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Rational and Irrational Bubbles: An Experiment

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Abstract

This paper proposes a theory of rational bubbles in an economy with finite trading opportunities. Bubbles arise because agents are never sure to be last in the market sequence. This theory is used to design an experimental setting in which bubbles can be made rational or irrational by varying one parameter. This complements the experimental literature on irrational bubbles initiated by Smith, Suchanek and Williams (1988). Our experimental results suggest that it is pretty difficult to coordinate on rational bubbles even in an environment where irrational bubbles flourish. Maximum likelihood estimations show that these results can be reconciled within the context of Camerer, Ho, and Chong (2004)'s cognitive hierarchy model, and Mc Kelvey and Palfrey (1995)'s quantal response equilibrium.

Keywords: Experiment, rational bubbles, irrational bubbles, cognitive hierarchy model, quantal response equilibrium.

1 Introduction

This paper presents an experimental investigation of speculative behavior in asset markets where both rational and irrational speculation can arise. Recent economic developments suggest that financial markets experienced various periods of bubbles and crashes. The dot com mania, and the subprime mortgage frenzy are frequently interpreted as evidence that asset prices on financial markets reach levels well above their fundamental value.¹ Likewise, Dutch Tulip, South Sea, and Mississippi are names often associated with the term bubble to refer to more ancient episodes of price run ups followed by crashes. However, to the extent that fundamental values cannot be directly observed in the field, it is very difficult to empirically demonstrate that these episodes actually correspond to mispricings.

To overcome this difficulty and study bubble phenomena, economists have relied on the experimental methodology: in the laboratory, fundamental values are induced by the researchers who can then compare them to asset prices. Starting with Smith, Suchanek and Williams (1988), many researchers document the existence of irrational bubbles in experimental financial markets. These bubbles are irrational in the sense that they would be ruled out by backward induction. The design created by Smith, Suchanek and Williams (1988) features a double auction market for an asset that pays random dividends in several successive periods. The subsequent literature shows that irrational bubbles also tend to arise in call markets (Van Boening, Williams, LaMaster, 1993), with a constant fundamental value (Noussair, Robin, Ruffieux, 2001), with lottery-like assets (Ackert, Charupat, Deaves, and Kluger, 2006), and tend to disappear when some traders are experienced (Dufwenberg, Lindqvist, and Moore, 2005), when there are futures markets (Porter and Smith, 1995) and no short-selling restrictions (Ackert, Charupat, Church, and Deaves, 2005). Lei, Noussair and Plott (2001) modify the Smith, Suchanek and Williams (1988) 's framework to show that, even when they cannot resell and realize capital gains, some participants still buy the asset at a price which exceeds the sum of the expected dividends to be distributed, a behavior consistent with risk-loving preferences or judgmental errors.

The objective of the present paper is to propose an experimental design in which rational and as well as irrational bubbles can be studied. To do so, we develop a theory of bubbles with a finite number of trading opportunities. This theory complements the analysis of Allen, Morris,

¹We define the fundamental value of an asset as the price at which agents would be ready to buy the asset given that they cannot resell it later. See Camerer (1989) and Brunnermeier (2008) for surveys on bubbles.

and Postlewaite (1993) in that it is simpler and features an economy where the bubble is common knowledge. This theory also complements the literature on rational bubbles that considers infinite trading opportunities via infinite time models (see, for example, Tirole (1985) for deterministic bubbles, and Blanchard (1979) and Weil (1987) for stochastic bubbles), via continuous trading models (see Allen and Gorton (1993)), or via clock games (see Abreu and Brunnermeir (2004)). We use our theoretical analysis in order to design a new experimental setting where rational bubbles can be studied. By appropriately modifying one parameter of the experimental design (in our case, by imposing a price cap), it is also possible to study irrational bubbles. Having such a unified framework is useful because it allows us to compare the formation of irrational and rational bubbles. It also enables us to study how our results on rational bubbles compare to the previous literature on irrational bubbles. Finally, our paper analyzes individual behavior (and not only market-level data) to better understand the sources of speculative behavior. This analysis is based on two models of bounded rationality: the cognitive hierarchy model developed by Camerer, Ho and Chong (2004), and the quantal response equilibrium of Mc Kelvey and Palfrey (1995). Studying individual behavior enables to better identify and study three different types of speculative behavior that might play a role in bubbles, namely irrational speculation (due to mistakes or erroneous beliefs), speculation on others' irrationality (betting on the fact that others are going to do mistakes), and speculation on others' rationality (betting on the fact that others are going to act rationally).

As noted above, our theory extends the analysis of rational bubbles to show that these can arise even with a finite number of trading opportunities. We consider a model where the market proceeds sequentially. The basic element that allows for the possibility of a rational bubble in this setting is that market participants are never sure to be last in the market sequence. This is achieved by including in the economy adequate uncertainty on participants' position in the market and on price paths. This idea is in the spirit of Allen and Gorton (1993). We complement their analysis by showing that limited liability and continuous trading are not required for a bubble to be sustainable at equilibrium. We then show that bubbles can be sustained as the outcome of a Bayesian Nash equilibrium.² Indeed, conditionally on observing a positive price at which they can buy the asset (which is assumed to have a zero fundamental value), each participant in the market sequence is better off entering the bubble. Thus, even in this zero-sum game,

²Our model of financial market can be viewed as a generalization of a constant-sum centipede game in which players do not know at which node of the game they play.

risk-neutral (or even risk averse) agents would be willing to participate. However, this situation does not create ex-ante surplus because the unconditional expected payoffs cannot be computed (the unconditional expectation of price levels as well as of profits and losses are infinite).

Our experimental design uses this theoretical analysis in order to empirically study bubbles. The previous theoretical environment cannot be implemented as it is in the laboratory because players are exposed to potentially infinite losses. In order to offer limited liability to our subjects, we thus devise a delegated portfolio management situation that is closely linked to the theoretical benchmark. We endow subjects one monetary unit that they can invest (and thus potentially loose). In the paper, we refer to the subjects as portfolio managers or managers. If the amount needed to buy the asset is higher than their endowment, managers receive capital from an outside financier, and profits and losses are shared in proportion of the initial stakes. We check that there exists a Bayesian Nash equilibrium of this game in which it is perfectly rational for managers to enter a bubble and for financiers to provide capital.³ If potential prices are capped, we are back to a situation similar to the one analyzed by the previous experimental literature in which bubbles cannot arise at equilibrium. This enables us to compare our results with the previous findings. It also allows us to study the impact of bounded rationality on bubble formation. Indeed, the higher the cap is, the higher is the number of iterated steps of reasoning that is required to reach equilibrium. By varying the level of the cap on prices, we thus vary the level of sophistication that is needed for the subjects to realize that it is not in their interest to enter into the bubble.

Our experimental protocol is as follows. We consider a setting with three market participants. Ex-ante, each subject has the same probability $\frac{1}{3}$ to be first, second or third in the sequence. Prices experience a tenfold increase at each trading date. The price proposed to the first trader in the market is a power of 10. This power is randomly determined according to a geometric distribution of parameter $\frac{1}{2}$. We run the experiment under different conditions. In one session, there is no cap on prices and there thus exists a bubble equilibrium. In other sessions, there is a cap K on the first price with K equals to 10,000, 100, or 1.

Our experimental results are as follows. First, we show that, when there is no price cap, bubbles are not observed for sure. There is a great deal of uncertainty as whether participants coordinate on the bubble equilibrium. Second, when there is a cap on the first price, we observe

³In our experiment, the price path is not left at the discretion of market participants. We thus do not have pricing results per se but instead focus on the decision to speculate by entering bubbles.

a lot of bubbles: the likelihood of a bubble is not significantly different whether there is a cap or not on the first price. This result shows that in our simple design, we are able to replicate the results found in the previous experimental literature. We complement this literature by showing, through a regression analysis, that the propensity for a subject to enter a bubble is positively related to the conditional probability to be first in the market sequence (from a participant's perspective), and to the number of reasoning steps needed to realize that subsequent participants can realize they are last. The propensity to enter a bubble is negatively related to the conditional probability to be last in the market sequence, and to risk aversion. These results suggest that subjects' decision to enter bubbles, if they are not in line with the level of rationality required to achieve Nash equilibrium, demonstrate some level of rationality.

To better understand subjects' behavior, we take two models of bounded rationality to the data: Camerer, Ho and Chong (2004)'s cognitive hierarchy (CH) model, and Mc Kelvey and Palfrey (1995)'s quantal response (QR) equilibrium. The CH model states that agents best-respond to mutually inconsistent beliefs. In particular, the model features players with different levels of sophistication: level-0 agents play randomly; level-1 agents believe that other players are level-0 and best-respond; type-2 agents believe that other players are level-1 or level-0, and best-respond,... The model considers that levels of sophistication are distributed according to a poisson distribution. The only free parameter in this model is thus the average level of sophistication. Maximum likelihood estimation indicates an average level of sophistication equal to 0.67 which is in line with the estimations of Camerer, Ho and Chong (2004) on other types of games.

The QR equilibrium also takes into account the fact that players make mistake but retains beliefs' consistency. Players' expectations are rational in the sense that they take into account the fact that others make mistakes. The only free parameter is the responsiveness of players' choice to the expected payoffs of the various actions: if it is 0, players choose uniformly among the available actions. If the responsiveness is infinite, players best-respond. Otherwise, players choose their actions stochastically with high expected payoff actions being more likely than low ones. Maximum likelihood estimation indicates an average responsiveness of 2.54 in line with previous estimations of Mc Kelvey and Palfrey (1995) on other types of games.

The CH model and the QR equilibrium both fit the data pretty well compared to the Nash equilibrium. These theories further appear to offer a good description of behavior both in rational and irrational bubbles. This suggests that it might be useful for market practitioners to include

limited sophistication and noisy responses in their analysis when thinking about the causes and consequences of bubbles in financial markets, and when trying to fight against these bubbles. Our analysis suggests that institutional features that reduce the required level of iterated reasoning necessary to find bubbles unprofitable might be useful in mitigating the development of such bubbles. Price caps (potentially temporary) can in this respect be useful to prevent the occurrence of bubbles.

Our experimental analysis is related to Brunnermeier and Morgan (2006) who study clock games both from a theoretical and an experimental standpoint. These clock games can indeed be viewed as metaphors of “bubble fighting” by speculators, gradually and privately informed of the fact that an asset is overvalued. Speculators do not know if others are already aware of the bubble. They have to decide when to sell the asset knowing that such a move is profitable only if enough speculators have also decided to sell. Their experimental investigation and ours share two common features. First, the potential payoffs are exogenously fixed, that is, there is a predetermined price path. Second, there is a lack of common knowledge over a fundamental variable of the environment. In Brunnermeier and Morgan (2006), the existence of a bubble is not common knowledge. In our setting, the existence of the bubble is common knowledge but traders’ position in the market sequence is not. One difference between our approach and theirs is the time dimension. The theoretical results tested by Brunnermeier and Morgan (2006) depend on the existence of an infinite time horizon. They implement this feature in the laboratory by randomly determining the end of the session. On the contrary, we design an economic setting in which there could be bubbles in finite time with finite trading opportunities, even if traders act rationally.

