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Identification of Technology Shocks in Structural VARs

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Abstract

The usefulness of SVARs for developing empirically plausible models is actually subject to controversies in macroeconomics. We propose a two–step SVARs based procedure which consistently estimates the effect of permanent technology shocks on aggregate variables. Simulation experiments from a standard business cycle model and a sticky prices model show that our approach outperforms standard SVARs. The two–step procedure, when applied to actual data, predicts a significant short–run decrease of hours after a technology improvement followed by a hump–shaped positive response. Additionally, the rate of inflation and the nominal interest rate displays a significant decrease after this shock.

Keywords: SVARs, long–run restriction, technology shocks, consumption to output ratio, hours worked

JEL Class.: C32, E32

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Introduction

Structural Vector Autoregressions (SVARs) have been widely used as a guide to evaluate and develop dynamic general equilibrium models. Given a minimal set of identifying restrictions, SVARs represent a helpful tool to discriminate between competing theories of the business cycle. For example, Galí (1999) uses long–run restrictions *à la* Blanchard and Quah (1989) in a SVAR model of labor productivity and hours and shows that the response of hours worked to a positive technology shock is persistently and significantly negative. This negative response of hours obtained from SVARs is then implicitly employed to discriminate among business cycle models (see Galí, 1999, Galí and Rabanal, 2004, Francis and Ramey, 2005 and Basu, Fernald and Kimball, 2006).

The usefulness of SVARs for building empirically plausible models has been subject to many controversies in quantitative macroeconomics (see e.g. Cooley and Dwyer, 1998). More recently, the debate about the effect of technology improvements on hours worked has triggered the emergence of several contributions concerned with the ability of SVARs to adequately measure the impact of technology shocks on aggregate variables.

Using Dynamic Stochastic General Equilibrium (DSGE) models estimated on US data as their Data Generating Process (DGP), Erceg, Guerrieri and Gust (2005) show that the effect of a technology shock on hours worked is not precisely estimated with SVARs. They suggest that part of their results originate from the difficulty to disentangle technology shocks from other shocks that have highly persistent, if not permanent, and sizeable effects on labor productivity and hours. Their findings also suggest to include in SVARs other variables with lower serial correlation.

Chari, Kehoe and McGrattan (2007b) simulate a prototypical business cycle model estimated by Maximum Likelihood on US data with structural shocks as well as measurement errors. They show that the SVAR model with a specification of hours in difference (DSVAR) or in quasi–difference (QDSVAR) leads to a negative response of hours under a business cycle model in which hours respond positively. Moreover, they show that a level specification of hours (LSVAR) does not uncover the true response of hours and implies a large upward bias. A significant part of their results originates from the inability of SVARs with a finite number of lags to properly capture the true dynamic structure of the model (see also Ravenna, 2007).[1] This problem can be eliminated if a relevant state variable is introduced in the SVAR model. Unfortunately, the lack of observability of such a variable (for example, capital stock and shocks) makes its use impossible. However, even if such a meaningful variable is virtually unobserved, we can always think about observable relevant instrumental variables that share approximatively the same dynamic structure.

Christiano, Eichenbaum and Vigfusson (2006) argue that SVARs are still a useful guide for

[1] The problem of missing state variable is especially problematic for VARs with long–run restrictions. The identification of technology shocks is then based on the long–run covariance matrix which depends on sum of the estimated VAR parameters. This is not the case with short–run restrictions.

developing models. They find that most of the disappointing results with SVARs in Chari, Kehoe and McGrattan (2007b) come from the values assigned to the standard errors of shocks in their economy. They notably show that when the model is more properly estimated, the standard error of the non–technology shocks is twice lower than the standard error of the technology shock. In such a case, the bias in SVARs with labor productivity and hours is strongly reduced. Evidence from their simulation experiments suggests using other variables which are less sensitive to the volatility of non–technology shocks and/or which contains a sizeable part of technology shocks.

In light of the above findings, we propose a simple alternative method to consistently estimate technology shocks and their short–run effects on aggregate variables. As an illustration and a contribution to the current debate, we concentrate our analysis on the response of hours worked. However, our empirical strategy can be easily implemented to other variables of interest. In the empirical part of the paper, we investigate the dynamic responses of the rate of inflation and the short–term nominal interest rate. Although imperfect, we maintain the labor productivity variable as a way to identify technology shocks using long–run restrictions. We argue that SVARs can deliver accurate results if more efforts are made concerning the choice of the stationary variables. More precisely, hours (or other highly persistent variables subject to empirical controversies about their stationarity) must be excluded from SVARs and replaced by any variable which presents better stochastic properties. The introduction of a highly persistent variable as hours worked in the SVARs confounds the identification of the permanent and transitory shocks and thus contaminates the corresponding Impulse Response Functions (IRFs). Following the previous quoted contributions, the selected variable must satisfy the following stochastic properties. First, the variable must display less controversies about its stationarity.[2] Second, the variable must behave more as a capital (or state) variable than hours worked do, so that a VAR model with a finite number of lags can more easily approximate the true underlying dynamics of the data. Third, the variable must contain a sizeable technology component and present less sensitivity to highly persistent non–technology shocks. The consumption to output ratio (in logs) is an promising candidate to fulfil these three requirements. The ratio is stationary and consequently displays less persistence than hours worked. Moreover, the consumption to output ratio represents probably a better approximation of the state variables than hours worked and appears less sensitive to transitory shocks. The first requirement can be directly found with actual data, since standard unit root tests reject the null hypothesis of an unit root. The two other requirements can be quantitatively (through numerical experiments) and analytically deduced from equilibrium conditions of DSGE models. In addition, Cochrane (1994) has already shown in SVARs that the consumption to output ratio allows to suitably characterize permanent and transitory components in GNP.

[2] Pesavento and Rossi (2005) and Francis, Owyang and Roush (2005) propose other methods to deal with the presence of highly persistent process.

The proposed approach consists in the following two steps. In a first step, a SVAR model which includes labor productivity growth and consumption to output ratio is considered to consistently estimate technology shocks using a long–run restriction. In the second step, the IRFs of hours (or any other aggregate variable under interest) at different horizons are obtained by a simple (univariate or multivariate) regression of hours on the estimated technology shock. We show that the IRFs are consistently estimated whether hours worked are projected in level or in difference in the second step. Consequently, our approach does not suffer from the specification choice of hours as in the standard SVAR approach. Our method can be viewed as a combination of a SVAR approach in the line of Blanchard and Quah (1989), Galí (1999) and Christiano, Eichenbaum and Vigfusson (2004) and the regression equation used by Basu, Fernald and Kimball (2006) in their growth accounting exercise.

To evaluate this proposed two–step approach, we perform simulation experiments using a standard business cycle model and a sticky prices model with a permanent technology shock and stationary preference and investment–specific shocks. The results show that our approach, denoted CYSVAR, performs better than the DSVAR and LSVAR models. In particular, the bias of the estimated IRFs is strongly reduced. In contrast with the results for the DSVAR and LSVAR models, we also show that the specification of hours (in level or in difference) does not matter. Moreover, the estimated technology shock using CYSVAR model is strongly correlated with the true technology shock while weakly with the non–technology shock. In other words, the estimated technology shock is not contaminated by other shocks that drive up or down hours worked. Consequently, the estimated response of hours obtained in the second step displays small bias. We complete these findings by calculating the coverage rate for the computed confidence interval. Apart the first difference specification in the second step, the coverage rate indicates that the confidence intervals include the true value of the IRFs with a probability close to the true one.

A central practical and legitimate question concerns the empirical usefulness of our two–step approach. Suppose that the variable under interest (hours worked, inflation, nominal interest rate in the empirical part of our paper) displays high level of serial correlation, such that its specification in a SVAR model with long run restriction is uncertain. In addition, level and first difference specifications may yield very different and controversial dynamic responses from SVARs models. Our empirical strategy proposes to eliminate this variable from the first step and then uses our procedure to consistently estimate the dynamic responses of this variable in a second step.

We then apply our two–step approach with US data. As a contribution to the current debate, we first investigate the dynamic responses of hours. The DSVAR and LSVAR specifications deliver conflicting results. In the DSVAR specification, hours significantly decrease in the short–

run whereas they display a positive hump pattern with the level specification. In contrast, the two–step approach provides the same dynamic responses whatever the specification of hours in the second step. Hours worked decrease in the short–run after a positive technology shock but display a positive hump–shaped response. Our results are in line with the previous empirical findings which show that hours fall significantly on impact (see Galí, 1999, Basu, Fernald and Kimball, 2006, Francis and Ramey, 2008) and display a positive hump pattern during the subsequent periods (see Christiano, Eichenbaum and Vigfusson, 2004 and Vigfusson, 2004). We also apply this methodology to the rate of inflation and the nominal interest rate and we find that these two nominal variables significantly decrease in the short–run after a positive technology shock.

The paper is organized as follows. In a first section, we present our two–step approach. The second section presents a prototypical business cycle model and documents in more details simulation experiments on SVARs. In section 3, we assess the robustness of our findings. In section 4, we present the empirical results. The last section concludes.

1 The Two–Step Approach

This section motivates the two–step approach and presents the practical implementation of this procedure in more details. It also discuss different models that can be used in the second step.

1.1 Motivations

The goal of our approach is to accurately identify the technology shocks in the first step using an adequate stationary variable in the SVAR model. A large part of the performance of the two–step approach depends on the time series properties of this variable. This latter can be interpreted as an instrument allowing to retrieve with more precision the true technology shock. The variable choice is motivated in part by simulation results in Erceg, Guerrieri and Gust (2005), Chari Kehoe and McGrattan (2007b) and Christiano, Eichenbaum and Vigfusson (2006). They show that, when hours worked are contaminated by an important persistent transitory component, the SVAR performs poorly in their experiments.

Chari, Kehoe and McGrattan (2007a) propose a method in order to account for economic fluctuations based on the measurement of various wedges. They assess what fraction of the output fluctuations can be attributed to each wedge separately and in combinations. For the postwar period, the efficiency and labor wedges are prominent to explain output movement. Investment wedge plays a minor role in the postwar period and especially at low frequencies of output fluctuations.

The results in Chari Kehoe and McGrattan (2007a) suggest that the observed fluctuations and persistence of hours worked depend on an important portion of the labor wedge. In contrast, in their prototypical economy, the consumption-output ratio is less dependent on labor wedge. As a consequence, the transitory component of the consumption-output ratio is then probably less important than the one corresponding to the permanent shock. According to this, the consumption-output ratio is a more promising variable to use in a SVAR model for identifying technology and non-technology and the associated dynamic responses than hours worked.

Cochrane (1994) also argues that the consumption to output ratio contains useful information to disentangle the permanent to the transitory component. This result can receive a structural interpretation using a simple permanent income model. This model implies that consumption is a random walk and that consumption and total income are cointegrated. Consequently, it follows from the intertemporal decisions on consumption that any shock to aggregate output that leaves consumption constant is necessary a transitory shock. The joint observation of output growth and the log of consumption to output ratio allows the econometrician to separate shocks into permanent and transitory components, as perceived by consumers. Moreover, in data, we can reject the unit root for this ratio and the empirical autocorrelation function is clearly less persistent that the one for hours. The introduction of a less persistent variable in level in the VAR also allows to minimize the problem of weak instruments raised by Christian, Eichenbaum and Vigfusson (2004) and Gospodinov (2006). So we decide to introduce this ratio as instrument to identify the technology shocks. With this identified shocks at the first step, we can then evaluate the impact of these shocks on a variable of interest (for example, hours) in the second step.

1.2 The approach

We now present in deeper details the two–step approach.

Step 1: Identification of technology shocks

We consider a VAR model which includes productivity growth $\Delta(y_t - h_t)$ and consumption to output ratio $c_t - y_t$ (in logs). For simplicity, we omit a vector of constant terms. We start by specifying a VAR(p) model in these two variables:

$$\begin{pmatrix} \Delta(y_t - h_t) \\ c_t - y_t \end{pmatrix} = \sum_{i=1}^{p} B_i \begin{pmatrix} \Delta(y_{t-i} - h_{t-i}) \\ c_{t-i} - y_{t-i} \end{pmatrix} + \varepsilon_t \tag{1}$$

where the white noise $\varepsilon_t = (\varepsilon_{1,t}, \varepsilon_{2,t})'$ and $\mathrm{E}(\varepsilon_t \varepsilon_t') = \Sigma$. Under usual conditions, this VAR(p) model admits a VMA(∞) representation

$$\begin{pmatrix} \Delta(y_t - h_t) \\ c_t - y_t \end{pmatrix} = C(L)\varepsilon_t$$

where $C(L) = (I_2 - \sum_{i=1}^{p} B_i L^i)^{-1}$. The SVAR model is represented by the following VMA(∞) representation

$$\begin{pmatrix} \Delta(y_t - h_t) \\ c_t - y_t \end{pmatrix} = A(L) \begin{pmatrix} \eta_t^T \\ \eta_t^{NT} \end{pmatrix}$$

where $\eta_t = (\eta_t^T, \eta_t^{NT})'$. η_t^T is period t technology shock, whereas η_t^{NT} is period t composite non–technology shock (see Blanchard and Quah (1989) and Faust and Leeper (1997) for a discussion on the conditions for valid shock aggregation in the small SVAR models). By normalisation, these two orthogonal shocks have zero mean and unit variance. The identifying restriction implies that the non–technology shock has no long–run effect on labor productivity. This means that the upper triangular element of $A(L)$ in the long run must be zero, *i.e.* $A_{12}(1) = 0$. In order to uncover this restriction from the estimated VAR(p) model, an estimator of the matrix $A(1)$ is obtained as the Choleski decomposition of the estimator for $C(1)\Sigma C(1)'$ resulting from the VAR. The structural shocks are then directly deduced up to a sign restriction:

$$\begin{pmatrix} \eta_t^T \\ \eta_t^{NT} \end{pmatrix} = C(1)^{-1} A(1) \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix}$$

Step 2: Estimation of the responses of hours to a technology shock

The structural infinite moving average representation for hours worked as a function of the technology shock and the composite non–technology shock[3] is given by:

$$h_t = a_1(L)\eta_t^T + a_2(L)\eta_t^{NT}. \tag{2}$$

Notice that we omit again a constant term in this equation. The coefficient $a_{1,k}$ $(k \geq 0)$ measures the effect of the technology shock at lag k on hours worked, *i.e.* $a_{1,k} = \partial h_{t+k}/\partial \eta_t^T$.

According to the debate on the right specification of hours worked, we first examine three univariate specifications to measure the impact of technology on this variable. In the first specification, hours series is projected in level on the identified technology shocks while in the second specification, hours series is projected in difference. Finally, in the third specification, the hours series is projected on its own first lag and the identified technology shocks.

