Free-Riding in Procurement Design

KLENIO BARBOSA

Free-Riding in Procurement Design

Klenio Barbosa[*][†]

December, 2009

Abstract

Low-powered contracts do not provide proper incentives to reduce cost; still empirical studies show that they are quite pervasive in public and private procurement. This paper argues that low-powered contracts arise due to a free-riding problem when the contractor enjoys economies of scale/scope working for different buyers. A buyer, offering a procurement contract to the contractor, does not fully internalize that higher-powered incentives provide cost reduction in the contractor's activities, benefiting other buyers. As a result, buyers offer lower-powered contracts than what would be designed by cooperative buyers. Strikingly, the higher the contractor's benefits from economies of scope/scale are, the lower the power of the procurement contracts will be. In addition, laws which force buyers to award fixed-price contracts can be welfare-enhancing.

JEL: H57; H83; L24.

Key Words: Free-Riding, Procurement, Multibuyers, Low-powered Contracts.

1 Introduction

Public and private transactions are governed by procurement contracts. They represent a large fraction of total economic activity, and are used to purchase goods and services. The value of public procurement transactions in EU countries is about 16 percent of their GDP, while in the United States it is around 20 percent. In the private sector, the value of transactions is even larger and is steadily increasing, due to the current trend towards outsourcing all non-core business activities.[1]

A typical procurement contract specifies a reimbursement fraction $a \in [0, 1]$ of the contractor's monetary expenditures C and a fixed fee b, which are paid by the buyer to the contractor. They jointly characterize the buyer's net transfer T to the contractor, which usually takes a linear[2] form $T = b + aC$.

According to the fraction of costs borne by the buyer a, the contracts are classified in: cost-plus contracts, $a = 1$; fixed-price contracts, $a = 0$; and incentive contracts, $a \in (0, 1)$. In cost-plus contracts, the contractor does not have incentives to reduce cost since all costs are borne by the buyer. By contrast, in the fixed-price and incentive contracts, the contractor is, respectively, the total and partial residual claimant for its cost saving, having incentives to engage in cost reduction activities. Due to their extremely low-powered incentive scheme, McAfee and McMillan (1986) argue that cost-plus contracts are never optimal.

Despite the fact that low-powered contracts (i.e., cost-plus) do not provide proper incentives to reduce cost, empirical studies show that they are quite pervasive in public and private procurement. For instance, Bajari, McMillan and Tadelis (2008) document that 43 percent of the contracts within private firms in the US building construction industry are cost-plus. Gagnepain and Ivaldi (2002) show that 24 percent of the contracts in the French communities public transportation industry are cost-plus. Based on this evidence, a natural question arises: Why are low-powered procurement contracts commonly used in these industries?

---

[1]See Dimitri, Piga and Spagnolo (2006), and Bajari and Tadelis (2006) for more details about the relevance of public and private procurement.

[2]Real-world contracts are often linear, but some have nonlinear features such as a ceiling on transfers from the buyer or a guarantee that the contractor will not lose money. Laffont and Tirole (1993) describe in details other complex procurement contracts, and Chiappori and Salanié (2000) provide a survey on such complex contracts.

The paper shows that low-powered contracts arise in equilibrium due to a free-riding problem in procurement design when a contractor (agent) has economies of scale/scope working for several buyers (principals), thereby enjoying positive externalities when providing goods or services (activities) for different buyers.

In a nutshell, low-powered contracts emerge in equilibrium because a buyer, when designing a procurement contract to an activity provider, does not fully internalize that eliciting higher-powered incentives induces cost reduction in the contractor's activities, which benefits other buyers. However, he internalizes all costs associated to such incentive scheme. Because buyer's private benefit of higher-powered contracts is lower than the public one, buyers offer lower-powered contracts that would be offered by cooperative buyers.

To understand the main ideas of this paper, consider an industry with several risk neutral buyers (principals) who all individually demand an activity from a single risk averse contractor (agent). The contractor faces risks and incurs production costs when performing activities. However, he can reduce costs by making effort, which is not observed by the buyers (moral hazard). The contractor also benefits from economies of scale/scope (i.e., positive externality) when performing activities for different buyers, which is represented by a spillover from the technology: a contractor, when making an effort to reduce cost in a certain activity for one of the buyers, also enjoys cost reduction in activities for other buyers.[3]

Each buyer designs a procurement contract that minimizes his own expenditures when contracting out an activity. As such, he faces a trade off between risk-sharing and incentives to reduce cost. On the one hand, when offering a high-powered incentive scheme, a buyer makes the contractor more residual claimant, thereby increasing the contractor's exposure to risk.[4] On the other hand, a high-powered incentive scheme induces the contractor to exert high effort, thereby reducing activity's costs. By solving this trade-off, buyers choose the power of the procurement contracts.

---

[3]The economies of scale/scope analyzed in this paper can be illustrated by the case of a certain company (i.e., contractor) which has several branches, and each of them engages in new methods to reduce cost and/or increase productivity. In this context, the economies of scale/scope take place when a new method developed by one of the branches reaches other parts of the company, through, for instance, spillover. Consequently, the whole company benefits from methods developed by any of its branches. This paper will give further detailed examples about the kind of economies of scale/scope is presented in the model.

[4]Indeed, it leads the risk averse contractor to demand high monetary compensation to perform the activity.

When the contractor benefits from economies of scale/scope, a free-riding effect is added to that trade-off. A buyer does not internalize all the benefits of eliciting high effort: high-powered incentive scheme also induces the contractor to reduce cost in other activities, thereby reducing other buyer's expenditures. Due to the public good feature of eliciting effort, in equilibrium buyers offer excessively low-powered contracts.

Two assumptions of the model are crucial to generate this free-riding effect: the same contractor performs activities for different buyers, and the contractor's economies of scale/sope in performing several activities. So, one may wonder if those two ingredients are present in the industries where low-powered contracts are pervasive: US building construction industry and French communities public transportation industry.

Empirical evidence suggests that those ingredients do exist in those industries. For instance, Bajari, McMillan and Tadelis (2008) show that 60 percent of contracts in the Northern California construction industry are done by the same firm. Gagnepain and Ivaldi (2002) show that 29 percent of the French urban transportation market belongs to one firm.[5] Based on these findings, it seems that there is some supporting evidence that the same contractor performs activities for different buyers in those industries.

Empirical studies testing the existence of economies of scale/scope, or even the existence of positive externalities within different activities in the building and public urban transportation industry are quite scarce.[6] However, Gagnepain and Ivaldi (2008) provide some evidence on the existence of such economies of scope/scale in the French transportation industry showing that contractors have engineer teams in each city which are responsible for research development in quality control, maintenance, and efficiency, whose new methods and procedures can be used by the entire company.

Some examples in those industries may also illustrate the kind of economies of scale/scope

---

[5] Gagnepain and Ivaldi (2002) document that VIA-GTI Company owns 29 % of the French market share, whereas other big companies like TRANSDEV and CGEA own, respectively, 15 % and 14 %. Market shares refer to the whole market, including Paris, Lyon, and Marseilles, the biggest cities of France.

[6] A set of empirical studies documents the existence of those economies and positive externalities in multi-product firms in other industries. Monteverde and Teece (1982), for instance, show that the existing positive externalities in the automobile production process, generated by nontransferable know-how and specific skills, explains the vertical structure of the US automobile industry. Additionally, Kim (1985) presents evidence on scale economies in multi-product firms in the US water sector. Furthermore, Fraquelli, Piacenza and Vannoni (2004) document the presence of scope and scale economies in multi-services providers of public utilities (gas, water and electricity) in Italy.

presented in this paper. In the public transportation industry, the spillover of new repairing methods and procedures inside Veolia Transportation Company describes well such economies.[7] For instance, when Veolia's mechanic team develops a new method for repairing its light rail in Lyon, part of this knowledge is reached by Veolia's mechanic team in Nice, where Veolia also provides the same kind of public transportation. Similarly, the Granite Construction example illustrates those economies in the building construction industry: when Granite's design team develops a new procedure on project design for working in San Francisco, its project team in San Diego will also learn this new procedure.[8]

This paper provides some empirical implications describing under which circumstances low-powered contracts should be more frequently used. In particular, it shows that cost-plus contracts are more likely to be pervasive in industries in which a contractor benefits from positive externality for running different activities (economies of scale/scope) than in industries in which this effect is nonexistent. In addition, it predicts that the higher the number of activities that a contractor performs for different buyers, the more likely cost-plus contracts are used.

In this paper, excessively low-powered incentive contracts arise as an inefficient allocation due to a free-riding problem. One may wonder if there is any policy that can be designed in order to improve the social allocation. The paper shows that when the number of activities performed by the same contractor for different buyers is sufficiently high, it is optimal for buyers to commit to rules that bind them to offer fixed-price contracts rather than choosing individually their own contracts. This policy recommendation is contrary to Bajari and Tadelis (2001)'s policy recommendation which suggests that laws forcing the US public entities to award fixed-price contracts by competitive bidding should be withdrawn from FAR (Federal Acquisition Rules).

■ **Related Literature.** A set of papers in the literature has analyzed the widespread use of cost-plus contracts in the US building construction and in the French urban transporta-

---

[7]Veolia Transportation is a leader in the transportation industry, and the largest private provider of multiple modes of transportation in the world.

[8]The Granite Construction Company is the leader of the 2005 and 2006 California Construction annual ranking of the largest general contractors in California.

tion industry.[9] Bajari and Tadelis (2001), for instance, argue that cost-plus contracts are more flexible and easier to renegotiate than the fixed-price under unanticipated contingencies.[10] Hence, cost-plus contracts are more likely to be prevalent in industries characterized by complex activities and frequently subject to incompleteness of contracts, like in building construction.[11] Bajari and Tadelis argue that low-powered contracts (i.e., cost-plus) are more often used than other contracts because they provide better allocation. In contrast, this paper argues that low-powered contracts emerge in equilibrium as an inefficient allocation due to a free-riding problem.

Laffont and Tirole (1986) formulate a principal-agent model of cost-based procurement and regulation and demonstrate that the principal can implement the optimal mechanism by offering a menu consisting of a continuum of linear contracts.[12] In particular, they show that cost-plus contracts are optimally designed for the less efficient contractors, whereas fixed-price are for the most efficient ones. Nevertheless, Gagnepain and Ivaldi (2002, 2008) provide evidence that such revelation mechanisms do not explain the widespread use of cost-plus contract in the French urban transportation industry. They show that the political aspects (i.e., the political side - left or right wing - and the weight given by politicians to costs - workers - and profits - shareholders) explain the pervasiveness of low-powered contracts in that industry.

This paper builds on Weitzman (1980) and McAfee and McMillan (1986) which primarily characterize the optimal procurement contract under moral hazard for a risk neutral buyer (principal) and a risk averse contractor (agent). Instead, in a multiprinicipals one agent setting this paper solves the trade off between inducing cost reduction and compensating the contractor for bearing risk.

Furthermore, this paper is also related to the literature on multi-contracting under moral

---

[9]Analyzing the US Air Force engine procurement contracts, Crocker and Reynolds (1993) find that cost-plus contracts are frequently used at the initial production stages of projects. They argue that in these initial stages unanticipated changes in projects are expected, therefore flexible contracts (i.e., cost-plus) are desired.

[10]Bajari and Tadelis (2001) illustrate the incompleteness of contracts in the building construction industry describing many anticipated contingencies in the site conditions occurred during the building of the Getty Center Art Museum in Los Angeles. Those events implied in several changes in the original project design.

[11]Bajari, McMillan and Tadelis (2008) find evidence for that showing that cost-plus contracts are more often used in complex task, which are more likely to be suffer anticipated contingencies.

[12]Laffont and Tirole wrote a series of papers which expand upon the insights of their basic model and the results are reported in Laffont and Tirole (1993).

hazard, started by Bernheim and Whinston (1986), and extended by Dixit (1996).[13] In Bernheim and Whinston, the agent takes one action (one dimension choice variable) which affects the payoff of all principals. Instead, in our paper the agent takes several actions, one action for each principal (multidimensional choice variable), and each action affects the other buyers' payoff through the positive externality in the technology. This difference allows this paper to provide empirical implications and policy recommendations which have not been addressed in the procurement literature.

In Dixit (1996), principals write contracts on all agent's outcome and the externality in contracting present in his model arises due to the public good nature of eliciting effort. In this paper, differently, principals contract only on part of agent's outcome.[14]

This paper also contributes to the literature on economies of scale/scope and the power of procurement contracts.[15] This link was primarily made by Rogerson (1992), who analyzes the contractor's behavior when he performs several activities that share common production costs - a reason pointed out by Teece (1990) for existence of economies of scale/scope. Rogerson shows that contractors optimally switch common costs to least powered contracts in order to get reimbursed for most of the production costs. Naturally, buyers anticipate this contractor's behavior, and design excessively high-powered contracts. Due to the difference in the nature of economies of scale/scope, Rogerson's model and this paper have different predictions about the relationship between power of contracts and the existence of such economies.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 characterizes the optimal cooperative contract, our benchmark case, which maximizes the net surplus of all buyers, taking into account all existing externalities associated to the contracting out problem. Section 4 characterizes the noncooperative contracting equilibrium which arises when buyers individually design their procurement contracts. Section 5 discuss the results

---

[13]Le Breton and Salanié (2003) apply the common agency framework to analyze a lobbying game in which special interest groups are not a priori organized or unorganized and the type of the politician is not common knowledge.

[14]Another difference is that Bernheim and Whinston (1986), and Dixit (1996) solve an intrinsic common agency problem, when the agent must accept or reject all contracts. Instead, this paper deals with a delegated common agency problem, when the agent can accept or reject any subset of contracts which are offered. Yet in the adverse selection multi-contracting setting, Martimort and Stole (2009) analyzes the market participation under delegated and intrinsic common agency games.

[15]This literature was originally developed by Willig (1979), Teece (1980), and Panzar and Willig (1981), and related to market structure and multiproduct industries by Bailey and Friedlaender (1982).

and formally presents the empirical implications of the paper. Section 6 provides a set of policy recommendations to improve the social welfare, and Section 7 analyzes the effect of competition among contractors on the power of equilibrium contracts. Section 8 concludes. Proofs of the propositions that are not in the text can be found in the appendix.

2 The Model

The model is an extension of the standard bilateral principal-agent problem with moral hazard to the case of many principals, where the principals are risk-neutral buyers and the agent is a risk-averse contractor.

