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« Consumption, Risk and Prioritarianism »

Matthew D. Adler and Nicolas Treich

Toulouse School of Economics

Consumption, Risk and Prioritarianism

Matthew D. Adler
Duke Law School

Nicolas Treich*
Toulouse School of Economics (LERNA-INRA)

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Abstract

Most economic problems combining risk and equity have been studied under utilitarianism. As an alternative, we study consumption decisions under risk assuming a prioritarian social welfare function. Under a standard assumption about the utility function (i.e., decreasing absolute risk aversion), there is always *more* current consumption under ex ante prioritarianism than under utilitarianism. Thus, a concern for equity (in the ex ante prioritarian sense) means less concern for the risky future. In contrast, under standard utility and social welfare functions, there is *less* current consumption under ex post prioritarianism than under utilitarianism.

Key words: Precautionary savings, utilitarianism, prioritarianism, discounting, climate change.
JEL: D81, I31, E21

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1 Introduction

In this paper, we examine the implications for consumption under risk of using a prioritarian social welfare function (hereafter SWF) of the form

$$\sum_t g(u(c_t)), \tag{1}$$

where $u(c_t)$ is the utility function of consumption c_t in period t, and where $g(.)$ is strictly increasing and strictly concave. More precisely, we derive interpretable conditions so that there is more or less consumption under prioritarianism compared to utilitarianism (i.e., the case where g is linear).

The concept of prioritarianism has its root in contemporary philosophy (Parfit 1991, Nagel 1995). Essentially, prioritarianism means that one must give priority to the less well off. While a utilitarian social planner maximizes the sum of utilities and is thus indifferent to the distribution of utilities, a prioritarian social planner maximizes the sum of a strictly concave transformation $g(.)$ of utilities, and thus gives greater priority to welfare changes affecting relatively worse-off individuals.[1] In economics, the form (1) has been extensively used in social choice and in the optimal taxation literature (Sen 1970, Lambert 1983, Kaplow 2008). It is also sometimes used in policy evaluation when "distributional weights" capture the nonlinearity in u and in g (Johansson-Stenman 2005, Adler 2013).

The use of a prioritarian SWF has an interesting moral dimension for the choice between risky prospects. Indeed, one can distinguish between ex ante and ex post prioritarianism. The ex ante prioritarian decision maker maximizes the sum of transformed expected utilities, while the ex post prioritarian decision maker maximizes the expectation of the sum of transformed utilities. As a result, the ex post prioritarian decision maker cares about the difference in utilities ex post, once the risk is resolved. In contrast, the ex

[1] In particular, unlike utilitarianism, prioritarianism leads to a strict preference for a mean-preserving contraction of utilities. In other words, a prioritarian SWF satisfies the Pigou-Dalton axiom: a non-leaky, non-rank-switching transfer of utility from someone at a higher utility level to someone at a lower level should be seen as an improvement. The other axioms leading to (1) are Pareto, anonymity, continuity and separability. See Adler (2012) for an extensive discussion of prioritarianism. Note that there exists an axiomatic foundation to prioritarianism based on an extension of Harsanyi's utilitarian impartial observer theorem and coined "generalized utilitarianism" (Grant et al. 2010). For a criticism of prioritarianism, see for instance Harsanyi (1975).

ante prioritarian decision maker cares about the difference in expected utilities ex ante, before the risk is resolved. While the choice between the ex ante and ex post criteria and its moral implications have been discussed extensively in the literature (Diamond 1967, Broome 1984, Fleurbaey 2010, Adler 2012, Fleurbaey and Bovens 2012), its economic and policy implications have not been thoroughly examined.[2]

Our results are relevant for the debate about climate change policy. One important issue in this debate concerns the intergenerational equity dimension. Another important issue concerns the risk dimension. Very often however, these issues are treated independently in the climate change literature. And when treated simultaneously, a utilitarian SWF has usually been assumed both in the theoretical and empirical literature (Stern 2007, Dasgupta 2008, Nordhaus 2008, Weitzman 2008, Gollier 2012).[3]

In this paper, we want to stress the importance and the richness of the ex ante/ex post prioritarian approach for economic problems that combine risk and equity dimensions. To do so, we consider a simple consumption model, often known as the cake eating problem. The model can be interpreted as follows. A decision maker must split a cake among different agents who come sequentially (e.g., among different generations). Under certainty, the problem is trivial, and the cake is equally shared (because the agents are identical). But the problem is that the size of the cake is unknown. If the decision maker is prioritarian rather than utilitarian, should he give more or less of the cake to the first agent, given that the remaining portion of the cake is unknown?

