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“Competition and the Hold-Up Problem: a Setting with Non-exclusive Contracts”

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Abstract

This work studies how the introduction of competition to the side of the market offering trading contracts affects the equilibrium investment profile in a bilateral investment game. By using a common agency framework, where contracts are not exclusive, we find that the equilibrium investment profile depends on the competitiveness of the equilibrium outcome. Full efficiency can only be implemented when the trading outcome is the most competitive. Moreover, lowering the outcome competitiveness is not always Pareto dominant for the parties offering the contracts, and larger social welfare can be obtained with low competitive equilibria.

Keywords: bilateral investment; hold-up; competition; Pareto dominance; social surplus.

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1 Introduction

In many economic situations, parties undertake relation-specific investment to increase potential gains from the relationship. Consider for instance an insurer that researches on possible contingencies to better suit the special needs for his contractor; or a seller that reduces the production cost of an intermediate good that is specific to a downstream producer. Even if the potential gains from trade increases, the ex-ante decision to undertake relation specific investment depends on the extent that the investing party can appropriate the gains arising from his investment. Economists have extensively studied this subject and have shown that efficient investment decisions fail to materialize whenever the investing party is not able to appropriate all the benefits generated from his specific investment. In other words, transaction-specific investments results into a fundamental transformation in market transactions, by reducing the field of available alternatives from a large number (in the ex-ante bargaining situation) to a small number (in the ex-post bidding situation), Williamson (1983). Then, economic agents get wrong investment incentives due to the problem of being "held-up", and decide to undertake lower levels of investment. This has detrimental effects on resource allocation and economic welfare.

The existence of the "hold-up" problem is generally traced to incomplete contracts, that is, the inability of parties to write contracts depending on all relevant and publicly available information.[1] The economic literature has mainly focused on two different approaches to solve this problem. The first approach, organization design, is closely related to the theory of the firm and searches for conditions which determine when transactions should be undertaken trough a price mechanism - market - or by fiat - firm. It also establishes provisions for asset ownership and dictates that the residual right of control should be given to the party who is more prompt to suffer from the ex-post opportunism by the other side, Hart (1995). The second is the long-term contract approach. It focuses on establishing contractual provisions such as default or option contracts, that can be enforced in case of disagreement, which relaxes potential conflicts of interests between the parties. However, the main caveat is that the previous solutions pre-assume the existence of economic institutions allowing for either a

[1]If specific investment was verifiable or enforceable ex-post, it will be in the interest of the contractual parties to write compensation schemes linked to investment. Grossman & Hart (1986), Grout (1984), Hart and Moore (1988) and Williamson (1985).

clear definition and allocation of property rights and/or the existence of a third party able to enforce ex-ante contracts. Hence, can we implement efficient levels of investment in a setting with weak economic institutions? In the present work, we propose a very simple mechanism consisting on introducing competition to one side of the market and allocating of the whole bargaining power to this competing side.

We consider an economy with a single large producer/ buyer who must use a specific input provided by different sellers. Consistent with a bilateral investing game, only one of the sellers is aware of the technology that enables him to reduce the cost of input production, and the producer/buyer invests to improve her valuation of the input by adapting her production process to this specific input. Our objective is to see how the introduction of competition to the side of the market offering trading contracts affects the equilibrium investment profile in a bilateral investment game. Since we are interested in a situation where there is competition in the market, we work in a set-up where the buyer can sign trading contracts with many sellers at the same time. We find evidence of this type of contracts in the cycling industry, where large brands of elaborated cycling components such as Shimano, Specialized and Trek buy raw materials and other simple components from different suppliers. Another example is provided by the financial sector, where a large firm establishes multiple banking relationships and customers also hold multiple credit cards from different networks.[2]

In a common agency framework, where trading contracts are not exclusive, we find that the equilibrium investment profile depends on the degree of competition in the trading game. By allowing active sellers to coordinate their out of equilibrium trading offers we show that the upper-bound on the transfer that each seller obtains depends on the "loss" on the trading surplus generated when the buyer excludes him from trade. This "loss" directly depends on the gains from trade that the rest of sellers are able to attain. Hence, the higher the number of sellers coordinating their out of equilibrium offers aimed at excluding any seller, the larger the gains from trade that can be generated and the lower is the equilibrium transfer of the former. The equilibrium outcome is more competitive the higher the number of active sellers coordinating their out of equilibrium offers.

We show that the equilibrium investment profile depends on this "intensive" degree of competition ex-post, which arises in the trading game once specific investment is made. We

[2] Open listing agreements is another example of non-exclusivity in the real estate market.

obtain that full efficiency can only be implemented when competition is the most intense. Here, the offering parties appropriate their marginal contribution to the trading surplus. In situations where the equilibrium outcome is less competitive, full efficiency is never guaranteed. The "hold-up" problem affects the receiving party while the offering party tends to over-invest. With low competition, each seller obtains more that his marginal contribution to the trading surplus and the gains appropriated by the investing seller are larger than the increase on the surplus generated by his investment. We also find a relationship between the equilibrium investment profile and the "extensive" degree of competition, which corresponds to the number of active sellers in the industry. The higher the number of active sellers, the equilibrium investment profile tends to efficiency regardless of the competitiveness of the trading outcome.

We further explore which level of competition leads to a larger ex-ante efficiency, and we analyze which degree of competition is preferred by the side of the market offering the trading contracts. In our model, the most preferred equilibrium is not always the one that makes competition less severe, since the investment profile depends on competition. In this regard, the purpose of an investing party might not longer be to appropriate as much gains from the relationship as possible, because this will have an effect on the investment decision of the other party, and so in the overall potential gains. In some situations, it is beneficial to distribute potential gains from trade evenly among different participants, and this is obtained when the outcome of the trading game is very competitive. Moreover, we show that in situations where the level of competition is low, the results are also influenced by the sensitivity of the equilibrium allocation on investment. When a seller is more efficient that the rest, due to his specific investment, he is indirectly putting a constraint on the transfers of the other sellers. An increase of investment of the seller entails a reduction of the amount traded by the non-investing sellers. If this effect turns out to be small, the incentives of the sellers are aligned and they prefer a more favorable partition of the surplus. Different degrees of competition are preferred when the previous effect is big. While the investing seller opts for an equilibrium where the investment is maximal, this is not the case for the non-investing sellers. This comes from the fact that investments are strategic complements, then a higher investment of the buyer entails larger investment of the seller which in turn creates lower trade for the non-investing sellers.

This strategic complementarity, also explains why the maximization of welfare is not always obtained under situations when the outcome is more competitive. The inefficiency created to one side of the market can restore efficiency to the other side, leading to larger potential gains from trade. Surprisingly, we get that lower competitive outcomes ex-post can lead to higher levels of welfare. Therefore, a competition authority should be careful in analyzing an industry where ex-ante specific investments are important, because promoting competitive outcomes might fail to maximize the total welfare that can be generated in the market.

The remaining of the paper is organized as follows. Section (2), briefly discusses the related literature. Before introducing the formal set-up of the model, in section (3), we provide a simple numerical example. Later, we proceed by solving the game backwards. Therefore, in section (5.1) we study the properties of our equilibrium allocation and the equilibrium transfers are characterized in section (5.2). In section (5.3), we obtain the equilibrium investment profile. Equilibria comparison is undertaken in section (6). We start by analyzing which is the Pareto dominant equilibrium by the offering parties and we continue by ranking equilibria in terms of welfare. Finally, section (7) concludes. All proofs are relegated to the appendix.

2 Related Literature

The present work builds on the literature on markets and contracts. In this literature instead of considering the impossibility of contracting on some states of nature or actions, there are limits on the number of parties that can be part of the same contract. In this paper, we use the most recent set-up where trading contracts are non-exclusive, and a common agent can freely sign multiple bilateral trading contracts with different parties.[3] The first theoretical work to consider a general model of contracting between one agent and multiple principals is due to Segal (1999). In a general framework, he shows that with the absence of direct externalities, the contracting outcome is efficient.[4] However, in a bidding game - where multiple principals propose trading contracts to the common agent - inefficiencies can arise from the coexistence of offers made by different parties. In this regard, Bernheim & Whinston (1986) show that

[3]Earlier studies have centered the analysis on exclusive contracts, this is the spirit of Akerlof (1970), Rothschild & Stiglitz (1976) and Aghion & Bolton (1987).

[4]There are no externalities when the principals' payoffs depend only on their own trade with the agent.

an equilibrium always exists and it is efficient in the absence of direct externalities.[5]

While it has been shown that under some mild conditions a unique efficient outcome always exists, it has been proven that there is multiplicity in the equilibrium payoffs, Chiesa & Denicolò (2009).[6] Restricting to non-linear schedules, the payoffs of the principals - the ones that offer the trading contracts - depend on the transfers or fixed payment that they obtain for the equilibrium amount traded. In a common agency framework, this is determined by the threat of any principal to be excluded from trade, and this threat pins down to which type of "latent" or out of equilibrium contracts are submitted by the rest of the principals. If the principals submit latent contracts such that there exists optimal collective replacement of a given principal, then the equilibrium transfers are truthful in the sense that each principal appropriates his marginal contribution to the surplus. Conversely, if the latent contracts submitted are such that any principal is unilaterally replaced by the most efficient seller, the equilibrium transfers are the ones where the rent of the common agent is minimized. The authors establish which strategies will support this last equilibrium. By giving structure to the out of equilibrium offers such that principals can coordinate their out of equilibrium trading contracts to exclude any other principal, we obtain a characterization of the equilibrium payoffs that lie between the two previous equilibria.

In a more recent paper, Chiesa & Denicolò (2012) undertake comparative statics of the two extreme equilibria. They show that the equilibrium where the rent of the buyer is minimized is Pareto dominant from the parties offering the contracts, and state that truthful strategies are not in general very attractive. This comes from the fact that the potential gains from trade are irrelevant of the distribution of rents and those who submit contracts always prefers an equilibrium where the distribution is more favorable to them. We challenge their finding by introducing a previous stage where parties can undertake specific self-investment before the contracting stage takes place. Moreover, by introducing an investment stage in our game, we are able to compare equilibria with regards to the welfare obtained. This analysis has not been carried out in the markets and contracts literature, where the different type of equilibria are only a different way to distribute the rents from trade, and welfare remains

[5]The authors consider an equilibrium where the principals submit global truthful schedules.

[6]Indeed, the authors show that the set of equilibrium payoffs is a semi-open hyper-rectangle. Additionally, Martimort & Stole (2009) show multiplicity of equilibria in a public common agency game and offer strategies for equilibrium refinement.

unchanged. Therefore, in our model, the redistribution of rents has implications on the investment decisions of the parties and on the final dimension of those gains.

In this regard, the present work is closely related to the "hold-up" literature where an early formulation is due to Klein, Crawford & Alchian (1978) & Williamson (1979, 1983). In those papers, the "hold-up" problem arises due to the fact that parties are unable to bargain over specific investment once they have been made as they are unverifiable. In our model the "hold-up" problem does not arise from non verifiability but from the fact that investments are not contractable. The literature concludes that in the absence of ex-ante contracts, investment is likely to be inefficiently low under any possible bargaining game, Grossman & Hart (1986) and Hart & Moore (1990). The literature has then centered in ways of designing a mechanism to restore the efficient levels of investment as in Aghion, Dewatripont & Rey (1994), Chung (1991) and Edlin & Reichelstein (1996). However, in our model ex-ante contracts are not considered. This relates to the recent literature on competition and the "hold-up" problem as in Cole, Mailath & Postlewaite (2001a, 2001b); Mailath, Postlewaite & Samuelson (2013); Felli & Roberts (2012); Makowski (2004) and Samuelson (2013). However, all those models consider a matching mechanism where, once investment has been undertaken, agents decide on the trading partner. Hence, investment works as a mechanism to increase the outside option giving higher incentives to invest. Departing from this literature, the offering part of the market competes by offering trading contracts to the monopolistic side.

3 Numerical example

Before presenting the formal model, we expose a simple numerical example that illustrates some of the results of the paper. Consider an economy with a common buyer and three competing sellers (i=1,2,3) producing an homogeneous good, whose utility and cost functions are respectively given by:

$$U(X \mid b) = 10 + (9 \times b + 1) \times \log(X) - K \times b;$$

$$C_1(x_1 \mid \sigma) = \frac{2x_1^2}{\sigma + 1} - \psi \times \sigma; \qquad C_i(x_i) = 2x_i^2 \text{ for } i = 2, 3,$$

where the buyer and seller 1 are able to undertake a discreet investment decision $b \in \{0,1\}$ and $\sigma \in \{0,1\}$, that increases the utility of consumption and reduces the cost of production respectively. The total amount traded is the sum of the individual amounts $X = x_1 + x_2 + x_3$, and the fixed cost of investment for the buyer and the seller are $K > 0$ and $\psi > 0$ respectively. Later in the paper, we elaborate more on our notion of competition and this depends on the proportion of the trading surplus that the sellers appropriate. For simplicity, we present here the results for the two extreme equilibria. Hence, the most competitive equilibrium is the one where each seller appropriates his marginal contribution to the trading surplus. In the least competitive equilibrium, the rent of the buyer is minimized. The following table presents the bounds for the fixed costs below which each party decides to invest. In brackets we represent the investment decision of the other party. The first column stands for the investment thresholds under efficiency, while the second and the third are for the highest and the lowest level of competition respectively.

Bounds	Efficient	Highest competition	Lowest competition
$K_{\{\sigma=0\}}$	5.71	0.244	0
$K_{\{\sigma=1\}}$	7.012	1.34	0.979
$\psi_{\{b=0\}}$	0.143	0.143	0.1614
$\psi_{\{b=1\}}$	1.43	1.43	1.614

Figure 1: Bounds below which the parties decide to invest. The brackets stand for the investment decision of the other side of the market.

Compared to efficiency, the common buyer decides to invest less often in equilibrium, and the likelihood of investment is larger whenever the trading outcome is more competitive. The contrary applies for the seller, whose incentive to invest is higher in a less competitive equilibrium, and the thresholds of investment in the most competitive equilibrium coincide with efficiency. Since the investment bounds increase with the investment of the other party, investment decisions are strategic complements. This complementarity leads to situations where, in a less competitive equilibrium, the buyer is more prompt to invest. This happens whenever the fixed cost of investment of the seller is $\psi \in (1.43, 1.614)$, which makes the investment threshold for the buyer to be larger in the least competitive equilibrium, i.e., $K^{LC}_{\{\sigma=1\}} = 0.979 > 0.244 = K^{HC}_{\{\sigma=0\}}$. Where LC and HC stands for the lowest and highest level of competition respectively.

In the table below, we expose the equilibrium payoffs of the sellers depending on the equilibrium investment profile. In the columns, we represent all the possible investment profiles and the rows stand for the two extreme equilibria considered. Because investment decisions depend on the level of competition ex-post, it might be beneficial for the sellers to end-up with a more competitive outcome, since this gives more incentives for the buyer to invest. The red numbers represent the payoffs for the investing seller, and we see that he is better with higher levels of competition if the buyer changes his investment decision, i.e., $\pi_1^{HC}(b=1,\sigma=1) = 3.465 > 0.375 = \pi_1^{LC}(b=0,\sigma=1)$. This is also the case for the non investing sellers even for an unchanged investment of the buyer $\pi_i^{HC}(b=1,\sigma=0) = 2.02 > 1.47 = \pi_1^{LC}(b=1,\sigma=1)$, represented in blue in the table. Here, a more competitive equilibrium gives higher payoffs since it reduces the investment of the competing seller.

$(b,\sigma) =$	$(0,0)$	$(1,0)$	$(0,1)$	$(1,1)$
$\pi_1^{HC}(b,\sigma)$	0.202	2.02	0.346	3.465
$\pi_i^{HC}(b,\sigma)$	0.202	2.02	0.143	1.43
$\pi_1^{LC}(b,\sigma)$	0.214	2.14	0.375	3.755
$\pi_i^{LC}(b,\sigma)$	0.214	2.14	0.147	1.47

Figure 2: Sellers' payoffs depending on the level of competition and ex-ante investment profile.

