WORKING PAPERS

N° TSE-483

March 2014

"Informative Advertisement of Partial Compatible Products"

Guillem Roig

Toulouse School of Economics

Informative Advertisement of Partial Compatible Products

Guillem Roig*

First draft: November 2011

March 26, 2014

Abstract

Product design and advertisement strategy have been theoretically studied as separate firms decisions. In the present paper, we look at the link between advertisement and product design and we analyze how firms' advertising decisions influence the market effect of product design. We consider a model of informative advertisement where two firms produce a bundle of complementary products which are partially compatible. A product design with more compatible components is associated with a larger intensity of advertisement. Higher compatibility reduces competition between firms, which incentivizes them to give factual information about their bundle. Like Matutes and Regibeau (1988), industry profit and total welfare is maximized with full product compatibility. However, contrary to them, we obtain that consumer surplus is not monotone with the level of product compatibility and its maximum is attained with partial compatibility. Moreover, because consumer surplus not only depends on the equilibrium prices but also on the intensity of advertisement, we find that for intermediate equilibrium levels of advertising, consumers prefer fully compatible components rather than full incompatibility. As a result, a more compatible product design benefits all the agents in the economy.

Keywords: Informative advertisement; product design; partial compatibility; welfare.

JEL classification: D21, D43, L13, L15

*Toulouse School of Economics. GREMAQ. Contact information: guillemroig182@gmail.com. I am grateful to my supervisor Jacques Crémer and Jiangli Dou for their help and support.

1 Introduction

Economists are well aware that product characteristics beneficial to consumers might end-up generating lower consumers surpluses. Even if consumers value more reliable and higher quality products, they may prefer situations where the previous features stay low. The explanation comes from how product characteristics affect market competition. Higher quality or product reliability, might reduce competition in the market by making the existing products more heterogeneous. This translates to higher equilibrium prices and hence lower consumer surpluses. A paradigmatic example is the situation of product compatibility. While making compatible the components of different products is welfare enhancing, because it allows for a better match between product components and consumers' preferences, compatibility also reduces the competition between assembled products. In this regard, Matutes & Regibeau (1988) show that consumers with heterogeneous tastes for different components are better-off in situations where components' compatibility is low. In their model, the equilibrium price effect through compatibility dominates the gains coming from a better match. Can we then achieve a situation where product characteristics that are intrinsically beneficial to consumers, generate also larger consumer surpluses?

In the present paper, we argue that an alignment of customers surplus and welfare can be achieved if we extend the firms' strategy set. The existing marketing literature has extensively studied the topic of product design, and its resulting affects on the strategic interaction among firms and its final repercussion on consumers. However, the decision on product design has been theoretically separated from the product advertisement strategy. In the present paper, we aim at filling this gap by linking product advertisement and design. We study how the strategy to advertise influences the market effect of product design.

In this regard, we consider a situation where consumers are not aware of product existence, and we explore what are the incentives to undertake informative advertisement when two firms produce a bundle of complementary products that are partially compatible. Firms do not take compatibility decisions, the level of compatibility is exogenous, but they decide on the intensity of advertising prior to consumers' purchasing decisions. We find that advertising increases both demand and competition, but the effect on the latter is reduced whenever components are more compatible. Therefore, we find that the equilibrium intensity of advertisement increases with the level of product compatibility. Our results coincide with Matutes & Regibeau (1988) regarding industry profit and welfare, where its maximum is attained with full product compatibility. However, we find opposite results with regards to consumer surplus. Since in our model, the level of product compatibility does not only affect prices but also the equilibrium intensity of advertisement, consumer surplus is not monotone with the level of product compatibility and its value is maximized when compatibility is partial. A larger level of advertisement generates extra demand an the possibility of consumers to "mix and match" components. Moreover, for some parameters of the model we obtain that consumers prefer perfect compatible products rather than full incompatibility. Therefore, the existence of advertisement makes firms and consumers better-off with compatibility and it works as a mechanism to align players' preferences over product compatibility.

The present work is related to the existing literature on informative advertising. In this literature, the role of advertisement is to convey factual information to consumers about the prices and the specification of the advertised product. Therefore, consumers in the economy have a passive role as they only learn about the existence of a product when they receive an advertisement from the firm. Most of the existing literature builds from the work of Butters (1977), where the advertisement process is specified by assuming that firms send independent advertisement messages and have no ability to target advertisement to consumers. Grossman & Shapiro (1984) employ this framework to compare private and social advertisement in a model of highly differentiated products. They find that equilibrium prices are decreasing with the level of advertisement and its equilibrium level is socially excessive. Soberman (2004) extends the previous model and shows that whenever the level of product differentiations is high, prices increase with the level of advertisement. By assuming an exogenous small level of product differentiation, we obtain that the equilibrium prices decrease with the intensity of advertisement. Finally, because consumers do not incur to shopping cost, the advertisement of prices does not have any effect in our model. In this way, we do not have the possibility that firms incur to a "loss leader" advertisement strategy as studied in Chen & Rey (2012) and Ellison (2005).

The remaining of the paper is organized as follows. In section (2), we present the set-up of the model, and in section (3) we derive consumers' demand and firms' profits. Later, in section (4) we obtain the general formulation of the equilibrium prices and intensity of advertisement.

We proceed with welfare analysis in section (5). Finally, we conclude in section (6). All proofs are in the appendix.

2 Model

We consider a model of informative advertisement and partial product compatibility. Following Matutes and Regibeau (1988), we have a duopolistic market where each firm $i = A, B$ produces and sells two maximally differentiated components x_i and y_i with zero cost of production. Firms are located in the extremes of a Hotelling square, firm A is at the origin $(0, 0)$ and firm B is located at $(1, 1)$. Consumers are uniformly distributed with mass normalized to 1. A consumer (x, y) has a preferred first component that is x away from firm's A first component, and a preferred second component that is y away from firm's A second component. Similarly, her preferred point and firm's B components are $(1 - x)$ and $(1 - y)$ away from each component respectively. Therefore, the distances between consumer (x,y) and the specification of the x and y component sold by either firm A or B are:

Definition 1. $d_A^x = |0 - x|, \quad d_A^y = |0 - y|, \quad d_B^x = |1 - y|, \quad \textit{and} \quad d_B^y = |1 - y|.$

We have a static game that consists of two stages. At stage 1, each firm decides on its intensity of advertisement $\phi_i \in [0, 1]$ and sets prices p_i^x and p_i^y for each component. The firms do this simultaneously and non-cooperatively. The intensity of advertisement ϕ_i represents the fraction of the target population that is exposed to the message of firm i. In this regard, consumers in our model are passive because they only learn about the existence of a product when they see an advertisement. We denote by $E(\phi, \alpha)$ the total expenditure of advertisement, and the parameter α represents its effectiveness. Therefore, the higher the effectiveness, the lower is the cost to reach a given fraction of consumers. We assume $E'_\phi(\cdot) > 0$; $E''_{\phi\phi}(\cdot) > 0$; $E'_\alpha(\cdot) < 0$; and $E''_{\phi\alpha}(\cdot) < 0$, and that the Inada conditions are satisfied.[1]

At stage 2, consumers make their purchasing decisions. They need a unit of each component to form a system which gives them a gross utility of V. If consumers do not receive any advertisement they are uninformed and cannot effectuate any purchase. If they receive one advertisement, they are captive consumers and can only effectuate a purchase from the

[1] Decreasing returns to scale on advertisement might be due to media saturation or heterogeneity of consumers on viewing ads.

firm of which they are aware. Finally, consumers that receive two advertisements are selective and can "mix and match" components produced by different firms. Their ability to "mix and match" depends on the degree of incompatibility between components, represented by the parameter $z \geq 0$, which represents the loss in utility for consumers who consume a bundle of components produced by different firms.[2]

Therefore, the utility of uninformed consumers is 0 and they are in the economy with a proportion of $(1-\phi_A) \times (1-\phi_B)$. The utility of a captive consumer (x, y) is

$$V - (d_i^x + d_i^y) - p_i^x - p_i^y \qquad \text{for} \quad i = A, B,$$

and their proportion in the economy is $\phi_A + \phi_B - 2\phi_A\phi_B$. Finally, the utility of selective consumers, in proportion $\phi_A \times \phi_B$, is the same as captive consumers unless when they buy form different firms. In this latter case, their utility is

$$V - \left(d_A^x + d_B^y\right) - p_A^x - p_B^y - z,$$

if they buy component x from firm A and component y from B.