Building the model on that framework, we consider an industry consisting of $n \geq 2$ buyers and a single contractor. Each buyer wants to procure a different and indivisible activity,[16] and the contractor is the only firm which owns the technology to perform those activities. The contractor may exert an unobservable effort (i.e., investment) in each activity to reduce production cost. Efforts have spillovers: the effort exerted in certain activity reduces also production cost of other activities. This effort spillover creates positive externality within activities.

2.1 Technology

■ **Production Cost.** The contractor incurs a production cost C_i to perform the activity for buyer-i. It is described by:

$$C_i(e_1, ..., e_n, \kappa, \varepsilon_i) = \beta - e_i - \frac{\kappa}{n-1} \sum_{j \neq i} e_j + \varepsilon_i, \qquad \text{for } i = 1, ..., n. \tag{1}$$

Equation (1) states that the contractor incurs a fixed cost $\beta > 0$ to perform activity-i but he can reduce it by making an effort e_i in activity-i. Furthermore, the contractor also enjoys some cost reduction in activity-i by making effort e_j in other activities $j \neq i$. Besides that, there is an unpredictable cost ε_i which realizes when the contractor performs the activity. For

---

[16]The presence of a multi-unit demand keeps the results unchanged, whereas it introduces a pricing decision problem, which is out of scope of this paper.

simplicity, we assume that ε_i's are independent and identically distributed (i.i.d.) according to a normal distribution with zero mean and variance σ^2, $N(0, \sigma^2)$. In addition, we assume that C_i is contractible and verifiable.

Clearly, the technology allows the contractor to benefit from positive externality when performing activities for different buyers: making an effort to reduce cost in a certain activity for one of the buyers, the contractor also reduces cost in other activities for the other buyers. This positive externality is captured by the parameter κ, which measures the sum of marginal impact of increasing effort in all other activities to reduce cost in the activity-i.[17] Let us assume that $\kappa \geq 0$, which means that the externality within activities is positive. When $\kappa = 0$, we have the standard procurement problem already analyzed by Weitzman (1980), and McAfee and McMillan (1986). In the case where $\kappa > 0$, we have the contribution of this paper.

As it turns out, the positive externality in the production process can also be interpreted as economies of scale (when the contractor performs the same activity for different buyers) or economies of scope (when the contractor performs different activities for different buyers), since the contractor can reduce the production cost per activity performing more than one activity.

The Veolia Transportation and Granite Construction Company examples, which were previously discussed, fit to the kind of positive externality (or economies of scale/scope) in this paper. Other examples like investment in training, know-how and ideas, which the knowledge can spill over the entire company, also illustrate this externality story.

■ **Effort Cost.** As discussed above, effort clearly has a social value of reducing production cost. However, it is costly. The contractor bears a cost for making them. For simplicity, we

---

[17]The totaly externality in activity-i is divided by $n-1$ in order to bound the externality effect of increasing n in the the production cost defined in (1). This assumption is not crucial for the result. It can be replaced by a less restrictive one which says that the positive externality is not strictly increasing in the number of activities n. For instance, the results hold if it is assumed that the production cost of the activity-i takes the following form

$$C_i(e_1, ..., e_n, \kappa, \varepsilon_i) = \begin{cases} \beta - e_i - \kappa \sum_{j=1, j\neq i}^n e_j + \varepsilon_i & , \qquad \text{if } n \leq h \\ +\infty & , \qquad \text{otherwise} \end{cases} \quad \text{with } h > 1.$$

assume that the cost of effort e_i has a quadratic form and is represented by

$$\psi(e_i) = \frac{1}{2\alpha}e_i^2. \tag{2}$$

The total contractor's effort cost for making effort in all activities is the sum of (2) over all n performing activities.[18] In expression (2), α measures the cost sensitivity to changes in effort. Note that, the higher α, the lower the marginal cost of exerting effort. In addition, let us assume that $\alpha > 0$, which implies that effort cost is positive.

Let us assume that effort is not observable. Hence, the only contractible variable is contractor's production cost.

2.2   Buyers and The Contractor

■ **Buyers.** All n buyers are risk neutral and each derives utility $v > \beta$ per activity performed.[19] Each buyer is indexed by i which belongs to the set of buyers $N = \{1, 2, ..., n\}$. Because effort is not contractible, they individually contract on the activity cost, defined in (1), offering the contractor a standard linear procurement contract.

The standard procurement contract offered by buyer-i to the contractor specifies a reimbursement fraction a_i of the contractor's monetary expenditures C_i in activity-i and a fixed fee b_i. They jointly characterize the buyer-i's net transfer T_i to the contractor, which takes the form

$$T_i(C_i) = b_i + a_i C_i, \ \text{with} \ a_i \in [0, 1]. \tag{3}$$

In this linear setting, contracts are fully characterized by the (b_i, a_i). Buyers, being risk neutral, are assumed to design contracts to maximize their own expected payoff, which is the utility derived from the activity minus the expected payment to the contractors, $v - E(T_i(C_i))$:

$$V(b_i, a_i, C_i) = v - b_i - a_i E(C_i). \tag{4}$$

---

[18]It is assumed that there is no substitutability or complementarity between efforts in the total effort cost function. Clearly, it is quite natural that efforts are complementary within similar activities performed by the same contractor: making an investment (effort) to reduce cost in activity-i, may reduce the cost of doing a similar investment (effort) in an activity-j. If such complementarity is assumed, the results of this paper are even stronger. Hence, for simplicity, it was assumed the effort cost function described in (2).

[19]This assumption guarantees that it is optimal for all buyers to procure for an activity.

■ **The Contractor.** The contractor is a risk averse firm with constant absolute risk aversion, whose preferences are represented by an CARA utility function described by

$$u(x) = -exp\{-rx\}, \tag{5}$$

where x is the total monetary transfer that contractor receives from buyers minus the production and effort costs for performing activities, and $r \geq 0$ is the coefficient of absolute risk aversion. The contractor is assumed to be risk averse because he faces shocks when undertaking activities.[20]

In order to describe the total expected contractor's payoff-utility when he performs activities for several buyers, let us first characterize the payoff-utility in a given activity-i. Then we sum over all activities to obtain his total payoff-utility.

On the one hand, the contractor earns a monetary transfer $T_i = b_i + a_i C_i$ from buyer-i for performing the activity-i. On the other hand, the contractor incurs production cost C_i, defined in (1), and an effort cost for exerting effort in the activity-i, defined in (2). Accordingly, the expected contractor's payoff for performing activity-i is:

$$E\left[u\left(b_i - (1 - a_i)C_i - \frac{e_i^2}{2\alpha}\right)\right]. \tag{6}$$

Since the contractor has CARA utility function, his expected payoff in equation (6) can be written as follows:

$$E\left[-exp\left\{-r\left[b_i - (1 - a_i)C_i - \frac{e_i^2}{2\alpha}\right]\right\}\right]. \tag{7}$$

From expression (1), note that C_i is composed by a sum of deterministic variables, ($e_1, ..., e_n$, κ), and a shock ε_i, which is a random variable normally distributed with zero mean and variance σ^2. In order to compute the expected value of $\exp(\varepsilon_i)$, we rely on the property of the normal distributions that: when a random variable ε_i is normally distributed with zero mean and variance σ^2, then $E[\exp(\varepsilon_i)]$ is equal to $exp(\frac{\sigma^2}{2})$. Accordingly, the expected contractor's

---

[20]The existence of risk averse contractors has been documented by Kawasaki and McMillian (1987), Asanuma and Kikutani (1992) and Yun (1999) when analyzing the relationship between manufacturers and contractors in Japan and Korea. Experimental works (see Kagel (1995) for a survey) and econometric studies (Athey and Levin (2001)) identify risk averse firms analyzing their behaviors in auctions. Salanié (1990) and Laffont and Rochet (1998) analyze the issue of risk aversion in a principal agent model with adverse selection.

payoff for performing activity-i is:

$$EU(b_i, a_i) = \left[- exp\left\{ - r\left[b_i - (1 - a_i)E[C_i] - \frac{e_i^2}{2\alpha} - \frac{r(1 - a_i)^2\sigma^2}{2}\right]\right\}\right]. \tag{8}$$

In equation (8), note that, the lower the reimbursement fraction of the contractor's cost in activity-i a_i, the more the contractor residual claimant for its cost saving. Since a_i lies in the interval between 0 and 1, then $(1 - a_i)$ measures the power of the procurement contract.

In the case that the contractor performs a subset I of the N existing activities, he will receive monetary transfers for performing those I activities, but he will also incur production and effort costs. Hence, his expected payoff will be:

$$E\left[u\left(\sum_{i \in I} b_i - (1 - a_i)C_i - \frac{e_i^2}{2\alpha}\right)\right]. \tag{9}$$

Applying to expression (9) the same algebraic manipulations employed in (6) and (7), we obtain the total contractor's expected payoff for performing the I activities:

$$EU(\{(b_i, a_i)\}_{i \in I}) = \left[- exp\left\{ - r\sum_{i \in I}\left[b_i - (1 - a_i)E[C_i] - \frac{e_i^2}{2\alpha} - \frac{r(1 - a_i)^2\sigma^2}{2}\right]\right\}\right], \tag{10}$$

where $\{(b_i, a_i)\}_{i \in I}$ are, respectively, the fixed fees and reimbursement fractions of the I activities.

■ **The Procurement Contracts.** The model described above involves two seemingly ad hoc assumptions, which coincides with the assumption in Holmstrom and Milgrom (1991) multitask paper. The more obvious one is that the contract that parties sign specifies a transfer which is a linear function of production cost. The second assumption is more conventional and therefore less likely to be noticed, but is no less troubling. It is the assumption that the contractor is required to make a single, once-and-for-all choice of how he will allocate this efforts during the relationship without regard to the arrival of performance information over time. A remarkable fact, which was established by Holmstrom and Milgrom (1987), is that these two simplifying assumptions are exactly offsetting in this model where the contractor has CARA preferences and the shocks are normally distributed. That is, the optimal linear

11

contract analyzed in this model coincides with the optimal contract in a principal-agent problem in which the agent (contractor): (i) chooses efforts continuously over time, and (ii) can observe his accumulated production cost before acting.

In view of its underlying assumptions, the model seems especially well suited for analyzing contracts between buyers and contractors represented by cost-reimbursement schemes paid over a short period, like a month, a quarter, or perhaps a year, in industries in which production costs are cumulative result of persistent efforts over time.

2.3   Sequence of Decisions and Events

Figure 1 describes the timing of the game. The game begins at date 0, when each buyer individually makes a take-it-or-leave-it offer of a procurement contract to the contractor.[21] Since the contracts are restricted to be linear, as described in (3), each buyer's offer can be represented by the fixed fee b_i and the reimbursement fraction a_i. With abuse of notation, let $N = \{1, 2, ..., n\}$ be the set of all contracts, and $\mathcal{P}(N)$ be the set of all subsets of the N, where I is a typical element of $\mathcal{P}(N)$.

At date 1, the contractor accepts or refuses any individual offer, having the right to accept all contracts in N, or any subset of contracts $I \in \mathcal{P}(N)$. At date 2, the contractor chooses an effort e_i to make in each activity-i, which belongs to I, in order to maximize his total expected utility-profit.

When the contractor accepts I contracts, he chooses efforts $\{e_i\}_{i \in I}$ which maximizes this expected payoff. In this case, his expected payoff is characterized by (10). Hence, the optimal efforts are the ones that solve the following problem:

$$\max_{\{e_i\}_{i \in I}} -exp\left\{ -r \sum_{i \in I} \left[b_i - (1 - a_i)E[C_i] - \frac{e_i^2}{2\alpha} - \frac{r(1 - a_i)^2\sigma^2}{2} \right] \right\} \tag{11}$$

where $\{(b_i, a_i)\}_{i \in I}$ are, respectively, the fixed fees and reimbursement fractions of the I contracts accepted by the contractor.

---

[21]The main results of this paper hold under less extreme assumption on how the buyers and the contractor reach an agreement over the terms of the contracts. However, it is important that all bargaining power is not fully allocated to the contractor.

The optimal effort in the activity-i is characterized by

$$\frac{e_i}{\alpha} = (1 - a_i) + \frac{\kappa}{n-1} \sum_{j \neq i} (1 - a_j), \text{ for all } i, j \in I, \qquad (IC)$$

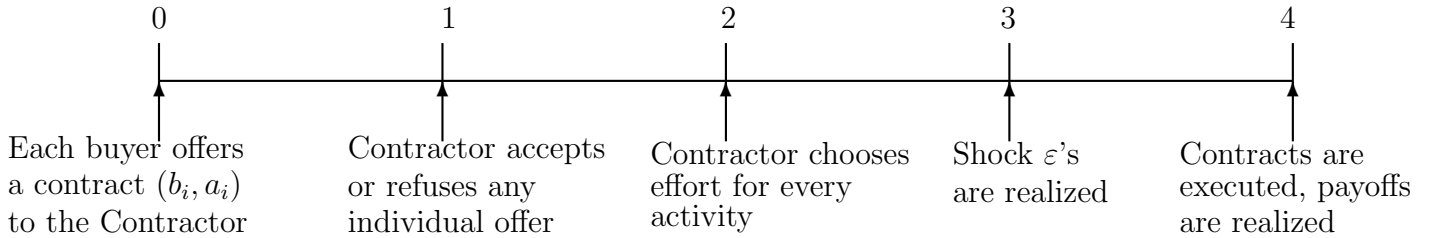which satisfies the necessary and sufficient conditions of program (11).

According to equation above, the contractor optimally chooses the effort in activity-i which equalizes its marginal cost to its marginal benefit. The marginal cost of increasing e_i is equal to $\frac{e_i}{\alpha}$, which is computed by deriving the effort cost in (2) with respect to e_i. The marginal benefit is the effect of increasing e_i on the total nonreimbursed production cost. This marginal effect corresponds to the sum of the marginal effect of e_i on nonreimbursed cost of activity-i, which is $(1 - a_i)$, and its marginal effect on nonreimbursed cost of the other activities, which is $\frac{\kappa}{n-1} \sum_{j \neq i} (1 - a_j)$. Note that the higher the power of the contracts $(1 - a_i)$ is, the higher the efforts e_i's will be.

As it turns out, equation (IC) corresponds to the incentive compatibility constraint in the multiprincipals one agent framework. It says that the contractor (agent) optimally chooses efforts given the incentive schemes $\{a_i\}_{i \in I}$ provided by the buyers (principals). Note that, if no incentives are provided ($a_i = 1$ for all $i, j \in I$), then all efforts are equal to zero. This is quite natural: if all costs are borne by the buyers (i.e., contracts are cost-plus), then the contractor does not have incentives to reduce cost.