Because the investment equilibrium profile depends on the degree of competition ex-post, the offering parties may prefer lower levels of competition and welfare may be maximized with lower levels of ex-post competition. To illustrate this last point, consider that the fixed costs of investment of the seller and the buyer are between $\psi \in (1.43, 1.614)$ and $K \in (0.244, 0.979)$ respectively. Notice that, for this range of cost parameters, both equilibria are inefficient since efficiency requires only the buyer to invest. Nevertheless, in the most competitive equilibrium nobody invests, and in the least competitive both parties invest. To see which equilibrium performs better, we compare the welfare obtained in both equilibria $W^{HC} = TS_{\{b=\sigma=0\}}$ and $W^{LC} = TS_{\{b=\sigma=1\}} - K - \psi$. Where $TS_{\{b,\sigma\}}$ stands for the trading surplus that can be generated for a given investment profile (b,σ), and this is equal to the utility from consumption minus the cost of production. The difference in welfare is then

$$W^{HC} - W^{LC} = TS_{\{b=\sigma=0\}} - (TS_{\{b=\sigma=1\}} - K - \psi) = 9.35 - (16.5 - K - \psi) = -7.15 + K + \psi,$$

and for the extreme values of the investment costs, i.e. $\bar{\psi} = 1.61$ and $\bar{K} = 0.97$, we get

$$W^{HC} - W^{LC} = -7.15 + 1.61 + 0.97 = -4.53 < 0.$$

Therefore, for the investment costs considered, the lowest degree of competition does better than the highest, and inefficiency of one side of the market restores efficiency to the other side.[7]

4 Model

We consider a bilateral investment game where a monopolistic buyer trades with many ex-ante identical sellers. In our model, there are N exogenous sellers indexed by $i \in \{1, ..., N\}$, who produce an homogeneous specific input for the buyer. We have a game consisting of two

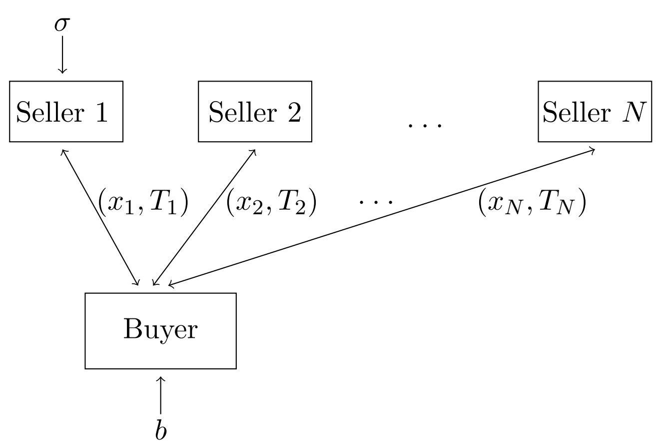


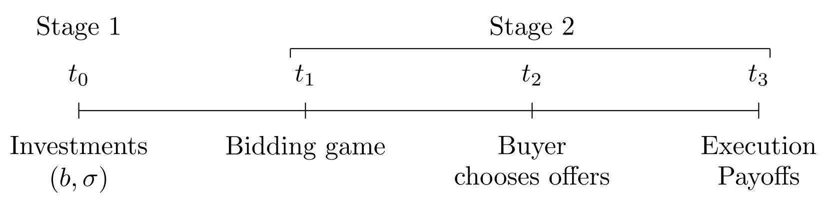
Figure 3: Bilateral investment game with N competing sellers.

stages that are played sequentially. In stage one, specific investment takes place. Here, only seller 1 can invest in a cost-reducing technology, that allows him to produce more efficiently. The amount of investment is a continuous variable $\sigma \geq 0$, with a convex cost $\psi(\sigma)$. The buyer

[7]The calculation of the social surplus and equilibrium allocation are available upon request to the author.

also undertakes specific investment to enhance her valuation of the total amount traded. She takes a binary decision whether or not to invest $b = \{0, 1\}$, and incurs a fixed costs of K. This binary investment is consistent with the buyer's decision on whether to adapt his production process to the input provided by the sellers. We assume that investing parties do not have any budget constraint because they are not financially restrained on the amount of investment they take.

In stage two, each seller trades with the common buyer. As in Chiesa & Denicolò (2009), we consider a bidding game in which trade is modeled as a first-price auction in which the sellers simultaneously submit a menu of contracts and the buyer then chooses the quantity she purchases from each. A typical trading contract is a pair $m_i = (x_i, T_i)$, where $x_i \geq 0$ is the quantity seller i is willing to supply and $T_i \geq 0$ is the corresponding total payment or transfer from the buyer towards seller i. Because trade is voluntary, we require that the null contract is always offered in equilibrium, i.e. $m_i^0 = (0, 0)$. The moves of the game are then.



Our model is one of complete information, hence the equilibrium concept is sub-game perfect Nash (SPNE). Even if investment is observable, it is not contractable and this is because investment cannot be enforced by a third party.[8] We proceed by analyzing the payoffs and the surplus generated from trade.

4.1 Payoffs and trading surplus

The payoffs of the buyer and the sellers are quasi-linear in transfers.[9] The buyer obtains

$$\Pi(b) = U\left(X \mid b\right) - \sum_{i=1}^{N} T_i - K \times b, \tag{4.1}$$

[8] Additionally, the amount of trade that a single seller offers to the buyer cannot be made conditional on the amount of trade that the buyer undertakes with other sellers. Hence, our model is one of private agency.

[9] This assumption means that all parties have a constant marginal utility for money. Furthermore, this allows us both to technically reduce the complexity of the problem and focus our analysis on welfare comparison.

where $X = \sum_{i=1}^{N} x_i$ is the total quantity traded, and the seller's 1 payoff is

$$\pi_1(\sigma) = T_1 - C\left(x_1 \mid \sigma\right) - \psi(\sigma). \tag{4.2}$$

For the rest of the sellers, the payoff does not directly depend on the investment profile and it is equal to

$$\pi_i = T_i - C\left(x_i\right), \quad \text{for all } i \neq 1. \tag{4.3}$$

Finally, given the vectors of investment (b, σ) the maximal trading surplus is

$$TS^*(b, \sigma) = \max_{x_1, \ldots, x_n} \left[U(x_1 + \ldots + x_n \mid b) - C(x_1 \mid \sigma) - \sum_{i \neq 1} C(x_i) \right], \tag{4.4}$$

and $\mathbf{x}^* = (x_1^*, \ldots, x_N^*)$ is the vector of quantities that solves the problem. For later use, we denote by $X^*_{\{-H\}} = \sum_{i \notin H} x_i^*$, the sum of the efficient quantities without taking the quantities of the subset of sellers in H. We finish by stating the assumptions regarding the utility and costs functions:

1. $U'_x(\cdot) > 0,\ U''_{xx}(\cdot) < 0,\ U(X \mid b = 1) > U(X)$ and $U'_x(X \mid b = 1) > U'_x(X)$,

2. $C'_x(\cdot) > 0,\ C''_{xx}(\cdot) > 0,\ C'_\sigma(\cdot) < 0,\ C''_{x\sigma}(\cdot) < 0,\ \psi'_\sigma(\sigma) > 0$ and $\psi''_{\sigma\sigma}(\sigma) > 0$,

3. $\lim_{X \to 0} U'_x(\cdot) = +\infty$, $\lim_{X \to \infty} U'_x(\cdot) = 0$, $\lim_{x_i \to 0} C'_x(\cdot) = 0$ and $\lim_{x_i \to \infty} C'_x(\cdot) = +\infty$.

5 Analysis

We solve the model backwards and we obtain the sub-game perfect Nash equilibrium (SPNE). We first state the equilibrium of stage two, represented by an equilibrium allocation and transfer, i.e. (x^e, T^e). After describing the properties of the equilibrium allocation, we characterize a subset of the equilibrium transfers by assuming that sellers are able to coordinate their out of equilibrium offers. Later, we characterize the equilibrium investment profile. Finally, we undertake equilibrium comparison where we rank equilibria with regards to Pareto dominance and welfare.

5.1 Allocation in the trading game

The equilibrium allocation in the trading game depends on the investment decisions undertaken at stage one of the game. Here, we characterize the equilibrium allocation for a given vector of investment (b, σ). Because the production cost of each seller only directly depends on the amount of input he produces, his individual payoff is not directly affected by the trading contracts submitted by all other sellers. Hence, given the contracts of all other sellers each seller effectively plays a bilateral trading game with the buyer where he has the whole bargaining power. Hence, when submitting a contract, he maximizes the potential gains from trade that can be generated between him and the buyer.[10]

$$U\left(\bar{X}^*_{-i} + x^*_i \mid b\right) - \sum_{j \neq i} T_j - C(x^*_i \mid \sigma) > U\left(\bar{X}^*_{-i} + \hat{x}_i \mid b\right) - \sum_{j \neq i} T_j - C(\hat{x}_i \mid \sigma); \text{ for any } \hat{x}_i \geq 0.$$

In words, seller i does not profit by deviating from x^*_i to any $\hat{x}_i$. Since this holds true for every seller $i \in N$. Consequently, the efficient allocation is indeed a Nash equilibrium defined by the following system of equations:

$$\begin{aligned} U'_x(X^* \mid b) &= C'_x(x^*_1 \mid \sigma) \qquad \text{for} \quad i = 1, \\ U'_x(X^* \mid b) &= C'_x(x^*_i) \qquad\quad\; \text{for all } i \neq 1, \end{aligned} \tag{5.1}$$

where the marginal utility of consumption equals the marginal costs of production.

We proceed to analyze how the efficient allocation changes with the investment undertaken at stage one. The result is shown in the following lemma, whose proof, presented in the appendix, page 39, comes directly from the previous system of equations.

Lemma 1. *In an equilibrium with the efficient allocation:*
i) for a given investment of the buyer, an increase on the investment by seller 1 rises the amount of trade between the buyer and seller 1, but decreases the amount of trade with all other sellers. The total amount traded increases.

$$\frac{dx^*_1}{d\sigma} > 0; \quad \frac{dx^*_j}{d\sigma} < 0 \;\; \textit{for all } j \neq 1 \quad \textit{and} \quad \frac{\partial}{\partial\sigma} X^* > 0.$$

[10] The first and the second conditions are called "individual excludability" and "bilateral efficiency" in the market and contracts literature Bernheim & Whinston (1996) and Segal (1999).

ii) For a given investment of the seller, the amount of trade for each seller increases.

$$x_i^*(1, \sigma) > x_i^*(0, \sigma) \qquad \forall i \in N.$$

For a given investment of the buyer, the higher the investment undertaken by seller 1 the more efficient he becomes with respect to the other sellers. In the trading stage, the buyer substitutes trading from the other sellers to seller 1. Nevertheless, this substitution effect is of second order, since the economy in aggregate is more efficient, the total amount traded is higher. We denote by the "allocative externality" the magnitude on the decrease of trade to the other sellers, and this depends on the primitives of the economy.[11] With regard to the second part of the lemma, as long as the investment of the seller does not change, the relative efficiency of the sellers stays the same, and if the buyer decides to invest, she trades a higher amount with every seller. In this regard, the investment of the buyer works as a public good and all sellers benefit from her investment.

5.2 Equilibrium transfers

By restricting attention to the efficient allocation, we proceed to characterize the equilibrium transfers. The literature on markets and contracts has shown that in the absence of restrictions on the trading offers, there is multiplicity in the equilibrium transfers. Here, we focus on a subset of equilibrium transfers which are obtained by assuming that sellers are able to coordinate their out of equilibrium offers aimed at excluding any seller i from trade. The following two definitions are crucial to characterize the equilibrium transfers.

Definition 1. ***(Coordination)*** *A set H of sellers coordinate their offers if the gains from*

[11] In our model we have assumed that sellers produce products that are homogeneous. However, the degree of substitutability will have a strong effect on the externality that investment by one seller creates to the equilibrium allocation of others. With perfect homogenous products, the buyer can perfectly substitute products from sellers and we expect that the indirect externality coming from investment of seller 1 is big. In our model the degree of substitutability depends on the primitives of the economy, that is, on the convexity of the cost function. Conversely, if products have some degree of heterogeneity, the buyer will not reduce much the amount that she trades with other sellers after an increase of investment by seller 1. Therefore, the degree of the indirect externality will depend on the product substitutability. I am thankful to professor Sánchez-Pagués for this observation.

trade that can be generated between them and the buyer is the largest.

$$V_H\left(X^*_{-\{H\}} \mid b\right) = \max_{\{x_j\}_{j\in H}} \left[U\left(X^*_{-\{H\}} + \sum_{j\in H} x_j \mid b\right) - \sum_{j\in H} C(x_j \mid \sigma)\right]. \tag{5.2}$$

We denote by $\tilde{x}_j(\cdot \mid H)$ the quantity that solves the problem in expression (5.2). By assuming that sellers are able to coordinate their offers we define the loss of exclusion of a given seller i.

Definition 2. ***(Exclusion loss)*** *Those are the trading gains that cannot be realized due to the exclusion from trade of a given seller.*

The loss of exclusion for a given seller is directly related to the gains than can be attained by the rest of the sellers, and those gains depend on the number of sellers coordinating their out of equilibrium offers.[12]

Following Chiesa & Denicolò (2009), we know that the difference of the gains from trade that can be generated between the sellers in J_i and the common buyer with and without any seller i are equal to

$$V_{J_i}\left(X^*_{-\{J_i\}} \mid b\right) - V_{J_i}\left(X^*_{-\{J_i,i\}} \mid b\right),$$

where

$$V_{J_i}\left(X^*_{-\{H\}} \mid b\right) = \max_{\{x_j\}_{j\in J_i}} \left[U\left(X^*_{-\{H\}} + \sum_{j\in J_i} x_j \mid b\right) - \sum_{j\in J_i} C(x_j)\right].$$

With simple algebra, and assuming that the sellers in J_i are able to coordinate their out of equilibrium offers, we obtain that the "loss" of exclusion of seller i by the set J_i is equal to

$$L_i\left(J_i\right) = \left(U\left(X^* \mid b\right) - \sum_{j\in J_i} C(x^*_j)\right) - V_{J_i}\left(X^*_{-\{J_i,i\}} \mid b\right), \tag{5.3}$$

where

$$V_{J_i}\left(X^*_{-\{J_i,i\}} \mid b\right) = \max_{\{x_j\}_{j\in J_i}} \left[U\left(X^*_{-\{J_i,i\}} + \sum_{j\in J_i} x_j \mid \tilde{x}_i = 0, b\right) - \sum_{j\in J_i} C(x_j)\right]. \tag{5.4}$$

[12] In the literature of markets and contracts, this out of equilibrium contracts are called "latent" contracts and those are the offers or trading contracts that are never accepted by the buyer, but effectuate a constraint on the equilibrium transfer of sellers.

As before, $\tilde{x}_j(\cdot \mid J_i)$ is the amount of trade that solves the previous expression. The "loss" of exclusion is computed by putting equal to zero the trading quantity of seller i, keeping constant the production of the seller not in J_i, and choosing optimally the quantities of the sellers belonging to J_i.[13] The convexity of the cost function, makes it straightforward to see that the "loss" of exclusion $L_i(J_i)$ is weakly decreasing in $J_i : J_i \supset J_i' \Longrightarrow L_i(J_i') \leq L_i(J_i)$.[14] That is, the more the number of sellers coordinating their out of equilibrium offers, the larger is the trading surplus that they can generate and the lower is the "loss" of exclusion. Regarding the quantities that are submitted in the out of equilibrium offers, we introduce the following lemma that will be useful for the rest of the paper.

Lemma 2. *For any investment profile (b, σ) and any J_i, the total amount traded is higher when all sellers effectuate trade with the buyer.*

$$X^*(b,\sigma) > X^*_{-\{J_i,i\}}(b,\sigma) + \sum_{j \in J_i} \tilde{x}_j(b,\sigma \mid J_i), \quad \textit{for any } \ i \in N.$$

and

$$\tilde{x}_j(b,\sigma \mid J_i') > \tilde{x}_j(b,\sigma \mid J_i) > x_i^*(b,\sigma); \quad \forall j \in J, J' \textit{ and } J' \subset J.$$

The formal proof is in the appendix, page 39. From to the convexity of the cost function, the increase of the total amount traded, due to the extra seller, always dominates the increase on the amount traded from the subset of sellers J_i. It is immediate to see that the individual amount that any seller $j \in J_i$ submits in his out of equilibrium contract, is larger than his efficient amount. Because those offers aim at excluding one seller from trade, they have to offer a larger amount to the buyer.