Let us now present in more details these three specifications. In the first one, we regress the logs of hours worked on the current and past values of the identified technology shocks $\widehat{\eta}_t^T$ in the first-step:

$$h_t = \sum_{i=0}^{q} \theta_i \widehat{\eta}_{t-i}^T + \nu_t \tag{3}$$

where $q < +\infty$ and $\widehat{\eta}_t^T$ denotes the estimated technology shocks obtained from the SVAR model in the first step. ν_t is a composite error term that accounts for non–technology shocks and the remainder technology shocks. A standard OLS regression provides the estimates of the population

[3] In typical DSGE models, non–technology shocks correspond to preference, taxes, government spending, monetary policy shocks and so on. When the number of stationary variables in the SVAR model is small respective to the number of these shocks and without additional identification schemes, these shocks are not identifiable. For our purpose, this identification issue does not matter since we only focus on the dynamic response of hours to a (permanent) technology shock.

responses of hours to the present and lagged values of the technology shocks, $\widehat{a}_{1,k} = \widehat{\theta}_k$. Hereafter, we refer to this approach as LCYSVAR. According to the debate on the appropriate specification of hours, this variable is regressed in first difference on the current and past values of the identified technology shocks. Hereafter, we refer to this approach as DCYSVAR. The response of hours worked to a technology shock is now estimated from the regression:

$$\Delta h_t = \sum_{i=0}^{q} \tilde{\theta}_i \widehat{\eta}^T_{t-i} + \tilde{\nu}_t. \tag{4}$$

As hours are specified in first difference, the estimated response at horizon k is obtained from the cumulated OLS estimates, $\widehat{\widehat{a}}_{1,k} = \sum_{i=0}^{k} \widehat{\widehat{\theta}}_i$. Finally, an interesting avenue is to adopt a more flexible approach by freely estimating the autoregressive parameter of order one for hours. This lets the data discriminate between the presence of an unit root in the stochastic process of hours worked. Hereafter, we refer to this approach as CYSVAR–AR. The response to a technology shock is now estimated from the regression of hours on one lag of itself and lags of the technology shock:

$$h_t = \rho h_{t-1} + \sum_{i=0}^{q} \tilde{\tilde{\theta}}_i \widehat{\eta}^T_{t-i} + \tilde{\tilde{\nu}}_t. \tag{5}$$

The estimated response at horizon k is obtained from the OLS estimates of ρ and θ_i $(i = 1, ..., q)$, $\widehat{\widehat{\widehat{a}}}_{1,k} = \sum_{i=0}^{k} \widehat{\rho}^i \, \widehat{\widehat{\widehat{\theta}}}_{k-i}$. This last specification calls various comments. First, equation (5) is more flexible than (3) and (4) since it allows to freely estimate the autoregressive parameter of order one for hours. Therefore, it lets the data select the appropriate time series representation of hours worked. The LCYSVAR and DCYSVAR specifications are in fact restricted versions of the third specification with the autoregressive parameter ρ fixed to zero or to one. Second, it imposes that the dynamic responses of hours to various aggregates shocks shares the same root. It does not mean that the shape of IRFs are the same since they are obtained from the autoregressive parameter and the $MA(q)$ representation of these shocks. Notice that this is the case in most of DSGE models where the variables of interest share the same dynamics implied by the state variables (for example, the capital stock in the simple model), but differs in their sensitivity to shocks that hit the economy. The regression equation (5) simply accounts for these features. In the following proposition, we show that the OLS estimators of the effect of technology shocks are consistent estimators of the true ones for the three specifications (see the proof in Appendix A).

Proposition 1 *Assume the infinite moving average representation (2) for hours worked and consider the estimation of the finite VAR in the first step as defined in (1) and the three projections (3), (4) and (5) in the second step. The OLS estimators $\widehat{a}_{1,k}$, $\widehat{\widehat{a}}_{1,k}$ and $\widehat{\widehat{\widehat{a}}}_{1,k}$ converge in probability to $a_{1,k}$ for the three specifications, $\forall k$.*

In Proposition 1, the property of consistency is derived under the assumption that hours worked follow a stationary process. While the specification of hours in difference could provide a good statistical approximation of this variable in small sample, hours worked per capita are bounded and therefore the stochastic process of this variable cannot have a unit root asymptotically. By definition, the consistency property of an estimator is an asymptotically concept so only the asymptotic behavior of hours worked is of interest. Consequently, the consistency of the OLS estimators for the three specifications is derived only under the assumption that hours worked per person is a stationary process. It is worth noting that the specification of hours (level or first difference) does not asymptotically matter. However, the small sample behavior of the three estimators associated to the three specifications can differ.

We can complement the two–step approach by considering a VAR model in the second step. The VAR model includes the labor productivity growth, hours worked (either in level and first difference) and the consumption to output ratio[4]

$$\begin{pmatrix} \Delta\left(y_t - h_t\right) \\ h_t \\ c_t - y_t \end{pmatrix} = \sum_{i=1}^{p} D_i \begin{pmatrix} \Delta\left(y_{t-i} - h_{t-i}\right) \\ h_{t-i} \\ c_{t-i} - y_{t-i} \end{pmatrix} + \zeta_t \tag{6}$$

This reduced form VAR is estimated to retrieve the residuals $\widehat{\zeta}_t$. We then regress these residuals $\widehat{\zeta}_t$ on the technology shock series $\widehat{\eta}_t^T$ identified from the first step and then we obtained the IRFs to the technology shocks by inverting the VAR model. Hereafter, we refer to this approach as CYSVAR–VAR. Notice however that there exist no specific reason to assume that aggregate data are generated by a VAR. This is especially true if we consider a subset of the variables included in the VAR model. Moreover, most of business cycle models admits a VARMA representation. From this view, the specification (5) appears more able to capture an ARMA structure than (6).

Other regression models can be considered in the second step. First, we can successively regress hours on each estimated technology shocks instead of considering a given block of the shocks, as in regressions (3), (4) or (5) (see Chang and Sakata, 2007). This approach, similar to ours, does not impose any parametric restrictions on the IRFs. Notice also that the resulting estimator is asymptotically equivalent to those of Proposition 1. Second, the CYSVAR–AR can include additional lags of hours worked in order to better capture their persistence. Third, we can estimate an ARMA model on hours, regress the residuals on the technology shock series $\widehat{\eta}_t^T$ and then compute the IRFs by inverting the model. The advantage of this approach is its parsimony compared (3), (4) or (5). Moreover, it accounts for the possible ARMA structure of hours in DSGE models. Fourth, following Jordà (2005), we can run local projections at the second

[4]In most of our simulation experiments and the empirical applications, the VAR model includes three variables. When we simulate the business cycle models with two shocks, the VAR model includes only two variables (productivity growth and hours).

step using a VAR model including labor productivity growth and hours worked (either in level and first difference). This approach as proven to be more flexible than VARs and represents a natural alternative to estimating IRFs. All these models have been investigated in our simulation experiments. As they deliver very similar results, we do not report the results to save space.[5]

Finally, the two–step procedure is not only used to measure the effect of technology shocks on hours worked (or any other variable of interest) but also to hypothesis testing about the significance of these responses. The approach raises two practical econometric issues. First, confidence intervals in the second step must account for the uncertainty resulting from the first step estimation. This is usually called the *generated regressors problem*. Second, the residuals in the second step can be serially correlated in practice. This is especially true for the regression (3) with hours in level. Confidence intervals of IRFs are computed using a bootstrap procedure which accounts for the generated regressors problem and serial correlation. We draw randomly and jointly the fitted VAR residuals from the first step and the residuals from the second step. For the second step residuals, we use a block bootstrap method with an optimal block length chosen according to the data-based procedure proposed by Politis and White (2004). In practice we implement the correction of this procedure such proposed in Patton, Politis and White (2008). It is important to use the same ordering of the random draws for the first and the second steps to preserve the correlation structure of the whole system of equations. For each random draw, the first-step VAR is estimated and the corresponding identified technology shocks are retrieved. The second step projection is then estimated to obtain the bootstrapped impulse responses conditioning on the technology shocks identified at the first step and the random draws from the residuals of this second step (with the same ordering of the random draw). The percentile confidence intervals of the effect of technology shocks is constructed by repeating this bootstrap procedure for 1000 random draws.

2 Simulation Experiments from a Business Cycle Model

This section briefly presents the model and its parametrisation and then documents in more details simulation experiments on SVARs.

2.1 The Business Cycle Model

We consider a standard business cycle model that includes three shocks. The utility function of the representative household is given by

$$E_t \sum_{i=0}^{\infty} \beta^i \left(\log\left(C_{t+i}\right) + \psi \ \chi_{t+i} \ \log\left(1 - H_{t+i}\right)\right)$$

[5]All the simulation experiments with the above mentioned approaches are available from the authors upon request.

where $\beta \in (0,1)$ denotes the discount factor and $\psi > 0$ is a time allocation parameter. E_t is the expectation operator conditional on the information set available at time t. C_t and H_t represent consumption and labor supply at time t. The labor supply H_t is subjected to a preference shock χ_t, that follows a stationary stochastic process.

$$\log(\chi_t) = \rho_\chi \log(\chi_{t-1}) + (1-\rho_\chi)\log\bar{\chi} + \sigma_\chi \varepsilon_{\chi,t}$$

where $\bar{\chi} > 0$, $|\rho_\chi| < 1$, $\sigma_\chi > 0$ and $\varepsilon_{\chi,t}$ is *iid* with zero mean and unit variance. As noted by Galí (2005), this shock can be an important source of fluctuations as it accounts for persistent shifts in the marginal rate of substitution between goods and work (see Hall, 1997). Such shifts capture persistent fluctuations in labor supply following changes in labor market participation and/or changes in the demographic structure. Additionally, this preference shock allows us to simply account for other distortions on the labor market, labelled labor wedge in Chari, Kehoe and McGrattan (2007a).

The representative firm use capital K_t and labor H_t to produce a final good Y_t. The technology is represented by the following constant returns–to–scale Cobb–Douglas production function

$$Y_t = K_t^\alpha \left(Z_t H_t\right)^{1-\alpha}$$

where $\alpha \in (0,1)$. Z_t is assumed to follow an exogenous process of the form

$$\log(Z_t) = \log(Z_{t-1}) + \gamma_z + \sigma_z \varepsilon_{z,t}$$

where $\sigma_z > 0$ and $\varepsilon_{z,t}$ is *iid* with zero mean and unit variance. In the terminology of Chari, Kehoe and McGrattan (2007a), $Z_t^{1-\alpha}$ in the production function corresponds to the efficiency wedge. This wedge may capture for instance input-financing frictions. Capital stock evolves according to the law of motion

$$K_{t+1} = (1-\delta)\, K_t + I_t$$

where $\delta \in (0,1)$ is a constant depreciation rate. Finally, the final output good can be either consumed or invested

$$Y_t = C_t + I_t$$

The model is thus characterized by two time varying wedges, *i.e.* the efficiency and labor wedges, that summarize a large class of mechanisms without having to explicitly specify them.

To analyze the quantitative implications of the model, we first apply a stationary–inducing transformation for variables that follow a stochastic trend. Output, consumption, investment and government consumption are divided by Z_t, and the capital stock is divided by Z_{t-1}. The approximate solution of the model is computed from a log–linearisation of the stationary equilibrium conditions around the deterministic steady state.

The parameter values are familiar from business cycle literature (see Table 1). We set the capital share to $\alpha = 0.33$ and the time allocation parameter $\psi = 3.60$, such that households spend 20% of their time endowment to market activity. We choose the discount factor so that the steady state annualized real interest rate is 3%. We set the depreciation rate $\delta = 0.025$. The growth rate of Z_t, namely γ_z, is equal to 0.0036. The parameters of the two forcing variables (Z_t, χ_t) are borrowed from previous empirical works with US data. The standard–error σ_z of the technology shock is equal to 1% (see Prescott, 1986, Burnside and Eichenbaum, 1996, Chari, Kehoe and McGrattan, 2007b and Christiano, Eichenbaum and Vigfusson, 2006). We choose alternative values (0.90;0.95;0.99) for the autoregressive parameter ρ_χ of the preference shock. Previous estimations (see Chari, Kehoe and McGrattan, 2007b and Christiano, Eichenbaum and Vigfusson, 2006) suggest value between 0.95 and 0.99, but we add $\rho_\chi = 0.90$ for a check of robustness. Finally, the standard error of this shock σ_χ takes three different values (0.005;0.01;0.02). These values roughly summarize the range of previous estimates (see Erceg, Guerrieri and Gust, 2005, Chari, Kehoe and McGrattan, 2007b, and Christiano, Eichenbaum and Vigfusson, 2006). The alternative calibrations summarize previous estimates which use different datasets and estimation techniques. They allow us to conduct a sensitivity analysis and to evaluate the relative merits of different approaches for various calibrations of the forcing variables.

2.2 Simulation Results

We assess each approach from the distribution of IRFs based on Monte–Carlo experiments. We generate 1000 data samples from the business cycle model. Every data sample consists of 200 quarterly observations and corresponds to the typical sample size of empirical studies. In order to reduce the effect of initial conditions, the simulated samples include 200 initial points which are subsequently discarded in the estimation. For every data sample, we estimate VAR models with four lags as in Erceg, Guerrieri and Gust (2005), Chari, Kehoe and McGrattan (2007b), and Christiano, Eichenbaum and Vigfusson (2006).

For each experiment, we investigate the reliability of different SVARs approaches to identify of technology shocks and their aggregate effects: a LSVAR model with labour productivity growth and hours in level; a DSVAR models with labour productivity growth and hours in first difference; a LCYSVAR approach in which the SVAR model includes labour productivity growth and consumption to output ratio in the first step and hours in level are regressed on the estimated technology shock in the second step. The DCYSVAR and CYSVAR–AR approaches are the same in the first step, but they consider hours in first difference and once lagged hours in the second step. The CYSVAR–VAR approach includes in the second step labour productivity growth and hours in level. In the second step of the CYSVAR approach (LCYSVAR and DCYSVAR), we consider current and twelve lagged values of the identified (in the first step) technology shocks. For the

CYSVAR–AR, we include current and four lagged values of the shock and for the CYSVAR–VAR, we set four lags in the regression. We have also investigated the sensitivity of the results to the number of lags. All the empirical findings are left unaffected.

Figures 1 and 2 display the responses of hours for each SVARs in our baseline calibration ($\rho_\chi = 0.95$ and $\sigma_z = \sigma_\chi = 0.01$). The solid dashed represents the response of hours in the model, whereas the solid line corresponds to the estimated response from SVARs.

The responses of hours obtained from a LSVAR model displays a large upward bias, as the estimated response on impact is almost twice the true response and is persistently above the true response. These results are in the line with those of Chari, Kehoe and McGrattan (2007b). The confidence intervals are very large and therefore not informative.

The response of hours obtained from the DSVAR model displays a large downward bias, and it is persistently negative. This result is again similar to Chari, Kehoe and McGrattan (2007b) who show that the difference specification of hours adopted by Galí (1999), Galí and Rabanal (2004) and Francis and Ramey (2005) can lead to mistaken conclusions about the effect of a technology shock. Note that a DSVAR model is obviously misspecified under the business cycle model considered here, as it implies an over–differentiation of hours. The first difference specification of hours can create distortions and lead to biased estimated responses. However, Chari, Kehoe and McGrattan (2007b) show that SVARs with hours in quasi–difference, consistent with the business cycle model, display similar patterns.

Consider now the CYSVAR approach. Figure 2 shows that this approach delivers reliable estimates of the response of hours. The bias is small, especially in comparison with the ones from the DSVAR and LSVAR. Another interesting result is that the four CYSVAR approaches deliver very similar results. Therefore, our two–step approach does not suffer from the specification of hours, contrary to the DSVAR and LSVAR. It is worth noting that these small sample experiments support the asymptotic results of Proposition 1.

