Regulation Effects on Investment Decisions in Two-Sided Market Industries: The Net Neutrality Debate

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Abstract

This paper studies, using a two-sided market framework, the impact of regulation on platform's pricing scheme, on investment decisions, on network users' decision to join the network, and on welfare. We take a monopoly platform that serves a continuum of vertically differentiated buyers and sellers that, after deciding to enter, will start to trade. The profit-maximizing platform can only charge a different entry fee to all network users. If profit-maximizing platform cannot charge sellers, i.e. Net Neutrality regulation, will be more network users, more investment, and welfare is higher. If profit-maximizing platform cannot charge buyers there will be more investment than without the regulation. If on top of not charging buyers, network effects make sellers' trade surplus and buyers' trade surplus close, then less network users will be excluded, and welfare will be higher with the regulation than with the profit-maximizing platform. If network effects make sellers' surplus high enough compared to buyers' surplus, welfare is higher with the profit-maximizing platform. Finally, we show that welfare when a profit-maximizing platform cannot charge sellers, is higher than when he cannot charge buyers.

Keywords: investment incentives, two-sided markets, regulation, net neutrality, pricing scheme

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1 Introduction

Consider a platform who owns a network, and that actively discriminate among network users. The platform chooses network's quality, and the higher the quality, the higher network users trade will be. This paper studies the impact of limiting the platform's discriminatory policies on network users. Our interest in such markets (e.g. the telecommunications, the postal, the energy, the gas, the rail, the water) is multi-folded. A first reason is that network-based markets have become essential for many country's economic growth, and in particular for the service industry. Another reason is that market participants must coordinate in some way to both expand and upgrade the network infrastructure. And third, is usual to find that downstream firms, in each of these markets, are engaged in R&D activities.

To simplify the exposition, this paper focus on the telecommunication industry, and in particular on the broadband market. More precisely, we study how limiting platform's discrimination policies toward network users (e.g. content providers and end-users) will affect the optimal pricing scheme and investment level, and the mass of network users willing to join the network. In addition, we make welfare comparisons to rank the several regulation scenarios. To further motivate this paper, results will prove to be useful for a well known debate in the broadband market called the "Network Neutrality Debate", or in short the "NN debate".

The NN debate is as follows. Platforms (e.g. Verizon or AT&T in USA, France Telecom in France, or Telefónica in Spain) started providing broadband access to both content providers and end-users without discriminating because the cost of doing so was very high; on top of this, in the US the FCC did not allowed broadband owners to discriminate. As a consequence, competition in the content providers' side lead to well renowned projects (e.g. Google, Amazon, eBay, among others) and internet's value added was maximized. Around 2005 in the US, the FCC changed the internet transmission category from "telecommunication services" to "information services". Also by that time platforms had upgraded their technology and were able to either identify the sender and the receiver of each information package, and/or to classify the content of each information package. As a consequence, by 2005 platforms in the US could discriminate, and also were legally allowed, among broadband users. Finally, mainstream NN debate arguments are the following. On one hand, platforms argue they need to be allowed to efficiently use and price their network because they are the ones making the investment; not allowing them to do so will imply lower investment levels. On the other hand, NN regulation advocates say platform discrimination will distort their innovation incentives, and ultimately internet value added will be reduced.

Rigorous economic analysis of Net Neutrality Debate is rather new[1]. Within the emerging literature differences arise mainly in two aspects: in what NN Regulation imply, and in what Internet's Valued Added (IVA) mean. Moreover, although one could argue telecommunication industry is not one-sided[2], only few attempts try to fully use a two-sided framework. This paper contributes on all three aspects.

Hermalin & Katz (2007) consider a profit-maximizing platform that helps connect a continuum of vertically differentiated content providers, and end users. Without any restriction the platform will offer a continuum of vertically differentiated services to content providers; NN regulation will force platforms to offer the same quality service to all content providers. The authors provide sufficient and necessary conditions such that a profit-maximizing platform excludes strictly more content providers than a profit-maximizing platform with the single-quality restriction. On the other hand, welfare analysis is ambiguous because under the single-quality restriction, while new

[1] Literature in other fields, e.g. law and policy making, is thicker. See Ganley & Allgrove (2006), Felten (2006), Han, Litan & Singer (2007), Han & Wallsten (2006), and Baumol et al. (2007).

[2] See Rochet & Tirole (2006), and Jullien & Calliaud (2002) for the basics on two-sided market literature.

middle market content providers will receive a higher quality service, top content providers service will be reduced. The main difference of our model is that end users are also vertically differentiated, and that the profit-maximizing platform can invest; then our approach generalizes the one of Hermalin & Katz (2007).

Economides & Tåg (2007), using a two-sided market framework, emphasize more on the platform's pricing strategy, and NN regulation is understood as setting a zero access fee to content providers. They consider a profit-maximizing platform that connect a continuum of differentiated content providers and end-users. The authors show a profit-maximizing platform will set a positive access fee to content providers, then more exclusion will arise in this market. They also find end-users access fee will be reduced, then less exclusion will arise in the other market. Welfare analysis is unambiguous and favor imposing NN regulation. The main difference from our model is that, in addition to studying the NN regulation effects on content providers' and end users' network exclusion, and on the pricing scheme, we study the impact on the platform's investment incentives.

Recently a couple of papers have focused their attention on NN regulation effects on platform investment incentives. Choi & Kim (2008) is the first paper that gets to NN debate's core; that is, IVA is jointly determined by platform's and content providers' investments. The authors sketch a model, using the queuing theory, in which one profit-maximizing platform help end-users and two content providers to meet; the platform is able to offer a "fast lane" service to either both content providers. Short term results compare situations were no "fast lane" services can be offered, to a situation were such services can be offered to only one content provider; Choi & Kim argue content providers are worst, platforms are not necessarily in worst shape, and welfare comparison is ambiguous. Long term results focus on regulation impact on platform's investment incentives; the authors cannot discard NN regulation will boost platform's investment[3]. Our paper differs in three aspects. We fully embrace the two-sidedness nature of the industry, we let network users decide to join the network, and we let network users to trade.

Cheng et al. (2006) also study NN regulation impact on platform's investment incentives; here regulation force platforms to offer content providers the same service. The authors use a game theoretic model with one platform, two content providers fully horizontally differentiated, and a set of consumers indexed by a preference parameter on the unit interval. They study how each player will be affected by imposing NN regulation, and also how platform's decision to upgrade networks capacity will be affected. Results are that content providers are worse off, platforms are naturally better off, and end-users effect is ambiguous; they show welfare is unambiguously higher only if one, not both, content provider choose to pay to upgrade their service; and finally, they obtain the platform will invest more in capacity because this imply network congestion levels will decrease. Our paper have two main differences with this approach. First, while we fully use a two-sided market approach, Cheng et al. (2006) assume end-users are always fully served; second, ad-hoc assumptions about content provider parameters are avoided.

As we showed, not even in theoretically based approaches to the NN debate one finds consensus about what NN regulation is. This paper considers a simple, but yet comprehensive, view where NN regulation imply that the profit-maximizing platform is forced to charge content providers a zero access fee; we will call this *level zero NN regulation*. To keep things simple, we also assume platforms can only charge an access fee to all network users. Exploiting the two-sidedness nature of platforms we study other regulatory scenario; we will call them *non-standard NN regulation*. In the first scenario, NN regulation force the profit-maximizing platform to charge end-users a zero

[3]NN regulation impact on content providers investment are also analyzed. As expected, assuming content provider's investment is relationship specific, a standard hold up problem arise and content providers have less incentives to invest as platforms can appropriate some of their rents. The authors argue that only in the presence of commitment mechanism platforms will have incentives to not appropriate rents, and consequently content providers investment incentives will not be distorted.

access fee. In a last scenario, the profit-maximizing platform is replaced with welfare-maximizing platform that is publicly funded and charges no access fee[4].

Our paper contribute in several aspects. Until very recently[5] studies about NN debate either assumed the only measure of internet value added was the mass of content providers joining the network, or was platforms investment. We argue that internet value added is coming from both content provider's and platform's investments. In the model we will distinguish NN regulation short term effects, from those in the long term; while the former are captured by changes in platform's pricing decisions, and in content providers and end-users decision to join the network, the long term effects are captured by the effects on content providers and platforms investment incentives. In this paper we only allow platforms to make investments.

Our modeling strategy is also a novelty. Although is natural to think that the broadband market is not one-sided, our model is the first one that study the short term, and the long term effects of NN regulation using a full two-sided market framework. Several implications follow. With this model we are able to see network effect's role in NN debate. But more importantly, we point out that a regulator may find ways, different from only forcing platforms to not distort content provider's behavior, to guarantee internet value added distortion is minimized. For example, it might be the case lower distortions arise if platforms are allowed to *distort enough* both content providers and end-users behavior. Finally, a limitation of our model is that we assume content providers are not big enough to influence the mass of end-users that will decide to join the network.

Results tend to support those in favor of the regulation. On one hand, if the profit-maximizing platform is forced to charge content providers a zero access fee, then less content providers and end-users will be excluded, more investment will be observed, and welfare will be higher than with the profit-maximizing platform. On the other hand, if the regulatory agency cannot implement the previous regulation, but can force profit-maximizing platforms to charge end-users a zero access fee, then more investment will be observed than with profit-maximizing platform. If on top, network effects make content providers' trade surplus and end-user's trade surplus close enough, then the profit-maximizing platform forced to charge end-users a zero access fee will also exclude less network users, and naturally, welfare will be higher than with the profit-maximizing platform. But if network effects make content providers trade surplus high enough compared to end-users trade surplus, then the profit-maximizing platform will exclude less end-users, will exclude more content providers, and welfare will be higher than with the profit-maximizing platform force to charge end-users no access fee.

Additional insights arise by comparing the situation were content providers pay no access fee, with the one were end-users pay no access fee. We found there is a trade-off between network users exclusion and investment. In particular, less content providers and end-users will be excluded if the profit-maximizing platform is forced to charge content providers no entry fee, but more investment will be observed if the profit-maximizing platform is forced to charge end-users no entry fee. Moreover, we find welfare is higher if profit-maximizing platform is forced to charge content providers a zero access fee.

Finally, in the appendix we studied the possibility a profit-maximizing platform is replaced by a welfare-maximizing platform that is publicly funded and that charges no entry fee. We find that, because content providers have some market power, the free platform will exclude less content providers, but will exclude more end-users than the profit-maximizing platform. If on top, network effects make content providers trade surplus high enough compared to end-users' trade surplus, then the free platform will invest more than the profit-maximizing platform to try and attract content providers to the network, but the cost of doing so will be significant and welfare will be

[4] To make exposition straightforward, this case is in the appendix.

[5] See Choi & Kim (2008).

lower compared to the profit-maximizing platform. On the other hand, if network effects make content providers trade surplus and end-users trade surplus close enough, then the free platform will be less willing to invest, in order to attract content providers, than the profit-maximizing platform. The free platform will turn out to invest less, and welfare will be higher compared to the profit-maximizing platform.

The remainder of the paper is as follows. Section two describes the model. In section three we study three scenarios that will help us analyze the NN debate; in particular, results for level zero NN debate will be shown, then results for non-standard NN debate . Last section concludes.

2 Model

2.1 Environment

Consider three players, the platform, the buyers and the sellers. Buyers and sellers have common interests, i.e. they want to do business, but they must do it through the platform. In addition, the platform is aware of this when choosing a investment level that will benefit the rest of the players. In particular, investment can be understood as quality improvements to the physical network that jointly reduces sellers' cost and boost buyers' usage experience. Investment main characteristic is that platform will receive no direct benefits from investing, all of them will indirectly come from improvements in buyers' and sellers' network use. Finally, the platform charge a once time only access fee, not necessarily the same, to buyers and sellers.

The model focuses on cases where platform decisions affect buyers and sellers trade surplus; indeed, higher levels of investment will make sellers more efficient and will increase buyers utility. Several industries fit this framework. The telecommunication industry represent a natural example, and the easiest one is the internet. Here, owners of the physical network are the platforms, and the network users are the content providers (sellers) and the end-users (buyers). Other examples are the electricity industry, and the gas industry. As before, owners of the physical network are the platforms, and the network users are the electricity generators (sellers) and the real economy (buyers). The only difference in the gas industry is that sellers are now the gas extraction companies.

Network users, i.e. buyers and sellers, have to make two decisions: to join the network, and if they decide to enter, the level of trade. In the first decision, buyers and sellers must individually decide to join or not the network, they will enter if the expected trade surplus is higher than the entry cost. In the second decision, network users that enter will do business. To simplify, the platform will choose his investment level before knowing who joins the network.

For convenience of exposition we will focus on the internet industry. Then, sellers will be named as content providers, and buyers as end-users.

Buyers. There is a continuum of consumers with two sources of private information, one for joining the network (η), and another for using the network (θ_b). Two reasons motivate this assumption: one theoretical, and another rather technical. In the first place, it makes no sense assuming end-users are homogeneous when deciding to join or not the network; for the internet example, the learning curve for someone in its fifties is very much steeper than the one from a teenager[6]. On technical grounds, by using one source of private information we are combining two things, the valuation of using the network, and the cost of learning how to use the network. Thus, by using

[6]For the electricity industry, as well for the gas industry, households or factories have the sunk cost of adapting their machines to the new source of energy.

two sources of private information we are isolating these effect, and we are making analysis simpler. To simplify, let $\theta_b \sim U_{[\underline{\theta}_b, \overline{\theta}_b]}$ and $\eta \sim U_{[0,1]}$, and $2\underline{\theta}_b > \overline{\theta}_b$

On top, we will concentrate on the case platform and content providers cannot obtain information of θ_b by observing end-users entry decision. Then, end-users' entry decision will not be affected by the potential signalling of their usage valuation because η is uninformative about θ_b. To achieve this, on one hand we will assume both sources of private information are independent; on the other hand, end-users will discover their valuation use type θ_b after joining the network.

Each end-user's consumption level r from a particular θ_s-type content provider, who sets a nonlinear price $\Gamma(r, \theta_s)$, is the solution to,

$$\max_r \left\{ \theta_b (\theta_s Q)^\alpha \left(\frac{r^{1-\alpha}}{1-\alpha} \right) - \Gamma(r, \theta_s) \right\} \tag{1}$$

where Q is platform's investment level, and r is the consumption level from θ_s-type content provider. Finally, notice the end-user already knows both content provider's private information (i.e. $\theta_s \sim U_{[\underline{\theta}_b, \bar{\theta}_b]}$), and his own valuation (i.e. θ_b).

End-users derive utility from consumption levels $r(\theta_s)$ from content providers that join the network (e.g. $\theta_s \geq \hat{\theta}_s$, where $\hat{\theta}_s$ is the threshold type), and from the rest of goods he consumes (e.g. y)[7],

$$\begin{aligned} \mathbf{U}^b(\theta_b) &= \int_{\hat{\theta}_s}^{\bar{\theta}_s} \theta_b (\theta_s Q)^\alpha \left(\frac{r(\theta_s)^{1-\alpha}}{1-\alpha} \right) g_{\theta_s}(\theta_s) d\theta_s + y \\ &= E_{\theta_s}(N_s \mathfrak{U}^b(\theta_b) \mid \theta_s \geq \hat{\theta}_s) + y \end{aligned}$$

where $\mathfrak{U}^b(\theta_b)$ is the trade surplus from making business with one content provider that join the network, and $g_{\theta_s}(\theta_s)$ is θ_s's pdf.

Notice platform's investment level directly affects end-users' utility. In particular, higher levels of investment increases the marginal utility of each consumption unit $r(\theta_s)$. For example, say $r(\theta_s)$ is the number of downloaded videos, then by investing more end-users will be able to download more videos in the same amount of time. In the case of the electricity industry, say $r(\theta_s)$ be the bimonthly demand of kW from a household, then higher investment levels allow energy supply to be of a higher quality.

End-users' decision to join the network consider the expected trade surplus, and the costs of joining in. In particular, the expected trade surplus acknowledges the platform will exclude some content providers based on their type (θ_s), and the costs include the access fee (t_b) he pays to the platform, and the idiosyncratic cost (η). A end-user will join in if the expected trade surplus exceed the entry costs, that is

$$\mathbb{E}_{\theta_b, \theta_s}[N_s \mathfrak{U}^b(\theta_b) - t_b(\theta_b) - \eta \mid \theta_s \geq \hat{\theta}_s] \geq 0 \tag{2}$$

where θ_s is the content provider's private information.

Assume all private information random variables (e.g. θ_s, θ_b, η) are fully independent. On one hand, before we assumed end-users' random variables θ_b and η are fully independent; but on the

[7]This functional form parallels the one used by Hermalin & Katz (2007), i.e.

$$\mathbf{U}^b = \int_{supp(\theta_s)} \int_0^{r(\theta_s)} u\left(\frac{z}{\theta_s Q}\right) f(\theta_s) dz \theta_s + y$$

where $u(\cdot)$ is the marginal utility function. While in our case we have a continuum of vertically differentiated content providers and end-users, Hermalin & Katz (2007) have a continuum of vertically differentiated content providers and one household.

other hand, end-users' private information parameters are fully independent from content provider's private information parameter θ_s. This assumption, given that end-users' learn θ_b after joining the network, is rather important because eliminates the possibility content providers learn end-users' private information after observing his entry decision. Equivalently this assumption avoids having content providers with too much market power, and capable to influence the mass of end-users joining the network.

Sellers. There is a continuum of content provider's with one source of private information, i.e. $\theta_s \sim U_{[\underline{\theta}_s, \bar{\theta}_s]}$, that captures the fact that not every content provider is equally valuable for all end-user; content providers privately learn their type before choosing to enter the network. This formulation is saying that content providers privately know the quality of their product before joining the network. Finally, in this model each content provider behaves as a monopolist, but they are small enough to not affect the mass of end-users joining the network.

Every content provider designs the optimal non-linear tariff for all end-users. Given that all private information random variables are fully independent, and also because sellers reveal their type when they choose to join the network, there is no loss of generality in using the Revelation Principle[8]. The optimal non-linear for every content providers is the solution of,

$$\begin{aligned} & \max_{\{\Gamma(\theta_b), r(\theta_b)\}} && N_b \mathbb{E}_{\theta_b} \left[\Gamma(\theta_b) - \frac{C_s r(\theta_b)}{Q} \right] \\ & (IC) && \theta_b = arg \max_{\tilde{\theta}_b} \left[\theta_b \int_0^{r(\tilde{\theta}_b)} \left(\frac{z}{\theta_s Q} \right)^{-\alpha} dz - \Gamma(\tilde{\theta}_b) \right] \\ & (RC) && \theta_b \int_0^{r(\theta_b)} \left(\frac{z}{\theta_s Q} \right)^{-\alpha} dz - \Gamma(\theta_b) \geq 0 \forall \theta_b \end{aligned} \tag{3}$$

where N_b, which is taken as a parameter, is the mass of end-users joining the network, $r(\theta_b)$ is consumption level of end-user θ_b, $\Gamma(\theta_b)$ is the associated nonlinear tariff scheme of content provider θ_s, Q is platform's investment level, and C_s is same marginal cost to all S's. Incentive compatibility constraints (IC), and rationality constraints (RC) have the usual interpretation. Notice private information parameter (e.g. θ_s) is not in the cost function, and will come from end-users utility function (equation (1)).

Should be remarked incentive compatibility constraint (IC) and rationality constraint (RC) are applied to all θ_b-type end-users. This comes from assuming all private information random variables are independent, and from assuming end-users learn their type after joining the network. We are abstracting from two situations. First, from when content providers acknowledge platforms exclude end-users based on type (e.g. θ_b), and consequently include in (IC) the probability a particular end-user will not be excluded. And second, from when content providers choose to target only a specific segment of end-user, and consequently restrict (RC) to only those θ_b in that segment.

Each content provider's decision to join the network must compare the trade surplus from making business with each end-users ($\mathfrak{U}^s(\theta_s)$), the number of end-users joining the network N_b, with the transfer (t_s) he makes to the platform to join the network. Any content provider will join in if the expected revenues exceed the entry fee, that is

$$N_b \mathbb{E}_{\theta_b} \left[\Gamma - \frac{C_s r}{Q} \right] - t_s = N_b \mathfrak{U}^s(\theta_s) - t_s \geq 0$$

[8]See Bolton & Dewatripont (2003).

