^\alpha}$	+	+	=/+	+	-	+	=/-	r_{01}	=	=	=	=	=	=	=
$c_{11}^{h^\alpha}$	=	+	=	-	-	-	-	r_{11}	=	=	=	=	=	=	=
$c_{21}^{h^\alpha}$	+	-	+	=	-	=/-	=/+	r_{21}	=	=	=	=	=	=	=
$c_{02}^{h^\alpha}$	+	+	=/+	+	+	+	=	r_{02}	=	=	=	=	=	=	=
$c_{12}^{h^\alpha}$	=	-	=	=	+	=/-	+	r_{12}	=	=	=	=	=	=	=
$c_{22}^{h^\alpha}$	+	-	+	=	+	=/-	=	r_{22}	=	=	=	=	=	=	=
$c_{01}^{h^\beta}$	+	-	=/-	-	+	-	=	$\bar{r}_\alpha$	+	-	-	-	-	-	-
$c_{11}^{h^\beta}$	=	+	=	+	+	+	+	$\bar{r}_\beta$	+	+	-	+	-	+	-
$c_{21}^{h^\beta}$	-	+	+	=	+	=/+	=	$\pi_{0\alpha\beta}$	=	=	=	=	=	=	=
$c_{02}^{h^\beta}$	-	-	=/-	-	-	-	=/+	$\pi_{1\alpha\beta}$	=	+	=	+	-	+	=
$c_{12}^{h^\beta}$	=	+	=	=	-	=/+	-	$\pi_{2\alpha\beta}$	-	+	+	+	-	=/+	=
$c_{22}^{h^\beta}$	-	+	-	=	-	=/+	=/-	$TB(01)$	+	-	-	-	=	-	=
Utility α	+	+	+	+	+	=/+	=/+	$TB(11)$	=	+	=	+	=	+	=
Utility β	+	=/+	=/+	=/+	+	=/+	=/+	$TB(21)$	-	+	+	+	=	+	=
μ_{01}^{α}	=	=	=	=	=	=	=	d_2^α	+	-	-	-	+	-	+
μ_{11}^{α}	=	=	=	=	=	=	=	μ_1^α	-	=	+	-	-	-	-
μ_{21}^{α}	=	=	=	=	=	=	=	d_1^β	-	+	+	+	+	+	+
μ_{02}^{β}	=	=	=	=	=	=	=	μ_2^β	+	-	-	-	-	-	-
μ_{12}^{β}	=	=	=	=	=	=	=	K_{11}	+	+	-	+	+	+	+
μ_{22}^{β}	=	=	=	=	=	=	=	K_{12}	-	-	=	-	+	-	+
p_{01}	-	-	=/-	-	-	=/-	=/-	q_{01}	+	-	=/-	-	+	-	=/+
p_{11}	=	-	=	-	-	-	-	q_{11}	=	+	=	+	+	+	+
p_{21}	+	-	-	=	-	=/-	=	q_{21}	-	+	+	=	+	=/+	=/-
p_{02}	-	-	=/-	-	-	=/-	=/-	q_{02}	+	+	=/+	+	+	+	=/-
p_{12}	=	=	=	=	-	=/+	-	q_{12}	=	=	=	=	+	=/-	+
p_{22}	-	+	-	=	-	=/+	=	q_{22}	+	-	+	=	+	=/-	=/+
$b_{02}^{h^\alpha}$	=/-	+	=/+	+	-	+	=/-	$b_{012}^{h^\alpha}$	=	=	=	=	=	=	=
$b_{12}^{h^\alpha}$	=	=	=	=	=	=	=	$b_{112}^{h^\alpha}$	=/-	-	=/+	-	-	-	-
$b_{22}^{h^\alpha}$	=	=	=	=	=	=	=	$b_{212}^{h^\alpha}$	+	=	-	+	+	=/+	+
$b_{01}^{h^\beta}$	+	-	-	-	-	-	=/-	$b_{021}^{h^\beta}$	=	=	=	=	=	=	=
$b_{11}^{h^\beta}$	=	=	=	=	=	=	=	$b_{121}^{h^\beta}$	=/-	+	=/+	+	-	+	-
$b_{21}^{h^\beta}$	=	=	=	=	=	=	=	$b_{221}^{h^\beta}$	-	+	=/+	+	+	=/+	+

