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ETIENNE BILLETTE DE VILLEMEUR, RICHARD RUBLE

AND BRUNO VERSAEVEL

Ecole d'économie de Toulouse (TSE) - Manufacture des Tabacs
Aile Jean-Jacques Laffont - 21, allée de Brienne - 31000 TOULOUSE
Tél : +33 (0) 5 61 12 85 89 - Fax : +33 (0) 5 61 12 86 37 - **www.TSE-fr.eu** - contact@TSE-fr.eu

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Etienne Billette de Villemeur, Toulouse School of Economics, IDEI & GREMAQ, France

Richard Ruble, EMLYON & CNRS, GATE, France*

Bruno Versaevel, EMLYON & CNRS, GATE, France

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Abstract

We show that the standard analysis of vertical relationships transposes directly to investment timing. Thus, when a firm undertaking a project requires an outside supplier (e.g. an equipment manufacturer) to provide it with a discrete input, and if the supplier has market power, investment occurs too late from an industry standpoint. The distortion in firm decisions is characterized by a Lerner index, which is related to the parameters of a stochastic downstream demand. When feasible, vertical restraints restore efficiency. For instance, the upstream firm can induce entry at the correct investment threshold by selling a call option on the input. Otherwise, competition may substitute for vertical restraints. In particular, if two firms are engaged in a preemption race downstream, the upstream firm sells the input to the first investor at a discount that is chosen in such a way that the race to preempt exactly offsets the vertical externality, and this leader invests at the optimal market threshold.

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1 Introduction

In real option models of investment, the cost of the investment (strike price) is often tacitly taken to reflect economic fundamentals closely. This assumption seems reasonable when the investment consists of R&D that is performed largely in-house, or in industries such as real estate development that may rely on competitive outside contractors. However, there are many other cases in which a firm wishing to exercise an investment option depends on an outside firm with market power to provide it with a discrete input (e.g., a key equipment) it needs, before starting producing and selling. Thus, an electricity producer may buy a nuclear plant from an outside firm, an oil company that decides to drill offshore must acquire a platform from a specialized supplier, an aeronautics firm will coordinate aircraft development with engine manufacturers, or a pharmaceutical firm will need a factory constructed to very exact specifications. In addition, strategic issues may arise if several firms seek to exercise related investment options, and call upon the same supplier.

This paper uses advances in real options games to build a model of vertical relationships in which the cost of a firm's investment is endogenous.[1] We adopt similar specifications to models by Boyer, Lasserre and Moreaux [1], Mason and Weeds [13], and Smit and Trigeorgis [17], incorporating an upstream equipment supplier that prices with market power. In so doing, the supplier generally delays exercise of the option relative to the optimal exercise threshold for the industry.

We show that the standard analysis of vertical relationships translates directly to investment timing, with investment trigger replacing price as the decision variable of the downstream firm. Thus, an industry earns lower profits under separation than under integration because of a vertical effect akin to double marginalization, which causes the downstream firm to unduly delay its investment decision. This distortion increases with both market growth and volatility. In contrast with the standard real option framework, greater volatility decreases firm value, both upstream and downstream, near the exercise threshold.

If feasible (for example, because the upstream firm has information regarding the stochastic

[1]For recent surveys of game theoretic real options models, see Boyer, Gravel, and Lasserre [2], and Huisman, Kort, Pawlina, and Thijssen [9]. Among economic extensions of real option models, Grenadier and Wang [8] comes closest to our work here, as it studies the effect of agency issues on option exercise (albeit, in a corporate governance framework). Moreover, Lambrecht, Pawlina, and Teixeira [11] and Patel and Zavodov [14] develop alternative approaches to investment options in vertical structures.

final demand), vertical restraints that take the form of an option or down payment restore the industry optimum. In contrast with existing real option models however, increased volatility does not necessarily increase firm value, because of the simultaneous presence of two effects – the option value of delay is balanced by a greater markup choice by the upstream firm.

When habitual vertical restraints are not feasible, the upstream firm benefits from the presence of a second downstream firm, although this possibly occurs at the expense of aggregate industry profits. The race between downstream firms to preempt one another exactly balances the incentive to delay caused by the upstream firm's mark-up, so the leader invests at the optimal integrated threshold, whereas the follower invests at the separation threshold (for duopoly profits), a type of "no distortion at the top" result. The leader receives a discounted price. The difference between this price and the follower's price decreases when the volatility rises. The leader's investment threshold, which has a closed-form expression, together with the follower's threshold, increase with volatility.

The remainder of the paper is organized as follows. In Section 2 we describe the model, with one upstream supplier and one downstream firm, and investigate the basic vertical externality. This is done by comparing the equilibrium outcomes in the integrated case, which we use as a benchmark, with the outcomes of the separated case. In Section 3, we discuss the introduction of vertical restraints that restore the industry optimum. In Section 4, we introduce a second downstream firm and compute the preemption equilibrium, before comparing the investment threshold and pricing outcomes with the single-firm case. Final remarks appear in Section 5. All the proofs are in the appendix.

2 The Basic Vertical Externality

The flow profit resulting from investment is $Y_t \pi_M$ where π_M is an instantaneous monopoly profit, and $Y_t > 0$ is a random shock assumed to follow a geometric Brownian motion with drift $dY_t = \alpha Y_t dt + \sigma Y_t dZ_t$. The non-negative parameters α and σ represent the market's expected growth rate and volatility, and Z_t is a standard Wiener process. A lowercase $y = Y_t$ is used to denote the current level of Y_t, and y_i denotes an investment trigger, which is a decision variable. The cost of production of the discrete input is I, the discount rate $r > \alpha$ is common to both the upstream and the downstream firms, and to rule out degenerate solutions it is assumed that $\frac{\sigma^2}{2} < \alpha$.

2.1 Integrated Case

Suppose that a single firm produces the discrete input, decides at what threshold y_i to invest, for a current market size $y \leq y_i$, and earns the subsequent flow profit. The value of such a firm is:

$$V(y, y_i, I) = \left(\frac{y}{y_i}\right)^{\beta} \left(\frac{\pi_M}{r-\alpha} y_i - I\right), \tag{1}$$

where $\beta \equiv \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}$, a standard expression in real option models (see Dixit and Pindyck [4]), is referred to in what follows as a discounting term.[2] We will use the property that β is monotone decreasing in α and in σ. For notational compactness, and to facilitate the ranking of equilibrium outcomes throughout the paper, define the function $\gamma(r, \alpha, \sigma)$ by:

$$\gamma \equiv \frac{\beta}{\beta - 1}.$$

Note that $\gamma > 1$, with $\frac{\mathrm{d}\gamma}{\mathrm{d}r} < 0$, $\frac{\mathrm{d}\gamma}{\mathrm{d}\alpha}, \frac{\mathrm{d}\gamma}{\mathrm{d}\sigma} \geq 0$.

Differentiating (1) gives the value-maximizing investment trigger, $y^* = \gamma \frac{r-\alpha}{\pi_M} I$, which serves as a benchmark throughout the analysis. Then the value of the firm that invests at y^* is:

$$V(y, y^*, I) = \frac{\gamma}{\beta} \left(\frac{y}{y^*}\right)^{\beta} I. \tag{2}$$

It is assumed throughout that the current market size y at $t = 0$ is positive and sufficiently small relative to I so that it is not profitable to invest immediately.

2.2 Separated Case

Suppose that the input production and investment decisions are made by distinct firms. We suppose that the upstream firm, as an input producer on the intermediate market, does not observe Y_t at (almost) any date t.[3] It therefore chooses an input price $p_U \geq I$ that is independent of the random shock. The downstream firm is assumed to be a price-taker in the intermediate

[2] The term $\left(\frac{y}{y_i}\right)^{\beta}$ in (1) reads as the expected discounted value, measured when $Y_t = y$, of receiving one monetary unit when Y_t reaches y_i for the first time. In the certainty case $\sigma = 0$, $\beta = \frac{r}{\alpha}$ and $\left(\frac{y}{y_i}\right)^{\beta} = e^{-r(t_i - t)}$, which is the standard continuous time discounting term under certainty.

[3] We relax this assumption only in Section 3 which discusses possible vertical restraints.

market.[4] Given p_U, it observes the final market shock, and decides when to invest, at y_i. To establish the equilibrium in (y_i, p_U) we proceed by backwards induction.

At the current market size y, the downstream firm's value is:

$$V(y, y_i, p_U) = \left(\frac{y}{y_i}\right)^{\beta} \left(\frac{\pi_M}{r-\alpha} y_i - p_U\right), \tag{3}$$

all $y \leq y_i$. The downstream value-maximizing investment trigger is $y_D(p_U) = \gamma \frac{r-\alpha}{\pi_M} p_U$, which is increasing in the input price p_U, with $y_D(I) = y^*$.

At the current market size y, the upstream firm's value is:

$$W(y, p_U) = \left(\frac{y}{y_D(p_U)}\right)^{\beta} (p_U - I), \tag{4}$$

all $y \leq y_D$. Given $y_D(p_U)$, the upstream firm maximizes $W(y, p_U)$ by setting $p_U^* = \gamma I$. In what follows, for compactness denote $y_D(p_U^*)$ by y_D^*. We find:

Proposition 1 *In the separated case, there is a unique equilibrium characterized by:*

$$y_D^* = \gamma^2 \frac{r-\alpha}{\pi_M} I \text{ and } p_U^* = \gamma I. \tag{5}$$

Substituting back into (3) and (4), the firm values in the separated equilibrium case are:

$$V(y, y_D^*, p_U^*) = \gamma \left(\frac{y}{y_D^*}\right)^{\beta} \frac{I}{\beta-1} \text{ and } W(y, p_U^*) = \left(\frac{y}{y_D^*}\right)^{\beta} \frac{I}{\beta-1}, \tag{6}$$

where the downstream value-maximizing investment trigger y_D^*, for an upstream value-maximizing input price p_U^*, takes the closed-form expression in (5).

2.3 Comparison and Comparative Statics

In the separated case, the upstream firm introduces a distortion, we refer to as a negative externality, by charging a price p_U above the cost I. This is analogous to the baseline model of vertical externality[5], the investment trigger substituting for the final price as the downstream decision variable.

[4] As in Tirole [18], this is "for simplicity" only that we "assume that the manufacturer chooses the contract" (p. 173), and the outside option of the downstream firm is normalized to zero.

[5] See Tirole [18] for a description.

In fact, this model is formally identical to the baseline model with price choices, a final demand $Q = aP^{-b}$, and a constant marginal cost of production c, taking $y_i \equiv P$, $a \equiv \frac{\pi_M}{r-\alpha} y^{\beta}$, $b \equiv \beta$, $c \equiv \frac{r-\alpha}{\pi_M} I$.