Platform. When platform meet end-users and content providers he is unaware of their private information. That is, he ignores if content providers are good or not (i.e. θ_s), and end-users fixed entry cost to join the network (i.e. η); on top of this, platform ignores too if end-users will enjoy the internet (i.e. θ_b). Entry fees do not depend on private information because of the independence assumption, and because platform invest before end-users learn θ_b. Consequently, platform's profit function is

$$\Pi = t_b N_b + t_s N_s - \phi Q \tag{4}$$

where $\phi > 0$ is the marginal cost of investment, t_i for $i \in \{s, b\}$ is the entry fee for i, and N_i for $i \in \{s, b\}$ is the mass of i's that choose to enter the network.

Timing. This model assume platforms invest before knowing the mass of end-users and content providers that will join the network. Once end-users and content providers decide to join in or not, they will use the network as an environment to do business. Timing is as follows: (1) end-users privately learn η, content providers privately learn θ_S; (2) the platform create the mechanism and choose investment, that is, he choose (t_b, t_s, Q); (3) content providers and end-users decide to join or not; (4) end-users learn θ_s, and privately learns θ_b; (5) content providers create the mechanism, that is, they creates the non-linear tariff $\Gamma(\theta_b)$; (6) end-users choose their consumption level (i.e., r); (7) Ends.

2.2 Network Users Surplus & Demands Platform's Face

End-users and Content Providers Relationship. We assume each content provider builds an optimal nonlinear tariff scheme for two reasons. First, as we are interested in the regulation effects on the optimal end-users and content providers exclusion levels, and on the optimal investment levels, forcing content providers to use linear tariff or a two part tariff schemes could distort the results by arguing content providers are no behaving optimally. Second, after computing trade surpluses rather than obtaining uneasy to interpret expressions, we obtain nice expression to work with. The optimization program for solving the optimal nonlinear tariff can be found in equation (3).

Given this model parametrization, the expected trade surpluses are determined by three factors: the platform investment level, the distribution function of θ_b, and the distribution function of θ_s. The following lemma summarizes these results.

Lemma 1. *Given the optimization program for a θ_s-type content providers in (3), the expected trade surplus for content providers and end-users are,*

$$\mathbb{E}_{\theta_s,\theta_b}(N_s \mathfrak{U}^b(\theta_b) \mid \ \theta_s \geq \hat{\theta}_s) = N_s \bar{K}_b(G_{\theta_b}, C_s, \alpha) \mathbb{E}_{\theta_s}(\theta_s \mid \theta_s \geq \hat{\theta}_s) Q^{\frac{1}{\alpha}} \tag{5}$$

$$N_b \mathfrak{U}^s(\theta_s) = N_b \bar{K}_s(G_{\theta_b}, C_s, \alpha) \theta_s Q^{\frac{1}{\alpha}} \tag{6}$$

and,

$$\bar{K}_s(\cdot) \geq \bar{K}_b(\cdot) \tag{7}$$

To simplify notation, from hereon we will refer $\bar{K}_i(G_{\theta_b}, C_s, \alpha)$ for $i \in \{b, s\}$, with $\bar{K}_i$ for $i \in \{b, s\}$. Network effects are captured by $\bar{K}_i$, for $i \in \{s, b\}$. In particular, while end-users network effect on content providers is captured by $\bar{K}_s$, content provider's network effect on end-users is captured by $\bar{K}_b$.

Let us do some minor comments. Observing equations (5) and (6), higher levels of investment are beneficial for both end-users and content providers; this is natural as investment (e.g. a higher network quality) simultaneously increases end-users valuation and decreases content providers marginal cost. A similar effect is produced by the expected content provider's type $\mathbb{E}_{\theta_s}(\theta_s)$.

More relevant is the role of the mass of consumers, i.e. N_b. Notice from equation (6) that all content providers assign the same value for each end-user, i.e. $\bar{K}_s\theta_s Q^{\frac{1}{\alpha}}$. This is explained by two reasons. First, assuming content providers cannot affect the mass of end-users joining then network force them to take as given the mass of end-users joining the network; here we are ruling out having big players such as Google or Yahoo[9]. Second, the independence of all private information random variables, and the fact end-users learn θ_b after joining the network, avoid the possibility η could affect the distribution of θ_b.

The ratio of the network effects, i.e. $\frac{\bar{K}_b}{\bar{K}_s}$, represent a measure of how trade surplus is divided between content providers and end-users. Equation (7) show content provider's trade surplus share is bigger than the end-users, this comes from assuming each content provider being a monopoly. As a consequence, if the ratio is close to one, then trade surplus is evenly shared, but if ratio is close to zero, then trade surplus is mostly appropriated by content providers.

Platform's Demands. Under this environment content providers and end-users entry decision are interrelated. The number of end-users joining in (N_b) will equal the mass of end-users whose expected trade surplus outweigh the entry fee (t_b) and the idiosyncratic entry fee (η). On the other hand, the number of content providers joining in (N_s) will equal the mass of content providers whose type (θ_s) is above a threshold value $\hat{\theta}_s$, i.e. $N_s = Prob(\theta_s \geq \hat{\theta}_s) = \frac{\overline{\theta}_s - \hat{\theta}_s}{\Delta\theta_s}$ where $\Delta\theta_s = \overline{\theta}_s - \underline{\theta}_s$. This threshold type content provider characterizes from having a zero rent. The following system of equations will define (N_b, N_s),

$$N_b = Prob\left\{N_s\mathbb{E}_{\theta_s,\theta_b}\left[\mathfrak{U}^b(\theta_b) \mid \theta_s > \hat{\theta}_s\right] - t_b \geq \eta\right\} \quad (8)$$

$$0 = \mathfrak{U}^s(\hat{\theta}_s) - t_s \quad (9)$$

Some remarks. Notice equation (8) show end-users internalize the fact not every content-provider will join the network. Equation (9) defines the threshold type $\hat{\theta}_s$ as the content provider that receives zero rent.

Existence and Uniqueness . Although this point is rather technical, we want to stress that whenever an equilibrium exist it is also unique. Two scenarios will be studied: a corner solution, and the interior solution. In the first one, see Figure (1), a corner solution is obtained because all content providers join the network, e.g. $N_s = 1$; indeed, the equilibrium is situated at point A on Figure (1).

The other case is an interior solution, e.g. $N_s < 1$. Here, on top of guaranteeing existence, we might face one or two equilibrium, see Figure (2). If more than one equilibrium arises, one will be unstable, and the other will be stable. Indeed, in Figure (2) the two equilibria are situated at points B and D, and while B is unstable, point D is stable. In addition, moving the hyperbole to the north-west corner one might keep only the stable equilibrium.

Lemma 2. *If an equilibrium exist its unique. Moreover, equilibrium is a corner solution if* $\Omega(t_b) > t_s$*, and equilibrium is interior if* $\bar{\Upsilon}(t_b) \geq t_s > \underline{\Upsilon}(t_b)$*. See appendix for details.*

[9]A natural extension should consider content providers that do internalize their impact on the mass of consumers. For example, abstracting from big players like Google, the market for software developers heavily use blogs or non-profit consumer's associations (i.e. see www.ohloh.net); then, small software developers can have a big impact on the mass of consumers through these associations. For the electricity or gas industries, this is clearer because power generator usually have some market power.

3 Results

We propose studying four scenarios: (1) profit-maximizing platform, (2) profit-maximizing platform forced to charge content providers a zero access fee (i.e. $t_s = 0$), (3) profit-maximizing platform forced to charge end-users a zero access fee (i.e. $t_b = 0$), and (4) a free publicly funded platform[10]. Although in all scenarios the platform chooses the optimal investment level, this is not true for the entry fees. In the first scenario a profit-maximizing platform chooses the optimal entry fees. In the next scenario a profit-maximizing platform is forced to only charge end-users. In the third scenario a profit-maximizing platform is now forced to only charge content providers. Finally, in the last scenario we replace a profit-maximizing platform with a publicly funded platform that maximizes content providers and end-users expected trade surplus.

We pursue several objectives. In the first set of results we want to give a theoretically founded answer to the NN debate; we will call it Level Zero NN Debate. That is, assuming only platforms invest and each content provider is understood as an innovation, we will compare the decisions from a profit-maximizing platform (e.g. scenario (1)) with the ones from a profit-maximizing platform forced to let in all content providers (e.g. scenario (2)). Consequently, we argue the standard NN debate is represented by comparing scenarios (1)-(2).

In a second set of results we propose alternative regulation scenarios for protecting internet value added from platform's incentives to discriminate; we will call it Non-Standard NN Debate. In particular we compare the decisions from a profit-maximizing platform (e.g. scenario (1)) with the ones from a profit maximizing platform forced charge end-users a zero access fee (e.g. scenario (3)). On the other hand, at the appendix we compare it also with the decisions from a publicly funded platform that maximizes total expected trade surplus (e.g. scenario (4)). Consequently, we argue the non-standard NN debate is represented by comparing scenarios (1)-(3), and scenarios (1)-(4).

In the last set of results we will compare two regulation scenarios. In particular, we will compare the case a profit-maximizing platform is forced to charge content providers a zero access fee (e.g. scenario (2)), with the case he is forced to charge end-users a zero access fee (e.g. scenario (3)).

3.1 For Level Zero NN Debate

As we argued at the beginning of this article, is impossible to say there is a consensus about what Network Neutrality regulation mean. In this article NN regulation means forcing a profit-maximizing platform to charge content providers a zero access fee; consequently, the platform will obtain his revenues from end-users' access fees. Notice this interpretation of the regulation implies the platform is unable to obtain revenues from content providers under the NN regulation. The reason for calling this interpretation as *level zero* is precisely that the least one expects to happen under NN regulation is that all content providers are allowed to join in; assuming platforms obtain revenues from content providers under the regulation will complicate the analysis.

We will compare the mass of content providers and end-users joining the network, the platform investment level, and the welfare coming from a profit-maximizing platform, with the ones coming from a profit-maximizing platform forced to charge content providers a zero access fee. In both situations platforms can choose end-users' entry fee and the investment level.

[10]This case is fully developed at the appendix.

Profit-Maximizing Platform

Platform's optimization program is atypical given both end-users and content providers have private information. On one hand, end-users two sources of private information are uninformative to the platform. Indeed, notice that θ_b is revealed after the platform picks a level of investment, and η is independent from all other private information parameters and enters as a fixed cost in the utility function. Consequently, the platform cannot extract information to set a different t_b for each η, and will choose only one t_b.

On the other hand, the platform will find optimal to set only one t_s for all θ_s. From equations (8) and (9) we observe that N_b and N_s are interrelated (i.e. by fixing N_b also N_s its fixed), and given that the platform finds optimal to set only one t_b, and that he has only one instrument to control end-users, he will find optimal also to choose only one t_s.

The optimization program, using equations (8)-(9), is

$$\max_{N_b,N_s,Q} \quad \left[(\overline{\theta}_s - \Delta\theta_s N_s)\left(\frac{\overline{K}_b}{2} + \overline{K}_s\right) + \frac{\overline{K}_b\overline{\theta}_s}{2}\right] N_s N_b Q^{\frac{1}{\alpha}} - N_b^2 - \phi Q$$

and the solution (N_s^*, N_b^*, Q^*) is,

$$\begin{aligned} N_s^* &= \frac{\overline{\theta}_s}{\Delta\theta_s}\left(\frac{\overline{K}_b + \overline{K}_s}{\overline{K}_b + 2\overline{K}_s}\right) \\ N_b^* &= \frac{1}{4}\left[8\alpha\phi\left(\frac{\Delta\theta_s}{\overline{\theta}_s^2}\frac{\overline{K}_b + 2\overline{K}_s}{(\overline{K}_b + \overline{K}_s)^2}\right)^{\alpha}\right]^{\frac{1}{2-\alpha}} \\ Q^* &= \left[8\alpha\phi\left(\frac{\Delta\theta_s}{\overline{\theta}_s^2}\frac{\overline{K}_b + 2\overline{K}_s}{(\overline{K}_b + \overline{K}_s)^2}\right)^{2}\right]^{\frac{\alpha}{2-\alpha}} \end{aligned}$$

Minor comments should be made. It must be the case that N_s^* and N_b^* lie between the unit internal, so assuming that the support of θ_s, i.e. $\Delta\theta_s$, is big enough the mass of content providers will be in the desired range. In particular, Figure (4) plot the optimal mass of content providers (e.g. N_s^*) in terms of two ratios: the ratio of network effects (e.g. $\frac{\bar{K}_s}{\bar{K}_b} \in [1, \infty]$), and the ratio of the support bounds of θ_s (e.g. $\frac{\bar{\theta}_s}{\underline{\theta}_s} \in [1, \infty]$). We obtain its enough to assume $\bar{\theta}_s > 3\underline{\theta}_s$ such that $N_s^* \in [0, 1]$[11].

Lemma 3. *A profit-maximizing platform will charge a positive fee to content providers, and will subsidize end-users if $\bar{K}_b$ is far enough from $\bar{K}_s$. In particular,*

1. $t_s^* > 0$
2. $t_b^* < 0$ *if* $\frac{\bar{K}_b}{\bar{K}_s}$ *is far enough from one, and viceversa*

Platform's pricing scheme reflect the presence of network effects. The first result is explained from the fact that content providers have no extra fixed entry costs like end-users. In particular, if content-providers face an additional entry barrier (e.g. obtain funding for start running the project) platforms might end-up subsidizing content providers. In other words, assume some content providers were credit constrained, then not all content providers are willing to join the network even

[11] Additional conditions should be included for $N_b^* \in [0, 1]$. We obtained its enough to avoid having very low levels of expected content providers type, i.e. $E_{\theta_s}(\theta_s)$; or if this is not possible, its also enough to avoid having high levels of quality cost provision, i.e. ϕ.

without an access fee. Given this, the platform could subsidize those credit constrained content providers if network effects are *high enough*.

Second bullet of Lemma (3) establish conditions that determine end-users entry fee. We argue the platform will subsidize end-users if $\bar{K}_b$ is *far enough* from $\bar{K}_s$. This have two interpretations. On one hand, if content providers' network effect on end-users (e.g. $\bar{K}_b$) is low enough compared to end-users' network effect on content providers (e.g. $\bar{K}_s$), then the platform will subsidize end-users to increase revenues from this market. On the other hand, if network effects are such than content providers obtain a bigger trade surplus share, then the platform will subsidize end-users to incentive them to join the network. Intuitively, if content providers are getting most of the trade surplus, then the profit-maximizing platform will incentive end-users to join in by subsidizing them.

Profit-Maximizing Platform under Content Providers' Free Entry

NN regulation is interpreted as forcing the profit-maximizing platform to charge content providers a zero access fee. Under this setup internet's value added is jointly determined by content providers and by the platform. Indeed, interpreting each content provider represents an innovation, total trade surplus will be determined by the platform's investment level and by the number of content providers that join the network[12]. Compared to all previous literature, our model studies how platform investment incentives will be affected by NN regulation under the presence of network effects.

By setting $t_s = 0$ all content providers will join the network, i.e. $\hat{\theta}_s = \underline{\theta}_s \rightarrow N_s^{E1} = 1$; superscript "E1" will design the optimal level of any choice variable under this environment. Notice this result does not depend on the platform's investment level or on a network effect variable because content providers have no extra idiosyncratic access fee like end-users.

One final remark before continuing. The fact content providers have no idiosyncratic entry fee implies the *mass* of innovations arriving into the network, i.e. the mass of content providers that join in is independent from the network's quality, i.e. platform's investment level, when content providers have no entry fee. If we eliminate this assumption this will no longer be true; that is, the *mass* of innovations arriving into the market will always depend on the network's quality.

The optimization program is,

$$\max_{N_b, Q} \quad N_b \bar{K}_b \mathbb{E}_{\theta_s}(\theta_s) Q^{\frac{1}{\alpha}} - N_b^2 - \phi Q$$

Yielding the optimal levels of $(N_s^{Ex1}, N_b^{Ex1}, Q^{Ex1})$,

$$\begin{aligned} N_s^{E1} &= 1 \\ N_b^{E1} &= \frac{1}{2}\left[\frac{2\alpha\phi}{(\bar{K}_b \mathbb{E}_{\theta_s}(\theta_s))^{\alpha}}\right]^{\frac{1}{2-\alpha}} \\ Q^{E1} &= \left[\frac{2\alpha\phi}{(\bar{K}_b \mathbb{E}_{\theta_s}(\theta_s))^{2}}\right]^{\frac{\alpha}{2-\alpha}} \end{aligned}$$

Lemma 4. *A profit-maximizing platform forced to charge content providers a zero access fee, i.e. $t_s = 0$, will charge end-users a positive entry fee, i.e. $t_b^{E1} > 0$.*

Pricing scheme changes when profit-maximizing platforms are forced to charge content providers zero access fee. Rationale is the following, as platforms looses money from connecting more content

[12] This model could be easily extended to let also content providers invest. For example, this could be done assuming content providers invest in a time period zero, and that such investment shapes the distribution of θ_s's. The impact of NN regulation should consider also the changes in the distribution of θ_s.

providers, platforms are willing to compensate by not subsidizing end-users, and start charging them a positive access fee. In other words, platforms will minimize the loss coming from the *extra* content providers by only connecting those end-users with *high enough* valuation.

We found that results depend on two key variables: the width of θ_s's support (e.g. $\Delta\theta_s$), and the ratio of network effects (e.g. $\frac{\bar{K}_b}{\bar{K}_s} \in [0,1]$). While the former is related to the variety of content providers available to end-users (i.e. the bigger the support, the more possible values θ_s can take), the latter is related to the distribution of total trade surplus between content providers and end-users. Moreover, we found that as long as end-users can encounter a *wide enough* variety of content providers, it is possible to obtain sharp conclusions no matter how trade surplus is distributed among content providers and end-users.

Start considering the case where the profit-maximizing platform must serve, by law or by technological reasons, a fixed number of end-users. Notice that while platform's marginal revenues of investment are higher because more content providers join in, marginal costs remain unchanged. Then, the profit-maximizing platform will invest more when they are forced to let in all content providers.

Now consider the case where the profit-maximizing platform no longer chooses the network quality, but keeps the right to block the the entry of both content providers and end-users. In this situation we are facing a platform that only affect content providers' and end-users' trade surplus through the number of network users joining in, consequently internet value added only will come from the number of content providers (i.e. innovation) available to end-users. If the regulatory agency force the profit-maximizing platform to let in all content providers, the platform will exclude more end-users as a way to balance the losses coming from the extra content providers they must serve.

Results from previous paragraphs are summarized in the following lemma.

Lemma 5. *As content providers have no idiosyncratic access fee, always will be the case* $N_s^{E1} > N_s^*$.

1. *Fix an arbitrary level of mass of end-users, e.g.* $\tilde{N}_b$*, and let platforms choose* N_s *and* Q*. Then,* $Q^{E1}(\tilde{N}_b) > Q^*(\tilde{N}_b)$.
2. *Fix an arbitrary level of investment, e.g.* $\tilde{Q}$*, and let platforms choose* N_s *and* N_b*. Then,* $N_b^*(\tilde{Q}) > N_b^{E1}(\tilde{Q})$.

Now consider the big picture were the profit-maximizing platform controls all choice variables. Assume the network effects are such that end-users' trade surplus is very close to the one of content providers, i.e. $\frac{\bar{K}_b}{\bar{K}_s} \approx 1$. Given that content providers have no idiosyncratic entry cost, is immediate that all content providers will join in, and network effects will incentive end-users also to join in too; the fact end-users' trade surplus is similar to the one content providers obtain will reinforce the previous argument. The platform knows more content providers and end-users will join in under the regulation, then as marginal revenues of investment increases but marginal cost remains unchanged, the profit-maximizing platform under the regulation will invest more than the profit-maximizing platform.

On the other hand, assume network effects are such that content providers' trade surplus greatly exceed the one from end-users, i.e. $\frac{\bar{K}_b}{\bar{K}_s} << 1$. As before, with a zero access fee all content providers will join because they have no idiosyncratic entry fee. From end-users point of view things are more complicated because with the regulation the platform will no longer subsidize them, but network effects will compensate because more content will be available in the network if they decide to join in. From platforms perspective, with the regulation they are loosing money by serving more content providers, the good part of this is that end-users are now more willing to join in. The last positive effect of the regulation on the platform is that they do not have to subsidize content providers, then regulation will reduce platform's opportunity cost of serving extra end-users. In the overall, we show network effects dominate, then more end-users will join in. Like in the previous paragraph, given that more content providers and end-users will join with the regulation, the platform will also invest more while the marginal revenues of investment increases, the marginal costs remain unchanged.