Finally, since firms cannot differentiate consumers that have seen both advertisements or only their own; price discrimination is not feasible.

3 Demand and Payoffs

With the utility functions of consumers, we can derive the form of the demand function. This is obtained by identifying the consumer that is indifferent between two alternatives. In order to simplify the calculations, we make the following assumption regarding the gross utility of consumption.

Assumption 1. *The gross utility of consumption V is large enough to ensure full market coverage.*[3]

The assumption implies that consumes will always effectuate a purchase. Hence, the indif-

[2]Consider for instance the installation of some extra plug-ins for a certain software to work with a specific hardware.

[3]Later in the paper, we show that for an equilibrium intensity of advertisement $\hat{\phi}$ and prices, this is sufficient

ferent selective consumers are represented in the figure below. Because the characteristics of the consumers are uniformly distributed, the demand is just the area of the regions represented in the figure.

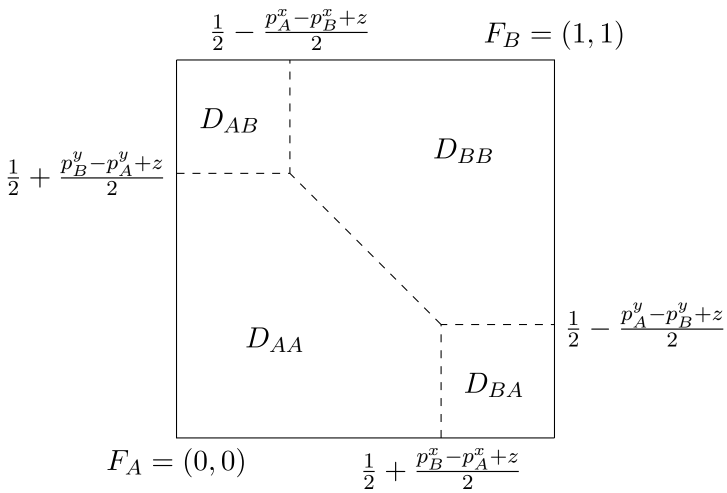


Figure 1: Demand for selective customers, where dashed lines represent the set of indifferent consumers.

They are analytically given by:

$$D_{A,A} = \frac{1}{4}\left[(1 + p_B^x - p_A^x + z)(1 + p_B^y - p_A^y + z) - 2z^2\right],$$
$$D_{A,B} = \frac{1}{4}\left[(1 + p_B^x - p_A^x - z)(1 - p_B^y + p_A^y - z)\right],$$

where the first and the second under-script stands for the identity of the firm for the first and second component respectively. We refer to the appendix in page 22 for the calculation of the demand functions.

Finally, the profit of each firm is the revenue obtained from each group of consumers minus the total expenditure of advertising. Hence, firm A obtains

$$\begin{aligned}\pi_A = (\phi_A \times \phi_B) \times \left[(p_A^x + p_A^y) \times D_{A,A} + p_A^x \times D_{A,B} + p_A^y \times D_{B,A}\right] \\ + \phi_A \times (1 - \phi_B) \times \left(p_A^x + p_A^y\right) - E(\phi_A, \alpha).\end{aligned} \tag{3.1}$$

to have

$$\frac{2 \times \left(2 + \hat{\phi} z\right)}{\hat{\phi} \times (1 + z)} \leq V \leq \frac{4 - \hat{\phi} \times \left(\hat{\phi} - 2z\left(1 - \hat{\phi}\right)\right)}{(1 - \hat{\phi}) \times \hat{\phi} \times (1 + z)}.$$

The first line are the revenues obtained by consumers who are aware of both bundles, and the second are the ones obtained by the consumers that are only know the bundle produced by firm A. Having defined the profit function, we proceed to obtain the equilibrium price and intensity of advertisement.

4 Equilibrium

Firms make simultaneous decisions to choose the intensity of advertisement and the price for each component.[4] From the previous demand functions, and the assumption of full coverage, we observe a conflict between the pricing strategies for each group of consumers: because of the different competitive pressure, firms want to set low prices to selective consumers and high prices to those consumers that are captive. Due to the fact that firms cannot price discriminate, the market equilibrium price is a compromise between those two conflicting interests, and a pure strategy equilibrium may fail to exist.

The following proposition characterizes the equilibrium in prices and the intensity of advertisement for a symmetric Nash equilibrium in pure strategies and full coverage of consumers.

Proposition 1. *In a symmetric equilibrium with full coverage, the price of each component and the intensity of advertisement is implicitly defined by the following system of equations* $(\hat{p}, \hat{\phi})$

$$\hat{p} = \frac{(2-\hat{\phi})}{\hat{\phi}(1+z)}, \tag{4.1}$$

$$E_{\phi}(\hat{\phi}, \alpha) = \frac{(2-\hat{\phi})^2}{\hat{\phi}(1+z)} = \hat{p} \times (2-\hat{\phi}), \tag{4.2}$$

whenever the equilibrium intensity of advertisement is $\hat{\phi} < 1$. *The equilibrium price is* $\hat{p}(z) = \frac{1}{1+z}$, *when the equilibrium intensity of advertisement is* $\hat{\phi} = 1$.

The previous expressions are easily obtained from the first order condition of prices and intensity of advertisement. We relegate the calculation in the appendix page 18, where we verify that no firm wants to deviates from the proposed equilibrium.

[4] In our model consumers do not have shopping costs and they are passive. Only firms' advertisement gives them information about the existence of the product and market prices. If consumers had shopping costs, it might not be optimal for the firms to disclose information about prices. At this regard, an optimal strategy is to advertise one of the components at a loss in order to attract consumers.

The expressions in the proposition give an implicit solution of the problem and later, we provide an explicit solution by making use of a specific advertising cost function. Condition (4.1) tells us that the equilibrium price of each component decreases in both the level of incompatibility and the intensity of advertisement. When incompatibility rises, selective consumers tend to buy both components from the same firm. Because in equilibrium, firms compete for the bundle rather than for separate components, competition is intensified and the equilibrium price drops. Similarly, when the intensity of advertisement increases, the relative importance of selective consumers with respect to captive consumers rises, and since competition occurs only with respect to selective consumers, prices fall.

Condition (4.2) states that the marginal cost of advertising equals its marginal benefit.[5] The intensity of advertisement increases with the effectiveness parameter α and decreases with the level of incompatibility. Because with a larger effectiveness, advertisement becomes cheaper, a higher proportion of consumers are informed about the product. Since incompatibility increases competition for selective consumers, and advertisement intensifies competition even further, firms decide to reach a lower proportion of consumers in order to keep competition milder.

