# Pass-through as an Economic Tool*

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**Abstract**

Pass-through rates play an analogous role in imperfectly competitive markets to elasticities under perfect competition. Log-curvature of demand links the pass-through of cost and production shocks to the division between consumer and producer surplus. Therefore in a wide range of single-product, symmetric multi-product, two-sided markets and merger analysis models knowledge of simple qualitative properties of the pass-through rate sign, and full knowledge of it quantifies, many comparative statics. Most functional forms for theoretical and empirical analysis put unjustified ex-ante restrictions on pass-through rates, a limitation our *Adjustable-pass-through* (Apt) demand and *Constant Pass-through Demand System* (CoPaDS) avoid.

*A few of the results here were originally circulated as parts of other papers: "The Price Theory of Two-Sided Markets", "Double Marginalization, Vulnerability and Two-Sided Markets" and "Double Marginalization in One- and Two-Sided Markets". No results overlap with current drafts of other papers, except where explicitly cited. We are grateful to the Economic Analysis Group at the Antitrust Division of the United States Department of Justice, the University of Chicago Becker Center on Price Theory, the Toulouse School of Economics, el Ministerio de Hacienda de Chile which hosted us on visits while we conducted this research. The Milton Fund supported the last stage of this project and financed the excellent research assistance provided by Ahmed Jaber, Rosen Kralev, Dan Sacks and especially Yali Miao. We also appreciate the helpful comments and advice on this research supplied by many colleagues, particularly Gary Becker, Jeremy Bulow, Xavier Gabaix, Faruk Gul, Joe Farrell, Jerry Hausman, James Heckman, Kevin Murphy, Aviv Nevo, Ariel Pakes, Bill Rogerson, José Scheinkman, Carl Shapiro, Andrei Shleifer, Jean Tirole and seminar participants at el Banco Central de Chile, the Justice Department, Northwestern University, Princeton University, Stanford's Graduate School of Business, University of California Berkeley's Haas School of Business and University of Chicago. All confusion and errors are our own.

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Elasticities of supply and demand play fundamental roles in the analysis of competitive markets. Will raising a tax increase or decrease revenue? It depends on the elasticities of supply and demand. As Harberger (1964) first argued and Chetty (Forthcoming) surveys, elasticities are sufficient statistics for a wide range of welfare analysis in competitive markets.

For a monopolist, however, the elasticity of demand determines the level, rather than the comparative statics, of price. The slope of elasticities therefore takes the place of its level in imperfectly competitive markets. This log-curvature of demand determines how the firm's optimal mark-up varies with cost and therefore the *pass-through rate*[1], at which a firm finds it optimal to pass-through increases in that cost to consumers.

Under generalizations of many common demand systems, changes in other firms' prices affect firm behavior much as changes in cost do, a feature used to identify many empirical industrial organization models (Berry et al., 1995). How (absolute) pass-through compares to one therefore determines, among other things, whether Bertrand competition exhibits strategic substitutes or complements and whether the entry of a competitor raises or lowers equilibrium prices (Section III). Yet virtually all commonly used demand forms place undue restrictions on pass-through rates (Subsection V.A). These bias policy analysis as, for example, the magnitude of merger effects, and even their sign in two-sided markets, are driven by pass-through (Subsections IV.A-B). We therefore propose functional forms that allow flexible pass-through (Subsections IV.B and V.B), as well as deriving testable and policy-relevant predictions that hold under weak, non-parametric assumptions (Section II).

Section I develops pass-through in the simplest monopoly context. The common assumption of log-concave demand is equivalent to pass-through less than one-for-one (cost-absorbing), but an equally tractable, testable second-order condition allows for arbitrary (positive) pass-through rates. We show pass-through to be a unit-less measure of the inverse concavity of monopoly profits about their optimum, applying equally to production and pricing. In particular pass-through is high when the monopolist is nearly indifferent over her price because large infra-marginal consumer surplus tempts her to raise price. Thus the ratio of consumer to producer surplus at the monopoly optimal price is the average value of the pass-through rate above that optimal price. Throughout the paper we maintain *Signed Pass-through Assumptions* (SPAs), obeyed by nearly all common demand functions, requiring that the pass-through rate stays on the same side of one as prices change and that it is monotonic in price/cost. Though empirical evidence is sparse, pass-through rates above one appear fairly common, though those below one seem somewhat more frequent.

We apply pass-through to a range of oligopoly models, beginning in Section II with a

[1]We use "pass-through rate" and "pass-through" interchangeably. This always refers to absolute pass-through not the elasticity of price with respect to cost.

simple example, the two-firm Cournot (1838)-Spengler (1950) model of vertical monopolies (double marginalization). The pass-through rate determines theoretically through strategic effects, and observed pass-through identifies empirically, the relationship among all firm and industry mark-ups and profits within and across industrial organizations (sequential, simultaneous and integrated). These results generalize to determine the effects of changing industrial organizations within a broad *Generalized Cournot-Stackelberg vertical monopolies model* we propose, where an arbitrary number of firms act in arbitrary sequence. Furthermore, by their duality (Sonnenschein, 1968) with quantity competition, the same results apply to the corresponding generalization of the Cournot (1838) and von Stackelberg (1934) models of symmetric linear cost quantity competition when the *quantity pass-through rate* (Section I.C), also identified by cost shocks, is substituted for the pass-through rate.

Section III shows how the results generalize to symmetric multi-product oligopoly models with linear costs, with either price interactions in the spirit of Bertrand (1883) or quantity interactions in the spirit of Cournot (1838). We call a demand system *horizontal* if changes to other firms' prices (quantities) enter as effects on willingness to pay (purchase) but demand is otherwise arbitrary. Horizontality implies that an increase in, say, a competitor's price have the opposite effect on a firm's optimal mark-up to an increase in that firm's cost. Therefore equilibrium own (individual firm) and industry-wide pass-through rates lie on the same side of one as primitive (demand curvature) pass-through and, together with the form of oligopoly (substitutes or complements, Bertrand or Cournot), determine whether there are strategic complements or substitutes. This helps resolve a basic confound in oligopoly theory highlighted by Fudenberg and Tirole (1984) and Bulow et al. (1985): nearly any result and its opposite can be obtained by assuming strategic complements or substitutes. It also addresses theoretical disputes about the effect of entry or mergers on prices (Chen and Riordan, 2008) and the effect of competition on own and industry pass-through rates (Farrell and Shapiro, 2008). Using formulae derived by Gabaix et al. (2009), we show that the same results hold for a wide range of discrete choice and auction models, including the (non-parametric) Berry et al. (1995) model, when there are a large number of symmetric firms. In these discrete choice models, pass-through rates are determined by the log-curvature of the idiosyncratic variations in consumer preferences underlying the model.

Section IV overviews two other applications from our previous work and the work of others. The crucial policy implications of the Rochet and Tirole (2003)(RT2003) and independent-valuations Hermalin and Katz (2004) models of two-sided markets turn on pass-through and the model can be tested using exogenous cost variations (Weyl, 2009). Pass-through allows the extension of the double marginalization problem to the RT2003 model of two-sided markets (Weyl, 2008); the greater complexity of this model actually enhances its predictive power. Froeb et al. (2005) and Farrell and Shapiro (2008) demonstrate that pass-through determines the magnitude of unilateral merger effects under generalized Bertrand oligopoly. We generalize linear demand to a class of *Horizontal Constant Pass-through Demand Systems* (HCoPaDS) that are tractable for merger analysis and flexible on pass-through rates. We use these to explore when mergers providing local incentives for higher prices lead to higher prices in equilibrium as is commonly assumed.

Section V shows that (at least the single-product forms of) demand functions typically used in industrial organization severely and implausibly restrict the pass-through rate and its slope, directly imposing, rather than empirically measuring, answers to the theoretical questions which turn on these. This may be problematic in more general oligopoly models because of the connection between log-curvature of idiosyncratic variations and pass-through. We propose an alternative *Adjustable-pass-through* (Apt) demand form (distribution of idiosyncratic variations) allowing simultaneously extreme tractability and flexibility regarding pass-through and its slope. Section VI concludes by discussing directions for future research.

Given its length, the sections of the paper are designed to be largely independent of one another. Most formal results are established in appendices that appear separately from this paper, along with mathematical software for estimating and manipulating Apt demand, at http://www.people.fas.harvard.edu/∼weyl/research.htm.

# I. Monopoly Pricing and Pass-through

## A. Basics

Consider a monopolist facing consumer demand $Q(\cdot)$ (assumed decreasing and thrice continuously differentiable) and constant marginal cost of production $c$. We maintain our (relatively innocuous) assumption of smooth demand and our (quite strong) assumption of constant marginal cost[2] throughout the paper. The monopolist's familiar first-order condition is :

$$m \equiv p - c = \mu(p) \equiv -\frac{Q(p)}{Q'(p)} = \frac{p}{\epsilon(p)} \tag{1}$$

where $\epsilon(p)$ is the elasticity of demand. We refer to $\mu$, the ratio of price to elasticity of demand or the inverse hazard rate of demand, as the firm's *market power*. Note that $m$, which we refer to as the firm's mark-up, is an absolute, not relative, terms: $m = p - c$.

[2]This separates the demand side analysis here from the relatively independent supply side analysis which should be added to it in many applications. See our discussion at the end of Subsection III.A.

## B. Second-order conditions and pass-through

A common condition ensuring the sufficiency sufficiency of equation (1) for optimization is that demand is log-concave ($\log(Q)'' < 0$), which is equivalent to market power being decreasing. However this condition is grossly sufficient[3] for this purpose. More importantly it restricts the pass-through rate[4], the amount a monopolist finds it optimal to raise prices in response to a small increase in cost. Implicit differentiation shows that a monopolist's optimal absolute, not relative ($\frac{dp}{dc}\frac{c}{p}$), *pass-through rate* (or *pass-through* for short) of linear cost is given by

$$\rho \equiv \frac{dp}{dc} = \frac{1}{1-\mu'} \tag{2}$$

In what follows, where not otherwise explicitly indicated, we use *pass-through* to refer to the primitive property of demand $\frac{1}{1-\mu'}$, which determines the optimal absolute pass-through rate of a linear cost monopolist, rather than the actual equilibrium pass-through rate in a particular industry. As can easily be seen from equation (2), log-concavity (convexity) is equivalent to pass-through being less (greater) than 1-for-1. We will therefore generally refer to log-concave demand as "cost-absorbing" (e.g. linear) and log-convex demand as "cost-amplifying" (e.g. constant elasticity), using the terminology of Rochet and Tirole (2008). A much weaker condition than cost absorbtion that makes equation (1) sufficient for the monopolist's optimization is that $\mu'(p) < 1$ for all $p$[5]. It assumes that as mark-up increases, the marginal incentive to increase mark-up declines. We therefore refer to this condition as "mark-up contraction" (MUC). We call $\mu'(p) \leq 1$ for all $p$ weak MUC. The main testable implication of this assumption is that a firm facing a binding price control will choose to charge at the controlled price.

**Theorem 1.** *If demand exhibits MUC then any solution to equation (1) is the monopolist's optimal price and for any cost a monopolist facing price ceiling (floor) below (above) her unconstrained optimum will always choose to charge a price at that ceiling (floor). Conversely if $Q$ fails to satisfy weak MUC, even at single point, then for some cost*

1. *there is a solution to (1) which is not optimal.*

2. *there is a price ceiling (or floor) below (above) the monopolist's unconstrained optimal*

[3]For an extensive discussion of the properties of log-concave functions (and particularly probability distributions), see Bagnoli and Bergstrom (2005). The authors also discuss a wide variety of economic applications, including many prominent papers in industrial organization, where log-concavity is assumed.