The vertical externality in the model may be gauged as follows. The trigger in the separated case, $y_D^* = \gamma y^*$, is greater than in the integrated case, which is itself greater than the social optimum.[6] In static models of oligopoly, the Lerner index is often used as a measure of market power. For the upstream firm, we have:

$$L_U \equiv \frac{p_U^* - I}{p_U^*} = \frac{1}{\beta}. \tag{7}$$

Formally, β plays the same role as the (absolute value of) the elasticity of demand in a monopoly model. By analogy, although it does not represent a price-cost margin, one may define a measure of the dynamic dimension of the downstream firm's market power as:

$$L_D \equiv \frac{y_D^* - y^*}{y_D^*} = \frac{1}{\beta}. \tag{8}$$

Note from (7) and (8) that L_D and L_U are fully characterized by β, and are impacted in the same proportions by a higher growth rate or a greater volatility.[7] Note also that $\frac{V\left(y, y_D^*, p_U^*\right)}{W\left(y, p_U^*\right)} = \frac{p_U^*}{I} = \frac{y_D^*}{y^*} = \gamma$, implying that, in the separated case, the relative firm values have the same sensitivity to a change in β, or any of its constitutive parameters. The distortion in the two firms' decisions and in their resulting values, vis-à-vis the integrated case, is monotone decreasing in the discounting term, with L_U, L_D, and $\frac{V\left(y, y_D^*, p_U^*\right)}{W\left(y, p_U^*\right)}$ tending to 0 when β tends to infinity. Another important magnitude is the relative joint value under separation and integration, that is $\frac{V\left(y, y_D^*, p_U^*\right) + W\left(y, p_U^*\right)}{V\left(y, y^*, I\right)} \equiv \Delta(\beta)$, where $\Delta(\beta) = (1+\gamma)\,\gamma^{-\beta}$. As this expression satisfies $\Delta(\beta) \in \left(\frac{2}{\mathrm{e}}, 1\right)$, the industry value is lower under separation than under integration as is to be expected, and since $\Delta'(\beta) < 0$, in contrast with the distortion in upstream and downstream choices, the distortion in the separated and integrated payoffs *decreases* with market growth and volatility in this model. This is because, as explained further below, factors other than the vertical externality also affect firm values.

To summarize:

[6] The consumer surplus is maximized when the investment occurs at $t = 0$.

[7] These expressions are comparable to those in Dixit, Pindyck, and Sødal [5].

Proposition 2 *The industry value is lower under separation than under integration. The distortion in firm decisions, as measured by L_U and L_D, is increasing in market growth rate and volatility, whereas the distortion in separated and integrated payoffs is decreasing in market growth rate and volatility.*

For the sensitivity analysis of firm choices, we find in particular that the effect on y_D^* and p_U^* of a change in the growth rate and volatility, is univocal:

$$\frac{\mathrm{d}y_D^*}{\mathrm{d}\alpha} > 0, \quad \frac{\mathrm{d}y_D^*}{\mathrm{d}\sigma} > 0, \quad \frac{\mathrm{d}p_U^*}{\mathrm{d}\alpha} > 0, \quad \frac{\mathrm{d}p_U^*}{\mathrm{d}\sigma} > 0.$$

We also evaluate the effect of α and σ on the two firms' respective value. A change in these parameters does not only impact the magnitude of the direct externality, as there is also a real option effect. To see that, focus first on α, and consider the upstream value. For notational simplicity, let $V^* \equiv V\left(y, y_D^*, p_U^*\right)$. By the envelope theorem, $\frac{\partial V}{\partial y_D^*} = 0$, and we find that:

$$\frac{\mathrm{d}V^*}{\mathrm{d}\alpha} = \frac{\partial V^*}{\partial \alpha} + \left(\frac{\partial V^*}{\partial \beta} + \frac{\partial V}{\partial p_U^*}\frac{\partial p_U^*}{\partial \beta}\right)\frac{\mathrm{d}\beta}{\mathrm{d}\alpha} \tag{9a}$$

$$= V^*\left(\frac{\beta}{r-\alpha} + \left(\frac{1}{\beta} + \ln\frac{y}{y_D^*}\right)\frac{\mathrm{d}\beta}{\mathrm{d}\alpha}\right) > 0, \tag{9b}$$

all $y \leq y_D^*$. The direct effect in (9a) is positive. The two terms between brackets, which describe the indirect effect, have opposite signs because a higher growth rate increases the investment option's value, but simultaneously raises the input price. However, the magnitude of the latter term is limited, so that the option value effect dominates the vertical effect. The sensitivity analysis is similar for the upstream firm, whose value in equilibrium $W^* \equiv W\left(y, p_U^*\right)$ has an analogous form. Specifically, the effect of greater market growth on upstream value is univocal, as we find $\frac{\mathrm{d}W^*}{\mathrm{d}\alpha} = W^*\left(\frac{\beta}{r-\alpha} + \left(\frac{1}{\beta-1} + \ln\frac{y}{y_D^*}\right)\frac{\mathrm{d}\beta}{\mathrm{d}\alpha}\right) > 0$ (see Appendix 6.4).

In contrast with this, the effect of volatility on firm values is *not* univocal. Taking first the case of upstream value, and noting again that $\frac{\partial V}{\partial y_D^*} = 0$, we find that:

$$\frac{\mathrm{d}V^*}{\mathrm{d}\sigma} = \left(\frac{\partial V^*}{\partial \beta} + \frac{\partial V}{\partial p_U^*}\frac{\mathrm{d}p_U^*}{\mathrm{d}\beta}\right)\frac{\mathrm{d}\beta}{\mathrm{d}\sigma} \tag{10a}$$

$$= V^*\left(\ln\frac{y}{y_D^*} + \frac{1}{\beta}\right)\frac{\mathrm{d}\beta}{\mathrm{d}\sigma}. \tag{10b}$$

A change in σ has two opposite indirect effects on V^*. The first term in (10a) is the real option effect: greater volatility (hence a lower β) has a positive impact on the downstream value. The second term is the vertical effect: greater volatility raises the upstream's optimal price p_U^*, lowering the downstream value. The net effect depends, in particular, on the current market size y. The right-hand side term between parentheses reveals that, at low market sizes, the real option effect dominates and the downstream firm benefits from greater volatility, whereas at higher market sizes, which are closer to the investment trigger y_D^*, the real option effect is less important and the vertical effect tends to dominate, the crossover occurring at $y_D^* \exp\left(-\frac{1}{\beta}\right) \equiv \hat{y}$, which is lower than y_D^*. For large initial market sizes, greater uncertainty thus reduces firm value, which stands in contrast with many real option models. Figure 1 illustrates the behavior of $V\left(y, y_D^*, p_U^*\right)$ over $[0, y_D^*]$ for several levels of β.

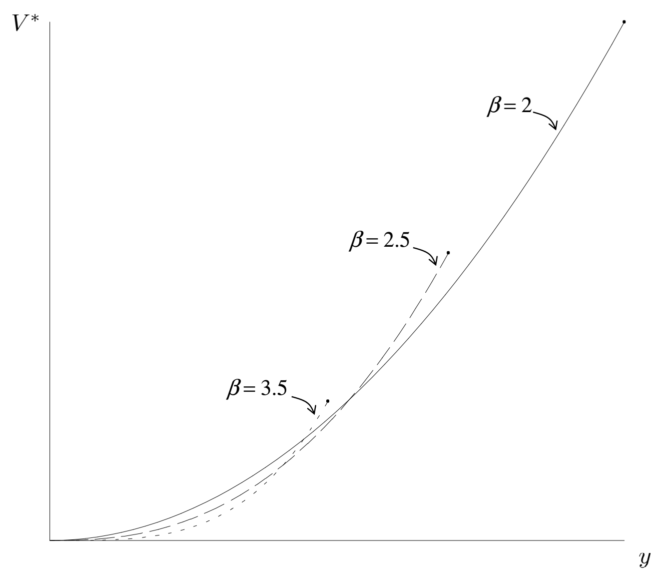

Figure 1: Downstream value $V^* = V\left(y, y_D^*, p_U^*\right)$, for $y \leq y_D^*$, with $\frac{r-\alpha}{\pi_M} = I = 1$, and $\beta = 2$ (solid), 2.5 (dash), 3.5 (dots). For large initial market sizes, greater uncertainty or growth (i.e., a lower β) reduces firm value.

Similarly, the effect of greater volatility on the value of the upstream firm is also ambiguous. As $\frac{\mathrm{d}W^*}{\mathrm{d}\sigma} = W^*\left(\ln \frac{y}{y_D^*} + \frac{1}{\beta-1}\right)\frac{\mathrm{d}\beta}{\mathrm{d}\sigma}$, the crossover occurs at a lower threshold than for the upstream firm, that is at $y_D^* \exp\left(-\frac{1}{\beta-1}\right) \equiv \check{y} < \hat{y}$. Thus, both firms benefit from greater volatility at low enough market sizes, and both are harmed by volatility at high enough market sizes, but there exists a range of market sizes $(\check{y}, \hat{y})$ over which the two firms have divergent preferences with respect to volatility.

The following proposition summarizes these results, making use of the inherent elasticity form in expressions such as (9a) and (10a).

Proposition 3 *In the separated case, a higher growth rate or more volatility increase the upstream price and the downstream trigger. A higher growth rate increases upstream and downstream values, with* $0 < \varepsilon_{W^*/\alpha} < \varepsilon_{V^*/\alpha}$. *The effect of higher volatility on firm values depends on the market size:*

$$\begin{cases} 0 \leq \varepsilon_{W^*/\sigma} < \varepsilon_{V^*/\sigma} & \text{if } y \leq \check{y}; \\ \varepsilon_{W^*/\sigma} < 0 < \varepsilon_{V^*/\sigma} & \text{if } \check{y} < y < \hat{y}; \\ \varepsilon_{W^*/\sigma} < \varepsilon_{V^*/\sigma} \leq 0 & \text{if } \check{y} \leq y. \end{cases}$$

3 Vertical Restraints

In the baseline model of vertical externality, various contracting options or vertical restraints allow the separated structure to realize the integrated profit. Similar mechanisms apply here, although the interpretation is different because of the underlying dynamic nature of the model. We illustrate them by means of two simple examples.[8]

In Figure 2, the dashed line is the locus of the downstream firm's optimal responses to given upstream prices, $y_D(p_U)$. With the chosen parameters (that is, $\beta = 2$ and $I = \frac{\pi_M}{r-\alpha} = 1$), the separation outcome of Section 2.2 is $(y_D^*, p_U^*) = (4, 2)$. For a given y below the benchmark trigger $y^* = 2$, we may graph the isovalue curves of both firms in the plane (y_i, p_U). The convex curves are the upstream isovalues, whereas the downstream isovalue curves are concave. The ordering of the curves follows from the monotonicity of the value functions V and W in p_U. Because p_U^* maximizes $W(y, p_U)$, the point (y_D^*, p_U^*) lies at a tangency of an upstream isovalue with the locus $y_D(p_U)$. To illustrate, when $y = 1$, say, the firm values in the separated case are $V(1, y_D^*, p_U^*) = \frac{1}{8}$ and $W(1, p_U^*) = \frac{1}{16}$, whereas the integrated value is $V(1, y^*, I) = \frac{1}{4}$.

[8] Note though that certain contractual arrangements, such as a maintenance contract, may not have bearing on the vertical externality. In the present model the distortion in investment timing arises because there is a mark-up on the overall cost of the input. It is independent of how this investment expense is allocated in time.

