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"Dynastic Accumulation of Wealth"

Emmanuel Thibault and Arianna Degan

Dynastic Accumulation of Wealth

Arianna Degan* and Emmanuel Thibault†

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Abstract: Why do some dynasties maintain the fortune of their founders while others completely squander it in few generations? To address this question, we use a simple deterministic microfounded model based on two main elements: the "hunger for accumulation" and the "willingness to exert effort". Contrary to models with capital market imperfections, our setting points to the crucial role of our two key ingredients, rather than of initial wealth or transitory shocks to wealth or inflation, on the long-run process of wealth accumulation within a family lineage. In addition, in a context with heterogeneous dynasties, we show that our model can provide a novel interpretation for some macroeconomics issues such as the demise of the rich bourgeoisie, class structure, social mobility, and wealthy inequalities. For example it predicts that those who take the effort to innovate and take advantage of new profitable opportunities are agents who are neither too poor nor too rich. Obviously, this simple framework is a great starting point for further empirical analysis.

Keywords: Intergenerational accumulation, Social mobility, Wealth inequality, Spirit of capitalism, Effort choice.

JEL classification: D 10, D 91, E 21, D 64.

*University of Quebec at Montreal (UQAM).

†Toulouse School of Economics (GREMAQ-IDEI, University of Perpignan, CDED). Email: emmanuel.thibault@univ-tlse1.fr

"The parent who leaves his son enormous wealth generally deadens the talents and energies of the son, and tempts him to lead a less useful and less worthy life than he otherwise would."

Andrew Carnegie 1891.

1 Introduction

It is highly rare for a family fortune to last more than three generations. It is so much so that the well-known adage, often attributed to Andrew Carnegie, "Shirtsleeves to shirtsleeves in three generations" exists in different versions worldwide.[1] Its message is supported by a recent study by Cochell and Zeeb (2005) who found that 6 out of 10 families lose their fortunes by the end of the second generation and 9 out of 10 by the end of the third. Nevertheless, the idea of building a family legacy that lasts long after you are gone is not only appealing but possible. In fact there are many famous families, like the Rockefellers or the Rothschilds, that have built impressive patrimonies which have lasted, or are likely to last, more than 100 years. However, there is also an important proportion of families who do not receive any patrimony and/or may not be able to build one to start with.[2]

The main goal of this paper is twofold: (i) to propose a microeconomic framework that is able to explain the observed variety of wealth accumulation patterns within a family lineage,[3] and (ii) to use the proposed setting in contexts with heterogenous dynasties to provide a new explanation for some macroeconomic issues such as class structure, social mobility, the demise of the rich bourgeoisie, and wealth inequalities.

The two key ingredients of the model we propose are the "hunger for accumulation" and the "willingness to exert effort". The first is a parameter related to Max Weber's theory of the "spirit of capitalism", wherein accumulating wealth has value in itself. The inclusion of direct preferences over transmitted wealth typically takes the formulation of either "spirit

[1] Precisely this adage comes from an antique chinese proverb, "Rice paddy to rice paddy in 3 generations". The japanese, indian, and british versions are, respectively, "Kimono to kimono in 3 generations", "Sandals to sandals in 3 generations", and "Clogs to clogs in 3 generations".

[2] According to Kennickell (2006) only 20% of families receives any inheritance (40% at the top 5% of the wealth distribution). However, Kopczuk and Lupton (2007) estimates that about 75% of a representative sample of elderly single households has a desire to leave an estate with positive net worth. The magnitude of this desire is both statistically and economically significant.

[3] There is little works on the analysis of long-run wealth dynamics within a dynasty. Arrondel and Grange (2006) provide a review of this literature as well as a study based on French data from 1800.

of capitalism" or "pure joy of giving". Although these two specifications are, as it will be discussed later, extremely similar, the "spirit of capitalism" has been used by many authors who have tried to explain growth[4] and/or savings (see, recently, Carroll 2000, De Nardi 2004, Galor and Moav 2006 or Pestieau and Thibault 2012). Consistent with empirical evidence, differently from "pure joy of giving", it generates the properties that the average propensity to bequest is an increasing function of wealth and that the wealth held by an individual does not always have an inherited component.

The second key ingredient is related to the introduction of a choice variable for effort which allows taking into account the predisposition towards working or, alternatively, the entrepreneurial attitude. According to recent findings in psychology (see Bowles and Gintis 2002) people have different beliefs about the determinants of an individual's social status. Some think that the rich are rich because of "hard work" and the poor are poor because of "laziness". Others think that the rich (poor) are rich (poor) because of (lack of) "luck" or "family money or connections". In our setting the willingness to exert effort will play a crucial role in the interaction between inherited wealth, effort (labor) supply, and transmitted wealth.[5] Such interplay reveals in which circumstances these different beliefs can be justified.

At the beginning of the 20th century the liberal economist Franck Knight pointed out three main determinants of the accumulation and transmission of wealth: heritage, effort, and luck. Although our basic setting is based only on heritage and effort, a simple form of luck will be considered as an extension. Modeling luck typically requires the use of stochastic processes, which makes the analysis of the dynamics extremely complicated. In fact, works that follow this direction focus on the distribution of wealth in the steady-state. By abstracting from this form of luck we can explicitly study the dynamics of wealth accumulation. Consequently our approach can be viewed as complementary to those studies of wealth accumulation that consist of the calibration of stochastic growth models or of theoretical models with human capital considerations or imperfect credit markets.

Our basic setup considers the problem of different members of a given family lineage. Each member of a dynasty chooses how much to consume, how much wealth to leave to the next generation, and whether to exert effort. Even in the absence of any source of uncertainty

[4]Recently, Galor and Michalopoulos (2012) advance the hypothesis that a Darwinian evolution of entrepreneurial spirit plays a significant role in the process of economic development and in the time path of inequality within and across societies.

[5]Using data for Swedish men, Björklund, Jäntti and Roemer (2012) reveal that several circumstances are important for long-run inequality, but that variations in individual effort account for the most part of that inequality.

or capital market imperfections, our framework allows generating a (large) variety of long-run dynamics (hereafter LRD) of wealth and effort choice. For instance, contrary to the "pure joy of giving", the "spirit of capitalism" allows generating both the dynamics of the numerous households with near zero wealth (i.e., from a certain period, all generations work and choose not to transmit any wealth) and the one symbolized by the famous "Shirtsleeves to shirtsleeves" adage (i.e., from a certain period, there is periodically one generation who does not work and completely squanders all the wealth accumulated by the preceding generations).

Our microeconomic setting is then extended in three different directions. First, consistent with Kaldorian facts, we analyze a context were the wage is allowed to grow at an exogenous fixed rate. Second, since inflation has been considered by many economists one of the leading causes of the disappearance of the rich bourgeoisie, we consider the effects on the LRD of episodes of inflation. Third, we study the effects of negative and positive shocks that concern wealth directly. Wars and epidemics are examples of negative shocks. The emergence of a lottery winner or sport superstar within a family lineage corresponds to positive shocks. The incorporation of such exogenous shocks allows our model to account for "luck" as a determinant of the wealth accumulation process and transmission.

Both the analysis of the basic model and of the different extensions point to the crucial role of the willingness to exert effort and the hunger for accumulation, rather than of initial wealth or transitory shocks to wealth or inflation, in generating the patterns of wealth accumulation within a family lineage. Interestingly, the accumulation of wealth may be slowed down by the the existence of a member of the dynasty who builds an extremely big patrimony, and the presence of positive shocks is another way our model can generate predictions that are consistent with the famous "Shirtsleeves to shirtsleeves" adage.

Lastly and more importantly, we introduce two frameworks with heterogenous dynasties to use our microfounded model to explain some macroeconomic issues such as the disappearance of the rich bourgeoisie, the existence of dilapidators, the evolution of wealth inequalities, social mobility and class structure.

First, considering a context with two types of dynasties differing with respect to their hunger for accumulation, we show that our model provides a simple deterministic alternative to the sophisticated model of Matsuyama (2006) for the endogenous emergence of a stratified society, wherein inherently (almost) identical agents may endogenously separate into the rich and the poor. While there are cases where wealth inequalities can indefinitely increase in the long run, we identify situations where even without government intervention inequalities are temporarily reduced.

Finally, using a setting with fully heterogenous dynasties, we show that our model provides a possible interpretation for the demise of the rich bourgeoisie and the end of a class struggle which is consistent with the recent explanations of Galor and Moav (2006) and of Doepke and Zilibotti (2005 and 2008) rather than with those based on capital markets imperfections.[6] According to our model the high wages during the 20th century might have caused the switch from a two-class very unequal society to a three-class society where inherited wealth is always positive and the difference between the rich and the other classes are reduced. Social mobility is higher, as even dynasties with zero or little wealth can, depending on their hunger for accumulation, in the long-run belong to any social class. Similarly, in the medium-run it is likely to observe new capitalists emerging from the middle and lower classes. Therefore, differently from models with capital market imperfections, our model predicts that those who take the effort to innovate and take advantage of new profitable opportunities are agents who are neither too poor nor too rich.

The paper is organized as follows. Section 2 introduces the model. Section 3 is devoted to the characterization of the possible types of long-run dynamics. Section 4 studies three extensions of the model to take account of Kaldorian facts, episodes of inflation, and exogenous temporary shocks to wealth. Section 5 introduces heterogeneity across altruistic dynasties to study the implications of our model with respect to the formation and dissolution of a stratified society and the demise of the rich bourgeoisie. Finally, Section 6 concludes. All proofs are gathered in the Appendix.

2 The Model

We consider one dynasty composed of successive generations of agents, each living one period and giving birth to a child. The problem faced by an agent, member of this dynasty, is the same in each period: given initial wealth, he has to decide whether to work, how much to consume, and how much to leave as end-of-period wealth.

Formally, we let $t = 0, ..., \infty$ denote the time. The agent at time t has initial (inherited) non-negative wealth $X_t = R_t x_t \geq 0$, where $x_t \geq 0$ is[7] the wealth left by the previous

[6] Veneziani (2007) also analyses exploitation and class formation in a dynamic context. He proves that differential ownership of (scarce) productive assets is an inherent feature of a capitalist economy, while exploitation tends to disappear in the long run.

[7] For simplicity there is only one risk-free asset in the economy and there is no explicit capital. Since each agent lives only one period and has to repay his debt within his lifetime, there is no active role for credit markets.

generation and R_t is the asset gross rate of return. We consider a binary choice variable for effort (labor), $e_t \in \{0,1\}$: agents can provide either minimal effort (normalized to 0), or some fixed positive level (normalized to 1).[8] When the agent exerts effort (works) he receives an exogenous wealth (wage) w_t. The agent lifetime disposable income, $\Omega_t = w_t e_t + R_t x_t$, is allocated between consumption c_t and end-of-period wealth x_{t+1}.

2.1 - Preferences.

The preferences of an agent born in t are defined over his period consumption, c_t, his end-of-period wealth, x_{t+1}, which will be transferred to his offspring, and his effort level, e_t. Such preferences are represented by the following utility function:

$$U(c_t, x_{t+1}, e_t) = (1-\beta)\ln c_t + \beta \ln(\varepsilon + x_{t+1}) - \xi e_t \tag{1}$$

where $\beta \in (0,1)$, $\varepsilon \geq 0$ and $\xi \geq 0$.

Equation (1) is quite general, as it imbeds different specifications used in the literature. When $\xi = 0$ and $\varepsilon = 0$ we recognize the "pure joy of giving" (or "warm-glow") approach used by, for example, Galor and Zeira (1993) and Aghion and Bolton (1997). The case with effort ($\xi = 1$ and $\varepsilon = 0$) is treated by Piketty (1997). When $\varepsilon > 0$ and $\xi = 0$ we recognize the "spirit of capitalism" specification used recently by De Nardi (2004) or Galor and Moav (2006).

Our specification of the utility derived from wealth allows us to interpret the parameter β alternatively as a degree of dynastic altruism[9] or as the hunger for dynastic accumulation. Notice also that restricting to $\varepsilon > 0$ is equivalent to ruling out the condition of infinite marginal utility of wealth at $x_{t+1} = 0$. It will be shown later that it allows having a richer set of dynamics of wealth accumulation and these dynamics fit better the data.

