How many wives? On the evolution of preferences over polygyny rates[*]

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September 6, 2015

Abstract

What determines a society's polygyny rate? Since grassroots individual preferences likely influence marital norms, understanding what determines these preferences is key to understanding polygyny patterns. I use evolutionary logic to derive these preferences from first principles, assuming that each adult's ultimate goal is to maximize reproductive success. Males face a quantity-quality trade-off: given a fertility rate, an increase in the number of wives may reduce average child success enough for the male's reproductive success to decrease. In some ecologies males strictly prefer monogamy over polygyny. Even in ecologies where they don't, they may prefer monogamy to low degrees of polygyny.

Keywords: economics of the family, monogamy, polygyny, reproductive success, evolution

JEL codes: D13, C73

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[*] Support by Agence Nationale de la Recherche (Chaire d'Excellence ANR-12-CHEX-0012-01) is gratefully acknowledged. I am grateful to Donald Cox and Hillard Kaplan for extensive comments and discussions. I also thank Alexis Chaine, Hanna Kokko, François Salanié, Jonathan Stieglitz, and seminar participants at IAST for comments.

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1 Introduction

Some of the oldest known written records, such as the Code of Hammurabi, show that humans have devised rules and customs to govern mating practices for at least 4000 years. Marital norms have varied over space and time, but in broad terms polygyny has been legal at some time in most societies, monogamy has gained in prevalence over time, and polyandry has been virtually non-existent. Although rules imposing monogamy on all males appeared already in ancient Greece and Rome (Scheidel, 2008), polygyny is still legal in about one fourth of all countries, mostly in Africa and Asia (see, e.g., Zeitzen, 2008). Marital norms affect economic outcomes (Tertilt, 2005, 2006) as well as the extent to which males and females contribute to the genetic material of future generations (Lippold et al., 2014). But what drives the design of marital norms in the first place? Since grassroots individual preferences are likely to influence the way in which marital norms develop, understanding the mechanisms behind these preferences is key to understanding the patterns of monogamy versus polygyny.

Existing theories about polygyny rates have highlighted several factors, besides a skewed sex ratio (Thornton, 1983). First, male heterogeneity, be it in wealth or in genes, is expected to favor polygyny, either because females are willing to share the preferred males with other females (Becker, 1974, 1991, Grossbard, 1976, Low, 1990, Bergstrom, 1994, Lagerlöf, 2005), or because richer males can afford to secure agricultural labor by acquiring wives (Boserup, 1970); by contrast, female heterogeneity is expected to reduce polygyny, by making attractive females more costly for males (Gould, Moav, and Simhon, 2008). Second, the polygyny rate in a population may depend on the females' willingness to "trade" faithfulness for a lower polygyny rate (Kokko and Morrell, 2005, Fortunato and Archetti, 2010, Gavrilets, 2012). Third, monogamy is favored by between-group competition if it leads to a reallocation of resources from wasteful activties to paternal investment (Henrich, Boyd, and Richerson, 2012). Finally, the degree of polygyny in a society may be expected to depend on whether females are free to choose or are coerced into polygynous marriages (Gibson and Mace, 2007, Winking et al., 2013).

These theories all take preferences over polygyny rates as given and focus on deriving equilibrium outcomes. Furthermore, and importantly, it is generally assumed that if males could choose freely, they would choose to have several wives. In this paper I push the theory one step further by analyzing how natural selection shapes *preferences* over polygyny rates in the first place, absent factors such as heterogeneity and unfaithfulness. It is indeed necessary to understand these preferences if we want to understand how hard males are willing to fight to get more than one wife.

In a nutshell, I use evolutionary logic to address the following questions: If the ultimate goal of individuals is to maximize reproductive success, how many wives would a man have if he could freely choose? Would females prefer a different polygyny rate? And how do male and female preferences over polygyny rates depend on exogenously given factors of the ecology in which the population at hand evolves? Finally, which polygyny rate would be in the best interest of the population as a whole?

In order to highlight the trade-offs that may appear above and beyond those already mentioned in the literature as described above, I rule out male and female heterogeneity, unfaithfulness, as well as group competition, by assumption. The model instead focuses on the role played by behaviors within the household (Becker, 1991). In the model, a man and his wife/wives have a given time endowment each, and in line with basic evolutionary logic each adult is assumed to seek to maximize his or her own reproductive success. The reproductive success of a female is defined as the expected number of her children that survive to sexual maturity,[1] while that of the male is the sum of the reproductive successes of his wives. The probability that a child survives to adulthood depends on the quantities of food, protection, and care that the child receives, where food is produced both by the male and the females, protection is produced by the male, and care is produced by the females.

The first step of the analysis consists in characterizing the equilibrium behaviors of the adults in a household whose composition (number of wives and number of children per wife) is given. Having characterized the equilibrium time allocations, I use them to write male and female reproductive success as functions of the *reproductive culture* (the polygyny rate and the fertility rate). For any given ecology, these functions measure male and female preferences over reproductive cultures (assuming that their ultimate goal is to maximize own reproductive success). The second step of the analysis consists in analyzing the shape of these functions, and in determining how this shape depends on the ecology.

In the model there are nine exogenously given parameters which together form *the ecology*. In sum, the analysis delivers insights as to how preferences over polygyny and fertility rates may depend on the ecology not only directly, but also indirectly through the effects of the ecology on the division of labor.[2] Some of the ecological parameters pertain to the relative

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[1] While mating success of offspring also matters, it is not modeled here.

[2] The quest for insights as to how ecological factors affect the evolution of mating systems is common

importance of food, protection, and care in determining child survival, thus allowing me the flexibility to compare, for instance, environments where shelter is more important than in others, perhaps because of harsh weather conditions or the prevalence of predators. Other ecological parameters measure the marginal returns to time spent on different tasks. Finally, two of the ecological parameters measure the extent to which protection and care are public goods, food being a pure private good. Below, these parameters will be referred to as "degrees of publicness."

While the goods produced by the male are assumed to be shared equally among all the children in the household, for the goods produced by the females in polygynous households two alternative sharing arrangements are considered: under "private property" a female's outputs are shared (equally) among her own children only, while under "household property" they are shared (equally) among all the children in the household.[3] Furthermore, for completeness, the preferred fertility rate is also determined.

I find that female and male preferences over fertility rates are identical; this is not surprising, since for a given number of wives, a man's reproductive success simply equals that of each wife multiplied by the number of wives. There is a *fertility quantity-quality trade-off*, which implies that reproductive success is decreasing in fertility for sufficiently high fertility rates. The analysis reveals how the trade-off depends on the ecology. First, the preferred fertility rate depends on the degrees of publicness of protection and care: a lower degree of publicness of protection or care means that a given amount of protection or care benefit children less, which worsens the quantity-quality trade-off and reduces the preferred fertility rate. Second, the preferred fertility rate depends on the marginal return on child survival to time spent on care, protection, and food production. This effect is more subtle than the first one, and it is driven by how these marginal returns affect equilibrium time allocations. As an illustration, consider an increase in the marginal return to food production. Such an increase would lead to a reallocation of time from care and protection to food production. Since food is a private good, if care and protection are somewhat public this time reallocation worsens the quantity-quality trade-off and reduces the preferred fertility rate.

