The Use of Information in Relational Contracts

Preliminary Draft

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June 4, 2009

Abstract

We investigate the use of information in repeated principal-agent game and report three results. First, consistent with Kandori (1992), garbling signals within each period hurts the efficiency of the game. Second, contrary to Abreu, Milgrom, and Pearce (1991), bundling signals across periods and then fully revealing them never increases the efficiency of the game. Third, and most importantly, we construct an intertemporal garbling of signals that transforms the repeated game into one with private information. The main finding of the paper is that in the transformed game, there exists a belief-based pure-strategy equilibrium that can be more efficient than the optimal equilibrium in the original game with imperfect public monitoring.

1 Introduction

This paper studies the use of information in repeated principal agent games, i.e. relational contracts. The prevalence and importance of relational contracts, contracts enforced not by the rule of the court but rather by the self-interests of the participating parties in concern of future contracts, have been emphasized both inside and outside the economics literature. The existing theoretical literature on relational contracts, see for example MacLeod and Malcomson (1988), Levin (2003), and Fuchs (2007), has focused on the efficiency of the relational contract taking the information structure as fixed. Little is known about how information structure affects the efficiency of relational contracts.

The role of information in repeated-game (without transfers) has received considerable attention from economists. The two most influential papers in this literature are Abreu, Milgrom, and Pearce (1991) and Kandori (1992).[1] Kandori (1992) shows that in a repeated game with imperfect public monitoring, the efficiency of the game is weakly increased if the commonly observed public signal of the output becomes more informative in the sense of Blackwell. Kandori (1992) also provides conditions under which the efficiency of the game can be strictly increased. Abreu, Milgrom, and Pearce (1991) (AMP hereafter) show that, when the players play strongly-symmetric strategies and their discount factors approach 1, the efficiency of the game can be enhanced through bundling signals across several consecutive periods and then fully revealing them at the end of these periods.

In this paper, we investigate the role of information in relational contracts. We show that the logic of Kandori (1992) developed in repeated game without transfers carries through to relational contracts: the efficiency of the relational contracts is weakly enhanced if the signals are more informative in the sense of Blackwell. On the other hand, contrasting AMP's finding, we show that bundling signals across periods and fully revealing them every T periods decreases the efficiency of the relational contract. While these two results would appear to suggest the efficiency of the relational contract increases when the signals become more informative and are revealed more frequently, our main result shows that this is not true.

[1]Kandori and Obara (2006) show that when the discount factor is close to one, reducing observability allows for asymmetric punishment in repeated games with imperfect public monitoring. This can expand the equilibrium payoff set.

In our main result, we construct a signal garbling process according to which information is linked intertemporally and is revealed partially. In this process, future signals are informative of and affected by past (unobservable) outputs. Intertemporal garbling then implies that, when the agent deviates from the equilibrium strategy, he will have a different belief from the principal about the probability distribution of future signals. In other words, the relational contract under intertemporal garbling, unlike relational contract with imperfect public monitoring, doe not have a recursive formulation that allows current and future payoffs to be separated. With this information structure, however, we can still construct a belief-based pure-strategy equilibrium. For some parameters, this equilibrium is more efficient than any equilibrium under any information structure without intertemporal garbling.

Our setup can be viewed is a simplified version of Levin's (2003), except that we allow the public signal in each period to potentially depend on the entire history of previous outputs. A principal and an agent trade repeatedly until at least one of them decides to terminate the relationship. Every period the principal offers a contract specifying some court enforceable fixed wage and a relational bonus. Production technology is that if the agent exerts a fixed positive effort, then with a positive probability the output is positive, and if the agent does not exert effort, then the probability of a positive output is lower (or for simplicity, zero in part of our analysis).

If we restrict attention to information garbling within each period, then Kandori's (1992) result that more precise signals in the sense of Blackwell lead to a larger equilibrium payoff set continues to hold in the context of relational contracts. The intuition is that noisier signals are less indicative of effort, so the principal has to pay a larger bonus upon observing a good signal in order to induce effort. Requiring the principal to pay a larger bonus leads to a stronger incentive for her to renege, rendering the relational contract harder to sustain.

AMP's result that the equilibrium payoff set of a repeated game may be expanded when public signals are pooled and revealed once every multiple periods does not generalize to relational contracts with public monitoring for the following reason.[2] The idea behind their finding is that pooling signals across periods allows players to coor-

[2] Fuchs (2007) shows that with private monitoring, similar signal pooling helps reduce the probability of inefficient termination (a form of money burning). One way to see why such signal pooling does not enhance efficiency in repeated principal-agent relationships with public monitoring is that there is no inefficient termination when monitoring is public.

dinate on punishment more efficiently, punishing only when the worst possible signals are realized and punishing harshly in these realizations. Such arrangement lowers the overall inefficiency due to punishment because the likelihood ratio in the test for deviation is highest at the worst signals. Applying AMP's insight to a relational contract with T-period signal pooling would be to pay a bonus to the agent at the end of every T periods except when the worst outcomes are realized. To make the punishment in the worst possible outcomes harsh, the bonus has to be large. This is problematic, however, because the maximal bonus the principal is willing to give is constrained by the present discount value of the future surplus of the relationship which remains unchanged. In summary, reducing the frequency of signal revelation necessarily increases the maximum bonus required to induce effort from the worker, and this increase makes the principal's incentive to pay out the bonus harder to sustain.

Nevertheless, a closer investigation suggests that an alternative form of signal garbling can address a limitation of relational contract under full revelation of (imperfect) signal. Here we describe the limitation and explain how it can be addressed. When the signal is perfectly indicative of the output (but not the effort) and it is revealed in each period, the bonus required to induce effort from the worker is decreasing in the probability of success. In other words, when a success is highly unlikely even if the agent puts in effort, the bonus needs to be very large to induce effort. But such large bonus hurts the incentive of the principal, who will not find it incentive compatible to pay out the bonus to the agent if the bonus exceeds the future surplus of the relationship. Therefore, a relational contract is hard to sustain when the probability of success is small.

An alternative way to provide incentive to the worker we propose is to break the total reward for success into two parts: a) a lowered bonus to be paid out immediately and b) a higher future continuation payoff for the agent. In this way, the principal's incentive to renege on the bonus is weakened (as long as he does not know about the agent's higher continuation payoff which implies more bonus will be required in the future). For the agent, a success not only brings bonus in the immediate future, but also increases the likelihood that future bonus will be paid out. In particular, we construct an equilibrium in which the bonus is paid out based on a garbled signal which is generated with the following garbling process, as illustrated in Figure 1.

Rule-brief

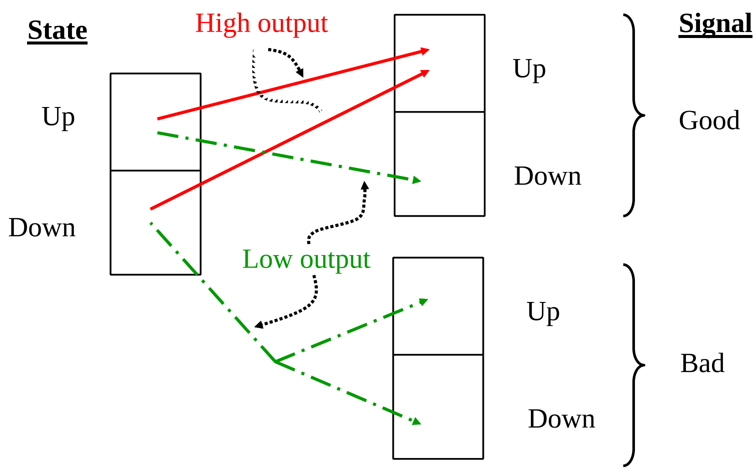

Figure 1: Signal Garbling Process

1.*pdf*

In each period, the garbled signal may be *good* or *bad*, but given any signal, there are two secrete states: *up* and *down*. Players only observe the garbled signal but do not know the state within the signal. If the output is a success, a *good* signal is publicly observed and the state is *up*. If the output is a failure, there are two cases. If the state is *up* in the previous period, then a *good* signal is publicly observed and the state is *down*. If the state is *down* in the previous period, then a *bad* signal is publicly observed and the *up* state is generated with a fixed probability. This signal garbling process may be interpreted as the consequence of a specific exogenous signal generating process in which the information about the output is gradually released. Alternatively, this can be viewed as the evaluation of the agent's performance written by a supervisor and given to the principal who does not directly observe the worker's production.

Moving the state to *up* following a high output regardless of the current state in the garbling process is certainly intended to provide additional incentive for the agent to put in effort. The disincentivizing forces of such scheme are, however, that a) when the agent is in the *up* state, he will be rewarded a bonus regardless of the outcome of production, and b) when the state is low, the agent will be moved to the *up* state with a positive probability even when the output is low. It will become clear in our formal analysis that these disincentivizing parts of the signal garbling process are needed for maintaining the stationarity of the process and ensuring that both the principal and agent never know which state they are in on the equilibrium path. One part of our equilibrium construction is to show that when the probability of success in production becomes small, the disincentivizing forces become less important will be eventually dominated by the incentivizing force. The basic intuition is that when the probability of success is small, then the (stationary) probability that players are in the *up* state is small. Therefore, the probability that disincentivizing bonus reward in (a) is given out in a small probability and the increased chance of being in the *up* state in (b) is also small.

With such signal generating process, we construct an equilibrium in which the principal pays out a bonus whenever the garbled signal is good. The agent's strategy is to put in effort whenever he believes that his probability of being in the up state is weakly small than a threshold. On the equilibrium path, the agent always puts in effort and he believes his probability of being in the up state is exactly equal to the threshold. For all discount factors, this equilibrium performs better than the imperfect public equilibrium without signal garbling when the probability of success is small. And its degree of improvement is increasing in the discount factor of the agent.