As already pointed out, one of the novelties of this paper is the introduction of effort in a "spirit of capitalism" setting. The parameter ξ represents the willingness to exert effort or, in other words, the cost of effort. It is a key parameter in our setting, as it drives the response

[8]The discretization of effort (labor) e, introduced by Shapiro and Stiglitz (1985) and used by Piketty (1997), is often used when labor income is not necessarily the only source of wealth (see, e.g., Diamond and Mirrlees 1978 in a context of social insurance). This is also a classical tool in the principal-agent literature. Here, e can be viewed as the effort to exert by an agent endowed (by a principal) with wealth x to obtain an additional wealth w (rather than a pure number of worked hours). Importantly, our results are not due to the fact that e_t is a discrete variable rather than a continuous one. Indeed, we show Appendix G that Propositions 1 and 2 hold when we only assume $e_t \geq 0$.

[9]Remark that a non-degenerate wealth distribution is also obtained by Dutta and Michel (1998) in a setting with imperfect altruism and linear price.

of effort choice to wealth. Such response will, in turn, determine the different typologies of dynastic wealth dynamics. While everybody would agree that such parameter is (at least weakly) positive, its magnitude is controversial. Our theoretical model allows analyzing the possible effects of wealth on effort and wealth transmission as a function of ξ.

2.2 - Optimal choices of effort and wealth transmission.

The following two propositions characterize the optimal choices of effort and wealth transmission of an agent living in t.

Proposition 1 *An agent living in t transmits to his child an increasing proportion of his disposable income Ω_t, which is independent of prices w_t and R_t. In particular:*

$$x_{t+1} = \begin{cases} 0 & if \quad \Omega_t \leq \sigma\varepsilon \\ \beta\Omega_t - (1-\beta)\varepsilon & if \quad \Omega_t > \sigma\varepsilon \end{cases}, \tag{2}$$

where $\sigma = 1/\beta - 1$.

Proof - See Appendix A.

Proposition 1 tells us that the agent leaves a positive wealth only when lifetime income is sufficiently high. Otherwise an agent may be captured into a poverty trap, wherein he is so poor (inherited wealth and wage are both very low) that he consumes all of his resources without leaving any wealth to his successor. Because in our model the propensity to save can be defined as the ratio $x_{t+1}/\ \Omega_t$, Proposition 1 tells us that, coherent with empirical evidence (see, e.g., Galor 2000, Dynan, Skinner and Zeldes 2004, or Galor and Moav 2006, footnote 36), the propensity to save is zero for individuals with low lifetime income and is increasing in lifetime income otherwise. On this dimension our microfounded formulation is more empirically relevant than those of the standard literature on distributional dynamics with credit-rationing, where each agent leaves an exogenous fraction of his total income to the next generation (see, e.g., Aghion and Bolton 1997, Piketty 1997 or Matsuyama 2000).

For what concerns the effort choice, while it is obvious that when an agent inherits no wealth he decides to work (otherwise he would have zero consumption), when inherited wealth is positive he may decide not to work.

Proposition 2 *There exists a positive threshold $\mathcal{X}_t$, increasing in w_t but independent of R_t, such that an agent living in t decides not to exert effort if and only if his inherited wealth $X_t = R_t x_t$ is greater than $\mathcal{X}_t$. Hence:*

$$e_t = \begin{cases} 1 & if \quad X_t \leq \mathcal{X}_t \\ 0 & if \quad X_t > \mathcal{X}_t \end{cases}.$$

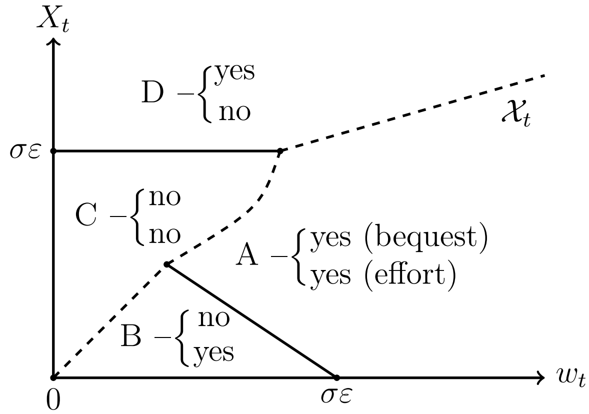

Figure 1: Optimal decisions as a function of the wage and inherited wealth.

Proof - See Appendix B.

Let $\mu_1 = (1-\beta)\left(1 - 1/e^{\xi/(1-\beta)}\right)/\beta$ and $\mu_2 = \left(e^{\xi} - 1\right)/\beta$, where $\mu_1 < \sigma$ and $\mu_1 < \mu_2$. As shown in Appendix B, there exist three thresholds $X_t^{\sharp}$, $X_t^{\star}$ and $\overline{X}$ satisfying $X_t^{\sharp} \leq \sigma\varepsilon - w_t \leq X_t^{\star} \leq \sigma\varepsilon \leq \overline{X}_t$ such that:

$$\mathcal{X}_t = \begin{cases} X_t^{\sharp} & \text{if} \quad w_t \leq \mu_1\varepsilon \\ X_t^{\star} & \text{if} \quad \mu_1\varepsilon < w_t < \mu_2\varepsilon \\ \overline{X}_t & \text{if} \quad w_t \geq \mu_2\varepsilon \end{cases} .$$

Figure 1 provides a very useful graphical representation of the optimal decisions about whether to transmit a positive wealth and/or to work as a function of the wage and inherited wealth, as implied by Propositions 1 and 2.

The dashed line $\mathcal{X}_t$ is increasing. It is linear for low and high wages and it is convex for intermediate wages. Above $\mathcal{X}_t$, that is in regions C and D, an agent living in t decides not to work. The decision about wealth transmission x_{t+1} is determined by the line $X_t = \sigma\varepsilon$, above which (region D) the agent transmits positive wealth and below which (region C) he leaves zero wealth. Below $\mathcal{X}_t$, that is in regions A and B, the agent decides to work. In these regions the decision about x_{t+1} is determined by the line $X_t = \sigma\varepsilon - w_t$. To its left (region B) the agent leaves no wealth and to the right (region A) he leaves positive wealth.

From the above results it follows that transmitted wealth x_{t+1} may be non monotonic in inherited wealth X_t. For instance for wages between $\mu_1\varepsilon$ and $\min\{\mu_2\varepsilon, \sigma\varepsilon\}$, when inherited wealth is zero (region B), the agent leaves no bequest to his successor. As the level of initial wealth X_t increases, eventually we reach region A, where the agent leaves positive wealth. However, as initial wealth increases further we eventually enter into region C, where the agent goes back to transmitting no wealth.

It is important to notice that when $\varepsilon = 0$, the relevant part of Figure 1 becomes the upper right region ($\sigma\varepsilon$ collapses to zero), where only regions A and D exist. That is, in presence of pure joy of giving agents always leave a positive wealth to the successive generation. Such a simpler formulation although more tractable does not take into consideration the empirical evidence about the existence of a large proportion of (poor) agents who do not leave positive bequests because their lifetime income is barely enough to finance their own consumption.

Finally, remark that our results are not due to the fact that e_t is a discrete variable rather than a continuous one. Indeed, we show Appendix G that Propositions 1 and 2 hold when we only assume $e_t \geq 0$.

3 Dynamics of dynastic wealth

In this section we characterize the full dynamics of wealth from generation to generation within a dynasty. As common in the closely related literature, for simplicity we assume that R_t and w_t are constant through time.[10] A straightforward application of Propositions 1 and 2 leads to the following dynamic equation of wealth.

Proposition 3 *The evolution of dynastic wealth is given by the following dynamic equation:*

$$X_{t+1} = \begin{cases} 0 & \text{if} \quad X_t \leq \sigma\varepsilon - w \text{ or } X^* < X_t \leq \sigma\varepsilon \\ \beta R[w + X_t - \sigma\varepsilon] & \text{if} \quad \sigma\varepsilon - w < X_t \leq X^* \text{ or } \sigma\varepsilon < X_t \leq \overline{X} \\ \beta R[X_t - \sigma\varepsilon] & \text{if} \quad X_t > \overline{X} \end{cases} . \tag{3}$$

The dynamic equation of dynastic wealth is determined by three branches. In the first branch wealth is equal to zero. In the other two branches the evolution of wealth is defined by an arithmetic-geometric progression which depends on whether the agent is exerting effort. Proposition 3 provides a compact description of the evolution of wealth. The reader can find its complete characterization as a function of wage in Appendix C.

3.1 - Typologies and properties of long-run dynamics.

Since our model is deterministic, for any given initial wealth X_0 we can trace all the sequence, X_1, X_2, ... of dynastic wealth. Although initial wealth X_0 might affect the behavior of the first generations, we can find some regularities in the behavior of future generations starting

[10] As Galor and Zeira (1993) we relax in Section 4.1 the standard assumption of constant wages and consider the case where wages grow at a constant rate. In Section 4.2, where we consider episodes of inflation, we implicitly study the effects of changes in R.

from a certain period which is not necessarily too far away. We have to clarify this point because, in general, the evolution of wealth and work behavior across members of the same dynasty do not exhibit stationary patterns in our model. The characterization of the above mentioned regularities, which we refer to as "long-run dynamics", is the object of this section.

Before defining and formally characterizing the types of LRD that can emerge in our setting, some clarifications about the terminology that we will use are in order. We breakdown the class of individuals who inherit a positive wealth and do not work into three sub-categories. We define as: (a) *rentiers* those agents who do not work but nevertheless transmit a level of wealth greater than the wealth they have inherited; (b) *dilapidators* those agents who do not work and transmit a (strictly positive) level of wealth which is lower than the one they have inherited; and (c) *ruiners* those agents who receive positive wealth but neither work or leave positive wealth.

The general typologies of LRD that can be observed in a society and their properties are summarized in Table 1. We assign to each type of LRD a composite name composed of two parts. The first indicates the type of long-run wealth that the LRD allows reaching. It is ZERO if dynastic wealth converges towards or periodically becomes zero; FINI or CYCL if accumulated wealth is positive and finite, and INFI if it grows unboundedly. The second part indicates the long-run working status of the dynasty. It is WORK if in the long run each member of the dynasty works, RENT if in the long run no member of the dynasty works, and MIX if there exists a mix of workers and non-workers (*dilapidators* or *rentiers*). It follows that a LRD is said to be:

• ZERO-WORK if, from a certain period, all generations work and choose not to transmit any wealth.

• FINI-WORK if, from a certain period, all generations work and transmitted wealth monotonically converges towards a positive finite value.

• CYCL-WORK if, from a certain period, all generations work but the finite wealth accumulated does not converge towards a unique value.

• ZERO-MIX if, from a certain period, there is periodically one generation who does not work and completely squanders all the wealth accumulated by the preceding generations.

• FINI-MIX if, from a certain period, there are infinite successive runs of generations who work and build up an upper-bounded patrimony and of generations who do not work and squander part of their initial wealth.

• INFI-MIX if, from a certain period, there are infinite successive runs of generations who work and build up a patrimony which tends to infinity and of generations who do not work and squander part of their initial wealth.

• INFI-RENT if, from a certain period, no generation works and transmitted wealth increases monotonically towards infinity.

Type of LRD	Long-run wealth	Dynamics of accumulation	Long-run existence of			
			workers	dilapidators	ruiners	rentiers
ZERO-WORK	Zero	Constant	Yes	No	No	No
FINI-WORK	Finite	Monotone	Yes	No	No	No
CYCL-WORK	Finite	Cyclical	Yes	No	No	No
ZERO-MIX	Zero/Finite	Cyclical	Yes	Possible	Yes	No
FINI-MIX	Finite	Cyclical	Yes	Yes	No	No
INFI-MIX	Infinite	Cyclical	Yes	Yes	No	No
INFI-RENT	Infinite	Monotone	No	No	No	Yes

Table 1: Typologies and properties of long-run dynamics (LRD).

In a ZERO-WORK LRD, the dynasty is caught into a poverty trap, where in the long-run all generations work and consume all of their wage without transmitting any wealth. In a FINI-WORK or CYCL-WORK LRD all generations work but wealth from generation to generation either converges monotonically to a positive finite value $\widetilde{X}$ (in the case of a FINI-WORK LRD) or has different finite accumulation modes (in the case of an CYCL-WORK LRD). An interesting feature of the MIX types of LRD is that wealth fluctuates. In a ZERO-MIX LRD the cycles are regular and there exists periodically one generation, the *ruiner*, who completely squanders all the wealth. This *ruiner* could appear after a sequence of generations who cumulated (an increasing) positive wealth or could appear after a sequence of *dilapidators*. Both in the FINI-MIX and the INFI-MIX LRD the cycles need not to be regular and wealth is always bounded away from zero, implying the existence of *dilapidators* but the absence of *ruiners*.