To evaluate the size of the bias, Table 2 reports the cumulative absolute bias between the average response in SVARs and the true response over different horizons.[6] Since quantitative results are rather similar, we report to save space only simulation results with the CYSVAR–AR and CYSVAR–VAR approaches. Our benchmark calibration corresponds to the second panel in Table 2 when $\rho_\chi = 0.95$ and $\sigma_\chi/\sigma_z = 1$. We also obtained a large bias with DSVAR and LSVAR models (both on impact and for different horizons). However, the CYSVAR–AR and CYSVAR–VAR deliver very reliable results compared with DSVAR and LSVAR. We also investigate other calibration of (ρ_χ, σ_χ). When the standard error σ_χ of the non–technology shock is smaller, the

[6] This measure is defined as $cmd(k) = \sum_{i=0}^{k} |irf_i(model) - irf_i(svar)|$ where k denotes the selected horizon, $irf_i(model)$ the RBC impulse response and $irf_i(svar) = (1/N)\sum_{j=1}^{N} irf_i(svar)^j$ the mean of impulse responses over the N simulation experiments obtained from a SVAR model. In fact, the *cmd* measures the area of the bias up to the horizon k.

accuracy of the LSVAR and DSVAR models increases (see the cases where $\sigma_\chi/\sigma_z = 0.5$) and the LSVAR model and the CYSVAR–AR and CYSVAR–VAR approaches deliver very similar results. Conversely, when the standard error σ_χ of the preference increases, the LSVAR and DSVAR models poorly identify the effect of a technology shock on hours (see the cases $\sigma_\chi/\sigma_z = 2$). In this latter case, the CYSVAR approach tends to over–estimate the true effect of the technology shock, but the cumulative absolute mean bias remains small compared to the LSVAR and DSVAR models. Table 2 displays another interesting result: when the persistence of the preference shock increases from 0.9 to 0.99, the bias decreases. For the DSVAR model, this result can be partly explained by a decrease in distortions created by over–differentiation. For the CYSVAR approach, the bias reduction mainly originates from the effect of the preference shock on hours and consumption to output ratio. It is worth noting that the CYSVAR–AR delivers a smaller bias than the CYSVAR-VAR in most of cases.

To better understand these last results, we investigate the effect of ρ_χ and σ_χ on the structural autoregressive moving average representation of hours and consumption to output ratio. For our baseline calibration ($\rho_\chi = 0.95$, $\sigma_z = \sigma_\chi = 0.01$), we obtain:

$$\begin{aligned}\log(H_t) &= \text{cst} + 0.3536\frac{1}{(1-0.9622L)}\sigma_z\varepsilon_{z,t} - 1.5240\frac{(1-0.9759L)}{(1-0.9622L)(1-0.95L)}\sigma_\chi\varepsilon_{\chi,t} \\ \log(C_t) - \log(Y_t) &= \text{cst} - 0.4220\frac{1}{(1-0.9622L)}\sigma_z\varepsilon_{z,t} + 0.8180\frac{(1-0.9928L)}{(1-0.9622L)(1-0.95L)}\sigma_\chi\varepsilon_{\chi,t},\end{aligned}$$

where cst is an appropriate constant. The non–technology component is larger for hours than for consumption to output ratio. In this case, the preference shock accounts for 91% of variance of hours, whereas it represents 63% of the variance of the ratio. Moreover, the persistence of hours generated by the preference shock is more pronounced. This can be seen from the ARMA(2,1) representation of hours and consumption to output ratio. The two series display the same autoregressive parameters (0.9622 and 0.95), which are associated to the dynamics of the deflated capital K/Z and the persistence of the preference shock. However, the moving average parameter differs. In the case of hours, the parameter is equal to -0.976, whereas it is -0.993 for the consumption to output ratio. Figure 3 illustrates this property and reports the autocorrelation function of these two variables due to the preference shock. We see that the autocorrelations of the consumption to output ratio are smaller than the ones of hours. The labour wedge has therefore a greater impact in terms of volatility and persistence on hours than on consumption to output ratio. When the standard error of the preference shock is reduced ($\sigma_\chi = 0.005$), its contribution to the variance decreases, it becomes 73% for hours and 30% for the consumption to output ratio. In this case, SVARs have less difficulty to disentangle technology shocks from other shocks that have highly persistent, if not permanent effects on labour productivity. This explains why SVARs can properly uncover the true IRFs of hours to a technology shock.

To assess the effect of a highly persistent preference shock, we now set $\rho_\chi = 0.99$. This situation is of quantitative interest as Christiano, Eichenbaum and Vigfusson (2006) obtain values for this parameter between 0.986 and 0.9994. In this case, the ARMA representation becomes:

$$\begin{aligned}\log(H_t) &= \text{cst} + 0.3536\frac{1}{(1-0.9622L)}\sigma_z\varepsilon_{z,t} - 1.2710\frac{(1-0.9737L)}{(1-0.9622L)(1-0.99L)}\sigma_\chi\varepsilon_{\chi,t} \\ \log(C_t)-\log(Y_t) &= \text{cst} - 0.4220\frac{1}{(1-0.9622L)}\sigma_z\varepsilon_{z,t} + 0.5167\frac{(1-0.9960L)}{(1-0.9622L)(1-0.99L)}\sigma_\chi\varepsilon_{\chi,t}.\end{aligned}$$

The roots of moving average and the autoregressive parameters related to the preference shock in the expression of the consumption to output ratio are very similar,[7] so its dynamics can be approximated by a first order autoregressive process:

$$(\log(C_t)-\log(Y_t)) \simeq \text{cst} + 0.9622(\log(C_{t-1})-\log(Y_{t-1})) - 0.4220\sigma_z\varepsilon_{z,t} + 0.5167\sigma_\chi\varepsilon_{\chi,t}.$$

The consumption to output ratio behaves like the deflated capital. Conversely, hours do not share this property and finite autoregressions cannot properly uncover its true dynamics. This is illustrated in Figure 4 which reports the autocorrelation function of hours, consumption to output ratio and capital deflated by the total factor productivity. As emphasized by Chari, Kehoe and McGrattan (2007b), one of the problem with a SVAR model is that it does not include capital–like variable. In the model, the corresponding relevant state variable is $\log(K_t/Z_{t-1})$. Since Z_t is not observable in practice and K_t is measured with errors, we cannot include $\log(K_t/Z_{t-1})$ in SVARs. As can be seen from Figure 4, the autocorrelation functions of (C/Y) and (K/Z) are very close, but the ones of hours differ sharply.

This latter result suggests that the consumption to output ratio can be a good proxy of the relevant state variable when shocks to labour supply are very persistent or non-stationary. Conversely, hours cannot display this pattern. Highly persistent or non–stationary labour supply shocks is of course debatable but empirical works support this specification in small sample (see Gali, 2005, Christiano, Eichenbaum and Vigfusson, 2006 and Chang, Doh and Schorfheide, 2007). Chang, Doh and Schorfheide (2007) have shown that a DSGE model with a permanent shock to labour supply fits the data better. However, this result only holds for a frictionless version of their model. When the model includes labour adjustment costs, a version with stationary preference shock must be preferred. To better understand the results under a close to non–stationary labour supply, we report in appendix B some calculations about the dynamic behavior of the consumption to output ratio and hours for an economy with non stationary labour supply shocks. We notably show that when preference shocks follow a random walk (and thus hours are

[7] When we set $\rho_\chi = 0.999$, this finding is strengthened. Regarding only the effect of the preference shock, the reduced form of the consumption to output ratio is $\log(C_t) - \log(Y_t) = 0.3733(1-0.9993L)(1-0.9622L)^{-1}(1-0.999L)^{-1}\sigma_\chi\varepsilon_{\chi,t}$.

non–stationary), the consumption to output ratio follows an autoregressive process of order one with an autoregressive parameter exactly equal to the one of the deflated capital. Conversely, the growth rate of hours follows an ARMA process which can be poorly approximated by finite autoregressions. As pointed out in introduction, this problem is essentially problematic for SVARs with long–run restrictions, because the estimated VAR parameters enter in the computation of the long–run covariance matrix. Note that a SVAR model with long–run restrictions that includes labour productivity growth and the consumption to output ratio is valid whatever the process (stationary or non-stationary) of the hours series. The CYSVAR approach allows us to abstract from the very sensitive specification choice of hours in SVARs.

Simulation results for the cumulative absolute bias are completed with a measure of uncertainty about the estimated effect of the technology shocks. We thus compute the cumulative Root Mean Square Errors (RMSE) at various horizons.[8] The RMSE accounts for both bias and dispersion of the estimated IRFs. The results are reported in Table 3. Simulation experiments for different calibrations show again that the CSVAR approach provides smaller RMSE than the LSVAR and DSVAR models. This result comes essentially from the smaller bias with CSVAR. The large RMSE of DSVAR mainly originates from the large bias. In consequence, DSVAR model displays IRFs that are strongly biased but more precisely estimated. In contrast, LSVAR model displays smaller bias of IRFs but larger dispersion than DSVAR. The CSVAR approach presents the smallest bias on estimated IRFs and the estimated responses are more precisely estimated in comparison with LSVAR. These results from RMSE suggest favoring CYSVAR to LSVAR and DSVAR.

Finally, to judge the identification of the structural shocks, we compare the correlation between the estimated shock and the true shock for the different model's parametrisations. First, we compute the correlation between the estimated (from SVARs) and the true technology shocks, namely: $Corr(\varepsilon_z, \widehat{\eta}^T)$, where ε_z denotes the true technology shock and $\widehat{\eta}^T$ is the estimated technology shock from SVARs in the first step. Second, we compute $Corr(\varepsilon_\chi, \widehat{\eta}^T)$, the correlation between the estimated technology shock and non–technology shock ε_χ of the business cycle model. The idea is that if any method is able to consistently estimate the technology shock, we must obtain $Corr(\varepsilon_z, \widehat{\eta}^T) \approx 1$ and $Corr(\varepsilon_\chi, \widehat{\eta}^T) \approx 0$. These correlations are reported in the last column of Table 2. The CYSVAR approach always delivers the highest $Corr(\varepsilon_z, \widehat{\eta}^T)$. This correlation is relatively high, as it always exceeds 0.9 and it is not very sensitive to changes in $(\sigma_z, \rho_\chi, \sigma_\chi)$. Conversely, this correlation is lower in the case of the DSVAR model and it decreases dramatically with the volatility of the preference shock. The LSVAR delivers better results that the DSVAR, but it never outperforms the CYSVAR approach. Concerning the correlation between the iden-

[8]This measure is defined as $crmse(k) = \sum_{i=0}^{k} rmse_i$ where k denotes the selected horizon, $rmse_i = ((1/N)\sum_{j=1}^{N}(irf_i(model) - irf_i(svar)^j)^2)^{1/2}$ the RMSE at horizon i, $irf_i(model)$ the RBC impulse response function of hours and $irf_i(svar)^j$ the $SVAR$ impulse responses function of hours for the j^{th} draw and N is the number of simulation experiments.

tified technology shocks and the true preference shocks, the CYSVAR approach always delivers the lowest correlation (in absolute value). In the case of the DSVAR model, this correlation is large and positive This allows to explain why the DSVAR model underestimates the response of hours. Indeed, the estimated technology shock is contaminated by the preference shock. Hours worked persistently decrease after this shock in the model. It follows that the DSVAR model erroneously concludes that hours drop after a technology shock. A similar result applies in the case of the LSVAR model: the correlation between the estimated technology shock and the true non–technology shock is negative. This explains why the LSVAR model over–estimates the effect of a technology shock. In contrast, the CYSVAR approach does not suffer from this contamination.

3 Robustness

We now investigate three robustness issues of the two–step approach. We first consider an additional shock in the baseline model. Second, we connect the simulation part with the estimation part by considering a sticky prices model in which hours worked decrease in the short–run after a technology improvement. Third, we assess the accuracy of confidence intervals obtained from bootstrap techniques by computing the coverage rate for each models in the second step.

3.1 Results from the three shock model

The baseline model is augmented with a third shock. The capital stock is now assumed to evolve according to the law of motion

$$K_{t+1} = (1-\delta)\, K_t + x_t I_t$$

The variable x_t represents a disturbance to the investment–specific technology process and is assumed to follow a first order autoregressive process

$$\log(x_t) = \rho_x \log(x_{t-1}) + \sigma_x \varepsilon_{x,t}$$

where $|\rho_x| < 1$, $\sigma_x > 0$ and $\varepsilon_{x,t}$ is *iid* with zero mean and unit variance. The calibration of (ρ_x, σ_x) is reported in Table 1. In the benchmark experiment, we set $\rho_x = 0.95$ and $\sigma_x = 0.01$. In this case, the productivity shock accounts for tiny portion of aggregate fluctuations. It represents 6% of hours and 13% of the consumption to output ratio. This roughly corresponds to the two shock version of the model when $\sigma_z = 0.01$ and $\sigma_\chi = 0.02$, which constitute the worst case for each approach. Consequently, we analyze another case where technology shock accounts for a sizeable part of fluctuations. In this second situation, we set $\sigma_z = 0.01$ and $\sigma_\chi = \sigma_x = 0.005$. With this new parametrisation of the standard–error, we obtain that the technology shock represents 20% of the variance of hours and 38% of the consumption to output ratio.

We first investigate the reliability of SVARs which include two variables (labour productivity growth and hours for the LSVAR and DSVAR models; labour productivity growth and consumption to output ratio for the two–step approach). Figures 5 and 6 display the responses of hours for each approach using our baseline calibration ($\rho_\chi = \rho_x = 0.95$, $\sigma_z = \sigma_\chi = \sigma_x = 0.01$). As in the case of two shocks, the response of hours obtained from the DSVAR model is downward biased (see Figure 5) and persistently negative. The response of hours from the LSVAR model is upward biased and the CYSVAR approach delivers again more reliable results. This is confirmed in the first panel of Table 4. For the two values of σ_χ and σ_x, the CYSVAR approach outperforms the DSVAR and LSVAR models.

We also assess the DSVAR and LSVAR models when they include three variables (labour productivity growth, hours and consumption to output ratio). SVAR models that include three variables deliver better results (see Figure 6) . The downward bias of the DSVAR is strongly reduced, as the response on impact becomes positive. Moreover, the upward bias of the LSVAR decreases. The introduction of the consumption to output ratio in the VARs helps to disentangle the permanent shock from the transitory ones, as argued by Cochrane (1994). In our experiments, the CYSVAR approach still outperforms the DSVAR and LSVAR models (see Table 4). Moreover, the CYSVAR approach always delivers the smallest RMSE at each horizon. We also report in Table 4 the correlation between the estimated technology shock and the true shock of the business cycle model. We do not report the correlation with individual stationary shocks as we cannot separately identify each of them. The CYSVAR approach delivers in most of cases the highest $Corr(\varepsilon_z, \eta^T)$.

3.2 Simulation Experiments from a Sticky Prices Model

We now consider a sticky prices model. As illustrated by Galí (1999), this model generates a negative response of hours because the aggregate short–run dynamics is essentially driven by the demand side. Facing a positive productivity improvement, hours worked may decrease because firms can satisfy the given level of demand with less labour input. The model (not expound here to save space) is borrowed from Ireland (2003). The real part of the model is identical to the baseline model, apart the imperfect competition on the good market. Price setting by a local monopolist is subject to a quadratic adjustment cost. Different from Ireland (2003), the model includes the two previous shocks, *i.e.* a permanent productivity shock and a persistent shock that shift labour supply every periods. All the parameters are similar to those of the previous model and are calibrated according to Table 1. Two additional parameters in the sticky prices model (the mark–up and price adjustment cost parameters) are set to 20% and 20, respectively. Our findings appears not sensitive to the calibration of these two parameters.

