Optimal external debt and default

Bernardo Guimaraes*

October 2008

Abstract

This paper analyses a small open economy that wants to borrow from abroad, cannot commit to repay debt but faces costs if it decides to default. I study the optimal debt contract in this world in order to quantify the impact of shocks on the incentive compatible level of debt. Debt relief generated by severe output shocks is no more than a couple of percentage points. In contrast, shocks to world interest rates can generate debt relief one order of magnitude higher. Debt reduction prescribed by the model following the interest rate hikes of 1980-81 accounts for over half of the debt forgiveness obtained by the main Latin American countries through the Brady agreements.

KEYWORDS: sovereign debt; default; optimal contract; world interest rates.

JEL CLASSIFICATION: F34.

1 Introduction

What generates sovereign default? Which shocks are behind the episodes of debt crises we observe? The answer to the question is crucial to policy design. If we want to write contingent contracts,[1] build and operate a sovereign debt restructuring mechanism,[2] or an international lender of last resort,[3] we need to know which economic variables are subject to shocks that significantly increase incentives for default or could take a country to "bankruptcy".

I investigate this question by studying a small open economy that wants to borrow from abroad, cannot commit to repay debt but faces costs if it decides to default. There is an incentive compatible level of sovereign debt — beyond which greater debt triggers default — and it fluctuates with economic conditions. I quantify the impact of shocks on

*London School of Economics, Department of Economics, b.guimaraes@lse.ac.uk. I thank Giancarlo Corsetti, Nicola Gennaioli, Alberto Martin, Silvana Tenreyro, Alwyn Young and, especially, Francesco Caselli for helpful comments, several seminar participants for their inputs and Zsofia Barany and Nathan Foley-Fisher for excellent research assistance financed by STICERD.

[1]Borensztein and Mauro (2004) argue for GDP-indexed debt. Kletzer et al (1992) defend indexing debt payments to commodities prices.

[2]Krueger (2002) and Bolton and Jeanne (2007) argue for a sovereign debt restructuring mechanism.

[3]Fisher (1999) argues for an international lender of last resort.

the incentive compatible level of debt. More specifically, I study the optimal debt contract in this world; get analytical results on the optimal debt relief in response to different types of economic shocks; and compare the results to real world outcomes.

A framework to quantify the impact of different shocks on the incentive compatible level of debt serves two purposes: (i) it tell us which shocks are most likely to trigger crises; and (ii) it offers a way to contrast theoretical models of sovereign default with observed data. The size of default is a key variable our models should be able to explain when we look at default episodes. The debt relief obtained in an optimal contract is a natural theoretical counterpart.

One important result from this framework is that shocks to domestic productivity (or output) do not generate sizable fluctuations in the incentive compatible level of debt. Following a severe output shock, debt relief of a couple of percentage points would restore incentive compatibility. This is consistent with some empirical evidence (Tomz and Wright, 2007), although most of the recent quantitative models on debt and default focus on output shocks.[4] This result suggests that the literature has been focusing on the wrong type of shocks.

On the other hand, shocks to risk-free 'world' real interest rates are much more important. Debt relief in response to reasonable fluctuations in world interest rates is an order of magnitude higher than that generated by shocks to output. That is because a change in interest rates from 1% to 2% doubles the cost of servicing debt while a 5% fall in output reduces by just 5% the cost of not repaying. The large effect of shocks to world interest rates on debt default is present in some empirical work (Uribe and Yue, 2006).

This result is borne out in the data from the Latin American debt crisis of the 1980's, which I use to compare the model's prediction of optimal debt relief with the observed reduction in debt. I find that the increase in world interest rates at the beginning of the 1980's can solely account for over half of the debt forgiveness obtained by the main Latin American countries through the Brady agreements.

The model builds on the literature of endogenous sovereign debt and default (Eaton and Gersovitz (1981), Arellano (2008)).[5] One of the key assumptions in the model is that if a country repudiated its debt, it would be excluded from capital markets and incur an output loss.[6] The assumption of such costs is a simple way of modeling the costs that could be imposed on a country that defaults. Debt repudiation might inhibit foreign direct

[4] For example, Arellano (2008), Aguiar and Gopinath, (2006) and Yue (2006).

[5] It is also related to the literature on consumers' and firms' debt. In particular, the optimal contract in this model bears resemblance to that in Albuquerque and Hopenhayn (2004).

[6] The output cost of debt repudiation, as modeled here, is present in Cohen and Sachs (1986), Bulow and Rogoff (1989) and Arellano (2008), to name a few.

investment and undermine a country's capacity to obtain beneficial deals in multi-lateral organisations such as the WTO. In addition, creditors can threaten countries that might repudiate debt with sanctions such as the loss of access to short-term trade credit and seizure of assets.[7]

In reality, however, observed punishment for default is arguably tame and temporary. But that is, at least in part, because debtor and creditors renegotiate the debt and, sooner or later, a new agreement is reached. Instead of modeling the complicated renegotiation process,[8] I derive the optimal contingent debt contracts.[9] In the model, it is never optimal to renege on the contract, so the default cost is never paid in equilibrium.

In practice, we do not observe explicit contingent contracts. However, premium rates on borrowing and occasional debt reduction are observed, so contracts are contingent *de facto* even if not written as such. That makes sense: following a negative shock, creditors have an incentive to renegotiate debt down to a level where it is incentive compatible for the country to repay. In fact, the optimal contract yields the same results as the assumption that lenders have all the bargaining power — a usual implicit assumption in the literature — and they promptly agree to debt relief if it is optimal for them.

I begin with an endowment economy, where borrowers' impatience drives debt decisions. Then I move to a model with capital, where borrowing occurs due to differences in the marginal productivity of capital. Models with endogenous decision of debt repayment and capital accumulation are not very tractable analytically. However, for some limiting cases, I can derive simple and intuitive analytical solutions for the level of debt and debt relief. The method consists of taking first order approximations of the Bellman equations. I also solve the model numerically for non-limiting cases and show that the analytical solutions are good approximations.

The results on debt relief for both economies are very similar. Debt reduction occurs when the economy switches to a state with a lower incentive compatible level of debt. Debt relief predicted by the model depends positively on the magnitude of the shocks and the persistence of states. Importantly, the output cost of defaulting, a variable which is

[7] For a discussion of such costs, see Bulow and Rogoff (1989), English (1995), Sturzenegger and Zettelmeyer (2006) and Tomz (2007).

[8] Renegotiation in models of sovereign debt is studied by Bulow and Rogoff (1989), Fernandez and Rosenthal (1990) and Yue (2006). Kovrijnykh and Szentes (2007) study the return from debt overhang to the credit market.

[9] Sovereign debt was also analysed as an (implicitly) contingent claim by Grossman and Van Huyck (1988), Atkeson (1991) and Calvo and Kaminsky (1991) among others. Grossman and Van Huyck show that an equilibrium in which "excusable" default is allowed without sanctions can be sustained. Alfaro and Kanczuk's (2005) quantitative analysis builds on Grossman and Van Huyck. Calvo and Kaminsky (1991) take the optimal contract approach to study whether the small default premium paid by Latin American countries in the 1970's would be compatible with large debt reductions in the 1980's.

difficult to measure, has no significant effect on debt relief as a fraction of the outstanding debt. So the conclusions on the quantitative effects of shocks do not rest on any particular value of this variable.

There are two key differences between the similar results for both economies. First, in the economy with capital, differences in the consumption-savings decisions for different states of the economy could influence the optimal debt contract. However, I show they produce no first-order effects on the incentive compatible level of debt, as they have only second order effects on the value functions. Second, growth prospects influence the level of debt and debt relief, but those effects are not large. The amount of capital influences debt relief in both directions, rendering the result ambiguous.[10] Therefore, the conclusions about the impact of shocks on the incentive compatible level of debt are basically the same in both economies.

Section 2 studies debt relief in the endowment economy, Section 3 adds capital accumulation to the model, Section 4 contrasts the results with data from the Latin American debt forgiveness under the Brady Plan, Section 5 concludes. Proofs and numerical exercises are in the appendix.

2 Endowment economy

In this section, I consider a discrete time model of an open endowment economy that can borrow from abroad, but cannot commit to repay its debts. The focus of the paper is sovereign debt, so all results in this paper are derived from the point of a view of a domestic planner.[11]

The economy is populated by a continuum of infinitely lived agents whose preferences are aggregated to form the usual representative agent utility function:

$$\sum_{t=0}^{\infty} \beta^t u(c_t)$$

where $\beta \in (0, 1)$ is the subjective time discount factor, c_t is consumption at time t and $u(.)$ is the felicity function that satisfies the Inada conditions.

[10] In the numerical examples, debt relief depends negatively on the level of capital.

[11] In the standard Ramsey model, the central planner solution and the decentralised equilibrium are the same. However, that is not true without commitment to repay debt. The distinction between the central planner solution and the decentralised allocation is analysed by Kehoe and Perri (2004) and Jeske (2006). Kehoe and Perri (2004) show that the central planner solution can be decentralised if the central government is in charge of deciding about defaulting or not and taxes capital income to counteract an externality of capital accumulation. The logic behind Jeske's argument is similar and he finds that capital controls may be welfare improving.

In this Section, I assume that β is very low. The recent quantitative models of default assume very low values of β (Arellano (2008) uses $\beta = 0.82$/year, even lower values appear in Aguiar and Gopinath (2006) and Yue (2006)). Such extreme impatience is often interpreted as the discount rate of the policy maker, different from the population. In the next Section, the assumption of a very low β is removed.

I model debt default costs as an instantaneous permanent fall in output and loss of access to capital markets. The permanent fall specifically captures the loss that a country suffers by taking an antagonistic position towards the rest of the world and never repaying a cent of its debts. I denote by γ the fraction of output lost due to default, so output is given by:

$$y_t = \begin{cases} y_t \text{ , if it has never defaulted} \\ (1-\gamma)y_t \text{ , if it has ever defaulted} \end{cases}$$

In the model this is out-of-equilibrium behaviour, which corresponds to never observing such action in reality. For this reason, it is difficult to obtain an estimate of γ, but the main results of this paper do not depend on the value of γ.

The country can issue only one-period debt (d_t). The price of debt is denoted q_t.

There is a continuum of risk-neutral lenders that, in equilibrium, lend to the country as long as the expected return on their assets is not lower than the risk-free interest rate in international markets, r^*. The price of a bond that delivers one unit of the good next period with certainty, $(1+r^*)^{-1}$, is denoted q^*. There is a maximum amount of debt the country can contract that prevents it from running Ponzi schemes but it is never reached in equilibrium.

The economy's flow budget constraint is then given by:

$$c_t = \begin{cases} y_t - d_t + q_t d_{t+1} \text{ , if it has never defaulted} \\ (1-\gamma)y_t \text{ , if it has ever defaulted} \end{cases}$$

Two stochastic versions of the model are considered: (1) stochastic r^* and (2) stochastic y. There are 2 possible states, $s_t \in \{h, l\}$, and the probability of switching states is ψ. $\Pr(s_t \neq s_{t-1}) = \psi$, $\Pr(s_t = s_{t-1}) = 1 - \psi$. $\psi \leq 0.5$.

2.1 Stochastic world interest rates

Here, I analyse the optimal debt contract for an economy with fixed per-period endowment, but fluctuations in world interest rates, r^*, lead to fluctuations in the price of risk-free debt, q^*. In a stochastic world, the incentive compatible level of debt fluctuates. If debt goes above its incentive compatible level, the debtor prefers not to repay it, but then both creditors and debtors have incentives to renegotiate. I solve for the optimal contingent contract, which specifies the country's debt at each possible state. The country has the choice of honouring its debt or defaulting, but the default option is never taken in equilibrium.

The price of a riskless bond in international markets is q^{*h} in the high state and q^{*l} in the low state, $q^{*h} > q^{*l}$. Denote by $d^h$and d^l the repayment conditional on high and low state, respectively, and $\Delta d = d^h - d^l$. A risk-neutral creditor that lends $q^* d'$ must get an expected repayment equal to d'. Thus if $s_{t-1} = h$, repayments at time t are given by $d^h = d' + \psi \Delta d$ and $d^l = d' - (1 - \psi)\Delta d$. Likewise, if $s_{t-1} = l$, repayments at time t are $d^l = d' - \psi \Delta d$ and $d^h = d' + (1 - \psi)\Delta d$.

In each period, the central planner chooses between repaying or defaulting. Each option yields a different value function and the planner chooses the maximum of the two:

$$V^i(d) = \max \left\{ V^i_{pay}(d), V_{def} \right\} \text{ , and } i \in \{l, h\}.$$

If the country repays, it also chooses d' and Δd. The value functions are conditional on repayment are:[12]

$$\begin{aligned}
V^h_{pay}(d) &= \max_{d', \Delta d} \left\{ u(y - d + q^h(d')d') + \beta \left[(1 - \psi)V^h(d' + \psi \Delta d) + \psi V^l(d' - (1 - \psi)\, \Delta d) \right] \right\} \\
V^l_{pay}(d) &= \max_{d', \Delta d} \left\{ u(y - d + q^l(d')d') + \beta \left[(1 - \psi)V^l(d' - \psi \Delta d) + \psi V^h(d' + (1 - \psi)\, \Delta d) \right] \right\}
\end{aligned}$$

In case of default, the value function in both states is:

$$V_{def} = u((1 - \gamma)y) + \beta V_{def} = \frac{u((1 - \gamma)y)}{1 - \beta}$$

It makes no difference whether foreign interest rates are low or high if the country is excluded from international financial markets.

In equilibrium:

- If contracts can be written and enforced contingent on q^*, debt repudiation is never optimal, so d^h and d^l will always be incentive compatible. Thus, in equilibrium, $q^h(d') = q^{*h}$ and $q^l(d') = q^{*l}$. A contract specifying no debt in one of the states is strictly better than the case of repudiating debt at that state because, given that the expected payment must be the same, the debt payments in the alternative state will

[12] I follow the literature (Eaton and Gersovitz (1981), Arellano (2008)) and assume that the debtor fully repays debt provided it is incentive compatible to do so. As noted by Fernandez and Rosenthal (1990), we are assuming that all bargaining power lies with the creditors (ex-post). The optimal contract can be implemented if creditors immediately reduce the level of debt to its incentive compatible level when it goes above it, so it allows creditors to extract the maximum incentive compatible payment from the lenders. Thus the assumption of an optimal contract can be seen as an extension of the usual assumption about bargaining power.

be identical so the only difference between the cases will be the loss in output when debt is repudiated.