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in the biology literature (see, e.g., Bateman, 1948, Orians, 1969, Emlen and Oring, 1977, Clutton-Brock and Vincent, 1991, as well as Kokko and Jennions, 2008, and the references therein). To the best of my knowledge, however, in this literature parental care is a one-dimensional variable (while here parents provide three goods to their offspring) and focus is often on the effects of spatial and temporal availability of mates (features that are disregarded here).

[3] Household property may be interpreted as a form of allomothering (Hrdy, 2009).

Turning to preferences over polygyny rates, I find that female and male preferences typically differ, and that females prefer a smaller polygyny rate than males. Surprisingly, males do not necessarily want to simply maximize the number of wives, although there is no cost associated with "acquiring" wives. Depending on the ecology, male reproductive success may be increasing, decreasing, or non-monotonic in the polygyny rate. This is because males face a *polygyny quantity-quality trade-off*: while a greater number of wives implies a direct gain for the male in the form of a greater number of children, it also entails an indirect loss in the form of a decrease in the probability that each child survives to sexual maturity. The trade-off appears not only because the goods the male provides to the children get divided among a larger number of children when the number of wives increases; it also depends on how division of labor is affected.

Thus, the more the protection that the male provides to this children is a private good, the more severe is the trade-off. If the degree of privateness is high enough, male reproductive success is non-monotonic in the polygyny rate, and males prefer low polygyny rates to monogamy but high polygyny rates to monogamy (in the extreme case, the polygyny quantity-quality trade-off is so severe that males strictly prefer monogamy to polygyny). If the degree of privateness is low enough, male reproductive success never decreases in the polygyny rate.

The analysis further reveals that the severity of the polygyny quantity-quality trade-off depends on whether the male spends time on food production. If he does, then an increase in the polygyny rate leads him to spend less time on producing food and more time on producing protection. Indeed, because food is a fully private good, the male is better off producing more of the somewhat public good protection as the number of children he has to raise grows. For high enough polygyny rates, the male devotes all his time to protection, and a further increase in the polygyny rate does not affect the division of labor. The benefit of adding a wife then outweighs the cost of sharing protection among a larger number of offspring, and male reproductive success increases. In sum, the polygyny quantity-quality trade-off is severe for low polygyny rates when the male devotes time to food production, but it disappears for high enough polygyny rates.

Turning now to females, they also suffer from the decrease in the child survival probability that an increase in the polygyny rate entails. However, unless the care provided by females is a fully private good, a female may also benefit from polygyny if her children can benefit from the care provided by the other wives, which is the case under household property.

Accordingly, I find that under household property females prefer polygyny over monogamy if the degree of publicness of care is sufficiently high, while under private property females always prefer monogamy over polygyny.

The model is silent as to how males and females are matched together to form households, the key idea being that understanding how natural selection shapes preferences is a necessary first building block in our quest to understanding household formation processes. While analysis of such processes is thus left for future research, I discuss the potential ramifications of my results for this issue based on (1) efficiency, where a reproductive culture is efficient if it maximizes the average reproductive success in the population at hand, and (2) male preferences over polygyny rates and the ensuing incentives for males to fight for mating success.

This paper rests on the idea, delineated by Bergstrom (1996) and Robson (2001, 2002), that economists may obtain valuable insights about human motivation by including evolutionary forces in their models.[4] It is closely related to the growing literature on preference evolution (see, e.g., Dekel, Ely, and Yilankaya, 2007, Heifetz, Shannon, and Spiegel, 2007, Rayo and Becker, 2007, Robson and Samuelson, 2011, Alger and Weibull, 2010, 2013). This paper is, however, the first to study the formation of preferences over polygyny rates.

The model is described in the next section. Equilibrium time allocations and analysis of preferences over fertility rates and polygyny rates are derived in Sections 3 and 4. Efficiency is analyzed in Section 5, while Section 6 discusses some implications of the results, and Section 7 concludes. All the proofs are in the Appendix.

2 Model

Consider a population in which each individual lives for at most two periods; in the first period as a non-productive and non-reproductive child, and in the second period as a productive and reproductive adult. Assume that the sex ratio is balanced at birth (as evolutionary theory would predict, Fisher, 1930). Each adult female lives in a household that consists of

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[4] Note that the theoretical models in this literature, including the one proposed here, are silent as to whether traits are genetically or culturally determined.

one adult male and his $k \geq 1$ wives, each of whom gives birth to $n \geq 1$ children,[5] where $\omega \equiv (k, n)$ is the *reproductive culture*. If married males have more than one wife, some males are single. However, the process by which the lucky males are picked is not modeled. The aim is instead to understand how male and female *preferences* over reproductive cultures may have been shaped by natural selection. The modeling strategy is as follows.

Based on evolutionary logic, I assume that the goal of each adult is to maximize own reproductive success, which is measured by the number of the adult's offspring that survive to sexual maturity.[6] Since the number of children per female is given, what matters for reproductive success is the probability that each child survives to sexual maturity, a probability that depends on the productive behaviors of the adults in the household. First, I characterize equilibrium productive behaviors of the male and the female(s) in a given household; these behaviors may depend on k and n. Second, the equilibrium productive behaviors are used to express reproductive success of the male as well as the female(s) in the household, as a function of reproductive cultures. These expressions represent male and female preferences over reproductive cultures. Ultimately, the goal is to understand how these preferences depend on the *ecology*, i.e., the exogenously given environment in which the population is taken to have evolved over a long time; in the model the ecology will be summarized by a vector $\boldsymbol{\xi}$ of parameters, a vector that will be formally defined below.

The probability that a child survives to sexual maturity is taken to depend on the quantities of food, protection, and care that it receives.[7] I assume that there are two sources of food, which will be referred to as hunted and gathered food.[8] Letting G_i and H_i denote the amounts of gathered and hunted food that child i receives, and P_i and C_i the amounts of protection and care that (s)he receives, I assume that the probability that the child (whether

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[5] The term "wife" is used for convenience only, since marriage has no function *per se*. The key assumption is that both males and females engage in parental care, which is a reasonable assumption for humans and some other species (see Alger and Cox, 2013 for a review of the biology literature on parental care).

[6] While mating success of offspring who have survived to sexual maturity also matters for an adult's reproductive success, it is beyond the scope of this paper.

[7] The terms protection and care should be interpreted broadly. Thus, protection may include shelter construction and maintenance, active protection against predators, as well as the transmission of human capital pertaining to such activities. Likewise, care may include the production and mending of clothes, storytelling, as well as the transmission of knowledge about social rules, plants, and animals.

[8] The vocabulary used here is tailored to hunter-gatherer societies, but the model can readily be reinterpreted to fit horticultural and agricultural societies, and even market economies.

male or female) survives to sexual maturity is

$$a\left(n\right)\cdot S\left(C_i,G_i,H_i,P_i\right),$$

where a is a decreasing function of n that reflects the physical toll of child-bearing, and

$$S\left(C_i,G_i,H_i,P_i\right)=P_i^{\lambda}\cdot C_i^{\delta}\cdot\left(\rho G_i+H_i\right)^{\tau}, \tag{1}$$

where $\rho>0$ and $\lambda,\delta,\tau\in(0,1]$. Thus, food, protection, and care are all essential goods: food is useless unless some protection and some care is provided, and *vice versa*. Furthermore, gathered food and hunted food are perfectly substitutable, except for the fact that gathered food may be more important than hunted food (if $\rho>1$), equally important (if $\rho=1$), or less important (if $\rho<1$). The parameters λ, δ, and τ measure the importance of protection, care, and total food intake, respectively. Adequate parameter restrictions will be imposed below to ensure that $S\left(C_i,G_i,H_i,P_i\right)$ always takes a value between 0 and 1. Furthermore, below it will be assumed that $a\left(n\right)=\max\left\{0,1-bn\right\}$ for $b\in(0,1)$, where $1/b$ can be thought of as the maximum number of children a female can have before she dies with certainty (in which case the children die since the mother's inputs are essential).