The key to this construction is to specify the action of the agent off the equilibrium path. In the above-constructed signal generating process, the signal in each period can depend on the entire history of past outputs, and thus it may depend on the entire private history of past actions of the agent. In other words, while the principal always forms one (equilibrium) belief, the agent's belief of the probability he is in an up state can depend on his entire private history of efforts. When it is possible for the principal and the agent to form different beliefs (following the agent's deviation), checking one-stage deviation no longer guarantees that a strategy profile constitutes an equilibrium. To check that the agent's strategy is an optimal response to the principal's strategy, one needs to check multi-stage deviations as well. Since this is an infinitely repeated game, checking such multi-stage deviations can be difficult[3].

In our construction, conditional on the current period's output, the signal is completely determined by the previous period's state the agent was in. In other words, with this two-state construction, the only payoff relevant belief of the agent is the

[3] The need to check multi-stage deviation also appears in Abreu, Milgrom, and Pierce (1991), and Fuchs (2007). These two papers consider T-period review strategies, so there is no need to check deviations that exceed T-stages. In contrast, there is a priori no upper bound in the number of stages of deviation.

probability that he is in an up state. This allows for a recursive formulation of the agent's value function with the state variable being the (privately known) probability that the agent is in an up state. With this recursive representation, we show that the optimal action of the agent is to put in effort if and only this probability falls weakly below a threshold.

In standard relational contracts with imperfect public monitoring, one way to maximize the enforceable bonus payment from the principal is to give the agent his individually rational continuation payoff after each bonus payout. Such arrangement will not be sustainable in our current setup with intertemporally garbled signals. This is because the agent can shirk and after privately knowing being punished by placed on the low state, quits the game, rendering the punishment of low state ineffective. We show that this incentive problem can be resolved by always postponing the bonus payment to be paid out as part a higher base wage offered by the principal in the following period. Since now the base wage is made contingent of the previous period's signal, it suggests that the optimal relational contract with intertemporal signal garbling may be necessarily nonstationary.

For the rest of the paper, we set up the model in Section 2. We analyze the model in Section 3, with our main result presented in Subsection 3.3 and some generalization relegated to the Appendix. In our discussion in Section 4, we show that the "belief-free" approach does not enhance efficiency and that signal garbling helps enhance efficiency only when the probability of success in production is not equal to half. Section 5 concludes.

2 Setup

Time is discrete and indexed by $t \in \{1, 2, ..., \infty\}$.

2.1 Players

There's one principal and one agent. Both are risk neutral, infinitely lived, and have a common discount rate of δ. The agent's per period outside option is $\underline{u}$; the principal's per period outside option is $\underline{\pi}$.

2.2 Production

If the principal and the agent engages in production together in period t, the agent chooses effort $e_t \in \{0, 1\}$. The cost of effort is given by $c(0) = 0$ and $c(1) = c$. The output is binary: $Y_t \in \{0, y\}$. We assume that

$$\begin{aligned} \Pr(Y_t &= y|e_t = 1\} = p; \\ \Pr(Y_t &= y|e_t = 0\} = q, \end{aligned}$$

where $1 > p > q \geq 0$.

We make the standard assumption in the literature that the relationship has a positive surplus if the agent puts in effort and a negative one if the agent does not put in effort. More formally, we assume that

$$py - c > \underline{u} + \underline{\pi} > qy.$$

2.3 Information Structure

In each period t, after the output is realized, the principal and the agent both observe a *public signal* $s_t \in \mathbf{S}$, the set of possible signals. The s_t is produced by the following the signal generating function:

$$S_t : \prod_{j=1}^{t} Y_j \to \Delta\mathbf{S},$$

where $\Delta\mathbf{S}$ stands for the probability distribution on the set of possible signals.

In addition to the public signal, the agent also knows his effort level e_t. The principal does not know the agent's effort. [4]

The signal generating function provides a framework for modelling different information structure. Since the main focus of the paper is to study the effect of

[4] In a more general setting, we may also assume that both the principal and the agent observe some private information about the output in addition to the public signal. The analysis in this setting, however, is likely to be complicated. Fuchs (2007) considers a model where the principal has private information about the output and the agent does not. Fuchs (2007) shows the the optimal relational contract can be implemented by a termination contract, but the full characterization of the termination contract remains unknown.

information structure on the efficiency of the relational contract, we provide below several examples of commonly observed information structure and show how the signal generating function can be used to model them.

Example 1 (Perfect Observability of Outputs)

A common information structure is that the output **in each period** can be observed without error. This is also the standard modeling assumption in relational contracts; see for example Malcomson and Macleod (1988) and Levin (2003). To model this information structure, we let the set of signals be $\mathbf{S} = \{0, y\}$. In each period t, we let the signal $s_t = y_t$, the output in period t. More formally, the signal generating function is given by

$$\Pr(S_t(y_1, ..., y_t) = y_t) = 1, \text{ for all } \{y_1, ..., y_t\}.$$

Example 2: (T-period Revelation)

Another common information structure is that the outputs are revealed perfectly every T periods and no information about the outputs is known in between. This information structure has received considerable attention from the literature; see for example Abreu, Milgrom, Pearce (1991) and Fuchs (2007). To model this information structure, we let $\mathbf{S} = \{0, y\}^T \cup \{N\}$, where N stands for no information. When $t \neq nT$ for each $n \in N$, the signal $s_t = N$. When $t = nT$, $s_t = (y_{(n-1)T+1}, ..., y_{nT})$. More formally, when $t \neq nT$, the signal distribution function is given by

$$\Pr(S_t(y_1, ..., y_t) = N) = 1.$$

When $t = nT$,

$$\Pr(S_t(y_1, ..., y_t) = (y_{(n-1)T+1}, ..., y_{nT})) = 1, \text{ for all } \{y_1, ..., y_t\}.$$

The information structures above have both received considerable attention from the literature. The next two examples are information structures that are common in life but less studied.

Example 3: (Delayed Information Revelation)

In many economic situations, information of the outcome is not readily available. For example, the time it takes to collect the information creates such delay. But even if information collection can be done in real time, information about the "right outcome" may not be known immediately after the actions are taken. For example, computer system can track the sales of each item, but the real sales figure should also account for the returns from the customers, and this can take place weeks after the salesperson made the sale.

The simplest case of delayed information is that output is always revealed with k period of delay. To model this case, we let $\mathbf{S} = \{0, y\} \cup \{N\}$, where again N stands for no information. In period $t = 1$, the signal $s_1 = N$, and in period $t > 1$, the signal $s_t = y_{t-1}$. More formally, the signal generating function is given by

$$\begin{aligned}
\Pr(S_1(y_1, ..., y_t) &= N) = 1, \text{ for } t < k \\
\Pr(S_t(y_1, ..., y_t) &= y_{t-k}) = 1, \text{ for } t > 1 \text{ and all } \{y_1, ..., y_t\}.
\end{aligned}$$

Example 4: (Partial Information Revelation)

In many economic situations, information about outputs is only revealed partially. For example, when a building or bridge is finished, information about its reliability can take years or decades to be revealed. That how well the building stands up in an earthquake is, in most cases, not known. Costs of collecting information also prevents the information from being completely revealed. For instance, when senior executives decide the bonus of a worker at the end of the year, it is difficult and very costly to know the performance of the worker on each single day. Instead, the senior executives often base the bonus of the worker on a crude performance evaluation measure (often supplied by some middle-level manager), say a performance grade, which misses many information of the worker's performance.

One example that captures partial information revelation is the following. Let the set of the signal be $S = \{Success, Failure\}$. In period t, the signal $s_t = Success$ if more than half of the previous outcomes $y = Y$, and $s_t = Failure$ otherwise. More formally,

$$\begin{aligned}
\Pr(S_t(y_1, ..., y_t) &= Success) = 1, \text{ if } \sum_{j=1}^{t} y_j > \frac{ty}{2} \\
\Pr(S_t(y_1, ..., y_t) &= Success) = 0, \text{ if } \sum_{j=1}^{t} y_j \leq \frac{ty}{2}
\end{aligned}$$

Note that in this example, the signal in each period only captures the average of the existing outputs. Consequently, the exact output in many periods may not be publicly known. More interestingly and importantly here, even if the output y_t in period t may not be known, it has an impact on all future signals. Moreover, while this seems to suggest that the signal generating process is very complicated and may require an infinite memory, simplification is possible here by having the right "state variable". In the example above, even if the signal s_t depends on all of the previous t outputs, it can be determined by the output in period t (y_t) and the average output in all previous periods (the state variable).

2.4 Timing

The timing goes as follows. At the beginning of period t, the principal offers a contract that consists of a fixed wage w_t. The agent chooses whether to accept or not: $d_t \in \{0, 1\}$. If the agent rejects, the principal and the agent receive their outside options. If the agent accepts, he chooses e_t. The signal s_t is realized and the principal pays out $W_t \geq w_t$.

Note that the timeline above does not explicitly specify a contingent bonus payment and when it should be paid out. The specification above is sufficiently broad that it nests several possibilities. For example, in Levin (03), there is a contingent bonus (b_t) paid out at the end of each period. In this case, we have that (on the equilibrium path)

$$W_t = w_t + b_t.$$

As another example, in Fong and Li (09a), the bonus is paid out at the beginning of next period as part of the efficiency wage. To model this case,

$$\begin{aligned} W_t &= w_t; \\ w_{t+1} &= \underline{w} + \frac{b_t}{\delta}, \end{aligned}$$

where $\underline{w}$ is the wage paid to the worker if the output is 0.

Malcomson and MacLeod (1989) and (1998) show that the timing of the bonus (pay of performance vs efficiency wage) does not affect the efficiency of the relational contract when the outputs are perfectly observed. As we will see below, however, the timing of the bonus will matter when the outputs are only partially revealed.

2.5 Strategy and Equilibrium Concept

2.5.1 History

We denote $h_t = \{w_t, d_t, s_t, W_t\}$ as public events that happen in period t. Denote $h^t = \{h_n\}_{n=0}^{t-1}$ as a public history path at the beginning of period t. $h^1 = \Phi$. Let $H^t = \{h^t\}$ be the set of public history paths till time t. Finally, define $H = \cup_t H^t$ as the set of public histories. The principal only observes the public history. For the agent, at the beginning of period t, he also observes his past actions $e^t = \{e_j\}_{j=1}^{t-1}$. Denote $H_A^t = H^t \cup \{e^t\}$ as the set of agent's private history at the beginning of period t.