The main result of the paper is that the answer is opposite depending on whether the decision maker uses an ex ante or an ex post prioritarian approach. Under standard assumptions on the form of the utility function $u(.)$ and on the prioritarian SWF transformation $g(.)$, the decision maker should always give *more* to the first agent under ex ante prioritarianism than under utilitarianism, but always *less* under ex post prioritarianism. We also show that this result is robust to a situation in which the decision maker may learn the size of the cake after the first decision has been made.

[2] Exceptions include Ulph (1982), Adler, Hammitt and Treich (2012) and Fleurbaey and Bovens (2012), all in the context of mortality risk policies.

[3] Some exceptions include Johansson-Stenman (2000), Ha Duong and Treich (2004), Bommier and Zuber (2008), Roemer (2008), Traeger (2012), Dietz and Asheim (2012) and Fleurbaey and Zuber (2013).

2 A simple consumption model

We consider two periods, and assume that the utility function u is identical across the two periods, with u strictly increasing, strictly concave and thrice differentiable. Under utilitarianism, the optimal consumption in the first period, denoted c^U, is defined by

$$c^U = \arg\max_c u(c) + Eu(\widetilde{w} - c),$$

where $\widetilde{w}$ is a random variable representing risk over the size of the "cake" (i.e., for instance over wealth, the stock of an exhaustible resource, or the greenhouse gases total emissions target).[4] We assume that $\widetilde{w}$ has a lower bound $w_{\inf} > 0$. Since utility is strictly increasing, we have directly introduced into the optimization program the fact that the cake will be fully consumed. The first order condition (hereafter FOC) of this program gives[5]

$$u'(c^U) - Eu'(\widetilde{w} - c^U) = 0. \tag{2}$$

Note that the assumption $u'(0) = +\infty$ is sufficient to ensure that it is never optimal to run the risk of consuming entirely the cake before the final period. Also, note that we have assumed implicitly that the remaining cake, $\widetilde{w} - c$, is not productive. Moreover, recall that the utility is the same in both periods, and observe that there is no discounting.[6] These assumptions are made for simplicity, and will play no role in the Propositions until Section 6.

It is well known from the precautionary savings literature that consumption is reduced under risk, i.e. $c^U \leq \frac{E\widetilde{w}}{2}$, if and only if (hereafter iff) the decision maker is "prudent" $u''' \geq 0$ (Leland 1968, Kimball 1990).[7] Indeed, under prudence, the marginal utility of wealth is higher under risk, and thus it makes sense to transfer more wealth into the future when the risk will be

[4] A similar basic two-period consumption model has often been used in the literature on climate change to illustrate the effect of growth risk on discounting (Weitzman 2009, Gollier 2012, Millner 2013).

[5] Second order conditions will be satisfied throughout the paper, and thus will not be discussed.

[6] Thus, the only source of heterogeneity across the two periods comes from the risk over the future that makes future consumption risky. Note also that assuming no discounting throughout will ensure that the utilitarian and prioritarian SWFs respect anonymity.

[7] Technically, the result holds iff $Eu'(\widetilde{w} - c) \geq u'(E\widetilde{w} - c)$ for all $\widetilde{w}$, namely iff marginal utility is convex by Jensen inequality.

faced. Note that the restriction $u''' > 0$ is necessary for the common decreasing absolute risk aversion (hereafter DARA) hypothesis, and is usually accepted in the risk theory literature (Gollier 2001).

3 Ex ante prioritarianism

Under ex ante prioritarianism (hereafter EAP), optimal consumption is defined by

$$c^{EAP} = \arg\max_c g(u(c)) + g(Eu(\widetilde{w} - c)),$$

where g is strictly increasing, strictly concave and thrice differentiable. Note that the decision maker maximizes the sum of transformed expected utilities, consistent with an ex ante approach. The optimal level of consumption is characterized by the following FOC:

$$f(c^{EAP}) \equiv g'(u(c^{EAP}))u'(c^{EAP}) - g'(Eu(\widetilde{w} - c^{EAP}))Eu'(\widetilde{w} - c^{EAP}) = 0. \quad (3)$$

There is more consumption under EAP than under utilitarianism, i.e. $c^{EAP} \geq c^U$, or equivalently $f(c^U) \geq 0$, which by using (2) holds iff

$$u(c^U) \leq Eu(\widetilde{w} - c^U).$$

This leads to the following result.