Therefore, the level of incompatibility has two opposite effects on the equilibrium price. The direct effect is that a higher degree of incompatibility makes the selective market more competitive and the equilibrium price of each component falls. The indirect effect comes from advertisement. Since advertisement works as mechanism to increase competition, the equilibrium intensity of advertisement decreases with the level of incompatibility. Consequently, the relative importance of captive consumers is larger and prices increase. The following lemma shows that the direct effect is of first order and an increase of incompatibility unambiguously reduces the equilibrium market price. The formal proof is in page 20 in the appendix.

Lemma 1. *An increase of incompatibility decreases the market equilibrium prices:*

$$\frac{d\hat{p}}{dz} \leq 0.$$

We proceed with welfare analysis, and we see that consumers obtain a larger surplus with

[5] A unique equilibrium exists. Since the cost function is convex and the right hand side is decreasing with the intensity of advertisement, there exists only one value of ϕ such that both expressions cross.

higher levels of product compatibility. The reason is that whenever firms need to inform about the existence of the product, consumers' surplus is not only affected by equilibrium prices but also by the intensity of advertisement.

5 Welfare

In order to make the analysis tractable, we assume that the reservation price of consumers is such that the equilibrium stated in proposition 1 exists.

Total welfare in our model is the sum of both industry profit and consumer surplus.

$$W = \Pi + CS.$$

Firms' profits is the sum of the revenues obtained from selective and captive consumers minus the expenditure on advertisement:

$$\pi(z,\alpha) = \frac{1}{2}\hat{\phi}^2(\cdot) \times 2\hat{p}(\cdot) + \hat{\phi}(\cdot) \times \left(1 - \hat{\phi}(\cdot)\right) \times 2\hat{p}(\cdot) - E(\phi,\alpha) = \frac{\left(2 - \hat{\phi}(\cdot)\right)^2}{1+z} - E(\phi,\alpha), \quad (5.1)$$

and due to symmetry, the total industry profit is $2\pi(z,\alpha)$. Consumer surplus is equal to

$$CS(z,\alpha) = \hat{\phi}(\cdot) \times \left(2 - \hat{\phi}(\cdot)\right) \times (V - 2\hat{p}(\cdot)) - IC(z,\alpha) - PC(z,\alpha), \quad (5.2)$$

where $IC(z,\alpha)$ and $PC(z,\alpha)$ are now defined.

Incompatibility costs, $IC(z,\alpha)$, are incurred only by selective consumers who "mix and match" components from different firms and equal

$$IC(z,\alpha) = \hat{\phi}^2(\cdot) \times (D_{AB} + D_{BA}) \times z = \frac{z}{2}(1-z)^2 \times \hat{\phi}^2(\cdot), \quad (5.3)$$

and preference costs, $PC(z,\alpha)$, come from the fact that consumers cannot perfectly match their preferences with the actual systems in the market. On average, selective consumers have a lower preference cost than captive consumers, and this is because the former are able to "mix and match" components from both firms. This equals to

$$PC(z,\alpha) = \hat{\phi}^2(\cdot) \times \left(\frac{1+z^2}{2}\right) + 2\hat{\phi}(\cdot) \times (1-\hat{\phi}(\cdot)), \tag{5.4}$$

where the first part corresponds to selective consumers and the second to captive consumers. The derivation of the preference costs is not straightforward and the interested reader is referred to the appendix page 22. By introducing expressions (5.3) and (5.4) into consumer surplus we obtain

$$\begin{aligned} CS(z,\alpha) = \hat{\phi}(\cdot) \times \Big[\left(2-\hat{\phi}(\cdot)\right) \times (V - 2\hat{p}(\cdot)) - \hat{\phi}(\cdot) \times \left(\frac{z}{2}(1-z)^2 + \frac{1+z^2}{2}\right) \\ - 2\left(1-\hat{\phi}(\cdot)\right) \Big], \end{aligned} \tag{5.5}$$

and adding this with the industry profit, yields total welfare

$$W(z,\alpha) = \hat{\phi}(\cdot) \times \left(2-\hat{\phi}(\cdot)\right) \times V - \hat{\phi}^2(\cdot) \times \left(\frac{1+z^2}{2} + \frac{z}{2}(1-z)^2 + \frac{(2-2\hat{\phi})}{\hat{\phi}(\cdot)}\right) - \frac{2\hat{\phi}^2(\cdot)}{\alpha}. \tag{5.6}$$

We proceed to obtain a closed solution of the model by assuming a specific form of the expenditure of advertisement.

5.1 Explicit Solution

We assume a particular form for the cost of advertisement

$$E(\phi,\alpha) = \frac{\phi^2}{\alpha}. \tag{5.7}$$

Differentiating this expression with respect to ϕ and substituting into (4.2), the equilibrium intensity of advertisement is equal to

$$\hat{\phi}(z,\alpha) = \min\left\{\frac{2\alpha}{\alpha+\sqrt{2\alpha(1+z)}}, 1\right\}. \tag{5.8}$$

Hence, for $\alpha < 2(2+z) = \bar{\alpha}(z)$ there is partial information. By substituting the equilibrium investment to all previous expressions, we obtain the following results.

In the figure below, we see how the equilibrium intensity of advertisement and prices evolve with the level of incompatibility.

	$\alpha < 2(1+z) = \bar{\alpha}(z)$	$\alpha \geq \bar{\alpha}(z)$
θ	$\frac{2\alpha}{\alpha+\sqrt{2\alpha(1+z)}}$	1
$\hat{p}(z,\alpha)$	$\frac{\sqrt{2}}{\sqrt{\alpha(1+z)}}$	$\frac{1}{1+z}$
$\Pi(z,\alpha)$	$\frac{8\alpha}{\left(\alpha+\sqrt{2\alpha(1+z)}\right)^2}$	$\frac{2}{1+z} - \frac{2}{\alpha}$
$IC(z,\alpha)$	$\frac{2\alpha^2(1-z)^2 z}{\left(\alpha+\sqrt{2\alpha(1+z)}\right)^2}$	$\frac{z(1-z)^2}{2}$
$PC(z,\alpha)$	$\frac{2\alpha\left[2\sqrt{2\alpha(1+z)}+\alpha(z^2-1)\right]}{\left(\alpha+\sqrt{2\alpha(1+z)}\right)^2}$	$\frac{1+z^2}{2}$
$CS(z,\alpha)$	$\frac{2\alpha\left[-8+2(V-1)\sqrt{2\alpha(1+z)}+\alpha(1-z)(1+z^2)\right]}{\left(\alpha+\sqrt{2\alpha(1+z)}\right)^2}$	$\frac{2V(z+1)-(5+2z)-z^4}{2(1+z)}$
$SW = \Pi + CS$	$\frac{2\alpha\left[-4+2(V-1)\sqrt{2\alpha(1+z)}+\alpha(1-z)(1+z^2)\right]}{\left(\alpha+\sqrt{2\alpha(1+z)}\right)^2}$	$\frac{2V(z+1)-(1+2z)-z^4}{2(1+z)} - \frac{2}{\alpha}$

Figure 2: Intensity of advertisement, equilibrium price and welfare indicators. The column on the left stands for a situation where the effectiveness of advertisement is such that there is partial advertisement, i.e $\hat{\phi} < 1$ and the one on the right stands for full informed market i.e. $\hat{\phi} = 1$.

The dashed lines represent a situation with full advertisement for low values of incompatibility. In this case, the decrease in price is more severe than when we have an equilibrium with partial advertisement. When the market is not fully informed, an increase of incompatibility creates a decrease on the intensity of advertisement, and this boosts the proportion of captive consumers in the economy. This last effect, smooths the reduction in prices.[6]

If we turn to the analysis of welfare, we see that with full advertising, the results are similar to Matutes and Regibeau (1988). In such a case, both industry profit and welfare attains its maximum with full product compatibility and consumer surplus is maximized with full incompatibility $CS\left[z = 0 \mid \alpha > \bar{\alpha}(z)\right] < CS\left[z = 1 \mid \alpha > \bar{\alpha}(z)\right]$. Here, all results are driven by equilibrium prices, because all consumers are informed, the intensity of advertisement does not have any effect on the total demand.