Results from previous paragraphs are summarized in the following proposition.

Proposition 1. *A profit-maximizing platform forced to charge content providers zero access fee will exclude less content providers and end-users, and will invest more than a profit-maximizing platform; in particular,* $N_s^{E1} > N_s^*$, $N_b^{E1} > N_b^*$, $Q^{E1} > Q^*$.

The welfare function, which is the last item we consider, is the sum of end-users and content providers trade surplus, less end-users idiosyncratic entry cost and network quality provision cost[13], and can be understood as a measure of internet's value added.

Proposition (1) anticipates welfare comparison may be ambiguous. Indeed, if the profit-maximizing platform forced to charge content providers a zero access fee will exclude less network users, and if on top will invest more than a profit-maximizing platform, we conclude total trade surplus will be higher under regulation, but also the sum of end-users idiosyncratic entry cost and network quality provision cost will be higher under regulation. In short, the welfare benefits of imposing the regulation over the profit-maximizing platform may or may not outweigh its welfare costs.

The following proposition shows that for any value of the ratio of network effects (i.e. $\frac{\bar{K}_b}{\bar{K}_s} \in [0,1]$), and of the support of θ_s, the welfare benefits of forcing profit-maximizing platforms to charge content providers a zero access fee, will outweigh the welfare costs. Finally, we should remark that no matter how network effects distribute trade surplus between content providers and end-user, the presence of network effect will suffice to guarantee welfare increases when the profit-maximizing platform do not exclude content providers.

Proposition 2. *Welfare is higher when profit-maximizing platform is forced to charge content providers a zero access fee compared to the welfare from a profit-maximizing platform, i.e.* $\mathcal{W}^{E1} > \mathcal{W}^*$.

3.2 Non-Standard NN Debate

Recognizing platform environment is not one-sided opens the possibility internet's value added can be protected by different ways. That is, just like in Taxation Theory is possible to achieve one goal by different ways, if market is two-sided nothing prevents NN regulation goals to be achieved using different restrictions on platforms. In this case we investigate if internet's value added is better protected by forcing platforms to set zero access fee to end-users, instead than over content providers. For example, assume the platform sells a package of phone, internet and television, all using the same internet cable, to end-users for a low fixed fee[14]; doing so is as letting almost all end-users join the network. Then, the regulator could let platforms discriminate content providers, and concentrate on guaranteing end-users achieve a good and cheap service.

[13] In particular, the welfare function used is

$$\mathcal{W}(N_s, N_b, Q) = N_b \mathbb{E}_{\theta_s,\theta_b}[U^b(\theta_b, \theta_s)] + N_s \mathbb{E}_{\theta_b,\theta_s}[U^s(\theta_b, \theta_s)] - \int_0^{N_b} \eta d\eta - \phi Q$$

Notice platform profits are not included, just like in Hermalin & Katz (2007), because they are not part of internet's value added, they belong to the industry profits.

[14] In France this is indeed the case because end-users pay 30 euros for these three services.

Profit-Maximizing Platform under End Users' Free Entry

This regulation scenario, although it eliminates the double marginalization, distorts the mass of S's joining the network. Before going into the details, as it is expected by now, platform will choose a positive access fee over content providers given they earn no money from the end-users market; the immediate effect is a reduction in the mass of content providers joining the network. But on top of that, end-users will be less attracted to join in because they will obtain less content, thus there is a reinforcing effect over content providers that reduces even more the mass of content providers joining the network; from hereon we will name this as *reinforcing effect*[15].

The main difference of the regulation scenario with the previous one is that imposing a zero entry fee for end-users does not imply all end-users will join in, i.e. $N_b^{E2} < 1$; superscript "E2" designs the optimal level of the choice variables under this environment. Indeed, imposing $t_b = 0$ on equation (8) will not guarantee that all end-users will join the network. The reason for this is that end-users have an idiosyncratic entry cost, e.g. a learning cost for the case of internet or a technology upgrading cost for the energy industry, that do not depend on the platform's entry fee. The consequence of it will be that network effects will not be as strong as before. For example, in the previous case all content providers join in with the regulation, now not every end-user will respond to the regulation, and consequently the power of the network effects decreases.

Using equations (8)-(9) the optimization program and first order conditions are,

$$\max_{N_s, Q} \quad \bar{K}_b \bar{K}_s \frac{Q^{\frac{2}{\alpha}}}{2} (2\overline{\theta}_s N_s - \Delta\theta_s N_s^2)(\overline{\theta}_s N_s - \Delta\theta_s N_s^2) - \phi Q$$

Thus, we obtain

$$\begin{aligned}
N_s^{E2} &= \frac{\bar{\theta}_s}{\Delta\theta_s} 0.61 \\
N_b^{E2} &= \frac{1}{2}\left[(\overline{K}_b(2-\beta))^{1-\alpha} \left(\frac{\Delta\theta_s}{\overline{\theta}_s^2 \beta} \right)^{\alpha} \left(\frac{\alpha\phi}{\overline{K}_s(1-\beta)} \right) \right]^{\frac{1}{2-\alpha}} \\
Q^{E2} &= \left[\frac{\alpha\phi}{\overline{K}_b \overline{K}_s \frac{\overline{\theta}_s^4}{\Delta\theta_s^2} \beta^2 (2-\beta)(1-\beta)} \right]^{\frac{\alpha}{2-\alpha}} \\
t_s^{E2} &= N_b^{Ex2} \bar{K}_s^{Ex2} (Q^{Ex2})^{\frac{1}{\alpha}} \bar{\theta}_s (1-\beta)
\end{aligned}$$

As expected, content provider's access fee is positive. Finally, conditions for having $N_s^{E2}, N_b^{E2} \in [0,1]$ are similar to those that guarantee $N_s^*, N_b^* \in [0,1]$.

As before, assume the profit-maximizing platform must serve a fixed number of end-users. When the regulation is enforced platform's revenues are reduced because although the same number of end-users join in, the platform receives no entry fee from them. Then, the platform must compensate his loses by raising content providers entry fee, naturally more content providers will be excluded with the regulation. But the platform cannot afford to loose too many content providers, then he will raise the investment level.

On the other hand, assume the profit-maximizing platform can only affect end-users' and content providers' trade surplus through the number of network users that join in; in other words, the platform cannot choose his investment level. We obtained that if network effects are such that trade surplus is not too unequally divided (i.e. $\frac{\bar{K}_b}{\bar{K}_s}$ high), then more end-users will join in because

[15] This also can be understood as a global network effect on content providers. This effect is different from $\bar{K}_b$ and $\bar{K}_b$, and is always less or equal to one. For the level zero NN debate, the reinforcing effect is equal to $\frac{\bar{K}_b + \bar{K}_s}{\bar{K}_b + 2\bar{K}_s}$.

before the regulation platform was charging them a positive entry fee, and now there is no entry fee. This effect makes content providers more willing to join in. The platform acknowledges this and chooses content provider's entry fee such that they are not discouraged to join in.

If network effects are such that content providers retain most of trade surplus share, then we know the profit-maximizing platform will subsidize end-users. When regulation force the platform to charge zero entry fee, less end-users will join the network; this will discourage content providers to join in. As the platform cannot invest more to attract content providers, the overall effect is that less content providers and end-users will join the network

Theses results are summarized in the following lemma.

Lemma 6. *Statement is two-fold,*

1. *Fix an arbitrary level of end-users, e.g.* $\tilde{N}_b$*, and let platforms choose* N_s *and* Q*.*
   - $Q^{E2}(\tilde{N}_b) > Q^*(\tilde{N}_b)$ *and* $N_s^*(\tilde{N}_b) > N_s^{E2}(\tilde{N}_b)$
2. *Fix an arbitrary level of investment, e.g.* $\tilde{Q}$*, and let platforms choose* N_s *and* N_b*.*
   - *If* $\frac{\bar{K}_b}{\bar{K}_s}$ *is high enough (i.e.* ≈ 0.6*), then* $N_b^{E2}(\tilde{Q}) > N_b^*(\tilde{Q})$ *and* $N_s^{E2}(\tilde{Q}) > N_s^*(\tilde{Q})$*; otherwise* $N_b^{E2}(\tilde{Q}) < N_b^*(\tilde{Q})$ *and* $N_s^{E2}(\tilde{Q}) < N_s^*(\tilde{Q})$*.*

Two results are obtained when we do a full comparison. The first one is that no matter how network effects shape trade surplus distribution between content providers and end-users, the profit-maximizing platform forced to charge end-users a zero access fee will invest more than the profit-maximizing platform. Assume the ratio of network effects is such that the profit-maximizing platform charges a positive entry fee to end-users. After imposing the regulation more end-users will join in, so the platform will compensate his losses by increasing content providers' entry fee. Finally, the platform will try to make more attractive the network to content providers by investing more on quality[16]. On the other hand, if network effects are such that the profit-maximizing platform subsidizes end-users, after imposing the regulation less end-users will join in; this will discourage content providers to enter. Notice that the regulation incentive the platform to attract content providers; platforms achieve this by investing more, and not by increasing too much content providers' entry fee. Indeed this is feasible because the platform now uses the money of the subsidy to invest more without having to increasing that much content providers' entry fee.

The second result is that platform's exclusion of network users depends on the ratio of network effects. In case the ratio is high, but not close to one, the profit-maximizing platform will charge end-users a positive entry fee, thus when the regulation is implemented end-users will face a lower entry cost. Moreover, as mentioned in the previous paragraph, under the regulation the profit-maximizing platform will also invest more. Then, the profit-maximizing platform forced to charge end-users a zero access fee will exclude less content providers than a profit-maximizing platform because still he finds content providers attractive. This will no longer hold if content providers were unattractive.

A special arise when the ratio of network effect is close to one. In this situation content providers' and end-users' trade surplus are close, this imply that content providers' market power is not reflected into their trade surplus. A profit-maximizing platform that acknowledges this will certainly face a countervailing effect because they will no longer see content providers as attractive as before. In particular, as the profit-maximizing platform without the regulation will not subsidize

[16]This even hold in case the platforms do not find content providers very attractive, e.g. when content providers' and end-users equally share total trade surplus. Investment is a vehicle to compensate the fact less end-users will join the network, and does not imply more content providers will enter.

end-users, when it arrives more of them will join the network. Given that there is more investment under the regulation, platform will now exclude more content provider than the profit-maximizing platform because content providers are no longer attractive.

The last situation is when network effects are such that content providers keep the biggest share of total trade surplus. As a profit-maximizing platform will subsidize end-users, when regulation is implemented less of them will join the network. In addition, as it was mentioned before, the profit-maximizing platform's opportunity cost of serving an extra content providers decreases after regulation is implemented. In the overall, platforms will opt to serve more content providers because the share of total trade surplus they keep is significant compared to the one from end-users. Finally, from content providers' point of view, the fact their share in total trade surplus is high, and given there is more investment under regulation, more than compensates the fact less end-users will enter.

The following proposition summarizes last paragraph's ideas.

Proposition 3. *Comparing profit-maximizing and E2 programs,*

1. $N_s^* > N_s^{E2}$ *if* $\frac{\bar{K}_b}{\bar{K}_s}$ *close to one, and* $N_s^* < N_s^{E2}$ *otherwise.*
2. $Q^{E2} > Q^*$
3. $N_b^{E2} > N_b^*$ *if* $\frac{\bar{K}_b}{\bar{K}_s}$ *high enough (i.e.* ≥ 0.6*), and* $N_b^{E2} < N_b^*$ *otherwise.*

Welfare comparison also depends on the ratio of network effects. In case network effects are such that the profit-maximizing platform charge end-users a positive entry fee, and abstract a moment from the presence of countervailing incentives, the previous proposition say that under the regulation less content providers and end-users are excluded, and more investment is observed. Consequently, under the regulation total trade surplus, e.g. welfare revenue, is higher, but also welfare costs will be higher. We are able to show that by forcing the profit-maximizing platform to charge end-users a zero access fee, the change in welfare revenues outweigh the increase in welfare costs[17].

Now assume network effects are such that the profit-maximizing platform subsidizes end-users. After imposing the regulation less end-user will join the network. But the platform, for whom content providers are very attractive, have now a lower opportunity cost of serving extra content providers. Then, the platform must invest more hopping to attract end-users to the network. In overall, its likely that welfare revenues increase with the regulation because more content providers (whose impact on total trade surplus is higher) join in, and there is more investment; but also welfare costs are higher with the regulation. We obtain that the increase in welfare costs exceed the increase in welfare revenues. Intuitively this happen because the profit-maximizing platform forced to charge end-users a zero access fee must do a big effort to keep being attractive to content providers.

Proposition 4. *Welfare comparisons between the profit-maximizing and E2 programs. Welfare when the profit-maximizing platform is forced to charge end-users a zero access fee is higher (lower) than the one with a profit-maximizing platform, if the ratio of network effects is high (low) enough. In particular,* $W^{E2} \gtrless W^*$ *if* $\frac{\bar{K}_b}{\bar{K}_s}$ *is high (low), i.e.* ≈ 0.55.

[17] In the presence of countervailing incentives the difference is that now less content providers will join the network. The result do not change because the effect of having more end-users, who have a similar share in total trade surplus as content providers, and more investment exceed the negative effect of having less content providers.

3.3 Comparing Regulation Scenarios

This part is devoted to compare two regulation scenarios[18]. There are many reasons a regulatory agency is interested in comparing two different scenarios; mainly, is very costly in terms of time and resources (or sometimes its impossible) to run several regulatory scenarios, and then make a comparison. For the example of this paper, is unclear from an ex-ante point of view if its better to ban content providers' entry fee, or end-users entry fee. In particular, we are interested in comparing the scenario were a profit-maximizing platform is forced to charge content providers zero access fee, with the scenario were a profit-maximizing platform is forced to charge end-users a zero access fee.

Two lessons are obtained. First, there is a trade-off between exclusion of network users and investment. So, if the regulatory agency prefers less exclusion, and do not care much about investment, he should eliminate content providers entry fee. On the other hand, if the regulatory agency prefers having more investment, and cares less about exclusion, then he must eliminate end-users entry fee. And second, if the regulatory agency cares only for the effects on the welfare, the agency should force the profit-maximizing platform charge content providers no entry fee.

Before comparing choice variables' optimal levels lets do an exercise similar to that in Lemma (5)[19]. In case the profit-maximizing platform cannot modify network's quality, e.g. $Q = \tilde{Q}$, we find that less content providers are excluded if the profit-maximizing platform is forced to charge them a zero access fee than if they charge end-users a zero access fee, i.e. $N_s^{E1}(\tilde{Q}) > N_s^{E2}(\tilde{Q})$. We also find that less end-users will be excluded if the profit-maximizing platform is forced to charge them a zero access fee, i.e. $N_b^{E2}(\tilde{Q}) > N_b^{E1}(\tilde{Q})$; this is explained because end-users tariff is positive when content providers face no entry fee, and is zero in the other case.

Now assume that for some reason, e.g. legal or technological, the profit-maximizing platform serve a fixed number of end-users, e.g. $N_b = \tilde{N}_b$. Less content providers will be excluded if the profit-maximizing platform is forced to charge them a zero access, than if he is forced to charge end-users no entry fee, i.e. $N_s^{E1} > N_s^{E2}$. Then, if network effects are such that content providers keep an important share of total surplus, i.e. $\frac{\bar{K}_b}{\bar{K}_s}$ low, the profit-maximizing platform will finds them very attractive. As a consequence, the profit-maximizing platform forced to charge content providers a zero access fee will have to make less effort (investment) to persuade content providers to join the network, i.e. $Q^{E2}(\tilde{N}_b) > Q^{E1}(\tilde{N}_b)$. On the other hand, if network effects are such that content providers' and end-users' trade surplus are close, i.e. $\frac{\bar{K}_b}{\bar{K}_s}$ high, the profit-maximizing platform will find content providers less attractive[20]. Consequently, the profit-maximizing platform forced to charge end-users a zero access fee will invest less than if he is forced to charge content providers a zero access fee, i.e. $Q^{E2}(\tilde{N}_b) < Q^{E1}(\tilde{N}_b)$.

Comparing choice variables' optimal levels, we identified a trade-off between exclusion and investment. We find that for almost any combination of network effects the profit-maximizing platform forced to charge content providers a zero access fee will exclude less content providers and end-users, but will invest less than if he is forced to charge end-users a zero access fee. To explain it assume the status quo is when the profit-maximizing platform is force to charge content providers zero access fee. If he is forced to charge end-users a zero access fee, the platform will charge content providers a positive fee; as a consequence, less content provider are willing to enter, but more end-users are willing to join in. In addition, in the status quo the platform invested to attract end-users, in the new situation he invest to attract content providers; as platform's value

[18] In the appendix we also compare a profit-maximizing platform that cannot charge content-providers, and a welfare-maximizing platform publicly financed and that charges no access fee.

[19] See appendix, under the name of Remark (1), for a formal statement and proof.

[20] As we saw before, platform faces countervailing incentives when content providers and end-users equally share total trade surplus.

more content providers than end-users due to their market power, investment is higher in the new situation than in the status quo. End-users acknowledge there will be less investment if the profit-maximizing platform is forced to charge then a zero access fee. In the overall, more content providers and end-users will join the network, but less investment will be observed in the status quo, i.e. $N_s^{E1} > N_s^{E2}$, $N_b^{E1} > N_b^{E2}$, and $Q^{E1} < Q^{E2}$.

A special case arise if network effects are such that content providers keep almost all total trade surplus. If the profit-maximizing platform is forced to charge content providers a zero access fee, end-users will not find the network attractive because they know if they join in their trade surplus will be very small, and on top they know content providers do not even have to pay the access fee. The profit-maximizing platform acknowledges this and decides to invest more to make the network more attractive to end-users. In the overall, more content providers and end-users will join in the network, and also more investment will be observed in the status quo, i.e. $N_s^{E1} > N_s^{E2}$, $N_b^{E1} > N_b^{E2}$, and $Q^{E1} > Q^{E2}$.

The following proposition summarizes these ideas.

Proposition 5. *Comparing E1 and E2 programs,*

1. $N_s^{E1} > N_s^{E2}$ *and* $N_b^{E1} > N_b^{E2}$.
2. *If* $\frac{\bar{K}_b}{\bar{K}_s}$ *is almost zero, then* $Q^{E1} > Q^{E2}$. *In any other case,* $Q^{E1} < Q^{E2}$.

The effect of having less exclusion of network users is of higher order than having less investment. As it was mentioned, for almost any combination of network effect the profit-maximizing platform forced to charge content providers a zero access fee will exclude less network users, but will invest less than if he is forced to charge end-users a zero access fee. We obtained the less-exclusion effect dominate the less-investment effect, and consequently welfare is higher if the profit-maximizing platform is force to charge content providers a zero access fee. Intuition is as follows, as end-users have an idiosyncratic access fee, but content providers do not have it, the impact of no access fee will be bigger for content providers[21]. On the other hand, benefits from having higher investment levels when end-users have no access fee are low because the platform has to make an important effort in investment to attract content providers to join the network. Finally, while the less-exclusion effect is high, the negative effect from a lower investment is low; then, welfare is higher when the profit-maximizing platform is force to charge content providers a zero access fee. The following proposition formalizes these ideas.

Proposition 6. *Welfare is higher when the profit-maximizing platform is forced to charge content providers zero access fee, than when the profit-maximizing platform is forced to charge end-users zero access fee; in particular,* $\mathcal{W}^{E1} > \mathcal{W}^{E2}$.

4 Conclusion

We are interested in two-sided market industries where the platform takes investment decisions that affect network users' trade surplus. Our question is about the effects of regulating the platform's pricing scheme on the investment level, on the number of network users that decide to join the network, and on the welfare. We propose a profit-maximizing platform that helps connect a continuum of vertically differentiated content providers (sellers), and a continuum of vertically differentiated end-users (buyers). Three scenarios were studied: a profit-maximizing platform, a

[21] Additionally, content providers trade surplus is higher than the one from end-users because of market power they have.

profit-maximizing platform that cannot charge content providers, a profit-maximizing platform that cannot charge end-users.

Results are the following. If the regulatory agency only cares about platform's investment level, our finding support Google and Amazon's arguments in the Net Neutrality Debate . Indeed, we find a profit-maximizing platform will invest less than a profit-maximizing platform forced to either charge content providers a zero access fee, or charge end-users a zero access fee. But if the regulatory agency care only about the mass of network users that join in, results are a bit more elaborated. We find that if network effects are such that content providers' and end-users' trade surplus are close enough, then the profit-maximizing platform will exclude more content providers and end-users than the profit-maximizing platform forced to either charge content providers, or end-users a zero access fee. On the other hand, if network effects are such that content providers trade surplus exceeds enough end-users trade surplus, then the profit maximizing platform will exclude more content providers, but less end-users than the profit-maximizing platform forced to charge content-providers a zero access feel.

