TSE working paper series

10-154

Research Group: *Environmental Economics and Natural Resources*

April, 2010

Testing Value vs Waiting Value in Environmental Decisions under Uncertainty

GIUSEPPE ATTANASI AND ALDO MONTESANO

Ecole d'économie de Toulouse (TSE) - Manufacture des Tabacs
Aile Jean-Jacques Laffont - 21, allée de Brienne - 31000 TOULOUSE
Tél : +33 (0) 5 61 12 85 89 - Fax : +33 (0) 5 61 12 86 37 - **www.TSE-fr.eu** - contact@TSE-fr.eu

Toulouse School of Economics

Testing Value vs Waiting Value

in Environmental Decisions under Uncertainty*

Giuseppe ATTANASI (Toulouse School of Economics, LERNA) †

Aldo MONTESANO (Bocconi University, Milan) ‡

April 2010

*We thank participants at the 2006 Meeting on Game Theory and Practice dedicated to development, natural resources and the environment in Zaragoza, the FUR 2006 in Rome, the AFSE 2008 in Toulouse and a seminar at University of Padua, for useful comments and suggestions. Financial support by the Chair "Georges Meyer" in Mathematical Economics at Jean-Jacques Laffont Foundation (TSE) and by the French National Research Agency (ANR) through the project on "Fair Environmental Policies" is gratefully acknowledged.

†Corresponding author: Chair "Georges Meyer" in Mathematical Economics, Toulouse School of Economics, 21 allée de Brienne, 31000 Toulouse. Phone: +33(0)561128713. Fax: +33(0)561128520. Email: attanasi@cict.fr

‡Department of Economics, Bocconi University, via Roentgen, 1 20136 Milan. Email: aldo.montesano@unibocconi.it

Testing Value vs Waiting Value in Environmental Decisions under Uncertainty

Abstract This paper introduces the concept of the *Testing Value* into the analysis of environmental decisions under uncertainty and irreversibility. This value emerges in situations where the probability of receiving information concerning future economic benefits and costs of development depends on the level of development carried out. We show that when information may be acquired also exogenously, the Testing Value could push a risk-neutral decision maker to preserve more in the present and eventually in the future. The reason is that the Testing Value often leads to a only partial development of the environmental asset; on the contrary, the *Waiting Value* (a generalization of the quasi-option value *à la* Arrow and Fisher (1974)) always leads to corner solutions. Although its existence stems from endogenous information, surprisingly enough, the Testing Value is positively related to the probability of acquiring information exogenously.

JEL references: C61; D81; Q32.

Keywords: Testing value; Waiting value; Exogenous and Endogenous Information; Irreversibility.

1 Introduction

The issue of irreversibility and uncertainty in environmental decisions has been broadly analyzed by economic theorists. Since the first definition of the quasi-option value given by Arrow and Fisher (1974), the key concept has been developed in several articles, including Henry (1974), Freixas and Laffont (1984), Hanemann (1989) and Fisher (2000).

The concept of quasi-option value is introduced by Arrow and Fisher (1974) in a two-period model of the choice of the optimal preservation level of a natural resource. Development can take place either "now" or "in the future" but, once undertaken, the resource cannot be restored to its original state of preservation. The future benefits of preservation and development are uncertain. The expected net benefits of preservation in the future period are conditional upon the current choice. They also assume risk neutrality of the Decision Maker (DM henceforth) and independent learning (exogenous information). The latter implies that the DM can receive information about the future benefits of her current choice only by letting time pass. Information before the future choice is acquired by "waiting": it is independent from the current choice.

More specifically, there are two alternative information scenarios. In the first scenario, exogenous information is available with certainty between the current and the future choice. In this scenario, the prospect of future information is fully recognized and explicitly incorporated in the current decision. In the second information scenario, information is not available before the future decision. Therefore, the DM sets both the current and the future level of development without knowing the realized values of the future net benefits.

Consider an extension of the first information scenario of Arrow and Fisher (1974). Assume that exogenous information is available with a given probability $q \in [0, 1]$: q is the probability that information will be acquired exogenously between the current and the future decision. Thus, $q \in [0, 1]$ in the first scenario and $q = 0$ in the second one.

Denote by c_τ the amount of environmental resource preserved at time $\tau = 1, 2$, where $\tau = 1$ and $\tau = 2$ indicate "now" and "in the future" respectively. The total amount of environmental resource is normalized to 1, so that $c_1 \in [0, 1]$ and $c_2 \in [0, c_1]$ because of irreversibility. Let $EV_{exo}(c_1, c_2)$ be the expected value of the two-period net benefits of preserving c_1 at $\tau = 1$ and c_2 at $\tau = 2$ in the "*exo*genous information" scenario and $EV_{no}(c_1, c_2)$ the same expected value in the "*no* information" scenario. Under specific assumptions over $EV(c_1, c_2)$, Arrow and Fisher (1974) show that the current optimal development decision is confined to a binary choice between full development ($c_1^* = 0$) and no development at all ($c_1^* = 1$).[1] Let $EV(c_1) := \max_{c_2 \in [0, c_1]} EV(c_1, c_2)$. Define the *Q*uasi-*O*ption *V*alue *à la* Arrow-Fisher (QOV henceforth) as

$$QOV = \left[EV_{exo|q=1}(1) - EV_{exo|q=1}(0)\right] - \left[EV_{no}(1) - EV_{no}(0)\right]$$

where $q = 1$ because in the Arrow-Fisher framework *exo*genous information is available "with certainty" between $\tau = 1$ and $\tau = 2$. It can be rewritten as

$$QOV = \left[EV_{exo|q=1}(1) - EV_{no}(1)\right] - \left[EV_{exo|q=1}(0) - EV_{no}(0)\right]$$

and interpreted as a correction factor: $\left[EV_{exo|q=1}(1) - EV_{no}(1)\right]$ is the value of "certain" ($q = 1$) exogenous information conditional on having chosen to preserve the whole environmental area at $\tau = 1$; similarly, $\left[EV_{exo|q=1}(0) - EV_{no}(0)\right]$ is the value of "certain" ($q = 1$) exogenous information conditional on having chosen to destroy the whole environmental area at $\tau = 1$. The QOV is the difference between these two values. Nonetheless, irreversibility creates a choice asymmetry: if the DM decides to develop everything

[1] Henry (1974) assumes that development is indivisible, so that $c_1 \in \{0, 1\}$. Arrow and Fisher (1974) instead consider a continuous choice set and assume that the expected benefits are $EV(d_1, d_2, \theta) = B_1(d_1) + B_2(d_1 + d_2, d_2; \theta)$, with $B_1(\cdot) + B_2(\cdot)$ being linear functions of the development levels $d_1 = 1 - c_1$, $d_1 + d_2 = 1 - c_2$ and d_2; uncertainty is represented by the random variable θ. The maximization of the expected benefits leads to a corner solution with $c_1^* \in \{0, 1\}$, which coincides with Henry's choice set.

now ($c_1 = 0$), the decision cannot be reversed in the future and any subsequent information she may receive has no economic value. Hence, $EV_{exo|q=1}(0) = EV_{no}(0)$ and the expression of the quasi-option value *à la* Arrow-Fisher becomes

$$QOV = EV_{exo|q=1}(1) - EV_{no}(1) \tag{1}$$

Since exogenous information is not "dangerous", it is $EV_{exo|q=1}(1) \geq EV_{no}(1)$, and so the QOV is always non-negative. This does not mean that developing now should never be optimal: it may happen that $EV_{exo|q=1}(1) < EV_{exo|q=1}(0)$, in that case $(c_1^*)_{exo|q=1} = 0$. This rather means that the case for preservation is strenghtened when one recognizes the prospect of further information about the future consequences of development: the amount of environmental resource preserved when there is exogenous information is equal or greater, i.e. $(c_1^*)_{exo|q=1} \geq (c_1^*)_{no}$.[2] This means that learning favors more flexible decisions ("irreversibility effect").[3]

A broader look at (1) shows that the QOV can be thought of as a particular "Waiting Value", i.e. a value emerging when the DM "stands by" in the present, moving her decision to the future, when exogenous information may be available. Broadly speaking, it is the money the DM is willing to pay in order to shift the decision from now to the future. Conrad (1980) suggests that the QOV is identical to the unconditional expected value of information at $\tau = 1$. Hanemann (1989) subsequently clarifies that this identity holds only if the value of information is conditional to having set $c_1 = 1$. Miller and

[2] As in Arrow and Fisher (1974) and Henry (1974), assume that $c_1^* \in \{0, 1\}$ and that $EV(c_1, c_2)$ are addictive with respect to the two-period benefits, i.e. $EV(c_1, c_2) = B_1(c_1) + EV[B_2(c_1, c_2)]$. It follows that $(c_1^*)_{exo|q=1} < (c_1^*)_{no}$ is impossible. This inequality would require $(c_1^*)_{exo|q=1} = 0$ and $(c_1^*)_{no} = 1$, hence respectively $B_1(1) + EV\left[B_2(1, (c_2^*)_{exo|q=1})\right] \leq B_1(0) + EV[B_2(0, 0)]$ and $B_1(1) + EV[B_2(1, 1)] \geq B_1(0) + EV[B_2(0, 0)]$, which would imply $EV\left[B_2(1, (c_2^*)_{exo|q=1})\right] \leq EV[B_2(1, 1)]$, i.e. $(c_2^*)_{exo|q=1} = 1$, which is incompatible with $(c_1^*)_{exo|q=1} = 0$.

[3] Epstein (1980) and later Hanemann (1989) and Ulph and Ulph (1995) show it is not always the case that $(c_1^*)_{exo|q=1} \geq (c_1^*)_{no}$. In particular, Epstein (1980) states a set of sufficient conditions on the expected net benefits function so that the Arrow and Fisher's (1974) irreversibility effect appears. Graham-Tomasi (1995), Mäler and Fisher (2006) and Salanie and Treich (2009) further clarify this specific issue.