Our contribution stands for analyzing the case where there is an equilibrium with partial

[6]We have already mentioned that the competition effect is of first order with comparison to the change in the consumers' composition. We compare how prices changes with the level of incompatibility in both equilibria. We observe that the change of prices with the level of incompatibility is larger with a full informed equilibrium.

$$\frac{\partial p\left(z,\alpha \mid \hat{\phi} < 1\right)}{\partial z} = -\frac{2}{\alpha(1+z)^2} > -\frac{1}{(1+z)^2} = \frac{\partial p\left(z,\alpha \mid \hat{\phi} = 1\right)}{\partial z} \to \alpha > 2.$$

and this is always the case for most of the parameters of efficiency considered.

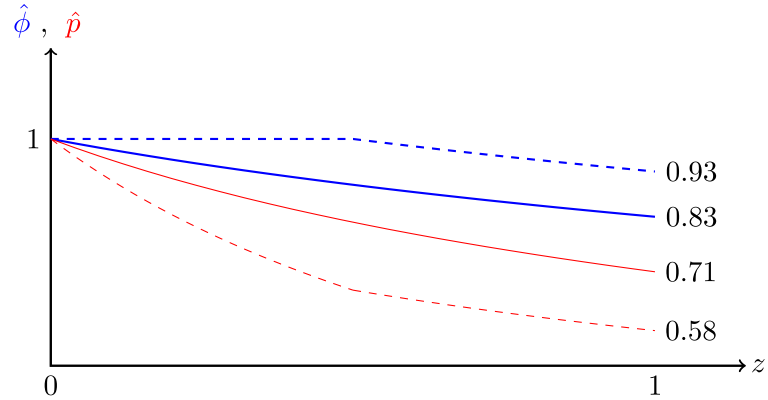


Figure 3: Equilibrium price and intensity of advertisement as a function of the level of incompatibility z. The thick line represents a situation where the parameter of advertisement efficiency is equal to $\alpha = 2$, and the dashed line stands for $\alpha = 3$.

advertisement and not all consumers are aware of the existence of both systems. In order to perform this analysis, for the rest of the paper we consider that $V = 6$

$$\alpha(z) \geq 1/2(1+z) = \underline{\alpha}(z) \tag{5.9}$$

In the appendix, page 23, we show that this is necessary to obtain a pure strategy equilibrium in prices.

Industry profit does not depend on the reservation price of consumers, and this decreases with the level of incompatibility. Here, we distinguish two effects. When incompatibility increases so does the equilibrium price, and because the intensity of advertisement also decreases, demand is consequently reduced. However, the fall of profits is more accentuated in a situation when all consumers are informed about the products, since the reduction of prices in this case is larger. In general, higher efficiency of the advertising technology entails lower industry profits. With a higher effectiveness of advertisement, firms advertise more in equilibrium, and the proportion of selective consumers increases, brining about a more intense competition. This competition effect dominates the increase in demand and the reduction of the cost of advertisement coming from a more efficient advertising technology.

With regards to consumer surplus, the following proposition states the main result of the paper. This reveals that consumers are better-off with an intermediate level of product compatibility.

Proposition 2. *With $V = 6$ and (5.9) holding, consumer surplus attains its maximum for an intermediate level of product compatibility whenever the advertising efficiency belongs to $(\hat{\alpha}, \bar{\alpha})$, and $\hat{\alpha}$ is defined by*

$$10\sqrt{\hat{\alpha}} - \hat{\alpha} + 2\sqrt{\hat{\alpha}^3} = 8.$$

Moreover, whenever $\alpha \in \left(\hat{\hat{\alpha}}, \bar{\alpha}\right)$, where $\hat{\hat{\alpha}}$ is defined by

$$\frac{-8 + 10\sqrt{2\hat{\hat{\alpha}}} + \hat{\hat{\alpha}}}{\left(\sqrt{2} + \sqrt{\hat{\hat{\alpha}}}\right)^2} = \frac{4\left(-2 + 5\sqrt{\hat{\hat{\alpha}}}\right)}{\left(2 + \sqrt{\hat{\hat{\alpha}}}\right)^2},$$

consumers are better with complete product compatibility than full incompatibility.[7]

The formal proof is in the appendix, page 21. Whenever the market is partially informed, the level of incompatibility affects not only the equilibrium price, but also the total demand as well as the demand composition in the market. While with a low level of incompatibility, the decrease of the equilibrium price dominates the decrease in the intensity of advertisement, for high levels of incompatibility, the reduction in the level of advertisement, and the subsequent increase in the preference costs, dominates the effect of the price decrease. Consequently, the average consumer is worse-off. In general, consumers prefer to be better informed about the products offered in the market at the expense of higher equilibrium prices. In the figure 4 below, we illustrate how consumers' surplus varies with z, the level of incompatibility and as shown in the proposition, its maximum is attained with an intermediate level of compatibility.

The level of incompatibility also plays a role the evolution of both incompatibility and preference costs. For low values of incompatibility, the total incompatibility cost increases with incompatibility as selective consumers who "mix and match" pay this cost. For large values of incompatibility, the total incompatibility costs decrease, because there are less selective consumers in the market, due to lower advertisement, and also a smaller proportion of these consumers decide to "mix and match". Preference costs are an increasing function on the level of incompatibility. A lower intensity of advertisement, due to an increase of incompatibility, reduces the proportion of selective consumers while the proportion of captive consumers is increased. Because captive consumers have on average a larger preference costs, the total

[7] There is only one $\hat{\alpha}$ and $\hat{\hat{\alpha}}$ that solve the previous equations. See page 21 in the appendix for the formal proof.

preference cost in the economy is increased. A lower proportion of selective consumers will "mix and match" components, and the preference costs are accordingly increased.

Furthermore, we observe that increases in the advertising effectiveness unambiguously increases consumers surplus. Not only prices are lower in equilibrium, but also consumers are better informed about the existing products, because the intensity of advertisement is bigger.

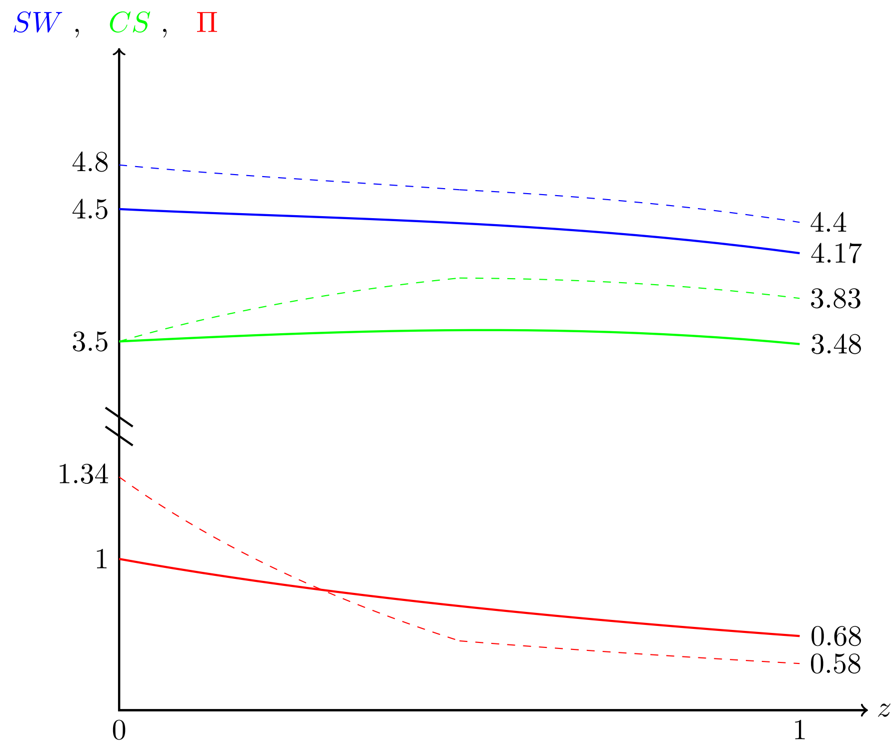


Figure 4: Industry profit, consumer surplus and social welfare as a function of the level of incompatibility z. The thick line represents a situation where the parameter of advertisement efficiency is equal to $\alpha = 2$, and the dashed line stands for higher efficiency $\alpha = 3$.