Notice that our setting can generate different degrees of inter-generational mobility. This is characterized by the changes in the working and wealth accumulation behaviors among members of the same dynasty in correspondence to the different LRD. In 4 out of 7 LRD the status of the members of a dynasty is invariant. However, mobility exists in all the three LRD of type MIX.

3.2 - Characterizations of the long-run dynamics.

The determination of the LRD of a dynasty hinges on the analysis of the non-trivial branches of the dynamic equation (3). For instance, it is important to notice that when the wage is sufficiently appealing (i.e., $w > \sigma\varepsilon$), the wealth accumulated by an infinite sequence of workers tends towards infinity if $\beta R > 1$ and towards $\widetilde{X} = \beta R[w - \sigma\varepsilon]/(1 - \beta R)$ if $\beta R < 1$. When $\beta R < 1$, an agent deciding not to work always transmits a level of wealth which is lower than the one he had inherited. It is this type of decumulation of wealth that gives rise to a succession of generations who decide not to work and dilapidate part or the integrality of a given inheritance. Conversely, when $\beta R > 1$, an agent deciding not to work transmits a level of wealth greater than the one he had inherited if and only if the latter is greater than $\widehat{X} = \beta R\sigma\varepsilon/(\beta R - 1)$. It is this type of wealth accumulation that makes the emergence of *rentiers* possible.

Using these results, we now characterize the types of LRD generated by our model. We distinguish such characterization according to the relative values of the interest rate and the hunger for accumulation, i.e. $\beta R < 1$ or $\beta R > 1$. Table 2 summarizes our results (see Propositions 4 and 5 in Appendix D). Notice that the basic model generates only five of the seven LRD showed in Table 1. The LRD of type CYCL-WORK and INFI-MIX will emerge from the extensions of the model considered in the next sections. The LRD in Table 2 that are underlined correspond to those not obtainable starting from $X_0 = 0$.

	$\beta R < 1$	$\beta R > 1$
$w \leq \mu_1\varepsilon$	ZERO-WORK	ZERO-WORK <u>INFI-RENT</u>
$\mu_1\varepsilon < w \leq \sigma\varepsilon$	ZERO-WORK	ZERO-WORK <u>FINI-MIX</u> <u>INFI-RENT</u>
$\sigma\varepsilon < w \leq \mu_2\varepsilon$	FINI-WORK FINI-MIX ZERO-MIX	FINI-MIX ZERO-MIX INFI-RENT
$w > max\{\sigma\varepsilon, \mu_2\varepsilon\}$	FINI-WORK FINI-MIX	INFI-RENT FINI-MIX

Table 2: Characterization of the LRDs in the "spirit of capitalism" setting.

When $\beta R < 1$, the intuition behind our results is the following. When the wage is at a subsistence level ($w \leq \sigma\varepsilon$), eventually all members of the dynasty choose to work and to not transmit any wealth. This pattern holds for any level of initial wealth. Clearly it holds when inherited wealth is small, as the agent is forced to work and to allocate the totality of income to consumption. Interestingly, it holds also for higher level of wealth. In fact, when initial wealth is sufficiently high ($X_0 > \sigma\varepsilon$), due to the low wage, the initial generation and potentially some of the successive ones decide not to work and to finance consumption with inherited wealth. Therefore, in a finite time the latter becomes zero and stays zero thereafter. This ZERO-WORK LRD entails a poverty trap and is consistent with the empirical evidence that consumption tracks current income and that many households do not receive an inheritance.

For higher wages ($w > \sigma\varepsilon$) the LRD can be of three types: FINI-WORK, FINI-MIX or ZERO-MIX. In order for a FINI-WORK LRD to exist, the limiting value $\widetilde{X}$ of wealth accumulated by an infinite sequence of workers must be no greater than the threshold $\mathcal{X}$ above which individuals choose not to work. When $X_0 < \mathcal{X}$ all generations work and wealth converges to $\widetilde{X}$ monotonically from below. When $X_0 > \mathcal{X}$ the initial generations decide not to work and decumulate wealth. However, once $\widetilde{X} < X_t < \mathcal{X}$, all future generations work and wealth monotonically decreases towards $\widetilde{X}$.

The LRD is of type MIX whenever $\widetilde{X}$ is greater than $\mathcal{X}$. This is because, once there is a member of the dynasty who does not work (there is always some when $\widetilde{X} > \mathcal{X}$), transmitted wealth becomes lower than inherited wealth. Wealth decreases through time until it becomes too low for the following generation to decide not to work. Of the two types of MIX LRD a ZERO-MIX could exist only when $\sigma\varepsilon < w < \mu_2\varepsilon$. In fact the wage w must be greater than $\sigma\varepsilon$, otherwise we would have a ZERO-WORK LRD, and lower than $\mu_2\varepsilon$, otherwise transmitted wealth could never become zero. A FINI-MIX LRD exists only if once wealth becomes positive it never goes back to zero. This is clearly the case when $w > \max\{\sigma\varepsilon, \mu_2\varepsilon\}$ and it could happen also when $\sigma\varepsilon < w < \mu_2\varepsilon$.

Different from the previous case, the types of LRD obtained when $\beta R > 1$ always depend on the level of initial wealth, X_0. When the wage is relatively low ($w \leq \mu_1\varepsilon$) a ZERO-WORK LRD emerges for low levels of inherited wealth. This is because, although for each dollar bequeathed the next generation receives more than 1 dollar, the level of wealth is too low to consider leaving a big portion of it to the next generation. Transmitted wealth decreases over time and eventually becomes zero. Conversely, an INFI-RENT LRD is obtained for high levels of inherited wealth. All members of the dynasty will choose not to work and nevertheless, because of the high interest rate and/or hunger for accumulation, will leave increasing bequests. When $\mu_1\varepsilon < w \leq \sigma\varepsilon$, in addition to the ZERO-WORK and to the INFI-RENT LRD it is also possible to have a FINI-MIX LRD. When $w > \max\{\sigma\varepsilon, \mu_2\varepsilon\}$ we find the same type of MIX LRD as with $\beta R < 1$. In addition, while with $\beta R < 1$ it was only possible to have a FINI-WORK LRD, with $\beta R > 1$ it is only possible to have INFI-RENT LRD.

At this point we have a wider understanding of the role played by the parameter ε. Our formulation, when $\varepsilon > 0$, generates five possible typologies of wealth dynamics. Conversely, pure joy of giving, where $\widehat{X} = 0$ and $\varepsilon = 0$, delivers only three of them, which are summarized in Table 3.

	$\beta R < 1$	$\beta R > 1$
$w > 0$	FINI-WORK FINI-MIX	INFI-RENT

Table 3: Characterization of the LRDs in the "pure joy of giving" setting.

To summarize, in our framework, contrary to joy of giving, spirit of capitalism allows generating ZERO-WORK LRD (i.e., the numerous households with near zero wealth) and ZERO-MIX LRD (i.e., in line with the "Shirtsleeves to shirtsleeves in three generations" adage).

A graphical representation of the results of this section makes the study of the role of prices (i.e., w and R) and of the effort parameter (i.e., ξ) easier and will be at the basis of the comparative static results implicit in the analysis of the next sections. It also allows pointing out the ability of our framework to explain part of the labor choices and wealth dynamics observed in our contemporary societies. However, due to the many configurations that can emerge in equilibrium and in order not to distract the attention of the reader from the most relevant results, such a graphical treatment (Figures 4 to 8) and its tedious derivation are provided in Appendix E.

4 Extensions

The analysis of the effort and wealth accumulation choices of Section 2 and the characterization of the LRD of the basic model of Section 3, allows determining, which conditions guarantee the emergence and the persistence or disappearance of *rentiers* within a family lineage. The understanding of such conditions is a pre-requisite for the study of some macro issues related to the distribution of wealth. We reserve such analysis to Section 5, where we study contexts with heterogenous dynasties. In this section we consider some extensions of the basic model and their implications for the emergence, the persistence or disappearance of *rentiers* within a family lineage and more in general for the LRD within a dynasty.

Our analysis so far has assumed, as most theoretical models studying the dynamics of wealth accumulation and distribution, that both wages and interest rates are constant over time. We start by relaxing this assumption. In fact, according to Kaldor (1957), who summarizes into stylized facts a number of empirical regularities in the growth process in industrialized countries, the (real) rate of return on investment R is roughly constant over long periods of time while the real wage w grows at a positive constant rate over time. To verify the robustness of our results to this fact, we consider in subsection 4.1 the case where wages grow at a constant rate.

After 1957, year of the influential article by Kaldor, strong inflation waves have hit many countries, such as, for example, the US, UK and France, contradicting the Kaldor argument that real interest rates are roughly constant. As a second extension we therefore consider, in subsection 4.2, the impact of inflation on the behavior of the members of a dynasty as well as on wealth accumulation.

In our model, from the perspective of a member of a dynasty, inflation is nothing but a shock to the return on inherited wealth. We consider in subsection 4.3 positive and negative shocks that concern wealth directly. Wars and epidemics are examples of negative shocks to wealth that affect all dynasties. Winning a lottery or being the immediate descendent of a sport/movie star or of a successful self-made business man are examples of positive idiosyncratic shocks to wealth that hit specific dynasties. The incorporation of such exogenous shocks allows our model to account for "luck" as a determinant of the wealth accumulation process and transmission. Indeed, although we explicitly chose to abstract from luck we do recognize that luck is an important determinant of the process of wealth accumulation. It is for example a crucial determinant to explain the appearance of "self made (wo)men" and the "reverse of fortunes".

4.1 - Exogenous wage growth.

In this subsection we partially relax the assumption of fixed prices and assume, coherently with Kaldorian facts, that (real) wages grow at a constant positive rate γ, i.e., $w_t = (1+\gamma)^t w_0$. Under this assumption both thresholds $\mathcal{X}_t$ and $\widetilde{X}$ increase over time and grow towards infinity. As it is derived in Appendix F, we find that, there is always a period T' after which the wealth accumulated by a sequence of workers is greater than both $\widehat{X}$ and $\mathcal{X}_{T'+t}$.[11] Moreover, since the wage w_t eventually grows towards infinity, in all types of LRD wealth must tend towards infinity as well. Consequently, for each initial positive wage ($w_0 > 0$), its rate of growth ($\gamma > 0$), and initial wealth ($X_0 \geq 0$), there are only two possible LRD: when $\beta R > 1$ the economy grows unboundedly towards an INFI-RENT LRD, and when $\beta R < 1$ the economy converges towards a INFI-MIX LRD. The convergence towards the infinite wealth dynamics is monotonic in the first case but not in the second.

These results concern the long run only. However, as in the basic model, they do not necessarily characterize the non-monotonic pattern that wealth can potentially follow in a

[11] The fact that wages grow over time does not affect the dynamic of wealth accumulation of a sequence of agents who do not work, $X_{t+1} = \beta R[X_t - \sigma\varepsilon]$. Therefore $\widehat{X}$ is independent of w.

shorter time span. In fact, the medium-term dynamics that can emerge in the context of exogenously growing wages can be compatible with a variety of dynastic behaviors observed in the real world. For example, even in the case of an INFI-RENT LRD we can find along the way both *ruiners* and *dilapidators*.

We conclude by two important and interesting remarks. First, since the higher the wage the higher the incentive to work, our model predicts that, when $\beta R < 1$, wage growth triggers the disappearance of *rentiers* within each dynasty. Second, if we were to consider heterogeneous dynasties, the model would also imply the disappearance of *rentiers* at the top of the wealth distribution. This is because when the wage increases so does the level of wealth which is needed to be at the top of the distribution. Therefore, while dynasties who choose to work accumulate bigger and bigger patrimonies, dynasties that continue to be *rentiers* may no longer appear at the top of the distribution. These two implications, captured by our model, are supported by empirical evidence pointed out by Piketty and Saez (2006): "top income shares have increased substantially in English speaking countries but not at all in continental Europe countries or Japan. This increase is due to an unprecedented surge in top wage incomes starting in the 1970s and accelerating in the 1990s. As a result, top wage earners have replaced capital income earners at the top of the income distribution in English speaking countries."

4.2 - Inflation.

