TSE working paper series

09-023

Research Group: *Environmental Economics and Natural Resources*

March 27, 2009

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ANDRE GRIMAUD, BERTRAND MAGNÉ AND LUC ROUGE

Ecole d'économie de Toulouse (TSE) - Manufacture des Tabacs
Aile Jean-Jacques Laffont - 21, allée de Brienne - 31000 TOULOUSE
Tél : +33 (0) 5 61 12 85 89 - Fax : +33 (0) 5 61 12 86 37 - **www.TSE-fr.eu** - contact@TSE-fr.eu

Toulouse School of Economics

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André Grimaud[1], Bertrand Magné[2] and Luc Rouge[3]

27th March 2009

[1]Toulouse School of Economics (IDEI and LERNA), Manufacture des Tabacs, 21 Allée de Brienne, 31000 Toulouse, France, and Toulouse Business School. E-mail: grimaud@cict.fr

[2]International Energy Agency. E-mail: Bertrand.MAGNE@iea.org

[3]Corresponding author. Université de Toulouse, Toulouse Business School, 20 Bd Lascrosses, 31068 Toulouse Cedex 7, France. E-mail: l.rouge@esc–toulouse.fr Tel: +33 5 61 29 48 20 Fax: +33 5 61 29 49 94

Abstract

We study how the availability of an abatement technology affects the optimal use of polluting exhaustible resources, and optimal climate policies. We develop a Romer endogenous growth model in which the accumulated stock of greenhouse gas emissions harms social welfare. Since the abatement technology allows reducing the effective pollution for each unit of resource use, extraction and pollution are partially disconnected. Abatement accelerates the optimal extraction pace, though it may foster CO2 emissions for the early generations. Moreover, it is detrimental to output growth. Next, we study the implementation of a unit tax on carbon emissions. Contrary to previous results of the literature, its level here matters, as it provides the right incentives to abatement effort. When it is measured in final good, the optimal (Pigovian) carbon tax is increasing over time, while it is constant when expressed in utility. Moreover, it can be interpreted ex-post as a decreasing ad-valorem tax on the resource. Finally, we study the impact of the climate policy on the decentralized equilibrium: in particular, it fosters both the intensity and the rate of carbon abatement. In the near-term, it spurs research and output growth, while decreasing output level.

Keywords: abatement, endogenous growth, polluting non-renewable resources.

JEL classification: O32, O41, Q20, Q32

1 Introduction

The exploitation of fossil resources raises two concerns: the first one is scarcity, because fossil resources are exhaustible by nature, the second one is related to greenhouse gases (GHG) emission associated to their combustion.

Numerous models deal with this double issue. Some of them are placed in the context of partial equilibrium (e.g. Sinclair (1992), Withagen (1994), Ulph and Ulph (1994), Hoel and Kverndokk (1996) or Tahvonen (1997)) whereas some others tackle this issue in general equilibrium growth frameworks (Stollery (1998), Schou (2000, 2002), Grimaud and Rouge (2005, 2008), Groth and Schou (2007)). Two main questions are addressed: the socially optimal outcome on the one hand, and, on the other hand, its implementation in a decentralized economy along with the impacts of environmental policies. It is generally shown that postponing the resource extraction, and thus polluting emissions, is optimal. In addition, model recommendations in terms of environmental policy are less unanimous. For instance, Sinclair (1992) advocates a decreasing ad valorem tax on resource use, whereas Ulph and Ulph (1994), among others, show that such a tax may not always be optimal, especially when the pollution stock partially decays over time. Considering the sole endogenous growth models with polluting exhaustible resources, with the exception of Schou (2000, 2002) for whom no environmental policy is required, results generally exhibit a decreasing optimal carbon tax (see Grimaud and Rouge (2005, 2008) or Groth and Schou (2007)). Moreover, as in Sinclair (1992), a change of the tax level only has redistributive effects and does not alter the model dynamics, e.g. neither the extraction nor the pollution emission time-paths.

A common feature of those papers lies in the fact that, when no alternative (backstop) energy, like solar, is considered, reducing carbon emissions necessarily means extracting less resource. Indeed, a systematic link between resource extraction and pollution emission, in the form of a simple functional relation (e.g. linear), is generally made. It is therefore equivalent to tax either the pollution stream or the resource use itself. Nevertheless, it is well known that abatement technologies, allowing to reduce emissions for a given amount of extracted resource, exist. In particular, the possibility of capturing and sequestering some fraction of the carbon dioxide arising from fossil fuel combustion has recently caught a lot of attention, reinforced by its recent demonstrated viability (for an overview, see IPCC special report (2005)). This process, often labelled as CO_2 capture and storage (CCS), consists of separating the carbon dioxide from other flux gases during the process of energy production; once captured, the gases are then being disposed into various reservoirs[1]. Despite the numerous uncertainties still surrounding the sizable deployment of carbon capture technologies, especially with regard to the ecological consequences of massive carbon injection, this technological option has become promising for the fossil energy extractive industry. One important issue is that taking such abatement technology into account partially breaks the aforementionned link between resource extraction and carbon emissions.

Many authors have developed growth models that featured pollution and abatement. In particular the impact of environmental policies on economic growth has been much studied; for a survey on this question, see for instance Ricci (2007). In most of these models, pollution is a by-product of the production, or capital, and it does not result from the use of non-renewable resources. It is generally shown that positive long term growth is compatible with decreasing emissions, when technical progress is fast enough. However, Gradus and Smulders (1993), or Grimaud (1999) show that there is a trade-off between environmental quality and economic growth. Other contributions have studied the links between carbon abatement, optimal climate policy and technical change. In particular, Goulder and Mathai (2000) show that the presence of induced technical change generally lowers the time profile of optimal carbon taxes. Moreover, efforts in $R\&D$ shift part of the abatement from the present to the future. In a close framework,

[1]The sequestration reservoirs include depleted oil and gas fields, depleted coal mines, deep saline aquifers, oceans, trees and soils. Those various deposits differ in their respective capacities, their costs of access or their effectiveness in storing the carbon permanently.

Gerlagh et al. (2008) study the link between innovation and abatement policies under certain assumptions, in particular, the fact that patents can have a finite lifetime; we refer to some of their results later in the text. In these studies, *final* (or *effective*) carbon emissions are endogenous as there is an abatement activity with dedicated technical progress. Furthermore, the authors use partial equilibrium frameworks in which *baseline* emissions are exogenous.

The present paper considers the availability of such abatement technology in the context of a general equilibrium model with endogenous growth and a polluting exhaustible resource. Our aim is to assess how some results of the literature recalled above, namely in terms of optimal policy, are modified in such a framework. In particular, we study the optimal properties of the economy, and we analyse the impact of a climate policy on the decentralized equilibrium and the design of the optimal policy instruments.

We develop a Romer endogenous growth model in which the production of final goods requires the input of an extracted resource, whose stock is available in limited quantities. Furthermore, this resource use generates polluting emissions, interpreted as GHG emissions, whose flow in turn damages the environment - the quality index of environment is here considered as a stock. Notice that the environment features partial natural regeneration capacity. Finally, the index of environmental quality enters the utility function as an argument and thus allows gauging how pollution accumulation affects the welfare. But the main novelty of the model lies in the consideration of the availability of an abatement technology, which, via some effort, allows for the partial reduction of CO_2 release. Then, we distinguish between the total potential CO_2 emission associated to one unit of fossil resource (referred to as total carbon content per unit of resource in the remainder) and the effective emission, i.e. the remaining pollution fraction left after CO_2 removal. The implication in terms of climate change policy is then straightforward: the first best outcome can only be restored by taxing the pollution but not by taxing the resource itself[2].

Our main results can be summarized as follows. The availability of abatement technology speeds up the optimal pace of resource extraction while relaxing the environmental constraint. Additionally, it modifies the emissions time-path of GHG. In the long term, the pollution level decreases without ambiguity. But, if the preference for environmental quality is not high enough, the pollution level may increase in the short-term. In this case, the following counter-intuitive result emerges: the introduction of a carbon abatement technology leads to an increase of CO_2 emissions. Lastly, the availability of such a technology is detrimental to output growth because of the acceleration in resource extraction combined with a negative effect on R&D effort.