Finally, welfare always decreases with the level of incompatibility. Because prices are just a transfer from consumers to firms an increase in incompatibility unambiguously decreases welfare.[8] Incompatibility does not only create a higher preference cost as the proportion of consumers who "mix and match" is decreased, but in our model it also decreases the equilibrium intensity of advertisement. Less advertisement, creates a reduction of demand as the proportion of uninformed consumers increases.

Therefore, whenever firms have to undertake advertisement to inform about the existence

[8] This is the case as we are working with full coverage. Otherwise, the equilibrium price would have an effect on social welfare as it has an effect on total demand.

of the products, all agents have similar preferences regarding the level of product compatibility. We have shown that for some parameters firms and consumers are better-off with full product compatibility. Moreover, efficient advertising in our model works as a way to increase potential demand but it also fosters competition. This second effect is reduced by making products compatible. Hence, we have found two instruments that firms might use to increase profits. One is to increase compatibility, the other is to agree on having an inefficient advertising technology.

6 Conclusion

In this paper, we have shown that intrinsic beneficial characteristics, such as product compatibility, create larger consumer surpluses if we extend the firms' strategy set. In our model, higher product compatibility generates lower competition which translate to higher prices. However, lower competition incentivizes firms to convey information about their product, which generates a demand increase and a better match between products and consumers' preferences. In contrast with previous literature, we find that consumer surplus is non-monotonic on the level of incompatibility and it has an inverse U-shaped form. A competition authority should then be careful about possible actions aiming at increasing the consumer surplus such as increasing market competition. Larger competition between firms might be achieved by making existing products less compatible. However, we have seen that increasing the level of competition might have detrimental effects to consumers, because they are less informed about the purchasing possibilities in the market.

Following the existing literature on informative advertising, we have considered that consumers are passive as they do not engage into any active search to find about the products offered in the market. Furthermore, consumers do not incur to any shopping costs when they commute to the firm to effectuate a purchase. This assumption simplifies the analysis tremendously, because the decision of firms referring to what component and what price to advertise is not relevant. However, if consumers experienced some positive shopping costs it might be in the interest of the firms to practice some sort of "loss leader" advertising strategy as studied in Chen & Rey (2012). We believe that as long as a "loss leader" strategy softens competition, the intensity of advertisement will be larger in equilibrium. However, the results on consumer surplus are not clear. Prices will be larger in equilibrium but consumers will also be more informed about the consumption alternatives in the marketplace. In the current model, we have also considered that consumers are distributed uniformly over the unit square. This assumption implies that consumers do not have brand preferences. By relaxing this assumption, we believe that competition for selective consumers would be heavily reduced implying also a boost on the level of advertisement.

Moreover, firms in the model are not able to perform price discrimination, consumers that purchase the bundled product pay the sum of the pice for each component. An interesting extension is to consider how our results change if we allow firms to effectuate a discount for those consumers that buy both components from the same firm. We conjecture that allowing for this strategy gives an extra instrument to firms to increase competition. Therefore, a decrease on the equilibrium price might bring about a lower intensity of advertisement, and this might be detrimental to consumers.

Finally, we have assumed that the level of compatibility and the effectiveness of advertisement are exogenous in the model. It might be interesting to consider endogenous compatibility choice by firms. Firms would like to have full compatibility since this reduces the competition for selective consumers. Therefore, in a previous stage, firms decide on how to make their components compatible, or at least until which degree. With regards to the effectiveness of advertisement, we have seen that this has the strategic effect of a prisoners' dilemma. The more effective advertisement is, the more firms increase the intensity of advertisement and the more fierce competition becomes. Therefore, in a stage before the market game, firms might tacitly collude on using an ineffective advertising technology.

References

[1] Bagwell K., “The Economic Analysis of Advertising,”*Handbook of Industrial Organization,Volume 3, Chapter 28, 2007.*

[2] Chen, Z. and P. Rey, “Loss Leading as an Exploitative Practice, ”American Economic Review, American Economic Association, vol. 102(7) (2012), pages 3462-82, December.

[3] Ellison G. “A Model of Add-on Pricing, ”The Quarterly Journal of Economics, MIT Press, vol. 120(2) (2005), pages 585-637, May.

[4] Grossman M. G. and Shapiro C., “Informative Advertising with Differentiated Products,” *The Review of Economic Studies, Vol. 51, No. 1 (Jan., 1984), 00. 63-81.*

[5] Hamilton F. S., “Informative Advertising in Differentiated Oligopoly Markets,” *International Journal of Industrial Organization, Vol. 27, Issue 1, 2009, pp. 60-69.*

[6] Matutes C. and Regibeau P., “Mix and Match”: Product Compatibility without Network Externalities,” *The Rand Journal of Economics, Vol. 19, No. 2 (Summer, 1988), pp. 221-234.*

[7] Meurer M. and Stahl O. D, “Informative Advertising and Product Match,” *International Journal of Industrial Organization 12 (1994) 1-19.*

[8] Soberman A. D., “Research Note: Additional Learning and Implications on the Role of Informative Advertising,” *Management Science, Vol. 50, No. 12, December 2004, pp.1774-1750.*

[9] Stegeman M., “Advertising in Competitive Markets,” *The American Economic Review, Vol. 81, No. 1 (Mar., 1991), pp. 210-223.*

Appendices

A Appendix

Proof of proposition 1: We start by assuming that a pure strategy in prices exist, and later we derive the necessary conditions to ensure that no firm wants to deviate form this proposed equilibrium.