Lad (1984) underline that this option value stems from the flexibility of the sequential decision when information can arrive before $\tau = 2$, whereas irreversibility per se accounts only for its "quasi" qualification. Therefore, in the Arrow-Fisher framework, among the two optimal decisions, only $c_1^* = 1$ preserves flexibility in the exogenous information scenario, and the QOV is the value of such flexibility. This value is a "quasi"-option because it vanishes if $c_1^* = 0$. The cost of this option is the economic loss generated by the non-development of the resource at $\tau = 1$.

Following this line of reasoning, one can define a more general *Waiting Value* (WV henceforth), relying on the difference between the expected value of the optimal flexible sequential decision and that of the optimal fixed once-and-for-all decision, i.e.

$$WV = EV_{exo}((c_1^*)_{exo}) - EV_{no}((c_1^*)_{no}) \tag{2}$$

where $EV_{exo}(c_1)$ and $EV_{no}(c_1)$ are, respectively, the expected value of the net benefits of preserving $c_1 \in [0,1]$ at $\tau = 1$, when exogenous information arrives with probability $q \in [0,1]$ before $\tau = 2$, and when no information arrives at all ($q = 0$). We indicate with $(c_\tau^*)_{exo}$ and $(c_\tau^*)_{no}$ the optimal level of preservation at $\tau = 1, 2$ in the "*exo*genous information" and in the "*no* information" scenario, respectively. The DM chooses $(c_2)_{exo}$ at $\tau = 2$ after having received, with a given probability $q \in [0,1]$, information about the relative benefits of the second period (flexible sequential decision). Instead, in the "*no* information" scenario, where there is no possibility of acquiring information before $\tau = 2$, for the DM it is equivalent to choose $(c_2)_{no}$ already at $\tau = 1$, by setting $(c_2)_{no} = (c_1)_{no}$ (fixed once-and-for-all decision).

Our definition of the WV differs from the Arrow-Fisher QOV in three aspects. First, in the Arrow-Fisher framework, exogenous information is "certain" ($q = 1$), while in our formulation $q \in [0,1]$. Second, in the QOV, it is $(c_1)_{exo} = (c_1)_{no} = 1$, while in the WV it can be $(c_1^*)_{exo} \neq (c_1^*)_{no}$, $(c_1^*)_{exo} \neq 1$ and $(c_1^*)_{no} \neq 1$. Third, the arguments of the WV

are the DM's optimal choices of $(c_1)_{exo}$ and $(c_1)_{no}$ (i.e., $(c_1^*)_{exo}$ and $(c_1^*)_{no}$). These two choices are taken in two situations that are identical except for the information available. On the contrary, the argument of the QOV is the hypothetical value $(c_1)_{exo} = (c_1)_{no} = 1$. Therefore, $QOV = WV$ only if $q = 1$ and $(c_1^*)_{exo} = (c_1^*)_{no} = 1$. In section 4.1 we show that the QOV has to be intended as an upper bound of the WV, since $QOV \geq WV$.

As discussed above, the conclusions drawn by Arrow and Fisher (1974), Henry (1974) and the related literature on the optimality of a complete preservation of an environmental resource when its development is irreversible are derived in a very particular framework of independent learning, i.e. with exogenous information about the net benefits of preservation. This result does not hold if the information is endogenous (dependent learning).

Miller and Lad (1984) and Freeman (1984) show that if information concerning future effects of the irreversible depletion of an environmental resource can be obtained only by carrying out depletion itself at $\tau = 1$, then it is optimal to develop (at least) a small portion of the environmental asset in the current period. In other words, the policy of postponing the choice in order to enable the DM to profit from the incoming information is sub-optimal when this information is endogenous.

Consider a third information scenario. Freeman (1984) assumes that full information is provided by any amount of development, i.e. for any $c_1 \in [0, 1)$. Moreover, no exogenous information arrives. Define $\pi_{endo} \in [0, 1]$ as the probability of acquiring information endogenously in Freeman's scenario. Let $EV_{endo|\pi_{endo}=1}(c_1)$ denote the expected value of the two-period net benefits of preserving c_1 at $\tau = 1$ when only *endo*genous information is available and it arrives "with certainty", i.e. $\pi_{endo} = 1$. Fisher and Hanemann (1987) clarifies that $EV_{endo|\pi_{endo}=1}$ is equivalent to the expected value function in the "*no* information" scenario, EV_{no}, in the event that no development is undertaken, and to that in the "*exo*genous information" (with certainty) scenario, $EV_{exo|q=1}$, in the event that any development is undertaken. In symbols,

$$EV_{endo|\pi_{endo}=1}(c_1) = \begin{cases} EV_{no}(1) & \text{if} \quad c_1 = 1 \\ EV_{exo|q=1}(c_1) & \text{if} \quad c_1 \in [0,1) \end{cases}$$

Several results follow from this specific formulation of the problem. First of all, in Freeman's "*endo*genous information" scenario it can never be optimal to preserve the whole amount of the environmental resource in the current period, i.e. $(c_1^*)_{endo} \neq 1$. Secondly, there is again a corner solution for the level of development at $\tau = 1$, in the sense that one either develops fully now, i.e. $(c_1^*)_{endo} = 0$, or engages in an infinitesimal amount $\varepsilon > 0$ of development, i.e. $(c_1^*)_{endo} = 1-\varepsilon$. Finally, Fisher and Hanemann (1987) introduce a *QOV of the minimum feasible development* ($\varepsilon - development$), defined as

$$QOV_{\varepsilon} = EV_{endo||\pi_{endo}=1}(1-\varepsilon) - EV_{no}(1-\varepsilon) \tag{3}$$

which always is non-negative.

Notice that this specific definition raises some problems about the meaning of QOV_{ε}. First of all, as in the QOV, the argument of the QOV_{ε} is a hypothetical level of preservation. Moreover, the hypothetical preservation level $c_1 = 1-\varepsilon$ is generally[4] not optimal in the "*no* information" scenario, where instead $(c_1^*)_{no} \in \{0, 1\}$: only total preservation or total development is an optimal choice in $\tau = 1$ when there is no possibility to obtain information between $\tau = 1$ and $\tau = 2$. Finally, the specificity of Freeman's (1984) and Fisher and Hanemann's (1987) results raises doubts as to whether the environmental policy implications of the endogenous information arrival could depend on the precise manner in which development generates information, i.e., on the form of the information production function.

For all these reasons, we try to model endogenous information in a more general

[4]In the "no information" scenario there can be an internal solution in $\tau = 1$ (and so also in $\tau = 2$) only when the current net benefit and the expected future net benefit have opposite sign and are equal in absolute value. In this trivial case, each preservation level $c_1 \in [0, 1]$ leads to the same expected value of the two-period net benefits (see section 3.3.3).

framework (section 2). There is a large number of environmental problems in which the possibility of acquiring information endogenously could depend on the size of the development the DM chooses to perform. In the case of oil extraction in a country, for example, there may be uncertainty about whether and where the land contains oil in commercial quantities. If this is the case, it is likely that the uncertainty can be solved by undertaking some development. But it is doubtless that if you drill the land (by destroying a part of the natural resource), the deeper you drill the higher the probability of discovering whether there is or is not an oil well in that country. Another example: if you "destroy" only a few trees of a forest, little information is obtained about the possible extinction of a particular species. If you keep on destroying a larger portion of the forest, you can obtain greater information about the pervasive effects of the development activity.

Thus, in many environmental decision problems it seems plausible that, if information about future net benefits of preservation depends on the current development of the natural resource, the probability of obtaining information in the future must be increasing in the level of development currently carried out, i.e. inversely related to c_1. We make this assumption in our model. Moreover, differently from Fisher-Hanemann's QOV_{ε}, when dealing with endogenous information we introduce a *Testing Value* (TV henceforth), defined as

$$TV = EV_{exo\&endo}((c_1^*)_{exo\&endo}) - EV_{exo}((c_1^*)_{exo}) \tag{4}$$

where $EV_{exo\&endo}(c_1)$ is the expected value of the two-period net benefits of preservation when there is both *exo*genous (arriving with probability q) *and endo*genous (arriving with probability λ, depending negatively on c_1) information;[5] $(c_1^*)_{exo\&endo}$ is the opti-

[5]Notice that in our more general formulation of dependent learning, even when choosing to destroy the entire amount of the environmental resource in $\tau = 1$, it can happen that no endogenous information arrives before $\tau = 2$.

mal preservation level in the current period, under the "*exo*genous *and endo*genous" information scenario. Again, it can be that $(c_1^*)_{exo\&endo} \neq (c_1^*)_{exo}$.

According to our definition, the TV is the additional value attached to endogenous information, additional with respect to information arriving exogenously. It is the gain the DM obtains when she can receive, with some probability, information regarding future benefits also by developing in the current period (compared to the case in which there is only independent learning *à la* Arrow-Fisher).[6]

The QOV_ε in (3) becomes a particular TV that emerges when the following four conditions are simultaneously satisfied: exogenous information is completely absent ($q = 0$); the level of information endogenously arriving is the same for every $c_1 \in [0, 1)$; information arrives with certainty ($\pi_{endo} = 1$) for every $c_1 \in [0, 1)$; the optimal choice at $\tau = 1$ in both information scenarios (without exogenous information) should be $(c_1^*)_{endo} = (c_1^*)_{no} = 1 - \varepsilon$. As anticipated above, it can never be $(c_1^*)_{no} = 1 - \varepsilon$. Thus, leaving aside the last condition $(c_1^*)_{endo} = (c_1^*)_{no} = 1 - \varepsilon$, whenever $(c_1^*)_{endo} = 1 - \varepsilon$, it should be that $(c_1^*)_{no} = 0$.