If the regulatory agency only cares for the welfare effects, results show welfare is higher if the profit-maximizing platform is forced to charge content-providers a zero access fee. On the other hand, assume that because of technological or political reasons the regulatory agency cannot force the profit-maximizing platform to charge content providers no entry fee, but can force them to charge end-users a zero access fee. We find welfare if higher is network effects are such that content providers and end-users trade surplus are close enough, otherwise welfare is greater if no regulation is imposed.

The regulatory agency might want to know if its better to force the profit-maximizing platform to charge content providers a zero access fee, or instead charge end-users no access fee. We find there is a trade-off between exclusion of network users, and platform's investment. In particular, less content providers and end-users are excluded if the profit-maximizing platform is forced to charge end-users a zero access fee, but less investment will be observed than if the platform is forced to charge end-users a zero access fee. Now, in terms of welfare, it is better to force the profit-maximizing platform to charge content providers no access fee.

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Appendix

Free Platform

Policy makers from time to time consider the possibility of forbidding the access of profit-maximizing firms to industries that have a significant impact in the overall economy, and which are also difficult to regulate. Here we are interested in comparing the profit-maximizing platform with a publicly funded welfare-maximizing platform. As the public platform, from hereon called public platform, use the financing to set entry fees equal to zero, then we do not obtain the first best levels of investment, and of network users' entry decision.

Setting both entry fees equal to zero guarantee all content providers join the network, but this is not the case for end-users because of their idiosyncratic entry cost (η). Indeed, imposing $t_i = 0$ for $i \in \{s, b\}$ on equations (8)-(9) guarantee that free platform's demands are $N_s^{fp} = 1$ and $N_b^{fp} = \bar{K}_b \mathbb{E}_{\theta_s}(\theta_s) Q^{\frac{1}{\alpha}}$.

The objective function of the free platform is the welfare function. Welfare revenues are nothing else than the sum of expected trade surplus from those content providers and end-users that join the network; welfare costs are the idiosyncratic cost from end-users that join the network, and the cost of investment. The optimization program will be,

$$\max_{Q} \quad N_b^{fp} \mathbb{E}_{\theta_s,\theta_b}[U^{b,sb}(\theta_b,\theta_s)] + N_s^{fp} \mathbb{E}_{\theta_b,\theta_s}[U^{s,sb}(\theta_b,\theta_s)] - \int_0^{N_b} \eta d\eta - \phi Q$$

$$\max_{Q} \quad \frac{\bar{K}_b}{2} (\mathbb{E}_{\theta_s}(\theta_s))^2 (\bar{K}_b + 2\bar{K}_s) Q^{\frac{2}{\alpha}} - \phi Q$$

and the optimal levels of $(N_s^{fp}, N_b^{fp}, Q^{fp})$ will be,

$$\begin{aligned} N_s^{fp} &= 1 \\ N_b^{fp} &= \left[\frac{\alpha \phi \bar{K}_b^{1-\alpha}}{(\bar{K}_b + 2\bar{K}_s) \mathbb{E}_{\theta_s}(\theta_s)^{\alpha}} \right]^{\frac{1}{2-\alpha}} \\ Q^{fp} &= \left[\frac{\alpha \phi \bar{K}_b^{-1}}{(\bar{K}_b + 2\bar{K}_s)(\mathbb{E}_{\theta_s}(\theta_s))^2} \right]^{\frac{\alpha}{2-\alpha}} \end{aligned}$$

A comparison of all choice variables from both programs show the ratio of network effects is a key variable. Assume network effects are such that the profit-maximizing platform subsidizes end-users, i.e. $\frac{\bar{K}_b}{\bar{K}_s}$ high. If we replace the profit-maximizing platform with the free platform, at the same time all content providers will join in, and end-users are less willing to enter. From the free platform's point of view, measures must be taken to attract content providers to join the network because their share on the total surplus is high; in particular, the public platform will invest more. In short, if we replace the profit-maximizing platform with the free platform, more content providers and end-users will join the network, and more investment will be observed.

Now assume the network effects are such that the profit-maximizing platform do not subsidizes end-users. By replacing the profit-maximizing platform with a free platform, still all content providers will join the network, but now end-users are more willing to join in. From the free platform's point of view is no longer necessary to keep the same investment level because more network users are willing to enter. In the overall, we obtain end-users value more the decrease in the network's quality than the absence on entry cost, more content providers will enter, and more investment will be observed.

The following lemma summarizes previous paragraph's ideas.

Lemma 7. *Comparing free platform and profit-maximizing programs,*

- *The free platform will exclude less content providers, but will exclude more end-users, than the profit-maximizing platform; in particular,* $N_s^{fp} > N_s^*$ *and* $N_b^* > N_b^{fp}$.
- *The free platform will invest more than the profit-maximizing platform if the ratio of network effects is low enough, and viceversa. In particular,* $Q^{fp} \gtrless Q^*$ *if* $\frac{\bar{K}_b}{\bar{K}_s}$ *low (high).*

Welfare comparison in this case is more elaborated because, besides depending on the ratio of network effects, results also depend on the support of content providers private information random variable[22], i.e. $\Delta\theta_s$. Assume that network effects are such that the profit-maximizing platform charge end-users a positive entry fee, i.e. $\frac{\bar{K}_b}{\bar{K}_s}$ high,

[22] Until now we just needed that the support $\Delta\theta_s$ is big enough to guarantee that N_s^* and N_s^{E2} are strictly less than one.

and assume the support $\Delta\theta_s$ is high. From previous lemma we know that the free platform exclude less end-users, more content providers, and invest less than with the profit-maximizing platform. We obtain that if end-users face a higher variety (i.e. $\Delta\theta_s$ high), and quantity, of content providers with the free platform, this effect will dominate the negative effects of having less investment and less end-users

On the other hand, assume ratio of network effects is low, and assume the support is also low. Previous lemma showed the free platform exclude less content provider, more end-users, and invest more than the profit-maximizing platform. We obtained that if the reduced mass of end-users that join the network face a low variety of content providers (i.e. $\Delta\theta_s$ low), this negative effect on the welfare will dominate any other positive effect. In particular, will dominate the effect of having higher investment levels, and more content providers joining the network.

The following lemma summarizes last two paragraph's ideas.

Lemma 8. *Welfare comparison between the free platform and profit-maximizing programs depend on two variables:* $\Delta\theta_s$ *and* $\frac{\bar{K}_s}{K_b}$.

- *If* $\Delta\theta_s$ *and* $\frac{\bar{K}_s}{K_b}$ *are high, then the welfare for the free platform is higher than for the profit-maximizing platform, i.e.* $W^{fp} > W^*$.
- *If* $\Delta\theta_s$ *is low, or if* $\frac{\bar{K}_s}{K_b}$ *is low, then the welfare for the profit-maximizing platform is higher than for the free platform, i.e.* $W^{fp} < W^*$.

Proofs

Proof of Lemma 1

Proof. First bullet. The optimization program for any content provider θ_s that join the network is,

$$\begin{aligned}
&\max_{\{\Gamma(\theta_b), r(\theta_b)\}} && N_b \mathbb{E}_{\theta_b}\left[\Gamma(\theta_b) - \frac{C_s r(\theta_b)}{Q}\right] \\
&(IC) && \theta_b = arg\max_{\tilde{\theta}_b}\left[\theta_b \int_0^{r(\tilde{\theta}_b)} \left(\frac{z}{\theta_s Q}\right)^{-\alpha} dz - \Gamma(\tilde{\theta}_b)\right] \\
&(RC) && \theta_b \int_0^{r(\theta_b)} \left(\frac{z}{\theta_s Q}\right)^{-\alpha} dz - \Gamma(\theta_b) \geq 0 \forall \theta_b
\end{aligned}$$

Using the standard first order approach to solve the program, the optimal consumption level for end-user θ_b is $r(\theta_b)^{**}$, and the optimal nonlinear pricing scheme is $\Gamma(\theta_b)^{**}$,

$$\begin{aligned}
r^{**}(\theta_b) &= \theta_s Q^{\frac{1+\alpha}{\alpha}} \left(\frac{2\theta_b - \overline{\theta}_b}{C_s}\right)^{\frac{1}{\alpha}} \\
\Gamma^{**}(\theta) &= \frac{\theta_s Q^{\frac{1}{\alpha}}}{(1-\alpha) C_s^{\frac{1-\alpha}{\alpha}}}\left[\theta_b(2\theta_b - \overline{\theta}_b)^{\frac{1-\alpha}{\alpha}} - \frac{\alpha}{2}((2\theta_b - \overline{\theta}_b)^{\frac{1}{\alpha}} - (2\underline{\theta}_b - \overline{\theta}_b)^{\frac{1}{\alpha}})\right]
\end{aligned}$$

Replacing $r(\theta_b)^{**}$ and $\Gamma(\theta_b)^{**}$ into the objective functions, the trade surplus for content providers and end-users that join the network is

$$\begin{aligned}
N_s \mathfrak{U}^b(\theta_b) &= \overbrace{\frac{\alpha}{2(1-\alpha)} \frac{1}{C_s^{\frac{1-\alpha}{\alpha}}}((2\theta_b - \overline{\theta}_b)^{\frac{1}{\alpha}} - (2\underline{\theta}_b - \overline{\theta}_b)^{\frac{1}{\alpha}})}^{K_b(\bar{\theta}_b, \underline{\theta}_b, C_s, \alpha)} \theta_s Q^{\frac{1}{\alpha}} \\
&= K_b(\bar{\theta}_b, \underline{\theta}_b, C_s, \alpha)\theta_s Q^{\frac{1}{\alpha}} \\
N_b \mathfrak{U}^s(\theta_s) &= N_b \mathbb{E}_{\theta_b}\left[\overbrace{\frac{1}{(1-\alpha)C_s^{\frac{1-\alpha}{\alpha}}}\left[\frac{(2\theta_b - \overline{\theta}_b)^{\frac{1-\alpha}{\alpha}}}{2}(\overline{\theta}_b + (1-\alpha)(\overline{\theta}_b - 2\theta_b)) + \frac{\alpha}{2}(2\underline{\theta}_b - \overline{\theta}_b)^{\frac{1}{\alpha}}\right]}^{K_s(\bar{\theta}_b, \underline{\theta}_b, C_s, \alpha)}\right] \theta_s Q^{\frac{1}{\alpha}} \\
&= N_b \theta_s Q^{\frac{1}{\alpha}} \mathbb{E}_{\theta_b}\left[K_s(\bar{\theta}_b, \underline{\theta}_b, C_s, \alpha)\right] \\
&= N_b \overline{K}_s(U_{[\underline{\theta}_b, \bar{\theta}_b]}, C_s, \alpha)\theta_s Q^{\frac{1}{\alpha}}
\end{aligned}$$

Finally, the expected trade surplus end-users and content providers use for deciding to join the network are,

$$\begin{aligned}
\mathbb{E}_{\theta_s,\theta_b}(N_s\mathfrak{U}^b(\theta_b) \mid \theta_s \geq \hat{\theta}_s) &= N_s\mathbb{E}_{\theta_b,\theta_s}[K_b(\bar{\theta}_s,\underline{\theta}_s,C_s,\alpha)\theta_s Q^{\frac{1}{\alpha}} \mid \theta_s \geq \hat{\theta}_s] \\
&= N_s\bar{K}_b(U_{[\underline{\theta}_b,\bar{\theta}_b]},C_s,\alpha)\mathbb{E}_{\theta_s}(\theta_s \mid \theta_s \geq \hat{\theta}_s)Q^{\frac{1}{\alpha}} \\
N_b\mathfrak{U}^s(\theta_s) &= N_b\overline{K}_s(U_{[\underline{\theta}_b,\bar{\theta}_b]},C_s,\alpha)\theta_s Q^{\frac{1}{\alpha}}
\end{aligned}$$

Second bullet. Directly comparing one obtains,

$$\begin{aligned}
\bar{K}_s(U_{[\theta_b,\bar{\theta}_b]},C_s,\alpha) \gtrless \bar{K}_b(U_{[\theta_b,\bar{\theta}_b]},C_s,\alpha) \quad &\Leftrightarrow \quad \mathbb{E}_{\theta_b}\left[\frac{1}{(1-\alpha)}\frac{1}{C_s^{\frac{1-\alpha}{\alpha}}}\left(\frac{(2\theta_b-\bar{\theta}_b)^{\frac{1-\alpha}{\alpha}}}{2}(\bar{\theta}_b+(1-\alpha)(\bar{\theta}_b-2\theta_b))+\frac{\alpha}{2}(2\underline{\theta}_b-\bar{\theta}_b)^{\frac{1}{\alpha}}\right)\right] \\
&\gtrless \mathbb{E}_{\theta_b}\left[\frac{\alpha}{2(1-\alpha)}\frac{1}{C_s^{\frac{1-\alpha}{\alpha}}}((2\theta_b-\bar{\theta}_b)^{\frac{1}{\alpha}}-(2\underline{\theta}_b-\bar{\theta}_b)^{\frac{1}{\alpha}})\right] \\
&\Leftrightarrow \quad \mathbb{E}_{\theta_b}\left\{\frac{1}{(1-\alpha)}\frac{1}{C_s^{\frac{1-\alpha}{\alpha}}}\left[(2\theta_b-\bar{\theta}_b)^{\frac{1-\alpha}{\alpha}}(\bar{\theta}_b-\theta_b)+\alpha(2\underline{\theta}_b-\bar{\theta}_b)^{\frac{1}{\alpha}}\right]\right\} > 0
\end{aligned}$$

As $\theta_b \in U_{[\underline{\theta}_b,\bar{\theta}_b]}$, $\underline{\theta}_b > 0$, and $2\underline{\theta}_b > \bar{\theta}_b$ result follows. □

Proof of Lemma 2

Proof. Equations (8)-(9) can be reexpressed as,

$$R_b(N_s): \qquad N_b = N_s\frac{\bar{K}_b}{2}Q^{\frac{1}{\alpha}}(2\bar{\theta}_s - \Delta\theta_s N_s) - t_b \tag{10}$$

$$R_s(N_b): \qquad 0 = N_b\bar{K}_s Q^{\frac{1}{\alpha}}(\bar{\theta}_s - \Delta\theta_s N_s) - t_s \tag{11}$$

It is straightforward to show equation (10) is strictly concave in N_s, and equation (11) is a hyperbole in N_s.

First bullet: Corner solution. Replacing $N_s = 1$ into $R_b(N_s)$ and $R_s(N_b)$ one obtains $\hat{N}_b$ and $\check{N}_b$ respectively. Corner solution is obtained when $\hat{N}_b > \check{N}_b$, this imply that $\bar{K}_s\bar{K}_b Q^{\frac{2}{\alpha}}\underline{\theta}_s E_{\theta_s}(\theta_s) > t_s + t_b K_s Q^{\frac{1}{\alpha}}\underline{\theta}_s \Rightarrow \Omega(t_b) = \bar{K}_s Q^{\frac{1}{\alpha}}\underline{\theta}_s[\bar{K}_b Q^{\frac{1}{\alpha}}\mathbb{E}_{\theta_s}(\theta_s) - t_b] > t_s$.

Second bullet: Interior solution. The upper bound $\bar{\Upsilon}(t_b)$ is obtained when $R_b(N_s)$ and $R_s(N_b)$ are tangent; the coordinate for this point is $(\underline{\underline{N}}_s, \underline{\underline{N}}_b) = \left(\frac{\bar{\theta}_s}{\Delta\theta_s} - \frac{1}{\Delta\theta_s}\left(\frac{t_s\Delta\theta_s}{\bar{K}_b\bar{K}_s Q^{\frac{2}{\alpha}}}\right)^{1/3}, \frac{\bar{K}_b}{2\Delta\theta_s}Q^{\frac{1}{\alpha}}\left[\bar{\theta}_s^2 - \left(\frac{t_s\Delta\theta_s}{\bar{K}_b\bar{K}_s Q^{\frac{2}{\alpha}}}\right)^{2/3}\right] - t_b\right)$. Now, replace $(\underline{\underline{N}}_s, \underline{\underline{N}}_b)$ into $R_s(N_b)$ and solve for $\underline{\underline{t}}_s$; naturally, $\underline{\underline{t}}_s \equiv \bar{\Upsilon}(t_b)$. The lower bound $\underline{\Upsilon}(t_b)$ is obtained by introducing $N_s = 1$ into $R_b(N_s)$ and $R_s(N_b)$, and obtaining the values of N_b, namely $\bar{\bar{N}}_b$ and $\tilde{\tilde{N}}_b$. The lower bound arise when $\bar{\bar{N}}_b > \tilde{\tilde{N}}_b$. In particular,

$$\begin{aligned}
\bar{\Upsilon}(t_b) &= \left\{\left(\frac{\bar{K}_b\bar{K}_s Q^{\frac{2}{\alpha}}}{\Delta\theta_s}\right)^{\frac{2}{3}}\frac{\bar{\theta}_s^2}{3} - \frac{2}{3}t_b\left(\frac{\Delta\theta_s}{\bar{K}_b Q^{\frac{1}{\alpha}}}\right)^{\frac{1}{3}}(\bar{K}_s Q^{\frac{1}{\alpha}})^{\frac{2}{3}}\right\}^{\frac{3}{2}} \\
\underline{\Upsilon}(t_b) &= \bar{K}_b\bar{K}_s Q^{\frac{2}{\alpha}}\underline{\theta}_s\mathbb{E}_{\theta_s}(\theta_s) - t_b
\end{aligned}$$

□

Proof of Lemma 3

Proof. Replacing (N_s^*, N_b^*, Q^*) in equations (10)-(11) is trivial to obtain,

$$\begin{aligned}
t_s^* = N_b^*\bar{K}_s(Q^*)^{\frac{1}{\alpha}}\bar{\theta}_s\frac{2\bar{K}_s}{\bar{K}_b+2\bar{K}_s} \quad &\Rightarrow \quad t_s^* > 0 \\
sgn(t_b^*) = sgn\left[\bar{K}_b\frac{\bar{K}_b+3\bar{K}_s}{\bar{K}_b+\bar{K}_s}(\bar{K}_b+2\bar{K}_s)^{-2\frac{1-\alpha}{2-\alpha}} - \frac{(\bar{K}_b+2\bar{K}_s)^{\frac{\alpha}{2-\alpha}}}{2}\right] \quad &\Rightarrow \quad t_b^* \gtrless 0 \leftrightarrow \frac{\bar{K}_b}{\bar{K}_b+\bar{K}_s} \gtrless \frac{1}{2}\frac{\bar{K}_b+2\bar{K}_s}{\bar{K}_b+3\bar{K}_s} \\
&\Leftrightarrow \quad t_b^* \gtrless 0 \leftrightarrow \bar{K}_b^2 + 3\bar{K}_b\bar{K}_s - 2\bar{K}_s^2 \gtrless 0
\end{aligned}$$

Figure (3) show that $\bar{K}_b^2 + 3\bar{K}_b\bar{K}_s - 2\bar{K}_s^2 < 0$ for $\frac{\bar{K}_b}{\bar{K}_s}$ close to zero, consequently $t_b^* < 0$. Naturally, for $\frac{\bar{K}_b}{\bar{K}_s}$ close to one, then the profit-maximizing platform will charge content providers a positive access fee. □

Proof of Lemma 4

Proof. Using $(N_s^{E1}, N_b^{E2}, Q^{E1})$ on equation (11) one obtains,

$$t_b^{E1} = \frac{1}{8}\frac{(2\alpha\phi)^{\frac{1}{2-\alpha}}}{(\bar{K}_s\mathbb{E}_{\theta_s}(\theta_s))^{\frac{\alpha}{2-\alpha}}} > 0$$