We derive the expression of the Pigovian carbon tax. Contrary to results obtained in a context without abatement, as in Sinclair (1992) or Grimaud and Rouge (2005, 2008) for instance, the tax level here matters and especially allows for setting the optimal abatement effort level. We give a full interpretation of this optimal tax level, we study its properties -namely the impact of a more efficient $R\&D$ sector, and we show that, though this tax is constant when it is expressed in utility, it is an increasing function of time when it is measured in final good. Moreover, this tax can be expressed ex-post as a decreasing ad-valorem tax on the resource.

Finally, we study the impact of the climate policy on the decentralized economy's trajectories. We show that an increase in this tax fosters the intensity and the rate of carbon abatement, while decreasing effective pollution per unit of carbon content. It also leads the economy to postpone resource extraction. In the near-term, this climate policy spurs research and output growth, but reduces output level.

The remainder of the paper is organized as follows. We present the model as well as the social optimum in section 2 and we portray the decentralized equilibrium in section 3. In

[2] Here we assume that the regulator is able to fully measure the greenhouse gases emissions. This may not be systematically the case: While emission data is fairly reliable in industrialized countries, collecting accurate data on industrial activities from developing regions and deducting the emissions may prove more difficult.

section 4, we compare both market and optimal outcomes. We then characterize the optimal policy instruments, and we analyze the effects of a climate policy on the decentralized economy. Conclusive remarks are given in section 5.

2 Model and Optimal Paths

2.1 The model

At each date $t \in [0, +\infty)$, the final output is produced using the range of available intermediate goods, labor and a flow of resource. The production function is

$$Y_t = \left(\int_0^{A_t} x_{it}^{\alpha} di \right) L_{Yt}^{\beta} R_t^{\gamma}, \ \alpha + \beta + \gamma = 1, \tag{1}$$

where x_{it} is the amount of intermediate good i, L_{Yt} the quantity of labor employed in the production sector, and R_t is the flow of non-renewable resource. A_t is a technological index which measures the range of available innovations. The production of innovations writes

$$\dot{A}_t = \delta L_{At} A_t, \, \delta > 0, \tag{2}$$

where L_{At} is the amount of labor devoted to research, and δ is the efficiency of $R\&D$ activity.

To each available innovation is associated an intermediate good produced from the final output:

$$x_{it} = y_{it}, \ i \in [0, A_t]. \tag{3}$$

Pollution is generated by the use of the non-renewable natural resource within the production process. In case of no abatement, the pollution flow would be a linear function of resource use: hR_t, where $h > 0$. In this way, hR_t can be seen as the carbon content of resource extraction or, equivalently, as maximum potential pollution at time t. Nevertheless, firms can abate part of this carbon so that the actual emitted flow of pollution is

$$P_t = hR_t - Q_t, \tag{4}$$

where Q_t is the amount of carbon that is extracted from the potential emission flow. We assume that Q_t is produced from two inputs, the pollution content hR_t and dedicated labor L_{Qt}, according to the following Cobb-Douglas abatement technology[3]:

$$Q_t = (hR_t)^\eta L_{Qt}^{1-\eta},\ 0 < \eta < 1, \text{ if } L_{Qt} < hR_t \tag{5}$$

and

$$Q_t = hR_t, \text{ if } L_{Qt} \geq hR_t.$$

For any given hR_t, the total cost of labor, $L_{Qt} = Q_t^{1/(1-\eta)}(hR_t)^{-\eta/(1-\eta)}$, is an increasing and convex function of Q_t. The marginal and average labor costs, respectively $\partial L_{Qt}/\partial Q_t = [1/(1-\eta)]\, Q_t^{\eta/(1-\eta)}(hR_t)^{-\eta/(1-\eta)}$ and $L_{Qt}/Q_t = Q_t^{\eta/(1-\eta)}(hR_t)^{-\eta/(1-\eta)}$, are also increasing functions of Q_t. The Cobb-Douglas form allows simple analytical developments. Given any quantity of potentially emitted carbon hR_t, it is the effort in terms of labor only that enables pollution abatement. Of course, one could also consider physical capital for instance; however, this would yield further computational complexity as it would add another state variable. Our abatement technology is such that the fraction of abated carbon, Q_t/hR_t, is comprised between 0 and 1. The pollution flow is fully abated as soon as $L_{Qt} = hR_t$[4][5].

The non-renewable resource is extracted from an initial finite stock S_0. At each date t, a

[3] More generally, one could have considered the technology $Q_t = (hR_t)^\eta(\xi L_{Qt})^{1-\eta}$, $0 < \eta < 1$, if $L_{Qt} < hR_t/\xi$ and $Q_t = hR_t$, if $L_{Qt} \geq hR_t/\xi$, with $\xi > 0$. Here we normalize ξ at one.

[4] In Appendix 1, we make an assumption on parameters so that this corner solution never occurs.

[5] Note that, contrary to Goulder and Mathai (2000) or Gerlagh et al. (2008) for instance, we do not consider technical progress in abatement. Of course, such assumption would be more realistic, but, in this endogenous growth framework, it would also make our computations much more complex. We leave this for future research.

flow $-\dot{S}_t$ is extracted. This implies the standard following law of motion:

$$\dot{S}_t = -R_t. \tag{6}$$

In this case, there are no extraction costs, as it is the case in most endogenous growth models with polluting non-renewable resources (see for instance Schou (2000, 2002), Grimaud and Rouge (2005) or Groth and Schou (2007)). Such costs could be modelled here following Andre and Smulders (2004), for instance. In this case, the flow $-\dot{S}_t$ is extracted, and a proportion

$$R_t = -\dot{S}_t/(1+\mu_t), \ \mu_t > 0, \tag{7}$$

is supplied on the market, while $-\dot{S}_t\mu_t/(1+\mu_t)$ vanishes, where $\mu_t/(1+\mu_t)$ is the unit cost of extraction in terms of resource. We will later on denote by $\hat{\mu}_t$ the term $\dot{\mu}_t/(1+\mu_t)$. $\hat{\mu}_t < 0$ means that the unit cost of extraction is decreasing over time because of technical progress that increases exploration efficiency. Conversely, $\hat{\mu}_t$ can be positive if we consider that exploitable reserves are getting less accessible despite better drilling results. A consequence of such extraction costs on the path of the resource owner's rent is presented in section 4.1[6].

The flow of pollution (P_t) affects negatively the stock of environment (E_t). We assume $E_t = E_0 - \int_0^t P_s e^{\theta(s-t)} d_s$, with $E_0 > 0$, and θ is the (supposed constant) positive rate of regeneration. This gives the following law of motion[7]

$$\dot{E}_t = \theta(E_0 - E_t) - P_t. \tag{8}$$

[6]Our main results are obtained in the case of constant unit cost of extraction. This allows to avoid heavy computational complexity. For general optimal solutions in the presence of extraction costs à la André and Smulders (2004) in a model with no abatement, see for instance Grimaud and Rouge (2008). Using data on the prices of fossil fuels over the last century, Gaudet (2007) shows that, despite high volatility, these prices remained approximatively constant, or at most weakly increased. In our framework, this advocates for $\hat{\mu}_t \leq 0$, as we show below (see section 3.1.2).

[7]As Gerlagh et al. (2008) point out, environmental dynamics in the presence of greenhouse gases are more complex. However, such formulation is standard in the literature.

Production flow Y_t is used for consumption (C_t) and for the production of intermediate goods:

$$Y_t = C_t + \int_0^{A_t} y_{it} di. \tag{9}$$

Population is assumed constant, normalized at one, and each individual is endowed with one unit of labor. Thus we have:

$$1 = L_{Yt} + L_{At} + L_{Qt}. \tag{10}$$

The household's instantaneous utility function depends on both consumption, C_t, and the stock of environment E_t[8]. The intertemporal utility function is:

$$U = \int_0^{+\infty} [\ln C_t + \omega E_t] \, e^{-\rho t} dt, \;\; \rho > 0 \text{ and } \omega \geq 0. \tag{11}$$

Note that, contrary to Aghion and Howitt (1998) for instance, the instantaneous marginal utility of the stock of environment, ω, is constant. In the case of strong damages to the environment, it may be more realistic to consider that this marginal utility is increasing (think of catastrophic events). Nevertheless, this assumption allows simple computations in this general equilibrium model.