The rest of the paper is organized as follows. Next section presents a model in which bubbles arise at equilibrium in an economy with finite trading opportunities. Section 3 details the experimental design. Results are in Section 4. Section 5 concludes and provides potential extensions.

2 A theory of rational bubbles with finite trading opportunities

The objective of this section is to show that bubbles can emerge in a financial market with perfectly rational traders and finite trading opportunities. Consider a financial market in which trading proceeds sequentially. There are T agents, referred to as traders. Traders’ position in the

market sequence is random with each potential ordering being equally likely. Traders can trade an asset that is issued by agent 0, referred to as the issuer.⁴ The first trader in the sequence is offered to buy the asset at a price P_1 . If he does so, he proposes to resell at price P_2 to the second trader. More generally, the i -th trader in the sequence, $i \in \{1, \dots, T-1\}$, is offered to buy the asset at price P_i and resell at price P_{i+1} to the $i+1$ -th trader. Traders take the price path as given, with $P_i > 0$ for $i \in \{1, \dots, T\}$. Finally, the last trader in the sequence is offered to buy the asset at price P_T but is unable to resell. If the i -th trader buys the asset and is able to resell it, his payoff is $P_{i+1} - P_i$. If he is unable to resell the asset, his payoff is $-P_i$. For simplicity, we consider that if a trader refuses to buy the asset, the market process stops. Without loss of generality, we assume that the asset is worth zero. Traders are risk neutral and have an initial wealth denoted by W_t , $t \in \{1, \dots, T\}$.⁵ As a benchmark, consider the case in which traders have perfect information, that is, each trader t knows his position in the sequence and this is common knowledge. In this perfect information benchmark, it is straightforward to show that no trader will accept to buy the asset except at a price of 0 which corresponds to the fundamental value of the asset. Indeed, the last trader in the queue, if he buys, ends up with $W_T - P_T$ which is lower than W_T . Since he knows that he is the last trader in the queue, he prefers not to trade. By backward induction, this translates into a no-bubble equilibrium. This result is summarized in the next proposition.

Proposition 1 *When traders know their position in the market sequence, the unique perfect Nash equilibrium involves no trade.*

Let's now consider what happens when traders do not initially know their position in the market sequence, and this is common knowledge. We model this situation as a Bayesian game. The set of players is $\{1, \dots, T\}$. The set of states of the world is Ω which includes the $T!$ potential orderings. ω refers to a particular ordering. The set of actions is identical for each player t and is denoted by $A = \{B, \emptyset\}$ in which B stands for buy and \emptyset for refusal to buy. The set of signals that may be observed by player t is the set of potential prices denoted by P . Denote by $\omega_i^t \subset \Omega$ the set of orderings in which trader t 's position in the market sequence is i . The signal function of player t is $\tau(t) : \omega_i^t \rightarrow P_i$, in which P_i refers to the price that is proposed to the i -th trader

⁴The potential bubbles that may arise in our environment can be interpreted as Ponzi schemes, and the issuer of the asset as the scheme organizer.

⁵In our model, traders might end up with negative wealth.

in the market sequence. The price path P_i is defined as follows. The price P_1 proposed to the first trader in the sequence is random and is distributed according to the probability distribution $g(\cdot)$ on P .⁶ Other prices are determined as $P_{i+1} = f_i(P_i)$, with $f_i(\cdot) : P \rightarrow P$ being a strictly increasing function that controls for the explosiveness of the price path. A strategy for player t is a mapping $S_t : P \rightarrow A$ in which $S_t(P_t)$ indicates what action is chosen by player t after observing a price P_t . We denote by $S_{-t}(P_t)$ the actions chosen by players other than t when this player t has observed P_t . Player t indeed understands that the next player in the market sequence will observe $f_t(P_t)$, and that he chooses $S_{t+1}(f_t(P_t))$. Using the signal function, players may learn about their position in the market sequence. A strategy profile $\{S_1^*, \dots, S_T^*\}$ is a Bayesian Nash equilibrium if the following individual rationality (IR) conditions are satisfied:

$$\mathbb{E} [\pi(S_t^*(P_t), S_{-t}^*(P_t)) | P_t] \geq \mathbb{E} [\pi(S_t(P_t), S_{-t}^*(P_t)) | P_t], \forall t \in \{1, \dots, T\}, \text{ and } \forall P_t \in P.$$

$\pi(S_t(P_t), S_{-t}^*(P_t))$ represents the payoff received by player t given that he chooses action $S_t(P_t)$ and that other players choose actions $S_{-t}^*(P_t)$.

We now study under what conditions there exists a bubble equilibrium $\{S_1^* = B, \dots, S_T^* = B\}$. The crucial parameter a player t has to worry about in order to decide whether to enter a bubble is the conditional probability to be last in the market sequence, $\mathbb{P}(\omega \in \omega_T^t | P_t)$. The IR condition can be rewritten as:

$$(1 - \mathbb{P}[\omega \in \omega_T^t | P_t]) \times (W_t + f_t(P_t) - P_t) + \mathbb{P}[\omega \in \omega_T^t | P_t] \times (W_t - P_t) \geq W_t, \forall t \in \{1, \dots, T\}, \text{ and } \forall P_t \in P$$

$$\Leftrightarrow (1 - \mathbb{P}[\omega \in \omega_T^t | P_t]) \times f_t(P_t) \geq P_t, \forall t \in \{1, \dots, T\}, \text{ and } \forall P_t \in P.$$

If $\mathbb{P}[\omega \in \omega_T^t | P_t] = 1$ for some P_t , the IR condition is not satisfied and the bubble equilibrium does not exist. This is for example the case when the support of the distribution $g(\cdot)$ is bounded above by a threshold K . Indeed, a trader upon observing $P_t = f_1 \circ \dots \circ f_{T-1}(K)$ knows that he is last and refuses to trade. Backward induction then prevents the existence of the bubble equilibrium. The IR function is also not satisfied if the signal function $\tau(t)$ is injective. Indeed, by inverting the signal function, players, including the one who is last in the sequence, learn what their position is. These results are summarized in the following proposition.

⁶One can consider that this first price P_1 is chosen by Nature or by the issuer according to a mixed strategy characterized by $g(\cdot)$.

Proposition 2 *The no-bubble equilibrium is the unique Bayesian nash equilibrium if i) the signal function is injective, ii) the first price is randomly distributed on a support that is bounded above, iii) the price path is not explosive enough, or iv) the probability to be last in the market sequence is too high.*

We now propose an environment where the IR condition derived above is satisfied. Consider that the set of potential prices is defined as $P = \{m^n \text{ for } m > 1 \text{ and } n \in \mathbb{N}\}$, that is, prices are powers of constant $m > 1$. Also, assume that $g(P_1 = m^n) = (1 - q)q^n$, that is, the power n follows a geometric distribution of parameter $q \in (0, 1)$. Finally, we set $f_i(P_i) = f(P_i) = m \times P_i$. Given these assumptions, upon being proposed to buy at a price of m^2 , a player can be first in the market sequence with probability $\frac{1}{8}$, second with probability $\frac{1}{4}$, and third with probability $\frac{1}{2}$. Also, if there are T players on the market, the probability that a player t is last in the sequence, conditional on the price P_t that he is proposed, is computed by Bayes' rule: $\mathbb{P}[\omega \in \omega_T^t | P_t = m^n] = \frac{\mathbb{P}[P_t = m^n | \omega \in \omega_T^t] \times \mathbb{P}[\omega \in \omega_T^t]}{\mathbb{P}[P_t = m^n]} = \frac{(1-q)q^{n-(T-1)} \times \frac{1}{T}}{\sum_{j=n-(T-1)}^n (1-q)q^j \times \frac{1}{T}} = \frac{1-q}{1-q^T}$ if $n \geq T - 1$, and $\mathbb{P}[\omega \in \omega_T^t | P_t = m^n] = 0$ if $n < T - 1$. Under our assumptions, Bayes' rule implies that i) the conditional probability to be last in the market sequence is either 0 if the proposed price is smaller than m^{T-1} , and ii) if the proposed price is equal to or higher than m^{T-1} , this conditional probability does not depend on the level of the price that is proposed to the players.⁷ The IR condition can be rewritten:

$$\left(\frac{q - q^T}{1 - q^T} \right) \times m \geq 1, \forall t \in \{1, \dots, T\}, \text{ and } \forall P_t \in P.$$

This condition is less restrictive when there are more traders present on the market. This condition is equivalent to $m \geq \frac{1-q^T}{q-q^T}, \forall t \in \{1, \dots, T\}, \text{ and } \forall P_t \in P$.

There thus exists an infinity of price paths that sustain the existence of a bubble equilibrium. Obviously, there always exists a no-bubble equilibrium.⁸ Indeed, if players anticipate that other

⁷We implicitly assume here that players cannot observe if transactions occurred before they trade. However, we do not need such a strong assumption. For example, if each transaction was publicly announced with a probability strictly smaller than a threshold, our results would still hold. This threshold should be small enough such that the likelihood of being last in the sequence does not get too high.

⁸When there exists a bubble-equilibrium in pure strategies, there can also exist mixed-strategies equilibria in which traders enter the bubble with a positive probability that is lower than 1. We have characterized these equilibria for the two-player case. They involve peculiar evolutions of the probability to enter the bubble depending on the price level that is observed.

players do not enter the bubble, then they are better off refusing to trade. These results are summarized in the next proposition.

Proposition 3 *If i) the T traders are equally likely to be last in the market sequence, ii) the price P_1 proposed to the first trader in the sequence is randomly chosen in powers of m according to a geometric distribution, and iii) $P_t = m \times P_{t-1}$, $\forall t \in \{2, \dots, T\}$, there exists a bubble Bayesian Nash equilibrium if and only if $m \geq \frac{1-q^T}{q-q^T}$, $\forall t \in \{1, \dots, T\}$, and $\forall P_t \in P$. There always exist a no-bubble equilibrium.*

Our results hold even if one introduces randomness in the underlying asset payoff, and (potentially random) payments at interim dates. In Appendix A, we show that our results can still hold if traders are risk averse. One could be tempted to interpret our results as an inverse-Hirshleifer effect: going from perfect to imperfect information seem to imply a creation of gains from trade in our setting even with risk-neutral agents. However, note that it is not possible to compute the ex-ante welfare created by the game of imperfect information. Indeed, this would require computing the unconditional expectation of the price P_1 that is proposed to the first trader in the market sequence. Given our assumptions, this expectation is given by the expression $\mathbb{E}[P_1] = \lim_{x \rightarrow +\infty} \sum_{n=0}^{n=x} \frac{(1-q)[1-(qm)^{x+1}]}{1-qm}$. This sum converges if and only if $qm \leq 1$. This inequality is in conflict with the IR condition according to which $m \geq \frac{1-q^T}{q-q^T} > 1$. This implies that the only games in which the ex-ante welfare is well-defined are the games where only the no-bubble equilibrium exists. This makes it hard to conclude that the imperfect information game is actually creating welfare even if interim (that is, knowing the proposed price), all traders are strictly better off entering the bubble if they anticipate that other traders are also going to do so.