Inflation has been considered by many economists as one of the leading causes of the disappearance of *rentiers*. In fact, inflation reduces real interest rates and, as a consequence, it slows down wealth accumulation. In the last chapter of "The General Theory", Keynes foresees the ultimate fall of the rate of return to capital to zero and calls this situation the "Euthanasia of the rentier".

To analyze the effect of inflation, we can assume that at a certain period κ a strong inflation rate i hits the economy. Using Fisher equation, the real interest rate R can be written as $R^i = R/(1+i)$. By reducing the real return on transmitted wealth, inflation has a confiscatory effect that reduces accumulated wealth and, in turn, the possibility of having *rentiers*. In fact, as a result of inflation the member of a dynasty at κ receives wealth $X^i_\kappa = R^i x_\kappa$ which is lower than $X^0_\kappa = R^0 x_\kappa$, the one he would have received without inflation. Independent of the wealth X_{t-1} received by its predecessor, the higher the inflation the lower the chances that the member in κ will behave as a *rentier*. In particular, when $\mathcal{X} < X^0_\kappa$ and $X^i_\kappa < \mathcal{X}$, inflation slows down the possible appearance of *rentiers* (if $X_{\kappa-1} < \mathcal{X}$) or it makes *rentiers* disappear (if $X_{\kappa-1} > \mathcal{X}$).

Since, in our context episodes of hyperinflation have only transitory effects, they can explain the phenomenon of the Euthanasia of *rentiers* in the short and medium run only. It follows that, when the disappearance of *rentiers* is due uniquely to episodes of inflation, and these are followed by episodes of low inflation, we would expect the dynasty to rebuild its wealth and *rentiers* to reappear. Our model can capture the phenomenon of the Euthanasia of *rentiers* in the long run in case of persistent inflation, that is, when from a period κ the real interest rate becomes $R_t = R^i = R/(1-i)$. In this case, it can be shown that the confiscatory effect of inflation is such that there always exists an inflation rate above which both inherited wealth and the relative real interest rate become so low ($\beta R^i < 1$ and $X^i_\kappa < \mathcal{X}$) that all members of the dynasty eventually become workers (i.e., the LRD becomes either FINI-WORK or ZERO-WORK).[12] The result that persistent episodes of hyperinflation makes the survival of *rentiers* very hard is supported by empirical evidence about, for example, the disappearance of *rentiers* in France during the 20th century. Indeed, according to Piketty (2003), "one must bear in mind that inflation did act as a powerful capital tax. The French consumer price index was multiplied by a factor of more than 100 between 1914 and 1950, which means that bondholders were fully expropriated by inflation. The same process applied, in a less extreme way, to real estate owners and landlords. Rent control was severe during both world wars, and the real value of rents was divided by 10 between 1913 and 1950. Further, the 1914-50 inflationary process was something entirely new for the economic agents of the time."

4.3 - Luck and bad luck.

Obviously, the depletion of the fortunes of *rentiers* is not uniquely explained by inflation. It could be due to other types of shocks such wars or the Great Depression.

The behavior of a dynasty reacts not only to unforeseen shocks hitting the whole economy but also to idiosyncratic shocks within each dynasty. This is the case for example when within a dynasty an *exceptional* member (a creator, successful businessman, a TV or sport star, a lottery winner, etc.) appears whose cumulated wealth reaches *abnormally* high levels. The incorporation of such exogenous transitory shocks therefore allows our model to account for "luck", which, as advocated by Frank Knight, is one of the three determinants of the wealth accumulation process and transmission.

The transitory shocks in our economy can affect the four fundamental exogenous parameters of the model (i.e., w, R, ξ, β and ε) or directly inherited wealth X_T at a certain period T. Formally, a transitory shock corresponds to assuming that at a certain time T

[12] Since $\mathcal{X} > 0$ is independent of R^i and for all finite x_κ, $\lim_{i\to+\infty} X^i_\kappa = Rx_\kappa/(1+i) = 0$, there exists $i^\star$ and T such that for all $i > i^\star$ and $t > T$, we have $X^i_t < \mathcal{X}$.

one parameter changes to return to its initial value at time $T' > T$. It should be noticed that the LRD obtained after such a temporary shock is equivalent to the one obtained in correspondence of the initial parameters and a new (initial) wealth $X_{T'}$. It follows that, in order to understand the impact of transitory shocks it is enough to study the effect of an unexpected change in inherited wealth.

Figure 2 illustrates a case where wealth receives a particularly big positive shock, caused, for example, by the emergence of a self-made man (like Bill Gates). On the LHS of Figure 2 there is the situation where without the positive shock the economy would converge to a FINI-WORK LRD. The RHS of Figure 2 shows the situation in presence of a positive shock. In particular, when the member of the fourth generation makes, thanks to the exogenous positive shock, a big fortune (i.e., $X_4 > \mathcal{X}$) the fifth generation chooses not to work and dilapidates part of inherited wealth. The next generation also chooses not to work and ruins all the wealth (i.e., $X_6 = 0$). From then on, all the following generations work and accumulate wealth, meaning that the long-run dynamics has not been affected by the temporary shock. However, paradoxically, the accumulation of wealth has been slowed down by the positive shock to wealth. In fact starting from the sixth period, each member of the dynasty is richer when he belongs to the (otherwise equal) dynasty that hasn't received the shock than when he belongs to the dynasty that did receive the shock.

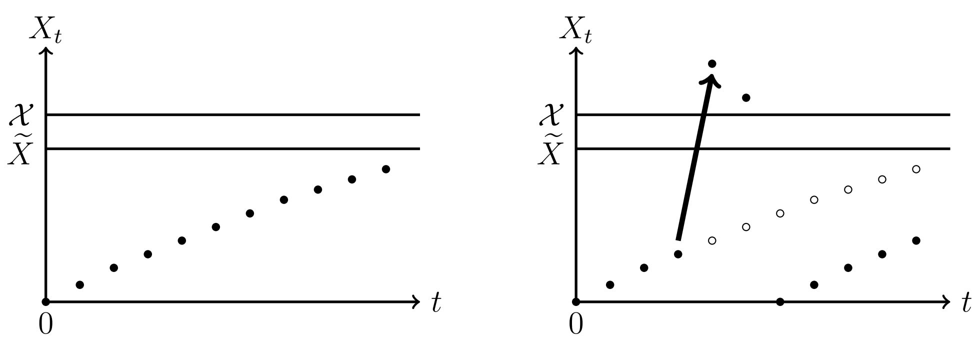

Figure 2: An illustration of the "Shirtsleeves to shirtsleeves in three generations" adage.

This example illustrates that the presence of positive shocks is another way our model can generate predictions that are consistent with the the "Shirtsleeves to shirtsleeves in three generations" adage as well as with the more recent evidence that 6 out of 10 families loose their fortunes by the end of the second generation and 9 out of 10 by the end of the third (see Cochell and Zeeb, 2005).

According to our model, the existence of a member of the dynasty (a self-made man) who builds an extremely big patrimony may not be enough to guarantee the prosperity of all future members of the dynasty. This is because the hunger for accumulation β and the effort cost ξ, rather than initial wealth, are the crucial determinants of the LRD. When $\beta R < 1$, initial wealth can only lead to a switch between a FINI-MIX and a ZERO-MIX LRD. Indeed, the LRD is independent of initial wealth except when we have a dynamic of type MIX. However, wealth still plays a very important role when $\beta R > 1$, where a sufficient increase in wealth always allows switching from a ZERO-WORK, or a FINI-MIX, or a ZERO-MIX LRD to an INFI-RENT LRD. Indeed, an INFI-RENT LRD can emerge once the positive shock to wealth is such that $X_{T'} > \widehat{X}$.

The situations with negative shocks to wealth, due for example to wars, epidemics or to reverse of fortunes, can be analyzed in the same way. Consider for example the case where a negative shock causes the ruin of a dynasty (that is wealth suddenly becomes zero).[13] In this case, if the wage w is sufficiently high (i.e., $w > \sigma\varepsilon$) the LRD remains INFI-RENT. The wealth depletion only makes the short-term wealth accumulation process to restart from zero but it does affects its long run dynamics.

To summarize, we have shown in subsections 4.1, 4.2 and 4.3 that wage growth, episodes of inflation, and other shocks all provide a possible explanation for the erosion of fortunes and disappearance of *rentiers* in the short and medium term. However, our analysis suggests that by and large episodes of inflation or other shocks do not affect the long-run wealth accumulation process but only the speed of convergence towards its LRD. Therefore, although these factors explain the disappearance of *rentiers*, in the absence of other "shocks" they do not provide an explanation for its persistence.

We will show in the next sections that our model can also account for the persistent disappearance of *rentiers* once we extend the basic model to consider an economy populated by heterogenous dynasties.

5 Class structure and the demise of the rich bourgeoisie

In this section we introduce heterogeneous dynasties to use our micro-founded model to study more in detail some macro issues related to the distribution of wealth and in particular at its top quantiles. Namely, we will focus on the endogenous emergence or dissolution of a class society and the demise of the rich bourgeoisie, and the evolution of wealth inequalities.

We show in subsection 5.1. that our model provides a simple deterministic alternative to

[13]This sudden depletion of wealth can also be the consequence of a positive shock leading the following generation to deplate their wealth (see the example described in Figure 2).

the sophisticated model of Matsuyama (2006) for the endogenous emergence of a stratified society, wherein inherently identical agents may endogenously separate into the rich and the poor. In subsection 5.2 we use our model to provide a possible interpretation for the demise of the rich bourgeoisie and the end of a class struggle, which is consistent with the recent explanations by Galor and Moav (2006) and Doepke and Zilibotti (2005 and 2008) rather than with those based on capital markets imperfections.

5.1 - Endogenous class society.

In our model the social class attained by a dynasty (characterized by the long-run working behavior and pattern of wealth) depends in big part on the hunger for accumulation. We study the role of the parameter β on the possibility of social stratification and on the evolution of inequality. Obviously, dynasties can be heterogeneous with respect to other characteristics. As long as heterogeneity concerns the effort cost ξ and/or the wage w, the results derived in Appendix E and summarized in Figures 4-8 can be directly used to infer the type of social stratification that can emerge.

We consider two dynasties, indexed by a and b, with zero initial wealth and different hunger for accumulation, $\beta_a > \beta_b$. We focus only on relatively high wages, that is $w > \max\{\sigma\varepsilon, \mu_2\varepsilon, (R-1)\varepsilon\}$. Since both dynasties start with zero initial wealth, the first member of each dynasty works. However, in the long run the working behavior of the two dynasties may differ, depending on their hunger for accumulation relative to the two thresholds β_1 and β_2, where $\beta_1 = [R\varepsilon + \overline{X}]/[R(w + \varepsilon + \overline{X})] < 1/R$ corresponds[14] to the solution of $\overline{X} = \widetilde{X}$ and $\beta_2 \geq 1/R$ corresponds to the minimum β such that starting from $X_0 = 0$ it is possible to converge to an INFI-RENT LRD.

Hunger β_b and β_a $(\beta_b < \beta_a)$	LRD of Dynasty b / Dynasty a
$\beta_b < \beta_a < \beta_1$	FINI-WORK / FINI-WORK
$\beta_b < \beta_1 < \beta_a < \beta_2$	FINI-WORK / FINI-MIX
$\beta_b < \beta_1 < \beta_2 < \beta_a$	FINI-WORK / INFI-RENT
$\beta_1 < \beta_b < \beta_a < \beta_2$	FINI-MIX / FINI-MIX
$\beta_1 < \beta_b < \beta_2 < \beta_a$	FINI-MIX / INFI-RENT
$\beta_2 < \beta_b < \beta_a$	INFI-RENT / INFI-RENT

Table 4: Endogenous social stratification.

Table 4 summarizes the social class reached by each of the two dynasties in the long run for any possible value of their hunger for accumulation. First, notice that whenever the

[14]The assumption that $w > (R-1)\varepsilon$ guarantees that $\beta_1 < 1/R$.

hunger for accumulation is greater than β_1, some or all members of a dynasty will not belong to the working class (as it is instead the case in a FINI-WORK LRD). Table 4 also shows that in three out of six configurations there is an endogenous emergence of social stratification. In fact, although both dynasties start with the same initial wealth and wage opportunities, due to a different hunger for accumulation they end up in different social classes. The striking result is that such endogenous stratification is possible even when the two dynasties are almost identical (i.e., the differences in their hunger for accumulation is infinitesimal).[15]

We now turn the attention to the evolution of wealth inequality between the two dynasties. Since the wealth accumulated by the first dynasty a is higher than the one accumulated by the first generation of dynasty b, wealth inequality always emerges in the short run. If none of the dynasties ever exits the working-class, in the long run wealth inequality increases towards the finite value $\widetilde{X}_a - \widetilde{X}_b$. However, when at least one of the two dynasties has an INFI-RENT LRD, inequality can indefinitely increase in the long run.