[4]As Jeremy Bulow pointed out to us, this is equivalent to marginal revenue curve sloping down more steeply than inverse demand.

[5]This is the same as marginal revenue declining in quantity or $\frac{1}{Q}$ being convex in price. This regularity condition is commonly used in auction theory (Myerson, 1981).

*price given that cost such that the monopolist chooses a constrained price strictly below (above) that ceiling (or floor).*

*Proof.* See appendix *Monopoly* Section I.

MUC is the weakest condition ensuring global "first-orderness" of the monopoly's problem. Because it has no grounding in consumer theory it is a strong restriction on demand functions. It is, however a reasonable methodological commitment over the range in which one analyzes monopoly problems in most applications for two reasons.

First, from a theoretical perspective it amounts to a natural extension of simplifying assumptions made for the sake of tractability, such as differentiability of demand. Many results derived on the basis of such assumptions, which are used to simplify and therefore make analysis more transparent, can easily be generalized (Milgrom and Shannon, 1994; Amir and Grilo, 1999; Amir and Lambson, 2000) and they are therefore typically technical conveniences rather than substantive restrictions. Second, in empirical applications cost, price and quantity information over a limited range is commonly used for estimation. It seems difficult, if not impossible[6], to make predictions about discontinuously different outcomes based on such data. Thus the plausibility of structural empirics implicitly rest on these assumptions of endogenous continuity of firm behavior, based on unimodality of consumer valuations. Whether or not such an approach is plausible in most markets is largely an open question[7], though one that is empirically testable by Theorem 1.

## C. Pass-through as an elasticity

If a monopolist faces a rather rigid "price the market will bear" then her optimal price is very sharply defined and increases in cost will move her optimal price significantly. On the other hand, if the monopolist is close to indifferent between a range of prices, a small increase in cost can cause a dramatic shift in optimal price[8]. This can be seen formally by noting that (at the monopoly optimal price)

$$\rho = \frac{1}{-\frac{d^2\pi}{dm^2}\frac{m^2}{\pi}} \tag{3}$$

[6] An analyst would require a estimate of third-order properties of demand, so that the rate at which firm best response curves return to a fixed point could be measured.

[7] Evidence of multi-modality in the distribution of consumer preferences in supermarkets presented by Burda et al. (2008), based on much more flexible, non-parametric estimation than is typically employed, is therefore worrying.

[8] Joe Farrell suggests that when pass-through is high it may also be unpredictable if firms are imperfect profit maximizers. One way to think about pass-through is as the inverse of the friction on a plane. Costs gives the firm a shove and it slides longer, but also less predictably, the higher pass-through is.

Thus pass-through is exactly the inverse of the second-order elasticity of profits[9] with respect to mark-up. This provides a simple way to interpret the theme running through this paper, that pass-through is the analog of elasticity in monopoly problems. In choosing their optimal price, monopolists take first-order effects (elasticities of demand) into account. Therefore the *level* of price elasticity is replaced by its own *elasticity*.

The fact that pass-through is an elasticity (a unit-less measure), rather than a derivative, implies that it also determines the comparative statics of the monopolist's production. Imagine a (pseudo-)monopolist choosing an optimal quantity to produce, given that there exists some exogenous quantity[10] $\tilde{q}$ of the good already available. Let $q^\star \equiv \tilde{q} + q_M$, the monopolist's optimal production, be the total industry production given that the monopolist optimizes. By duality with the pricing problem, the monopolist's first-order conditions are

$$q^\star - \tilde{q} = -\frac{P(q) - c}{P'(q)} \equiv \kappa(q) \tag{4}$$

where $P(q) \equiv Q^{-1}(q)$ and $\kappa$ is the *market capacity*. The natural quantity analog of the pass-through rate is the *quantity pass-through rate*[11]

$$\rho_q \equiv \frac{dq^\star}{d\tilde{q}} \tag{5}$$

**Theorem 2.** *$\rho(p^\star) = \rho_q(q^\star)$ when the optima are for the same values of $c$ and $\tilde{q}$.*

*Proof.* By duality $\rho_q = \frac{1}{1-\kappa'}$. So we just need to show that at the monopoly optimal price/production $\kappa' = \mu'$. To see this note

$$\kappa' = \frac{m''m}{(m')^2} - 1 = -m(Q')^2 \cdot \frac{Q''}{(Q')^3} - 1 = -\frac{mQ''}{Q'} - 1 = \frac{Q''Q}{(Q')^2} - 1 = \mu'$$

where the first equality follows from differentiation, the second from the inverse function theorem (as $m' = P'$), the third from equation (1), the fourth from the definition of market power and the final from differentiation again.

[9] We suspect that, given the role of second derivatives in statistical discrimination problems (Fisher, 1922; Chernoff, 1959), that the pass-through is likely related to the optimal degree of price experimentation by a monopolist. An interesting topic for future research is to understand the relationship between pass-through and experimentation, as this may provide a way of generating testable implications of optimal experimentation, either with or without rational expectations.

[10] While this "exogenous quantity" may appear an artificial construct, it has, as shown below, a very natural interpretation in the context of quantity competition. In particular$\rho_q - 1$ determines the strategic complementarity (if it is positive) or substitutability (if negative) in quantity competition. The first work to (implicitly) link pass-through rates and the strategic interactions in Cournot competition was Seade (1986).

[11] In appendix *Apt Demand* Subsection II.B we briefly discuss the equivalent of MUC for the production problem, which we assume in analyzing the production problem.

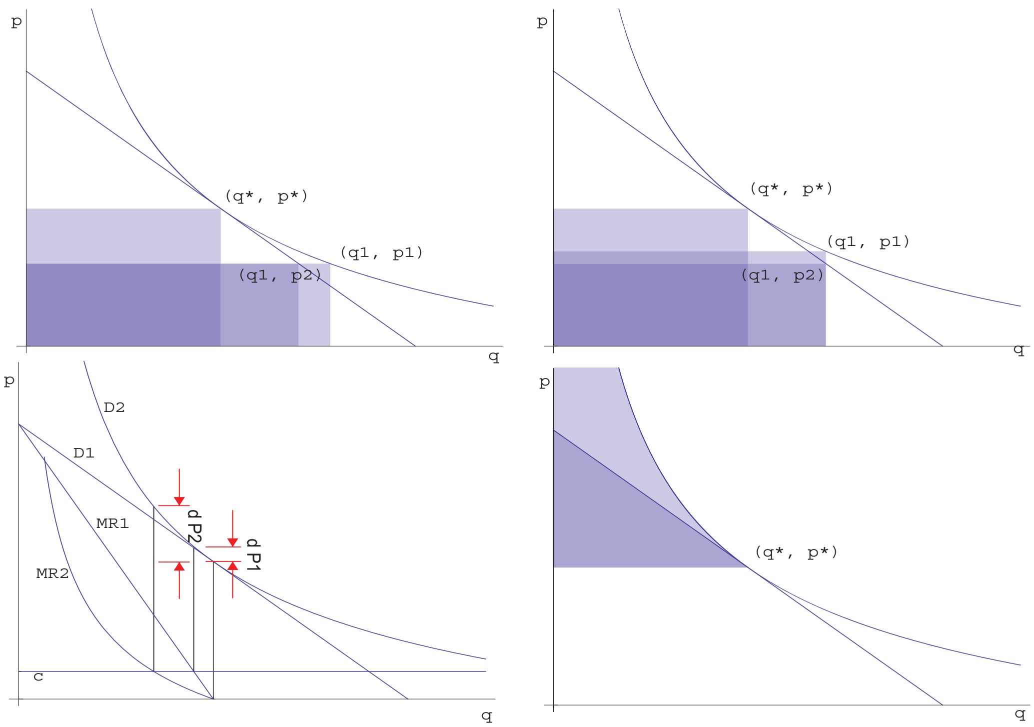


Figure 1: A low $Q_1(p) = 1 - p$ and high $Q_2(p) = \frac{2}{(1+2p)^2}$ constant pass-through demand function. High pass-through leads profits to die off more slowly if either price (Panel 1) or quantity (Panel 2) changes away from the optimum compared to low pass-through. It also leads pass-through of an increase in cost to price to be higher, as can be seen by drawing the corresponding marginal revenue curves (Panel 3). Finally it leads to higher consumer surplus as shown in Panel 4.

A graphical intuition for the proof is provided in Figure 1. The first three panels in the figure show two different constant pass-through demand functions, one (linear demand) with a low pass-through rate and one (translated constant elasticity) with a high pass-through rate with the same monopoly optimal price and consumer demand at that price $(q^{\star}, p^{\star})$. The first two panels demonstrate that a change in price or quantity away from this optimum will lead profits to die off much more quickly when pass-through is low than when it is high. The third panel shows the associated marginal revenue curves for each demand and the price increase for each resulting from a small increase in cost (from 0), showing how high pass-through results from slow fall-off in profits as prices change.

## D. Pass-through and the division of surplus

A monopolist will be close to indifferent over a range of prices if there is large, infra-marginal consumer surplus relative to the profits she is earning; she will then be conflicted between chasing this surplus by charging a higher price and continuing to serve a larger market. The pass-through rate is therefore closely related to the division of surplus between consumers and producers at monopoly optimal prices[12]. Graphical intuition for this is supplied in Figure 1. The last panel illustrates the fairly obvious point that high pass-through leads to high consumer surplus under constant pass-through demand. The result holds more generally.

Formally, when price $p$ is charged and $\overline{p}$ is the maximum price (possibly $\infty$) at which demand is strictly positive, consumer surplus is given by

$$V(p) \equiv \int_p^{\overline{p}} Q(q)dq$$

and producer surplus, the monopolist's profits, is $\mu(p)Q(p)$ by her first-order conditions. Therefore the ratio of consumer to producer surplus is

$$r(p) \equiv \frac{V(p)}{Q(p)\mu(p)}$$

The following theorem states that this division of surplus[13] is given by an average of the pass-through rate over prices above the monopolist's optimal price.

**Theorem 3.** *Assume MUC and* limit MUC, *that* $\lim_{p\to\infty} \mu'(p) < 1$. *Then*

$$r(p) = \overline{\rho}(p) \equiv \int_p^{\overline{p}} \lambda(q;p)\rho(q)dq$$

[12]This reasoning also suggests, as pointed out to us by colleagues at the University of Chicago, the only compelling reason we know of why we may not expect to find high pass-through rates in real industries with market power. If consumer surplus is large relative to profits and a firm has significant market power, it is likely to try to price discriminate to capture some of that surplus. We may therefore expect that industries that would have high pass-through rates under uniform pricing will not use uniform pricing at all. This is an interesting empirical hypothesis and suggests further investigation of the relationship between pass-through rates and price discrimination. It also suggests a connection (Bulow and Roberts, 1989), explored only sparsely below, of pass-through to auction theory.

[13]One might also wonder if the size of deadweight loss relative to monopoly profits or consumer surplus is related to pass-through. In appendix *Monopoly* Section III we show that for constant pass-through demand, deadweight loss due to monopoly $d = \left(\rho^{\frac{-\rho}{\rho-1}} - \rho - 1\right)\pi$ where $\pi$ is the monopolist's maximum profits. It is easy to show based on this that the ratio of consumer surplus to deadweight loss ($\frac{\rho\pi}{d}$), as does the ratio of deadweight loss to monopoly profits ($\frac{d}{\pi}$, as pass-through grows. Intuitively as pass-through increases both consumer surplus and deadweight loss grow relative to monopoly profits as the tails of demand get larger, but consumer surplus grows more rapidly as it is not "truncated" at costs as deadweight loss is. We also conjecture that increasing (decreasing) pass-through leads to larger consumer surplus relative to deadweight loss than does constant pass-through. Unfortunately, we have not yet established any result that holds beyond the constant pass-through class, though we are actively working on this, and therefore we do not report these results in the text. However if confirmed, this reinforces the argument that high pass-through should create incentives for price discrimination, as it also increases the relative size of the monopoly profits lost to deadweight loss.