Because in our model trade is voluntary and the sellers have the whole bargaining power, the equilibrium transfer of any seller i is the maximal monetary amount such that the common buyer is indifferent between trading or excluding him from trade.[15] Hence, the equilibrium

[13] Chiesa & Denicolò (2009) consider the case where the set of sellers is a singleton, and we extend the case for any number of sellers. Using a similar methodology it can be shown the existence of the strategies that will support the equilibrium transfers.

[14] In general this will be strictly decreasing.

[15] Chiesa & Denicolò (2009) talk about the "threat" that a given seller is replaced from trade. They state that the upper-bound of the transfer that each seller can ask for supplying the efficient amount x^* depends on the threat of being excluded from trade, and this is related on how aggressively any other seller bids for quantities that are larger than the efficient ones.

transfer for any seller i cannot be greater than the "loss" of exclusion $L_i(J_i)$ for any J_i as the buyer will decide not to trade with him. This cannot be lower, as seller i has a profitable deviation to increase it. Hence, we center attention to the case where the equilibrium transfer equals to this "loss" of exclusion $T_i^e(J_i) = L_i(J_i)$.

Now, we can easily obtain the equilibrium payoffs in the trading game, which are stated in the following proposition.

Proposition 1. *For a given J_i, and an investment profile (b, σ), the equilibrium payoffs of the sellers are:*

$$\pi_1(b, \sigma \mid J_1) = TS^*(b, \sigma) - \tilde{TS}_{-1}(b \mid J_1) - \psi(\sigma); \quad \textit{for } i = 1, \tag{5.5}$$

$$\pi_i(b, \sigma \mid J_i) = TS^*(b, \sigma) - \tilde{TS}_{-i}(b, \sigma \mid J_i); \qquad \textit{for } i \neq 1, \tag{5.6}$$

and the one of the common buyer is

$$\Pi(b, \sigma \mid J) = TS^*(b, \sigma) - \sum_i \left(TS^*(b, \sigma) - \tilde{TS}_{-i}(b, \sigma \mid J_i)\right) - K \times b, \tag{5.7}$$

where $\tilde{TS}_{-i}(b, \sigma \mid J_i)$ is the maximal trading surplus that can be generated when a subset of sellers J_i coordinate their out of equilibrium offers.
ii) $\partial\left(\tilde{TS}_{-i}(b, \sigma \mid J_i)\right)/\partial J_i > 0$, and for $|J_i| < N - 1$ each seller obtains more than his marginal contribution to the trading surplus.

The proof is presented in the appendix, page 40. When all active sellers coordinate their out of equilibrium offers i.e. $J_i = N \setminus \{i\} = \bar{J}_i$, each seller appropriates his marginal contribution to the surplus and the trading gains are evenly distributed to all players. In this case, we consider that the equilibrium outcome is competitive since in this equilibrium, and for a given investment profile, the partition of the trading surplus is the one that minimizes the payoffs of the side of the market offering the trading contracts.[16] For a lower number of sellers coordinating their out of equilibrium offers i.e. $|J_i| < N \setminus \{i\}$, the distribution of the

[16] In this case, the equilibrium payoffs coincide with the so called truthful equilibrium. A strategy is said to be truthful relative to a given action if it truly reflects the sellers' marginal preference for another action relative to the given action. However, in a framework of private common agency with no direct externalities, truthful means that each principal can ask for payments that differ from his true valuations of the proposed trade only by a constant.

gains from trade is biased in favor of the sellers and the rent of the common buyer is reduced. In those cases, we consider that the equilibrium outcome is less competitive, because each seller appropriates more than his marginal contribution to the surplus.

Having identified the equilibrium payoffs, we proceed to study stage one of the game when investment decision are taken.

5.3 Investment profile

We begin this section by characterizing the efficient investment profile and we proceed with the equilibrium. Efficiency serves as a benchmark to compare with equilibrium, and allows us to see whether full efficiency can be restored in situations where ex-ante contracts are not considered. We see that the decisions to invest of both sides of the market depend on the competitiveness of the equilibrium outcome, because this has a direct effect on the part of the trading surplus that each party is able to appropriate.

5.3.1 Efficient investment

The efficient vector of investment is the one that arises when the investing parties appropriate all the gains coming from their own investment. The efficient investment is then uniquely characterized by the solution of the following system of equations

$$\psi'_\sigma(\sigma_{\mathbf{E}}) = -C'_\sigma\left(x_1^*(b,\sigma_{\mathbf{E}}^b) \mid \sigma_{\mathbf{E}}^b\right), \quad \forall\, b; \tag{5.8}$$

$$K \begin{cases} \leq TS^*(1,\sigma_{\mathbf{E}}^1) - TS^*(0,\sigma_{\mathbf{E}}^0) - \left(\psi(\sigma_{\mathbf{E}}^1) - \psi(\sigma_{\mathbf{E}}^0)\right) \equiv K_{\mathbf{E}} & \text{then } b = 1 \\ > K_{\mathbf{E}} & \text{then } b = 0. \end{cases} \tag{5.9}$$

The seller sets the level of investment such that the marginal reduction of the production costs equals the marginal cost of investment. Similarly, the buyer invests if the fixed cost of investment K is lower than the increase on welfare, represented by the threshold $K_{\mathbf{E}}$. A characteristic of the efficient investment profile, that also carries over in equilibrium, is that investments are strategic complements. Hence the more one of the parties invests, the higher are the incentives of the other party to increase investment. Here, this result comes from a variant of super-modularity. From lemma 1, we know that the investment of one party always increases the total amount of trade, and through this the trade allocation, the value of investment by one party increases the marginal return to the other party's investment.[17]

5.3.2 Equilibrium investment profile

In equilibrium, as the investing party does not appropriate all the benefits coming from investment, the implementation of the efficient investment profile is not always possible. Interestingly, we see that efficiency can only be implemented whenever the equilibrium outcome of the trading game is the most competitive. In the analysis that follows, we consider both the "intensive" and "extensive" degree of competition. The former takes into account how many sellers coordinate their out of equilibrium offers aimed at excluding any other seller from trade. The latter, considers how the equilibrium investment profile depends on the number of active sellers in the industry.

Definition 3. ***("Intensive" Competition)*** *An equilibrium outcome is more competitive the lower the partition of the trading surplus that a seller can appropriate. Hence, for a given number of sellers N and a given investment profile, the most competitive equilibria is when $|J_i| = N - 1 = \bar{J}_i$. The least competitive is when $|J_i| = 1 = \underline{J}_i$.*

5.3.3 "Intensive" competition

The ex-post level of competition depends on how many sellers coordinate their out of equilibrium offers. The larger the number of those sellers, the higher are the gains from trade that can be obtained excluding any seller i, and the lower is his "loss" of exclusion. The equilibrium investment decisions are best-response actions, and the following definition states an equilibrium in the investing game.

[17] We have proven in lemma 1 that the amount traded with each seller increases if the buyer is investing, this implies that for a given level of seller's investment we have $x_1^*(1,\sigma) > x_1^*(0,\sigma)$ and together with assumption $C''_{x\sigma}(\cdot) < 0$ we obtain that the right hand side of (5.8) increases with the level of investment of the buyer.

$$\partial(rhs) = -C'_\sigma(x_1^*(1,\sigma) \mid \sigma) + C'_\sigma(x_1^*(0,\sigma) \mid \sigma) = -\int_{x_1^*(0,\sigma)}^{x_1^*(1,\sigma)} C''_{x\sigma}(\tau)d\tau > 0,$$

where the above is true due to lemma 1. A similar argument can be used to see that the investment threshold of the buyer increases with the investment of the seller. In the case that the investment of the buyer is continuous i.e. $b \geq 0$, we have investment complementarity if the function $TS^*(b,\sigma)$ is super-modular in (b,σ) (i.e $TS^*_{b\sigma}(b,\sigma) > 0$; see *Donald Topkins 1978).*

Definition 4. *The vector* (b^e_J, σ^e_J) *constitutes an equilibrium, if and only if:*

$$b^e_J \in arg \max_b \; \Pi\left(b, \sigma^e_J \mid J\right),$$
$$\sigma^e_J \in arg \max_\sigma \; \pi_1\left(b^e_J, \sigma \mid J\right).$$

Because the equilibrium payoff depends on the set of sellers belonging to J, we obtain a direct link between the degree of competition in the trading game and the equilibrium investment profile. It this regard, we are interested to know whether the efficient investment profile can be implemented in equilibrium. This result is introduced in the following proposition.

Proposition 2. *The efficient investment profile is implementable if and only if the outcome of the trading game is the most competitive i.e.* $J_i = N \setminus \{i\} = \bar{J}_i$.

Investment decisions depend on how each party appropriates the gains coming from investment. In the most competitive equilibrium, the gains from trade are evenly distributed and each seller obtains his marginal contribution of the trading surplus. As a result, the investing seller appropriates the increase of the trading surplus coming from his investment. This is never the case when the outcome of the trading game is less competitive. In this case, each seller obtains more than his marginal contribution of the trading surplus, which distorts the incentives to invest efficiently. Hence, given the investment decision of the buyer, the seller always invests efficiently in the most competitive equilibrium. Because the buyer takes the efficient level of investment, under some values for the fixed cost of investment, we obtain the result stated in the proposition. We refer to the appendix for the formal proof, page 41.

From the previous result, we can easily characterize the investment profile when the equilibrium outcome of the trading game is the most competitive. This is introduced in the following corollary whose proof is also relegated to the appendix, page 42.

Corollary 1. *In the most competitive equilibrium, when the buyer fails to take the efficient level of investment, the equilibrium investment profile is characterized by underinvestment.*

The previous two results state that the investment decision of the seller, in the most competitive equilibrium, is constrained efficient. For a given investment of the common buyer, the seller always takes the efficient investment decision. However, when the buyer fails to make the efficient investment decision, the equilibrium investment profile is characterized by the "hold-up" problem, and both parties underinvest. Downward distortions of investment arise because of strategic complementarity, a lower investment of the buyer creates lower potential gains from trade, and this makes also the investing seller to lower the level of investment. In the figure below, we compare this equilibrium investment profile from the efficient one. The line in red represents the region where inefficiencies occur.

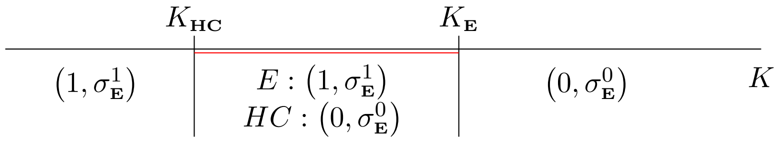


Figure 4: Equilibrium investment profile when the equilibrium outcome is the most competitive. In the horizontal line there is the fixed cost of investment of the buyer and K_{HC}, K_E stand for the investing thresholds for the most competitive equilibrium and efficiency respectively. Full efficiency is implemented whenever $K \notin (K_{HC}, K_E)$.

We proceed to study what are the characteristics of the investment when the equilibrium outcome is less competitive i.e. when the number of sellers coordinating their out of equilibrium offers is lower, i.e. $|J_i| < N \setminus \{i\}$. The result is stated in the following proposition.

Proposition 3. *When the equilibrium outcome is not the most competitive, i.e.* $|J| < N - 1$, *we obtain that:*

i) for a given investment of the buyer, the magnitude of seller's over-investment depends on the level of ex-post competition and the degree of the "allocative externality" $dx^*_m/d\sigma$, *and this is equal to*

$$\gamma(J) = -\sum_{m \notin J,1} \left(\int_{X^*}^{X^*_{-\{J,1\}} + \sum_{j \in J} \tilde{x}_j(J)} U''_{xx}(\tau) d\tau \right) \frac{dx^*_m}{d\sigma}.$$

Over-investment decreases with the level of "intensive" competition, $\partial\gamma(J)/\partial J < 0$.

ii) When the buyer's investment decision is not efficient, this is characterized by underinvestment, and for a given investment of the seller, the region of costs below which the buyer invests increases with the competitiveness of the equilibrium outcome, $\partial K_J(\bar{\sigma})/\partial J > 0$.

Again the formal proof is in the appendix, page 43. Contrary to the case where competition is the most severe, the investment of the seller is distorted upwards. This is due to the fact that he does not only appropriate all the direct gains coming from his investment, but also part of the payoffs from the other sellers.

For a fixed investment of the buyer, the magnitude of over-investment $\gamma(J)$ decreases with J. In other words, the lower is the level of competition - which implies a smaller J - the distortion of investment will be larger. With the same investment of the buyer, the investment of the seller is monotonically decreasing with the level of competition. Additionally, the amount of over-investment depends on how the equilibrium allocation of the non investing sellers changes with respect to the investment of the former. The larger is this "allocative externality" the more the seller over-invests. This is because the "loss" of exclusion depends on his investment profile through the allocation of the out of equilibrium offers that remains unchanged for the sellers who do not coordinate their offers. The larger the investment of the seller the more costly it will be to exclude him, and he consequently obtains a larger transfer at the expense of the other sellers.

Regarding the investment threshold of the buyer, she decides to invest if the fixed cost of production are lower than the following threshold

$$K(J) \equiv TS^*(1, \sigma^1(J)) - TS^*\left(0, \sigma^0(J)\right) - \sum_{i \in N} \left(T^1(J) - T^0(J)\right).$$

It is easy to see that, whenever the seller's investment remains unchanged, this threshold is increasing with the level of competition. Higher competition entails a larger partition of the trading surplus that goes to the buyer and she has larger incentives to invest. However, in equilibrium the investment threshold of the buyer also depend on the investment undertaken by the seller, and due to investment complementarity, this will be positively affected by lower levels of competition. The change of this threshold with regards to the level of "intensive" competition is given by

$$\frac{d\left[K(J)\right]}{dJ} = \underbrace{\frac{\partial k}{\partial J}}_{+} + \underbrace{\frac{\partial K(J)}{\partial \sigma} \times \frac{\partial \sigma}{\partial J}}_{-},$$

whose final sign depends on how the investment of the seller is affected by competition, and this is represented by $(\partial\gamma(J)/\partial J)$.

With regards to the equilibrium investment profile, we introduce the following corollary. Now, ex-ante inefficiencies may arise to both sides of the market.

Corollary 2. *Whenever the buyer takes the efficient investment decisions the seller over-invests.*

i) If the investment decision of the buyer is not efficient, the inefficiency created is two-sided:

A) the buyers underinvests, and

B) the seller over-invests or underinvest depending on how investment affects the efficient allocation of the non investing sellers. Over-investment always appears in equilibrium if the "allocative externality" is sufficiently big, i.e.

$$-\frac{dx_m^*}{d\sigma} > \frac{\int_{x_1^*(1,\sigma_{\boldsymbol{E}}^1)}^{x_1^*(0,\sigma_{\boldsymbol{E}}^0)} C''_{x\sigma}(\tau)d\tau}{(N \setminus \{1\} - J) \times \int_{X^*(0,\sigma_{\boldsymbol{J}}^0)}^{X^*_{-\{J,m\}}(0,\sigma_{\boldsymbol{J}}^0)+\sum_{j\in J}\tilde{x}_j(0,\sigma_{\boldsymbol{J}}^0|J)} U''_{xx}(\tau)d\tau} = \lambda(J).$$

The formal proof is the appendix, page 45, and the figure below represents the equilibrium investment decisions and its differences from the efficient one. One red line represents a situation where inefficiency occurs to only one side of the market, and two lines represents inefficiencies arising to both sides.