The remaining part of the paper is structured as follows. In section 2 we present our two-period model of environmental choice under uncertainty and irreversibility. In section 3 we analyze the DM's maximization problem and optimal environmental choices in the different information scenarios. In section 4 we calculate the WV and the TV as functions of the parameters of the environmental decision problem. Moreover, we show the main features of the TV in comparison to those of the WV. In section 5 we conclude by discussing some policy implications of our theoretical predictions.

[6] Obviously, if $(c_1^*)_{exo\&endo} = 1$, there is only exogenous information, then $EV_{exo\&endo} \equiv EV_{exo}$, $(c_1^*)_{exo\&endo} = (c_1^*)_{exo}$ and $TV = 0$. Attanasi and Montesano (2008) show that if there is strategic interaction between two decision makers, the TV can be positive even for the decision maker choosing $(c_1^*)_{exo\&endo} = 1$.

2 The Model

2.1 Assumptions and notation

Consider a two-period model of environmental decision. The risk neutral DM chooses the amount of environmental resource to preserve at two subsequent *times* ($\tau = 1, 2$). We call *period* 1 the time period between $\tau = 1$ and $\tau = 2$ and *period* 2 the time period after $\tau = 2$. At $\tau = 1$ the DM chooses the amount of environmental resource to be preserved in *period* 1, i.e. until $\tau = 2$. At $\tau = 2$ she chooses the amount of resource to be preserved in *period* 2. Given the assumption that development is irreversible, the DM's options at $\tau = 2$ are constrained by the decision taken at $\tau = 1$. Normalizing the level of the environmental resource to 1, $c_1 \in [0, 1]$ denotes the amount preserved at $\tau = 1$. By irreversibility, the amount preserved at $\tau = 2$ cannot be greater than c_1.

We define the two-period expected net benefits adopting the same separable and linear functional form used by Arrow and Fisher (1974). Let the net benefit be directly proportional to the amount of preserved resource, with b_1 representing the net benefit per unit of resource preserved in period 1.[7] We assume that the current net benefit from preservation is known to the DM at $\tau = 1$ and it is negative, i.e. $b_1 < 0$; thus, the unique incentive to choose $c_1 \neq 0$ at $\tau = 1$ is given by the possibility to obtain a positive future net benefit from preservation in period 2.[8]

This future net benefit is uncertain, depending on two possible states of the world. With probability π, the state is revealed to the DM in period 1, i.e. before she takes her decision at $\tau = 2$. With probability $1 - \pi$, the DM does not know the state of the world when she chooses the optimal level of c_2 at $\tau = 2$: this state will be revealed

[7] In Dasgupta and Heal (1979) and Chichilnisky and Heal (1993), b_τ represents the benefit of preservation in period τ, with $\tau = 1, 2$. We interpret it as the difference between the benefit of preservation and the benefit of development in period τ.

[8] We choose not to contemplate in the analysis the case $b_1 = 0$, since it makes the choice of c_1 irrelevant concerning the net benefit in period 1.

in period 2, after this decision has been taken. We indicate with b_2^j the net benefit per unit of resource still preserved in period 2, when the state of the world is s^j,with $j = u, f$. The future net benefit from preservation is negative if the state of the world is s^u ($u$nfavorable state), and positive if the state of the world is s^f ($f$avorable state), i.e. $b_2^u < 0, b_2^f > 0$. We indicate with $p \in [0, 1]$ the probability of the unfavorable state s^u.

Let us normalize the net benefits in terms of b_2^f by defining $x := -\frac{b_1}{b_2^f}, y := -\frac{b_2^u}{b_2^f}$ and by putting $b_2^f = 1$. Notice that both x and y are positive, with x representing the relative weight of the loss from preservation in period 1 with respect to the gain in the favorable state of the world in period 2; and y representing the relative weight of the loss from preservation in period 2 in the unfavorable state of the world, with respect to the gain in the favorable one.

We indicate with c_2 the amount of environmental resource preserved at $\tau = 2$ when the state of the world has not been revealed in period 1 and with c_2^j the amount of environmental resource preserved at $\tau = 2$ when the DM knows the revealed state of the world is s^j. The structure of the decision problem is represented in figure 1. It can be summarized as follows:

- $\tau = 1$: the DM chooses the amount of the resource to be preserved in *period* 1;
- *period* 1: the state of the world is either revealed or not;
- $\tau = 2$: the DM chooses the amount of the resource to be preserved in *period* 2;
- *period* 2: the state of the world is revealed, given it was not revealed in *period* 1.

Therefore, when the DM receives information in period 1, in $\tau = 2$ she is in the upper part of the decision tree and she chooses the optimal preservation level at $\tau = 2$, $(c_2^u)^*$ or $(c_2^f)^*$, knowing the state of the world, s^u or s^f, respectively. Otherwise, in $\tau = 2$ she is in the lower part of the decision tree and the optimal choice at $\tau = 2$, c_2^*, is independent from the state of the world, that will be revealed only in period 2.

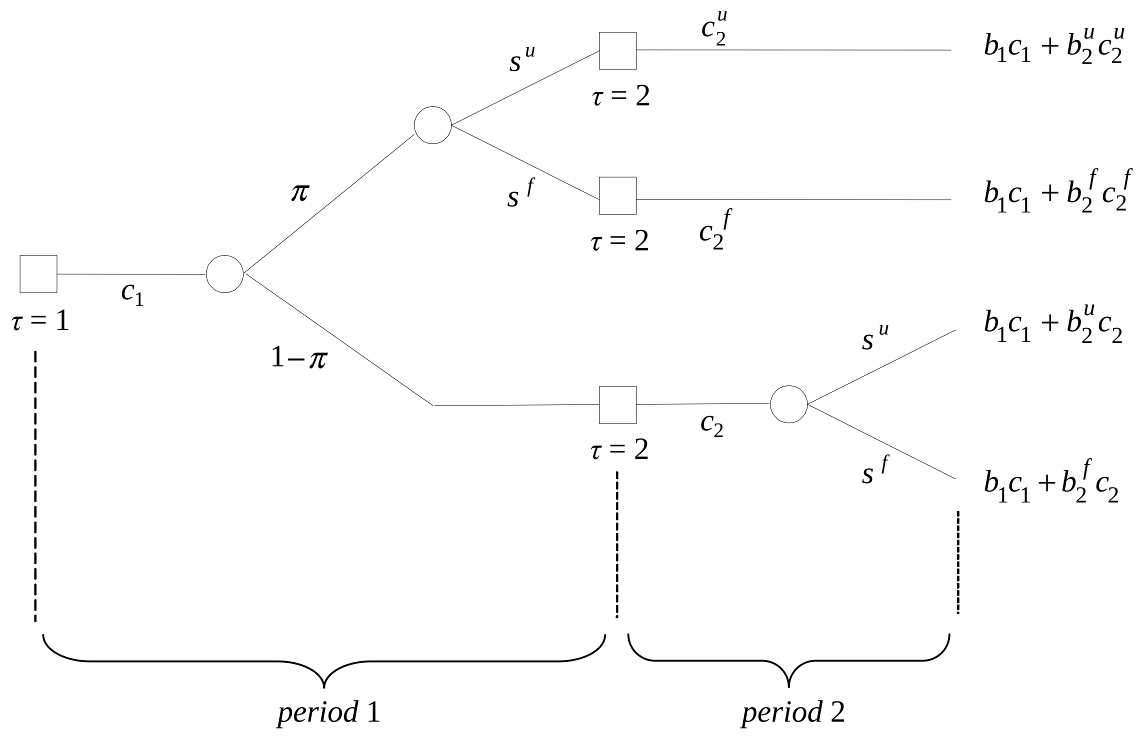

Figure 1. The decision tree

2.2 Modelling uncertainty

In our framework, when choosing at $\tau = 1$, the DM does not know if the state of the world will be known or not when she will choose again at $\tau = 2$. The key parameter is $\pi \in [0, 1]$, the probability that the state will be revealed in period 1, i.e. the probability that information (exogenous and/or endogenous) will arrive before $\tau = 2$. We distinguish different "degrees of certainty" of receiving information: it can arrive with certainty ($\pi = 1$), with some probability ($\pi \in [0, 1]$) or may certainly not arrive ($\pi = 0$).

Moreover, according to the components inside π, we distinguish different "kinds of information". Information can be (only) exogenous, (only) endogenous, or both. In the first case, π does not depend on $(1-c_1)$, the amount of environmental resource developed at $\tau = 1$. In the second case, π depends only on $(1 - c_1)$, the level of development. Here, as anticipated in section 1, we assume that in case of dependent learning the probability of information arrival depends on the level of development carried out. In the third case, a part of the information arrives exogenously and the rest arrives according to $(1 - c_1)$; hence, $\pi = q + \lambda f(1 - c_1)$, with $q \in [0, 1]$ being the probability of acquiring exogenous information and $\lambda f(1 - c_1) \in [0, 1 - q]$ being the probability of acquiring endogenous information through the information production function $f : [0, 1] \to [0, 1]$, with $f'(\cdot) > 0$, i.e. strictly decreasing in c_1. In particular, we analyze the linear case $f(1 - c_1) = 1 - c_1$.

We identify the third case, in which both *exo*genous *and endo*genous information may occur, as the general case, namely (*exo&endo*), represented by

$$\begin{aligned} \pi &= q + \lambda(1 - c_1) \qquad \text{for} \quad c_1 \in [0, 1] \\ &\quad \text{with} \quad q \in [0, 1]\,, \quad \lambda \in [0, 1 - q]\,. \end{aligned}$$

The other relevant information scenarios are derived by imposing specific restrictions on the key parameters:

(*exo*) only *exo*genous information: $\lambda = 0$ for $c_1 \in [0, 1]$;

(*endo*) only *endo*genous information: $q = 0$ for $c_1 \in [0, 1]$;

(*no*) *no* information: $\lambda = q = 0$, for $c_1 \in [0, 1]$.