Proposition 1 *There is more consumption under EAP than under utilitarianism iff u is DARA.*

Proof: Under DARA, $-u'$ is more risk averse than u. Namely, we have $u = \phi(-u')$ with ϕ strictly increasing and convex. This leads to

$$\begin{aligned} Eu(\widetilde{w} - c^U) &= E\phi(-u'(\widetilde{w} - c^U)) \\ &\geq \phi(-Eu'(\widetilde{w} - c^U)) \\ &= \phi(-u'(c^U)) \\ &= u(c^U), \end{aligned}$$

which proves the inequality above. We now show the necessity. If u is not DARA, then ϕ is locally concave, and the above inequality can be reversed for a well chosen $\widetilde{w}$. Therefore, consumption under EAP can then be lower than under utilitarianism. Q.E.D

The intuition is that, assuming utilitarianism, under DARA (and thus under prudence) the reduction in current consumption due to risk implies that the future expected utility is higher than the current utility. This in turn gives an incentive to increase consumption in the first period under prioritarianism. This result shows that under a standard assumption on the utility function, prioritarianism leads to more, and not less, current consumption.[8] In other words, this result indicates that a concern for equity (in the EAP sense) means less concern for the risky future. If one views ex ante fairness as socially desirable (Diamond 1967, Epstein and Segal 1991), our model perhaps illustrates a surprising implication of this view.

Note also that under constant absolute risk aversion (hereafter CARA), we have $u(c^U) = Eu(\widetilde{w} - c^U)$ leading to $c^U = c^{EAP}$ and thus to $u(c^{EAP}) = Eu(\widetilde{w} - c^{EAP})$. Namely, under CARA, the (expected) utilities are equal across the two periods both under utilitarianism and under prioritarianism.

4 Ex post prioritarianism

Under ex post prioritarianism (hereafter EPP), optimal consumption is defined by

$$c^{EPP} = \arg\max_{c} g(u(c)) + Eg(u(\widetilde{w} - c)).$$

Note that the decision maker now maximizes the expectation of transformed utilities, consistent with an ex post approach. The FOC is given by

$$k(c^{EPP}) \equiv g'(u(c^{EPP}))u'(c^{EPP}) - Eg'(u(\widetilde{w} - c^{EPP}))u'(\widetilde{w} - c^{EPP}) = 0. \quad (4)$$

Similar as before, there is less consumption under EPP than under EAP iff $k(c^{EAP}) \leq 0$. In the following Proposition, we derive a sufficient condition for this inequality.

Proposition 2 *There is less consumption under EPP than under EAP when $g''' \geq 0$.*

[8] Since there is more consumption under EAP than under utilitarianism, one may wonder whether it is possible that there is more consumption under EAP than under certainty (under either utilitarianism or prioritarianism). It is straightforward to show that this is never the case under $u''' \geq 0$, and that there is thus always precautionary savings.

Proof: Let $\widetilde{w} - c^{EAP} \equiv \widetilde{z}$. Then observe that $k(c^{EAP}) \leq 0$ iff

$$g'(Eu(\widetilde{z}))Eu'(\widetilde{z}) \leq Eg'(u(\widetilde{z}))u'(\widetilde{z}).$$

Observe now that $Eg'(u(\widetilde{z}))u'(\widetilde{z}) = Eg'(u(\widetilde{z}))Eu'(\widetilde{z}) + Cov(g'(u(\widetilde{z})), u'(\widetilde{z}))$, and since $g'(u(z))$ and $u'(z)$ are both decreasing in z, the covariance term is positive. Therefore the result holds if $g'(Eu(\widetilde{z})) \leq Eg'(u(\widetilde{z}))$, which is the case iff $g''' \geq 0$ by Jensen inequality. Q.E.D.