2.2 Welfare

2.2.1 Characterization of optimal paths

Now we characterize the socially optimal trajectories of the economy. The results are given in Appendix 1, where we fully depict the optimal transition time-paths of all variables in the case of no extraction costs. The main findings are summarized in the following Proposition 1. We drop time subscripts for notational convenience (upper-script ^{o} stands for social optimum and

[8] It would be equivalent to assume that utility is a decreasing function of the pollution stock $X_t = X_0 + \int_0^t P_s e^{\theta(s-t)} d_s$. From this expression, one gets the law of motion $\dot{X}_t = \theta(X_0 - X_t) + P_t$ and we have the following correspondence: $X_t - X_0 = E_0 - E_t$. In this context, we could also consider a target carbon concentration $X_t \leq \bar{X}$ for all t, as an alternative to our damage function.

g_X is the rate of growth of any variable X).

Proposition 1 *At the social optimum:*

(i) In the case of strictly positive environmental preference ($\omega > 0$), due to the presence of the environmental stock E, the economy is always in transition and asymptotically converges towards the case where pollution does not matter ($\omega = 0$).

(ii) The extraction flow, R^o, decreases over time (i.e. $g_R^o < 0$); moreover, strictly positive environmental preference slows down the process. As the optimal flows of abatement (Q^o) and of pollution (P^o) are proportional to R^o, they also decrease over time.

(iii) Labor in production, L_Y^o, is constant over time. Labor in abatement, L_Q^o, is proprotional to the flow of extraction, R^o, and thus follows the same dynamics (i.e. $g_{L_Y}^o = g_R^o$). Therefore, labor in research, L_A^o, increases over time and converges to $1 - L_Y^o$ as time goes to infinity.

All optimal levels and growth rates are given in Appendix 1.

Proof. See Appendix 1. ■

2.2.2 General comments

Let us give some comments on formulas (35)-(43) and let us first consider the case where $\omega = 0$, i.e., the environmental quality does not affect the household's utility. Here, the econony immediately jumps to its steady-state. From (35), (36), (37) and (39), we can see that $L_{Qt}^o = 0$, $Q_t^o = 0$ and $L_{At}^o = 1 - \beta\rho/\delta(1-\alpha)$: no abatement is undertaken, and the efforts dedicated to production and $R\&D$ are constant. Moreover, B becomes nil and (42) implies $g_{Rt}^o = -\rho$. Since there is no abatement, $P_t^o = hR_t^o$ (from (40)): this means that the total carbon content of each unit of extracted resource is emitted. Hence, the growth rate of pollution is constant, as the growth rate of extraction.

Finally, one also easily obtains from (43) that the growth rate of output, g_{Yt}^o, is equal to $\delta - \rho$, as in more general endogenous growth models with non-polluting non-renewable resources (see for example Grimaud and Rouge (2003)). In addition, it will be shown later that the optimal outcome of this economy when $\omega = 0$ is identical to the decentralized outcome of an economy where no climate policy is implemented and research is optimally funded.

We now turn to the case where $\omega > 0$. Contrary to the preceding case, the economy is now always in transition. From (38), R_t^o also decreases over time but g_{Rt}^o is now greater than $-\rho$. In other words, when the environmental quality affects the household's utility, the social planner postpones resource extraction (see Withagen (1994) for a similar result in a partial equilibrium context). As L_{Qt}^o, Q_t^o and P_t^o are linear functions of R_t^o, they exhibit similar dynamics: they decrease over time and so do their growth rates. This also implies that the fraction of captured emissions, i.e. Q_t^o/P_t^o, remains constant over time. Note that L_Y^o is also constant over time (see (35)). As we show in Appendix 1 (i), this results from an arbitrage condition in the allocation of labor between production and research activity. The social optimum is achieved when a marginal increase in labor in any of these activities entails the same increase in intertemporal utility. Hence, the remaining flow of labor is split between abatement activity and research. As L_{Qt}^o decreases over time, L_{At}^o increases: as the effort in abatement gets lower and lower, $R\&D$ investment rises.

When t tends to infinity, $g_{Rt}^o = g_{L_{Qt}}^o = g_{Q_t}^o = g_{P_t}^o$ tends to $-\rho$. At the same time, L_{Qt}^o decreases down to 0, L_{At}^o tends to $1 - \beta\rho/\delta(1-\alpha)$ and g_{Yt}^o tends to $\delta - \rho$. Those asymptotic values are identical to the ones in the steady state where $\omega = 0$ depicted above. The resource is asymptotically exhausted and thus the pollution flow tends to zero. That is the reason why, at infinity, the socially optimal time-path converges to the steady-state of an economy where pollution does not matter anymore.

2.2.3 Impact of abatement on optimal paths

In order to study the impact of carbon abatement on the socially optimal paths, we are going to compare the social optimum with abatement (depicted above) with the social optimum without abatement. We denote by $X_t^{o\varnothing}$ the optimal level of any variable X_t when no abatement technology is available - X_t^o still standing for the optimal value in the abatement case. We give the optimal levels and growth rates in the no-abatement case in Appendix 2.

Proposition 2 *Introducing abatement alters the optimum results as follows:*

(i) Resource extraction is faster (i.e. $g_{Rt}^o < g_{Rt}^{o\varnothing}$): more resource is extracted in the early stages, and less in the future.

(ii) The short and long-run effects on pollution may differ. In the short-run, the increase in resource extraction (see (i) above) favors pollution augmentation whereas abatement activity steers towards the opposite outcome: the overall effect is ambiguous. In the long run, since resource extraction diminishes and a part of emissions is abated, the pollution flow decreases without ambiguity.

(iii) Economic growth is lower (i.e. $g_{Yt}^o < g_{Yt}^{o\varnothing}$).

Standard models with non-renewable resources show that the optimal extraction is less fast when pollution is taken into account. Here, we can see that abatement allows to partially relax this environmental constraint. The speed up of resource extraction ($g_{Rt}^o < g_{Rt}^{o\varnothing}$) is depicted in Figure 1. As formulated in the above proposition, the impact of abatement on the optimal pollution paths is less obvious. The pollution level P is equal to $hR - Q$. Let us first consider the near-term. Two opposite effects drive the pollution path. An extraction effect fosters hR, and an abatement effect fosters Q. One needs to know which effect dominates. We have shown in Appendix 1 that $Q_t^o = hR_t^o \left(\rho\omega(1-\eta)/\delta(\rho+\theta)\right)^{(1-\eta)/\eta}$. This means that, for a given R_t^o, the higher ω, the higher is Q_t^o. In other words, the more the household values environment, the higher is the fraction of abated carbon. Hence, for high values of ω, abatement is intensive, and the abatement effect tends to be the strongest. Thus pollution is lower in the abatement case. If ω is low, i.e., the representative household is less sensitive to environmental quality, the abatement effect is low, and it is dominated by the extraction effect. Thus, the introduction of a carbon abatement technology induces higher pollution level. We thus have the counter-intuitive case in which abatement leads to a simultaneous increase in resource extraction and pollution in the near-term.

In the long-term, abatement unambiguously induces lower pollution. Indeed, we have shown that extraction decreases; thus, whatever the amount of abated carbon, pollution decreases. Figure 2 provides an illustration of these results.

Let us now turn to the effect of abatement on optimal growth. First, $L_{Qt}^{o\varnothing}$ and $Q_t^{o\varnothing}$ are obviously nil. Moreover, $L_Y^o = L_Y^{o\varnothing} = \beta\rho/\delta(1-\alpha)$ (see equation (32) in Appendix 1, and Appendix 2). This implies $L_{At}^o < L_{At}^{o\varnothing}$: the amount of labor devoted to $R\&D$ is higher in the "no-abatement case" as there is no need to use labor for abatement. So there is a first research effect which is detrimental to growth. In addition, the aforementioned extraction effect also holds growth back. Thus, we have the following inequality: $g_{Yt}^o = \delta L_{At}^o + (\gamma/(1-\alpha))g_{Rt}^o < g_{Yt}^{o\varnothing} = \delta L_{At}^{o\varnothing} + (\gamma/(1-\alpha))g_{Rt}^{o\varnothing}$, that is, carbon abatement is detrimental to economic growth. We have seen that the amount of labor in production is unchanged by the introduction of the abatement technology, and that resource extraction is increased in the near-term. If we consider a sufficiently short period of time during which the reduced growth of knowledge does not overcome these two former effects, then the production level is fostered. Hence, in an economy with abatement technology, early generations consume more at the optimum. In other words, their "sacrifice" is reduced.