Each firm chooses the intensity of advertisement and the level of prices to maximize its profit given by expression (3.1). The equilibrium is obtained by the solution of the system of the first order conditions given by:

$$\begin{aligned}
\frac{\partial \pi_i}{\partial p_i^x} = 0 \rightarrow & \phi_i \phi_j \left[D_{i,i'} + D_{i,j} + (p_i^x + p_{i'}^y) \times \frac{\partial D_{i,i'}}{\partial p_i^x} + p_i^x \times \frac{\partial D_{i,j}}{\partial p_i^x} + p_{i'}^y \times \frac{\partial D_{j,i}}{\partial p_i^x} \right] \\
& + \phi_i (1-\phi_j) \left[D_i^c + (p_i^x + p_{i'}^y) \times \frac{\partial D_i^c}{\partial p_i^x} \right] = 0, \quad \text{for } i = i' \text{ and; } i \neq j \\
\frac{\partial \pi_i}{\partial p_{i'}^y} = 0 \rightarrow & \phi_i \phi_j \left[D_{i,i'} + D_{j,i'} + (p_i^x + p_{i'}^y) \times \frac{\partial D_{i,i'}}{\partial p_{i'}^y} + p_{i'}^y \times \frac{\partial D_{j,i'}}{\partial p_{i'}^y} + p_i^x \times \frac{\partial D_{i,j}}{\partial p_{i'}^y} \right] \\
& \phi_i (1-\phi_j) \left[D_i^c + (p_i^x + p_{i'}^y) \times \frac{\partial D_i^c}{\partial p_{i'}^y} \right] = 0, \quad \text{for } i = i' \text{ and; } i \neq j \\
\frac{\partial \pi_i}{\partial \phi_i} = 0 \rightarrow & \phi_j \left[(p_i^x + p_{i'}^y) \times D_{i,i'} + p_i^x \times D_{i,j} + p_{i'}^y \times D_{j,i'} \right] + (1-\phi_j)(p_i^x + p_{i'}^y) \times D_i^c = E_{\phi_i}(\phi_i, \alpha).
\end{aligned}$$

Because we are restricting attention to the case where captive consumers are fully covered, we get that the demand for the selective consumers is equal to $D_i^c = 1$ and the partial derivatives are zero i.e. $\partial D_i^c / \partial p_i^x = \partial D_i^p / \partial p_{i'}^y = 0$. Therefore, by adding this constraints and the demand forms in both pricing expressions, we obtain:

$$\begin{aligned}
\frac{\partial \pi_i}{\partial p_i^x} = 0 \rightarrow \frac{\phi_i \left[(1-\phi_j) + (p_j^x - 2p_i^x)\phi_j + (p_j^y - 2p_{i'}^y)\phi_j z \right]}{2} = 0 \rightarrow p_i^x = \frac{1}{2} \left(p_j^x - \frac{\phi_j - 2}{\phi_j (1+z)} \right) \\
\frac{\partial \pi_i}{\partial p_{i'}^y} = 0 \rightarrow \frac{\phi_i \left[(1-\phi_j) + (p_j^y - 2p_{i'}^y)\phi_j + (p_j^x - 2p_{i'}^x)\phi_j z \right]}{2} = 0 \rightarrow p_{i'}^y = \frac{1}{2} \left(p_j^y - \frac{\phi_j - 2}{\phi_j (1+z)} \right),
\end{aligned}$$

and with the conditions for p_j^x and p_j^y we obtain that the prices for each component is the same

$$p_i^x = p_{i'}^y = p_i = \frac{\phi_i (4 - 3\phi_j) + 2\phi_j}{3\phi_i \phi_j (1+z)}; \qquad p_j^x = p_{j'}^y = p_j = \frac{\phi_j (4 - 3\phi_i) + 2\phi_i}{3\phi_i \phi_j (1+z)}.$$

Because, the cost of advertisement is the same for both firms, they also set the same intensity of advertisement $\hat{\phi}_i = \hat{\phi}_j = \hat{\phi}$. As a result, the equilibrium price for each component is

$$p_i = p_j = \frac{2 - \hat{\phi}}{\hat{\phi}(1+z)}.$$

By substituting the equilibrium prices and the demands in the first order condition of each firm with respect to the intensity of advertisement, we obtain that the equilibrium intensity of advertisement is

$$E_{\phi}(\phi,\alpha) = \hat{\phi} \times \left[\frac{4-\hat{\phi}}{\hat{\phi}(1+z)} \times D_{i,i'} + \frac{2-\hat{\phi}}{\hat{\phi}(1+z)} \times D_{i,j} + \frac{2-\hat{\phi}}{\hat{\phi}(1+z)} \times D_{j,i'}\right] + (1-\hat{\phi}) \times \left(\frac{4-\hat{\phi}}{\hat{\phi}(1+z)}\right)$$
$$= \hat{\phi} \times \left(\frac{4-\hat{\phi}}{2\hat{\phi}(1+z)}\right) + (1-\hat{\phi}) \times \left(\frac{4-\hat{\phi}}{\hat{\phi}(1+z)}\right) = (2-\hat{\phi}) \times \left(\frac{2-\hat{\phi}}{\hat{\phi}(1+z)}\right).$$

Finally, to be sure that the previous conditions constitute an equilibrium, we need to verify that no firm wants to deviate from the proposed equilibrium and all consumers are covered. Due to the different incentives for pricing the captive and selective consumers there might exist an equilibrium in mixed strategies. Each firm has a monopoly position over their captive consumers and face competition only over the selective ones. Therefore the mixed strategy equilibrium appears due to the trade-off from the incentive to extract the whole reservation price form her captive consumers, or undercut the price to attract selective consumers. The following lemma establishes the reservation value for consumers such that no firm wants to deviate from the prices stated in proposition 1 and all consumers are covered.

Lemma 2. *There equilibrium of the game is in pure strategies as stated in proposition 1 if consumer's reservation price is* $V \in \left[\underline{V}(z,\alpha), \bar{V}(z,\alpha)\right]$ *and*

$$\underline{V}(z,\alpha) = \frac{2 \times \left(2+\hat{\phi}z\right)}{\hat{\phi} \times (1+z)}; \qquad \bar{V}(z,\alpha) = \frac{4-\hat{\phi} \times \left(\hat{\phi} - 2z\left(1-\hat{\phi}\right)\right)}{(1-\hat{\phi}) \times \hat{\phi} \times (1+z)},$$

where the equilibrium intensity of advertisement $\hat{\phi}(z,\alpha)$ *is implicitly defined in equation* (4.2)*.*

We then need to verify that, with the proposed equilibrium price, all captive consumers purchase the good and that no firm can obtain a larger profit by extracting the whole surplus from his captive consumers. In our model as both the equilibrium price and intensity of advertisement depend on the level of incompatibility, so is the range for the reservation price of consumers that allows for a pure strategy equilibrium. Hence we obtain a full coverage of captive consumers if their net utility from consumption is not negative $V \geq 2 + 2 \times \hat{p}(\hat{\phi}, z)$, and by using the equilibrium price given by expression (4.1) we obtain

$$V \geq 2 + \frac{2(2-\hat{\phi})}{\hat{\phi}(1+z)} = \frac{2 \times \left(2+\hat{\phi}z\right)}{\hat{\phi} \times (1+z)} = \underline{V}(z,\hat{\phi}).$$

It is not profitable to extract the whole surplus from captive consumers by setting a price of each component equal to $p = (V-2)/2$ if

$$\frac{1}{2}\hat{\phi}^2 2\hat{p}(\cdot) + \hat{\phi}(1-\hat{\phi})2\hat{p}(\cdot) - \frac{\hat{\phi}}{\alpha} \geq \hat{\phi}(1-\hat{\phi})2 \times \left(\frac{V-2}{2}\right) - \frac{\hat{\phi}}{\alpha} \rightarrow (2-\hat{\phi})\hat{p}(\cdot) \geq (1-\hat{\phi})(V-2d)$$

$$V \leq \frac{(2-\hat{\phi})^2 + 2(1-\hat{\phi})\hat{\phi}(1+z)}{(1-\hat{\phi})\hat{\phi}(1+z)} = \frac{4-\hat{\phi}\times\left(\hat{\phi}-2z\left(1-\hat{\phi}\right)\right)}{(1-\hat{\phi})\times\hat{\phi}\times(1+z)} = \bar{V}(z,\alpha).$$

Hence, we work with the case where the gross utility from consumption is $V \in \left[\underline{V}(z,\alpha), \bar{V}(z,\alpha)\right]$. When the incompatibility increases, the equilibrium price of each component decreases and this makes the upper and the lower bound to decrease. For any value of effectiveness of investment $\alpha \in R_+$ and level of incompatibility $z \in (0,1)$ we obtain

$$\bar{V}(z,\alpha) = \frac{4-\hat{\phi}\times\left(\hat{\phi}-2z\left(1-\hat{\phi}\right)\right)}{(1-\hat{\phi})\times\hat{\phi}\times(1+z)} \geq \frac{2\times\left(2+\hat{\phi}z\right)}{\hat{\phi}\times(1+z)} = \underline{V}(z,\hat{\phi}) \rightarrow \frac{4-\hat{\phi}\times\left(\hat{\phi}-2z\left(1-\hat{\phi}\right)\right)}{(1-\hat{\phi})} \geq 2\times\left(2+\hat{\phi}z\right)$$
$$\rightarrow \hat{\phi} < \frac{4}{1+z},$$

and the above is always the case since $\hat{\phi} \in [0,1]$.