The amounts of protection, care and food that a child receives depends on the efforts put into the production of these goods by the adults in the household to which she belongs, and on how these goods are shared among the children. While two alternative sharing regimes will be considered, there is only one production regime. Technology is given within an individual's lifetime, and labor is the only variable production input; labor will be measured in units of time devoted to work, and I normalize the lifetime time budget of each adult to 1. For simplicity, adults do not consume anything of what they produce,[9] and interactions between households are disregarded. Furthermore, divorce and unfaithfulness are ruled out by assumption, females are assumed to have identical production abilities, and likewise for men.

In general, then, each adult allocates his or her time between four activities: hunting, gathering, protecting, and caring.[10] While it would be reasonable to assume decreasing

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[9] The amount of food produced by a parent in the model can be thought of as the surplus of food that remains after (s)he has consumed the minimum amount of food (s)he requires to survive.

[10] To apply the model to a market setting, assume that each adult allocates his or her time between work inside and outside the home, and that the amounts of food, protection and care quantities that a child receives may be purchased in the marketplace or produced in the home.

returns to labor on any given day, the scale of the analysis at hand—the entire time spent in adulthood—calls for the opposite assumption, namely, non-decreasing returns to labor in any given activity. Indeed, the skills required for successful hunting and gathering are by no means trivial, and there are thus gains to specialization. Now, hunting and protecting from predators may require similar skills, but these activities cannot be carried out at the same time. Furthermore, there may be economies of scope between gathering and caring, especially if it is possible to engage in both activities at the same time; for instance, an adult who is gathering food may bring a sleeping child on her back, or let the child play next to her. Because breast-feeding ties a mother to her baby and berries do not run away from crying babies, it will be assumed throughout that females specialize in caring and gathering, while males specialize in protecting and hunting. Let x_j denote the time that female $j = 1, ..., k$ devotes to gathering. The time that she spends on care is $1 - \phi x_j$, where the exogenously given parameter $\phi \in (0, 1)$ measures the extent to which a female may gather food while providing care. Letting y denote the time that the male devotes to hunting, $1 - y$ is the time he devotes to protecting the children.

Assuming for simplicity that the marginal returns to efforts are constant,[11] and letting $\theta > 0$ denote the marginal return to male effort devoted to hunting, the total amounts of hunted food and protection in the household are

$$H\left(y\right) = \theta y \tag{2}$$

and

$$P\left(y\right) = 1 - y, \tag{3}$$

respectively. Likewise, let $g\left(x_j\right) = \gamma x_j$ denote the total amount of food gathered by a woman spending time x_j on gathering, where $\gamma > 0$, and $c\left(x_j\right) = 1 - \phi x_j$ the total amount of care that she produces. The production functions g and c are the same for all the women. At the household level, the total amount of food gathered by women is

$$G\left(\boldsymbol{x}\right) = \sum_{j=1}^{k} g\left(x_j\right) = \gamma \sum_{j=1}^{k} x_j, \tag{4}$$

where $\boldsymbol{x} = (x_1, ..., x_k)$ is the vector of all the female gathering efforts, and the total amount

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[11] Constant returns to effort are perfectly compatible with specialization, if, for instance, the marginal return to male effort devoted to gathering and to caring is strictly lower than that of a female, and the marginal return to female effort devoted to hunting and to protecting is strictly lower than that of the male.

of care provided is

$$C(\boldsymbol{x}) = \sum_{j=1}^{k} c_j(x_j) = \sum_{j=1}^{k} (1 - \phi x_j). \tag{5}$$

Turning now to how food, protection, and care produced by the parents is allocated among the children in the household, there is an important distinction to be made between food on the one hand, and care and protection on the other. Indeed, while food is a private good, care and protection may be public goods; for instance, a wall around the village protects all the children equally well, and a class about poisonous plants may benefit several children simultaneously. Let an index $i = 1, 2, ..N$ refer to the i'th child in the household, where $N = kn$ is the total number of children. Consider the goods produced by the male first. Ruling out any favoritism by the father by assumption, these goods are assumed to be divided equally among all the children in the household, whether they are private or public.[12] Since food is a private good,

$$H_i(y) = \frac{H(y)}{N} = \frac{\theta y}{N}. \tag{6}$$

By contrast, protection may be a public good. Formally, let

$$P_i(y) = \frac{P(y)}{N^{\alpha}} = \frac{1 - y}{N^{\alpha}}, \tag{7}$$

where $\alpha \in [0, 1]$ measures the extent to which protection is a public good: the smaller is α, the more protection is public in nature. In the extreme case where $\alpha = 0$, protection is a pure public good, and each child receives the full benefit of the total amount of protection produced by the father: $P_i(y) = P(y)$. At the other extreme, if $\alpha = 1$, protection is a fully private good, and the total amount of protection is simply divided among the N offspring: $P_i(y) = P(y)/N$. By a slight abuse of language, henceforth α will be referred to as the degree of publicness of protection, where a *decrease* in the value of α will be interpreted as an *increase* in the degree of publicness.

For female production the situation is more complex as soon as there is more than one wife, since each female is then the mother of a subset of the children in the household. In particular, the amount of power that a female has over the distribution of the food and care she produces may matter. Two extreme cases will be considered: private property and household property. Under private property, the goods produced by a woman are her own and she distributes them among her own children only. In this case, a child i whose mother

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[12] This assumption is not restrictive, as will be seen below (see Footnote 15).

provides effort x_j receives the amount of food

$$G_i(\boldsymbol{x}) = \frac{g(x_j)}{n} = \frac{\gamma x_j}{n}, \tag{8}$$

and the amount of care

$$C_i(\boldsymbol{x}) = \frac{c(x_j)}{n^\beta} = \frac{1-\phi x_j}{n^\beta}, \tag{9}$$

where $\beta \in [0,1]$ measures the extent to which care is a public good. If $\beta = 0$, care is a pure public good, so that $C_i(\boldsymbol{x}) = c(x_j)$, while if $\beta = 1$, it is a private good, in which case $C_i(\boldsymbol{x}) = c(x_j)/n$. As I did for protection, henceforth β will be referred to as the degree of publicness of care, where a decrease in the value of β will be interpreted as an increase in the degree of publicness.

Under household property, the goods produced by a female are divided equally among all the children in the household. Thus, the amounts of gathered food and care received by child i write

$$G_i(\boldsymbol{x}) = \frac{G(\boldsymbol{x})}{N} = \frac{\gamma \sum_{j=1}^{k} x_j}{N}, \tag{10}$$

and

$$C_i(\boldsymbol{x}) = \frac{C(\boldsymbol{x})}{N^\beta} = \frac{\sum_{j=1}^{k}(1-\phi x_j)}{N^\beta}, \tag{11}$$

respectively. Again, if care is a public good ($\beta = 0$), $C_i(\boldsymbol{x}) = C(\boldsymbol{x})$, whereas if it is a private good ($\beta = 1$), $C_i(\boldsymbol{x}) = C(\boldsymbol{x})/N$.[13]

All the elements are in place for reproductive success of the adults in a household to be defined. Accounting for the physical toll that child-bearing inflicts on women,[14] the reproductive success of a female, whose n children receive the amounts of food and care (C_i, G_i, H_i, P_i), writes

$$n \cdot a(n) \cdot S(C_i, G_i, H_i, P_i). \tag{12}$$

The reproductive success of a male is simply the sum of the reproductive successes of his k wives. Finally, I assume that $\rho\gamma + \theta \leq 1$; as will be seen below, this ensures that S takes a value between 0 and 1.