2.5.2 Strategy and Payoff

In period t, the following functions capture the actions of the players.

- The wage offer of the principal is given by

$$w_t : H^{t-1} \rightarrow R.$$

- Agent's acceptance decision satisfies

$$d_t : H_A^{t-1} \times \{w_t\} \rightarrow \{0, 1\},$$

  where the second component in the cross-product denotes the set of wage offers.

- Agent's effort decision satisfies

$$e_t : H_A^{t-1} \times \{w_t\} \to \{0,1\}$$

- Total compensation function satisfies

$$W_t : H^{t-1} \times D_t \times S_t \to R$$

The pure strategy of the agent is given by

$$s^A = \{d_t, e_t\}_{t=1}^{\infty}.$$

And the pure strategy of the principal is given by

$$s^P = \{w_t, W_t\}_{t=1}^{\infty}.$$

We can also allow the principal and the agent to play mixed strategies. We restrict our attention to pure strategy partly to distinguish ourselves from the literature of repeated game with private monitoring, where mixed strategies play a crucial role.

Take a strategy profile (s^A, s^P). The expected payoff of the agent following a private history h_A^t and w_t is given by

$$U(h_A^t, w_t, s^A, s^P) = E[\sum_{\tau=t}^{\infty} \delta^{\tau-t}\{\underline{u} + 1_{\{d_\tau=1\}}(-ce_\tau + W_\tau - \underline{u})\}|h_A^t, s^A, s^P].$$

Similarly, we can define $U(h_A^t, w_t, d_t, s^A, s^P)$, the expected payoff of the agent following his acceptance decision in period t.

The expected payoff of the principal following a private history h_A^t is given by

$$\begin{aligned}\pi(h_A^t, s^A, s^P) = \quad & E[\sum_{\tau=t}^{\infty} \delta^{\tau-t}\{\underline{\pi} + 1_{\{d_\tau=1\}}(y(q+(p-q)e_\tau) \\ & -W_\tau - \underline{\pi})\}|h_A^t, s^A, s^P].\end{aligned}$$

Since the principal does not observe the private history of the agent, we define

$$\Pi(h^t, s^A, s^P) = E_{\mu^P}[\pi(h_A^t, s^A, s^P)|h^t]$$

as his expected payoff following public history h^t. Here, the expectation is taken over all possible private history of the agent (h_A^t) according to the principal's belief (μ^P) conditional on observing public history h^t.

We also denote $\pi(h_A^t, w_t, d_t, s_t, s^A, s^P)$ as the principal's expected payoff in period t following agent's private history h_A^t, the principal's wage offer, the agent acceptance decision d_t, and the signal s_t. We define $\Pi(h^t, w_t, d_t, s_t, s^A, s^P)$ similarly.

2.5.3 Perfect Bayesian Equilibrium

The solution concept we use here is Perfect Bayesian Equilibrium (PBE). A PBE in this model consists of the strategy of the principal (s^{*P}), the strategy of the agent (s^{*A}), the belief of the principal (μ^P), and the belief of the agent μ^A such that

- following any history $\{h_A^t, w_t\}$, and $\{h_A^t, w_t, d_t\}$,

$$\begin{aligned} U(h_A^t, w_t, s^{*A}, s^{*P}) &\geq U(h_A^t, w_t, \widetilde{s}^A, s^{*P}); \\ U(h_A^t, w_t, d_t, s^{*A}, s^{*P}) &\geq U(h_A^t, w_t, d_t, \widetilde{s}^A, s^{*P}); \end{aligned}$$

- following any history h and $\{h^t, w_t, d_t, s_t\}$,

$$\begin{aligned} \Pi(h^t, s^{*A}, s^{*P}) &\geq \Pi(h^t, s^{*A}, \widetilde{s}^P); \\ \Pi(h^t, w_t, d_t, s_t, s^{*A}, s^{*P}) &\geq \Pi(h^t, w_t, d_t, s_t, s^{*A}, \widetilde{s}^P). \end{aligned}$$

- the beliefs are consistent with σ^* and are updated with Bayes rule whenever possible.

In this setting, since only the agent has private information, the belief of the agent is degenerate. The principal knows the public history, and his belief of the agent's private history is again degenerate whenever the action of the agent is consistent with the equilibrium play. When the agent's action does not conform to the equilibrium play, we may assume the principal believes that the agent has never put in effort in the past.

Note that while the principal and the agent will share the same belief along the equilibrium path, this is not true if the agent deviates. When the agent deviates, his belief of his actions and thus the output distribution in the past is different from the equilibrium belief of the principal. Since the future signals can depend on the realization of past outputs, the agent's belief of the signal distribution in the future will be different from the principal as well (even if the agent follows the equilibrium strategy in the future). This differences in beliefs imply that checking one-stage deviation will no longer be sufficient to guarantee that a strategy profile is a PBE.

3 Analysis

In this section, we study how the information structure affects the efficiency of the relational contract. In Section 3.1, we show that garbling of information within a period worsens the efficiency of relational contracts. This result is consistent with a related result in repeated game; see Kandori (1992). In Section 3.2, we show that the T-period review contracts, in which the signals of performance becomes public every T periods, is strictly worse than the contract in which information about output is fully revealed in each period. This result contrasts with the finding in the repeated game literature in which bundling information across periods can expand the equilibrium payoff set; see Abreu, Milgrom, and Pearce (1991). While the preceding results seem to suggest that the efficiency of the relational contract increases with the informativeness of the signal and the frequency at which the signal is disseminated, we show in Section 3.3 that the efficiency of the relationship can be enhanced through some form of intertemporal garbling of signals that transforms the game between the principal and agent into one of private monitoring.

3.1 Within-Period Information Garbling

In this subsection, we restrict attention to garbling information within each period and we study how the efficiency of the relationship is affected by the informativeness of the signal. We focus on the two-signal case, which eases the analysis and helps highlight the intuition of the result. The general case with multiple action and multiple signals is analyzed in the appendix.

Let the set of signals be $S = \{0, y\}$. And we start with the case in which the signal is perfectly informative of the output, i.e. $s_t = y_t$. Note that even if the signal is perfectly informative, the output still does not correspond one-to-one to the effort level. In particular, this information structure is a special case of relational contract with imperfect public monitoring studied by Levin (2003). Levin (2003) shows that the optimal relational contract with public monitoring can be implemented by a sequence of stationary contracts.

In the stationary contract, the principal pays out a base wage w in each period, and he also pays out a bonus b if the high output is realized. Note that to induce the effort from the worker, we need the bonus to be big enough such that

$$\begin{aligned} w - c + pb &\geq w + qb \\ b &\geq \frac{c}{p-q}. \end{aligned} \tag{1}$$

With the bonus big enough so that the agent will put in effort per period, the principal can lower the base wage of the worker to his outside option, i.e.

$$w - c + pb = \underline{u}.$$

In this way, the principal can capture the entire surplus of the relationship.

Finally, since the bonus is non-contractible, we need to check that the principal is willing to pay the bonus. It is incentive compatible for the principal to pay the bonus if the future gain of doing so exceeds the short-term loss of paying the bonus. We may assume without loss of generality that if the principal fails to pay the bonus the two parties will receive their outside options forever. This implies that for the principal to pay the bonus, we need

$$b \leq \frac{\delta(py - c - \underline{u} - \underline{\pi})}{1-\delta}, \tag{2}$$

where $\frac{\delta(py-c-\underline{u}-\underline{\pi})}{1-\delta}$ is the discounted expected future surplus of the relationship, which is completely captured by the principal.

Note that equation (1) and (2) combined implies that an relational contract can induce effort in this setting if and only if

$$\frac{c}{p-q} \leq \frac{\delta(py - c - \underline{u} - \underline{\pi})}{1-\delta}. \tag{3}$$

In other words, the incentive cost should be smaller than the discounted expected future surplus.

Now suppose that instead of having signal as being a perfect indicator of the output, the signal is noisy instead. In particular, we assume that

$$\begin{aligned}
\Pr(s_t &= y|Y_t = y) = \theta_1 \\
\Pr(s_t &= 0|Y_t = y) = 1 - \theta_1 \\
\Pr(s_t &= 0|Y_t = 0) = \theta_2 \\
\Pr(s_t &= y|Y_t = 0) = 1 - \theta_2,
\end{aligned}$$

where $\theta_1 > \frac{1}{2}$ and $\theta_2 > \frac{1}{2}$ so that the signal is indicative of the true output. This information structure is a garbling of the perfect signal. In other words, the garbled signal is less informative than the perfect signal in the sense of Blackwell.

With this information structure, it can be shown that the optimal contract can again be implemented by a sequence of stationary contracts. In the stationary contract, a bonus b' is paid out when a signal $s_t = y$ is realized. To induce the agent to put in effort, we need

$$\begin{aligned}
-c + (p\theta_1 + (1-p)(1-\theta_2))b' &\geq (q\theta_1 + (1-q)(1-\theta_2))b' \\
b' &\geq \frac{c}{(p-q)(\theta_1 + \theta_2 - 1)}.
\end{aligned}$$

Now the principal can again set the wage to capture the entire surplus of the relationship. In this case, the incentive constraint of the principal to pay the bonus is again given by

$$b' \leq \frac{\delta(py - c - \underline{u} - \overline{\pi})}{1-\delta}.$$

Combining the two equations above, we have that, with noisy signals, the necessary and sufficient condition to induce effort in a relational contract is given by

$$\frac{c}{(p-q)(\theta_1 + \theta_2 - 1)} \leq \frac{\delta(py - c - \underline{u} - \overline{\pi})}{1-\delta}.$$

It is clear from the expression above that, as along as $\theta_1 < 1$, or $\theta_2 < 1$, i.e. the signal does not reflect the output perfectly, we have

$$\frac{c}{(p-q)(\theta_1+\theta_2-1)} > \frac{c}{(p-q)},$$

so the condition for sustaining effort is strictly more stringent here.