*where* $\lambda(q;p) \equiv \frac{\nu(q)}{\int_p^{\overline{p}} \nu(r)dr}$ *and* $\nu(p) \equiv \frac{Q(p)}{\rho(p)}$*; thus* $\int_p^{\overline{p}} \lambda(q;p)dq = 1$.

*Proof.* See appendix *Monopoly* Section III.

This result has a number of useful, if trivial, corollaries. If pass-through is globally above (below) some threshold $k$, then so is the consumer-to-producer surplus ratio. For example, globally cost-absorbing demand always has greater producer than consumer surplus at monopoly optimal prices. If pass-through is globally increasing (decreasing) then the consumer-to-producer surplus ratio is always above (below) pass-through at monopoly optimal prices. It makes quantitative the classic qualitative link (Prékopa, 1971; An, 1998) between log-curvature of a distribution and the log-curvature of its survivor function (the fatness of its upper tail)[14]; in fact, Theorem 7 in appendix *Monopoly* Section III shows that if $\overline{p}$ is finite (demand is "tailless") demand is cost-absorbing above some price and if $\overline{p} = \infty$ (consumer values are unbounded above) pass-through cannot be bounded below 1 for large prices. This helps calibrate intuitions about pass-through rates: they will tend to be high when the distribution of consumer valuations is fat-tailed and low when it is thin-tailed or reaches a choke point. Furthermore the formula is useful even if the monopolist does not have linear cost, as market power is commonly measured directly in empirical work. Beyond monopoly it applies in a common oligopoly context considered in the empirical literature on the surplus generated by new products. If we consider (Hausman, 1997; Gentzkow, 2007) the surplus created by a new good holding fixed the prices of all other goods, then the formula is valid under oligopoly so long as the pass-through rate is the primitive rate driven by the log-curvature of demand, the *short-term own pass-through* we discuss in Subsection III.A.

While the ratio of consumer to producer surplus will only be exactly the pass-through rate only when this is constant, one might think that, in the absence of data to the contrary, we have little reason to expect pass-through rates to vary systematically with prices and therefore $Q(p)\mu(p)\rho(p)$ might be a reasonable guess for consumer surplus. Conversely if one believes that pass-through rates are systematically non-constant, this result should lead one to reject demand functions typically used, as they nearly all have pass-through rates that are on the same side of unity globally (see Section V) and many common demand functions, those in the Bulow and Pfleiderer (1983) class including linear, negative exponential and constant elasticity, assume literally constant pass-through rates.

[14] Another way to see this is that the "tail index" of a demand function from extreme value theory (Resnick, 1987) is just the limiting value of $\mu'$, which is monotonically increasing in $\rho$.

## E. Signed Pass-through Assumptions

The assumption that demand is globally either log-concave or log-convex seems, therefore, to be a natural assumption. In fact, an even more robust feature of demand functions commonly used is that their pass-through rates are strictly monotone (or constant) in price, suggesting also that such monotonicity is a natural, weak restriction to place on demand functions. We maintain these *Signed Pass-through Assumptions* (SPAs) throughout the paper.

**Assumption 1.** *Demand is either globally cost-absorbing (log-concave), globally cost-amplifying (log-covex) or globally constant mark-up (log-linear). Demand has either globally increasing pass-through (concave inverse hazard rate), globally decreasing pass-through (convex inverse hazards) or constant pass-through (linear inverse hazards) as a function of cost/price.*

Section V shows that these assumptions are satisfied by nearly all common statistical distributions and demand functions. Whether this calls into question typical demand functions and distributions, or bolsters Assumption 1, is left to the reader to decide. The full force of the assumption is not needed for most results in the paper; it can typically be replaced by assuming properties about some average of pass-through rates (or their slope) over some relevant range of prices. These weaker assumptions can be seen as relatively mild strengthening of boundedness of higher-order effects conditions used to justify tradition linear approximation methods used throughout economics. However because the exact assumptions needed for particular results[15] would vary in a confusing way across examples if they were maximally relaxed, for expositional clarity we maintain Assumption 1 throughout.

## F. What are pass-through rates in the real world?

Whether relaxing log-concavity of demand to allow for cost amplification is a mere theoretical exercise or of potential empirical relevance depends on whether pass-through rates greater than unity are common in real oligopolies. Empirical studies based on imposed functional forms are of little use as the two most common functional forms used in the monopoly case, linear and constant elasticity, assume ex-ante that demand is respectively cost-absorbing and cost-amplifying. Unfortunately reduced-form evidence on pass-through rates is sparse and where it exists it is rarely firm-specific, plausibly in monopolized industries or for firms with a single product.

Barzel (1976) reports cigarette industry pass-through rates rather than firm (much less monopoly) pass-through rates. However, Section III argues that these may plausibly be on

[15] Ilya Segal suggested that Assumption 1 may be necessary for the results here to hold robustly, perhaps across a wide range of cost levels. Because we are a bit uncertain as to exactly how to formalize this claim, we have not attempted to derive such a result; however we suspect its spirit is correct.

the same side of 1. Therefore Barzel's finding that pass-through rates are significantly above 1 provides some evidence for cost amplification. The pass-through of broader sales taxes have been extensively studied. Haig and Shoup (1934) interviewed shop-owners and found they reported cost absorption, while more quantitative studies by Besley and Rosen (1998a,b) found cost amplification and Poterba (1996) found essentially constant mark-ups. Macro-level exchange rate shocks are typically partially absorbed in the short run, but mark-ups are close to constant in the longer-term (Menon, 1995; Campa and Goldberg, 2005). Evidence on product-and-firm specific pass-through rates is mixed. Ashenfelter et al. (1998) and Besanko et al. (2001) find significant cost absorption of firm-specific cost shocks, but pass-through is still non-negligable, ranging form 15-40% as an elasticity (always below pass-through rates if there is positive market power). In the context of multi-product semi-monopoly (a major retailers), the informal, accounting-based study of Chevalier and Curhan (1976) found a mix of cost absorption, constant mark-ups and cost amplification while the econometric approach of Besanko et al. (2005) reports that on approximately 70% of products own-brand pass-through elasticities are below unity and on approximately 30% they are greater than unity. Einav et al. (2008) provide a clean non-parametric estimate of a particular monopoly demand curve (for insurance provided through employers) which appears concave and therefore appears to have a pass-through less than a half.

Empirical evidence on firm-specific pass-through rates for single-product firm is patchy at best. There appears to be little empirical evidence and at best weak theoretical arguments pointing towards pass-through rates being typically below one. Our best guess is that cost absorption is somewhat more common than cost amplification, perhaps as much as twice as common, but cost amplification hardly seems rare. As far as we know we have no evidence at all on how pass-through rates change with prices. Thus, at least at this stage, theory should accommodate all possibilities allowed by Assumption 1 and assumptions restricting these cases, as made by common demand forms (Subsection V.A), are implausible. Further empirical work measuring pass-through rates and their slopes is badly needed.

# II. The Generalized Cournot-Stackelberg Model

This section studies applications of pass-through to single product models, in particular perfect complements Bertrand oligopoly (vertical monopolies) and perfect substitutes Cournot competition. We begin by considering the two firm double marginalization problem and then generalize to many firms and Cournot competition.

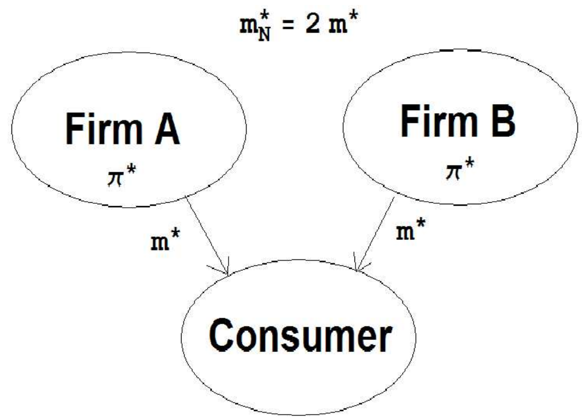


Figure 2: The Nash industrial organization

## A. The Cournot-Spengler model

The classic Cournot-Spengler double marginalization model has two equivalent formulations. In the first (Cournot, 1838) two monopolists sell goods that are perfect complements in consumption. In the second (Spengler, 1950) one firm sells an input to a second firm which sells to a consumer. The only difference between these models is that in Cournot's the assembly is performed by the consumer and in Spengler's it is performed by the downstream firm. The firms have total linear cost between them $c_I$, the division of which we will show is irrelevant to the outcomes of interest. The natural benchmark against which to judge the monopolistic vertically separated organizations that Cournot, Spenler and we consider is that of a single vertically integrated monopolist Integrated who sets her markup $m_I^\star$ to solve equation (1), plugging in $c_I$ for $c$.

The separated organization envisioned by Cournot is shown in Figure 2. The two firms simultaneously choose prices to charge consumers for whom the goods are perfect complements in consumption. The first-order condition for each firm $i$ is

$$m_i^\star = \mu(m_i^\star + m_j^\star + c_I) \tag{6}$$

At equilibrium each firm earns profits $\pi^\star$ and the total mark-up in the industry is $m_N^\star \equiv 2m^\star$.

Cournot's problem can also be formulated in Spengler's physical organization, shown in Figure 2, so long as the firms chose their mark-ups simultaneously. Thus it is the Nash timing that distinguishes the situation in Figure 2 (which we therefore call the Cournot-Nash organization) from that pictured in Figure 3 where, as Spengler originally assumed, Upstream commits to its price before Downstream chooses its price. Again, Cournot's physical organization combined with price leadership in the spirit of von Stackelberg (1934) by one

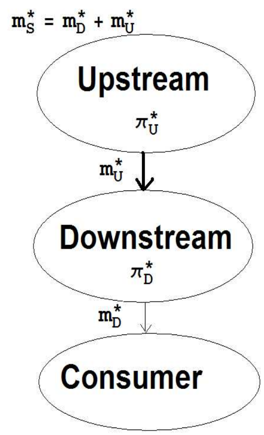


Figure 3: The Spengler-Stackelberg industrial organization

will yield the same outcomes as the organization in Figure 3. For any choice of mark-up $m_U$ by Upstream, the optimal mark-up of Downstream is given, as Upstream's mark-up is just like an increased cost for Downstream, by

$$m_D^\star = \mu(m_U + m_D^\star + c_I) \tag{7}$$

Taking this into account, Upstream maximizes her profits $m_U D\big(m_U + m_D(m_U) + c_I\big)$ according to the first-order condition

$$m_U^\star = \frac{\mu\big(m_U^\star + m_D(m_U^\star) + c_I\big)}{\rho\big(m_U^\star + m_D(m_U^\star) + c_I\big)} \tag{8}$$

Equation (8) resembles Downstream's first-order condition, but takes into account the strategic effect of Upstream's choice on Downstream. Under cost absorbtion ($\mu' < 1$), mark-ups are strategic substitutes in the sense of Bulow et al. (1985): one firm raising its mark-up, which is equivalent to imposing a tax on the other firm, induces the other firm to absorb this increase and lower its mark-up. Conversely under cost amplification, mark-ups are strategic complements. At equilibrium Upstream earns profits $\pi_U^\star$, the Downstream earns profits $\pi_D^\star$ and the total markup charged by the two firms is $m_S^\star \equiv m_U^\star + m_D^\star$.