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[13] Note that while under household property the females are assumed to live in the household at the same time, under private property a possible alternative interpretation is that the male engages in serial polygyny.

[14] Here the physical toll is modeled as a scaling factor: *ceteris paribus*, the more children a woman has, the smaller is the survival probability of each of her children. Alternatively, one could let the physical toll have an impact on female productivity, γ, and possibly on the extent to which the female may gather food while caring, ϕ. This is left for future research.

This completes the description of the setup. In the extremely long run, everything in life is endogenous. However, the speed at which different features of a human society evolve differs. In the model, certain features will be viewed as being fixed, others will be thought of as evolving at a slow rate, and still others as being the result of individual choice. First, the production functions for food, protection, and care, the degree of publicness of protection and care, and the child survival probability function, are assumed to be exogenous and fixed, and we refer to the associated set of parameters as the ecology. Formally, then, *the ecology* is the vector $\boldsymbol{\xi} = (\alpha, \beta, \gamma, \delta, \theta, \lambda, \rho, \tau, \phi)$. The ecology determines how parental time allocations are transformed into offspring success. Second, the polygyny rate, k, as well as the fertility rate, n, are taken to be exogenously given for individual human beings, but subject to evolutionary pressure in the longer term. Thus, from the perspective of the individual adult, both the ecology and the reproductive culture are given, and together they will be referred to as the *reproductive environment*, formally denoted $(\boldsymbol{\xi}, \omega)$. Finally, the time allocations, i.e., the vector $\boldsymbol{x} = (x_1, ..., x_k)$ of female time allocations, and the male time allocation y, are the outcome of individual choice, where the goal of each individual consists in maximizing his or her reproductive success. The vector $(\boldsymbol{x}, y)$ will be referred to as the *household time allocation*. Any allocation whereby all females choose the same time allocation, x, will be called *female-symmetric*, and will be denoted (x, y).

The aim of the analysis is to understand whether, and if so how, the ecology affects preferences over reproductive cultures. For this purpose, I first characterize equilibrium household time allocations for a given reproductive ecology $(\boldsymbol{\xi}, \omega)$, and then determine preferences over reproductive cultures, assuming that adults use the associated equilibrium household time allocation.

3 Analysis: Private property

Here, focus is on male and female behaviors and the ensuing preferences over reproductive cultures under the assumption that each female shares the food and care she produces equally among her own children. Each female $j = 1, ..., k$ chooses the time allocation x_j that maximizes her reproductive success (see (12)), taking the male's time allocation as given, and the male chooses the time allocation y that maximizes his reproductive success, taking the females' time allocations as given. Formally, the situation at hand is a simultaneous-move game with $k+1$ players; each female player $j = 1, ..., k$ has strategy x_j and strategy set $[0, 1]$, while the male player has strategy y and strategy set $[0,1]$. To complete the description of the game, it is necessary to define each player's payoff as a function of the strategy profile $(\boldsymbol{x}, y)$, where $\boldsymbol{x} = (x_1, ..., x_k)$. Starting with the females, the reproductive success of female j, and hence her payoff in the game, writes

$$F(x_j, y) = n \cdot a(n) \cdot s(x_j, y), \tag{13}$$

where $s(x_j, y)$, which obtains by replacing H_i, P_i, G_i,and C_i in $S(C_i, G_i, H_i, P_i)$ with the expressions in (6), (7), (8), and (9), writes:

$$s(x_j, y) = \left(\frac{1-y}{(kn)^{\alpha}}\right)^{\lambda} \cdot \left(\frac{1-\phi x_j}{n^{\beta}}\right)^{\delta} \cdot \left(\frac{\rho\gamma x_j}{n} + \frac{\theta y}{kn}\right)^{\tau}. \tag{14}$$

Note that the assumption $\rho\gamma + \theta \leq 1$ implies that $s(x_j, y, n, k, \boldsymbol{\xi})$ takes a value between 0 and 1 for all $x_j, y \in [0,1]$, all $n \geq 1$ and all $k \geq 1$.

Hence, the reproductive success of the male writes

$$M(\boldsymbol{x}, y) = \sum_{j=1}^{k} F(x_j, y) = \sum_{j=1}^{k} a(n) \cdot n \cdot s(x_j, y), \tag{15}$$

where $\boldsymbol{x} = (x_1, ..., x_k)$.

3.1 Equilibrium time allocations

Here I fix the reproductive culture and the ecology, and derive the (Nash) equilibrium time allocations. The trade-off between the time allocated to food production and that allocated to protection and care production, respectively, depends both on the ecological parameters and on the polygyny rate. As an illustration of how it may depend on the polygyny rate, Figure 1 shows the value of $s(x_j, y)$, for $k = 1$ (the top graph) and for $k = 4$. Focusing first on the trade-offs that appear under monogamy, it appears from the graph that in this ecology both the male and the female would choose to devote part of their time to food production (the star indicates the equilibrium time allocations). By contrast, if the male has four wives, although the ecology is the same the graph suggests that the male would choose to devote all his time to protection (the thick dot indicates the equilibrium time allocations). The probability of survival per child would also be lower than under monogamy.

The following proposition shows that there exists a unique Nash equilibrium time allocation:

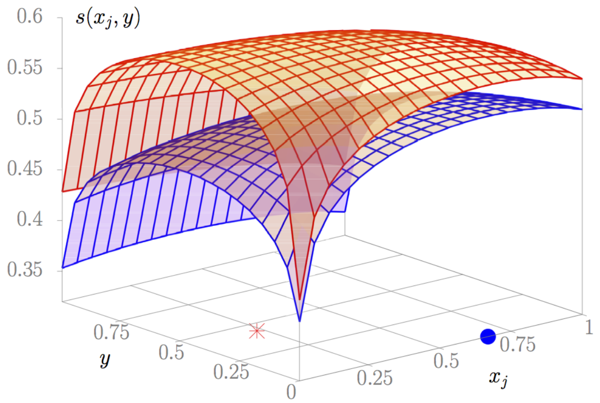

Figure 1: $s(x_j, y)$ for $n = 5$, $\lambda = \delta = \tau = 0.1$, $\phi = 0.75$, $\rho = \theta = 1$, $\gamma = 0.5$, $\alpha = 0.4$, and $\beta = 1$. The top graph is for $k = 1$ and the bottom one for $k = 4$.