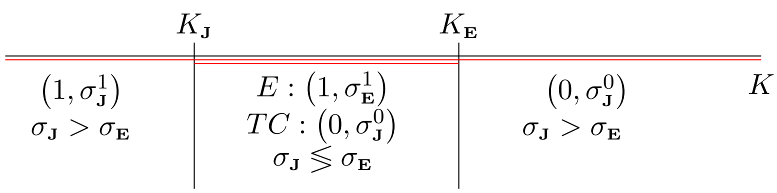


Figure 5: Equilibrium investment profile when the competition ex-post is not the most competitive. In the horizontal line, there is the fixed cost of investment for the buyer and K_J, K_E stand for the investing thresholds in equilibrium and efficiency respectively. Full efficiency is never implemented and it can be double-sided when the fixed cost of investment of the buyer is in $K \in (K_J, K_E)$.

In order to give some clarity of the results, we proceed to represent graphically the equilibrium investment profile depending on the degree of "intensive" competition. On the upper part of the figure 6, picture a) and b) we represent the equilibrium investment of the seller and this crucially depends on the level of competition ex-post and the degree of the "allocative externality" that he creates to the other sellers. This does not only determines the slope of the curve represented in the figure, but also the investment decision of the buyer. We see that whether the investment threshold of the buyer is monotone or not, depends on this "allocative externality". In general, when the effect that the investing seller has on the equilibrium amount traded of the other sellers is small, a more unfavorable partition of the surplus, coming from a lower competitive outcome, dominates the constraining effect that the investing seller creates on

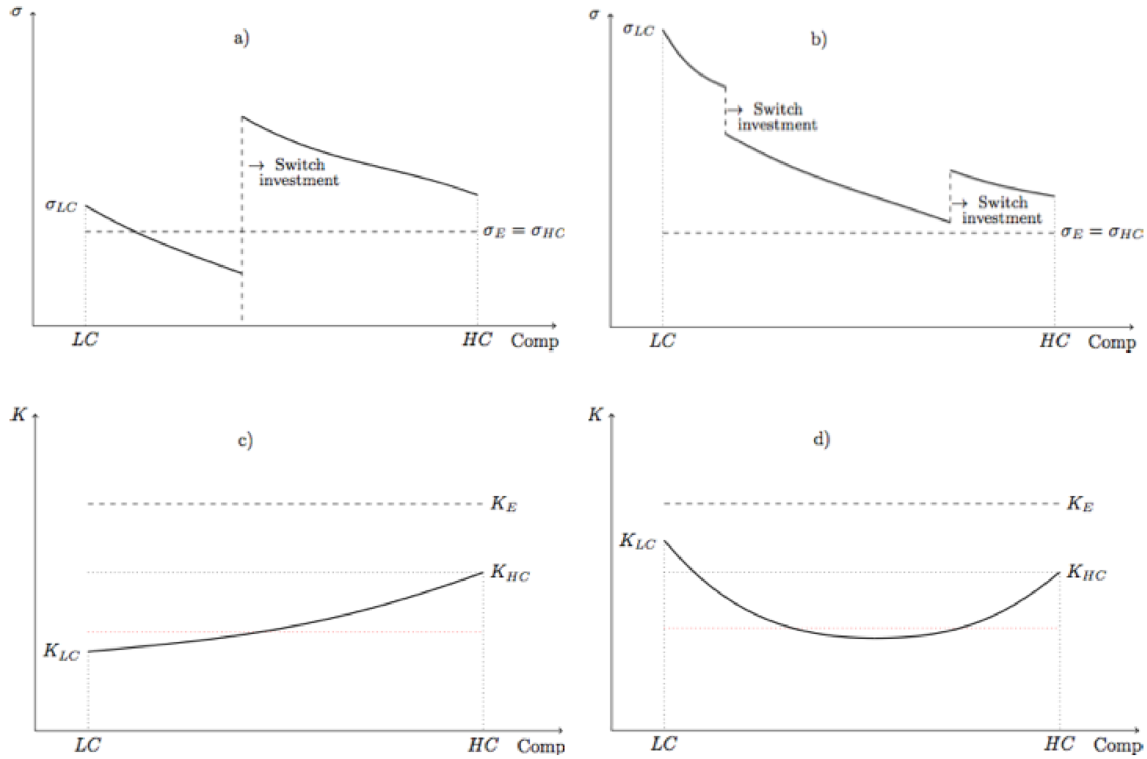


Figure 6: Equilibrium investment profile of the seller and the buyer depending on the level of ex-post competition. The fixed investment cost of the buyer is represented by the red line in pictures c) and d). The left represents a situation with a moderate "allocative externality", and on the right this is large.

the equilibrium transfers of the others $d\left[K(J)\right]/dJ > 0 \rightarrow |\partial k/\partial J| > |\partial K(J)/\partial\sigma \times \partial\sigma/\partial J|$. In this case, the investment threshold of the buyer is monotonically decreasing with lower competitive outcome as it is stated in proposition 3, and represented in picture c). As a result, a lower level of competition entails that the buyer mat decide to switch his investment decision from investment to non-investment, which generates the jump downwards on the investment of the seller as represented in a). Conversely, whenever the "allocative externality" is large, the constraint created to the transfers of the non-investing sellers dominates the more unfavorable partition of the trading surplus. Here, lower competition makes the buyer to undertake a positive level of investment, that will not come about with higher levels of competition. This is represented in picture d), where for low levels of competition the buyer decides to invest.

So far we have established how the competitiveness of the outcome in the trading game affects the investment decisions of both sides of the market. We now proceed to study how the equilibrium investment profile is affected by the "extensive" degree of competition, in order words, on the number of active sellers in the industry.

5.3.4 "Extensive" competition

In this section, we study how the equilibrium investment profile depends on the number of active sellers in the economy. "Ceteris paribus", the larger the number of sellers, the lowers are the cost of exclusion, and hence the lower the transfer that each seller obtains in equilibrium. The trading amount of the excluded seller can be easily substituted by trading more with the rest, and the higher their number, the smaller is the "loss" of trade caused by exclusion. Unsurprisingly, this has an effect on the equilibrium investment profile and the result is stated in the following proposition.

Proposition 4. *Regardless of the level of ex-post competition, full efficiency is implemented provided that the number of active sellers is "sufficiently" large.*

The derivation is in the appendix, page 46, and this comes from the fact that, as the number of sellers increases, each seller is able to appropriate less from the trading surplus, as the "loss" of exclusion decreases with the number of active sellers. When the number of sellers is large, each seller is only able to appropriate his marginal contribution of the trading surplus regardless of the equilibrium outcome. The buyer also invests efficiently, with a larger number of active sellers, each one of them appropriates less from the trading gains generated by the investment of the buyer, and she invests efficiently no matter the fixed costs of investment. The link between unilateral investment decisions and the number of active sellers is illustrated in the figure 7.

The picture illustrates that with one active seller, we have a situation of a bilateral monopoly. Because we consider a game where the seller has the whole bargaining power, the buyer is completely "held-up" and she never invests. Conversely, the investment decision of the seller is the efficient one. With more than one seller, we have a situation where the buyer decides to invests, this is because sellers start to compete for the trading contracts and the former is able to appropriate part of the benefits coming from his investment. With low levels of competition, the seller over-invests as he gets more than his marginal contribution to

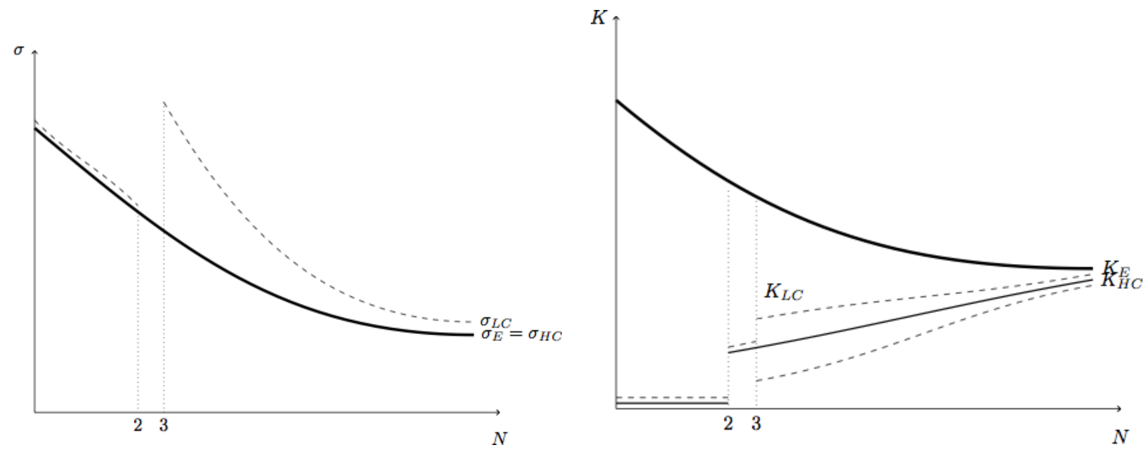


Figure 7: Unilateral investment decisions as a function of number of active sellers. On the left, the investment of the seller and on the right, the threshold below which the buyer decides to invest. The thick solid line stands for the efficient investment profile, the solid line represents a situation where competition is the most severe. The dashed line is the one corresponding to the lowest level of competition.

the surplus. However, since competition increases with a larger number of sellers, in the limit, each seller is only able to obtain his marginal contribution of the trading surplus. Regardless of the equilibrium in the trading game, the buyer is able to get all the benefits coming for her investment with a sufficiently large number of sellers.

With the link between investment and competition, we proceed to undertake equilibrium comparison. We start by introducing the concept of Pareto optimality and we indicate which equilibrium is the one preferred by the parties offering the trading contracts. Later, we depart from the analysis of surplus distribution, and we establish which equilibrium performs best in terms of welfare.

6 Comparison of equilibria

We see that both the concept of Pareto dominance and equilibrium ranking in terms of welfare crucially depend on how competitive the outcome in the trading game is. Moreover, we see that the magnitude of the "allocative externality" that the investing seller creates to the rest of the sellers is of the utmost importance.

6.1 Pareto optimality

In this section, we analyze which equilibrium gives higher payoffs to the side of the market offering the trading contracts. It is obvious that with a given investment profile, the offering parties always prefer a situation where the outcome is less competitive. When the trading surplus remains unchanged, the sellers prefer a setting where the distribution of the surplus is more favorable to them.[18] However, since in our model the equilibrium investment profile depends on the level of competition ex-post, low equilibrium outcomes may not always be preferred. For the investing seller, the trade-off is whether a more favorable distribution has an effect on the investing decision of the buyer. For the non-investing sellers, in addition, there is also the investment decision of the seller and how this affects their equilibrium allocation. The result is presented in the following proposition.

Proposition 5. *Whenever the "allocative externality" is "small" such that* $\partial K_J/\partial J > 0$:
i) the least competitive equilibrium is Pareto dominant for the sellers if the investment decision of the buyer is equilibrium invariant,
ii) otherwise, Pareto dominance is attained with an intermediate level of competition.

Whenever the "allocative externality" is "big" such that $\partial K_J/\partial J < 0$*, the least competitive equilibrium is never Pareto dominant,*
i) while it is always preferred for the investing seller,
ii) the non-investing sellers are always better-off with the most competitive equilibrium.

The formal proof is relegated to the appendix, page 48, and we see that this result comes at no surprise. For convenience we have defined the "allocative externality" of being either "big" or "small" depending on whether the investment threshold of the buyer either increases or decreases with regards to the level of ex-post competition. In figure 8 and 9, we give a graphical interpretation of the result stated in the proposition. While figure 8 represents a situation where the "allocative externality" is small, figure 9 stands for the opposite case.

Because in equilibrium the investment of one party is affected by the other one, and those are strategic complements, we are sure that the investing seller and the buyer are always

[18] This is the case in Chiesa & Denicolò (2009, 2012). They state that the minimum rent equilibrium - the least competitive equilibrium - is Pareto dominant. This result comes from the fact that as long as parties do not invest, the potential trading surplus stays the same, all the sellers are identical and an equilibrium of the trading game represents only a split of the surplus.

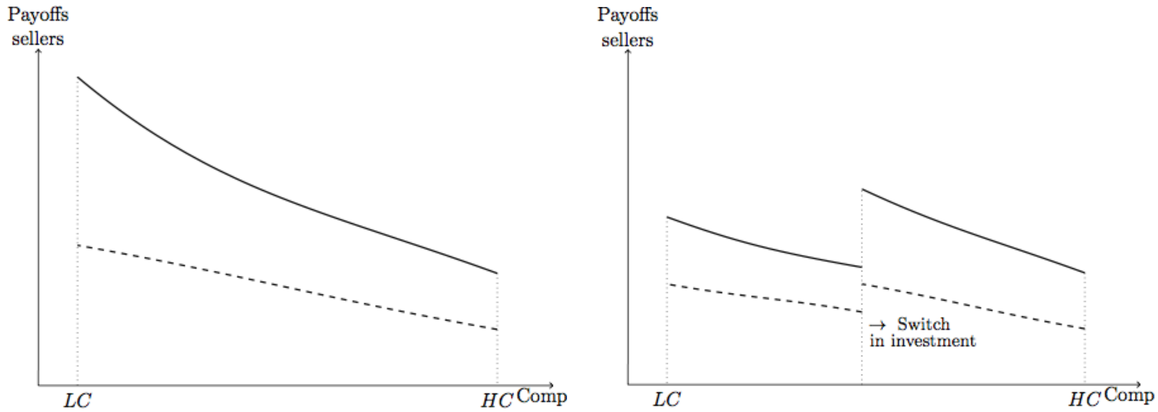


Figure 8: Payoffs of the sellers as a function of the level of ex-post competition. It depicts a situation where the "allocative externality" is small. The black line represents the payoff of the investing seller and the dashed line is the one for the non-investing sellers. The picture on the left stands for a situation where the equilibrium investment of the buyer is equilibrium invariant.

better-off the higher the investments in equilibrium. However, this is not always the case for the non investing sellers where the investment of the buyer and the seller go in opposite directions. The seller who is investing obtains a higher payoff as the investment is superior in a less competitive equilibrium when the investment of the buyer is equilibrium invariant. For the non-investing sellers, they will prefer a division of the surplus that is more favorable as long as the investment of the seller is not very different in all possible equilibria. As we have seen, an increase in the level of investment by the seller creates an "allocative externality" to the other sellers as they trade less with the common buyer. This negative externality then dominates a more favorable partition of the surplus when the difference of investments of the seller is large.

Therefore, Pareto dominance of a less competitive equilibrium is not robust when we introduce specific investment by the trading partners. The intuition behind this result is that under some situations and because both parts of the market are undertaking specific investment, it might be better to agree to a even distribution of the potential gains from trade between all parties than a more asymmetric one, since the latter might induce one of the parties to withdraw from investment.

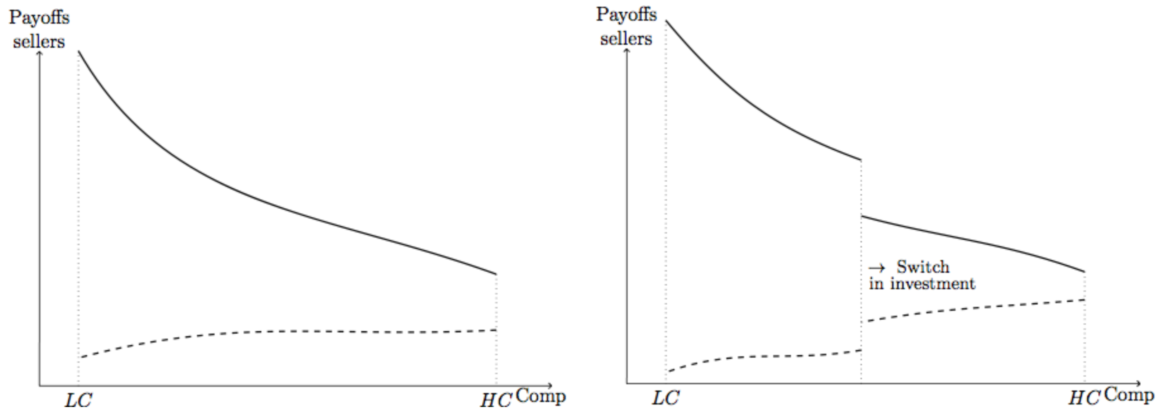


Figure 9: Payoffs of the sellers as a function of the level of ex-post competition. It represents a situation where the "allocative externality" is large. The black line depicts the payoff of the investing seller and the dashed line is the one for the non-investing sellers. The picture on the left stands for a situation where the equilibrium investment of the buyer is equilibrium invariant.