The basic theory of double marginalization states that starting from either separated industrial organization a merger to monopoly will reduce total industry prices and increase total industry profits as the merger leads firms to internalize the negative externalities across firm of high prices. This says nothing, however, about either comparing the two separated organizations or the mark-ups or profits of individual firms within or across organizations. In fact, $4! = 24$ rankings of firm mark-ups, $3! = 6$ rankings of firm profits, 2 rankings of

| | $\rho < 1$ | | $\rho > 1$ | |
|---|---|---|---|---|
| | Cost absorption | | Cost amplification | |
| | Decreasing pass-through | | Decreasing pass-through | |
| | $m_U^\star$ | | $m^\star$ | |
| $\rho'$ | $\vee$ | $\pi_U^\star$ | $\vee$ | $\pi_D^\star$ |
| $\wedge$ | $m_I^\star < m_N^\star < m_S^\star$ | $\vee$ | $m_D^\star$ | $\vee$ |
| 0 | $\vee$ | $\pi^\star$ | $\vee$ | $\pi_U^\star$ |
| | $m^\star$ | $\vee$ | $m_U^\star$ | $\vee$ |
| | $\vee$ | $\pi_D^\star$ | $\vee$ | $\pi^\star$ |
| | $m_D^\star$ | | $m_I^\star < m_S^\star < m_N^\star$ | |
| | Cost absorption | | Cost amplification | |
| | Increasing pass-through | | Increasing pass-through | |
| | $m_I^\star < m_N^\star < m_S^\star$ | | $m^\star$ | |
| $\rho'$ | $\vee$ | $\pi_U^\star$ | $\vee$ | $\pi_D^\star$ |
| $\vee$ | $m_U^\star$ | $\vee$ | $m_D^\star$ | $\vee$ |
| 0 | $\vee$ | $\pi^\star$ | $\vee$ | $\pi_U^\star$ |
| | $m^\star$ | $\vee$ | $m_I^\star < m_S^\star < m_N^\star$ | $\vee$ |
| | $\vee$ | $\pi_D^\star$ | $\vee$ | $\pi^\star$ |
| | $m_D^\star$ | | $m_U^\star$ | |

Table 1: Comparing mark-ups and profits among firms within and across organization of the Cournot-Spengler double marginalization model

Stackelberg versus Nash mark-ups and 4 ranges of values for pass-through rates and slope are possible. However as Table 1 summarizes, simply imposing SPAs narrows these 1152 possible patterns to only 4: if we know how whether demand is cost-absorbing or cost-amplifying and whether pass-through is increasing or decreasing in cost we obtain a full ranking of firm and industry mark-ups and profits within and across industrial organizations. Furthermore it is easy to show[16] that if the first-order effects of cost shocks on prices can be estimated (starting from any organization) the underlying pass-through rate can recovered and only the ranking of $m_U^\star$ and $m_I^\star$ remain ambiguous; if in addition a second-order effect can be estimated, or the first-order effects on prices at both levels starting from the Spengler-Stackelberg organization can be observed, then even this ambiguity is resolved.

Most of the action in Table 1 happens across the vertical line (from the left hand side to the right hand). This is where demand moves from being cost-absorbing on the left to cost-amplifying on the right and therefore mark-ups move from being strategic substitutes to complements. Knowledge of this implies all but one of the comparisons in Table 1.

Does a Nash firm or Integrated charge a higher mark-up? The only difference between their incentives is that a Nash firm's consumers are double marginalized. If it is optimal

[16] See appendix *Generalized Cournot-Stackelberg Models* Section II.

for her to absorb this tax on consumers the Nash firm will choose a lower mark-up than Integrated; if it is optimal to amplify it, Nash will charge a higher mark-up than Integrated.

Does Upstream or Downstream charge a higher mark-up? Both face the same demand and therefore have the same market power. The only difference in their incentives is that Upstream's mark-up affects Downstream choice, giving Upstream an incentive to do whatever induces Downstream to reduce her mark-up. When mark-ups are strategic substitutes (complements) this involves Upstream increasing (decreasing) her mark-up and thus charging a higher (lower) mark-up than Downstream. Because both face the same end-demand, this further determines the comparison of their profits.

All of the rest of the results that shift across the vertical line follow similar logic, as developed formally in appendix *Generalized Cournot-Stackelberg Models* Section I. The one comparison that varies across the horizontal line (from top to bottom) is between $m_U^\star$ and $m_I^\star$. This is closely related to the fact pass-through, rather than elasticity, determines the comparative statics of monopoly: in the Stackelberg organization the effects of cost changes are filtered through *two layers* of firm optimization, third-order properties of demand become relevant. However this also means that under the Spengler-Stackelberg equilibrium, the slope of pass-through is observable in the first-order pass-through behavior of the upstream firm[17] as shown in appendix *Generalized Cournot-Stackelberg Models* Section II.

More precisely, in the case of cost absorption, there are two incentives facing the Upstream firm. On the one hand she would like to increase her mark-up, relative to what Integrated would charge, as she has a strategic incentive to induce Downstream to decrease her mark-up. On the other hand Upstream has an incentive to partially absorb Downstream's mark-up which Integrated does not confront; this leads Upstream to decrease her mark-up relative to what Integrated would charge. The first strategic incentive is marginal: by how much, on the margin, does a small increase in Upstream's price induce Downstream to reduce her price? The second incentive, on the other hand, depends on the average rate at which Upstream should absorb Downstream's mark-up. Unsurprisingly, the relative size of the average versus the marginal effect depends on whether pass-through is increasing or decreasing in cost.

## B. The GCS vertical monopolies model

For expositional clarity we focused in the previous subsection on the two firm case. However our results generalize to an arbitrary number of firms acting in arbitrary sequence.

[17]This point was first made to us by Kevin Murphy and, as shown in appendix *Generalized Cournot-Stackelberg Models* Section II, extends to the GCS model: the information about pass-through rates that one needs to sign comparisons within and across industrial organizations are empirically identified as long as one can observe the the first-order effects of cost on prices at all layers starting from the organization with the largest number of layers.

A good has many necessary components (all are jointly perfect complements in consumption), each produced by a firm monopolizing the production of that component. The firms are divided into $K$ groups. Each group $k = 1, \ldots, K$ has $N_k$ firms. The total linear cost of production of all firms is $c$. The pricing game has $K$ rounds. In the first round, the $N_K$ firms in group $K$ simultaneously choose their mark-ups $m_{i_K}$ for $i_K = 1, \ldots N_K$. In the $k$th round, for $k = 2, \ldots, K-1$ the $N_{K-k+1}$ firms in group $K-k+1$ choose their mark-ups taking as given the mark-ups of all firms in groups $k' > K-k+1$. Finally in round $K$ all firms in group 1 simultaneously choose their mark-up and each firm $i_k$ receives a payoff $\pi_{i_k} = m_{i_k} Q\left(c + \sum_{k=1}^{K}\sum_{i=1}^{N_k} m_{i_k}\right)$. The basic set-up is pictured in Figure 4. For example in the case when $N_k = 1$ for all $k$, one considered by Anderson and Engers (1992) in the case of quantity competition, this game is often thought of as firm $1_K$ selling an input to firm $1_{k-1}$ who transforms this input into another intermediate input and sells this to firm $1_{k-2}$ and so forth until it reaches firm $1_1$ which produces a final output selling it to a final consumer. We refer to this model as the *Generalized Cournot-Stackelberg (GCS) vertical monopolies model.* As far as we know, this generalization of classic Cournot (1838), von Stackelberg (1934) and Spengler (1950) models is novel to this paper and we are there first to provide any general results on it. It nests all of the common industrial organizations of vertical monopolies. For example, the Spengler-Stackelberg organization from the previous section is the special case of $K = 2, N_k = 1$ and the Cournot-Nash organization when $K = 1$ and $N_1 = 2$.

Because the results for the GCS vertical monopolies model are so general, our broad theorems on it are somewhat cumbersome formal statements. Therefore instead of describing them in detail here, we have left their formal statement and proof to appendix *Generalized Cournot-Stackelberg Models* Section I. Instead we discuss the results informally, showing how they naturally generalize those for the simple two-firm case[18].

Let $p_k^\star$ be the total price after the mark-ups of all firms weakly above $k$ have been included; thus $p_1^\star$ is the final price to consumers and the total cost of all firms is $p_{K+1}^\star = c$. Let $\rho_k \equiv \frac{dp_k^\star}{dp_{k+1}^\star}$ and assume that the SPAs apply to $\rho_k$ for all $k$. That is we assume that each $\rho_k$ stays globally on one side of unity and each is globally monotone, though they need not be consistent across different $k$. Let $m_k^\star$ be the equilibrium mark-up of a firm at the $k$th

[18]Like the two-firm case, most of the comparisons here involve comparisons across two levels of the chain or comparisons across industrial organizations differing only in that firms have changed positions across two levels of the chain. For results that go much beyond these, one would need to combine the pass-through rates at various levels of the chain or perhaps even consider higher-order properties of pass-through. However we also suspect that there are some further results that can be characterized using simple properties of pass-through beyond what we have developed here. We conjecture, but given space constraints have not attempted to show, that it is possible to rank all mark-ups and profits within and across industrial organizations by observing the effects of a shock to cost on prices at all levels in whichever industrial organization being compared has a larger number of firms

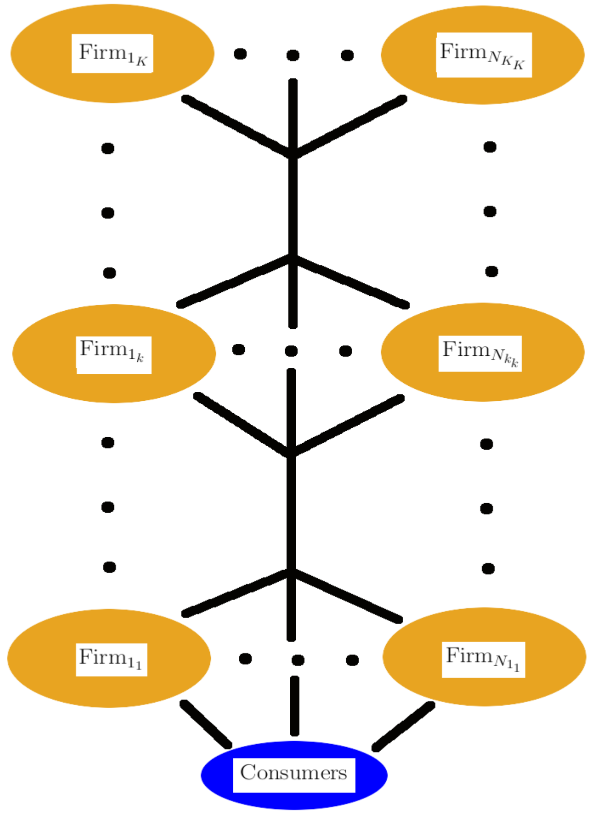


Figure 4: The GCS vertical monopolies model

level. We now list the results following the two-firm comparisons which they generalize.

- $m_I^\star$ v. $m^\star$: Entry of an additional firm at level $K$ will lead to higher (lower) $m_K^\star$ if $\rho_K > (<)1$.

- $m_U^\star$ v. $m_D^\star$: $m_{k+1}^\star > (<) m_k^\star \iff \rho_k < (>)1$.