3 Decentralized Economy

Now that we have characterized the optimal dynamics, we study the equilibrium trajectories of the decentralized economy. This will namely enable us to study the impacts of a climate policy as well as to compute the optimal level of the policy tools. Since we study a Romer model, there are two first basic distortions with respect to the optimum: the standard public good character of knowledge and the monopolistic structure of the intermediate sector. Moreover, a third distortion arises from polluting emissions which damage the stock of environment. Hence we introduce three economic tools: a unit subsidy to the use of intermediate goods, a research subsidy, and a tax on pollution. Note that this climate policy does not consist of a tax on the polluting resource, as in Grimaud and Rouge (2005, 2008) or Groth and Schou (2007). Indeed, the basic externality is polluting emissions and, as abatement technology is available, a tax on these emissions and a tax on the polluting resource are no more equivalent. As will be shown below, this tax on carbon emissions has two main effects: it leads to postponing extraction (as in the models without abatement possibility). It also yields incentives to produce optimal efforts in carbon abatement at each time t.

3.1 Agents' behaviour

The price of the final good is normalized at one, and w_t, p_{it}, p_{Rt}, and r_t are, respectively, the wage, the price of intermediate good i, the price of the non-renewable resource, and the interest rate on a perfect financial market. We drop time subscripts for notational convenience.

3.1.1 Household

The representative household maximizes (11) subject to her budget constraint $\dot{b} = rb + w + \pi - C + T$, where b is her total wealth, π represents total profits in the economy and T is a lump-sum subsidy (or tax). One gets the following standard Ramsey-Keynes condition:

$$g_C = r - \rho. \tag{12}$$

3.1.2 Non-renewable resource sector

On the competitive natural resource market, the maximization of the profit function $\int_t^{+\infty} p_{Rs} R_s e^{-\int_t^s r_u du} ds$, subject to $\dot{S}_s = -R_s$, $S_s \geq 0$, $R_s \geq 0$, $s \geq t$, yields the standard equilibrium "Hotelling rule":

$$\frac{\dot{p}_R}{p_R} = r, \tag{13}$$

which states that the rent of the resource's owner is equal to the interest rate. As usual, the transversality condition is $\lim_{t\to+\infty} S_t = 0$.

If we consider extraction costs, for instance the à la André and Smulders formulation (see (7)), one gets $\dot{p}_R/p_R = r + \hat{\mu}$. This means that if technical progress reduces the cost of access to exploitable resource stocks, i.e. $\hat{\mu} < 0$, then $\dot{p}_R/p_R < r$ (which seems rather realistic, as shown by Gaudet (2007), for instance); if the decrease in extraction costs is sufficiently fast, we can even have $\dot{p}_R/p_R < 0$. Obviously, the reverse occurs when extraction costs increase.

3.1.3 Final sector

The final sector maximizes the following profit function:

$$\pi_Y = \left(\int_0^A x_i^\alpha di\right) L_Y^\beta R^\gamma - w(L_Y + L_Q) - p_R R - \tau h(R - h^{\eta-1} R^\eta L_Q^{1-\eta}) - \int_0^A p_i(1-s) x_i di,$$

where τ is a unit tax on polluting emissions P (i.e., $hR-(hR)^{\eta}L_Q^{1-\eta}$) and s is a unit subsidy to the use of intermediate goods. The first-order conditions of this program are:

$$\frac{\partial \pi_Y}{\partial x_i} = \alpha x_i^{\alpha-1} L_Y^{\beta} R^{\gamma} - p_i(1-s) = 0, \text{ for all } i \tag{14}$$

$$\frac{\partial \pi_Y}{\partial L_Y} = \beta Y/L_Y - w = 0, \tag{15}$$

$$\frac{\partial \pi_Y}{\partial R} = \gamma Y/R - p_R - \tau h(1-\eta h^{\eta-1} R^{\eta-1} L_Q^{1-\eta}) = 0, \tag{16}$$

$$\text{and } \frac{\partial \pi_Y}{\partial L_Q} = -w + \tau h^{\eta}(1-\eta) R^{\eta} L_Q^{-\eta} = 0. \tag{17}$$

3.1.4 Intermediate and research sectors

Innovations are protected by infinitely lived patents, which gives rise to a monopoly position in the intermediate sector. The profit of the i^{th} monopolist is $\pi_i^m = (p_i - 1)x_i(p_i)$, where $x_i(p_i)$ is the demand for intermediate good i by the final sector (see (14)). Hence, the price chosen by the monopolist is

$$p_i \equiv p = 1/\alpha, \text{ for all } i. \tag{18}$$

As a result, quantities and profits are symmetric. One gets

$$x_i \equiv x = \left(\frac{\alpha^2 L_Y^{\beta} R^{\gamma}}{1-s} \right)^{1/(1-\alpha)} \tag{19}$$

and

$$\pi_i^m \equiv \pi^m = \frac{1-\alpha}{\alpha} x. \tag{20}$$

The market value of a patent is $V_t = \int_t^{+\infty} (\pi_s^m + \sigma_s) e^{-\int_t^s r_u du} ds$, where σ_s is a subsidy to research aimed at correcting the standard distortion caused by the intertemporal spillovers. Note that Barro and Sala-i-Martin (2003), for instance, consider a direct subsidy to labor in research;

our assumption alleviates computational complexity in the context of polluting non-renewable resources and abatement. Differentiating this equation with respect to time gives

$$r = g_V + \frac{\pi^m + \sigma}{V}, \tag{21}$$

which states that bonds and patents have the same rate of return at equilibrium.

The profit function of the research sector is $\pi^{RD} = V\delta A L_A - w L_A$. Free-entry in this sector leads to the standard zero-profit condition :

$$V = \frac{w}{\delta A}. \tag{22}$$

3.2 Equilibrium

The preceding first-order conditions enable us to determine the equilibrium in the decentralized economy, that is, the set of quantities, prices and growth rates at each date. All equilibrum levels and growth rates are given in Appendix 3. As we mentionned above, the three basic distortions concern research and polluting emissions. Recall that, in the present model, there is no directed technical change[9], in particular in the abatement technology; we do not study the links between the climate policy and research subsidies -for such analysis in a partial equilibrium framework, see for instance Goulder and Mathai (2000) or Gerlagh et al. (2008). In order to focus on the climate policy, we assume here that research is optimally funded; in other words, both subsidies s and σ are set at their optimal levels (also given in Appendix 3).

For obvious reasons, it is impossible to study all types of carbon tax profiles. We will limit our analysis to a specific type. In proposition 4, we will show that the optimal carbon tax is a linear function of Y. Then, we focus here on the impact of a climate policy consisting of a tax growing at the same rate as output: $\tau_t = aY_t$ (where a is constant).

[9] For an endogenous growth model with a stock of pollution and directed technical change, see for instance Grimaud and Rouge (2008).

The main findings concerning the equilibrium are summarized in the following Proposition. We drop time subscripts for notational convenience (upper-script ^{e} stands for equilibrium).

Proposition 3 *At the equilibrium in the decentralized economy:*

(i) The economy is always in transition.

(ii) The flow of resource extraction, R^e, as well as the flows of abated carbon, Q^e, and of pollution, P^e, decrease over time.

(iii) Labor in final good production, L_Y^e, is constant over time. Labor devoted to abatement activity, L_Q^e, is proportional to the flow of resource extraction, R^e, and thus follows the same dynamics: $g_{L_{Qt}}^e = g_{Rt}^e < 0$. Therefore, labor devoted to research, L_A^e, increases over time and converges to the constant level $1 - L_Y^e$ as time goes to infinity.

