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“Strategic Inaccuracy in Bargaining”

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Abstract

This paper studies a buyer-seller game with pre-trade communication of private horizontal taste from the buyer followed by a take it or leave it offer by the seller. The amount of information transmitted improves the gains from trade, but also determines how this surplus will be shared between the two. Lack of commitment to a price creates a hold-up problem and a trade off between efficiency and rent extraction. In this setting, coarse information arises due to the concerns on the terms of the transaction. As the preferences get less important, information transmission becomes less precise. It is shown that in the buyer optimal equilibria of the static and dynamic games, the messages sent are just informative enough to ensure trade. In the dynamic game, the buyer is always better off sending informative messages only at the first period, implying no gains from gradual revelation of information.

Keywords: information; cheap-talk; bargaining; buyer-seller relation
JEL classification: C72; D83

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1 Introduction

Communication plays an important role in bilateral relations in the presence of private information. In settings where there is room for negotiation, the parties want to be strategic when revealing their preferences. A buyer may not want to reveal her preference over different options when facing an opportunistic seller, in order to avoid losing information rents. Among the goods that have the same intrinsic value, the seller could propose a different price depending on how much he estimates the valuation of the buyer is. By providing only an estimate of her preferred option, the buyer could avoid giving up all bargaining power to the seller. This paper addresses the issue of how a buyer can strategically reveal her preferences to an uncommitted seller via pre-trade communication.

There are cases when a buyer chooses how much information to reveal. When going to a real estate agency in search of a house, you could either provide a certain neighborhood you are searching for, or a broader range of the city where you would be willing to reside. When you describe a very specific area, the selling agent can show houses that are expensive or on which he has a higher profit, knowing that you are restricted to that neighborhood. On the other hand, if you do not reveal your specific preference over neighborhoods, he is better off showing you houses of more reasonable prices from different areas in order to increase the probability that you actually end up buying. Being very specific about preferences can make you gain time but lose information rents.

This paper considers a buyer searching for a good or service and has private information about her taste or need and should communicate with the seller. The seller will come up with an offer as a response to the buyer's message. However, as the seller has lack of commitment to a price, this creates a hold up problem: knowing what the buyer values, the seller can better extract her surplus. Then the tradeoff faced by the buyer is the one between finding a better fit good and being charged a higher price.

In this setting, the sharing of the gains from trade depends on the extent of information revelation. The buyer could either wait for the seller to make offers or provide information about her taste. If the buyer provides no information, she might end up not getting a relevant offer. On the other hand, the more the buyer is precise about her preferences, the higher the price that the seller can ask her.

To study this situation, I introduce information transmission by pre-trade cheap talk into a buyer-seller game and show that coarse information arises due to the conflict on the terms of the transaction. Bargaining happens on the horizontal dimension, among goods that have the same intrinsic value. The lack of commitment by the seller to a price implies that any information on the buyer's taste serves as a tool for rent extraction. The seller is free to make any offer while the buyer holds private information, which can be seen as a way to study how much information the buyer would like to reveal in the presence of extreme hold-up. The amount of information provided improves the match between the good and the buyer's type, but provides a higher share of the surplus to the seller. Hence there is a clear trade off between efficiency and rent extraction.

The main result is the characterization of the buyer optimal equilibrium after showing the existence of a continuum of equilibria. The multiplicity comes from the wide range of possibilities of communication, which is usual in cheap talk models. After showing that any equilibrium should consist of monotone partitions, meaning the buyer types pool in intervals, I search for the equilibria which are the best for the buyer from an ex-ante point of view, and call these buyer-optimal. The buyer decides how much information to provide, and although there exists more informative equilibria, including the perfect revelation, the buyer is better off under equilibria in which information is less precise even if that may decrease the chances of trade.

I explore both one period and dynamic settings. The buyer-optimal equilibrium in the one period game is the one having the coarsest information structure (the least number of intervals) that covers the market (trade takes place for all types). In this equilibrium, the buyer's messages are just precise enough to guarantee trade: partition intervals are the largest subject to the constraint that the seller doesn't exclude any type. In other words, the buyer always benefits from being imprecise as long as the seller's offer is still acceptable.

In the two period game, the buyer's optimal strategy is to send informative messages only at the first period and babble in case next period is reached, as this induces the seller to make more favorable offers and reduce the expected delay in trade. Hence, it is not optimal to gradually reveal information for the owner of information in this setting.[1] Now, in a pooling interval in period 1, there are some types of the buyer who will accept the offer and others who will refuse and move on to the next period. Once the next period is reached, the seller's prior about the buyer's type is updated. The seller can ask for a higher price in period 1 compared to the static game and leave out some types to be served the next period, as he can get more information and differentiate among the types over time. Two informational benefits of second period for the seller are identified: the first one is that the interval of possible types of the buyer shrinks when period 2 is reached, and the second one is the possibility of another informative signal in the coming period.

When the only informative signals are sent at period 1 and period 2 messages are babbling, the seller no longer enjoys the second type of informational benefit. Still, the total information provided is enough to ensure trade over the two periods.

[1] This is in contrast to Horner and Skrzypacz (2013), who, although in a different setting, find that the owner of verifiable and valuable information will prefer gradual revelation.

In this setting, the seller and the buyer are both worse off when the discount factor is high enough and there is expected delay. When the discount factor is low, the buyer optimal equilibrium is the no-delay equilibrium in which the period 1 intervals are fine enough that the seller doesn't exclude any type and period 2 is never reached. This is due to delay becoming so costly that the buyer is better off revealing more information in order to avoid it. The no-delay equilibrium gives higher profits to the seller and results in higher welfare compared to the one period game.

To eliminate inefficient equilibria in the dynamic setting, a *subdivision* condition is provided which is the separation of types who reject the offer in a given period and are initially connected in an interval into two or more intervals by sending separate messages in the next period. It is shown that any equilibrium with subdivision is Pareto dominated. The dynamic model is tractable as the threshold types in one period are also threshold types in the next period in case of rejecting the current offer, hence their incentive compatibility constraint is satisfied. Then, due to the monotonicity of strategies, the incentive compatibility of the types inside the interval is also satisfied.

Finally, when the game is extended to infinite horizon, it is shown that it is always more optimal that the only informative signals are sent at period 1. Sending babbling messages at $t > 1$ decreases expected delay: the period 1 intervals are finer when future messages are babbling and the seller excludes less types in each period. For any equilibrium which has informative signals in future periods, there is another one found by modifying the signal structure such that the only informative signals are sent at period 1 and does strictly better in terms of buyer and seller surplus. In the dynamic game, the discount factor is what prevents the seller from finding the good that perfectly fits the buyer's taste and extracting all her surplus.

The reason for non gradualism in information revelation is that if information is eventually going to be revealed, it is always more efficient to reveal it at period 1 instead of later, as there is a time cost of delaying trade and the seller will leave out more types today if he expects more information to arrive tomorrow. When future messages are uninformative, the seller is more eager to sell earlier, which means a lower price and less expected delay. The uncertainty is not on the extent of the information the buyer is willing to provide, but on the information itself. Hence, the buyer is better off resolving this uncertainty in the beginning and waiting for the seller to make offers but not providing more information in the coming periods. Finally, the results of the dynamic game suggests that the owner of information does not gain from gradually revealing information in this setting.

1.1 Related Literature

Horner and Skyrypacz (2013) study a dynamic problem between a firm and an agent who has valuable and verifiable information. The lack of commitment leads to a hold up problem: the firm has to pay for the information before knowing if it is valuable, as once the information is provided the firm would not be willing to pay at all. Hence, the seller of information benefits from the gradual revelation of his information.

There is a recent literature about the seller incentives to disclose horizontal match information, such as Anderson and Renault(2006), Sun (2011) and Koessler and Renault (2012). The departure from this literature is that in this paper the focus is on the buyer's incentive to provide information in a bargaining setting.

This paper is also related to the literature on strategic information transmission, pioneered by Crawford and Sobel (1982). However, the conflict of interest on the outcome present in their paper and the literature that follows does not arise here, and instead the conflict is due to the sharing of the surplus. Golosov et al. (2013) study the dynamic version of the Crawford and Sobel model in which the seller takes a decision every period[2] and Krishna and Morgan (2004) show that adding an ex-ante communication stage improves the outcome of the communication game. Levy and Razin (2007) show that linking decisions together may reduce communication due to spillover effects, specifically that the conflict of interest on one issue may impede communication on another issue.[3]

Finally, the paper is related to the bargaining literature. The setting of this paper can be seen as an ex ante stage in which a good to be bargained is being picked among many, using the information provided by the buyer. As there is no vertical uncertainty, the same good is offered only once. The reason I rule out vertical uncertainty is in order to highlight the preference revelation aspect, and in presence of vertical differentiation the buyer would not have an incentive to reveal that she has a high valuation. In the extension part, I show this by considering a simple one period example by introducing uncertainty on the vertical valuation.

The rest of the paper is organized as follows: Section 2 explains the general model, section 3 solves for the equilibrium of the one period game, section 4 studies the two periods setting, section 5 studies the infinite game, section 6 provides an extension, and section 7 concludes. The omitted proofs and solutions can be found in the Appendix at the end of the paper.

[2] They show that full revelation is possible by constructing a non-monotonic partition equilibrium, in which far away types pool initially and separate later on. In contrast, in this paper there is a one time decision and it is costly to delay trade. Hence, although full revelation could be supported as an equilibrium, it is inferior for the buyer from an ex-ante point of view.

[3] This relates to the current paper as two dimensions are being agreed upon, however the information transmission happens only on one dimension and there is a bargaining stage that follows the message.

2 The Model

There are two players, a buyer (B) and a seller (S) who interact in order to agree on the one time exchange of a good or service. The buyer privately observes her type θ, a random variable in $[0, 1]$ which determines her horizontal taste for the good. The seller can offer from a continuum of goods, $y \in [0, 1]$. The utility of the buyer from a good y is $U(\theta, y) = k - f(|\theta - y|)$, where f, the cost of mismatch between the good and the buyer's type, is strictly convex in $|\theta - y|$[4], $U_{12} > 0$ (the preferred good of the buyer is increasing in his type), $U_{22} < 0$ (there is a single good that perfectly fits each type of the buyer).

The seller is assumed indifferent among different goods and his valuation is normalized to zero, which will imply that trade is always optimal. The seller would like to find the best fit for the buyer order in order to get the highest profits. Outside options are zero. Vertical heterogeneity in buyer valuation is studied as an extension, and the main part of the paper focuses on the horizontal taste parameter and the seller's incentive to charge different prices for goods having the same intrinsic value.

First, the buyer sends a costless message, $m \in M$ to the seller, who responds by an offer consisting of a good $y(m)$ and a transfer $\tau(m)$. If B accepts the offer, the game ends with realized payoffs $U(\theta, y) - \tau$ and τ. The maximum utility, k, is what the buyer can get from a good that perfectly matches her taste. It is small enough that in the absence of information the market will not be covered, in other words the seller's offer will exclude some types of the buyer.

The total surplus from trade is maximized when $y = \theta$, in case every type gets their most preferred good. The crucial assumption is that the seller cannot commit to a price schedule ex-ante. If he could, then he would extract the maximum rent, k through a truthful revelation mechanism.

2.1 Equilibrium Analysis

The solution concept that will be used is Perfect Bayesian Equilibrium. Given θ, $\mu(m|\theta)$ is the signaling strategy of the buyer. The seller's strategy is an offer $(y(m), \tau(m))$, to which the buyer responds by $\sigma_\theta(y, \tau) \in 0, 1)$ [5]. Strategies that constitute a PBE satisfy:

- for any $\theta \in (0, 1)$, if $\mu(m|\theta) > 0$, then $U(\theta, y(m)) - \tau(m) \geq U(\theta, y(m')) - \tau(m')$ for all $m' \in M$. (B's message maximizes her utility among feasible messages given S's optimal response.)

[4] the results continue to hold for any f as long as it is not too concave, shown in the lemma 3

[5] due to the assumption that when the buyer is indifferent between accepting the offer or not, she will accept

- for any m, $(y(m), \tau(m)) = \arg\max_{y,\tau} \tau \int_0^1 \sigma_\theta(y,\tau)\rho(\theta|m)d\theta$ where

$$\rho(\theta|m) = \frac{\mu(m|\theta)f(m)}{\int_0^1 \mu(m|t)f(t)dt}$$

(S makes the offer which maximizes his expected profits as a response to the message sent by B.)

2.2 Some Properties and Definitions

This section introduces some properties that will be used throughout the paper.

Definition 1. *A monotone partition equilibrium is one in which the buyer type space* $[0,1]$ *is divided into* n *intervals with boundary points* $0 = a_1 < a_2 < .... < a_n = 1$*, and* $\mu(m|\theta) = \mu(m|\theta')$ *if* $\theta, \theta' \in (a_i, a_{i+1})$ *and if* $\mu(m|\theta) > 0$ *for all* $\theta \in (a_i, a_{i+1})$ *then* $\mu(m|\theta') = 0$ *for all* $\theta' \in (a_j, a_{j+1})$ *for any* $j \neq i$*.*

In a monotone partition equilibrium, each message is sent by types that are connected in intervals and no other type outside these interval sends the same message. This is called uniform signaling (Crawford and Sobel, 1982). In this type of equilibrium, fewer number of intervals is equivalent to coarser communication. From now on, I will denote the message sent by types in an interval i as m_i and say that interval i sends message m_i. Hence, upon receiving a message m_i, the seller knows that θ is found in interval i.

Before restricting attention to monotone partition equilibria, I will rule out the possibility of non-monotone partitions. A non-monotone partition has types in separate intervals that pool in their messages[6]. As each buyer type will have a strictly preferred message, I restrict attention to pure strategies of the buyer.

Lemma 1. *If* $\mu(m|\theta_1) = \mu(m|\theta_2) = 1$*, then* $\mu(m, \theta_3) = 1$ *for all* $\theta_3 \in (\theta_1, \theta_2)$*: equilibrium strategies are monotone in any game in which trade happens with certainty.*

Hence, there cannot exist any non-monotone partition equilibria in which all buyer types accept an offer. [7] As the paper focuses on efficient equilibria in which trade happens for all types, attention will be restricted to monotone partitions. Throughout the paper, the term an interval is served or covered will be used to say that the offer made by the seller as a response to the message sent by that interval is accepted by all the types.