- $m_I^\star$ v. $m_U^\star$: Suppose there is a lone "leader" firm acting in group $K$. If $\tilde{N}$ firms now act after the leader but before all other firms, the leader charges mark-up $\tilde{m}_{K+1}^\star > (<) m_K^\star$ in the new equilibrium if and only if $\rho_K' < (>)0$.

- $m_U^\star$ v. $m^\star$: A firm moving from level $K$ to level $K+1$, with $N_K > 1$, charges a higher (lower) mark-up after the move if and only if $\rho_K < (>)1$.

- $m_S^\star$ v. $m_N^\star$: Imagine moving one of the $N_k$ firms currently acting at level $k$ up to acting with the firms at level $k+1$ for any $k$ with $N_k > 1$. This will lead to higher (lower) final prices to consumers[19] if and only if $(\rho_k - 1)(N_k - 1 - \rho_k N_{k+1}) < (>) 0$.

Despite the greater complexity of the GCS model, using pass-through still yields obtain significant results. Each $\rho_k$, as well as $\rho_k'$ for all $k < K$, can be observed if there is a shock to cost at any level by observing what happens to prices moving between levels. In fact, the comparison of mark-ups of firms at various stages (bullet 2 above) immediately identifies whether demand is cost-absorbing or cost-amplifying at all stages below $K$.

Beyond its direct importance the vertical monopolies model is closely related to a number of economic problems of recent interest. This suggests a number of potential applications of our results which we briefly discuss here. If industrial organizations shift and pass-through can be measured, as in Mortimer (2008), they can be used to test the model. Such testing has been a topic of substantial interest in recent years (Villas-Boas, 2007) and has been forced to rely on restrictive (convex, but cost-absorbing) functional form restrictions for identification, a restriction that our results potentially relax. Pass-through can be used to determine policy makers' preferred organization as well as to predict the organization likely to emerge endogenously(Amir and Grilo, 1999). The reasoning can be adapted (Subsection IV.B) to analyze mergers between firms that produce complements in some markets and substitutes in others. They can be used to analyze vertical relationships among tax authorities (see Keen (1998) for a survey) or the tax relationship between a regulator and a monopolist[20]. They can be used to analyze the effects of commitment on the pricing of monopolists selling goods with inter-temporal complementarities[21] (Klemperer, 1987; Murphy and Becker, 1988). Finally, Martimort and Stole (2009) establish a tight connection between delegated common agency games and the vertical monopolies problem.

## C. GCS quantity competition

Sonnenschein (1968) notes that vertical monopolies are just the dual of (symmetric linear cost) quantity competition: in the former the quantity is a function of the sum of mark-ups, while in the second the mark-up is a function of the sum of quantities. Therefore all our results[22] above apply equally to the quantity competition GCS model, where groups of firms sequentially choose production levels, *mutatis mutandis*: mark-ups become quantities, quantities mark-ups, pass-through becomes quantity pass-through, etc. Quantity pass-through

[19]Note that when $k = K$, $N_{k+1} = 0$ so, as in the two-firm case, cost-abosrption vs. amplification on its own determines whether sequentiality or simultaneity leads to higher prices.

[20]Thanks to Bill Rogerson for this application.

[21]This was pointed out to us by Kevin Murphy.

[22]Some of these results, in the special case of two firms and the context of quantity competition, were first established by Dowrick (1986), Amir and Grilo (1999) and Amir and Lambson (2000), though the connection to pass-through was not recognized.

rates can also be measured qualitatively by comparing relative productions of firms and/or quantitively using exogenous industry cost shocks appropriately (see appendix Section III).

# III. Multiple Products

In this section we consider how our results may generalize to industries with multiple products, as these are of interest in many applications. Here we discuss only symmetric linear cost oligopolies (with either substitutes or complements). This setting is restrictive, though dominant in the recent theoretical literature on pass-through with multiple firms (Anderson et al., 2001). Despite their limited purview we believe the results provide some insight into how the logic of pass-through extends to industries with many products.

## A. Horizontal demand systems

We begin by discussing the case in which the reasoning behind this extension is most transparent. We refer to this case as *Horizontal Demand Systems* (HDSs). Horizontality means, when firms take others' prices as given in the spirit of Bertrand (1883), that changes in other firms' prices affect residual demand by uniformly shifting consumers willingness to pay for the good or in the Cournot case shifts in other firms' production affect residual inverse demand by horizontal translation. We focus on the Bertrand case, only briefly noting the analogous results for generalized quantity oligopoly.

Formally consider an industry with $N$ firms each producing a single good and let $\mathbb{Q} : \mathbb{R}^N \to \mathbb{R}^N$ be the (smooth) demand function, with the interpretation that if firm $i$ charges price $p_i$ and other firms charge prices $\mathbf{p}_{-i}$ demand for $i$'s goods will be $Q_i(p_i, \mathbf{p}_{-i})$.

**Definition 1.** $\mathbb{Q}$ *is a* symmetric horizontal demand system *(SHDS) for a Bertrand oligopoly if for all i* $Q_i(p_i, \mathbf{p}) = \tilde{Q}\left(p_i - g[\mathbf{p}_{-i}]\right)$ *for some* $g : \mathbb{R}^{N-1} \to \mathbb{R}$ *and* $\tilde{Q} : \mathbb{R} \to \mathbb{R}$.

Symmetric horizontal inverse demand systems are defined *mutatis mutandis* for Cournot (quantity) oligopoly by the Cournot-Bertrand duality (Singh and Vives, 1984). HDSs includes the linear demand system and a generalization of this, Horizontal Constant Pass-through Demand System we propose in Section IV.B. However HDSs are much more general than HCoPaDS as it imposes no functional form on $\tilde{Q}$ nor on $g$.

We assume that general equilibrium would be unique: the Jacobian of $\mathbb{Q}$ at any price vector is Hicksian (Hicks, 1939; Gale and Nikaido, 1965). A grossly sufficient condition for this is Slutsky symmetry of demand, nearly always assumed for practical purposes in industrial organization applications by quasi-linear utility specifications or by the fact that the good is a small part of income when income effects are allowed (Berry et al., 1995). We also assume $\tilde{Q}$ (as a single-product demand function) satisfies MUC and SPAs and that the matrix of cross-partials of firm profits are globally Hicksian (equilibrium is unique)[23].

Consider the equilibrium of the Bertrand game when all firms begin with symmetric costs (i.e. $c_i = c$ for all $i$). Let $\mathbf{p}^\star$ be the (unique, symmetric) equilibrium price vector. There are five natural notions of pass-through in this context.

1. *Short-run own* (Sop): this is the effect of an increase in one firm's cost, holding other firm prices fixed: $\rho_i \equiv \left.\frac{\partial p_i^\star}{\partial c_i}\right|_{\mathbf{p}_{-i}\text{constrained}}$. MUC guarantees this is positive. This is also the pass-through rate that each firm would have if it were not competing with (or being double marginalized by) other firms and the pass-through rate for an industry cost shock if the industry were cartelized or merged to monopoly.

2. *Short-run cross* (Sxp): this is the effect of an increase in one firm's cost on another firm's price in the population sub-game where the prices of all firms not included in this pair are held fixed $\rho_{ij} \equiv \left.\frac{\partial p_j^\star}{\partial c_i}\right|_{\mathbf{p}_{-i,j}\text{constrained}}$. The sign corresponds to direct strategic substitutability or complementarity (Fudenberg and Tirole, 1984; Bulow et al., 1985).

3. *Long-run own* (Lop): this is the effect of an increase in one firm's cost on its own equilibrium price $\rho_i^{eq} \equiv \frac{dp_i^\star}{dc_i}$. Stability ensures this is positive under symmetry.

4. *Long-run cross* (Lxp): this is the effect of an increase in one firm's cost on the equilibrium price of another firm $\rho_{ij}^{eq} \equiv \frac{dp_j^\star}{dc_i}$. $\sigma_{ij} \equiv \frac{\rho_{ij}^{eq}}{\rho_i^{eq}}$ is a reasonable measure of equilibrium strategic substitutability or complementarity. While this need not in general have the same sign as $\rho_{ij}$, symmetry guarantees that it does (Athey and Schmutzler, 2001).

5. *Industry*: this is the effect of an increase in $c$, the level of all firm's cost, on each firm's price $\rho_I = \frac{dp_i^\star}{dc}$ which is the same (positive) number by symmetry for all firms.

**Theorem 4.** *Under the assumptions maintained in this subsection, $\rho_i - 1, \rho_i^{eq} - 1$ and $\rho_I - 1$ all have the same signs and these are the same (opposite) as the signs of $\rho_{ij}$ and $\rho_{ij}^{eq}$ if the goods are complements (substitutes). Therefore the entry of a new firm into the industry will reduce (increase) prices if there is cost absorption and substitutes (complements) or cost-amplificaiton and complements (substitutes). Similarly a merger (with no efficiencies) between two firms producing substitutes (complements) will raise (lower) the merging firm prices and raise the prices of the other firms under cost absorption but lower them under*

[23] Our results continue to hold if this is only true locally and one considers comparative statics of the local equilibrium, but formally dealing with multiple equilibria and defining the local equilibrium here would be cumbersome.

*cost-amplificaiton. Analogous results hold for Cournot oligopoly and are stated formally in appendix* Multiple Products *Section III.*

*Proof.* See appendix *Multiple Products* Sections I-II.

The intuition behind this result can be seen by considering the first-order condition for particular firm. By analogy with equation (1), the first-order conditions for firm $i$ is

$$p_i^\star - c_i = \tilde{\mu}\left(p_i - g[\mathbf{p}_{-i}]\right) \tag{9}$$

where firms' market power $\tilde{\mu} \equiv -\frac{\tilde{Q}}{\tilde{Q}'}$. Pass-through is less than 1 when market power declines in price. When all prices rise, demand for each product must fall. Thus cross-effects on market power cannot dominate own-effects. The slope of market power therefore determines pass-through in the short-term, the long-term and for the industry as a whole. Furthermore by the assumption of an HDS, the effect of an increase in another firm's price on my market power is in the same relation to the effect of an increase in my price as are the effects of each on demand. Thus if there are substitutes (complements) the slope of market power with respect to another firm's price is the opposite (same) as its slope with respect to my price.

This shows that the conventional wisdom (Eaton and Grossman, 1986; Rasmusen, 2006), shown by the case of $\rho < 1$ in Table 2, on the relationship between form of oligopoly and strategic effects is valid in this setting if and only if demand is cost-absorbing. The general falsity of this conventional wisdom (Fudenberg and Tirole, 1984; Bulow et al., 1985) created a damning ambiguity in the theory of oligopoly: one can obtain nearly any result or its opposite by the appropriate assumptions on demand. However, at least in this simple context, this ambiguity is resolved by the fact that observing either short or long-run effects of any one of a variety of cost shocks on even a single price identifies these qualitative effects. The entry of a new firm is effectively like the price of another firm falling from infinity and a merger like those prices rising (falling) in the case of substitutes (complements). Therefore pass-through rates allow us to predict the effect of competition of either kind on prices[24], a topic of recent theoretical interest (Chen and Riordan, 2008; Gabaix et al., 2009). Furthermore, Fudenberg and Tirole (1984) and Bulow et al. (1985), and a large literature they sparked, show that a wide range of issues in oligopoly theory turn on the distinction between strategic complements and substitutes, the ambiguity of which these results begin to resolve.

[24]It is also simple to allow for the entry of a firm producing a complement when all other firms produce substitutes (a good complementary to any one of the competing products), or a substitute when all other firms produce complements (an alternative to the multimarginalized product), so long as the effects are symmetric. The results are intuitive: entry of a substitute or has the same qualitative effect as it would have if other goods had the same utility interaction.