Result follows. □

Proof of Lemma 5

Proof. Fix $N_b = \tilde{N}_b$: After observing the optimal mass of content providers is unchanged, i.e. $N_s^* = N_s^*(\tilde{N}_b)$, the profit-maximizing program is,

$$\begin{aligned}
\max_{Q} \quad & \frac{(\bar{K}_b+\bar{K}_s)^2}{(\bar{K}_b+2\bar{K}_s)}\frac{\bar{\theta}_s^2}{\Delta\theta_s}\frac{\tilde{N}_b}{2}Q^{\frac{1}{\alpha}} - \tilde{N}_b^2 - \phi Q \\
\Rightarrow \quad & Q^*(\tilde{N}_b) = \left[\frac{2\alpha\phi}{\tilde{N}_b}\frac{\Delta\theta_s}{\bar{\theta}_s^2}\frac{\bar{K}_b+2\bar{K}_s}{(\bar{K}_b+\bar{K}_s)^2}\right]^{\frac{\alpha}{1-\alpha}}
\end{aligned}$$

Similarly, the optimal mass of content providers is unchanged, i.e. $N_s^{E1} = N_s^{E1}(\tilde{N}_b)$, the profit-maximizing program when forcing platform to charge content providers a zero access fee is,

$$\begin{aligned}
\max_{Q} \quad & \tilde{N}_b\bar{K}_bQ^{\frac{1}{\alpha}}\mathbb{E}_{\theta_s}(\theta_s) - \tilde{N}_b^2 - \phi Q \\
\Rightarrow \quad & Q^{E1}(\tilde{N}_b) = \left[\frac{\alpha\phi}{\tilde{N}_b}\frac{1}{\bar{K}_b\mathbb{E}_{\theta_s}(\theta_s)}\right]^{\frac{\alpha}{1-\alpha}}
\end{aligned}$$

Then,

$$\begin{aligned}
Q^{E1}(\tilde{N}_b) \gtrless Q^*(\tilde{N}_b) \quad & \Leftrightarrow \quad \frac{\alpha\phi}{\tilde{N}_b}\frac{1}{\bar{K}_b\mathbb{E}_{\theta_s}(\theta_s)} \gtrless \frac{2\alpha\phi}{\tilde{N}_b}\frac{\Delta\theta_s}{\bar{\theta}_s^2}\frac{\bar{K}_b+2\bar{K}_s}{(\bar{K}_b+\bar{K}_s)^2} \\
& \Leftrightarrow \quad \frac{1}{4}\frac{\bar{\theta}_s^2}{\Delta\theta_s}\frac{1}{\mathbb{E}_{\theta_s}(\theta_s)} \gtrless \frac{1}{2}\frac{\bar{K}_b(\bar{K}_b+2\bar{K}_s)}{(\bar{K}_b+\bar{K}_s)^2} \\
& \Leftrightarrow \quad \frac{\bar{\theta}_s^2}{\Delta\theta_s}\frac{1}{2(2\bar{\theta}_s-\Delta\theta_s)} \gtrless \underbrace{\frac{1}{2}\frac{\bar{K}_b(\bar{K}_b+2\bar{K}_s)}{(\bar{K}_b+\bar{K}_s)^2}}_{<1} \qquad (12)
\end{aligned}$$

Lhs of equation (12) indeed is less than one by assuming the support of θ_s is wide enough; in particular we assumed $\bar{\theta}_s > 3\underline{\theta}_s$. Moreover, notice the partial derivative of lhs wrt $\underline{\theta}_s$ is $\frac{2\bar{\theta}_s^2\underline{\theta}_s}{(\Delta\theta_s(\bar{\theta}_s+\underline{\theta}_s))^2} > 0$, then lhs of equation (12) increases as $\Delta\theta_s$ decreases. In other words, as $\Delta\theta_s$ decreases lhs of equation (12) approaches to one, and viceversa. Indeed, figure (5) confirms this fact.

One can show rhs of equation (12) is less than one. In addition, the gap betweem 1 and the rhs is decreasing in $\frac{\bar{K}_b}{\bar{K}_s} \in [0,1]$; indeed, in Figure (6) the gap is decreasing in the network effects ratio, e.g. $g\left(\frac{\bar{K}_b}{\bar{K}_s}\right) = 1 - rhs = 1 + \left(\frac{\bar{K}_b}{\bar{K}_s}\right)^2 + 2\frac{\bar{K}_b}{\bar{K}_s} \Rightarrow g'(\cdot) > 0$. Thus, as $\frac{\bar{K}_b}{\bar{K}_s}$ approaches to 1, rhs will move toward one, and viceversa.

Two variables solve equation's (12) inequality: the width of θ_s's support (e.g. $\Delta\theta_s$), and the ratio of network effects (e.g. $\frac{\bar{K}_b}{\bar{K}_s} \in [0,1]$). For this case we show in Figure (7) that the lhs is always greater than the rhs. This result is robust to any level of $\bar{\theta}_s$. Result follows.

Fix $Q = \tilde{Q}$: The profit-maximizing program yield,

$$\begin{aligned}
\max_{N_s,N_b} \quad & \left[(\bar{\theta}_s - \Delta\theta_sN_s)\left(\frac{\bar{K}_b}{2}+\bar{K}_s\right) + \frac{\bar{K}_b\bar{\theta}_s}{2}\right]N_sN_b\tilde{Q}^{\frac{1}{\alpha}} - N_b^2 - \phi\tilde{Q} \\
\Rightarrow \quad & N_s^*(\tilde{Q}) = \frac{\bar{\theta}_s}{\Delta\theta_s}\left(\frac{\bar{K}_b+\bar{K}_s}{\bar{K}_b+2\bar{K}_s}\right) \\
& N_b^*(\tilde{Q}) = \frac{1}{4}\frac{\bar{\theta}_s^2}{\Delta\theta_s}\tilde{Q}^{\frac{1}{\alpha}}\frac{(\bar{K}_b+\bar{K}_s)^2}{\bar{K}_b+2\bar{K}_s}
\end{aligned}$$

On the other hand, the profit-maximizing program, when forcing platform to charge content providers a zero access fee, yield,

$$\begin{aligned}
\max_{N_b} \quad & N_b \bar{K}_b \tilde{Q}^{\frac{1}{\alpha}} \mathbb{E}_{\theta_s}(\theta_s) - N_b^2 - \phi Q \\
\Rightarrow \quad & N_s^{E1}(\tilde{Q}) = 1 \\
& N_b^{E1}(\tilde{Q}) = \frac{\bar{K}_b}{2} \mathbb{E}_{\theta_s}(\theta_s) \tilde{Q}^{\frac{1}{\alpha}}
\end{aligned}$$

Then one obtains,

$$\begin{aligned}
N_b^{E1}(\tilde{Q}) \gtrless N_b^*(\tilde{Q}) \quad & \Leftrightarrow \quad \frac{1}{2} \frac{\bar{K}_b(\bar{K}_b + 2\bar{K}_s)}{(\bar{K}_b + \bar{K}_s)^2} \gtrless \frac{1}{4} \frac{\bar{\theta}_s^2}{\Delta\theta_s} \frac{1}{\mathbb{E}_{\theta_s}(\theta_s)} \\
& \Leftrightarrow \quad \underbrace{\frac{1}{2} \frac{\bar{K}_b(\bar{K}_b + 2\bar{K}_s)}{(\bar{K}_b + \bar{K}_s)^2}}_{<1} \gtrless \underbrace{\frac{\bar{\theta}_s^2}{\Delta\theta_s} \frac{1}{2(2\bar{\theta}_s - \Delta\theta_s)}}_{<1}
\end{aligned}$$

Notice above's inequality is the opposite obtained in equation (12), then we conclude the lhs is lower than the rhs for any value of $\Delta\theta_s$ and $\frac{\bar{K}_b}{\bar{K}_s}$. Result follows. □

Proof of Proposition 1

Proof. First bullet.

$$N_s^{E1} \gtrless N_s^* \quad \Leftrightarrow \quad 1 \gtrless \frac{\bar{\theta}_s}{\Delta\theta_s} \underbrace{\frac{\bar{K}_b + \bar{K}_s}{\bar{K}_b + 2\bar{K}_s}}_{<1}$$

recall we need to assume here the support of random variable θ_s, i.e. $\Delta\theta_s$, is big enough to guarantee $N_s \in [0, 1]$. Given this assumption is immediate that $N_s^{E1} > N_s^*$.

Second bullet.

$$\begin{aligned}
Q^{E1} \gtrless Q^* \quad & \Leftrightarrow \quad \frac{\bar{\theta}_s^2}{\Delta\theta_s} \frac{1}{4\mathbb{E}_{\theta_s}(\theta_s)} \gtrless \frac{\bar{K}_b(\bar{K}_b + 2\bar{K}_s)}{2(\bar{K}_b + \bar{K}_s)^2} \\
& \Leftrightarrow \quad \frac{\bar{\theta}_s^2}{\Delta\theta_s} \frac{1}{2(2\bar{\theta}_s - \Delta\theta_s)} \gtrless \underbrace{\frac{\bar{K}_b(\bar{K}_b + 2\bar{K}_s)}{2(\bar{K}_b + \bar{K}_s)^2}}_{<1} \qquad (13)
\end{aligned}$$

Like in Lemma (5) is possible to solve the inequality of equation (13); in particular, we showed the lhs is greater than the rhs for any level of $\Delta\theta_s$ and $\frac{\bar{K}_b}{\bar{K}_s}$.

Last bullet.

$$\begin{aligned}
N_b^{E1} \gtrless N_b^* \quad & \Leftrightarrow \quad 2\left(\frac{\bar{\theta}_s^2}{\Delta\theta_s} \frac{1}{4\mathbb{E}_{\theta_s}(\theta_s)}\right) \gtrless \frac{\bar{K}_b(\bar{K}_b + 2\bar{K}_s)}{(\bar{K}_b + \bar{K}_s)^2} \\
& \Leftrightarrow \quad \underbrace{\frac{\bar{\theta}_s^2}{\Delta\theta_s} \frac{1}{2(2\bar{\theta}_s - \Delta\theta_s)}}_{<1} \gtrless \underbrace{\frac{1}{2} \frac{\bar{K}_b(\bar{K}_b + 2\bar{K}_s)}{(\bar{K}_b + \bar{K}_s)^2}}_{<1}
\end{aligned}$$

Above's equation is the same of equation (12), there we showed that the lhs is greater than the rhs. Then, $N_b^{E1} > N_b^*$ and result follows. □

Proof of Proposition 2

Proof. The welfare function is,

$$\begin{aligned}
\mathcal{W}(N_s, N_b, Q) \quad & = \quad N_b \mathbb{E}_{\theta_s,\theta_b}[N_s \bar{K}_b \theta_s Q^{\frac{1}{\alpha}}] + N_s \mathbb{E}_{\theta_s,\theta_b}[N_b \bar{K}_s \theta_s Q^{\frac{1}{\alpha}}] - \int_0^{N_b} \eta d\eta - \phi Q \\
& = \quad N_s N_b \mathbb{E}_{\theta_s}(\theta_s) Q^{\frac{1}{\alpha}} (\bar{K}_b + \bar{K}_s) - \frac{N_b^2}{2} - \phi Q
\end{aligned}$$

Replacing (N_s^*, N_b^*, Q^*) and $(N_s^{E1}, N_b^{E1}, Q^{E1})$ one obtains,

$$\mathcal{W}(N_s^*, N_b^*, Q^*) \gtrless \mathcal{W}(N_s^{E1}, N_b^{E1}, Q^{E1}) \quad \Leftrightarrow \quad \mathbb{E}_{\theta_s}(\theta_s)(\bar{K}_b + \bar{K}_s)A \gtrless \frac{B}{4} + \frac{C}{\alpha} \tag{14}$$

$$A = \frac{\bar{\theta}_s}{2\Delta\theta_s}\left(\frac{\bar{K}_b + \bar{K}_s}{\bar{K}_b + 2\bar{K}_s}\right)\left[16\left(\frac{\Delta\theta_s}{\bar{\theta}_s^2}\frac{\bar{K}_b + 2\bar{K}s}{(\bar{K}_b + \bar{K}_s)^2}\right)^{2+\alpha}\right]^{\frac{1}{2-\alpha}} - \left[\frac{1}{(\bar{K}_b\mathbb{E}_{\theta_s}(\theta_s))^{2+\alpha}}\right]^{\frac{1}{2-\alpha}}$$

$$B = \frac{1}{4}\left[4\left(\frac{\Delta\theta_s}{\bar{\theta}_s^2}\frac{\bar{K}_b + 2\bar{K}_s}{(\bar{K}_b + \bar{K}_s)^2}\right)^{\alpha}\right]^{\frac{2}{2-\alpha}} - \left[\frac{1}{(\bar{K}_b\mathbb{E}_{\theta_s}(\theta_s))^{\alpha}}\right]^{\frac{2}{2-\alpha}}$$

$$C = \left[4\left(\frac{\Delta\theta_s}{\bar{\theta}_s^2}\frac{\bar{K}_b + 2\bar{K}_s}{(\bar{K}_b + \bar{K}_s)^2}\right)^{2}\right]^{\frac{\alpha}{2-\alpha}} - \left[\frac{1}{(\bar{K}_b\mathbb{E}_{\theta_s}(\theta_s))^{2}}\right]^{\frac{\alpha}{2-\alpha}}$$

Signs are determined by,

$$sgn(A) \gtrless 0 \quad \Leftrightarrow \quad \left[\frac{1}{2}\left(\frac{\bar{K}_b + \bar{K}_s}{\bar{K}_b + 2\bar{K}_s}\right)^{2-\alpha}\right]^{\frac{1}{2+\alpha}}\frac{\bar{K}_b}{2}\frac{\bar{K}_b + 2\bar{K}_s}{(\bar{K}_b + \bar{K}_s)^2} \gtrless \frac{\bar{\theta}_s}{4\mathbb{E}_{\theta_s}(\theta_s)}\left(\frac{\bar{\theta}_s}{\Delta\theta_s}\right)^{\frac{2\alpha}{2+\alpha}}$$

$$sgn(B) \gtrless 0 \quad \Leftrightarrow \quad \frac{\bar{K}_b}{2}\frac{\bar{K}_b + 2\bar{K}_s}{(\bar{K}_b + \bar{K}_s)^2} \gtrless \frac{1}{4}\frac{\bar{\theta}_s^2}{\Delta\theta_s}\frac{1}{\mathbb{E}_{\theta_s}(\theta_s)}$$

$$sgn(C) \gtrless 0 \quad \Leftrightarrow \quad sgn(B) \gtrless 0$$

Notice the inequality that determines $sgn(B)$ and $sgn(C)$ is the same as in equation (12). Then, we conclude that $sgn(B) < 0$ and $sgn(C) < 0$. To obtain sharp results one should confirm $sgn(A) > 0$, but in Figure (8) we show this is impossible[23].

The sign of term A depends on two variables, e.g. $\Delta\theta_s$ and $\frac{\bar{K}_b}{\bar{K}_s}$. Figure (8) shows the effect of θ_s's support dominates de effect the effect of the ratio of network effects. Then, $sgn(A) < 0$ too and the welfare comparison is ambiguous. Moreover, when we take equation (14) to the computer we obtain that welfare is always higher when the profit-maximizing platform is forced to charge content providers a zero access fee[24]. Indeed, figure (9) shows $\mathcal{W}^{E1} > \mathcal{W}^*$ for any value of $\underline{\theta}_s \in [0, \bar{\theta}_s]$ and $\frac{\bar{K}_b}{\bar{K}_s} \in [0, 1]$. Result follows.

Proof of Lemma 6

Proof. <u>Fix $N_b = \tilde{N}_b$</u>. The optimal levels of content providers and investment from the profit-maximizing platform are,

$$N_s^*(\tilde{N}_b) = \frac{\bar{\theta}_s}{\Delta\theta_s}\left(\frac{\bar{K}_b + \bar{K}_s}{\bar{K}_b + 2\bar{K}_s}\right)$$

$$Q^*(\tilde{N}_b) = \left[\frac{2\alpha\phi}{\tilde{N}_b}\frac{\Delta\theta_s}{\bar{\theta}_s^2}\frac{(\bar{K}_b + 2\bar{K}_s)}{(\bar{K}_b + \bar{K}_s)^2}\right]^{\frac{\alpha}{1-\alpha}}$$

The corresponding optimal levels from the profit-maximizing platform forced to charge end-users zero access fee are,

$$\max_{N_s, Q} \quad N_s\tilde{N}_b\bar{K}_sQ^{\frac{1}{\alpha}}(\bar{\theta}_s - \Delta N_s) - \alpha Q$$

$$\Rightarrow \quad N_s^{E2}(\tilde{N}_b) = \frac{\bar{\theta}_s}{2\Delta\theta_s}$$

$$Q^{E2}(\tilde{N}_b) = \left[\frac{4\alpha\phi}{\tilde{N}_b\bar{K}_s}\frac{\Delta\theta_s}{\bar{\theta}_s^2}\right]^{\frac{\alpha}{1-\alpha}}$$

The comparing,

$$Q^{E2}(\tilde{N}_b) \gtrless Q^*(\tilde{N}_b) \quad \Leftrightarrow \quad 1 \gtrless \underbrace{\frac{\bar{K}_s(\bar{K}_b + 2\bar{K}_s)}{2(\bar{K}_b + \bar{K}_s)^2}}_{<1}$$

Result follows.

[23] This result is robust for any value of α.
[24] This result is robust for any value of α.

Fix $Q = \tilde{Q}$. Optimal level of end-users and content providers joining the network with a profit-maximizing platform are,

$$\max_{N_s, N_b} \quad \left[(\bar{\theta}_s - \Delta\theta_s N_s) \left(\frac{\bar{K}_b}{2} + \bar{K}_s \right) + \frac{\bar{K}_b \bar{\theta}_s}{2} \right] N_s N_b \tilde{Q}^{\frac{1}{\alpha}} - N_b^2 - \phi Q$$

$$\Rightarrow \quad N_b^*(\tilde{Q}) = \frac{\bar{\theta}_s}{\Delta\theta_s} \left(\frac{\bar{K}_b + \bar{K}_s}{\bar{K}_b + 2\bar{K}_s} \right)$$

$$N_b^*(\tilde{Q}) = \frac{1}{4} \frac{\bar{\theta}_s^2}{\Delta\theta_s} \tilde{Q}^{\frac{1}{\alpha}} \frac{(\bar{K}_b + \bar{K}_s)^2}{(\bar{K}_b + 2\bar{K}_s)}$$

Optimal levels from a profit-maximizing platform force to charge zero access fee are,

$$N_s^{E2}(\tilde{Q}) = \frac{\bar{\theta}_s}{\Delta\theta_s} \beta$$

$$N_b^{E2}(\tilde{Q}) = \frac{\bar{\theta}_s^2}{\Delta\theta_s} \beta \bar{K}_b \frac{\tilde{Q}^{\frac{1}{\alpha}}}{2} (2 - \beta)$$

Comparing one obtains,

$$N_b^{E2}(\tilde{Q}) \gtrless N_b^*(\tilde{Q}) \quad \Leftrightarrow \quad 4\beta(2-\beta) \gtrless \underbrace{\frac{2(\bar{K}_b + \bar{K}_s)^2}{\bar{K}_b(\bar{K}_b + 2\bar{K}_s)}}_{>1}$$

In figure (10) we obtain $N_b^{E2}(\tilde{Q}) < N_b^*(\tilde{Q})$ if $\beta < 0.45$ or if $\frac{\bar{K}_b}{\bar{K}_s} < 0.45$. If any of both conditions fail, then for several combinations of $(\beta, \frac{\bar{K}_b}{\bar{K}_s})$ will be that $N_b^{E2}(\tilde{Q}) > N_b^*(\tilde{Q})$; in particular, if $\beta > 0.45$ one need $\frac{\bar{K}_b}{\bar{K}_s}$ is *high enough*, or if $\frac{\bar{K}_b}{\bar{K}_s} > 0.4$ one need β is high enough.

For this model, as $\beta = \frac{4.88}{8} = 0.61$, we obtain $N_b^{E2}(\tilde{Q}) > N_b^*(\tilde{Q})$ if $\frac{\bar{K}_b}{\bar{K}_s}$ is *high enough*; and $N_b^{E2}(\tilde{Q}) < N_b^*(\tilde{Q})$ if $\frac{\bar{K}_b}{\bar{K}_s}$ is *not high enough*. □

Proof of Proposition 3

Proof. First bullet. Comparing optimal values one obtains,

$$N_s^{E2} \gtrless N_s^* \quad \Leftrightarrow \quad \underbrace{\frac{\bar{K}_s}{\bar{K}_b}}_{>1} \gtrless \frac{(1-\beta)}{2\beta - 1}$$

We obtain in figure (11) that $N_s^{E2} > N_s^*$ if $\beta > \frac{2}{3}$, and $N_s^{E2} < N_s^*$ if $\beta < \frac{1}{2}$. Finally, for $\beta \in [\frac{1}{2}, \frac{2}{3}]$, if $\frac{\bar{K}_b}{\bar{K}_s}$ is close to zero and β close to $\frac{2}{3}$, then $N_s^{E2} > N_s^*$; and if $\frac{\bar{K}_b}{\bar{K}_s}$ is close to one and β close to $\frac{1}{2}$, then $N_s^{E2} < N_s^*$.