The intuition for this result is straightforward. When the signals are noisy, they are less indicative of effort, so it requires a larger bonus to induce effort. But the larger bonus makes the principal more likely to renege, and it follows that efforts are harder to sustain in equilibrium.

While the analysis above was based on a two-action, two-output setting, the intuition carries over to more general settings. For example, the idea that noisy signals require larger bonus is directly related to the idea that larger prizes are required to induce effort in a tournament setting with continuous effort and outputs. We also perform a similar analysis in a multiple signal, multiple action setting in the appendix.

3.2 Bundling T Periods

In this subsection, we analyze how the efficiency of the relational contract is affected when the signals are not revealed in each period, but rather are bundled together and revealed once every several periods.

In particular, we assume that the information becomes public every T periods and no information is revealed in between. Let $S = \{0, Y\}^T \cup \{N\}$, where N stands for no information. When $t \neq nT$ for each $n \in \mathbb{N}$, the signal $s_t = N$. When $t = nT$, $s_t = (y_{(n-1)T+1}, ..., y_{nT})$. More formally, when $t \neq nT$, the signal distribution function is given by

$$\Pr(S_t(y_1, ..., y_t) = N) = 1.$$

When $t = nT$,

$$\Pr(S_t(y_1, ..., y_t) = (y_{(n-1)T+1}, ..., y_{nT})) = 1, \text{ for all } \{y_1, ..., y_t\}.$$

In this game, it is straightforward to show that the optimal contract can be implemented as a sequence of stationary contracts. In particular, the bonus will be paid out at the end of each T periods, and there is a bonus function $B(y_{(n-1)T+1}, ..., y_{nT})$ that maps $\{0, y\}^T$ to R^+ for all n.

Now define the maximum bonus the principal ever pays out as

$$B_{\max} = \max_{\{(y_{(n-1)T+1}, ..., y_{nT})\}} \{B(y_{(n-1)T+1}, ..., y_{nT})\}.$$

To induce the principal to pay out this bonus, we need that

$$B_{\max} \leq \frac{\delta(py - c - \underline{u} - \underline{\pi})}{1 - \delta}.$$

In other words, the bonus cannot be larger than the discounted expected future surplus.

Now consider the agent's incentive to exert effort. Let $\{e^*_{nT+1}, e^*_{nT+2}, ...e^*_{(n+1)T}\}$ be the equilibrium effort of the agent from period $nT + 1$ to $(n + 1)T$. For the agent to find it incentive compatible to exert effort in period $nT + 1$, it is necessary that he does not find it profitable to shirk in that period:[5]

$$\begin{aligned} c \leq E[\delta^{T-1} B(y_{nT+1}, ..., y_{(n+1)T}) | e_{nT+1} &= 1, e^*_{nT+2}, ...e^*_{(n+1)T}] \\ -E[\delta^{T-1} B(y_{(nT+1}, ..., y_{(n+1)T}) | e_{nT+1} &= 0, e^*_{nT+2}, ...e^*_{(n+1)T}]. \end{aligned}$$

Now

$$\begin{aligned} & E[B(y_{nT+1}, ..., y_{(n+1)T}) | e_{nT+1} = 1, e^*_{nT+2}, ...e^*_{(n+1)T}] \\ = \; & pE[B(y, ..., y_{(n+1)T}) |, e^*_{nT+2}, ...e^*_{(n+1)T}] + (1 - p) E[B(0, ..., y_{(n+1)T}) |, e^*_{nT+2}, ...e^*_{(n+1)T}]. \end{aligned}$$

Similarly,

$$\begin{aligned} & E[B(y_{nT+1}, ..., y_{(n+1)T}) | e_{nT+1} = 0, e^*_{nT+2}, ...e^*_{(n+1)T}] \\ = \; & qE[B(y, ..., y_{(n+1)T}) |, e^*_{nT+2}, ...e^*_{(n+1)T}] + (1 - q) E[B(0, ..., y_{(n+1)T}) |, e^*_{nT+2}, ...e^*_{(n+1)T}]. \end{aligned}$$

Therefore, the expected benefit of putting effort in period $nT + 1$ while keeping other

[5] The sufficient condition would be that the optimal deviation, which potentially involves changing efforts in subsequent periods, is also not profitable.

periods' efforts fixed is given by

$$\begin{aligned}
& (p-q)\delta^{T-1}(E[B(y,...,y_{(n+1)T})|, e^*_{nT+2}, ...e^*_{(n+1)T}] - E[B(0,...,y_{(n+1)T})|, e^*_{nT+2}, ...e^*_{(n+1)T}]) \\
\leq \; & (p-q)\delta^{T-1}B_{\max}.
\end{aligned}$$

It follows that a necessary condition to induce effort in period $nT+1$ is that

$$\frac{c}{p-q} \leq \delta^{T-1}B_{\max}.$$

It is also immediate that a necessary condition to induce effort in any period $nT+k$ is given by

$$\frac{c}{p-q} \leq \delta^{T-k}B_{\max}. \tag{4}$$

Since (4) is the easiest to satisfy for $k=T$, the necessary condition for some effort to be exerted in some period is $c/(p-q) \leq B_{\max}$. Combining with the incentive constraint of the principal, a necessary condition for inducing effort in any period is given by

$$\frac{c}{p-q} \leq \frac{\delta(py - c - \underline{u} - \overline{\pi})}{1-\delta},$$

and this is exactly the necessary and sufficient condition for inducing effort in the case in which information is revealed in each period. In other words, bundling periods together cannot help sustain cooperation in the relational contract.

3.3 Intertemporal Garbling with Partial Revelation

In the previous two subsections, the analysis seems to suggest that the relational contract is more efficient when the signals are more precise and are revealed more frequently. In this section, we show that when the success probability is low, the efficiency of the relationship can be enhanced through linking information intertemporally but not fully revealing them. To keep the analysis tractable, we look at the special case that $q=0$.

To describe the signal generating process, we imagine that there is a supervisor who privately observes the output and then publicly announces a garbled signal of his observation. Every period, the supervisor reports whether the (garbled) signal is *good* or *bad*. The report of a *good* signal can be viewed as a recommendation for the principal to pay the agent a bonus although the principal will pay only if it is incentive compatible to do so. Apart from that, the supervisor is also required to *privately* keep tract of a state which may be *up* or *down*. It is important that the supervisor never discloses the state to the principal or the agent. We also assume the supervisor has no interest in the game and can be asked to garble the signal in any way the principal would like him to, with the restriction that the agent is fully aware of the garbling process.

We further restrict our attention to a specific garbling process which is described as follows. If the output is high, then regardless of the state, the supervisor publicly announces the signal *good* and privately moves the state to *up*. Conditional of the state being *up*, if the output is low, he will publicly announce *good* but at the same time move the state to *down*. If both the state is *down* and the output is low, then he will announce *bad* and at the same time he will randomize and move to the *up* state with probability ρ^* and to the *down* state with probability $1 - \rho^*$, where $\rho^* \in [0, 1]$ is the unique probability that satisfies

$$\rho^* = \frac{p}{p + (1-p)\rho^*}.$$

The following figure illustrates how different outputs and previous states lead to different signals and states. It is similar to Figure 1 except the probability of each path is labeled.

Rule

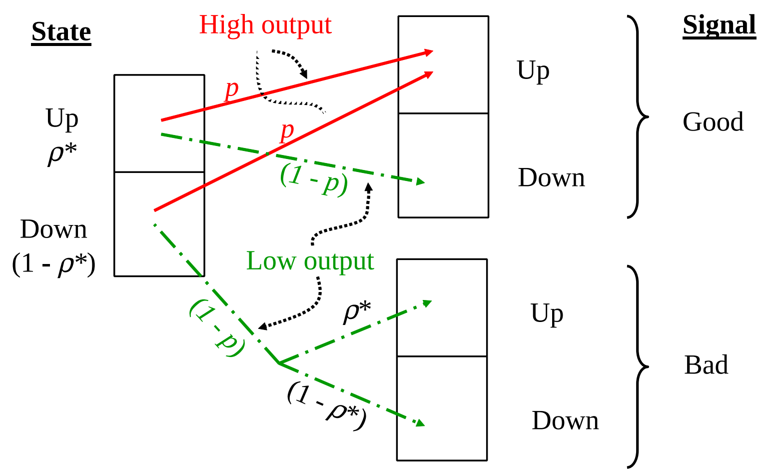

Figure 1: Reporting Rule

$_2.pdf$

It can be verified that if in period 1 the probability that the state is *up* is ρ^*, the agent puts in effort, and the period 1 signal is good, then the conditional probability that the state is *up* in period 2 is again ρ^*. Now recall that that if the period 1 signal is bad, the probability that the state is *up* in period 2 is also ρ^* (by the definition of the signal generating process). Essentially, this signal generating process ensures that if the initial probability that the state is *up* is $\rho^*$and the agent always puts in effort, then the conditional probability that the state is *up* in any subsequent periods is always ρ^* regardless of history of signals.

The discussion above suggests the following "strategy" may be an equilibrium. In each period $t > 1$, the principal offers a wage $w_t \in \{w_b, w_g\}$, such that w_b is offered if the signal in the previous period is bad and w_g is offered otherwise. The agent always put in effort. The probability that the state is up is ρ^* in period 1. And period 1 wage is chosen such that the payoffs of both parties inside the relationship exceed their outside options.

This equilibrium is stationary in the sense that if the agent always puts in effort, then at the beginning of any period t, the probability that the state is up in is always ρ^*, just as in period 1, so it's as if the history does not matter and the relationship starts anew in period t.

This stationarity, however, holds only on the equilibrium path when the agent always puts in effort. If the agent shirks in some period t, then his (correct) subjective probability of the up-state in future periods $t' > t$ will not be ρ^* anymore. Yet the principal's (incorrect) subjective probability of the up-state remains at ρ^*. In other words, if the agent ever deviates, the principal and the agent will no longer share the same belief about the game, and the stationarity of the equilibrium is lost.