Table 5: Comparative Statics For Real Shocks

Columns 1-3 display the effects of a marginal increases in the GDP. Column 4 shows the consequences of an increase in the coefficient of risk aversion by UK residents while column 5 shows the effects an increase in global risk aversion. Column 6 is an increase in the severity of regulation in the and finally column 7 is an increase in the severity of regulation globally.

in the US. In both scenarios welfare improves due to the higher aggregate product.

Increase in UK Risk Aversion

Higher risk aversion raises the real interest rate and so agents optimse by spreading more of their consumption to the future lowering the UK yield. A higher coefficient of relative risk aversion also increases the elasticity between imports and home goods so we see greater exports and an increase in imports. The lower prices in the UK encourage Americans to import more goods and also export more causing prices in the US to fall. The expected inflation then causes the yield in the US to rise. The main implication of this exercise is through the OTV condition in state 1 where the higher risk aversion induces agents to deliver more so delivery rates rise. In the US, the greater British imports and resulting stronger pound induce greater exports from the US however from the OTV this causes lower delivery rates and more default. There is an unambiguous gain in welfare for the UK from this and a minor gain for the US caused by a higher delivery rate on its asset offset by higher dead-weight loss of default and an ambiguous change in allocation.

Increase in Global Risk Aversion

In this setting we see an unambiguous pareto improvement following an increase in global risk aversion. This is supported by higher volumes of trade, and hence an improved allocation, higer delivery rates (and less debt), and hence lower dead-weight costs of default.

Increase in the Severity of UK Regulation

A higher default penalty results in the marginal rate of substitution in the UK in state 1 being below the (new) level of regulation so the delivery rate improves. The higher repayment of debt is supported by higher exports so there is an increase in the trade balance in state 1 and a fall in the price level. In the US there is greater consumption of imports and an incentive to export. HOwever as as the marginal rate of substitution there cannot change due to regulation then we see an increase in the default rate there.

The stronger UK trade balance in state 1 is then offset by a trade deficit in period 0. In the UK the lower real interest rate there pushes down the yield there supported by greater US asset purchases and lower UK debt levels. For the US, the US trade surplus in period 0 causes the real interest rate there to rise so we see a higher yield. Overall we see an improvement in global welfare due to greater imports by the US and a lower default rates in the UK.

Increase in the Severity of Global Regulation

As in a unilateral increase in regulation, a global increase in regulation will lower delivery rates and increase trade in state 1. The intertemporal effects of this are ambiguous and so the main welfare implication rest on the improved allocation and lower default rates so we see a pareto improvement.

8 References

Ball, Laurence and David Romer, 1991, Sticky Prices as Coordination Failures, American Economic Review 81:539-552.

Ball, Laurence and David Romer, 1990, Real Rigidities and the Non-Neutrality of Money, Review of Economic Studies 57: 183-203.

Blanchard, Oivier and Nobuhiro Kiyotaki, 1987, Monopolistic Competition and the Effects of Aggregate Demand, American Economic Review 77:647-666.

Dubey, P. and Geanakoplos, J., 1992, The Value of Money in a Finite-Horizon Economy: a Role for Banks, in P. Dasgupta, D. Gale et al. (eds), Economic Analysis of Market and Games. Cambridge: M.I.T. Press.

Dubey, P., Geanakoplos, J., 2003a, Inside and outside money, gains to trade, and IS-LM. Economic Theory 21, 347-397

Dubey, P., Geanakoplos, J., 2003b, Monetary equilibrium with missing markets, Journal of Mathematical Economics 39, 585-618

Dubey, P., Geanakoplos, J., 2005, Default and Punishment in General Equilibrium, Econometrica, vol. 73(1), pages 1-37, 01

Dubey, P., Geanakoplos, J., 2006, Real Determinacy with Nominal Assets and Outside Money, Economic Theory, Vol. 27, pp. 79-106.