In addition, equation (IC) describes how the incentive schemes provided by different buyers are connected in this model. In particular, it shows that when buyer-i sets high-powered incentive scheme (low a_i) to induce high effort and reduce production cost in his activity-i, he is also eliciting high effort in all other j activities, thereby reducing production cost of the activities for all other j buyers. This link between incentive schemes through the incentive compatibility constraint (IC) is the source of free-riding problem analyzed in this model: A buyer, when designing his contract, does not internalize that when inducing high effort in his activity, he is also providing incentives to reduce cost in activities for other buyers.

Finally, after making the effort choice, shocks ε_i realize in each activity at date 3. At date 4, contracts are executed, and payoffs are realized.

Figure 1: Timing of events

0	1	2	3	4
Each buyer offers a contract (b_i, a_i) to the Contractor	Contractor accepts or refuses any individual offer	Contractor chooses effort for every activity	Shock ε's are realized	Contracts are executed, payoffs are realized

3 Optimal Cooperative Contract: The Benchmark

The analysis of a free-riding problem requires a benchmark, which is useful for making allocation and welfare comparisons. The benchmark in this paper is the optimal cooperative contract. It is defined as the contract that maximizes the total surplus of all buyers, taking into account all externalities associated to this procurement problem. Indeed, this is the most appropriate benchmark since the free-riding problem analyzed in this paper arises because buyers do not completely internalize all the benefits of eliciting effort from the contractor.

Additionally, the cooperative contracting problem addressed in this section is useful to analyze situations in which a consortia of buyers offers a contract to a firm who has to perform activities for all associated members, and also the contracting out problem of a national or state government who has to contract a firm to provide public services/goods in several local communities.[22]

In the cooperative contracting problem, buyers cooperatively choose I activities that they find profitable to contract for, and the terms of the contracts of those activities $\{(b_i, a_i)\}_{i \in I}$. The optimal contract is the one that maximizes the sum of all buyers net expected payoff. It is formally defined as follows:

Definition 1 *The optimal cooperative contract is defined by* $(I^c, \{(b_i^c, a_i^c)\}_{i \in I^c})$ *which solves*

---

[22]The aim of this section is to characterize the optimal contract when the buyers cooperative contract out some activities. It does not state when buyers should organize themselves in consortia, or when the public good/services decision should be centralized. For that, it will be necessary to have a theory which describes the costs and benefits of coordinating (centralizing) buyers (local public services/goods), which is not the objective of this paper.

following problem:

$$\max_{I\in\mathcal{P}(N),\{(b_i,a_i)\}i\in I} E\Big[\sum_{i\in I}(v - b_i - a_i C_i)\Big]$$

s.t.

$$a_i \in [0,1]$$

$$\frac{e_i}{\alpha} = (1 - a_i) + \frac{\kappa}{n-1}\sum_{j\neq i}(1 - a_j) \quad i,j \in I \qquad (IC)$$

$$\max_{\{e_i\}_{i\in I}} E\Big[u\Big(\sum_{i\in I} b_i - (1 - a_i)C_i - \frac{e_i^2}{2\alpha}\Big)\Big] \geq U(0) \qquad (IR)$$

where I^c are the activities the buyers find profitable to contract for, and $\{(b_i^c, a_i^c)\}_{i\in I^c}$ are, respectively, the fixed fees and reimbursement fractions of those I^c activities.

The objective function in the problem above is composed by the sum of all buyers utility for the I activities, minus their total transfer to the contractor. The first constraint of the problem says that buyers choose the reimbursement fraction of the procurement contracts in an interval which goes from zero to one. When $a = 1$, the optimal contract is cost-plus, whereas when $a = 0$, it is a fixed-price contract. It is an incentive contracts, if $a \in (0,1)$. Expression (IC) is the incentive compatibility constraint, and says that buyers take into account that the contractor optimally chooses the effort in each activity. The last constraint, defined as (IR), is the participation constraint and says that the contractor must be better off accepting the I contracts than rejecting all of them. In the case that the contractor rejects all I contracts, he does not incur any cost, however he receives no monetary transfers. Note that there is an underlining assumption associated to expression (IR): buyers make an all-or-nothing offer to the contractor. It means that the contractor should accept to perform all I activities, otherwise no contract will be offered to him.

Solving the cooperative contracting problem, buyers face a trade off between earning benefits from inducing cost reduction and bearing the costs to compensate the contractor to bear some risk. Accordingly, if buyers want to induce cost reduction in activity-i, they must make the contractor more residual claimant, thereby lowering their a_i. However, this would increase contractor's exposure to risk, inducing him to demand a higher fixed fee b_i to per-

15

form the activity. By solving this trade-off, they optimally choose the fixed fees b_i's and the reimbursement fractions a_i's of the optimal contract.

The optimal contract $(I^c, \{(b_i^c, a_i^c)\}_{i \in I^c})$ which solves the cooperative contracting problem is characterized in the following proposition:

Proposition 1 *The **optimal cooperative contract** $(I^c, \{(b_i^c, a_i^c)\}_{i \in I^c})$ is characterized by:*

$$I^c = N \tag{12}$$

$$a_i^c = \begin{cases} 1 - \dfrac{\alpha(1+\kappa)^2}{\alpha n [\frac{2\kappa}{n-1} + \frac{\kappa^2(n-2)}{(n-1)^2}] + [\alpha(1-\frac{\kappa}{n-1})^2 + r\sigma^2]} \in (0,1) & , \ \textit{if } r\sigma^2 \geq \kappa(n-2) \\ 0 & , \ \textit{otherwise} \end{cases} \tag{13}$$

and

$$\sum_{i=1}^{n} b_i^c = n \left[(1 - a_i^c)\beta - \frac{(\alpha(1+\kappa)^2 - r\sigma^2)(1 - a_i^c)^2}{2} \right] \tag{14}$$

for all $i \in I$, where b_i^c and a_i^c are, respectively, the fixed fee and the reimbursement fraction of the optimal contract for activity-i.

Proposition 1 shows that buyers will optimally ask the contractor to perform all activities, $I^c = N$. This result comes directly from the assumption that $v - \beta > 0$, described in Section 2.2, which assumes symmetry between activities. In addition, it assumes that the net expected utility of each activity is always positive, even though the contractor does not exert any effort to reduce cost. Indeed, if all activities are symmetric and, independently of the efforts, all activities individually give positive net utility surplus for the buyers, then they will optimally demand all of them.

Additionally, Proposition 1 demonstrates that if the risk aversion and shock's variance are sufficiently high such that their product, $r\sigma^2$, is higher than a certain threshold $\kappa(n-2)$, then buyers optimally choose an incentive contract, whose reimbursement fraction a_i^c of the contracts is between 0 and 1. Otherwise, they choose fixed-price contracts, $a_i^c = 0$.

The optimal contract has a single solution for the the sum of all fixed fees, which is characterized in expression (14). However, due to the symmetry between activities, the fixed

fees $\{b_i^c\}_{i \in I^c}$ of the optimal contract are not uniquely determined. In particular, many different combinations of b_i's that satisfy expression (14) are solutions for the cooperative contracting problem described in Definition 1.

According to Proposition 1, the power of the optimal contract $(1 - a_i^c)$, implicitly described by (13), depends on the externality effect κ. The following corollary analyzes the relationship between those variables.

Corollary 1 *The optimal cooperative contract has the following properties:*

- *The power of the optimal contract $(1 - a_i^c(\kappa))$ is monotonically increasing in the externality effect, κ :*

$$\frac{\partial(1 - a_i^c(\kappa))}{\partial \kappa} \geq 0 \tag{15}$$

- *In the presence of the externality effect, $\kappa > 0$, the power of the optimal contract $(1 - a_i^c(\kappa))$ is higher than the power of the optimal contract $(1 - a_i^c(0))$ when the externality effect is nonexistent, $\kappa = 0$:*

$$(1 - a_i^c(\kappa)) > (1 - a_i^c(0)). \tag{16}$$

Corollary 1 shows that the higher the externality effect (measured by the parameter κ), the higher the power of procurement contracts. The economic intuition for these results is straightforward: The higher the positive externality within activities, the higher the effort's impact in reducing overall contractor's cost. Hence, the lower the buyer's transfer to compensate the contractor for the activity's cost. Since high-powered contracts induce higher effort, they will be the most profitable incentive schemes for buyers.

In addition to that, Corollary 1 demonstrates that the optimal cooperative contract with externality, $\kappa > 0$, provides higher-powered incentive schemes than the optimal contract without externality, $\kappa = 0$.

4 Noncooperative Contracting Equilibrium

The focus of this section is to analyze the free-riding problem in procurement design. This externality in contracting emerges in our model because buyers do not completely internalize all the benefits of eliciting effort from the contractor.[23]

To analyze the strategic interaction between procurement designers (buyers), we look at the Nash equilibrium in contract design. In this context, each buyer individually designs a procurement contract that maximizes his expected payoff, taking the other buyers' contracts as given. This equilibrium concept is formally defined as follows:

Definition 2 *The contract profile* $(I^{nc}, \{(b_i^{nc}, a_i^{nc})\}_{i \in I^{nc}})$ *is a **noncooperative contracting equilibrium** if and only if*
(i) For any given contract, $(I, \{(b_i, a_i)\}_{i \in I})$, *the contractor*

- *optimally chooses efforts in the I activities:*

$$e_i = \alpha \left[(1 - a_i) + \frac{\kappa}{n-1} \sum_{j \neq i} (1 - a_j) \right] \quad i, j \in I \qquad (IC)$$

- *is better off accepting the I contracts than any other J subsets of contracts:*

$$\max_{\{e_i\}_{i \in I}} E\left[u\left(\sum_{i \in I} b_i - (1 - a_i)C_i - \frac{e_i^2}{2\alpha} \right) \right] \geq \max_{\{e_i\}_{j \in J, \forall J \in \mathcal{P}(N)}} E\left[u\left(\sum_{j \in J} b_j - (1 - a_j)C_j - \frac{e_j^2}{2\alpha} \right) \right] \; (IR^{nc})$$

(ii) Given $(b_{-i}^{nc}, a_{-i}^{nc})$, *then buyer-i maximizes his expected payoff:*

$$(b_i^{nc}, a_i^{nc}) \in \arg\max_{(b_i, a_i)} E\left[v - b_i - a_i C_i \right]$$

$$s.t. \qquad a_i \in [0, 1].$$

This equilibrium definition corresponds to Perfect Nash Equilibrium in contracts. By this

---

[23]Yardstick competition, a cost comparison mechanism proposed by Shleifer (1985), may mitigate the free-riding problem analyzed in this paper. However, as Rogerson (2003) argues, such complex contracts are difficult and costly to implement. Consequently, buyers and contractors usually prefer to write simple contracts which have low informational requirements, as the ones characterized in this section.

definition, $(I^{nc}, \{(b_i^{nc}, a_i^{nc})\}_{i \in I^{nc}})$ is a noncooperative contracting equilibrium if the contract (b_i^{nc}, a_i^{nc}) maximizes the buyer-i expected payoff, $v - b_i - a_i C_i$, taking into account that other buyers are choosing the contracts $(b_{-i}^{nc}, a_{-i}^{nc})$. In addition, it has to satisfy to two other conditions to be an equilibrium contract. First, constraint (IC) is the incentive compatibility constraint, which states that the contractor optimally chooses the effort in each activity in I. Secondly, constraint (IR^{nc}) is the participation constraint of the game. It states that the contractor must be better off accepting the I contracts than any other J subsets of contracts, which includes rejecting all contracts. Note that, if contractor rejects all contracts, then he receives zero monetary transfer and does not incur any cost.

Proposition 2, described below, characterizes a **noncooperative contracting equilibrium** of this model:

Proposition 2 *The contract profile $(I^{nc}, \{(b_i^{nc}, a_i^{nc})\}_{i \in I^{nc}})$ described below is a **noncooperative contracting equilibrium**:*

$$I^{nc} = N \tag{17}$$

$$a_i^{nc} = 1 - \frac{\alpha\left(1 + \frac{\kappa^2}{n-1}\right)}{\alpha(1 + \kappa^2) + r\sigma^2}, \tag{18}$$

$$b_i^{nc} = (1 - a_i^{nc})\beta - \frac{\alpha(1 + \kappa)^2(1 - a^{nc})^2}{2} + \frac{r\sigma^2(1 - a^{nc})^2}{2}, \tag{19}$$

for all $i \in I$, where b_i^{nc} and a_i^{nc} are, respectively, the fixed fee and the reimbursement fraction of the contract for activity-i.

The equilibrium contracts characterized in Proposition 2 have some properties which are described as follows:

Proposition 3 *The equilibrium described in Proposition 2 has the following properties:*

- *The power of the noncooperative equilibrium contracts $(1 - a_i^{nc}(\kappa))$ is lower than the*

power of the optimal cooperative contract $(1 - a_i^c(\kappa))$,

$$(1 - a_i^{nc}(\kappa)) \le (1 - a_i^c(\kappa)), \tag{20}$$

holding with equality if and only if there is no externality, i.e. $\kappa = 0$;

- *the power of the noncooperative equilibrium contracts* $(1 - a_i^{nc}(\kappa))$ *is decreasing in n:*

$$\frac{\partial(1 - a_i^c(\kappa))}{\partial n} \le 0 \tag{21}$$

- *It is the unique equilibrium with n-contracts accepted;*

- *If* $\kappa \in [0, 1]$, *then it is more profitable than exclusive contracts. Hence, there is no ex-ante and no ex-post incentive toward exclusive dealing agreements.*

The first result in Proposition 3 says that the equilibrium procurement contracts have lower-powered incentive schemes than the optimal cooperative contracts. This conclusion is quite intuitive: A buyer, who individually offers a procurement contract for a contractor, does not fully internalize that eliciting higher-powered incentives provides cost reduction in the contractor's activities, which benefits other buyers. Hence, in equilibrium buyers offer lower-powered procurement contracts than what would be offered if buyers cooperatively designed their contracts.

To investigate in more details this effect, let us first look at the case in which κ is equal to zero. Clearly, in this case there is no externality effect in the production cost, and consequently, there is no free-riding problem. Therefore, the reimbursement fraction of the procurement contract in the optimal cooperative contracting problem and in noncooperative contracting equilibrium are the same.