In this framework, the scenario "information arriving with certainty" can be derived by imposing $\lambda = 1 - q$: information arrives with certainty if $q = 1$ or if $q < 1$ and $c_1 = 0$. This condition in subcase (*exo*) implies $q = 1$ and in subcase (*endo*) implies $\lambda = 1$ and $c_1 = 0$. This scenario has been frequently analyzed in the environmental option values literature: it enables us to make comparisons and to show that our results also hold under the restriction of information arriving with certainty.

3 Optimal Preservation Choices

3.1 Two preliminary results

In this section, for each information structure described in section 2.2, we find the DM's optimal preservation level at $\tau = 1$, c_1^*, and at $\tau = 2$ when the state of the world is not revealed in period 1, c_2^*. First of all, we state two basic results which hold independently from the kind of information structure we deal with, i.e. independently from the way in which π is defined. Referring to figure 1:

Result 1. If the state of the world is revealed in period 1, then $(c_2^u)^* = 0$ and $(c_2^f)^* = c_1$.

Result 2. If the state of the world is not revealed in period 1,

- case (i): if the expected second-period net benefit of preservation $(pb_2^u + (1-p)b_2^f)$ is positive, i.e. $y \in \left(0, \frac{1-p}{p}\right)$, where $y = -\frac{b_2^u}{b_2^f}$, with $b_2^f = 1$ (see section 2.1), then ${c_2}^* = c_1$;

- case (ii): if the expected second-period net benefit of preservation is negative, i.e. $y \in \left(\frac{1-p}{p}, +\infty\right)$, then ${c_2}^* = 0$;

- case (iii): if the expected second-period net benefit of preservation is null, i.e. $y = \frac{1-p}{p}$, then ${c_2}^* \in [0, c_1]$.

3.2 General case ($exo\&endo$): both Exogenous and Endogenous Information

Let us write and solve the DM's utility maximization problem in the general case, in which both exogenous and endogenous information are available with some probability (respectively, with $q \in [0,1]$ and $\lambda(1-c_1) \in [0, (1-q)(1-c_1)]$) in period 1. Given *Result* 1, the realized payoffs are as indicated in figure 2.

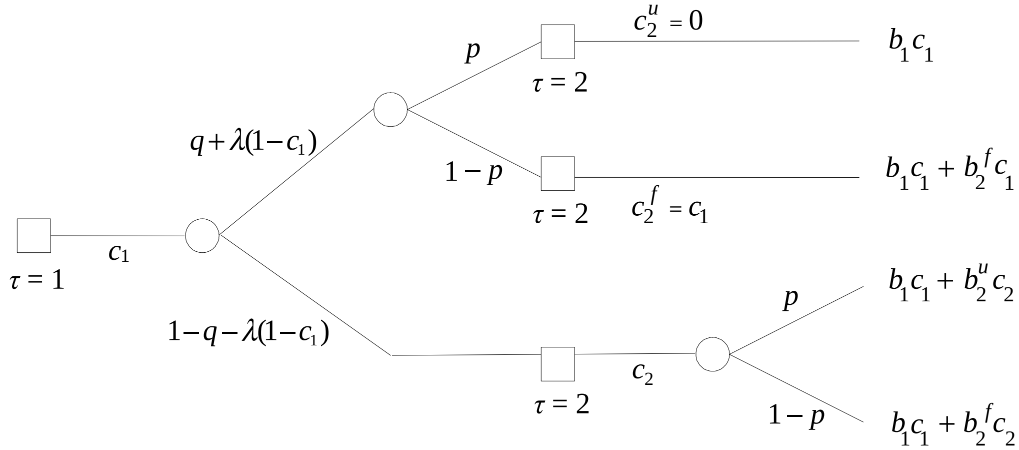

Figure 2. Exogenous and Endogenous information scenario

The DM's expected value of net benefits of preservation in both periods is:

$$\begin{aligned} EV_{exo\&endo}(c_1, c_2 |\, c_2^u = 0, c_2^f = c_1) &= [q + \lambda(1-c_1)]\left[b_1 + (1-p)b_2^f\right] c_1 + \\ &\quad \{1 - [q + \lambda(1-c_1)]\}\left[b_1c_1 + pb_2^u c_2 + (1-p)b_2^f c_2\right] \end{aligned}$$

By analyzing the lower part of the compound lottery in figure 2, one can distinguish three cases, according to the expected value of the second-period net benefit (*Result* 2).

Case (i). Given that $y \in \left(0, \frac{1-p}{p}\right)$, the optimal preservation level at $\tau = 2$ when the state of the world is not known is $(c_2^*)_{exo\&endo} = (c_1^*)_{exo\&endo}$, with

$$(c_1^*)_{exo\&endo} = \begin{cases} 1 & \text{if} \quad y \in \left(0, \frac{1-p}{p}\frac{1}{1-q+\lambda}\right) \text{ and } \\ & \qquad x \in (0, 1-p-(1-q+\lambda)py) \\ \frac{(1-p-x)-(1-q-\lambda)py}{2\lambda py} & \text{if} \quad x \in [\max\{0, 1-p-(1-q+\lambda)py\}, \\ & \qquad 1-p-(1-q-\lambda)py] \\ 0 & \text{if} \quad x \in (1-p-(1-q-\lambda)py, +\infty) \end{cases}$$

and the optimal expected value function is:

$$EV^*_{exo\&endo}(x,y) = \begin{cases} (1-p-x)-(1-q)py & \text{if} \quad \begin{array}{l} y \in \left(0, \frac{1-p}{p}\frac{1}{1-q+\lambda}\right) \text{ and} \\ x \in (0, 1-p-(1-q+\lambda)py) \end{array} \\ \frac{[(1-p-x)-(1-q-\lambda)py]^2}{4\lambda py} & \text{if} \quad \begin{array}{c} x \in [\max\{0, 1-p-(1-q+\lambda)py\}, \\ 1-p-(1-q-\lambda)py] \end{array} \\ 0 & \text{if} \quad x \in (1-p-(1-q-\lambda)py, +\infty) \end{cases}$$

Case (ii). Given that $y \in \left(\frac{1-p}{p}, +\infty\right)$, the optimal preservation level at $\tau = 2$ when the state of the world is not known is $(c_2^*)_{exo\&endo} = 0$, and

$$(c_1^*)_{exo\&endo} = \begin{cases} 1 & \text{if} \quad q > \lambda \text{ and } x \in (0, (1-p)(q-\lambda)] \\ \frac{(1-p)(q+\lambda)-x}{2(1-p)\lambda} & \text{if} \quad x \in ((1-p)\max\{0, q-\lambda\}, (1-p)(q+\lambda)] \\ 0 & \text{if} \quad x \in [(1-p)(q+\lambda), +\infty) \end{cases}$$

and the optimal expected value function is:

$$EV^*_{exo\&endo}(x,y) = \begin{cases} (1-p)q-x & \text{if} \quad q > \lambda \text{ and } x \in (0, (1-p)(q-\lambda)] \\ \frac{[(1-p)(q+\lambda)-x]^2}{4(1-p)\lambda} & \text{if} \quad x \in ((1-p)\max\{0, q-\lambda\}, (1-p)(q+\lambda)] \\ 0 & \text{if} \quad x \in [(1-p)(q+\lambda), +\infty) \end{cases}$$

Case (iii). Given that $y = \frac{1-p}{p}$, the previous results on $(c_1^*)_{exo\&endo}$ and on $EV^*_{exo\&endo}(x,y)$ apply. The only difference is that in this case $(c_2^*)_{exo\&endo} \in [0, (c_1^*)_{exo\&endo}]$.

In general, notice that for $y \in \left(\frac{1-p}{p}, +\infty\right)$, $(c_1^*)_{exo\&endo} = 1$ only if $q > \lambda$, while for $y \in \left(0, \frac{1-p}{p}\right)$ it can be that $(c_1^*)_{exo\&endo} = 1$ also if $q < \lambda$.

3.3 Specific information scenarios

Let us specify the previous results in the three subcases introduced in section 2.2.

3.3.1 Subcase (exo): Only Exogenous Information

Since $\lambda = 0$, the DM at $\tau = 1$ knows that, independently from the preservation level chosen at $\tau = 1$, with probability $q \in [0, 1]$ she will know the realized value of the net benefit b_2^j in period 1. The optimal levels of preservation are:[9]

$$((c_1^*)_{exo}, (c_2^*)_{exo}) = \begin{cases} (1,1) & \text{if} \quad y \in \left(0, \frac{1-p}{p}\right) & \text{and} \quad x \in (0, 1-p-(1-q)\,py) \\ (0,0) & \text{if} \quad y \in \left(0, \frac{1-p}{p}\right) & \text{and} \quad x \in (1-p-(1-q)\,py, +\infty) \\ (1,0) & \text{if} \quad y \in \left(\frac{1-p}{p}, +\infty\right) & \text{and} \quad x \in (0, (1-p)q) \\ (0,0) & \text{if} \quad y \in \left[\frac{1-p}{p}, +\infty\right) & \text{and} \quad x \in [(1-p)q, +\infty) \end{cases}$$

and the optimal expected value function $EV_{exo}^*(x, y)$ is equal to:

$$\begin{cases} (1-p-x)-(1-q)py & \text{if} \quad y \in \left(0, \frac{1-p}{p}\right] & \text{and} \quad x \in (0, 1-p-(1-q)\,py) \\ 0 & \text{if} \quad y \in \left(0, \frac{1-p}{p}\right] & \text{and} \quad x \in (1-p-(1-q)\,py, +\infty) \\ (1-p)q-x & \text{if} \quad y \in \left[\frac{1-p}{p}, +\infty\right) & \text{and} \quad x \in (0, (1-p)q] \\ 0 & \text{if} \quad y \in \left[\frac{1-p}{p}, +\infty\right) & \text{and} \quad x \in [(1-p)q, +\infty) \end{cases}$$

When $q = 1$ (exogenous information arrives with certainty), we have the traditional case of Arrow and Fisher (1974), as defined in section 1. However, in section 4.1 we clarify that their definition of QOV as in (1) restricts the analysis to the subset of pairs (x, y) for which $(c_1^*)_{exo} = (c_2^*)_{exo} = 1$.