- V^h_{pay} and V^l_{pay} are decreasing in d, but V_{def} is independent of d. As in equilibrium there is no debt repudiation, $V^i = V^i_{pay}$, so d' must be such that $V^i_{pay}(d') \geq V_{def}$ for $i \in \{l, h\}$. The borrowing constraint is binding if the optimal choice of d' would be different if default were not an option.

- If the borrowing constraint is not binding, then differentiating the value functions with respect to Δd yields
$$\frac{\partial V^h_{pay}(d)}{\partial d} = \frac{\partial V^h_{pay}(d)}{\partial d}$$
Differentiating the value functions with respect to d, we obtain:
$$c^h = c^l$$
Thus, if the borrowing constraint is not binding, then:
$$\begin{aligned} q^{*h}u'(c^h_t) &= \beta\left[\psi u'\left(c^h_{t+1}\right) + (1-\psi)u'\left(c^l_{t+1}\right)\right] \Rightarrow q^{*h}u'(c_t) = \beta u'\left(c_{t+1}\right) \\ q^{*l}u'(c^l_t) &= \beta\left[\psi u'(c^l_{t+1}) + (1-\psi)u'\left(c^h_{t+1}\right)\right] \Rightarrow q^{*l}u'(c_t) = \beta u'\left(c_{t+1}\right) \end{aligned}$$
As $\beta < q^{*l}$ and $\beta < q^{*h}$, consumption is decreasing in time, so debt is increasing. Therefore, assuming a sufficiently low value of β, the borrowing constraint will eventually bind.

- If the borrowing constraint is binding at both future states then $V^h_{pay}(d^h) = V_{def} = V^l_{pay}(d^l)$. That leads to:
$$c^h = c^l = (1-\gamma)y$$
When the borrowing constraint is binding, the best feasible choice makes consumption constant and independent of the state. The borrowing constraint will be binding forever.

In sum, if β is smaller than q^{*h} and q^{*l}, consumption will eventually be limited to the point where the country cannot contract more debt in either state. From that point on, the borrowing constraint will always be binding at both states. I focus on the case when the country has already reached this constraint, as this is the interesting one for the study of default.

Denote $\bar{q} = (q^{*h} + q^{*l})/2$. The following proposition establishes the value of Δd:

Proposition 1 *If the borrowing constraint is binding in both states, Δd is given by:*

$$\frac{d^h - d^l}{d^h} = \frac{q^{*h} - q^{*l}}{1 - q^{*l} + 2\psi\bar{q}} \quad , \quad \frac{d^h - d^l}{d^l} = \frac{q^{*h} - q^{*l}}{1 - q^{*h} + 2\psi\bar{q}} \tag{1}$$

Proof. *If the borrowing constraint is binding in both states,*

$$\begin{aligned} V^h_{pay}(d^h) &= u(y - d^h + q^{*h}\left[(1-\psi)d^h + \psi d^l\right]) + \beta V_{def} \\ V^l_{pay}(d^l) &= u(y - d^l + q^{*l}\left[(1-\psi)d^l + \psi d^h\right]) + \beta V_{def} \end{aligned}$$

so $V^h_{pay}(d^h) = V^l_{pay}(d^l)$ implies:

$$y - d^h + q^{*h}\left[(1-\psi)d^h + \psi d^l\right] = y - d^l + q^{*l}\left[(1-\psi)d^l + \psi d^h\right]$$

which yields the claim. ■

Debt relief depends on: (i) the magnitude of interest rate fluctuations and (ii) the persistence of the interest rate process. In the i.i.d. case, $\psi = 0.5$, debt relief when the state switches from h to l is approximately equal to $\left(q^{*h} - q^{*l}\right)$. In the other extreme, as $\psi \to 0$, debt reduction is much higher: $\left(q^{*h} - q^{*l}\right)/(1 - q^{*l})$, as it has to compensate for all the expected future loss brought on by the fall in q^*. Hence, higher persistence implies higher difference between d^h and d^l.

The output cost γ has no effect on $\Delta d/d$. It is important to determine the level of d, but it does not influence the ratio between the incentive compatible levels of debt at both states.

2.2 Stochastic endowment

In this section, I fix the world interest rates at r^* and allow y to fluctuate between y^h in the high state and y^l in the low state, $y^h > y^l$. A risk-neutral creditor that lends $q^* d'$ must get an expected repayment equal to d'. Denote by d^h and d^l the repayment conditional on high and low state, respectively, and $\Delta d = d^h - d^l$. The value functions conditional on repayment are:

$$\begin{aligned} V^h_{pay}(d) &= \max_{d',\Delta d}\left\{u(y^h - d + q^h(d')d') + \beta\left[(1-\psi)V^h(d' + \psi\Delta d) + \psi V^l(d' - (1-\psi)\Delta d)\right]\right\} \\ V^l_{pay}(d) &= \max_{d',\Delta d}\left\{u(y^l - d + q^l(d')d') + \beta\left[(1-\psi)V^l(d' - \psi\Delta d) + \psi V^h(d' + (1-\psi)\Delta d)\right]\right\} \end{aligned}$$

where $V^i(d) = \max\left\{V^i_{pay}(d), V^i_{def}(\gamma)\right\}$ and i denotes the state. Should the country go into default, the value functions are:

$$\begin{aligned} V^h_{def} &= u((1-\gamma)y^h) + \beta\left[(1-\psi)V^h_{def} + \psi V^l_{def}\right] \\ V^l_{def} &= u((1-\gamma)y^l) + \beta\left[(1-\psi)V^l_{def} + \psi V^h_{def}\right] \end{aligned}$$

As there is no debt repudiation in equilibrium, $q^h(d') = q^l(d') = q^*$.

As in the case of stochastic world interest rates, if β is sufficiently low, the country's borrowing constraint will eventually bind in both states and it will be limited in every state to borrowing just enough to retain incentive compatibility. So, $V_{pay}^h(d^h) = V_{def}^h$ and $V_{pay}^l(d^l) = V_{def}^l$.

The following proposition establishes the value of Δd:

Proposition 2 *If the borrowing constraint is binding in both states, Δd is given by:*

$$\frac{d^h - d^l}{\bar{d}} = \frac{(1-q^*)}{1-q^*(1-2\psi)} \frac{y^h - y^l}{\bar{y}} \tag{2}$$

where $\bar{d} = \left(d^h + d^l\right)/2$ and $\bar{y} = \left(y^h + y^l\right)/2$.

Proof. *Consider the case $y = y^h$. If the borrowing constraint is binding and debt is at its maximum level, then:*

$$V_{pay}^h(d^h) = u(y^h - d^h + q^*\left[\psi d^l + (1-\psi)d^h\right]) + \beta\left[(1-\psi)V_{def}^h + \psi V_{def}^l\right]$$

Making $V_{pay}^h(d^h) = V_{def}^h$, we get:

$$d^h - q^*\left[\psi d^l + (1-\psi)d^h\right] = \gamma y^h \tag{3}$$

Analogously:

$$d^l - q^*\left[\psi d^h + (1-\psi)d^l\right] = \gamma y^l \tag{4}$$

Subtracting (4) from (3), we get:

$$\left(d^h - d^l\right)\left[1 - q^*(1-2\psi)\right] = \gamma(y^h - y^l)$$

Summing (4) and (3) and manipulating, we get:

$$\gamma = \frac{\bar{d}}{\bar{y}}(1-q^*)$$

Substituting the value of γ in the above equation, we get the claim. ■

As before, larger fluctuations and more persistent states imply higher debt relief and γ has no effect on $\Delta d/\bar{d}$.

2.3 Contrasting stochastic q^* and stochastic y

In order to contrast debt relief in the cases of stochastic interest rates and endowments, we need to contrast the numerators of Equations (1) and (2), as the denominators are virtually the same. The key distinction is that the numerator of Equation 1 is the difference between interest rates in both states, while the numerator of Equation 2 is the relative change in endowment multiplied by $(1 - q^*)$, the present discounted interest rate. A reasonable range for the numerator of Equation 1 (stochastic interest rates) is between 2% and 4%. On the other hand, a reasonable range for endowment fluctuations is from 2% to 5%, which in combination with a range of average real interest rates from 1% to 3% gives a range for the numerator of Equation 2 (stochastic technology) of 0.02% to 0.15%. This is one or two orders of magnitude below what we get from fluctuations in world interest rates.

Even permanent fluctuations in output would not generate sizable debt relief. The effect of a permanent output fall on debt relief can be found by taking the limit $\psi \to 0$. In that case, we get

$$\frac{d^h - d^l}{\bar{d}} = \frac{y^h - y^l}{\bar{y}}$$

so a permanent 2% fall in output leads to a fall of 2% in the incentive compatible level of debt.

Fluctuations in q^* alter the cost of servicing debt. Shocks to y change the present value of output losses due to default. Both these changes affect the incentive compatible level of debt and the amount of debt relief depends on how the incentive compatible level of debt is affected by fluctuations in y and q^*. There is a great distinction in quantitative effects of shocks to y and q^* because an increase in world interest rates from 1% to 2% doubles the cost of servicing debt while a 5% fall in the endowment reduces by 5% the loss due to default.

3 Debt and default in a growth model

The study of external debt and default is closely related to the question of why capital does not flow from rich to poor countries. One proposed explanation is that the risk of default prevents larger capital inflows in emerging economies (Reinhart and Rogoff (2004) and Reinhart, Rogoff and Savastano (2003)). Alternative explanations emphasise differences in productivity (Lucas (1990)) and question whether the marginal productivity of capital is really higher in poor countries (Caselli and Feyrer (2007)). Yet, most of the recent work on debt and default building on Eaton and Gersovitz (1981) focuses on risk sharing.[13]

[13] Cohen and Sachs (1986) present a growth model in which debt is repaid only if it is incentive compatible to do so, but assume a linear production function and have no uncertainty. They also analyse a numerical example with decreasing returns to capital which is essentially the deterministic model of this section. Marcet and Marimon (1992) and Kehoe and Perri (2004) also study economies with capital accumulation.

In this Section, I introduce capital accumulation in the model. The domestic country borrows because its marginal product of capital is higher, not because of risk sharing or impatience. The assumption of a low β is replaced with $\beta = q^*$. In the case of stochastic world interest rates, q^* fluctuates around β.

The main conclusion is that the impact of output and interest-rate shocks on the incentive compatible level of debt are very similar to their impact in the endowment economy.

The model follows the structure of the previous section, but with capital, k_t, depreciating at rate δ and output a function solely of the level of capital, as labour is normalised to 1:

$$y_t = \begin{cases} A_t.f(k_t) \text{ , if it has never defaulted} \\ A_t(1-\gamma).f(k_t) \text{ , if it has ever defaulted} \end{cases}$$

The economy's flow budget constraint is then given by:

$$c_t + k_{t+1} = \begin{cases} A_t.f(k_t) + (1-\delta)k_t - d_t + q_t d_{t+1} \text{ , if it has never defaulted} \\ A_t(1-\gamma).f(k_t) + (1-\delta)k_t \text{ , if it has ever defaulted} \end{cases}$$

3.1 Deterministic Model

I begin with a deterministic model, where the value functions are given by:

$$V(k,d) = \max\{V_{pay}(k,d), V_{def}(k,\gamma)\}$$

and:

$$\begin{aligned} V_{pay}(k,d) &= \max_{k',d'} \{u(Af(k) + (1-\delta)k - k' - d + qd') + \beta V(k',d')\} \\ V_{def}(k,\gamma) &= \max_{k'} \{u((1-\gamma)Af(k) + (1-\delta)k - k') + \beta V_{def}(k')\} \end{aligned}$$

I assume that decisions about k' and d' are made simultaneously and lenders can observe k' before taking their lending decisions (or condition their decisions on k'). As noted by Cohen and Sachs (1986), the country would otherwise have an incentive to borrow d' but then invest less, consume more and default on its debt.[14]

An equilibrium is a $\{k_t\}_{t=0}^{\infty}$, $\{d_t\}_{t=0}^{\infty}$ and $\{q_t\}_{t=0}^{\infty}$ such that the central planner maximises the value function $V(k,d)$ at every period and lenders are indifferent between the domestic and risk-free bond.

The following results hold in an equilibrium with no uncertainty:

[14] To see this, note that in the optimal plan $V_{pay}(k',d') = V_{def}(k',\gamma)$ and $u'(c) = \beta\frac{\partial V_{pay}}{\partial k'}(k',d')$. But $\frac{\partial V_{pay}}{\partial k'}(k',d') > \frac{\partial V_{def}}{\partial k'}(k')$, so if the country has already borrowed d' and hasn't committed to k', a marginal decrease in k' leads to an increase in today's utility that is bigger than the decrease in tomorrow's value. This moral hazard problem is studied by Atkeson (1991).

1. $q = q^*$, a constant. As there is no uncertainty, $q = q^*$ if the country will repay and $q = 0$ otherwise. The choice $d' = 0$ is strictly better than any choice d' such that $q = 0$ because that yields the same amount of consumption today and more production next period (by avoiding the $\gamma Af(k)$ output loss). So, in equilibrium, I obtain the no-default condition: $V_{pay}(k, d) \geq V_{def}(k, \gamma)$.

2. $d^{\max}(k, \gamma)$ is the maximum level of incentive compatible debt and is increasing in γ. Since by differentiating the value function we obtain that V_{pay} is decreasing in d and V_{def} is decreasing in γ, $V_{pay}(k, d^{\max}) = V_{def}(k, \gamma)$. Thus, an increase in γ implies an increase in $d^{\max}(k, \gamma)$.

3. If k' is below the steady state level of capital, k^*, then $d' = d^{\max}$. In the steady state, $k' = k = k^*$ and the marginal productivity of capital, $mpk = Af'(k^*) - \delta$, equals the marginal cost of renting an extra unit of capital, r^*. In this case, the country has no incentive to change the level of its debt, because capital is at the optimal level and smooth consumption can be achieved by always choosing $d' = d$. In contrast, if $k < k^*$, $mpk > r^*$ and d cannot be smaller than $d^{\max}$, otherwise the no-default condition would not bind, so the country could borrow an extra unit at r^*, invest it and obtain a greater return than r^* next period.