The next section will use these theoretical results in order to design an experimental setting where the existence of a bubble equilibrium depends on an institutional parameter of the economy. This allows us to study bubble formation in the laboratory in a context where bubbles can be individually rational for all the participants.

3 Experimental design

This section proposes a simple experimental design in which bubbles can arise at equilibrium. Based on the previous theoretical analysis, this design features a sequential market for an asset whose fundamental value is 0. There are three traders on the market.⁹ Trading proceeds sequentially. Each trader is assigned a position in the market sequence and can be first, second or third with the same probability $\frac{1}{3}$. Traders are not told their position in the market sequence but get some information when observing the price at which they can buy the asset. Prices are exogenously given and are powers of 10. For simplicity, in this experimental design, we do not include the issuer of the asset. The first trader is offered to buy at a price $P_1 = 10^n$. In the baseline experiment, the power n follows a geometric distribution of parameter $\frac{1}{2}$, that is $\mathbb{P}(n = j) = \frac{1}{2}^{j+1}$, with $j \in \mathbb{N}$. The geometric distribution is useful from an experimental point of view because it is simple to explain and implies that the probability to be last in the market sequence conditional on the proposed price is constant. This probability is equal to 0 if the proposed price is 1 or 10, and is equal to $\frac{4}{7}$ otherwise.¹⁰ If he decides to buy the asset, trader i in the market sequence proposes the asset to the next trader at a price $P_{i+1} = 10P_i$. In order to avoid participants from discovering their position in the market sequence by hearing other subjects making their choices, we propose simultaneously to the first, second, and third traders to buy the asset. Once P_1 has been randomly determined, the first, second and third traders are simultaneously offered prices of P_1 , P_2 , and P_3 , respectively.¹¹

If we stopped here, participants' net payoffs (that is, their gains and losses relative to their initial wealth) would be as illustrated in Panel A of Figure 1. Payoffs depend on the various traders' decisions. For example, if the first and the second traders buy while the third one refuses to buy, payoffs are $P_2 - P_1$, $-P_2$, and 0, respectively. Except for the case in which the first trader refuses to buy (so that the bubble does not start), each potential market outcome of the game

⁹We could have designed an experiment with only two traders per market. However, this would have required higher payments for bubbles to be rational. Indeed, the conditional probability to be last would be higher. We could also have chosen to include more than three traders per market. We decided not to do so in order to have a high number of markets. This is useful from an econometric perspective since markets constitute statistically independent observations.

¹⁰The probabilities to be first, second or third conditional on the prices, which are computed using Bayes' rule, are given to the participants in the Instructions.

¹¹When a trader does not accept to buy the asset, subsequent traders end up with their initial wealth whatever their decisions.

translates into a loss for one of the market participants (the last one in the market sequence).¹² Since the first price is distributed on an unbounded support, this loss can be very large. This feature is unappealing because experimental subjects cannot be asked to pay large amounts of money. We thus introduce limited liability in a way that does not affect subjects incentive to enter into bubbles. To do so, we rely on a delegated portfolio situation that we refer to as the manager/financier game (as opposed to the trader game that we considered above). Each trader in the previous game is replaced by an asset manager who is endowed with an initial wealth of 1 euro. If additional capital is required in order to buy the asset at price P_t , this additional capital (that is, $P_t - 1$) is provided by an outside financier. We assume that each manager is financed by a different financier. Net payoffs (potential gains and losses) are then divided between the manager and the financier according to their share in the initial capital: $\frac{1}{P_t}$ for the manager, and $\frac{P_t-1}{P_t}$ for the financier. Consequently, if the manager decides to buy the asset at price P_t , his loss in case the next trader refuses to trade is:

$$\frac{1}{P_t} \times (-P_t) = -1.$$

The manager has invested 1 euro (along with the $P_t - 1$ euros of the financier) in order to buy an asset at a price of P_t but he is unable to resell the asset which has a liquidation value of 0. He ends up with a final wealth of 0 since he has lost 1 euro due to the fact that the bubble bursted after he entered. Likewise, if the manager decides to buy the asset at price P_t , his gain in case the next trader accepts to buy is:

$$\frac{1}{P_t} \times (P_{t+1} - P_t) = \frac{1}{P_t} \times (10P_t - P_t) = 9.$$

When the manager is able to resell the asset, he gets $\frac{1}{P_t}$ percent of the proceed $P_{t+1} = 10P_t$ and thus ends up with a final wealth of 10 which corresponds to 1 euro of initial wealth plus a gain of 9 euros. Overall, whatever the price at which the manager buys, he can loose 1 euro or win 9 euros. If he anticipates that other managers buy the asset, it is in a manager's best interest to also buy the asset if the following individual rationality (IRm) condition is satisfied to buy the asset if and only if:

$$\frac{3}{7}U_m(W_m + 10) + \frac{4}{7}U(W_m) \geq U(W_m + 1),$$

¹²At each outcome of the game (except if the first trader refuses to buy), the total payoff is equal to $-P_1$. This aggregate loss corresponds to the gain of the issuer of the asset (who is not part of the experiment).

where $U_m(\cdot)$ is a manager's utility function and W_m his initial wealth. For a bubble equilibrium to exist, the IRm condition has to be satisfied for all managers on the market. It is straightforward to show that there exists functions $U_m(\cdot)$ for which the IRm condition hold for all W_m .

This IRm condition in the manager/financier game echoes the IR condition that prevails for the trader game presented in the previous section (see appendix A, for an analysis of the trader game with risk averse agents). The strategic incentives that agents face in both games are similar except that in the trader game potential gains or losses are potentially infinite. The fact that we have finite gains and losses for the manager/financier game is extremely useful from an experimental point of view since it enables the experimenter to assign limited liability to subjects.

In order to show that the manager/financier game is meaningful, we now check that the financier has an interest in providing capital to the manager. The individual rationality of the outside financier (IRf) is written as:

$$\frac{3}{7}U_f(W_f + f_t(P_t) - P_t - 10) + \frac{4}{7}U_f(W_t - P_t + 1) \geq U_f(W_t), \forall t \in \{1, \dots, T\}, \text{ and } \forall P_t \in P,$$

where $U_f(\cdot)$ is a financier's utility function and W_f his initial wealth. For a bubble equilibrium to exist, the IRf condition has to be satisfied for all financiers on the market. It is straightforward to show that there exist functions $U_f(\cdot)$ for which the IRf condition holds for all W_f . In order to limit subjects' potential losses in our experiment, the outside financiers are not part of the experiment. The timing of the game in which subjects are involved is illustrated in Panel B of Figure 1 (the payoffs of the financiers who are not part of the experiment are also indicated for completeness). Notice that the sum of managers' and financiers' payoffs in the manager/financier game equals the payoffs of the traders in the trader game.

The timing of the game as it was presented to subjects in our experiment is given in Figure 2.¹³ In this game, subjects face strategic incentives that are identical to the one faced by a trader who has to decide to enter into a bubble. In order to study how formation of bubbles is influenced by their rationality, we also focus on an experimental design where there is a cap K on the first price. This will allow us to relate our work to the previous experimental literature

¹³This timing does not correspond to the extensive form of the game. Indeed, it leaves aside the issue of which player is first, second, or third. The extensive form game is provided in Appendix B for the two-player case.

on bubbles initiated by Smith et al. (1988) that focuses on irrational bubbles. Indeed, as shown in the previous section, in this design, bubbles are irrational in the sense that they would be ruled out by backward induction. The cap on the first price translates into a cap on the highest potential price in the experiment. Upon being proposed this price, a subject should understand that he is last in the market sequence and, consequently, should refuse to buy. Anticipating this refusal, subjects who are proposed lower prices should also refuse to buy. At equilibrium, the bubble never starts. However, given the experimental results on centipede games (see Mc Kelvey and Palfrey (1992) or Kawagoe and Takizawa (2009), for example), we expect that this will not happen in our experiment (because of altruism or failures in backward induction). Varying the level of the cap K then offers potentially interesting comparative statics because it controls the number of iterated steps of reasoning that are needed in order to reach the Nash equilibrium. When the proposed price is $P = 100K$, a subject knows that he is last and there is no iterated step of reasoning. When the proposed price is $P = 10K$, a subject knows that he is not first in the market sequence (he can be second or third). At equilibrium, he has to anticipate that the next trader in the market sequence (if any) would not accept to buy the asset. One step of iterated reasoning is thus needed to derive the equilibrium strategy. More generally, when the proposed price is $1 \leq P \leq 100K$, the required number of iterated steps of reasoning is $\log_{10}(\frac{100K}{P})$. In order to study whether this required number of iterated steps of reasoning could affect bubble formation, we have chosen to experimentally study cases in which K equals 1, 100, and 10,000.

The experimental protocol runs as follows. Our experiment includes a total of 93 subjects. Subjects are in the last year of the Bachelor in Business Administration at the University of Toulouse. We have four sessions with 21 to 24 subjects per session. Each subject participates in only one session. Each session includes only one replication of the trading game.¹⁴ Subjects' risk aversion is measured thanks a procedure which is inspired from Laury and Holt (2002). We adjust their questionnaire in order to match the set of possible decisions to the decisions subjects may actually face in our experiment.¹⁵ The experiment is run with paper and pencil. Subjects are given the conditional probabilities to be first, second, and third given the price they are proposed. The instructions for the case where $K = 10,000$ are in Appendix C.

¹⁴Because we only consider a single replication of the experiment, we thus cannot discuss learning issues that are left for future research.

¹⁵The questionnaire is composed of 14 decisions. For each decision i , subjects may choose between the riskless option A, which is to receive 1 euro for sure, or the risky option B, which is to receive 10 euros with probability $\frac{i}{14}$, or 0 euro with probability $\frac{14-i}{14}$.

Our experimental design is summarized in Table 1.