Espinoza R. and Tsomocos, D.P, 2008, Asset Prices in an Exchange Economy with Money and Trade, OFRC fe-xx, University of Oxford, Saïd Business School.

Espinoza, R., C.A.E. Goodhart and D.P. Tsomocos, 2007, Endogenous State Prices, Liquidity, Default, and the Yield Curve, Oxford Financial Research CenterWorking Paper No. 2007-FE-01, and Financial Markets Group discussion paper 583 (FMG, LSE).

Friedman, Milton A., 1953, The Case for Flexible Exchange Rates, in Essays in Positive Economics. Chicago: University of Chicago Press

Geanakoplos, J. D. Tsomocos, D. P., 2002, International finance in general equilibrium, Research in Economics, Elsevier, vol. 56(1), pages 85-142, June

Goodhart, C.A.E., Sunirand, P., Tsomocos,D.P., A model to analyse financial fragility, Econ Theory 27, 107-142 (2006)

Kareken, J. and N. Wallace. 1981, On the Indeterminacy of Equilibrium Exchange Rates, Quarterly Journal of Economics, 86: 207-22.

Lucas, Robert Jr., 1982, Interest rates and currency prices in a two-country world, Journal of Monetary Economics, Elsevier, vol. 10(3), pages 335-359.

Obstfeld, Maurice and Kenneth Rogoff, 1995, Exchange rate dynamics redux, Journal of Political Economy. 102, 624-660.

Shubik, M. and Tsomocos, D. P., 1992, A Strategic Market Game with a Mutual Bank with Fractional Reserves and Redemption in Gold (A Continuum of Traders), Journal of Economics, Vol 55(2), pp. 123-150.

Shubik M. and C. Wilson, 1997, The Optimal Bankruptcy Rule in a Trading Economy Using Fiat Money, Journal of Economics, Vol. 37 (3-4), pp. 337-354.

Tsomocos, D.P., 2008, Generic Determinacy and Money Non-Neutrality of International Monetary Equilibria, Journal of Mathematical Economics, 44 (7-8), 866-887, 2008.

9 Appendix

Proof of Theorem 1

Let $M^* \equiv M^\gamma + \sum_{h\in H}\sum_{s\in S^*} m_s^h$ be the total quantity of money appearing in the economy of each country. For $h \in H$ and $\varepsilon > 0$ let

$$\sum_\varepsilon^h = \left\{ \begin{array}{c} (x^h, q^h, b^h, \mu^h, \bar{\mu}^h, d^h, \bar{d}^h, \theta^h, \phi^h, D^{hj} \\ \in \mathbb{R}_+^{L\times S^*} \times \mathbb{R}_+^{L\times S^*} \times \mathbb{R}_+^{L\times S^* + C(C-1)\times S^*} \times \mathbb{R}_+^{J^*} \times \mathbb{R}_+^{J^*} \times \mathbb{R}_+^{(S^*\times(L+1))\times J^*} : \\ 0 \le x^h \le 2A1, \varepsilon \le q_{sl}^h \le \frac{1}{\varepsilon}, \varepsilon m_s^h \le b_{sl}^h \le \frac{1}{\varepsilon}, \varepsilon m_s^h \le \mu_s^h \le \frac{1}{\varepsilon}, \varepsilon m_0^h \le \bar{\mu}_s^h \le \frac{1}{\varepsilon}, \\ 0 \le d_s^h \le M^*, 0 \le \bar{d}^h \le M^*, \varepsilon m_s^h \le \theta_j^h \le \frac{1}{\varepsilon}, \varepsilon \le \phi_j^h \le \frac{1}{\varepsilon}, \varepsilon m_s^h \le D_{sj}^h \le \frac{1}{\varepsilon} \end{array} \right.$$

that is both compact and convex.