Figures 7 and 8 display the responses of hours worked for each approaches. Now, under the sticky prices model, hours decrease in the short–run. The LSVAR and DSVAR models still deliver conflicting results. The LSVAR model tends to over–estimate the true response, whereas the DSVAR model delivers a more pronounced negative response in the short–run. At the opposite, the two–step approach yields very accurate responses, whatever the regression used in the second step. Again, this approach does not suffer from the specification of hours, contrary to standard SVARs. Table 5 reports the absolute bias and the RMSE. The DSVAR model works very well on impact but the bias increases very quickly with the horizon. The LSVAR model and the CYSVAR approach work equally well on impact, but as the horizon increase the LSVAR is more and more inaccurate. Again, the CYSVAR outperforms LSAVR and DSVAR models, both in terms of bias and dispersion (RMSE).

3.3 Coverage Rate and the Accuracy of Confidence Intervals

We finally assess the accuracy of the confidence interval estimators. As previously mentioned, these estimators are obtained from a bootstrap procedure which accounts for generated regressors problem and serial correlation. As in Christiano, Eichenbaum and Vigfusson (2006), we proceed as follows. For a given random realisation of the DGP, we estimate each specification of the the two–step approach (level, difference, AR and VAR). From the resulting residuals, we use the estimated parameters from the two–step procedure to generate 1000 data sets. We then estimate each specification and then compute the associated population of IRFs. For each artificial data set, the confidence intervals are defined as the top 2.5 percent and bottom 2.5 percent of the estimated IRFs. We then assess the accuracy of the confidence interval estimators using the coverage rate. For a given realisation of the DGP, we verify if the true IRF lies inside the computed confidence interval and we count for this. The rate corresponds also to the fraction of times, across the 1000 data sets simulated from the DGP, that the confidence interval contains the true IRFs of hours. If the confidence intervals were perfectly accurate, the coverage rate would be 95 percent. A coverage rate that exceeds 95 percent means that the computed confidence intervals are too large, leading to accept too often the model. Conversely, a coverage rate less than 95 percent indicates that the computed confidence intervals are too small, leading to reject too often the model.

Figure 9 reports the coverage rate for each approach under the frictionless model with two shocks under the baseline calibration (see section 2). This figure indicates that the coverage rate is roughly 95 percent for the CYSVAR-AR and CYSVAR-VAR. So, with these two models, the confidence intervals include the true IRF with a probability close to the true one. The results are similar with level specification, except on impact response. With the difference specification, the coverage rate is lower especially on impact. The bootstrapped confidence intervals with the specification (4) appears too small for short horizons and empirical testing must be conducted with caution.

4 Application of the Two–Step Approach

We now apply the two-step methodology with US data. Except for the Federal Fund rate, the data cover the sample period 1948Q1-2003Q4. We first study the dynamic responses of hours work to technology shocks. Second, we investigate the effects of these shocks on the rate of inflation and the nominal interest rate.

4.1 The Dynamic Responses of Hours Worked

We first present results for the IRFs of hours to technology shocks. In the first step, the VAR model includes the growth rate of labour productivity and the log of consumption to output ratio. Labour productivity is measured as the non farm business output divided by non farm business hours worked. Consumption is measured as consumption on nondurables and services and government expenditures. The consumption to output ratio is obtained by dividing the nominal expenditures by nominal GDP. In the second step, the log level h_t (see equations (3) and (5)) and the growth rate of hours Δh_t (see equation (4)) are projected on the estimated technology shocks. Hours worked in the non farm business sector are converted to per capita terms using a measure of the civilian population over the age of 16. The period is 1948Q1-2003Q4.

We also compare the estimation results with our two–step approach to those obtained from the estimation of SVAR models. These SVAR models include growth rate of labour productivity, the log of consumption to output ratio and either the log level of hours (LSVAR) or the growth rate of hours (DSVAR). In each of the SVAR models, we identify technology shocks as the only shocks that can affect the long-run level of labour productivity. The lag length p for each VAR model (1) is obtained using the Hannan–Quinn criterion. For each estimated model, we also apply a LM test to check for serial correlation. The number of lags p is 4. For the two-step procedure, we include in the second step the current and twelve past values of the identified technology shocks in the first step, *i.e.* $q = 13$ in (3), (4) and (5).

In order to assess the dynamic properties of hours worked and consumption to output ratio (in logs), we first compute their autocorrelation functions (ACFs). Figure 10 reports these ACFs for lags between 1 and 15. As this figure makes clear, the autocorrelation functions of hours worked always exceed those of the consumption to output ratio. Additionally, these ACFs decay at a slower rate. We also perform Augmented Dickey Fuller (ADF) test of unit root. For each variable, we regress the growth rate on a constant, lagged level and four lags of the first difference. The ADF test statistic is equal to -2.74 for hours and -2.93 for the consumption to output ratio. This hypothesis cannot be rejected at the 5 percent level for hours, whereas it is rejected at the 5 percent level for the consumption to output ratio. These findings suggests that the consumption to output ratio is less persistent than hours.

The estimated IRFs of hours after a technological improvement are reported in Figure 11. The upper panel shows the well known conflicting results of the effect of a technology shock on hours worked between LSVAR and DSVAR specifications.[9] The LSVAR displays a positive hump–shaped response whereas DSVAR implies a decrease in hours. We obtained (from bootstrap techniques) wide 95% confidence intervals in the LSVAR specification, such that the estimated IRFs of hours are not significantly different from zero at any horizon. For the DSVAR specification, the impact response is more precisely estimated (not significant at 95%, but significant at 90%), but as the horizon increase the negative response is not significantly different from zero. In these SVARs, including the consumption to output ratio does not help to reconcile the two specifications.

In contrast, the two-step approach delivers the same picture whether hours are specified in level, first difference, included a lagged term in the regression or specified in a VAR model (see the bottom panel of Figure 11). In the very short run, the IRFs of hours are very similar and when the horizon increases the positive response is a bit more pronounced when hours are taken in level rather than in first difference, with the lagged hours or included in a VAR model. On impact, hours worked decrease, but after five periods the response becomes persistently positive and hump–shaped. The bottom panel of Figure 11 reports also the 95 percent confidence interval obtained from bootstrap techniques. As previously mentioned, these confidence intervals account for the *generated regressor problem* and the serial correlation of the errors term in equations (3), (4), (5) and (6). The confidence interval is wide when we consider hours in level. Consequently, these response cannot be used to discriminate among business cycle theories and for model building. When hours are projected in first difference, the dynamic response are more precisely estimated and hours significantly drop on impact. However, our previous simulations suggest that the coverage rate associated with this approach is inadequate, especially in the very short–run. Consequently, we should use this finding with caution. The case of lagged hours in equation (5) delivers almost similar confidence intervals, *i.e.* the response of hours on impact differs significantly from zero. Our simulation experiments has indicated that this approach delivers accurate confidence intervals (the coverage rate is close to 95%). When the horizon increases, the IRFs are less precisely estimated. The VAR specification in the second step provides similar results, but we can not reject that the response is zero on impact at 95%. Notice that when we consider 90% confidence intervals both the AR and VAR specification predicts a significant decrease of hours in the short–run. Finally, we report the dynamic responses of the LSAVR, DSVAR and two–step VAR in the last part of figure 11. This figure shows that our two step–step procedure partially solves the conflicting results about the right specification of hours in SVARs. Notably, our findings are in line with those of previous empirical papers which obtain that hours fall significantly on impact (see Galí, 1999, Basu, Fernald and Kimball, 2006, Francis and Ramey,

[9]Christiano, Eichenbaum and Evans (2004) also obtain conflicting results in larger SVARs.

2008), but display a hump–shaped positive response during the subsequent periods (see Vigfusson, 2004).

4.2 The Dynamic Responses of Inflation and Nominal Interest Rate

We now illustrate the potential of our two–step approach by looking at the dynamic responses of the inflation rate and the short–term nominal interest rate after a technology shock. These two variables are known to display high level of serial correlation and some empirical studies have found that they can be characterized by an integrated process of order one.[10] Therefore, we use these two variables to illustrate the consequence of the specification choice (level *versus* first difference) in SVARs.

We first investigate the response of the inflation rate. The measure of inflation is obtained using the growth rate of the GDP deflator. The estimated IRFs of the inflation rate after a technological improvement are reported in Figure 12. As previously, the upper panel reports the estimated dynamic responses obtained from LSAVR and DSVAR specifications. The LSVAR model includes labour productivity growth, the inflation rate and the log of consumption to output ratio. The DSVAR model includes the same variables but inflation is considered in first difference. As this figure shown, the specification of the inflation rate matters. In the DSVAR specification, the rate of inflation responds very little to identified technology shocks. Conversely, the response of inflation in the LSVAR model is persistently negative.

The two-step approach provides similar IRFs according to the specification of the inflation rate in the second step (see the bottom panel of Figure 12). With the level specification, the dynamic responses are more pronounced but the four specifications of the inflation rate in the second step provide the same shape for the responses. In all cases, the inflation rate decreases on impact and steadily goes back to its long–run value. The bottom panel of Figure 12 reports also the 95 percent confidence interval. Contrary to hours worked, the confidence interval appears less sensitive to the specification of inflation in the second step. In each regression, the inflation rate significantly decreases in the short–run. Note that the effect of a technology improvement has no long–lasting effect on inflation since the response is almost zero after two years. Our finding are again in the line of Basu, Fernald and Kimball (2006). It also complement their results by providing dynamic responses at quarterly frequency.

We now investigate the effect of technology shocks on the short–run nominal interest rate,

[10] The empirical results offered in the literature are mixed, depending on the the econometric technique used. Recent contributions on trend inflation specifies actual inflation as a sum of a random walk and a stationary noise (see Stock and Watson, 2007, Cogley, Primicery and Sargent, 2008). In Juselius (2006), cointegrated VAR models include the inflation rate and the nominal interest rate in first difference. In the context of permanent technology shocks, Galí (1999) considers a DSVAR model with the inflation rate in first difference and a cointegration between the nominal interest rate and the inflation rate. See also King, Plosser, Stock and Watson (1991) for further evidence of the non–stationarity of these two nominal variables in cointegrated VAR models.

measured with Federal Fund rate. This rate is available for a shorter sample 1954Q1–2003Q4. Since much of business cycle literature is concerned with post–1959 data, we follow Christiano, Eichenbaum and Vigfusson (2004) and therefore consider a second sample period given by 1959Q1–2003Q4. The dynamic responses of the nominal interest rate after a technological improvement are reported in Figure 13. In the upper panel, we report the IRFs obtained from LSVAR and DSVAR specifications. The LSVAR model includes now labour productivity growth, the nominal interest rate and the log of consumption to output ratio. The DSVAR model includes the same variables but the nominal interest rate is now specified in first difference. We obtain that the specification of the nominal interest rate modify the dynamic responses of this variable. In particular, the DSVAR specification implies a permanent long–run decrease, whereas it steadily goes back to its long–run value in the LSVAR specification.

With the two-step approach, the shape of the IRFs is not altered by the specification of the nominal interest rate in the second step (see the bottom panel of Figure 12). However, the dynamic responses with the level specification are more pronounced than the ones obtained from the three other specifications (as for the rate of inflation). In the bottom panel of Figure 13, we also report the 95 percent confidence interval. For the four specifications in the second step, we obtain a persistent and significant decrease in the Fed Fund rate. These empirical results with quarterly frequency data are again similar to those of Basu, Fernald and Kimball (2006).

5 Conclusion

This paper proposes a simple two–step approach to consistently estimate a technology shock and the response of aggregates variables that follows a technology improvement. In a first step, a SVAR model with labour productivity growth and consumption to output ratio allows us to estimate the technology shock. In a second step, the response of hours is obtained by a simple regression of hours on the estimated technology shock. When applied to artificial data generated by business cycle models (RBC and sticky prices), our approach replicates more closely the model IRFs. Importantly, the results are invariant to the specification of hours in the second step. The two–step approach, when applied to actual data, predicts a short–run decrease of hours after a technology improvement, as well as a delayed and hump–shaped positive response. In addition, the rate of inflation and the nominal interest rate displays a significant decrease after a positive technology shock.

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Appendix

A Proof of Proposition 1

The consistency of the second step estimators depends on the consistency of the autoregressive coefficients in the first step. The consistency of the the autoregressive coefficients ensures the consistency of the estimated technology shocks. Two cases are of interest: *i)* the data are generated by a finite VAR or *ii)* the data are generated by an infinite VAR. When the data are generated by a finite VAR, the VAR estimators in the first step are consistent for a number of lags included in the VAR greater or equal to the true ones. For data generated by an infinite VAR, Lewis and Reinsel (1985) show that a finite order k fitted VAR to a realisation T provides consistency and asymptotic normality of the estimated autoregressive coefficients assuming that $k \to \infty$ at some rate as $T \to \infty$. In particular, they show the consistency for k function of T such that $k^2/T \to 0$ as $k, T \to \infty$. Now, consider the first specification in the second step. The convergence in probability is established by standard arguments. First, the estimator $\widehat{a}_{21,k}$ is centered to the true value by direct straightforward implications of the orthogonality of the permanent and the transitory shocks and by the fact that those shocks are serially uncorrelated. Second, it is easy to show that the variance of the OLS estimator converges to zero. The convergence in probability follows. Let us now examine the second and the third specifications. One can always rewrite the infinite moving average representation as follows:

$$\begin{aligned} h_t - \tilde{\rho} h_{t-1} &= (a_1(L) - \tilde{\rho} a_1(L)L)\, \eta_t^T + \sum_{l=1}^{s} (a_{2,l}(L) - \tilde{\rho} a_{2,l}(L)L)\, \eta_{l,t}^{NT} \\ &= \theta_1(L)\eta_t^T + \theta_2(L)\eta_t^{NT}. \end{aligned}$$

The structural moving average coefficients corresponding to the impact of the technology shocks on hours can thus be retrieved by the following relationship $a_{1,k} = \sum_{j=0}^{k} \tilde{\rho}^j \theta_{1,k-j}$. First, consider the AR(1) specification for our second step. We can easily show for this case that the OLS estimators of $\hat{\theta}_{1,k}$ converges in probability to $\theta_{1,k}$. For a given estimator $\widehat{\rho}$, a consistent estimator $\hat{a}_{1,k}$ is thus guaranteed by the consistency of $\hat{\theta}_{1,k}$. In fact, we only needs to suppose that the OLS estimator of $\tilde{\rho}$ is bounded in probability, namely $\sqrt{T}\,(\hat{\rho} - \tilde{\rho}) = O_p(1)$ for some $\tilde{\rho} \in \mathbb{R}$ (see Andrews and Mohanan (1992) for a similar argument in a different context). Finally for the case with the hours in difference, this corresponds to fix $\tilde{\rho}$ to 1. The OLS estimator of $\theta_{1,k}$ is

$$\hat{\theta}_{1,k} = \frac{\sum_{t=q+1}^{T} \eta_{t-k}^T \Delta h_t}{\sum_{t=q+1}^{T} \left(\eta_{t-k}^T\right)^2}$$

The estimator $\hat{a}_{1,k}$ is given by the cumulative sum of the $\hat{\theta}_{1,k}$, namely:

$$\hat{a}_{1,k} = \frac{\sum_{t=q+1}^{T} \left(\eta_t^T h_t - \eta_t^T h_{t-1} + \eta_{t-1}^T h_t - \eta_{t-1}^T h_{t-1} + \cdots + \eta_{t-k}^T h_t - \eta_{t-k}^T h_{t-1}\right)}{\sum_{t=q+1}^{T} \left(\eta_{t-k}^T\right)^2}$$

By the stationarity hypothesis for h_t, $\frac{1}{T}\sum \eta_{t-k}^T h_t \xrightarrow{p} \gamma_k$ for all k not depending on t where γ_k is the covariance function between η_t^T and h_t. Moreover, $\frac{1}{T}\sum \eta_t^T h_{t-1} \xrightarrow{p} 0$ and $\frac{1}{T}\sum_{t=q+1}^{T} \left(\eta_{t-k}^T\right)^2 \xrightarrow{p} 1$, the consistency result follows.