Is the restriction $g''' \geq 0$ plausible? At least, this restriction is technically more plausible than the opposite $g''' < 0$. In fact, it can be shown that if $g'(u) > 0$ and $g''(u) < 0$ and if $g'''(u)$ has the same sign for all $u > 0$, then it must be that $g'''(u) > 0$ (Menegatti 2001). Indeed a positive, decreasing and concave g' would have to cross the origin at some point (thus contradicting $g' > 0$), as illustrated in Figure 1. Note that the standard Atkinsonian function, i.e. $g(u) = (1-m)^{-1}u^{1-m}$ with $u > 0$ and $m > 0$ (and $g(u) = \log u$ for $m = 1$), always displays $g''' \geq 0$. An alternative is the negative exponential function, i.e. $g(u) = -e^{-u}$, which also displays $g''' \geq 0$.

< INSERT FIGURE 1 >

Thus under commonly used SWFs, there is less consumption under EPP than under EAP. The next objective is to examine whether there could be less consumption under EPP than under utilitarianism. We know the answer in the CARA case. Indeed, under u CARA, we have seen that consumption under EAP is equal to that under utilitarianism. Therefore Proposition 2 indicates that consumption under EPP is also lower than under utilitarianism under CARA when $g''' \geq 0$. But we would want to sign the comparison in the general case. The answer is given in the following Proposition.

Proposition 3 *There is less consumption under EPP than under utilitarianism iff*

$$\frac{u'''(w)}{-u''(w)} \leq 3\frac{-u''(w)}{u'(w)} + \{\frac{g'''(u(w))}{-g''(u(w))}\}u'(w). \tag{5}$$

Proof: Let us define $v(.) = g(u(.))$. We want to examine under which conditions we have: $u'(c^U) - Eu'(\widetilde{w} - c^U) = 0$ implies $v'(c^U) - Ev'(\widetilde{w} - c^U) \leq 0$.

Now let $-v'(.) = \varphi(-u'(.))$ with φ strictly increasing and concave. Then

$$\begin{aligned} -Ev'(\widetilde{w} - c^U) &= E\varphi(-u'(\widetilde{w} - c^U)) \\ &\leq \varphi(-Eu'(\widetilde{w} - c^U)) \\ &= \varphi(-u'(c^U)) \\ &= -v'(c^U). \end{aligned}$$

Conversely, if φ is locally convex, then it is possible to find a well chosen $\widetilde{w}$ so that the inequality above is reversed. Therefore the necessary and sufficient condition is that $-v'$ is more concave than $-u'$, or $\frac{v'''}{-v''} \geq \frac{u'''}{-u''}$ given that v is itself more concave than u. This condition is provided in the theorem 3.4 in Eeckhoudt and Schlesinger (1994), which yields (5). Q.E.D

As shown in Table 1, our analysis so far has permitted to compare current consumption levels under utilitarianism, EAP and EPP. Note that the comparison is unambiguous under $g''' \geq 0$, except for the pair (c^U, c^{EPP}) which depends on a complex condition (5).

u CARA	u DARA
$c^{EAP} = c^U \geq c^{EPP}$ under $g''' \geq 0$	$c^{EAP} \geq c^U$, $c^{EAP} \geq c^{EPP}$ under $g''' \geq 0$, $c^U \geq c^{EPP}$ iff (5)

Table 1: Consumption levels under utilitarianism, EAP and EPP.

Why is condition (5) so complex? Denoting $v(.) = g(u(.))$, the proof shows that the comparison between EPP and utilitarianism depends on how a change in preference from u to v affects precautionary savings. More precisely, it depends on whether more risk aversion, i.e. $\frac{-u''}{u'} \leq \frac{-v''}{v'}$, leads to more prudence, i.e. $\frac{u'''}{-u''} \leq \frac{v'''}{-v''}$. This last implication explains why condition (5) involves the third derivatives of both u and g as one needs to compute v'''. Although it sounds intuitive that a more risk averse agent should be more prudent, this is not always the case (Eeckhoudt and Schlesinger 1994). For instance, one may change the degree of risk aversion of a quadratic utility function without affecting the degree of prudence.[9]

Nevertheless, one can show that this condition is satisfied for most commonly used utility and social welfare functions. First, in the CARA case, we

[9] Moreover, here is an example where the condition (5) does not hold for some wealth levels. Take $g(u) = -e^{-u}$ and $u(w) = (1-\gamma)^{-1}w^{1-\gamma}$, then the condition is violated iff wealth is below $\widehat{w} = (1-2\gamma)^{\frac{1}{1-\gamma}}$.