4. If $\gamma = 0$, no debt can be sustained. If $\gamma = 0$, any positive discounted stream of repayment is worse than defaulting, so the maximum incentive compatible positive discounted stream of repayment is zero.[15]

5. If $\gamma = 1$, we obtain full commitment. If $\gamma = 1$ with no default, in the steady state, the country obtains consumption equal to $Af(k^*) - \delta k^* - r^* d^*$ which is positive because $d^* \leq k^*$, $Af'(k^*) - \delta = r^*$ and $f(k^*) > k^* f'(k^*)$. Hence the first best, full commitment equilibrium is incentive compatible.[16]

Using these results, I now derive the path of debt in the neighbourhood of $\gamma = 0$. I will detail two observations that are used to derive it.

First, consider k'_p and d' such that $V_{pay}(k'_p, d') = V_{def}(k'_p, \gamma)$ and $k'_p \leq k^*$. Then there exists some $k \leq k'_p$ and d such that the country is indifferent between "repaying and choosing (k'_p, d')" and "defaulting and choosing (k'_d)". The value functions will be

[15] This result is due to the absence of uncertainty in the model. It has already been shown in the literature that, with uncertainty, there may be debt in equilibrium even in the absence of output costs (Eaton and Gersovitz, 1981).

[16] Again, this is a result due to the absence of uncertainty in the model. It has already been shown in the literature that with uncertainty and incomplete asset markets, the full commitment equilibrium may not be the first best equilibrium because the possibility of default makes debt somewhat contingent (Zame, 1993).

equivalent in this case and are given by:

$$
\begin{aligned}
V_{pay}(k,d) &= \max_{k'_p,d'} \left\{u(y+(1-\delta)k-k'_p-d+qd')+\beta V_{pay}(k'_p,d')\right\} \\
V_{def}(k,\gamma) &= \max_{k'_d} \left\{u((1-\gamma)y+(1-\delta)k-k'_d)+\beta V_{def}(k'_d,\gamma)\right\}
\end{aligned}
$$

Second, from result 4, we know that the value functions when $\gamma = d = d' = 0$ are identical and have a common optimal level of capital:

$$
\begin{aligned}
V_0(k) &= \max_{k'} \left\{u(y+(1-\delta)k-k')+\beta V_{def}(k',0)\right\} \\
&= \max_{k'} \left\{u(y+(1-\delta)k-k')+\beta V_{pay}(k',0)\right\}
\end{aligned}
$$

Then, from result 3, we always have $d' = d^{\max}$ and therefore the no default condition will always bind: $V_{pay}(k, d^{\max}) = V_{def}(k,\gamma)$. Using the first observation, we rewrite these value functions in the form above. Then by taking a linear approximation of $V_{pay}(k,d)$ and $V_{def}(k,\gamma)$ around $V_0(k)$ and manipulating the linearised expressions, we can find d and γ that equate $V_{pay}(k,d)$ and $V_{def}(k,\gamma)$ without solving for the value functions. The expression for d turns out to be a good approximation for its true value if γ is not more than a few percentage points. This is because when $\gamma \to 0$, the optimal choice of capital is independent of the decision about defaulting — which allows us to get the analytical results. As γ moves away from 0, that is no longer true, however the impact on the value function of reoptimising the level of capital due to a 1% or 2% fall in productivity is very small, and so is its impact on the maximum incentive compatible level of debt.

The results do not depend on the functional forms of utility or production, which have only second order effects.

Proposition 3 *In this economy, for a very small γ:*

1. *In steady state:*

$$d^* = \frac{\gamma y^*}{1-q^*} \tag{5}$$

2. *For $y_t < y^*$:*

$$\frac{d_{t+1}}{y_t} = \frac{\gamma}{1-q} + \frac{\Delta d_{t+1}}{(1-q)y_t}$$

and:

$$d_t = q.d_{t+1} + (1-q)\frac{\gamma y_t}{1-q}$$

Proof: see appendix.

Part (1) of the proposition shows that in the steady state, the country keeps repaying its debt if the interest payment, $d^*(1-q^*)$, is not greater than the output loss, γy^*, due to default. The debt as proportion of GDP is, to a first order approximation, equal to $\gamma/(1-q^*)$. Positive debt with no uncertainty arises in equilibrium to finance convergence. If $\gamma = 1\%$ and $q^* = 0.98$ ($r^* \approx 2\%$), the debt-GDP ratio is 50%.

The level of debt is proportional to the output loss and inversely proportional to the risk-free interest rate. Note that a change in interest rate from 1% to 2% has the same impact on d as a 50% decrease in GDP. The intuition for the different impacts of A and q^* on the incentive compatible level of debt (Equation 5) is similar to the reasons for their distinct effects on the optimal contract in Section 2 (Equations 1 and 2).

Part (2) of the proposition shows that for $y_t < y^*$, the condition for default reduces to a comparison between output losses and resources paid to foreign agents *in the present period*. But the increase in debt is endogenously determined by considering that the country will be indifferent in the next period between repaying and defaulting — so, ultimately, debt at period t is obtained by backward induction from the steady state level of debt.

For $y_t < y^*$, the absolute level of debt is increasing over time because the present value of the output loss due to default is increasing as output rises to its steady state level. The debt-GDP ratio is decreasing over time, because positive capital inflows generate greater incentive for the country to repay, and capital inflows are decreasing over time.

The proposition also shows that in equilibrium the country must experience net outflows of resources on the path of convergence. Debt is increasing (financial account is in surplus) but the increase is smaller than the interest paid on its debt. So, even though the current account is in deficit, the country is a net exporter of goods.

In the appendix, I present a numerical example that confirms the analytical expression is a good approximation for the results and illustrates convergence in this economy.

3.2 Stochastic world interest rates

Here, I analyse the optimal debt contract for an economy with fixed technology, A, and fluctuations in world interest rates, r^*, which lead to fluctuations in the price of risk-free debt, q^*. Except for capital accumulation and the assumption that q^{*h} and q^{*l} are close to β, the model is identical to that of section 2.1.

The value functions conditional on repayment are:

$$\begin{aligned} V_{pay}^h(k,d) &= \max_{k',d',\Delta d}\left\{u(c)+\beta\left[(1-\psi)V^h(k',d'+\psi\Delta d)+\psi V^l(k',d'-(1-\psi)\,\Delta d)\right]\right\} \\ V_{pay}^l(k,d) &= \max_{k',d',\Delta d}\left\{u(c)+\beta\left[(1-\psi)V^l(k',d'-\psi\Delta d)+\psi V^h(k',d'+(1-\psi)\,\Delta d)\right]\right\} \end{aligned}$$

where $c = Af(k)+(1-\delta)k-k'-d+q^i(k',d')d'$, $V^i(k,d) = \max\left\{V_{pay}^i(k,d), V_{def}^i(k,\gamma)\right\}$ and $i \in \{l,h\}$.

In the event of default, the value function in both states is:

$$V_{def}(k,\gamma) = \max_{k'}\left\{u((1-\gamma)Af(k)+(1-\delta)k-k')+\beta V_{def}(k',\gamma)\right\}$$

As in equilibrium there is no debt repudiation, $V^h = V_{pay}^h$ and $V^l = V_{pay}^l$. The optimal Δd will depend on k, d and s and on whether the borrowing constraint is binding or not (that is, whether the country would borrow more in the absence of commitment problems). The following propositions formalize that.

Define $mpk = Af'(k)-\delta$ and $r^{*i} = 1/q^{*i}-1$.

Proposition 4 *If the borrowing constraint is not binding,* Δd *is chosen to make*

$$\frac{\partial V_{pay}^h(k',d^h)}{\partial d} = \frac{\partial V_{pay}^l(k',d^l)}{\partial d}$$

Proof: see appendix.

Proposition 5 *If the borrowing constraint is binding, so that* $mpk > r^{*i}$*, and* $q^{*h}-q^{*l}$ *is arbitrarily small, then* Δd *is chosen to make* $V_{pay}^h(k',d^h) = V_{pay}^l(k',d^l)$.

Proof: see appendix.

If the country's borrowing constraint is not binding, Δd is chosen to equate the marginal value functions in both states. When the constraint is binding, the optimal contract equates the value functions in both states in order to ensure there are no further gains from transferring debt across states. If $V_{pay}^h(k',d^h)$ and $V_{pay}^l(k',d^l)$ do not coincide, the country can always borrow more by transferring debt across states. In equilibrium, if the borrowing constraint is binding, $V_{pay}^h(k',d^h) = V_{pay}^l(k',d^l) = V_{def}(k',\gamma)$.

In order to expand its borrowing possibilities, a country with a high marginal productivity of capital chooses to make debt in each of the future states as high as possible, respecting the incentive compatibility constraints.

3.2.1 The value of Δd

For analytical convenience, I consider that $dq^* = q^{*h}-q^{*l}$ is sufficiently small and work with linear approximations, which implies we are not considering the effects on the value function of reoptimising the choice of k' when the country changes state.

In addition, I temporarily consider an alternative process for q^* that I denote by the ξ-process, as opposed to the ψ-process that we described above. At time $t = 0$, $q_0^* = q^{*\xi}$; from time $t = 1$ on, there is a constant probability at each period that q^* permanently goes to $\bar{q}$. So, for $t > 0$:

- if $q_{t-1}^* = q^{*\xi}$, $\Pr(q_t^* = q^{*\xi}) = \xi$ and $\Pr(q_t^* = \bar{q}) = 1 - \xi$;
- if $q_{t-1}^* = \bar{q}$, $q_t^* = \bar{q}$.

The value function at (k, d) if $q_t = q^{*\xi}$ is:

$$V^\xi(k, d, q^{*\xi}) = \max_{k', d', d'^\xi} \left\{ u\left(c\right) + \beta \left[(1-\xi) V^{\det}(k', d') + \xi V^\xi(k', d'^\xi, q^{*\xi}) \right] \right\}$$

where $c = Af(k) + (1-\delta)k - k' - d + q^{*\xi}(d'(1-\xi) + \xi d'^\xi)$ and $V^{\det}$ is the value function in the model with no uncertainty.

The ξ-process and the ψ-process are related using the following lemma:

Lemma 6 *Define $\bar{q} = (q^{*h} + q^{*l})/2$ and denote by $V^h(k, d, q^{*h})$ the value function for the ψ-process. Then $V^h(k, d, q^{*h}) = V^\xi(k, d, q^{*h})$ if $\xi = 1 - 2\psi$.*

Proof: see appendix.

Compare the following two cases when q^* follows the ξ-process: (1) $q^* = q^{*\xi}$ and debt is d_0^ξ and (2) $q^* = \bar{q}$ and debt is d_0. We want to find the values of d_0^ξ and d_0 that make the country indifferent between both cases in order to determine Δd using proposition 5. By taking a linear approximation of $V^\xi(k, d^\xi, q^{*\xi})$ around $V^{\det}(k, d_0)$ and using the indifference condition that $V^\xi(k, d^\xi, q^{*\xi}) = V^{\det}(k, d_0)$, we get the following lemma:

Lemma 7 *Indifference between both states, $V^\xi(k, d^\xi, q^{*\xi}) = V^{\det}(k, d_0)$, implies:*

$$\begin{aligned} & u'(c_0)\left(d_0^\xi - d_0\right) \\ = \; & \sum_{t=0}^{\infty} (\beta\xi)^t \, u'(c_t) \left(q^{*\xi} - \bar{q}\right) d_{t+1} + \sum_{t=0}^{\infty} (\beta\xi)^t \left[u'(c_t)\bar{q} + \beta \frac{\partial V(k_{t+1}, d_{t+1})}{\partial d} \right] (d_{t+1}^\xi - d_{t+1}) \end{aligned} \tag{6}$$

where d_{t+1}^ξ is debt contracted at time t if $q_t^* = q^{*\xi}$ and d_{t+1} is debt contracted at time t if $q_t^* = \bar{q}$.

Proof: see appendix.

Suppose that $q^{*\xi} > \bar{q}$. The first line in the above expression shows the utility cost of having higher debt. The second line shows the utility benefit of borrowing at a lower rate, taking into account the probability of cheaper borrowing in future periods, plus the benefit of being able to borrow more due to lower interest rates. If the borrowing constraint is binding, then

$$u'(c_t)\bar{q} > -\beta\frac{\partial V(k_{t+1}, d_{t+1})}{\partial d}$$

which means that the benefit of borrowing an extra unit this period is greater than the cost of holding an extra unit of debt next period.

From Lemma 7, we can write:

$$\frac{d_0^\xi - d_0}{d_0} > \sum_{t=0}^{\infty} (\beta\xi)^t \frac{u'(c_t)}{u'(c_0)} \left(q^\xi - \bar{q}\right) \frac{d_{t+1}}{d_0}$$

It is convenient to consider Equation 6 as we approach the steady state of the deterministic economy. Formally, first I take the limit of small shocks ($q^{*h} - q^{*l} \to 0^+$) and then I consider the limit of small differences in the marginal product of capital ($mpk - r^* \to 0^+$).

In the limit of small shocks, as k approaches k^*, the borrowing constraint stops binding, c_t and d_t approach their steady state, constant, values, and we get::

$$\frac{d_0^\xi - d_0}{d_0} = \frac{q^\xi - \bar{q}}{1 - \beta\xi} \tag{7}$$

And that leads to the following proposition:

Proposition 8 *Consider a deterministic steady state around,* $\bar{k}$, $\bar{d}$ *and* $\bar{q}$, *such that* q^{*h} *and* q^{*l} *are close to* $\bar{q} = \beta$ *and* $\bar{q} = \left(q^{*h} + q^{*l}\right)/2$. *Then, a linear approximation around the steady state will satisfy* $V(\bar{k}, d^h, q^{*h}) = V\left(\bar{k}, d^l, q^{*l}\right)$ *when:*

$$\frac{d^h - d^l}{\bar{d}} = \frac{q^{*h} - q^{*l}}{1 - \bar{q}(1 - 2\psi)} \tag{8}$$

where $\bar{d} = \left(d^h + d^l\right)/2$.