For this model $\beta = \frac{4.8}{8} = 0.61$, then $N_s^* > N_s^{E2}$ if $\frac{\bar{K}_b}{\bar{K}_s}$ is close to 1, otherwise will be $N_s^* < N_s^{E2}$.

Second bullet. Comparing the optimal values one obtains,

$$Q^{E2} \gtrless Q^* \quad \Leftrightarrow \quad \frac{1}{32\beta^2(2-\beta)(1-\beta)} \gtrless \underbrace{\frac{\bar{K}_s}{\bar{K}_b}}_{>1} \left(\underbrace{\frac{\bar{K}_b(\bar{K}_b + 2\bar{K}_s)}{2(\bar{K}_b + \bar{K}_s)^2}}_{<1} \right)^2$$

Figure (12) shows that $Q^{E2} > Q^*$ for any value of $\beta \in [0,1]$ and $\frac{\bar{K}_b}{\bar{K}_s} \in [0,1]$.

Third bullet. Comparing optimal values one obtains,

$$N_b^{E2} \gtrless N_b^* \quad \Leftrightarrow \quad \frac{(2-\beta)^{1-\alpha}}{2^{1+2\alpha}\beta^{\alpha}(1-\beta)} \gtrless \underbrace{\frac{\bar{K}_s}{\bar{K}_b}}_{>1} \left(\underbrace{\frac{\bar{K}_b(\bar{K}_b + 2\bar{K}_s)}{2(\bar{K}_b + \bar{K}_s)^2}}_{<1} \right)^{\alpha}$$

Figure (13) shows $N_b^{E2} > N_b^*$ if $\alpha < 1$ and $\frac{\bar{K}_b}{\bar{K}_s}$ *close enough* to one. On the other hand, if $\alpha < 1$ and $\frac{\bar{K}_b}{\bar{K}_s}$ *not close enough* to one, there are several combinations $(\beta, \frac{\bar{K}_b}{\bar{K}_s})$ such that $N_b^* > N_b^{E2}$.

For this model $\beta = \frac{4.88}{8} = 0.61$, will obtain $N_b^{E2} > N_b^*$ if $\frac{\bar{K}_b}{\bar{K}_s}$ is high enough (e.g. $\frac{\bar{K}_b}{\bar{K}_s} \approx 0.6$), and will obtain the opposite result otherwise. □

Proof of Proposition 4

Proof. Again welfare function is,

$$\mathcal{W}(N_s, N_b, Q) \quad = \quad N_s N_b \mathbb{E}_{\theta_s}(\theta_s) Q^{\frac{1}{\alpha}} (\bar{K}_b + \bar{K}_s) - \frac{N_b^2}{2} - \phi Q$$

Replacing (N_s^*, N_b^*, Q^*) and $(N_s^{E2}, N_b^{E2}, Q^{E2})$ one obtains,

$$\mathcal{W}(N_s^*, N_b^*, Q^*) \gtrless \mathcal{W}(N_s^{E2}, N_b^{E2}, Q^{E2}) \quad \Leftrightarrow \quad \mathbb{E}_{\theta_s}(\theta_s)(\bar{K}_b + \bar{K}_s)A \gtrless \frac{\bar{\theta}_s}{4}B + \frac{2\bar{\theta}_s}{\alpha}C \tag{15}$$

$$A \quad = \quad \frac{64^{\frac{1}{2-\alpha}}}{2}\left(\frac{\bar{K}_b + 2\bar{K}_s}{\bar{K}_b + \bar{K}_s}\right)^{\frac{2\alpha}{2-\alpha}}\left(\frac{1}{\bar{K}_b + \bar{K}_s}\right)^{\frac{2+\alpha}{2-\alpha}} - \beta\left[\frac{(\bar{K}_b(2-\beta))^{1-\alpha}}{\bar{K}_s(1-\beta)}\frac{1}{\bar{K}_b\bar{K}_s\beta^{2+\alpha}(2-\beta)(1-\beta)}\right]^{\frac{1}{2-\alpha}}$$

$$B \quad = \quad \frac{1}{4}\left[8\left(\frac{\bar{K}_b + 2\bar{K}_s}{(\bar{K}_b + \bar{K}_s)^2}\right)^{\alpha}\right]^{\frac{2}{2-\alpha}} - \left[\frac{(\bar{K}_b(2-\beta))^{1-\alpha}}{\bar{K}_s\beta^{\alpha}(1-\beta)}\right]^{\frac{2}{2-\alpha}}$$

$$C \quad = \quad \left[8\left(\frac{\bar{K}_b + 2\bar{K}_s}{(\bar{K}_b + \bar{K}_s)^2}\right)^{2}\right]^{\frac{\alpha}{2-\alpha}} - \left[\frac{1}{\bar{K}_b\bar{K}_s\beta^{2}(2-\beta)(1-\beta)}\right]^{\frac{\alpha}{2-\alpha}}$$

Signs are determined by,

$$sgn(A) \gtrless 0 \quad \Leftrightarrow \quad \left(\frac{\bar{K}_b}{2}\frac{\bar{K}_b + 2\bar{K}_s}{(\bar{K}_b + \bar{K}_s)^2}\right)^{2\alpha}\left(\frac{\bar{K}_b}{\bar{K}_b + \bar{K}_s}\right)^{2-\alpha}\left(\frac{\bar{K}_s}{\bar{K}_b}\right)^{2} \gtrless \frac{2^{2-3\alpha}}{64\beta^{2\alpha}(2-\beta)^{\alpha}(1-\beta)^2}$$

$$sgn(B) \gtrless 0 \quad \Leftrightarrow \quad \frac{\bar{K}_s}{\bar{K}_b}\left(\frac{\bar{K}_b}{2}\frac{\bar{K}_b + 2\bar{K}_s}{(\bar{K}_b + \bar{K}_s)^2}\right)^{\alpha} \gtrless \frac{1}{2^{2\alpha+1}}\frac{(2-\beta)^{1-\alpha}}{\beta^{\alpha}(1-\beta)}$$

$$sgn(C) \gtrless 0 \quad \Leftrightarrow \quad \frac{\bar{K}_s}{\bar{K}_b}\left(\frac{\bar{K}_b}{2}\frac{\bar{K}_b + 2\bar{K}_s}{(\bar{K}_b + \bar{K}_s)^2}\right)^{2} \gtrless \frac{1}{32\beta^2(2-\beta)(1-\beta)}$$

By figure (12) we have that $sgn(C) < 0$, and by figure (13) for this model (e.g. $\beta = \frac{4.88}{8} = 0.61$) we have that $sgn(B) < 0$ if $\frac{\bar{K}_b}{\bar{K}_s}$ is high enough (e.g. $\frac{\bar{K}_b}{\bar{K}_s} \approx 0.6$), and $sgn(B) > 0$ otherwise. Finally, figure (14) show $sgn(A)$ depend on the ratio of network effects, on β and α. For α low, as $\beta = 0.61$, then $sgn(A) < 0$ if $\frac{\bar{K}_b}{\bar{K}_s}$ is high, and $sgn(A) > 0$ otherwise; if α is closer to one, then $sgn(A) < 0$ if $\frac{\bar{K}_b}{\bar{K}_s}$ is high or low, and $sgn(A) > 0$ otherwise.

Sign of equation (15) depend on two variables: the support of θ_s, and the ratio of the network effects. Also we know the support of both variables, while for the former must hold that $\bar{\theta}_s > 3\underline{\theta}_s$, for the latter is the unit interval. In figure (15) we show that if $\frac{\bar{K}_b}{\bar{K}_s} > 0.55$ then $\mathcal{W}^{E2} > \mathcal{W}^*$, and otherwise will be $\mathcal{W}^{E2} < \mathcal{W}^*$. □

Proof of Lemma 6

Proof. First bullet: Comparing the optimal levels,

$$N_s^{fp} \gtrless N_s^* \quad \Leftrightarrow \quad 1 \gtrless \frac{\bar{\theta}_s}{\Delta\theta_s}\left(\frac{\bar{K}_b + \bar{K}_s}{\bar{K}_b + 2\bar{K}_s}\right) < 1$$

Result follows.

Second bullet: Comparing the optimal levels,

$$N_b^{fp} \gtrless N_b^* \quad \Leftrightarrow \quad \left[\frac{(\bar{K}_b\mathbb{E}_{\theta_s}(\theta_s))^{1-\alpha}\alpha\phi}{(2\bar{K}_s + \bar{K}_b)\mathbb{E}_{\theta_s}(\theta_s)}\right]^{\frac{1}{2-\alpha}} \gtrless \frac{1}{4}\left[8\alpha\phi\left(\frac{\Delta\theta_s}{\bar{\theta}_s^2}\frac{\bar{K}_b + 2\bar{K}_s}{(\bar{K}_b + \bar{K}_s)^2}\right)^{\alpha}\right]^{\frac{1}{2-\alpha}}$$

$$\Leftrightarrow \quad \frac{\bar{\theta}_s^2}{\Delta\theta_s}\frac{1}{4\mathbb{E}_{\theta_s}(\theta_s)} \gtrless \left(\frac{2\bar{K}_s + \bar{K}_b}{2^{1-\alpha}\bar{K}_b}\right)^{\frac{1}{\alpha}}\left(\frac{\bar{K}_b(\bar{K}_b + 2\bar{K}_s)}{2(\bar{K}_b + \bar{K}_s)^2}\right)$$

Figure (21) show that $N_b^* > N_b^{fp}$ for any value of $\Delta\theta_s$, given that condition $\bar{\theta}_s > 3\underline{\theta}_s$ hold, and $\frac{\bar{K}_b}{\bar{K}_s} \in [0, 1]$. Result is robust for any value of $\alpha \in [0, 1]$.

Third bullet: Comparing the optimal levels,

$$Q^{fp} \gtreqless Q^* \quad \Leftrightarrow \quad \left[\frac{\alpha\phi}{\bar{K}_b(2\bar{K}_s+\bar{K}_b)(\mathbb{E}_{\theta_s}(\theta_s))^2}\right]^{\frac{\alpha}{2-\alpha}} \gtreqless \left[8\alpha\phi\left(\frac{\Delta\theta_s}{\bar{\theta}_s^2}\frac{\bar{K}_b+2\bar{K}_s}{(\bar{K}_b+\bar{K}_s)^2}\right)^2\right]^{\frac{\alpha}{2-\alpha}}$$
$$\Leftrightarrow \quad \frac{\bar{\theta}_s^2}{\Delta\theta_s}\frac{1}{4\mathbb{E}_{\theta_s}(\theta_s)} \gtreqless 2^{\frac{1}{2}}\left(\frac{2\bar{K}_s+\bar{K}_b}{\bar{K}_b}\right)^{\frac{1}{2}}\left(\frac{\bar{K}_b(\bar{K}_b+2\bar{K}_s)}{2(\bar{K}_b+\bar{K}_s)^2}\right)$$

Figure (22) show that $Q^{fp} > Q^*$ for low enough values of $\frac{\bar{K}_b}{\bar{K}_s}$, otherwise will happen that $Q^{fp} < Q^*$. □

Proof of Lemma 8

Proof. Again welfare function is,

$$\mathcal{W}(N_s, N_b, Q) \quad = \quad N_s N_b \mathbb{E}_{\theta_s}(\theta_s) Q^{\frac{1}{\alpha}}(\bar{K}_b+\bar{K}_s) - \frac{N_b^2}{2} - \phi * Q$$

Replacing (N_s^*, N_b^*, Q^*) and $(N_s^{fp}, N_b^{fp}, Q^{fp})$ one obtains,

$$\mathcal{W}(N_s^*, N_b^*, Q^*) \gtreqless \mathcal{W}(N_s^{fp}, N_b^{fp}, Q^{fp}) \quad \Leftrightarrow \quad \mathbb{E}_{\theta_s}(\theta_s)(\bar{K}_b+\bar{K}_s)A \gtreqless \frac{B}{2}+\frac{C}{\alpha}$$
$$A \quad = \quad \frac{\bar{\theta}_s}{\Delta\theta_s}\left(\frac{\bar{K}_s+\bar{K}_b}{\bar{K}_b+2\bar{K}_s}\right)\frac{1}{4}\left[(8\alpha\phi)^2\left(\frac{\Delta\theta_s}{\bar{\theta}_s^2}\frac{\bar{K}_b+2\bar{K}_s}{(\bar{K}_b+\bar{K}_s)^2}\right)^{2+\alpha}\right]^{\frac{1}{2-\alpha}} - \left[\frac{(\alpha\phi)^2}{(\bar{K}_b+2\bar{K}_s)^2(\mathbb{E}_{\theta_s}(\theta_s))^{2+\alpha}\bar{K}_b^{\alpha}}\right]^{\frac{1}{2-\alpha}}$$
$$B \quad = \quad \frac{1}{16}\left[8\alpha\phi\left(\frac{\Delta\theta_s}{\bar{\theta}_s^2}\frac{\bar{K}_b+2\bar{K}_s}{(\bar{K}_b+\bar{K}_s)^2}\right)^{\alpha}\right]^{\frac{2}{2-\alpha}} - \left[\frac{\alpha\phi\bar{K}_b^{1-\alpha}}{(\bar{K}_b+2\bar{K}_s)(\mathbb{E}_{\theta_s}(\theta_s))^{\alpha}}\right]^{\frac{2}{2-\alpha}}$$
$$C \quad = \quad \left[8\alpha\phi\left(\frac{\Delta\theta_s}{\bar{\theta}_s^2}\frac{\bar{K}_b+2\bar{K}_s}{(\bar{K}_b+\bar{K}_s)^2}\right)^2\right]^{\frac{\alpha}{2-\alpha}} - \left[\frac{\alpha\phi}{\bar{K}_b(\bar{K}_b+2\bar{K}_s)(E_{\theta_s}(\theta_s))^2}\right]^{\frac{\alpha}{2-\alpha}}$$

Signs are determined by,

$$sgn(A) \gtreqless 0 \quad \Leftrightarrow \quad \frac{\bar{K}_b(\bar{K}_b+2\bar{K}_s)}{2(\bar{K}_b+\bar{K}_s)^2} \gtreqless \left(\frac{\bar{\theta}_s^2}{\Delta\theta_s}\frac{1}{4\mathbb{E}_{\theta_s}(\theta_s)}\right)\left(\frac{\Delta\theta_s}{\bar{\theta}_s}\right)^{\frac{2-\alpha}{2+\alpha}}$$
$$sgn(B) \gtreqless 0 \quad \Leftrightarrow \quad \left(\frac{\bar{K}_b(\bar{K}_b+2\bar{K}_s)}{2(\bar{K}_b+\bar{K}_s)^2}\right)\left(\frac{\bar{K}_b+2\bar{K}_s}{2^{1-\alpha}\bar{K}_b}\right)^{\frac{1}{\alpha}} \gtreqless \frac{\bar{\theta}_s^2}{\Delta\theta_s}\frac{1}{4\mathbb{E}_{\theta_s}(\theta_s)}$$
$$sgn(C) \gtreqless 0 \quad \Leftrightarrow \quad \left(\frac{\bar{K}_b(\bar{K}_b+2\bar{K}_s)}{2(\bar{K}_b+\bar{K}_s)^2}\right)2^{\frac{1}{2}}\left(\frac{\bar{K}_b+2\bar{K}_s}{\bar{K}_b}\right)^{\frac{1}{2}} \gtreqless \frac{\bar{\theta}_s^2}{\Delta\theta_s}\frac{1}{4\mathbb{E}_{\theta_s}(\theta_s)}$$

Figure (21) guarantee that $sgn(B) > 0$, and figure (22) show $sgn(C) < 0$ for low values of $\frac{\bar{K}_b}{\bar{K}_s}$, and $sgn(C) > 0$ otherwise. Also can be shown $sgn(A) < 0$ for low values of $\frac{\bar{K}_b}{\bar{K}_s}$, and $sgn(A) > 0$ otherwise.

Finally, figure (23) shows that $\mathcal{W}^* > \mathcal{W}^{fp}$ for low values of $\Delta\theta_s$ or $\frac{\bar{K}_b}{\bar{K}_s}$. On the other hand, $\mathcal{W}^* < \mathcal{W}^{fp}$ for high values of $\Delta\theta_s$ and $\frac{\bar{K}_b}{\bar{K}_s}$. □

Remark 1. *Two intermediate results given* $\beta = 0.61$,

1. *Fix an arbitrary level of mass of B's, e.g.* $\tilde{N}_b$, *and let platform choose* N_s *and* Q.
   - *If* $\frac{\bar{K}_b}{\bar{K}_s} < 0.5$, *then* $Q^{E2}(\tilde{N}_b) > Q^{E1}(\tilde{N}_b)$; *and if* $\frac{\bar{K}_b}{\bar{K}_s} > 0.6$, *then* $Q^{E2}(\tilde{N}_b) < Q^{E1}(\tilde{N}_b)$.
   - *If* $\frac{\bar{K}_b}{\bar{K}_s} \in [0.5, 0.6]$. *Will obtain* $Q^{E2}(\tilde{N}_b) < Q^{E1}(\tilde{N}_b)$ *if network effect is close to 0.5 and* θ_s*'s support is low as possible; and* $Q^{E2}(\tilde{N}_b) < Q^{E1}(\tilde{N}_b)$ *if network effect is close to 0.6 and the support as big as possible.*
2. *Fix an arbitrary level of investment, e.g.* $\tilde{Q}$, *and let platforms choose* N_s *and* N_b.
   - $N_b^{E2}(\tilde{Q}) > N_b^{E1}(\tilde{Q})$.

Proof. Fix $N_b = \tilde{N}_b$. The optimal level of choice variables from the profit-maximizing platform forced to charge content providers zero access fee will be,

$$\begin{aligned} \max_{Q} \quad & \tilde{N}_b \bar{K}_b Q^{\frac{1}{\alpha}} \mathbb{E}_{\theta_s}(\theta_s) - \tilde{N}_b^2 - \phi Q \\ \Rightarrow \quad & N_s^{E1}(\tilde{N}_b) = 1 \\ & Q^{E1}(\tilde{N}_b) = \left[\frac{\alpha\phi}{\tilde{N}_b} \frac{1}{\bar{K}_b \mathbb{E}_{\theta_s}(\theta_s)} \right]^{\frac{\alpha}{1-\alpha}} \end{aligned}$$

The corresponding values from a profit-maximizing platform forced to charge end-users zero access fee will be,

$$\begin{aligned} \max_{N_s, Q} \quad & N_s \tilde{N}_b \bar{K}_s Q^{\frac{1}{\alpha}} (\bar{\theta}_s - \Delta\theta_s N_s) - \phi Q \\ \Rightarrow \quad & N_s^{E2}(\tilde{N}_b) = \frac{\bar{\theta}_s}{2\Delta\theta_s} \\ & Q^{E2}(\tilde{N}_b) = \left[\frac{4\alpha\phi}{\tilde{N}_b \bar{K}_s} \frac{\Delta\theta_s}{\bar{\theta}_s^2} \right]^{\frac{\alpha}{1-\alpha}} \end{aligned}$$

Comparing,

$$\begin{aligned} Q^{E2}(\tilde{N}_b) \gtrless Q^{E1}(\tilde{N}_b) \quad \Leftrightarrow \quad & \frac{\bar{\theta}_s^2}{\Delta\theta_s} \frac{1}{4\mathbb{E}_{\theta_s}(\theta_s)} \gtrless \frac{\bar{K}_b}{\bar{K}_s} \\ \Leftrightarrow \quad & \underbrace{\frac{\bar{\theta}_s^2}{\Delta\theta_s} \frac{1}{2(2\bar{\theta}_s - \Delta\theta_s)}}_{<1} \gtrless \underbrace{\frac{\bar{K}_b}{\bar{K}_s}}_{<1} \end{aligned}$$

Figure (16) shows $\Delta\theta_s$ plays no role if $\frac{\bar{K}_b}{\bar{K}_s} < 0.5$ or if $\frac{\bar{K}_b}{\bar{K}_s} > 0.6$; in particular, if $\frac{\bar{K}_b}{\bar{K}_s} < 0.5$ then $Q^{E2}(\tilde{N}_b) > Q^{E1}(\tilde{N}_b)$, and if $\frac{\bar{K}_b}{\bar{K}_s} < 0.6$ then $Q^{E2}(\tilde{N}_b) < Q^{E1}(\tilde{N}_b)$. On the other hand, if $\frac{\bar{K}_b}{\bar{K}_s} \in [0.5, 0.6]$ one obtain that $Q^{E2}(\tilde{N}_b) > Q^{E1}(\tilde{N}_b)$ if the ratio of network effects is close to 0.5 and $\Delta\theta_s$ is as low as possible (guaranteeing condition $\bar{\theta}_s > 3\underline{\theta}_s$ hold); and $Q^{E2}(\tilde{N}_b) < Q^{E1}(\tilde{N}_b)$ if the ratio of network effects is close to 0.6 and $\Delta\theta_s$ is as big as possible.