Proposition 1 *For any reproductive environment* $(\boldsymbol{\xi}, \omega)$ *there exists a unique Nash equilibrium time allocation,* $(\boldsymbol{x}^*, y^*)$*, which, moreover, is female-symmetric.*

The proof of this proposition as well as the derivation of the expressions for the equilibrium time allocations are provided in the appendix. The full expressions are somewhat involved because corner solutions arise for some parameter values. It is, however, possible to save somewhat on notation by letting $\mu = \theta / (\rho\gamma)$; this ratio measures the importance of food contributed by the male relative to that contributed by the females, for child success. Let x^* denote the time allocation chosen by each females. Two cases arise: first, if $\frac{\tau}{\delta+\tau} \leq \phi$,

$$(x^*, y^*) = \begin{cases} \left(\frac{\tau}{\phi(\delta+\tau)}, 0\right) \text{ if } \mu \leq \frac{k\lambda}{\phi(\delta+\tau)} \\ \left(\frac{(\lambda+\tau)k-\delta\phi\mu}{k\phi(\lambda+\delta+\tau)}, \frac{\mu\phi(\delta+\tau)-\lambda k}{\mu\phi(\lambda+\delta+\tau)}\right) \text{ if } \mu \in \left[\frac{k\lambda}{\phi(\delta+\tau)}, \frac{k(\lambda+\tau)}{\delta\phi}\right] \\ \left(0, \frac{\tau}{\lambda+\tau}\right) \text{ if } \mu \geq \frac{k(\lambda+\tau)}{\delta\phi}. \end{cases} \tag{16}$$

Thus, all the food is produced by the females if the relative importance of male food production, μ, is small enough, while the opposite is true if μ is high enough. For intermediate values of μ both the male and the females produce food. A qualitatively similar but slightly more involved result is obtained for $\frac{\tau}{\delta+\tau} > \phi$, in which case:

$$(x^*, y^*) = \begin{cases} (1,0) \text{ if } \mu \leq \frac{k\lambda}{\tau} \\ \left(1, \frac{\tau}{\lambda+\tau} - \frac{k\lambda}{\mu(\lambda+\tau)}\right) \text{ if } \mu \in \left[\frac{k\lambda}{\tau}, \frac{k[\lambda+\tau-\phi(\lambda+\delta+\tau)]}{\delta\phi}\right] \\ \left(\frac{(\lambda+\tau)k-\delta\phi\mu}{k\phi(\lambda+\delta+\tau)}, \frac{\mu\phi(\delta+\tau)-\lambda k}{\mu\phi(\lambda+\delta+\tau)}\right) \text{ if } \mu \in \left[\frac{k[\lambda+\tau-\phi(\lambda+\delta+\tau)]}{\delta\phi}, \frac{k(\lambda+\tau)}{\delta\phi}\right] \\ \left(0, \frac{\tau}{\lambda+\tau}\right) \text{ if } \mu \geq \frac{k(\lambda+\tau)}{\delta\phi}. \end{cases} \tag{17}$$

The following comparative statics results are immediate:

Proposition 2 Ceteris paribus*, x^* is non-decreasing in λ, τ, and k, and non-increasing in δ, μ, and ϕ, while y^* is non-decreasing in δ, μ, τ, ϕ, and non-increasing in λ and k. Furthermore, neither x^* nor y^* depend on α, β, or n.*

These results are intuitive. First, if food becomes more important (i.e., if τ increase), the marginal benefit from producing food for both males and females, and therefore leads both the male and the females to spend more time on food production. Likewise, if protection becomes more important (λ increases), the male spends less time and the female more time on food production (the opposite occurs if care becomes more important, i.e., if δ increases). Second, an increase in the economies of scope between the two female activities, ϕ, decreases the cost for females of allocating time away from caring, and hence makes females allocate more time to food production. This in turn entails a decrease in the marginal effect of male food production on child success, which makes him reduce the time allocated to food production. Third, an increase in the impact of time spent by the male on food production relative to that of females, μ, makes the male (female) spend more (less) time on food production. Finally, an increase in the number of wives, k, leads females to allocate more time and the male less time to food production. Since an increase in k implies that the male's food output is shared among a larger number of children, it implies that each unit of time that a female spends on food production has a larger relative impact on the total amount of food that each child receives (it has the same effect as a decrease in μ).

To see why neither the degree of publicness of protection (β) or care (α), nor the fertility rate (n) matters for the equilibrium time allocations, note that the expression in (14) may be written

$$s(x_j, y) = \frac{1}{(kn)^{\lambda\alpha} n^{\delta\beta} n^{\tau}} \cdot (1-y)^{\lambda} (1-\phi x_j)^{\delta} \cdot \left(\rho\gamma x_j + \frac{\theta y}{k}\right)^{\tau}. \tag{18}$$

While the first term affects the value of $s(x_j, y)$, it does not affect the trade-off between food, protection, and care that is implied by the remaining terms.

Proposition 2 shows that changes in some ecological parameters have opposite effects on male and female time allocations, and thus an *a priori* ambiguous effect on total food intake per child. It can be shown, however, that the effects are in fact non-ambiguous:

Proposition 3 Ceteris paribus, *the total amount of food that each child receives is decreasing in* k*,* n*, and* δ*,* λ*, and* ϕ*, and increasing in* ρ*,* γ*,* θ*, and* τ*.*

In sum, given a reproductive culture (k, n), the total amount of food that each child receives increases if it becomes relatively more beneficial to devote time to food production rather than to protection or care (i.e., if ρ, γ, θ, or τ increases, or if δ, λ, or ϕ decreases). Furthermore, given an ecology $\boldsymbol{\xi}$, the total amount of food that each child receives decreases if the fertility rate increases, which is natural and expected. The detrimental effect of an increase in the polygyny rate on average food intake per child is less obvious, but it is readily understood by noting that an increase in k is akin to a decrease in $\theta / (\rho\gamma)$, i.e., in the marginal impact of the time that the male spends on food production relative to that of a female.

3.2 Preferences over reproductive cultures

I now turn to the heart of the analysis, namely, female and male preferences over reproductive cultures (k, n). These preferences will be determined under the assumption that for any given reproductive culture, adults choose the corresponding equilibrium time allocations (see (16) and (17)). Combining (13), (15), and (18), and noting that the equilibrium time allocations are functions of k and $\boldsymbol{\xi}$, female and male reproductive success may be written as functions of the reproductive culture and the ecology. Letting $\tilde{F}$ and $\tilde{M}$ denote these functions:

$$\tilde{F}(n, k, \boldsymbol{\xi}) = n \cdot a(n) \cdot s\left(x^*(k, \boldsymbol{\xi}), y^*(k, \boldsymbol{\xi})\right) \tag{19}$$

and

$$\tilde{M}(n, k, \boldsymbol{\xi}) = k \cdot n \cdot a(n) \cdot s\left(x^*(k, \boldsymbol{\xi}), y^*(k, \boldsymbol{\xi})\right). \tag{20}$$

For expositional convenience, the results will be stated without regard to the fact that n and k are integers (although $n \geq 1$ and $k \geq 1$ are imposed).

The reader will have kept in mind that if $k > 1$ and the sex ratio at sexual maturity is balanced, there are males whose reproductive success is nil. It is therefore necessary to distinguish between preferences of males behind the veil of ignorance as to who will be lucky enough to marry, and preferences of males who have already been selected to get married. I begin with the preferences of lucky males, and return to the preferences behind the veil of ignorance towards the end of this section.