Proof: see appendix.

Equation 8 is very similar to equation 1, and for small fluctuations of q^*, they are exactly the same. There are two differences between the case with capital accumulation and the endowment economy. One is that, with capital accumulation, the optimal decision on consumption and savings depends on the state, which influences the value function. But because of the envelope theorem, around the point of maximum, those differences in the choice of k have only second order effects on the value function. It follows that they have no first order impact on the incentive compatible level of debt.

The second difference is that growth prospects have an influence on the incentive compatible level of debt. Proposition 8 shows that in the steady state, debt relief for economies with and without capital coincide. But lemma 7 shows that, outside the steady state, $\Delta d/\bar{d}$ depends not only on the the magnitude of interest rate fluctuations and the persistence of the interest rate process, but also on the current level of capital — and its marginal productivity. The lower is the level of capital, the greater are the marginal productivity of capital and the difference between $u'(c)\bar{q}$ and $-\beta\frac{\partial V(k',d')}{\partial d}$, which contribute to increase Δd: a switch to the low state that prevents the country from borrowing is more punitive when capital is lower. A lower level of capital also implies lower consumption and, therefore, higher marginal utilities, so present consumption is more important and higher costs of borrowing in the future are less relevant, which induces a decrease in Δd. Lastly, a higher ratio between future and present debt increases the importance of future costs of borrowing, which induces an increase in Δd. Thus the overall effect cannot be deduced from the formula. In the numerical examples, Δd is slightly decreasing in k, implying the effect of the borrowing constraint predominates.

The analysis has focused on the two-state case, but the same insights apply if we consider more general processes. The next proposition considers the case of an autoregressive process for q^*.

Proposition 9 *Suppose that q^* follows an AR(1) process:*

$$q^*_{t+1} - \bar{q} = \zeta(q^*_t - \bar{q}) + \varepsilon_{t+1}$$

and $Var(\varepsilon_t)$ is arbitrarily small. If the economy is close to its steady state ($k \simeq k^*$), $V(k, d^1, q^{*1}) = V(k, d^2, q^{*2})$ for any $\{q^{*1}, q^{*2}\}$ close to $\bar{q}$ and $\{d^1, d^2\}$ when:

$$\frac{d^1 - d^2}{\bar{d}} = \frac{q^{*1} - q^{*2}}{1 - \beta\zeta}$$

where $\bar{d}$ is the level of debt in the deterministic model when $q = \bar{q}$.

Proof: see appendix.

3.3 Stochastic technology

In this section, I consider fixing the world interest rates at r^* and allowing for fluctuations in A. Productivity is A^h in the high state and A^l in the low state, $A^h > A^l$. It is assumed $q^* = \beta$. The value functions conditional on repayment are:

$$\begin{aligned} V^h_{pay}(k,d) &= \max_{k',d',\Delta d} \left\{u(c^h) + \beta\left[(1-\psi)V^h(k', d' + \psi\Delta d) + \psi V^l(k', d' - (1-\psi)\,\Delta d)\right]\right\} \\ V^l_{pay}(k,d) &= \max_{k',d',\Delta d} \left\{u(c^l) + \beta\left[\psi V^h(k', d' + (1-\psi)\Delta d) + (1-\psi)V^l(k', d' - \psi\Delta d)\right]\right\} \end{aligned}$$

where $c^i = A^i f(k) + (1-\delta)k - k' - d + q(k', d')d'$, $V^i(k, d) = \max\left\{V^i_{pay}(k, d), V^i_{def}(k, \gamma)\right\}$ and i denotes the state.

In case of default, the value functions are:

$$\begin{aligned} V^h_{def}(k, \gamma) &= \max_{k'}\left\{u((1-\gamma)A^h f(k) + (1-\delta)k - k' + \beta\left[(1-\psi)V^h_{def}(k', \gamma) + \psi V^l_{def}(k', \gamma)\right]\right\} \\ V^l_{def}(k, \gamma) &= \max_{k'}\left\{u((1-\gamma)A^l f(k) + (1-\delta)k - k' + \beta\left[(1-\psi)V^l_{def}(k', \gamma) + \psi V^h_{def}(k', \gamma)\right]\right\} \end{aligned}$$

As before, if the country's borrowing constraint is binding, it borrows up to a debt limit in each possible future state that ensures it remains incentive compatible for it to repay. This result is stated in the following proposition:

Proposition 10 *If the borrowing constraint is binding, so that $mpk > r^*$, and $A^h - A^l$ is arbitrarily small, then Δd is chosen to make $V^h_{pay}(k', d^h) = V^h_{def}(k', \gamma)$ and $V^l_{pay}(k', d^l) = V^l_{def}(k', \gamma)$.*

Proof: see appendix.

The proof is analogous to the proof of Proposition 5.

As before, we need to obtain an expression for Δd. The analogy to Proposition 8 for the case of stochastic technology requires the additional assumption that γ is arbitrarily small and yields the following result:

Proposition 11 *Consider a deterministic steady state, $\{\bar{k}, \bar{d}, \bar{A}\}$, such that A^h and A^l are close to $\bar{A} = \left(A^h + A^l\right)/2$. Then, for arbitrarily small γ, a linear approximation around the steady state will satisfy$V^h_{pay}(k', d^h) = V^h_{def}(k', \gamma)$ and $V^l_{pay}(k', d^l) = V^l_{def}(k', \gamma)$ when:*

$$\frac{d^h - d^l}{\bar{d}} = \frac{(1-q^*)}{1 - q^*(1-2\psi)} \frac{A^h - A^l}{\bar{A}} \tag{9}$$

where $\bar{d} = \left(d^h + d^l\right)/2$.

Proof: see appendix.

Equation 9 is exactly the same as equation 2.

4 The Latin American debt crisis of the 1980's

In this section, I contrast the predictions of the model with data from the Latin American debt crisis of the 1980's. The results show that the interest rate shock at the beginning of the 1980's can account for a large part of the observed debt relief.

4.1 Observed debt relief

External shocks were important factors in the Latin American debt crisis of the 1980's. As noted by Diaz-Alejandro (1984), countries with different policies and distinct economies ended up in similar crises in the beginning of the 1980's, facing problems that in 1979 would have been considered unlikely.

One key external shock was the increase in US real interest rates, shown in Figure 1 (from Dotsey et al, 2003). In contrast to the 1970's when real interest rates were around 0%, in the 1980's they were around 4%. Such large increase in US real interest rates makes the The Latin American crisis a convenient case to evaluate the model's predictions.[17]

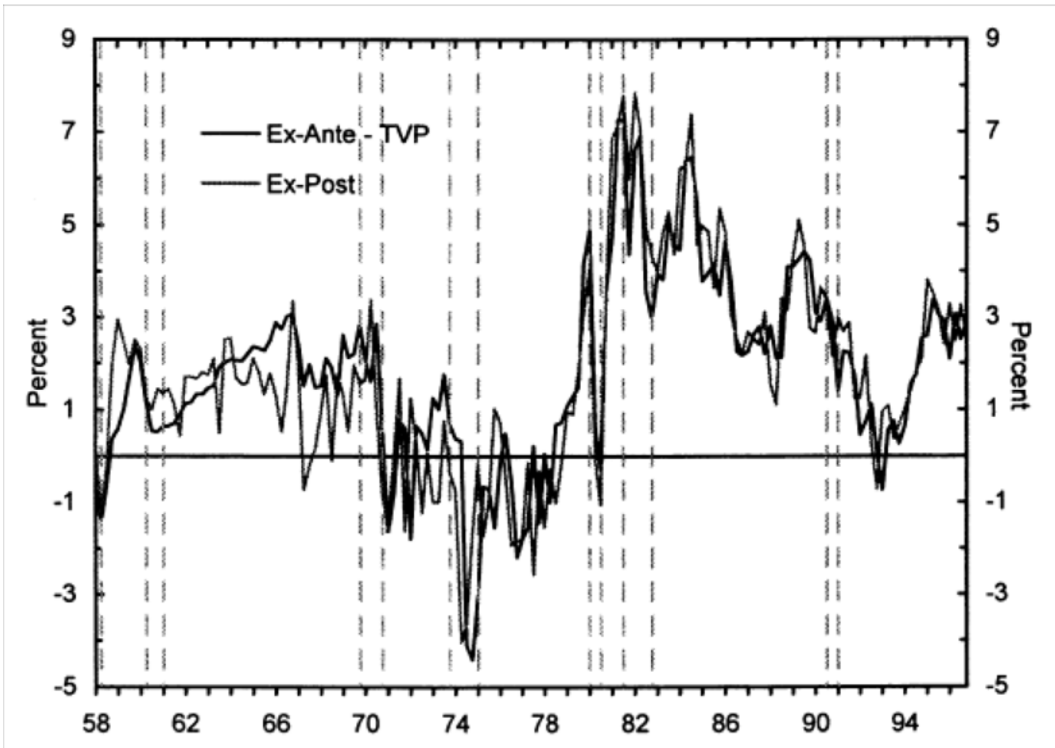

Figure 1: US real interest rates

In the beginning of the 1980's, the prices of Latin American bonds in secondary markets collapsed, capital flows to those economies dried or reverted and the fast process of economic growth of the 1970's stopped. After countless IMF missions, several debt reschedules and some attempts of debt renegotiation (including the Baker Plan), came the Brady agreements, starting in 1989. In the period between 1989 and 1994, most of

[17] There are other cases in which interest rate increases in the US contributed to crises in other countries. For example, the sharp increase in US interest rates in 1994 is sometimes mentioned as one of the factors that almost led Mexico to default in December 1994 (see Calvo, Leiderman and Reinhart, 1996).

the main Latin American countries got some debt relief. Table 1 shows the debt relief following the Brady Plan agreements as a percentage of the outstanding long term debt in the main Latin American countries. With the exception of Venezuela, at 20%, the other four countries are around 30%.[18]

Table 1: Debt relief – Brady plan agreements, Cline (1995)

Venezuela	20%
Brazil	28%
Argentina	29%
Mexico	30%
Uruguay	31%

As the Brady agreements did not cover all forms of external debt, the figures in Table 1 should be seen as upper bounds (Cline, 1995).

In the model, the optimal contract prescribes automatic instantaneous debt relief in order to compensate for unexpected increases in interest rates. In reality, debt relief came ten years later, and what happened in those ten years had some influence on the final agreement. Despite the delay, it is worth comparing the debt relief prescribed by the model and the Brady agreements because the latter were in fact the relief solution to the crisis.

4.2 Debt relief according to the model

If the borrowing constraints of the Latin American countries were binding in 1979, then the interest rate rise at the beginning of the 1980's would bring debt, d, above its incentive compatible level. In this section, I compute the optimal debt reduction prescribed by the model and compare it to the data.

Consider the model calibrated to represent the 1970's and 1980's. Suppose that world interest rates may be either 0% or 4% a year and that each state lasts for an average of 10 years: $q^{*h} = 1.00$, $q^{*l} = 1.04^{-1}$, $\beta = 1.02^{-1}$ and $\psi = 0.10$. Equation 8 yields the optimal debt relief given a switch from the high state to the low:

$$\frac{\Delta d}{\bar{d}} = \frac{1.00 - 1.04^{-1}}{1 - 1.02^{-1}(1 - 2 \times 0.10)} = 0.178$$

[18] As noted by Cline (1995), the initial approach for dealing with the problem of debt overhang was aimed both at reducing debt and providing new loans, but "for practical purposes the Brady Plan has been all forgiveness and no new money" (Cline, 1995, page 236). Indeed, according to the model, if the amount of debt exceeds its incentive compatible level, new money will not be made available.

That implies a spread over treasury of 1.8% when the state is high but debt relief of 18% when the state switches to low and interest rates jump from 0% to 4%. Data from 1973-80 show a spread over treasury of 1.4% for Brazil and Argentina and 1.1% for Mexico (Calvo and Kaminsky, 1991).

$\Delta d/\bar{d}$ is not significantly affected by the length of debt contracts. If we work with a period of 5 years instead of one year, $q^{*h} = 1.00$, $q^{*l} = 1.04^{-5}$, $\beta = 1.02^{-5}$ and $\psi = 0.50$, we still obtain $\left(d^h - d^l\right)/\bar{d} = 0.178$.

The result holds for any $\gamma > 0$ and is robust to other interest rates processes. Using the auto-regressive process and assuming a half-life of 3 years for the interest rate increase, the AR-1 coefficient, ζ, would be 0.79. A jump in real interest rates from 1% a year to 6% a year would then imply even greater debt relief: $\Delta d/\bar{d} = 22.5\%$.

In sum, the decrease in the level of debt predicted by the model in response to an interest rate increase of the magnitude observed in the data exceeds half the debt relief of the Brady agreements. A substantial debt reduction following the increase in US interest rates would have been part of an optimal contract drawn up in the 1970's.[19]

On the other hand, a negative productivity shock would not generate results of similar magnitude. A huge 10% reduction in productivity, assuming an average persistence of 10 years ($\psi = 0.10$) and world interest rates of 2% a year would imply debt reduction slightly *below 1%* according to Equation 9.

Numerical results, presented in the appendix, show that:

1. The formula in Proposition 8 is a good approximation for $\Delta d/d'$ provided the borrowing constraint is binding in both states of the economy, $mpk > r^{*l}$. While that holds, the level of capital makes little difference on debt relief.

2. If the borrowing constraint is binding and the level of capital is substantially below its steady state level, then there are large gains from debt contracts designed to make $V^h(k', d^h) = V^l(k', d^l)$;

3. Regarding the Latin American debt crisis, if the marginal productivity of capital in those countries was not lower than $r^{*l} = 4\%$, the analytical results are good approximations. Therefore, the sharp interest rate rise at the beginning of the 1980's would imply debt relief of more than half of the observed reduction obtained through

[19] It can be argued that the welfare implications of borrowing and whether a contract that maximises borrowing is truly optimal depend on the differences in marginal productivity of capital. The results in Caselli and Feyrer (2007) suggest that the marginal productivity of capital was indeed higher in poor countries in the 1970's. In the context of the Latin American debt crisis, the task of measuring productivity of capital is complicated by the fact that a substantial part of the debt went to financing government investment, often in infrastructure, on which return is not easily measured.

the Brady agreements. However, lower marginal productivity of capital (combined with low adjustment costs for capital) reduces the amount of debt relief prescribed by the model.