TABLE 1

Session	Number of Subjects	cap on initial price, K	Equilibrium
1	24	1	no-bubble
2	21	100	no-bubble
3	24	10,000	no-bubble
4	24	∞	no-bubble or bubble

4 Results

4.1 Market behavior

We first start by analyzing overall market behavior. At this aggregate level, we can measure the frequency as well as the magnitude of bubbles. The frequency of bubbles is defined as the proportion of replications where the first trader accepted to buy the asset. The magnitude of bubbles is referred to as large if all three traders accepted to buy the asset, medium if the first two traders accepted, and small if only the first trader accepted. Figure 3 presents the results per session.

Figure 3 shows that there are bubbles in an environment where backward induction is supposed to shut down speculation, namely when there exists a price cap. This is in line with the previous experimental litterature cited in the introduction and contradicts our Proposition 2 (ii). Figure 3 helps us identifying three types of speculation. First, we observe large bubbles even in situations where the existence of a cap enables some subjects to perfectly infer that they are last in the market sequence. A possible explanation is related to bounded rationality.¹⁶ It is possible that some traders make mistakes and buy when they are last in the market sequence. Second, this observation gives rise to a second motive for speculation: if initial traders in the sequence anticipate that some participants will behave as described above, it may become optimal for them to buy the asset. In this case, participants rationally speculate on others' irrationality (betting on the fact that others are going to do mistakes). Third, we observe bubbles when there is no price

¹⁶An alternative explanation could be related to social preferences. However, extreme altruism would be required in order for a subject to be willing to loose in order to let other subjects gain. We thus do not focus on this interpretation.

cap, that is, when there exists a bubble equilibrium. This suggests that participants speculate on others' rationality (betting on the fact that others are going to act rationally).

Figure 3 also shows that bubbles are not more likely when there is no price cap. This indicates that traders fail to perfectly coordinate on the bubble equilibrium. Proposition 3 indicates that this could be due to strategic uncertainty because there always exists a no-bubble equilibrium, or to risk aversion because if traders are sufficiently risk averse, it is not beneficial to enter the bubble. An additional interpretation is that the possible existence of irrational traders, who may not buy when it would be rational, increases the risk of entering the bubble for rational traders and may prevent them from doing so.

4.2 Individual behavior

To gain more insights on bubbles formation, we now study individual decisions to buy the over-valued asset. Figure 4 plots our entire data set. For each session, each bar represents the number of times a given price has been proposed. Within each bar, the dark grey part corresponds to buy decisions while the white part corresponds to refusals to trade. Figure 4 complements the results found in the previous subsection. Let's first focus on the case where there is no cap on the first price. In this treatment, participants who are proposed prices of 1 or 10 buy the asset while those who are proposed higher prices tend to refuse to trade quite often. This pattern is consistent with Nash equilibrium if one takes into account subjects' risk aversion: when they are proposed a price of 1 or 10, subjects are sure not to be last, whereas when they are proposed higher prices, they have 4 chances out of 7 to be last. Their level of risk aversion may thus become binding. When prices are above 100, if participants coordinate on the same equilibrium, their decisions should be the same for all price levels. We cannot reject this equality using a Wilcoxon rank sum test that compares the proportion of buy decisions after observing prices of 100, 1,000, and 10,000 to this proportion after observing higher prices.

The high probabilities to buy in the sessions where there is a price cap are inconsistent with Nash equilibrium. However, we keep observing the same pattern as in the situation where there is no cap on the initial price. In the former cases though, higher prices are informative on a trader being last in the sequence. To investigate this result, Figure 5 reports the probability to buy, conditional on participants' inference on their position. On the one hand, the probability to buy decreases with the likelihood that a trader is last in the market sequence. Traders buy very

often when they are sure to be first and sure not to be last, while they buy half of the time when they cannot infer their position. This is consistent both with the fact that they face a more risky decision and with the fact that traders are reluctant to coordinate on the bubble equilibrium. Also, this indicates that there are some elements of rationality in subjects' decisions. On the other hand, this result holds whether bubbles are irrational or not. In particular, the propensity of subjects to enter a bubble is extremely large when they know that they are first or second, even in a situation in which there exists a price cap. This result contradicts the predictions of Nash equilibrium, so that rationality does not appear to be perfect.

These results suggest that the number of steps of iterated reasoning which are needed to derive the equilibrium strategy may help explaining our descriptive statistics, in situations where there exists no bubble equilibrium. We therefore run a probit regression that tries and explains the propensity to buy the overvalued asset as a function of several variables: the number of steps of iterated reasoning needed to derive the equilibrium strategy, the coefficient of risk aversion, the probability to be first conditional on the observed price, and the probability to be last conditional on the observed price. Conditional probabilities are computed following Bayes' rule¹⁷. A probit regression is adequate because we try to explain a dummy variable: either the subject chooses to buy the asset (a decision coded as 1) or not (coded as 0).

The results are in Table 2 (presented at the end of the paper) and show that, apart from the level of risk aversion, all the explanatory variables have statistically significant coefficients. The sign of these coefficients is intuitive: a subject's propensity to buy the overvalued asset increases with his probability to be first in the market sequence and with the number of steps of iterated reasoning needed to derive the equilibrium strategy, and decreases with his probability to be last, and with his risk aversion. The risk aversion coefficient is not significant due to a spurious correlation with the probability to be third. This correlation is spurious since subjects have been randomly assigned a position in the experiment, independently from their answers to the questionnaire aiming at measuring their risk aversion coefficient. We therefore report in Table 2 the results of the regression excluding the conditional probability to be last. Risk aversion is now significantly and negatively related to the propensity to enter into the bubble.

¹⁷Remember that these probabilities were given to subjects during the experiment.

4.3 Fitting models of bounded rationality

Our results so far suggest that some players may have bounded rationality. The fact that some agents either play randomly or make errors may induce rational players to enter a bubble even if they would not do so in presence of rational traders only. Likewise, these agents with bounded may induce rational players not to enter bubbles even if they would do so in presence of rational traders only. To account for these phenomenon, we extend our analysis by estimating models that explicitly incorporate bounded rationality: the cognitive hierarchy model and the quantal response equilibrium.

In cognitive hierarchy (CH) theories, traders best-respond to mutually inconsistent beliefs. Traders differ in their level of sophistication s , and each player believes he understands the game better than the other players. Specifically, level-0 traders play randomly, level-1 traders believe that other traders are level-0, and level- s traders believe that other traders are a mixture of $s - 1$, $s - 2$, , 0, and best-respond to this belief. Cognitive hierarchy models posit decision rules for players doing s steps of thinking, that reflect this iterated process of strategic thinking. As in Camerer, Ho and Chong (2004), we assume that traders' types are distributed according to a Poisson distribution. let τ denote the average level of sophistication. For each parameter τ and each level of price P , we find the best-response of a risk-neutral level-1 trader who considers that the next trader is a level-0 player observing $P \times 10$. After one iteration, we find the best-response of a risk-neutral level-2 player who considers that the next trader, observing $P \times 10$, is a level-0 player with probability $\frac{\exp(-\tau) \times \tau^0}{0!}$, and a level-1 player with probability $\left(1 - \frac{\exp(-\tau) \times \tau^0}{0!}\right)$. We use a similar iterative process to find the best-response of a level- s player who considers that the next trader, observing $P \times 10$, is a level- j player with probability $\frac{\exp(-\tau) \times \tau^j}{j!}$ for $j < s - 1$, and a level- $(s - 1)$ player with the complementary probability. This process is fully described in Appendix D for the case where the price cap is $K = 1$. This enables us to determine the likelihood function under the assumption that subject are risk-neutral. We then estimate the parameter τ by maximum likelihood in each session and for the entire dataset.

Results are reported in Table 3 (presented at the end of the paper). The fit of the CH-Poisson model is compared with the fit of Nash equilibrium, by session and overall. The first two lines describe our data, namely, the group size, and the observed average probability to buy. The middle three lines show the predictions of the Nash equilibrium under a similar assumption of risk-

neutrality, the log-likelihood of this model, and its mean squared deviation.¹⁸ The mean choices are generally far off from the Nash equilibrium; the probability to buy is too low when there exists a bubble-equilibrium, and too high when it does not exist. The five next lines report the estimate of the parameter τ , the predictions of the cognitive hierarchy model for this value of the estimate, and the corresponding log-likelihood and mean squared deviation. We further compute the 90 percent confidence interval for τ estimated from a randomized resampling (bootstrap) procedure using 10,000 simulations. We estimate an overall average level of sophistication of 0.67, which is consistent with the estimates reported in Camerer, Ho and Chong (2004). This suggests a high proportion of level-0 players, that is, almost 50%. Interestingly, what drives this result is not really the fact that traders enter too much into bubbles when they should not. Indeed since there is only around 10% of subjects who buy when they are last in the market sequence and could understand that. This would suggest a proportion of level-0 players equal to 20%. What explains the high estimated proportion of level-0 players is rather the fact that subjects do not buy as much as expected by the cognitive hierarchy model (with a higher average sophistication level) when there is no cap on the initial price or when the cap is large. The reason why subjects do not buy as much as expected by the risk-neutral cognitive hierarchy model can be related to risk aversion. This suggests that the average level of sophistication may be underestimated due to the risk-neutrality assumption.

The Poisson CH model retains best-response (except for level-0 players), but it weakens equilibrium (that is, belief-choice consistency). McKelvey and Palfrey (1995) propose an alternative approach, which retains equilibrium but weakens best-response. In their Quantal Response Equilibrium, players may make mistakes. However, the likelihood of these errors depends on the impact of such errors on players' expected utility. More specifically, the following logit specification of the error structure is assumed, so that, if the buy decision yields an expected profit of u_B while the no buy decision yields an expected profit of u_\emptyset , the probability to buy writes:

$$\Pr(B) = \frac{e^{\lambda u_B}}{e^{\lambda u_B} + e^{\lambda u_\emptyset}}$$

This enables us to determine the likelihood function under the assumption that subject are risk-neutral (Appendix D below describes how this probability to buy is computed conditionally on

¹⁸We consider that traders coordinate on the bubble equilibrium when there is no cap on the initial price. The no-bubble Nash equilibrium has a lower log-likelihood. In order to compute these likelihoods, we assume that players choose non-equilibrium strategies with a probability of 0.0001.

the proposed price for the case where the price cap is $K = 1$). We then estimate the parameter λ , which we refer to as responsiveness to expected payoffs, by maximum likelihood in each session and for the entire dataset. Responsiveness is inversely related to the level of errors made by subjects. The results are reported in the last five lines of Table 3. We estimate an overall average responsiveness of 2.54, which is consistent with the results of McKelvey and Palfrey (1995). The QRE seems to fit better our data than the cognitive hierarchy model, especially when there is a cap on the initial price. This result seems to contradict those of Kawagoe and Takizawa (2008), who compare the goodness of fit of both models in laboratory experiments of the centipede game. To further investigate this issue, we compare in Figure 7 the probability to buy conditional on the proposed price, in the CH model and in the QRE, with our observations. What the QRE seems to better capture in our data is the drop in the probability to buy for prices larger than $P = 100$. In the CH model, this pattern either does not characterize the expected outcome (see the case in which there is no cap on the initial price), or captures it less intensively and with a lag (see the cases in which there is a cap at $K = 100$ or $K = 10,000$). In the QRE however, agents' mistakes have the feature that costlier (in terms of expected payoff) mistakes are less likely. This model is thus able to capture the drop in players' expected utility of buying related to the fact that they are proposed a price $P \geq 100$ in which case the conditional probability to be third is greater than or equal to $\frac{4}{7}$ instead of a price of 1 or 10 in which case their conditional probability to be third is zero. This feature is present in our design and is different from the centipede games analysed by Kawagoe and Takizawa (2008).