Focusing first on fertility, the following result obtains (the proof is in the appendix):[15]

Lemma 1 *For each ecology $\boldsymbol{\xi}$ and each polygyny rate k, there exists a unique number of children per female that maximizes both the female and the male reproductive success:*

$$n^* = \max\left\{1, \frac{1-\lambda\alpha-\delta\beta-\tau}{2-\lambda\alpha-\delta\beta-\tau}\cdot\frac{1}{b}\right\}. \tag{21}$$

Several features of this result are noteworthy. First, when it comes to fertility, female and male interests are aligned, because the male's reproductive success is simply k times that of each female in the household. Second, more is not always better, because resources are limited: for any given polygyny rate $k \geq 1$, an increase in fertility means that each child gets less food (see Proposition 3), and less (or as much) protection and care. The ensuing decline in average child success eventually outweighs the direct, positive impact of fertility on reproductive success. This trade-off may be so severe that having one child per female maximizes reproductive success. When the preferred number of children exceeds one, it is a fraction $\frac{1-\lambda\alpha-\delta\beta-\tau}{2-\lambda\alpha-\delta\beta-\tau}$ of the maximum number of children that a female can get $(1/b)$, and it varies as follows with the ecology:

Proposition 4 *If $n^* = \frac{1-\lambda\alpha-\delta\beta-\tau}{2-\lambda\alpha-\delta\beta-\tau}\cdot\frac{1}{b} > 1$, then it is decreasing in α, β, δ, λ, and τ.*

To see why these results are intuitive, suppose that protection and/or care becomes a more private good, i.e., α and/or β increases. Then there is less protection and/or care per child, and the preferred fertility rate declines. The parameters δ, λ, and τ have the same qualitative effect, because an increase in any of these parameters is as if the corresponding good became more private in nature. Finally, the preferred fertility rate does not depend on the relative effect of food produced by the male compared to that produced by the females $(\theta/(\rho\gamma))$, nor on the economies of scope in female production (ϕ), because while these parameters affect the equilibrium time allocations, they do not affect how food and care is being shared among the household's children.

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[15] Together with the strict concavity of s, this result implies that the assumption that all the children in the household receive an equal share of the goods produced by the parents is not restrictive (see Footnote 12) if the adults can perfectly control the number of children that each female gives birth to.

Turning now to preferences over polygyny rates, and starting with females, the following result obtains:

Lemma 2 *For each ecology* $\boldsymbol{\xi}$ *and each fertility rate* n, $\tilde{F}(n,1,\boldsymbol{\xi}) > \tilde{F}(n,k,\boldsymbol{\xi})$ *for all* $k > 1$, *unless* $\alpha = y^*(1,\boldsymbol{\xi}) = 0$, *in which case* $\tilde{F}(n,1,\boldsymbol{\xi}) = \tilde{F}(n,k,\boldsymbol{\xi})$ *for all* $k \geq 1$.

The intuition behind this result is clear: for any given fertility rate $n \geq 1$, an increase in the number of wives implies that each child gets a smaller fraction of male outputs, which reduces average child success, and thus also female reproductive success. The only exception to this rule arises in the (arguably inexistent) case where the totality of the male output is a pure public good: this occurs if the male produces no food ($y^*(1,\boldsymbol{\xi}) = 0$) and protection is a pure public good ($\alpha = 0$).

Turning now to males, there are two opposing effects: on the one hand, an increase in the number of females has a negative impact on average female reproductive success, as shown in Lemma 2; on the other hand, given an average female reproductive success, adding a female to the household has a positive impact on male reproductive success. The following proposition shows that the former effect may outweigh the latter effect at all levels of polygyny.

Lemma 3 *For each ecology* $\boldsymbol{\xi}$ *and each fertility rate* n, $\tilde{M}(n,1,\boldsymbol{\xi}) \geq \tilde{M}(n,k,\boldsymbol{\xi})$ *for all* $k \geq 1$ *if and only if* $\alpha = 1/\lambda$. *The inequality holds strictly for all* $k > 1$ *if, moreover,* $y^*(1,\boldsymbol{\xi}) > 0$.

In ecologies where the male produces only protection and no food (i.e., if $y^*(k,\boldsymbol{\xi}) = 0$ for all k, which is true if and only if $y^*(1,\boldsymbol{\xi}) = 0$), where protection is not at all public in nature ($\alpha = 1$), and where there are constant returns to protection ($\lambda = 1$), there are constant returns to "polygyny" scale (the term $k^{1-\lambda\alpha}$ in (46) reduces to 1), and hence $\tilde{M}(n,1,\boldsymbol{\xi}) \geq \tilde{M}(n,k,\boldsymbol{\xi})$. However, in ecologies where $\alpha = 1/\lambda$ and the male devotes some time to food production under monogamy ($y^*(1,\boldsymbol{\xi}) > 0$), $\tilde{M}(n,1,\boldsymbol{\xi}) > \tilde{M}(n,k,\boldsymbol{\xi})$: the first-order effect of an increase in k on the amount of food that each child gets from the father then outweighs the second-order effects of its impact on the time allocations. The net result is a decrease in male reproductive success, for any given fertility rate.

The first part of Lemma 3 further implies that as soon as protection is a somewhat public good, i.e., if $\alpha < 1$, for any fertility rate there exists some polygyny rate that gives the male a higher reproductive success than monogamy. Figure 2, which plots male reproductive success

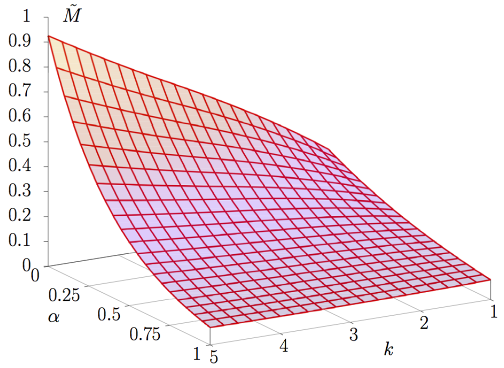

Figure 2: $\tilde{M}\left(n,k,\boldsymbol{\xi}\right)$, for $\lambda=\delta=0.8$, $\tau=0.3$, $\phi=0.75$, $\rho=1$, $\gamma=0.1$, $\theta=0.6$, $\beta=1$, $b=0.05$, and $n=5$

as a function of k and α, shows a case where male reproductive success is decreasing in k over the range $k=1$ to $k=5$ for large values of α, although it increases in k for small values of α. As is shown next, this result is general: if protection is not *too* public, monogamy maximizes male reproductive success *locally*, in the sense that an increase from one female per household to slightly more than one would have a negative impact on male reproductive success. Let

$$\hat{\alpha}\equiv\frac{\rho\gamma x^{*}\left(1,\boldsymbol{\xi}\right)+\left(1-\tau\right)\theta y^{*}\left(1,\boldsymbol{\xi}\right)}{\lambda\left[\rho\gamma x^{*}\left(1,\boldsymbol{\xi}\right)+\theta y^{*}\left(1,\boldsymbol{\xi}\right)\right]}. \tag{22}$$

Then:

Lemma 4 *For each ecology* $\boldsymbol{\xi}$ *and each fertility rate* n*, if* $\alpha\in\left(\hat{\alpha},1/\lambda\right)$ *there exists* $\hat{k}_1>1$ *and* $\hat{k}_2\geq\hat{k}_1$ *such that for any* $k\in[1,\hat{k}_1)$*,* $\tilde{M}\left(n,1,\boldsymbol{\xi}\right)>\tilde{M}\left(n,k,\boldsymbol{\xi}\right)$*, while for any* $k>\hat{k}_2$*,* $\tilde{M}\left(n,k,\boldsymbol{\xi}\right)>\tilde{M}\left(n,1,\boldsymbol{\xi}\right)$*. Otherwise,* $\tilde{M}\left(n,k,\boldsymbol{\xi}\right)\geq\tilde{M}\left(n,1,\boldsymbol{\xi}\right)$ *for all* $k\geq1$*, and there exists* $\hat{k}\geq1$ *such that* $\tilde{M}\left(n,k,\boldsymbol{\xi}\right)>\tilde{M}\left(n,1,\boldsymbol{\xi}\right)$ *for all* $k>\hat{k}$*.*

Figure 3 illustrates this and the preceding lemma, by showing male reproductive success as a function of the polygyny rate. The three curves, which are based on the same fertility rate and the same ecological parameters except α, represent the three cases that may arise depending on whether $\alpha=1/\lambda$ (the lower curve), $\alpha\in\left(\hat{\alpha},1/\lambda\right)$ (the middle curve), or $\alpha<\hat{\alpha}$ (the upper curve). In sum, then, for a given ecology and a given fertility rate, male reproductive success may be decreasing, non-monotonic, or increasing in the polygyny rate.