Proof of lemma 1: We calculate the first order condition of the equilibrium price given by expression (4.1) with respect to the level of incompatibility.

$$\frac{d\hat{p}\left(\hat{\phi}(z),z\right)}{dz} = \frac{-\frac{\partial\hat{\phi}(z)}{\partial z}\times\left(\hat{\phi}(z)\times(1+z)\right) - \left[\left(\frac{\partial\hat{\phi}(z)}{\partial z}\times(1+z)+\hat{\phi}(z)\right)\left(2-\hat{\phi}(z)\right)\right]}{\left(\hat{\phi}(z)\times(1+z)\right)^2}$$
$$= \frac{\overbrace{-\hat{\phi}(z)\times\left(2-\hat{\phi}(z)\right)}^{\text{direct effect}} + \overbrace{\left[-\frac{\partial\hat{\phi}(z)}{\partial z}\times 2(1+z)\right]}^{\text{indirect effect}}}{\left(\hat{\phi}(z)\times(1+z)\right)^2}.$$

The second part of the numerator represents the indirect effect that incompatibility has on prices and this depends on how an increase of incompatibility affects the equilibrium level of advertisement. This is positive as the intensity of advertisement decreases with the level of incompatibility $\partial\hat{\phi}(z)/\partial z < 0$. The first part of the numerator stands for the direct effect and this is always negative. We obtain that the direct effect dominates and the expression above is negative if the decrease on the equilibrium intensity of advertisement with incompatibility is relatively mild, i.e, $-\frac{\partial\hat{\phi}(z)}{\partial z} < \frac{\hat{\phi}(z)\times(2-\hat{\phi}(z))}{2(1+z)}$. To show that this is always the case, we differentiate equation (4.2) with respect to the level of incompatibility.

$$0 = \frac{-2\times\left(2-\hat{\phi}(z)\right)\times\frac{\partial\hat{\phi}(z)}{\partial z}\times\left(\hat{\phi}(z)\times(1+z)\right)-\left[\left(\frac{\partial\hat{\phi}(z)}{\partial z}\times(1+z)+\hat{\phi}(z)\right)\times\left(1-\hat{\phi}(z)\right)^2\right]}{\hat{\phi}(z)\times(1+z)^2}$$

$$\rightarrow -\frac{\partial\hat{\phi}(z)}{\partial z} = \frac{\left(2-\hat{\phi}(z)\right)\times\hat{\phi}(z)}{\left(\hat{\phi}(z)+2\right)\times(1+z)},$$

and buy introducing this in the previous expression, this is easy to verify that

$$\frac{\left(2-\hat{\phi}(z)\right)\times\hat{\phi}(z)}{\left(\hat{\phi}(z)+2\right)\times(1+z)} < \frac{\hat{\phi}(z)\times\left(2-\hat{\phi}(z)\right)}{2(1+z)} \iff 4-2\hat{\phi}(z) < \left(2-\hat{\phi}(z)\right)\times\left(\hat{\phi}(z)+2\right) \iff \hat{\phi}(z) < 2.$$

Hence, since $\hat{\phi} \in [0,1]$ the above is always true and we have shown that the equilibrium price decreases with the level of product incompatibility.

Proof of proposition 2: We restrict our analysis to the case with partial informed equilibrium and hence the advertising efficiency parameter is $\alpha \in (\underline{\alpha}, \bar{\alpha})$. To obtain that the maximum consumer surplus is obtained with an intermediate level of product compatibility, we calculate the derivative of the consumer surplus with respect to z.

$$\frac{dCS(z,\alpha)}{dz} = \frac{2\alpha^2\left[8\sqrt{2}+\sqrt{\alpha(1+z)}\times(z\alpha(2-3z)-\alpha-10)+\sqrt{2}\alpha\times\left(3-2z\left(z+z^2-1\right)\right)\right]}{\sqrt{\alpha(1+z)}\times\left(\alpha+\sqrt{2\alpha(1+z)}\right)^3},$$

and this is a continuous function with respect to both the intensity of advertisement (α) and the level of incompatibility. To show that the function is concave in z for the range of the effectiveness of advertisement considered, we do not rely on the second derivative as its expression is complicated but rather we calculate the sign of the derivative at both extremes of the level of incompatibility. Hence, we obtain:

$$\left.\frac{dCS(z,\alpha)}{dz}\right|_{z=0} = -\frac{2\left(10\sqrt{\alpha}-8\sqrt{2}-3\sqrt{2}\alpha+\alpha^{3/2}\right)}{(\sqrt{2}+\sqrt{\alpha})^3},$$

$$\left.\frac{dCS(z,\alpha)}{dz}\right|_{z=1} = \frac{2\left(8-10\sqrt{\alpha}+\alpha-2\alpha^{3/2}\right)}{(2+\sqrt{\alpha})^3}.$$

It is easy to see that for any $\alpha \in (\underline{\alpha}, \bar{\alpha})$ we get that the derivative evaluated at $z=0$ is always positive. Hence, in order to have a concave function with respect to z we need that the derivative evaluated at $z=1$ to be negative. This is the case when $8-10\sqrt{\alpha}+\alpha-2\alpha^{3/2}<0$ and this occurs whenever $\alpha > \hat{\alpha}$ where $\hat{\alpha}$ is the solution $10\sqrt{\hat{\alpha}}-\hat{\alpha}+2\sqrt{\hat{\alpha}^3}=8$. To verify that the threshold $\hat{\alpha}$ is inside the range of the values of α considered, we see that the right hand side $\daleth(\alpha) = 10\sqrt{\alpha} - \alpha + 2\sqrt{\alpha^3}$ is increasing in α since $\frac{\partial \daleth(\alpha)}{\partial \alpha} = \frac{5+3\alpha}{\sqrt{\alpha}} - 1 > 0$. Finally, by evaluating the the function at the extremes $\daleth(\underline{\alpha}) = 10\sqrt{\frac{1}{4}} - \frac{1}{4} + 2\sqrt{\frac{1}{4}^3} < 8$ and $\daleth(\bar{\alpha}) = 10\sqrt{4} - 4 + 2\sqrt{4^3} > 8$ we obtain that $\hat{\alpha}$ is in the interval considered.

To show that consumers are better off with full product compatibility that incompatibility we just need to compare the value of the consumer social welfare at both extremes.