Secondly, let us analyze the case in which $\kappa > 0$. It corresponds to the case in which the externality effect exists and, consequently, the free-riding problem in procurement design is present. Doing so, let us illustrate it in a simple case where the number of activities-buyers is equal to two, $n = 2$. To perform the analysis, we start with a key observation: in both situations, optimal cooperative contract and noncooperative contracting equilibrium,

the contractor is indifferent between accepting and rejecting all contracts. This means that the contractor's participation constraint is binding in both these cases. Consequently, it corresponds to say that the total transfer received by the contractor is equal to his total costs, which are composed by production costs, effort costs and contractor's desutility for bearing risk:

$$
\begin{aligned}
T_1 + T_2 = \ & E[C_1(e_1(a_1,a_2), e_2(a_1,a_2), \kappa, \varepsilon_1)] + E[C_2(e_2(a_1,a_2), e_1(a_1,a_2), \kappa, \varepsilon_2)] \\
& + \frac{1}{2\alpha}\left(e_1^2(a_1,a_2) + e_2^2(a_1,a_2)\right) + \frac{r\sigma^2}{2}\left[(1-a_1)^2 + (1-a_2)^2\right]
\end{aligned}
$$

First, note that in the expression above, the efforts e_1 and e_2 are function of the reimbursement fraction of the procurement contracts, a_1 and a_2. They come from the incentive compatibility constraint described in (IC), which says that the power of contracts affects the contractor's effort decision.

Now, let us see the forces which make the optimal cooperative contract different from the noncooperative contract. In the optimal cooperative contract, buyers jointly choose $(b_i, a_i)_{i \in I}$ such that minimize the total transfers, $T_1 + T_2$.

In contrast, in the noncooperative contracting equilibrium, each buyer chooses b_i and a_i that minimize its own transfer. For instance, buyer-1 minimizes

$$
\begin{aligned}
T_1 = \ & E[C_1(e_1(a_1,a_2), e_2(a_1,a_2), \kappa, \varepsilon_1)] + E[C_2(e_2(a_1,a_2), e_1(a_1,a_2), \kappa, \varepsilon_2)] \\
& + \frac{1}{2\alpha}\left(e_1^2(a_1,a_2) + e_2^2(a_1,a_2)\right) + \frac{r\sigma^2}{2}\left[(1-a_1)^2 + (1-a_2)^2\right] - T_2.
\end{aligned}
$$

Taking T_2, which is equal to $b_2 + a_2 E[C_2(e_2(a_1,a_2), e_1(a_1,a_2), \kappa, \varepsilon_2)]$, off his problem, buyer-1 does not fully internalize that by eliciting higher effort from the contracting (i.e., lowering a_1), he will reduce the contractor's production cost.

Proposition 3 also provides other interesting results. It says that the higher the number of activities n performed by the same contractor, the lower the power of the equilibrium contract $(1 - a_i^{nc}(\kappa))$. In addition, it states that the noncooperative contracting equilibrium described in Proposition 2 is the unique equilibrium when the N contracts are accepted. Furthermore, it shows that those equilibrium contracts are more profitable for buyers than

exclusive contracts if the externality effect is not so big, $\kappa \in [0,1]$. An implication is that buyers and the contractor have no ex-ante or no ex-post bilateral incentives to write exclusive dealing contracts. Intuitively, this result says that despite the fact that the free-riding effect leads to low-powered contracts and, consequently, low effort, those negative effects do not offset the benefits of having positive externality for providing activities for several buyers.[24]

At this time, it is worth recalling Corollary 1 and comparing the effect of the externality on the power of contracts in the optimal cooperative contracts with the noncooperative equilibrium contracts. Corollary 1 says that the higher externality effect κ is, the higher the power of optimal cooperative contracts will be. This result comes from the fact that the higher the positive externality within activities, the higher the impact of an effort in reducing overall contractor's cost. Hence, it is profitable for buyers to increase the power of the optimal cooperative contracts.

In contrast, in the noncooperative equilibrium these cooperative statics are not straightforward. These results follow from the fact that there are two opposite forces determining the power of the contracts in equilibrium: the positive externality effect and the free-riding effect. The positive externality effect is the same one discussed in Corollary 1: The higher the positive externality within activities is, the higher the power of contracts will be.[25] However, the existence of the free-riding problem puts pressure to lower the power of the equilibrium contracts. Intuitively, the higher the externality effect κ is, the lower the private contractor's benefit for eliciting effort from the contractor will be. Hence, the lower the power of the contracts. In particular, this free-riding effect increases with the number of activities n.

Corollary 2 analyzes the effect of externality, measured by κ, on the power of equilibrium contracts $(1 - a_i^{nc})$.

Corollary 2 *The power of the noncooperative equilibrium procurement contracts* $(1 - a_i^{nc})$

---

[24]In contrast, Martimort (1996) shows that exclusive dealing can be optimal in a multi-contracting environment with adverse selection. In particular, he demonstrates that depending on the extent of the adverse selection problem and on the complementarily or substitutability of their brands, manufacturers prefer to use either a common or an exclusive retailer.

[25]A buyer, who takes as given the other buyers' contracts, has more incentive to induce the contractor's effort when the externality effect κ is high. It happens because: the higher κ, the higher the reduction in the contractor's total cost for eliciting more effort is, and the lower is the buyer's fixed fee transfer to the contractor will be. Hence, the higher the benefit of high-powered contracts.

have the following relationship with the positive externality effect (economies of scope/scale), measured by κ:

- *When the number of buyers contracting with the same contractor is sufficiently low, $n \leq 2 + \frac{r\sigma^2}{\alpha}$, then the higher the contractor's positive externality for performing different activities, the higher the power of equilibrium procurement contracts: $\frac{\partial(1-a_i^{nc})}{\partial\kappa} \geq 0$;*

- *When the number of buyers contracting with the same contractor is sufficiently high, $n > 2 + \frac{r\sigma^2}{\alpha}$, then the higher the contractor's positive externality, the lower the power of equilibrium procurement contracts: $\frac{\partial(1-a_i^{nc})}{\partial\kappa} < 0$.*

Corollary 3 and 4 conclude the section providing additional results on the power of procurement contracts in equilibrium.

Corollary 3 *When the number of buyers (principals) contracting with the same contractor is sufficiently high, $n \geq 2 + r\sigma^2$, then the noncooperative equilibrium contracts with externality, $\kappa > 0$, have lower-powered incentive schemes than the optimal cooperative contract without externality, $\kappa = 0$,*

$$(1 - a_i^{nc}(\kappa)) \leq (1 - a_i^c(0)).$$

Corollary 4 *If $n \geq 3$, then each buyer's expected transfer in the noncooperative equilibrium increases with the number of buyers (principals),*

$$\frac{\partial T_i^{nc}}{\partial n} > 0.$$

Corollary 3 says that, even in the presence of positive externality, the equilibrium contracts have lower incentive power than the optimal contract when the positive externality is nonexistent. Corollary 4 says that the higher the number of activities performed by the same contractor for all buyers, the higher the transfers paid by each buyer. Consequently, the lower the buyer's expected payoff. It implies that the free-riding effect does damage the buyer's payoff, and in particular, that negative impact increases with the number of buyers. In addition, it shows that negative free-riding effect dominates the benefits of having the contractors performing several activities (i.e., the positive externality).

The negative relationship between buyer's transfer and the number of buyers, described in Corollary 4, has narrow connections to some results in the public good literature. In particular, it is related to Mailath and Postlewaite (1990) and Hellwig (2003)'s result that when the number of agents who contribute for a public good increases, the provision of public goods decreases. The common feature between this paper and the public good literature comes from the fact that eliciting effort in a multi-contracting setting has a public good nature: a buyer, who gives incentives to the contractor to reduce cost in his own activity, is at the same time contributing to reduce cost in activities for other contractors.

5 Empirical Implications

The main contribution of this paper is to explain how the existence of a positive externality creates free-riding problem in procurement design, which leads to excessive low-powered contracts in different industries. A testable model on the power of procurement contract has to deliver conditions under which low-powered contracts are more likely to be used. Our model yields potentially testable implications describing circumstances under which low-powered contracts should be used more frequently.

5.1 Externality in production, and economies of scope/scale: the likelihood of low-powered contracts

An empirical implication which can be derived from this paper refers to the relationship between κ and the power of procurement contracts. It can be obtained directly from Corollary 2, and it is formally stated in the following implication:

Implication 1 *Consider the industries in which economies of scope and scale in the production process are prevalent, $\kappa > 0$. When the number of buyers contracting with the same contractor is sufficiently low, $n \leq 2 + \frac{r\sigma^2}{\alpha}$, the higher contractors' economies of scope/scale κ in a given industry, the more likely the use of high-powered procurement contracts. On the other hand, when the number of buyers contracting with the same contractor is sufficiently high, $n \geq 2 + \frac{r\sigma^2}{\alpha}$, then this relationship is inverted: the higher the contractors' economies of*

scope/scale κ, the more likely the use of low-powered procurement contracts.

This implication comes from that fact that two different forces determine the power of procurement contracts: the externality effect and the free-riding effect. The externality effect offsets the free-riding effect for low values of n, establishing a negative relationship between the externality parameter κ and the power of the equilibrium contracts. Conversely, for high values of n, the free-riding effect dominates, which makes the power of the equilibrium contracts decrease with κ.

Another implication can be easily derived from Corollary 3. In summary, this corollary suggests: when the number of buyers contracting with the same contractor is sufficiently high, then the noncooperative equilibrium contracts in the presence of externality in the production process (economies of scale/scope) have lowered-powered incentive schemes than the optimal cooperative contracts when such economies are nonexistent.

In order to derive an empirical implication from this result, let us first note that the optimal cooperative contract without externality coincides with the noncooperative contract without externality. It is quite natural since the source of inefficiency in multi-contracting is the externality. Therefore, the empirical implication is formally expressed as follows:

Implication 2 *We consider industries in which the number of buyers contracting with the same contractor is sufficiently high, $n \geq 2 + r\sigma^2$. Then, industries in which economies of scale/scope in the production process are prevalent ($\kappa > 0$) shall have lower-powered procurement contracts than industries in which externalities of this kind are nonexistent ($\kappa = 0$).*

Other papers in the literature have related economies of scale/scope to the power of procurement contracts. For instance, Rogerson (1992) and (1994) analyze the contractor's behavior when he performs several activities which share common production costs. Rogerson shows that the contractor optimally swifts common costs to least powered contract in order to get reimbursed for most of the production costs. Naturally, an implication of Rogerson's result is that contract designers will anticipate the contractor's behavior and design excessively high-powered contracts in a way to avoid the overhead allocation of costs.

As it turns out, Rogerson (1992) and (1994) provide different empirical implications con-

cerning the relationship between the power of procurement contracts and the intensity of economies of scale/scope in certain industries: Rogerson's models suggest that this relationship should be positive in order to prevent from contractor's cost overhead behavior. By contrast, this paper, by Corollary (2), argues that the same relationship should be negative when the number of buyers contracting with the same contractor is sufficiently high.

Clearly, these opposite empirical implications come from the different nature of the economies of scale/scope addressed in this paper and Rogerson's. In Rogerson's papers, the economies of scale/scope arises due to the existence of common assets, inputs and workers, which are used in different activities of the contractors. As such, it suggests that there should be a positive relationship between the power of procurement contractors and the share of common cost used by the contractor in his different activities.

In contrast, in this paper the economies of scale/scope emerge due to the existence of common methods or procedures which are developed by a contractor's branch (i.e., activity), and can be used for all other contractor's activities. Hence, it implies that we should observe in the data a negative relationship between the power of procurement contractors and the new methods and procedures developed by the contractors.

5.2 Incomplete Contracts, renegotiation and the number of buyers: on the pervasiveness of cost-plus contracts

In some industries, only two types of procurement contracts are observed: fixed-price and cost-plus contracts. That is, for instance, the case of the US building construction industry, described by Bajari, McMillan and Tadelis (2008), and the French public urban transportation, highlighted by Gagnepain and Ivaldi (2002).

The literature has already discussed the reasons why we only observe those extreme contracts. Bajari and Tadelis (2001), relying on Townsend (1979), and Gale and Hellwig (1985), argue that the existence of measurement costs (i.e., auditing or verification cost) explains the choice of fixed-price contract ($a = 0$) to any other cost-sharing contract with $a \in (0, 1]$. Basically, fixed-price contracts do not require the measurement of production costs, whereas any cost-sharing contract requires such measurement, which is costly. This leads to a non-

convexity in the cost of measuring and monitoring product costs. An immediate implication of the existence of auditing cost is that fixed-price contracts will dominate contracts that are close to them.

On the other hand, the incompleteness of contracts explains the choice of cost-plus contract ($a = 1$) to any other procurement contracts with $a \in [0, 1)$. As argued by Bajari and Tadelis (2001), cost-plus contracts are more flexible and easier to renegotiate under noncontratactable contingencies. Hence, when renegotiation is costly, cost-plus contracts dominate any other contracts with low-powered incentive scheme where $a \in [0, 1)$.

The focus of this section is to derive conditions under which cost-plus contracts arise in industries plagued by incomplete contracts and costly renegotiations. In order to provide empirical implications for those industries, let us add some additional assumptions to the standard model described in section 2.

■ **Incomplete Contracts and Renegotiation.** Assume that there are some states of the nature in which contracts are incomplete. It occurs with probability $\mu \in (0, 1)$. It can happen because some contingencies happen (e.g., site conditions in the building construction industry) and a buyer decides to change the demand or project after the contract has been signed with the contractor, and the contractor had optimally chosen the effort. Under incompleteness, buyer and contractor renegotiate the contract. Following Bajari and Tadelis (2001), let us assume that cost-plus contract ($a = 1$) have zero renegotiation cost, whereas the renegotiation of any other contract[26] with $a \in [0, 1)$ costs K.

In this extended model, let us derive the conditions in which cost-plus contracts emerge in equilibrium. A sufficient condition is that each buyer is better off relying on cost-plus than relying in the equilibrium contracts described in Section 4.[27]

---

[26]The renegotiation process can take different ways, which will depend, among other things, on the bargain power of buyers and the contractor. In each different renegotiation process, the cost will be different. For simplicity, we do not model the renegotiation process. That is the reason why we consider the negotiation cost as an exogenous variable.

[27]A sufficient and necessary condition for an equilibrium with cost-plus contracts is that each buyer is better off relying on cost-plus than deviation offering another contracts. If all buyers, except buyer-i, offer a cost-plus contracts, then the contractor effort will be $e_i^* = \alpha(1 - a_i) < e_i^{nc}$, and $e_j^* = \frac{\kappa\alpha(1-a_i)}{n-1} < e_j^{nc}$. Given these effort levels, the cost will be $C_i^* > C_i^{nc}$, and then the net transfer to the contractor when deviating T_i^* is lower than the net in equilibrium T_i^{nc} described in Section 4. For this reason it is sufficient to compare the buyer's payoff under cost-plus contracts with the equilibrium contracts described in Section 4.