[6]this definition is borrowed from Golosov et al. 2014

[7]In case some types are already not getting an offer, then they might as well be indifferent to sending other messages which will also give them 0 surplus.

In addition, the following condition for the boundary types in each interval to be indifferent between the offers induced by the two messages should be satisfied:

$$U(a_i, y(m_{i-1})) - \tau(m_{i-1}) = U(a_i, y(m_i)) - \tau(m_i)$$

for all i.

3 One Period Game

First, I consider the game which ends after only one round of communication and offer. Due to the assumption that k is sufficiently small, if the interval which sends a message is not fine enough, then the seller's offer will exclude some types, in other words trade will not take place for these types. Given that there is a single offer, the buyer is willing to accept it as long as $U(\theta, y) - \tau \geq 0$ as his outside option is 0. As $U(\theta, y) = k - f(\theta - y)$ is decreasing as θ moves away from y, and τ is constant, then the types for whom $U(\theta, y) - \tau \geq 0$ are located on a connected line.

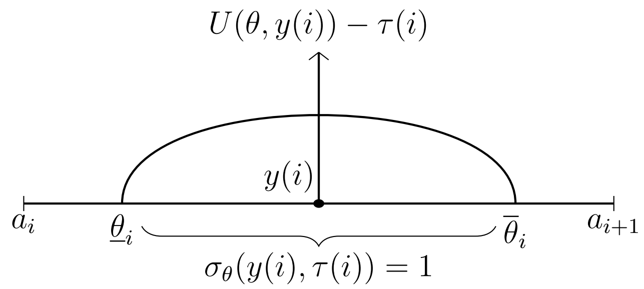
Definition 2. *The threshold types $\underline{\theta}_i$ and $\overline{\theta}_i$ are the types in interval i with the lowest utility among those who accept the offer:* $(\underline{\theta}_i, \overline{\theta}_i) = \arg\min_\theta U(\theta, y_i)$ *such that* $\sigma_\theta(y_i, \tau_i) = 1$.

The threshold types are the highest and lowest types in an interval who accept the offer. The types located in $(\underline{\theta}_i, \overline{\theta}_i)$ will accept, while those located outside the interval reject the offer made as a response to the message sent by the types in interval i, as their utilities from the offer are negative. The boundary types coincide with the threshold types when the whole interval is being served. As the threshold types get 0 surplus, $\tau_i = k - f(\underline{\theta} - y_i) = k - f(\overline{theta} - y_i)$ and by the symmetry around 0 of f, $y_i - \underline{\theta}_i = \overline{\theta}_i - y_i$, which leads to $y_i = \frac{\underline{\theta}_i + \overline{\theta}_i}{2}$. In case the whole interval is served, $y_i = \frac{a_i + a_{i+1}}{2}$. Figure 1 displays the response to a given offer (y_i, τ_i) after a message sent by an interval i and the utilities of the types in that interval.

Proposition 1. *Any monotonic partition can be observed as an equilibrium.*

Proof. I will prove this by showing that no type has an incentive to deviate in a monotone partition equilibrium. Take a monotone partition, and an interval i sending the message m_i. Upon receiving this message, it is optimal for the seller to always leave zero surplus to the boundary types a_i and a_{i+1}: either these types are excluded or they are the threshold types who accept the offer with 0 surplus as they are the farthest from y_i. If the threshold types are located strictly inside the interval, $a_i < \underline{\theta}_i$ and $a_{i+1} > \overline{\theta}_i$, then $U(\theta, y_i) - \tau_i < 0$ for the boundary types. In

Figure 1: response of interval i to offer $(y(i), \tau(i))$



either case, for the types located in the consecutive intervals, $\theta < a_i$ and $\theta > a_{i+1}$, $U(\theta, y_i) - \tau_i < 0$ as f increases with the distance from the good y_i, so these types do not have an incentive to deviate and pool with the interval i. As the same holds for any interval i, this partition does form an equilibrium partition. $\square$

Now, I will find the largest interval that the seller is willing to serve. This is the interval that the seller's offer will cover in the babbling equilibrium. A babbling equilibrium is one in which all buyer types $\theta \in (0, 1)$ send the same message.

Definition 3. *$x^* < 1$ is the largest interval that the seller's offer will cover in any equilibrium.*

Upon receiving an uninformative message, the seller responds by an offer that will be accepted by types $\theta \in (\underline{\theta}, \overline{\theta})$ where $\overline{\theta} - \underline{\theta} = x^*$, and $x^* < 1$ due to the assumption that k is small enough. $x^* = \int_0^1 \sigma_\theta(y^*, \tau^*)d\theta$ where y^* and τ^* are given by the seller's problem:

$$(y^*, \tau^*) = \underset{y \in \Theta, \tau \in \Re^+}{\arg\max}\, \tau \int_0^1 \sigma_\theta(y, \tau) d\theta$$ [8]

Hence, any interval such that $x \leq x^*$ will be covered.

Lemma 2. *In any interval $(\underline{\theta}_i, \overline{\theta}_i)$ where $\sigma_\theta(y_i, \tau_i) = 1$ for all θ, the expected buyer surplus is strictly positive: $\int_{\underline{\theta}_i}^{\overline{\theta}_i} [U(\theta, y_i) - \tau_i] d\theta > 0$.*

Proof. The threshold types in any interval i get 0 surplus: $k - f(\overline{\theta}_i - y_i) = k - f(y_i - \underline{\theta}_i) = \tau_i$ where $y_i = \frac{\overline{\theta}_i + \underline{\theta}_i}{2}$. As $U_{12} > 0$, $U(\theta, y_i) - \tau_i > 0$ for all $\theta \in (\underline{\theta}, \overline{\theta})$ which leads to $\int_{\underline{\theta}_i}^{\overline{\theta}_i} (U(\theta, y_i) - \tau_i) d\theta > 0$. $\square$

[8]For a given (y^*, τ^*), there is an interval $(\underline{\theta}, \overline{\theta})$ for which $U(\underline{\theta}, y^*) - \tau^* = U(\overline{\theta}, y^*) - \tau^* = 0$ and $U(\theta, y^*) - \tau^* \geq 0$. Then, outside this interval, for $\theta < \underline{\theta}$ and $\theta > \overline{\theta}$, $U(\theta, y^*) - \tau^* < 0$. y^* can be any y that satisfies $\overline{\theta} - \underline{\theta} = x^*$ where $y^* = \frac{\overline{\theta} + \underline{\theta}}{2}$. Hence, x^* is the largest pooling interval such that there is no exclusion.

Lemma 2 will be used for making welfare comparisons among equilibria. In any interval, the buyer surplus is decreasing as θ moves away from y_i, with the type $\theta = y_i$ getting the highest surplus. Hence, the types closer to the middle of the interval get information rents that are increasing in their distance from the threshold types. Next proposition establishes the existence of a fully revealing equilibrium.

Proposition 2. *There exists an equilibrium which is fully revealing in which each θ sends $m(\theta)$ inducing the offer (θ, k) and leaving 0 surplus to the buyer. The expected buyer surplus is strictly positive in any other less informative equilibria.*

Proof. First, consider the fully revealing equilibrium. Each type of the buyer has no incentive to deviate, as in case of sending another message $m(\theta')$, the seller's offer will be (θ', k) which gives utility $U(\theta, \theta') < k$ and would be rejected. Hence, this is indeed an equilibrium. Now let us consider another equilibrium in which some positive measure of buyer types pool, $\theta \in (\underline{\theta}, \overline{\theta})$ where $0 < \overline{\theta}) - \underline{\theta} < x^*$. By definition 2 this interval is covered by the seller's offer, and by lemma 2, the buyer surplus in this interval is strictly positive. This completes the proof that the buyer surplus is strictly positive in any less informative equilibria. □

Fully revealing equilibrium yields the highest social welfare as every type is getting the good which is the perfect fit and also the highest seller profit while leaving zero rents to the buyer. Having no commitment power, the seller cannot force the buyer to fully reveal her type and will propose a lower price in case the information about the buyer's type is less precise. A lower price means a larger interval that is covered. Given that the seller optimal equilibrium is trivially perfect revelation, I will focus on the other extreme which is the buyer optimal equilibrium. This is the equilibrium which is superior to any other one in terms of the expected buyer surplus, in other words from an ex-ante perspective. Communication happens to the extent that the buyer reveals information. While moving from fully revealing equilibrium to less informative ones, buyer surplus increases while the seller profit decreases, as the next lemma shows.

Lemma 3. *The expected surplus of the buyer is increasing and the seller's surplus is decreasing in the lengths of the partition intervals as long as no type is excluded inside each interval.*

Proof. Let us take an interval i inside which all buyer types are accepting the offer. As the boundary types get 0 surplus, $\tau_i = U(a_i, y_i) = U(a_{i+1}, y_i)$ so $U(\theta, y_i) - \tau_i = f(y_i - a_i) - f(\theta - y_i)$. Then, the expected, or per buyer utility of the types that belong to interval i is:

$$\int_{a_i}^{a_{i+1}} [f(y_i - a_i) - f(\theta - y_i)] d\theta \tag{1}$$

as $y_i = \frac{a_i + a_{i+1}}{2}$ and f is symmetric around 0, this simplifies to $2[(y_i - a_i)f(y_i - a_i) - \int_{a_i}^{y_i} f(y_i - \theta)d\theta]$. Then, replacing for simplicity $a_i = 0$ and $\frac{a_{i+1} - a_i}{2} = x$ we get $f(y_i - a_i) = f(x)$, and equation 1 simplifies to:

$$2 \int_0^x (f(x) - f(\theta))d\theta \tag{2}$$

When the partition rule is x, meaning there are $\frac{1}{x}$ intervals of size x on the $(0,1)$ line (disregarding the last asymmetric interval for the moment, and to be later shown that the presence of a single asymmetric interval doesn't affect the optimality of the partition):

$$x^* = \arg\max_x \frac{2}{x} \int_0^x (f(x) - f(\theta))d\theta$$

the derivative wrt x gives:

$$f'(x) - \frac{f(x)}{x} + \int_0^x \frac{f(\theta}{x^2} d\theta \tag{3}$$

then, as $xf'(x) > f(x)$ and $\int_0^x \frac{f(\theta}{x^2} d\theta > 0$, equation 3 is always positive for any x for convex f, (indeed holds also for most concave functions f), so the expected buyer surplus is increasing in the interval length as long as no type is being excluded. When the number of intervals decreases subject to the constraint that there is no exclusion, the total buyer surplus always increases. Then, the expected buyer surplus increases in the length of the partition intervals for $x \leq x^*$. The seller surplus on the other hand is decreasing in the length of the partition intervals for $x < x^*$. The price $\tau_i = k - f(\frac{x}{2})$ is decreasing as x increases, in other words as the interval $(a_{i+1} - a_i)$ widens, until reaching x^*. Hence, the seller's profit increases as the number of intervals increases, while the equilibrium partition becomes more informative. □

Lemma 3 displays the conflict between the buyer and the seller in terms of surplus sharing. The buyer benefits from larger intervals (coarser information) as she enjoys information rents, but if messages are too imprecise then the seller ends up excluding some types.

4 The buyer-optimal equilibrium of the one period game

This section solves for the buyer-optimal equilibrium of the one period game. Lemma 3 showed that the expected buyer surplus is increasing as the partition intervals get wider as long as there is no exclusion. In addition, definition 3 defined

x^* to be the largest interval such that no type is excluded from trade. Then, it is intuitive to reach the conclusion that the buyer optimal equilibrium in the one period game should follow the partition rule x^*, which means $\lfloor \frac{1}{x^*} \rfloor$ intervals of x^* and there may be a single interval having different size. The next lemma shows that the presence of this interval does not affect the optimality of the partition rule.

Lemma 4. *Among the equilibria in which trade happens for all types, the buyer preferred one has the least number of intervals, in which all but one interval is of size x^*.*

When there are 2 intervals which are both finer than x^*, it is always surplus enhancing when one enlarges at the expense of the other until reaching the threshold x^*. This means, if x^* is the optimal partition rule, then there can be one and only one interval which is finer. Finally, I need to show that in the buyer-optimal equilibrium there is no exclusion before concluding that x^* is the optimal partition rule.

Proposition 3. *The buyer-optimal equilibrium of the one period game has the minimum information revelation such that trade is ensured for all types, and has the partition rule x^*.* [9]

Proof. First, in the completely uninformative equilibrium, an interval of $x^* = \overline{\theta} - \underline{\theta}$ buyer types get served by definition 3. For the threshold types $\overline{\theta}$ and $\underline{\theta}$, $U(\theta, y) - \tau = 0$. Now consider another equilibrium in which everything else constant, the types slightly below, $\theta \in (\underline{\theta} - \epsilon, \underline{\theta})$ send a separate message, m', inducing the offer $(y(m'), \tau(m'))$ such that $\forall \theta \in (\underline{\theta} - \epsilon, \underline{\theta})$, $\sigma_\theta(y(m'), \tau(m')) = 1$ (as $\epsilon < x^*$, this interval is covered). As $\underline{\theta}$ is a threshold type, $U(\underline{\theta}, y(m')) - \tau(m') = 0$ which means for $\theta > \underline{\theta}$, $U(\theta, y(m')) - \tau(m') < 0$. So the messages and surpluses of the types in the initial interval x^* do not change. For the new types being served, by lemma 2, $\int_{\underline{\theta}-\epsilon}^{\underline{\theta}} (U(\theta, y(m')) - \tau(m'))d\theta > 0$. Hence this new equilibrium is superior to the initial one in terms of buyer surplus. Repeating the same procedure, any equilibrium where some types are excluded is dominated by a more informative equilibrium in which more types are served, until reaching an equilibrium in which trade happens for all types. The first part of the proof is done. Second part consists of showing that moving to more informative equilibria lowers buyer surplus. Lemma 4 showed that any equilibrium partition that has more than 1 interval

[9]This partition equilibrium looks like:

- $\left\langle \frac{1}{x^*} \right\rangle$ intervals of size x^* (where $\left\langle \frac{1}{x^*} \right\rangle$ is the biggest integer smaller than $\frac{1}{x^*}$)
- if $(1 - \left\langle \frac{1}{x^*} \right\rangle x^*) > 0$, another interval of size $1 - \left\langle \frac{1}{x^*} \right\rangle x^*$

of length smaller than x^* is inferior in terms of buyer surplus. This completes the proof that the partition rule x^* dominates any other more or less informative equilibria in terms of buyer surplus □

Proposition 3 says that the equilibrium which is buyer-optimal has the largest intervals subject to the constraint that there is no exclusion. In other words, as larger intervals imply less informative equilibria, messages are just informative enough for trade to take place with probability one. The buyer does not benefit from being more informative, as no type is being excluded and the expected buyer surplus is the highest possible under this rule. In addition, she does not benefit from being less informative either, because in any equilibrium which is less informative, some types will be excluded from trade. Hence, the buyer benefits from being imprecise as long as she is still allocated a good.