$\rho < 1$

| | Substitutes | Complements |
|---|---|---|
| Bertrand | Strategic complements | Strategic substitutes |
| Cournot | Strategic substitutes | Strategic complements |

$\rho > 1$

| | Substitutes | Complements |
|---|---|---|
| Bertrand | Strategic substitutes | Strategic complements |
| Cournot | Strategic complements | Strategic substitutes |

Table 2: Strategic effects and pass-through rates with Horizontal Demand Systems

This reasoning is quantitative, as well as qualitative, and implies a number of hopefully now-intuitive comparative statics on the effects of interaction strength (competition or complementing), pass-through rates and industry diffuseness on Sop, Sxp, Lop, Lxp and industry pass-through. Let the strength of interaction $s \equiv \left|\sum_{j \neq i} \frac{\partial g}{\partial p_j}\right|$; in the case of substitutes $s \leq 1$ as an increase in prices must (weakly) reduce each firm's demand; in the case of complements $s \leq N$ as goods cannot be more than perfect substitutes. We consider the comparative statics of Lop $\rho_i^{eq}$, equilibrium strategic effects $\sigma_{ij}$ and industry pass-through $\rho_I$ with respect to adjusting Sop $\rho_i$, the number of firms $N$ and $s$, each while keeping fixed the others.

**Theorem 5.** *$\rho_i^{eq}$ and $\rho_I$ are increasing in $\rho_i$. $\rho_i^{eq}$ is increasing in $s$ and decreasing in $N$. $\rho_I$ is constant in $N$ and when goods are substitutes (complements) is increasing (decreasing) in $s$ under cost-absorpton and decreasing (increasing) in $s$ under cost amplification. $\sigma_{ij}$ is decreasing (increasing) in $\rho_i$, so long as $\rho_i < 2$, and when goods are substitutes (complements), is increasing in absolute value in $s$ and decreasing in absolute value in $N$.*

*As $N$ becomes large, $\rho_i^{eq}$ approaches $\rho_i$ and $\sigma_{ij}$ approaches $0$. Under substitutes as $s$ approaches $1$, $\rho_i^{eq}$ approaches a finite limit $\tilde{\rho}_i^{eq} > \rho_i$ which is on the same side of $1$ as $\rho_i$, $\sigma_{ij}$ approaches some finite number of the appropriate sign and $\rho_I$ approaches $1$.*

*Proof.* See appendix *Multiple Products* Sections I-II.

Thus the quantitative degree of strategic effects follow the same logic as their qualitative direction. Furthermore all pass-through rates move in the same direction quantitatively as well as qualitatively. The theorem provides, as far as we know, the first attempt to formally characterize the relationship between the degree of differentiation and pass-through rates in multi-product industries, a central topic in antitrust analysis (Farrell and Shapiro, 2008). Conventional wisdom (Besanko et al., 2001; Kim and Cotterill, 2008) is that strong competition leads to low Lop and industry pass-through near 1. Our theorem indicates that, with constant marginal cost, only the second of these intuitions is correct: increased differentiation *reduces* pass-through. Intuitively, competition increases strategic interactions[25];

[25] Increasing the number of firms, while holding $s$ constant, decreases the strength of such strategic inter-

because strategic effects are symmetric here, by the LeChatelier Principle (Samuelson, 1947) the stronger such effects are, the stronger the elasticity of reaction (pass-through) to changes in cost. However, in work in progress, Weyl shows that when marginal costs are not constant as assumed here, the relative elasticity of supply and demand (is the industry more competitive than it is close to constant marginal cost?) becomes crucial. If demand is much more elastic than supply, competition does drive down pass-through[26]. Nonetheless, Theorem 5 makes clear the delicacy of common intuitions about pass-through rates and establishes new ones regarding the relationship between pass-through rates and strategic effects[27].

## B. Discrete choice models

The results above provide basic intuitions about how pass-through can be used for analysis of multi-product industries. However while HDSs (especially linear demand) are often used in theoretical analysis, they are only occasionally applied empirically. Far more common are discrete choice models of demand such as the logit (McFadden, 1974; Werden and Froeb, 1994) and mixed logit models (Berry et al., 1995). These models are challenging to analyze theoretically and little is therefore known about their general implications for the comparative statics of oligopoly. Until recently the most comprehensive results were due to Perloff and Salop (1985) who derive, for a few special distributions and under the assumption of independent consumer valuations across products, the effects of entry on prices.

Intuitively discrete choice models are closely tied to HDSs. Standard discrete choice models imply that the difference between the utility (and choice probabilities) of two goods depend on the prices of those goods only through the difference between the prices. However, given the complex choice probabilities arising from the discreteness of maximization and integration over consumer heterogeneity, the general validity of this intuition is unclear.

Luckily Gabaix et al. (2009) have recently made progress in understanding the oligopoly implications of fairly general discrete choice models with a large number of symmetric firms. They obtain analytic expressions for demand and its derivatives when there are a large number of symmetric firms[28] in the non-parametric Berry et al. (1995) (BLP) model, a generalization of mixed logit allowing arbitrary value distributions. It is straightforward to show that these formulae follow the same patterns for pass-through and strategic effects that HDSs do. Thus Theorems 4 and 5 apply there as well, suggesting that they may

actions by making them more diffuse along the lines of monopolistic competition, and thus has the opposite effect of increasing $s$.

[26]This sort of effect was first noted by Bishop (1968) in the context of monopoly.

[27]Preliminary research available on request indicates that these are robust to non-constant marginal cost.

[28]Or, presumably, when there are clusters of large numbers of similar firms between which most substitution takes place.

apply more generally to symmetric oligopoly where demand is generated by discrete choice. Furthermore, the log-curvature of idiosyncratic consumer variations[29] of goods determines the log-curvature of demand in own price and therefore whether demand is cost-absorbing or cost-amplifying, there are strategic complements or substitutes, etc. Thus functional form assumptions that impose the log-curvature of idiosycratic variations, which we show in Section V.A are often made, are strongly restrictive at least in this simple settings[30].

**Theorem 6.** *Theorems 4 and 5 apply to the Gabaix et al. (2009) non-parametric BLP model with a large number of symmetric firms. Furthermore in the Gabaix et al. (2009) model demand is cost-absorbing (amplifying) in the Sop sense near the symmetric equilibrium if the distribution of (idiosyncratic) valuations are log-concave (convex). If a distribution of valuations has uniformly weakly lower (higher) pass-through rate than another, it induces in both models a weakly lower (higher) Sop.*

*Proof.* See appendix *Multiple Products* Section IV.

Theorem 6 ties together symmetric discrete choice models and symmetric horizontal demand systems. Furthermore it helps provide intuition behind the Gabaix et al. (2009) results on the effects of entry on prices. Theorem 6 also shows how Gabaix et al. (2009)'s results apply to any other issue where strategic complements vs. substitutes is crucial, providing an empirical test for their conditions both qualitatively and quantitatively. Furthermore Gabaix et al. (2009) show how their results apply to the comparatives statics of surplus in large auctions, suggesting a role for pass-through in auction theory.

# IV. Other Applications

## A. Two-sided markets

A recent topic in industrial organization has been so-called "two-sided markets", industries with network effects that occur *between* two distinct groups of consumers. Typical examples are firms serving as a platform for transactions (payment cards), two-sided services (advertising, website access, video game playing) or matching (dating clubs or websites).

One of the most influential models of two-sided markets is that proposed by Rochet and Tirole (2003)(RT2003) and analyzed by Weyl (2009). Consider a credit card company, call

[29] By definition in the standard mixed logit model, idiosyncratic variations are Type I Extreme Value. As shown in Subsection V.A Type I Extreme Value distributions are log-concave and therefore generate cost absorption.

[30] It is possible, though unlikely, that the marginal restrictiveness of these assumptions would decline as a result of moving to a more complex model with asymmetries across firms. Consideration of this is an important topic for future research.

it Visa, which charges a per-transaction price to card-carying consumers and card-accepting merchants. If exogenously the price Visa charges to merchants rises, this provides it with a greater incentive to encourage consumers to use cards at stores by reducing price. In fact, the defining feature of the RT2003 model is that this is the *only* cross effect between pricing to the two sides of the market: each dollar earned on one side of the market (per-transaction) acts as a cross-subsidy of exactly one dollar to the other side of the market.

Therefore the economics of the RT2003 model turn on the rate at which the firm finds it optimal to pass-through subsidies from one side to a reduction in prices to consumer on the other. For example, competition tends to reduce "overall prices" (the sum of prices on the two sides of the market) when demand on both sides is cost-absorbing; if demand on one side is cost-amplifying, competition lowering prices on one side will be more than offset by an increase in prices on the other. On the normative side, firms only internalize the benefits that marginal (not average) consumers gain from more partners joining on the other side of the market, as they cannot price discriminate. The relative size of this infra-marginal surplus relative to the internalized mark-up are given by the pass-through rate according to the logic of Section I.D. Therefore, as shown by Weyl (2009), the overall price is a good gauge of welfare when both demands are cost-absorbing and a poor gauge when one demand is cost-amplifying[31]. Exogenous cost variations allow the distinction between these cases to be identified and the model to simultaneously be tested.

The RT2003 model's multiplicative demand specification has been used as a building block in many other studies. For example, the main results of Hermalin and Katz (2004) turn on the distinction between cost absorption and cost amplification for the same reasons as in the RT2003. Unsurprisingly, models of platforms as competing groups of vertical monopolies (Carrillo and Tan, 2009) also largely turn on pass-through rates for reasons more closely related to our results in Section II.

Because the same properties of pass-through identify many simple models, more complicated ones need not be exponentially more complex (under-identified). In fact, Weyl (2008) provides an example of a case where the opposite happens, and the same assumptions allow exponentially more *simplicity* (testing). There I study a model combining the vertical and two-sided markets aspects of platforms to analyze vertical integration of intermediaries in two-sided markets, such as a merger between card-issuing banks and debit clearing networks or video game and joystick producers. Combining two problems which are identified and testable by SPAs leads to a model that is more easily testable than either of its components, offering hope that our approach may apply in more complex, realistic models.

[31] Both demand being cost-amplifying violates second-order conditions.

## B. Merger analysis

The two central elements of static merger analysis in differentiated product industries are the evaluation of the anticompetitive effect of merging firms internalizing diverted profits and offsetting efficiencies (Shapiro, 1996). Both of these shift costs faced by firms. Horizontal mergers tend to be anticompetitive as they increase the opportunity cost of sales faced by a firm, as after the merger it must take into account the lost sales cannibalized from the sale of a substitute product. Efficiencies, which may offset these anticompetitive effects, are reductions in firms' marginal costs as a result of productive synergies. As Froeb et al. (2005) and Farrell and Shapiro (2008) argue that, under Bertrand competition, once the relative size of efficiencies and cannibalization, and therefore the sign of the price effect (Werden, 1996) of the merger tied down, the magnitude is determined by the pass-through rate.

Thus shocks to the marginal costs of the merging firms alone[32], coupled with a measurement of efficiencies, are sufficient to estimate a local approximation to the static Bertrand merger effects: they yield estimates of the relevant pass-through rates (own and cross) and (nearly)[33] elastiticities. This provides a non-parametric (local) foundation for merger analysis that avoids rampant sensitivity of merger analysis to the functional form (see the following section) used in the analysis, even given a collection of measured elasticities and cross-elasticities (Crooke et al., 1999). Nonetheless these analyses ignore interactions between the anticompetitive effects on the two goods: as one good's price rises, cannibalization changes. These effects are systematically related to pass-through rates, as shown below.