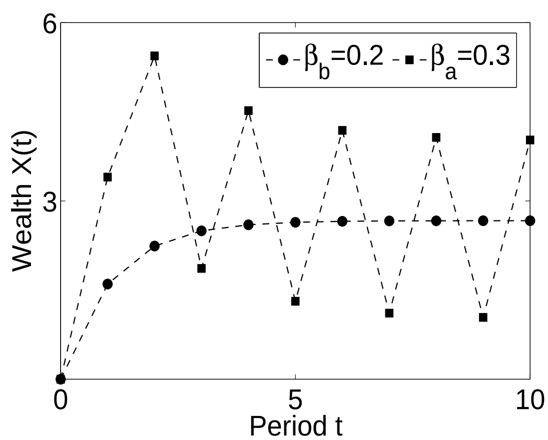

Figure 3: Dynamics of dynastic wealth when $\varepsilon = 1$, $\xi = 1$, $w = 8$ and $R = 2$.

Interestingly there exist configurations where, even without the intervention of the government, inequalities are temporarily reduced. This can be the case when the LRD of dynasty a (with the greatest hunger for accumulation) is of type FINI-MIX and the *dilapidators* of dynasty a appear during a period where the members of dynasty b work and accumulate wealth. This case is depicted in Figure 3, for values of parameters $\varepsilon = 1$, $\xi = 1$, $w = 8$ and $R = 2$.

5.2 - The demise of the rich bourgeoisie.

We now use our framework to explain the existence and the demise of the 19th century's European class structure: the rich bourgeoisie and the poor proletariat. To this purpose, let

[15] Considering $\epsilon \sim 0$, this is the case if $\beta_a = \beta_1 - \epsilon/2$ and $\beta_b = \beta_1 + \epsilon/2$ or if $\beta_a = \beta_2 - \epsilon/2$ and $\beta_b = \beta_2 + \epsilon/2$.

us consider an economy with a continuum of dynasties, heterogeneous with respect to initial wealth X_0, effort cost ξ, and hunger for accumulation β.

A possible explanation for the existence of a capitalist-workers class structure before the industrial revolution could be the prevalence of very low "subsistence" wages. In this case, corresponding in our model to $w < \mu_1\varepsilon$, society is inevitably divided into two classes (see Table 2): the poor workers and the rich *rentiers* (or capitalists) with a ZERO-WORK and an INFI-RENT LRD, respectively. The first class includes agents with low initial wealth (i.e., $X_0 < \widehat{X}$) and those with low hunger for accumulation (i.e., $\beta < 1/R$). The second class includes agents who have both a sufficiently high hunger for accumulation and high initial endowment (i.e., $\beta > 1/R$ and $X_0 > \widehat{X}$). In this society class mobility is non-existent, as the two possible LRD depend on initial wealth and in a ZERO-WORK LRD wealth never grows, and wealth inequalities exacerbate over time.

According to our model, the presence of relatively high wages during the 20th century could explain why a capitalist-worker class structure did not survive. It is consistent with the theory of Galor and Moav (2006) for which during the 20th century we assisted to a wage increase because "The capitalists found it beneficial to support universal publicly financed education, which enhanced the participation of the working class in the process of human and physical capital accumulation, and led to a widening of the middle class and to the eventual demise of the capitalists-workers class structure". While wage is exogenous in our model, it delivers implications coherent with this theory. In fact when wages are high $w > \max\{\sigma\varepsilon, \mu_2\varepsilon\}$, in our economy there are three social classes (see Table 2) characterized by dynasties with FINI-WORK, FINI-MIX, and INFI-RENT LRD. In this new society agents are richer, as not only the wage is higher but inherited wealth is always positive in the long run. For the same reason, economic differences between the rich *rentiers* and the other social classes are reduced. Furthermore, initial endowment is less important in determining the evolution of wealth. In fact even dynasties with zero or little initial wealth can, in the long run and depending on their hunger for accumulation, belong to any social class. As a consequence, in the medium run social mobility is higher and we are likely to observe new capitalists emerging from the middle and lower classes.

Our model can therefore account for both the passage from a two-class very unequal society to a three-class less unequal society and the decreasing importance over time of initial wealth (and, hence, for example, of landowners) just by considering the wage increase observed in the 20th century.

Notice that in our setting extremely wealthy agents (see Proposition 2) as well as agents with high cost of effort (see Figures 4-8 in Appendix E) will choose not to work (or innovate). It follows that the prediction of our model are also consistent, under appropriate assumptions on the correlation between wealth and the cost of effort, with Doepke and Zilibotti (2005 and 2008). According to them, the decline of the bourgeoisie is the result of the endogenous choice of the rate of time impatience, which conducts the middle class to become patient and more willing to engage in new costly but profitable opportunities.[16] Different from models with capital market imperfections (see, e.g., Banerjee and Newman 1993, Galor and Zeira 1993 or Matsuyama 2006), our model predicts that those who take the effort to innovate and take advantage of new profitable opportunities are agents who are neither too poor nor too rich.

6 Conclusions

This paper proposes a microfounded model to study the accumulation and transmission of wealth within a dynasty. Despite being deterministic the model can generate a variety of long-run patterns of wealth and effort choice. For instance, it can explain why some dynasties are captured into a poverty trap, why some other dynasties present *dilapidators* and *ruiners* who give rise to patterns of wealth as the one celebrated in the adage attributed to Andrew Carnegie, and why some dynasties consist of *rentiers*, who cumulate patrimonies that are meant to last indefinitely. For its focus on the dynamics of wealth accumulation the paper should be considered as a contribution complementary to those studies of wealth accumulation that consist of the calibration of stochastic growth models or of theoretical models with human capital considerations or imperfect credit market.

Importantly, we also explained some macroeconomic features of social mobility and class structure as well as the existence and the demise of the rich bourgeoisie using frameworks which do not need to appeal to capital markets imperfections.[17] For example, in a setting

[16]They focus on the puzzle of the rapid decline of the bourgeoisie: "why did the upper classes prove unable to exploit the new opportunities arising with industrialization, in spite of their superior wealth and education? Economic theories of wealth inequality often appeal to capital market imperfections: poor individuals may be unable to finance otherwise profitable investment projects, and are therefore forced to enter less productive professions. According to this theory, when new technological opportunities arise, the rich (who are least constrained by credit market imperfections) should be the first beneficiaries. Indeed, this theory should be highly relevant for the British Industrial Revolution, because wealth inequality was quite extreme and financial markets shallow by modern standards. Yet, we know now that the old rich did not do well at all, and were overtaken by a new economic elite that rose from the middle classes."

[17]The interested reader can find some examples of introducing wage and inheritance taxation in our

with fully heterogeneous dynasties our model provides an explanation for the demise of the rich bourgeoisie that predicts that those who take the effort to innovate and take advantage of new profitable opportunities are agents who are neither too poor nor too rich. Finally, our model provides a simple deterministic alternative to recent sophisticated literature for the endogenous emergence of a stratified society, wherein inherently (almost) identical agents may endogenously separate into the rich and the poor.

To summarize, our analysis reveals that the main factors determining the long-run dynamics of wealth and effort are the intensity of preferences towards wealth accumulation and the predisposition towards working or, alternatively, the entrepreneurial attitude.[18] In most cases the interplay between these two preference parameters, rather than initial wealth, transitive shocks to wealth, inflation, or capital market imperfections determine the long-run process of wealth accumulation and transmission within a family lineage and the evolution of wealth inequalities.

Obviously, our analysis is a great starting point for further empirical analysis. It would be interesting to carefully calibrate the model and discuss the relevance and plausibility of the chosen parameter values for specific countries or historical periods. Depending on the period or the country of study, one may need to introduce further extensions into the model. For instance, right now the interest rate is exogenous. But in the long run, it must be determined endogenously together with the wage rate and the capital accumulated by a country. Another possibility would thus be, for instance, to select a specific event or situation (e.g. a specific country) and apply our model, with a careful discussion of the parameters, to capture the actual behavior of the economy, highlighting which of the ingredients of our model (spirit of capitalism, etc.) is (or is not) fundamental to understand the empirical facts.

framework in Degan and Thibault (2008). In particular, using a "savers-spenders" approach popularized by Mankiw (2000), we show that inheritance taxation can provide an explication for the fact that the big fortunes that have been depleted have never completely rebuilt. In addition, even in those cases where inheritance taxation slows down the wealth accumulation process, it can lead in certain periods to a higher transmitted wealth than in contexts without taxation and can therefore redistribute wealth and lifetime income intergenerationally. We also show that, in a context with heterogeneous dynasties, the effect of a simple form of redistributive labor income tax strictly depend on the behavior of the richest dynasties without taxation, on wealth and the tax rate. Then, we illustrate a situation where the labor income tax can lead to a CYCL-WORK LRD for the poorest dynasties.

[18] Mookherjee and Napel (2007) also argue that when occupational mobility is sought to be explained by heterogeneity of talent (or investment cost, or ex post income uncertainty), long run macroeconomic outcomes become less history dependent.

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Appendix

Appendix A – Proof of Proposition 1.

An agent chooses e_t and x_{t+1} in order to maximize (1) subject to the budget constraint $\Omega_t = w_t e_t + R_t x_t$. Therefore, given x_t, an agent maximizes: $\phi(x_{t+1}, e_t) = (1-\beta)\ln\big[w_t e_t + R_t x_t - x_{t+1}\big] + \beta \ln\big[\varepsilon + x_{t+1}\big] - \xi e_t$. It follows that the desired bequests $\check{x}_{t+1}$ satisfies: $\phi'_x(\check{x}_{t+1}, e_t) = -(1-\beta)/(\Omega_t - \check{x}_{t+1}) + \beta/(\varepsilon + \check{x}_{t+1}) = 0$. Hence: $\check{x}_{t+1} = \beta\Omega_t - (1-\beta)\varepsilon$. Taking into account the non-negativity bequest constraint $x_{t+1} = \max\{\check{x}_{t+1}, 0\}$ we obtain (2). □

Appendix B – Proof of Proposition 2.

According to Proposition 1, we have:

$$x_{t+1} = \begin{cases} 0 \quad \text{if} \quad (X_t \leq \sigma\varepsilon \ \ \text{and} \ \ e_t = 0) \ \ \text{or} \ \ (w_t + X_t \leq \sigma\varepsilon \ \ \text{and} \ \ e_t = 1) \\ \beta[w_t + X_t - \sigma\varepsilon] \quad \text{if} \quad (w_t + X_t > \sigma\varepsilon \ \ \text{and} \ \ e_t = 1) \\ \beta[X_t - \sigma\varepsilon] \quad \text{if} \quad (X_t > \sigma\varepsilon \ \ \text{and} \ \ e_t = 0) \end{cases}.$$

When $X_t < \sigma\varepsilon - w_t$, the end-of-period wealth x_{t+1} is zero independent of the effort choice. In order to choose effort then the agent compares the utility he derives when he does not work $\phi(0,0)$ with the utility he derives when he works $\phi(0,1)$. Since $\phi(0,0) = (1-\beta)\ln X_t + \beta \ln\varepsilon$ and $\phi(0,1) = (1-\beta)\ln(w_t + X_t) + \beta\ln\varepsilon - \xi$, we have that $\phi(0,0) > \phi(0,1)$ if and only if $\xi > (1-\beta)\ln(1 + w_t/X_t)$. Then, when $X_t < \sigma\varepsilon - w_t$, $e_t = 0$ if and only if $X_t > w_t/(\tilde{e} - 1)$, where $\tilde{e} = e^{\xi/(1-\beta)}$. Taking into consideration that $w_t/(\tilde{e} - 1) \leq \sigma\varepsilon - w_t$ if and only if $w_t \leq \mu_1\varepsilon$, where $\mu_1 = \sigma(1 - 1/\tilde{e})$, it follows that when $X_t < \sigma\varepsilon - w_t$, $e_t = 0$ if and only if:

$$X_t > X_t^\sharp = \begin{cases} w_t/(\tilde{e} - 1) & \text{if } w_t < \mu_1\varepsilon \\ \sigma\varepsilon - w_t & \text{if } w_t \geq \mu_1\varepsilon \end{cases}.$$

When $X_t > \sigma\varepsilon$, the end-of-period wealth x_{t+1} is positive independent of the effort choice. Similarly to the above case, in order to choose effort the agent compares the utility he derives when he does not work $\phi(x_+, 0)$ with the one he derives when he works $\phi(x_+, 1)$.