The two firms can be made better off by reaching a contractual agreement that yields a greater total value than in (1), the equilibrium of the separated case. The most direct value-maximizing contract specifies both the investment trigger and the price. This is analogous to resale price maintenance in the standard vertical framework. For simplicity, assume that the upstream manufacturer chooses the contract, and proposes it to the other firm, at any given $y \leq y^*$. The contract proposal must satisfy the constraints that the downstream buyer earns no less than in the separation outcome with no vertical restraint. Should the upstream firm's current offer be rejected by the downstream firm, the upstream firm could not credibly commit *not* to sell the specific input at p_U^* at a future date when the trigger y_D^* is reached. It follows that the downstream firm's reservation value is $V\left(y, y_D^*, p_U^*\right)$. The upstream firm can appropriate all benefits on top of the latter downstream reservation level by dictating the trigger y^*, so that the total industry value is maximized, before charging the price for which the downstream participation constraint is exactly satisfied.

Formally, for any $y \leq y^*$, and by slightly abusing notation[9] to introduce y_i as an argument of the function W, the upstream firm's problem is:

$$\max_{y_i, p_U} W\left(y, y_i, p_U\right)$$

$$\text{s.t.} \qquad p_U \leq \overline{p}_U\left(y_i\right), \tag{11a}$$

$$p_U \geq \underline{p}_U\left(y_i\right), \tag{11b}$$

where $\underline{p}_U\left(y_i\right)$ is defined by $W\left(y, y_D^*, p_U^*\right) = W\left(y, y_i, \underline{p}_U\left(y_i\right)\right)$, and $\overline{p}_U\left(y_i\right)$ by $V\left(y, y_D^*, p_U^*\right) = V\left(y, y_i, \overline{p}_U\left(y_i\right)\right)$. It is clear from Figure 2 that the first constraint is equivalent to $V\left(y, y_i, p_U\right) \geq V\left(y, y_D^*, p_U^*\right)$. Total value maximization implies that $y_i = y^*$, and the upstream supplier maximizes its share of total value by charging $\overline{p}_U\left(y^*\right)$. With the parameter values that we use in our example, the two participation constraints reduce to $\frac{5}{4} \leq p_U \leq \frac{3}{2}$, and the input supplier chooses $\overline{p}_U\left(2\right) = \frac{3}{2}$.[10]

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[9]In this example, we define $W\left(y, y_i, p_U\right) = \left(\frac{y}{y_i}\right)^{\beta}\left(p_U - I\right)$, all $y \leq y_i$.

[10]The constraints (11a-11b) are compatible whenever $\underline{p}_U\left(y_i\right) \leq \overline{p}_U\left(y_i\right)$, which always holds if $y_i = y^* < y_D^*$. This results from continuity of $V\left(y, y_i, p_U\right)$ and $W\left(y, y_i, p_U\right)$ in y_i and p_U, together with $V\left(y, y_i, p_U\right)$ being monotone increasing in y_i on $\left[y^*, y_D^*\right]$, and decreasing in p_U, whereas $W\left(y, y_i, p_U\right)$ is monotone decreasing in y_i, and increasing in p_U. Obviously, the participation constraint of the input supplier, who writes the contract, can be omitted. However, when the bargaining power is more evenly distributed among the parties, the price p_U can be chosen anywhere in the interval $\left[\underline{p}_U\left(y^*\right), \overline{p}_U\left(y^*\right)\right]$.

Proposition 4 *Suppose that* $y \leq y^*$*. In a contract analogous to resale price maintenance, the upstream firm chooses the investment trigger* y^* *and charges the input price* $\overline{p}_U(y^*)$*, as defined by* $V(y, y_D^*, p_U^*) = V(y, y^*, \overline{p}_U(y^*))$*. The downstream value is the same as in the separation outcome, and the upstream value is* $W(y, y^*, \overline{p}_U(y^*)) > W(y, y_D^*, p_U^*)$*.*

One caveat is that the implementation of this contract requires the upstream firm to continuously monitor y until the market size reaches y^*, which may be costly. This was not needed in the separated case, with no vertical restraint. Nor is it clear that y^* is an easily verifiable contract provision, or that such contracts are used in practice.

A contractual alternative is to set the equivalent of a two-part tariff. In this case, for any $y \leq y^*$, the integrated value is realized by means of an up-front option offered to the downstream firm on the specific input at an exercise price, p_U.[11] We know from Section 2.2 that the input buyer maximizes its private value by exercising the option when Y_t reaches the barrier $y_D(p_U)$. As in the previous contractual example, the objective of the upstream supplier is to induce the choice of the efficient investment trigger by the input buyer, and to appropriate the value in excess of the downstream reservation level $V(y, y_D^*, p_U^*)$. This can be done through a transfer payment, t_U, made at y, which we interpret here as the option premium. This contract also corresponds to a non-refundable deposit on the specific input.

The upstream problem is then:

$$\max_{p_U, t_U} W(y, p_U) + t_U$$

$$\text{s.t.} \quad t_U \leq V(y, y_D(p_U), p_U) - V(y, y_D^*, p_U^*), \tag{12a}$$

$$t_U \geq W(y, p_U^*) - W(y, p_U). \tag{12b}$$

With the joint-value maximizing input price $p_U^* = I$, the downstream firm chooses to invest when $Y_t = y_D(I) = y^*$. The transfer payment is chosen under the condition that the downstream firm's participation constraint is exactly satisfied.[12] With the same parameter values as in Figure

[11] Airlines typically buy options to purchase planes conditioned on air traffic volumes.

[12] The constraints (12a-12b) are compatible whenever $V(y, y_D(p_U), p_U) + W(y, p_U) \geq V(y, y_D^*, p_U^*) + W(y_D^*, p_U^*)$, which always holds if $p_U = I$. As in the case of the previous vertical restraint, the participation constraint of the upstream firm, which writes the contract, can be omitted. We keep it here in order to describe the range of possible transfer payments when the bargaining power is less asymmetrically distributed.

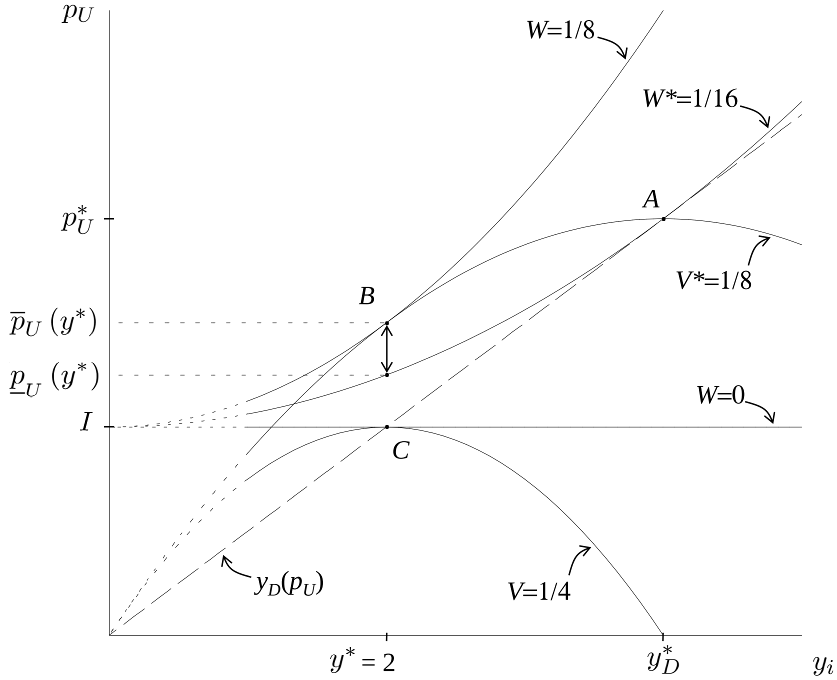

Figure 2: Upstream and downstream isovalues ($\beta = 2$, $y = I = \frac{\pi_M}{r-\alpha} = 1$). Point A describes the separated equilibrium, the upstream firm charges $p_U^* = 2$, and the downstream firm enters at $y_D^* = 4$. Points B and C describe joint-value maximizing contracts, as chosen by the upstream firm under the constraint that the downstream firm earns no less than $V^* = \frac{1}{8}$. In both contracts the upstream firm chooses the investment level $y^* = 2$. In B, the input price is $\frac{3}{2}$, and in C it is I, so that the upstream supplier takes no margin. In the latter case the supplier can sell an up-front option with strike I and specify a transfer payment $t_U^* = \frac{1}{8}$, resulting in a downstream value $V\,(1, y^*, I)$ equal to the reservation level $V^* = \frac{1}{8}$.

2, (12a) and (12b) reduce to $\frac{1}{16} \leq t_U \leq \frac{1}{8}$, hence the upstream firm chooses $t_U^*(1) = \frac{1}{8}$, as paid by the downstream firm when the market size is $y = 1$.

Proposition 5 *Suppose that* $y \leq y^*$*. In a contract analogous to a two-part tariff, the upstream firm charges the price* I*, and chooses the transfer* $t_U^*(y) = V(y, y^*, I) - V(y, y_D^*, p_U^*)$*. The downstream value is the same as in the separation outcome, and the upstream value is* $W(y, I) + t_U^*(y) > W(y, p_U^*)$*.*

This kind of contract imposes a smaller informational requirement on the upstream firm, although it does require it to have an estimate of the current market size y at the date at which the option is written.

If the contracting alternatives, as described in this section, are not available to the upstream firm, the presence of a downstream firm may act as a substitute. As it results in earlier investment, the race to preempt downstream counteracts the "double marginalization" distortion, at least for the first firm that invest. It leads downstream firms to delay investments when faced with a single price for the input. This point is taken up in the next section.

4 Downstream Duopoly

In this section the structural assumptions are those of Section 2, except that on the intermediate market the upstream firm faces two downstream buyers, that also compete on the final market. We build on the analysis of Fudenberg and Tirole [6] (preemption), Boyer, Lasserre, and Moreaux [1], Grenadier [7], and Mason and Weeds [13] (preemption under uncertainty).[13] Now Y_t describes an industrywide shock, so that the flow profits are $Y_t \pi_M$ (monopoly) if a single firm has entered the final market, and $Y_t \pi_D$ (duopoly profits) if both firms have invested, with $\pi_D < \pi_M$.

The upstream firm is constrained to a single instrument, that is the spot price of the specific input, but it may charge different prices at different dates (intertemporal price discrimination). It may thus condition the spot price on the information it receives regarding the demand of downstream firms (in particular, how many firms are present). In what follows p_{U_L} denotes the spot price charged to the first firm to invest (the "leader"), and p_{U_F} denotes the spot price for the second firm (the "follower"). We also assume that the upstream supplier cannot make commitments at one date regarding prices at some future date.

In the absence of strong positive technological externalities at the downstream stage (if total duopoly profits are lower than monopoly profits), the integrated optimum from the industry's

[13] A comprehensive discussion of these contributions can be found in Chevalier-Roignant and Trigeorgis [3].

viewpoint is for a single downstream firm to be active. However, in the separated case, and in the absence of sufficient other instruments, we have seen that the downstream firm invests too late. Therefore the upstream firm may find it profitable to allow a second firm into the market. Then the race to preempt downstream, as it typically results in earlier investment, can counteract the "double marginalization" distortion, and thereby functions as a substitute for the vertical restraints examined in Section 3.[14]

4.1 Equilibrium

The underlying strategies of the downstream firms are the "simple" mixed strategies defined in Fudenberg and Tirole [6],[15] which consist of (augmented) distributions of investment thresholds, conditional on the number of downstream firms to have already invested. In order to determine the equilibrium, it suffices to determine two investment triggers, y_P and y_F, at which the leader and the follower invest, respectively. In equilibrium, the identity of the leader and follower are indeterminate, in that either firm effectively invests first, with equal probability. The latter trigger results from standard arguments: once the leader has invested, the subgame between the upstream firm and the follower is identical to that in Section 2.2.