To avoid this problem without restricting the pass-through rate and thereby biasing the

[32]Often obtaining data providing even such a limited number of cost shocks is prohibitively difficult, especially if the merging firms sell many products. Despite this, we believe this way of viewing mergers is useful. First, it provides a thought experiment: elasticities based on some combination of introspection, informal knowledge of the industry and measures of pass-through rates in other industries can help fill in unmeasurable parameter values. Second, in practice shocks to firm costs are often "synthesized" through variation in competitive conditions (Berry et al., 1995; Nevo, 2001). Focusing this approach on synthesizing shocks to a subset of costs, rather than all costs, might improve the efficiency of (and burden of assumptions necessary for) estimation of the relevant magnitudes by reducing the number of parameters that must be estimated. This would, of course, require demand systems that allow flexible pass-through rates; Subsection V.A raises doubts that typical demand systems for structural empirical analysis pass this hurdle.

[33]This not quite correct. Such a shock does not literally identify the effects of changing each price on demand for other goods, because other firm prices change as well. Thus the diversion that a Bertrand firm considers in its decision making is not quite estimated. However, the approximation is likely to be quite good for several reasons. First, if firms instead of playing a Bertrand game follow the generalized Bertrand analog of Bresnahan (1981)'s notion of consistent conjectures then the local approximation is precisely correct. Second, Lxp is second-order relative to Lop, especially in an industry with many firms, and thus the estimated elasticities likely give a close approximation to those that are relevant. Finally because only the ratio of the cross-price demand slope to own-price demand slope, and not their level, is relevant to measuring the diversion, only systematic differences in the effects of changes in other prices on these demands could bias calculations of cannibalization. We doubt such differences are likely to have a first-order effect. However, formal investigation of these properties is an important area for future research in merger analysis.

results, a natural approach is to formulate a tractable demand system which is known to allow flexible elasticities and pass-through rates[34]. The most straight-forward way to formulate such a demand system is to assume that (all) pass-through rates are constant in (all) prices, the natural multi-product extension of the Bulow and Pfleiderer (1983) constant pass-through class of demand functions. In appendix *Constant Pass-through Demand Systems* Section I we show that these *Constant Pass-through Demand Systems* (CoPaDS) take the form

$$Q_i(p_i, \mathbf{p}_{-i}) = f^i(\mathbf{p}_{-i}) \left( [1-\rho_i] \left[ \tilde{p}_i - p_i + \sum_{j \neq i} \beta_{ji} p_j \right] \right)^{\frac{\rho_i}{1-\rho_i}} \tag{10}$$

where $\tilde{p}_i, \rho_i$ and $\beta_{ji}$ are parameters that can be adjusted to set, respectively, the own-price elasticity of demand, the Sop and the relationship between Sxp and Sop, while a smooth, positive function $f_i$ is used to obtain arbitrary levels and cross-price elasticities as a function of other firm prices given own-price. In addition to their flexibility, these demand systems have the attractive property, shown in appendix *Constant Pass-through Demand Systems* Section I, that solutions to oligopoly pricing are a simple linear matrix algebra problem $\mathbf{p}^{\star} = \mathbf{K}(\mathbf{c} + \alpha)$ where $\mathbf{c}$ is the vector of costs, $\mathbf{K}$ is a matrix of parameters determined by the values of $\rho_i$ and $\beta_{ji}$ and $\alpha$ the $N$-vector with typical entry $\alpha_i = \frac{\tilde{p}_i \rho_i}{1-\rho_i}$. This implies analytic global comparative statics of prices with respect to all parameters, making sensitivity analysis of any result highly transparent, a unique solution (so long as $\mathbf{K}$ is non-singular) and makes the imposition of stability conditions easy through the restriction of $\mathbf{K}$.

However these demand systems have two significant disadvantages. The first is that, except in the special linear case, they typically violate Slutsky symmetry; they are therefore difficult to rationalize from consumer preferences, limiting their usefulness in welfare analysis[35]. More generally CoPaDS can give strange answers outside of the local area in which they are calibrated and are therefore best thought of as extensions the local methods of Froeb et al. (2005) and Farrell and Shapiro (2008). These problems may be less severe than they at first appear, however, given that we are mostly interested in such local approximations (merger impacts are typically reasonably small) and to the extent that they are not the local data used to estimate these effects are likely to be insufficient[36]. Local Slutsky symmetry

[34]Of course this approach has the significant disadvantage of requiring estimates of *all* elasticities, cross-elasticiites and pass-through rates and not merely those of the merging firms. However techniques for measuring elasticities have advanced significantly in recent years and once a matrix of elasticities and cross-elasticities have been measured, a single cost shock to *any combination of goods* suffices to measure all pass-through rates in HCoPaDS. This property is, of course, a result of the restrictive horizontal nature of HCoPaDS and therefore comes at a cost of its own.

[35]We are currently working to formulate a Slutsky symmetric flexible demand system that overcomes this difficult, at the cost, of course, of the tractability of CoPaDS.

[36]See our discussion in Subsection I.B above for more details.

can easily be imposed so for small changes in prices, using the Harberger approximation that consumer welfare loss from a change in prices is $\mathbf{Q}(d\mathbf{p})^\top$ may still be quite accurate, even if the demand system does not globally obey the assumptions justifying this approximation.

A more serious challenge is that, because of the freedom introduced by $f_i$, CoPaDS is not really a parametric class of demand functions. The simplest way to overcome this problem is to consider the special case of CoPaDS that is horizontal in the sense of Subsection III.A: the *Horizontal Constant Pass-through Demand System* (HCoPaDS) given by

$$Q_i(p_i, \mathbf{p}_{-i}) = \lambda_i \left( [1-\rho_i] \left[ \tilde{p}_i - p_i + \sum_{j \neq i} \beta_{ji} p_j \right] \right)^{\frac{\rho_i}{1-\rho_i}} \tag{11}$$

This demand systems still nests linear demand as a special case and allows arbitrary pass-through rates, though it draws a direct connection between the relative size of own- and cross-price elasticities and the relative size of Sop and Sxp as HDSs naturally do (Subsection III.A). It has the additional useful feature that a post-merger equilibrium when two firms merge is well-approximated by the solution to a low-order polynomial equation as long as no $\rho_i$ is close to 0 or 1 (see appendix *Constant Pass-through Demand Systems* Section II). Furthermore because of its tractability, it allows the investigation of the demand properties needed to support typical assumptions used in applied policy analysis.

We provide one example here. Farrell and Shapiro (2008) assume that if a merger creates a local incentive for price increases on both products (*Upward Pricing Pressure* [UPP] in their terminology), both products' prices will rise in equilibrium. Clearly this holds if the two prices are complements (supermodular) for the firm and prices across firms are strategic complements or if the two good are symmetric. However many have argued that prices are likely substitutes across goods within a firm (Hausman, 1997), as higher prices reduce market shares and therefore cannibalization across products. We are not aware of any conditions on demand sufficient for the assumption to be valid. We provide an example in the HCoPaDS class where this fails (even for the local optimum) and then offer fairly restrictive, assumptions about demand within HCoPaDS ensuring that such a *strictly anticompetitive merger* raises both prices. We focus on the case of two firms merging to monopoly, allowing us to focus on the failure of complementarity within the merged firm.

**Example 1.** *Consider the limit case when $\rho_i \to 1$ and demand is exponential. The goods are symmetric pre-merger with constant marginal cost $c_i = 1$ and demand $Q^i = e^{.2p_j - p_i}$; Slutsky symmetry is thus locally satisfied. Pre-merger optimal prices of the firms are $p_i^\star = 2$. The merger creates efficiencies in the pricing of good 1, reducing its marginal cost of production to $\tilde{c}_1 = .81$ but not in the production of good 2 whose marginal cost remains at 1. Then, by*

*Farrell and Shapiro (2008)'s formula the UPP is the net of diversion and efficiencies:*

$$UPP_1 = -\frac{Q_1^2}{Q_1^1}(p_2 - c_2) + \tilde{c}_1 - c_1 = .2\frac{e^{.2p_i - p_j}}{e^{.2p_j - p_i}}1 - .19 = .01 > 0$$

$$UPP_2 = .2 \cdot 1.19 = .238 > 0$$

*Thus the merger is strictly anticompetitive. But post-merger first-order conditions are*

$$\tilde{p}_1^\star = 1.81 + .2(\tilde{p}_2^\star - 1)e^{1.2(\tilde{p}_1^\star - \tilde{p}_2^\star)}$$

$$\tilde{p}_2^\star = 2 + .2(\tilde{p}_1^\star - .81)e^{1.2(\tilde{p}_2^\star - \tilde{p}_1^\star)}$$

*Again, HCoPaDS can give implausible predictions outside a local range of prices. We therefore restrict post-merger prices for the goods to be in the range* $[1, 2.5]x[1, 2.5]$. *Over this range, profits are concave as shown in the appendix Section III, so the the solution to the above first-order conditions is the unique (interior) local optimum. Solving numerically, post-merger optimal prices are* $\tilde{p}_2^\star = 2.38$ *but* $\tilde{p}_1^\star = 1.98 < 2$. *Thus prices fall for one good, though a local approximation indicates consumer welfare falls by* $.36 \cdot e^{-1.6} = \$.07$.

Despite price changes being fairly large (20% in the case of good 2), the Farrell and Shapiro (2008) local approximation performs reasonably well, predicting $p_2^\star = 2.24$ and $p_1^\star = 2.01$ especially when compared to the $p_2^\star = 2.12$ and $p_1^\star = 2$ that would have obtained from linear demand, though it clearly understates the welfare harms.

**Proposition 1.** *A strictly anticompetitive merger-to-monopoly between two firms with horizontal demand that*

1. *is weakly concave in its own price ($\rho_i \leq \frac{1}{2}$ for both i),*
2. *has both diversion ratios* $-\frac{\frac{dQ^j}{dp_i}}{\frac{dQ^i}{dp_i}}$ *less than* 1
3. *and has the property that consumption of both goods falls when both prices rise*

*leads to higher equilibrium prices for both products.*

*Proof.* See appendix *Constant Pass-through Demand Systems* Section III.

Thus if pass-through is sufficiently low for both products, a strictly anticompetitive merger increases both prices, though the magnitude of the effects are small.

# V. Functional Forms and Pass-through

The distinction between cost absorption and cost amplification organizes the comparative statics of many industrial organization models. In sequential or dynamic models, the slope of pass-through plays an important role as well. Because we have little solid basis for generally determining which case holds (Subsection I.F), it is important that functional forms used for both theoretical and empirical analysis are flexible along these dimensions. We argue that currently-available functional forms fail this test and offer a new functional form, *Adjustable pass-through* (Apt) demand, solving this problem while maintaining tractability.

## A. A taxonomy of functional forms

Common functional forms in industrial organization fall into three categories:

1. Most common in theoretical work and empirical analysis of single-product industries are the Bulow and Pfleiderer (1983) *constant pass-through* class of demand functions, which include linear (uniform), constant elasticity (Pareto) and constant mark-up (negative exponential) demands as special cases. This class is highly tractable, yielding linear solutions to monopoly problems, and allows flexible pass-through in the rare cases when authors estimate or analyze across the whole class. However, it is defined by constant pass-through and therefore imposes the slope of pass-through.

2. A common class of demand functions, used in both single- and multi-product empirical analysis, is those based on statistical distributions. These are more often used as building blocks for demand systems (McFadden, 1974; Berry et al., 1995) than as direct demand functions. However from Subsection III.C, we may worry that log-curvature assumptions on idiosyncratic variations impose pass-through restrictions.