Let the typical element of $\sum_\varepsilon^h$ be $\sigma^h \in \sum_\varepsilon^h$. Define $B_\varepsilon^h(\eta) = B^h(\eta) \cap \sum_\varepsilon^h$. Also let $\sigma = (\sigma^1, ..., \sigma^H) \in \sum_\varepsilon = X_h \in H \sum_\varepsilon^h$. Define the map $\Psi_\varepsilon : \sum_\varepsilon \to N$, where

$$N = h = (p, \pi, r, \bar{r}, \psi, K) \in \mathbb{R}_{++}^{LS^*} \times \mathbb{R}_{++}^{C(C-1)\times S^*} \times \mathbb{R}_{++}^{C\times S^*} \times \mathbb{R}_{++}^{C} \times \mathbb{R}_{++}^{J} \mathbb{R}_{++}^{S^*\times J^*}$$

and Ψ is defined by equation (i)-(ix). In addition define (η, σ) to be an $\varepsilon - IMED$ iff $\eta = \Psi_\varepsilon(\sigma)$ and (x), (i.e. (a)$\sigma^h \in Arg \max_{\sigma^h \in B_\varepsilon^h(\eta)} \Pi^h(x^h(\sigma^h))$). Note also that all elements of $\Psi_\varepsilon(\sigma) = \eta$ are continuous functions of σ, since in each market some agents are bidding (offering) strictly positive amounts and repayments are bounded away from 0 by the Inactive Market Hypothesis.

Furthermore, define

$$G : N \Rightarrow \underset{h\in H}{X} \sum_\varepsilon^h = \sum_\varepsilon, \text{where}$$

$$Gh = \sigma^h \in Arg \max_{\sigma^h \in B_\varepsilon^h(\eta)} \Pi^h(xh(h))$$

and

$$G = \underset{h\in H}{X} G^h.$$

Finally, let $F = Go\Psi : \sum_\varepsilon \Rightarrow \sum_\varepsilon$. G is convex-valued since $\sigma \to u^h(x^h(\sigma^h))$ is concave. Recall, $\sigma^h \to x^h(\sigma^h)$ is linear, and that $B_\varepsilon^h(\eta)$ is convex. Since Ψ is a function, $F = Go\Psi$ is also convex valued. Moreover, if ε is sufficiently small, G is non-empty, since $m_s^h > 0\ \forall h \in H$. When $\varepsilon > 0$, $p, \pi, r, \bar{r}, \psi, K > 0$, and since $e^h \neq 0$, $B_\varepsilon^h(\eta)$ for $h \in H$ is a continuous correspondence. Hence, by the Maximum Theorem, G is compact-valued and upper semicontinuous, and therefore so is F. Note that since we have restricted the domain of Ψ to $\sum_{\varepsilon}$ and since for each good and money, some $h \in H$ has a strictly positive endowment, the restriction Ψ to strictly positive prices, and interest rates strictly greater than -1 is legitimate. The same applies for K's since an external agent always guarantees a minimum repayment $\varepsilon > 0$ by the Inactive Markets Hypothesis. Finally, observe that the total amount of money is bounded above. Commodity prices, $p_{sl} \leq (M^* + |L \times S^*|)/\varepsilon \leq 2M^*/\varepsilon \; \forall sl$ and in each country. Thus, $p_{sl}K^j_{sl}A^j_{sl}$ is bounded above and so the external agent never delivers more $\dfrac{|L \times S^*|\varepsilon^2 2M^*}{2M^* \times \varepsilon}$ units of money. Thus the total amount of money is $M^* + (|L \times S^*| + |J|)_{\varepsilon} \leq 2M^*$.

Step 1: An $\varepsilon - IMED$ exists for any sufficiently small $\varepsilon > 0$.

Proof: The map F satisfies all the conditions of the Kakutani fixed point theorem, and therefore admits a fixed point $F(\sigma) \ni \sigma$ that satisfies (i)-(x) for an ε - IMED.

For every small $\varepsilon > 0$, let $(\eta(\varepsilon), \sigma_{\varepsilon})$ denote the corresponding ε- IMED.