By relying on a cost-plus contractor, a buyer knows he will never pay the renegotiation cost. However, he will anticipate that the contractor will choose all efforts equal to zero, having production cost C_i equals to β. Since in equilibrium with only cost-plus contracts $a_i = 1, \forall i$, the contractor does not bear any risk and makes zero effort, buyer's transfer to the contractor will be equal to the production cost of the activity which is β. So, buyer's expected payoff for offering cost-plus contracts will be:

$$v - \beta. \tag{22}$$

Instead, if all buyers offer the equilibrium contracts described in Section 4, then, with probability $1 - \mu$, they will not have to renegotiate the contracts, and each one will have a payoff equal to $v - T_i^{nc}$. However, with probability μ, they will renegotiate the contracts, and each one ends up with payoffs equal to $v - T_i^{nc} - K$. Therefore, buyer's expected payoff for offering the equilibrium contracts described in Section 4 will be:

$$v - T_i^{nc} - \mu K. \tag{23}$$

Expressions (22) and (23) give us the necessary ingredients to verify when that cost-plus contracts will be emerge in equilibrium. That will occur when buyer's payoff for choosing $v - \beta$ is greater than $v - T_i^{nc} - \mu K$, buyer's payoff for choosing the equilibrium contract. After some simple algebra, we can show that the cost-plus contracts will emerge in equilibrium when the renegotiation cost is sufficiently high:

$$K \geq \overline{K}, \text{where } \overline{K} = \frac{\beta - T_i^{nc}}{\mu}. \tag{24}$$

The following proposition formally characterizes the condition for cost-plus contracts to emerge in equilibrium in a model with incomplete contracts and renegotiation.

Proposition 4 *If $K \geq \overline{K}$, where $\overline{K}$ is defined in (24), then cost-plus contracts are equilibrium contracts in a model with incomplete contracts and renegotiation.*

Note that, through expression (24), whenever buyer's transfer in equilibrium T_i^{nc} decreases,

so will the threshold $\overline{K}$. This result is quite intuitive since by decreasing T_i^{nc}, the benefit of offering the equilibrium contract also reduces.

In order to test the result characterized above, an econometrician will need information about the renegotiation cost to proceed the analysis. However, in many situations the renegotiation cost is not observable.[28] In general, the only information that the econometrician has is that K is distributed according to a certain density function $G(.)$.

With the information described above, an econometrician derives that the probability of cost-plus contracts emerging in equilibrium is equal to:

$$Prob(K \geq \overline{K}) = 1 - G(\overline{K}) \tag{25}$$

Indeed, a testable model on power of procurement contracts must establish what increases or reduces the probability of observing cost-plus contracts, described in (25). In particular, it has to characterize what changes $\overline{K}$, and the direction of the change. This model does so by using the result from Corollary 3. Combining the Corollary 3 with the implication discussed above, we can derive the following implication:

Implication 3 *The higher the number of buyers per contractor, n, the more likely it is that the cost-plus contract will be chosen.*

Bajari, McMillan and Tadelis (2008) provide some evidence which is consistent with this implication, showing that the higher the number of contracts carry out by a contractor, the higher the probability that cost-plus contracts will be offered to this contractor.

Bajari, McMillan and Tadelis (2008) offer a different explanation for such empirical finding, based on Banerjee and Duflo (2000). Banerjee and Duflo argue that in industries in which the services (or goods) are very complex and the quality of the final product is very different from one contractor to another, then buyers would not offer contracts based on competitive bidding for fixed-price contractors. Instead, buyers optimally award an activity to the contractor who has built a good reputation over time, and they negotiate a cost-plus contract which will lead

---

[28]Bajari, McMillan and Tadelis (2008) uses the complexity of tasks as an instrument for renegotiation cost since the more complex the task are, the more likely that contract will be incomplete.

to a high quality service or good. Relying on this explanation, Bajari, McMillan and Tadelis argue that the number of jobs done by a contractor is a measure of reputation. Therefore, it explains the positive relationship between the number of jobs done by a contractor and the probability that cost-plus contracts will be offered to the contractor.

6 Policy Recommendations

In this paper, excessively low-powered incentive contracts emerge as an inefficient allocation due to a free-riding problem: buyers offer lower-powered contracts than cooperative buyers.

One may wonder about the possible solutions for this problem. For the public sector, a natural policy recommendation is the centralization of local and state government purchases. Centralizing the purchase of services, with positive externalities among themselves, will induce the procurer to internalize a big portion of the benefit of eliciting effort.

Nevertheless, centralization in public sector is never an easy policy to implement, since it is usually accompanied by a reduction of local government's power.[29] Alternatively, national authorities could create mechanisms which induce local governments to buy goods and public services together. Inducing them to design procurement contract jointly can potentially make them to internalize possible existing externalities in contracting out for services or goods. An example of this kind is the Brazilian System of Price Registration, a procurement system which allows different branches of the national Brazilian government (administrative units, public institutes, hospitals, universities, and others) to buy jointly different goods. However, it has been used only to buy simple goods, which are more likely to be optimally purchased through fixed-price contracts. In the spirit of this paper, this system should be extended to encompass the purchase of public services, where positive externalities are more likely to be present.[30]

As in the public sector, the private sector should also be aware of free-riding in procurement and try to design mechanisms to overcome it. Agreements and consortia of buyers should

---

[29]Hart and Holmstrom (2009) analyze the trade off between autonomy and coordination faced by an organization which sees some benefits from integrating a set of activities.

[30]The Brazilian System of Price Registration (Sistema Brasileiro de Registro de Preços) is regulated by the Law n° 8.666 created in 1993 and has been widely used in several branches of the Brazilian government.

be encouraged, since they may induce buyers to design procurement contracts jointly, and internalize all existing positive externalities among different activities.[31]

Nevertheless, solutions of these kind are not always easy to implement and rely strongly on the capacity of buyers to coordinate themselves. One may wonder if there is any simple rule that can be designed to improve the social allocation.

Proposition 5 *If $r\sigma^2 \leq \overline{r\sigma^2}$, then there exists $\overline{n}$ such that for all $n \geq \overline{n}$ buyers are better off committing to fixed-price contracts, $a = 0$, than relying on linear contracts, $a \in (0,1)$.*

Proposition 5 states that laws in the public sector that bind public entities to offer fixed-price contracts rather than choosing individually their own contracts can be welfare-enhancing. Binding public entities to offer fixed-price contracts ($a = 0$) prevents a buyer from free-riding behavior, since under the fixed-price contracts buyers internalize all the benefit of eliciting effort. However, it transfers too much risk for the contractor, who requires a high compensation for bearing risk of the activity. When the risk and the variance of the shock are not so high, expressed by the condition $r\sigma^2 \leq \overline{r\sigma^2}$ above, then buyers are better off jointly relying on fixed-price contracts.

The result in Proposition 5, discussed in the previous paragraph, provides the theoretical foundations for the following policy recommendation:

Policy Recommendation *Laws that force public sector to award fixed-price contracts competitive bidding should be adopted in industries in which:*

- *there are a lot of buyers and a few contractors;*

- *risk borne by contractors is low enough.*

Bajari and Tadelis (2001), and Bajari, McMillan and Tadelis (2008) argue that cost-plus contract are more efficient than fixed-price contracts under certain circumstances (i.e., under incompleteness of contracts and high costs of renegotiation). They advice the laws which

---

[31]Of course, regulators should be aware that promoting agreements between buyers in noncompetitive markets can also give them additional incentive to collude: Buyers, whose activities are more likely to be affected by externality effects, are also the same ones who compete for the same consumers.

oblige the US public entities to award fixed-price contracts by competitive bidding should be withdrawn from FAR (Federal Acquisition Rules).

Note that Bajari and Tadelis (2001), and Bajari, McMillan and Tadelis (2008) have a policy recommendation which is contrary to the one in this paper. Naturally, these opposite guidelines come from differences in accounting for the pervasiveness of low-powered procurement contracts in certain industries. Bajari and Tadelis, and Bajari, McMillan and Tadelis argue that low-powered contracts (i.e., cost-plus) are more often used than other contracts because they provide better allocation. In contrast, this paper argues that low-powered contracts emerge in equilibrium as an inefficient allocation due to a free-riding problem.

7 Competition between Contractors

In the basic model, described in Section 2, there is only one contractor providing services for all buyers. However, in the real world, many potential contractors compete for provision of goods and services for different buyers. Hence, it will be interesting to analyze how competition between contractors impacts the power of incentive schemes in equilibrium.

To introduce competition between contractors, we make a modification in the basic model, based on McAfee and McMillan (1986). In this extension, potential contractors compete for activities bidding for the fixed fee b_i of the procurement contracts offered by buyers.[32]

This extended model will be called **Procurement Auction Design Game**. Here are the additional assumptions to the basic model and the new timing of the game:

■ **Potential Contractors.** Assume that there is a continuum of potential contractors, each one is indexed by its fixed cost β. In addition, assume that β is distributed according to a density function $F(\beta)$, with $\frac{\partial F(\beta)}{\partial \beta} = f(\beta)$, and $f(\beta) \neq 0$ for all β in the interval $[\underline{\beta}, \overline{\beta}]$. For simplicity, assume that the contractor's fixed cost β is public information.

■ **Sequence of Events.** Figure 2 describes the timing of the Procurement Auction Design

---

[32]Laffont and Tirole (1987, 1988) suggest a different approach to model competition among contractors. In particular, they consider the auctioning for incentive contracts of an indivisible project among several contractors. Contractors have private information about their future cost at the bidding stage, and the selected firm ex post invests in cost reduction.

Game. At date 0, each buyer individually chooses and posts the reimbursement fraction $a_i \in [0,1]$ of a contract awarded to the contract winner. At date 1, every contractor bids for the fixed fee b_i of each contract. The contractor who bids for the lowest fixed fee of buyer-i'contract b_i, will perform the activity for the buyer-i. At date 2, buyer-i awards activity-i to the winner, and, at date 3, the contract winner(s) choose(s) the effort e_i to make in each activity in order to maximize his expected utility profit. After making the effort choice, at date 4 the shocks in each activity ε_i realize. Contracts are executed, and payoffs are realized at date 5.

As it turns out, this extended model is a dynamic game with perfect information, and can be divided in two sequential stages. The first one is the Procurement Design stage, where buyers choose $a_i \in [0,1]$ and award the activity to the contract winner. It corresponds to date 0 in Figure 2. The second one is the Procurement Auction stage, where contractors compete and make effort in the activities. It is represented by dates 1 to 5 in Figure 2.

Figure 2: Timing of Procurement Auction Design Game
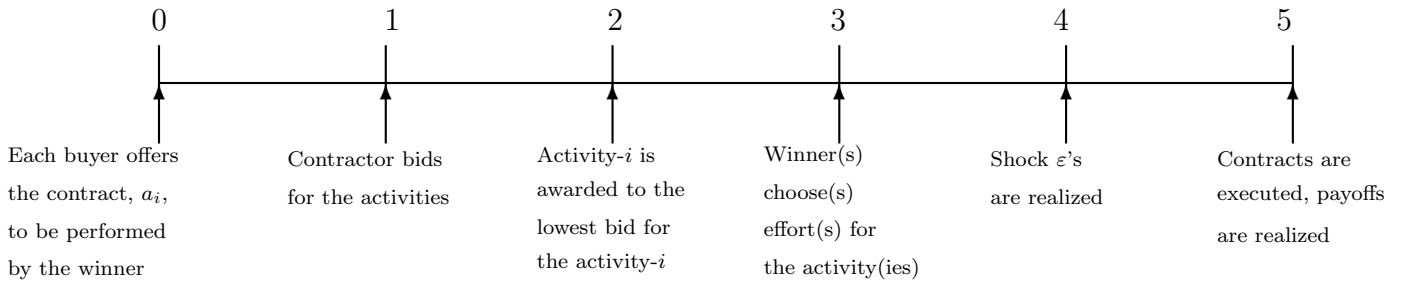
■ **Sub-game Perfect Nash Equilibrium.** For games of this kind, the relevant equilibrium concept is Sub-Game Perfect Nash Equilibrium (SPNE). This equilibrium can be characterized by the backward induction. Doing so, we first derive the Nash Equilibrium in the Procurement Auction stage, and then proceed to characterize the Nash equilibrium in the Procurement Design stage.

Nash Equilibrium in the Procurement Auction stage: In this stage, contractors bid for the right to perform the activity for each buyer. Since there is no private information, and the

only source of asymmetry among contractors is the fixed cost β, the kind of competition which takes place at this stage of the game is the standard Bertrand competition between a continuum of asymmetric agents.

As such, contractors fiercely compete for the n activities and, in equilibrium, they make zero profit. As a result, the sum of all bids of certain contractors $\beta \in [\underline{\beta}, \overline{\beta}]$ will be equal to total cost of performing all activities, which is expressed by:

$$\sum_{i=1}^{n} b_i(\beta) = \sum_{i=1}^{n} \left[(1 - a_i)E[C_i(\beta)] + \frac{e_i^2}{2\alpha} + \frac{r(1 - a_i)^2\sigma^2}{2} \right] \tag{26}$$

The equilibrium outcome of this competition is that the most efficient contractor, the one whose $\beta = \underline{\beta}$, wins all procurement auction, and performs every activity for all buyers. Due to the presence of the positive externality within activities, many bidding strategies for each activity leads to the total bidding equilibrium strategy described in (26). To proceed to the Procurement Design Stage, we have to pick one of them. So, let us take bidding strategy equilibrium which the bid for the activity-i is equal the contractor's cost bearing in activity-i:

$$b_i(\beta) = (1 - a_i)E[C_i(\beta)] + \frac{e_i^2}{2\alpha} + \frac{r\sigma^2(1 - a_i)^2}{2} \tag{27}$$

It is easy to see that the bidding strategy in (27) satisfies to total bidding strategy equilibrium requirement described in (26).

Nash Equilibrium in the Procurement Design stage: In this stage, each buyer individually chooses the optimal reimbursement fraction of his procurement contract, a_i, taking into account the bidding strategy equilibrium from the Procurement Auction stage, described in (27). The buyer's problem and the equilibrium associated to this strategic interactions among them is formally defined in the following definition:

Definition 3 *The contract profile* $(I^{pa}\{(b_i^{pa}, a_i^{pa})\}_{i \in I^{pa}})$ *is a **procurement auction design contracting equilibrium** if*

$$(b_i^{pa}, a_i^{pa}) \ \text{solves} \ \max_{b_i, a_i} E\left[v - b_i - a_i C_i \right]$$

s.t.

$$a_i \in [0, 1], \ (IC), \ and \ (27)$$

for given $(b^{pa}_{-i}, a^{pa}_{-i})$, *for all* $i \in I^{pa}$.