The difference in beliefs between the principal and the agent off the equilibrium implies that the game cannot be decomposed into "today" and "future", and the one-stage deviation principle does not apply here. To check a strategy profile is an equilibrium, one needs to check multi-stage deviations. One example of multi-stage deviation that's of particular concern here is that the agent may shirk this period and rejects the contract next period. Rejection is possible since the agent's subjective probability that the state is up will be smaller than ρ^*, and his assessment of the value of the relational contract will be smaller than that of the principal's.

The key to verifying that the proposed strategy is an equilibrium is by noticing that, fixing the play of the principal, there is a sufficient statistic that determines the agent's play. In particular, the distribution of future signals (in periods $t' > t$) is affected by the past play only through the conditional probability that the state is up in period t. It follows that the agent's subjective probability that the state is up is a state variable that determines his future plays. This observation implies that the agent's maximization problem, i.e. choosing his strategies as a best response to the principal's strategy, can be formulated recursively with the state variable being his subjective probability that the state is up. This greatly simplifies the analysis.

More formally, define the random wage

$$\widetilde{w}_t = \begin{cases} w_b & \text{if } s_{t-1} = bad \\ w_g & \text{if } s_{t-1} = good \end{cases}$$

for $t > 1$, where

$$\begin{aligned} w_b &= (1-\delta)\underline{u} + c - \frac{p}{\rho^*}B; \\ w_g &= w_b + B; \\ B^* &= \frac{c/p}{\left(\frac{1}{\rho^*} - \frac{\rho^*}{p} + \delta\frac{(1-\rho^*-\delta(1-\rho^*)\rho^*)}{1-p\delta^2(1-\rho^*)}\right)}. \end{aligned}$$

The choice of the value of these variables will become here in the proof. Essentially, these numbers are chosen such that a) the agent is indifferent from working and shirking along the equilibrium path, and b) the principal captures all of the surplus in the relationship.

Theorem 1: *The following strategies constitute an equilibrium if*

$$B^* \leq \frac{\delta\left(y - c - \underline{u} - \pi\right)}{1 - \delta}$$

Principal: Offers $w_1 = w_b$. If the agent has always accepted the contract for all periods before t, $(\prod_{\tau<t} d_\tau = 1)$ and the principal has offered $w_\tau = \widetilde{w}_\tau$ for all $\tau < t$, then offers

$$w_t = \widetilde{w}_t.$$

Otherwise, offers $w_t = \underline{u} - 1$.

Always offers $W_t = w_t$.

Agent: Chooses $d_t = 1$ if a) $w_1 = w_b$ and $w_\tau = \widetilde{w}_\tau$ for all $\tau < t$ or b) $w_t > \underline{u}$. Chooses $d_t = 0$ otherwise.

Chooses $e_t = 0$ if a) $w_1 \neq w_b$ or $w_\tau \neq \widetilde{w}_\tau$ or $d_\tau \neq 1$or some $\tau < t$ or b) the agent believes that the probability he is in the up-state is strictly bigger than ρ^*.

The belief of the agent is calculated via Bayes rule. The probability that the state is up in period 1 is equal to ρ^*.

Proof. To check that this is an equilibrium, we first check that the agent's strategy is a best response to the principal's strategy and then show that the principal's strategy is a best response to the agent's.

To check that the agent's strategy is a best response to the principal's strategy following any history, we take the following steps. First, we note that the equilibrium strategy of the principal is rather passive (offer w if the signal is bad and $w + \frac{B}{\delta}$ if the signal is good). Therefore, the agent's optimal strategy is the solution to the following dynamic decision making problem: there is a machine that gives out w if the last-period signal is bad and gives out $w + \frac{B}{\delta}$ if the last-period signal is good. The signal generating process is describe in the graph above, and the initial condition being the agent is in the up state with probability ρ^*. If the agent chooses whether to play against the machine and whether to put in effort each period. If the agent chooses not to play against the machines, he receives his outside option $\underline{u}$ forever.

In the decisionmaking problem above, suppose the agent has always accepted to play against the machine up till and including period t, then fixing the strategy of the agent from period t on, his payoff is determined by the past effort only through ρ_t, the probability that the agent is in an up state at the beginning of period t.

This observation implies that we can define the following value function. Let $\tilde{V}(\rho, w, B)$ be the agent's maximal payoff if he has just accepted to play against the machine, the machine pays out w this period (and then pay out in the future according to the specification of the signals, i.e. w after the bad signal and $w+\frac{B}{\delta}$ after the good signal), and the probability that the agent is in the up-state is ρ. Then we have the following value function

$$\begin{aligned}\tilde{V}(\rho,w,B) \quad = \quad & \max\{w-c+(p+(1-p)\rho)\delta\max\{\underline{u},\frac{B}{\delta}+\tilde{V}(\frac{p}{p+(1-p)\rho},w,B)\} \\ & +(1-(p+(1-p)\rho))\delta\max\{\underline{u},\tilde{V}(\rho^*,w,B)\}, \\ & w+\rho\delta\max\{\underline{u},\frac{B}{\delta}+\tilde{V}(0,w,B)\}+(1-\rho)\delta\max\{\underline{u},\tilde{V}(\rho^*,w,B)\}\}.\end{aligned}$$

The first term in the expression is the expected payoff of the agent if he puts in effort. The term $p+(1-p)\rho$ is the probability that the signal is good this period given effort, and $\frac{p}{p+(1-p)\rho}$ is the conditional probability that the agent is in the up state given effort and good signal. We have the max operator to include the possibility that the agent may choose not to play against the machine next period. Similarly, the second term is the agent's expected payoff if he does not put in effort.

The functional equation above is somewhat complicated, but for our purpose, it can be simplified as follows. First, it is clear that changing the value of w will only affect the value function by a constant, so WLOG we can normalize $\underline{u}$ to zero. Second, we can choose w^* and B^* such that $\tilde{V}(\rho^*, w^*, B^*) = 0$. Finally, if $\frac{B^*}{\delta}+\tilde{V}(\rho, w^*, B^*) \geq 0$, for all ρ, then the agent will never take the outside option. Assuming this holds, we have the following relaxed value function

$$\begin{aligned}V(\rho) \quad = \quad & \max\{w^*-c+\delta(p+(1-p)\rho)(\frac{B^*}{\delta}+V(\frac{p}{p+(1-p)\rho})), \\ & w^*+\delta\rho(\frac{B^*}{\delta}+V(0))\},\end{aligned}$$

where $V(\rho) = \tilde{V}(\rho^*, w^*, B^*)$.

This relaxed functional equation has a unique solution with explicit formula for some p and δ.[6] In particular, we have

$$V(\rho) = \begin{cases} \frac{1-\delta(1-p)\rho^*}{1-\delta^2(1-p)\rho^{*2}} B^* & \text{for } \rho \geq \rho^* \\ \frac{(1-p)(1-\delta\rho^*)}{1-\delta^2(1-p)\rho^{*2}} B^* & \text{for } \rho < \rho^* \end{cases},$$

and this is supported by

$$\begin{aligned} B^* &= \frac{c/p}{(\frac{1}{\rho^*} - \frac{\rho^*}{p} + \delta\frac{(1-\rho^*-\delta(1-\rho^*)\rho^*)}{1-p\delta^2(1-\rho^*)})}; && (5) \\ w^* &= c - \frac{p}{\rho^*}B, && (6) \end{aligned}$$

and the effort choice that the agent puts in effort if and only if $\rho \leq \rho^*$.

To check that above is indeed the real value function, we check a) $V(\rho^*) = 0$ given the effort function,b) V satisfies the functional equation given the effort, c) the effort choice specified above is optimal, and d) $\frac{B^*}{\delta} + V(\rho) \geq 0$.

To check a), we note that, according to the effort choice,

$$V(\rho^*) = w^* - c + \delta(p + (1-p)\rho^*)(\frac{B^*}{\delta} + V(\rho^*)).$$

With the specification of w^*, it is clear that $V(\rho^*) = 0$.

To check b) that V satisfies the functional equation given the effort, we can use direct substitution. Alternatively, we note that according to the effort choice, for $\rho > \rho^*$,

$$V(\rho) = w^* + \delta\rho(\frac{B^*}{\delta} + V(0)).$$

And for $\rho < \rho^*$,

$$\begin{aligned} V(\rho) &= w^* - c + [p + (1-p)\rho][B^* + \delta V(\frac{p}{p + (1-p)\rho})] \\ &= w^* - c + [p + (1-p)\rho]\left[B^* + \delta(B^* + \delta V(0))(\frac{p}{p + (1-p)\rho} - \rho^*)\right] \\ &= w^* - c + [(p + (1-p)\rho][B^* - \delta\rho^*(B^* + \delta V(0))] + \delta(B^* + \delta V(0))p. \end{aligned}$$

These two expressions imply that V is piecewise linear in ρ with slope $B^* + \delta V(0)$ for $\rho > \rho^*$ and $(1-p)[B^* - \delta\rho^*(B^* + \delta V(0))]$ for $\rho < \rho^*$. It follows that the whole functional equation (given effort) is satisfied as long as the end points ($V(0)$, $V(\rho^*_-)$,

[6]Note that the right hand side of this functional equation is a contraction mapping, so there is always a unique solution of V.