5 Conclusion

This paper explores speculative behavior in a laboratory experiment. The objective is to be able to better understand the various types of speculation that may be observed during bubble episodes: irrational speculation (due to mistakes or erroneous beliefs), rational speculation on others' irrationality (betting on the fact that others are going to do mistakes), and rational speculation on others' rationality (betting on the fact that others are going to act rationally). In order to design an experimental environment (that necessarily induces a finite number of trading opportunities) in which bubbles can occur at equilibrium, a theory is developed that extends the insights of Tirole (1982), Allen and Gorton (1993), and Abreu and Brunnermeier (2003). The idea is to model financial markets as a game in which agents trade an asset sequentially.

When agents do not know at which position they are in the market sequence, it can be in their interest to enter the bubble. We design an experimental setting based on this insight. Our design includes several treatments that defer by only one parameter, namely the level of a cap on prices. When there is no cap (or an infinite cap), there exists a bubble equilibrium. When there is a cap, there is only a no-bubble equilibrium. However, the higher the cap is, the higher is the number of iterated steps of reasoning that is needed to reach equilibrium.

Our results show that bubbles are frequently observed whether there is a price cap or not. This relates to the previous literature on bubbles initiated by Smith, Suchanek and Williams (1988) that shows that bubbles arise even when theory predicts they are irrational. We complement this literature by showing that, when bubbles can be expected in theory, they do not always materialize. We also show that the decision to speculate and enter into a bubble is positively related to the likelihood to be first in the market sequence, and to the number of iterated steps of reasoning required to rule out bubbles. This decision is negatively related to risk aversion and to the likelihood to be last in the market sequence. We reconcile these results thanks to Camerer, Ho, and Chong (2004)'s cognitive hierarchy (CH) model. In this model, players have different levels of sophistication and best respond to lower-level players' behavior. The average sophistication level estimated in our data through maximum likelihood is in line with the ones estimated by Camerer, Ho, and Chong (2004). Our results are also in line with the Quantal Response Equilibrium developed by McKelvey and Palfrey (1995). In this model, players make mistakes but costlier mistakes are more likely. Overall, both the cognitive hierarchy model and the quantal response equilibrium capture two of our main experimental observations: we observe bubbles when there is a price cap and we do not observe more bubbles when there is no price cap. This is because both models enable one to model the following two features. First, the presence of irrational traders, who buy when it is not beneficial, may create a rationale motive for speculation, even if speculation would be ruled out by backward induction if all traders were rational. Second, the existence of irrational traders, who do not buy when it would be beneficial, decrease the expected payoffs of entering the bubble for rational traders, so that buying may become suboptimal.

The experimental setting proposed in the present paper opens several avenues of research. It may indeed be used to study the manipulability and controllability of speculative markets. In particular, it could be interesting to study whether the occurrence of bubbles (rational and

irrational) vary with the number of traders, the introduction of risk in the underlying asset payoff, the level of transparency (one could proxy for transparency by setting a non-null probability that a trade is publicly announced). It would also be fruitful to study what is the impact of learning on bubble formation. Indeed, one can expect that irrational bubbles would be less likely when traders are experienced, whereas rational bubbles would be more frequent. Finally, it would be interesting to extend this setting to cases in which the price path and the timing are left at the discretion of traders. This would allow testing whether traders are able to coordinate on a price path and a timing that sustains rational bubbles.

6 Appendix

Appendix A: Bubble equilibrium with risk aversion

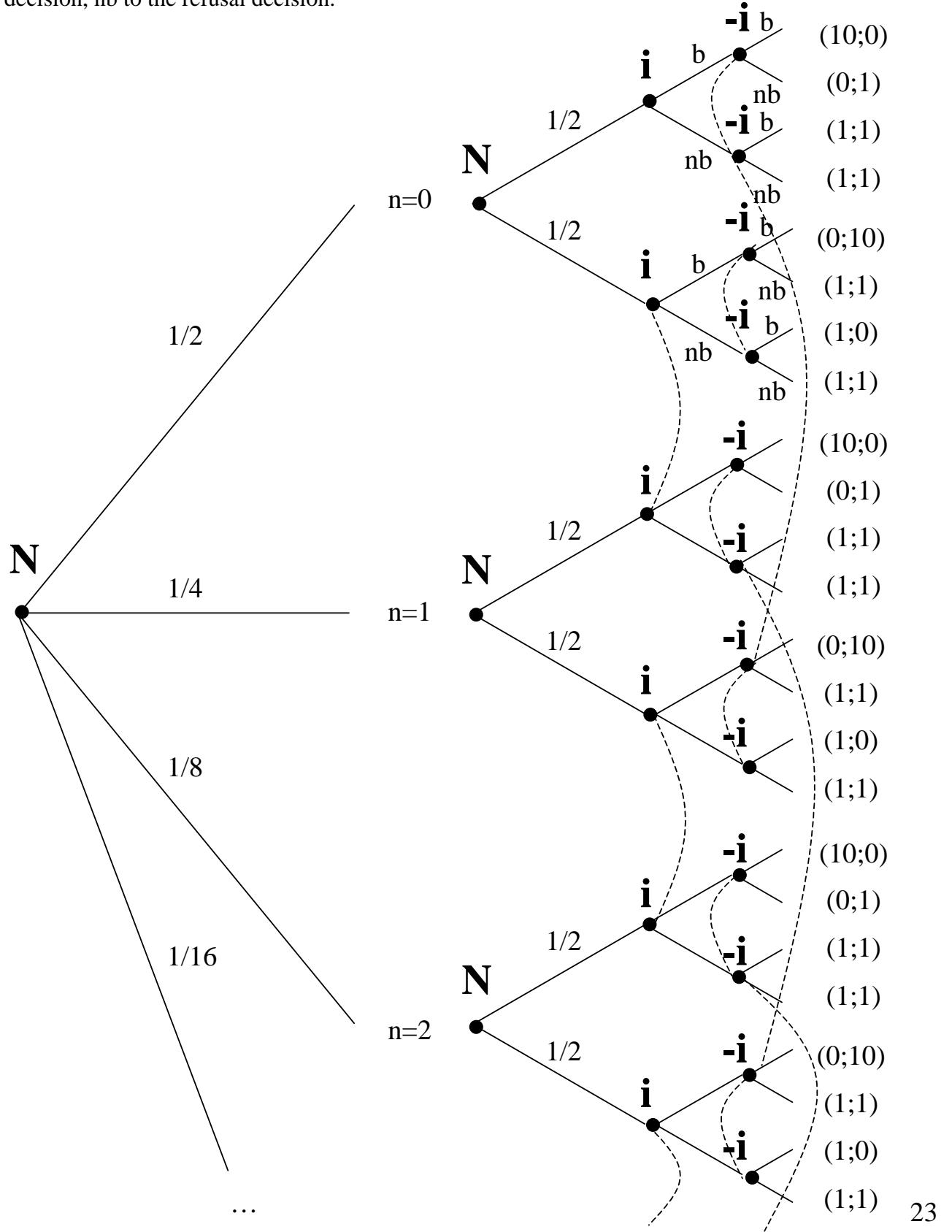
Consider the environment in which a bubble-equilibrium exists when players are risk-neutral. We now show that a bubble equilibrium can still exist if players are risk averse. The environment is as follows. There are T players. The set of potential prices is defined as $P = \{P_n = m^n \text{ for } m > 0 \text{ and } n \in \mathbb{N}\}$. P_1 is randomly determined following a geometric distribution: $g(P_1 = m^n) = (1 - q) q^n$ with $q \in (0, 1)$. Finally, the price path is defined as $P_{i+1} = m \times P_i$ for $i \in [1, \dots, T - 1]$. For simplicity, we assume that utility functions are piecewise linear with a kink at agents' initial wealth, that is player t 's utility function is: $U_t(x) = x \mathbb{1}_{x \leq W_t} + [W_t + (1 - \gamma_t)(x - W_t)] \mathbb{1}_{x > W_t}$, where $\gamma_t \in [0, 1]$ is a measure of player t 's risk aversion. The IR condition is now written as:

$$\begin{aligned} & \left(\begin{array}{l} (1 - \mathbb{P}[\omega = \omega_T^t | P_t]) \times U_t(W_t + f_t(P_t) - P_t) \\ + \mathbb{P}[\omega = \omega_T^t | P_t] \times U_t(W_t - P_t) \end{array} \right) \geq U_t(W_t), \forall t \in \{1, \dots, T\}, \text{ and } \forall P_t \in P \\ \Leftrightarrow & \left(\begin{array}{l} (1 - \mathbb{P}[\omega = \omega_T^t | P_t]) \times [W_t + (1 - \gamma_t)(f_t(P_t) - P_t)] \\ + \mathbb{P}[\omega = \omega_T^t | P_t] \times (W_t - P_t) \end{array} \right) \geq W_t, \forall t \in \{1, \dots, T\}, \text{ and } \forall P_t \in P. \\ \Leftrightarrow & \gamma_t \leq 1 - \frac{\mathbb{P}[\omega = \omega_T^t | P_t] \times P_t}{(1 - \mathbb{P}[\omega = \omega_T^t | P_t])(f_t(P_t) - P_t)}, \forall t \in \{1, \dots, T\}, \text{ and } \forall P_t \in P. \\ \Leftrightarrow & \gamma_t \leq 1 - \frac{(1 - q)}{(q - q^T)(m - 1)}, \forall t \in \{1, \dots, T\}. \end{aligned}$$

This inequality indicates that, if players are not too risk averse, there exists a bubble equilibrium. It also shows that, when m gets larger, the range of risk aversion for which a bubble equilibrium exists is larger.

Appendix B: Extensive form of the game with two players

At each node, Nature (N), player i or player $-i$ choose an action. $(x;y)$ represents the payoffs; x for player i , and y for player $-i$. Dotted lines relate nodes that are observationally equivalent. b refers to the buy decision, nb to the refusal decision.



Appendix C: Instructions for the case where $K = 10,000$

Welcome to this market game. Please read carefully the following instructions. They are identical for all participants. Please do not communicate with the other participants, stay quiet, and turn off your mobile phone during the game. If you have questions, please raise your hand. An instructor will come and answer.