The buyer's problem in this procurement design is similar to the buyer's problem in Definition (2). The only difference is that in Definition (3) the bidding strategies, represented by constraint (27), replace the participation constraint (IR^{nc}) in Definition (2).

The next proposition compares the power of the procurement auction equilibrium contracts with the noncooperative equilibrium contracts discussed in Proposition (2):

Proposition 6 *If* $\kappa \in (0, 1)$, *then the power of the procurement contracts in the procurement auction design game* $(1 - a^{pa}_i)$ *is lower than the power of the noncooperative equilibrium contracts* $(1 - a^{nc}_i)$,

$$(1 - a^{pa}_i) < (1 - a^{nc}_i), for \ all \ i. \tag{28}$$

This result says that in the presence of competition of contractors for the fixed fee, the free-riding effect becomes even stronger. To illustrate this, let us again, for simplicity, take $n = 2$. As we have discussed in Section 4, buyer-1 minimizes

$$
\begin{aligned}
T_1 \ = \ & E[C_1(e_1(a^{pa}_1, a^{pa}_2), e_2(a^{pa}_1, a^{pa}_2), \kappa, \varepsilon_1)] + E[C_2(e_2(a^{pa}_1, a^{pa}_2), e_1(a^{pa}_1, a^{pa}_2), \kappa, \varepsilon_2)] \\
& + \frac{1}{2\alpha}\left(e_1^2(a^{pa}_1, a^{pa}_2) + e_2^2(a^{pa}_1, a^{pa}_2)\right) + \frac{r\sigma^2}{2}\left[(1 - a^{pa}_1)^2 + (1 - a^{pa}_2)^2\right] - T_2.
\end{aligned}
$$

where T_2 is equal to $b_2(a^{pa}_1, a^{pa}_2) + a^{pa}_2 E[C_2(e_2(a^{pa}_1, a^{pa}_2), e_1(a^{pa}_1, a^{pa}_2, \kappa), \varepsilon_2)]$. Note that, unlike the noncooperative contracting problem, the contractor's bid for the fixed fee of activity-2, $b_2(a^{pa}_1, a^{pa}_2)$, depends on the reimbursement fraction of the contractor, and that is the reason for the free-riding problem to be stronger in the Procurement Auction Design Game.

The explanation for this is quite intuitive: When buyer-1 decreases a^{pa}_1, he elicits higher effort, and provides higher incentives to reduce cost. Conceivably, lower cost will be translated into a lower bid for activity-1 and 2. As the expression above shows, buyer-1 does not enjoy any profit from lower bids in activity-2, $b_2(a^{pa}_1, a^{pa}_2)$. Hence, he elicits less incentive to perform higher effort that he does in the noncooperative contracting equilibrium.

As it turns out, the result in Proposition 6 comes from the sequential nature of the Procurement Auction Design Game described in this section. To understand the reason for that, first note that in the last stage of the game, *in the procurement auction stage*, contractors choose their bids taking as given the power of all contracts. In particular, the higher the power of any contract, the higher the bid of each contractor for each activity, as demonstrated in equation (27). In the first stage of the game, *in the procurement design stage*, buyers choose the power of the contracts. When a certain buyer chooses the power of his contracts, he internalizes that by increasing the power of his contract, the bid for his activity increases. However, he doesn't internalize that by increasing the power of his contract, he will induce the firms to bid more aggressively for other activities. This new effect, added to the free-riding effect discussed in section 4, jointly explains why the inefficiency is stronger when contractors bid for the fixed fee.

Note that, in the equilibrium described in Section 4, the fixed fees correspond to the bids described in this section. However, unlike this section, in the noncooperative equilibrium buyers simultaneously choose the fixed fees and the power of the contracts.

8   Conclusions

This paper analyzes free-riding in procurement design, which provides an explanation for the pervasiveness of low-powered contracts in certain industries, such as the US building construction and French public bus transportation industry. We argue that buyers do not provide high enough incentive schemes because they do not completely internalize all the benefits of eliciting contractor effort.

The model predicts that low-powered contracts are more likely to be pervasive in industries in which a contractor benefits from positive externality when running different activities than in industries in which this effect is nonexistent, and in industries in which the number of activities that a contractor performs for different buyers is sufficiently high. These implications could be the subject of an interesting empirical test that will shed light on contracts that have been used in other industries.

Since excessively low-powered incentive schemes emerge as an inefficiency, this paper pro-

vides some policy recommendation to improve the social allocation. In particular, it recommends that under certain circumstances laws that bind local governments to award fixed-price contracts by competitive bidding should be used. It may overcome the free-riding problem, and increase the social welfare. Hence, unlike Bajari and Tadelis (2001), this paper argues that in some industries cost-plus contract should not be allowed.

A potential interesting extension of this work would be the analysis of this multi-contracting problem in an adverse selection setting. Martimort and Stole (2009) solve a similar problem under private, intrinsic and delegated agency. The introduction of economies of scape/scope presented in this paper to the standard Laffont and Tirole (1986) model, can potentially bring new insights about how contracts are used in the practice.

References

Asanuma, Banri and Tatsuya Kikutani, 1992, Risk Absorption in Japanese Subcontracting: A Microeconometric Study of the Automobile Industry, *Journal of the Japanese and International Economies*, 6, 1-29.

Athey, S. and J. Levine, 2001, Information and Competition in US Forest Service Timber Auctions, *Journal of Political Economy*, 109, 375-417

Bailey, Elizabeth E. and Ann F. Friedlaender, 1982, Market Structure and Multiproduct Industries, *Journal of Economic Literature*, 20(3), 1024-1048

Bajari, Patrick and Steven Tadelis, 2001, Incentives versus Transaction Costs: A Theory of Procurement Contracts , *The RAND Journal of Economics*, 32(3), 387-407

——————————————, 2006, Incentives and Award Procedures: Competitive Tendering vs. Negotiations in Procurement , in Dimitri Nicola, Gustavo Piga and Giancarlo Spagnolo (eds.), *Handbook of Procurement*, Cambridge (UK): Cambridge U.P.

Bajari, Patrick, Robert McMillan and Steven Tadelis, 2009, Incentives and Award Procedures: Competitive Tendering vs. Negotiations in Procurement, *Journal of Law, Economics, and Organization*, 25(2):372-399

Banerjee, Abhijit and Esther Duflo, 2000, Reputation Effects and the Limits of Contracting: A Study of the Indian Software Industry, *Quarterly Journal of Economics*, 115, 989-1018

Bernheim, B Douglas and Michael D. Whinston, 1986, Common Agency, *Econometrica*, 54(4), 923-42

Carpineti, Laura, Gustavo Piga and Matteo Zanza, 2006, The Variety of Procurement Practice: Evidence from Public Procurement, in Dimitri Nicola, Gustavo Piga and Giancarlo Spagnolo (eds.), *Handbook of Procurement*, Cambridge U.P., Cambridge (UK)

Chiappori, Pierre-André and Bernard Salanié, 2000, Testing Contract Theory: A Survey of Some Recent Work, *Proceedings of the World Congress of the Econometric Society*

Crocker, Keith J. and Kenneth J. Reynolds, 1993, The Efficiency of Incomplete Contracts: An Empirical Analysis of Air Force Engine Procurement, *The Rand Journal of Economics*, 24(1), 126-146

Dimitri, Nicola, Gustavo Piga and Giancarlo Spagnolo, 2006, Handbook of Procurement, Cambridge U.P., Cambridge (UK)

Dixit, Avinash, 1996, Special-Interest Lobbying and Endogenous Commodity Taxation, *Eastern Economic Journal*, 22(4), 375-388

Fraquelli, Giovanni, Massimilliano Piacenza and David Vannoni, 2004, Scope and Scale Economies in Multi-utilities: Evidence from Gas, Water and Eletricity, *Applied Economics*, 36, 2045-2057

Gagnepain, Philippe and Marc Ivaldi, 2002, Incentive regulatory policies: the case of public transit systems in France, *The RAND Journal of Economics*, 33 (4), 605629

————————————, 2008, Contract choice, incentives, and political capture in public transit, *Working paper, Toulouse School of Economics*

Gale, Douglas and Martin Hellwig, 1985, Incentive Compatible Debt Contracts: The One Period Problem, *Review of Economic Studies,* 52, 647-663

Hart, Oliver and Bengt Holmstrom, 2009, A Theory of Firm Scope, *forthcoming, The Quarterly Journal of Economics*

Hellwig, Martin F., 2003, Public-Good Provision with Many Participants, *Review of Economic Studies*, 70(3), 589-614

Holmstrom, Bengt and Paul Milgrom, 1987, Aggregation and Linearity in the Provision of Intertemporal Incentives, *Econometrica*, 55 (2), 303-28

————————————, 1991, Multitask Principal-Agent Analysis: Incentive Contracts, Asset Ownership, and Job, *Journal of Law, Economics, and Organization*, 7 (Special Issue), 24-52

Kagel, John H., 1995, Auctions: A Survey of Experimental Research, in John H. Kagel and Alvin E. Roth (eds.), *Handbook of Experimental Economics*, Princeton University Press, Princeton (US)

Kawasaki, Seiichi and John McMillan, 1987, The Design of Contracts: Evidence from Japanese Subcontracting. *Journal of the Japanese and International Economies*, 1, 327349.

Kim, Youn, 1987, Economies of Scale in Multi-Product Firms: An Empirical Analysis, *Economica*, 54(214), 185-206

Laffont, Jean-Jacques and Jean-Charles Rochet, 1993, Regulation of a Risk Averse Firm, *Games and Economic Behavior*, 25(2), 149-173

Laffont, Jean-Jacques and Jean Tirole, 1986, Using Cost Observation to Regulate Firms, *Journal of Political Economy*, 94, 614-641.

Laffont, Jean-Jacques and Jean Tirole, 1987, Auctioning Incentive Contracts, *Journal of Political Economy*, 95(5), 921-937

Laffont, Jean-Jacques and Jean Tirole, 1988, Repeated Auctions of Incentive Contracts, Investment, and Bidding Parity with an Application to Takeovers, *The Rand Journal of Economics* 19(4), 516-537

Laffont, Jean-Jacques and Jean Tirole, 1993, A Theory of Incentives in Procurement and Regulation, MIT Press

Le Breton, Michel and François Salanié, 2003, Lobbying under Political Uncertainty, *Journal of Public Economics*, 87, 2589 2610

Mailath, George and Andrew Postlewaite, 1990, Asymmetric Information Bargaining Problems with Many Agents, *Review of Economic Studies* 57, 351-360.

McAfee, R. Preston and John McMillan, 1986, Bidding for Contracts, a Principal-Agent Analysis, *The Rand Journal of Economics*, 17(5), 326-338.

Martimort, David, 1996, Exclusive Dealing, Common Agency, and Multiprincipals Incentive Theory, *The Rand Journal of Economics* 27(1), 1-31

Martimort, David, 2007, Multi-Contracting Mechanism Design, Advances in Economic Theory, in Blundell, Newey and Persson (eds.), *Proceedings of the World Congress of the Econometric Society*, Cambridge University Press

Martimort, David and Lars Stole, 2009, Market Participation under Delegated and Intrinsic Common Agency Games, *The Rand Journal of Economics* 40(1), 78-102.

Monteverde, Kirk and David J. Teece, 1982, Supplier Switching Costs and Vertical Integration in the Automobile Industry, *The Bell Journal of Economics*, 13(1), 206-213

Panzar, John C., and Robert D. Willig, Economies of Scope, *The American Economic Review Papers and Proceedings*, 71(2), 268-272

Rogerson, William P., 1992, Overhead Allocation and Incentives for Cost Minimization in Defense Procurement, *The Accounting Review*, 67 (4), 671-690

————————————, 1994, Economic Incentives and the Defense Procurement Process, *The Journal of Economic Perspectives*, 8 (4), 65-90

————————————, 2003, Simple Menus of Contracts in Cost-Based Procurement and Regulation, *The American Economic Review*, 93 (3), 919-926

Salanié, Bernard, 1990, Sélection Adverse et Aversion pour Le Risque, *Annales d'Économie et de Statistique*, 18, 131-149

Shleifer, Andrei, 1985, A Theory of Yardstick Competition, *The Rand Journal of Economics*, 16, 319-327

Taylor, Curtis R. and Steven N. Wiggins, 1997, Competition or Compensation: Supplier Incentives under the American and Japanese Subcontracting Systems, *The American Economic Review*, 87(4), 598-618

Teece, David J., 1980, The Economies of Scope and The Scope of the Entreprise, *Journal of Economics Business and Organization*, 1(3), 223-247

Townsend, Robert, 1979, Optimal Contracts with Costly State Verification, *Journal of Economic Theory*, 21, 265-293

Weitzman, Martin L., 1980, Efficient Incentive Contracts, *The Quarterly Journal of Economics*, 94(4), 719-730

Willig, Robert D., 1979, Multiproduct Technology and Market Structure, *The American Economic Review Papers and Proceedings*, 69 (2), 346-351

Yun, Mikyung, 1999, Subcontracting Relations in the Korean Automotive Industry: Risk Sharing and Technological Capability, *International Journal of Industrial Organization*, 17, 81-108.

Appendix: Proofs of Propositions

Proof of Proposition 1 *Part 1*: Since that effort in each activity is restricted to positive, $e_i \geq 0, \forall i$, then expected production cost of each activity defined in (1) has a upper bound which is β. Therefore, wherever is the contractor's effort choices, $E(C_i) \leq \beta$. Because $v - \beta > 0$, then $v - E(C_i) > 0$. It means that any activity contributes positively to increases the total net surplus, even though the contractor exerts zero effort. Hence, the buyers will optimally demand all activities. That is the reason why is optimality to have $I^c = N$.