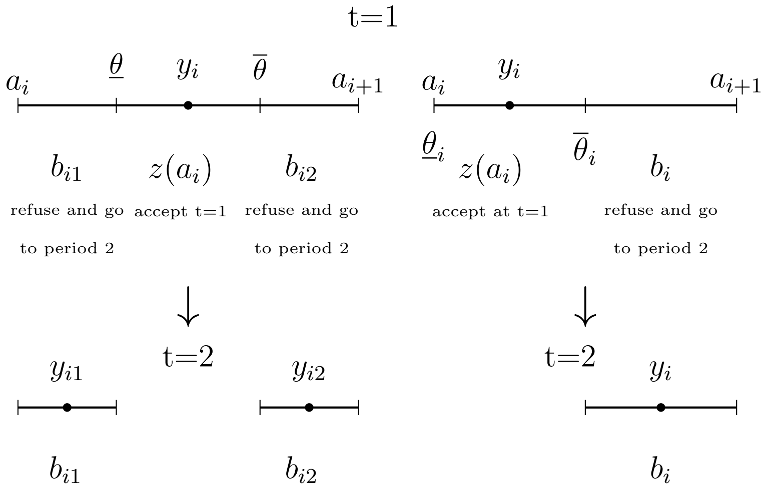
Appendix B solves for the equilibrium partition rule using the quadratic utility example: $U(\theta, y) = k - (y - \theta)^2$, and finds $x^* = 2\sqrt{\frac{k}{3}}$, As the intrinsic value k increases, the pooling intervals become wider, in other words less information is transmitted in equilibrium. When preferences are less important compared to the intrinsic value of the good, the buyer can be less precise and still get a good. In addition, as f' increases, meaning as f becomes more convex, x^* decreases. When the cost of mismatch increases faster, more informative messages are necessary in order to ensure trade. While moving from fully informative equilibrium to less informative ones, the seller surplus is decreasing and the buyer surplus is increasing, while total welfare is decreasing.

5 Dynamics of the game: two period case

In this section, the dynamics of the game are explored by introducing a second round of communication and offer. This case also provides the intuition for extension to the infinite horizon game. The only modification to the one period game is that now when period 1 offer is rejected, there is a second round of communication and offer, and surplus is discounted by δ. The type of the buyer does not change when the game moves to the next round, but she sends a second message. The discount factor is what makes delay costly. The buyer's strategy is now denoted as $q(m_1|\theta)$, $q(m_2|\theta)$ and the seller's strategy $h(m_1), h(m_2, m_1)$. The seller's period 2 offer now depends on both the period 1 and period 2 messages.

To understand the dynamics of the game, look at a period 1 interval i of length x_i sending the message m_i. The seller's best response is an offer (y_i, τ_i) such that at least some types in the interval accept, which will be denoted as an interval $z(x_i)$ with the threshold types $(\underline{\theta}_i, \overline{\theta}_i)$. Depending on the seller's offer, there may be some types, $\theta \in b_i$ where $b_i = x_i \setminus z(x_i)$ who reject the offer. In case the

Figure 2: Dynamics of the game



game moves on to period 2, the seller knows that the buyer is located in interval i outside $(\underline{\theta}_i, \overline{\theta}_i)$, as in case θ were found in this region then the offer would be accepted at period 1. If $\underline{\theta}_i = a_i$ or $\overline{\theta}_i = a_{i+1}$ the seller puts probability one to the buyer being found in the single interval b_i. Otherwise, there are 2 separated intervals, $b_{i1} = (a_i, \underline{\theta})$ and $b_{i2} = (\overline{\theta}, a_{i+1})$ inside which the buyer could be found, so the seller's belief is $\theta \in b_{i1} \cup b_{i2}$ at the beginning of period 2. The period 2 offer strategy of the seller is similar to the one period game given that it is the last period. Figure 2 gives a demonstration of these dynamics, restricted the case in which there is no exclusion over the 2 periods and no division of intervals that refuse the period 1 offer. It will be shown throughout this section that the buyer optimal equilibrium has this structure.

The presence of a second period gives the seller the opportunity to better identify the buyer's taste and differentiate among the types over the two periods. Given identical partition intervals, the seller always charges a higher price in period 1 of the 2 period game compared to what he charges in the single offer game, as he can make another offer at period 2 in case the period 1 offer is rejected. The rejection also gives information about the buyer's type, as it leads the interval of possible types to shrink. Hence, even if the period 2 message is uninformative, the seller has better information about the buyer's type due to the period 1 message and the fact that the period 1 offer has been rejected. To sum up, the presence of period 2 gives two informational benefits to the seller: the shrinking of the interval in which the buyer can be found, and another possibly informative message in period 2.

Now I define the equilibrium in which the game never moves to period 2, which is called the no-delay equilibrium.

Definition 4. *A no-delay equilibrium is one in which for each message m_i sent at period 1, the seller's offer (y_i, τ_i) is such that $\sigma_\theta(y_i, \tau_i) = 1$ for all $\theta \in (a_i, a_{i+1})$ and for all i: any buyer type in any interval accepts the period 1 offer and the probability of reaching period 2 is zero. I call the no-delay threshold interval length x^{nd} such that whenever $x \leq x^{nd}$ for all intervals, trade happens with no delay.*

In the no-delay equilibrium, the period 1 partition intervals are fine enough that the seller does not have an incentive to exclude any type and move on to period 2 with positive probability. It is trivial that $x^{nd} < x^*$, as x^* is the largest interval served in the one period game when there is no next period, and in case there is a second period, the seller benefits from leaving out some types to the next period. The threshold no-delay interval length x^{nd} is constant for any game of T periods, as once $x \leq x^{nd}$ in any period, trade will take place with certainty and in any other case, there will always be delay for some types. If the seller does not find it profitable to exclude any buyer type when there is only one period of game left to play, he would not exclude any type when there is more than one periods left either and conversely, if he excludes some types when there is only one period left, he would also do so when there are more periods left.

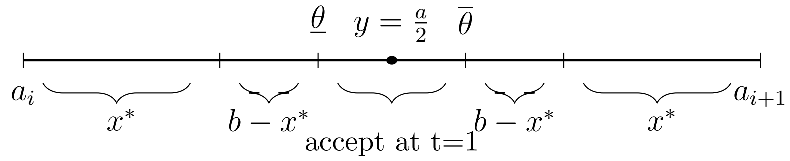
Another result that carries on from the static case is that partitions are monotone. It is easy to replicate lemma 12 for the dynamic case. Take two types θ_1 and θ_3 which send message m at period 1 and $\theta_2 \in (\theta_1, \theta_3)$ that sends another message m'. In case trade happens for these types over the 2 periods, then it means that θ_2 prefers the present value of his allocation to that of θ_1 and θ_3. Then using the same argument as in lemma 12, it concludes that there cannot be a non monotone signaling rule.

Lemma 5. *A threshold type at period* 1 *will also be the boundary type in her interval at period* 2 *in case of rejecting the period* 1 *offer, and hence will get* 0 *surplus from the offer in either period.*

Proof. At period 1, $\sigma_\theta = 1$ for $\theta \in (\underline{\theta}_i, \overline{\theta}_i)$. So, in case period 1 offer is rejected, at period 2, $\underline{\theta}_i$ will be the higher boundary type of the interval $(a_i, \underline{\theta_i})$, and $\overline{\theta}_i$ is the lower boundary type of the interval $(\overline{\theta}_i, a_{i+1})$. The seller's offer at period 2 will leave 0 surplus to the boundary types in case they get served, or they will reject the offer and still get 0. Then, as by definition the threshold types are indifferent to accepting the offer in either period, they are left 0 surplus in the period 1 offer as well. □

Lemma 5 ensures that the dynamic incentive compatibility of the threshold types is satisfied. By the monotonicity of strategies, the dynamic incentive compatibility of all the types inside the interval $(\underline{\theta}_i, \overline{\theta}_i)$ is satisfied once the dynamic incentive compatibility of the threshold types is satisfied. Hence, when the threshold types are indifferent to accepting the offer in a period, the types inside the

Figure 3: subdivision



interval are strictly better off doing so. Then, the types inside an interval are not affected by the change in the strategies of the types in the neighboring intervals, as long as the threshold types are indifferent between the offers over the two periods. So, it follows that the types inside a period 1 interval that accept the period 1 offer will be found in connected intervals.

Definition 5. *A subdivision happens when the types inside an interval i that reject the period* 1 *offer and are connected separate in period* 2 *into at least* 2 *intervals sending separate messages.*

Subdivision enriches the possible period 2 partitions. However, it will be shown that subdivision is always dominated hence we will restrict attention to efficient equilibria with no subdivision.

Least informative partition at $t = 2$: Let us find the least informative partition that ensures trade in case period 2 is reached. For an interval b_i of types that reject the period 1 offer, the least informative partition at period 2 which ensures trade has the partition rule x^*, which means if $b_i \geq x^*$, the interval b_i subdivides into n intervals of measure x^* and a final interval of measure $b_i - nx^*$, where x^* is the largest interval that can be served in period 2. In case $b_i \leq x^*$, the types in interval b_i pool in their message. Hence, as in the one period game, x^* denotes the least informative partition rule that ensures trade at period 2.

5.1 Equilibria of the two period game

In this section I explore all the kinds of equilibria that can arise under different information structures. As in the one period game, a continuum of equilibria could arise. I restrict attention to equilibria with no exclusion among which I will solve for the buyer optimal one, which will be characterized in proposition 6. The seller's period 1 offer depends on the buyer's strategy in period 2. Hence, the period 1 offer is the one which is optimal given the message of the buyer in period 1 and her strategy in case the game moves to period 2. In return, the buyer optimal strategy is the one that maximizes the overall utility of the types that accept the offer in period 1 and in period 2. The seller's offer (y_i, τ_i) at period 1 determines which buyer types in interval i accept his offer and which ones reject. Next step is to solve for the seller's offer in period 1 as a function of the period 1 and period 2 information structures.

The condition for all types to be served[10] without a subdivision determines the candidate buyer optimal equilibria, as equilibria having subdivision are Pareto dominated. There is a maximum period 1 interval length above which some types will be excluded from trade in case there is no subdivision at period 2, as the intervals will be too large and the seller will exclude some types. Then, period 1intervals should be fine enough for there to be no exclusion at the end of the game.

If the seller is serving types of measure x, by equation **??**, $\tau = k - f(\frac{x}{2})$, as the threshold types found at a distance of $\frac{x}{2}$ are getting 0. I will restrict attention to a single interval, as given the period 1 message, the seller disregards the other intervals and the solution is identical for any given interval.

5.1.1 When period 2 is informative

Definition 6. *Period* 2 *is informative if the buyer types in a period* 1 *interval which refuse the period* 1 *offer and are disconnected send at least 2 separate messages at period 2.*

Lemma 6. *If the period* 2 *strategy of the buyer is informative, then in period* 1*,* $y_i = \frac{a_i+a_{i+1}}{2}$ *is a weakly dominant offer for the seller as a response to a message* m_i*, so that the types which refuse his offer will be found in* 2 *separated and equal intervals.*

Lemma 6 says that when period 2 is informative, the seller is better off when the types excluded from period 1 offer are found in two separate and equal intervals. This is because the seller prefers smaller intervals, and having 2 identical intervals at period 2 is the best for the seller when these intervals do not pool together. The problem of the seller is to choose z which is the measure of types served at period 1, hence choosing at the same time b, each of the two separate intervals that are excluded at period 1. As we restrict attention to equilibria with no subdivision, each interval b will be served at period 2.

$$(z^*, b^*) = \arg\max_{z,b} z(k - f(\frac{z}{2})) + 2\delta b(k - f(\frac{b}{2}))$$

subject to:

$$z + 2b = x$$

[10]It follows from the one period game that any equilibrium in which some types are excluded is Pareto dominated. To see how this applies to 2 period game: if some types are excluded at the end of 2 periods, then these types could have at period 2 separated by sending another message and get an acceptable offer at period 2, without affecting the allocation of other types.

The FOC gives: $k - f(\frac{z}{2}) - zf'(\frac{z}{2}) = \delta(k - f(\frac{b}{2}) - zf'(\frac{b}{2}))$ if $z^*, b^* > 0$, which means as $\delta < 1$ and f is convex, $b \leq z$, and $z(x) \geq \frac{x}{3}$. The largest period 1 interval such that all types are served with no subdivision is $x = 3x^*$, which gives $z(x) = b = x^*$.

5.1.2 When period 2 is babbling

Now I consider the equilibrium in which only the first period messages are informative.