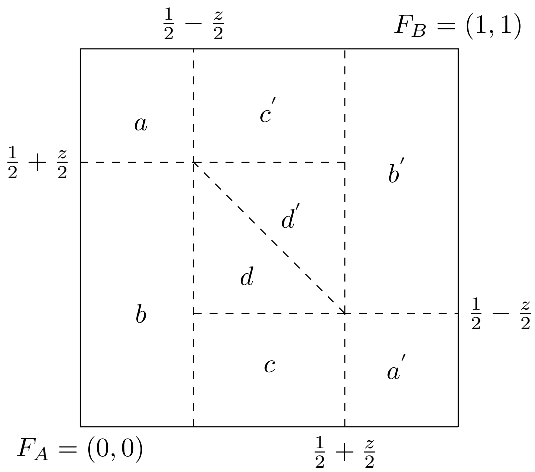
$$CS\left(z = 0 \mid \alpha < \bar{\alpha}(z)\right) = \frac{-8 + 10\sqrt{2\alpha} + \alpha}{\left(\sqrt{2} + \sqrt{\alpha}\right)^2} \geq \frac{4\left(-2 + 5\sqrt{\alpha}\right)}{\left(2 + \sqrt{\alpha}\right)^2} = CS\left(z = 1 \mid \alpha < \bar{\alpha}(z)\right),$$

and it can be seen that this is the case whenever $\alpha > \hat{\hat{\alpha}}$ where $\hat{\hat{\alpha}}$ is the solution $\frac{-8+10\sqrt{2\hat{\hat{\alpha}}}+\hat{\hat{\alpha}}}{\left(\sqrt{2}+\sqrt{\hat{\hat{\alpha}}}\right)^2} - \frac{4\left(-2+5\sqrt{\hat{\hat{\alpha}}}\right)}{\left(2+\sqrt{\hat{\hat{\alpha}}}\right)^2} = 0$. Finally, we need to verify that the values of the threshold is inside the range and by applying the same procedure as before we prove the claim.

Derivation of demand: When the gross utility of consumption V is large enough such that all costumers are covered we can easily obtain the demands for selective consumers by first obtaining the indifferent consumer (x,y) as represented in figure 1 in the main text. Since we have assumed that consumes are uniformly distributed, the demand is just the area for the different regions.

$$\begin{aligned}
D_{i,j} &= \left(\frac{1}{2} - \frac{p_i^x - p_j^x + z}{2}\right) \times \left(1 - \left(\frac{1}{2} + \frac{p_j^y - p_i^y + z}{2}\right)\right) = \left(\frac{1 - p_i^x + p_j^x - z}{2}\right) \times \left(\frac{1 - p_j^y + p_i^y - z}{2}\right) \\
&= \frac{1}{4}\left[(1 + p_j^x - p_i^x - z)(1 - p_j^y + p_i^y - z)\right] \quad \text{for} \quad i \neq j. \\
D_{i,i'} &= \left(\frac{1}{2} + \frac{p_j^x - p_i^x + z}{2}\right) \times \left(\frac{1}{2} + \frac{p_j^y - p_{i'}^y + z}{2}\right) - \frac{\left[\frac{1}{2} + \frac{p_j^x - p_i^x + z}{2} - \left(\frac{1}{2} - \frac{p_i^x - p_j^x + z}{2}\right)\right]^2}{2} \\
&= \left(\frac{1 - p_j^x - p_i^x + z}{2}\right) \times \left(\frac{1 + p_j^y - p_{i'}^y + z}{2}\right) - \frac{z^2}{2} \\
&= \frac{1}{4}\left[(1 + p_j^x - p_i^x + z)(1 + p_j^y - p_{i'}^y + z) - 2z^2\right] \quad \text{for } i = i' \text{ and}; i \neq j.
\end{aligned}$$

Derivation of preference costs: The average preference cost of the captive consumer for each component is $v_x = v_y = 1/2$ and the average preference cost for the consumption of both components is $PC_c = 1$. To obtain the average preference cost of the selective consumers depends on the region that a given consumer belongs to, and this is represented in the figure below.



In this table, we compute the mass of each region (m_i) and the average distance for each component iv_x and iv_y for $i=a,b,c,d,a^{'},b^{'},c^{'},d^{'}$.

Point	Proportion	Averages	Total: $(v_x+v_y)\times d\times m$
a	$m_a=\frac{(\lambda-z)^2}{4\lambda^2}$	$av_x=av_y=\frac{\lambda-z}{4\lambda}$	$\frac{(\lambda-z)^3}{8\lambda^2}$
b	$m_b=\frac{\lambda^2-z^2}{4\lambda^2}$	$bv_x=\frac{\lambda-z}{4\lambda},\quad bv_y=\frac{\lambda+z}{4\lambda}$	$\frac{\lambda^2-z^2}{8\lambda}$
c	$m_c=\frac{z(\lambda-z)}{2\lambda^2}$	$cv_x=\frac{1}{2},\quad cv_y=\frac{\lambda-z}{4\lambda}$	$\frac{z(\lambda-z)(3\lambda-z)}{8\lambda^2}$
d	$m_d=\frac{z^2}{2\lambda^2}$	$dv_x=dv_y=\frac{1}{2}$	$\frac{z^2}{2\lambda}$
$a^{'}$	$m_{a^{'}}=\frac{(\lambda-z)^2}{4\lambda^2}$	$a^{'}v_x=a^{'}v_y=\frac{\lambda-z}{4\lambda}$	$\frac{(\lambda-z)^3}{8\lambda^2}$
$b^{'}$	$m_{b^{'}}=\frac{\lambda^2-z^2}{4\lambda^2}$	$b^{'}v_x=\frac{\lambda+z}{4\lambda}\quad b^{'}v_y=\frac{\lambda-z}{4\lambda}$	$\frac{\lambda^2-z^2}{8\lambda}$
$c^{'}$	$m_{c^{'}}=\frac{z(\lambda-z)}{2\lambda^2}$	$c^{'}v_x=\frac{\lambda-z}{4\lambda}\quad c^{'}v_y=\frac{1}{2}$	$\frac{z(\lambda-z)(3\lambda-z)}{8\lambda^2}$
$d^{'}$	$m_{d^{'}}=\frac{z^2}{2\lambda^2}$	$d^{'}v_x=d^{'}v_y=\frac{1}{2}$	$\frac{z^2}{2\lambda}$

$$PC_s=\frac{(1-z)^3}{4}+\frac{1-z^2}{4}+\frac{z(1-z)(3-z)}{8}+\frac{z^2}{2}=\frac{1+z^2}{2}.$$

Therefore, we obtain that the preference cost in the economy is equal to

$$PC=\hat{\phi}^2\times PC_s+2\hat{\phi}(1-\hat{\phi})PC_c=\hat{\phi}^2\times\frac{1+z^2}{2}+2\hat{\phi}(1-\hat{\phi}).$$

Bounds of V: Here, with a reservation price of $V=6$ we determine the value of α such that we have an equilibrium in pure strategies as stated in lemma 2. Therefore, we need that the reservation price of consumers be inside the interval $6\in\left[\underline{V},\bar{V}\right]$.

$$6\geq\underline{V}=\frac{2\times\left(2+\hat{\phi}z\right)}{\hat{\phi}\times(1+z)}\rightarrow\hat{\phi}\geq\frac{2}{3+2z}\rightarrow\frac{2\alpha}{\alpha+\sqrt{2\alpha(1+z)}}\geq\frac{2}{3+2z}\rightarrow\alpha\geq\frac{1}{2(1+z)}$$

$$6 \leq \bar{V} = 2 + \frac{(2-\hat{\phi})^2}{(1-\hat{\phi})\hat{\phi}(1+z)} \rightarrow \hat{\phi}\left(8 - 5\hat{\phi} + 4z - 4\hat{\phi}z\right) \leq 4,$$

and it is easy to see that the second condition is fulfilled for any equilibrium advertising intensity $\hat{\phi}$ and we only need to be sure that the effectiveness of advertisement is above the value represented in the first condition.