5 Concluding remarks

Fluctuations in world interest rates can have a strong impact on the incentive compatible level of sovereign debt. In particular, the increase in US interest rates in the beginning the 1980's can account for more than half of the debt relief obtained by the main Latin American countries through the Brady agreements.

However, such debt relief came with a 10-year delay. By analysing the optimal contract, this paper does not consider the costly bargaining process that follows the announcement of a sovereign default. Some recent policy prescriptions focus on such bargaining costs: the IMF's sovereign debt restructuring mechanism (SDRM) is an important example (Krueger (2002)). The collective action clauses (CAC's) aim to allow creditors to quickly reduce the level of debt when it goes above its incentive compatible level, which is a way to implement the optimal contract studied in this paper.

Another alternative would be to make debt contingent on world real interest rates (which could be implemented through real interest rate swaps). In the case of the Latin American debt crisis of the 1980's, having such kind of debt would have avoided 10 years of costly bargaining. However, while the real interest rate shock of 1980 can be seen as a policy decision, regular movements on world real interest rates might be correlated with variables that affect the incentive compatible level of debt. A thorough evaluation of all the practical implications of debt contingent on world real interest rates is beyond the scope of this paper, but deserves attention.[20]

Recent quantitative models of sovereign debt aim at explaining the recent debt crisis in Argentina using output shocks. However, according to the model in this paper, output fluctuations do not have a sizable effect on the incentive compatible level of debt — nothing remotely close to the observed debt reduction of 71%.[21]

[20] In principle, at least, the idea of conditioning debt payments to world interest rates sounds appealing. Many of the problems related to GDP-indexed bonds do not apply to contracts contingent on world interest rates: (i) there is no moral hazard: the country's actions do not affect its debt; (ii) there are no major measurement problems, danger of misreporting, data revisions, lag in data announcements, we only need to estimate expected inflation in the relevant developed countries; and (iii) while countries' GDP's are positively correlated, r^* and y^* are negatively correlated. As shown by Neumeyer and Perri (2005), interest rates and output are positively correlated in developed countries but negatively correlated in emerging markets.

[21] The negative correlation between probability of default and output does not tell us about causality, because a high probability of default has a negative impact on output of emerging economies through its impact on interest rates (Neumeyer

Another usual suspect is the terms of trade. Foley-Fisher (2007) applies the method developed in this paper to study the effect of terms of trade shocks on the incentive compatible level of debt. He finds they have a higher impact than output shocks, but lower impact than world real interest rates.

Last, fluctuations in the cost of default (γ) could be important. This cost could be influenced by the political environment.[22] But as noted by Sturzenegger and Zettelmeyer (2006), the importance of the various default cost channels remains unknown and a challenge for future research.

A Proofs

While proving the propositions in this paper, I often use the following method and I refer to it as a first order Taylor approximation. Consider the following Value function:

$$V(x,y) = \max_{a,b,c}\{u(x,y,a,b,c) + \beta V(x'(a,b,c), y'(a,b,c))\}$$

So the values of a, b, c are chosen, which determine the next period's x, y (denoted as a convention by x', y'). Take the function that is to be maximised:

$$f(x,y,a,b,c) = u(x,y,a,b,c) + \beta V(x'(a,b,c), y'(a,b,c))$$

Denote by a^*, b^* and c^* the maximising values of $V(x,y)$ and by $\tilde{a}^*$, $\tilde{b}^*$ and $\tilde{c}^*$ the maximising values of $V(\tilde{x},\tilde{y})$. Then $V(x,y) = f(x,y,a^*,b^*,c^*)$ and $V(\tilde{x},\tilde{y}) = f(\tilde{x},\tilde{y},\tilde{a}^*,\tilde{b}^*,\tilde{c}^*)$. Now, if (x,y) and $(\tilde{x},\tilde{y})$ are sufficiently close, we can take the approximation of the maximand function with respect to the variables to be chosen:

$$\begin{aligned} f(x,y,a^*,b^*,c^*) &\approx f(\tilde{x},\tilde{y},\tilde{a}^*,\tilde{b}^*,\tilde{c}^*) + \sum_{z=x,y,a,b,c} \frac{\partial f(\tilde{x},\tilde{y},\tilde{a}^*,\tilde{b}^*,\tilde{c}^*)}{\partial z}(z^* - \tilde{z}^*) \\ &= V(\tilde{x},\tilde{y}) + \sum_{z=x,y,a,b,c}\left(\frac{\partial u(\tilde{x},\tilde{y},\tilde{a}^*,\tilde{b}^*,\tilde{c}^*)}{\partial z} + \beta\frac{\partial V(\tilde{x}',\tilde{y}')}{\partial z}\right)(z^* - \tilde{z}^*) \end{aligned}$$

If there is a binding constraint on the possible values of a variable, then its maximised value will be determined by the constraint. Otherwise, the envelope theorem applies and the derivative of f with respect to that variable will be zero.

This method is different from the general first order Taylor approximation: the original function $V(x,y)$ is not a function of the variables with respect to which the approximation is done. This is why the maximand function has to be defined.

and Perri, 2005).

[22] If politics leads to overborrowing and default, restrictions on foreign borrowing or the imposition of capital controls may be welfare enhancing. Reasoning along these lines, Reinhart and Rogoff (2004) argue in favour of limiting the ability of poor countries to contract debt.

A.1 Proposition 3

Proof. The value functions in case of repayment and in case of default are maximised at (k'_p, d'_p) and (k'_d) respectively, which means that:

$$\begin{aligned} V_{pay}(k,d) &= u(y + (1-\delta)k - k'_p - d + qd'_p) + \beta V_{pay}(k'_p, d'_p) \\ V_{def}(k,\gamma) &= u(y + (1-\delta)k - k'_d - \gamma y) + \beta V_{def}(k'_d, \gamma) \end{aligned}$$

When $d = d' = 0$ and $\gamma = 0$, the value functions are identical in the two cases, $V_0(k)$. It is maximised by choosing $k' = k'_o$.

Consider (k, d, γ) such that the country is indifferent between repaying and choosing (k'_p, d') or defaulting and choosing (k'_d), which means that $V_{pay}(k,d) = V_{def}(k,\gamma)$. Approximate the functions that are to be maximised,

$$\begin{aligned} f_{pay}(d, k', d') &= u(y + (1-\delta)k - k' - d + qd') + \beta V_{pay}(k', d') \\ f_{def}(k', \gamma) &= u(y + (1-\delta)k - k' - \gamma y) + \beta V_{def}(k', \gamma) \end{aligned}$$

around $V_0(k)$. The first order Taylor approximation of these functions around $V_0(k)$ with respect to (k', d, d') or (k', γ) respectively yields

$$\begin{aligned} f_{pay}(d, k', d') &= V_{pay}(k,0) + \frac{\partial f_{pay}(0, k'_o, 0)}{\partial d}(d - 0) + \frac{\partial f_{pay}(0, k'_o, 0)}{\partial d'}(d' - 0) \\ &\quad + \frac{\partial f_{pay}(0, k'_o, 0)}{\partial k'}(k' - k'_o) + O_{k,p} \\ &= V_0(k) + u'(c_o)(-d + qd') + \beta \frac{\partial V_{pay}(k'_o, 0)}{\partial d'} d' + O_{k,p} \end{aligned}$$

$$\begin{aligned} f_{def}(k', \gamma) &= V_{def}(k,0) + \frac{\partial f_{def}(k'_o, 0)}{\partial \gamma}(\gamma - 0) + \frac{\partial f_{def}(k'_o, 0)}{\partial k'}(k' - k'_o) + O_{k,d} \\ &= V_0(k) - u'(c_o)\gamma y + \beta \frac{\partial V_{def}(k'_o, 0)}{\partial \gamma}\gamma + O_{k,d} \end{aligned}$$

Where I used that since $\frac{\partial V_{pay}}{\partial k'} = \frac{\partial V_{def}}{\partial k'} = 0$ due to the Envelope condition for the unconstrained maximization of V_{pay} and V_{def} with respect to k', their evaluation $\frac{\partial V_{pay}(k,0)}{\partial k'} = \frac{\partial V_{def}(k,0)}{\partial k'} = 0$ as well. The optimal consumption with no borrowing, no punishment is denoted by $c_o = y + (1-\delta)k - k'_o$. Furthermore, $\lim_{(d,d',k') \to (0,0,k'_o)} \frac{O_{k,p}}{\|d,d',k'\|^2} = 0$ and $\lim_{(\gamma,k') \to (0,k'_o)} \frac{O_{k,p}}{\|\gamma,k'\|^2} = 0$.

Note that $V_{pay}(k,d) = V_{def}(k,\gamma) \iff f_{pay}(d, k'_p, d') = f_{def}(k'_d, \gamma)$. Using the first order Taylor expansions at points (d, k'_p, d') and (k'_d, γ), we get:

$$u'(c_o)(-d + qd' + \gamma y) + \beta \left(\frac{\partial V_{pay}(k'_o, 0)}{\partial d'} d' - \frac{\partial V_{def}(k'_o, 0)}{\partial \gamma}\gamma \right) + (O_{k,p} - O_{k,d}) = 0.$$

The last part is to show that $\frac{\partial V_{pay}(k'_o, 0)}{\partial d'} d' - \frac{\partial V_{def}(k'_o, 0)}{\partial \gamma}\gamma$ is approximately zero.

If $k < k^*$ then the country borrows the maximum level of incentive compatible debt. This debt level makes the country indifferent between repaying and defaulting at (k'_p, d'):

$$V_{pay}(k'_p, d') = V_{def}(k'_p, \gamma)$$

First order Taylor approximations of $V_{pay}(k'_p, d')$ and $V_{def}(k'_p, \gamma)$ around $V_o(k'_o)$ with respect to only k', d' and γ yield:

$$\begin{aligned} V_{pay}(k'_p, d') &= V_0(k'_p) + \frac{\partial V_{pay}(k'_o, 0)}{\partial k'}.(k'_p - k'_o) + \frac{\partial V_{pay}(k'_o, 0)}{\partial d'}.d' + O_{k'}(d'^2) \\ V_{def}(k'_p, \gamma) &= V_0(k'_p) + \frac{\partial V_{def}(k'_o, 0)}{\partial k'}.(k'_p - k'_o) + \frac{\partial V_{def}(k'_o, 0)}{\partial \gamma}.\gamma + O_{k'}(\gamma^2) \end{aligned}$$

$V_{pay}(k'_p, d') = V_{def}(k'_p, \gamma)$ implies:

$$\frac{\partial V_{pay}(k'_p, 0)}{\partial d'}.d' - \frac{\partial V_{def}(k'_p, 0)}{\partial \gamma}.\gamma \approx 0$$

Using this last equation the difference of the original Taylor expansions simplifies to:

$$u'(c_o)\left(-d + qd' + \gamma y\right) + (O_{k,p} - O_{k,d}) \approx 0$$

$u'(c_o) \neq 0$, and $(O_{k,p} - O_{k,d})$ is very small near $(d, d', k') = \left(0, 0, k'_p\right)$ and $(\gamma, k') = (0, k'_d)$ imply that

$$-d + qd' + \gamma y = 0 \Longleftrightarrow d = qd' + \gamma y$$

Which yields the second part of the claim. In steady state, $d = d'$, and we get the first part of the claim. ■

A.2 Proposition 4

Proof. Suppose $s = h$. If the borrowing constraint is not binding, the first order condition with respect to Δd yields:

$$\beta(1-\psi)\frac{\partial V^h_{pay}(k', d' + \psi\Delta d)}{\partial d}\psi - \beta\psi\frac{\partial V^l_{pay}(k', d' - (1-\psi)\Delta d)}{\partial d}(1-\psi) = 0$$

which yields the claim. If $s = l$, a similar expression is obtained. ■

A.3 Proposition 5

Proof. Suppose $s = h$ and $k', d', \Delta d$ are such that $V^h_{pay}(k', d^h) > V^l_{pay}(k', d^l)$. If the borrowing constraint is binding, the country borrows up to the incentive compatible level (and there is no debt repudiation in equilibrium), so $V^l_{pay}(k', d^l) = V_{def}(k')$. By increasing Δd by dD, V^l_{pay} increases and V^h_{pay} decreases. The borrowing constraint is no longer binding, so the country can increase k' by some dk' and d' by dk'/q^{*h} while still respecting the borrowing constraint.

Consumption in the present period is unchanged. $V^l(k', d^l)$ changes by

$$u'(c'_L). \left[dk'.(mpk - r^{*h}) + (1-\psi).dD\right]$$

and $V^h(k', d^h)$ changes by

$$u'(c'_H). \left[dk'. \left(mpk - r^{*h}\right) - \psi.dD\right]$$

where c'_L and c'_H are consumption next period in the low and high state, respectively.

So the change in $V^h(k, d)$ equals:

$$\beta. \left\{ \left[\psi u'(c'_L) + (1-\psi)u'(c'_H)\right] dk'(mpk - r^{*h}) + \psi(1-\psi)dD\left[u'(c'_L) - u'(c'_H)\right] \right\}$$

Now, if $q^{*h} - q^{*l}$ is small enough, $(u'(c'_L) - u'(c'_H))$ is small and the change in $V^h(k, d)$ is positive because $mpk > r^{*h}$.

Similar expressions can be derived if $V^l_{pay}(k', d^l) > V^h_{pay}(k', d^h)$ and/or if $s = l$. ■

A.4 Lemma 6

Before proving lemma 6, we need an auxiliary result:

Lemma 12 *Consider the model with 2 states, h and l, and probability of changing state equal to ψ. Define $\bar{q} = (q^h + q^l)/2$. In a first order approximation,*

$$V(k, d, \bar{q}) = \left[V(k, d, q^h) + V(k, d, q^l)\right] \div 2$$

Proof. *Proof: Comes directly from a Taylor expansion of $V(\bar{k}, \bar{d}, q^h)$ and $V(\bar{k}, \bar{d}, q^l)$ around $V(\bar{k}, \bar{d}, \bar{q})$.* ■

We are ready to prove lemma 6.