Definition 7. *A babbling strategy in period* 2 *is one in which all the buyer types in an interval that reject the period 1 offer send the same period 2messages.*

Lemma 7. *If the buyer's strategy is to babble at period 2, then given a message m_i, the seller weakly prefers (y_i, τ_i) such that either $\underline{\theta}_i = a_i$ or $\overline{\theta}_i = a_{i+1}$ so that the types which reject the period* 1 *offer are found in a single interval.*

If the excluded types are found in a single interval, at period 2 the seller knows $\theta \in b_i = x_i \setminus z_i$ and he will serve at most x^* measure of types. In case the types that refuse his offer are found in 2 separated intervals b_{i1} and b_{i2}, the seller knows that $\theta \in b_{i1} \cup b_{i2}$, so he may find it optimal to serve only one of the intervals, meaning trade may not occur for a positive measure of types. In case he wants to sell to some types in both intervals, he has to charge a lower price than he would if they were found in a single interval.[11] At period 1, the seller chooses z and b taking into account that the excluded types will pool in their period 2 messages (again, focusing on the case where b will not subdivide in period 2):

$$(z^*, b^*) = \arg\max_{z,b} z(k - f(\frac{z}{2})) + \delta b(k - f(\frac{b}{2}))$$

subject to:

$$z + b = x$$

The FOC gives: $k - f(\frac{z}{2}) - zf'(\frac{z}{2}) = \delta(k - f(\frac{b}{2}) - zf'(\frac{b}{2}))$ when $z^*, b^* > 0$, which gives $b \leq z$ and $z(x) \geq \frac{x}{2}$ for f convex. The maximum value x can take so that there is no exclusion is $x = 2x^*$ which leads to $z(x) = b = x^*$. Then, for all $a \leq 2x^*$, $z(a) \geq b$.

[11] The reason he is weakly better off is that there can be some cases in which the seller will be indifferent between having the excluded types in a single interval or two separated intervals. For example if the measure of buyer types excluded is $b \leq x^*$, then he is strictly better off when these types are found in a single interval, as he will choose to serve all of them. In case they are separated, then either he will serve only one interval or charge a lower price to serve all the types. However, in case $b > x^*$, as the seller will sell only to x^* measure of types at period 2, he is indifferent to having a single or 2 separate intervals as long as one of the intervals measures at least x^*.

5.1.3 No-delay equilibrium

Take an equilibrium which is babbling at period 2. In case $\frac{\partial}{\partial z}[(k - f(\frac{z}{2}))z + \delta(k - f(\frac{x-z}{2}))(x - z)] > 0$ for $z(x) = x$, then no type is excluded at period 1 and there is no expected delay (the condition is identical when period 2 is informative). The maximum x such that there is no delay is found by replacing $z(x) = x$ in the derivative and setting it to zero, which gives:

$$k(1-\delta) = f(\frac{x}{2}) + \frac{x}{2}f'(\frac{x}{2})$$

the right hand side is increasing in x whereas the left hand side is constant, hence there is a single x^{nd} such that this equation holds with equality. For all $x \leq x^{nd}$, the game has no delay, and for all $x > x^{nd}$, there is expected delay.

The last type of equilibrium is the one which is babbling at period 1 and informative at period 2, which is shown to be dominated in the Appendix.

Proposition 4. *In any dynamic game, buyer surplus is always inferior to the surplus in the buyer-optimal equilibrium of the one period game.*

Proof. Proposition 3 showed that x^* is the interval length in the equilibrium which maximizes the buyer surplus in the one period (single offer) game. In the 2 periods game, given z_i for all i the intervals which get the offer at $t = 1$, and b_i for all i the intervals that get the offer at $t = 2$ in each interval i, the expected surplus of the buyer using equation 2 is:

$$\left[\sum_{i=1}^{n_z} 2\int_0^{f(\frac{z_i}{2})} (f(\frac{z_i}{2}) - f(\theta))d\theta + \sum_{i=1}^{n_b} 2\delta \int_0^{f(\frac{b_i}{2})} (f(\frac{b_i}{2}) - f(\theta))d\theta\right] \tag{4}$$

where $\sum_{i=1}^{n_z} z_i + \sum_{i=1}^{n_b} b_i \leq 1$. Hence, $\sum_{i=1}^{n_z} z_i \leq 1$. As the seller will never serve any interval larger than x^*, $z_i \leq x^*$ for all i. The surplus would be maximized if $z_i = x^*$ for all i and $\sum_{i=1}^{n_z} z_i = 1$: only if no type were excluded and the game ended at period 1 with certainty, the surplus would be equal to the surplus of the one period game. However, when x^* is the interval length at period 1, then $z(x) < x^*$ whether period 2 is informative or babbling, and $z(x) = x^*$ only when $x = 2x^*$ in the babbling equilibrium, and when $x = 3x^*$ in the informative equilibrium. So, a positive measure of types are excluded at period 1, implying there exists $b_i > 0$. Then, $\sum_{i=1}^{n_z} z_i < 1$, and due to the discount factor δ, surplus is always lower than in the static game. Finally, among the equilibria in which no type is excluded at period 1, $x = x^{nd}$ is the largest interval length, which also gives lower surplus because $x^{nd} < x^*$. Then it follows that the buyer surplus in the 2 period game is always lower than in the buyer optimal equilibrium of the one period game. This generalizes to more than 2 periods, as the seller even has more incentives to exclude types when there are more periods to be played. □

5.2 The buyer-optimal equilibrium

The buyer-optimal equilibrium is the best equilibrium from an ex-ante point of view as it maximizes the surplus over buyer types, as well as the best equilibrium once period 2 is reached, as it maximizes the expected surplus of the buyer types that have probability to reach period 2. Hence, in case the buyer types could choose to coordinate on an equilibrium before knowing their type, this would be the one.

Lemma 8. *Among the equilibria in which there is positive expected delay, the one which has the period* 2 *partition rule* x^* *is the buyer-optimal one. When* b_i *is the interval of types in period 1 interval* x_i *that reject the period* 1 *offer, following is the optimal partition rule in period* 2*:*

- *if* $b_i \leq x^*$*, the types in* b_i *pool in their messages.*
- *if* $b_i > x^*$*, then the types in* b_i *divide into* n *intervals of length* x^* *and a final interval of* $b_i - nx^*$ *by sending separate messages.*

As in the one period game, the surplus maximizing equilibrium for the buyer is one in which the partition intervals are fine enough to ensure that no type is excluded at the end of the game. If period 2 is informative, then the types who reject the period 1 offer will send a period 2 message which is just precise enough that trade happens with certainty. On the other hand, if period 2 messages are babbling, then the period 1 partition has to be fine enough so that trade still happens with certainty over the 2 periods.

Proposition 5. *In the buyer-optimal equilibrium of the two period game, trade happens for any type either at period 1 or at period 2.*

Proof. This proposition is a direct result of lemma 8. Either trade happens at $t = 1$ or otherwise at period 2, due to lemma 8, $b_i \leq x^*$ for all i, hence there is no exclusion in period 2 in case it is reached. □

Now that I concluded there is no exclusion, in the expression of the buyer surplus in proposition 4, I can replace $\sum_{i=1}^{n_z} z_i + \sum_{i=1}^{n_b} b_i = 1$.

Lemma 9. *Any equilibrium in which messages are informative at period* 1 *and a subdivision happens at period* 2 *is Pareto dominated by another equilibrium with no subdivision and a more informative period* 1 *partition.*

This is proven by showing that the intervals which subdivide at period 2 would have been better off in another equilibrium in which they had separated themselves at period 1. In the buyer-optimal equilibrium, if information is provided in period 1, it should be sufficient to make sure trade is ensured without a subdivision over the 2 periods. Lemma 9 tells us that equilibria with subdivision can be disregarded in the search for the buyer-optimal equilibrium. Together with lemma 8, it leads to the condition $b_i \leq x^*$ for all i in order for trade to be ensured at period 2 without a subdivision. After showing that there is no subdivision, in the surplus function of the buyer I now replace $b_i = \frac{x_i - z_i(x_i)}{2}$ in case period 2 is informative, and $b_i = x_i - z(x_i)$ in case period 2 is babbling. Given a partition rule of x at period 1, the expected buyer surplus can be written as:

$$\left\langle \frac{1}{x} \right\rangle 2\int_0^{\frac{z(x)}{2}} (f(\frac{z(x)}{2}) - f(\theta))d\theta + 2\delta 2\int_0^{\frac{x-z(x)}{4}} (f(\frac{x-z(x)}{4}) - f(\theta))d\theta +$$
$$2\int_0^{\frac{z(x')}{2}} (f(\frac{z(x')}{2}) - f(\theta))d\theta + 2\delta 2\int_0^{\frac{x'-z(x')}{4}} (f(\frac{x'-z(x')}{4}) - f(\theta))d\theta \quad (5)$$

for the equilibrium with $t = 2$ informative, and:

$$\left\langle \frac{1}{x} \right\rangle 2\int_0^{\frac{z(x)}{2}} (f(\frac{z(x)}{2}) - f(\theta))d\theta + \delta 2\int_0^{\frac{x-z(x)}{2}} (f(\frac{x-z(x)}{2}) - f(\theta))d\theta +$$
$$2\int_0^{\frac{z(x')}{2}} (f(\frac{z(x')}{2}) - f(\theta))d\theta + \delta 2\int_0^{\frac{x'-z(x')}{2}} (f(\frac{x'-z(x')}{2}) - f(\theta))d\theta \quad (6)$$

for the equilibrium with $t = 2$ babbling, where $x' = 1 - x\left\langle \frac{1}{x} \right\rangle$ is the final asymmetric interval. Now, by an argument similar to the single offer game, I will conclude that the presence of this last asymmetric interval does not affect the overall optimality of the partition rule.

Lemma 10. *The optimal partition rule a^* is given by:*

$$a^* = \arg\max_x \frac{1}{x}\left[2\int_0^{\frac{z(x)}{2}} (f(\frac{z(x)}{2}) - f(\theta))d\theta \right.$$
$$\left. + 2\delta 2\int_0^{\frac{x-z(x)}{2}} (f(\frac{x-z(x)}{2}) - f(\theta))d\theta \right] \quad (7)$$

when $t = 2$ is informative, and:

$$a^* = \arg\max_x \frac{1}{x}\left[2\int_0^{\frac{z(x)}{2}} (f(\frac{z(x)}{2}) - f(\theta))d\theta \right.$$
$$\left. + \delta 2\int_0^{\frac{x-z(x)}{2}} f(\frac{x-z(x)}{2}) - f(\theta))d\theta \right] \quad (8)$$

when $t = 2$ is babbling.

Next proposition characterizes the buyer-optimal equilibrium of the two period game.

Proposition 6. *In the buyer-optimal equilibrium of the 2 period game, period* 1 *messages are informative,*

- *for $\delta \geq \hat{\delta}$, partition intervals at period* 1 *are of length $2x^*$:*
  - *half of the types in each interval are served at period* 1 *and the rest at period* 2
  - *period 2 messages are babbling*
- *for $\delta < \hat{\delta}$, no-delay equilibrium with intervals of x^{nd}:*
  - *the game ends at period* 1 *with certainty.*

A short sketch of the proof is as follows: by lemma 9 and lemma 8, I know that in equilibrium trade will happen for any type of the buyer with no subdivision. The candidate equilibria either have informative messages in both periods, or informative messages in period 1 and babbling messages at period 2. The equilibria which have babbling messages at period 1 can be shown to be dominated. The surplus functions in lemma 10 have 2 maximal points. The first one is when $z(x)$ takes its maximum value subject to delay. In this kind of equilibrium, $z(x) = x^*$ when $x = 2x^*$ in the babbling equilibrium and when $x = 3x^*$ in the informative equilibrium. This shows that more types are getting the period 1 offer in the babbling equilibrium at the same price than in the informative equilibrium. The second maximum is achieved when period 1 interval takes its largest value subject to no-delay, hence $x = x^{nd}$. When $\delta < \hat{\delta}$, the no-delay equilibrium dominates the equilibria with delay, and for $\delta \geq \hat{\delta}$ the babbling equilibrium dominates the informative one.(The extended proof can be found in Appendix B.)

In the no-delay region $\delta \leq \hat{\delta}$, buyer surplus is at first decreasing in δ, as the partition intervals get finer in order to satisfy this condition. As waiting is too costly in this region, the buyer is better off in the equilibrium having finer pooling intervals. When $\delta > \hat{\delta}$, the buyer optimal strategy is less informative, and the intervals are the largest possible subject to the constraint that all types get served over the 2 periods. The intervals are constant in δ and the surplus increases in δ only because the cost due to delay is lower. In the equilibrium with delay, the buyer and seller surpluses are both lower than in the one period game.

The seller surplus has a jump down at $\delta = \hat{\delta}$ when shifting to the equilibrium with delay. When $\delta > \hat{\delta}$, the seller surplus is always lower than in the static game, as the buyer's period 1 message is less informative and there is expected delay. On the other hand, when $\delta < \hat{\delta}$, his surplus is even higher than in the static game, due to the finer partition intervals and no delay in trade. In this region, the match between the good and the buyer improves compared to the single offer game.

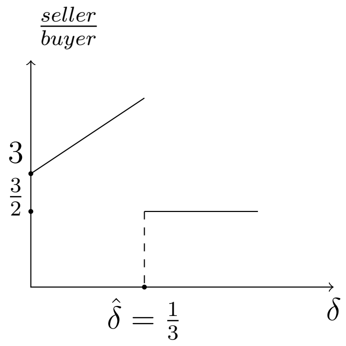
The result says that the buyer is better off providing information only at period 1 when there are 2 periods to play. Babbling strategy serves is a way to discipline the seller and increases the possibility of trade at period 1. The seller sells to more types at period 1. In return, the period 1 partition has to be informative enough that trade still happens with certainty as a result of the 2 periods. When the the seller knows the period 1 message is the only information he will get, he has less incentive to delay by asking a higher price than when he expects an informative message at period 2.

Having 2 rounds first appears to be an advantage for the seller who can better identify the type of the buyer due to the opportunity to make a second offer in case the period 1 offer is rejected. This advantage is less in the equilibrium which is babbling in period 2. Knowing that he will not receive another informative message, the seller is more eager to trade at period 1 compared to the game in which period 2 is informative. To conclude, revealing information gradually is not profitable for the buyer who is the owner of information. The result when $\delta > \hat{\delta}$ suggests that the introduction of a second period does not improve information transmission compared to the single period game and in addition results in expected delay. However, when $\delta < \hat{\delta}$, information transmission and overall welfare increase. Hence, the second round of play is beneficial for the gains from trade only in case delay is very costly.