In what follows, when the current market size is y, the value of a follower that invests at a threshold y_F and pays a price p_U is:

$$F\left(y, y_F, p_{U_F}\right) = \left(\frac{y}{y_F}\right)^{\beta}\left(\frac{\pi_D}{r-\alpha}y_F - p_{U_F}\right). \tag{13}$$

By the same arguments as in Section 2.2, the optimal second spot price for the upstream firm is $p^*_{U_F} = \gamma I$, and the optimal follower investment threshold is $y^*_F = \gamma^2 \frac{r-\alpha}{\pi_D} I$. Compared with the case where the specific input is produced internally (Boyer, Lasserre, and Moreaux [1], Mason and Weeds [13]), the follower invests at a level of y that is γ times higher, and has lower value by a factor of $\gamma^{1-\beta}$.

Remark 1 $F\left(y, y^*_F, p^*_{U_F}\right)$ *does not depend on* (p_{U_L}, y_P).

[14] If an exclusive dealing clause is allowed, an upstream firm that is able to implement resale price maintenance or price the downstream option contract can potentially use the threat of downstream duopoly, altering the terms discussed in Section 3: it offers the downstream firm exclusivity but benefits from a lower reservation value which corresponds to the ex-ante downstream value in a preemption equilibrium, as displayed in (23).

[15] See also Huisman, Kort, and Thijssen [10].

Indeed, what the firm takes into account when it chooses an investment trigger, as a follower, is the profit flow it may expect in the future. This flow is *not* impacted by the investment cost of the leader, nor by its exact investment date.

To determine the preemption threshold y_P, given p_{U_L}, it is necessary to refer to the value of a firm that invests immediately at the current market size y, given that its rival invests optimally as a follower. Let $L(y, p_{U_L})$ denote this value, which has a different form from the $V(\cdot)$ expressions is the rest of the paper:

$$L(y, p_{U_L}) = \frac{\pi_M}{r-\alpha} y - p_{U_L} - \left(\frac{y}{y_F^*}\right)^\beta \frac{\pi_M - \pi_D}{r-\alpha} y_F^*. \tag{14}$$

Although this function is commonly used in preemption models, it is also useful to consider a more general expression of (14), that is $\tilde{L}(y, y_L, y_F^*, p_{U_L}) = \left(\frac{y}{y_L}\right)^\beta \left(\frac{\pi_M}{r-\alpha} y_L - p_{U_L}\right) - \left(\frac{y}{y_F}\right)^\beta \frac{\pi_M - \pi_D}{r-\alpha} y_F$. The function $\tilde{L}(y, y_L, y_F^*, p_{U_L})$ measures the value, at the current market size y, of a firm that is free to invest at y_L as a leader.[16] We have $L(y, p_{U_L}) = \tilde{L}(y, y_L, y_F, p_{U_L})$ when the constraint $y_L \equiv y$ is imposed, and $y_F = y_F^*$.

Remark 2 $\arg\max_{y_L} \tilde{L}(y, y_L, y_F^*, I) = \{y^*\}$.

In other words, when it incurs the "true" cost of investment $p_{U_L} = I$, a firm that is free to choose y_L invests at the same date as in the integrated case (with a single firm). This is another illustration of the "myopic" behavior as coined by Leahy [12].

The analysis of the investment game based on the functions (13) and (14) closely follows that of existing models. The threshold y_P, which is defined by $L(y_P(p_{U_L}), p_{U_L}) = F(y_P\left(p_{U_L}), y_F^*, p_{U_F}^*\right)$, is a function of p_{U_L}. We define $y_P^* \equiv y_P(p_{U_L}^*)$, where $p_{U_L}^*$ denotes the upstream supplier's value-maximizing price. We find:

Proposition 6 *In the separated case with two downstream firms, there is a unique equilibrium characterized by:*

$$\text{(i) downstream triggers} \quad : \quad y_P^* = \gamma \frac{r-\alpha}{\pi_M} I, \qquad y_F^* = \gamma^2 \frac{r-\alpha}{\pi_D} I, \tag{15}$$

$$\text{(ii) upstream prices} \quad : \quad p_{U_L}^* = \left(1 - \Gamma\left(\beta, \frac{\pi_M}{\pi_D}\right)\right)\gamma I, \qquad p_{U_F}^* = \gamma I, \tag{16}$$

with $\Gamma\left(\beta, \frac{\pi_M}{\pi_D}\right) \equiv \left(\gamma \frac{\pi_M}{\pi_D}\right)^{1-\beta} - \left(\gamma \frac{\pi_M}{\pi_D}\right)^{-\beta} \in \left(0, \frac{1}{\beta}\right)$, $\frac{\mathrm{d}\Gamma\left(\beta, \frac{\pi_M}{\pi_D}\right)}{\mathrm{d}\beta} < 0$, *and* $\frac{\mathrm{d}\Gamma\left(\beta, \frac{\pi_M}{\pi_D}\right)}{\mathrm{d}\frac{\pi_M}{\pi_D}} < 0$.

[16] See Reinganum [15].

The intuition for this result is that, in a preemption equilibrium, rent equalization implies that, for any investment cost chosen by the upstream, including $p_{U_L} = I$, the leader's value is pegged on the follower payoff, that is $F\left(y, y_F^*, p_{U_F}^*\right)$, which does not depend on p_{U_L} (Remark 1). By raising its price p_{U_L} above I, the upstream firm increases the cost of leading the sequence of investments, and thereby shifts out the preemption equilibrium trigger $y_P\left(p_{U_L}\right)$. It also appropriates any additional monetary gain on top of the constant share $\tilde{L}\left(y, y_P\left(p_{U_L}\right), y_F^*, p_{U_L}\right) = F\left(y, y_F^*, p_{U_F}^*\right)$ retained by the downstream leader. Therefore, the supplier's value-maximizing strategy is to set y_P equal to the investment trigger y^*, as this trigger maximizes the joint value of the two vertically related units. This is the same investment trigger as the one chosen by the leader when it incurs the "true" cost I (Remark 2).

4.2 Rankings and Comparative Statics

The comparison of investment thresholds and input prices, across the single-firm and two-firm scenarios, and the integrated case, follows directly from the expressions in (5) and (15). The comparative statics are also similar in nature to those of Section 2.

Proposition 7 *In a preemption equilibrium, downstream triggers and upstream prices satisfy the following rankings:*

$$y_P^* = y^* < y_D^* < y_F^* \text{ and } I < p_{U_L}^* < p_{U_F}^* = p_U^*. \tag{17}$$

Moreover, a higher market growth rate, a lower interest rate, and lower volatility, result in higher triggers $\{y_P^*, y_F^*\}$ *and higher downstream prices* $\left\{p_{U_L}^*, p_{U_F}^*\right\}$ *with:*

$$\varepsilon_{p_{U_F}^*/\beta} < \varepsilon_{p_{U_L}^*/\beta} < 0. \tag{18}$$

In the downstream duopoly case, the upstream supplier induces an investment threshold for the first firm, via the price p_{U_L}, that is identical to the investment threshold in the integrated case (2.1), that is y^*, analogously to a "no discrimination at the top" result. The threat of preemption among downstream firms thus has the effect of a vertical restraint, insofar as it induces investment at the correct trigger for the first firm. The race to be first exactly counterbalances the incentive that the leader would otherwise have to delay, if its investment date resulted from the optimization of an investment threshold. This substitute for vertical restraint does not represent an industry first-best however if, as it may be assumed, the presence of a second firm reduces industry profit ($\pi_D < \pi_M/2$).

To illustrate, in Figure 3 the two solid curves refer to the separated case. The quasi concave one represents $\tilde{L}\left(y, y_L, y_F^*, p_{U_L}^*\right)$, that is the value of the leader as a function of y_L, and measured at a given y (specifically, $y = 1$) provided that the follower invests at the optimal threshold y_F^*, and for an upstream value-maximizing price $p_{U_L}^*$ (with $\beta = 2$, $I = r - \alpha = \pi_M = 2\pi_D = 1$). The other solid curve is a graph of $F\left(y, y_F, p_{U_F}^*\right)$, which has the same expression as in (13). Note that, when $y = y_F^*$, the leader value is higher since $p_{U_L}^* = \frac{13}{8} < 2 = p_{U_F}^*$. The preemption threshold y_P^* is determined by the condition that firms are indifferent at that point between investing as a leader or waiting to invest as a follower. In this figure, the dashed curve represents the upstream firm's optimization problem. It describes the reference (or "true") leader value, based on the actual investment cost I (i.e., $p_{U_L} = I$) for all possible $y_L \leq y_F^*$, and for $y_F = y_F^*$ (i.e., $p_{U_F} = p_{U_F}^* > I$). This is the graph of $\tilde{L}(y, y_L, y_F^*, I)$, which reaches a maximum when $y_L = y^*$.

We also represent the separated case with two integrated downstream firms (i.e., $p_{U_L} = p_{U_F} = I$). We know from existing models with similar specifications (e.g., Boyer, Lasserre, and Moreaux [1], Mason and Weeds [13]) that, in that case, in a preemption equilibrium the leader invests first at $\tilde{y}_P$, which is strictly less than $y^* = \gamma\frac{r-\alpha}{\pi_M}I$, as computed in Section 2.1. Then the follower invests at $\tilde{y}_F = \gamma\frac{r-\alpha}{\pi_D}I$. In Figure 3, the two dotted curves represent $\tilde{L}(y, y_L, \tilde{y}_F, I)$ and $F(y, y_F, I)$. It is straightforward to check that $\tilde{y}_P < y_P^* < \tilde{y}_F < y_F^*$.[17]

A noteworthy feature of this specification compared with similar real option games is that the solution, in the preemption scenario, is analytic. The closed-form expression of y_P^* facilitates the comparative statics, which are consistent with the interpretation of the model given in Section 2.3. Recall from (7-8) that the parameter β is analogous to an elasticity of demand in the intermediate market, and lower elasticity (greater β) results in greater vertical distortion, with higher prices and triggers. Although these features are robust to the introduction of a second downstream firm, an additional remark is warranted. In the downstream duopoly case, the supplier sells the specific input to the first entrant at a discount $\frac{p_{U_L}^*}{p_{U_F}^*} = 1 - \Gamma\left(\beta, \frac{\pi_M}{\pi_D}\right) < 1$. As observed in Proposition 6, this discount increases with β. The net effect of a greater β on the first spot price, expressed here

[17] The comparison of $\tilde{y}_F$ with y_D^* is less straightforward in that it depends on the ratio $\frac{\pi_D}{\pi_M}$.

in terms of elasticities, is therefore a priori ambiguous. By definition,

$$\varepsilon_{p_{U_L^*}/\beta} = \underbrace{\varepsilon_{p_{U_F^*}/\beta}}_{<0} + \underbrace{\varepsilon_{\left(1-\Gamma\left(\beta,\frac{\pi_M}{\pi_D}\right)\right)/\beta}}_{>0}, \tag{19}$$

that is to say a greater β increases the follower price on the one hand, but also induces the upstream firm to emphasize price discrimination by increasing the spread in prices on the other. The net effect can be shown to be negative, but the comparative static in the preemption game is thus not a direct corollary of the bilateral monopoly case. Therefore, according to Proposition 7, changes in market conditions (as captured by the parameter β) have a greater impact on the follower input price than the leader input price.