As an appreciation for your presence today, you receive a participation fee of 5 euros. In addition to this amount, you can earn money during the game. The game will last approximately half an hour, including the reading of the instructions.

Exchange process

To play this game, we form groups of three players. Each player is endowed with one euro which can be used to buy an asset. Your task during the game is thus to choose whether you want to buy or not the asset. This asset does not generate any dividend. If the asset price exceeds one euro, you can still buy the asset. We indeed consider that a financial partner (who is not part of the game) provides you with the additional capital and shares profits with you according to the respective capital invested. The market proceeds sequentially. The first player is proposed to buy at a price P_1 . If he buys, he proposes to sell the asset to the second player at a price which is ten times higher, $P_2 = 10 \times P_1$. If the second player accepts to buy, the first player ends up the game with 10 euros. The second player then proposes to sell the asset to the third trader at a price $P_3 = 10 \times P_2 = 100 \times P_1$. If the third player buys the asset, the second player ends up the game with 10 euros. The third player does not find anybody to whom he can sell the asset. Since this asset does not generate any dividend, he ends up the game with 0 euro. This game is summarized in the following figure¹⁹.

At the beginning of the game, players do not know their position in the market sequence. Positions are randomly determined with one chance out of three for each player to be first, second or third.

Proposed prices

The price P_1 that is proposed to the first player is random. This price is a power of 10 and is determined as follows:

¹⁹See Figure 2 in the present paper.

Price P_1	Probability that this price is realized
1	1/2 (50%)
10	1/4 (25%)
100	1/8 (12.5%)
1,000	1/16 (6.3%)
10,000	1/16 (6.3%)

Players decisions are made simultaneously and privately. For example, if the first price $P_1 = 1$ has been drawn, the prices that are simultaneously proposed to the three players are: $P_1 = 1$ for the first player, $P_2 = 10$ for the second player, and $P_3 = 100$ for the third player. Identically, if the first price $P_1 = 10,000$ has been drawn, the prices that are simultaneously proposed to the three players are: $P_1 = 10,000$ for the first player, $P_2 = 100,000$ for the second player, and $P_3 = 1,000,000$ for the third player.

The prices that you are been proposed can give you the following information regarding your position in the market sequence:

- if you are proposed to buy at a price of 1, you are sure to be first in the market sequence;
- if you are proposed to buy at a price of 10, you have one chance out of three to be first and two chances out of three to be second in the market sequence. You are sure not to be last;
- if you are proposed to buy at a price of 100 or 1,000, you have one chance out of seven to be first, two chances out of seven to be second, and four chances out seven to be last in the market sequence;
- if you are proposed to buy at a price of 10,000, you have one chance out of four to be first, one chance out of four to be second, and two chances out four to be last in the market sequence.
- if you are proposed to buy at a price of 100,000, you have one chance out of two to be second, and one chance out of two to be third. In this case, you are sure not to be first in the market sequence.
- if you are proposed to buy at a price of 1,000,000, you are sure to be last in the market sequence.

In order to preserve anonymity, a number will be assigned to each player. Once decision will be made, we will tell you (anonymously) the group to which you belong, your position in the

market sequence, if you are proposed to buy, and your final gain.

Do you have any question?

Appendix D: Solving and estimating models of bounded rationality

Consider the environment in which the cap on the initial price is equal to $K = 1$. We derive the conditional probabilities to buy for risk-neutral traders observing prices of $P = 1$, $P = 10$ and $P = 100$ respectively, in the Poisson-cognitive hierarchy model of Camerer et al. (2004), and in the logit-quantal response equilibrium model of Mc Kelvey and Palfrey (1995). These probabilities are then used to estimate the deep parameter of each model thanks to maximum likelihood technics.

Cognitive hierarchy model

Consider first the case of a trader observing a price $P = 100$. This trader perfectly infers from this observation that he is third in the sequence. Consequently, in the CH model, he only buys if he is a level-0 player. Given that there is a fraction $f(0) = \frac{\tau^0 \times \exp(-\tau)}{0!}$ of such traders in the population, and given that these traders buy randomly with probability $\Pr(B|P = 100, \tilde{S} = 0) = \frac{1}{2}$, the probability to observe a trader buying at a price of $P = 100$ is:

$$\Pr(B|P = 100) = \frac{1}{2} \exp(-\tau)$$

Consider now the case of a trader observing a price $P = 10$. This trader perfectly infers from this observation that he is second in the sequence.

- If he is a level-0 player, he buys with probability $\Pr(B|P = 10, \tilde{S} = 0) = \frac{1}{2}$.
- If he is a level- s player with $s \geq 1$, he thinks that the next player observing the price $P_3 = 10 \times P_2$ is a level-0 with probability $f(0) = \exp(-\tau)$, a level-1 with probability $f(1) = \tau \times \exp(-\tau)$, ..., and a level- $s-1$ player with the truncated probability $1 - \sum_{i=0}^{s-2} f(i)$. Consequently, his expected profit if he buys writes:

$$E\Pi_{s \geq 1}(B|P = 10) = \frac{f(0)}{\sum_{i=0}^{s-1} f(i)} \times \frac{1}{2} \times 10.$$

He is strictly better off buying if and only if $\sum_{i=0}^{s-1} \frac{\tau^i}{i!} < 5$. This condition depends on s and on τ . Consider for instance the case of a level-1 player. He thinks that the next player observing the price $P_3 = 10 \times P_2$ is a level-0 player with probability 1, and that such a trader would buy with probability $\frac{1}{2}$. Consequently, his expected profit if he buys is $E\Pi_{s=1}(B|P = 10) = \frac{1}{2} \times 10$ which is strictly larger than his profit if he does not buy, that is, 1. So level-1 players buy with probability $\Pr(B|P = 10, \tilde{S} = 1) = 1$. But if he is a level-2 player, he thinks that the next player is a level-0

with probability $f(0)$ and a level-1 with the complementary probability $1 - f(0)$. Consequently, his expected profit if he buys decreases to $E\Pi_{s=2}(B|P=10) = \frac{f(0)}{\sum_{i=0}^1 f(i)} \times \frac{1}{2} \times 10$. This is lower than the expected payoff of a level-1 trader since a level-2 perceives that there is a lower proportion of level-0 players who would buy when observing $P = 100$. Consequently, a level-2 player would only enter for lower levels of τ than level-1 players, namely for $\tau < 4$.

Now, $\sum_{i=0}^{s-1} \frac{\tau^i}{i!}$ is increasing in s and $\lim_{s \rightarrow \infty} \sum_{i=0}^{s-1} \frac{\tau^i}{i!} = e^\tau$. If $\tau \leq \ln(5)$, then the inequality $\sum_{i=0}^{s-1} \frac{\tau^i}{i!} < 5$ holds for all s : the proportion of level-0 players who buy after observing a price $P_3 = 100$ is sufficiently high to induce all higher level players to buy when they observe $P_2 = 10$. If $\tau > \ln(5)$ however, there exists a $s_1^* \geq 1$ such that s_1^* -level players buy, that is, $\sum_{i=0}^{s_1^*-1} \frac{\tau^i}{i!} < 5$, but not level- $s_1^* + 1$ players who have a more accurate perception of the distribution of lower-level types, that is, $\sum_{i=0}^{s_1^*-1} \frac{\tau^i}{i!} \geq 5$.

Finally, given the distribution of players' types, the probability to buy conditionnal on the price being $P = 10$ writes:

$$\Pr(B|P=10) = \exp(-\tau) \times \left[\frac{1}{2} + \sum_{s=1}^{\infty} \left(\frac{\tau^s}{s!} \times 1_{\sum_{i=0}^{s-2} \frac{\tau^i}{i!} < 5} \right) \right]$$

To compute this probability as a function of τ , we use the following process.

i) If the condition $\tau \leq \ln(5)$ is satisfied, then

$$\text{For } \tau \leq \ln(5) : \Pr(B|P=10) = 1 - \frac{1}{2} \times \exp(-\tau)$$

ii) Otherwise, we define s_1^* and:

$$\text{For } \tau > \ln(5) : \Pr(B|P=10) = \exp(-\tau) \times \left(\sum_{s=0}^{s_1^*} \frac{\tau^s}{s!} - \frac{1}{2} \right),$$

$$\text{with } s_1^* \in \mathbb{N}^* \text{ such that: } \sum_{i=0}^{s_1^*-1} \frac{\tau^i}{i!} \leq 5 \text{ and } \sum_{i=0}^{s_1^*} \frac{\tau^i}{i!} > 5.$$

Consider finally the case of a trader observing a price $P = 1$. This trader perfectly infers from this observation that he is first in the sequence.

- If he is a level-0 player, he buys with probability $\Pr(B|P=1, \tilde{S}=0) = \frac{1}{2}$.

- If he is a level- s player with $s \geq 1$, he thinks that the next player observing the price $P_2 = 10 \times P_1$ is a mixture of level-0, level-1 ... level- $s-1$ players. Now, depending on the value

of τ , although level- i players would not buy at date 3 for $i > 0$, we have shown above that they may be willing to buy at date 2 if they anticipate that they would be able to resell to third level-0 players sufficiently frequently. The incentive to buy of a sophisticated player is thus higher after observing 1 than after observing 10. His expected profit writes:

$$E\Pi_{s \geq 1}(B|P = 1) = \left(\frac{f(0)}{\sum_{i=0}^{s-1} f(i)} \times \frac{1}{2} + \frac{\sum_{i=1}^{s-1} \Pr(B|P = 10, \tilde{S} = i)}{\sum_{i=0}^{s-1} f(i)} \times 1_{s>1} \right) \times 10.$$

Again, this expected profit depends on the value of τ . Straightforward manipulations lead to the following condition for a level- s player to buy when observing $P = 1$:

$$4 + \sum_{j=1}^{s-1} \left(\frac{\tau^j}{j!} \times \left(10 \times 1_{\frac{5}{\sum_{i=0}^{j-1} \frac{\tau^i}{i!}} > 1} - 1 \right) \right) \times 1_{s>1} > 0$$

Now, if the condition $\tau \leq \ln(5)$ is satisfied, then the inequality becomes $4 + \left(9 \times \sum_{j=1}^{s-1} \frac{\tau^j}{j!} \right) \times 1_{s>1} > 0$, which always holds. If $\tau > \ln(5)$, s_1^* , defined above, is such that level- $s_1^* + 1$ players would not buy when observing $P_2 = 10$. But for $s > s_1^*$, we can rewrite the first element as $4 + 9 \sum_{j=1}^{s_1^*-1} \frac{\tau^j}{j!} - \sum_{j=s_1^*}^{s-1} \frac{\tau^j}{j!}$, which is decreasing in s . Given that

$$\lim_{s \rightarrow \infty} 4 + 9 \sum_{j=1}^{s_1^*-1} \frac{\tau^j}{j!} - \sum_{j=s_1^*}^{s-1} \frac{\tau^j}{j!} = 5 + 10 \sum_{j=1}^{s_1^*-1} \frac{\tau^j}{j!} - \exp(\tau),$$

depending on the value of τ , there may exist a $s_2^* > s_1^*$ such that the condition () is not satisfied.