3. The Almost Ideal Demand System (AIDS) of Deaton and Muellbauer (1980) with constant expenditures has been used in many applications (Hausman, 1997).

The pass-through properties of the first class are immediate. Table 3 provides a taxonomy, established formally in appendix *Taxonomy of Functional Forms*, of the pass-through properties of the single product, homogeneous consumer version of the second class[37] and a single-product version of AIDS described in the appendix.

Of course, there is no reason why a particular class should fall into one of the four categories permitted by the SPAs: different parameter values and/or prices might well lead

[37]The reader should understand by a probability distribution $F$ a demand function $D(p) = A\big(1 - F[p]\big)$; because pass-through is scale-invariant, all categorization hold for arbitrary positive $A$.

| | $\rho < 1$ | $\rho > 1$ | Price-dependent | Parameter-dependent |
|---|---|---|---|---|
| $\rho'$ $\wedge$ 0 | | | AIDS with $b < 0$ | |
| $\rho'$ $\vee$ 0 | Normal (Gaussian)<br>Logistic<br>Type we Extreme Value (Gumbel)<br>Laplace<br>Type III Extreme Value (Reverse Weibull)<br>Weibull with shape $\alpha > 1$<br>Gamma with shape $\alpha > 1$ | | Type II Extreme Value (Fréchet) with shape $\alpha > 1$ | |
| Price-dependent | | | | |
| Parameter-dependent | | | | |
| Does not globally satisfy MUC | | Type II Extreme Value (Fréchet) with shape $\alpha < 1$<br>Weibull with shape $\alpha < 1$<br>Gamma with shape $\alpha < 1$ | | |

Table 3: A taxonomy of some common demand functions

to different pass-through rates and slopes. Table 3 allows for violations of SPAs. However, strikingly many commonly used distributions *do* turn out to be simply classifiable according to this taxonomy. This perhaps provides a vague justification for the SPAs we use. More persuasively they show how in problems where the level and slope of pass-through are crucial, many commonly-used demand functions are implausibly restrictive, at least in the single-product case. If the distinction between cost absorption and cost amplification (or between increasing and decreasing pass-through) determines the effects of a policy, then assuming demand is of almost any of the common forms[38] is not an innocent simplifying assumption for computational purposes or even a questionable structural restriction. Instead it drives the analysis of an "empirically estimated" model entirely independent of the data.

## B. Adjustable-pass-through (Apt) demand

These restrictions can be eliminated, without sacrificing tractability, by generalizing the constant pass-through demand class to allow flexibility in the slope as well as level of pass-through. This leads naturally to a form for demand that we call *Adjustable-pass-through* (Apt). Apt demand takes a few different forms, depending on the parameter values, which are shown in Table 4. These parallel the forms of the constant pass-through class, which may be transformed-linear or translated constant elasticity depending on pass-through rates,

[38]With the exception of AIDS . And even this is price dependent (neither is globally cost-amplifying). That is while AIDS *does* have pass-through rates that can be either cost-absobing or cost-amplifying it *does not* allow these to be flexible once the elasticity and level of demand have been tied down. That is, while it is first-order flexible it is not flexible on pass-through given these first order properties. In this way it suffers from the same defects as, say, constant elasticity demand (Bulow and Pfleiderer, 1983).

| Name | Parameter values | Demand form |
|---|---|---|
| Limiting cost absorption | $\overline{\rho} < 1$ | $\lambda\left([1-\overline{\rho}]\sqrt{\overline{p}-p}-2\overline{\rho}\alpha\right)^{\frac{2\overline{\rho}}{1-\overline{\rho}}}$   $p < \overline{p} - \frac{4\alpha^2\overline{\rho}^2}{(1-\overline{\rho})^2}1_{\alpha>0}$<br>$0$   $\overline{p} - \frac{4\alpha^2\overline{\rho}^2}{(1-\overline{\rho})^2}1_{\alpha>0} \leq p$ |
| Cost-absorbing constant limiting mark-up | $\overline{\rho} = 1, \alpha < 0$ | $\lambda e^{-\frac{\sqrt{\overline{p}-p}}{\alpha}}$   $p < \overline{p}$<br>$0$   $\overline{p} \leq p$ |
| Constant mark-up | $\overline{\rho} = 1, \alpha = 0, \mu > 0$ | $\lambda e^{-\frac{p}{\mu}}$ |
| Cost-amplifying constant limiting mark-up | $\overline{\rho} = 1, \alpha < 0$ | $\infty$   $p \leq \underline{p} - \alpha^2$<br>$\frac{2\lambda\alpha^2}{e(p-\underline{p}+\alpha^2)}$   $\underline{p} - \alpha^2 < p \leq \underline{p} + \alpha^2$<br>$\lambda e^{\frac{\sqrt{p-\underline{p}}}{\alpha}}$   $\underline{p} + \alpha^2 < p$ |
| Limiting cost amplification | $\overline{\rho} > 1$ | $\infty$   $p \leq \underline{p} + \alpha^2\overline{\rho}\left(\frac{4\overline{\rho}}{(1-\overline{\rho})^2}1_{\alpha>0} - 1_{\alpha<0}\right)$<br>$\lambda\left(\overline{\rho}[\overline{\rho}+1]\right)^{-\frac{\overline{\rho}+1}{\overline{\rho}-1}}\left(\alpha^2\right)^{-\frac{1}{\overline{\rho}-1}}\left(p-\underline{p}+\alpha^2\overline{\rho}\right)^{-1}$   $\underline{p} - \overline{\rho}\alpha^2 1_{\alpha<0} < p \leq \underline{p} + \overline{\rho}^2\alpha^2 1_{\alpha<0}$<br>$\lambda\left([\overline{\rho}-1]\sqrt{p-\underline{p}}-2\overline{\rho}\alpha\right)^{-\frac{2\overline{\rho}}{\overline{\rho}-1}}$   $\underline{p} + (\overline{\rho}\alpha)^2\left(1_{\alpha<0} + \frac{4}{(1-\overline{\rho})^2}1_{\alpha>0}\right) < p$ |

Table 4: The forms of Apt demand $Q(p)$ for various parameter values, all with $\overline{\rho}, \lambda > 0$

which it generalizes. Apt demand has a number of other attractive properties which we state and establish formally in appendix *Apt Demand*. Apt demand

1. nests as special cases all previous demand forms for which the monopoly problem can be explicitly solved (constant pass-through class);

2. is weakly positive, monotone decreasing and smooth almost everywhere that matters;

3. satisfies MUC and gives quadratic solutions to monopoly pricing for all cost levels;

4. satisfies SPAs, including in the GCS extensions discussed in Subsection II.B;

5. can match arbitrary combinations of levels, elasticities, pass-through rates and a wide range of slopes thereof, making it more flexible than any common demand form;

6. is easy to estimate based on a second-order regression of prices and quantities on cost shocks, or a third order regression of quantities on prices;

7. has a simple closed form consumer surplus and thus can easily be derived from the utility maximization of a representative consumer;

8. always gives simple, explicit solutions for final price and any mark-up in the GCS vertical monopolies model, leaving flexible the relevant pass-through rates and slopes;

9. can therefore be used to compare, in a mechanical yet quite general way, a wide range of equilibrium outcomes across and within industrial organizations.

While Apt demand is convenient for monopoly pricing and Bertrand games, but less tractable for production decisions and Cournot games. We therefore propose another demand form, *Apt inverse demand*, where mark-up as a function of quantity takes the Apt form. Apt demand can also form the basis of a class of statistical distribution by interpreting the demand as the survivor function of a distribution of valuations[39]; as far as we know this is the first class of statistical distributions allowing flexibility in the first two derivatives of inverse hazard rates[40]. A computational toolkit accompanying this paper[41] allows researchers to easily estimate, manipulate and predict using Apt (inverse) demand.

# VI. Conclusion

This paper argues that primitive properties of demand which determine the pass-through rate a monopolist would choose play an important role in a wide range of industrial organization models. We demonstrate that reformulating the comparative statics of these models in terms of pass-through rates makes apparent connections between many predictions, increasing their empirical content under weak assumptions. Finally we have show that the lack of understanding of the role played by pass-through has lead theoretical and empirical analysis to impose functional forms that restrict pass-through rates in implausible ways and have proposed a tractable classes of demand functions and systems avoiding these restrictions.

Nonetheless, our results directly empirically relevant only in applications where simple models are plausible. For many applications our results leave out important features. We are actively working to extend our approach to address these shortcomings. Extensions in progress include allowing for imperfect supply elasticities, mutliproduct firms, asymmetries in multi-product industries, vertical product and consumer differentiation and discrete choice models with arbitrary numbers of firms. Other promising extensions are empirical tests based on micro level data (using the links between log-curvature of valuation distributions and demand), incorporation of information on product characteristics and income distributions and discrete choice demand systems employing pass-through-flexible random coefficient and idiosyncratic variation distributions.

Another important direction for future research is the investigation of the relevance of results here to contexts beyond static oligopoly. A few classic topics in industrial organization seem clearly related. Demand-side incentives for collusion depend on the relative value of the gains to monopoly and the incentives for defection, which are independent of elasticities (Ivaldi et al., 2003) but seem clearly tied to pass-through rates or their slope. Given the connection between dynamic oligopoly investments, Stackelberg competition and strategic

[39]It is therefore easy also to derive Apt demand from a statistical distribution of consumer valuations.

[40]We hope this may be of some use in other fields, such reliability theory (Barlow and Proschan, 1975) and statistics (Cox, 1972) where hazard rates play an important role.

[41]Designed by our outstanding research assistant Yali Miao.

effects established by Fudenberg and Tirole (1984) and Bulow et al. (1985), pass-through rates are likely to prove useful in analyzing at least some models of dynamic oligopoly. Finally given the tight connection between pass-through rates, the division of surplus and common agency games, we suspect that they have applications to the study of price discrimination. Of course price discrimination is itself intimately related (Bulow and Roberts, 1989) to auction theory, and this combined with the results of Gabaix et al. (2009) described above, suggests pass-through may be applicable to the analysis of auctions. Futher afield, pass-through may be connected to optimal tax design, given the connections between optimal income taxation and optimal price discrimination and the important role Saez (2001) shows log-curvature plays in optimal tax theory[42]. Recent work by Gopinath and Itskhoki (Forthcoming) shows that the primitives used here also link exchange rate pass-through to the frequency of firms' price adjustments, suggesting the tools developed here may be useful in international macroeconomics.

We hope future research will relax the assumptions underlying the results derived here. The SPAs proposed in Subsection I.E are certainly much stronger than needed for most results and obscure natural quantitative information: if pass-through is significantly, rather than slightly, below unity this should clearly count for something. More careful statistical formulation of assumptions for particular results would therefore be helpful for empirical applications. Furthermore, because the results here help remove some of the ancillary assumptions typically used for identifying industrial organization models, they help expose the maintained economic assumptions to falsification. This holds out hope of confirming underlying economic theory, rejecting it in favor of alternative models or relaxing assumptions needed for identification, such as knowledge by the firm of the true demand system.

Finally, if our results were ever to be used in policy applications this would clearly provide firms with incentives to distort their pass-through rates. Such a critique in the spirit of Lucas (1976) applies to many, if not all, standard approaches in empirical industrial organization. We hope the transparency of the approach here might eventually allow for the formulation of policy-oriented empirical approaches that take into account such firm incentives.

[42] This was first pointed out to us by Jerry Hausman.

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