The graph displays the ratio of buyer and seller surpluses calculated by using the following quadratic utility:

$$U^B = k - (\theta - y)^2 \tag{9}$$

- $\hat{\delta} = \frac{1}{3}$
- for $\delta > \frac{1}{3}$, buyer surplus $\frac{k(1+\delta)}{9}$ and seller surplus $\frac{k(1+\delta)}{6}$.
- for $\delta < \frac{1}{3}$, buyer surplus $\frac{2k(1-\delta)}{9}$ and seller surplus $\frac{2k+\delta}{3}$



6 The infinite horizon game

This section generalizes the game to one in which the rounds of communication and offer continue until trade takes place. If trade takes place at period T, then the buyer's utility is $\delta^{T-1}(U(\theta, y) - \tau)$ and the seller's utility $\delta^{T-1}\tau$ from a period 0 point of view. The seller's information at period t and hence the offer depends on the history which of the past messages plus the current message $h_t = \{m_s\}_{s=0}^{t}$. Many properties of the two periods game carry on to the infinite horizon.

I will focus on a single interval, hence an interval at period t, x_{it} will be shortened as x_t, and z_{it} which is the interval of types that accept the offer at period t, will be shortened as z_t and b_t will denote the set of types excluded in period t inside an interval i, which again may be a single or two separate intervals.

As already stated in the two period game, in any period in which all the partition intervals satisfy $x \leq x^{nd}$, trade will happen with certainty. This generalizes to infinite horizon game: if given an interval the seller has no incentives to exclude any type in the first period when there is a second period, then he would not have any incentive to exclude when there are an infinite number of periods left either. As x^{nd} is the largest interval such that there is no-delay, then for any interval $x > x^{nd}$, he will also serve at least x^{nd} measure of types but there will be expected delay. Hence, in any period, for any interval $x > x^{nd}$, some types are not served: $z_t \geq x^{nd}$ and $b_t > 0$.

This section we will make use of the more specific utility function:

$$U^B = k - (\theta - y)^2 \tag{10}$$

Although the results continue to hold under the more general utility function, this specification makes computation easier in this section.

Remark 1. *The no-subdivision condition of the two periods game carries on to the infinite horizon game: in the buyer-optimal equilibrium there will be no subdivision in any future period $t > 1$.*

This was proven in the 2 periods game by lemma 9. Now, I generalize it to the infinite game. By making use of the lemma 9, any equilibrium in which subdivision happens at $t-1$ Pareto dominates equilibria in which subdivision happens at period t. Repeating the same argument until period 1, I conclude that there is no subdivision in any period $t \geq 2$.

Two kinds of equilibria which satisfy the no subdivision condition are of interest. The first one is the equilibrium in which the messages are informative in each period. Using lemma 6, if future periods are informative, the seller's weakly optimal offer is the one in which the types that reject the period 1 offer will be found in 2 separated and equal intervals in period 2, which means the seller sets $y_i = \frac{a_i + a_{i+1}}{2}$. In the infinite horizon game, this is still relevant: the types in an interval that reject the period 1 offer are found in 2 separate intervals. Continuing in this fashion, from a period 1 point of view, there are 2^{t-1} intervals of z_t that are served at each period t in the informative case. The second type of equilibrium is the babbling one in which the only informative messages are sent at period 1 and the types that reject the offer pool in each coming period. In the babbling equilibrium, by lemma 7, a single interval z_t is served at each t. Hence, from a period 1 point of view, there is only 1 interval z_t served in each period in the babbling equilibrium.

Given a period 1 message sent by a partition interval x_1, if the game lasts at most T periods, at each t, x_t is the length of the pooling interval while z_t is the measure of types that accept the offer. The intervals follow $x_{t+1} = x_t - z_t$ in the babbling equilibrium and $x_{t+1} = \frac{x_t - z_t}{2}$ in the informative equilibrium. Given the first period partition and the future message strategy of the buyer, the seller determines the future offers $(y_t, \tau_t)_{t=0}^{T}$.

Lemma 11. *If trade happens at latest at period T, $z_1 \geq z_2 \geq .... \geq z_T$ and so $p_1 \leq p_2 \leq p_3 .... \leq p_T$.*

Proof. This is found by taking the seller's problem which finds that the game will end at latest at period T. Upon receiving a message sent by an interval x at period 1 and given whether or not the future periods are informative, the seller's problem is to maximize his profit by determining z_t for each t, and hence T which is the last period in which trade can take place for some types. The discounted expected surplus of the seller in the informative case is:

$$\Pi^i(z_t) = \sum_{t=1}^{\infty} 2^{t-1} \delta^{t-1} z_t (k - f(\frac{z_t}{2}))$$

subject to:

$$\sum_{t=1}^{\infty} 2^{t-1} z_t = x$$

The FOC wrt z_t gives:

$$k - f(\frac{z_t}{2}) - \frac{z_t}{2} f'(\frac{z_t}{2}) \le \frac{\lambda}{\delta^{t-1}}$$

As $\delta < 1$, it follows that $z_1 \geq z_2 \geq z_3 ... \geq z_T$, where T denotes the latest period that can be reached. There is a period T such that $x_T = z_T \leq x^{nd}$ such that in case this period is reached, trade will take place in that period with certainty.

Second, in case all periods $t > 1$ are babbling, the surplus given the first period interval x is:

$$\Pi^b(z_t) = \sum_{t=1}^{\infty} \delta^{t-1} z_t (k - f(\frac{z_t}{2}))$$

subject to:

$$\sum_{t=1}^{\infty} z_t = x$$

The FOC wrt z_t:

$$(k - f(\frac{z_t}{2}) - \frac{z_t}{2} f'(\frac{z_t}{2}) = \frac{\lambda}{\delta^{t-1}}$$

which leads to and $z_1 > z_2 > ... > z_T$ where again T is the latest period which can possibly be reached. Realize that the FOC are identical in the informative and babbling cases. In case $z_1 = x^*$, then $z_1 = z_2 = ... = z_{T-1} = x^*$ as $(k - f(\frac{x_t^*}{2}) - \frac{x_t^*}{2} f'(\frac{x_t^*}{2}) = 0$, and for all other values, $z_1 > z_2 > ... > z_T$. $\square$

If we use the quadratic utility, the relation $z_{t+1} = \sqrt{\frac{z_t^2 - \frac{4}{3}k(1-\delta)}{\delta}}$ is satisfied for all $t < T$ and leads to $z_1 = \sqrt{\frac{4k(1-\delta^{T-1})}{3} + \delta^{T-1} z_T^2}$.

Given a period 1 interval x and the signaling rule in the future periods, the seller's problem determines that the game will end at latest at period T which satisfies $x - \sum_{i=1}^{T-1} z_t \leq x^{nd}$ in the babbling equilibrium and $\frac{x - \sum_{i=1}^{T-1} 2^{T-1} z_t}{2} \leq x^{nd}$ in the informative equilibrium. The final period T is such that $x_T \leq x^{nd}$ for all any interval: the length of the interval is below the no-delay interval and the game ends as the seller will make an offer which will be accepted by any type that can be present at period T.

I rule out equilibria in which one future period is informative and the next one is babbling, as they can easily be shown to be dominated. As in any given period t there is possibly an infinite number rounds left, it cannot be optimal for the buyer to send babbling messages at a period t and informative ones at $t + 1$.

Proposition 7. *Any equilibrium strategy which has informative messages in any period $t > 1$ is dominated for any buyer type and for the seller by another one which is informative only at period 1 and babbling in all future periods $t > 1$.*

(Proof in Appendix A)

The proof proceeds by taking any informative equilibrium and showing that there exists a corresponding babbling equilibrium which also lasts for T periods and has identical z_t for all $t \in (1, T)$ and a finer period 1 partition rule, meaning $x^b < x^i$, which gives higher buyer surplus. In other words, any informative equilibrium can be transformed into another one in which all information is provided in period 1 instead of in later periods and provides a weakly higher surplus to any buyer type. This equilibrium is also better for the seller as there is less delay.

Given that in an informative equilibrium the seller anticipates that the buyer will send informative messages in the future, the buyer is always better off in an equilibrium in which all information is revealed today and at once. The seller faces no uncertainty about the extent of information that the buyer is willing to provide, but about the information itself, so the buyer is better off resolving this uncertainty at the beginning. If the seller anticipates that he will get more information tomorrow, he asks for a higher share of the surplus today. In the equilibrium which is babbling in the future periods, trade happens earlier in expectation compared to the informative one.

7 Extensions: Uncertainty on the value of k

An interesting question to ask is how would the information transmission and outcome be modified if uncertainty about the value of k is introduced, especially whether truthful revelation by the high value buyer could be observed. In this section, a single offer game with two types is considered. The type of the buyer now consists of two variables: (k, θ) where k can take a high or low value: $\overline{k}$ or $\underline{k}$ with equal probabilities. As expected, the high valuation types will always have an incentive to pool with the low valuation ones on the vertical dimension as long as the low types are not excluded by the seller.

Proposition 8. *There is no equilibrium in the one period game in which high valuation types separate themselves from low types types and low types are not excluded by the seller.*

Proof. This is proven by contradiction. First, assume an equilibrium exists where the types with $\underline{k}$ and $\overline{k}$ separate. Hence, there are 2 separate partitions, where the partition of $\overline{k}$ types has intervals of length $\overline{a}$ and those of $\underline{k}$ has $\underline{a}$ with $\overline{a} > \underline{a}$. Look at a threshold type $(\overline{k}, \overline{a_i})$ in interval i whose surplus is 0 when truthfully revealing her type by pooling with the $\overline{k}$ types in interval i. However by pooling with the $(\underline{k}, \overline{a_i})$ type, she can ensure a utility of at least $\overline{k} - \underline{k}$ (as the worst case would be that $\underline{k}$ type is also a threshold type). Hence, in a separating equilibrium, the threshold types with $\overline{k}$ should get positive surplus upon revealing that they are high type. However, this is not compatible with the seller's optimality: he will leave 0 surplus to the threshold types with $\overline{k}$ once he receives this message, as he has no commitment power. So, there is no equilibrium in which the high and low types follow different partition rules with $\overline{a} > \underline{a}$. To conclude, unless $\underline{k}$ types are excluded by the seller, the $\overline{k}$ types always have an incentive to pool with them. $\square$

The buyer's expected utility is maximized in the partition equilibrium with x^* as long as $\underline{k}$ types are not excluded. In this equilibrium, $\overline{k}$ and $\underline{k}$ types pool together in intervals of length x^* which is a function of $\overline{k}$. However, if the difference $\overline{k} - \underline{k}$ is high enough, the seller will then exclude the $\underline{k}$ types. Below are the three possible equilibria that may arise under this condition. This part again makes use of the following utility for the buyer:

$$U(\theta, y) = k - (\theta - y)^2 \tag{11}$$

The first two cases arise when the seller excludes $\underline{k}$ types in case $x^* = 2\sqrt{\frac{\overline{k}}{3}}$ partition is played. This happens as long as:

$$\overline{k} > \frac{7}{5}\underline{k}$$

[12] Then, there are 2 different partitions that can arise under this condition:

- If $\frac{7}{15}\overline{k} < \underline{k} < \frac{5}{7}\overline{k}$, the high and low types pool in a partition such that no type will be excluded. This partition interval is given by $2\sqrt{2\underline{k} - \overline{k}}$. The expected surplus of the buyer in this equilibrium is $\frac{5\underline{k}-\overline{k}}{6}$. [13]

- If $\underline{k} \leq \frac{7}{15}\overline{k}$. Now, $\underline{k}$ has become so low that it is better that these types are excluded to maximize the buyer surplus. Then $\overline{k}$ types separate themselves and play a less informative partition which is $x^* = 2\sqrt{\frac{\overline{k}}{3}}$ which is the optimal partition such that no $\overline{k}$ type is excluded. Only the high types are served, which gives an expected buyer surplus of $\frac{\overline{k}}{9}$.

  The last case arises if $\overline{k} - \underline{k}$ is not too big, meaning if:

$$\overline{k} \leq \frac{7\underline{k}}{5}$$

[12] in case both types are served, $\tau = \underline{k} - \frac{\overline{k}}{3} = \pi$. In case only $\overline{k}$ are served, $\tau = \overline{k} - \frac{\overline{k}}{3}$ and $\pi = \frac{\overline{k} - \frac{\overline{k}}{3}}{2}$. Comparing the two gives the result.

[13] $2\sqrt{2\underline{k} - \overline{k}}$ is the highest interval such that the lower types are not excluded, found by: $\frac{\overline{k} - \frac{x^2}{4}}{2} \leq \underline{k} - \frac{x^2}{4}$.

Now, $\overline{k}$ and $\underline{k}$ are close enough that even when the partition $x^* = 2\sqrt{\frac{\overline{k}}{3}}$ is played no low type is excluded.[14]

To sum up, in case of two possible values of k, either the low types are excluded, or there is pooling and the high types are always better off pooling with the low types. There is no equilibrium in which $\overline{k}$ types separate, which concludes that there is no information provided on the vertical attribute.

There is one assumption which would allow to have a separation on the vertical value. This would be the case when the buyers do not know their horizontal taste parameter before they reveal their vertical valuation, and they realize it afterward. Then, it will be possible to have an equilibrium in which there is separation on the vertical dimension.

8 Conclusion

This paper studies how a buyer can strategically reveal her preferences to a seller who is not committed to a price. By introducing cheap talk communication in a buyer seller game, it provides a framework for bargaining over different options under private preferences. As communication occurs to the extent that the buyer is reveals her information, the equilibria that are focused on, and which are interesting, are the ones which maximize the ex-ante buyer surplus. It is shown that the buyer is better off withholding information as long as that trade happens with certainty. In the single offer game, the buyer optimal equilibrium partition is the coarsest one that ensures trade for all types of the buyer. When extended to several rounds, the buyer is better off restricting information revelation to the first period and sending babbling messages afterward. When the seller knows that future messages are informative he is less eager to trade today and prefers to ask for a higher price, as it is more attractive to delay trade when he knows that he will get more information tomorrow. As the uncertainty is not on the extent of information revelation but on the information itself, the buyer is better off resolving the uncertainty in the beginning. Finally, in this game, the gradual revelation of information is not profitable for the owner of information.

Both buyer and seller surpluses are lower in the dynamic case compared to the static one in case there is expected delay. However, when the discount factor is very low, a more informative strategy is played in order to avoid delay which then becomes very costly. In this case, the outcome turns out to be even more efficient

[14] when $2\sqrt{\frac{\overline{k}}{3}}$ is the partition rule, serving both types is better than serving only high types iff $\overline{k} \leq \frac{7\underline{k}}{5}$

than in the static game, as more information is revealed and trade happens without delay.