Proof. See Appendix 3. ■

Let us now consider the case in which there is no climate policy (i.e. $\tau = 0$ at each date). The economy immediatly jumps to its steady-state, where the amount of labor devoted to abatement is nil (see formula (45)): $L_Q^e = 0$, which means that no carbon is abated ($Q^e = 0$). This, in turn, implies that the total potential emission is released in the atmosphere, i.e. $P^e = hR^e$. Moreover, labor used in the production of the final good, L_Y^e, is constant, and thus labor devoted to the research sector, $L_A^e = 1 - L_Y^e$ is also constant. Here also, this property stems from an arbitrage condition in the allocation of labor between production and research activities. The flow of extraction at date t is $R_t^e = \rho S_0 e^{-\rho t}$. This implies $g_R^e = -\rho$ for all t. This latter case corresponds to the optimum without environmental preference ($\omega = 0$).

We now compare the equilibrium growth rate of resource extraction (g_R^e) in the absence of climate policy to its optimal level. Combining the previous results with those given in Proposition 1, we obtain the following inequalities:

$$g_R^e = -\rho < g_{Rt}^o < g_{Rt}^{o\varnothing}.$$

Recall that $g_{Rt}^{o\varnothing}$ is the optimal growth rate of extraction in the case of no available abatement technology (defined in section 2.2.3). First, $g_R^e < g_{Rt}^{o\varnothing}$ means that, in an economy in which no abatement technology is available, resource extraction in the laissez-faire economy is too fast, compared to the socially optimal path. For a similar result in a partial equilibrium context, see Withagen (1994). Nevertheless, introducing abatement into the analysis leads to two complementary results. The inequality $g_R^e = -\rho < g_{Rt}^o$ is an extension of the previous result: even if abatement is possible, it is optimal to postpone extraction, relative to what is done in the decentralized laissez-faire equilibrium. However, the inequality $g_{Rt}^o < g_{Rt}^{o\varnothing}$ states that in the case of abatement, the optimal extraction path is less restrictive than in the absence of such technology. In other words, abatement partially relaxes the environmental constraint. As we stated earlier, the sacrifice of earlier generations is reduced.

4 Climate policy

We first determine the Pigovian carbon tax; then we can link our results to the existing literature, in particular partial equilibrium models. Furthermore, our general equilibrium framework enables us to study the impact of this climate policy on the economic variables (resource extraction, abatement, polluting emissions, $R\&D$, output...).

4.1 Optimal climate policy

Comparing the optimal levels of the variables to their levels at the decentralized equilibrium (see Appendix 1 and 3), we obtain the following result which gives the design of the optimal (Pigovian) carbon tax.

Proposition 4 *At each date t, $\tau_t^o = \frac{\omega(1-\alpha)}{\rho+\theta} Y_t$ is the level of the carbon tax that implements the socially optimal path.*

First, note that $\tau_t^o = \zeta_t e^{\rho t}(1-\alpha)Y_t$, where ζ_t is the co-state variable associated to E_t, the stock of environment, in the social planner program (see Appendix 1, formula (33)). As we commented earlier, here the tax level matters, contrary to standard results of the literature (see Sinclair (1992), Grimaud and Rouge (2005, 2008), Groth and Schou (2007) for instance). This comes from the fact that we have introduced an abatement option - in other words, if our model did not feature abatement, the tax level would not matter. Indeed, when abatement technology is available, the social planner has to give the right signal in terms of social costs of pollution to firms, so as to induce the optimal effort in abatement.

The optimal value of this carbon tax can be interpreted as follows. If we use the non-specified expression of the utility function, $U(C,E)$, the optimal tax is equal to $\frac{1}{U_C}\int_t^{+\infty} U_E e^{-(\rho+\theta)(s-t)}ds$. Indeed, using (11), we can see that $\int_t^{+\infty} U_E e^{-(\rho+\theta)(s-t)}ds = \frac{\omega}{\rho+\theta}$, and $1/U_C = C = (1-\alpha)Y$. Thus, it is obvious that the optimal tax is the sum of discounted social costs of one unit of carbon emitted at date t, for all (present and future) times, measured in final good. This expression of the optimal carbon tax can be linked to the ones obtained in partial equilibrium frameworks: see for instance Hoel and Kverndokk (1996, formula (17)), Goulder and Mathai (2000, formula (13)) or Gerlagh et al. (2008, formula (18)).

Since the abatement effort results from profit maximization by firms, we also have $\tau = (\partial Y/\ \partial L_Y)/(\partial Q/\ \partial L_Q)$. Indeed, $\partial Y/\ \partial L_Y = \beta Y/L_Y$ and $\partial Q/\ \partial L_Q = (1-\eta)Q/L_Q$. Using (35), (36) and (39), we get τ^o as expressed in the proposition: in this model, increasing abatement leads to a decrease in output through a labor transfer from the final good sector to the abatement one. This expression of the optimal tax means that the optimal carbon tax is the cost of one unit of abated carbon, measured in final good[10].

When it is expressed in utility, this optimal tax is equal to $\omega/(\rho+\theta)$. First, note that it is an increasing function of parameter ω, which measures how the representative household values the

[10] Goulder and Mathai (2000) provide a similar expression (see equation (11) in their paper).

environment. It is a decreasing function of the psychological discount rate ρ: the more people care about the present (relative to future times), the lower the optimal climate tax is, since future environmental damages are less taken into account. This tax is also a decreasing function of the rate of environmental regeneration, θ. In other words, when the environment has a higher regeneration capacity, a given flow of pollution has less overall negative impact, which implies a lower tax. Moreover, the tax is constant under this form, in particular because we have assumed that the marginal utility of environment, ω, is constant. However, when it is measured in final good, the tax increases over time and grows at the same rate as output. Indeed, economic growth being positive, the marginal utility of consumption decreases over time. Thus, the amount of final good that will compensate the household for the emission of one unit of carbon increases over time. Observe that the Pigovian tax is increasing though utility is a linear function of E; a convex functional form would probably reinforce this result - see for instance the discussion on this issue in Goulder and Mathai (2000, p.34).

Furthermore, the optimal carbon tax, which in particular leads the decentralized economy to postponing resource extraction, can be interpreted ex-post as a decreasing ad valorem tax on the resource. Here we can make a link with standard literature in the case of no abatement (see Sinclair (1992), Grimaud and Rouge (2005, 2008) or Groth and Schou (2007)). When the optimal tax is implemented, the "total" (i.e., including the price of the resource and the carbon tax) unit price paid by users for the resource increases less fast than the unit price perceived by owners of the resource -whose growth rate is the interest rate. That is why extraction is postponed. Ex-post, this has the same effect as a decreasing ad valorem tax. Indeed, the "total" price paid by firms is $p_R R + \tau^o h(R - h^{\eta-1} R^\eta L_Q^{1-\eta}) = p_R R \left[1 + (\tau^o h/p_R)(1 - (L_Q/hR)^{1-\eta})\right]$ (see the profit of the final sector in section 3.1.3). Using (36) and $\tau^o = \omega(1-\alpha)Y/(\rho+\theta)$ (see the proposition above), this price is given by

$$p_R R \left[1 + \left(1 - \left(\frac{\omega\rho(1-\eta)}{\delta(\rho+\theta)} \right)^{(1-\eta)/\eta} \right) \frac{\omega(1-\alpha)hY}{(\rho+\theta)p_R} \right].$$

Since $g_Y = r - \rho$ and $g_{p_R} = r$, the ratio Y/p_R decreases over time. Thus, this expression can be written as $p_R R(1+\xi)$ where ξ can be interpreted as an ad valorem tax on the resource, which is decreasing over time.

Finally, an increase in δ, that is, the productivity of research activities, diminishes the optimal tax level in the near term. Indeed, parameter B increases and thus g_R^o increases (from (38) and (42)); therefore R^o decreases in the short-term. Hence, Y^o decreases, since L_Y^o is constant and A^o is a state-variable. Given the expression of τ^o in the proposition, the result is straightforward. This means that a more efficient $R\&D$ sector allows to partially relax the climate tax burden.

4.2 Impact of the climate policy

Let us now study the impact of the climate policy on the equilibrium paths of this economy.

Proposition 5 *An increase in the ratio τ/Y has the following effects:*

(i) Resource extraction and carbon emissions decrease at a lower pace, and so does the effort in abatement, as well as abatement activity itself (i.e.: g_R^e, g_P^e, $g_{L_Q}^e$ and g_Q^e increase).

(ii) The intensity of effort in abatement (L_{Qt}^e/Q_t^e), the effort by unit of carbon content (L_{Qt}^e/hR_t^e), as well as the instantaneous rate of abatement (Q_t^e/hR_t^e), all increase.