---

[9] In the limit cases, we find that

$$\begin{cases} (c_1^*)_{exo} = 1, (c_2^*)_{exo} \in [0,1] & \text{if} \quad y = \frac{1-p}{p} & \text{and} \quad x \in (0, (1-p)q) \\ (c_1^*)_{exo} = (c_2^*)_{exo} \in [0,1] & \text{if} \quad y \in \left(0, \frac{1-p}{p}\right) & \text{and} \quad x = 1-p-(1-q)\,py \\ (c_1^*)_{exo} \in [0,1], (c_2^*)_{exo} \in [0, (c_1^*)_{exo}] & \text{if} \quad y = \frac{1-p}{p} & \text{and} \quad x = (1-p)q \\ (c_1^*)_{exo} \in [0,1], (c_2^*)_{exo} = 0 & \text{if} \quad y \in \left(\frac{1-p}{p}, +\infty\right) & \text{and} \quad x = (1-p)q \end{cases}$$

3.3.2 Subcase (*endo*): Only Endogenous Information

Since $q = 0$, the DM can obtain information in period 1 with probability $\lambda(1-c_1)$, where $\lambda \in [0,1]$, only if at $\tau = 1$ she destroys a portion of the environmental resource. Hence, when choosing at $\tau = 1$, she knows that the probability of information arriving in period 1 depends negatively on c_1. The optimal levels of preservation $\left((c_1^*)_{endo}, (c_2^*)_{endo}\right)$ are equal to:[10]

$$\begin{cases} (1,1) & \text{if } \; y \in \left(0, \frac{1-p}{p}\frac{1}{1+\lambda}\right) & \text{and} \;\; x \in (0, 1-p-(1+\lambda)py] \\ \frac{(1-p-x)-(1-\lambda)py}{2\lambda py} \cdot (1,1) & \text{if } \; y \in \left(0, \frac{1-p}{p}\right] & \text{and} \;\; \begin{array}{c} x \in [\max\{0, 1-p-(1+\lambda)py\}, \\ 1-p-(1-\lambda)py] \end{array} \\ (0,0) & \text{if } \; y \in \left(0, \frac{1-p}{p}\right] & \text{and} \;\; x \in [1-p-(1-\lambda)py, +\infty) \\ \left(\frac{1}{2} - \frac{x}{2(1-p)\lambda}, 0\right) & \text{if } \; y \in \left[\frac{1-p}{p}, +\infty\right) & \text{and} \;\; x \in (0, (1-p)\lambda] \\ (0,0) & \text{if } \; y \in \left[\frac{1-p}{p}, +\infty\right) & \text{and} \;\; x \in [(1-p)\lambda, +\infty) \end{cases}$$

and the optimal expected value function $EV_{endo}^*(x,y)$ is equal to:

$$\begin{cases} (1-p-x)-py & \text{if } \; y \in \left(0, \frac{1-p}{p}\frac{1}{1+\lambda}\right] & \text{and} \;\; x \in (0, 1-p-(1+\lambda)py] \\ \frac{[(1-p-x)-(1-\lambda)py]^2}{4\lambda py} & \text{if } \; y \in \left(0, \frac{1-p}{p}\right] & \text{and} \;\; \begin{array}{c} x \in [\max\{0, 1-p-(1+\lambda)py\}, \\ 1-p-(1-\lambda)py] \end{array} \\ 0 & \text{if } \; y \in \left(0, \frac{1-p}{p}\right] & \text{and} \;\; x \in [1-p-(1-\lambda)py, +\infty) \\ \frac{[(1-p)\lambda - x]^2}{4(1-p)\lambda} & \text{if } \; y \in \left[\frac{1-p}{p}, +\infty\right) & \text{and} \;\; x \in (0, (1-p)\lambda] \\ 0 & \text{if } \; y \in \left[\frac{1-p}{p}, +\infty\right) & \text{and} \;\; x \in [(1-p)\lambda, +\infty) \end{cases}$$

Notice that when the relative weight of the loss from preservation in period 2 in the unfavorable state of the world with respect to the gain in the favorable one is high enough $\left(y > \frac{1-p}{p}\right)$, the expected value of the second-period net benefit of preservation is negative. In this case, it is never optimal to preserve completely the environmental

[10] If $y = \frac{1-p}{p}$ and $x \in [0, (1-p)\lambda]$, then $(c_1^*)_{endo} = \frac{1}{2} - \frac{x}{2(1-p)\lambda}$ and $(c_2^*)_{endo} \in [0, (c_1^*)_{endo}]$.

resource in either of the two periods. In particular, it is never optimal to preserve a positive amount of the resource at $\tau = 2$ and the highest possible amount of the resource that it is optimal to preserve at $\tau = 1$ is limited by half. Instead, when the expected value of the second-period net benefit of preservation is positive, preservation of the whole amount of the resource can occur, thus renouncing the possibility to obtain information.

In our framework, when $q = 0$, information can "arrive with certainty" ($\lambda(1-c_1) = 1$) only under the condition that all the resource is destroyed at $\tau = 1$ ($c_1 = 0$). This differs from Freeman's endogenous information scenario, the one in which Fisher and Hanemann (1987) define the QOV_ε as in (3). Their assumption that full information is provided by any amount of development would imply in our endogenous information setting that $\pi = 1$ for every $c_1 \neq 0$.

3.3.3 Subcase (no): No Information

Since $\pi = 0$, the DM cannot obtain information in period 1. Hence, it is equivalent for her to choose c_1 and c_2 simultaneously at $\tau = 1$. The optimal preservation levels are:[11]

$$((c_1^*)_{no}, (c_2^*)_{no}) = \begin{cases} (1,1) & \text{if} \quad y \in \left(0, \frac{1-p}{p}\right] \quad \text{and} \quad x \in (0, 1-p-py) \\ (0,0) & \text{if} \quad y \in \left(0, \frac{1-p}{p}\right] \quad \text{and} \quad x \in (1-p-py, +\infty) \\ (0,0) & \text{if} \quad y \in \left[\frac{1-p}{p}, +\infty\right) \end{cases}$$

and the optimal expected value function is:

$$EV_{no}^*(x,y) = \begin{cases} (1-p-x)-py & \text{if} \quad y \in \left(0, \frac{1-p}{p}\right] \quad \text{and} \quad x \in (0, 1-p-py] \\ 0 & \text{if} \quad y \in \left(0, \frac{1-p}{p}\right] \quad \text{and} \quad x \in [1-p-py, +\infty) \\ 0 & \text{if} \quad y \in \left[\frac{1-p}{p}, +\infty\right) \end{cases}$$

[11]If $x = 1-p-py$, then $(c_1^*)_{no} \in [0,1]$ and $(c_2^*)_{no} \in [0, (c_1^*)_{no}]$. More precisely, this happens when the expected value of the second period net benefit of preservation is positive and equal to the absolute value of the first period net benefit, i.e. $pb_2^u + (1-p)b_2^f = -b_1$, given that $b_1 < 0$.

4 Environmental option values analysis

In this section, we express the WV and the TV as functions of the decision problem parameters, using the results on the DM's optimal behavior obtained in sections 3.2 and 3.3. Then, we describe the main features of each of the two option values and their effects on the DM's optimal behavior.

First of all, let us represent graphically the results on the optimal preservation level at $\tau = 1$ and at $\tau = 2$ in the four information scenarios, when the state of the world is not revealed in period 1. Consider again figure 1. The DM chooses c_1 optimally, taking into account the possibility that information could emerge in period 1.

If this occurs, when choosing at $\tau = 2$, the DM knows the state of the world (she is in the upper part of the decision tree). Then, by *Result* 1, the optimal preservation levels are $(c_2^u)^* = 0$ and $(c_2^f)^* = c_1^*$: she develops everything if the unfavorable state of the world comes out in period 1 and preserves everything she has preserved at $\tau = 1$ otherwise. Therefore, in this case, the optimal choice at $\tau = 2$, namely $(c_2^j)^*$ for $j = u, f$, is uniquely determined by c_1^* in every scenario except the "*no* information" one, where, obviously, it is not possible to obtain information in period 1.

If information does not emerge, the DM is in the lower part of the decision tree in figure 1. Also in this case the possibility of acquiring information (exogenously and/or endogenously) in period 1 influences the DM's choice at $\tau = 2$, since, by irreversibility, the choice of c_1 constrains the set of possible c_2.

We compare the DM's behavior in the different information scenarios, by indicating, for each of them, the optimal pair (c_1^*, c_2^*). Recall that c_2^* is the optimal amount of environmental resource preserved at $\tau = 2$ given that information does not arrive in period 1.

Let us look at figures 3-7, where the values of (c_1^*, c_2^*) are indicated for all possible pairs of relative benefits $(x, y) \in \mathbb{R}^2_{++}$, for each of the four information scenarios introduced in section 2.2. When we move towards the south-east (north-west) of the set of possible pairs of relative benefits, preservation in both periods becomes more (less) convenient.

Figures 3, 4 and 5, in which, respectively, the three information scenarios (*no*), (*exo*) and (*endo*) are represented, are drawn for the same values of the relevant parameters, namely $(p, q, \lambda) = (1/2, 1/2, 1/2)$. In figures 6 and 7 the (*exo&endo*) scenario is represented, with $(p, q, \lambda) = (1/2, 1/2, 1/2)$ and $(p, q, \lambda) = (1/2, 1/2, 1/3)$ respectively. Figure 8 represents the difference in terms of (c_1^*, c_2^*) between the (*exo&endo*) scenario represented in figure 7 and the (*exo*) scenario represented in figure 4.