There are some possible extensions to the paper that would be of interest. One interesting case would be to look at a setting when the seller also has some private information. Another one would be to consider a case when the buyer has to first reveal her vertical type (high or low value buyer) before realizing her horizontal valuation, in which case it might be possible to find a truthful revelation by the buyer on her vertical type as well.

Appendix A

Proof of Lemma 1:

Proof. Suppose there exists a non-monotone partition such that there are θ_1 and θ_3 who send the same message m, and θ_2 sending a different message, m' and all types accepting the offer they receive. If $(y(m), \tau(m))$ is the offer induced by message m, then the following condition holds for θ_1 and θ_3:

$$U(\theta, y(m)) - \tau(m) \geq U(\theta, y(m')) - \tau(m') \tag{12}$$

and $U(\theta_3, y(m')) - \tau(m') \geq U(\theta_3 2, y(m)) - \tau(m)$. Assume WLOG that $y(m) > \theta_2$. Then we have $U(\theta_2, y(m)) > U(\theta_1, y(m))$, as U is decreasing in distance. Then $U(\theta_2, y(m_2)) - \tau(m_2) \geq U(\theta_2, y(m)) - \tau(m) > U(\theta_1, y(m)) - \tau(m)$. First, assume $y(m_2) > y(m)$. Then, $U(\theta_3, y(m_2)) > U(\theta_3, y(m))$. As $U(\theta_2, y(m_2)) - \tau(m_2) \geq U(\theta_2, y(m)) - \tau(m)$ and $U(\theta_2, y(m)) > U(\theta_2, y(m_2))$, we conclude $\tau(m_2) < \tau(m)$. Then it should be the case that $U(\theta_3, y(m_2)) - \tau(m_2) > U(\theta_3, y(m)) - \tau(m)$, which means θ_3 would in the beginning prefer to send the message m_2. Second, if $y(m_2) < y(m)$, then $U(\theta_1, y(m_2)) > U(\theta_1, y(m))$ and $U(\theta_2, y(m_2)) - \tau(m_2) > U(\theta_2, y(m)) - \tau(m)$ which results in $U(\theta_2, y(m_2)) - U(\theta_2, y(m)) > \tau(m_2) - \tau(m)$, and due to $U_{11} > 0$, $U(\theta_1, y(m_2)) - U(\theta_1, y(m)) > U(\theta_2, y(m_2)) - U(\theta_2, y(m)) > \tau(m_2) - \tau(m)$, which finally results in $U(\theta_1, y(m_2)) - \tau(m_2) > U(\theta_1, y(m)) - \tau(m)$. □

Proof of Lemma 6:

Proof. I already know by lemma 3 that the buyer's expected utility is increasing in the lengths of the intervals as long as there is no exclusion. Least number of intervals is equivalent to largest interval lengths, and x^* is the largest possible interval length subject to no exclusion. In case $\frac{1}{x^*}$ is not an integer, then there are $n-2$ intervals of size x^* and 2 consecutive intervals i and $i+1$ respectively of lengths x_1 and x_2 with $x_2, x_1 \leq x^*$ and $x_1 + x_2 = x > x^*$. The surplus of buyer types in these intervals, using the fact that the threshold types are at distance $\frac{x_i}{2}$, is:

$$2\int_0^{\frac{x_1}{2}} (f(\frac{x_1}{2}) - f(\theta))d\theta + 2\int_0^{\frac{x_2}{2}} (f(\frac{x_2}{2}) - f(\theta))d\theta \tag{13}$$

subject to:

$$x_1 + x_2 = x \tag{14}$$

the partial derivatives of 13 are: $f(\frac{x_1}{2})f'(\frac{x_1}{2})$ and $f(\frac{x_1}{2})f'(\frac{x_1}{2})$. As f is convex, then equation 13 is always increasing when one interval widens at the expense of the other. However, as no exclusion requires that $x_1, x_2 \leq x^*$, then optimal partition will have the last two intervals having length x^* and $x_1 + x_2 - x^*$. □

Proof of Lemma 6**:**

Proof. Let us look at a case when the seller excludes at period 1 a total measure c of types in an interval i, which can be a single or 2separated intervals. Let us assume that c consists of 2 separated intervals of lengths b_{i1} and b_{i2}, with $b_{i1} + b_{i2} = c$. Restrict attention to $b_{i1}, b_{i2} \leq x^*$ so all the types can be served with no subdivision. As we know that as long as $b_{i1}, b_{i2} \leq x^*$, the buyer optimal equilibrium suggests no subdivision. The seller's expected profit when t=2 is informative, so that the types in b_{i1} and b_{i2} send separate messages at $t = 2$ and both intervals are covered, will be:

$$b_{i1}(k - f(\frac{b_{i1}}{2})) + (c - b_{i1})(k - f(\frac{c - b_{i1}}{2}))$$

FOC wrt b_{i1}:

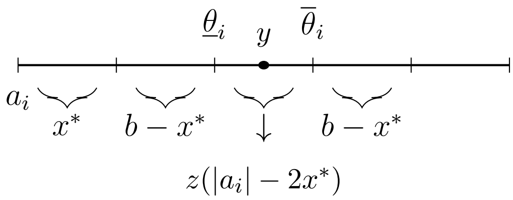
$$f(\frac{c - b_{i1}}{2}) - f(\frac{b_{i1}}{2}) + \frac{c - b_{i1}}{2} f'(\frac{c - b_{i1}}{2}) \frac{b_{i1}}{2} f'(\frac{b_{i1}}{2}) = 0$$

$c = 2b_{i1}$ satisfies the equality, so $b^*_{i1} = b^*_{i2} = \frac{c}{2}$. If the seller is going to exclude a total of c measure of types, it is weakly optimal that these types are found in two separated intervals of equal length when next period will be informative. $\square$

Proof of Lemma 7**:**

Proof. I restrict attention to the case when there is no informative messages at period 2. If $\underline{\theta}_i > a_i$ and $\overline{\theta}_i < a_{i+1}$, given that the game moves to period 2, the buyer may be in either of the 2 separated intervals: $\theta \in b_{i1} = (\overline{\theta}_i, a_{i+1})$ or $\theta \in b_{i2} = (a_i, \underline{\theta}_i)$ where $b_{i1} + b_{i2} = x_i \setminus z_i$. In case $\underline{\theta}_i = a_i$ or $\overline{\theta}_i = a_{i+1}$, the buyer is known to be in a single interval, $(\overline{\theta}, a_{i+1})$ or $(a_i, \underline{\theta}))$ where $\underline{\theta} - a_i = a_{i+1} - \overline{\theta} = x_i \setminus z_i$. First possibility is that $x_i \setminus z_i \leq x^*$. This means at period 2, the seller would serve the whole interval if they were found in a single interval by offering $(\frac{b_i}{2}, k - f(\frac{b_i}{2}))$, which gives him profits $k - f(\frac{b_i}{2})$. In case of 2 separated intervals, the seller either has to charge lower if he wants to include all types or has to exclude some types. If he wants to sell to all types, the profit is $(k - f(\frac{a_i + a_{i+1}}{2})) * (x_i \setminus z_i)$ otherwise some types are excluded which gives profits lower than $k - f(\frac{b_i}{2})$. Second, in case $x_i \setminus z_i > x^*$ and there is a single interval, the seller will charge $(k - f(\frac{x^*}{2}))$ and x^* measure of types will accept, giving him profits of $\frac{x^*}{x_i - z_i}(k - f(\frac{x^*}{2}))$ whereas when there are 2 separated intervals, he either chooses to sell to one of the intervals b_{i1} or b_{i2} which means he makes lower profits in case $max(b_{i1}, b_{i2}) < x^*$, or he has to offer a good located in $(\underline{\theta}_i, \overline{\theta}_i)$ and hence sell to $x < x^*$ measure of types. So he will always get weakly lower profits than when the types are found in a single interval. Hence, we conclude that the seller is weakly better off when the excluded types are found in a single interval. $\square$

Figure 4: interval i



Proof of Lemma 8:

Proof. First, I show that this strategy is optimal once $t = 2$ is reached. If the game moves to period 2, the surplus maximizing partition rule for the types in b_i is with intervals of x^* as in the one period game. So, this strategy is optimal once period 2 is reached.

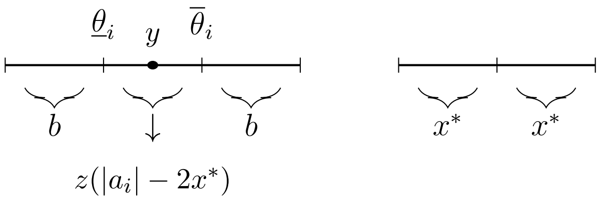
Next let us check whether this strategy is also optimal from an ex-ante point of view, by verifying whether the buyer would be better off under a more informative period 2 partition rule $x' < x^*$. When $b_i < x'$, this strategy is the same as the partition rule x^*, as there is no subdivision. In case $x' < b_i < 2x'$, b_i subdivides into 2 intervals of x' and $b_i - x' < x'$. As this increases the seller's period 2 profit, the only change in the seller's strategy would be charging higher and excluding more types at period 1. Plus in case the game moves to period 2, a more informative partition leads to lower expected utility for the buyer given that no type is excluded in either case, by lemma 3. Hence, the expected surplus of buyer types that are served in period 1 and also at period 2 are both lower, and the possibility of reaching period 2 is higher which also leads to lower surplus due to delay. So, more informative partitions do not improve the buyer surplus.

Finally, let us verify that committing to a less informative period 2 strategy is not profitable either. A less informative strategy means that at $t = 2$, if $b_i > x^*$, all the types pool in their messages. As the seller's offer covers at most x^* measure of types in a given interval, there is $b_i - x^* > 0$ of types for whom trade will not happen. Assume WLOG that these types are located on the left end of the initial interval. Then, by separating and sending a different message at period 2, they wouldn't affect the allocation of the x^* interval of types, plus the expected surplus of these types would be positive. $\square$

Proof of Lemma 9:

Proof. Figure 5 demonstrates 2 intervals i with a subdivision happening at period 2. Look at the intervals of measure x^*. When these two intervals are pooling with interval a_i, they are excluded at period 1 and get the period 2 offer, hence the

Figure 5: interval i without subdivision



expected surplus over these intervals is:

$$2\delta \int_0^{\frac{x}{2}} (f(\frac{x}{2}) - f(\theta))d\theta \tag{15}$$

Now, consider another partition where these intervals were found consecutively on the right side of interval i and separate from the interval i and adopt a strategy of pooling together in period 1 and babbling at period 2. By the seller's problem and the equation **??**, x^* would be served at period 1 and the other x^* at period 2. Then, the surplus over the interval $2x^*$ will be:

$$(1+\delta)2 \int_0^{\frac{x}{2}} (f(\frac{x}{2}) - f(\theta))d\theta \tag{16}$$

then, (16) > (15). From the seller's problem, lemma 5 and lemma **??**, we know that $z_i = z(x_i - 2x^*)$ which is not influenced by the change in the strategy of these two subintervals. In addition, all the types inside the intervals x^* are weakly better off as well: half of the types get the same offer but now at period 1 instead of period 2. Hence, this new partition which has 2 intervals of lengths $x_i - 2x^*$, $2x^*$ and dominates the previous partition. To conclude, any equilibrium partition with subdivision is dominated by another one which is more informative at period 1 and has no subdivision at period 2. $\square$

Proof of Lemma 10

Proof. This proof makes use of the quadratic utility $U^B = k - (\theta - y)^2$ where given an interval z, $\tau = k - \frac{z^2}{4}$ and buyer surplus $2\int_0^{\frac{z}{2}} (\frac{z^2}{4} - f(\theta))d\theta = \frac{1}{6}z^3$. Look at the surplus under partition rule a:

$$\frac{1}{6}\left[\left\langle \frac{1}{a} \right\rangle (z(a)^3 + 2\delta(\frac{a - z(a)}{2})^3) + z(a')^3 + 2\delta(\frac{a' - z(a')}{2})^3\right]$$

a^* is the interval length which maximizes the surplus over the interval $a^*(\left\langle \frac{1}{a^*} \right\rangle)$. Let us take the asymmetric interval of length $a' = 1 - a^* \left\langle \frac{1}{a^*} \right\rangle$ and the neigbour interval of length a^*, and look for an improvement. Call $a^* + a' = c$, and suppose that this whole interval is divided into two intervals of a_k and a_{k+1} with $z_k = z(a_k)$ and $z_{k+1} = z(c - a_k)$, where wlog $a_k > a_{k+1}$. The expected surplus of the types in these two intervals is:

$$\frac{1}{6}\left(z_(a_k)^3 + 2\delta(\frac{a_k - z(a_k)}{2})^3 + (z(c-a_k))^3 + 2\delta(\frac{c - a_k - z(c-a_k)}{2})^3\right)$$

maximizing wrt a_k and rearranging gives:

$$z'(a_k)\left[z(a_k)^2 - \frac{\delta(a_k - z(a_k))^2}{2}\right] - z'(c-a_k)\left[z(c-a_k)^2 - \frac{\delta(c - a_k - z(c-a_k))^2}{2}\right] + \frac{\delta}{2}[(a_k - z(a_k))^2 - (c - a_k - z(c-a_k))^2 = 0$$

First, as $a_k > c - a_k$, and $z(a)$ is an increasing and convex function, $z'(a_k) > z'(c - a_k)$. In addition, $z(a_k)^2 - \frac{\delta(a_k - z(a_k))^2}{2} > z(c-a_k)^2 - \frac{\delta(c-a_k-z(c-a_k))^2}{2}$. So, the first two expressions give a positive value. Finally, as $a_k - z(a_k) > c - a_k - z(c - a_k))$, the third expression is also positive. Hence, the whole expression is positive as long as $a_k > c - a_k$, which means that surplus is increasing when one interval is increased at the expense of the other, subject to the constraint that $a_k \leq a^*$. It concludes that there exists an optimal partition rule a^*. □

Proof of proposition 6

Proof. This is proven by characterizing all the candidate equilibria. By using lemma 8 and 9, I restrict attention to equilibria where trade happens for all types without a subdivision. This leaves the following candidate equilibria:

1. period 1 is informative and:
   - period 2 is informative
   - period 2 is babbling
2. period 1 is babbling and period 2 is informative.