Step 2: At any $\varepsilon - IMED$, $r_s(\varepsilon), \bar{r}(\varepsilon), p(\varepsilon) \geq 0 \forall s \in S^*$.
Proof: By Propositions 1 and 2.

Step 3: At any ε-IMED $\exists I, Z < \infty \ni r_s(\varepsilon), \bar{r}(\varepsilon) < I$ and $\bar{\mu}(\varepsilon), \mu_s(\varepsilon) \leq Z, s \in S^*, h \in H$.

Proof: Suppose that $r_s(\varepsilon) \to \infty$. Then $\exists h \in H$ such that $\mu^h_s(\varepsilon) \to \infty$ and consequently $D^h_{sN(s)} = (\mu^h_s - M^*) \to \infty$. Then, since $\lambda >> 0$, $\dfrac{\nabla \Pi^h_{sl}}{p_{sl}(\varepsilon)} << D^h_{sN(s)}$. Thus, h could have been better off by reducing $\mu^h_s(\varepsilon)$ by Δ , a contradiction. Similarly, for $\bar{r}(\varepsilon), \mu^h_s(\varepsilon)$.

Step 4: For any ε-IMED $\exists K > 0 \ni p_{s^*l}(\varepsilon) > K$, $\pi_{s\alpha\beta}(\varepsilon) > K \; \forall s \in S^*, s \in S, \alpha, \beta \in C, l \in L^{h^\alpha}$.

Proof: Suppose that $p_{s^*l}(\varepsilon) \to 0$ for some $s \in S^*$, $l \in L^{h^\alpha}$. Then choose $h \in H^{h^\alpha}$. He could have borrowed Δ more to buy $\Delta/p_{s^*l}(\varepsilon) \to \infty$. His net gain in utility would be $\left(\dfrac{\nabla \Pi^h_{s^*l}}{p_{s^*l}} - \lambda^{hj}_{s^*l} r_s\right) > 0$ since $\lambda^{hj}_{s^*l} r_s < \infty$ and by (A3), $\pi^h(0,,Q,,0) > u^h(A1)$ with Q in the s^*lth place. Thus, $p_{s^*l} \; \nabla \Pi^h_{s^*l}/\lambda^{hj}_{s^*l} r_s > K > 0$.

Similarly an agent from country $\beta \neq \alpha$ with $m^h_{0\beta} > 0$ ($m^h_{s\beta} > 0$) can purchase $m^h_{0\beta}/(\pi_{s\beta\alpha}p_{sl})$ ($m^h_{s\beta}/(\pi_{s\beta\alpha}p_{sl})$) units of good sl for $s \in S$ and $l \in L^{h^\alpha}$ by hoarding his β-money, exchanging it for α-money and then purchasing good l. If $\pi_{s\beta\alpha}p_{sl} \to 0$ then he could purchase an infinite amount of good sl, contradicting the fact that we are at an ε-IMED. Thus $\pi_{s\beta\alpha}p_{sl} > K$. As $p_{sl} > K$ and $\pi_{s\beta\alpha}p_{sl} > K$ then $\pi_{s\beta\alpha} > K$.

Step 5: For any ε-IMED $\exists \Gamma \ni \psi^h_j(\varepsilon) < \Gamma, \forall j \in H, h \in H$.

Proof: Suppose that for some $j \in J$, $\psi_j^h(\varepsilon) \to \infty$ and $K_{sl}^j A_{sl}^j > 0$, for some $l \in L$. Then h can deliver at most $\phi_j^h A_{sl}^j \max_{\substack{1 \le l \le L \\ s \in S^*}} \sum_{h \in H} e_{sl}^h = \bar{e}$, and therefore his disutility from default would be $(\lambda_j^{hj} p_{sl} \phi_j^h A_{sl}^j - M^*)/p_s g_s < u(A1)$. Otherwise, $\sigma^h(\varepsilon)$ are not optimal.