6.2 Welfare

Here, we rank equilibria according to the welfare obtained, and this equals the total gains generated from trade minus the costs of investment

$$W^*(b,\sigma) = TS^*(b,\sigma) - K \times b - \psi(\sigma).$$

From the previous analysis we have seen that ex-ante inefficiencies are more prompt to emerge whenever the ex-post competition is less severe. Hence, the most competitive equilibrium, in general, performs better in terms of welfare. However, we have also established that investments in our setting are strategic complements. Therefore, under some parameters, we find that decreasing the level of ex-post competition may entail larger welfare whenever the "allocative externality" created to the non-investing sellers is sufficiency big. Hence, the inefficiencies in investment, arising to the side of the seller, might work as a mechanism to restore the efficient investment of the buyer. As long as this latter investment has an important contribution to the trading surplus, this is welfare enhancing. The following theorem states this result.

Theorem 1. *When in the most competitive equilibrium, the investment decision of the buyer is not efficient and the "allocative externality" is sufficiently "big", welfare is maximized with an intermediate level of competition. Otherwise, the maximum welfare is always attained with* *the highest level of competition.*

The formal proof of the theorem is in the appendix, page 50 and the graphical interpretation of the result is in figure 10. We see that welfare is monotonically increasing with the level of competition ex-post in situations where the investment decision of the buyer is equilibrium invariant. The figure presents jumps whenever the investment decision of the buyer depends on the competitiveness of the equilibrium outcome. In this case, we find that welfare is maximized with an intermediate level of competition if the decision of the buyer switches from non-investing to investing when the equilibrium outcome is less competitive. In the situation when the buyer is taking the efficient investment decision in the most competitive equilibrium, any change of the investment of the buyer translates to a lower level of welfare.

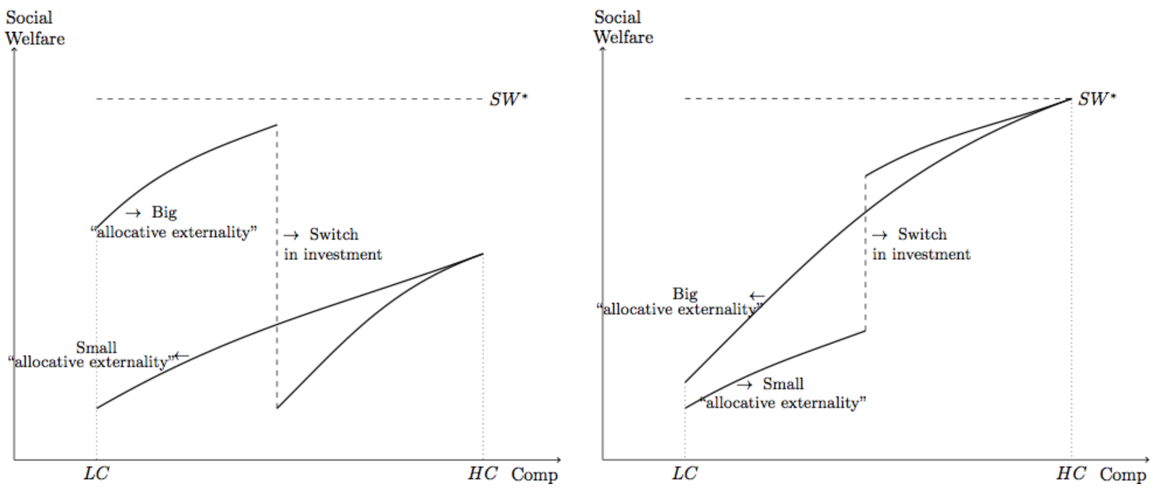


Figure 10: Welfare as a function of the level of competition. The figure on the right stands for the situation where the investment of the buyer in the higher level of competition coincides with the efficient one, and the figure on the left is when the contrary occurs. Jumps in the curves stand for a switch in the investing decision of the buyer and a higher slope of the curves represented larger levels of the "allocative externality".

7 Conclusion

In this paper, we have seen that by introducing competition to the side of the market who has the whole bargaining power, the "hold-up" problem can be solved without the existence of ex-ante contracts. Hence, organizing the economic activity to a certain way can circumvent situations were economic institutions do not allow for designing or enforcing ex-ante contracts.

Full ex-ante efficiency can only be achieved when the outcome of the trading game is the most competitive. In this case, sellers coordinate their out of equilibrium offers in order to minimize the "loss" arising when a seller is excluded from trade. Here, each seller appropriates his marginal contribution to the trading surplus which gives the incentives to invest efficiently.

Moreover, with the introduction of specific investment to both sides of the market, the equilibrium played in the trading game is not only a way to redistribute rents between the sellers and the buyer, but it has also an effect on the size of the potential gains from trade. In previous analysis, it has been stated that an equilibrium where the competition is maximal is not necessarily very attractive from the part of the market offering the trading contracts. This is because the offering part can "tacitly" coordinate to reduce competition in order to obtain a more favorable partition of the potential gains from trade. However, in the current work we have seen that, in general, an equilibrium with higher competition displays a more efficient investment profile which implies a larger surplus. Yet, we obtained that if the efficient trading allocation is very sensitive to the investment of the seller, lower levels of ex-post competition, might perform better than higher competition in terms of welfare. This is possible in our model due to the fact that specific investment is undertaken by both sides of the market and they are strategic complements. We have shown that with a large "allocative externality" the investing seller invests more and the "loss" of exclusion of the remaining sellers might be smaller. As a result, the common buyer decides to invest in a lower competitive equilibrium bringing about a larger welfare.

Therefore, one question that deserves attention is what equilibrium is more likely to arise. This issue has already been addressed in the literature but there does not exist a clear answer.[19] Yet, in our model this is a question of great importance due to its affects on welfare. Despite the fact that we can not be sure of the equilibrium played in the trading game, we think that an external player might induce some set of equilibria to be played. This might be the case in markets that has been recently liberalized, where an external player, in order to maximize welfare, has to ensure that real competition exists in the market. In our model, inducing a particular type of equilibrium has to do with the number of sellers who coordinate their out of equilibrium transfers. Hence, and a third party might be able to induce one

[19]Some of the works addressing equilibrium selection are Martimort & Stole (2009) and Klemperer & Meyer (1989).

equilibrium or another by imposing restrictions on the number of contracts submitted or on putting obstacles to coordination.

It is also important to see if the present results go trough if we relax some of our assumptions. In the model, we considered the case that only one of the sellers knows the new technology and can undertake specific investment to reduce the cost of production. A natural extension of the model is to consider the case where all sellers have knowledge of a technology to reduce the production costs. However, even if unilateral investment decisions are easy to obtain and coincide with the ones obtained in this paper, the characterization of the equilibrium investment profile seems daunting. This is so because the investment of the buyer between the sellers are strategic complements, while the ones among sellers are strategic substitutes. However, we suspect that the strategic substitutability among sellers' investment is of second order. Hence, an increase in the investment of the buyer makes all sellers to invest more in equilibrium.

Another extension is to consider a setting without a monopolistic buyer. In this case, non-exclusivity also comes from the fact that a seller can sign multiple trading contracts with different buyers. In such a case, a buyer differentiates and creates an indirect externality to the others if she decides to invest. We believe that the equilibrium menus offered are complicated to obtain, and we conjecture that the competitive advantage that the buyer gets with respect to the rest might induce him to over-invest.

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A Appendix

Lemma 3. *The total gains from trade are bigger when the buyer invests, that is, $TS^*(1,\sigma^1) > TS^*(0,\sigma^0)$.*

Proof. This is easy to see because:

$$\begin{aligned}
TS^*(1,\sigma^1) &= U(X^*(1,\sigma^1) \mid b=1) - C(x_1^*(1,\sigma^1) \mid \sigma^1) - \sum_{i\neq 1} C(x_i^*(1,\sigma^1)) \\
&> U(X^*(1,\sigma^0) \mid b=1) - C(x_1^*(1,\sigma^1) \mid \sigma^1) - \sum_{i\neq 1} C(x_i^*(1,\sigma^1)) \\
&= U(X^*(1,\sigma^0) \mid b=1) - U(X^*(1,\sigma^0)) + U(X^*(1,\sigma^0)) - C(x_1^*(1,\sigma^1) \mid \sigma^1) - \sum_{i\neq 1} C(x_i^*(1,\sigma^1)) \\
&\geq U(X^*(1,\sigma^0) \mid b=1) - U(X^*(1,\sigma^0)) + TS^*(0,\sigma^0) \\
\implies \quad & TS^*(1,\sigma^1) - TS^*(0,\sigma^0) \geq U(X^*(1,\sigma^0) \mid b=1) - U(X^*(1,\sigma^0)) > 0.
\end{aligned}$$

The first inequality comes from efficiency and the last inequality comes by assumption $U(X^* \mid b = 1) - U(X^*) > 0 \quad \forall X$. □

Lemma 4. *The total gains from trade are is bigger with a higher investment of the seller, $TS^*(b,\sigma\prime) > TS^*(b,\sigma)$ for $\sigma\prime > \sigma$.*

Proof. We consider the case where $b = 0$ but the case where $b = 1$ is analogous.

$$\begin{aligned}
TS^*(0,\sigma) &= U(X^*(0,\sigma)) - C(x_1^*(0,\sigma) \mid \sigma) - \sum_{i\neq 1} C(x_i^*(0,\sigma)) \\
&< U(X^*(0,\sigma)) - C(x_1^*(0,\sigma) \mid \sigma\prime) - \sum_{i\neq 1} C(x_i^*(0,\sigma)) + U(X^*(0,\sigma\prime)) - U(X^*(0,\sigma\prime)) \\
&< U(X^*(0,\sigma)) - U(X^*(0,\sigma\prime)) + TS^*(0,\sigma\prime) \\
\implies \quad & TS^*(0,\sigma\prime) - TS^*(0,\sigma) \geq U(X^*(0,\sigma\prime)) - U(X^*(0,\sigma)) = \int_{X^*(0,\sigma)}^{X^*(0,\sigma\prime)} U_x'(\tau)d\tau > 0,
\end{aligned}$$

where the strict inequality comes from lemma 1 that $X^*(0,\sigma\prime) > X^*(0,\sigma)$ for any $\sigma\prime > \sigma$. □

Lemma 5. *The increase on the total gains from trade by an extra seller are higher when the buyer is investing:*

$$TS^*(1,\sigma^1) - TS^*_{-i}(1,\sigma^1) \geq TS^*(0,\sigma^0) - TS^*_{-i}(0,\sigma^0) \quad \textit{for} \quad i \neq 1.$$

Proof. We will make explicit use of lemma 3. Observe that the previous expression is equivalent to

$TS^*(1,\sigma^1) - TS^*(0,\sigma^0) \geq TS^*_{-i}(1,\sigma^1) - TS^*_{-i}(0,\sigma^0)$ and by lemma 3 we know that

$$TS^*(1,\sigma^1) - TS^*(0,\sigma^0) \geq U(X^*(1,\sigma^1) \mid b=1) - U(X^*(1,\sigma^1)) = \underline{D}.$$

We proceed by obtaining the upper bound of the difference $TS^*_{-i}(1,\sigma^1) - TS^*_{-i}(0,\sigma^0)$:

$$\begin{aligned}
&TS^*_{-i}(1,\sigma^1) = U\left(\sum_{j\neq 1} \tilde{x}_j(1,\sigma^1 \mid \bar{J}) \mid b=1\right) - C\left(\tilde{x}_1(1,\sigma^1 \mid \bar{J}) \mid \sigma^1\right) - \sum_{j\neq i,1} C\left(\tilde{x}_j(1,\sigma^1 \mid \bar{J})\right) \\
&\leq U\left(\sum_{j\neq i} \tilde{x}_j(1,\sigma^0 \mid \bar{J}) \mid 1\right) - C\left(\tilde{x}_1(1,\sigma^0 \mid \bar{J}) \mid \sigma^0\right) - \sum_{j\neq i,1} C\left(\tilde{x}_j(1,\sigma^0 \mid \bar{J})\right) \\
&= U\left(\sum_{j\neq i} \tilde{x}_j(1,\sigma^0 \mid \bar{J}) \mid b=1\right) - U\left(\sum_{j\neq i} \tilde{x}_j(1,\sigma^0 \mid \bar{J})\right) + U\left(\sum_{j\neq i} \tilde{x}_j(1,\sigma^0 \mid \bar{J})\right) \\
&- C\left(\tilde{x}_1(1,\sigma^0 \mid \bar{J}) \mid \sigma^0\right) - \sum_{j\neq i,1} C\left(\tilde{x}_j(1,\sigma^0 \mid \bar{J})\right) \\
&\leq U\left(\sum_{j\neq i} \tilde{x}_j(1,\sigma^0 \mid \bar{J}) \mid b=1\right) - U\left(\sum_{j\neq i} \tilde{x}_j(1,\sigma^0 \mid \bar{J})\right) + TS^*_{-i}(0,\sigma^0) \\
&\implies \quad TS^*_{-i}(1,\sigma^1) - TS^*_{-i}(0,\sigma^0) \leq U\left(\sum_{j\neq i} \tilde{x}_j(1,\sigma^0 \mid \bar{J}) \mid b=1\right) - U\left(\sum_{j\neq i} \tilde{x}_j(1,\sigma^0 \mid \bar{J})\right) = \overline{D}.
\end{aligned}$$

Where the first two inequalities come from efficiency and it is easy to see that $\underline{D} - \overline{D} > 0$ as

$$\begin{aligned}
\underline{D} - \overline{D} &= U(X^*(1,\sigma^0) \mid b=1) - U(X^*(1,\sigma^0)) - \left[U\left(\sum_{j\neq i} \tilde{x}_j(1,\sigma^0 \mid \bar{J}) \mid b=1\right) - U\left(\sum_{j\neq i} \tilde{x}_j(1,\sigma^0 \mid \bar{J})\right)\right] \\
&= U(X^*(1,\sigma^0) \mid b=1) - U\left(\sum_{j\neq i} \tilde{x}_j(1,\sigma^0 \mid \bar{J}) \mid b=1\right) - \left[U(X^*(1,\sigma^0)) - U\left(\sum_{j\neq i} \tilde{x}_j(1,\sigma^0 \mid \bar{J})\right)\right] \\
&= \int_{\sum_{j\neq i} \tilde{x}_j(1,\sigma^0 \mid \bar{J})}^{X^*(1,\sigma^0)} \left(U'_x(\tau \mid b=1) - U'_x(\tau)\right) d\tau > 0,
\end{aligned}$$

which is positive by lemma 2 and assumption $U'_x(\tau \mid b=1) > U'_x(\tau)$. □

Lemma 6. *The increase on the total welfare given by an extra seller is higher when the buyer is investing,*

$$TS^*(1,\sigma^1) - TS^*_{-1}(1) - \psi(\sigma^1) \geq TS^*(0,\sigma^0) - TS^*_{-1}(0) - \psi(\sigma^0).$$

Proof. We are going to proceed by contradiction. Take that the net profit of the seller when the agent invest is strictly lower than when he does not invest.

$$TS^*(1,\sigma^1) - TS^*_{-1}(1) - \psi(\sigma^1) < TS^*(0,\sigma^0) - TS^*_{-1}(0) - \psi(\sigma^0),$$

and by the optimality of the investment decision of the seller we have shown that $\sigma^1 > \sigma^0$ but then the seller could reduce the amount of investment when the agent invest and set $\sigma^1 = \sigma^0$ but the we have that $\psi(\sigma^1) = \psi(\sigma^0)$ and the previous expression is

$$TS^*(1,\sigma^0) - TS^*_{-1}(1) < TS^*(0,\sigma^0) - TS^*_{-1}(0),$$

but this contradicts what we have proven in lemma 5. $\square$

Lemma 7. *The difference obtained in the gains from trade from collective replacement to any other replacement undertaken by $J < N-1$ is higher if the buyer is investing.*

$$TS^*_{-1}(1) - \tilde{TS}_{-1}(1,\sigma^1_{\boldsymbol{J}} \mid J) > TS^*_{-1}(0) - \tilde{TS}_{-1}(0,\sigma^0_{\boldsymbol{J}} \mid J).$$