This proof makes use of the quadratic utility $U^B = k - (\theta - y)^2$.

Period 1 informative

subcase 1: period 2 babbling

Restricted to the case where $a \leq 2x^*$, the condition for the market to be covered over the two periods given no subdivision at period 2. As x^* is the highest interval that the seller covers, then $2x^*$ is the upper bound on the period 1 partition. By using lemma 10, the partition rule that gives the highest surplus to the buyer when playing an informative strategy at period 1 and babbling at period 2 is:

$$a^* = \arg\max_{a_i} \sum_{i=1}^{n_i} z(a_i)^3 + \delta(a_i - z(a_i))^3 \tag{17}$$

subject to:

$$\sum_{i=1}^{n_i} a_i = 1 \tag{18}$$

the FOC wrt a_i gives:

$$\left[z(a_i)^2(3z'(a_i)a_i - z(a_i)) + \delta(a_i - z(a_i))^2(2a_i - 3a_i z'(a_i) + z(a_i))\right] = \lambda \tag{19}$$

First, a_i's should be identical among i as $z(a_i)^2(3z'(a_i)a_i - z(a_i)) + \delta(a_i - z(a_i))^2(2a_i - 3a_i z'(a_i) + z(a_i)) = \lambda$for all i. $z(a) = a$ as long as $a \leq a^{nd}$, so until the no-delay partition, $z'(a) = 1$. At $a = 2\sqrt{\frac{k(1-\delta)}{3}}$, $z'(a) = 0$, which means a^{nd} gives a local maximum. As long as $z(a) = a$, the surplus is increasing in a. First candidate is then the maximum value $z(a)$ can take such that $a = z(a)$, which is a^{nd}. Second, when $a > 2\sqrt{\frac{k(1-\delta)}{3}}$, $\frac{1}{2} < z'(a) < 1$ and $z''(a) > 0$, meaning $z(a)$ is increasing in a, and together with the result that the buyer surplus is increasing in the intervals, the second local maximum is when $z(a)$ takes its highest value, meaning $a = 2x^*$. The overall expected surplus of the buyer can be written as:

$$\frac{1}{6}a^2$$

which gives $\frac{2}{9}k(1-\delta)$ in the no-delay equilibrium and $\frac{k}{9}(1+\delta)$ when period 2 is reached with positive probability.

$$\frac{k}{9}(1+\delta) > \frac{2}{9}k(1-\delta)$$

if $\delta > \frac{1}{3}$, and for $\delta < \frac{1}{3}$ the no-delay equilibrium gives the highest surplus.

To conclude, the best partition equilibrium when only period 1 is informative is:

1. when $\delta > \frac{1}{3}$:
   - the partition rule $a^* = 2x^*$
   - half of the types in each interval accepting the offer at period 1: $z = x^*$
   - the other half refusing and moving on to period 2: $b = x^*$

2. when $\delta < \frac{1}{3}$

   - no-delay partition interval: $2\sqrt{\frac{(1-\delta)k}{3}}$

The seller's surplus is: $(k - \frac{x^{*2}}{4})(\frac{1+\delta}{2}) = \frac{k}{3}(1+\delta)$ when $\delta > \frac{1}{3}$ and $\frac{(2-\delta)k}{3}$ when $\delta > \frac{1}{3}$.

subcase 2: period 2 informative

In order for trade to be ensured without any subdivision, the partition intervals should satisfy $a \leq 3x^*$ (in period 1 at most x^* types can be served, and at period 2, there will be 2 separate intervals of x^*.) If $a > 3x^*$, then as $z \leq x^*$, $b = \frac{a-z}{2} > x^*$, so some types would be excluded in case of no subdivision. The equilibrium surplus of the buyer from a partition rule a when period 2 is informative is:

$$\frac{1}{a}(\frac{z(a)^3 + \delta\frac{(a-z)^3}{4}}{6}) \tag{20}$$

derivating wrt z:

$$\frac{3}{6a}z'(a)(z(a)^2 - \delta\frac{(a-z)^2}{4})$$

as $z'(a) > 0$, this expression is always positive as long as $z > \frac{a}{3}$. By replacing $z(a)$ from the seller's problem, the condition simplifies to:

$$96K > 0$$

hence, $z(a) > \frac{a}{3}$ is satisfied. Compared to the previous section, where the surplus is maximized when $z(a)$ takes its maximum value, the only difference here is that $(a - z(a))$ is multiplied by $\frac{1}{2}$. Also, $z'(a) > 0$ and $\frac{\partial^2 z}{\partial^2 a} = 8k[1-\delta][6-\frac{3}{2}\delta] > 0$. Then optimal z is given by the highest value of a under the constraint $a - z(a) \leq 2x^*$. Given that subdivision is suboptimal, the highest value a can take is $3x^*$, in which case $z(a) = x^*$ and $b = x^*$.

Then, the partition rule that gives the highest buyer surplus when both periods are informative is:

- $a^* = 3x^*$
- $z(a) = x^*$
- $b_1 = b_2 = x^*$

comparing case 1 and case 2:

Let us compare the buyer surplus, where given an initial partition interval a_i, z_i are the intervals served at t=1 and b_i the intervals served at $t = 2$. In the babbling case:

$$\sum_{i=1}^{n^b}({z_i}^3 + \delta {b_i}^3) \tag{21}$$

where $\sum_{i=1}^{n}(z_i + b_i) = 1$ In the informative case:

$$\sum_{i=1}^{n^i} {z_i}'^3 + 2\delta {b_i}'^3 \tag{22}$$

where $\sum_{i=1}^{n}({z_i}' + 2{b_i}') = 1$

Then I have $z_i = {z_i}' = x^*$ and $b_i = b_i' = x^*$, and $a^i = 3x^*$ and $a^{i\prime} = 2x^*$ which leads to $n^b = \frac{3}{2}n^i$. Replacing n^i and n^b in the surplus functions, it is trivial to conclude that the surplus in the babbling equilibrium is higher.

Babbling at period 1

All types send the same period 1 message, hence there is a single interval of length 1. At period 1, by lemma 6 the seller offers y, τ such that $z(1) = x^*$ interval of types accept. Between the equilibria where $t = 1$ is babbling, by using lemma 8 which says that trade should take place with certainty, the buyer optimal one is informative at t=2 and has the partition rule x^*. Let us show that this equilibrium is dominated. Assume that $y = \frac{1}{2}$ and that the types $\theta \in (0, 2x^*)$ are excluded at period 1 and in period 2 separate into intervals of x^*. The expected surplus of types in these two intervals is $\frac{1}{6}(2\delta x^{*3})$. Consider another equilibrium in which these types separate themselves at period 1, and babble at period 2. This doesn't affect the surplus of other types, due to lemma 5 and lemma **??**. From the seller's problem, this means one of the intervals $(0, x^*)$ or $(x^*, 2x^*)$ will be served at $t = 1$ and the other one at $t = 2$. The overall surplus of these types would be higher than before: $\frac{1}{6}x^{*3}(1 + \delta) > \frac{1}{6}2\delta x^{*3}$. In fact, the surplus of each buyer type is weakly higher, and for some it is strictly higher. Finally, playing babbling at period 1 cannot be the buyer-optimal strategy. Now we conclude that the surplus maximizing equilibrium strategy for the buyer over all candidate equilibria is the one where period 1 is informative and period 2 is babbling in case it is reached. □

Proof of proposition 7

Proof. The FOC of the seller gives the identical $\frac{z_t}{z_{t+1}}$ under the informative and babbling cases for $z_t, z_{t+1} > 0$. However, in the informative strategy the number of z_t's in each initial interval evolve by 2^{t-1} whereas there is single z_t at each t

in the babbling case. In addition, as z_t is decreasing in t, for the same interval a more types accept the good in early periods in the babbling case compared to the informative one. Take an informative equilibrium with $z_t > 0$ until period T. Then, consider a babbling equilibrium with the same z_t for all t which also lasts at most T periods. This means ${z_t}^b = {z_t}^i$ for all t where b denotes babbling, and i denotes informative equilibrium. Using the quadratic utility, the surplus from the informative equilibrium:

$$\frac{\sum_{t=1}^{T} \delta^{t-1} 2^{t-1} {z_t}^3}{6 \sum_{t=1}^{T} 2^{t-1} z_t}$$

subject to:

$$\sum_{t=1}^{T} 2^{t-1} z_t = a^i$$

where a^i is the first period interval length in the informative case. The FOC with respect to z_t gives $\frac{3\delta^{t-1}2^{2t-2}z_t^2 \sum z_t - \delta^{t-1}2^{2t-2}z_t^3}{(6\sum_{t=1}^{T} 2^{t-1} z_t)^2}$ which is positive and decreasing in t. The surplus from the babbling equilibrium:

$$\frac{\sum_{t=1}^{T} \delta^{t-1} {z_t}^3}{6 \sum_{t=1}^{T} z_t}$$

subject to:

$$\sum_{t=1}^{T} z_t = a^b$$

The FOC with respect to z_t gives $\frac{3\delta^{t-1}z_t^2 \sum z_t - \delta^{t-1}z_t^3}{(6\sum_{t=1}^{T} z_t)^2}$ which is also positive and decreasing in t. Hence the surplus would be higher when earlier z_t would be higher. Next I will show that the period 1 intervals a are finer in the babbling equilibrium: $\sum_{t=1}^{T} z_t < \sum_{t=1}^{T} 2^{t-1} z_t$, meaning $a^b < a^i$ and so $n^b = \frac{1}{a^b} > \frac{1}{a^i} = n^i$: there are more intervals at period 1 in babbling equilibrium than in the informative one. Now the surplus in the informative case:

$$\frac{n^i [\sum_{t=1}^{T} \delta^{t-1} 2^{t-1} {z_t}^3]}{6}$$

subject to:

$$n_i \sum_{t=1}^{T} 2^{t-1} z_t = 1$$

The surplus for the babbling case:

$$\frac{n^b [\sum_{t=1}^{T} \delta^{t-1} {z_t}^3]}{6}$$

subject to:

$$n_b \sum_{t=1}^{T} z_t = 1$$

where $n^i < n^b$. As z_t's fixed by the seller's problem are identical and decreasing in t, the surplus functions are maximized when z_1 gets the highest coefficient. If $\sum_{t=1}^{T} n^b z_t = \sum_{t=1}^{T} n^i 2^{t-1} z_t$, then there is t^* such that for $t < t^*$, $n^b > n^i 2^{t-1}$ and for $t > t^*$, $n^b < n^i 2^{t-1}$. As the surplus function is higher when the z_t's with smaller t have higher coefficients, I conclude that the surplus in babbling equilibrium is higher. So, in the babbling equilibrium a higher proportion of types are getting the earlier offers compared to the informative equilibrium at the same price, and there are more intervals in the period 1 partition meaning a more informative equilibrium. The proportion of types accepting offers in each period in the informative equilibrium to the babbling one is increasing in t. So, any informative equilibrium is dominated by a babbling equilibrium in terms of the expected buyer surplus. Finally, in the buyer optimal equilibria, the only informative messages are sent at period 1. $\square$

Appendix B

This section considers the case of quadratic utility.

Solving for the partition in the section 4

Following the proposition 3, I want to find the partition having the minimum number of intervals such that trade is ensured. Given a message sent by the interval i, the weakly optimal offer is $y_i = \frac{a_i + a_{i+1}}{2}$, [15] at a price τ such that $\theta \in (\underline{\theta}_i, \overline{\theta}_i)$ will accept the offer where the boundary types are located at equal distance from y, satisfying $\underline{\theta}_i - a_i = a_{i+1} - \overline{\theta}_i$. Using the indifference condition of the boundary types, $\tau = k - (\frac{a_i + a_{i+1}}{2} - \underline{\theta}^i)^2$. The excluded types are $\theta \in (a_i, \underline{\theta}_i)$ and $(\overline{\theta}_i, a_{i+1})$

seller's problem

I first solve the seller's problem, given the buyers's signaling rule. WLOG I look at a pooling interval $(0, x)$. When the seller chooses $y = \frac{x}{2}$, $\theta \in (\underline{\theta}, \overline{\theta})$ will be accepting the offer. The indifference condition of the threshold types is given by $k - (\underline{\theta} - \frac{x}{2})^2 - \tau = 0$. The total demand can be written as: $(x - 2\underline{\theta})$. The problem of the seller is then to choose $\underline{\theta}$ and τ:

[15] In case the whole interval will be covered, this allows him to charge the maximal price. If not, there would be other values of y which would give at most the same profits.

$$\underline{\theta}^* = \arg\max_{\underline{\theta}} (k - (\frac{x}{2} - \underline{\theta})^2)(x - 2\underline{\theta})$$

where the first part is the price and the second one is the probability of the buyer accepting the offer, in other words the demand. The FOC wrt $\underline{\theta}$ is:

$$\frac{3}{2}x^2 + 6\underline{\theta}^2 - 6x\underline{\theta} - 2k = 0$$

as $x \geq 2\underline{\theta}$, we know $6\underline{\theta}^2 - 6x\underline{\theta} < 0$. Then the solution becomes:

- if $\frac{3}{2}x^2 - 2k > 0$, then $\underline{\theta}^* = \frac{x}{2} - \sqrt{\frac{k}{3}}$
- otherwise, $\underline{\theta}^* = 0$, meaning that the whole interval is served.

Then, the highest x which guarantees trade, meaning $\underline{\theta} = 0$ so that no type is excluded, is:

$$x^* = 2\sqrt{\frac{k}{3}}$$

Next part shows that x^* is indeed the buyer's optimal partition partition rule.

buyer's problem

The surplus of types in a partition interval of measure x, taking into account $\underline{\theta}$ determined by the seller's offer, is:

$$\int_{\underline{\theta}}^{x-\underline{\theta}} \frac{[(\frac{x}{2} - \underline{\theta})^2 - (\theta - \frac{x}{2})^2]d\theta}{x}$$

There are 2 cases to consider: $\frac{x}{2} - \sqrt{\frac{k}{3}} \geq 0$ and $\frac{x}{2} - \sqrt{\frac{k}{3}} < 0$.