Similarly, suppose that $A_{s,m}^j > 0$. Again h can deliver at most $\phi_j^h A_{S,m} \le \bar{e}$ and then his disutility from default would be $(\phi_j^h A_{S,m})/p_s g_s \le u(A1)$. Otherwise, he would not have optimized.

Step 6: For all $h \in H, d_s^h, \bar{d}^h, b_{sl}^h, b_j^h, u^h \le 2M^* \le 2M^*$ and $|KA| < \varepsilon$.

Proof: All variables are constrained by the total amount of money present in the economy and KA by assumption.

Step 7: For all $h \in H$, $\sigma_\varepsilon^h = \underset{\sigma_\varepsilon^h \in B^h(\eta(\varepsilon))}{\arg\max}\ U^h(x^h(\sigma^h))$, for sufficiently small $\varepsilon > 0$.

Proof: From steps 2-6 and the budget constraints (3^h), (8^h) of 3.1 and (1h) of 3.2, the ε-constraint is not binding thus concavity of payoffs guarantees the optimality of $\sigma^h(\varepsilon)$. IMED(η,σ) will be constructed by taking the limit of $\varepsilon - IMED$ $(\eta(\varepsilon),\sigma(\varepsilon))$, as $\varepsilon \to 0$. This is achieved by taking limits of sequences of ε and subsequences of subsequences.

Step 8: If for some $\bar{s}\bar{l}, p_{\bar{s}\bar{l}} \to \infty \forall s \in S, l \in L$. Also if $\theta_j(\varepsilon) \to \infty$ or $p_{0l}(\varepsilon) \to \infty$ then then $p_{sl}(\varepsilon) \to \infty\ \forall l \in L,\ s \in S^*$.

Proof: Some h owns $e_{\bar{s}\bar{l}}^h > 0$. If $p_{sl}(\varepsilon)$ stays bounded on some subsequence, then by borrowing very large $\bar{\mu}^h$ or μ_0^h if $s = 0$, h can use it to buy Q units of sl. Then since $\bar{r}(\varepsilon)$, $r_s(\varepsilon) < I$, h can sell Δ of $\bar{s}\bar{l}$ acquire $\Delta p_{\bar{s}\bar{l}}(\varepsilon) \to \infty$ to defray his loan and improve his utility, a contradiction.

If $\theta_j(\varepsilon) \to \infty$ for some $j \in J$ and $p_{sl}(\varepsilon) < \infty$, let h borrow $\Delta\theta_j(\varepsilon)/(1+r_0(\varepsilon))$ and buy $\Delta\psi^j(\varepsilon)/(1+r_0(\varepsilon))p_{sl}(\varepsilon)$ of sl and improve his utility. If $p_{0\bar{l}}(\varepsilon) \to \infty$, as previously argued then $p_{0l}(\varepsilon) \to \infty$,$\forall l \in L$. Then, by selling Δ of $o\bar{l}$ h can acquire $\Delta p_{0l}(\varepsilon) \to \infty$. If any of $p_{sl}(\varepsilon) \nrightarrow \infty$, $s \in S$ then by inventorying money he can improve upon his utility.

Step 9: $\exists K > 0 \ni p_{sl}(\varepsilon)/p_{sk}(\varepsilon) < k,\ \forall l,k \in L,\ s \in S$.

Proof: Suppose the opposite. Then take h with $e_{sl}^h > 0$. Let him reduce Δ of his sales of sl and lose $\Delta(\Pi^h(A1) - \Pi^h(0))$ at most. Then he could buy more sk buy borrowing $p_{sl}(\varepsilon)/(1+r_s(\varepsilon))$ and sell Δ of sl. His net gain in utility would be

$$\Delta(\varepsilon)\left\{\frac{p_{sl}(\varepsilon)}{(1+r_s(\varepsilon)p_{sk}(\varepsilon)}\left(\nabla\Pi_{sk}^h(x^h)\right) - (\Pi(A1) - \Pi(0)\right\} > 0$$

since $p_{sl}(\varepsilon)/p_{sk}(\varepsilon) \to \infty$ and by step 3, $r_s(\varepsilon) < I$.