Proof. First, we need to show that, close to the deterministic steady state $(k = \bar{k}, d = \bar{d})$, $V(\bar{k}, \bar{d}, q^h) = V^{\xi}(\bar{k}, \bar{d}, q^h)$ if $\xi = 1 - 2\psi$.

$$V^{\xi}(k, d, q^h) = \max_{k', d', \Delta d} \left\{ \begin{array}{c} u(Af(k) + (1-\delta)k - k' - d + q^h d') \\ +\beta\left[(1-\xi)V^{\det}(k', d' - \xi\Delta d) + \xi V^{\xi}(k', d' + (1-\xi)\Delta d, q^h)\right] \end{array} \right\}$$

Near the deterministic steady state, choosing the optimal (d', k') instead of $(\bar{d}, \bar{k})$ has only second order effect on the value function V^{ξ}. As a first order approximation, we can write:

$$\begin{aligned} V^{\xi}(\overline{k}, \overline{d}, q^h) &= u(c^h) + \beta\left[(1-\xi)V^{\det}(\overline{k}, \overline{d} - \xi\Delta d) + \xi V^{\xi}(\overline{k}, \overline{d} + (1-\xi)\Delta d, q^h)\right] \\ V^{\det}\left(\overline{k}, \overline{d}\right) &= u(\overline{c}) + \beta V^{\det}(\overline{k}, \overline{d}) \end{aligned}$$

where $c_h = Af(\overline{k}) - \delta\overline{k} - \overline{d}(1-q^h)$ and $\overline{c} = Af(\overline{k}) - \delta\overline{k} - \overline{d}(1-\overline{q})$.

Taking a first order Taylor approximation of $f^{\xi}(\overline{k},\overline{d},q^h,\Delta d) = V^{\xi}(\overline{k},\overline{d},q^h)$ around $V^{\det}\left(\overline{k},\overline{d}\right)$ ($q^h = \overline{q}$ and $\Delta d = 0$) with respect to $q, \Delta d$, we get:

$$\begin{aligned} V^{\xi}(\overline{k},\overline{d},q^h) &\approx u\left(\overline{c}\right) + u'(\overline{c})(q^h - \overline{q})\overline{d} + \beta\left(V^{\det}(\overline{k},\overline{d}) + (1-\xi)\frac{\partial V^{\det}(\overline{k},\overline{d})}{\partial d}(-\xi)(\Delta d - 0)\right) \\ &\quad + \beta\xi\left(\frac{\partial V^{\xi}(\overline{k},\overline{d},\overline{q})}{\partial d}(1-\xi)(\Delta d - 0) + \frac{\partial V^{\xi}(\overline{k},\overline{d},\overline{q})}{\partial q}(q^h - \overline{q})\right) \\ &= u\left(\overline{c}\right) + u'(\overline{c})(q^h - \overline{q})\overline{d} + \beta V^{\det}(\overline{k},\overline{d}) + \beta(q^h - \overline{q})\xi\frac{\partial V^{\xi}(\overline{k},\overline{d},\overline{q})}{\partial q} \end{aligned}$$

As a simple first order Taylor approximation, $\frac{\partial V^{\xi}(\overline{k},\overline{d},\overline{q})}{\partial q}(q^h - \overline{q}) = V^{\xi}(\overline{k},\overline{d},q^h) - V^{\xi}(\overline{k},\overline{d},\overline{q})$, so the above approximation can be written as:

$$V^{\xi}(\overline{k},\overline{d},q^h) = u\left(\overline{c}\right) + u'(\overline{c})(q^h - \overline{q})\overline{d} + \beta V^{\det}(\overline{k},\overline{d}) + \beta\xi\left[V^{\xi}(\overline{k},\overline{d},q^h) - V^{\xi}(\overline{k},\overline{d},\overline{q})\right]$$

Since $V^{\xi}(\overline{k},\overline{d},\overline{q}) = V^{\det}(\overline{k},\overline{d}) = u\left(\overline{c}\right) + \beta V^{\det}(\overline{k},\overline{d})$,

$$V^{\xi}(\overline{k},\overline{d},q^h) - V^{\det}(\overline{k},\overline{d}) = u'(\overline{c})(q^h - \overline{q})\overline{d} + \beta\xi\left[V^{\xi}(\overline{k},\overline{d},q^h) - V^{\det}(\overline{k},\overline{d})\right]$$

so

$$V^{\xi}(\overline{k},\overline{d},q^h) - V^{\det}(\overline{k},\overline{d}) = \frac{u'(\overline{c})(q^h - \overline{q})\overline{d}}{1-\beta\xi} \tag{10}$$

Now, note that, as a first order approximation:

$$\begin{aligned} V(\bar{k},\bar{d},q^h) - V^{\det}(\bar{k},\bar{d}) &= u'(\bar{c})\left(q^h - \bar{q}\right)\bar{d} + \beta\left[(1-\psi)V(\bar{k},\bar{d},q^h) + \psi V(\bar{k},\bar{d},q^l) - V^{\det}(\bar{k},\bar{d})\right] \\ &= u'(\bar{c})\left(q^h - \bar{q}\right)\bar{d} + \beta(1-2\psi)\left[V(\bar{k},\bar{d},q^h) - V^{\det}(\bar{k},\bar{d})\right] \end{aligned}$$

the last equality follows from lemma 12. Then:

$$V(\bar{k},\bar{d},q^h) - V^{\det}(\bar{k},\bar{d}) = \frac{u'(\bar{c})\left(q^h - \bar{q}\right)\bar{d}}{1-\beta(1-2\psi)} \tag{11}$$

If $\xi = 1 - 2\psi$, Equations (10) and (11) imply that $V(\bar{k},\bar{d},q^h) = V^{\xi}(\bar{k},\bar{d},q^h)$.

Now, we complete the proof by induction. Away from the steady state, we have:

$$V^{\xi}(k,d,q^h) = u(Af(k) + (1-\delta)k - k' - d + q^h d) + \beta\left[\xi V^{\xi}(k',d',q^h) + (1-\xi)V^{\det}(k',d')\right]$$

and

$$V(k,d,q^h) = u(Af(k) + (1-\delta)k - k' - d + q^h d) + \beta\left[(1-\psi)V(k',d',q^h) + \psi V^{\det}(k',d')\right]$$

If $V^{\xi}(k',d',q^h) = V(k',d',q^h)$, then, using lemma 12, we can write:

$$V(k,d,q^h) = u(Af(k) + (1-\delta)k - k' - d + q^h d) + \beta\left[\xi V^{\xi}(k',d',q^h) + (1-\xi)V^{\det}(k',d')\right]$$

and thus $V^{\xi}(k,d,q^h) = V(k,d,q^h)$. ■

A.5 Lemma 7

Proof. Subscripts denote time (k_0 is capital at time 0). The superscript ξ for $t > 0$ means that at time $t-1$, when the variable (k or d) has been chosen, $q^* = q^{*\xi}$.

$$V^\xi(k_0, d_0^\xi, q^\xi) = \max_{k_1^\xi, d_1^\xi, \Delta d} \left\{ \begin{array}{c} u(Af(k_o) + (1-\delta)k_o - k_1^\xi - d_0^\xi + q^{*\xi} d_1^\xi) \\ +\beta[(1-\xi)V^{\det}(k_1^\xi, d_1^\xi - \xi\Delta d) + \xi V^\xi(k_1^\xi, d_1^\xi + (1-\xi)\Delta d, q^{*\xi})] \end{array} \right\}$$

again define the function to be maximised:

$$\begin{aligned} f(k_0, d_0^\xi, q^\xi, k_1^\xi, d_1^\xi, \Delta d) &= u(Af(k_o) + (1-\delta)k_o - k_1^\xi - d_0^\xi + q^{*\xi} d_1^\xi) \\ &\quad +\beta[(1-\xi)V^{\det}(k_1^\xi, d_1^\xi - \xi\Delta d) + \xi V^\xi(k_1^\xi, d_1^\xi + (1-\xi)\Delta d, q^{*\xi})] \end{aligned}$$

and take a Taylor approximation with respect to $k_1, d_0, d_1, q, \Delta d$ around $q = \overline{q}$, $d = d_0$, $\Delta d = 0$ that is when $f() = V^{\det}(k_0, d_0)$.

$$\begin{aligned} & V^\xi(k_0, d_0^\xi, q^{*\xi}) = f(k_0, d_0^\xi, q^{*\xi}, k_1^\xi, d_1^\xi, \Delta d) \\ \approx\ & u(Af(k_o) + (1-\delta)k_o - k_1 - d_0 + \overline{q}d_1) + \beta\left[(1-\xi)V^{\det}(k_1, d_1) + \xi V^\xi(k_1, d_1, \overline{q})\right] \\ & + \frac{\partial f(k_0, d_0, \overline{q}, k_1, d_1, 0)}{\partial k_1}(k_1^\xi - k_1) + \frac{\partial f(k_0, d_0, \overline{q}, k_1, d_1, 0)}{\partial d_0}(d_0^\xi - d_0) \\ & + \frac{\partial f(k_0, d_0, \overline{q}, k_1, d_1, 0)}{\partial d_1}(d_1^\xi - d_1) + \frac{\partial f(k_0, d_0, \overline{q}, k_1, d_1, 0)}{\partial q}(q^{*\xi} - \overline{q}) \\ & + \frac{\partial f(k_0, d_0, \overline{q}, k_1, d_1, 0)}{\partial \Delta d}(\Delta d - 0) \\ =\ & V^{\det}(k_0, d_0) + \beta\xi[V^\xi(k_1, d_1, \overline{q}) - V^{\det}(k_1, d_1)] + u'(c_0)[-(d_0^\xi - d_0) + \overline{q}(d_1^\xi - d_1) + d_1(q^{*\xi} - \overline{q})] \\ & + \beta\frac{\partial V^{\det}(k_1, d_1)}{\partial d_1}\left\{(1-\xi)[(d_1^\xi - d_1) - \xi(\Delta d - 0)]\right\} \\ & + \beta\frac{\partial V^\xi(k_1, d_1, \overline{q})}{\partial d_1}\left\{\xi[(d_1^\xi - d_1) + (1-\xi)(\Delta d - 0)]\right\} + \beta\xi\frac{\partial V^\xi(k_1, d_1, \overline{q})}{\partial q}(q^{*\xi} - \overline{q}) \end{aligned}$$

Where I used that $V^{\det}(k_0, d_0) = u(Af(k_o) + (1-\delta)k_o - k_1 - d_0 + \overline{q}d_1) + \beta V^{\det}(k_1, d_1)$.

Note that as a first order Taylor approximation:

$$\begin{aligned} V^\xi(k_1, d_1, q^{*\xi}) &= V^\xi(k_1, d_1, \overline{q}) + \beta\left[(1-\xi)\frac{\partial V^{\det}(k_1, d_1)}{\partial q} + \xi\frac{\partial V^\xi(k_1, d_1, \overline{q})}{\partial q}\right](q^{*\xi} - \overline{q}) \\ &= V^\xi(k_1, d_1, \overline{q}) + \beta\xi\frac{\partial V^\xi(k_1, d_1, \overline{q})}{\partial q}(q^{*\xi} - \overline{q}) \end{aligned}$$

When $q = \overline{q}$, $V^{\det}(k_1, d_1) = V^\xi(k_1, d_1, \overline{q})$ and $\dfrac{\partial V^{\det}(k_1, d_1)}{\partial d_1} = \dfrac{\partial V^\xi(k_1, d_1, \overline{q})}{\partial d_1}$. Using these last equations I get:

$$\begin{aligned} V^\xi(k_0, d_0^\xi, q^{*\xi}) - V^{\det}(k_0, d_0) &= u'(c_0)[-(d_0^\xi - d_0) + \overline{q}(d_1^\xi - d_1) + d_1(q^{*\xi} - \overline{q})] \\ &\quad + \beta\frac{\partial V^\xi(k_1, d_1, \overline{q})}{\partial d_1}(d_1^\xi - d_1) + \beta\xi[V^\xi(k_1, d_1, q^{*\xi}) - V^{\det}(k_1, d_1)]. \end{aligned}$$

Analogously, for $t > 0$ and starting with equal initial debts at both states, we get:

$$\begin{aligned} V^{\xi}(k_t, d_t, q^{*\xi}) - V^{\det}(k_t, d_t) &= u'(c_t)[\overline{q}(d^{\xi}_{t+1} - d_{t+1}) + d_{t+1}(q^{*\xi} - \overline{q})] \\ &\quad + \beta \frac{\partial V^{\xi}(k_{t+1}, d_{t+1}, \overline{q})}{\partial d_{t+1}} (d^{\xi}_{t+1} - d_{t+1}) \\ &\quad + \beta\xi[V^{\xi}(k_{t+1}, d_{t+1}, q^{*\xi}) - V^{\det}(k_{t+1}, d_{t+1})] \end{aligned}$$

Recursive substitution leads to

$$\begin{aligned} & V^{\xi}(k_0, d^{\xi}_0, q^{*\xi}) - V^{\det}(k_0, d_0) = -u'(c_0)(d^{\xi}_0 - d_0) \\ & + \sum_{t=0}^{\infty} (\beta\xi)^t \left[u'(c_t)[\overline{q}(d^{\xi}_{t+1} - d_{t+1}) + d_{t+1}(q^{*\xi} - \overline{q})] + \beta \frac{\partial V^{\det}(k_{t+1}, d_{t+1})}{\partial d_{t+1}} (d^{\xi}_{t+1} - d_{t+1}) \right] \end{aligned}$$

Imposing $V^{\xi}(k_0, d^{\xi}_0, q^{*\xi}) = V^{\det}(k_0, d_0)$ we get the claim. ■

A.6 Proposition 8

Proof. Using Lemma 6 and Equation 7:

$$\begin{aligned} V(\bar{k}, d^h, q^{*h}) &= V(\bar{k}, \bar{d}, \bar{q}) \Rightarrow \\ V^{\xi}(\bar{k}, d^h, q^{*h}) &= V(\bar{k}, \bar{d}, \bar{q}) \Rightarrow \\ \frac{d^h - \bar{d}}{\bar{d}} &= \frac{q^{*h} - \bar{q}}{1 - \beta(1 - 2\psi)} \end{aligned}$$

.Analogously, $V(\bar{k}, d^l, q^{*l}) = V(\bar{k}, \bar{d}, \bar{q}) \Rightarrow$

$$\frac{d^l - \bar{d}}{\bar{d}} = \frac{q^{*l} - \bar{q}}{1 - \beta(1 - 2\psi)}$$

. Using both equations, we get the claim. ■

A.7 Proposition 9

Proof. Consider a Taylor approximation of $V^{AR}(\overline{k}, d^i, q^{*i})$ around the deterministic steady state $(V^{\det}(\overline{k}, \overline{d}, \overline{q}))$. The borrowing constraint is not binding, so choosing the optimal (d', k') instead of $(\overline{k}, \overline{d})$ has only second order effects on the value function V^{AR}. As a first order approximation, we can write:

$$\begin{aligned} V^{AR}(\overline{k}, d^i, q^{*i}) &= E\left(\sum_{t=0}^{\infty} \beta^t u\left(Af(k_t) + (1 - \delta)k_t - k_{t+1} - d^i_t + q^{*i} d^i_{t+1} \right) \right) \\ &\approx \sum_{t=0}^{\infty} \beta^t u(\overline{c}) + E\left(\sum_{t=0}^{\infty} \beta^t u'(\overline{c}) \overline{d}(q^{*i}_t - \overline{q}) \right) - u'(\overline{c})(d^i_0 - \overline{d}) \end{aligned}$$

Where $\overline{c} = Af(\overline{k}) - \delta\overline{k} - (1 - \overline{q})\overline{d}$.