Consider now the upstream value in the preemption equilibrium of Proposition 6, that is

$$\tilde{W}\left(y, p_{U_L}^*, p_{U_F}^*\right) = \left(\frac{y}{y_P\left(p_{U_L}^*\right)}\right)^{\beta}\left(p_{U_L}^* - I\right) + \left(\frac{y}{y_F^*}\right)^{\beta}\left(p_{U_F}^* - I\right). \tag{20}$$

This value can be visualized in Figure 3 by reinterpreting each term on the right hand side of the equality sign in (20) as follows. On the one hand, the supplier chooses p_{U_L}, shifting the leader value function, to maximize the difference between the reservation value that must be given to the leader and the reference leader value at the preemption trigger $y_P(p_{U_L})$. By charging exactly $p_{U_L}^* > I$, so that the leader invests at $y^* = y_P^* = y_P\left(p_{U_L}^*\right)$, the supplier appropriates the value differential

$$\tilde{L}\left(y, y^*, y_F^*, I\right) - \tilde{L}\left(y, y^*, y_F^*, p_{U_L}^*\right) = \left(\frac{y}{y_P\left(p_{U_L}^*\right)}\right)^{\beta}\left(p_{U_L}^* - I\right). \tag{21}$$

In addition, the upstream supplier earns the difference between the value that the follower would earn as an integrated firm, and the value it earns as a separate entity, as both measured at $y = y_F^*$, that is when the separated follower invests in the preemption equilibrium. Formally, by charging $p_{U_F}^* > I$, the supplier appropriates

$$F\left(y, y_F^*, I\right) - F\left(y, y_F^*, p_{U_F}^*\right) = \left(\frac{y}{y_F^*}\right)^{\beta}\left(p_{U_F}^* - I\right). \tag{22}$$

The magnitudes (21) and (22) are represented by the vertical arrows in Figure 3.

With respect to firm values, intuition suggests that, in a preemption equilibrium, a downstream firm is worse off, and the upstream firm is better off, than under bilateral monopoly since there

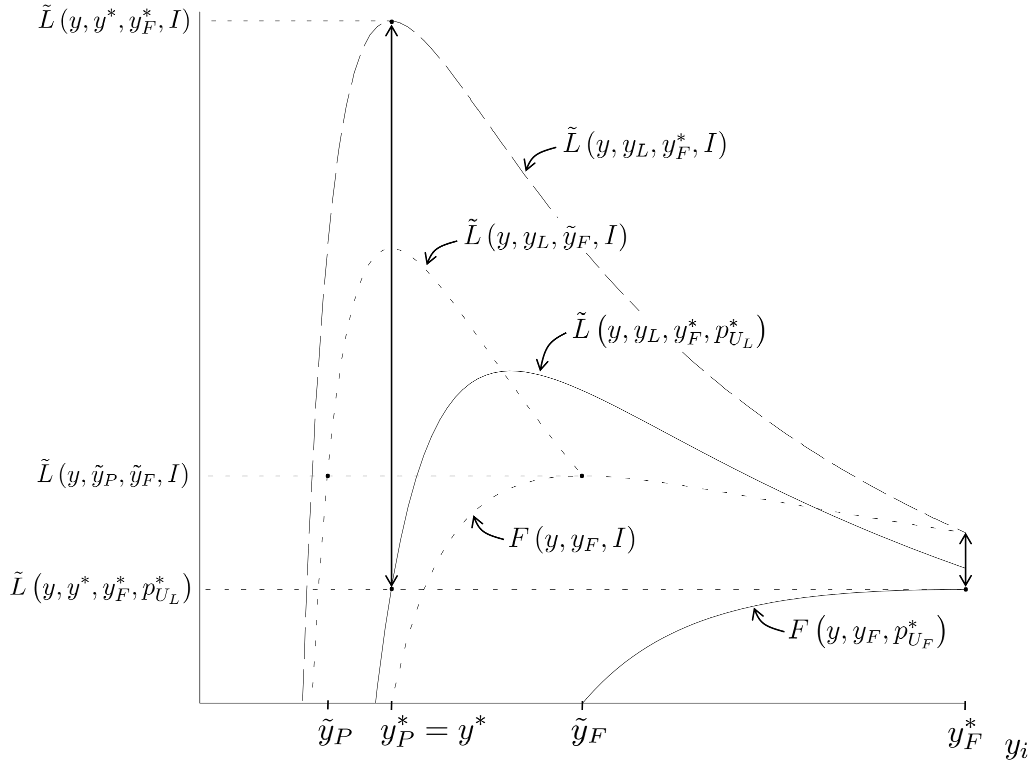

Figure 3: Leader and follower values at current market size $y = 1$ as a function of investment trigger y_i (with $\beta = 2$, $I = r - \alpha = \pi_M = 2\pi_D = 1$). The preemption trigger $y_P^* = y^*$ in the separated case maximizes the integrated leader value (that is, the leader value if $p_{U_L} = I$), and is greater than the trigger under preemption when downstream firms face the true investment cost. By charging $p_{U_L}^* > I$, the upstream firm appropriates the value differential $\tilde{L}(y, y^*, y_F^*, I) - \tilde{L}\left(y, y^*, y_F^*, p_{U_L}^*\right)$. By charging $p_{U_F}^* > I$, it also earns the difference $F(y, y_F^*, I) - F\left(y, y_F^*, p_{U_F}^*\right)$.

is competition downstream. The first of these comparisons is not immediate to verify, since the first downstream firm invests at a lower threshold $y_P^* < y_D^*$, but also faces a lower input price $p_{U_L}^* < p_U^*$. The closed-form expression of y_P^* is useful to resolve this ambiguity, as it allows us to exactly evaluate the downstream firm value, which we denote by $\tilde{V}\left(y, p_{U_L}^*, p_{U_F}^*\right)$, for all market sizes $y < y_P^*$. This value is given by:

$$\tilde{V}\left(y, p_{U_L}^*, p_{U_F}^*\right) = \frac{1}{2}\left(\frac{y}{y_P^*}\right)^{\beta}\left(\frac{\pi_M}{r-\alpha}y_P^* - p_{U_L}^*\right) + \left(\frac{y}{y_F^*}\right)^{\beta}\left(\frac{\pi_D - \frac{1}{2}\pi_M}{r-\alpha}y_F^* - \frac{1}{2}p_{U_L}^*\right). \quad (23)$$

The expression (23) reflects the fact that, ex-ante, a firm is equally likely to be a leader or a follower under preemption. The comparison of the equilibrium value of the upstream firm $\tilde{W}\left(y, p_{U_L}^*, p_{U_F}^*\right)$ when there are two downstream buyers, with $W\left(y, p_U^*\right)$, is not straightforward either. Although the upstream firm sells its input twice, the first sale is discounted and the second occurs at a further removed date. Industry value increases under duopoly only conditionally, since although the investment threshold of the first firm is more efficient, industry flow profits decrease with the investment of the second firm. Finally, the comparative statics of $\tilde{V}\left(y, p_{U_L}^*, p_{U_F}^*\right)$ and $\tilde{W}\left(y, p_{U_L}^*, p_{U_F}^*\right)$, which involve additional effects from those of $V\left(y, y_D^*, p_U^*\right)$ and $W\left(y, p_U^*\right)$, do not appear to have straightforward characterizations and are not explored further.[18] These results are summarized in the following proposition.

Proposition 8 *For all $y \leq y_P^*$, for all β and all $\pi_D < \pi_M$, downstream value is lower and upstream value is higher in a preemption equilibrium than under bilateral monopoly:*

$$\tilde{V}\left(y, p_{U_L}^*, p_{U_F}^*\right) < V\left(y, y_D^*, p_U^*\right) \text{ and } W\left(y, p_U^*\right) < \tilde{W}\left(y, p_{U_L}^*, p_{U_F}^*\right). \quad (24)$$

Moreover, for large enough $\left\{\beta, \frac{\pi_M}{\pi_D}\right\}$, total industry value is greater in a preemption equilibrium than under bilateral monopoly.

5 Discussion

In this paper, we have studied investment timing when firms depend on an outside supplier to provide a discrete input (e.g., a key equipment), developing a dynamic version of a heretofore

[18]See Section 6.8 in the Appendix: β may have either a monotone, or an ambiguous effect on total industry value, for instance.

static model. The upstream firm's mark-up depends on the stochastic process followed by downstream flow profits. A vertical externality arises because the upstream firm's pricing induces the downstream firm to delay the exercise of its investment option. This distortion, akin to a Lerner index, increases with both market growth and volatility. Downstream firm values are more sensitive to these parameters than upstream values, and in contrast with the standard real option framework, greater volatility decreases firm value near the exercise threshold. If the input supplier has sufficient information regarding downstream demand, it can induce optimal investment timing by means of standard vertical restraints. Otherwise, the upstream supplier benefits from the presence of a second downstream firm, which results in a preemption race and acts as a perfect substitute for vertical restraints. The input is then sold to the downstream leader at a discount which increases with volatility, the leader's price is less sensitive to market growth and volatility than the follower's, and the leader invests at the optimal threshold, resulting in greater industry profits when growth and volatility are sufficiently low.

In the first place, a strong qualitative prediction of the model is that, under duopoly, the first input is sold at a discount. In practice, we would expect that learning effects which decrease the upstream firm's production cost for the second input supplied, should reduce the apparent discount that is offered to the leader.

Second, with uncertainty, it is conceivable that the upstream and downstream firms have different costs of capital, and therefore different discounting terms β_U and β_D. In that case, the optimal preemption threshold induced by the upstream firm no longer has a closed form expression. The first-order condition has an additional term whose sign is given by $(\beta_U - \beta_D)$. If the upstream firm better diversifies its risk and thus has a lower cost of capital, a lower preemption threshold results.

Third, the assumption of a geometric Brownian motion for the stochastic market process can be relaxed. For example, Poisson jumps may be included to allow for greater risk. Many of the results here are unchanged, with the exception of those comparative static results that rely on the analytic expression of β.

Fourth, one may allow for upstream competition. If suppliers compete in prices and there is a single upstream firm, then the integrated optimum is restored. On the other hand, in an industry with two upstream and two downstream firms, upstream competition presumably results in a standard preemption race downstream, and the leader enters too early.

Finally, the analysis of this paper has adhered to the classical assumption that the contract terms are decided by the upstream firm. However, one could envisage that it is the downstream firms that have market power in the input market, and therefore write the contract, or alternatively that downstream firms may use some other device, such as the threat of reversion to a tacit collusion equilibrium if such exists.