have $\frac{u'''(w)}{-u''(w)} = \frac{-u''(w)}{u'(w)}$, and the condition (5) is always satisfied under $g''' \geq 0$, consistent with what we said above. Under the most common Atkinsonian SWF, $g(u) = (1-m)^{-1}u^{1-m}$ with $u > 0$ and $m > 0$, the inequality (5) reduces to

$$\frac{u'''(w)}{-u''(w)} \leq 3\frac{-u''(w)}{u'(w)} + (1+m)\frac{u'(w)}{u(w)}.$$

Interestingly, this last inequality exhibits three different utility curvature coefficients, namely the familiar degrees of risk aversion and of prudence, as well as the reciprocal of the degree of fear of ruin $\frac{u}{u'}$ (Foncel and Treich 2005). This also shows that under an Atkinsonian SWF the comparative statics analysis would not be affected by ratio-rescaling the utility function, namely by replacing u by au with $a > 0$; indeed, none of the curvature coefficients would be affected.[10] Moreover, take for instance a constant relative risk aversion (CRRA) utility function $u(w) = (1-\gamma)^{-1}w^{1-\gamma}$ with $\gamma \in (0,1)$; then the condition (5) is equivalent to $\frac{m(1-\gamma)+\gamma}{w} \geq 0$, and is always satisfied under our parametric assumptions.

Observe finally that the previous inequality is more likely to be satisfied when the "inequity aversion" parameter m increases. At the limit when m tends to infinity, i.e. for a Rawlsian-type SWF, the inequality (5) is always satisfied. This observation also provides an intuition for the result. Indeed, under EPP and a Rawlsian-type SWF, the decision maker's objective is to increase consumption in the worst state ex post (i.e., when $\widetilde{w} = w_{\text{inf}}$), as soon as the utility reached in that state is not higher than current utility. He thus essentially chooses consumption such that $u(c) \approx u(w_{\text{inf}} - c)$. This tends to yield less current consumption than under utilitarianism, given by $u'(c) = Eu'(\widetilde{w} - c)$, and to even less current consumption than under EAP (under a Rawlsian-type SWF), given by $u(c) \approx Eu(\widetilde{w} - c)$.

[10] This is not surprising since the Atkinsonian function is the only prioritarian SWF to display the ratio-rescaling invariance property (Bossert and Weymark 2004, Adler 2012). Namely, the ranking of prospects is unaffected by ratio-rescaling the utility function. Along similar lines, under a negative exponential SWF, i.e. $g(u) = -e^{-u}$, the ranking of prospects would not be affected by a change from u to $u+b$, and we can easily see that the inequality (5) would remain unaffected by such a change either.

5 A multi-periodic model

In this section, we briefly explore what happens in a specific multi-periodic model. Suppose first that we add only one period. Then the objective under utilitarianism becomes

$$\max_{c_1,c_2} u(c_1) + u(c_2) + Eu(\widetilde{w} - c_1 - c_2).$$

Note that perfect smoothing is optimal in the early periods $c_1 = c_2 = c$. The problem of finding optimal current consumption then becomes

$$c^U = \arg\max_c 2u(c) + Eu(\widetilde{w} - 2c). \tag{6}$$

Similarly, optimal consumptions under EAP and EPP are defined by

$$\begin{aligned} c^{EAP} &= \arg\max_c 2g(u(c)) + g(Eu(\widetilde{w} - 2c)), && (7)\\ c^{EPP} &= \arg\max_c 2g(u(c)) + Eg(u(\widetilde{w} - 2c)). && (8) \end{aligned}$$

It is easy to see then that this leads to similar comparative statics results as before. Considering more (than 3) periods would not change this result provided that perfect smoothing remains optimal in the early periods. However, the situation becomes more complex if learning is allowed, as we now show.