Fix $Q = \tilde{Q}$. The optimal level of choice variables from the profit-maximizing platform forced to charge content providers zero access fee will be,

$$\begin{aligned} \max_{N_b} \quad & N_b \bar{K}_b \tilde{Q}^{\frac{1}{\alpha}} \mathbb{E}_{\theta_s}(\theta_s) - N_b^2 - \phi Q \\ \Leftrightarrow \quad & N_s^{E1}(\tilde{Q}) = 1 \\ & N_b^{E1}(\tilde{Q}) = \frac{\bar{K}_b}{2} \mathbb{E}_{\theta_s}(\theta_s) Q^{\frac{1}{\alpha}} \end{aligned}$$

The corresponding optimal levels for the profit-maximizing platform forced to charge end-users zero access fee is,

$$\begin{aligned} N_s^{E2}(\tilde{Q}) &= \frac{\bar{\theta}_s}{\Delta\theta_s} \beta \\ N_b^{E2}(\tilde{Q}) &= \frac{\bar{\theta}_s^2}{\Delta\theta_s} \frac{\bar{K}_b}{s} \tilde{Q}^{\frac{1}{\alpha}} \beta(2-\beta) \end{aligned}$$

Comparing one obtains,

$$\begin{aligned} N_b^{E2}(\tilde{Q}) \gtrless N_s^{E1}(\tilde{Q}) \quad \Leftrightarrow \quad & \frac{1}{4} \frac{\bar{\theta}_s^2}{\Delta\theta_s} \frac{1}{\mathbb{E}_{\theta_s}(\theta_s)} \gtrless \frac{1}{4\beta(2-\beta)} \\ \Leftrightarrow \quad & \underbrace{\frac{\bar{\theta}_s^2}{\Delta\theta_s} \frac{1}{2(2\bar{\theta}_s - \Delta\theta_s)}}_{<1} \gtrless \frac{1}{4\beta(2-\beta)} \end{aligned}$$

Figure (17) shows variable $\Delta\theta_s$ is not relevant for $\beta < 0.25$ or for $\beta > 0.3$, in particular, if $\beta < 0.25$ then $N_b^{E1}(\tilde{Q}) > N_b^{E2}(\tilde{Q})$, and if $\beta > 0.3$ then $N_b^{E1}(\tilde{Q}) < N_b^{E2}(\tilde{Q})$. On the other hand, if β is between this thin interval, will obtain $N_b^{E1}(\tilde{Q}) > N_b^{E2}(\tilde{Q})$ if $\Delta\theta_s$ is as big as possible; and will obtain the opposite result if $\Delta\theta_s$ is as low as possible given that condition $\bar{\theta}_s > 3\underline{\theta}_s$ hold.

For this model, as $\beta = \frac{4.88}{8} = 0.61$, then $N_b^{E2}(\tilde{Q}) > N_b^{E1}(\tilde{Q})$. □

Proof of Proposition 5

Proof. First bullet. Lets compare N_s's optimal levels,

$$N_s^{E1} \gtrless N_s^{E2} \quad \Leftrightarrow \quad 1 > \frac{\bar{\theta}_s}{\Delta\theta_s}\beta$$

If support of θ_s is wide enough, then result follows.

Comparing N_b's optimal levels,

$$\begin{aligned} N_b^{E1} \gtrless N_b^{E2} \quad &\Leftrightarrow \quad \left(\frac{1}{4}\frac{\bar{\theta}_s}{\Delta\theta_s}\frac{\bar{\theta}_s}{\mathbb{E}_{\theta_s}(\theta_s)}\right)^{\alpha} 4^{\alpha}2(\beta(2-\beta))^{\alpha}\left(\frac{1-\beta}{2-\beta}\right) \gtrless \frac{\bar{K}_b}{\bar{K}_s} \\ &\Leftrightarrow \quad \left(\underbrace{\frac{\bar{\theta}_s^2}{\Delta\theta_s}\frac{1}{2(2\bar{\theta}_s-\Delta\theta_s)}}_{<1}\right)^{\alpha} 4^{\alpha}2(\beta(2-\beta))^{\alpha}\left(\frac{1-\beta}{2-\beta}\right) \gtrless \underbrace{\frac{\bar{K}_b}{\bar{K}_s}}_{<1} \end{aligned}$$

This comparison seems complicated since four variables are playing at the same time, e.g. $\Delta\theta_s$, α, β, and $\frac{\bar{K}_b}{\bar{K}_s}$. Simulations in figure (19) show $N_b^{E1} > N_b^{E2}$ for any parameter value, i.e. $\bar{\theta}_s > 3\underline{\theta}_s$ given $\bar{\theta}_s = 100$, and $\frac{\bar{K}_b}{\bar{K}_s} \in [0,1]$. Result is robust for any $\alpha \in [0,1]$, $\beta \in [0,1]$.

Second bullet. Comparing optimal levels,

$$\begin{aligned} Q^{E1} \gtrless Q^{E2} \quad &\Leftrightarrow \quad \left(\frac{1}{4}\frac{\bar{\theta}_s^2}{\Delta\theta_s}\frac{1}{\mathbb{E}_{\theta_s}(\theta_s)}\right)^{2} 8\beta^2(2-\beta)(1-\beta) \gtrless \frac{\bar{K}_b}{\bar{K}_s} \\ &\Leftrightarrow \quad \left(\underbrace{\frac{\bar{\theta}_s^2}{\Delta\theta_s}\frac{1}{2(2\bar{\theta}_s-\Delta\theta_s)}}_{<1}\right)^{2} 32\beta^2(2-\beta)(1-\beta) \gtrless \underbrace{\frac{\bar{K}_b}{\bar{K}_s}}_{<1} \end{aligned}$$

We show in Figure (18) that the only way $Q^{E1} > Q^{E2}$ is if $\frac{\bar{K}_b}{\bar{K}_s}$ almost equal to zero, in any other situation will obtain $Q^{E1} < Q^{E2}$. Moreover, this result is robust to any value of $\Delta\theta_s$ and β. □

Proof of Proposition 6

Proof. Again welfare function is,

$$\mathcal{W}(N_s, N_b, Q) \quad = \quad N_s N_b \mathbb{E}_{\theta_s}(\theta_s) Q^{\frac{1}{\alpha}}(\bar{K}_b + \bar{K}_s) - \frac{N_b^2}{2} - \phi * Q$$

Replacing $(N_s^{E1}, N_b^{E1}, Q^{E1})$ and $(N_s^{E2}, N_b^{E2}, Q^{E2})$ one obtains,

$$\mathcal{W}(N_s^{E1}, N_b^{E1}, Q^{E1}) \gtrless \mathcal{W}(N_s^{E2}, N_b^{E2}, Q^{E2}) \quad \Leftrightarrow \quad \mathbb{E}_{\theta_s}(\theta_s)(\bar{K}_b + \bar{K}_s)A \gtrless \frac{B}{4} + \frac{2}{\alpha}C \tag{16}$$

$$\begin{aligned} A &= \left[\frac{4}{(\bar{K}_b\mathbb{E}_{\theta_s}(\theta_s))^{2+\alpha}}\right]^{\frac{1}{2-\alpha}} \\ &\quad - \frac{\bar{\theta}_s}{\Delta\theta_s}\beta\left[\left(\frac{\Delta\theta_s}{\bar{\theta}_s^2\beta}\right)^{2+\alpha}\frac{(\bar{K}_b(2-\beta))^{1-\alpha}}{\bar{K}_b\bar{K}_s^2\beta^2(2-\beta)(1-\beta)}\right]^{\frac{1}{2-\alpha}} \\ B &= \left[\frac{2}{(\bar{K}_b\mathbb{E}_{\theta_s}(\theta_s))^{\alpha}}\right]^{\frac{2}{2-\alpha}} - \left[\left(\frac{\Delta\theta_s}{\bar{\theta}_s^2\beta}\right)^{\alpha}\frac{(\bar{K}_b(2-\beta))^{1-\alpha}}{\bar{K}_s(1-\beta)}\right]^{\frac{2}{2-\alpha}} \\ C &= \left[\frac{2}{(\bar{K}_b\mathbb{E}_{\theta_s}(\theta_s))^2}\right]^{\frac{\alpha}{2-\alpha}} - \left[\frac{1}{\bar{K}_b\bar{K}_s\frac{\theta_s^4}{(\Delta\theta_s)^2}}\beta^2(2-\beta)(1-\beta)\right]^{\frac{\alpha}{2-\alpha}} \end{aligned}$$

Signs are determined by,

$$sgn(A) \gtrless 0 \quad \Leftrightarrow \quad \left(\frac{\bar{\theta}_s^2}{\Delta\theta_s}\frac{1}{4\mathbb{E}_{\theta_s}(\theta_s)}\right)^{2+\alpha}\left(\frac{\bar{K}_s}{\bar{K}_b}\right)^2\left(\frac{\Delta\theta_s}{\bar{\theta}_s}\right)^{2-\alpha} \gtrless \frac{1}{4^{3+\alpha}\beta^{2\alpha}(2-\beta)^{\alpha}(1-\beta)^2} \tag{17}$$

$$sgn(B) \gtrless 0 \quad \Leftrightarrow \quad \frac{\bar{K}_s}{\bar{K}_b}\left(\frac{\bar{\theta}_s^2}{\Delta\theta_s}\frac{1}{4\mathbb{E}_{\theta_s}(\theta_s)}\right)^{\alpha} \gtrless \frac{(2-\beta)^{1-\alpha}}{(1-\beta)\beta^{\alpha}2^{1+2\alpha}} \tag{18}$$

$$sgn(C) \gtrless 0 \quad \Leftrightarrow \quad \frac{\bar{K}_s}{\bar{K}_b}\left(\frac{\bar{\theta}_s^2}{\Delta\theta_s}\frac{1}{4\mathbb{E}_{\theta_s}(\theta_s)}\right)^{2} \gtrless \frac{1}{32\beta^2(2-\beta)(1-\beta)} \tag{19}$$

Inequality of equation (18) is studied in Figure (19), then we conclude $sgn(B) > 0$. Inequality of equation (19) is studied in Figure (18), then we conclude that $sgn(C) > 0$ if the ratio of network effects is almost zero, otherwise $sgn(B) < 0$. We can show also that for this model (i.e. $\beta = 0.61$) $sgn(A) > 0$ for any value of α. Finally, to obtain sharper conclusions will be necessary to take equation (16) into the computer using $\beta = 0.61$, $\alpha \in [0,1]$, $\frac{\bar{K}_b}{K_s} \in [0,1]$, and pick $\underline{\theta}_s$ such that $\bar{\theta}_s > 3\underline{\theta}_s$ condition hold.

Figure (20) shows that $\mathcal{W}^{E1} > \mathcal{W}^{E2}$ for any value of $\frac{\bar{K}_b}{K_s} \in [0,1]$ and of $\Delta\theta_s$, given that condition $\bar{\theta}_s > 3\underline{\theta}_s$ hold. Result holds. □

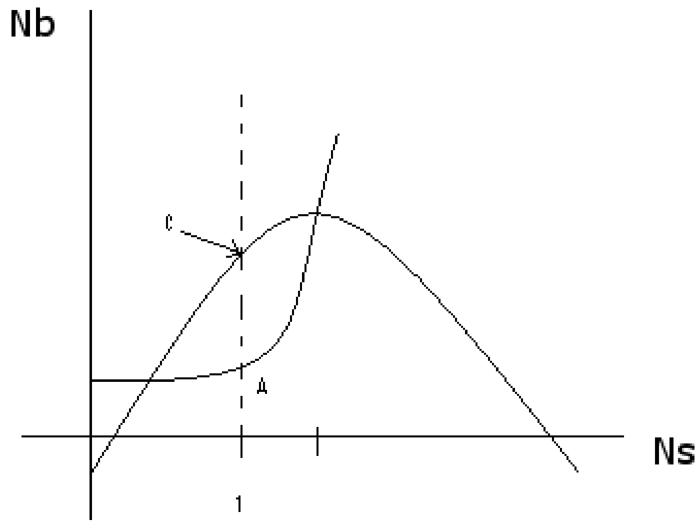

Figure 1: Existence and Uniqueness: Corner Solution Case

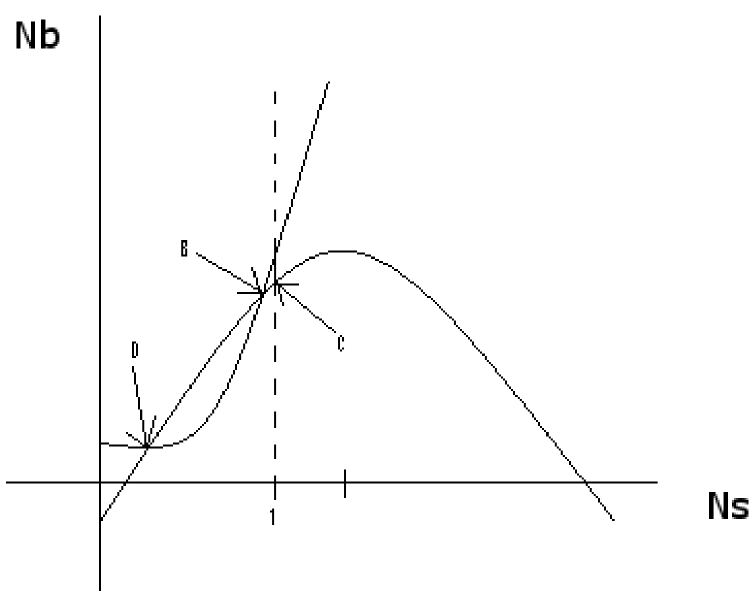

Figure 2: Existence and Uniqueness: Interior Solution Case

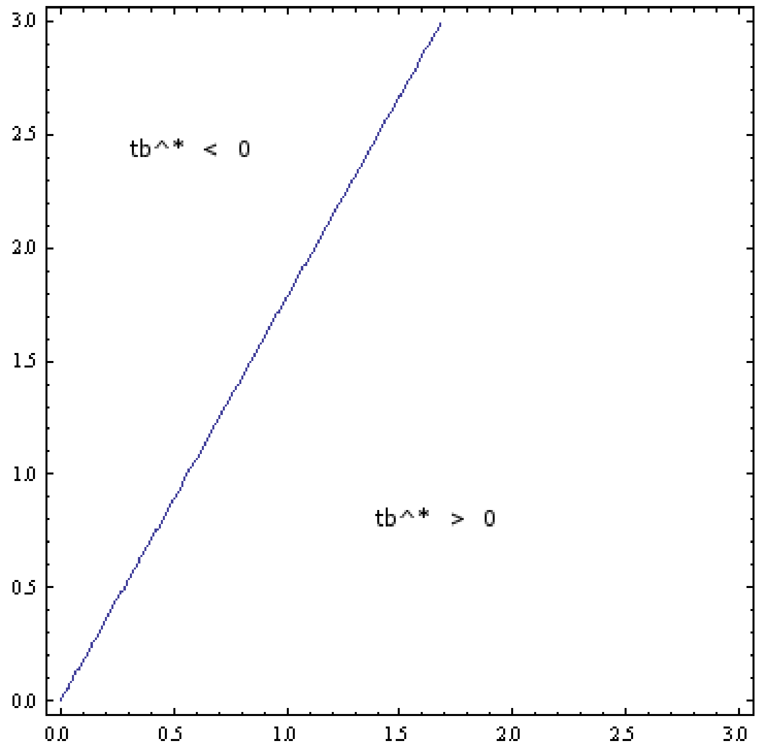

Figure 3: Conditions $t_s^* \gtrless 0$:Y-Axis $K_s \in [0, 3]$, X-Axis $K_b \in [0, 3]$

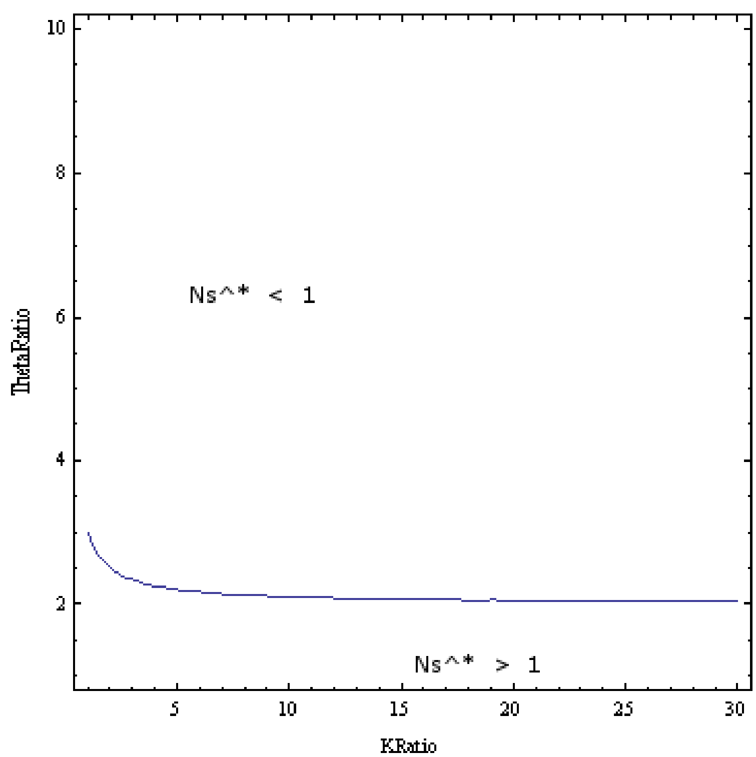

Figure 4: Conditions $N_s^* \in [0, 1]$: KRatio $= \frac{\bar{K}_s}{K_b}$, ThetaRatio $= \frac{\bar{\theta}_s}{\underline{\theta}_s}$

□

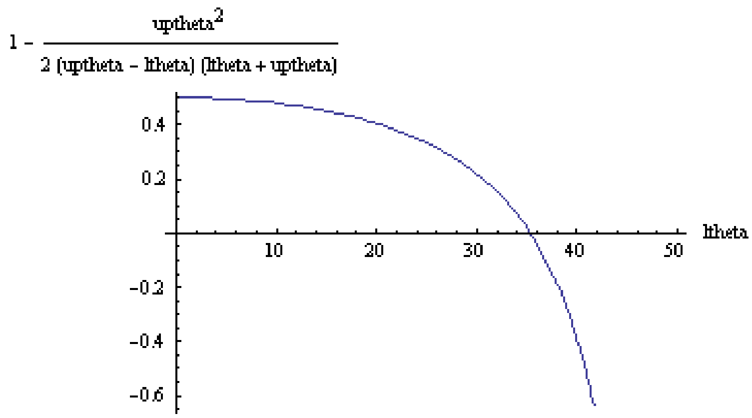

Figure 5: Lhs equation (13): fix $\bar{\theta}_s$=50

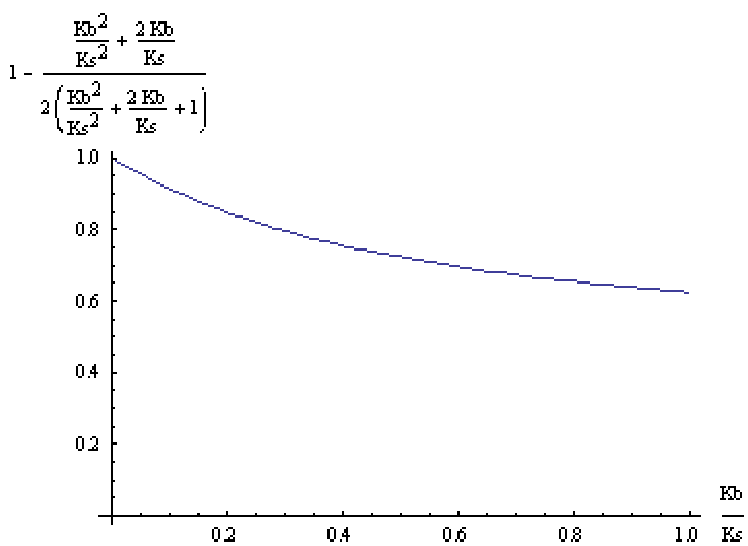

Figure 6: Rhs equation (13)