Looking at the middle part of this expression,

$$
\begin{aligned}
& E\left(\sum_{t=0}^{\infty}\beta^t u'(\overline{c})\overline{d}(q_t^{*i}-\overline{q})\right) \\
= & \; u'(\overline{c})\overline{d}\left[(q_0^{*i}-\overline{q})+E\left(\beta\left(\zeta(q_0^{*i}-\overline{q})+\varepsilon_1\right)+\beta^2\left(\zeta\left(\zeta(q_0^{*i}-\overline{q})+\varepsilon_1\right)+\varepsilon_2\right)+...\right)\right] \\
= & \; u'(\overline{c})\overline{d}\left[(q_0^{*i}-\overline{q})\sum_{t=0}^{\infty}(\beta\zeta)^t+E\left(\sum_{t=1}^{\infty}\beta^t\varepsilon_t\frac{1}{1-\beta\zeta}\right)\right] \\
= & \sum_{t=0}^{\infty}(\beta\zeta)^t u'(\overline{c})\overline{d}\left(q_0^{*i}-\overline{q}\right)=u'(\overline{c})\overline{d}\left(q_0^{*i}-\overline{q}\right)\frac{1}{1-\beta\zeta}
\end{aligned}
$$

So:

$$
V^{AR}(k,d^i,q^{*i})\approx\sum_{t=0}^{\infty}\beta^t u(\overline{c})+u'(\overline{c})\overline{d}\left(q_0^{*i}-\overline{q}\right)\frac{1}{1-\beta\zeta}-u'(\overline{c})(d_0^i-\overline{d})
$$

Now imposing $V^{AR}(k,d^1,q^{*1})=V^{AR}(k,d^2,q^{*2})$, we get:

$$
\begin{aligned}
& \sum_{t=0}^{\infty}\beta^t u(\overline{c})+u'(\overline{c})\overline{d}\left(q^{*1}-\overline{q}\right)\frac{1}{1-\beta\zeta}-u'(\overline{c})(d^1-\overline{d}) \\
= & \sum_{t=0}^{\infty}\beta^t u(\overline{c})+u'(\overline{c})\overline{d}\left(q^{*2}-\overline{q}\right)\frac{1}{1-\beta\zeta}-u'(\overline{c})(d^2-\overline{d}) \\
\Rightarrow & \; (d^2-\overline{d})-(d^1-\overline{d})=\frac{1}{1-\beta\zeta}\overline{d}\left(\left(q^{*2}-\overline{q}\right)-\left(q^{*1}-\overline{q}\right)\right) \\
\Rightarrow & \; d^2-d^1=\frac{1}{1-\beta\zeta}\overline{d}(q^{*2}-q^{*1}) \\
\Rightarrow & \; \frac{d^2-d^1}{\overline{d}}=\frac{q^{*2}-q^{*1}}{1-\beta\zeta}
\end{aligned}
$$

■

A.8 Proposition 10

Proof. Suppose $s=h$ and $k',d',\Delta d$ are such that $V_{pay}^h(k',d^h)>V_{def}^h(k',\gamma)$. If the borrowing constraint is binding, the country borrows up to the incentive compatible level (and there is no debt repudiation in equilibrium), so $V_{pay}^l(k',d^l)=V_{def}(k',\gamma)$. By increasing Δd by dD, V_{pay}^l increases and V_{pay}^h decreases. The borrowing constraint is no longer binding, so the country can increase k' by some dk' and d' by dk'/q^* while still respecting the borrowing constraint.

Consumption in the present period is unchanged. $V^l(k',d^l)$ changes by

$$
u'(c_L').\left[dk'.(A^l f'(k')-\delta k'-r^*)+(1-\psi).dD\right]
$$

and $V^h(k',d^h)$ changes by

$$
u'(c_H').\left[dk'.\left(A^h f'(k')-\delta k'-r^*\right)-\psi.dD\right]
$$

where c'_L and c'_H are consumption next period in the low and high state, respectively.

So the change in $V^h(k,d)$ equals β times:

$$dk'\left[\psi u'(c'_L)(A^l f'(k') - \delta k' - r^*) + (1-\psi)u'(c'_H)(A^h f'(k') - \delta k' - r^*)\right] + \psi(1-\psi)dD\left[u'(c'_L) - u'(c'_H)\right]$$

Now, if $A^h - A^l$ is small enough, $(u'(c'_L) - u'(c'_H))$ is small and the change in $V^h(k,d)$ is positive because the marginal productivity of capital is larger than r^*.

Similar expressions can be derived if $V^l_{pay}(k',d^l) > V^l_{def}(k',d^h)$ and/or if $s = l$. ■

A.9 Proposition 11

Proof. Consider that productivity follows the ξ-process, so that $A_0 = A^\xi$ and for $t > 0$:

- if $A_{t-1} = A^\xi$, $\Pr(A = A^\xi) = \xi$ and $\Pr(A = \bar{A}) = 1 - \xi$;
- if $A_{t-1} = \bar{A}$, $A_t = \bar{A}$.

The value function at (k,d) if $A_t = A^\xi$ is:

$$V^\xi(k,d,A^\xi) = \max_{k',d',d'^\xi}\left\{u(c) + \beta\left[(1-\xi)V^{\det}(k',d') + \xi V^\xi(k',d'^\xi,A^\xi)\right]\right\}$$

where $c = A^\xi f(k) + (1-\delta)k - k' - d + q^*(d'(1-\xi) + \xi d'^\xi)$ and $V^{\det}$ is the value function in the model with no uncertainty.

A Taylor approximation of $V^\xi(\overline{k},\overline{d},A^\xi)$ around the deterministic steady state $(V^{\det}(\overline{k},\overline{d}))$ yields:

$$V^\xi_{pay}(\overline{k},\overline{d},A^\xi) = u(\overline{c}) + u'(\overline{c})(A^\xi - \bar{A})f(k) + \beta(1-\xi)V^{\det}(\overline{k},\overline{d}) + \beta\xi V^\xi(\overline{k},\overline{d},A^\xi)$$

where $\overline{c} = \bar{A}f(\overline{k}) - \delta\overline{k} - (1-q^*)\overline{d}$.

So:

$$V^\xi_{pay}(\overline{k},\overline{d},A^\xi) - V^{\det}(\overline{k},\overline{d}) = u'(\overline{c})(A^\xi - \bar{A})f(k) + \beta\xi\left[V^\xi_{pay}(\overline{k},\overline{d},A^\xi) - V^{\det}(\overline{k},\overline{d})\right]$$

which yields:

$$V^\xi_{pay}(\overline{k},\overline{d},A^\xi) = V^{\det}(\overline{k},\overline{d}) + \frac{u'(\overline{c})(A^\xi - \bar{A})f(k)}{1-\beta\xi}$$

If d^ξ is close to $\overline{d}$, $V^\xi_{pay}(\overline{k},d^\xi,A^\xi)$ can be written as:

$$V^\xi_{pay}(\overline{k},d^\xi,A^\xi) = V^{\det}(\overline{k},\overline{d}) + \frac{u'(\overline{c})(A^\xi - \bar{A})f(k)}{1-\beta\xi} - u'(\overline{c})\left(d^\xi - \overline{d}\right) \tag{12}$$

Out of the equilibrium path, the value function conditional on default is:

$$V^\xi_{def}(k,\gamma,A^\xi) = \max_{k'}\left\{u(c) + \beta\left[(1-\xi)V^{\det}_{def}(k',\gamma) + \xi V^\xi_{def}(k',\gamma,A^\xi)\right]\right\}$$

where $c = (1-\gamma)A^\xi f(k) + (1-\delta)k - k'$ and $V^{\det}_{def}$ is the value function in the model with no uncertainty if the country decides to default.

A Taylor approximation of $V^{\xi}_{def}(\overline{k},\gamma,A^{\xi})$ around the deterministic steady state $(V^{\det}_{def}(\overline{k},\gamma))$ yields:

$$V^{\xi}_{def}(\overline{k},\gamma,A^{\xi}) = u(\overline{c}_d) + u'(\overline{c}_d)(A^{\xi}-\bar{A})(1-\gamma)f(k) + \beta(1-\xi)V^{\det}_{def}(\overline{k},\gamma) + \beta\xi V^{\xi}_{def}(\overline{k},\gamma,A^{\xi})$$

where $\overline{c}_d = (1-\gamma)\bar{A}f(\overline{k}) - \delta\overline{k}$, which yields:

$$V^{\xi}_{def}(\overline{k},\gamma,A^{\xi}) = V^{\det}_{def}(\overline{k},\gamma) + \frac{u'(\overline{c}_d)(1-\gamma)(A^{\xi}-\bar{A})f(k)}{1-\beta\xi} \tag{13}$$

Using an argument similar to Lemma 6, if $\xi = 1-2\psi$ and fluctuations of technology are small, $V^h(k,d,A^h) = V^{\xi}(k,d,A^h)$ and an argument similar to Proposition 10 shows that if the country is constrained, in the optimal contract, d^{ξ} and $\overline{d}$ are such that $V^{\xi}_{pay}(k',d^{\xi}) = V^{\xi}_{def}(k',\gamma)$ and $V^{\det}_{pay}(k',\overline{d}) = V^{\det}_{def}(k',\gamma)$. We want to know the values of d^{ξ} and $\overline{d}$ that make such equalities hold when we are close to the deterministic steady state.

Using Equations 12 and 13, $V^{\det}_{pay}(\overline{k},\overline{d}) = V^{\det}_{def}(\overline{k},\gamma)$ imply:

$$\begin{aligned}
V^{\xi}_{pay}(\overline{k},d^{\xi},A^{\xi}) &= V^{\xi}_{def}(\overline{k},\gamma,A^{\xi}) - \frac{u'(\overline{c}_d)(1-\gamma)(A^{\xi}-\bar{A})f(k)}{1-\beta\xi} \\
&\quad + \frac{u'(\overline{c})(A^{\xi}-\bar{A})f(k)}{1-\beta\xi} - u'(\overline{c})\left(d^{\xi}-\overline{d}\right) \\
&= V^{\xi}_{def}(\overline{k},\gamma,A^{\xi}) + \frac{\left[\gamma u'(\overline{c}) + u'(\overline{c}) - u'(\overline{c}_d)\right](A^{\xi}-\bar{A})f(k)}{1-\beta\xi} - u'(\overline{c})\left(d^{\xi}-\overline{d}\right)
\end{aligned}$$

From Proposition 3:

$$V^{\det}_{pay}(\overline{k},\overline{d}) = V^{\det}_{def}(\overline{k},\gamma) \Rightarrow \overline{d} = \frac{\gamma A f(\overline{k})}{1-q^*}$$

which implies $\overline{c}_d = \overline{c}$.

So $V^{\xi}_{pay}(\overline{k},d^{\xi},A^{\xi}) = V^{\xi}_{def}(\overline{k},\gamma,A^{\xi})$ if:

$$\begin{aligned}
\frac{(1-q^*)\overline{d}}{\bar{A}f(\overline{k})}u'(\overline{c})\frac{(A^{\xi}-\bar{A})f(k)}{1-\beta\xi} &= u'(\overline{c})\left(d^{\xi}-\overline{d}\right) \Rightarrow \\
\frac{(1-q^*)\overline{d}}{1-\beta\xi}\frac{(A^{\xi}-\bar{A})}{\bar{A}} &= d^{\xi}-\overline{d}
\end{aligned}$$

Using $\xi = 1-2\psi$ and substituting $\left(A^{\xi},d^{\xi}\right)$ for $\left(A^{h},d^{h}\right)$ and $\left(A^{l},d^{l}\right)$ we get 2 equations that relate debt and productivity at each of the two states. Combining both equations, we get Equation 9. ■

B Numerical examples

B.1 Deterministic model

In the numerical examples of this paper, specific utility and production functional forms are assumed as follows:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma} \quad , \quad f(k) = k^{\alpha}$$

I calibrate the specific parameters as follows: one period corresponds to one year. $A = 1$, $\alpha = 0.36$, $\beta = 1.02^{-1}$, $\sigma = 3$ and $\delta = 0.10$. The price of a riskless bond, q^* , equals β. The output loss in terms of default, $\gamma = 0.01$.

The numerical solution is obtained through value function iteration. The state space is discretised using grids for debt and capital but the planner can choose any point in the grid. From Figure 2, the numbers obtained in this solution are very similar to those from the analytical formulae using the path of y_t given by the numerical example.