Since $\phi(x_+,0) = (1-\beta)\ln[(1-\beta)(X_t+\varepsilon)] + \beta\ln[\beta(X_t+\varepsilon)]$ and $\phi(x_+,1) = (1-\beta)\ln[(1-\beta)(w_t+X_t+\varepsilon)] + \beta\ln[\beta(w_t+X_t+\varepsilon)] - \xi$, we have that $\phi(x_+,0) > \phi(x_+,1)$ if and only if $\xi > \ln[1+w_t/(X_t+\varepsilon)]$. Then, when $X_t > \sigma\varepsilon$, $e_t = 0$ if and only if $X_t > w_t/(e^{\xi}-1)-\varepsilon$. Since $w_t/(e^{\xi}-1)-\varepsilon > \sigma\varepsilon$ if and only if $w > \mu_2\varepsilon$, where $\mu_2 = (e^{\xi}-1)/\beta$, it follows that when $X_t > \sigma\varepsilon$, $e_t = 0$ if and only if:

$$X_t > \overline{X}_t = \begin{cases} \sigma\varepsilon & \text{if } w_t \leq \mu_2\varepsilon \\ w_t/(e^{\xi}-1)-\varepsilon & \text{if } w_t > \mu_2\varepsilon \end{cases}.$$

When $\sigma\varepsilon - w_t < X_t \leq \sigma\varepsilon$, the agent chooses between working and leaving a positive bequest and neither working nor leaving any bequest. His optimal choice then depends on the comparison between $\phi(x_+,1)$ and $\phi(0,0)$. We have that $\phi(0,0) > \phi(x_+,1)$ if and only if $\xi > (1-\beta)\ln[(1-\beta)(w_t+X_t+\varepsilon)/X_t] + \beta\ln[\beta(w_t+X_t+\varepsilon)/\varepsilon]$. Then, when $\sigma\varepsilon - w_t < X_t \leq \sigma\varepsilon$, $e_t = 0$ if and only if $\mathcal{A} = \varepsilon^{\beta}e^{\xi}/[\beta^{\beta}(1-\beta)^{1-\beta}] > \varphi(X_t) = (w_t+X_t+\varepsilon)/X_t^{1-\beta}$.

Since $\varphi'(X_t)$ has the same sign as $X_t - \sigma(w_t+\varepsilon)$, $\varphi(.)$ is decreasing on the interval $(\sigma\varepsilon - w_t, \sigma\varepsilon]$. Therefore, on this same interval, $\varphi(.)$ reaches its maximum at $\varphi(\sigma\varepsilon - w_t) = \varepsilon/[\beta(\sigma\varepsilon - w_t)^{1-\beta}]$ and its minimum at $\varphi(\sigma\varepsilon) = (\beta w_t + \varepsilon)/[\beta^{\beta}[(1-\beta)\varepsilon]^{1-\beta}]$. It follows that $\mathcal{A} > \varphi(\sigma\varepsilon - w_t)$ if and only if $w_t < \mu_1\varepsilon$ and $\mathcal{A} < \varphi(\sigma\varepsilon)$ if and only if $w_t > \mu_2\varepsilon$. Consequently, when $\sigma\varepsilon - w_t < X_t \leq \sigma\varepsilon$, $e_t = 0$ if and only if:

$$X_t > X_t^{\star} = \begin{cases} \sigma\varepsilon - w_t & \text{if } \; w_t \leq \mu_1\varepsilon \\ \text{Root of } [\mathcal{A} - \varphi(X_t)] & \text{if } \mu_1\varepsilon < w_t < \mu_2\varepsilon \\ \sigma\varepsilon & \text{if } \; w_t \geq \mu_2\varepsilon \end{cases}.$$

According to the previous thresholds, $e_t = 0$ if and only if: $X_t^{\sharp} < X_t \leq \sigma\varepsilon - w_t$, $\sigma\varepsilon - w_t \leq X_t^{\star} \leq X_t < \sigma\varepsilon$, and $\sigma\varepsilon \leq \overline{X}_t < X_t$. When $w < \mu_1\varepsilon$ we have that $X_t^{\sharp} < \sigma\varepsilon - w_t$, $X_t^{\star} = \sigma\varepsilon - w$, and $\overline{X}_t = \sigma\varepsilon$. Therefore, $e_t = 0$ if and only if $X_t > X_t^{\sharp}$. When $\mu_1\varepsilon < w < \mu_2\varepsilon$ we have that $X_t^{\sharp} = \sigma\varepsilon - w_t$, $\sigma\varepsilon - w < X_t^{\star} < \sigma\varepsilon$ and $\overline{X}_t = \sigma\varepsilon$. Then, $e_t = 0$ if and only if $X_t > X_t^{\star}$. When $w > \mu_2\varepsilon$ we have that $X_t^{\sharp} = \sigma\varepsilon - w_t$, $X_t^{\star} = \sigma\varepsilon$ and $\overline{X}_t > \sigma\varepsilon$. Then, $e_t = 0$ if and only if $X_t > \overline{X}_t$. It follows that $e_t = 0$ if and only if X_t is larger than the threshold $\mathcal{X}_t$ defined by:

$$\mathcal{X}_t = \begin{cases} X_t^{\sharp} = w_t/(e^{\xi/(1-\beta)}-1) & \text{if } \; w_t \leq \mu_1\varepsilon \\ X_t^{\star} = \text{Root of } \left\{\varepsilon^{\beta}e^{\xi}/[\beta^{\beta}(1-\beta)^{1-\beta}] - (w_t+X_t+\varepsilon)/X_t^{1-\beta}\right\} & \text{if } \; \mu_1\varepsilon < w_t < \mu_2\varepsilon \\ \overline{X}_t = w_t/(e^{\xi}-1)-\varepsilon & \text{if } \; w_t \geq \mu_2\varepsilon \end{cases},$$

where it should be noticed that $X_t^{\sharp} \leq \sigma\varepsilon - w_t < X_t^{\star} < \sigma\varepsilon \leq \overline{X}_t$. $\square$

Appendix C – Characterization of the wealth dynamics as a function of the wage.
According to Propositions 1 and 2 we can distinguish the five following dynamics of wealth accumulation:

(a) If $w < \mu_1\varepsilon$ then $X_{t+1} = \begin{cases} 0 \quad \text{if} \quad X_t \leq \sigma\varepsilon \\ \beta R\Big[X_t - \sigma\varepsilon\Big] \quad \text{if} \quad X_t > \sigma\varepsilon \end{cases}$

(b) If $\mu_1\varepsilon < w < \min(\sigma\varepsilon, \mu_2\varepsilon)$ then $X_{t+1} = \begin{cases} 0 \quad \text{if} \quad X_t \leq \sigma\varepsilon - w \text{ or } X^\star < X_t \leq \sigma\varepsilon \\ \beta R\Big[w + X_t - \sigma\varepsilon\Big] \quad \text{if} \quad \sigma\varepsilon - w \leq X_t \leq X^\star \\ \beta R\Big[X_t - \sigma\varepsilon\Big] \quad \text{if} \quad X_t > \sigma\varepsilon \end{cases}$

(c) If $\mu_2\varepsilon < w < \sigma\varepsilon$ then $X_{t+1} = \begin{cases} 0 \quad \text{if} \quad X_t \leq \sigma\varepsilon - w \\ \beta R\Big[w + X_t - \sigma\varepsilon\Big] \quad \text{if} \quad \sigma\varepsilon - w \leq X_t \leq \overline{X} \\ \beta R\Big[X_t - \sigma\varepsilon\Big] \quad \text{if} \quad X_t > \overline{X} \end{cases}$

(d) If $\sigma\varepsilon < w < \mu_2\varepsilon$ then $X_{t+1} = \begin{cases} 0 \quad \text{if} \quad X^\star < X_t \leq \sigma\varepsilon \\ \beta R\Big[w + X_t - \sigma\varepsilon\Big] \quad \text{if} \quad 0 \leq X_t \leq X^\star \\ \beta R\Big[X_t - \sigma\varepsilon\Big] \quad \text{if} \quad X_t > \sigma\varepsilon \end{cases}$

(e) If $w > \max(\sigma\varepsilon, \mu_2\varepsilon)$ then $X_{t+1} = \begin{cases} \beta R\Big[w + X_t - \sigma\varepsilon\Big] \quad \text{if} \quad 0 \leq X_t \leq \overline{X} \\ \beta R\Big[X_t - \sigma\varepsilon\Big] \quad \text{if} \quad X_t > \overline{X} \end{cases}$ $\square$

Appendix D – Characterization of the long run dynamics.

In order to characterize the types of LRD of our economy, we establish some property of the non trivial branches of the dynamic equation (3). In particular, after some easy but tedious calculations, we characterize in the following Lemma, the form of the k-th element, monotonicity, and convergence for each of the two branches. Let $\widetilde{X} = \beta R[w - \sigma\varepsilon]/(1 - \beta R)$ and $\widehat{X} = \beta R\sigma\varepsilon/(\beta R - 1)$:

<u>*Lemma 1*</u>

A – Let X_T ,..., X_{T+k} be a sequence such that for $t \in \{0, k\}$, $X_{T+t+1} = \beta R\Big[w + X_{T+t} - \sigma\varepsilon\Big]$.

a) For all $t \in \{0, k\}$, $X_{T+t} \equiv \Phi_{X_T}(t) = (\beta R)^t[X_T - \widetilde{X}] + \widetilde{X}$.

b) $\Phi_{X_T}(t + 1) - \Phi_{X_T}(t)$ has the sign of $(X_T - \widetilde{X})(\beta R - 1)$. Then, when $w \geq \sigma\varepsilon$, $\Phi_{X_T}(t + 1) - \Phi_{X_T}(t)$ is positive if $\beta R > 1$ and has the sign of $\widetilde{X} - X_T$ if $\beta R < 1$. When $w < \sigma\varepsilon$, $\Phi_{X_T}(t + 1) - \Phi_{X_T}(t)$ is negative if $\beta R < 1$ and has the sign of $X_T - \widetilde{X}$ if $\beta R > 1$.

c) $\lim\limits_{t\to+\infty} \Phi_{X_T}(t) = \widetilde{X}$ if $\beta R < 1$, $-\infty$ if ($\beta R > 1$ and $X_T < \widetilde{X}$) and $+\infty$ if ($\beta R > 1$ and $X_T > \widetilde{X}$).

B – Let X_T ,..., X_{T+k} be a sequence such that for $t \in \{0, k\}$, $X_{T+t+1} = \beta R\Big[X_{T+t} - \sigma\varepsilon\Big]$.

d) For all $t \in \{0, k\}$, $X_{T+t} \equiv \Psi_{X_T}(t) = (\beta R)^t(X_T - \widehat{X}) + \widehat{X}$.

e) $\Psi_{X_T}(t+1) - \Psi_{X_T}(t)$ is negative if $\beta R < 1$ and has the sign of $X_T - \widehat{X}$ if $\beta R > 1$.
f) $\lim_{t \to +\infty} \Psi_{X_T}(t) = \widehat{X} < 0$ if $\beta R < 1$, $-\infty$ if ($\beta R > 1$ and $X_T < \widehat{X}$) and $+\infty$ if ($\beta R > 1$ and $X_T > \widehat{X}$).

One last element to introduce before characterizing the LRD of our economy is $\Delta_{X_0}(t)$, which denotes the complete trajectory of wealth, starting from X_0, when wealth follows:

$$X_{t+1} = \begin{cases} \beta R[X_t + w - \sigma\varepsilon] & \text{if} \quad 0 \le X_t < \mathcal{X} \\ \beta R[X_t - \sigma\varepsilon] & \qquad X_t > \mathcal{X} \end{cases}$$

• A necessary and sufficient condition (hereafter, *nsc*) to obtain a ZERO-WORK LRD is that: (i) $\forall t$ such that $X_t = 0$ we have $X_{t+1} = 0$ and (ii) there exists a period T such that $X_T = 0$. According to App. C, (i) is satisfied if and only if $w < \sigma\varepsilon$. Then, a *nsc* to obtain a ZERO-WORK LRD is that $w < \sigma\varepsilon$ and (ii). When $w < \sigma$, according to App. C and Lemma 1, (ii) is satisfied $\forall X_0$ if $\beta R < 1$.