Part 2: Optimal $\{(b_i, a_i)\}_{i=1}^n$ Given that $I^c = N$, the cooperative contracting problem is:

$$\max_{\{(b_i, a_i)\}_{i=1}^n} E\Big[\sum_{i=1}^n (v - b_i - a_i C_i) \Big]$$

s.t.

$$a_i \in [0, 1]$$

$$e_i = \alpha \Big[(1 - a_i) + \frac{\kappa}{n-1} \sum_{j \neq i} (1 - a_j) \Big] \quad i, j = 1, ...n \tag{IC}$$

$$\max_{\{e_i\}_{i=1}^n} E\Big[u\Big(\sum_{i=1}^n b_i - (1 - a_i) C_i - \frac{e_i^2}{2\alpha} \Big) \Big] \geq U(0) \tag{IR}$$

Using expression (10), the (IR) constraint can be replaced by

$$b + \sum_{i=1}^n \Big[-(1 - a_i) E[C_i] - \frac{e_i^2}{2\alpha} - \frac{r(1 - a_i)^2 \sigma^2}{2} \Big] \geq 0, \tag{IR*}$$

where $b = \sum_{i=1}^n b_i$, and e_i and the optimal described in (IC).

Replacing the (IR) in the production cost definition in (1), and the replacing it in (IR*), we can rewrite the cooperative contracting problem as follows:

$$\max_{\{(b_i, a_i)\}_{i=1}^n} E\Big[\sum_{i=1}^n (v - b_i - a_i C_i) \Big]$$

s.t.

$$a_i \in [0, 1]$$

$$b + \sum_{i=1}^{n} \left\{ -(1-a_i)\left[\beta - \left[\alpha\left[(1-a_i) + \frac{\kappa}{n-1}\sum_{j\neq i}(1-a_j)\right]\right]\right] - \frac{\kappa}{n-1}\sum_{j\neq i}\left[\alpha\left[(1-a_i) + \frac{\kappa}{n-1}\sum_{j\neq i}(1-a_j)\right]\right] \right.$$

$$\left. + \frac{\left[\alpha\left[(1-a_i) + \frac{\kappa}{n-1}\sum_{j\neq i}(1-a_j)\right]\right]^2_i}{2\alpha} - \frac{r(1-a_i)^2\sigma^2}{2} \right\} \geq 0 \qquad\qquad (\text{IR}^*).$$

For the maximization problem, in the optimal constraint (IR^*) is binding. After replacing the bidding (IR^*) constraint in the objective function, we derive the first-order conditions of the problem.

The first-order conditions are:

$$(1+\kappa^2) = (1-a_i)\left[1 + \frac{\kappa^2}{n-1} + r\sigma^2\right] + \sum_{j\neq i}(1-a_j)\left[\frac{2\kappa}{n-1} + \frac{(n-2)\kappa^2}{(n-1)^2}\right], \forall i,j = 1,...,n$$

The n conditions above are equations of a system of equations. The $\{a_i\}_{i=1}^n$ which are solutions for this system of equations is the optimal reimbursement fraction of the contractor's production cost.

Solving it, we obtain

$$a_i^c = \begin{cases} 1 - \dfrac{\alpha(1+\kappa)^2}{\alpha n\left[\frac{2\kappa}{n-1} + \frac{\kappa^2(n-2)}{(n-1)^2}\right] + \left[\alpha(1-\frac{\kappa}{n-1})^2 + r\sigma^2\right]} \in (0,1) & , \text{if } r\sigma^2 \geq \kappa(n-2) \\ 0 & , \text{otherwise} \end{cases}$$

Replacing the optimal a_i^c's in the binding (IR^*) constraint, we obtain b. Since that $b = \sum_{i=1}^n b_i$, then we get

$$\sum_{i=1}^n b_i^c = n\left[(1-a_i^c)\beta - \frac{(\alpha(1+\kappa)^2 - r\sigma^2)(1-a_i^c)^2}{2}\right].$$

The first-order conditions are necessary and sufficient because the objective is concave in the choice variables.∎

Proof of Corollary 1 *Part 1*: From Proposition 1, in equation (13) we have

$$a_i^c = \begin{cases} 1 - \dfrac{\alpha(1+\kappa)^2}{\alpha n\left[\frac{2\kappa}{n-1} + \frac{\kappa^2(n-2)}{(n-1)^2}\right] + \left[\alpha(1-\frac{\kappa}{n-1})^2 + r\sigma^2\right]} \in (0,1) & , \text{if } \kappa \leq \frac{r\sigma^2}{n-2} \\ 0 & , \text{otherwise} \end{cases}$$

For the case which $\kappa \leq \frac{r\sigma^2}{n-2}$, we have

$$\frac{\partial(1-a_i^c(\kappa))}{\partial \kappa} = \frac{2(1+\kappa)\left[\frac{\alpha(1+\kappa)^2}{\alpha n\left[\frac{2\kappa}{n-1}+\frac{\kappa^2(n-2)}{(n-1)^2}\right]+\left[\alpha(1-\frac{\kappa}{n-1})^2+r\sigma^2\right]}\right] + (2+2\kappa)(1+\kappa)^2}{\left[\frac{\alpha(1+\kappa)^2}{\alpha n\left[\frac{2\kappa}{n-1}+\frac{\kappa^2(n-2)}{(n-1)^2}\right]+\left[\alpha(1-\frac{\kappa}{n-1})^2+r\sigma^2\right]}\right]}$$

After some algebraic manipulations, we get $\frac{\partial(1-a_i^c(\kappa))}{\partial \kappa} = 2(1+\kappa)r\sigma^2$, which is strictly positive. For the case which $\kappa \geq \frac{r\sigma^2}{n-2}$, then a_i^c is always equal to zero. Therefore, $\frac{\partial(1-a_i^c(\kappa))}{\partial \kappa} = 0$.

Part 2: From equation (13), note that when $\kappa = 0$, we are in the case that $\kappa \leq \frac{r\sigma^2}{n-2}$, which means that $(1-a_i^c) \in (0,1)$. In particular, $(1-a_i^c)$ reaches its lower bound, which is $1 - \frac{\alpha}{\alpha+r\sigma^2}$, when $\kappa = 0$.

When $\kappa > 0$, we can have two cases: (i) $\kappa \leq \frac{r\sigma^2}{n-2}$, and (ii) $\kappa > \frac{r\sigma^2}{n-2}$. In case (i), $\frac{\partial(1-a_i^c(\kappa))}{\partial \kappa} > 0$, which implies that $(1 - a_i^c(\kappa)) > (1 - a_i^c(0))$. In addition, $(1 - a_i^c(\kappa)) \in (0,1)$. In case (ii), $(1 - a_i^c(\kappa)) = 1$ which is greater than $(1 - a_i^c(\kappa))$ when $\kappa \leq \frac{r\sigma^2}{n-2}$. Therefore, $(1 - a_i^c(\kappa)) \geq (1 - a_i^c(0))$ for all κ. ∎

Proof of Proposition 2 *Part 1*: If buyers offer $\{(b_i^{nc}, a_i^{nc})\}_{i=1}^n$ as defined in (18) and (19), then the contractor will have payoff equals zero in the case that he accepts all the contract, and negative payoff if he rejects any of the n contract. To see that, just as (18) and (19) in expression (10), we obtain that $EU(\{(b_i, a_i)\}_{i \in I}) = U(0)$, if $I = N$, and $EU(\{(b_i, a_i)\}_{i \in I}) < U(0)$, if $I \neq N$. Note that, given the equilibrium contracts, the contractor is indifferent between accept all contracts and rejecting all contracts. In particular, he is worse-off accepting a subject of the contracts. It happens because if he accepts just a subset, he loses the positive externality, receiving a negative net payoff.

Part 2: Each buyer-i chooses (b_i, a_i) which minimizes his expected transfer, $T_i = b_i + a_i E[C_i]$, taking into account the other buyer's contracts, $(b_{-i}^{nc}, a_{-i}^{nc})$, described in (18) and (19), and (IC) and (IR^{nc}). Replacing (18) and (19), and (IC) in (IR^{nc}), buyer-i chooses (b_i, a_i) that make the (IR^{nc}) - buyer-i maximizing his expected payoff will not leave rent for the contractor. Given that, we obtain that

$$b_i = -\sum_{j\neq i} b_j^{nc} - \left[-(1-a_i)E[C_i] - \frac{e_i^2}{2\alpha} - \frac{r(1-a_i)^2\sigma^2}{2} \right] - \sum_{j\neq i} \left[-(1-a_j^{nc})E[C_j] - \frac{e_j^2}{2\alpha} - \frac{r(1-a_j^{nc})^2\sigma^2}{2} \right] \tag{29}$$

where $e_i = e_i(a_i, a_{-i}^{nc}), \forall i$ in the equation above are the optimal ones described in (IC).

Replacing the equation above in the objective function of the buyer-i, which is $T_i = b_i + a_i E[C_i]$, and

$$
\begin{aligned}
T_i \;=\; & -\sum_{j\neq i} b_j^{nc} - \left[-(1-a_i)E[C_i] - \frac{e_i^2}{2\alpha} - \frac{r(1-a_i)^2\sigma^2}{2} \right] \\
& -\sum_{j\neq i} \left[-(1-a_j^{nc})E[C_j] - \frac{e_j^2}{2\alpha} - \frac{r(1-a_j^{nc})^2\sigma^2}{2} \right] \\
& + a_i E[C_i].
\end{aligned}
$$

Buyer-i chooses a_i that minimizes the T_i described above. The first-order condition for the problem the buyer-i problem is

$$\alpha(1 + \frac{\kappa^2}{n-1}) = (1-a_i)[r\sigma^2 + \alpha(1+\kappa^2) \tag{30}$$

This equation gives the optimal choice of buyer-i given the other buyer $(b_{-i}^{nc}, a_{-i}^{nc})$, which is equal to

$$a_i^{nc} = 1 - \frac{\alpha\left(1 + \frac{\kappa^2}{n-1}\right)}{\alpha(1+\kappa^2) + r\sigma^2},$$

Replacing the optimal a_i^{nc} described above in (29), we obtain the

$$b_i^{nc} = (1 - a_i^{nc})\beta - \frac{\alpha(1+\kappa)^2(1-a^{nc})^2}{2} + \frac{r\sigma^2(1-a^{nc})^2}{2},$$

which is the one characterized in (19). ∎

Proof of Proposition 3 *Part 1*: From equations (13) and (18), then

$$(1 - a_i^c(\kappa)) - (1 - a_i^{nc}(\kappa)) = \frac{\alpha(1+\kappa)^2}{\alpha n[\frac{2\kappa}{n-1} + \frac{\kappa^2(n-2)}{(n-1)^2}] + [\alpha(1 - \frac{\kappa}{n-1})^2 + r\sigma^2]} - \frac{\alpha\left(1 + \frac{\kappa^2}{n-1}\right)}{\alpha(1+\kappa^2) + r\sigma^2}.$$

46

After some algebraic manipulations, we obtain

$$
\begin{aligned}
(1 - a_i^c(\kappa)) - (1 - a_i^{nc}(\kappa)) \ = \ & 2\kappa(n-1)r\sigma^2 + r\kappa^2\sigma^2(n-2) \\
& + \frac{\kappa^2}{(n-1)^2}(n^3 - 4n^2 + 6n - 4) \\
& + \frac{\kappa^4}{(n-1)^2}(2n^3 - 6n^2 + 5n - 1) + 2\kappa^3(n-1).
\end{aligned}
$$

which is equal or greater than zero for any $n \geq 2$. In particular, when $\kappa = 0$, $(1 - a_i^c(\kappa)) - (1 - a_i^{nc}(\kappa))$ is equal to zero.

Part 2: From equation (18), we have that $1 - a_i^{nc} = \frac{\alpha\left(1 + \frac{\kappa^2}{n-1}\right)}{\alpha(1+\kappa^2)+r\sigma^2}$. Deriving it with respect to n, we have that

$$
\frac{\partial(1 - a_i^c(\kappa))}{\partial n} = -\frac{\alpha\kappa^2}{(n-1)^2(\alpha(1+\kappa^2) + r\sigma^2)} \leq 0
$$

With strict inequality if $n > 2$, $\kappa > 0$ and $\alpha > 0$.

Part 3: In the equilibrium that the contractor will accept all contracts, $I^{nc} = N$, there is a unique $a_i, \forall i$ that solves the first-order conditions (30). Given that, the (19) is the unique symmetric b_i's that are equilibrium.

Part 4: In the exclusive dealing contract, a buyer and the contractor sign a contract in which the contract commits to perform activities only for that buyer. In the exclusive dealing contract, the optimal power of the contract is $(1 - a_i^{ED}) = (1 + r\sigma^2)^{-1}$, and the buyer expected transfer will be

$$
E[T_i^{ED}] = \beta - \frac{(1 - \alpha^{ED})(1 + \alpha^{ED})}{2} + \frac{r\sigma^2}{2}(1 - a^{ED})^2 \tag{31}
$$

From (3), we know that buyer-i expected transfer is $E[T_i(C_i)] = b_i + a_i E[C_i]$. Replacing the equilibrium values of b_i and a_i, characterized by (19) and (18) in that expected transfer expression, and also in the production cost expression in (1), we obtain the equilibrium buyer-i expected transfer which is

$$
E[T_i^{nc}] = \beta - \frac{(1+\kappa)^2}{2}\left[\frac{1 + \frac{\kappa^2}{n-1}}{1 + \kappa^2 + r\sigma^2}\right]\left[2 - \left[\frac{1 + \frac{\kappa^2}{n-1}}{1 + \kappa^2 + r\sigma^2}\right]\right] + \frac{r\sigma^2}{2}\left[\frac{1 + \frac{\kappa^2}{n-1}}{1 + \kappa^2 + r\sigma^2}\right]^2 \tag{32}
$$

We want to show that $E[T_i^{nc}] - E[T_i^{ED}] < 0$. Because, as will show in Corollary 4, that that $\frac{\partial E[T_i^{nc}(n)]}{\partial n} \geq 0, \forall n \geq 3$. It means that $E[T_i^{nc}(3)] \leq E[T_i^{nc}(3)] \leq ... \leq E[T_i^{nc}(n)]$. Therefore, it is sufficient to show that: (i) $E[\lim_{n \to \infty} T_i^{nc}(n)] - E[T_i^{ED}] < 0$, and (ii) $E[T_i^{nc}(2)] - E[T_i^{ED}] < 0$.