B A Business Cycle Model with Non–Stationary Hours

The model includes a random walk in productivity (Z_t) and non-stationary hours, due to a permanent preference shock (B_t). The intertemporal expected utility function of the representative household is given by

$$E_t \sum_{i=0}^{\infty} \beta^i \left\{ \log(C_{t+i}) - \chi(H_{t+i}/B_{t+i}) \right\},$$

where $\chi > 0$, $\beta \in (0,1)$ denotes the discount factor and E_t is the expectation operator conditional on the information set available as of time t. C_t is the consumption at t and H_t represents the household's labour supply. The labour supply is subjected to a preference shock B_t, that follows the stochastic process $\Delta \log(B_t) = \sigma_b \varepsilon_{b,t}$, where $\sigma_b > 0$, and $\varepsilon_{b,t}$ is *iid* with zero mean and unit variance. The representative firm uses capital K_t and labour H_t to produce the homogeneous final good Y_t. The technology is represented by the following constant returns–to–scale Cobb–Douglas production function $Y_t = K_t^{\alpha} (Z_t H_t)^{1-\alpha}$, where $\alpha \in (0,1)$. Z_t is assumed to follow an exogenous process of the form $\Delta\log(Z_t) = \sigma_z\varepsilon_{z,t}$, where $\varepsilon_{z,t}$ is *iid* with zero mean and unit variance. The capital stock evolves according to the law of motion $K_{t+1} = (1-\delta)\,K_t + I_t$, where $\delta \in (0,1)$ is the constant depreciation rate. Finally, the final good can be either consumed or invested $Y_t = C_t + I_t$. In this model, the labour supply shock B_t induces a stochastic trend into hours as well as into output, consumption, and capital. In addition, Z_t has a long-run impact on Y_t, C_t, K_t, and I_t. Accordingly, to obtain a stationary equilibrium, these variables must be detrended as $\breve{h}_t = H_t/B_t$, $\breve{y}_t = Y_t/(Z_tB_t)$, $\breve{c}_t = C_t/(Z_tB_t)$, $\breve{i}_t = I_t/(Z_tB_t)$ and $\breve{k}_{t+1} = K_{t+1}/(Z_tB_t)$. With these transformations, the approximate solution of the model is computed from a log–linearisation of the stationary equilibrium conditions around this deterministic steady state. It is important to notice that in our model, B_t has a long-run impact on H_t, as well as on Y_t and the above trending variables. At the same time, Z_t alone can have a long-run effect on labour productivity. Hence, this model is perfectly compatible with the identification assumptions used by Galí (1999).

The log–linearisation of equilibrium conditions around the deterministic steady state yields

$$\widehat{\breve{k}}_{t+1} = (1-\delta)(\widehat{\breve{k}}_t - \sigma_z\varepsilon_{z,t} - \sigma_b\varepsilon_{b,t}) + \frac{y}{k}\widehat{\breve{y}}_t - \frac{c}{k}\widehat{\breve{c}}_t \tag{7}$$

$$\widehat{\breve{h}}_t = \widehat{\breve{y}}_t - \widehat{\breve{c}}_t \tag{8}$$

$$\widehat{\breve{y}}_t = \alpha(\widehat{\breve{k}}_t - \sigma_z\varepsilon_{z,t} - \sigma_b\varepsilon_{b,t}) + (1-\alpha)\widehat{\breve{h}}_t \tag{9}$$

$$E_t\widehat{\breve{c}}_{t+1} = \widehat{\breve{c}}_t + \alpha\beta\frac{y}{k}E_t(\widehat{\breve{y}}_{t+1} - \widehat{\breve{k}}_{t+1} - \sigma_z\varepsilon_{z,t+1} - \sigma_b\varepsilon_{b,t+1}) \tag{10}$$

where $y/k = (1-\beta(1-\delta))/(\alpha\beta)$ and $c/k = y/k - \delta$. After substitution of (8) into (9), one gets

$$\widehat{\breve{y}}_t - \widehat{\breve{k}}_t = -\sigma_z\varepsilon_{z,t} - \sigma_b\varepsilon_{b,t} - \frac{1-\alpha}{\alpha}\widehat{\breve{c}}_t$$

Now, using the above expression, (7) and (10) rewrite

$$E_t\widehat{\breve{c}}_{t+1} = \varphi\widehat{\breve{c}}_t \quad \text{with} \quad \varphi = \frac{\alpha}{1-\beta(1-\alpha)(1-\delta)} \in (0,1) \tag{11}$$

$$\widehat{\breve{k}}_{t+1} = \nu_1\widehat{\breve{k}}_t - \nu_1(\sigma_z\varepsilon_{z,t} + \sigma_b\varepsilon_{b,t}) - \nu_2\widehat{\breve{c}}_t$$
$$\text{with } \nu_1 = \frac{1}{\beta\varphi} > 1 \quad \text{and} \quad \nu_2 = \frac{1-\beta(1-\delta(1-\alpha^2))}{\alpha^2\beta} \tag{12}$$

As $\nu_1 > 1$, (12) must be solved forward

$$\widehat{\breve{k}}_t = \sigma_z\varepsilon_{z,t} + \sigma_b\varepsilon_{b,t} + \left(\frac{\nu_2}{\nu_1}\right)\lim_{T\to\infty}E_t\sum_{i=0}^{T}\left(\frac{1}{\nu_1}\right)^i\widehat{\breve{c}}_{t+i} + \lim_{T\to\infty}E_t\left(\frac{1}{\nu_1}\right)^T\widehat{\breve{k}}_{t+T}$$

Excluding explosive pathes, *i.e.* $\lim_{T\to\infty}E_t\,(1/\nu_1)^T\,\widehat{\breve{k}}_{t+T} = 0$, and using (11), one gets the decision rule on consumption:

$$\widehat{\breve{c}}_t = \left(\frac{\nu_1-\varphi}{\nu_2}\right)\left(\widehat{\breve{k}}_t - (\sigma_z\varepsilon_{z,t} + \sigma_b\varepsilon_{b,t})\right) \tag{13}$$

After substituting (13) into (12), the dynamics of capital is given by:

$$\widehat{\breve{k}}_{t+1} = \varphi\left(\widehat{\breve{k}}_t - (\sigma_z\varepsilon_{z,t} + \sigma_b\varepsilon_{b,t})\right) \tag{14}$$

The persistence properties of the model is thus governed by the parameter $\varphi \in (0,1)$. The decision rules of the other (deflated) variables are similar to equation (13). Using (13) and (14), the consumption to output ratio is given by

$$\log(C_t) - \log(Y_t) = \nu_{cy}\left(-\frac{(\sigma_z\varepsilon_{z,t} + \sigma_b\varepsilon_{b,t})}{1-\varphi L}\right)$$

where $\nu_{cy} = \alpha(\nu_1 - \varphi - \nu_2)/\nu_2$. The latter expression shows that the consumption to output ratio follows exactly the same stochastic process (an autoregressive process of order one) as the deflated capital $\log(K_t/(Z_{t-1}B_{t-1}))$ in equation (14). The consumption to output ratio is thus an exact representation of the relevant state variable of the model. Notice than both shocks have a transitory effect on the ratio. Hours do not display a similar pattern. Using (8) and the above expression, the growth rate of hours is given by:

$$(1-\varphi L)\Delta\log(H_t) = \nu_{cy}\sigma_z\Delta\varepsilon_{z,t} + (1+\nu_{cy})\left(1 - \left(\frac{\varphi+\nu_{cy}}{1+\nu_{cy}}\right)L\right)\sigma_b\varepsilon_{b,t}$$

where $\Delta\log(H_t) = \Delta\widehat{\breve{h}}_t + \varepsilon_{b,t}$. The technology shock has no long–run effect on hours, whereas the preference shock increases hours permanently. More importantly, hours follow an ARMA(1,1) process, with an unit root in the moving average representation of the technology shock. It follows that finite autoregressions with long–run restrictions may be problematic in properly uncovering the true dynamics of hours.

Table 1: Calibrated Values

Deep Parameters		Shocks Parameters (benchmark)		Shocks Parameters (alternative)	
β	0.9926	σ_z	0.01	σ_χ/σ_z	[0.5;1;2]
α	0.330	ρ_χ	0.95	ρ_χ	[0.9;0.95;0.99]
δ	0.0250	σ_χ	0.01		
γ_z	0.0036	ρ_x	0.95		
ψ	3.600	σ_x	0.01	$\sigma_z/(\sigma_\chi, \sigma_x)$	[1;2]

Table 2: Simulation Results with two shocks: Cumulative Absolute Bias

			Horizon				Correlation	
ρ_χ	σ_χ/σ_z	Model	0	0 to 4	0 to 8	0 to 12	$Corr(\varepsilon_z, \eta^T)$	$Corr(\varepsilon_\chi, \eta^T)$
0.90	0.5	LSVAR	0.056	0.195	0.262	0.452	0.941	-0.101
		DSVAR	0.215	1.047	1.732	2.230	0.931	0.250
		CYSVAR–AR	0.022	0.123	0.355	0.649	0.953	0.013
		CYSVAR–VAR	0.022	0.106	0.294	0.565	0.953	0.013
	1	LSVAR	0.261	0.990	1.228	1.287	0.923	-0.198
		DSVAR	0.690	3.307	5.752	8.015	0.859	0.419
		CYSVAR–AR	0.070	0.228	0.326	0.540	0.949	-0.059
		CYSVAR–VAR	0.069	0.249	0.332	0.558	0.949	-0.059
	2	LSVAR	1.003	3.902	5.325	5.902	0.872	-0.365
		DSVAR	1.925	9.197	16.251	23.132	0.728	0.596
		CYSVAR–AR	0.4937	1.883	2.493	2.675	0.934	-0.180
		CYSVAR–VAR	0.036	0.186	0.352	0.550	0.934	-0.180
0.95	0.5	LSVAR	0.047	0.183	0.222	0.256	0.924	-0.104
		DSVAR	0.196	0.988	1.658	2.143	0.931	0.252
		CYSVAR–AR	0.035	0.168	0.331	0.503	0.950	0.033
		CYSVAR–VAR	0.035	0.169	0.331	0.513	0.950	0.033
	1	LSVAR	0.259	1.101	1.641	1.959	0.899	-0.218
		DSVAR	0.629	3.115	5.472	7.650	0.861	0.422
		CYSVAR–AR	0.035	0.130	0.149	0.194	0.943	-0.038
		CYSVAR–VAR	0.034	0.136	0.162	0.224	0.943	-0.038
	2	LSVAR	1.025	4.436	6.820	8.430	0.835	-0.407
		DSVAR	1.769	8.728	15.564	22.231	0.728	0.606
		CYSVAR–AR	0.420	1.794	2.738	3.385	0.927	-0.167
		CYSVAR–VAR	0.036	0.186	0.352	0.550	0.927	-0.167
0.99	0.5	LSVAR	0.002	0.006	0.038	0.108	0.874	-0.071
		DSVAR	0.158	0.812	1.359	1.726	0.936	0.238
		CYSVAR–AR	0.067	0.315	0.485	0.587	0.945	0.093
		CYSVAR–VAR	0.068	0.338	0.559	0.710	0.945	0.093
	1	LSVAR	0.158	0.737	1.280	1.792	0.842	-0.181
		DSVAR	0.510	2.559	4.498	6.260	0.874	0.405
		CYSVAR–AR	0.099	0.456	0.701	0.852	0.932	0.072
		CYSVAR–VAR	0.100	0.473	0.760	0.958	0.932	0.072
	2	LSVAR	0.760	3.570	6.065	8.243	0.788	-0.362
		DSVAR	1.499	7.474	13.348	19.057	0.738	0.610
		CYSVAR–AR	0.003	0.019	0.091	0.228	0.924	-0.007
		CYSVAR–VAR	0.036	0.186	0.352	0.550	0.924	-0.007

Table 3: Simulation Results with two shocks: Cumulative Root Mean Square Errors

			Horizon			
ρ_χ	σ_χ/σ_z	Model	0	[0:4]	[0:8]	[0:12]
0.90	0.5	LSVAR	0.184	0.824	1.317	1.731
		DSVAR	0.243	1.210	2.060	2.767
		CYSVAR–AR	0.169	0.782	1.271	1.695
		CYSVAR–VAR	0.168	0.7864	1.294	1.726
	1	LSVAR	0.434	1.835	2.727	3.318
		DSVAR	0.726	3.511	6.132	8.583
		CYSVAR–AR	0.341	1.517	2.286	2.825
		CYSVAR–VAR	0.340	1.529	2.354	2.932
	2	LSVAR	1.191	4.845	6.977	8.216
		DSVAR	1.968	9.454	16.709	23.7937
		CYSVAR–AR	0.813	3.448	5.014	5.950
		CYSVAR–VAR	0.812	3.474	5.110	6.085
0.95	0.5	LSVAR	0.203	0.960	1.593	2.105
		DSVAR	0.224	1.155	2.002	2.714
		CYSVAR–AR	0.168	0.824	1.366	1.809
		CYSVAR–VAR	0.168	0.832	1.408	1.878
	1	LSVAR	0.471	2.155	3.464	4.452
		DSVAR	0.663	3.319	5.869	8.256
		CYSVAR–AR	0.354	1.649	2.659	3.430
		CYSVAR–VAR	0.344	1.659	2.722	3.523
	2	LSVAR	1.245	5.559	8.791	11.170
		DSVAR	1.810	8.979	16.035	22.928
		CYSVAR–AR	0.840	3.719	5.925	7.584
		CYSVAR–VAR	0.803	3.7430	6.016	7.685
0.99	0.5	LSVAR	0.248	1.220	2.114	2.915
		DSVAR	0.185	0.983	1.724	2.350
		CYSVAR–AR	0.164	0.842	1.436	1.948
		CYSVAR–VAR	0.164	0.855	1.496	2.042
	1	LSVAR	0.520	2.519	4.359	6.015
		DSVAR	0.542	2.763	4.913	6.910
		CYSVAR–AR	0.354	1.776	3.030	4.118
		CYSVAR–VAR	0.353	1.786	3.097	4.223
	2	LSVAR	1.192	5.734	9.858	13.552
		DSVAR	1.535	7.715	13.824	19.777
		CYSVAR–AR	0.732	3.641	6.239	8.532
		CYSVAR–VAR	0.730	3.644	6.292	8.602