6 Learning

Learning is an important factor affecting risk policies (see for example the literature on climate change policy; Ulph and Ulph 1997 and Gollier, Jullien and Treich 2000). Unlike in the previous section, we now assume perfect learning between periods 2 and 3. That is, in period 2, the realization of $\widetilde{w}$ is known. Then the period 2 problem is made under certainty, and perfect smoothing is optimal in the future either under prioritarianism or under utilitarianism. Hence, viewed from the first period, optimal future (risky) consumption equals

$$c_2^* = \frac{\widetilde{w} - c_1}{2}.$$

Using obvious notations, the optimal consumption in the first period for utilitarianism and EPP are then defined as follows (the EAP case is treated later)

$$\begin{aligned} c^{UL} &= \arg\max_c u(c) + 2Eu(\frac{\widetilde{w}-c}{2}), \\ c^{EPPL} &= \arg\max_c g(u(c)) + 2Eg(u(\frac{\widetilde{w}-c}{2})). \end{aligned}$$

Note that the effect of learning under utilitarianism, and under prioritarianism, is given by comparing c^{UL} to c^U in (6) and c^{EPPL} to c^{EPP} in (8). It is not very difficult to show that learning usually increases consumption under utilitarianism and EPP compared to the no learning (i.e., "risk") case.[11] The intuition is that learning allows to better smooth consumption in the future, which thus increases future expected utility. As a result, there is more early consumption under learning because there is less need to worry about the future (Epstein 1980, Eeckhoudt, Gollier and Treich 2005).

Assuming learning, we now want to compare consumption under utilitarianism and EPP, i.e. c^{UL} and c^{EPPL}. This amounts to compare precautionary savings under $u(.)$ and $v(.) = g(u(.))$, and this comparison is direct from previous Proposition 3.

Proposition 4 *Under learning, there is less consumption under EPP than under utilitarianism iff (5) holds.*

The case of EAP is more difficult. Indeed, viewed from the first period, future utility equals $u(\frac{\widetilde{w}-c}{2})$ and is risky, which matters under EAP. Optimal consumption is given by

$$c^{EAPL} = \arg\max_c g(u(c)) + 2g(Eu(\frac{\widetilde{w}-c}{2})). \tag{9}$$

[11] Let us first compare consumption under learning and under risk assuming utilitarianism. Under learning, the FOC is given by $u'(c) - Eu'(\frac{\widetilde{w}-c}{2}) = 0$, while under risk it is given by $u'(c) - Eu'(\widetilde{w} - 2c) = 0$. Thus there is more current consumption under learning iff $Eu'(\widetilde{w} - 2c) \geq Eu'(\frac{\widetilde{w}-c}{2})$ given that $u'(c) - Eu'(\widetilde{w} - 2c) = 0$. Observe now $Eu'(\widetilde{w} - 2c) = \frac{1}{2}u'(c) + \frac{1}{2}Eu'(\widetilde{w} - 2c) \geq Eu'(\frac{\widetilde{w}-c}{2})$ by Jensen inequality and $u''' \geq 0$. This leads to the result that under prudence learning increases early consumption under utilitarianism. Note then that the role of learning under EPP is similar by just replacing u by $v = g(u)$ in the previous reasoning. But then observe that $g''' \geq 0$ ensures $v''' \geq 0$ so the result also carries over under $u''' \geq 0$ and $g''' \geq 0$. Note finally that both results rely on our assumptions that utility is homogenous, and that there is no discounting (which ensures anonymity).

The problem here is that the EAP criterion is time-inconsistent (Broome 1984, Adler and Sanchirico 2006). This relates to the fact that the intertemporal utility function in (9) is not linear in probabilities (Hammond 1983, Epstein and Le Breton 1992).[12] Indeed, unlike under utilitarianism or EPP, the optimization problem over the three periods cannot be formulated recursively under EAP.

Observe now that comparing, under learning, consumption under utilitarianism and under (time-inconsistent) EAP is similar as comparing those criteria under no learning. DARA is, again, the instrumental condition on the utility function that drives the analysis.

Proposition 5 *Under learning, there is more consumption under EAP than under utilitarianism iff u is DARA.*

Proof: We want to compare c^{EAPL} and c^{UL}. Respective FOCs equal $g'(u(c))u'(c) - g'(Eu(\frac{\widetilde{w}-c}{2}))Eu'(\frac{\widetilde{w}-c}{2}) = 0$ and $u'(c) - Eu'(\frac{\widetilde{w}-c}{2}) = 0$. Therefore we are done if we can show $u(c) \leq Eu(\frac{\widetilde{w}-c}{2})$ with $u'(c) = Eu'(\frac{\widetilde{w}-c}{2})$. By a similar reasoning as in the proof of Proposition 1, this holds iff DARA. Q.E.D.