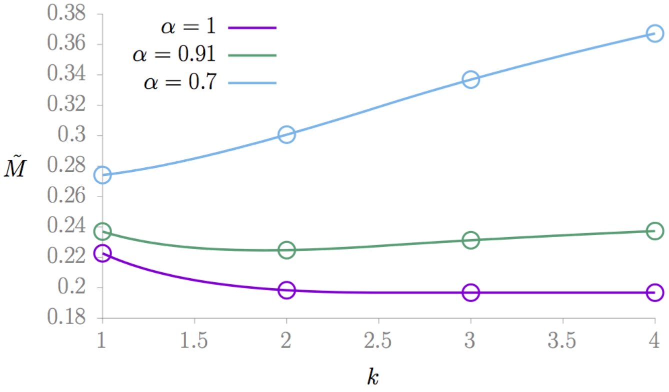

Figure 3: $\tilde{M}\left(n,k,\boldsymbol{\xi}\right)$, for $\lambda = 1$, $\delta = \tau = 0.5$, $\phi = 0.01$, $\rho = 1$, $\gamma = 0.1$, $\theta = 0.5$, $b = 0.05$, and $n = 2$

To understand why male reproductive success may be non-monotonic in the number of wives, consider the first-order effects of an increase in k on male reproductive success (see (46)). On the one hand, the effect on the amount of protection per child is positive if $\alpha < 1/\lambda$ (the term $k^{1-\lambda\alpha}$ in (46) increases); on the other hand, the effect on the amount of food per child is negative in ecologies where the male would devote some time to food production under monogamy, $y^*\left(1,\boldsymbol{\xi}\right) > 0$. The latter effect may outweigh the former effect if the former effect is small, which happens when α is large. However, even when α is large enough for this to occur at some levels of polygyny, it cannot happen for high enough levels of polygyny, because for high enough values of k the male devotes all his time to protection, and therefore the latter, negative effect of an increase in k on the amount of food per child, becomes nil.

The preceding results, which describe preferences over fertility rates for a given polygyny rate, and *vice versa*, can be combined to describe preferences over reproductive cultures $\omega = (k,n)$. Combining the fact that the preferred fertility rate does not depend on the polygyny rate (see Proposition 4) with Lemmas, 2, 3, and 4, the following propositions obtain:

Proposition 5 *Under private property, among all the possible reproductive cultures* $\omega = (k,n) \in [1,+\infty)$*, females prefer the fertility rate* $n = n^*$ *(see (21)) and they prefer monogamy* $(k = 1)$ *to polygyny* $(k > 1)$*, the latter preference being strict if* $\alpha \cdot y^*\left(1,\boldsymbol{\xi}\right) > 0$.

Proposition 6 *Under private property, among all the possible reproductive cultures* $\omega = (k,n) \in [1,+\infty)$*, conditional on mating successfully, males prefer the fertility rate* $n = n^*$

(see (21)), and they:
(i) prefer monogamy to any polygyny rate if $\alpha = 1/\lambda$ *(the preference being strict if* $y^*(1, \boldsymbol{\xi}) > 0$*);*
(ii) strictly prefer monogamy to low polygyny rates, and high polygyny rates to monogamy, if $(\hat{\alpha}, 1/\lambda)$*;*
(iii) (weakly) prefer any polygyny rate $k > 1$ *to monogamy if* $\alpha \in [0, \hat{\alpha})$*.*

Female reproductive success is determined by the number of children she gives birth to and their probability of survival. The male affects her reproductive success because of the food and protection he provides to her children. It is therefore not surprising that females always prefer monogamy to polygyny: sharing a husband, who has fixed time endowment, with other wives, is bound to reduce the probability of survival of her children in this setting where wives do not share the goods they produce with each other. By the same token, intuition might suggest that a male who can freely the number of wives as well as the number of children per wife, would simply choose to marry all the available females and let each wife have one child. But this is not true. Proposition 6 shows that as long as protection is a sufficiently private good, he would prefer to have a single wife rather than a small number of wives, although a large number of wives would be better than a single wife. Moreover, the preferred fertility rate is the same as for the females. These two properties are explained by two quantity-quality trade-offs: one pertaining to fertility (increasing the number of children lowers the probability of survival of each child), and one pertaining to polygyny (increasing the number of wives raises the number of children for the male, but it also lowers the probability of survival of each child).

The threshold value $\hat{\alpha}$ (see (22)), which determines whether male reproductive success declines or increases as he gets more than one wife, depends on the equilibrium time allocations. But since these depend on the ecology, $\hat{\alpha}$ is ultimately uniquely determined by the ecology. Figures 4 and 5 illustrate this, by showing how $\hat{\alpha}$ (measured on the vertical axis) depends on two ecological parameters. In Figure 4 the parameters that vary are those that measure the productivity in food production, γ (production by a female) and θ (production by the male). When γ is high and θ is low, the male devotes no time to producing the private good food, and all the time to producing the (partly public good) protection, which results in $\hat{\alpha} = 1$; in other words, in ecologies where females are very productive relative to the male in food production, males always prefer polygyny (even mild polygyny) over monogamy. By contrast, if the male is very productive relative to the females in food production (γ is low

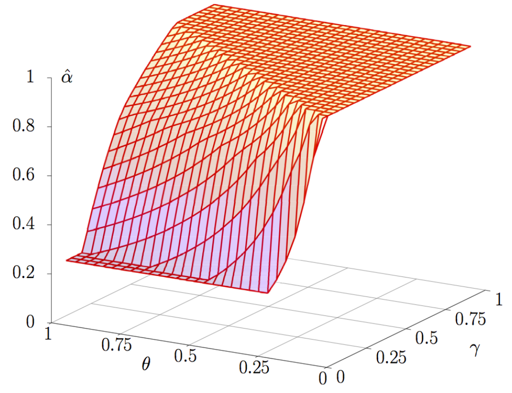

Figure 4: $\hat{\alpha}$ for $\lambda = 1$, $\delta = 0.5$, $\tau = 0.75$, $\phi = 0.5$, $\rho = 1$, $\alpha = \beta = 0.75$, and $n = 5$

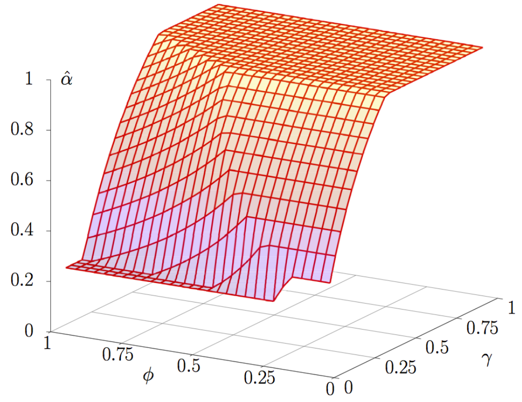

Figure 5: $\hat{\alpha}$ for $\lambda = 1$, $\delta = 0.5$, $\tau = 0.75$, $\rho = 1$, $\theta = 0.5$, $\alpha = \beta = 0.75$, and $n = 5$

and θ is high), the male spends much time on food production, which results in a low value of $\hat{\alpha}$ (here the lowest value of $\hat{\alpha}$ is around 0.25); thus, in such ecologies, males prefer monogamy over mild polygyny for almost any degree of publicness of protection.