(iii) Effective pollution by unit of carbon content (P_t^e/hR_t^e) decreases.

(iv) The effort in production (L_Y^e) remains unchanged.

(v) In the short-run, research is spurred: L_A^e and g_A^e both increase. Output growth (g_Y^e) is fostered, but the level of output (Y^e) decreases.

Assume $0 \leq \tau/Y \leq \delta(1-\alpha)/\rho(1-\eta)$. An increase in the ratio τ/Y has two basic effects: first, pollution gets more costly, which leads the economy to postpone extraction (g^e_{Rt} increases). Secondly, abatement activity becomes more profitable; hence the amount of labor by unit of carbon content (L^e_{Qt}/hR^e_t) increases. Therefore, Q^e_t/hR^e_t, that is, the instantaneous rate of abatement also increases. Simultaneously, effective pollution by unit of carbon content (P^e_t/hR^e_t) decreases. As abatement gets more profitable, the intensity of labor in this activity (L^e_{Qt}/Q^e_t) increases.

Let us now discuss the-short term effects of this climate policy on output's level and growth. First, as g^e_{Rt} increases, less resource is extracted in the early times; then, since labor devoted to output is unchanged, output level diminishes. Second, using (45) and (48), one can show that $\partial L^e_{Qt}/\partial t \leq 0$ if t is low enough, i.e., L^e_Q, the effort in abatement, decreases in the short-run. Then, as L^e_Y is unchanged, L^e_A and thus g^e_A both increase. Finally, output growth, $g^e_Y = g^e_A + (\gamma/(1-\alpha))g^e_R$, is fostered. This contrasts with many results of the literature in the context of endogenous growth models with environmental policy, which consider pollution as a by-product of output or capital - for a survey on this issue, see Ricci (2007). But empirical results in Bretschger (2007) confirm our result: increasing energy prices, and thus decreasing energy use foster output growth.

5 Conclusion

We have proposed a Romer endogenous growth model in which output is produced from a range of intermediate goods, labor and a polluting non-renewable resource. The aim of the paper was to study how previous results of the literature on growth and polluting non-renewable resources are modified when a carbon abatement technology is available -think of CCS, for instance. Here, part of the carbon flow that is emitted when the resource is used within the production process can be abated. This implies that, contrary to standard literature, pollution is dissociated from resource extraction. The remaining flow of carbon damages the state of the environment, which is harmful for household's utility.

We have fully characterized the optimal trajectories. We have shown how the abatement option speeds up the optimal resource extraction and thus helps to partially relax the environmental constraint, which reduces the sacrifice of early generations. Moreover, the path of GHG emissions is modified. In the long-run, emissions unambiguously decrease, but we have proved that pollution may increase in the near-term if environmental preferences are low. Finally, we showed that the availability of abatement technology is detrimental to growth.

We have also studied the decentralized economy. We characterized the optimal design of a unit tax on carbon. Here its level matters: it is equal to the sum of discounted social costs of one unit of carbon for all (present and future) generations -taking regeneration into account. Since abatement efforts are endogenously chosen by firms, it is also equal to the cost of one unit of abated carbon. Furthermore, this Pigovian tax is an increasing function of time when it is measured in final good, though it is constant when expressed in utility. However, it can be interpreted (ex-post) as a decreasing ad valorem tax on the resource: climate policy reduces the growth rate of the "total" resource price (i.e., the resource price including carbon tax). We have also shown that a more efficient $R\&D$ sector allows partially relaxing the climate tax burden.

More generally, the climate policy affects the decentralized economy as follows. It fosters the intensity and the rate of carbon abatement while decreasing effective pollution per unit of carbon content. Moreover, resource extraction is postponed. In the near-term, research and output growth are spurred, but output levels are lowered.

The decarbonization of the economy and the switch to renewable or non fossil fuel-based energy remain necessary (Gerlagh (2006)). In order to keep the model tractable, the availability of a clean and renewable energy source has not been introduced. This so-called backstop would not drastically alter the qualitative properties of our results. Nevertheless, it would be interesting to study the impact of the abatement option on the adoption timing of these alternative sources of energy. We can infer that the possibility to abate carbon emissions would delay the introduction of renewable energy. Indeed, the availability and use of abatement technologies may notably encourage a shift of electricity generation from natural gas to coal-based power plants thus favoring a coal renaissance (Newell et al. (2006)) over the next decades, while decreasing reliance on renewable energy sources.

Appendix

Appendix 1: Welfare

The social planner maximizes $U = \int_0^{+\infty} (\ln C_t + \omega E_t) e^{-\rho t} dt$ subject to (1)-(6) and (8)-(10). Here we assume $\hat{\mu} = 0$ for computational convenience. Moreover, we assume that $[\rho\omega(1-\eta)/\delta(\rho+\theta)]^{1/\eta} < 1$ (see equation (36)) in order to avoid a corner solution in which carbon emissions are fully abated, i.e. $L_Q = hR$. Thus, it is unnecessary to incorporate a Kuhn-Tucker condition for $L_Q \leq hR$. The Hamiltonian of the program is

$$\begin{aligned} H &= (\ln C + \omega E)e^{-\rho t} + \mu\delta A(1 - L_Y - L_Q) - \nu R + \zeta\left[\theta(E_0 - E) - h(R - h^{\eta-1}R^{\eta}L_Q^{1-\eta})\right] \\ &\quad +\varphi\left[(\int_0^A x_i^{\alpha} di) L_Y^{\beta} R^{\gamma} - C - \int_0^A x_i di\right], \end{aligned}$$

where μ, ν, ζ and φ are the co-state variables. The first order conditions $\partial H/\partial C = 0$ and $\partial H/\partial x_i = 0$,

$$e^{-\rho t}/C - \varphi = 0, \tag{23}$$

$$\alpha x_i^{\alpha-1} L_Y^{\beta} R^{\gamma} - 1 = 0, \text{ for all } i. \tag{24}$$

Note that this implies $x_i = x$, for all i. $\partial H/\partial L_Y = 0$, $\partial H/\partial L_Q = 0$ and $\partial H/\partial R = 0$ yield

$$-\mu\delta A + \varphi\beta Y/L_Y = 0, \tag{25}$$

$$-\mu\delta A + \zeta h^{\eta}(1-\eta)R^{\eta}L_Q^{-\eta} = 0, \tag{26}$$

$$\text{and } -\zeta h(1 - \eta h^{\eta-1}R^{\eta-1}L_Q^{1-\eta}) + \varphi\gamma Y/R - \nu = 0. \tag{27}$$

Moreover, $\partial H/\partial A = -\dot{\mu}$, $\partial H/\partial S = -\dot{\nu}$, and $\partial H/\partial E = -\dot{\zeta}$ yield

$$-\dot{\mu} = \mu\delta L_A + \varphi(x^{\alpha} L_Y^{\beta} R^{\gamma} - x), \tag{28}$$

$$-\dot{\nu} = 0, \tag{29}$$

$$\text{and } -\dot{\zeta} = \omega e^{-\rho t} - \zeta\theta. \tag{30}$$

i) Computation of L_Y.

(24) can be rewritten $Y = Ax/\alpha$. Since $Y = C + Ax$, one gets $C = (1-\alpha)Y$.

Dividing both hand sides of (28) by μ gives $-g_{\mu} = \delta L_A + (x^{\alpha} L_Y^{\beta} R^{\gamma} - x)\varphi/\mu$. The term between brackets can be rewritten as $Y/A - \alpha Y/A$, which is equal to $(1-\alpha)Y/A$. Moreover, from (25), we have $\varphi/\mu = \delta A L_Y/\beta Y$ and $g_{\mu} + g_A = g_{\varphi} + g_Y - g_{L_Y}$. Since (23) yields $g_{\varphi} = -\rho - g_C = -\rho - g_Y$, one gets $-g_{\mu} = g_A + \rho + g_{L_Y}$. Plugging these results in the first expression of $-g_{\mu}$, we obtain the following Bernoulli differential equation:

$$\dot{L}_Y = (\delta(1-\alpha)/\beta)L_Y^2 - \rho L_Y. \tag{31}$$

In order to transform this equation into a first-order linear differential equation, we consider the new variable $z = 1/L_Y$, which implies $\dot{z} = -\dot{L}_Y/L_Y^2$. The Bernoulli differential equation becomes $\dot{z} = \rho z - \delta(1-\alpha)/\beta$, whose solution is $z = e^{\rho t}\left[z_0 - \delta(1-\alpha)/\beta\rho\right] + \delta(1-\alpha)/\beta\rho$. Replacing z by $1/L_Y$ leads to $L_Y = \frac{1}{e^{\rho t}[1/L_{Y0} - \delta(1-\alpha)/\beta\rho] + \delta(1-\alpha)/\beta\rho}$.