Proof. By using the same procedure as in lemma 5 we obtain:

$$\begin{aligned}
&TS^*_{-1}(1) - \tilde{TS}_{-1}(1,\sigma^1_{\mathbf{J}} \mid J) - TS^*_{-1}(0) + \tilde{TS}_{-1}(0,\sigma^0_{\mathbf{J}} \mid J)\\
&\geq U\left(\sum_{j\neq i}\tilde{x}_j(\bar{J}) \mid b=1\right) - U\left(X^*_{-\{J,1\}} + \sum_{j\in J}\tilde{x}_j(J) \mid b=1\right) - \left[U\left(\sum_{j\neq i}\tilde{x}_j(\bar{J})\right) - U\left(X^*_{-\{J,1\}} + \sum_{j\in J}\tilde{x}_1(J)\right)\right]\\
&= \int_{X^*_{-\{J,1\}}+\sum_{j\in J}\tilde{x}_1(J)}^{\sum_{j\neq 1}\tilde{x}_j(\bar{J})} \left(U'_x(\tau \mid b=1) - U'_x(\tau)\right) d\tau > 0,
\end{aligned}$$

and this is positive by lemma 2 and by assumption $U(X \mid b=1) > U(X)$. Also by the facts that the investment of the seller makes the difference to increase. $\square$

Lemma 8. *When the buyer invests in J' but not in J when $J' \subset J$, the non-investing seller is always better in the most competitive equilibrium. For any $J' \subset J$ then:*

$$TS^*(0,\sigma^0_{\boldsymbol{J}}) - \tilde{TS}_{-i}(0,\sigma^0_{\boldsymbol{J}} \mid J) > TS^*(1,\sigma^1_{\boldsymbol{J'}}) - \tilde{TS}_{-i}(1,\sigma^1_{\boldsymbol{J'}} \mid J').$$

Proof. We proceed by contradiction, consider the case that

$$TS^*(0,\sigma^0_{\mathbf{J}}) - \tilde{TS}_{-i}(0,\sigma^0_{\mathbf{J}} \mid J) < TS^*(1,\sigma^1_{\mathbf{J'}}) - \tilde{TS}_{-i}(1,\sigma^1_{\mathbf{J'}} \mid J'),$$

but then as it has been shown before, it cannot be the case that $K_{J'} > K_J$, since in this case replacement by a lower number of sellers is more expensive and then it cannot be that with J' then $b = 1$ and at the same time that for J then $b = 0$. Then, we reach a contradiction. Finally, by the monotonicity of the investment of the seller we obtain that the maximum payoff for the non investing sellers is in the highest competitive equilibrium which is given by: $TS^*(0, \sigma^0_{\mathbf{HC}}) - TS^*_{-i}(0, \sigma^0_{\mathbf{HC}})$. $\square$

B Appendix

Proof of lemma 1: We start by proving how seller's investment affects the equilibrium allocation. We consider the case where $b = 0$ but this is analogous for $b = 1$. Differentiating the first-order conditions given in (5.1) for x_j^* with respect to σ we obtain

$$U''_{xx}(X^*) \times \sum_{h=1}^{N} \frac{dx_h^*}{d\sigma} = C''_{xx}(x_j^*) \times \frac{dx_j^*}{d\sigma}. \tag{B.1}$$

Since the left hand side is independent of j we find that all $dx_j^*/d\sigma$ have the same sign. Now suppose also $dx_1^*/d\sigma$ has that same sign. Then also the sum has that same sign and since $U''_{xx}(\cdot) < 0$ and $C''_{xx}(\cdot) > 0$ this leads to a contradiction. Now suppose $dx_1^*/d\sigma < 0$. The other signs therefore have to be positive. By (B.1) we find $\sum_{h=1}^{N} dx_h^*/d\sigma < 0$. But the first-order condition for x_1^*, differentiated with respect to σ is

$$U''_{xx}(X^*) \times \sum_{h=1}^{N} \frac{dx_h^*}{d\sigma} = C''_{xx}(x_1^* \mid \sigma) \times \frac{dx_1^*}{d\sigma} + C''_{x\sigma}(x_1^* \mid \sigma), \tag{B.2}$$

which would then have a positive left hand side and a negative right hand side due to $C''_{x\sigma}(\cdot) < 0$ - a contradiction.

We thus have shown the second and third point. Again by (B.1) the first claim follows from $\partial X^*/\partial\sigma = \sum_{h=1}^{N} dx_h^*/d\sigma$ and the level of the "allocative sensitivity" is implicitly characterized in expression (B.1).We proceed by analyzing the effect that the investment of the buyer has on the equilibrium allocation. Again, we are going to make use of the conditions for the equilibrium allocation represented in equation (5.1), and for a fixed investment of the seller we get

$$\begin{aligned} C'_x(x_1^* \mid \sigma) = U'_x(X^* \mid b=1) > U'_x(X^*) = C'_x(x_1^* \mid \sigma) \qquad &\text{for} \quad 1, \\ C'_x(x_i^*) = U'_x(X^* \mid b=1) > U'_x(X^*) = C'_x(x_i^*) \qquad &\text{for} \quad i \neq 1. \end{aligned}$$

The strict inequality is by assumption and by the convexity of the cost function we obtain the result.

Proof of lemma 2: We are going to consider the case where $b = 0$ but this is similar for $b = 1$. Also, consider any $J \subset N$. Whenever the investment profile is the same, we know that $\sum_{h \neq J_i, i} x_h^* = X^*_{-\{J_i,i\}}$, and the expression above is equivalent to $\sum_{j\in J_i} x_j^* + x_i^* > \sum_{j\in J_i} \tilde{x}_j(J_i)$. Therefore since $x_i^* > 0$ if $\sum_{j\in J_i} \left(x_j^* - \tilde{x}_j(J_i)\right) > 0$ we are done. Observe that for a given investment profile, if the above it is true, it has to be true for any $j \in J_i$, hence $x_j^* > \tilde{x}_j(J_i)$. If the contrary occurs, $x_j^* < \tilde{x}_j(J_i)$, then from the equilibrium allocation we have

$$U_x'\left(X^*_{-\{J_i,i\}} + \sum_{j\in J_i} \tilde{x}_j(J_i)\right) = C_x'(\tilde{x}_j(J_i)) > C_x'(x_j^*) = U_x'(X^*),$$

and by concavity of U we prove the claim. The previous also implies that for any $j \in J_i$ we have $\tilde{x}_j(J_i) > x_j^*$. Using the same procedure we can easily prove that for any $J \subseteq J'$ we have:

$$X^*_{-\{J_i',i\}} + \sum_{j\in J_i'} \tilde{x}_j(J_i') \geq X^*_{-\{J_i,i\}} + \sum_{j\in J_i} \tilde{x}_j(J_i),$$

and by using the same argument as before, we obtain that $\tilde{x}_j(J_i) \geq \tilde{x}_j(J_i')$.

Proof of proposition 1: The equilibrium transfer for seller 1 depend on the set J_1 and for a given investment profile this is equal to

$$T_1(J_1) = U\left(X^* \mid b\right) - \left(\max_{\{x_j\}_{j\in J_1}} \left[U\left(X^*_{-\{J_1,1\}} + \sum_{j\in J_1} x_j \mid \tilde{x}_1 = 0, b\right) - \sum_{j\in J_1} C(x_j)\right] + \sum_{j\in J_1} C(x_j^*)\right).$$

Operating we obtain

$$\begin{aligned}
T_1(J_1) &= U(X^* \mid b) - \sum_{j\in J_1} C(x_j^*) - \left[U\left(X^*_{-\{J_1,1\}} + \sum_{j\in J_1} \tilde{x}_j(J_1) \mid \tilde{x}_1 = 0, b\right) - \sum_{j\in J} C(\tilde{x}_j(J_1))\right] \\
&= U(X^* \mid b) - \sum_{j\in J_1} C(x_j^*) - \left[U\left(X^*_{-\{J_1,1\}} + \sum_{j\in J_1} \tilde{x}_j(J_1) \mid \tilde{x}_1 = 0, b\right) - \sum_{j\in J_1} C(\tilde{x}_j(J_1))\right] \\
&+ \left[\sum_{j\notin J_1,1} \left(C(x_j^*) - C(x_j^*)\right)\right] + \left[C(x_1^* \mid \sigma) - C(x_1^* \mid \sigma)\right] \\
&= TS^*(b,\sigma) - \left[U\left(X^*_{-\{J_1,1\}} + \sum_{j\in J_1} \tilde{x}_j(J_1) \mid \tilde{x}_1 = 0, b\right) - \sum_{j\in J_1} C(\tilde{x}_j(J_1)) - \sum_{j\neq J_1,1} C(x_j^*)\right] + C(x_1^* \mid \sigma) \\
&= TS^*(b,\sigma) - \tilde{TS}_{-i}(b \mid J_1) + C(x_1^* \mid \sigma).
\end{aligned}$$

By putting this to the payoff functions in (4.1) and (4.3), we show point (i) of the proposition by obtaining the equilibrium payoffs obtain the equilibrium payoffs of all the players.

We proceed to show point (ii). It states that for a given investment profile (b,σ) each sellers obtains more than his marginal contribution when they coordinate on reducing competition ex-post. This is equivalent to show that $TS^*_{-i}(b,\sigma) > \tilde{TS}_{-i}(b,\sigma \mid J_i)$. We consider the case where $b=0$ but $b=1$ is similar, and we take the equilibrium transfer for the investing seller.

$$\begin{aligned}
\tilde{TS}_{-1}(0 \mid J) &= U\left(X^*_{-\{J_1,1\}} + \sum_{j\in J_1}\tilde{x}_j(J_1)\right) - \sum_{j\in J_1} C(\tilde{x}_j(J_1)) - \sum_{j\neq J_1,1} C(x^*_j) \\
&= U\left(X^*_{-\{J_1,1\}} + \sum_{j\in J}\tilde{x}_j(J_1)\right) - \sum_{j\in J_1} C(\tilde{x}_j(J_1)) - \sum_{j\neq J,1} C(x^*_j) \\
&\quad + \left[U\left(\sum_{j\neq 1}\tilde{x}_j(\bar{J})\right) - U\left(\sum_{j\neq 1}\tilde{x}_j(\bar{J})\right)\right] \\
&\leq U\left(X^*_{-\{J_1,1\}} + \sum_{j\in J_1}\tilde{x}_j(J_1)\right) - U\left(\sum_{j\neq i}\tilde{x}_j(\bar{J})\right) + TS^*_{-1}(b) \\
\Longrightarrow TS^*_{-1}(b) - \tilde{TS}_{-1}(b \mid J_1) &\geq U\left(\sum_{j\neq 1}\tilde{x}_j(\bar{J})\right) - U\left(X^*_{-\{J_1,1\}} + \sum_{j\in J_1}\tilde{x}_j(J_1)\right) \\
&= \int_{X^*_{-\{J_1,1\}}+\sum_{j\in J_1}\tilde{x}_j(J_1)}^{\sum_{j\neq 1}\tilde{x}_j(\bar{J})} U'_x(\tau)d\tau > 0,
\end{aligned}$$

where the last inequality comes from lemma 2, and this can be applied for any seller i. From the above, it is also true for any i that for any $J' \supset J$ we obtain that $\tilde{TS}_{-i}(b,\sigma \mid J_i) < \tilde{TS}_{-i}(b,\sigma \mid J'_i)$.

Proof of proposition 2: As it will be clear later, depending on the fixed cost parameter the buyer undertakes the efficient investment. Therefore, in order to show existence of efficiency in the equilibrium investment profile, we pay attention to seller's investment. We first show the "if" part of the proposition. The payoff of the seller in the most competitive equilibrium is

$$\pi_1^{HC} = TS^*(b,\sigma) - TS^*_{-1}(b),$$

and the term $TS^*_{-1}(b)$ does not depend on the amount invested σ. Therefore using $TS^*(b,\sigma)$ given by expression (4.4) and by the envelope-theorem, the first-order condition for the seller 1 is given by

$$\psi'_\sigma(\sigma_{\mathbf{HC}}) = -C'_\sigma\left(x^*_1(b,\sigma^b_{\mathbf{HC}})|\sigma^b_{\mathbf{HC}}\right), \tag{B.3}$$

and this coincides with the efficient one obtained in expression (5.8). Because the seller receives the full marginal social surplus from his own investment, he becomes the residual claimant and invests efficiently. Therefore, whenever the investment decision of the buyer coincides with the efficient one i.e. $b_{\mathbf{HC}} = b_{\mathbf{E}}$ the equilibrium vector in the most competitive equilibrium is efficient.

To show the "only if" part, we take any $J \subset N$ and $J \neq N \setminus \{1\}$, and we obtain that the equilibrium payoffs of seller 1 is

$$\pi_1(b, \sigma \mid J_1) = TS^*(b, \sigma) - \tilde{TS}_{-1}(b, \sigma \mid J_1),$$

and calculating the first order condition and applying the envelope theorem we obtain that the equilibrium investment profile is characterized by

$$\psi'_\sigma(\sigma) = -C'_\sigma(x_1^*(b, \sigma^b)|\sigma^b) - \frac{\partial\left(\tilde{TS}_{-1}(b, \sigma \mid J_1)\right)}{\partial\sigma},$$

where the extra term depends on the investment of the seller from the allocation that remains unchanged $X^*_{-\{J,1\}}(b, \sigma)$. As a result, $\partial\left(\tilde{TS}_{-1}(b, \sigma \mid J_1)\right)/\partial\sigma \neq 0$ and this creates a distortion of the investment of the seller. Hence, we conclude that full efficiency is only implemented whenever the competition between sellers is the most severe.

Proof of corollary 1: The investment decision of the seller is the one in proposition (2) and the one for the buyer in the most competitive equilibrium is

$$K \begin{cases} \leq TS^*(1, \sigma^1_{\mathbf{E}}) - TS^*(0, \sigma^0_{\mathbf{E}}) - \kappa_{\mathbf{HC}} \equiv K_{\mathbf{HC}} & \text{then } b = 1 \\ > K_{\mathbf{HC}} & \text{then } b = 0 \end{cases}, \tag{B.4}$$

where the term $\kappa_{\mathbf{HC}}$ is the difference in the payoff of the sellers when the buyer decides to invest and it is equal to

$$\begin{aligned} \kappa_{\mathbf{HC}} &\equiv \pi_1^{HC}(1, \sigma^1_{\mathbf{E}}) - \pi_1^{HC}(0, \sigma^0_{\mathbf{E}}) + \sum_{i \neq 1}\left[\pi_i^{HC}(1, \sigma^1_{\mathbf{E}})) - \pi_i^{HC}(0, \sigma^0_{\mathbf{E}}))\right] \\ &= TS^*(1, \sigma^1) - TS^*_{-1}(1) - TS^*(0, \sigma^0) + TS^*_{-1}(0) \\ &+ \sum_{i \neq 1}\left[TS^*(1, \sigma^1) - TS^*_{-i}(1, \sigma^1) - TS^*(0, \sigma^0) + TS^*_{-i}(0, \sigma^0)\right]. \end{aligned}$$

The magnitude $\kappa_{\mathbf{HC}}$ then represents how much the sellers benefit from the investment of the buyer and are the gains that cannot be appropriated by the latter. By making an explicit use of the lemmas in appendix A we show that the appropriation of the gains by the sellers is bigger than the cost of investment $\kappa_{\mathbf{HC}} > \psi(\sigma^1_{\mathbf{E}}) - \psi(\sigma^0_{\mathbf{E}})$. We show that $\kappa > \psi(\sigma^1_{\mathbf{E}}) - \psi(\sigma^0_{\mathbf{E}})$ by splitting κ in two parts $A = \sum_{i \neq 1} \left[TS^*(1,\sigma^1) - TS^*_{-i}(1,\sigma^1) - TS^*(0,\sigma^0) + TS^*_{-i}(0,\sigma^0)\right]$ and $B = TS^*(1,\sigma^1) - TS^*_{-1}(1) - TS^*(0,\sigma^0) + TS^*_{-1}(0)$. In lemma 5, we show that $A > 0$ and in lemma 6 we show that $B > \psi(\sigma^1_{\mathbf{E}}) - \psi(\sigma^0_{\mathbf{E}})$.