Consider now all the possible values of the pair (c_1^*, c_2^*) in the four information scenarios represented in figures 3-7. We are able to distinguish five different regions in terms of (c_1^*, c_2^*):

- (w_1, w_2) region: the DM preserves everything (she *w*aits) at $\tau = 1$ and also at $\tau = 2$ when information does not arrive in period 1;

- (w_1, d_2) region: the DM preserves everything (she *w*aits) at $\tau = 1$ and *d*estroys everything at $\tau = 2$ when information does not arrive in period 1;

- (t_1, t_2) region: the DM preserves only a part of the resource (she *t*ests it) at $\tau = 1$ and preserves the same amount at $\tau = 2$ when information does not arrive in period 1;

- (t_1, d_2) region: the DM preserves only a part of the resource (she *t*ests it) at $\tau = 1$ and *d*estroys everything at $\tau = 2$ when information does not arrive in period 1;

- (d_1, d_2) region: the DM *d*estroys everything at $\tau = 1$, hence also at $\tau = 2$.

Notice that the two regions in which the DM "*t*ests" the environmental resource at $\tau = 1$ are possible only when there is endogenous information (fig. 5, 6, 7).

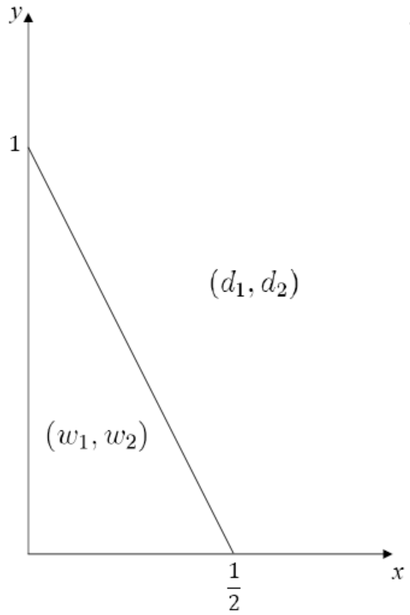

Figure 3. No Info,
$(p, q, \lambda) = \left(\frac{1}{2}, 0, 0\right)$

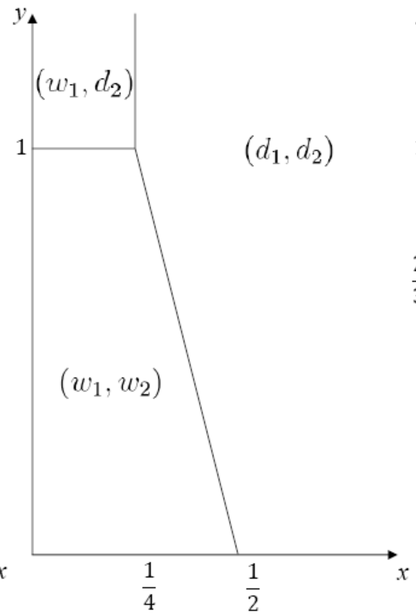

Figure 4. Exo Info,
$(p, q, \lambda) = \left(\frac{1}{2}, \frac{1}{2}, 0\right)$

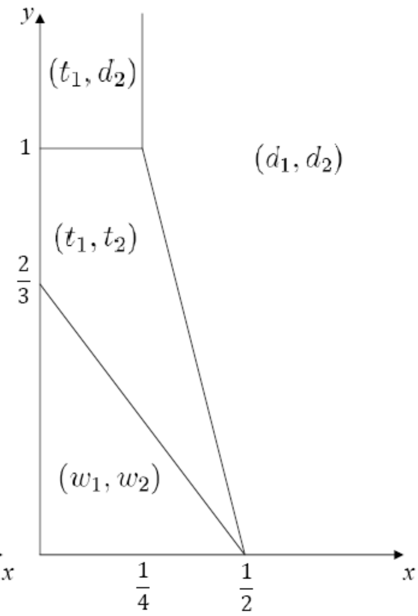

Figure 5. Endo Info,
$(p, q, \lambda) = \left(\frac{1}{2}, 0, \frac{1}{2}\right)$

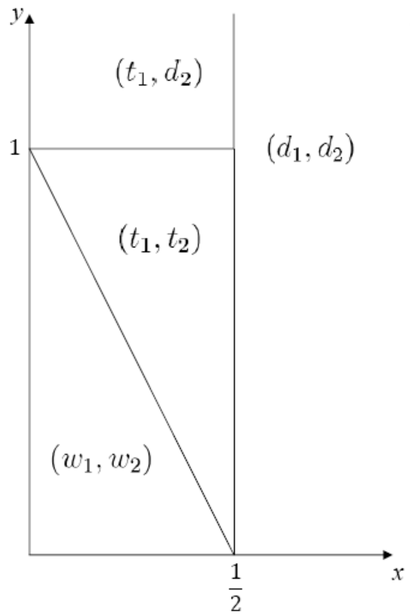

Figure 6. Exo&Endo Info,
$\lambda = q$, $(p, q, \lambda) = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$

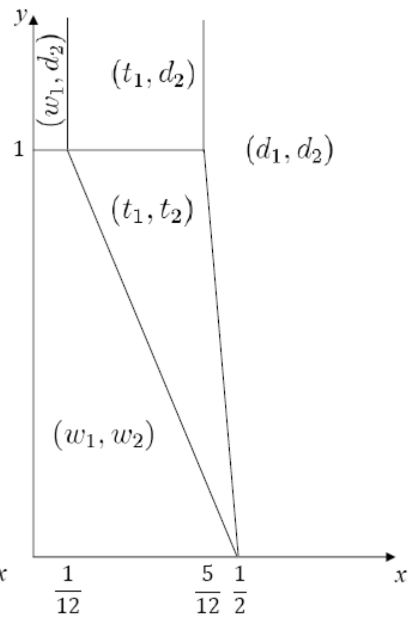

Figure 7. Exo&Endo Info,
$\lambda < q$, $(p, q, \lambda) = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{3}\right)$

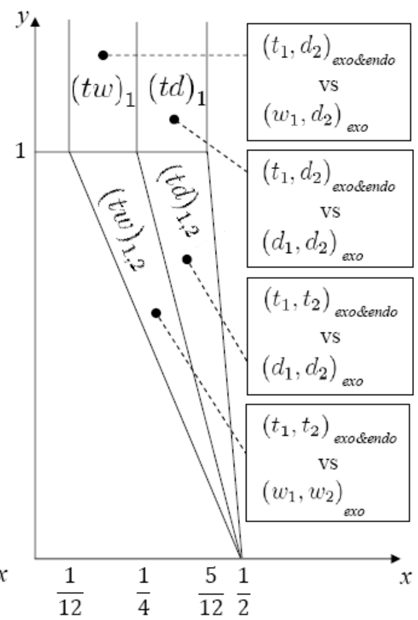

Figure 8. Exo&Endo (fig. 7)
vs Exo (fig. 4)

4.1 Analysis of the Waiting Value

Let us calculate the Waiting Value according to the way it has been defined in (2). We calculate the difference between the optimal expected value of net benefits of preservation in the (exo) scenario and in the (no) scenario. We are interested about how this difference varies according to x and y. In the regions in which this difference is positive, the WV is equal to

$$\begin{cases} qpy & \text{if} \quad y \in \left(0, \frac{1-p}{p}\right] \quad \text{and} \quad x \in (0, 1-p-py] \quad (ww)_{1,2} \\ (1-p-x)-(1-q)py & \text{if} \quad y \in \left(0, \frac{1-p}{p}\right] \quad \text{and} \quad x \in [1-p-py\ ,\ 1-p-(1-q)\,py] \quad (wd)_{1,2} \\ (1-p)q-x & \text{if} \quad y \in \left[\frac{1-p}{p}, +\infty\right) \quad \text{and} \quad x \in (0, (1-p)q] \quad (wd)_1 \end{cases}$$

and equal to zero otherwise.

Compare figure 4 to figure 3. The region $(ww)_{1,2}$:= "*w*aiting both at $\tau = 1$ and $\tau = 2$ with and without exogenous information" includes the values for (x, y) such that $(c^*_\tau)_{exo} = (c^*_\tau)_{no} = 1$ for $\tau = 1, 2$. The regions $(wd)_1$:= "*w*aiting instead of *d*estroying at $\tau = 1$" and $(wd)_{1,2}$:= "*w*aiting instead of destroying both at $\tau = 1$ and $\tau = 2$" include values for (x, y) such that the level of preservation is higher in the (exo) scenario, respectively at $\tau = 1$, and also at $\tau = 2$ when information does not arrive in period 1.

Notice that the WV, whenever positive, is increasing with the probability of receiving information exogenously; it is decreasing (or constant) with the level of x. As for the probability p of the unfavorable state, the WV is decreasing in it in $(wd)_1$ and $(wd)_{1,2}$ and increasing in $(ww)_{1,2}$, where both x and y are quite low; notice, however, that region $(ww)_{1,2}$ shrinks as long as p increases. In $(ww)_{1,2}$, when information does not arrive in period 1, the DM's optimal behavior is the same in both information scenarios. However, the WV is positive even in this region. This is because, when exogenous information arrives (with probability q) in period 1 and the unfavorable state (whose probability is p) is revealed, the DM develops the whole resource at $\tau = 2$, i.e. $(c_2^u)^* = 0$, hence obtaining the net benefit y, so that the WV is qpy. This advantage occurs in $(ww)_{1,2}$ and disappears when x and/or y becomes so large that we fall into $(wd)_{1,2}$: in this case, if information does not arrive in period 1 and the unfavorable state occurs in period 2, the decision to preserve at $\tau = 2$ is disadvantageous: it would have been better to develop the resource at $\tau = 2$, as done by the DM in the (no) scenario. Consequently, the WV is reduced by py, becoming $(1 - p - x) + qpy - py$.