Let us prove that (i) $E[\lim_{n \to \infty} T_i^{nc}(n)] - E[T_i^{ED}] < 0$. After some algebraic manipulations, we obtain

$$E[\lim_{n \to \infty} T_i^{nc}(n)] = \beta - \frac{(1+\kappa)^2}{2}\left(\frac{2}{1+\kappa^2+r\sigma^2}\right)\left(\frac{1+2\kappa^2+2r\sigma^2}{1+\kappa^2+r\sigma^2}\right) + r\sigma^2\left(\frac{1}{1+\kappa^2+r\sigma^2}\right)^2$$

Defining $\Phi(\kappa) = E[\lim_{n \to \infty} T_i^{nc}(n)] - E[T_i^{ED}]$, and after some algebraic manipulations, we obtain

$$\Phi(\kappa) = -\frac{1}{2}\left\{\frac{(1+\kappa)^2(1+2\kappa^2+2r\sigma^2)(1+r\sigma^2) - (1+\kappa^2+r\sigma^2)^2}{(1+r\sigma^2)(1+\kappa^2+r\sigma^2)^2}\right\}$$

Evaluation this expression at $\kappa = 0$, we obtain $\Phi(0) = 0$. In addition, we obtain that $\Phi(1)$ is equal to:

$$\Phi(1) = -\frac{1}{2(1+r\sigma^2)(2+r\sigma^2)}\left\{8 + 12r\sigma^2 + 3(r\sigma^2)^2\right\} < 0$$

To conclude the proof, we need to show that $\Phi(\kappa)$ is monotonic decreasing in κ, that is to show that $\frac{\partial \Phi(\kappa)}{\partial \kappa} > 0$. Computing this derivative, we obtain

$$\frac{\partial \Phi(\kappa)}{\partial \kappa} = -\frac{[(1+r\sigma^2)(1+\kappa^2+r\sigma^2)]}{2[(1+r\sigma^2)(1+\kappa^2+r\sigma^2)^2]^2}$$
$$+ \frac{[4\kappa(1+r\sigma^2)(1+\kappa^2+r\sigma^2)][(1+\kappa)^2(1+2\kappa^2+2r\sigma^2)(1+r\sigma^2) - (1+\kappa^2+r\sigma^2)^2]}{2[(1+r\sigma^2)(1+\kappa^2+r\sigma^2)^2]^2}.$$

which is negative for any $\kappa \in [0, 1]$.

Now, let us prove that (ii) $E[T_i^{nc}(2)] - E[T_i^{ED}] < 0$. Before, note that $E[T_i^{nc}(2)]$ is the equilibrium, which means that $E[T_i^{nc}(2)] = E[T_i^{nc}(2)(a_i^{nc}, a_{-i}^{nc})]$. Since a_i^{nc} is the optimal solution for buyer-i problem, then $E[T_i^{nc}(2)(a_i^{nc}, a_{-i}^{nc})] < E[T_i^{nc}(2)(a_i, a_{-i}^{nc})], \forall a_i$. In particular, $E[T_i^{nc}(2)(a_i^{nc}, a_{-i}^{nc})] < E[T_i^{nc}(2)(a^{ED}, a_{-i}^{nc})]$. Therefore, it is enough to show that

$E[T_i^{nc}(2)(a^{ED}, a_{-i}^{nc})] - E[T_i^{ED}].$

$$E[T_i^{nc}(2)(a^{ED}, a_{-i}^{nc})] = \beta - \frac{[(1 - a^{ED}) + \kappa(1 - a_i^{nc})][1 + a^{ED} + \kappa(1 - a_i^{nc})]}{2}$$
$$- \frac{[(1 - a_i^{nc}) + \kappa(1 - a^{ED})][1 + \kappa a^{ED} + \kappa - a_i^{nc})]}{2}$$
$$+ \frac{r\sigma^2}{2}(1 - a^{ED})^2 + + \frac{(1 - \kappa)^2(1 - a_i^{nc})^2}{2}.$$

Computing $E[T_i^{nc}(2)(a^{ED}, a_{-i}^{nc})] - E[T_i^{ED}]$, we get

$$E[T_i^{nc}(2)(a^{ED}, a_{-i}^{nc})] - E[T_i^{ED}] = -2\kappa(1 - a_i^{nc})(1 + a_i^{nc}) - \kappa^2(1 + a_i^{nc})(1 - a_i^{nc}) < 0. \blacksquare$$

Proof of Corollary 2 Deriving $(1 - a_i^{nc})$, which is implicity defined (18), with respect to κ, we obtain

$$\frac{\partial(1 - a_i^{nc})}{\partial \kappa} = -2\alpha\kappa\left[\alpha\left(1 - \frac{1}{n - 1}\right) - \frac{r\sigma^2}{n - 1}\right] \tag{33}$$

Expression (33) is positive if $n \geq 2 + \frac{r\sigma^2}{\alpha}$. This result says that, when the number of buyers contracting with the same contractor is sufficiently high, the higher is the externality effect, the lower is the lower power is the externality effects. This result is quite intuitive. When the number of buyers contracting with the same contractor is sufficiently high, the free-riding effect off set the fact that a higher κ increases the marginal benefit of eliciting contractor's effort (due to cost reduction), leading to lower power procurement contracts. $\blacksquare$

Proof of Corollary 3 From (18), we know that $(1 - a_i^{nc}) = \frac{\alpha\left(1 + \frac{\kappa^2}{n-1}\right)}{\alpha(1 + \kappa^2) + r\sigma^2}$. From (13), we know that when $\kappa = 0$, $(1 - a_i^c(0)) = 1 - \frac{\alpha}{\alpha + r\sigma^2}$. In particular, note that if $\kappa = 0$, then $(1 - a_i^c(0)) = (1 - a_i^{nc}(0))$. Therefore,

$$(1 - a_i^{nc}(\kappa)) - (1 - a_i^c(0)) = -1 + \frac{1}{n - 1} + \frac{r\sigma^2}{n - 1}.$$

It is positive if $n \geq \overline{n} = 2 + r\sigma^2$. $\blacksquare$

Proof of Corollary 4 From (3), we know that buyer-i expected transfer is $E[T_i(C_i)] = b_i + a_i E[C_i]$. Replacing the equilibrium values of b_i and a_i, characterized by (19) and (18) in that expected transfer expression, and also in the production cost expression in (1), we obtain

the equilibrium buyer-i expected transfer which is

$$E[T_i^{nc}(n, \kappa, \sigma)] = \beta - \frac{(1+\kappa)^2}{2}\left[\frac{1 + \frac{\kappa^2}{n-1}}{1 + \kappa^2 + r\sigma^2}\right]\left[2 - \left[\frac{1 + \frac{\kappa^2}{n-1}}{1 + \kappa^2 + r\sigma^2}\right]\right] + \frac{r\sigma^2}{2}\left[\frac{1 + \frac{\kappa^2}{n-1}}{1 + \kappa^2 + r\sigma^2}\right]^2 \quad (34)$$

The partial derivative of $E[T_i^{nc}(n, \kappa, \sigma)]$ with respect to n is equal to:

$$\frac{\partial E[T_i^{nc}(n, \kappa, \sigma)]}{\partial n} = \frac{-1}{2(n-1)^2(1 + \kappa^2 + r\sigma^2)^2}\left[-2r\kappa^2(1+\kappa)^2\sigma^2 + 2r\kappa^2\sigma^2\left(1 + \frac{\kappa^2}{n-1}\right)\right]$$

After some algebraic manipulation, we obtain the following expression for the derivative above:

$$\frac{\partial E[T_i^{nc}(n, \kappa, \sigma)]}{\partial n} = \frac{2r\kappa^3\sigma^2}{2(n-1)^3(1 + \kappa^2 + r\sigma^2)^2}\left[\kappa(n-3) + 2(n-1)\right],$$

with is positive whenever $\kappa \geq \overline{\kappa} = \frac{-2(n-1)}{n-3}$. Note that $\overline{\kappa} = \frac{-2(n-1)}{n-3}$ is negative for any $n \geq 3$. Since that we have assumed $\kappa \geq 0$, then we can conclude that if $n \geq 3$, then $\frac{\partial E[T_i^{nc}(n,\kappa,\sigma)]}{\partial n} \geq 0$. ∎

Proof of Propostion 4 In the text.∎

Proof of Proposition 5 . First, we compute the expected payoff of each buyer $(v - E[T_i^{FP}])$ when they commit to fixed-price contracts, $ai = 0, \forall i$. Then, we compute the expected payoff of each buyer in the noncooperative contracting equilibrium $(v - E[T_i^{nc}])$. To conclude, we provide the conditions that guarantees that $(v - E[T_i^{FP}]) > (v - E[T_i^{nc}])$.

Part 1 If $a_i = 0, \forall i$, then according to (IC), $e = e_i = (1 + \kappa), \forall i$. Then, in the symmetric equilibrium, the contractor will accept all the contracts $I = N$ if and only if $b_i = \beta - e(1 + \kappa) + \frac{r\sigma^2}{2} + \frac{e^2}{2}, \forall i$. Given that the buyer-$i$ expected transfer is $E[T_i(C_i)] = b_i + a_iE[C_i]$, and the production cost expression in (1), we obtain the buyer-i expected transfer when all buyer relies on fixed-price contracts.

$$E[T_i^{FP}(n, \kappa, \sigma)] = \beta + \frac{r\sigma^2}{2} - \frac{(1+\kappa)^2}{2}.$$

Part 2 In the noncooperative contracting equilibrium, the buyer-i expected transfer is

$$E[T_i^{FP}(n, \kappa, \sigma)] = \beta + \frac{r(1 - a_{nc}^i)^2\sigma^2}{2} - \frac{(1 - a_i^{nc})^2(1+\kappa)^2}{2}.$$

50

where a_{nc}^i are defined in (refnoncoopa).

Part 3 To conclude, we provide the conditions that guarantees that $(v - E[T_i^{FP}]) > (v - E[T_i^{nc}])$. For that, we need to show under which conditions $E[T_i^{FP}] \leq E[T_i^{nc}]$. After some algebraic manipulations, we obtain that

$$E[T_i^{FP}] - E[T_i^{nc}] = -\frac{a_i^{nc}}{2}(r\sigma^2(a_i^{nc} - 2) + (1 + \kappa)^2 a_i^{nc})$$

Therefore, to that the expression above is negative, it is sufficient to show that

$$(a_i^{nc} - 2) + (1 + \kappa)^2 a_i^{nc} \geq 0. \tag{35}$$

Replacing a_i^{nc}, defined in (18), in (35), we obtain that the left-hand side of the expression (35) is equal to

$$\psi(n) = \frac{-rn\sigma^2\kappa^2 - (n-1)r^2(\sigma^2)^2 - 2nr\sigma^2 + 2r\sigma^2}{(n-1)(1 + \kappa^2 + r\sigma^2)} + \frac{-2(1+\kappa)^2\kappa^2 + rn(1+\kappa)^2\sigma^2 - r\sigma^2(1+\kappa)^2}{(n-1)(1 + \kappa^2 + r\sigma^2)}.$$

Note that $\psi(2) = -\frac{r\sigma^2(r\sigma^2 + (1-\kappa)^2)}{(n-1)(1+\kappa^2+r\sigma^2)}$, which is negative. In addition, $\lim_{n \to \infty} \psi(n) = \infty$. Since that $\psi(n)$ is a continuous function, then the only conditions that we have to provide are the ones that guarantee that $\frac{\partial \psi(n)}{\partial n}$ monotonic positive. Doing so, we can guarantee that $\exists \bar{n}$ such that $\forall n \geq \bar{n} \ \psi(n)$, and consequently (35).

Deriving $\psi(n)$ with respect to n, we obtain

$$\frac{\partial \psi(n)}{\partial n} = -r^2\sigma^4 - r\sigma^2(1 - 2\kappa) + \kappa^2(1 + \kappa)^2.$$

Note that it is a quadratic equation in $(r\sigma^2)$. Solving that quadratic equation for $(r\sigma^2)$, we found that $\frac{\partial \psi(n)}{\partial n}$ is strictly positive if

$$r\sigma^2 \leq \overline{r\sigma^2} = \frac{(1 - 2\kappa)^2 + 2\kappa^2(1+\kappa)^2}{2} + \frac{(1 - 2\kappa)}{2}\sqrt{(1 + 2\kappa^2)^2 - 4\kappa(1+\kappa)(1 - 2\kappa)}. \blacksquare$$

Proof of Proposition 6 First we will derive the power of the procurement contracts in the procurement auction design game $(1 - a_i^{pa})$, then we will compare it with the the power of the noncooperative equilibrium contracts $(1 - a_i^{nc})$.

Part 1 - The power of the procurement contracts Without loss of generality, consider the case that $n = 2$. In the procurement auction design game, each buyer-i chooses a_i which minimizes his expected transfer, $T_i = b_i + a_i E[C_i]$, taking into account the contractor's bid. The equilibrium bidding strategies are described in (27) and (26). Replacing them in $T_i = b_i + a_i E[C_i]$, and given the (IC) constraint, we find that

$$
\begin{aligned}
T_i =\ & \beta - \frac{1}{2}[(1 - a_i) + \kappa(1 - a_j)][(1 + a_i) + \kappa(1 + a_j)] - \frac{1}{2}[(1 - a_j) + \kappa(1 - a_i)][(1 + a_j) + \kappa(1 + a_i)] + \\
& \frac{r\sigma^2(1 - a_i)^2}{2} + (1 - a_j) + \kappa(1 - a_i) + \kappa[(1 - a_i) + \kappa(1 - a_j)][(1 + a_i) + \kappa(1 + a_j)] + \\
& - \frac{[(1 - a_j) + \kappa(1 - a_i)]^2}{2}.
\end{aligned}
$$

Buyer-i chooses a_i that maximizes the T_i described above. The first-order condition for the problem the buyer-i problem is

$$
\kappa^2 + 1 = (1 - r\sigma^2)(1 - a_j) + \kappa(1 - a_i), \forall i, j \tag{36}
$$

Since that $n = 2$, to find the equilibrium a_1 and a_2 we have to solve the following system of equations

$$
\begin{aligned}
\kappa^2 + 1 &= (1 - r\sigma^2)(1 - a_2) + \kappa(1 - a_2) \\
\kappa^2 + 1 &= (1 - r\sigma^2)(1 - a_1) + \kappa(1 - a_2).
\end{aligned}
$$

The solution for the system of equation is:

$$
1 - a_i^{pa} = \frac{1 + \kappa^2}{1 + r\sigma^2 + \kappa}, \forall i
$$

which is the power equilibrium of the power of the procurement contracts in the procurement design game.

Part 2 - Comparing the power of the procurement contracts in the procurement auction design game $(1 - a_i^{pa})$ *with the power of the noncooperative equilibrium contracts* $(1 - a_i^{nc})$. For the case that $n = 2$, then

$$1 - a_i^{nc} = \frac{1 + \kappa^2}{1 + r\sigma^2 + \kappa^2}, \forall i$$

Making some algebraic manipulations, we can show that $(1 - a_i^{pa}) < (1 - a_i^{nc})$ if and if $\kappa^2 < \kappa$, which happens if and only if $\kappa \in (0, 1)$. ∎