$V(\rho^*_-)$ and $V(1)$) satisfies the functional equation. This can be checked by noting that

$$V(0) = \frac{-(1-p)\rho^*(1-\delta\rho^*)}{1-\delta^2(1-p)\rho^{*2}}B^*. \tag{7}$$

To check c) that the specified effort choice are optimal, we first need to make sure that for $\rho \leq \rho^*$,

$$\begin{aligned} & w^* + \rho(B^* + \delta V(0)) \\ \leq \; & w^* - c + [(p + (1-p)\rho)(B^* + \delta V(\frac{p}{p+(1-p)\rho})) \\ = \; & (1-p)(B^* - \delta\rho^*(B^* + \delta V(0)))(\rho - \rho^*), \end{aligned}$$

Note that the above is satisfied if

$$B^* + \delta V(0) \geq (1-p)(B^* - \delta\rho^*(B^* + \delta V(0))).$$

Let $x = B^* + \delta V(0)$, and define $T(x) = (1-p)(B^* - \delta\rho^* x)$, then the above can be rewritten as

$$T(x) \leq x.$$

We also want to make sure that for $\rho > \rho^*$, we have

$$\begin{aligned} & w^* + \rho(B^* + \delta V(0)) \\ \geq \; & w^* - c + [(p + (1-p)\rho)(B^* + \delta V(\frac{p}{p+(1-p)\rho})) \\ = \; & w^* - c + [(p + (1-p)\rho)(B^* + \delta(1-p)(B^* - \delta\rho^*(B^* + \delta V(0)))(\frac{p}{p+(1-p)\rho} - \rho^*))] \\ = \; & (1-p)(B^* - \delta\rho^*(1-p)(B^* - \delta\rho^*(B^* + \delta V(0))))(\rho - \rho^*). \end{aligned}$$

If we again have $x = B^* + \delta V(0)$ and $T(x) = (1-p)(B^* - \delta\rho^* x)$, then the slope of ρ in the expression above is given by $T(T(x))$, and we need

$$T(T(x)) \leq x.$$

Now note that $T(x)$ is an affine function of x with slope $-\delta\rho^*(1-p) > -1$. Let

x^* be such that $T(x^*) = x^*$, then

$$\begin{aligned}
(1-p)(B^* - \delta\rho^* x^*) &= x^* \\
x^* &= \frac{(1-p)B^*}{1+\delta\rho^*(1-p)}.
\end{aligned}$$

Now note that if $x \geq x^*$, then

$$T(x) \leq x.$$

Moreover, since the slope of $T(x)$ is equal to $-(1-p)\delta\rho^* > -1$, this implies that, for $x > x^*$,

$$\frac{T(x^*) - T(x)}{x - x^*} = \frac{x^* - T(x)}{x - x^*} < 1.$$

By the linearity of T, it follows that,

$$\frac{T(T(x)) - T(T(x^*))}{T(x^*) - T(x)} = \frac{T(x^*) - T(x)}{x - x^*} < 1$$

so that

$$T(T(x)) - T(T(x^*)) \leq x - x^*,$$

or

$$T(T(x)) \leq x.$$

The discussion above implies that, as long as

$$B^* + \delta V(0) = x \geq x^* = \frac{(1-p)B^*}{1+\delta\rho^*(1-p)},$$

the action profile is optimal. In other words, we need

$$V(0) \geq -\frac{1}{\delta}\left(\frac{p+\delta\rho^*(1-p)}{1+\delta\rho^*(1-p)}\right)B^*.$$

Recalling from (7) that

$$\begin{aligned}
V(0) &= \frac{-(1-p)\rho^*(1-\delta\rho^*)}{1-\delta^2(1-p)\rho^{*2}}B^* \\
&= \frac{p(1-\frac{1}{\rho^*}+\delta(1-\rho^*))}{1-p\delta^2(1-\rho^*)}B^*.
\end{aligned}$$

Now

$$\begin{aligned}
& \frac{p(1-\frac{1}{\rho^*}+\delta(1-\rho^*))}{1-p\delta^2(1-\rho^*)} + \frac{1}{\delta}(\frac{p+\delta\rho^*(1-p)}{1+\delta\rho^*(1-p)}) \\
= & \frac{1}{\delta\rho^*}\frac{\delta((\rho^*)^2(1-p)-p(1-\rho^*))+p\rho^*}{(1-p\delta^2(1-\rho^*))(1+\delta\rho^*(1-p))}
\end{aligned}$$

Now note that

$$(\rho^*)^2(1-p) = p(1-\rho^*),$$

so the expression above is always positive. And so the actions are optimal.

Finally, we need to check d) $\frac{B^*}{\delta}+V(\rho)\geq 0$ for all ρ. But this is immediate because from c) we see that

$$\frac{B^*}{\delta}+V(0)\geq \frac{(1-p)B^*}{1+\delta\rho^*(1-p)},$$

and it is easily checked that the slope of V is positive.

This shows that the proposed value function is truly the value function. And that the agent's strategy is optimal given the principal's strategy.

Now let us check that the principal's strategy is a best response to the agent's strategy following any history. From above, we know that the principal captures the entire surplus in this relationship. The maximal reneging temptation of the principal is given by $\frac{B^*}{\delta}$. But if the principal ever deviates, he loses the entire future surplus.[7] Therefore, as long as

$$\frac{B^*}{\delta}\leq \frac{py-c-\underline{u}-\underline{\pi}}{1-\delta},$$

the strategy of the principal above is an optimal response to the agent's equilibrium strategy. This finishes the proof. ■

In the equilibrium above, the agent is incentived to put in effort both because a high output not only leads to bonus today, but also puts the agent in the up state, and thus increasing the payoff of the agent in the future. The second effect gives advantage to equilibrium in Theorem 1 (in terms of providing incentive to the agent to work) compared to the equilibrium in imperfect public monitoring, where high output has no effect on the agent's future payoff. However, there are two disadvantages of the equilibrium in Theorem 1. First, paying bonus regardless of outcome of production whenever the agent is in the up state, which happens with probability ρ^*

[7] Note that if the principal ever offers a wage that's lower than what's specified in the equilibrium, the agent will stop putting in effort.

in equilibrium, weakens incentives to exert effort. Second, moving the agent to the up state with probability ρ^* following a failure in production in the down state also hurts effort incentives. However, the disincentivizing forces vanish as ρ^* goes to zero but the incentivizing force does not do. And ρ^* clearly goes to zero as we let p go zero (but at the same time also let B go to infinity so that pB is still comparable to the cost of effort c). This explains why intertemporally signal garbling may enhance efficiency as p is sufficiently small.

This observation leads to the main result of this section: when the probability of success is sufficiently small, the equilibrium constructed in Theorem 1 is sustainable for a wider range of discount factors compared to the case when the (imperfect) signal fully revealed every period.

In particular, if the output is perfectly revealed in each period, the necessary and sufficient condition to sustain an efficient equilibrium (in which effort is put in each period) is that

$$\frac{c}{p} \leq \frac{\delta\left(py - c - \underline{u} - \underline{\pi}\right)}{1-\delta}.$$

Theorem 1 and Corollary 1 implies that, when the signal is intertemporally garbled as in this section, the range to parameters to sustain an efficient equilibrium is expanded to

$$\frac{c}{p\left(1+\delta\right)} \leq \frac{\delta\left(py - c - \underline{u} - \underline{\pi}\right)}{1-\delta}$$

as $p \to 0$. Note that Corollary 1 is not just a result in the limit: for small enough p, the equilibrium in Theorem 1 is sustainable under a wider range of parameters, the limit gives the degree of improvement. In particular, as $p \to 0$, the intertemporal signal garbling cuts down the requirement on the size of the surplus by a factor of $1/\left(1+\delta\right)$ which goes to 50 percent as δ approaches 1.

Corollary 1: In the equilibrium constructed in Theorem 1,

$$\lim_{p\to 0} \frac{c}{pB(p)} = 1 + \delta.$$ [8]

[8] We write $B^*(p)$ to account for the dependence of B^* on p.

Proof. Following directly from (5),

$$\begin{aligned}\frac{c}{pB(p)} &= \frac{1}{\rho^*}-\frac{\rho^*}{p}+\delta\frac{(1-\rho^*-\delta(1-\rho^*)\rho^*)}{1-p\delta^2(1-\rho^*)}\\ &= \frac{\rho^*-p}{(1-p)\rho^*}+\delta\frac{(1-\rho^*-\delta(1-\rho^*)\rho^*)}{1-p\delta^2(1-\rho^*)},\end{aligned}$$

where we have used $(\rho^*)^2(1-p)=p(1-\rho^*)$ in simplifying $\frac{1}{\rho^*}-\frac{\rho^*}{p}$.

Now since

$$\begin{aligned}(1-p)\rho^{*2}+p\rho^*-p &= 0,\\ \rho^* &= \frac{-p+\sqrt{4p-3p^2}}{2}\end{aligned}$$

In other words, when p is small, ρ^* is roughly in the order of $\sqrt{p}$.

It is clear that as p goes to 0, both $\frac{\rho^*-p}{(1-p)\rho^*}$ and $\frac{(1-\rho^*-\delta(1-\rho^*)\rho^*)}{1-p\delta^2(1-\rho^*)}$ go to 1, so

$$\lim_{p\to 0}\frac{c}{pB(p)}=1+\delta.$$

■

The reason that this type of intertemporal garbling can do better than fully revealing the information every period, especially when the success probability is small, is the following. Under relational contract with perfect signals, the principal will be required to pay a big bonus (c/p) to the agent when the probability of success is small. Consequently, relational contract is hard to sustain because the bonus cannot exceed the expected discounted future surplus of the relationship.

By linking information intertemporally, the reward for high output is decomposed into two parts: the bonus at the end of this period, and a higher continuation payoff in the future. This decomposition of reward reduces the immediate bonus to be paid out and helps softens the incentive constraint of the principal. On the other hand, delaying the reward does have a cost: the absolute amount of total bonus paid out will be larger due to discounting and the higher continuation payoff of the agent makes it difficult to induce effort. Therefore, this intertemporal garbling can more easily outperform perfect signal when the success probability is extreme and the discount factor is high.

In particular, let β be the relative benefit of being in the up state instead of the down state. The payoff from exerting effort can be viewed as

$$[p + \rho^*(1 - p)]\, B + \delta[p + (1 - p)(1 - \rho^*)\rho^*]\beta$$

and the payoff from not exerting effort can be viewed as

$$\rho^* B + \delta(1 - \rho^*)\rho^* \beta.$$

In other words, the benefit of exerting effort or the difference in these payoffs is $p(1-\rho^*)B + \delta p\,[1 - (1-\rho^*)\,\rho^*]\,\beta$. As p goes to zero, ρ^* also goes to zero. In other words, both the equilibrium probability of receiving a bonus and the equilibrium probability of being in the up state are close to zero. On the other hand, once the agent is in the up state, he receives a bonus with probability one, as compared to p if he is in the down state. As p goes to zero, being in the *up* state increases the probability of getting a bonus from almost zero to one. Therefore, β goes to B as p goes to zero. This explain why the benefit of exerting effort can reach $(1+\delta)\,pB$, as compared to pB when the signal is not garbled. In other words, with intertemporal signal garbling, the same amount of bonus can provide a stronger incentive to put in effort.