Generally speaking, the WV reflects the main conclusion of Arrow and Fisher (1974) about the QOV as defined in (1): with information exogenously arriving, the WV leads the DM to choose a higher level of preservation of the environmental area for every τ. We extend this result to the case when exogenous information does not arrive with certainty, hence for every $q \in [0, 1]$. In fact, it is $({c_1}^*)_{exo} \geq ({c_1}^*)_{no}$ and $({c_2}^*)_{exo} \geq ({c_2}^*)_{no}$, independently from $q \in [0, 1]$. The first inequality is trivial. The second can be easily proven: because of irreversibility, it is always ${c_2}^* \leq {c_1}^*$, but since $({c_1}^*)_{exo} \geq ({c_1}^*)_{no}$, the DM in the (exo) scenario has, at $\tau = 2$, a larger choice interval, $(c_2^*)_{exo} \in [0, ({c_1}^*)_{exo}]$, with respect to the (no) scenario, where $(c_2^*)_{no} \in [0, ({c_1}^*)_{no}]$. Since the objective function is the same in each information scenario, the choice $({c_2}^*)_{no}$ is possible in both information scenarios and so $({c_2}^*)_{exo}$ cannot be lower than $({c_2}^*)_{no}$. This result can be observed by comparing figure 4 to figure 3.

Lastly, notice that in the (exo) scenario we generically have corner solutions for c_1 and c_2, for any $q \in [0, 1]$.[12] Recall that the WV is non-decreasing with respect to q. However, even for $q = 1$, the WV is never larger than the quasi-option value *à la* Arrow-Fisher, i.e. $0 \leq EV_{exo|q=1}\left((c_1^*)_{exo}\right) - EV_{no}\left((c_1^*)_{no}\right) = WV_{|q=1} \leq QOV = EV_{exo|q=1}(1) - EV_{no}(1)$.

[12]However, there are singular cases (specified in footnote 9) in which we can have indeterminate choices $(c_1^*)_{exo} \in [0, 1]$ and/or $(c_2^*)_{exo} \in [0, (c_1^*)_{exo}]$.

The two values coincide only if $(c_1^*)_{exo} = (c_1^*)_{no} = 1$ and $EV_{no}(1) = \max_{c_2 \in [0,1]} EV_{no}(1, c_2) = EV_{no}(1,1)$. This happens only in region $(ww)_{1,2}$, when both x and y are so low that it is convenient to preserve everything for every τ even in the (no) scenario.

4.2 Analysis of the Testing Value

The Testing Value has been defined in (4) as the difference between the optimal expected value of net benefits of preservation in the $(exo\&endo)$ scenario and in the (exo) scenario. Hence, defining our "Quasi-Option Value of *exo*genous *and endo*genous information" as

$$QOV_{exo\&endo} = EV^*_{exo\&endo} - EV^*_{no} \tag{5}$$

and taking into account definitions (2) and (4), we find that

$$QOV_{exo\&endo} = WV + TV \tag{6}$$

Thus, we introduce the Testing Value as an additional value of endogenous to exogenous information. This value is always non-negative. In the regions in which the TV is positive, it is equal to

$$\begin{cases} \frac{[(1-p-x)-(1-q+\lambda)py]^2}{4\lambda py} & \text{if } y \in \left(0, \frac{1-p}{p}\right] \text{ and } x \in \left(\max\{0, 1-p-(1-q+\lambda)py\}, 1-p-(1-q)py\right] & (tw)_{1,2} \\ \frac{[(1-p-x)-(1-q-\lambda)py]^2}{4\lambda py} & \text{if } y \in \left(0, \frac{1-p}{p}\right] \text{ and } x \in \left[1-p-(1-q)py, 1-p-(1-q-\lambda)py\right) & (td)_{1,2} \\ \frac{[(1-p)(q-\lambda)-x]^2}{4(1-p)\lambda} & \text{if } y \in \left[\frac{1-p}{p}, +\infty\right) \text{ and } x \in \left((1-p)\max\{0, q-\lambda\}, (1-p)q\right] & (tw)_1 \\ \frac{[(1-p)(q+\lambda)-x]^2}{4(1-p)\lambda} & \text{if } y \in \left[\frac{1-p}{p}, +\infty\right) \text{ and } x \in \left[(1-p)q, (1-p)(q+\lambda)\right] & (td)_1 \end{cases}$$

and equal to zero otherwise.

Let us concentrate on figure 8, where the difference in terms of (c_1^*, c_2^*) between the (*exo&endo*) scenario in figure 7 and the (*exo*) scenario in figure 4 is represented. The regions $(tw)_1$:= "*t*esting instead of *w*aiting at $\tau = 1$" and $(tw)_{1,2}$:="*t*esting instead of *w*aiting both at $\tau = 1$ and $\tau = 2$" include values for (x, y) such that the level of preservation is lower in the (*exo&endo*) scenario than in the (*exo*) scenario, respectively at $\tau = 1$, and also at $\tau = 2$ when information does not arrive in period 1. The regions $(td)_1$:= "*t*esting instead of *d*estroying at $\tau = 1$" and $(td)_{1,2}$:="*t*esting instead of *d*estroying both at $\tau = 1$ and $\tau = 2$" include values for (x, y) such that the level of preservation is higher in the (*exo&endo*) scenario than in the (*exo*) scenario, respectively at $\tau = 1$, and also at $\tau = 2$ when information does not arrive in period 1.

Intuitively, given that the TV is linked to endogenous information, it should be always increasing in the probability that this kind of information emerges (λ). However, this intuition is misleading. The TV depends positively on λ only for high values of x, i.e. when the acquisition of information through the development of the environmental resource at $\tau = 1$ is relatively costly (region $(td)_1$ and region $(td)_{1,2}$).[13] When instead x is low (region $(tw)_1$ and region $(tw)_{1,2}$), in the (*exo&endo*) scenario the DM faces a trade-off. On the one hand, the higher the probability to obtain information endogenously, the higher the "temptation" to develop in order to obtain information at $\tau = 1$. On the other hand, it would be optimal to preserve at $\tau = 1$, given that the cost of preservation at $\tau = 1$ is low and/or the benefit in the favorable state of the world at $\tau = 2$ is high. This is exactly what the DM does in the (*exo*) scenario. She does not face a trade-off when information is only exogenous: it is optimal to "wait" both from an economic point of view and from the point of view of acquiring and exploiting exogenous information.

[13] TV is increasing in λ in region $(td)_{1,2}$, since $TV = \frac{[(1-p-x)-(1-q-\lambda)py]^2}{4\lambda py}$ for $1 - p - x \in [(1 - q - \lambda)py, (1 - q)py]$, so that, putting $1 - p - x = (1 - q - \lambda)py + \eta\lambda py$ with $\eta \in [0,1]$, we find $TV = \frac{\eta^2\lambda py}{4}$. Analogously, in region $(td)_1$, putting $x = (1 - p)(q + \lambda) - \eta(1 - p)\lambda$ with $\eta \in [0,1]$.

This result is confirmed by looking at figure 8. The two regions in which, with only exogenous information, the level of preservation is higher at $\tau = 1$, $(tw)_1$, and for every τ, $(tw)_{1,2}$, compared to the scenario in which also endogenous information is potentially available, are also those in which the TV depends negatively on λ. Notice that as long as λ increases, both regions $(tw)_1$ and $(tw)_{1,2}$ enlarge towards the left in figure 8, hence the set of net benefits of preservation for which endogenous information leads to less preservation becomes greater. In particular, for $\lambda \geq q$ the region (w_1, d_2) in the north-west of figure 7 disappears. In fact, in the $(exo\&endo)$ scenario in figure 6, in which it is $\lambda = q = \frac{1}{2}$, it is never optimal to preserve everything at $\tau = 1$ when $y > 1$. In this case, in the second period the negative net benefit of preservation in the unfavorable state of the world is greater in absolute value than the positive net benefit in the favorable state of the world, given that both states are equally likely ($p = \frac{1}{2}$). The necessary condition for the region (w_1, d_2) to emerge in the $(exo\&endo)$ scenario is that $\lambda < q$; in other words, it has to be more likely that information in period 1 arrives exogenously rather than endogenously.

Notice, however, that for many values of the net benefits of preservation in the current period and in each of the two states of the world in the future, the possibility of acquiring information endogenously (added to the possibility of acquiring it exogenously) leads the DM to preserve more at $\tau = 1$ (and often also at $\tau = 2$) with respect to the case in which only exogenous information is potentially available. The reason is that endogenous information often leads to internal solutions, i.e. to a only partial development of the environmental resource. On the contrary, exogenous information alone generically leads to corner solutions, i.e. to destroy completely or preserve completely the environmental asset.[14]

Therefore, when the vector of relative net benefits of preservation belongs to region $(td)_1$ or to region $(td)_{1,2}$, the TV pushes the risk-neutral DM towards a higher level

[14] If the DM is risk averse, internal solutions could also emerge when information is only exogenous.

of preservation of the environmental resource. Given that x is high in both $(td)_1$ and $(td)_{1,2}$, acquiring information through destroying at $\tau = 1$ is relatively costly for the DM. Indeed, in correspondence of those values of x, the optimal choice both in the (no) and in the (exo) scenario would be destroying the environmental resource at $\tau = 1$, hence also at $\tau = 2$ (the DM does not face a trade-off). Instead, in the $(exo\&endo)$ scenario the DM optimally chooses to "test" the environmental resource at $\tau = 1$: she destroys only a small part of it, allowing herself to choose again (being potentially informed) whether to destroy or preserve at $\tau = 2$ everything she has not "tested" at $\tau = 1$. If information does not emerge in period 1 and y is high (region $(td)_1$), she destroys everything at $\tau = 2$, because the net benefits of preservation in the unfavorable state of the world are high (in absolute value) with respect to those in the favorable state of the world; if instead y is low (region $(td)_{1,2}$), at $\tau = 2$ she preserves everything she has preserved at $\tau = 1$.