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6 Appendix

6.1 Sensitivity of β to α and σ

The derivatives of β with respect to the growth and volatility parameters α and σ arise throughout the paper, and have the following expressions:

$$\frac{\mathrm{d}\beta}{\mathrm{d}\alpha} = \frac{-\beta}{\left(\beta - \left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)\right)\sigma^2} < 0, \tag{25}$$

$$\frac{\mathrm{d}\beta}{\mathrm{d}\sigma} = \frac{-2\left(r - \alpha\beta\right)}{\left(\beta - \left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)\right)\sigma^3} \leq 0. \tag{26}$$

With respect to the signs of these expressions, note that by assumption $\frac{1}{2} - \frac{\alpha}{\sigma^2} < 0$, and $r \geq \alpha\beta$, with equality only if $\sigma = 0$.

6.2 Sensitivity of y_D^* and p_U^* to α and σ

$$\begin{aligned}
\frac{\mathrm{d}y_D^*}{\mathrm{d}\alpha} &= \frac{\partial y_D^*}{\partial \beta}\frac{\mathrm{d}\beta}{\mathrm{d}\alpha} + \frac{\partial y_D^*}{\partial \alpha} = \gamma^2 \frac{\left(\sigma^2 + 2\alpha\right) I}{\left(\sigma^2 \left(2\beta - 1\right) + 2\alpha\right)} > 0 \\
\frac{\mathrm{d}y_D^*}{\mathrm{d}\sigma} &= \frac{\partial y_D^*}{\partial \beta}\frac{\mathrm{d}\beta}{\mathrm{d}\sigma} = -2y^* \left(\frac{\gamma}{\beta}\right)^2 \frac{\mathrm{d}\beta}{\mathrm{d}\sigma} > 0 \\
\frac{\mathrm{d}p_U^*}{\mathrm{d}\alpha} &= \frac{\partial p_U^*}{\partial \beta}\frac{\mathrm{d}\beta}{\mathrm{d}\alpha} = -I \left(\frac{\gamma}{\beta}\right)^2 \frac{\mathrm{d}\beta}{\mathrm{d}\alpha} > 0 \\
\frac{\mathrm{d}p_U^*}{\mathrm{d}\sigma} &= \frac{\partial p_U^*}{\partial \beta}\frac{\mathrm{d}\beta}{\mathrm{d}\alpha} = -I \left(\frac{\gamma}{\beta}\right)^2 \frac{\mathrm{d}\beta}{\mathrm{d}\sigma} > 0
\end{aligned}$$

6.3 Behavior of $\Delta\left(\beta\right)$

The expression $\Delta(\beta) = \left(\frac{\beta}{\beta-1}\right)^{-\beta}\left(1 + \frac{\beta}{\beta-1}\right)$ arises several times throughout the paper. To establish that $\Delta(1) = 1$, rewrite it as $\Delta(\beta) = \left(\frac{\beta}{\beta-1}\right)^{-\beta} + \left(\frac{\beta}{\beta-1}\right)^{1-\beta}$. Since $\lim_{\beta\to 1}\left(\beta - 1\right)^{\beta-1} = 1$, $\Delta(1) = 1$. To establish that $\lim_{\beta\to\infty}\Delta(\beta) = \frac{2}{\mathrm{e}}$, note that for the denominator, $\left(\frac{\beta}{\beta-1}\right)^{\beta} = \left(1 + \frac{1}{\beta-1}\right)^{\beta}$, and $\lim_{\beta\to\infty}\left(1 + \frac{1}{\beta-1}\right)^{\beta} = \lim_{z\to\infty}\left(1 + \frac{1}{z}\right)^{z}\left(1 + \frac{1}{z}\right) = \mathrm{e}$.

Finally, $\Delta'(\beta) = \frac{(\beta-1)^{\beta-1}}{\beta^{\beta}}\left(2 - (2\beta - 1)\ln\frac{\beta}{\beta-1}\right)$. Define $f_1\left(\beta\right) \equiv \frac{2}{2\beta-1} - \ln\frac{\beta}{\beta-1}$. Then, $\lim_{\beta\to\infty} f_1\left(\beta\right) = 0$, and $f_1'\left(\beta\right) = -\frac{4}{(2\beta-1)^2} + \frac{1}{\beta(\beta-1)} > 0$, so $f_1\left(\beta\right) < 0$, hence $\Delta'(\beta) < 0$. ■

6.4 Proof of Proposition 3

The effect of a change in σ on V^* and W^* follows directly from the expressions displayed in the main body of the paper, so that we need focusing on α only. We first sign $\frac{\mathrm{d}V^*}{\mathrm{d}\alpha}$. After simplification, computations yield, for all $y \leq y_D^*$:

$$\frac{\mathrm{d}V\left(y, y_D^*\left(p_U^*\right), p_U^*\right)}{\mathrm{d}\alpha} = \left(\frac{\beta}{r-\alpha} + \left(\frac{1}{\beta} + \ln\frac{y}{y_D^*}\right)\frac{\mathrm{d}\beta}{\mathrm{d}\alpha}\right) V\left(y, y_D^*, p_U^*\right). \tag{27}$$

Since $\left(\ln\frac{y}{y_D^*}\right)\left(\frac{\mathrm{d}\beta}{\mathrm{d}\alpha}\right) \geq 0$,

$$\frac{\mathrm{d}V\left(y, y_D^*\left(p_U^*\right), p_U^*\right)}{\mathrm{d}\alpha} \geq \left(\frac{\beta}{r-\alpha} + \frac{1}{\beta}\frac{\mathrm{d}\beta}{\mathrm{d}\alpha}\right) V\left(y, y_D^*, p_U^*\right). \tag{28}$$

Substituting for $\frac{\mathrm{d}\beta}{\mathrm{d}\alpha}$, $\frac{\mathrm{d}V\left(y,y_D^*\left(p_U^*\right),p_U^*\right)}{\mathrm{d}\alpha} > 0$ if $f_2(\alpha,r,\sigma) \equiv \beta\left(\beta - \left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)\right)\sigma^2 - (r-\alpha) > 0$. When $\alpha = r \equiv z$, $\beta = 1$, and therefore $f_2(z,z,\sigma) = \frac{1}{2} + \frac{z}{\sigma^2} > 0$. Moreover, $\frac{\mathrm{d}f_2(\alpha,r,\sigma)}{\mathrm{d}r} = \beta\left(\beta - \left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)\right)^{-1} > 0$. Therefore, $\frac{\mathrm{d}V\left(y,y_D^*\left(p_U^*\right),p_U^*\right)}{\mathrm{d}\alpha} > 0$ for all admissible parameter values.

Next, we sign $\frac{\mathrm{d}W^*}{\mathrm{d}\alpha}$. Similarly, after simplification, computations yield, for all $y \leq y_D^*$,

$$\frac{\mathrm{d}W\left(y,p_U^*\right)}{\mathrm{d}\alpha} = \left(\frac{\beta}{r-\alpha} + \left(\frac{1}{\beta-1} + \ln\frac{y}{y_D^*}\right)\frac{\mathrm{d}\beta}{\mathrm{d}\alpha}\right)W\left(y,p_U^*\right). \tag{29}$$

Again $\left(\ln\frac{y}{y_D^*}\right)\left(\frac{\mathrm{d}\beta}{\mathrm{d}\alpha}\right) \geq 0$ implies that

$$\frac{\mathrm{d}W\left(y,p_U^*\right)}{\mathrm{d}\alpha} \geq \left(\frac{\beta}{r-\alpha} + \frac{1}{\beta-1}\frac{\mathrm{d}\beta}{\mathrm{d}\alpha}\right)W\left(y,p_U^*\right). \tag{30}$$

Thus, $\frac{\mathrm{d}W\left(y,p_U^*\right)}{\mathrm{d}\alpha} > 0$ if $f_3(\alpha,r,\sigma) \equiv (\beta-1)\left(\beta - \left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)\right)\sigma^2 - (r-\alpha) > 0$. Taking $\alpha = r \equiv z$, $f_3(z,z,\sigma) = 0$, and $\frac{\mathrm{d}f_3(\alpha,r,\sigma)}{\mathrm{d}r} = (\beta-1)\left(\beta - \left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)\right)^{-1} > 0$. Therefore, $\frac{\mathrm{d}W\left(y,p_U^*\right)}{\mathrm{d}\alpha} > 0$ for all admissible parameter values.

It remains to rank $\varepsilon_{V^*/\alpha}$ and $\varepsilon_{W^*/\alpha}$. A simple reorganization of terms in (27) and (29), together with $\frac{1}{\beta} < \frac{1}{\beta-1}$, directly leads to $\varepsilon_{W^*/\alpha} < \varepsilon_{V^*/\alpha}$. ■

6.5 Proof of Proposition 6

The optimal follower investment threshold y_F^* and second spot price $p_{U_F}^*$ having been discussed in the text, only the first spot price $p_{U_L}^*$ and the preemption threshold y_P^* remain to be established.

First, we determine the upstream firm's strategy space to be of the form $[0,\overline{p}]$. The value function $F\left(y,y_F^*,p_{U_F}^*\right)$ is monotone increasing and convex and $L\left(y,p_{U_L}\right)$ is monotone increasing and concave in y. The choice of p_{U_L} is bounded because the equation $L\left(y,p_{U_L}\right) = F\left(y,y_F^*,p_{U_F}^*\right)$ must have at least one root in y, given p_{U_L}, for downstream firms to wish to invest first in the market. Setting $\frac{\mathrm{d}L\left(y,p_{U_L}\right)}{\mathrm{d}y} = \frac{\mathrm{d}F\left(y,y_F^*,p_{U_F}^*\right)}{\mathrm{d}y}$, we find that tangency occurs at $\overline{y} = \left(\frac{\pi_M}{\beta\pi_M - (\beta-1)\pi_D}\right)^{\frac{1}{\beta-1}} y_F^*$. Note that $\overline{y} < y_F^*$ so long as $\pi_D < \pi_M$. Then, $\overline{p}$ is defined implicitly by $L\left(\overline{y},\overline{p}\right) = F\left(\overline{y},y_F^*,p_{U_F}^*\right)$.