4 Discussion

The equilibrium constructed in Theorem 1 uses a two-state representation. It is natural to ask whether one can do better with more states. We conjecture that the answer is yes, but proving those strategies with more than two states are equilibrium is difficult. This is because one needs to check more than one-stage deviation in this setting, and the recursive structure in Theorem 1 becomes unwieldy when there are multiple states. One possible way in the literature to deal with this problem is the "belief-free" approach, in which the marginal benefit of having a high output is independent of which state the agent is in. Unfortunately, such approach cannot work here, as the next proposition shows.

Proposition 1 *If an information set has n states, and the difference in payoffs (between a high to low output) is independent of which state the agent is in, such information structure can do no better than perfect signals.*

Proof. Suppose that there are n states within an information set, with value $k_1 > k_2 > .... > k_n$. (The argument extends naturally to the case where n is infinity or represents a continuum.)

Let x_s^H be expected payoff following a high output when the agent is in state s. We define x_s^L accordingly.

Belief free requires that there exists a D such that

$$x_s^H - x_s^L = D \quad \text{for all } s.$$

Now note that

$$k_s = px_s^H + (1-p)x_s^L.$$

This implies that

$$\begin{aligned} x_s^H &= k_s + (1-p)D \\ x_s^L &= k_s - pD \end{aligned}$$

Now note that

$$x_1^H = k_1 + (1-p)D \leq b + \delta k_1,$$

where b is the per period bonus. This implies that

$$k_1 \leq \frac{b-(1-p)D}{1-\delta}$$

Also note that

$$x_n^L = k_n - pD \geq \delta k_n$$

This implies that

$$k_n \geq \frac{pD}{1-\delta}.$$

Since $k_1 > k_n$,

$$\begin{aligned} \frac{b-(1-p)D}{1-\delta} &\geq \frac{pD}{1-\delta} \\ b &\geq D. \end{aligned}$$

■

Since the intertemporal garbling creates improvement through exploiting the extremeness of the information, such linkage is less likely to be useful when the information content on the equilibrium path of perfect signal is more even. In fact, when $p = \frac{1}{2}$, revealing information perfectly is optimal.

Theorem 2: *When $p = 1/2$, the optimal information structure is given by $s_t = Y_t$ for all t.*

Proof. First recall that when $s_t = Y_t$ for all t, the necessary and sufficient condition for sustaining cooperation is given by equation (3):

$$\frac{c}{p-q} \leq \frac{\delta}{1-\delta}(py - c - \underline{u} - \underline{\pi}) \equiv S,$$

where without confusion in this proof, S denotes the surplus of the relationship (when the agent puts in effort each period.) We want to show that if the inequality above fails, it is impossible to construct an equilibrium in which the agent puts in effort. In particular, using standard argument as in Fuchs (2007), it suffices to show that there does not exist an equilibrium in which the agent always puts in effort (unless the relationship is terminated).

Consider an arbitrary information partition process. Pick one information set (h^t). Use x to denote the possible states within the information set. One interpretation of x is some output realizations y^t that falls into h^t.

Let $V(x)$ be the agent's continuation payoff in state x after e_{t+1} is put in but before y_{t+1} is realized and W_{t+1} is paid out. Let $V(x_i)$ be the agent's continuation payoff in state x after e_{t+1} is put in, y_{t+1} is realized but before W_{t+1} is paid out. Within each state x, we have $x_i \in \{x_y, x_0\}$, where x_y denotes that $Y_t = y$ is realized following x, and x_0 denotes that $Y_t = 0$ is realized.

Note that

$$V(x) = V(x) + p(V(x_y) - V(x)) + (1-p)(V(x_0) - V(x)).$$

And since the output Y_t is independent of the past state, we have $Cov(V(x_i) - V(x), V(x)) = 0$.

To induce effort, we need

$$E_x[V(x_y) - V(x_0)] = E_x[V(x_y) - V(x)] - E_x[V(x_0) - V(x)] \geq \frac{c}{p-q}.$$

This helps give a lower bound for $Var(V(x_i))$. In particular,

$$
\begin{aligned}
Var(V(x_i)) &= Var(V(x)) + Var(V(x_i) - V(x)) \\
&= Var(V(x)) + E_y[Var(V(x_i) - V(x))|Y] \\
&\quad + Var(E_x[V(x_i) - V(x)|Y]) \\
&\geq Var(V(x)) + E_y[Var(V(x_i) - V(x))|Y] \\
&\quad + p(1-p)(\frac{c}{p-q})^2 \\
&\geq Var(V(x)) + p(1-p)(\frac{c}{p-q})^2,
\end{aligned}
$$

where the first line follows because $Cov(V(x_i) - V(x), V(x)) = 0$, the second line uses the variance decomposition formula, the third line follows because $E_x[V(x_i) - V(x)|Y]$ is a binary value ($Y \in \{0, y\}$) such that with probability p its value is $E_x[V(x_y) - V(x)]$ and with probability $1-p$ its value is $E_x[V(x_0) - V(x)]$, and $E_x[V(x_y) - V(x)] - E_x[V(x_0) - V(x)] \geq \frac{c}{p-q}$.

Now let's provide an upper bound for $Var(V(x_i))$. Suppose a public signal $s(x_i)$ will be sent out after state x_i. Let $b(s)$ be the bonus paid out to the agent (at the end of the period) following signal s. This allows us to write

$$V(x_i) = b(s(x_i)) + \delta V_{s(x_i)}(x_i),$$

where $V_{s(x_i)}(x_i)$ is the continuation payoff of x_i, which goes to the information set by signal $s(x_i)$.

Note that for the principal to be willing to pay the bonus, we must have

$$\max_s\{b_s + \delta E_{x_i}[V_s(x_i)|s]\} - \min_s\{b_s + \delta E_{x_i}[V_s(x_i)|s]\} \leq S.$$

Because otherwise the expected payoff of the principal following some signal will be below his outside option.

Decomposing the variance on the signals, we have

$$
\begin{aligned}
Var(V(x_i)) &= Var(E[b_s + \delta V_s(x)|s]) + E[Var(b_s + \delta V_s(x_i)|s)] \\
&\leq \frac{1}{4}S^2 + \delta^2 E[Var(V_s(x_i)|s)].
\end{aligned}
$$

Now combining the upper and lower bound for $Var(V(x_i))$, we get that

$$\frac{1}{4}S^2 + \delta^2 E[Var(V_s(x_i)|s)] \geq Var(V(x)) + p(1-p)(\frac{c}{p-q})^2,$$

or equivalently,

$$E[Var(V_s(x_i)|s)] \geq \frac{1}{\delta^2}(Var(V(x)) + p(1-p)(\frac{c}{p-q})^2 - \frac{1}{4}S^2).$$

When $p = \frac{1}{2}$, and $\frac{c}{p-q} > S$, the inequality above implies that

$$E[Var(V_s(x_i)|s)] > \frac{1}{\delta^2}(Var(V(x)).$$

In particular, there will be one information set (associated with a signal) whose variance exceeds $\frac{1}{\delta^2}(Var(V(x))$. Now we can perform the same argument on this new information set, and we can construct a sequence of information set whose variance approaches infinity. This leads to a contradiction. ■

5 Conclusion

When probability of success is low, it is difficult to sustain efficient production with a relational contract because, to motivate the agent, it requires the principal to pay a large bonus payment upon observation of successful production, leading to a stronger incentive for her to renege. Our analysis showed that intertemporal garbling of the signals of the agent's success/failure can restore efficient production by reducing amount of bonus needed to be paid out by the principal. We believe this not only of theoretical interest.

There are many situations in which senior managers have to rely on supervisors/mid-level managers to monitor and evaluate employees' performances. Our result implies that there are situations in which it is suboptimal for an organization to require supervisors to write the most accurate year-end evaluations for their subordinates. In fact, it is better for the supervisor to privately keep track of the employee's performance in previous years and use that information to determine the employee's performance evaluation in the current year.

Due to intractability of relational contract with private monitoring, characterization of the optimal signal garbling process remains an open question and will be the focus of future research.

Appendix A: Extension to General Production Function and Signals

In this section, we generalize the production function and the signal structure to show that the results in Subsections 3.1-3.2 hold generally.

If the principal and the agent engage in production together, the agent chooses effort $e \in [0, \bar{e}]$, incurring an effort cost of $c(e)$. Assume that $c(0) = 0$, $c' > 0$ and $c'' > 0$. The outcome Y is a random variable distributed with the c.d.f. $F(\cdot|e)$, where $f(\cdot|e)$ exists, and the support of Y is independent of e.

We assume that there exists an effort level such that if this effort level can be induced in the relationship, then it is efficient to form the relationship. However, forming the relationship is less efficient than each player receiving his/her outside option if no effort can be induced in the relationship. In other words, there exists $e \in [0, \bar{e}]$ such that

$$\int_y yf(y|e)dy - c(e) > \underline{u} + \underline{\pi},$$

where $\underline{u}$ and $\underline{\pi}$ are respectively the agent and the principal's per-period outside option, and

$$\int_y yf(y|0)dy < \underline{u} + \underline{\pi}.$$

Define

$$u(e|Y) = (1-\delta)[w_0 - c(e) + \int \tilde{b}(y)f(y|e)dy] + \delta \int \tilde{u}(y)f(y|e)dy.$$

$$\begin{aligned}
v(y) &= -(1-\delta)[-y + w_0 + \tilde{b}(y)] + \delta\tilde{v}(y) \\
v(e|Y) &= \int -(1-\delta)[-y + w_0 + \tilde{b}(y) + \delta\tilde{v}(y)]f(y|e)dy
\end{aligned}$$

Note that every feasible payoff set is characterized by an upper bound on the total surplus s^* and can be written as the following:

$$W = \{(u, v) : u \geq \underline{u},\ v \geq \underline{\pi} \text{ and } u + v \leq s^*\}.$$

For any set $W \in R^2$, a vector $(e, b, \tilde{u}, \tilde{v})$ is called admissible with respect to W under Y if

(1) $(\tilde{u}(y), \tilde{v}(y)) \in W$ for all y in the support, and

(2) $u(e|Y) \geq u(e'|Y)$ for all $e' \in [0, \overline{e}]$ (agent's IC)

(3) $-(1-\delta)\tilde{b}(y) + \delta\tilde{v}(y) \geq \delta\underline{\pi}$ for all y in the support (principal's IC)

Let $B(W|Y)$ be defined by

$$B(W|Y) = \{(u, v)|(e, b, \tilde{u}, \tilde{v}) \text{ is admissible w.r.t. } W \text{ under } Y\}.$$

A payoff set W is self-generating under Y if $W \subseteq B(W|Y)$.