Figure 2 also shows the behaviour of capital in this economy, $\gamma = 0.01$, compared to the closed-economy case, $\gamma = 0$, and the full-commitment open-economy case, $\gamma = 1$. Without the possibility of default, the level of capital jumps to its steady state level and the marginal productivity of capital equals r^* in one period. The possibility of default makes convergence slower. Due to the initial capital inflow, the level of capital is higher in this economy than in the closed economy case until they converge. However, the closed economy slowly catches up, as the open economy will be experiencing net capital outflows (trade balance surpluses) during the whole history, as shown at Figure 2. Debt stabilises at 51% of GDP but reaches 60% of GDP at earlier stages.

A usual intuition is that financially open economies should converge faster to their steady states (Barro, Mankiw and Sala-i-Martin, 1995). In contrast, the equilibrium from this model shows that an indebted open economy would take *more* time to converge than a closed economy with the same level of capital. After the initial capital inflow, the country experiences net outflows of resources i.e. a positive trade balance. In addition, a closed economy that opens to capital flows would not converge significantly faster but, on the way towards the steady state, would have higher output than if it remained closed. In order to experience faster convergence, emerging economies need trade deficits and, as Proposition 3 shows, that does not occur in equilibrium.

B.2 Stochastic model

In this section, the accuracy of the analytical approximations used in Section 4.2 is tested using numerical simulations.

I want to obtain the values of $\Delta d/d'$ that make $V^h(k', d^h) = V^l(k', d^l)$ at every state (k, d). The numerical solution is obtained through value function iteration. The state space is discretised using grids for debt and capital but the planner can choose any point in the grid. At the beginning of each iteration, Δd is calculated to make $V^h(k', d^h) = V^l(k', d^l)$.

I use the same stylisation of the 1970's and 1980's to calibrate the model: one period

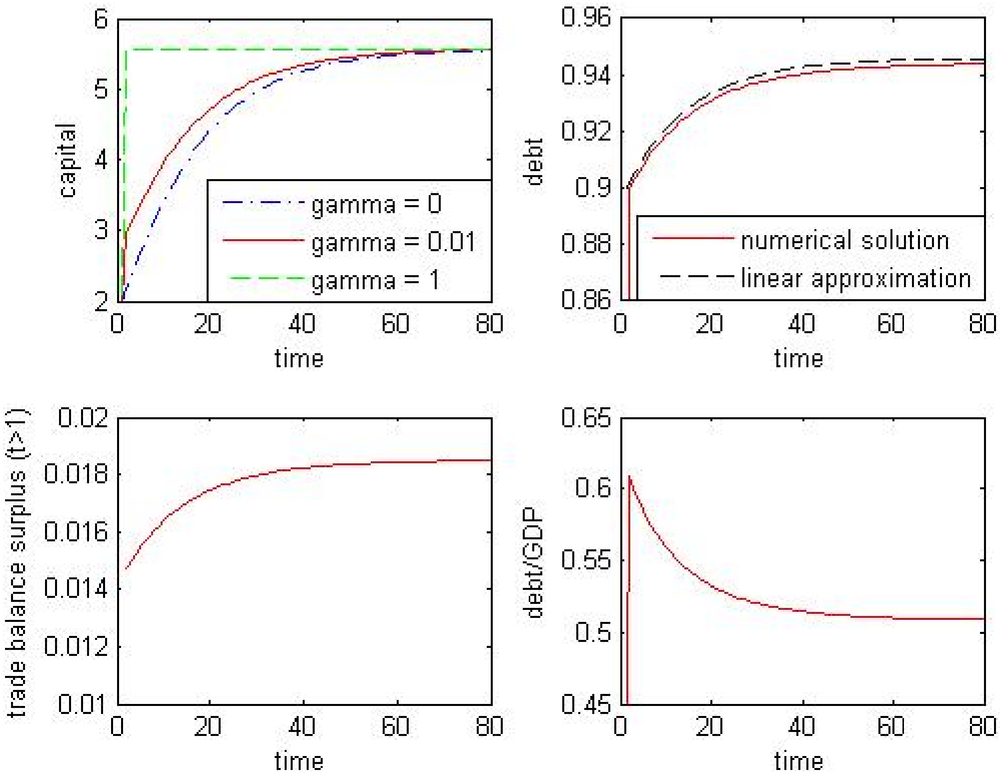

Figure 2: Deterministic model

correspond to one year, $\alpha = 0.36$, $\beta = 1.02^{-1}$, $\gamma = 0.01$, $\sigma = 3$ and $\delta = 0.10$. $A = 1$, and q^* fluctuates around β: $q^{*l} = 1.04^{-1}$ and $q^{*h} = 1.00$. I constrain $k' - k$ to lie in some interval — adjustment costs for capital are zero in that interval and infinity outside it. Figure 3 shows $\Delta d/d'$ as a function of the marginal productivity of capital if the borrowing constraint is binding and the state is high in two situations: (a) $k' - k \in (-0.10k, 0.10k)$ and (b) $k' - k \in (-0.05k, 0.10k)$.

The main results are as follows:

- For $mpk > 0.04 = r^{*l}$, the linear approximation works well: $\Delta d/d'$ is around 0.17 and gradually increasing in mpk. The possibility of borrowing an additional unit in the high state is worth slightly more to countries with high mpk.
- For mpk below $r^{*l} = 4\%$, $\Delta d/d'$ is considerably smaller. For lower values of mpk, when the state shifts to low, interest rates are higher than mpk, so the country sells capital and could even end up buying high-interest-rate foreign bonds. This sounds unrealistic because adjustment costs for capital would prevent such rapid capital

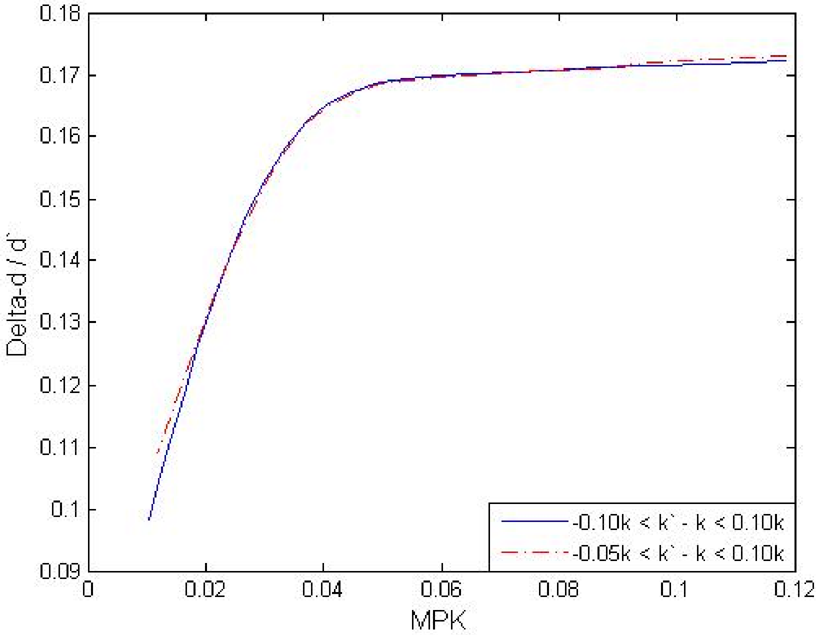

Figure 3: Debt relief

movement. Indeed, the optimal $\Delta d/d'$ for lower values of mpk are sensitive to the assumptions on adjustment costs. Figure 3 shows that when mpk is at its lowest value, $k' = k$, then $\Delta d/d'$ is around 0.10 with adjustment costs given by (a) and around 0.11 in case (b).

The optimality of $V^l(k', d^l) = V^h(k', d^h)$ is conditional on a binding borrowing constraint. If the constraint is not binding, then the condition becomes $\frac{\partial V^h_{pay}(k',d^h)}{\partial d} = \frac{\partial V^l_{pay}(k',d^l)}{\partial d}$ and the optimal contract in this case may prescribe very different values of Δd. The numerical simulations show that for an unconstrained country, with high levels of k, imposing debt contracts that imply $V^l(k', d^l) = V^h(k', d^h)$ at all states may be worse than trading unconditional bonds.

If the country is constrained, potentially there are substantial gains from debt contracts designed to make $V^h(k', d^h) = V^l(k', d^l)$. With adjustment costs given by (a), a country with $k = 2$ that holds its maximum incentive compatible level of debt and has access only to unconditional bonds would be indifferent between receiving a donation equal to 8.3% of its initial capital level and gaining access to those contingent contracts.

References

[1] Aguiar, Mark and Gopinath, Gita, 2006, "Defaultable debt, interest rates and the current account", *Journal of International Economics* 69, 64-83.

[2] Albuquerque, Rui and Hopenhayn, Hugo, 2004, "Optimal lending contracts and firm dynamics", *Review of Economic Studies* 71, 285-315.

[3] Alfaro, Laura and Kanczuk, Fabio, 2005, "Sovereign debt as a contingent claim: a quantitative approach", *Journal of International Economics* 65, 297-314.

[4] Atkeson, Andrew, 1991, "International lending with moral hazard and risk of repudiation", *Econometrica* 59, 1069-1089.

[5] Arellano, Cristina, 2008, "Default risk and income fluctuations in emerging economies", *American Economic Review*, forthcoming.

[6] Barro, Robert; Mankiw, N. Gregory and Sala-i-Martin, Xavier, 1995, "Capital mobility in neoclassical models of growth", *American Economic Review* 85, 103-115.

[7] Bolton, Patrick and Jeanne, Olivier, 2007, "Structuring and restructuring sovereign debt: the role of a bankruptcy regime", *Journal of Political Economy* 115, 901-924.

[8] Borensztein, Eduardo and Mauro, Paulo, 2004, "The case for GDP-indexed bonds", *Economic Policy* 2004, 165-216.

[9] Bulow; Jeremy and Rogoff, Kenneth, 1989, "A constant recontracting model of sovereign debt", *Journal of Political Economy* 97, 155-178.

[10] Calvo, Guillermo and Kaminsky, Graciela, 1991, "Debt relief and debt rescheduling", *Journal of Development Economics* 36, 5-36.

[11] Calvo, Guillermo; Leiderman, Leonardo and Reinhart, Carmen, 1996, "Inflows of Capital to Developing Countries in the 1990s", *Journal of Economic Perspectives* 10, n.2, 123-139.

[12] Caselli, Francesco and Feyrer, James, 2007, "The marginal product of capital", *Quarterly Journal of Economics* 122, 535-568.

[13] Cline, William, 1995, "International debt reexamined", Institute for International Economics.

[14] Cohen, Daniel and Sachs, Jeffrey, 1986, "Growth and external debt under risk of debt repudiation", *European Economic Review* 1986, 529-560.

[15] Diaz-Alejandro, Carlos, 1984, "Latin American debt: I don't think we are in Kansas anymore", *Brookings Papers on Economic Activity* 2:1984, 335-389.

[16] Dotsey, Michael; Lantz, Carl and Scholl, Brian, 2003, "The behavior of the real rate of interest", *Journal of Money, Credit and Banking* 35, 91-110.

[17] Eaton, Jonathan and Gersovitz, Mark, 1981, "Debt with potential repudiation: theoretical and empirical analysis", *Review of Economic Studies* 48, 289-309.

[18] English, William, 1996, "Understanding the costs of sovereign default: American state debts in the 1840's", *American Economic Review* 86, 259-275.

[19] Fernandez, Raquel and Rosenthal, Robert, 1990, "Strategic models of sovereign debt renegotiations", *Review of Economic Studies* 57, 331-349.

[20] Fischer, S., 1999, "On the need for an international lender of last resort", *Journal of Economic Perspectives* 13:4, 85-104.

[21] Foley-Fisher, Nathan, 2007, "Optimal debt and default in an economy with a natural resource endowment", mimeo.

[22] Grossman, Herschel and Van Huyck, John, 1988, "Sovereign debt as a contingent claim: excusable default, repudiation, and reputation", *American Economic Review* 78, 1088-1097.

[23] Jeske, Karsten, 2006, "Private international debt with risk of repudiation", *Journal of Political Economy* 114, 576-593.

[24] Kehoe, Patrick and Perri, Fabrizio, 2004, "Competitive equilibria with limited enforcement", *Journal of Economic Theory* 119, 184-206.

[25] Kletzer, Kenneth; Newbery, David and Wright, Brian, 1992, "Smoothing primary exporters' price risk: bonds, futures, options and insurance", *Oxford Economic Papers* 44, 641-671.

[26] Kovrijnykh, Natalia and Szentes, Balazs, 2007, "Equilibrium default cycles", *Journal of Political Economy* 115, 403-446.

[27] Krueger, Anne, 2002, "A New Approach to Sovereign Debt Restructuring", IMF.

[28] Lucas, Robert, 1990, "Why doesn't capital flow flow rich to poor countries", *American Economic Review* 80, 92-96.

[29] Marcet, Albert and Marimon, Ramon, 1992, "Communication, commitment and growth", *Journal of Economic Theory* 58, 219-249.

[30] Neumeyer, Pablo Andres and Perri, Fabrizio, 2005, "Business cycles in emerging economies: the role of interest rates", *Journal of Monetary Economics* 52, 345-380.

[31] Reinhart, Carmen and Rogoff, Kenneth, 2004, "Serial Default And The 'Paradox' Of Rich To Poor Capital Flows", *American Economic Review* 94, 52-58.

[32] Reinhart, Carmen; Rogoff, Kenneth and Savastano, Miguel, 2003, "Debt Intolerance", *Brookings Papers on Economic Activity* 1:2003, 1-74.

[33] Sturzenegger, Federico and Zettelmeyer, Jeromin, 2006, "Debt defaults and lessons from a decade of crises", MIT Press.

[34] Tomz, Michael, 2007, "Reputation and international cooperation", Princeton University Press,.

[35] Tomz, Michael and Wright, Mark, 2007, "Do countries default in 'bad times'?", *Journal of the European Economic Association* 5, 352-60.

[36] Uribe, Martin and Yue, Vivian, 2006, "Country spreads and emerging countries: who drives whom?", *Journal of International Economics* 69, 6-36.

[37] Yue, Vivian, 2005, "Sovereign default and debt renegotiation", mimeo.

[38] Zame, William, 1993, "Efficiency and the role of default when security markets are incomplete", *American Economic Review* 83, 1142-1164.