This implies that the threshold of the cost of the buyer below which she invests is lower compared to the efficient one $K_{\mathbf{HC}} < K_{\mathbf{E}}$. Thus, as the buyer cannot appropriate all the gains coming from his investment she underinvests whenever the fix cost of investment is $K \in (K_{\mathbf{HC}}, K_{\mathbf{E}})$ as $b_{\mathbf{HC}} \neq b_{\mathbf{E}} = 1$. Finally, since investments are strategic complements, this implies that the seller also underinvests in equilibrium, i.e. $\sigma_{\mathbf{HC}} < \sigma_{\mathbf{E}}$. Hence, as long as the investment of the seller increases the total amount traded as shown in lemma 1, the threshold for the investment of the buyer also increases with the investment of the seller. $\partial K_{HC}/\partial\sigma = -\partial\kappa_{\mathbf{HC}}/\partial\sigma > 0$. And this coincides with the change of the payoffs for the buyer.

$$\frac{\partial \Pi^{HC}(\cdot)}{\partial\sigma} = -\frac{\partial\kappa_{\mathbf{HC}}}{\partial\sigma} = -\sum_{i\neq 1}\int_{x_1^*(1,\sigma)+\tilde{x}_1(0,\sigma|\bar{J})}^{\tilde{x}_1(1,\sigma|\bar{J})+x_1^*(0,\sigma)} C''_{x\sigma}(\tau)d\tau > 0,$$

and this is positive by $\tilde{x}_1(1,\sigma \mid \bar{J}) + x_1^*(0,\sigma) > x_1^*(1,\sigma) + \tilde{x}_1(0,\sigma \mid \bar{J})$ and assumption $C''_{x\sigma}(\cdot) < 0$.

Proof of proposition 3: We start by showing point (i). From proposition 2 we know that the seller's investment fails to be efficient. Here, we show that there exist over-investment and we give it's magnitude. We take the first order condition for the seller. Then, from the equilibrium payoff of the seller 1 and the envelope condition, we obtain that for any $J_i < N \setminus \{i\}$

$$\begin{aligned}\psi'_\sigma(\sigma_{\mathbf{J}}) &= -C'_\sigma(x_1^*(b,\sigma_{\mathbf{J}}) \mid \sigma_{\mathbf{J}}) - \sum_{m\neq J,1}\left(U'_x\left(X^*_{-\{J,1\}} + \sum_{j\in J}\tilde{x}_j(J)\right) - C'_x(x_j^*)\right) \times \frac{dx_m^*}{d\sigma} \\ &= -C'_\sigma(x_1^*(b,\sigma_{\mathbf{J}}) \mid \sigma_{\mathbf{J}}) - \sum_{m\neq J,1}\left(U'_x\left(X^*_{-\{J,1\}} + \sum_{j\in J}\tilde{x}_j(J)\right) - U'_x(X^*)\right) \times \frac{dx_m^*}{d\sigma},\end{aligned} \tag{B.5}$$

where the transformation in the second line is due to the fact that, at the equilibrium allocation, marginal benefit equals marginal cost, i.e. $U'_x(X^*) = C'_x(x_j^*)$, $\forall j \in N$. Comparing this condition with the efficient one in (5.8), we see that the difference is the additional term is

$$\gamma(J) \equiv -\sum_{m\neq J,1}\left(U'_x\left(X^*_{-\{J,1\}} + \sum_{j\in J}\tilde{x}_j(J)\right) - U'_x(X^*)\right) \times \frac{dx_m^*}{d\sigma},$$

and by applying the fundamental theorem of calculus we get

$$\gamma(J) \equiv -\sum_{m\neq J,1}\left(\int_{X^*}^{X^*_{-\{J,1\}}+\sum_{j\in J}\tilde{x}_j(J)} U''_{xx}(\tau)d\tau\right)\times\frac{dx^*_m}{d\sigma}>0,$$

and the whole expression is positive. By lemma 2 and the concavity of U, we know that the part in brackets is positive. By lemma 1 we know that the amount traded with the sellers that are not investing is decreasing with the amount invested by the seller. Therefore, this term is strictly positive which means that the seller over-invests and it's magnitude depends on the "allocative sensitivity" that the investment of the seller creates to the non-investing sellers. In order to show that the degree of over-investment decreases with the level of competition, i.e. $\partial\gamma(J)/\partial J<0$, we calculate how the previous expression varies with an increase in J. By applying Leibniz rule we obtain

$$\begin{aligned}\frac{\partial\gamma(J)}{\partial J} &= \left(\int_{X^*}^{X^*_{-\{J,1\}}+\sum_{j\in J}\tilde{x}_j(J)} U''_{xx}(\tau)d\tau\right)\times\frac{dx^*_m}{d\sigma}\\ &- U''_{xx}\left(X^*_{-\{J,1\}}+\sum_{j\in J}\tilde{x}_j(J)\right)\times\underbrace{\frac{\partial\left(X^*_{-\{J,1\}}+\sum_{j\in J}\tilde{x}_j(J)\right)}{\partial J}}_{(+)}\times\frac{dx^*_m}{d\sigma}<0,\end{aligned}\tag{B.6}$$

and the sign is due to lemma 2.

Again point (ii) is more involved and the investment decision of the buyer is given by:

$$K\begin{cases}\leq TS^*(1,\sigma^1_{\mathbf{J}})-TS^*(0,\sigma^0_{\mathbf{J}})-\kappa_{\mathbf{J}}\equiv K_{\mathbf{J}} & \text{then } b=1\\ >K_{\mathbf{J}} & \text{then } b=0,\end{cases}\tag{B.7}$$

where the extra term $\kappa_{\mathbf{J}}$ is the difference in the payoff of the sellers when the buyer invests. Again this represents how much the sellers benefit from the investment of the buyer and those benefits can not be appropriate by the latter.

$$\begin{aligned}\kappa_{\mathbf{J}} &\equiv \pi_1\left(1,\sigma^1_{\mathbf{J}}\mid J_1\right)-\pi_1\left(0,\sigma^0_{\mathbf{J}}\mid J_1\right)+\sum_{i\neq1}\left[\pi_i\left(1,\sigma^1_{\mathbf{J}}\mid J_i\right)-\pi_i\left(0,\sigma^0_{\mathbf{J}}\mid J_i\right)\right]\\ &= TS^*(1,\sigma^1)-\tilde{TS}_{-1}(1,\sigma^1\mid\underline{J})-TS^*(0,\sigma^0)+\tilde{TS}_{-1}(0,\sigma^0\mid\underline{J})\\ &+\sum_{i\neq1}\left[TS^*(1,\sigma^1)-\tilde{TS}_{-i}(1,\sigma^1\mid\underline{J})-TS^*(0,\sigma^0)+\tilde{TS}_{-i}(0,\sigma^0\mid\underline{J})\right].\end{aligned}$$

And for a given investment of the seller, the threshold of the buyer is below the efficient one i.e. $K_{\mathbf{LC}}<K_{\mathbf{E}}$. Again, this comes from the fact that in equilibrium, the buyer is not able to appropriate all benefits coming from investment and the proof is the same as in corollary 1 and we do not repeat it

here. Moreover, with a given level of investment by the investing seller, each seller is able to appropriate a larger amount of the gains from trade as shown in proposition 1, and this reduces the rents of the buyer. Therefore, the incentives for the buyer to invest decreases the lower the level of competition ex-post.

Proof of corollary 2: The first point comes directly from proposition 3. Point A states that whenever the buyer does not take the efficient investment decision, this is characterized by underinvestment. We take the extreme case, and this is when the investment ex-post is the least competitive equilibrium. Hence, we have to show that $K_{LC} < K_E$ and hence

$$\begin{aligned}
K_{LC} \leq K_E \iff & TS^*(1,\sigma^1_{\mathbf{LC}}) - TS^*(0,\sigma^0_{\mathbf{LC}}) - \kappa_{LC} \leq TS^*(1,\sigma^1_{\mathbf{E}}) - TS^*(0,\sigma^0_{\mathbf{E}}) - \left(\psi\left(\sigma^1_{\mathbf{E}}\right) - \psi\left(\sigma^0_{\mathbf{E}}\right)\right) \\
\implies \quad & \psi\left(\sigma^1_{\mathbf{E}}\right) - \psi\left(\sigma^0_{\mathbf{E}}\right) \leq TS^*(1,\sigma^1_{\mathbf{E}}) - \tilde{TS}_{-1}(1,\sigma^1_{\mathbf{LC}} \mid \underline{J}) - \left(TS^*(0,\sigma^0_{\mathbf{E}}) - \tilde{TS}_{-1}(0,\sigma^0_{\mathbf{LC}} \mid \underline{J})\right) \\
& + \sum_{i\neq 1}\left[TS^*(1,\sigma^1_{\mathbf{E}}) - \tilde{TS}_{-i}(1,\sigma^1_{\mathbf{LC}} \mid \underline{J}) - \left(TS^*(0,\sigma^0_{\mathbf{E}}) - \tilde{TS}_{-i}(0,\sigma^0_{\mathbf{LC}} \mid \underline{J})\right)\right].
\end{aligned}$$

By using the same procedure as in lemma 5 we can see that in general the last part in brackets is positive. Therefore, to show the above, we need that

$$TS^*(1,\sigma^1_{\mathbf{E}}) - \tilde{TS}_{-1}(1,\sigma^1_{\mathbf{LC}} \mid \underline{J}) - \left(TS^*(0,\sigma^0_{\mathbf{E}}) - \tilde{TS}_{-1}(0,\sigma^0_{\mathbf{LC}} \mid \underline{J})\right) \geq \psi\left(\sigma^1_{\mathbf{E}}\right) - \psi\left(\sigma^0_{\mathbf{E}}\right).$$

Here we apply lemma 7 that states $\tilde{TS}_{-1}(1,\sigma^1_{\mathbf{LC}} \mid \underline{J}) < TS^*_{-1}(1) - TS^*_{-1}(0) + \tilde{TS}_{-1}(0,\sigma^0_{\mathbf{LC}} \mid \underline{J})$, and by introducing this the the previous expression we have that

$$\begin{aligned}
TS^*(1,\sigma^1_{\mathbf{E}}) - \tilde{TS}_{-1}(1,\sigma^1_{\mathbf{LC}} \mid \underline{J}) - \left(TS^*(0,\sigma^0_{\mathbf{E}}) - \tilde{TS}_{-1}(0,\sigma^0_{\mathbf{LC}} \mid \underline{J})\right) &> TS^*(1,\sigma^1_{\mathbf{E}}) \\
- \left[TS^*_{-1}(1) - TS^*_{-1}(0) + \tilde{TS}_{-1}(0,\sigma^0_{\mathbf{LC}} \mid \underline{J})\right] &- \left(TS^*(0,\sigma^0_{\mathbf{E}}) - \tilde{TS}_{-1}(0,\sigma^0_{\mathbf{LC}} \mid \underline{J})\right) \\
= TS^*(1,\sigma^1_{\mathbf{E}}) - TS^*_{-1}(1) - \left(TS^*(0,\sigma^0_{\mathbf{E}}) - TS^*_{-1}(0)\right) &> \psi\left(\sigma^1_{\mathbf{E}}\right) - \psi\left(\sigma^0_{\mathbf{E}}\right),
\end{aligned}$$

where the last inequality comes by lemma 6. Hence, in general the threshold of investment in the least competitive equilibrium is lower than the efficiency one.

To show point B we need to compare the right hand side of the expression determining the investment in the least competitive equilibrium (B.5) evaluated at $b = 0$, with the right hand side of expression determining the efficient investment (5.8) evaluated at $b = 1$.

$$Rhs^{LC}(b=0) = -C'_\sigma(x^*_1(0,\sigma^0_{\mathbf{J}}) \mid \sigma) - \sum_{m\neq \underline{J},1}\left(\int_{X^*}^{X^*_{-\{\underline{J},1\}}+\sum_{j\in\underline{J}}\tilde{x}_j(\underline{J})} U''_{xx}(\tau)d\tau\right)\frac{dx^*_m}{d\sigma}.$$

$$Rhs^E(b=1) = -C'_\sigma(x^*_1(1,\sigma^1_{\mathbf{E}}) \mid \sigma),$$

and we will have that the efficient investment is higher if

$$
\begin{aligned}
& Rhs^E(b=1) > Rhs^{LC}(b=0) \\
& \Longrightarrow -C'_\sigma(x_1^*(1,\sigma^1) \mid \sigma) > -C'_\sigma(x_1^*(0,\sigma^0) \mid \sigma) - \sum_{m \neq \underline{J},1} \left(\int_{X^*}^{X^*_{-\{\underline{J},1\}} + \sum_{j \in J} \tilde{x}_j(\underline{J})} U''_{xx}(\tau) d\tau \right) \frac{dx_m^*}{d\sigma} \\
& \Longrightarrow \int_{x_1^*(1,\sigma^1)}^{x_1^*(0,\sigma^0)} C''_{x\sigma}(\tau) d\tau > - \sum_{m \neq \underline{J},1} \left(\int_{X^*}^{X^*_{-\{\underline{J},1\}} + \sum_{j \in \underline{J}} \tilde{x}_j(\underline{J})} U''_{xx}(\tau) d\tau \right) \frac{dx_j^*}{d\sigma} \\
& \Longrightarrow -\frac{dx_j^*}{d\sigma} < \frac{\int_{x_1^*(1,\sigma^1)}^{x_1^*(0,\sigma^0)} C''_{x\sigma}(\tau) d\tau}{(N \setminus \{1\} - i) \times \int_{X^*}^{X^*_{-\{\underline{J},1\}} + \sum_{j \in \underline{J}} \tilde{x}_j(\underline{J})} U''_{xx}(\tau) d\tau},
\end{aligned}
$$

otherwise, the contrary occurs. That is, if the allocative sensitivity is large, the investing seller invests more than the efficiency level regardless of the investment decision of the buyer.

Proof of proposition 4: We proceed by construction and we consider the case when the number of active sellers tends to infinity. We consider first the investment decision of the seller and take the case where the distortion is maximal. Hence, if in this situation, investment tends to efficiency, so will be for any $J \subset N$. Second, we obtain that for any equilibrium, the investment threshold of the buyer tends to the efficient one.

Regarding the investment of the seller, we take the highest level of distortion and this is the one when competition ex-post is the least severe, or $|J| = 1 = \underline{J}$

$$
\gamma(N \mid \underline{J}) \equiv - \sum_{m \neq 1,i} \left(\int_{X^*(N)}^{X^*_{-\{1,i\}}(N) + \tilde{x}_i(N,\underline{J})} U''_{xx}(\tau) d\tau \right) \times \frac{dx_m^*}{d\sigma}. \tag{B.8}
$$

Observe that the magnitude of this object depends on the difference between the efficient amount traded and the one obtained with unilateral replacement which equals to $x_i^*(N) + x_1^*(N) - \tilde{x}_i(N, \underline{J}) > 0$. We now show that this difference tends to zero when the number of active sellers is arbitrarily large and so the expression within the brackets in (B.8) tends to zero. At this purpose we make use of the following two lemmas. The following lemma shows how the individual and aggregate amount of trade evolves with an increase of sellers.