This explains why disregarding the TV when the pair of relative net benefits belongs to regions $(td)_1$ and $(td)_{1,2}$ in figure 8, would mean underestimating the potential beneficial role of dependent learning in terms of both present and future preservation. Although its existence stems from endogenous information, surprisingly enough, the TV is positively related to the probability of acquiring information exogenously. Indeed, in all regions in which it is positive (hence also when additional endogenous information leads to less preservation) the TV is increasing in q.

This result stresses the complementarity between endogenous and exogenous information. Let us look for a moment at the problem from the other angle. When endogenous information is available, the possibility to acquire information also exogenously hampers the DM from destroying too much in order to acquire information endogenously. Therefore, when both endogenous and exogenous information are available, their complementarity seems to counterbalance their substitutability, hence reinforcing the effect of the TV on environmental preservation, at the same time moderating its natural "incentive to destroy" to get (endogenous) information.

The importance of exogenous information for the nature itself of the TV that we have defined in (4) is even more clear if we compare our TV to the one emerging without exogenous information. Consider the (*endo*) scenario, where only endogenous information is available (exogenous information is completely absent). Recall the definition of the $QOV_{exo\&endo}$ in (5) and the relation (6). Intuitively, because of the assumption of risk neutrality of the DM, one could define the Testing Value as the difference between the optimal expected value function in the (*endo*) scenario and in the (*no*) scenario, i.e.

$$TV' = EV^*_{endo} - EV^*_{no} \tag{7}$$

Given the results above and comparing figure 5 to figure 6, it is easy to see that generally $TV' \neq TV$: the latter depends on q, while the former is perforce independent. Notice that, compared to the (*no*) scenario, the potential emergence of only endogenous information leads to a lower level of resource preservation for every τ in some regions (x, y). This can be easily seen by comparing figure 5 to figure 3. Instead, we have shown that, when $\lambda \leq q$, the simultaneous presence of exogenous and endogenous information always leads the DM to preserve more with respect to the (*no*) scenario (compare figure 6 or 7 to figure 3). However, when $\lambda > q$, it is possible to destroy more with respect to the (*no*) scenario. This happens only if, when destroying everything at $\tau = 1$, endogenous information is more likely than exogenous information.

The difference between the nature itself of the TV and of the TV' is even more clear if one compares the (*endo*) scenario to the (*exo*) scenario, under the condition that the probability of getting information is the same in the two scenarios, i.e. $\hat{q} = \hat{\lambda}\,[1 - (\hat{c}_1^*)_{endo}]$, where $(\hat{c}_1^*)_{endo}$ is the optimal preservation level in the (*endo*) scenario given $\hat{\lambda}$. It is easy to check that for no triple (x, y, p) the preservation level in the (*endo*) scenario is larger than in the (*exo*) scenario (in figures 4 and 5 the limit case $q = \lambda$ is represented for the (*exo*) and the (*endo*) scenario respectively).

5 Conclusions

In this paper, we have extended Arrow and Fisher (1974) two-period model in order to analyze the effects of the simultaneous presence of endogenous and exogenous information on the optimal preservation choices of a risk-neutral decision maker. First of all, in this more general framework we replaced the quasi-option value (QOV) *à la* Arrow-Fisher with the $Waiting\ Value$ (WV) attached to the increase in expected utility due to the possibility of exogenously acquiring information about future net benefits of preservation. We have shown the main features of the WV and that it plays a role similar to the QOV in pushing the decision maker towards a higher level of preservation of the environmental area, in the case of exogenous information potentially arriving. The WV is the benchmark that we use to measure the importance of additional endogenous information in leading the decision maker to an even higher level of preservation with respect to the case in which only exogenous information is available. The main part of the paper is devoted to the analysis of this "unexpected" role of dependent learning.

Miller and Lad (1984), Freeman (1984) and Fisher and Hanemann (1987) have shown that in the Arrow-Fisher framework the policy of postponing the choice in order to enable the decision maker to profit from the incoming information is sub-optimal when this information is endogenous. In other words, if information can be acquired only by developing any portion of the environmental resource in the current period, the decision maker would never preserve the whole environmental resource in this period; moreover, it can happen that the level of preservation is lower than in the scenario where no information is available before the future choice. These intuitive conclusions should apply a fortiori in the "only endogenous information" scenario we adopt in this paper, in which we assume that the probability of information arrival is increasing in the level of development carried out. Nevertheless, we prove the counterintuitive result that the possibility of acquiring information both endogenously and exogenously could push the decision maker towards a higher level of preservation with respect to the case where information arrives only exogenously. By generalizing Freeman (1984) and Fisher and Hanemann (1987) analysis, we have introduced a $Testing\ Value$ (TV), defined as the additional value attached to endogenous information, when information arrives both exogenously and endogenously. It is the additional gain the decision maker obtains when she can receive information regarding future benefits by developing the environmental resource in the current period, compared to the case in which she can only wait for the information to arrive exogenously. We have shown that the TV is always non-negative and that, if the probability of acquiring endogenous information is not higher than the probability of acquiring it exogenously, it pushes the decision maker in the same direction as the WV (i.e. towards a higher level of preservation) compared to the case in which no information is available. Moreover, in many cases the TV pushes the decision maker towards preservation of environmental resources more than the WV does. With regard to the level of preservation in the "exogenous and endogenous" information scenario, we find that, compared to the scenario in which only exogenous information is available, in many cases accounting for additional endogenous information leads the decision maker to preserve more in both periods. The reason is that the TV can lead the decision maker to only partially develop the environmental resource; on the contrary, the WV generically leads to corner solutions. Finally, whether additional endogenous information leads to more or less preservation, the TV is increasing in the probability of acquiring information exogenously. This result is evidence of a form of complementarity between endogenous and exogenous information.

Our framework can be used to investigate some crucial environmental policy issues. When both exogenous and endogenous information are available, it is not obvious that preserving the whole amount of an environmental resource is the optimal choice.

Nonetheless, it is not obvious that the possibility of acquiring information endogenously leads to the development of a larger amount of the environmental resource. If it is the case, disregarding the TV would mean understating the potential positive effect of dependent learning in terms of both present and future preservation. We show that this happens especially when the cost of preservation in the current period is high. In this case, in the presence of only exogenous information, it should be optimal to destroy entirely the environmental resource. Instead, the possibility of acquiring information also endogenously and the consequent emergence of the TV leads the decision maker to destroy less of an environmental resource, with respect to the case in which she takes into account only the WV. When preservation in the current period is relatively costly, the choice to "test" the environmental resource, by destroying only a portion of it in the current period, maximizes the decision maker's intertemporal expected utility. Such a choice at the same time minimizes the amount of environmental resource destroyed in both periods: because of irreversibility, destroying a small part of the environmental resource in the present often induces the decision to destroy less in the future.

References

[1] Arrow, K. J. and A. C. Fisher (1974), "Environmental Preservation, Uncertainty and Irreversibility", *Quarterly Journal of Economics*, 89, 312-319.

[2] Attanasi, G. and A. Montesano (2008), "Competing for Endogenous Information in an Irreversible Environmental Resource Problem: a Game-Theoretic Analysis", *International Game Theory Review*, 10, 229-243.

[3] Chichilnisky, G. and G. Heal (1993), "Global Environmental Risks," *Journal of Economic Perspectives*, 7, 65-86.

[4] Conrad, J. M. (1980), "Quasi-Option Value and the Expected Value of Information", *Quarterly Journal of Economics*, 95, 812-820.

[5] Dasgupta, P. S. and G. M. Heal (1979), "Economic Theory and Exhaustible Resources", Cambridge, U.K., Cambridge University Press.

[6] Epstein, L. G. (1980), "Decision Making and the Temporal Resolution of Uncertainty", *International Economic Review*, 21, 269-283.

[7] Fisher, A. C. (2000), "Investment under Uncertainty and Option Value in Environmental Economics", *Resource and Energy Economics*, 22, 197-204.

[8] Fisher A. C. and W. M. Hanemann (1987), "Quasi-Option Value: Some Misconceptions Dispelled", *Journal of Environmental Economics and Management*, 14, 183-190.

[9] Freeman, A. M. (1984), "The Quasi-Option Value of Irreversible Development", *Journal of Environmental Economics and Management*, 11, 292-295.

[10] Freixas, X. and J. J. Laffont (1984), "On the Irreversibility Effect", in *Bayesian Models in Economic Theory*, Boyer, M. and R. Kihlstrom (eds.), North-Holland, Amsterdam, 105-113.

[11] Graham-Tomasi, T. (1995), "Quasi-Option Value", in *Handbook of Environmental Economics*, Bromley, D. W. (ed.), Blackwell, London, 594-614.

[12] Hanemann, W. M. (1989), "Information and the Concept of Option Value", *Journal of Environmental Economics and Management*, 16, 23-37.

[13] Henry, C. (1974), "Investment Decisions under Uncertainty: The 'Irreversibility Effect'", *American Economic Review*, 64, 1006-1012.

[14] Mäler, K. G. and A. C. Fisher (2005), "Environment, Uncertainty and Option Values", in *Handbook of Environmental Economics*, 2, Mäler, K. G. and J. R. Vincent (eds.), Elsevier, Amsterdam, 571-620.

[15] Miller, J. R. and F. Lad (1984), "Flexibility, Learning, and Irreversibility in Environmental Decisions: A Bayesian Approach", *Journal of Environmental Economics and Management*, 11, 161-172.

[16] Salanie, F. and N. Treich (2009), "Option Value and Flexibility: A General Theorem with Applications", LERNA, University of Toulouse, W.P. no. 09.12.288.

[17] Ulph, A. and D. Ulph (1997), "Global Warming, Irreversibility and Learning", *Economic Journal*, 107, 636-650.