Second, for $p_{U_L} \in [0,\overline{p}]$, a preemption equilibrium exists at the threshold $y_P\left(p_{U_L}\right)$ that verifies $L\left(y_P\left(p_{U_L}\right),p_{U_L}\right) = F\left(y_P\left(p_{U_L}\right),y_F^*,p_{U_F}^*\right)$. Specifically, $y_P\left(p_{U_L}\right)$ is defined implicitly by:

$$\frac{\pi_M}{r-\alpha}y_P - p_{U_L} - \left(\frac{y_P}{y_F^*}\right)^{\beta}\gamma\left(\gamma\frac{\pi_M}{\pi_D} - 1\right)I = 0. \tag{31}$$

The decision problem of the upstream firm can then be examined. Its value when the current market size is y is:

$$\tilde{W}\left(y, p_{U_L}, p_{U_F}^*\right) = \left(\frac{y}{y_P(p_{U_L})}\right)^\beta (p_{U_L} - I) + \left(\frac{y}{y_F^*}\right)^\beta \left(p_{U_F}^* - I\right). \tag{32}$$

From (31) we obtain an expression of $p_{U_L} - I$ that we plug into (32). This leads to:

$$\tilde{W}\left(y, p_{U_L}, p_{U_F}^*\right) = y^\beta \left(-Iy_P^{-\beta} + \frac{\pi_M}{r-\alpha} y_P^{1-\beta} - \gamma\left(\gamma\frac{\pi_M}{\pi_D} - 1\right) I y_F^{*-\beta}\right) + \left(\frac{y}{y_F^*}\right)^\beta \left(p_{U_F}^* - I\right). \tag{33}$$

Note that:

$$\tilde{W}\left(y, p_{U_L}, p_{U_F}^*\right) = V\left(y, y_P, I\right) + U\left(y, p_{U_F}^*\right), \tag{34}$$

where $U\left(y, p_{U_F}^*\right)$ is independent of y_P, and $V\left(y, y_P, I\right)$ is the integrated payoff (1) of Section 2.1. The upstream firm's decision problem is thus that of the integrated firm, and the first-order condition is satisfied at $y_P^* = \gamma\frac{r-\alpha}{\pi_M} I$. Substituting into (31) gives the optimal first downstream spot price $p_{U_L}^* = \left(1 - \Gamma\left(\beta, \frac{\pi_M}{\pi_D}\right)\right)\gamma I$, with $\Gamma\left(\beta, \frac{\pi_M}{\pi_D}\right) \equiv \left(\gamma\frac{\pi_M}{\pi_D}\right)^{1-\beta} - \left(\gamma\frac{\pi_M}{\pi_D}\right)^{-\beta}$.

Since $\gamma\frac{\pi_M}{\pi_D} > 1$, $\left(\gamma\frac{\pi_M}{\pi_D} - 1\right)\left(\gamma\frac{\pi_M}{\pi_D}\right)^{-\beta} = \Gamma\left(\beta, \frac{\pi_M}{\pi_D}\right) > 0$. Also, $\Gamma\left(\beta, 1\right) = \left(\frac{\beta-1}{\beta}\right)^{\beta-1}\frac{1}{\beta} < \frac{1}{\beta}$ and $\frac{\mathrm{d}\Gamma\left(\beta, \frac{\pi_M}{\pi_D}\right)}{\mathrm{d}\frac{\pi_M}{\pi_D}} = \beta\left(\frac{\pi_D}{\pi_M} - 1\right)\left(\gamma\frac{\pi_M}{\pi_D}\right)^{-\beta} < 0$ since $\pi_D < \pi_M$, so $\Gamma\left(\beta, \frac{\pi_M}{\pi_D}\right) < \frac{1}{\beta}$. The other derivative in the proposition is $\frac{\mathrm{d}\Gamma\left(\beta, \frac{\pi_M}{\pi_D}\right)}{\mathrm{d}\beta} = -\left(\frac{1}{\beta(\beta-1)} + \left(\frac{\pi_D}{\pi_M} - 1\right)\ln\frac{\pi_M}{\pi_D}\right)\left(\gamma\frac{\pi_M}{\pi_D}\right)^{-\beta} < 0$. In addition, it can be verified that $y_P^* < \overline{y}$ if and only if $\Gamma\left(\beta, \frac{\pi_M}{\pi_D}\right) < \frac{1}{\beta}$, so the equilibrium preemption trigger is in the admissible range.

Finally, as $p_{U_L}^*$ and $p_{U_F}^*$ are given under the assumption of price-taking by downstream firms, only the parameters of $L\left(y, p_{U_L}\right)$ and $F\left(y, y_F^*, p_{U_F}^*\right)$ are altered (specifically, the investment cost which is asymmetric for the first and second firm to invest), so Fudenberg and Tirole [6]'s argument applies to establish that downstream firms seek to invest immediately off the equilibrium path if no firm has entered yet when the market size reaches y_P^*. ■

6.6 Proof of Proposition 7

The rankings are direct. For the comparative statics, note that the functional form of the triggers y_P^* and y_F^* is similar to that of y_D^*, and that $p_{U_F}^* = p_U^*$. The calculations are similar to those of the bilateral monopoly case of Section 6.2. Determining that $\frac{\mathrm{d}p_{U_L}^*}{\mathrm{d}\beta} < 0$ is less straightforward. The argument is broken down into four steps.

6.6.1 Step 1: Limit of $\gamma^{\frac{\gamma}{\gamma-1}} - \gamma - \ln\gamma$

The limit at infinity of the expression $\gamma^{\frac{\gamma}{\gamma-1}} - \gamma - \ln\gamma$ arises in the evaluation of the sign of $\frac{\mathrm{d}p^*_{U_L}}{\mathrm{d}\beta}$. This indeterminate form can be evaluated as follows:

$$\begin{aligned}
\lim_{\gamma\to\infty}\left(\gamma^{\frac{\gamma}{\gamma-1}} - \gamma - \ln\gamma\right) &= \lim_{\gamma\to\infty}\left(\gamma\left(\exp\left(\frac{\ln\gamma}{\gamma-1}\right)\right) - \gamma - \ln\gamma\right)\\
&= \lim_{\gamma\to\infty}\left(\gamma\left(1+\frac{\ln\gamma}{\gamma-1}+\sum_{n=2}^{\infty}\frac{1}{n!}\left(\frac{\ln\gamma}{\gamma-1}\right)^n\right) - \gamma - \ln\gamma\right)\\
&= \lim_{\gamma\to\infty}\left(\frac{\ln\gamma}{\gamma-1}+\gamma\sum_{n=2}^{\infty}\frac{1}{n!}\left(\frac{\ln\gamma}{\gamma-1}\right)^n\right)\\
&= \lim_{\gamma\to\infty}\left(\frac{\ln\gamma}{\gamma-1}+\frac{\gamma}{\gamma-1}\sum_{n=2}^{\infty}\frac{1}{n!}\frac{(\ln\gamma)^2}{\gamma-1}\left(\frac{\ln\gamma}{\gamma-1}\right)^{n-2}\right) = 0,
\end{aligned}$$

where the last equality follows from the fact that $\lim_{\gamma\to\infty}\frac{\ln\gamma}{\gamma-1} = \lim_{\gamma\to\infty}\frac{(\ln\gamma)^2}{\gamma-1} = 0$, both by l'Hôpital's rule.

6.6.2 Step 2: Definition of $f_4\left(\beta, \frac{\pi_M}{\pi_D}\right)$

Recalling that $p^*_{U_L} = \left(1-\Gamma\left(\beta,\frac{\pi_M}{\pi_D}\right)\right)\gamma I$, define $z \equiv \gamma\frac{\pi_M}{\pi_D}$ so that $\Gamma\left(\beta,\frac{\pi_M}{\pi_D}\right)$ has the more compact expression $\Gamma\left(\beta,\frac{\pi_M}{\pi_D}\right) = z^{1-\beta} - z^{-\beta}$ in what follows. We wish to show that the derivative $\frac{\mathrm{d}p^*_{U_L}}{\mathrm{d}\beta} = -\gamma\frac{\mathrm{d}\Gamma\left(\beta,\frac{\pi_M}{\pi_D}\right)}{\mathrm{d}\beta}I - \left(1-\Gamma\left(\beta,\frac{\pi_M}{\pi_D}\right)\right)\frac{1}{(\beta-1)^2}I$ is negative. Since $\frac{\mathrm{d}z^{1-\beta}}{\mathrm{d}\beta} = \left(-\ln z + \frac{1}{\beta}\right)z^{1-\beta}$ and $\frac{\mathrm{d}z^{-\beta}}{\mathrm{d}\beta} = \left(-\ln z + \frac{1}{\beta-1}\right)z^{-\beta}$, after rearranging,

$$\frac{\mathrm{d}p^*_{U_L}}{\mathrm{d}\beta} = \left(-z^{\beta} - (\beta-2)\,z + \beta\,(\beta-1)\,(z-1)\ln z + (\beta-1)\right)\frac{z^{-\beta}I}{(\beta-1)^2}. \tag{35}$$

Therefore, $\frac{\mathrm{d}p^*_{U_L}}{\mathrm{d}\beta} < 0$ if and only if $f_4\left(\beta,\frac{\pi_M}{\pi_D}\right) \equiv z^{\beta} + (\beta-2)\,z - \beta\,(\beta-1)\,(z-1)\ln z - (\beta-1) > 0$. The rest of the proof is broken down into two steps. We first show that $f_4(\beta,1)$ is positive, and then that $\frac{\mathrm{d}f_4\left(\beta,\frac{\pi_M}{\pi_D}\right)}{\mathrm{d}\frac{\pi_M}{\pi_D}}$ is positive.

6.6.3 Step 3: $f_4(\beta, 1) > 0$

Evaluating,

$$f_4(\beta, 1) = \left(\frac{\beta}{\beta - 1}\right)^{\beta} - \beta \ln\left(\frac{\beta}{\beta - 1}\right) - \frac{1}{\beta - 1}. \tag{36}$$

First, recalling that $\frac{\beta}{\beta-1} = \gamma$ (so $\beta = \frac{\gamma}{\gamma-1}$),

$$\begin{aligned}\lim_{\beta\to 1} f_4(\beta, 1) &= \lim_{\gamma\to\infty}\left(\gamma^{\frac{\gamma}{\gamma-1}} - \gamma - \frac{\gamma}{\gamma - 1}\ln\gamma + 1\right) \\ &= \lim_{\gamma\to\infty}\left(\gamma^{\frac{\gamma}{\gamma-1}} - \gamma - \ln\gamma\right) + \lim_{\gamma\to\infty}\left(-\frac{\ln\gamma}{\gamma - 1}\right) + 1 \\ &= 1.\end{aligned}$$

Second, we show that $f_4(\beta, 1) = 0 \Rightarrow f_4'(\beta, 1) > 0$. Set $u = \beta \ln\left(\frac{\beta}{\beta-1}\right)$. Then $f_4(\beta, 1) = \mathrm{e}^u - u - \frac{1}{\beta-1}$, and $\frac{\mathrm{d}f_4(\beta,1)}{\mathrm{d}\beta} = u'(\mathrm{e}^u - 1) + \frac{1}{(\beta-1)^2}$, where $u' = \frac{1}{\beta}\left(u - \frac{\beta}{\beta-1}\right)$. Suppose that there exists a value β_0, and hence a value u_0, such that $f_4(\beta_0, 1) = 0$. At β_0, $\mathrm{e}^{u_0} = u_0 + \frac{1}{\beta_0-1}$. Substituting in, $\frac{\mathrm{d}f_4(\beta_0,1)}{\mathrm{d}\beta} = \frac{1}{\beta_0}\left(u_0 - \frac{\beta_0}{\beta_0-1}\right)\left(u_0 + \frac{2-\beta_0}{\beta_0-1}\right) + \frac{1}{(\beta_0-1)^2}$, or after simplification, $\frac{\mathrm{d}f_4(\beta_0,1)}{\mathrm{d}\beta} = \beta_0\left(\ln\frac{\beta_0}{\beta_0-1}\right)^2 - 2\ln\frac{\beta_0}{\beta_0-1} + \frac{1}{\beta_0-1}$. The polynomial in γ, $\beta_0\gamma^2 - 2\gamma + \frac{1}{\beta_0-1}$, has no roots. Therefore, $\frac{\mathrm{d}f_4(\beta_0,1)}{\mathrm{d}\beta} > 0$. Since $f_4(\beta, 1)$ is continuous over $(1, \infty)$, is positive in a neighborhood of 1, and may only cross the horizontal axis from below, $f_4(\beta, 1) > 0$.