Using transversality condition $\lim_{t \longrightarrow +\infty} \mu A = 0$, we show that L_Y immediately jumps to its steady-state level:

$$L_Y = \beta\rho/\delta(1-\alpha). \tag{32}$$

Indeed, using (25) it turns out that the transversality condition is only satisfied when $L_Y = L_{Y0} = \beta\rho/\delta(1-\alpha)$.

The optimal level of L_Y results from an arbitrage in the allocation of labor between production and research activities. The heuristic argument is the following. Let us suppose a marginal increase of labor in production, $\Delta L_{Yt} = 1$, at date t. This leads to an increase in production $\Delta Y_t = \beta Y_t / L_{Yt}$, which yields an increase in consumption $\Delta C_t = \beta C_t / L_{Yt}$. Finally one gets $\Delta U_t = \Delta C_t / C_t = \beta / L_{Yt}$. Assume now $\Delta L_{At} = 1$, at date t. This leads to an increase in knowledge, ΔA_s, and thus in production, ΔY_s, for all $s \geq t$. One gets $\Delta Y_s = (\partial Y_s / \partial A_s - x_s)\Delta A_s = x_s(1-\alpha)\Delta A_s / \alpha$. Since $A_s = A_0 e^{\int_0^s \delta L_{Au} du}$, we have $dA_s = A_s \delta dL_{At} = \delta A_s$, for all $s \geq t$. This yields $\Delta Y_s = (1-\alpha)\delta Y_s$, which gives $\Delta C_s = (1-\alpha)^2 \delta Y_s$. The increase in the instantaneous utility at s is thus $\delta(1-\alpha)$. Finally, the increase in the intertemporal utility is $\delta(1-\alpha)/\rho$. Equating both increases in the intertemporal utility leads to $L_Y = \beta\rho/\delta(1-\alpha)$.

ii) Computation of ζ.

The solution for equation (30) is $\zeta = e^{\theta t}(-\int_0^t \omega e^{-(\rho+\theta)s} ds + \zeta_0)$. Moreover, the transversality condition associated to E writes

$\lim_{t \longrightarrow +\infty} \zeta E = \lim_{t \longrightarrow +\infty} e^{\theta t} \left[-\int_0^t \omega e^{-(\rho+\theta)s} ds + \zeta_0 \right] \left[E_0 - \int_0^t P_s e^{\theta(s-t)} ds \right] = 0.$

Normalizing E_0 such that the second term between brackets is not nil, we obtain $\zeta_0 = \int_0^{+\infty} \omega e^{-(\rho+\theta)s} ds$, which gives $\zeta = e^{\theta t} \int_t^{+\infty} \omega e^{-(\rho+\theta)s} ds = e^{-\rho t} \int_t^{+\infty} \omega e^{-(\rho+\theta)(s-t)} ds$

$= e^{-\rho t} \int_0^{+\infty} \omega e^{-(\rho+\theta)u} du$. Finally, we get

$$\zeta = \omega e^{-\rho t} / (\rho + \theta). \tag{33}$$

ζ is the discounted value at $t = 0$ of the social cost of one unit of carbon emitted at date t, expressed in utility. This expression can be linked to the value of the optimal carbon tax at date t, measured in final good, in Proposition 4: $\tau^o = [\omega(1-\alpha)/(\rho+\theta)]\, Y = \zeta e^{\rho t}(1-\alpha) Y$.

iii) Computation of L_Q.

Using (33), (26) becomes $-\mu\delta A + \omega e^{-\rho t} h^\eta (1-\eta) R^\eta L_Q^{-\eta} / (\rho+\theta) = 0$. Using (23), (25) and

(32), we get $\mu\delta A = \delta e^{-\rho t}/\rho$. Plugging this result into the preceding one, we get

$$L_Q = \left[\frac{\rho\omega(1-\eta)}{\delta(\rho+\theta)}\right]^{1/\eta} hR. \tag{34}$$

iv) Computation of R.

Using (27), (33) and (34), we obtain $R = \frac{\gamma}{\varphi_0 e^{\rho t}+B}$, in which $B = \frac{(1-\alpha)\omega h}{\rho+\theta}\left[1-\eta\left(\frac{\rho\omega(1-\eta)}{\delta(\rho+\theta)}\right)^{(1-\eta)/\eta}\right]$.

Using the constraint $\int_0^{+\infty} R_t dt = S_0$, after some calculations we obtain $\varphi_0 = B/(e^{\frac{B\rho S_0}{\gamma}} - 1)$.

v) Computation of Q and P.

Plugging (34) into $Q = (hR)^\eta L_Q^{1-\eta}$, one gets $Q = \left(\frac{\rho\omega(1-\eta)}{\delta(\rho+\theta)}\right)^{(1-\eta)/\eta} hR$.

Then, using $P = hR - Q$, we have $P = \left[1-\left(\frac{\rho\omega(1-\eta)}{\delta(\rho+\theta)}\right)^{(1-\eta)/\eta}\right] hR$.

vi) Computation of x.

(1) can be rewritten as $Y = (Ax)x^{\alpha-1}L_Y^\beta R^\gamma$. Since $Ax = \alpha Y$ and using (32), we get $x = \alpha^{1/(1-\alpha)}(\beta\rho/\delta(1-\alpha))^{\beta/(1-\alpha)}R^{\gamma/(1-\alpha)}$.

vii) Computation of growth rates.

The growth rates directly follow from the log-differentiation of the preceding results.

In summary, one gets:

$$L_Y^o = \beta\rho/\delta(1-\alpha), \tag{35}$$

$$L_{Qt}^o = \left[\frac{\rho\omega(1-\eta)}{\delta(\rho+\theta)}\right]^{1/\eta} hR_t^o, \tag{36}$$

$$L_{At}^o = 1 - L_Y^o - L_{Qt}^o, \tag{37}$$

$$R_t^o = \frac{\gamma}{\varphi_0 e^{\rho t} + B}, \tag{38}$$

where $\varphi_0 = B/(e^{\frac{B\rho S_0}{\gamma}} - 1)$ and $B = \frac{(1-\alpha)\omega h}{\rho+\theta}\left[1 - \eta\left(\frac{\rho\omega(1-\eta)}{\delta(\rho+\theta)}\right)^{(1-\eta)/\eta}\right]$,

$$Q_t^o = \left(\frac{\rho\omega(1-\eta)}{\delta(\rho+\theta)}\right)^{(1-\eta)/\eta} hR_t^o, \tag{39}$$

$$P_t^o = \left[1 - \left(\frac{\rho\omega(1-\eta)}{\delta(\rho+\theta)}\right)^{(1-\eta)/\eta}\right] hR_t^o, \tag{40}$$

$$g_{At}^o = \delta L_{At}^o, \tag{41}$$

$$g_{Rt}^o = g_{L_{Qt}}^o = g_{Q_t}^o = g_{P_t}^o = \frac{-\rho}{1 + (e^{\frac{B\rho S_0}{\gamma}} - 1)e^{-\rho t}}, \tag{42}$$

$$g_{Yt}^o = g_{At}^o + (\gamma/(1-\alpha))g_{Rt}^o. \tag{43}$$

Appendix 2: Welfare in the no-abatement case

When no abatement technology is available, maximizing welfare leads to the following results (recall that we denote by $X_t^{o\varnothing}$ the optimal level of any variable X_t in this case):

$L_Y^{o\varnothing} = \beta\rho/\delta(1-\alpha)$, $L_A^{o\varnothing} = 1-\beta\rho/\delta(1-\alpha)$, $R_t^{o\varnothing} = \frac{\gamma}{\varphi_0^{\varnothing} e^{\rho t} + B^{\varnothing}}$, $g_R^{o\varnothing} = \frac{-\rho}{1+B^{\varnothing}/\varphi_0^{\varnothing} e^{\rho t}}$, $g_A^{o\varnothing} = \delta L_A^{o\varnothing}$, $g_Y^{o\varnothing} = \delta L_A^{o\varnothing} + (\gamma/(1-\alpha))g_R^{o\varnothing}$, where $\varphi_0^{\varnothing} = \frac{B^{\varnothing}}{e^{(B^{\varnothing}\rho S_0/\gamma)}-1}$ and $B^{\varnothing} = (1-\alpha)\omega h/(\rho+\theta)$.