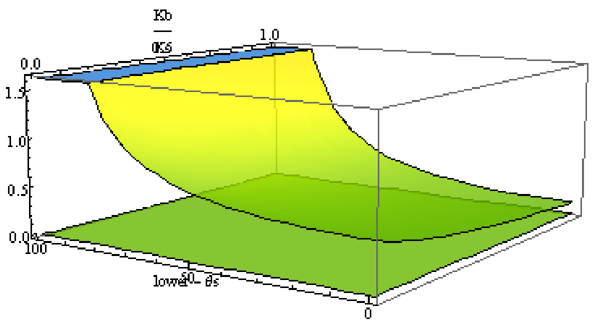

Figure 7: Inequality equation (12): $\frac{\bar{K}_b}{K_s} \in [0,1]$, $\underline{\theta}_s \in [0,100]$, $\bar{\theta}_s = 100$

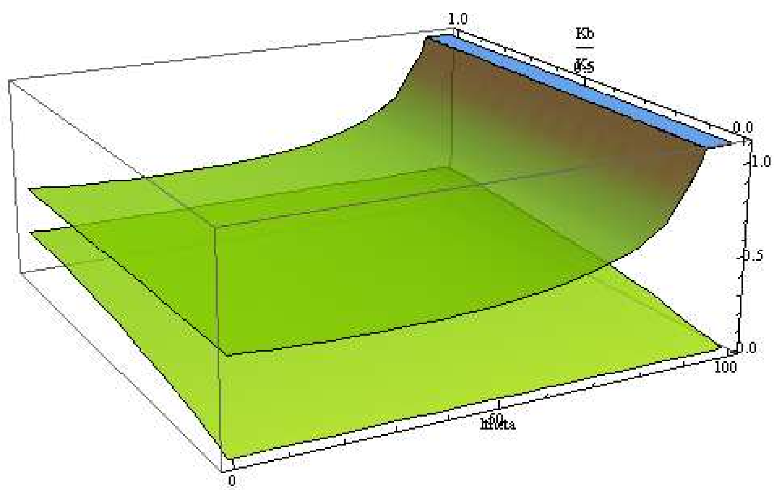

Figure 8: Welfare Comp. - Sgn(A): for $\bar{\theta}_s = 100$, $\frac{\bar{K}_b}{K_s} \in [0,1]$, $\underline{\theta}_s \in [0,100]$

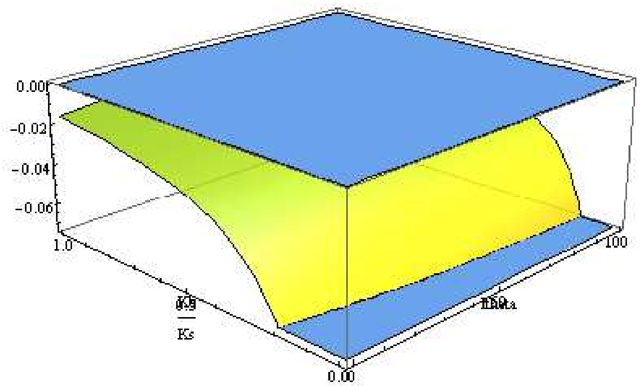

Figure 9: Welfare Comp: For $\bar{\theta}_s = 100$ graph $\mathcal{W}^* - \mathcal{W}^{E1}$

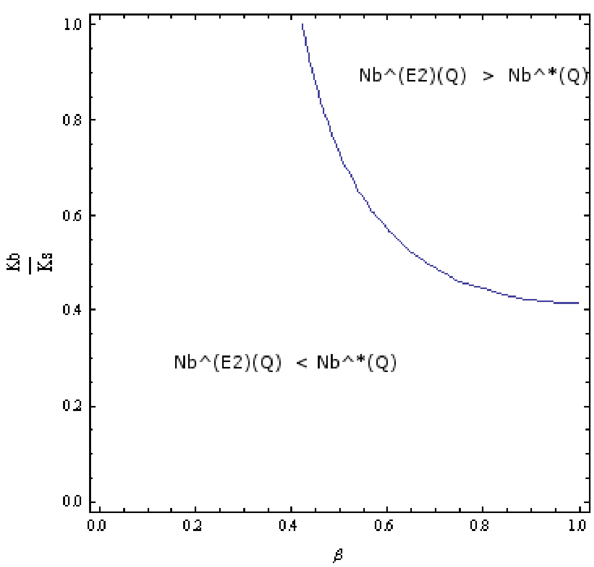

Figure 10: Determine $N_b^{E2}(\tilde{Q}) \gtrless N_b^*(\tilde{Q})$

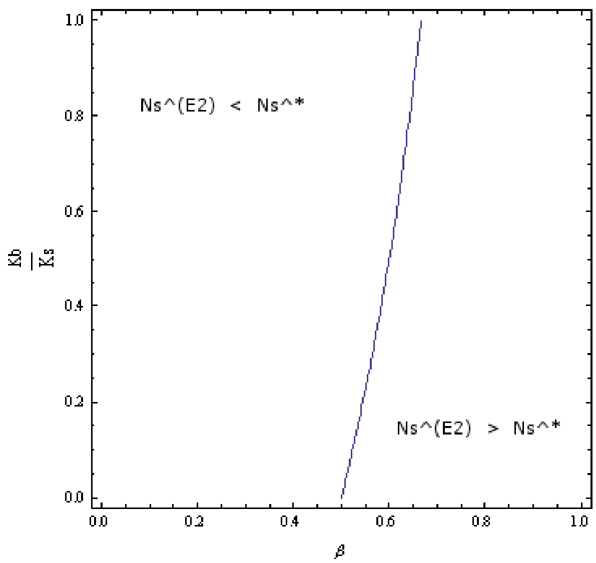

Figure 11: Determine $N_b^{E2} \gtrless N_b^*$

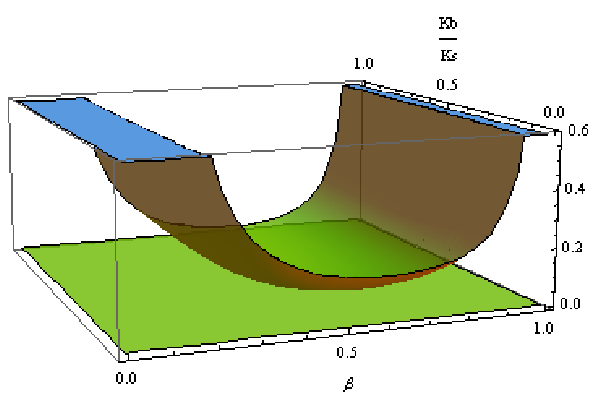

Figure 12: Graph $Q^{E2} - Q^*$

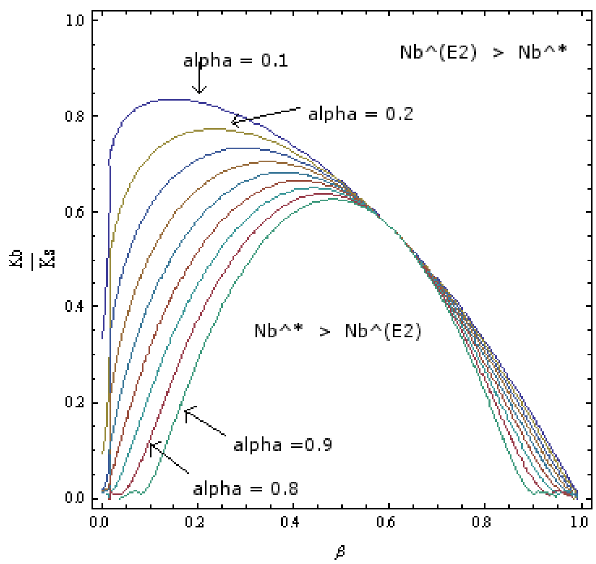

Figure 13: Determine $N_b^{E2} \gtrless N_b^*$

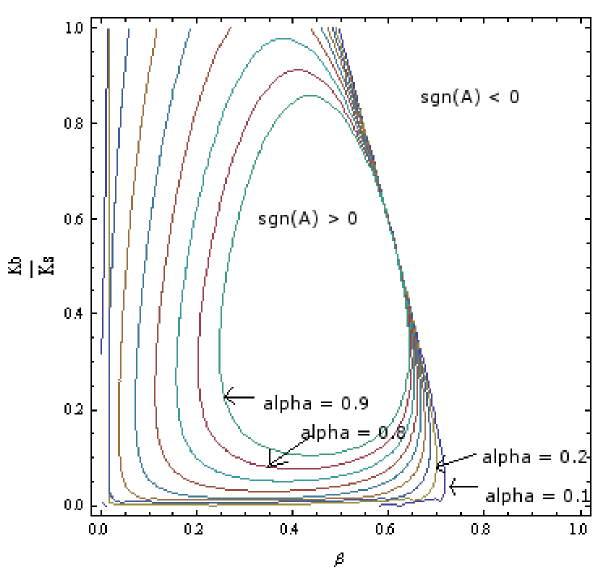

Figure 14: Lemma 9: Determine $sgn(A) \gtrless 0$

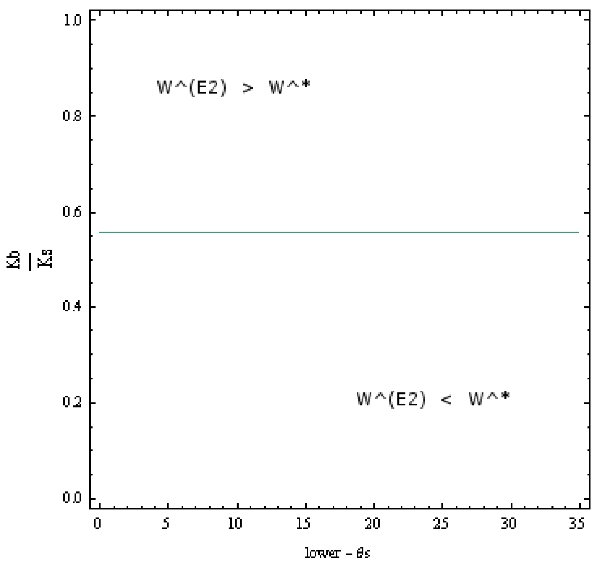

Figure 15: Welfare comparison: $\mathcal{W}^* \gtreqless \mathcal{W}^{E2}$

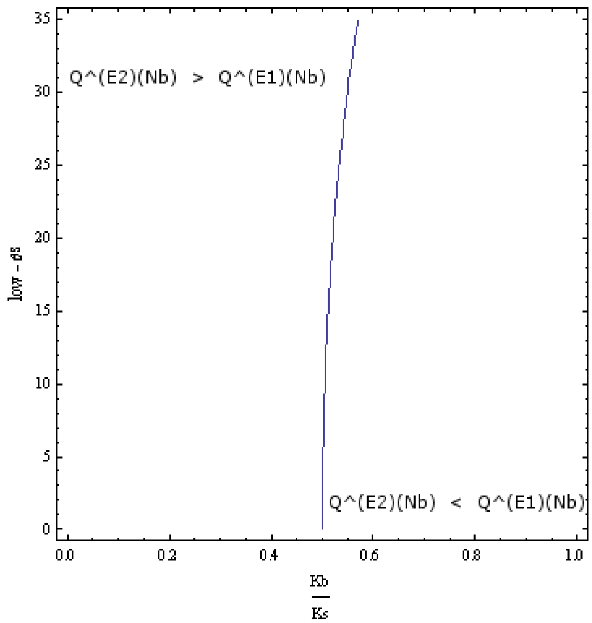

Figure 16: Lemma 9: Determine $Q^{E2}(\tilde{N}_b) \gtrless Q^{E1}(\tilde{N}_b)$

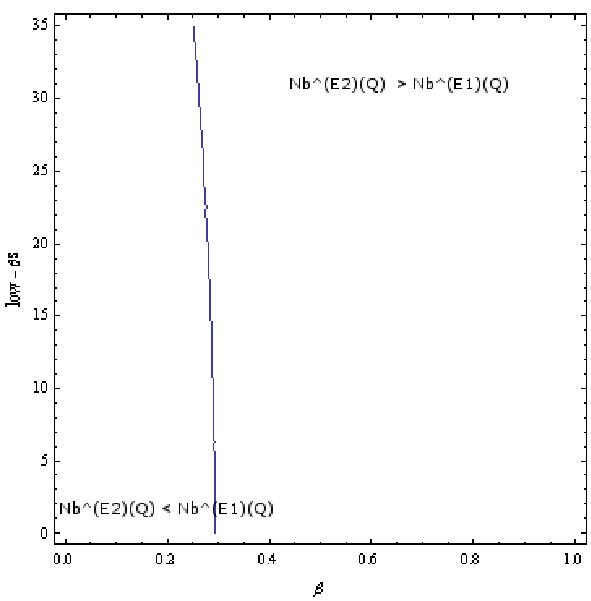

Figure 17: Lemma 9: Determine $N_b^{E2}(\tilde{Q}) \gtrless N_b^{E1}(\tilde{Q})$

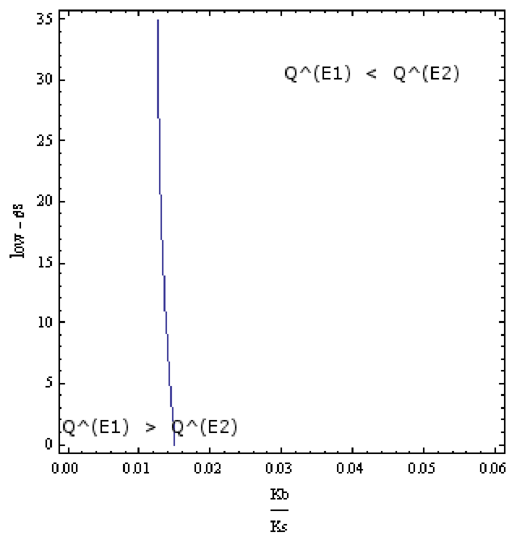

Figure 18: Lemma 10: Graph $Q^{E1} - Q^{E2}$

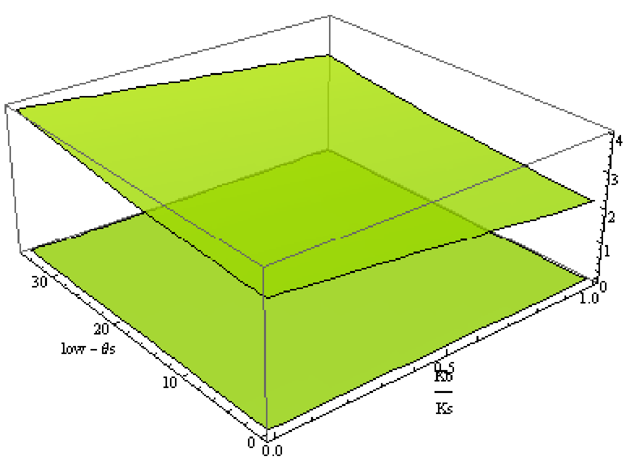

Figure 19: Lemma 10: Graph $N_b^{E1} - N_b^{E2}$

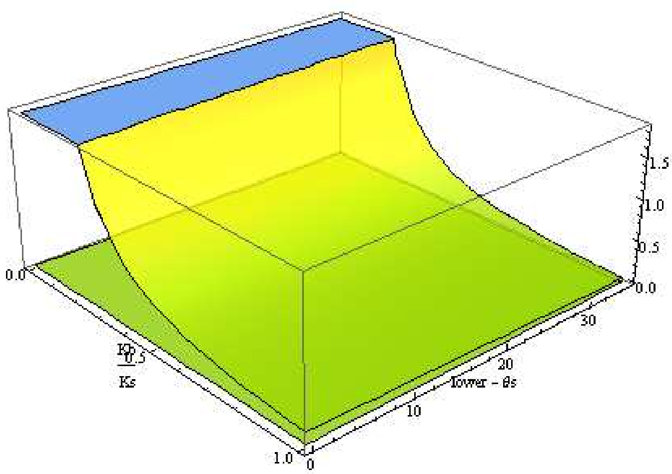

Figure 20: Welfare comparison: Graph $\mathcal{W}^{E1} - \mathcal{W}^{E2}$

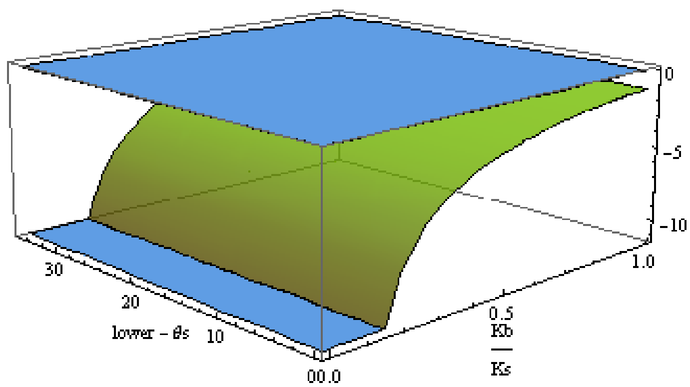

Figure 21: Lemma (7): Graph $N_b^{fp} - N_b^*$

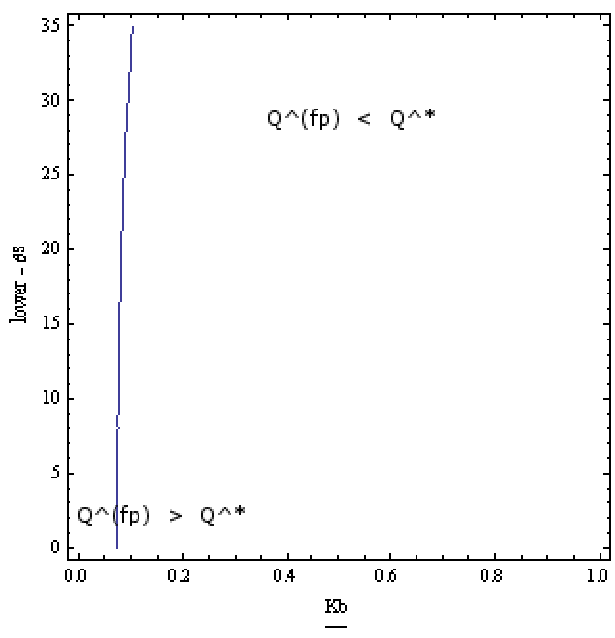

Figure 22: Lemma (7): Graph $Q^{fp} \gtrless Q^*$

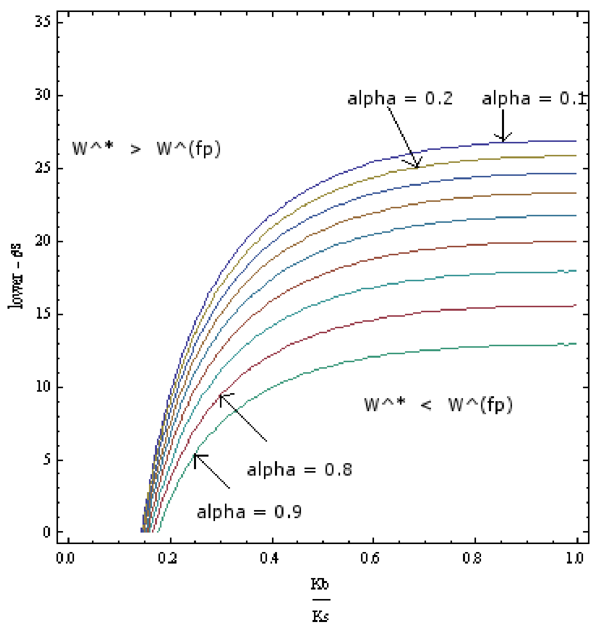

Figure 23: Welfare comparison: Graph $\mathcal{W}^* - \mathcal{W}^{fp}$