Step 10: $\exists K' > 0 \ni p_{0l}(\varepsilon)/p_{sl}(\varepsilon) < K', \forall s \in S^*, l \in L$.

Proof: If $s = 0$ then step 9 obtains. Otherwise, set $\Delta(4h)$ of Section 3.1 equal to $\Delta p_s l(\varepsilon)/1 + r_s(\varepsilon)$.

Step 11: $\theta_j(\varepsilon)/\sum\limits_{l \in L} p_{0l}(\varepsilon) \nrightarrow \infty, \forall j \in J$.

Proof: Suppose the contrary. Let h sell $\dfrac{\Delta}{(1+\bar{r}(\varepsilon))}$ of j and borrow $\dfrac{\Delta \bullet \theta_j(\varepsilon)}{(1+\bar{r}(\varepsilon))}$ more. Let him consume $\dfrac{\Delta \bullet \theta_j}{(1+\bar{r}(\varepsilon))p_{0l}(\varepsilon)}$ more ol for some $l \in L$ in $s = 0$. Then h can use the proceeds of the asset sale to defray the loan. His net gain of this action will be

$$\Delta\left(\frac{\theta_j}{(1+\bar{r}(\varepsilon)p_{0l}} - \frac{A^j_{sl}}{(1+\bar{r}(\varepsilon)}\right) > 0$$

since $\dfrac{\theta_j(\varepsilon)}{p_{0l}(\varepsilon)} \to \infty$.

Step 12: If $\theta_j(\varepsilon) \Big/ \sum\limits_{l \in L} p_{0l}(\varepsilon) \to 0$ then $K^j_{sl}A^j_{sl} = 0, \forall l \in L$ and $\sum\limits_{l \in L} p_{0l}(\varepsilon) \to \infty$, whenever $R^j_{sm} > 0$, $\forall s \in S$.

Proof: Suppose $R^j_{sl} > 0$ for some $s \in S$, $l \in L$. Choose $h \in H$ with $e^h_{0l} > 0$ for some $l \in L$. Let h sell $\dfrac{\Delta}{(1+\bar{r}(\varepsilon))}$ more of $0l$ and increase his loan by $\left(\dfrac{\Delta}{(1+\bar{r}(\varepsilon))}\right) p_{0l}$. Then he could purchase $\dfrac{\Delta \bullet p_{0l}}{(1+\bar{r}(\varepsilon))(1+r_0(\varepsilon))\theta_j(\varepsilon)}$ of j. Then, by borrowing in s and defraying his loan by asset deliveries he can improve his payoff, a contradiction. The same argument applies if $R^j_{sm} > 0$ and $\sum\limits_{l \in L} p_{0l}(\varepsilon) \nrightarrow \infty$.

Step 13: There exists $K \ni p_{sl}(\varepsilon) < K \forall s \in S^*, l \in L$.

Proof: Suppose the contrary and w.l.o.g. suppose that $p_{\bar{s}\bar{l}} \to \infty$ for some $\bar{s} \in S^*, \bar{l} \in L$. Since $p_{sl}(\varepsilon) = \dfrac{\sum_{h \in H} b^h_{sl}(\varepsilon)}{\sum_{h \in H} q^h_{sl}(\varepsilon)} \leq \dfrac{M^*}{\sum\limits_{h \in H} q^h_{sl}(\varepsilon)}$, it must necessarily be $q^h_{sl} \to_{\varepsilon \to 0} 0$ for all $s \in S^*$, $l \in L$ by step 8. At any ε -IMED, $\bar{r}(\varepsilon)r_s(\varepsilon) < \delta_s$ by step 3. Hence, at any ε-IMED, there are less than δ_s-gains to trade. By continuity, there are less than δ_s- gains to trade at $(e^h)_{h \in H}$. However, G-to-T hypothesis guarantees that there are more than δ_s-gains to trade $\forall s \in S$, a contradiction.