Appendix 3: Equilibrium

i) Computation of L_Y

In this paper, we focus on climate policy and its impacts on the economy. Hence we assume that research is optimally funded; in other words, we assume that both subsidies to research, s and σ, are set at their optimal levels. As in the standard case, the optimal level for the subsidy to the demand for intermediate goods, s, is $1-\alpha$. The optimal value of the subsidy to research σ is obtained in what follows.

Equation (14), in which $p_i(1-s) = 1$ (from (18)), can be rewritten $Y = Ax/\alpha$. Since $Y = C + Ax$, one gets $C = (1-\alpha)Y$, as it is the case at the optimum.

From (12) and (21), we have $r = \rho + g_C = g_V + \frac{\pi^m + \sigma}{V}$, where $g_C = g_Y$.

From (22) and (15), after log-differentiation, we get $g_V = g_w - g_A = g_Y - g_{L_Y} - g_A$. Moreover, from (15), (20) and (22), we obtain $\pi^m/V = \delta(1-\alpha)AxL_Y/\alpha\beta Y$; since $Ax = \alpha Y$, we get $\pi^m/V = \delta(1-\alpha)L_Y/\beta$. Plugging these two results into the expressions of r given above yields $\rho = -g_{L_Y} - g_A + \delta(1-\alpha)L_Y/\beta + \sigma/V$. It is now obvious that, if $\sigma/V = g_A = \delta L_A$, this Bernoulli differential equation is similar to equation (31) (given in Appendix 1) and therefore has the same solution (upper-script ^{e} stands for decentralized equilibrium):

$$L_Y^e = \frac{\beta\rho}{\delta(1-\alpha)}. \tag{44}$$

Here we can see that if research is optimally funded, then the amount of labor devoted to the production of final good immediately jumps to its optimal steady-state value[11].

ii) Computation of L_Q, Q and P.

From (15), (17) and (44), we have $Y\delta(1-\alpha)/\rho = \tau(1-\eta)(hR/L_Q)^\eta$. This yields

$$L_Q^e = \left[\frac{\tau\rho(1-\eta)}{\delta(1-\alpha)Y}\right]^{1/\eta} hR^e. \tag{45}$$

Plugging (45) into (5), we get

$$Q^e = \left[\frac{\tau\rho(1-\eta)}{\delta(1-\alpha)Y}\right]^{(1-\eta)/\eta} hR^e. \tag{46}$$

Finally, (46) and (4) yield

$$P^e = \left[1 - \left(\frac{\tau\rho(1-\eta)}{\delta(1-\alpha)Y}\right)^{(1-\eta)/\eta}\right] hR^e. \tag{47}$$

iii) Computation of R.

[11] The computation is similar to the one presented at the optimum (Appendix 1) if we use the transversality condition of the household's program.

Basically, R is obtained from (16). In order to express R as a function of time and of the climate policy, we need to rewrite three elements of this equation. First, L_Q/hR is obtained from (45). Secondly, using (12) in which $g_C = g_Y$, we get $Y = Y_0 e^{\int_0^t (r_u - \rho)du}$. Finally, from (13), we have $p_R = p_{R0} e^{\int_0^t r_u du}$. Plugging these three results into (16) yields $R^e = \frac{\gamma}{p_{R0}e^{\rho t}/Y_0 + \frac{h\tau}{Y}\left[1-\eta\left(\frac{\tau\rho(1-\eta)}{\delta(1-\alpha)Y}\right)^{(1-\eta)/\eta}\right]}$, where the constant p_{R0}/Y_0 is solution of the condition $\int_0^{+\infty} R_t^e dt = S_0$. For obvious reasons, we cannot compute this integral without assumptions on the ratio τ/Y. In fact, we show later that the optimal tax grows at the same rate as the output. Hence, in order to avoid computational complexity without limiting too much the scope of our study, we will now restrict our analysis to the set of constant τ/Y. In this case, we get

$$R^e = \frac{\gamma}{\psi_0 e^{\rho t} + G}, \tag{48}$$

where $\psi_0 = G/(e^{\frac{G\rho S_0}{\gamma}} - 1)$ and $G = \frac{h\tau}{Y}\left[1 - \eta\left(\frac{\tau\rho(1-\eta)}{\delta(1-\alpha)Y}\right)^{(1-\eta)/\eta}\right]$.

iv) Computation of the rates of growth.

The growth rates directly follow from the log-differentiation of the preceding results. We obtain

$$g_{At}^e = \delta L_{At}^e, \tag{49}$$

$$g_{Rt}^e = g_{L_{Qt}}^e = g_{Q_t}^e = g_{P_t}^e = \frac{-\rho}{1 + (e^{\frac{G\rho S_0}{\gamma}} - 1)e^{-\rho t}}, \tag{50}$$

$$g_{Yt}^e = g_{At}^e + (\gamma/(1-\alpha))g_{Rt}^e. \tag{51}$$

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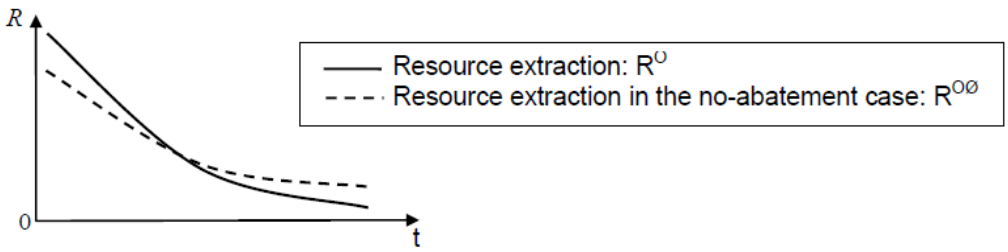

Figure 1: Optimal Resource Extraction

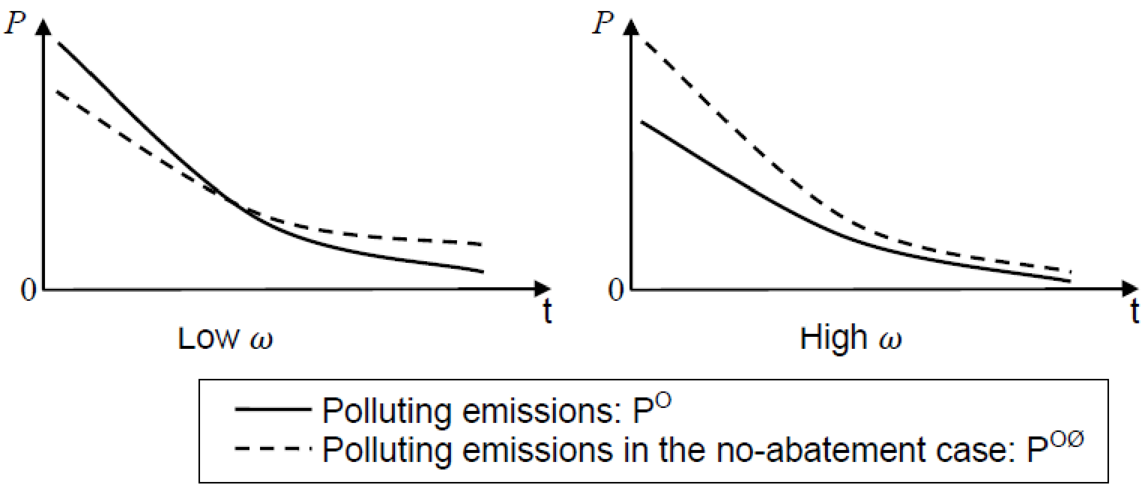

Figure 2: Optimal Polluting Emissions