A.1 Within-period Information Garbling

Now, consider a modified setup in which the output is X. Let $X \sim G(\cdot|e)$, where $g(\cdot|e)$ exists, such that

$$\int_y yf(y|e)dy = \int_x xg(x|e)dx \; \forall e \in [0, \overline{e}]. \tag{8}$$

This restriction is to preserve the overall productivity of the relationship. Furthermore, we assume that X is less informative than Y of the agent's effort in the sense of *quasi-garbling*. Following Kandori (1992), we impose that

$$\begin{aligned} \phi(x|y) &\geq 0 \text{ a.e. } x \text{ and } y \\ \int \phi(x|y)dx &= 1 \text{ a.e. } y \\ g(x|e) &= \int \phi(x|y)f(y|e)dy, \end{aligned} \tag{9}$$

and that the support of X is independent of e.

Proposition A (Kandori, 1992) Suppose X is a quasi-garbling of Y. Then if W is a compact self-generating set under X, it is also self-generating under Y.

Proof. Since W is self-generating under X, for any $w = (u, v) \in W$ there exists a vector $(e_X, \tilde{b}_X, \tilde{u}_X, \tilde{v}_X)$ which is admissible with respect to W under X and satisfies

$$u(e|X) = (1-\delta)[w_0 - c(e) + \int \tilde{b}_X(x)f(x|e)dy] + \delta \int \tilde{u}_X(x)f(x|e)dy.$$

$$\begin{aligned} v(x) &= -(1-\delta)[-x+w_0+\tilde{b}_X(x)]+\delta\tilde{v}_X(x) \\ v(e|X) &= \int -(1-\delta)[-x+w_0+\tilde{b}_X(x)+\delta\tilde{v}_X(x)]f(x|e)dx. \end{aligned}$$

Define $\tilde{u}_Y$ and $\tilde{b}_Y$ by

$$\begin{aligned} \tilde{u}_Y(y) &= \int \tilde{u}_X(x)\phi(x|y)dx \\ \tilde{v}_Y(y) &= \int \tilde{v}_X(x)\phi(x|y)dx \\ \tilde{b}_Y(y) &= \int \tilde{b}_X(x)\phi(x|y)dx. \end{aligned}$$

Then, for all $e \in [0, \overline{e}]$,

$$\begin{aligned} \int_y \tilde{u}_Y(y)f(y|e)dy &= \int_y \int_x \tilde{u}_X(x)\phi(x|y)dxf(y|e)dy \\ &= \int_x \tilde{u}_X(x)[\int_y \phi(x|y)f(y|e)dy]dx \\ &= \int_x \tilde{u}_X(y)g(x|e)dx \end{aligned}$$

and similarly

$$\int_y \tilde{v}_Y(y)f(y|e)dy = \int_x \tilde{u}_X(y)g(x|e)dx.$$

It is clear that $u(e|Y) = u(e|X)$ and $v(e|Y) = v(e|X)$. It follows that

(1) $(\tilde{u}(y), \tilde{v}(y)) \in coW$ for all y in the support, and

(2) $u(e|Y) \geq u(e'|Y)$ for all $e' \in [0, \overline{e}]$ (agent's IC)

(3) $-(1-\delta)\tilde{b}(y) + \delta\tilde{v}(y) \geq \delta\underline{\pi}$ for all y in the support (principal's IC).

Therefore, $u(e|Y)$and $v(e|Y)$ are admissible with respect to W under Y. Hence $W \subseteq B(W|Y)$. ■

A.2 T-period Bundling

Suppose signals (outputs) are released once every T periods. We call every T periods a stage. We reindex each period as $(i-1)T+\tau$ where $i \in \mathbb{N}$ and $\tau \in \{1, 2, ...T\}$, as the period in the τth period of the ith stage. We prove that if some efforts in the T periods of a stage, $\{\tilde{e}_1, \tilde{e}_2, ..., \tilde{e}_T\}$ are sustainable, then $\max\{\tilde{e}_1, \tilde{e}_2, ..., \tilde{e}_T\}$ can be supported by a fully revealing relational contract, i.e., when $T = 1$.

Suppose $\{\tilde{e}_1, \tilde{e}_2, ..., \tilde{e}_T\}$ are supported by $\tilde{b}(\mathbf{y}^T)$, where $\mathbf{y}^T \equiv \{y_1, y_2, ..., y_T\}$. Then $\max_{\mathbf{y}^T}\left\{\tilde{b}(\mathbf{y}^T)\right\}$ is no larger than the surplus of the relationship. Moreover, for these efforts to be sustainable, it requires that it is sequentially rational for the agent to exert the corresponding effort each period. In other words, $\tilde{e}_T$ solves

$$\max_{e_T} \int_{\mathbf{y}^T} \tilde{b}\left(\mathbf{y}^T\right) f(\mathbf{y}^T|\tilde{e}_1, \tilde{e}_2, ..., \tilde{e}_{T-1}, e_T) d\mathbf{y}^T + w_0 - c\left(e_T\right), \tag{10}$$

taking $\tilde{e}_1, \tilde{e}_2, ..., \tilde{e}_{T-1}$ as given.

Similarly, $\tilde{e}_{T-1}$ solves

$$\max_{e_{T-1}} \int_{\mathbf{y}^T} \delta\tilde{b}\left(\mathbf{y}^T\right) f(\mathbf{y}^T|\tilde{e}_1, \tilde{e}_2, ..., e_{T-1}, \tilde{e}_T(\tilde{e}_1, \tilde{e}_2, ..., e_{T-1})) d\mathbf{y}^T + w_0 - c\left(e_T\right) \tag{11}$$

taking $\tilde{e}_1, \tilde{e}_2, ..., \tilde{e}_{T-2}$ as given and anticipating $\tilde{e}_T$ to be the solution to (10).

More generally, $\tilde{e}_\tau$ solves for $\tau \in \{1, 2, ..., T-1\}$,

$$\max_{e_\tau} \int_{\mathbf{y}^T} \delta^{T-\tau}\tilde{b}\left(\mathbf{y}^T\right) f(\mathbf{y}^T|\tilde{e}_1, \tilde{e}_2, ..., e_\tau, \tilde{e}_{\tau+1}(\tilde{e}_1, ..., \tilde{e}_{\tau-1}, e_\tau), ..., \tilde{e}_T(\tilde{e}_1, ..., \tilde{e}_{\tau-1}, e_\tau) d\mathbf{y}^T + w_0 - c\left(e_T\right)$$

which is equivalent to solving

$$\max_{e_\tau} \int_{\mathbf{y}^T} \tilde{b}\left(\mathbf{y}^T\right) f(\mathbf{y}^T|\tilde{e}_1, \tilde{e}_2, ..., e_\tau, \tilde{e}_{\tau+1}(\tilde{e}_1, ..., \tilde{e}_{\tau-1}, e_\tau), ..., \tilde{e}_T(\tilde{e}_1, ..., \tilde{e}_{\tau-1}, e_\tau) d\mathbf{y}^T - \frac{c\left(e_T\right)}{\delta^{T-\tau}}. \tag{12}$$

Now, we define

$$\hat{b}(y_T) = \int_{\mathbf{y}^{-T}} \tilde{b}\left(\mathbf{y}^T\right) f(\mathbf{y}^T|\tilde{e}_1, \tilde{e}_2, ..., \tilde{e}_{T-1}, e_T) d\mathbf{y}^{-T}$$

and, for $\tau \in \{1, 2, ..., T-1\}$,

$$\hat{b}(y_\tau) = \int_{\mathbf{y}^{-\tau}} \tilde{b}\left(\mathbf{y}^T\right) f(\mathbf{y}^T|\tilde{e}_1, \tilde{e}_2, ..., e_\tau, \tilde{e}_{\tau+1}(\tilde{e}_1, ..., \tilde{e}_{\tau-1}, e_\tau), ..., \tilde{e}_T(\tilde{e}_1, ..., \tilde{e}_{\tau-1}, e_\tau) d\mathbf{y}^{-\tau}.$$

Now consider a one-period relational contract in which the principal pays the agent a bonus $\delta^{T-\tau}\hat{b}(y_\tau)$ in each period. Obviously, $\max_{y_\tau} \delta^{T-\tau}\hat{b}(y_\tau) \leq \max_{\mathbf{y}^T}\left\{\tilde{b}(\mathbf{y}^T)\right\}$. With such a contract, the agent solves

$$\delta^{T-\tau} \max_{y_\tau} \int_{y_\tau} \int_{\mathbf{y}^{-\tau}} \tilde{b}\left(\mathbf{y}^T\right) f(\mathbf{y}^T|\tilde{e}_1, \tilde{e}_2, ..., e_\tau, \tilde{e}_{\tau+1}(\tilde{e}_1, ..., \tilde{e}_{\tau-1}, e_\tau), ..., \tilde{e}_T(\tilde{e}_1, ..., \tilde{e}_{\tau-1}, e_\tau) d\mathbf{y}^{-\tau} dy_\tau - c(e_T).$$

Obviously, the solution is identical to that of (**??**). In other words, each of the efforts $\{\tilde{e}_1, \tilde{e}_2, ..., \tilde{e}_T\}$ can be induced in an unbundled relational contract. In particular, the unbundled relational contract inducing $\max\{\tilde{e}_1, \tilde{e}_2, ..., \tilde{e}_T\}$ every period weakly dominates the T-period relational contract.

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