Finally, given the distribution of players, the probability to buy conditionnal on the price being $P = 1$ writes:

$$\Pr(B|P = 1) = \exp(-\tau) \times \left(\frac{1}{2} + \tau + \sum_{s=2}^{\infty} \left(\frac{\tau^s}{k!} \times 1_{4 + \sum_{j=1}^{s-1} \left(\frac{\tau^j}{j!} \times \left(10 \times 1_{\frac{5}{\sum_{i=0}^{j-1} \frac{\tau^i}{i!}} > 1} - 1 \right) \right) > 0} \right) \right)$$

To compute this probability as a function of τ , we use the following process.

i) If the condition $\tau \leq \ln(5)$ is satisfied, then

$$\text{For } \tau \leq \ln(5) : \Pr(B|P = 1) = 1 - \frac{1}{2} \times \exp(-\tau)$$

ii) Otherwise, we define s_1^* as above.

ii-a) If the following condition is satisfied:

$$5 + 10 \sum_{j=1}^{s_1^*-1} \frac{\tau^j}{j!} - \exp(\tau) \geq 0$$

then:

$$\text{For } \tau \text{ such that } 5 + 10 \sum_{j=1}^{s_1^*-1} \frac{\tau^j}{j!} - \exp(\tau) \geq 0 : \Pr(B|P=1) = 1 - \frac{1}{2} \times \exp(-\tau)$$

ii-b) Otherwise, we defined s_2^* and:

$$\begin{aligned} \text{For } \tau \text{ such that } 5 + 10 \sum_{j=1}^{s_1^*-1} \frac{\tau^j}{j!} - \exp(\tau) &< 0 : \Pr(B|P=1) = \exp(-\tau) \times \left(\sum_{s=0}^{s_2^*} \frac{\tau^s}{s!} - \frac{1}{2} \right) \\ &\quad \text{with } s_2^* \in \mathbb{N}^* \text{ such that:} \\ 4 + 9 \times \sum_{j=1}^{k_1^*} \frac{\tau^j}{j!} - \sum_{j=k_1^*+1}^{k_2^*-1} \frac{\tau^j}{j!} &\geq 0 \text{ and } 4 + 9 \times \sum_{j=1}^{k_1^*} \frac{\tau^j}{j!} - \sum_{j=k_1^*+1}^{k_2^*} \frac{\tau^j}{j!} < 0 \end{aligned}$$

Quantal response equilibrium

Let $u_{i,B}$ be the expected payoff of a risk-neutral player if he buys after observing $P = m^{i-1}$, and $u_{i,\emptyset}$ his expected payoff if he does not buy. In the quantal response model, the probability with which the trader buys conditional on observing $P = P_i$ writes:

$$\Pr(B|P=P_i) = \frac{e^{\lambda u_{i,B}}}{e^{\lambda u_{i,B}} + e^{\lambda u_{i,\emptyset}}}$$

Consider first the case of a trader observing a price $P = 100$. This trader perfectly infers from this observation that he is third in the sequence. Consequently, his expected payoffs for buying and not buying respectively write:

$$u_{3,B} = 0$$

$$u_{3,\emptyset} = 1$$

The probability to buy is therefore:

$$\Pr(B|P=100) = \frac{1}{1 + e^\lambda}.$$

Consider now the case of a trader observing a price $P = 10$. This trader perfectly infers from this observation that he is second in the sequence. To compute his expected payoff from buying, he anticipates that the probability to buy of the third trader is not equal to zero. His expected payoffs for buying and not buying respectively write:

$$\begin{aligned} u_{2,B} &= \Pr(B|P = 10) \times 10 \\ u_{2,\emptyset} &= 1 \end{aligned}$$

The probability to buy is therefore:

$$\Pr(B|P = 10) = \frac{e^{\frac{10\lambda}{1+e^\lambda}}}{e^{\frac{10\lambda}{1+e^\lambda}} + e^\lambda} = \frac{1}{1 + e^{\lambda \times \left(1 - \frac{10}{1+e^\lambda}\right)}}.$$

Consider finally the case of a trader observing a price $P = 1$. This trader perfectly infers from this observation that he is first in the sequence. To compute his expected payoff from buying, he anticipates that the probability to buy of the second trader is not equal to zero. His expected payoffs for buying and not buying respectively write:

$$\begin{aligned} u_{1,B} &= \Pr(B|P = 1) \times 10 \\ u_{1,\emptyset} &= 1 \end{aligned}$$

The probability to buy is therefore:

$$\Pr(B|P = 1) = \frac{e^{10\lambda} \frac{e^{\frac{10\lambda}{1+e^\lambda}}}{e^{\frac{10\lambda}{1+e^\lambda}} + e^\lambda}}{e^{10\lambda} \frac{e^{\frac{10\lambda}{1+e^\lambda}}}{e^{\frac{10\lambda}{1+e^\lambda}} + e^\lambda} + e^\lambda} = \frac{1}{1 + e^{\lambda \left(1 - \frac{10}{1 + e^{\lambda \left(1 - \frac{10}{1+e^\lambda}\right)}}\right)}}.$$

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Figure 1 – Panel A: Timing of the market in the trader game

Market proceeds sequentially. Traders are equally likely to be first, second or third. The first, second and third traders are offered to buy at prices P_1 , P_2 , and P_3 respectively. This figure displays traders' net payoff, that is their gains or losses relative to their initial wealth. Traders can end up losing very large amount of money.

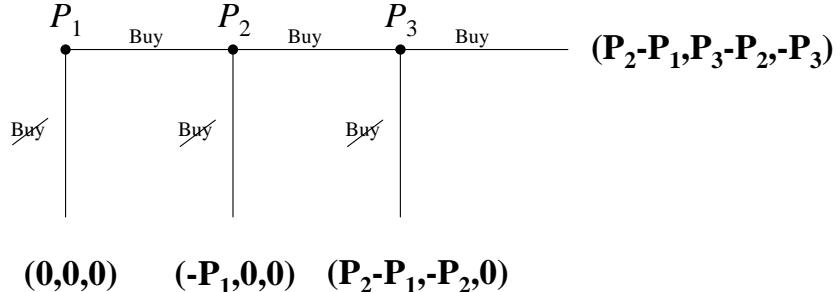


Figure 1 – Panel B: Timing of the market in the manager/financier game

Market proceeds sequentially. Portfolio managers can invest one monetary unit along with additional capital if needed coming from an outside financier. Managers are equally likely to be first, second or third. The first, second and third managers are offered to buy at prices P_1 , P_2 , and P_3 respectively. This figure displays managers' net payoff, that is their gains or losses relative to their initial wealth. Financiers' net payoffs are indicated for completeness but these agents are not part of the experiment.

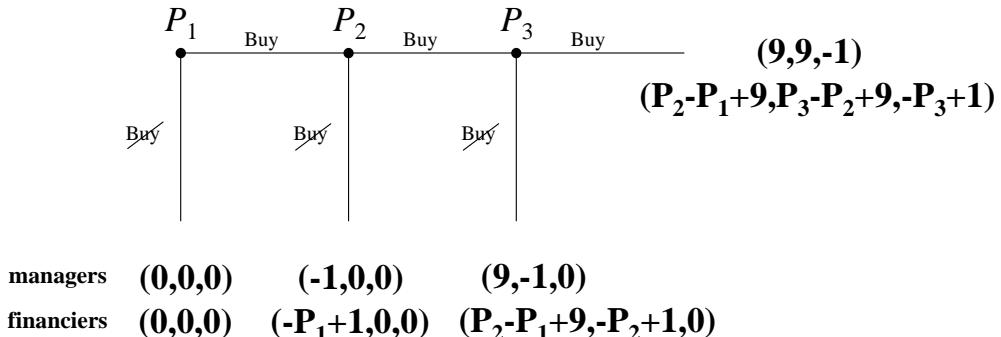


Figure 2: Timing of the market and payoffs in the experiment

Market proceeds sequentially. Subjects are endowed with one monetary unit that they can invest in a financial asset. If they do so, they receive ten monetary units if the next subject also decides to invest (the additional capital that is potentially needed to buy the asset is provided by outside financiers that are not part of the experiment). Subjects are equally likely to be first, second or third. The first, second and third subjects are offered to buy at prices P_1 , P_2 , and P_3 respectively. This figure displays subjects' payoff (if they refuse to buy they keep one monetary unit).

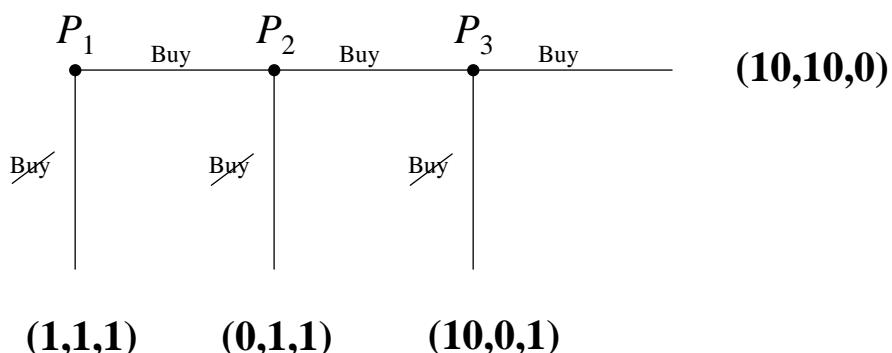


Figure 3: Probability to observe bubbles, depending on the cap on the initial price