7 Conclusion

In this paper, we have examined a simple consumption model under risk with a prioritarian social welfare function. We have shown that under standard assumptions on utility and social welfare functions (which include the familiar constant relative risk aversion utility and Atkinsonian social welfare functions), prioritarianism always leads to more current consumption under an ex ante approach, but to less current consumption under an ex post approach, than under utilitarianism.

Why is this result interesting? Many economic problems combine a risk and an equity dimension. Consider the general idea that future risks over climate change should justify less consumption of energy today. The traditional argument in the economics of discounting under utilitarianism relies

[12] Note that this nonlinearity might also imply a negative value of information (Wakker 1988). But we can show that this is not the case. Indeed the utility reached under learning (see (9)) is always higher than the one reached under risk (see (7)) iff $g(Eu(\frac{\widetilde{w}-c}{2})) \geq \frac{1}{2}(g(u(c)) + Eg(u(\frac{\widetilde{w}-c}{2}))$. This inequality always holds under g and u concave by simply applying twice the Jensen inequality.

on a precautionary savings motive. Our paper shows that this argument is reinforced by a moral prioritarian argument only under the ex post approach, but is weakened under the ex ante approach. Note also that the ex ante approach, although it respects individual expected utilities, has the additional drawback that it leads to time-inconsistent consumption rules.

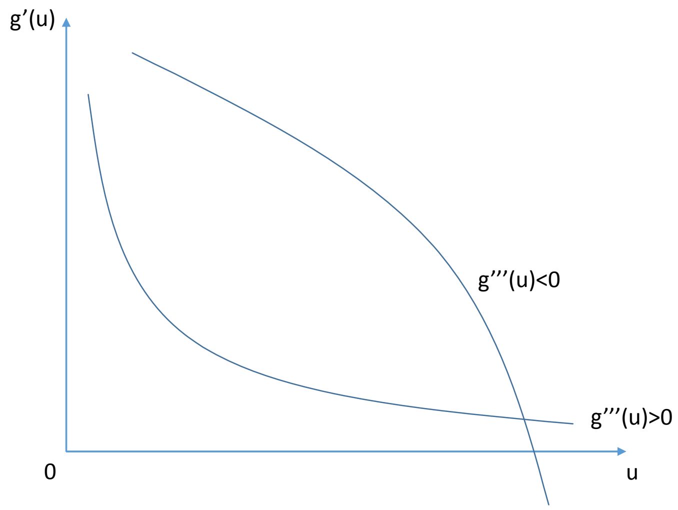

Figure 1:

This figure illustrates that the conditions $g'(u) > 0$, $g''(u) < 0$ together with $g'''(u) < 0$ for all $u > 0$ are mutually inconsistent.

References

Adler, M.D., 2012, Well-Being and Fair Distribution, Oxford University Press.

Adler, M.D., 2013, Cost-benefit analysis and distributional weights: An overview, mimeo.

Adler, M.D., J. Hammitt and N. Treich, 2012, The social value of mortality risk reduction: VSL vs. the social welfare function approach, mimeo.

Adler, M.D. and C.W. Sanchirico, 2006, Inequality and uncertainty: Theory and legal applications, University of Pennsylvannia Law Review 155, 279-377.

Bommier, A. and S. Zuber, S., 2008, Can preferences preferences for catastrophe avoidance reconcile social discounting with intergenerational equity?, Social Choice and Welfare 31, 415-434.

Bossert W. and J.A. Weymark, 2004, Utility in social choice, Handbook of Utility Theory 2, 1099-1177, Boston: Kluwer Academic.

Broome, J., 1984, Uncertainty and fairness, Economic Journal 94, 624-32.

Dasgupta, P., 2008, Discounting climate change, Journal of Risk and Uncertainty 37, 141-69.

Diamond P.A., 1967, Cardinal welfare, individualistic ethics, and interpersonal comparison of utility: Comment, Journal of Political Economy 75, 765-66.

Dietz S. and G.B. Asheim, 2012, Climate policy under sustainable discounted utilitarianism, Journal of Environmental Economics and Management 63, 321-335.

Eeckhoudt L. and H. Schlesinger, 1994, A precautionary tale of risk aversion and prudence, in Models and Experiments in Risk and Rationality (eds. Munier B. and M. Machina), Kluwer Academic Publishers.

Eeckhoudt, L., C. Gollier and N. Treich, 2005, Optimal consumption and the timing of the resolution of uncertainty, European Economic Review 49, 761-73.