Table 4: Simulation Results with Three Shocks

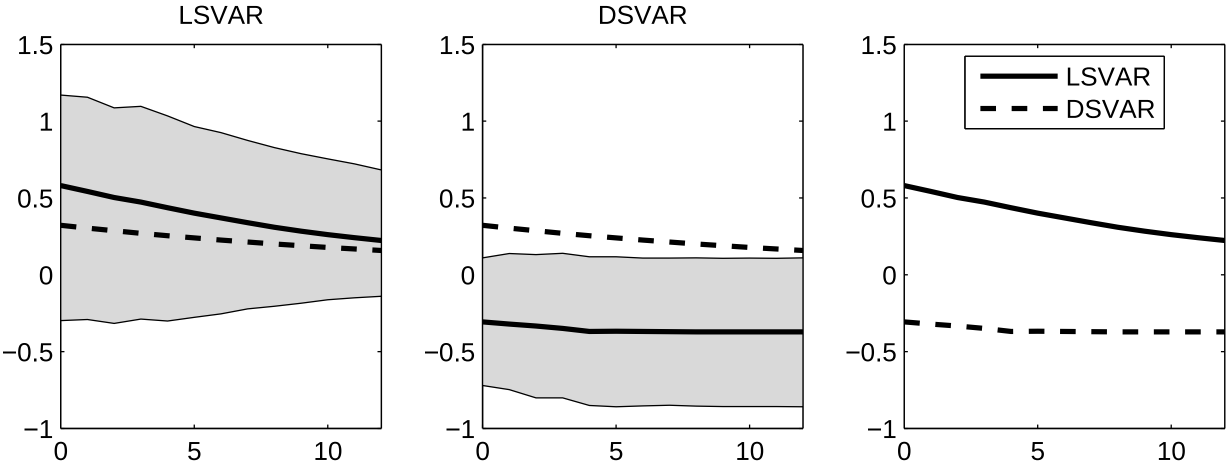
Average Cumulative Absolute Bias							
			Horizon				
Variables	$\sigma_z/(\sigma_\chi, \sigma_i)$	Model	0	0 to 4	0 to 8	0 to 12	$Corr(\varepsilon_z, \eta^T)$
$(y-h, h)$	1	LSVAR	0.592	2.513	3.748	4.450	0.847
		DSVAR	0.783	3.992	7.165	10.162	0.850
		CYSVAR–AR	0.333	1.383	2.008	2.358	0.836
		CYSVAR–VAR	0.316	1.272	1.779	2.005	0.836
	2	LSVAR	0.142	0.599	0.845	0.933	0.911
		DSVAR	0.261	1.332	2.300	3.083	0.922
		CYSVAR–AR	0.064	0.253	0.295	0.343	0.920
		CYSVAR–VAR	0.054	0.195	0.228	0.325	0.920
$(y-h, h, c-y)$	1	LSVAR	0.384	1.618	2.409	2.906	0.782
		DSVAR	0.320	1.323	1.718	1.828	0.850
		CYSVAR–AR	0.333	1.383	2.008	2.358	0.836
		CYSVAR–VAR	0.316	1.272	1.779	2.005	0.836
	2	LSVAR	0.065	0.268	0.359	0.376	0.844
		DSVAR	0.055	0.224	0.292	0.519	0.922
		CYSVAR–AR	0.064	0.253	0.295	0.343	0.920
		CYSVAR–VAR	0.054	0.195	0.228	0.325	0.920

Cumulative Root Mean Square Errors						
			Horizon			
Vairables	$\sigma_g/(\sigma_z, \sigma_\chi)$	Model	0	0 to 4	0 to 8	0 to 12
$(y-h, h)$	1	LSVAR	0.717	3.177	4.960	6.236
		DSVAR	0.821	4.208	7.570	10.765
		CYSVAR–AR	0.485	2.212	3.447	4.325
		CYSVAR–VAR	0.469	2.108	3.357	4.343
	2	LSVAR	0.261	1.202	1.932	2.491
		DSVAR	0.290	1.504	2.645	3.636
		CYSVAR–AR	0.194	0.929	1.497	1.941
		CYSVAR–VAR	0.187	0.893	1.494	2.018
$(y-h, h, c-y)$	1	LSVAR	0.615	2.732	4.297	5.490
		DSVAR	0.485	2.251	3.638	4.809
		CYSVAR–AR	0.485	2.212	3.447	4.325
		CYSVAR–VAR	0.469	2.108	3.357	4.343
	2	LSVAR	0.257	1.176	1.890	2.463
		DSVAR	0.197	0.970	1.669	2.335
		CYSVAR–AR	0.194	0.929	1.497	1.941
		CYSVAR–VAR	0.187	0.893	1.494	2.018

Table 5: Simulation Results with the Sticky Prices Model

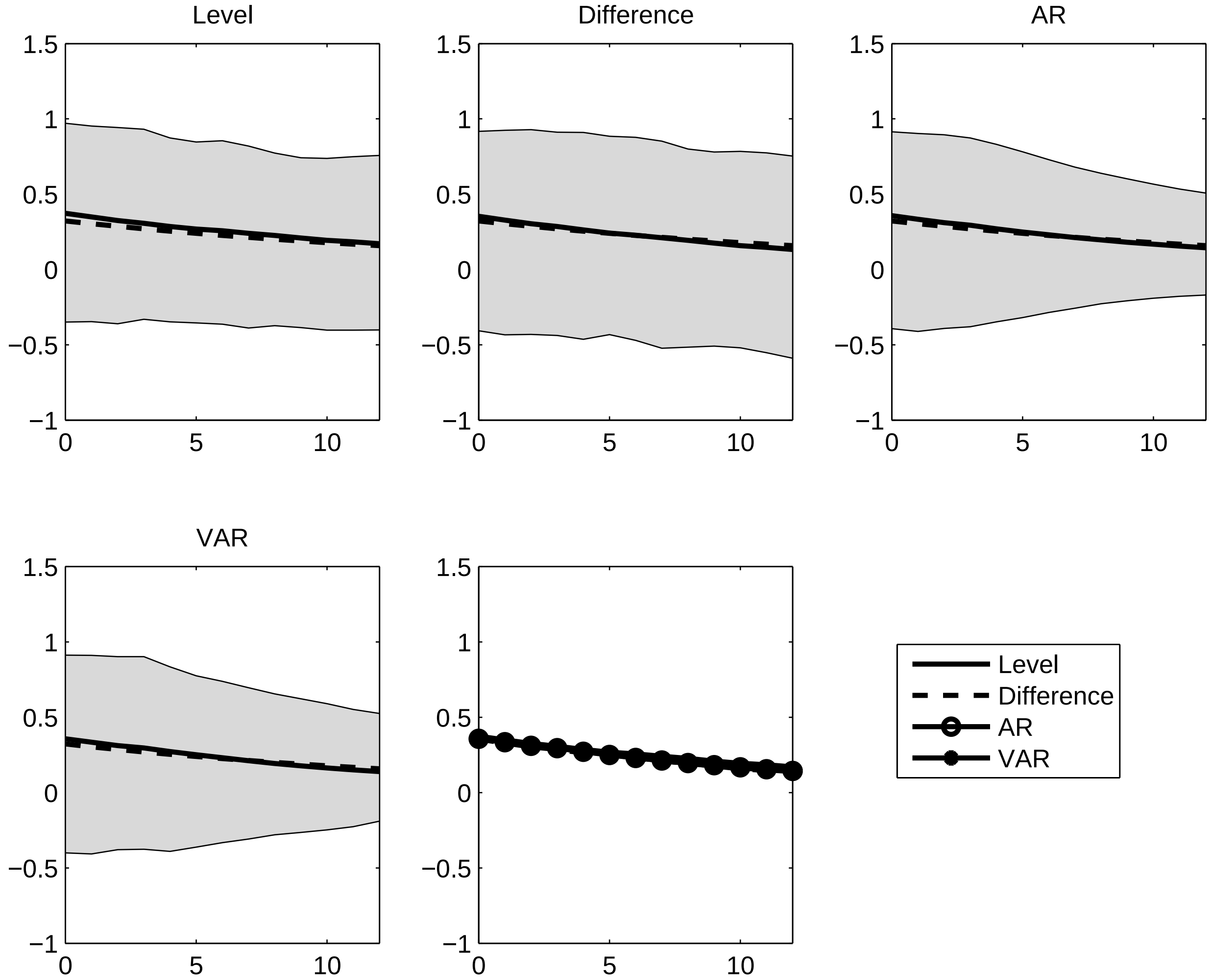
Average Cumulative Absolute Bias					
		Horizon			
Model	0	0 to 4	0 to 8	0 to 12	$Corr(\varepsilon_z, \eta^T)$
LSVAR	0.469	1.315	1.952	2.378	0.899
DSVAR	0.258	2.716	5.129	7.287	0.849
CYSVAR–AR	0.469	0.555	1.043	1.600	0.901
CYSVAR–VAR	0.400	0.984	1.423	1.693	0.901
Cumulative Root Mean Square Errors					
		Horizon			
Model	0	0 to 4	0 to 8	0 to 12	
LSVAR	0.691	2.306	3.500	4.376	
DSVAR	0.292	2.952	5.534	7.883	
CYSVAR–AR	0.679	1.763	2.492	3.128	
CYSVAR–VAR	0.567	1.785	2.714	3.384	

Figure 1: Simulation Results with with SVARs (Two Shocks)



Note: LSVAR and DSVAR models. The LSVAR model includes labour productivity growth and the log of hours. The DSVAR model includes labour productivity growth and the log of hours in first difference. The dashed line correspond to the true IRF of hours. The results are obtained from our benchmark calibration. The selected horizon for IRFs is 13. Confidence intervals are based the 95–percentile from 1000 Monte–Carlo experiments.

Figure 2: Simulation Results with with the Two–Step Approach (Two Shocks)



Note: Two–Step identification. The SVAR model in the first step includes labour productivity growth and the log of consumption to output ratio. In the second step, the dynamic responses of hours are obtained from equations (3), (4), (5) and (6). For this latter equation, the VAR model includes labour productivity growth, the log of hours and the log of consumption to output ratio. The dashed line correspond to the true IRF of hours. The results are obtained from our benchmark calibration. The selected horizon for IRFs is 13. Confidence intervals are based the 95–percentile from 1000 Monte–Carlo experiments.

Figure 3: Autocorrelation function (preference shock)

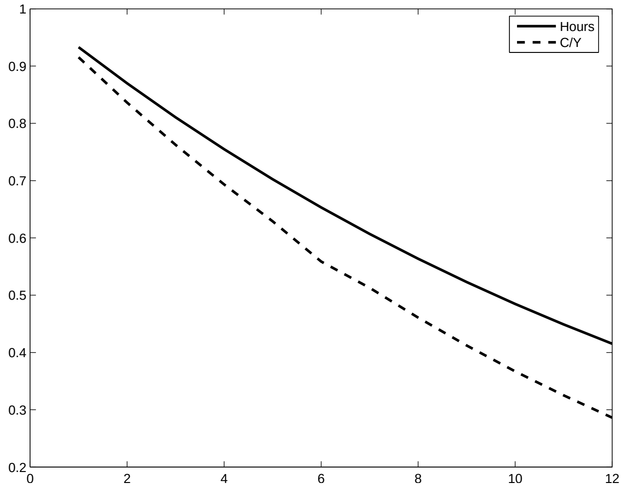


Figure 4: Autocorrelation function (technology and preference shock)

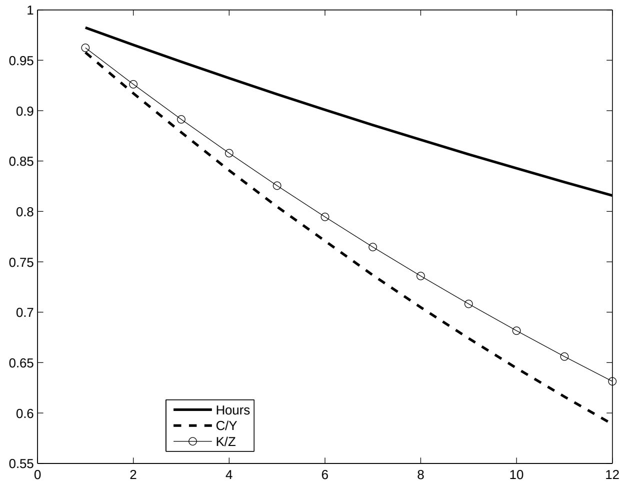
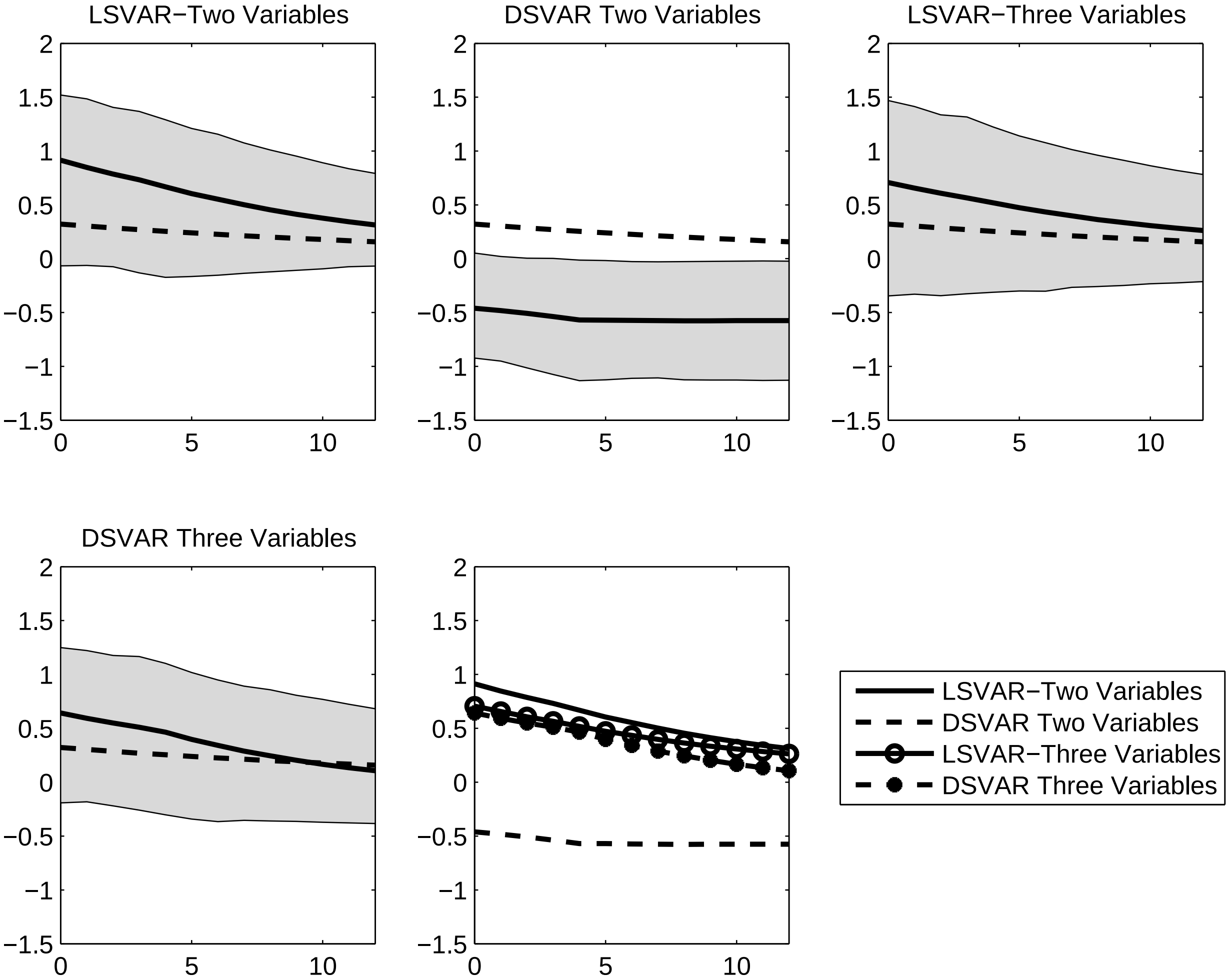
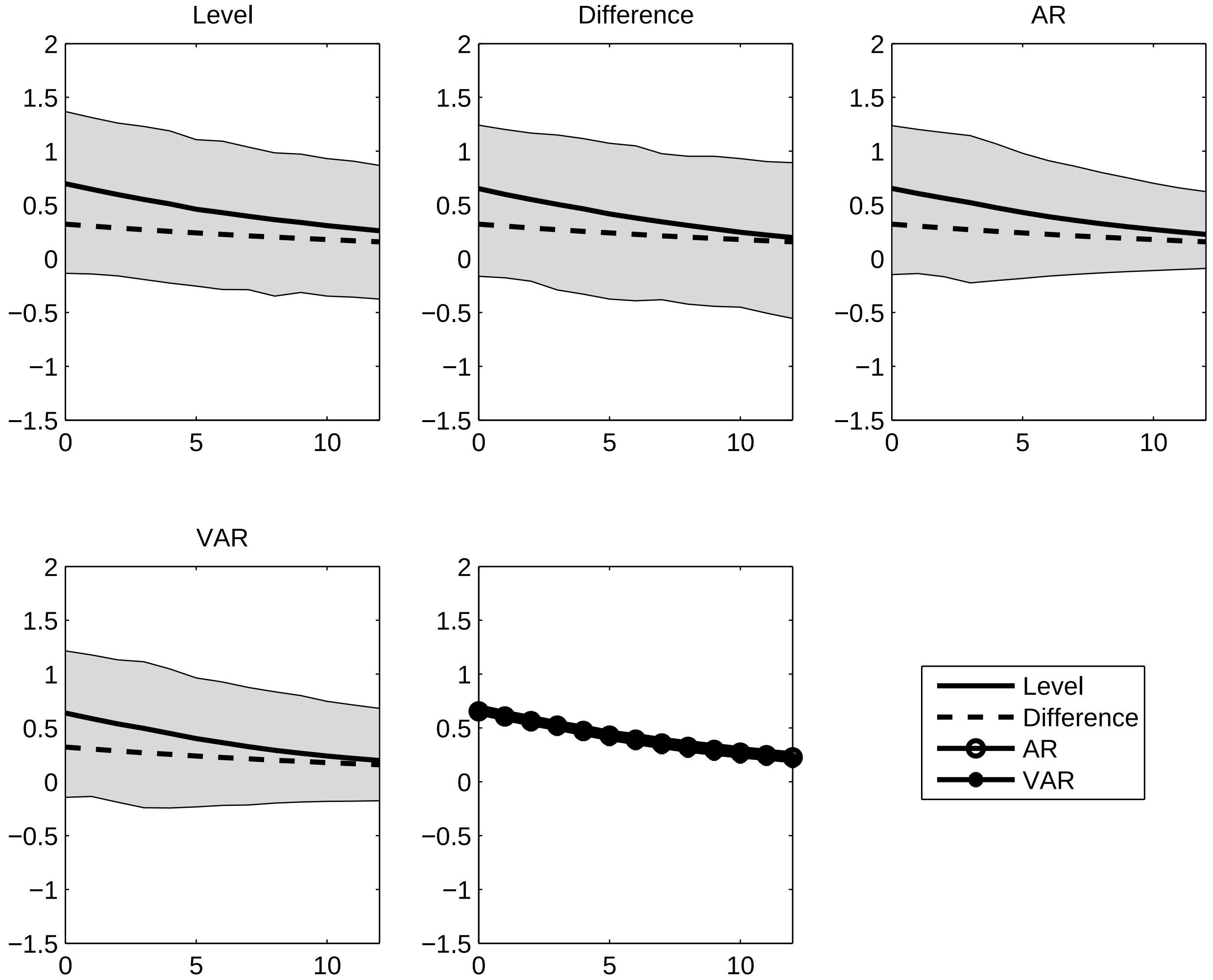