Case 1 : If $\frac{x}{2} - \sqrt{\frac{k}{3}} \geq 0$, then by the seller's problem, $\underline{\theta} = \frac{x}{2} - \sqrt{\frac{k}{3}}$. Replacing $\underline{\theta}$ in the buyer's surplus gives:

$$\frac{\int_{\frac{x}{2}-\sqrt{\frac{k}{3}}}^{\frac{x}{2}+\sqrt{\frac{k}{3}}} \left[\frac{k}{3} - z^2 + zx - \frac{x^2}{4}\right] dz}{x}$$

derivating this expression wrt x gives:

$$\frac{-53K\sqrt{k}}{9\sqrt{3}} < 0$$

which says that x should be as small as possible. However, as we have $\underline{\theta} = \frac{x}{2} - \sqrt{\frac{k}{3}}$ when $\theta \geq 0$, it should be that $\frac{x}{2} - \sqrt{\frac{k}{3}} \geq 0$. The smallest possible value x in this region is then:

$$x^* = 2\sqrt{\frac{k}{3}}$$

Next step is to check the interval $\frac{x}{2} - \sqrt{\frac{k}{3}} \leq 0$ which is when $\underline{\theta} = 0$.

Case 2 : Now we can replace $\underline{\theta} = 0$ in the buyer's surplus:

$$\frac{\int_0^x [\frac{x^2}{4} - z^2 + xz - \frac{x^2}{4}]dz}{x}$$

derivative wrt x gives $\frac{x}{3}$ which is always positive. Hence, the surplus increases in x up to $2\sqrt{\frac{k}{3}}$. This confirms the statement that the buyer gains from pooling as long as no type is excluded by the seller, meaning trade is ensured.

Finally the buyer-optimal partition rule is:

$$x^* = 2\sqrt{\frac{k}{3}} \tag{23}$$

As $\frac{1}{2\sqrt{\frac{k}{3}}}$ may not be an integer, following the proposition 3, the partition rule $x^* = 2\sqrt{\frac{k}{3}}$ will have:

- $\left\langle \frac{1}{2\sqrt{\frac{k}{3}}} \right\rangle$ intervals of length $2\sqrt{\frac{k}{3}}$
- if $(1 - \left\langle \frac{1}{2\sqrt{\frac{k}{3}}} \right\rangle (2\sqrt{\frac{k}{3}})) > 0$, a last interval of $1 - \left\langle \frac{1}{2\sqrt{\frac{k}{3}}} \right\rangle 2\sqrt{\frac{k}{3}}$.

The solution of seller's problem in Section ??

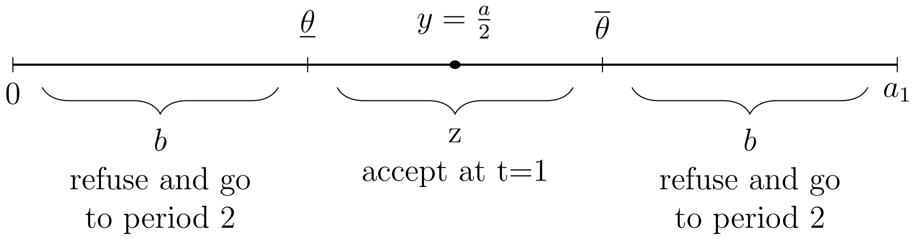
In a partition interval of a, $z(a)$ is the interval of types that accept the period 1 offer, and b the interval of types that reject, meaning $b = a \setminus z(a)$.

When period 2 is informative

Without subdivision

First we look at the case where the market is covered without a subdivision at $t = 2$, which requires $a \leq 2x^*$. Figure 6 demonstrates the interval a for this case.

Figure 6: interval a when $t = 2$ is informative



The seller's problem upon receiving a message from an interval a at period 1 and when period 2 will be informative:

$$z^*(a) = \arg\max_z z[k - (\frac{z}{2})^2] + \delta\frac{(a-z)}{2}[k - (\frac{a-z}{4}^2)] \tag{24}$$

where z is the interval that is served at period 1 and $\frac{a-z}{2}$ is each of the two separated intervals served at period 2. The whole interval a is served over the two periods in the buyer-optimal equilibrium, which means that the buyer's strategy is informative enough that trade is guaranteed. $z^*(a)$ is then found as:

$$z^*(a) = \frac{-3\delta a + 2\sqrt{8k[1-\delta][6-\frac{3}{2}\delta] + 9a^2\delta}}{12 - 3\delta} \tag{25}$$

for $a \geq 2\sqrt{\frac{k(1-\delta)}{3}}$. For $a \leq 2\sqrt{\frac{k(1-\delta)}{3}}$, $z^*(a) = a$ which means the game ends at period 1 and there will not be any delay.

The derivative $\frac{\partial z}{\partial a}$:

$$-3\delta + \frac{18a\delta}{\sqrt{8k[1-\delta][6-\frac{3}{2}\delta] + 9a^2\delta}}$$

which simplifies to:

$$3\delta(6a^2 - 8k[1-\delta]) \tag{26}$$

when $a \geq 2\sqrt{\frac{(1-\delta)k}{3}}$, which is the region where period 2 is reached with positive probability, $\frac{\partial z(a)}{\partial a} > 0$. In addition, $\frac{\partial^2 z}{\partial a} = 36\delta a > 0$ which makes z an increasing and convex function of a. Finally, each of the 2 excluded intervals, $b = \frac{a - z(a)}{2}$ is found as:

$$\frac{6a - \sqrt{8k[1-\delta][6-\frac{3}{2}\delta] + 9a^2\delta}}{12 - 3\delta}$$

The partial derivative $\frac{\partial b}{\partial a}$:

$$6 - \frac{9a\delta}{\sqrt{8k[1-\delta][6-\frac{3}{2}\delta] + 9a^2\delta}} > 0$$

as expected, $a - z(a)$ is also increasing in a. In addition, the condition $z \geq b$, meaning $z \geq \frac{a}{3}$ is satisfied as:

$$\sqrt{8k[1-\delta][6-\frac{3}{2}\delta] + 9a^2\delta} > a(2-\delta)^2$$

which simplifies in the end to:

$$8k[1-\delta][6-\frac{3}{2}\delta] > 4a^2 - 13\delta a^2 + \delta^2 a^2$$

and as $a \leq 3x^*$, replacing a with $x^* = 6\sqrt{\frac{k}{3}}$ gives:

$$96\delta k > 0$$

$b = \frac{a \backslash z(a)}{2}$, which is each excluded interval, is also increasing in a. $z(a) \geq b(a)$ and so $z(a) \geq \frac{a}{3}$

For $a \leq 3x^*$ trade takes place for all types of the buyer even without a subdivision. This threshold $3x^*$ is the highest value that the first period interval a can take such that $b \leq x^*$. Above this threshold, unless there is a subdivision among the types that reject the period 1 offer, trade will not happen with some types of the buyer.

With subdivision

In case $2x^* < a < 3x^*$, there can be a subdivision if the buyer plays an informative strategy at period 2. The x^* partition is played when period 2 is informative. In this case, when the pooling interval a is large enough, the seller makes an offer which will be accepted by $|z(a-2x^*)|$ measure of types and so each one of the two excluded intervals are $b = \frac{a-z(a-2x^*)}{2} > x^*$. Then, these types would subdivide at period 2. The interval a looks as in figure 7.

The profit of the seller from an interval a is now given as:

$$z(a-2x^*)\left[k - (\frac{z(a-2x^*)}{2})^2\right] + 2\delta(a-2x^*-z(a-2x^*))\left[k - (\frac{a-2x^*-z(a-2x^*)}{2})^2\right] + 2\delta x^*[k - (\frac{x^*}{2})^2] \quad (27)$$

Figure 7: subdivision

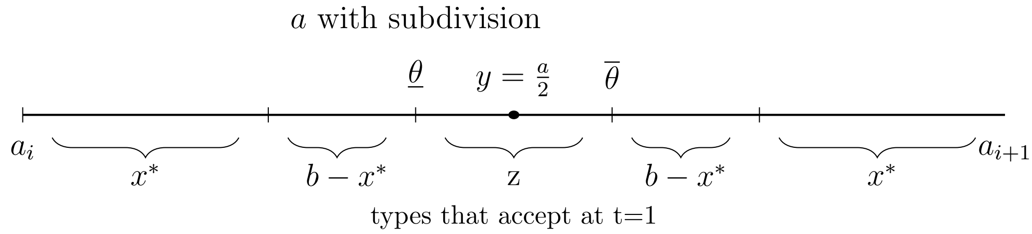
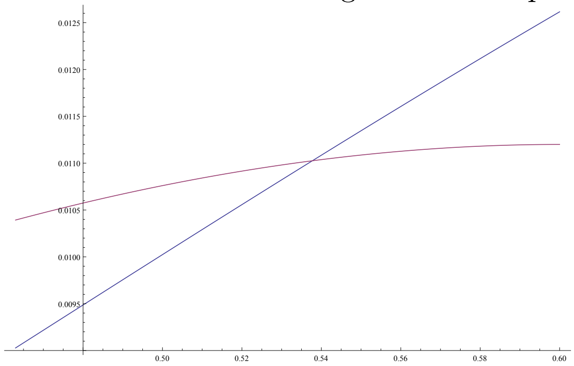


Figure 8: seller profits under 2 cases



where $x^* = 2\sqrt{\frac{k}{3}}$.

Indeed, there is a threshold a^* such that, for $a > a^*$, the seller is better off when subdivision happens, and for $a \leq a^*$, he would choose the offer such that there will be no subdivision given by equation 25 in the previous section, where the threshold a^* is $2x^* < a^* < 3x^*$.

Example:

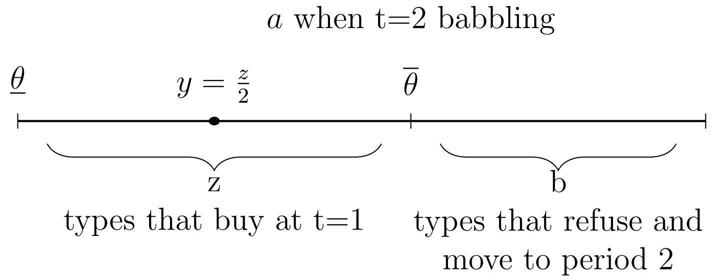
The figure 8 shows the expected surplus of the seller with subddivision and without subdivision, when $k = 0.03$, $\delta = 0.9$ and $x^* = 0.2$. The blue line represents the profits under subdivision, and the pink line under no subdivision. It is seen that for $a < 0.538$, the seller's profit is higher under no subdivision meaning he serves $z(a)$, whereas for $a > 0.538$, he is better off under subdivision: serving $z(a - 2x^*) = z(a - 0.4)$ at period 1 and excluding $a - z(a - 0.4)$ types. The values are $a = 0.538$, $z = 0.18$ and $b = 0.178581$, which confirm our findings.

When period 2 will be babbling

Without subdivision

When period 2 will be babbling, by lemma 7 the types excluded at period 1 should be found in a single interval. The condition for trade to happen for all types over

Figure 9: a when $t = 2$ is babbling



the 2 periods without a subdivision is $a \leq 2x^*$. Figure 9 demonstrates this case.

The problem of the seller is then:

$$z^*(a) = \arg\max_z z[k - (\frac{z}{2})^2] + \delta(a - z)[k - (\frac{a - z}{2})^2] \tag{28}$$

derivating wrt z gives:

$$z^*(a) = \frac{-\delta a + \sqrt{\delta a^2 + \frac{4}{3}(1-\delta)^2 k}}{1 - \delta} \tag{29}$$

for $a \geq 2\sqrt{\frac{k}{3}}$ meaning above the no-delay partition rule. For $a \leq 2\sqrt{\frac{k}{3}}$, $z(a) = a$ and it doesn't matter whether period 2 is informative or not. The no-delay partition rule is independant of period 2 information structure because when the seller decides whether to exclude the marginal type, it does not matter whether next period will be informative or babbling.

$b = \frac{a - z(a)}{2}$ is then:

$$\frac{a - \sqrt{\delta a^2 + \frac{4}{3}(1-\delta)^2 k}}{1 - \delta}$$

Let us show that z for all $a \leq 2x^* = 4\sqrt{\frac{k}{3}}$:

$$\frac{2\sqrt{\delta a^2 + \frac{4}{3}(1-\delta)^2 k} - a(1+\delta)}{1 - \delta} > 0$$

simplifying this condition:

$$\frac{16}{3}k(1-\delta)^2 > a^2[1 + \delta^2 - 2\delta]$$

As $[1 + \delta^2 - 2\delta] < 1$ and $a \leq 4\sqrt{\frac{k(1-\delta)}{3}} = 2x^*$, the condition is satisfied.

$z'(a) = -\delta + \frac{\delta a}{\sqrt{\delta a^2 + \frac{4}{3}(1-\delta)^2 k}} > 0$ and $z''(a) = \delta \left[\sqrt{\delta a^2 + \frac{4}{3}(1-\delta)^2 k} - \frac{\delta a^2}{\sqrt{\delta a^2 + \frac{4}{3}(1-\delta)^2 k}} \right] > 0$

$\frac{\partial z}{\partial a} > 0$ and $\frac{\partial^2 z}{\partial a} > 0$, meaning z is an increasing and convex function of a. Also, $z(a) \geq b(a)$ and so $z(a) \geq \frac{a}{2}$

With subdivision

In case $a > 2x^*$, then the seller has a choice of including only $z(a - x^*)$ at t=1 if a subdivision will be happening at period 2. However, remember that it is in the babbling equilibrium. Here babbling equilibrium means that the interval of length $b - x^*$ on the left would pool together with the $b - x^*$ on the right, and x^* on the left would pool with the interval x^* on the right. However, we know that when next period is babbling, the excluded types are found in a single separated interval. The interval a given period 1 offer looks as in the figure.

Then the surplus of the seller will be:

$$z(a-x^*)\left[k - (\frac{z[a-x^*]}{2})^2\right] + \delta(a - z(a-x^*))\left[k - (\frac{a - z(a-x^*)}{2})^2\right] + \delta x^* \left[k - (\frac{x^*}{2})^2\right] \tag{30}$$

As we show that subdivision is under optimal, this case is not going to occur in equilibrium.

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