Lemma 9. *For a given investment profile (b, σ) the amount that each seller trades with the buyer decreases with the number of active sellers, but the aggregate level of trade is higher.*

$$
x_i^*(\Delta N) < x_i^*(N) \quad \forall i \in N \quad \textit{and} \quad X^*(\Delta N) > X^*(N).
$$

Proof. The results comes directly from the concavity of the utility function and the convexity of the cost function. In order to ease notation, we do not consider investment. We define $\Delta N = N + 1$ and with a number of ΔN active sellers, the amount traded in equilibrium needs to satisfy

$$U'_x\left(\sum_{i=1}^{\Delta N} x_i^*(\Delta N)\right) = C'_x\left(x_i^*(\Delta N)\right).$$

We proof the claim by contradiction, assume that $x_i^*(\Delta N) \geq x_i^*(N)\ \forall\ i \in N$, and since $\Delta N > N$ we have that $\sum_i^{\Delta N} x_i^*(\Delta N) > \sum_i^N x_i^*(N)$ and by the concavity of $U(\cdot)$ and optimality it has to be the case that

$$C'_x\left(x_i^*(\Delta N)\right) = U'_x\left(\sum_{i=1}^{\Delta N} x_i^*(\Delta N)\right) < U'_x\left(\sum_{i=1}^{N} x_i^*(N)\right) = C'_x\left(x_i^*(N)\right) \quad \forall i \in N,$$

but the convexity of $C'_x(\cdot)$ implies that $x^*(\Delta N) < x^*(N)$, which leads to a contradiction. From the previous, we see that that $X^*(\Delta N) > X^*(N)$ comes directly. $\square$

Therefore as the number of seller increase, the amount $x_1^*(N)$ decreases and $\lim_{N\to\infty} x_1^*(N) \approx 0$. Regarding how the amount $\tilde{x}_i(N, \underline{J})$ evolves with the number of sellers, we know that this object is the solution of the function $V(X^*_{-\{\underline{J},i\}})$ introduced in expression (5.2) in the appendix. The properties of this function are introduced in the following lemma.

Lemma 10. *The function $V\left(X^*_{-\{\underline{J},i\}}\right)$ is well defined, strictly increasing and strictly concave in $X^*_{-\{\underline{J},i\}}$. The maximizer $\tilde{x}_i\left(X^*_{-\{\underline{J},i\}}\right)$ is decreasing in $X^*_{-\{\underline{J},i\}}$.*

Proof. That the function $V\left(X^*_{-\{\underline{J},i\}}\right)$ is well defined follows from the Inada conditions. By the envelop theorem we have $V'_x\left(X^*_{-\{\underline{J},i\}}\right) > 0$ and $V''_{xx}\left(X^*_{-\{\underline{J},i\}}\right) < 0$, which implies that the function is strictly increasing and strictly concave. By the implicit function theorem, we find that:

$$\frac{\partial \tilde{x}_i\left(X^*_{-\{\underline{J},i\}}\right)}{\partial X^*_{-\{\underline{J},i\}}} = \frac{U''_{xx}(\cdot)}{C''_{xx}(\cdot) - U''_{xx}(\cdot)} < 0.$$

Thus, it is decreasing with an increase of the unchanged equilibrium allocation due to the concavity of the utility function and the convexity of the cost function. $\square$

Hence, an increase of the number of sellers make $X^*_{-\{\underline{J},i\}}(N)$ increase as shown in lemma 9, and by the previous lemma we know that the amount $\tilde{x}_1(N, \underline{J})$ decreases. In the limit, we have that $\lim_{N\to\infty}\left[\tilde{x}_i(N, \underline{J})\right] \approx x_i^*(N)$. Thus, we have shown that with an arbitrarily number of active sellers, the difference between the upper and the lower integrand of (B.8) tends to zero. We conclude that the extra term causing the inefficiencies in the seller's investment disappear and this tends to efficiency $\lim_{N\to\infty}\left[\zeta(N)\right] \approx 0 \Longrightarrow \sigma_{\mathbf{LC}} \approx \sigma_{\mathbf{E}}$.

We now show that the investment thresholds of the buyer also converge whenever the number of active sellers is sufficiently large. The investment threshold for any $J \subset N$ is

$$K_J \equiv TS^*(1, \sigma^1_{\mathbf{J}}) - TS^*(0, \sigma^0_{\mathbf{J}}) - \kappa_{\mathbf{J}}.$$

From above, we know that the investment of the seller tends to efficiency $\sigma_{\mathbf{J}} \approx \sigma_{\mathbf{E}}$, which implies that the first part of the threshold also tends to efficiency $\lim_{N\to\infty} \left[TS^*(1, \sigma^1_{\mathbf{J}}) - TS^*(0, \sigma^0_{\mathbf{J}})\right] \approx TS^*(1, \sigma^1_{\mathbf{E}}) - TS^*(0, \sigma^0_{\mathbf{E}})$. By the same argument as before we can also show that the appropriation of gains from trade by the sellers coming from an investment of the buyer tends to zero when the number of sellers is arbitrarily large. Finally, the investing seller appropriation of investment tends to the private cost of investment, that is, $\lim_{N\to\infty} [\kappa_J] \approx \psi(\sigma^1_{\mathbf{E}}) - \psi(\sigma^0_{\mathbf{E}})$. Hence, with all things considered we have that $\lim_{N\to\infty} [K_J] \approx K_E$ and the equilibrium investment profile tends to efficiency.

Proof of proposition 5: We begin by considering the case where the "allocative sensitivity" is small. In this case, we have established that the investment threshold of the buyer is monotonically decreasing the lower the level of competition ex-post. The lower partition of the surplus appropriated by the buyer with lower competition dominates the higher investment of the seller. We start with the case that the investment of the buyer is the same regardless of the level of competition ex-post. To see that the investing seller is better-off with lower levels of competition we only need to verify that his investment increases the lower is the level of competition and this is the case since we know that $\partial\gamma(J)/\partial J < 0$. For the non-investing seller, it is easy to see that for any $J \subset N$ and $J \neq N \setminus \{i\}$ we obtain

$$\begin{aligned}
&TS^*(b, \sigma^b_{\mathbf{J}}) - \tilde{TS}_{-i}(b, \sigma^b_{\mathbf{J}} \mid J_i) > TS^*(b, \sigma^b_{\mathbf{E}}) - TS^*_{-i}(b, \sigma^b_{\mathbf{E}}) \\
&\qquad \Longrightarrow TS^*_{-i}(b, \sigma^b_{\mathbf{E}}) - \tilde{TS}_{-i}(b, \sigma^b_{\mathbf{J}} \mid J_i) > TS^*(b, \sigma^b_{\mathbf{E}}) - TS^*(b, \sigma^b_{\mathbf{J}}) \approx 0 \\
&\qquad \Longrightarrow \; TS^*_{-i}(b, \sigma^b_{\mathbf{E}}) - \tilde{TS}_{-i}(b, \sigma^b_{\mathbf{J}} \mid J_i) > 0.
\end{aligned}$$

The right hand side of the second line is close to zero due to the fact that when the "allocative sensitivity" is small, the investment of the seller is similar regardless to the equilibrium ex-post $\sigma^b_{\mathbf{J}} \approx \sigma^b_{\mathbf{E}}$. The third line is positive by point ii) in proposition 1. In a situation where the investment of the buyer depends on the equilibrium played ex-post and because the "allocative sensitivity" is small, we know that for any $J \subset N$ we have that the investment threshold in the most competitive equilibrium is the largest $K_{HC} > K_J$. Because the sensitivity is small then there exist $J \subset N$ and $\sigma^1_{\mathbf{J}} > \sigma^0_{\mathbf{LC}}$. This implies that the largest payoff of the investing seller is achieved with an intermediate level of competition. With regards to the non-investing sellers, we have that, the largest payoffs is achieved with an intermediate level of competition. By the same procedure as before, we know that

$TS^*(1,\sigma^1_{\mathbf{J}}) - \tilde{TS}_{-i}(1,\sigma^1_{\mathbf{J}} \mid J_i) > TS^*(1,\sigma^1_{\mathbf{E}}) - TS^*_{-i}(1,\sigma^1_{\mathbf{E}})$. Therefore, we have that the largest payoff is attained with a level of intermediate competition if

$$TS^*(1,\sigma^1_{\mathbf{J}}) - \tilde{TS}_{-i}(1,\sigma^1_{\mathbf{J}} \mid J_i) > TS^*(0,\sigma^0_{\mathbf{LC}}) - \tilde{TS}_{-i}(0,\sigma^0_{\mathbf{LC}} \mid \underline{J}). \tag{B.9}$$

Because the "allocative sensitivity" is small, we know that with a given investment of the buyer, the investment of the seller will be similar. Hence, by lemma 5 and proposition 1 we know that

$$\begin{aligned} TS^*(1,\sigma^1) - \tilde{TS}_{-i}(1,\sigma^1 \mid J_i) &> TS^*(0,\sigma^0) - \tilde{TS}_{-i}(0,\sigma^0 \mid J_i) \\ TS^*(0,\sigma^0) - \tilde{TS}_{-i}(0,\sigma^0 \mid \underline{J}) &> TS^*(0,\sigma^0) - \tilde{TS}_{-i}(0,\sigma^0 \mid J_i). \end{aligned}$$

By summing up both expressions we have

$$\tilde{TS}_{-i}(0,\sigma^0) < TS^*(1,\sigma^1 \mid \underline{J}) - \tilde{TS}_{-i}(1,\sigma^1 \mid J_i) + TS^*(0,\sigma^0) - 2\left[TS^*(0,\sigma^0) - \tilde{TS}_{-i}(0,\sigma^0 \mid J_i)\right],$$

and by putting this in equation (B.9) we obtain

$$\begin{aligned} TS^*(1,\sigma^1) - \tilde{TS}_{-i}(1,\sigma^1 \mid J_i) &> TS^*(0,\sigma^0) - TS^*(1,\sigma^1) + \tilde{TS}_{-i}(1,\sigma^1 \mid J_i) - TS^*(0,\sigma^0) \\ &\quad + 2\left[TS^*(0,\sigma^0) - \tilde{TS}_{-i}(0,\sigma^0 \mid J_i)\right] \\ &\Longrightarrow 2\left[TS^*(1,\sigma^1) - \tilde{TS}_{-i}(1,\sigma^1 \mid J_i)\right] > 2\left[TS^*(0,\sigma^0) - \tilde{TS}_{-i}(0,\sigma^0 \mid J_i)\right], \end{aligned}$$

where the last inequality holds again by lemma 5.

Whenever the "allocative sensitivity" is big, the proof is more involved. We have seen that if the "allocative externally" is big there exists a subset of agents that undertake collective replacement $J \subset N$ such that $K_J > K_{HC}$. For the investing seller it is easy to see that, as long as the investment is higher with a less competitive equilibrium his payoffs are also higher. This is always the case due to the complementarity of investment and that the buyer may decide to invest with a lower the degree of competition. For the non investing seller the proof is simple. In the case where the fixed costs of investment is such that the buyer invests in a lower competitive equilibrium we have that the non-investing sellers are better with the highest competitive equilibrium. Therefore, we have that

$$\begin{aligned} &TS^*(1,\sigma^1_{\mathbf{J}}) - \tilde{TS}_{-i}(1,\sigma^1_{\mathbf{J}} \mid J_i) < TS^*(0,\sigma^0_{\mathbf{E}}) - TS^*_{-i}(0,\sigma^0_{\mathbf{E}}) \\ &\Longrightarrow TS^*(1,\sigma^1_{\mathbf{J}}) - TS^*(0,\sigma^0_{\mathbf{E}}) < \tilde{TS}_{-i}(1,\sigma^1_{\mathbf{J}} \mid J_i) - TS^*_{-i}(0,\sigma^0_{\mathbf{E}}); \quad \forall J \in N, \end{aligned}$$

where the left hand side represents the whole gain coming from the investment of the buyer and this is positive. However, the right hand side is bigger and represents the gain in surplus when the buyer is investing whenever any seller $i \neq 1$ is excluded from trade and this is proved in lemma 8 in the appendix. Observe that whenever the investment decision of the buyer is equilibrium invariant, we also have that the maximal surplus is attained with the highest level of competition since

$$TS^*(1,\sigma^1_{\mathbf{E}}) - TS^*_{-i}(1,\sigma^1_{\mathbf{E}}) > TS^*(0,\sigma^0_{\mathbf{E}}) - TS^*_{-i}(0,\sigma^0_{\mathbf{E}}) > TS^*(1,\sigma^1_{\mathbf{J}}) - \tilde{TS}_{-i}(1,\sigma^1_{\mathbf{J}} \mid J_i); \quad \forall J \in N.$$

Proof of theorem 1: We start with the case when the allocative sensitivity is small. In the previous section we stated that with a small "allocative sensitivity" the investment threshold of the buyer is the biggest with the highest degree of competition, i.e $K_{HC} \geq K_J$ $\forall J \subset N$. This entails that the investment decision of the buyer is only efficient in any "tacitly" coordinating equilibrium whenever it is also in the most competitive equilibrium. Then, it is immediate to see that, because the investment decision of the seller in any "tacitly" coordinating equilibrium is inefficient as shown in proposition 2, we obtain that the highest level of social welfare is obtained when competition is the most severe.

We proceed by considering by analyzing when the "allocative sensitivity" is big. In this case, we have established that the investment threshold of the buyer in an equilibrium where sellers tacitly coordinate to bring competition down, might be above the one corresponding to the highest competitive equilibrium. Here, we show that there exists a situation where the social welfare is bigger with less competition. Therefore, in what follows, we consider the case where there exist a $J \subset N$ such that $K_{HC} < K_J$. We define the difference in net social surplus as

$$D(\cdot) = W^J(1,\sigma^1) - W^{HC}(0,\sigma^0) = TS^*(1,\sigma^1_{\mathbf{J}}) - K - \psi(\sigma^1_{\mathbf{J}}) - TS^*(0,\sigma^0_{\mathbf{E}}) + \psi(\sigma^0_{\mathbf{E}}).$$

Since we want to know if there exists a situation where a less competitive equilibrium does better, we take the lowest possible value of the cost of investment of the buyer, which is $K = K_{HC} = TS^*(1,\sigma^1_{\mathbf{E}}) - TS^*(0,\sigma^0_{\mathbf{E}}) - \kappa_{HC}$. By introducing this in the previous expression we obtain that the lower bound of the difference is given by:

$$\begin{aligned}
\bar{D}(\cdot) &= TS^*(1,\sigma^1_{\mathbf{J}}) - TS^*(1,\sigma^1_{\mathbf{E}}) + TS^*(0,\sigma^0_{\mathbf{E}}) + \kappa_{HC} - \psi(\sigma^1_{\mathbf{J}}) - TS^*(0,\sigma^0_{\mathbf{E}}) + \psi(\sigma^0_{\mathbf{E}}) \\
&= TS^*(1,\sigma^1_{\mathbf{J}}) - TS^*(1,\sigma^1_{\mathbf{E}}) + \kappa_{HC} - \big(\psi(\sigma^1_{\mathbf{J}}) - \psi(\sigma^0_{\mathbf{E}})\big) \\
&> TS^*(1,\sigma^1_{\mathbf{J}}) - TS^*(1,\sigma^1_{\mathbf{E}}) + \psi(\sigma^1_{\mathbf{E}}) - \psi(\sigma^0_{\mathbf{E}}) - \big(\psi(\sigma^1_{\mathbf{J}}) - \psi(\sigma^0_{\mathbf{E}})\big) \\
&= TS^*(1,\sigma^1_{\mathbf{J}}) - TS^*(1,\sigma^1_{\mathbf{E}}) - \big(\psi(\sigma^1_{\mathbf{J}}) - \psi(\sigma^1_{\mathbf{E}})\big),
\end{aligned}$$

where the first inequality comes from the proof of proposition 3. Therefore, we will obtain that the difference is positive, whenever the increase in the social surplus due to a higher investment of the seller is bigger than the cost, i.e. $TS^*(1,\sigma^1_{\mathbf{J}}) - TS^*(1,\sigma^1_{\mathbf{E}}) > \psi(\sigma^1_{\mathbf{J}}) - \psi(\sigma^1_{\mathbf{E}})$ and therefore, we additionally require that the effect of the investment of the seller in the social surplus is sufficiently big. Observe that because a lower degree of competition, that is, for $J' \subset J$, the investment of the seller is more inefficient and the investemnt of the buyer stays the same, the level of net social welfare is decreased. Consequently, the maximum is attained at J.