Step 14: There are $0 < k < K$ such that the exchange rates are bounded:

$$k < \pi_{s\alpha\beta}(\varepsilon) < K \quad \forall s \in S^*, \alpha, \beta \in C.$$

Proof: We showed in Step 4 that $\pi_{s\beta\alpha}(\varepsilon) > K \; \forall \alpha, \beta \in C$. But $\pi_{s\beta\alpha}(\varepsilon) = \pi^{-}1_{s\alpha\beta}(\varepsilon)$, hence $\pi_{s\beta\alpha}(\varepsilon)$ is bounded from above.

Step 15: $\eta = \lim_{\varepsilon \to 0} \eta(\varepsilon)$ and $\lim_{\varepsilon \to 0} (\eta(\varepsilon), \sigma(\varepsilon)) = (\eta, \sigma)$.

Proof: From the previous steps, $\eta(\varepsilon)$ is bounded in all components. The same applies for $\sigma(\varepsilon)$. Thus, a convergent subsequence can be selected that obtains (η, σ) in the limit. By continuity of $\Pi^h(\sigma^h)$, (η, σ) is a IMED, and the artificial upper and lower bounds on choices are irrelevant since they are not binding and payoff functions are concave in actions.

Proof of Proposition 1: From Proposition 8 and term structure of interest rates proposition, $r_s = 0, \forall s \in S^*$ and $\bar{r} = 0$. Then, from the definition of IMED and GEI the proposition follows immediately.

Proof of Proposition 2: Let $M^\gamma \to \infty$ and consider bounded asset trades. Then by choosing subsequences of further subsequences select a subsequence along that all relative σ's and η's converge. By Proposition 1, the limit of the last subsequence coincides with a GEI, a contradiction. Thus $\sum_{h \in H} \phi^{hj} \to \infty$. Thus by feasibility, $M^\gamma / \left\| \theta^j \right\| \to \infty$. Finally by the boundedness of η's $M^\gamma / \| p_{0l} \| \to \infty$. Similarly, Interiority of the maximum on $x_s^h \forall s \in S^*, h \in H$ guarantees bouded consumption.

Proof of Proposition 6:
(i) if $r_{s\alpha} < 0$ or $\bar{r}_\alpha < 0$ for any $s \in S^*$ and $\alpha \in C$ then agents could infinitely arbitrage the central bank and/or other lenders lenders.

(ii) if $(1 + \bar{r}_\alpha) < (1 + r_{0\alpha})\tilde{K}_{0\alpha}$ an agent endowed with fiat money could invest in the money market 1 unit of α currency and obtain $(1 + r_{0\alpha})\tilde{K}_{0\alpha}$ at the beginning of period 1. In the meantime he could borrow 1 unit in the bond market, using the proceeds for consumption and incurring a debt of $(1 + \bar{r}_\alpha)$ in the next period. If this is less than his earnings in the money market he has thus earned an arbitrage profit, that we rule out in equilibrium.

Proof of Proposition 8: Suppose $\theta_j < \lambda_1\theta_1 - (1 + r_{0\alpha})\lambda_2\theta_2$. Then, let a seller of asset j reduce his sale by ε and borrow $\varepsilon\lambda_2$ more. He can use the money obtained from the loan to buy $\varepsilon\lambda_2$ more of asset 2 and sell $\varepsilon\lambda_1$ more of asset 1. Then h has to deliver $\varepsilon(\lambda_1 A^1 - \lambda_2 A^2)$ less but he also receives ^{j} less . So, his net future deliveries remain unaffected. However, since $\varepsilon\lambda_1\theta_1 - \varepsilon(1 + r_{0\alpha})\lambda_2\theta_1 > \varepsilon\theta_j > 0$, he can pay back his loan and use his remaining savings to pay back an extra loan that he can use to increase his consumption, a contradiction with optimization. For the second part of the inequality, suppose$\theta_j > \lambda_1\theta_1 - (1/(1 + r_{0\alpha}))\lambda_2\theta_2$ and apply the reverse argument.