Figure 5: Simulation Results with with SVARs (Three Shocks)



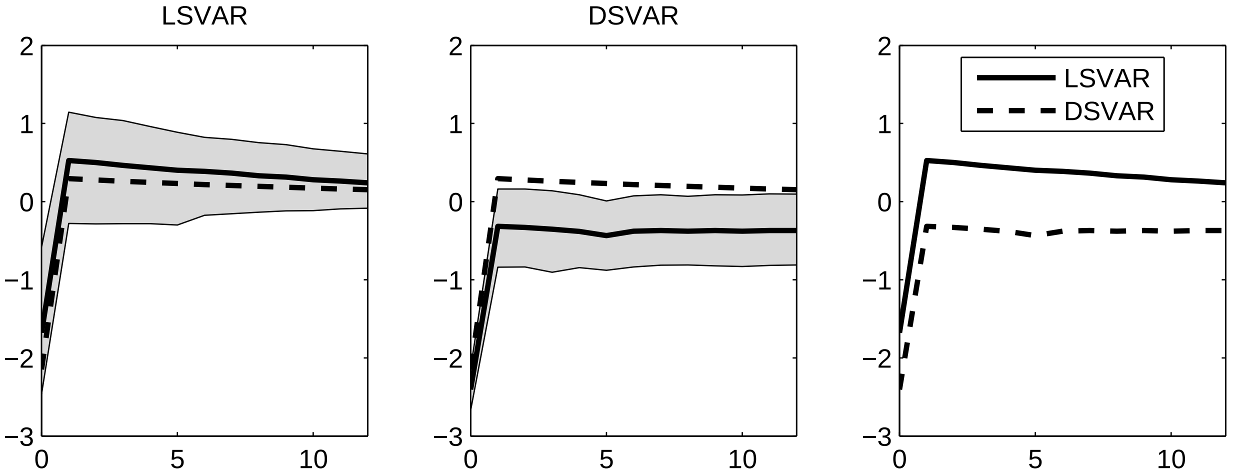
Note: LSVAR and DSVAR models. The Two Variable LSVAR model includes labour productivity growth and the log of hours. The Two variable DSVAR model includes labour productivity growth and the log of hours in first difference. The Three Variable LSVAR model includes labour productivity growth, the log of hours and the log of consumption to output ratio. The Three variable DSVAR model includes labour productivity growth, the log of hours in first difference and the log of consumption to output ratio. The dashed line correspond to the true IRF of hours. The results are obtained from our benchmark calibration and three shocks. Confidence intervals are based the 95–percentile from 1000 Monte–Carlo experiments.

Figure 6: Simulation Results with with the Two–Step Approach (Three Shocks)



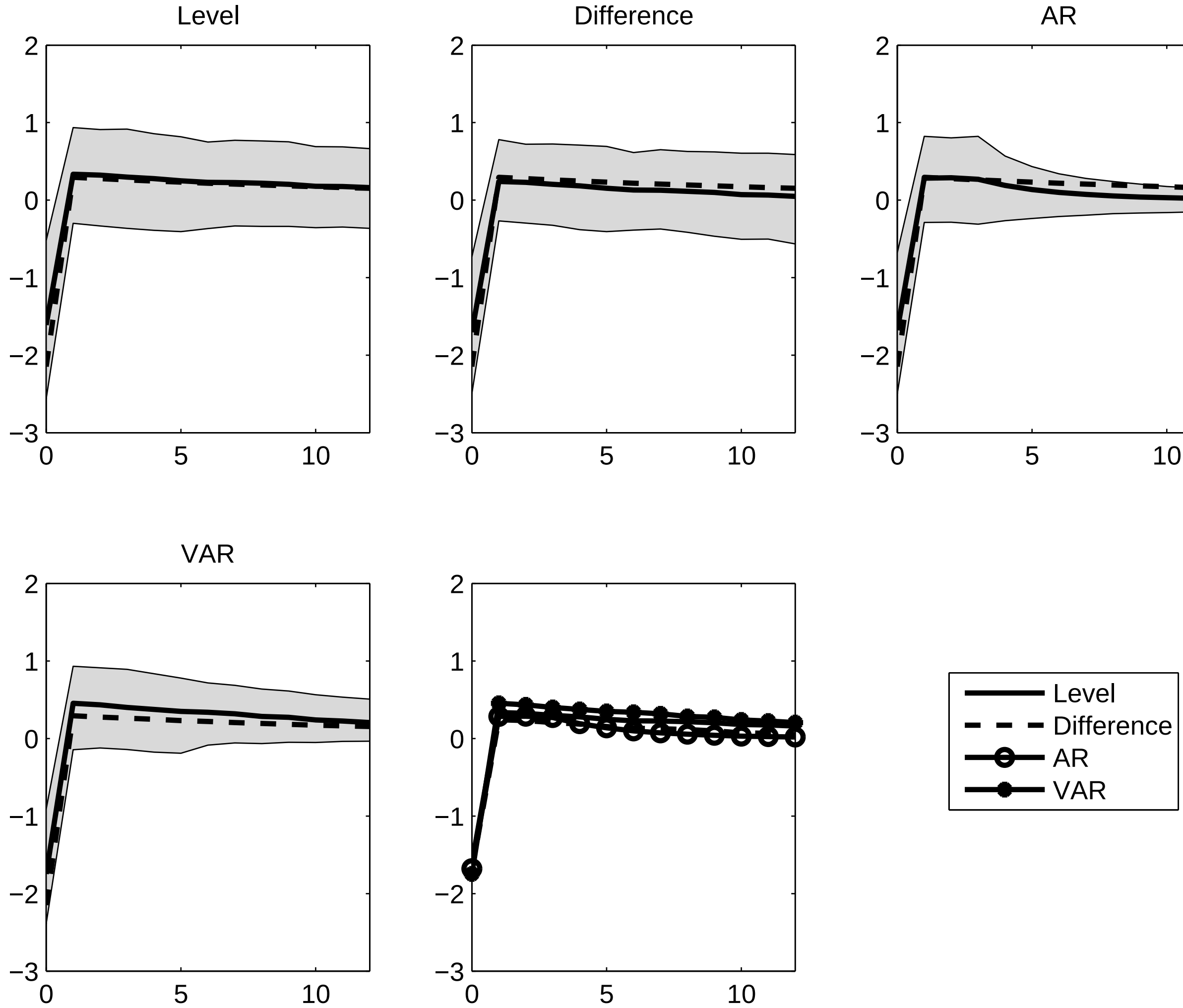
Note: Two–Step identification. The SVAR model in the first step includes labour productivity growth and the log of consumption to output ratio. In the second step, the dynamic responses of hours are obtained from equations (3), (4), (5) and (6). For this latter equation, the VAR model includes labour productivity growth, the log of hours and the log of consumption to output ratio. The dashed line correspond to the true IRF of hours. The results are obtained from our benchmark calibration and three shocks. The selected horizon for IRFs is 13. Confidence intervals are based the 95–percentile from 1000 Monte–Carlo experiments.

Figure 7: Simulation Results with with SVARs (Sticky Prices Model)



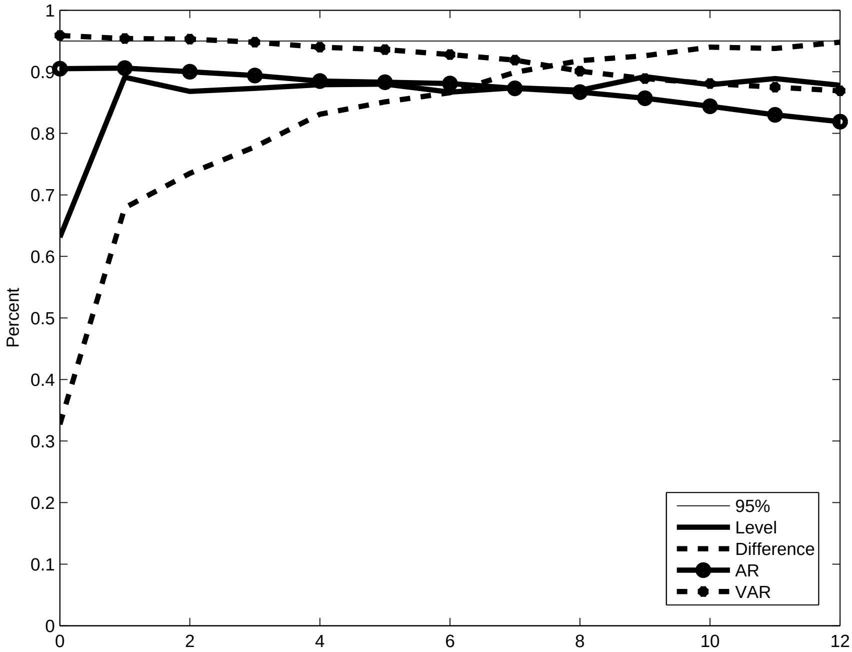
Note: DSVAR and LSVAR models. The LSVAR model includes labour productivity growth and the log of hours. The DSVAR model includes labour productivity growth and the log of hours in first difference. The dashed line correspond to the true IRF of hours. The results are obtained from our benchmark calibration and the sticky prices model. The selected horizon for IRFs is 13. Confidence intervals are based the 95–percentile from 1000 Monte–Carlo experiments.

Figure 8: Simulation Results with with the Two–Step Approach (Sticky Prices Model)



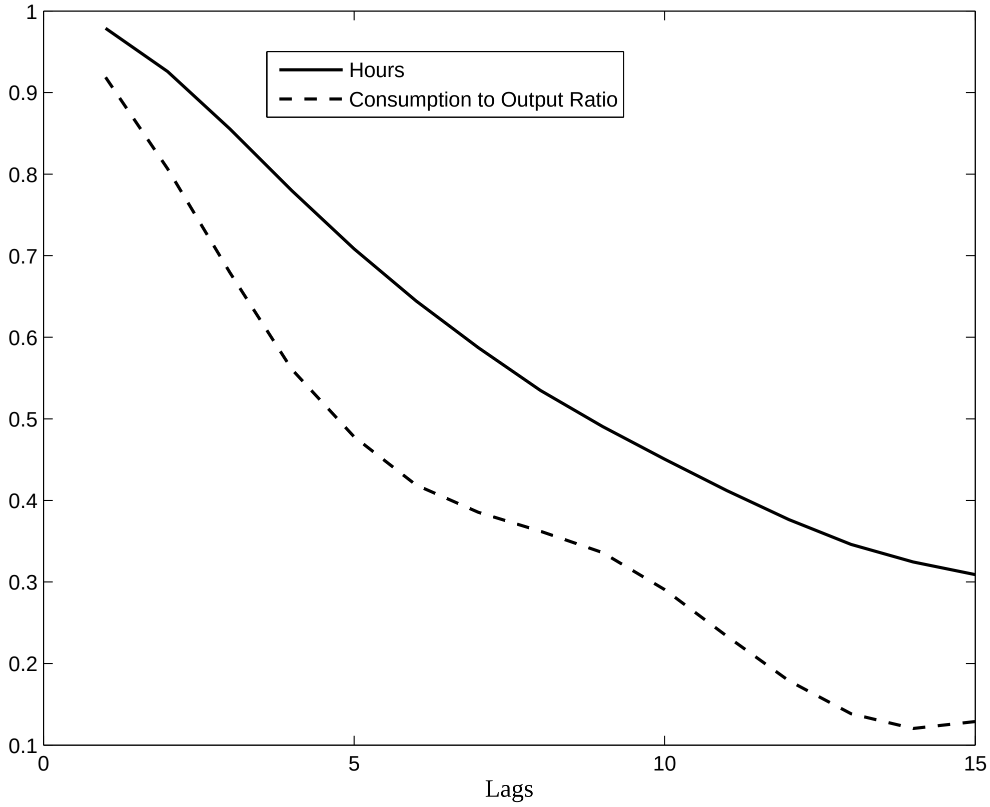
Note: Two–Step identification. The SVAR model in the first step includes labour productivity growth and the log of consumption to output ratio. In the second step, the dynamic responses of hours are obtained from equations (3), (4), (5) and (6). For this latter equation, the VAR model includes labour productivity growth and the log of hours. The dashed line correspond to the true IRF of hours. The results are obtained from our benchmark calibration and the sticky prices model. The selected horizon for IRFs is 13. Confidence intervals are based the 95–percentile from 1000 Monte–Carlo experiments.