Epstein, L.G., 1980, Decision-making and the temporal resolution of uncertainty, International Economic Review 21, 269-284.

Epstein, L.G. and M. Le Breton, 1993, Dynamically consistent beliefs must be bayesian, Journal of Economic Theory 61, 1-22.

Epstein, L.G. and U. Segal, 1991, Quadratic social welfare functions, Journal of Political Economy 100, 691-712.

Fleurbaey M., 2010, Assessing risky social situations, Journal of Political Economy 118, 649-80.

Fleurbaey M. and L. Bovens, 2012, Evaluating life or death prospects, Economics and Philosophy 28, 217-249.

Fleurbaey M. and S. Zuber, 2013, Discounting, risk, inequality: A general approach, mimeo.

Foncel J. and N. Treich, 2005, Fear of ruin, Journal of Risk and Uncertainty 31, 289-300.

Gollier C., 2001, The Economics or Risk and Time, The MIT Press.

Gollier C., 2012, Pricing the Planet's Future: The Economics of Discounting in an Uncertain World, Princeton University Press.

Gollier C., Jullien B. and N. Treich, 2000, Scientific progress and irreversibility: An economic interpretation of the Precautionary Principle, Journal of Public Economics 75, 229-53.

Grant S., Kajii A., Pollack B. and S. Safra, 2010, Generalized utilitarianism and Harsaniy's impartial observer theorem, Econometrica 78, 1939-71.

Ha Duong M. and N. Treich, 2004, Risk-aversion, intergenerational equity and climate change, Environmental and Resource Economics 28, 195-207.

Hammond P., 1983, Ex-post optimality as a dynamically consistent objective for collective choice under uncertaintyy, Social choice and welfare: Symposium on collective choices, Amsterdam: North-Holland-Publication.

Harsanyi, J.C., 1975, Nonlinear social welfare functions, Theory and Decision 6, 311-32.

Johansson-Stenman, O., 2000, On the value of life in rich and poor countries and distributional weights beyond utilitarianism, Environmental and Resource Economics 17, 299-310.

Johansson-Stenman, O., 2005, Distributional weights in cost-benefit analysis - Should we forget about them?, Land Economics 81, 337-52.

Kaplow L., 2008, The Theory of Taxation and Public Economics, Princeton University Press.

Kimball M.S., 1990, Precautionary savings in the small and in the large, Econometrica 58, 53-73.

Lambert P.J., 2001, The Distribution and Redistribution of Income, University of British Columbia Press.

Leland, H.E., 1968, Saving and uncertainty: The precautionary demand for saving, Quarterly Journal of Economics 82, 465-73.

Menegatti, M., 2001, On the conditions for precautionary saving, Journal of Economic Theory 98, 189-193.

Millner, A., 2013, On welfare frameworks and catastrophic climate risks, Journal of Environmental Economics and Management 65, 310-325.

Nagel, T., 1995, Equality and Partiality, Oxford University Press.

Nordhaus, W.D. 2008. A Question of Balance: Weighing the Options on Global Warming Policies. New Haven: Yale University Press.

Parfit, D., 1991, Equality or Priority, Lindley Lecture, University of Kansas. Reprinted in The Ideal of Equality (eds: M. Clayton and A. Williams), Houndmills: Palgrave 2000, 81-125.

Roemer, J.E., 2008, The ethics of intertemporal distribution in a warming planet, Environmental and Resource Economics 48, 363-390.

Sen, A.K., 1970, Collective Choice and Social Welfare, Holden-Day: San Francisco.

Stern, N., 2007, The Economics of Climate Change: The Stern Review. Cambridge: Cambridge University Press.

Traeger, C., 2012, Once upon a time preference - How rationality and risk aversion change the rationale for discounting, mimeo.

Ulph, A., 1982. The role of ex ante and ex post decisions in the valuation of life, Journal of Public Economics 18, 265-76.

Ulph, A. and D. Ulph, 1997, Global warming, irreversibility and learning, Economic Journal 107, 636-650.

Wakker, P., 1988, Nonexpected utility as aversion of information, Journal of Behavioral Decision Making 1, 169-75.

Weitzman, M.L., 2009, On modeling and interpreting the economics of catastrophic climate change, Review of Economics and Statistics 91, 1-19