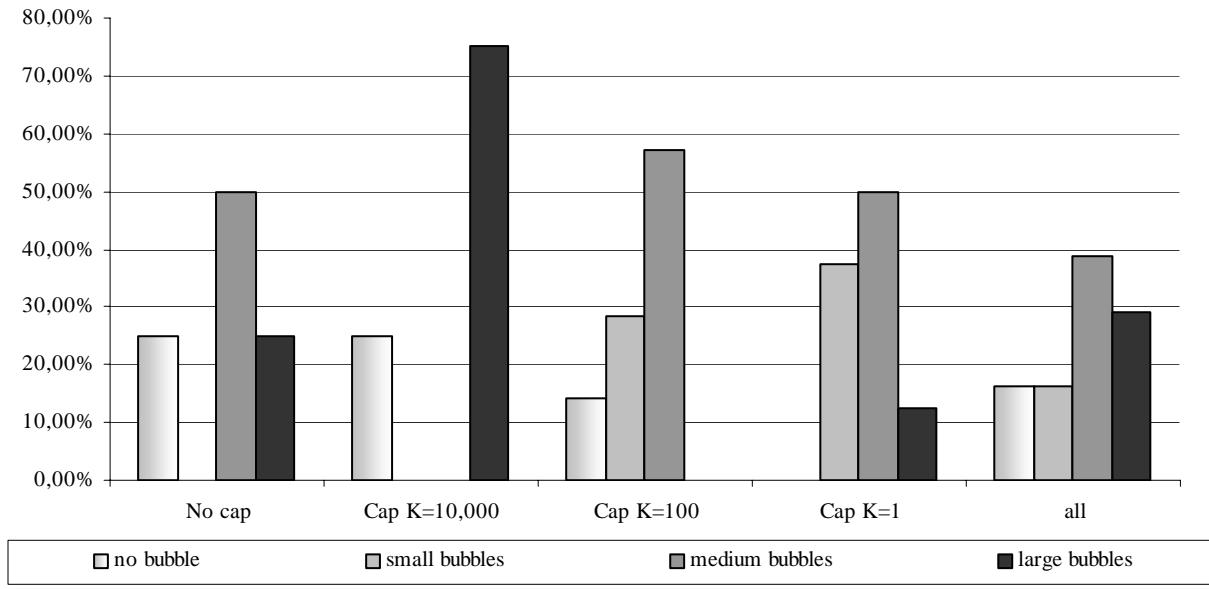


Figure 4

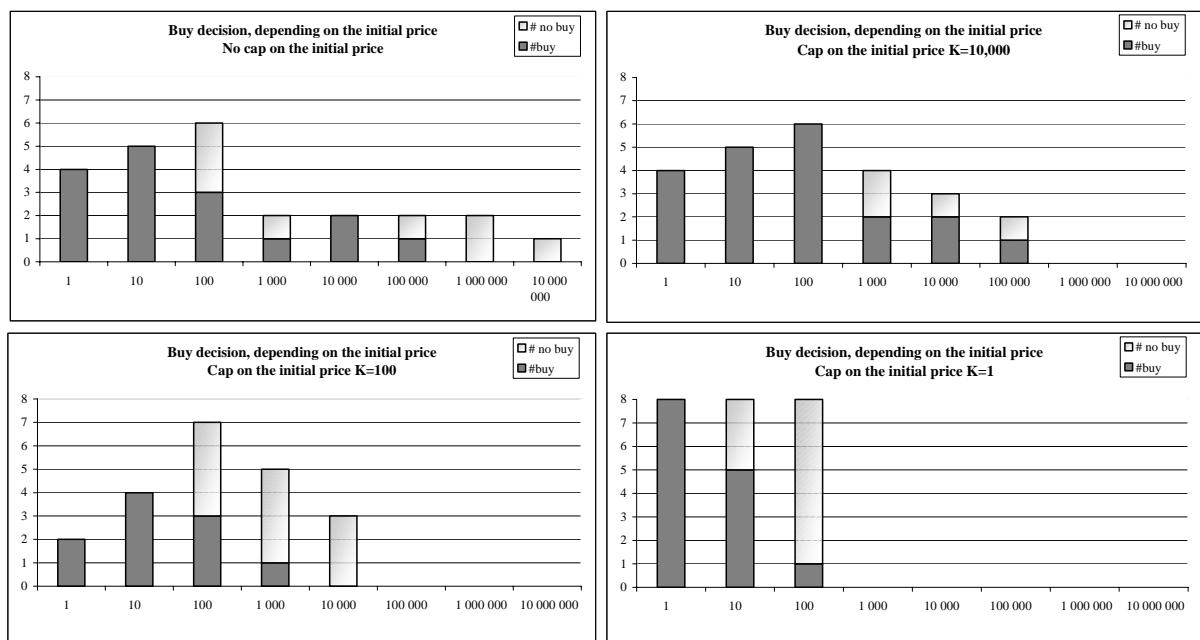


Figure 5: Probability to buy conditional on subjects' inferences

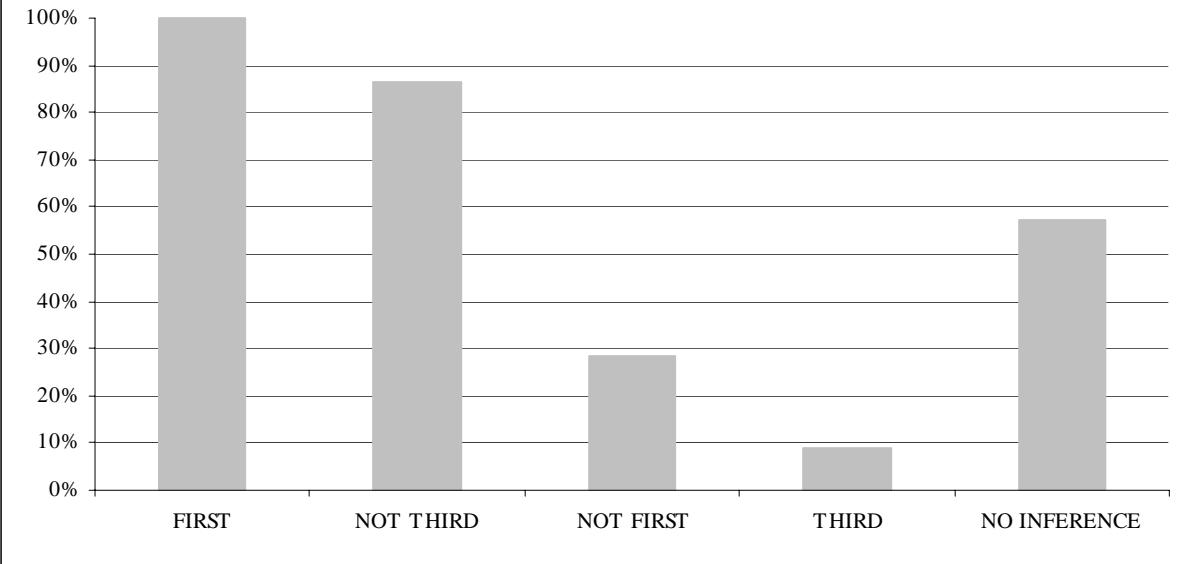


Figure 6

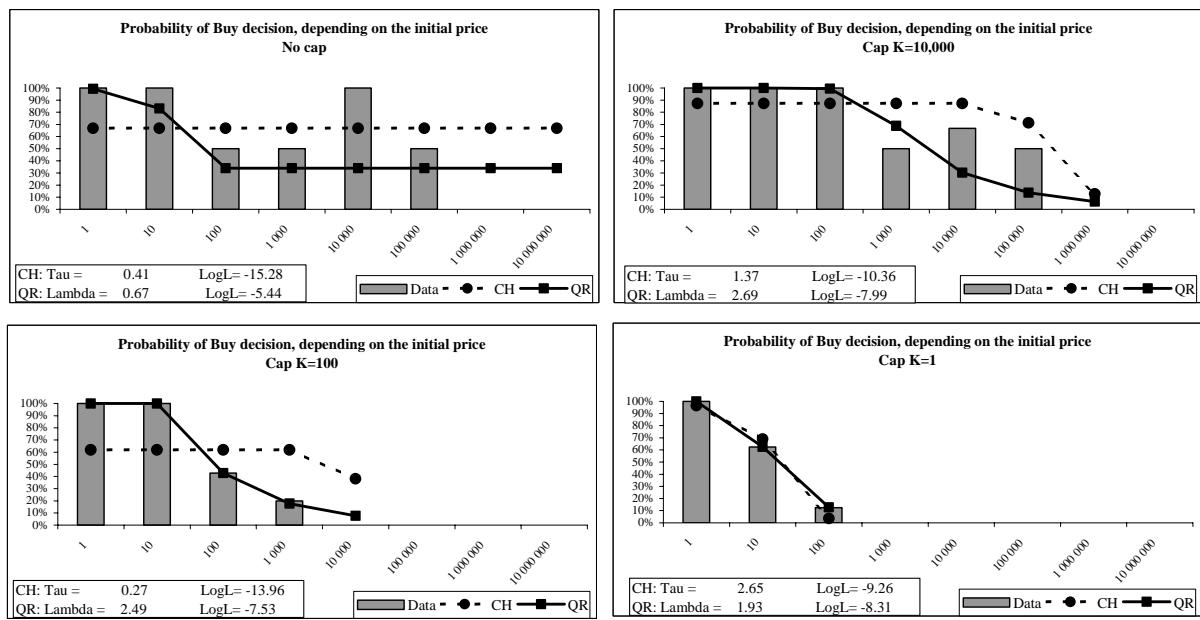


Table 2

Logit Regression on the Buy Decision

	Coefficient	Statistic	p-value	Coefficient	Statistic	p-value
Number of steps of iterated reasoning from maximal price	0.47	2.38	0.02	0.40	2.33	0.02
Risk aversion	-0.82	-1.48	0.14	-1.40	-3.02	0.00
Conditional probability to be first	1.96	1.54	0.12	2.54	2.24	0.03
Conditional probability to be third	-1.23	-2.12	0.03			
Log likelihood	-22.68			-25.28		
Number of observation	63			63		

Table 3

Comparison of fits of Nash, Cognitive Hierarchy and Quantal Response Equilibrium

Data		Session					
		No Cap	Cap K=10,000	Cap K=100	Cap K=1	All	
		Sample size	24	24	21	24	
Nash Equilibrium		Av. probability buy	67%	83%	48%	58%	
		Log L	-73,68	-184,21	-92,10	-128,95	
		Mean Squared Deviation	0,11	0,74	0,36	0,47	
		90% CI	[0 - 1.10]	[0.67 - 3.91]	[0 - 1.79]	[0.87 - 4]	
						[0.48 - 1.12]	
Cognitive Hierarchy		Tau	0,41	1,37	0,27	2,65	
		Av. probability buy	67%	86%	58%	56%	
		Log L	-15,28	-10,36	-13,96	-9,25	
		Mean Squared Deviation	0,000	0,042	0,116	0,004	
		90% CI	[0.56 - 2.52]	[1.77 - 2.86]	[0.54 - 2.83]	[0.46 - 1.16]	
						[0.51 - 2.74]	
Quantal Response		Lambda	0,67	2,69	2,49	1,93	
		Av. probability buy	55%	79%	48%	58%	
		Log L	-5,44	-7,99	-7,53	-8,31	
		Mean Squared Deviation	0,016	0,033	0,001	0,000	
		90% CI	[0.56 - 2.52]	[1.77 - 2.86]	[0.54 - 2.83]	[0.46 - 1.16]	
						[0.51 - 2.74]	