• To obtain a FINI-WORK LRD it is necessary that $\lim_{t \to +\infty} \Phi_{X_T}(t) = \widetilde{X} > 0$, i.e., that $\beta R < 1$ and $\widetilde{X} > 0$. Using Lemma 1, this is equivalent to requiring $\beta R < 1$ and $w \ge \sigma\varepsilon$. Another necessary condition is that $\widetilde{X} \le \mathcal{X}$. To see why this is the case, suppose the opposite was true, i.e. $\mathcal{X} < \widetilde{X}$. Since $\beta R < 1$, independent of X_0, there would exist a period in which wealth will be greater than $\mathcal{X}$. But then, the agents will choose to stop working, which (under $\beta R < 1$) would prevent wealth to converge towards $\widetilde{X}$. It follows that necessary conditions to obtain a FINI-WORK LRD are: $\beta R < 1$, $w \ge \sigma\varepsilon$, and $\widetilde{X} \le \mathcal{X}$. According to dynamics (d) and (e) of App. C, these necessary conditions are also sufficient.

• A *nsc* to obtain an INFI-RENT LRD is that $\beta R > 1$ and there exists a T such that $X_T > \widehat{X}$. According to Lemma 1, these conditions are sufficient because they guarantee the existence of a $T' \ge T$ such that $\lim_{t \to +\infty} X_{T'+t} = +\infty$. They are also necessary. Indeed, if $\beta R < 1$ $\lim_{t \to +\infty} X_t$ is finite. If $\beta R < 1$ and $\nexists\, T$ such that $X_T > \widehat{X}$, then $\lim_{t \to +\infty} X_t = -\infty$.

• An obvious necessary condition to obtain a ZERO-MIX LRD is the existence of a T such that $X_T = 0$. It is also necessary that when $X_T = 0$: (i) $X_{T+1} > 0$ and (ii) there exists an m such that $X_{T+m} = 0$. According to App. C, (i) implies $w \ge \sigma\varepsilon$ and (ii) implies $w < \mu_2\varepsilon$. It follows that to have a ZERO-MIX LRD, it is necessary to be in the case (d) of App. C. In such a setting, a *nsc* to have a ZERO-MIX LRD is that wealth eventually becomes zero both starting from $X_T = 0$ and from X_0. That is, there must exist t and t' such that $\Delta_{X_0}(t)$ and $\Delta_0(t') \in (X^\star, \sigma\varepsilon)$. We can find more specific conditions for the case with $\beta R < 1$. In fact, when $\widetilde{X} < X^\star$ it is impossible for a t such that $\Delta_0(t) \in (X^\star, \sigma\varepsilon)$ to exist. Conversely, according to Lemma 1, some t and t' such that $\Delta_{X_0}(t)$ and $\Delta_0(t') \in (X^\star, \sigma\varepsilon)$ always exist when $X^\star < \widetilde{X} < \sigma\varepsilon$.

• Since our model is deterministic, there exists a unique LRD for any given dynasty. If the long run accumulation is monotonic, wealth accumulated can only: be zero (and necessarily we have a ZERO-WORK LRD); converge to a finite value (and necessarily we have a FINI-WORK LRD); or grow towards infinite (and necessarily we have a INFI-RENT LRD). Among the LRD with non monotonic long-run accumulation, only two cases can arise, depending on whether accumulated wealth can be zero or not. The first one corresponds to a ZERO-MIX LRD whereas the second one corresponds to a FINI-MIX LRD. Then, a *nsc* to obtain a FINI-MIX LRD it that the necessary and sufficient conditions to obtain ZERO-WORK, FINI-WORK, INF-RENT and ZERO-MIX LRD are not satisfied.

Then, according to $\beta R < 1$ or $\beta R > 1$ we obtain:

Proposition 4 – *When* $\beta R < 1$ *the LRD is:*

• ZERO-WORK *if and only if* $w \leq \sigma\varepsilon$.

• FINI-WORK *if and only if* $\Big[\sigma\varepsilon < w < \mu_2\varepsilon$ *and* $\tilde{X} < X^\star\Big]$ *or* $\Big[w \geq \max\{\sigma\varepsilon, \mu_2\varepsilon\}$ *and* $\tilde{X} < \overline{X}\Big]$.

• ZERO-MIX *if and only if* $\Big[\sigma\varepsilon < w < \mu_2\varepsilon$ *and* $X^\star < \widetilde{X} < \sigma\varepsilon\Big]$ *or* $\Big[\sigma\varepsilon < w < \mu_2\varepsilon,\ \sigma\varepsilon < \widetilde{X}$ *and* $\exists\ t, t'$ *such that* $\Delta_{X_0}(t)$ *and* $\Delta_0(t') \in (X^\star, \sigma\varepsilon)\Big]$.

• FINI-MIX *if and only if* $\Big[\sigma\varepsilon < w < \mu_2\varepsilon,\ \sigma\varepsilon < \widetilde{X}$ *and* $\forall t > 0\ \Delta_{X_0}(t)$ *or* $\Delta_0(t) \in (0, X^\star) \cup (\sigma\varepsilon, +\infty)\Big]$ *or* $\Big[w \geq \max\{\sigma\varepsilon, \mu_2\varepsilon\}$ *and* $\overline{X} < \widetilde{X}\Big]$.

Proposition 5 – *When* $\beta R > 1$ *the LRD is:*

• ZERO-WORK *if and only if* $w \leq \sigma\varepsilon$ *and there exists a period* T *such that* $X_T = 0$.

• ZERO-MIX *if and only if* $[\sigma\varepsilon < w < \mu_2\varepsilon$ *and* $\exists\ t, t'$ *such that* $\Delta_{X_0}(t)$ *and* $\Delta_{X0}(t') \in (X^\star, \sigma\varepsilon)]$.

• FINI-MIX *if and only if* $\Big[\mu_1\varepsilon < w < \min\{\sigma\varepsilon, \mu_2\varepsilon\}$ *and* $\forall t \geq 0\ \Delta_{X_0}(t) \in (\max\{\sigma\varepsilon - w, \widetilde{X}\}, X^\star) \cup (\sigma\varepsilon, \widehat{X})\Big]$ *or* $\Big[\mu_2\varepsilon < w < \sigma\varepsilon$ *and* $\forall t \geq 0\ \Delta_{X_0}(t) \in (\max\{\sigma\varepsilon - w, \widetilde{X}\}, \widehat{X})\Big]$ *or* $\Big[\sigma\varepsilon < w < \mu_2\varepsilon$ *and* $\forall t \geq 0,\ \Delta_{X_0}(t) \in (0, X^\star) \cup (\sigma\varepsilon, \widehat{X})\Big]$ *or* $\Big[w \geq \max\{\sigma\varepsilon, \mu_2\varepsilon\}$ *and* $\forall t \geq 0 \Delta_{X_0}(t) \leq \widehat{X}\Big]$.

• INFI-RENT *if and only if there exists a period* T *such that* $X_T > \widehat{X}$. $\square$

Appendix E – Prices and long run dynamics.

In this Appendix we provide a graphical representation of the results of Propositions 4 and 5 in order to study the role of prices (w and R) and of the effort parameter ξ on the long run dynamics.[19]

E.1 - Effort cost, wage opportunity and Long Run Dynamics.

We start with the analysis of the type of LRD obtained as a function of the effort cost ξ of the members of the dynasty and the wage w prevailing in the economy. Throughout this

[19]A derivation of the graphical illustrations is available in the web version of Degan and Thibault (2008).

section we focus on the case where the wage w is greater than $\sigma\varepsilon$.[20] We distinguish three cases according to the value of βR.

Consider first the situation, depicted in Figure 4, where $\beta R < 1$. For low effort costs, independent of the wage, in the long run all generations work and transmit increasingly positive levels of wealth. Conversely for high effort costs, when the wage is low all generations are forced to work but, when the wage is high there are some generations who decide to stop working and to live with their inheritance. It is important to remark that what drives some of these (high wage) dynasties not to work is not a wealth effect. It is the fact that while the threshold $\mathcal{X}$ above which an agent decides not to work is decreasing in ξ, the limiting value of wealth $\widetilde{X}$ that can be accumulated by a dynasty of workers is independent of it. Therefore, while dynasties with low ξ work and can allow dynastic wealth to converge towards $\widetilde{X}$, dynasties with high ξ stop working before their wealth can approach $\widetilde{X}$ and (fully or partially) dilapidate their wealth.

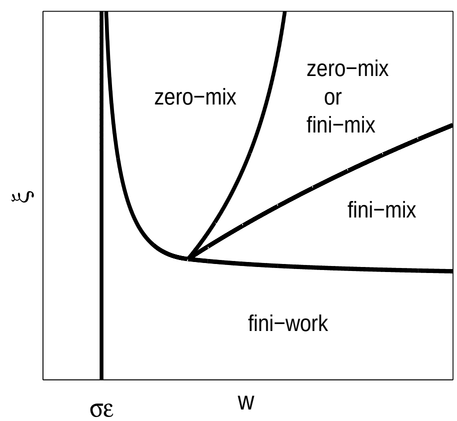

Figure 4: Case where $\beta R < 1$.

For given effort cost ξ, an increase in wage leads at the same time to an increase in the threshold $\mathcal{X}$ and in the limiting wealth $\widetilde{X}$. However, it can be shown that $\widetilde{X}$ increases faster than $\mathcal{X}$, making the existence of *dilapidators* and/or *ruiners* (i.e., the possibility that $\mathcal{X} < \widetilde{X}$) more plausible. Hence, an increase in wage allows going from a dynamics of type FINI-WORK to a dynamics of type MIX. In addition, among these MIX dynamics, there exist a (unique) wage below which the LRD is ZERO-MIX and above which it is FINI-MIX.

[20]This case is not restrictive. In fact when $\beta R < 1$ and $w < \sigma\varepsilon$, we have a ZERO-WORK LRD. This is also the case when $\beta R > 1$ and ($w < \mu_1\varepsilon$) or ($X_0 = 0$ and $\mu_1 < w < \sigma\varepsilon$). Considering only $w > \sigma\varepsilon$ avoids dealing with the possibility of a FINI-MIX and INFI-RENT LRD when $\mu_1\varepsilon < w < \sigma\varepsilon$. Cases that can emerge only for $\beta R > 1$ and some ranges of strictly positive X_0. Although the study of these cases is possible (and available from the authors upon request) it is extremely tedious and non-informative and is therefore omitted.

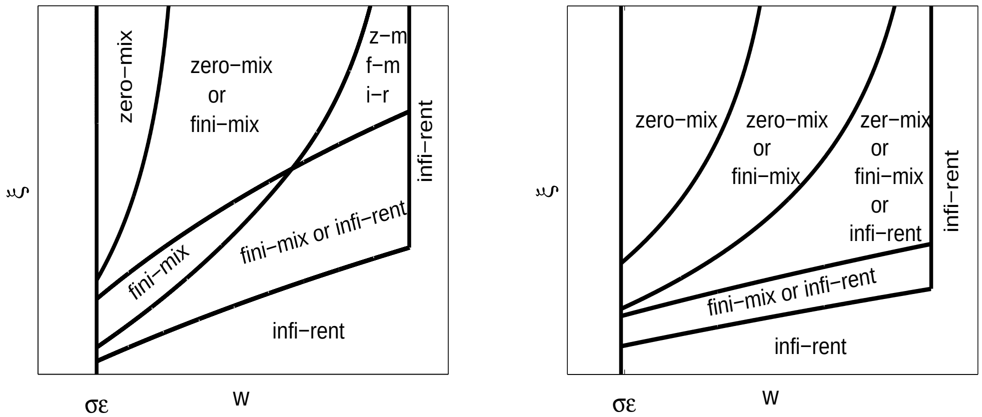

Figure 5: Cases where $\beta R > 1$.

Consider now $\beta R > 1$. We illustrate in Figure 5 the case where $1 < \beta R < 2$ (LHS) and $\beta R > 2$ (RHS), respectively, where it is assumed that $X_0 < \widehat{X}$.[21] As the cost of effort increases there is a possibility (when the wage is not too high, depending on initial wealth X_0) to fall into a LRD with *ruiners*. Similarly, the set of effort costs in correspondence to which we have an INFI-RENT LRD gets wider as the wage increases. This is because the higher w the higher the wealth accumulated by a chain of workers. Therefore, the threshold $\widehat{X}$ (independent of w) that inherited wealth has to pass in order to have an INFI-RENT LRD is easier to reach. In fact, for high wages this is the only LRD, independent of ξ.