Figure 9: Coverage Rates for Percentile-Based Confidence Intervals



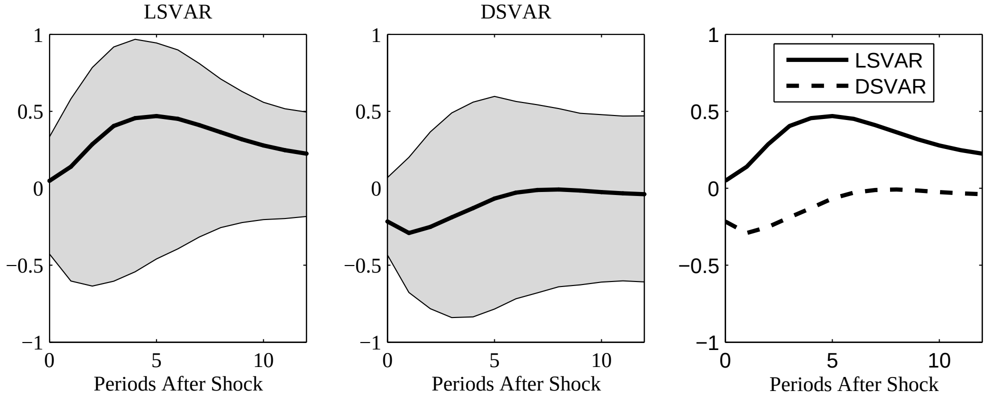
Note: Two–Step identification. The SVAR model in the first step includes labour productivity growth and the log of consumption to output ratio. In the second step, the dynamic responses of hours are obtained from equations (3), (4), (5) and (6).

Figure 10: ACFs of Hours and Consumption to Output Ratio

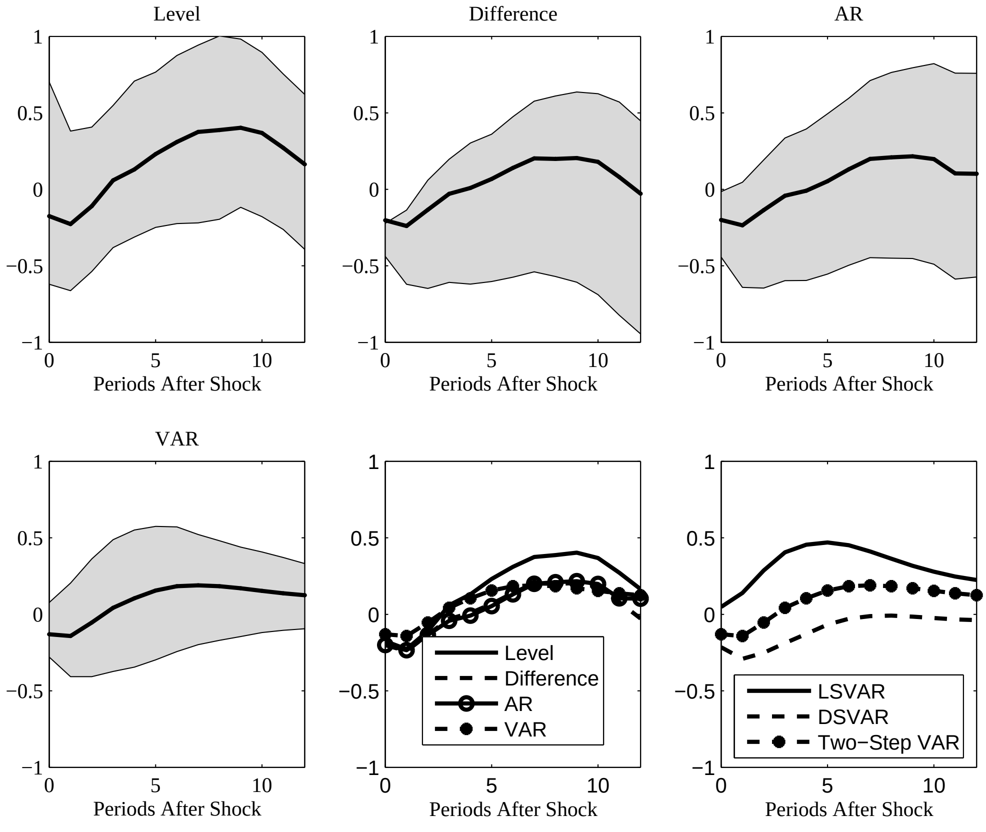


Note: NFB Sector data and Sample Period 1948Q1–2003Q4. All variables in logs.

Figure 11: IRFs of Hours Worked to a Technological Improvement

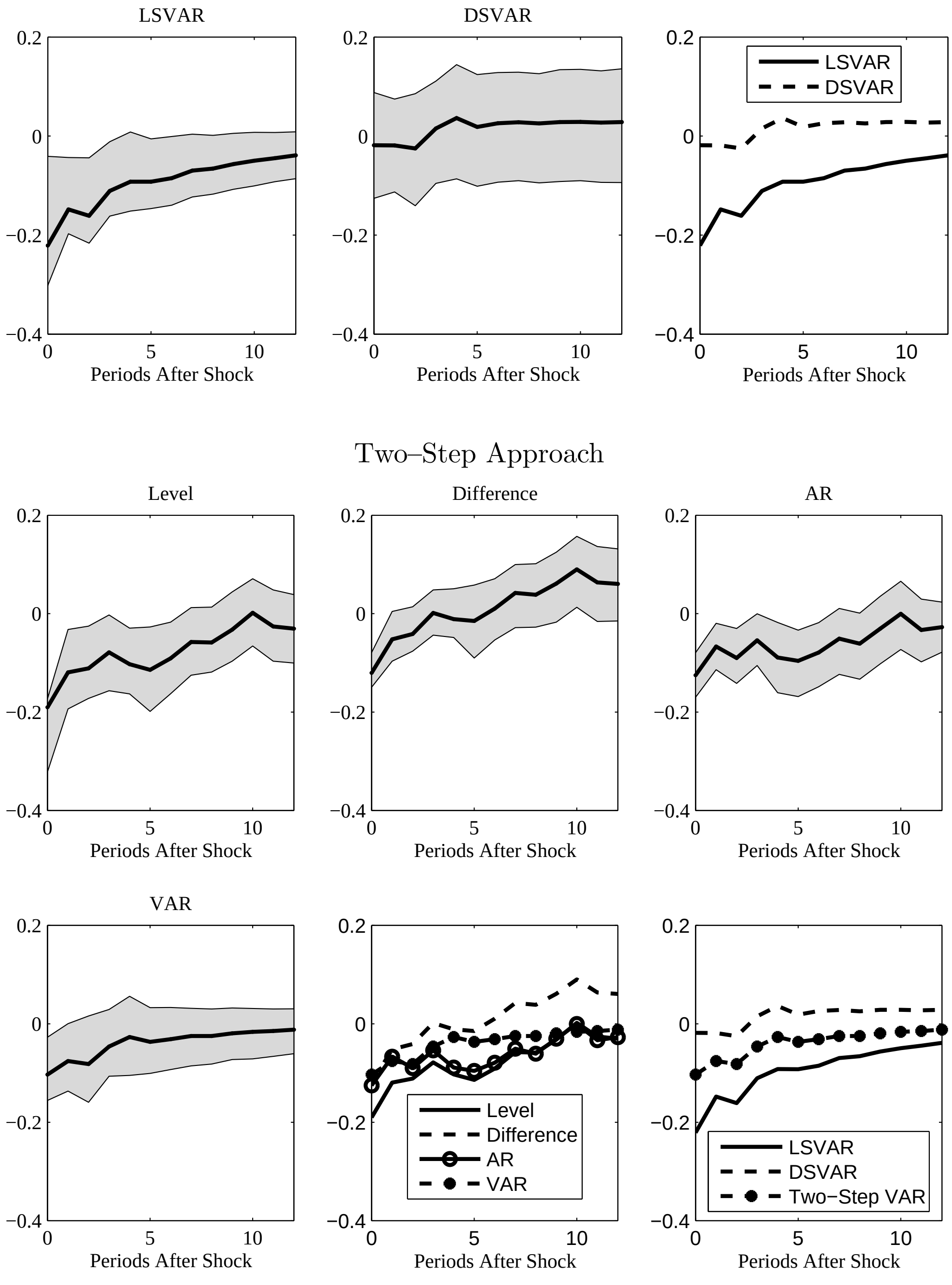


Two–Step Approach



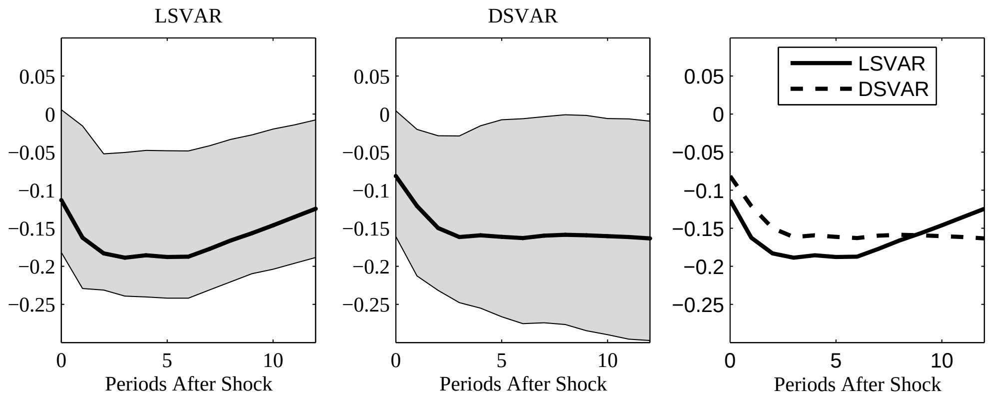
Note: LSVAR, DSVAR and two–step identification. The LSVAR model includes labour productivity growth, the log of hours and the log of consumption to output ratio. The DSVAR model includes labour productivity growth, the log of hours in first difference and the log of consumption to output ratio. For the two–step procedure, the SVAR model in the first step includes labour productivity growth and the log of consumption to output ratio. In the second step, the dynamic responses of hours are obtained from equations (3), (4), (5) and (6)). Top panel, IRFs computed from LSVAR and DSVAR specifications. Bottom panel, IRFs computed from two–step procedure (equations (3), (4), (5) and (6)). Non Farm Business Sector data and sample period 1948Q1–2003Q4. The selected horizon for IRFs is 13. 95 percent bootstrapped confidence interval shown.

Figure 12: IRFs of the Inflation Rate to a Technological Improvement

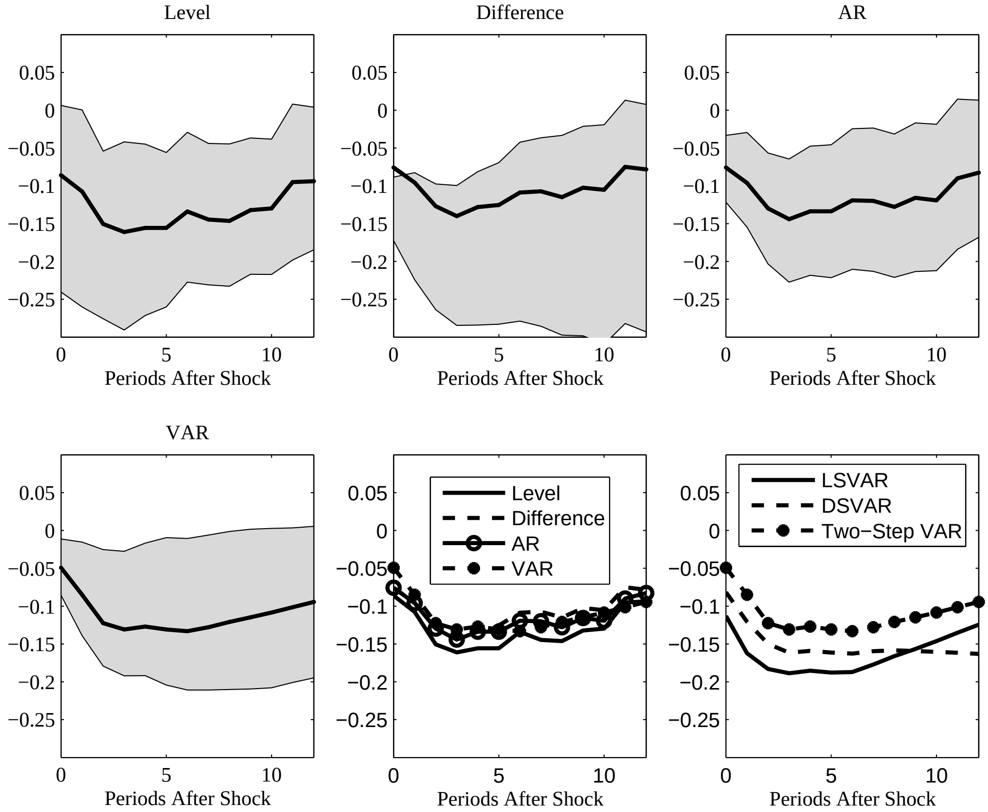


Note: LSVAR, DSVAR and two–step identification. The LSVAR model includes labour productivity growth, the inflation rate and the log of consumption to output ratio. The DSVAR model includes labour productivity growth, the inflation rate in first difference and the log of consumption to output ratio. For the two–step procedure, the SVAR model in the first step includes labour productivity growth and the log of consumption to output ratio. In the second step, the dynamic responses of the inflation rate are obtained from equations (3), (4), (5) and (6) after replacement of hours by the inflation rate. Top panel, IRFs computed from LSVAR and DSVAR specifications. Bottom panel, IRFs computed from two–step procedure (equations (3), (4), (5) and (6)). Non Farm Business Sector data and sample period 1948Q1–2003Q4. The selected horizon for IRFs is 13. 95 percent bootstrapped confidence interval shown.

Figure 13: IRFs of the Nominal Interest Rate to a Technological Improvement



Two–Step Approach



Note: LSVAR, DSVAR and two–step identification. The LSVAR model includes labour productivity growth, the nominal interest rate and the log of consumption to output ratio. The DSVAR model includes labour productivity growth, the nominal interest rate in first difference and the log of consumption to output ratio. For the two–step procedure, the SVAR model in the first step includes labour productivity growth and the log of consumption to output ratio. In the second step, the dynamic responses of the nominal interest rate are obtained from equations (3), (4), (5) and (6) after replacement of hours by the nominal interest rate. Top panel, IRFs computed from LSVAR and DSVAR specifications. Bottom panel, IRFs computed from two–step procedure (equations (3), (4), (5) and (6)). Non Farm Business Sector data and sample period 1959Q1–2003Q4. The selected horizon for IRFs is 13. 95 percent bootstrapped confidence interval shown.