6.6.4 Step 4: $\frac{\mathrm{d}f_4\left(\beta, \frac{\pi_M}{\pi_D}\right)}{\mathrm{d}\frac{\pi_M}{\pi_D}} > 0$

Evaluating,

$$\frac{\mathrm{d}f_4\left(\beta, \frac{\pi_M}{\pi_D}\right)}{\mathrm{d}\frac{\pi_M}{\pi_D}} = \frac{\beta}{\beta - 1}\left(\beta z^{\beta-1} - \beta(\beta - 1)\left(\ln z - \frac{1}{z}\right) - \beta^2 + 2\beta - 2\right). \tag{37}$$

Note first that:

$$\begin{aligned}\left.\frac{\mathrm{d}f_4\left(\beta, \frac{\pi_M}{\pi_D}\right)}{\mathrm{d}\frac{\pi_M}{\pi_D}}\right|_{\frac{\pi_M}{\pi_D}=1} &= \beta\left(\frac{\beta}{\beta - 1}\right)^{\beta} - \beta^2 \ln\frac{\beta}{\beta - 1} - \frac{\beta}{\beta - 1} \\ &= \beta f_4(\beta, 1) > 0,\end{aligned}$$

the last inequality following from Step 3 above. Evaluating, $\frac{\mathrm{d}^2 f_4\left(\beta,\frac{\pi_M}{\pi_D}\right)}{\left(\mathrm{d}\frac{\pi_M}{\pi_D}\right)^2} = \frac{\beta^3}{(\beta-1)z^2}\left(z^\beta - z - 1\right)$. It therefore suffices to show that the function $f_5\left(\beta,\frac{\pi_M}{\pi_D}\right) \equiv \left(\frac{\beta}{\beta-1}\frac{\pi_M}{\pi_D}\right)^\beta - \frac{\beta}{\beta-1}\frac{\pi_M}{\pi_D} - 1$ is positive for $\left(\beta,\frac{\pi_M}{\pi_D}\right) \in (1,\infty)\times[1,\infty)$. But $f_5(\beta,1) = \frac{2\beta-1}{\beta-1}\frac{1-\Delta(\beta)}{\Delta(\beta)} > 0$, and

$$\frac{\mathrm{d}f_5\left(\beta,\frac{\pi_M}{\pi_D}\right)}{\mathrm{d}\frac{\pi_M}{\pi_D}} = \frac{\beta}{\beta-1}\left(\beta\left(\frac{\beta}{\beta-1}\frac{\pi_M}{\pi_D}\right)^{\beta-1} - 1\right) > 0.$$

Since $\frac{\mathrm{d}f_4\left(\beta,\frac{\pi_M}{\pi_D}\right)}{\mathrm{d}\frac{\pi_M}{\pi_D}}$ is positive at $\frac{\pi_M}{\pi_D} = 1$ and increasing in $\frac{\pi_M}{\pi_D}$, it follows that $\frac{\mathrm{d}f_4\left(\beta,\frac{\pi_M}{\pi_D}\right)}{\mathrm{d}\frac{\pi_M}{\pi_D}} > 0$, and together with $f_4(\beta,1) > 0$, this implies that $\frac{\mathrm{d}p^*_{U_L}}{\mathrm{d}\beta}$ is negative. ■

6.7 Proof of Proposition 8

The downstream value in equilibrium is:

$$\tilde{V}\left(y,p^*_{U_L},p^*_{U_F}\right) = \frac{1}{2}\left(\frac{y}{y^*_P}\right)^\beta \frac{\beta}{\beta-1}\Gamma\left(\beta,\frac{\pi_M}{\pi_D}\right) I + \frac{1}{2}\left(\frac{y}{y^*_F}\right)^\beta \frac{\beta}{(\beta-1)^2}\left(1+\beta-\beta\frac{\pi_M}{\pi_D}\right) I. \quad (38)$$

Evaluating,

$$\tilde{V}\left(y,p^*_{U_L},p^*_{U_F}\right) - V\left(y,y^*_D,p^*_U\right) = \frac{(\beta-1)^{\beta-2}}{\beta^{\beta-1}}\left(\frac{y}{y^*_P}\right)^\beta\left(\left(\frac{\pi_D}{\pi_M}\right)^\beta - 1\right) I < 0. \quad (39)$$

The upstream value in equilibrium is:

$$\tilde{W}\left(y,p^*_{U_L},p^*_{U_F}\right) = \left(\frac{y}{y^*_P}\right)^\beta\left(1-\beta\Gamma\left(\beta,\frac{\pi_M}{\pi_D}\right)\right)\frac{I}{\beta-1} + \left(\frac{y}{y^*_F}\right)^\beta \frac{I}{\beta-1}. \quad (40)$$

Let $x \equiv \frac{\pi_M}{\pi_D}$, $x > 1$ since $\pi_M > \pi_D$. After simplification, $\tilde{W}\left(y,p^*_{U_L},p^*_{U_F}\right) > W\left(y,p^*_U\right)$ if and only if:

$$f_6(\beta,x) = \left(\left(\frac{\beta}{\beta-1}\right)^\beta - 1\right)x^\beta - \frac{\beta^2}{\beta-1}x + \beta + 1 > 0. \quad (41)$$

Note first that $f_6(\beta,1) = \left(\frac{\beta}{\beta-1}\right)^\beta - \frac{\beta}{\beta-1} > 0$. Next, we compute:

$$\frac{\mathrm{d}f_6(\beta,x)}{\mathrm{d}x} = \beta\left(\left(\frac{\beta}{\beta-1}\right)^\beta - 1\right)x^{\beta-1} - \frac{\beta^2}{\beta-1}. \quad (42)$$

Since $x > 1$,

$$\begin{aligned} \frac{\mathrm{d}f_6(\beta,x)}{\mathrm{d}x} &> \beta\left(\left(\frac{\beta}{\beta-1}\right)^\beta - 1\right) - \frac{\beta^2}{\beta-1} \\ &= \frac{\beta\left(2\beta-1\right)}{\beta-1}\frac{1-\Delta\left(\beta\right)}{\Delta\left(\beta\right)} > 0, \end{aligned} \tag{43}$$

where the last inequality follows because $\Delta\left(\beta\right) < 1$ (see section 6.3), and therefore $f_6(\beta,x) > 0$.

The industry value with two firms is:

$$2\tilde{V}\left(y, p^*_{U_L}, p^*_{U_F}\right) + \tilde{W}\left(y, p^*_{U_L}, p^*_{U_F}\right) = \left(\frac{y}{y^*_P}\right)^\beta \frac{I}{\beta-1} + \left(\frac{y}{y^*_F}\right)^\beta \frac{\beta}{(\beta-1)^2}\left(1+\beta-\beta\frac{\pi_M}{\pi_D}\right)I. \tag{44}$$

After simplification, $2\tilde{V}\left(y, p^*_{U_L}, p^*_{U_F}\right) + \tilde{W}\left(y, p^*_{U_L}, p^*_{U_F}\right) > V\left(y, y^*_D, p^*_U\right) + W\left(y, p^*_U\right)$ if and only if:

$$\left(\frac{y^*_D}{y^*_P}\right)^\beta \frac{\beta-1}{2\beta-1} + \left(\frac{y^*_D}{y^*_F}\right)^\beta \frac{1+\beta-\beta\frac{\pi_M}{\pi_D}}{2\beta-1} = \frac{1+\beta}{2\beta-1}\left(\frac{\pi_D}{\pi_M}\right)^\beta - \frac{\beta}{2\beta-1}\left(\frac{\pi_D}{\pi_M}\right)^{\beta-1} + \frac{1}{\Delta\left(\beta\right)} > 1. \tag{45}$$

Since $\frac{1}{\Delta(\beta)} > 1$ and $\lim_{\beta\to\infty}\frac{1}{\Delta(\beta)} = \frac{\mathrm{e}}{2}$ (section 6.3), (45) holds for β large enough and $\frac{\pi_D}{\pi_M}$ small enough. Moreover, the condition (45) is indeed violated for admissible parameter values. Let $f_7(\beta, \frac{\pi_M}{\pi_D}) \equiv \frac{1+\beta}{2\beta-1}\left(\frac{\pi_M}{\pi_D}\right)^{-\beta} - \frac{\beta}{2\beta-1}\left(\frac{\pi_M}{\pi_D}\right)^{1-\beta} + \frac{1}{\Delta(\beta)} - 1$. For a given β, this is minimized at $\frac{\pi_M}{\pi_D} = \frac{1+\beta}{\beta-1}$. Then, $f_7(\beta, \frac{1+\beta}{\beta-1}) = \frac{1}{2\beta-1}\left(\frac{\beta^\beta}{(\beta-1)^{\beta-1}} - \left(\frac{\beta-1}{\beta+1}\right)^{\beta-1}\right) - 1$, and $\lim_{\beta\to 1} f_7(\beta, \frac{1+\beta}{\beta-1}) = -1 < 0$. ■

6.8 Sensitivity of Firm Values to β under Preemption

The simplest expression to consider is the total industry value, whose comparative static behavior is independent of $\Gamma\left(\beta, \frac{\pi_M}{\pi_D}\right)$. We have:

$$\frac{\mathrm{d}\left(2\tilde{V}+\tilde{W}\right)}{\mathrm{d}\beta} = \left(\frac{y}{y^*_P}\right)^\beta \frac{\gamma}{\beta} I \ln\left(\frac{y}{y^*_P}\right) + \left(\frac{y}{y^*_F}\right)^\beta \left(\frac{\gamma}{\beta}\right)^2 I\left(2\gamma\frac{\pi_M}{\pi_D} + \beta\ln\left(\frac{y}{y^*_F}\right)\right). \tag{46}$$

Consider the value of this expression at the preemption threshold y^*_P, to find:

$$\left.\frac{\mathrm{d}\left(2\tilde{V}+\tilde{W}\right)}{\mathrm{d}\beta}\right|_{y=y^*_P} = \left(\frac{y^*_P}{y^*_F}\right)^\beta \left(\frac{\gamma}{\beta}\right)^2 \left(\gamma\frac{\pi_M}{\pi_D} - \beta\ln\left(\gamma\frac{\pi_M}{\pi_D}\right)\right) I. \tag{47}$$

This expression has the sign of $\frac{\gamma}{\beta}\frac{\pi_M}{\pi_D} - \ln\left(\gamma\frac{\pi_M}{\pi_D}\right) \equiv f_8(\beta, \frac{\pi_M}{\pi_D})$. However, for a given $\frac{\pi_M}{\pi_D}$, $\lim_{\beta\to 1} f_8(\beta, \frac{\pi_M}{\pi_D}) = \infty$ by l'Hôpital's rule, and $\lim_{\beta\to\infty} f_8(\beta, \frac{\pi_M}{\pi_D}) = -\ln\left(\frac{\pi_M}{\pi_D}\right)$. ■