E.2 - Interest rate, wage opportunity and Long Run Dynamics.

We use Propositions 4 and 5 simultaneously to consider the LRD as a function of w and R. Since there are too many possible configurations, we focus only on the case where initial wealth X_0 is zero (or, by continuity, sufficiently low). We distinguish three cases according to the value of ξ (represented in Figure 6, 7, and 8, respectively): low effort cost, $\xi < \ln(2-\beta)$; intermediate effort cost, $\ln(2-\beta) < \xi < \ln[1/\beta]$; and high effort cost, $\xi > \ln[1/\beta]$.[22]

When the wage is relatively low (i.e., $w < \sigma\varepsilon$), independent of the effort cost ξ the interest rate R does not affect the type of LRD, which is of type ZERO-WORK. Consider now wages greater than $\sigma\varepsilon$. When the cost of effort is low (Figure 6), the higher the wage the higher the possibility of having agents who do not work. In particular there exist two thresholds for the interest rate, R_1 and R_2 ($0 < R_1 < R_2$), such that: if R is lower than R_1 the LRD is of type FINI-WORK; if R is greater than R_2 the LRD is of type INFI-RENT; and when R is in between R_1 and R_2 the LRD is of type FINI-MIX.

[21] When $\beta R > 1$ and $X_0 > \widehat{X}$ there is only an INFI-RENT LRD.

[22] Alternatively, in terms of the hunger for accumulation these three cases correspond to $\beta < 2 - e^{\xi}$, $2 - e^{\xi} < \beta < e^{-\xi}$, and $\beta > e^{-\xi}$, respectively.

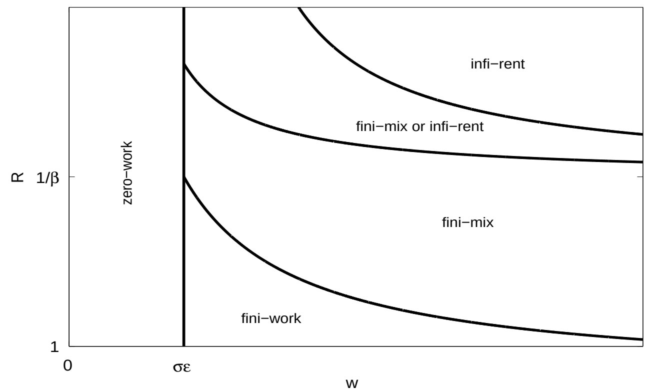

Figure 6: Case with low effort cost.

The possible switch from a FINI-WORK to a FINI-MIX LRD is due, different from subsection E.1, to a wealth effect. In fact, $\widetilde{X}$ is increasing in R but the threshold $\mathcal{X}$ is independent of it. Conversely, the possible switch from a MIX to an INFI-RENT LRD is not necessarily due to a wealth effect, as $\widehat{X}$ is decreasing in R. Intuitively, because the wealth of *rentiers* grows at a rate βR, the higher βR the lower the minimum initial wealth needed to obtain at a certain period a given level of wealth.

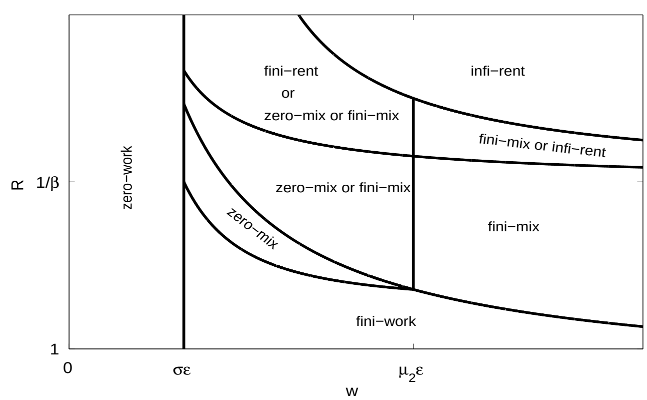

Figure 7: Case with intermediate cost effort.

The types of switches among LRD described above emerge also for intermediate effort costs (Figure 7) but only when the wage is sufficiently high, i.e. $w > \mu_2\varepsilon$. For intermediate wage levels, i.e. $\sigma\varepsilon < w < \mu_2\varepsilon$, changes in the interest rate can generate as well as eliminate *ruiners*. In fact when the wage is in this range, a direct switch from a FINI-WORK to a FINI-MIX LRD is no longer possible. As R increases the LRD must first move from FINI-WORK to ZERO-MIX. Only then it could go through a FINI-MIX and eventually become INFI-RENT.

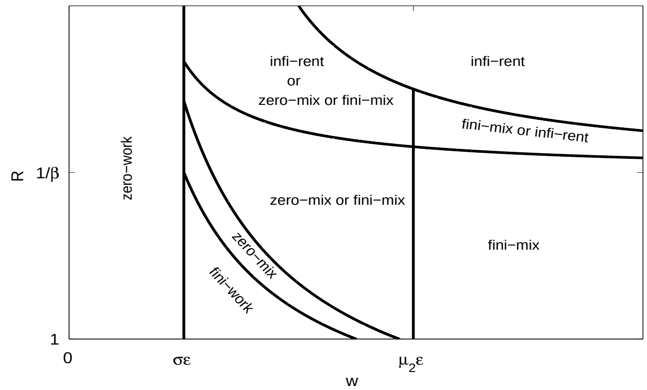

Figure 8: Case with high effort cost.

Ruiners can emerge because the actual value of wealth received from the previous generation may not be high enough to convince an agent at the same time not to work and to leave a positive wealth. For a given bequest, as the interest rate reaches a certain threshold, the agent is induced to transmit a positive wealth so that the initial *ruiner* at higher interest rates becomes a *dilapidator*. For analogous reasons, as the interest rate increases further, the *dilapidator* becomes a *rentier*.

The analysis of Figures 6-8 also point to the fact that the higher the effort cost the less plausible is to have a dynasty of all workers. In fact when the cost of effort is high (Figure 8) a FINI-WORK LRD exists only for a very small range (and low values) of w and R. In particular a FINI-WORK LRD exists only for wages strictly lower than $\mu_2\varepsilon$, implying that it is no longer possible to go directly from a FINI-WORK to a FINI-MIX LRD. In the remaining cases the effect of an increase in R is equivalent to the ones discussed above. □

Appendix F – Exogenous wage growth.

When wages grow at fixed positive rate γ, i.e. $w_t = (1+\gamma)^t w_0$, the threshold $\mathcal{X}_t$ is no longer constant over time. In fact, when at a certain period T the wage becomes sufficiently high (i.e. $w_T = (1+\gamma)^T w_0 > \max\{\mu_2\varepsilon, \sigma\varepsilon\}$), the threshold $\mathcal{X}_t$ is represented by $\overline{X}_t$ and in period $T+t$ is therefore $\mathcal{X}_{T+t}(w_T) = (1+\gamma)^t w_T/(e^{\xi}-1) - \varepsilon$. This threshold increases over time in a convex way and tends towards infinity. While the threshold $\mathcal{X}$ still exists, this is no longer the case for the finite threshold $\widetilde{X}$ that can be accumulated by an infinite sequence of workers when $\beta R < 1$.

In fact, starting from any given value of X_T and w_T, successive iterations of wealth according to the dynamic equation $X_{T+t+1} = \beta R[w_{T+t+1} + X_{T+t} - \sigma\varepsilon]$ lead to:

$$\begin{aligned} X_{T+t}(w_T) &= w_T[(1+\gamma)^{T+t}\beta R + (1+\gamma)^{T+t-1}(\beta R)^2 + ... + (1+\gamma)^{T+1}(\beta R)^t] \\ &\quad -\sigma\varepsilon[\beta R + (\beta R)^2 + ... + (\beta R)^t] + (\beta R)^t X_T(w_T) \end{aligned}$$

Independent of X_0, there always exists a date T' such that $X_{T'+t}$ converges towards $+\infty$ as t goes to $+\infty$. Therefore, independent of βR, the wealth accumulated by a infinite sequence of workers eventually goes to infinity. Importantly, there always exists a T'' such that $cx-d>0$, where $c=(1+\gamma)^{T''}w_0$, $x=\beta R$, and $d=w_0/(e^{\xi}-1)$. By letting $a=X_T\geq 0$, $b=\sigma\varepsilon$ and $y=1+\gamma$ it follows that:

$$\mathcal{J}_t = X_{T''+t}-\mathcal{X}_{T''+t}(w_{T''}) = ax^t - b(x+x^2+...+x^t)+y^t\left[cx-d+\frac{cx^2}{y}+\frac{cx^3}{y^2}+...+\frac{cx^t}{y^{t-1}}\right]+\varepsilon,$$

and, since $y>1$ and $cx>d$, $\lim_{t\to+\infty}\mathcal{J}_t=+\infty$. Consequently, even if the threshold of wealth above which an agent decides not to work increases and tends towards infinity, eventually the wealth accumulated by a sequence of workers is even greater, that is $X_{T''+t} > \mathcal{X}_{T''+t}$, and it is also greater than $\widehat{X}$. Hence, when wages grow at a fixed positive rate, independent of X_0, there are only two possible cases: a INFI-MIX LRD when $\beta R<1$ or a INFI-RENT LRD when $\beta R>1$. □

Appendix G – Proofs of Propositions 1 and 2 when e_t is continuous.

We show that the results of Section 2 hold when e_t is continuous.

Assume that an agent chooses e_t and x_{t+1} in order to maximize (1) subject to the budget constraint $\Omega_t = w_te_t + R_tx_t$ and $e_t\geq 0$. Therefore, given x_t, an agent maximizes: $\phi(x_{t+1},e_t) = (1-\beta)\ln\left[w_te_t+R_tx_t-x_{t+1}\right]+\beta\ln\left[\varepsilon+x_{t+1}\right]-\xi e_t$.

- It follows that the desired bequests $\check{x}_{t+1}$ satisfies: $\phi'_x(\check{x}_{t+1},e_t) = -(1-\beta)/(\Omega_t-\check{x}_{t+1}) + \beta/(\varepsilon+\check{x}_{t+1})=0$. Hence: $\check{x}_{t+1}=\beta\Omega_t-(1-\beta)\varepsilon$. Taking into account the non-negativity bequest constraint $x_{t+1}=\max\{\check{x}_{t+1},0\}$ we obtain (2). Then, Proposition 1 does not depend on the fact that e_t is a discrete or continuous variable.
- When e_t is a continuous variable, the desired effort $\check{e}_t$ satisfies: $\phi'_e(x_{t+1},\check{e}_t) = (1-\beta)w_t/(\Omega_t-x_{t+1})-\xi=0$. Merging this condition with the preceding, $\phi'_x(\check{x}_{t+1},e_t)=0$, allows to obtain $\tilde{x}_{t+1}=\beta w_t/\xi-\varepsilon$ and $\tilde{e}_t = 1/\xi-(\varepsilon+X_t)/w_t$. Then, using (2) and the two non-negative constraints $x_{t+1}\geq 0$ and $e_t\geq 0$, it is easy to show that:

$$e_t = \begin{cases} \max\left\{\dfrac{1-\beta}{\xi}-\dfrac{X_t}{w_t},0\right\} & \text{if} \quad w_t<\dfrac{\xi\varepsilon}{\beta} \\ \max\left\{\dfrac{1}{\xi}-\dfrac{(\varepsilon+X_t)}{w_t},0\right\} & \text{if} \quad w_t\geq\dfrac{\xi\varepsilon}{\beta} \end{cases}$$

Consequently, we have established that there exists a positive threshold $\mathcal{X}_t$ [equal to $(1-\beta)w_t/\xi$ if $w_t<\xi\varepsilon/\beta$ and to $w_t/\xi-\varepsilon$ if $w_t\geq\xi\varepsilon/\beta$], increasing in w_t but independent of R_t, such that an agent living in t decides not to exert effort if and only if his inherited wealth $X_t=R_tx_t$ is greater than $\mathcal{X}_t$. Then, Proposition 2 does not depend on the fact that e_t is a discrete or continuous variable. □