In Figure 5 the parameters that vary there are those that measure female productivity in food production, γ, and the economies of scope between food and care production by females, ϕ. The fundamental reason for why $\hat{\alpha}$ varies with these parameters is the same as in Figure 3, namely, these parameters affect the extent to which the male engages in the production of (the private good) food.

The male preferences derived above apply to lucky males. But since evolutionary theory predicts that at birth the expected reproductive success of females and males be equal, and that the sex ratio equal one (Fisher, 1930), the reproductive culture that maximizes the reproductive success of males at birth—and thus behind the veil of ignorance as to who will be lucky and who will be unlucky—coincides with the reproductive culture that maximizes female reproductive success. Thus, while at birth males would prefer monogamy (see Proposition 5), if lucky males prefer polygyny to monogamy, and if there is no institution that imposes monogamy, each adult male has an incentive to fight other males to secure his preferred number of wives. The results derived above provide insights as to how hard males would be willing to fight.

4 Analysis: Household property

Here I characterize male and female behaviors and the ensuing preferences over reproductive cultures under the assumption that each female shares the food and care she produces among all the children in the household.

Using a superindex H for "household property," the reproductive success of each female is

$$F^{H}\left(\boldsymbol{x},y\right)=n\cdot a\left(n\right)\cdot s^{H}\left(\boldsymbol{x},y\right), \tag{23}$$

where $s^{H}\left(\boldsymbol{x},y\right)$, which obtains by replacing H_i, P_i, G_i,and C_i in $S\left(C_i,G_i,H_i,P_i\right)$ with the expressions in (6), (7), (10), and (11), writes:

$$s^{H}\left(\boldsymbol{x},y\right)=\left(\frac{1-y}{\left(kn\right)^{\alpha}}\right)^{\lambda}\cdot\left(\frac{\sum_{j=1}^{k}\left(1-\phi x_j\right)}{\left(kn\right)^{\beta}}\right)^{\delta}\cdot\left(\frac{\rho\gamma\sum_{j=1}^{k}x_j}{kn}+\frac{\theta y}{kn}\right)^{\tau}. \tag{24}$$

It follows that the reproductive success of a male is

$$M^{H}\left(\boldsymbol{x},y\right)=k\cdot n\cdot a\left(n\right)\cdot s^{H}\left(\boldsymbol{x},y\right). \tag{25}$$

4.1 Equilibrium time allocations

The following proposition characterizes the set of Nash equilibrium time allocations:[16]

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[16] Formally, the game that is being analyzed is a simultaneous-move game with $k+1$ players, where each player has strategy set $[0,1]$, where the strategy of female $j=1,...,k$ is denoted x_j, the strategy of the male is denoted y, and the payoff to female j is given in (23) while the payoff to the male in (25).

Proposition 7 *For each reproductive environment* $(\boldsymbol{\xi}, \omega)$*, there exists a unique Nash equilibrium time allocation, which, moreover, is the same as under private property (see (16) and (17)).*

It may come as a surprise that the equilibrium time allocation is the same as under private property, since intuition suggests that under household property the females would have an incentive to free ride on each other. However, the result is readily explained by the fact that, by assumption, females must devote all of their time to either food or care production: in other words, there is no opportunity cost of time, and therefore no incentive to free ride.

Although the equilibrium time allocations are the same as under private property, reproductive success is generally higher under household property, since now each child receives a fraction k^{β} of the care produced by all the females, $k\left[1-\phi x^{*}\left(k, \boldsymbol{\xi}\right)\right]$. Formally, let $\tilde{F}^{H}$ and $\tilde{M}^{H}$ denote female reproductive success as a function the reproductive culture and the ecology:

$$\begin{aligned} \tilde{F}^{H}\left(n, k, \boldsymbol{\xi}\right) &= a\left(n\right) \cdot n \cdot s^{H}\left(x^{*}\left(k, \boldsymbol{\xi}\right), y^{*}\left(k, \boldsymbol{\xi}\right)\right) \\ &= k^{1-\delta\beta} \cdot \tilde{F}\left(n, k, \boldsymbol{\xi}\right), \end{aligned} \tag{26}$$

where $\tilde{F}$ is defined in (19). Likewise, let $\tilde{M}^{H}$ denote male reproductive success as a function of the reproductive culture and the ecology:

$$\begin{aligned} \tilde{M}^{H}\left(n, k, \boldsymbol{\xi}\right) &= k \cdot a\left(n\right) \cdot n \cdot s^{H}\left(x^{*}\left(k, \boldsymbol{\xi}\right), y^{*}\left(k, \boldsymbol{\xi}\right)\right) \\ &= k^{1-\delta\beta} \cdot \tilde{M}\left(n, k, \boldsymbol{\xi}\right), \end{aligned} \tag{27}$$

where $\tilde{M}$ is defined in (20). Thus, as soon as care is a somewhat public good ($\beta < 1$) and $\delta < 1$, reproductive success is higher than under private property. This will have implications for the preferences over reproductive cultures, to which I turn next.

4.2 Preferences over reproductive cultures

As under private property, I first analyze the preferences of lucky males and then comment on the preferences behind the veil of ignorance at the end of the section.

To begin, note that since the factor $k^{1-\delta\beta}$ in (26) and (27) does not affect preferences over the number of children, n^{*} (see (21)) remains the preferred fertility by the male as well as his k wives. By contrast, preferences over k are affected as soon as care is somewhat public ($\beta < 1$) and $\delta < 1$. Letting

$$\tilde{\alpha} \equiv \hat{\alpha} - \frac{1 - \delta (1 - \beta)}{\lambda} \tag{28}$$

the following result obtains (the proof is in the appendix).

Proposition 8 *Under household property, among all the possible reproductive cultures* $\omega = (k, n) \in [1, +\infty)$*, females prefer the fertility rate* $n = n^*$ *(see (21)), and they:*
(i) prefer monogamy to any polygyny rate if $\alpha \geq \delta (1 - \beta) / \lambda$*, the preference being strict if* $y^* (1, \boldsymbol{\xi}) > 0$*;*
(ii) strictly prefer monogamy to low polygyny rates, and high polygyny rates to monogamy, if $\alpha \in (\tilde{\alpha}, \delta (1 - \beta) / \lambda)$*;*
(iii) (weakly) prefer any polygyny rate $k > 1$ *to monogamy for any* $\alpha \in [0, \tilde{\alpha}]$*.*
For any $\alpha < \delta (1 - \beta) / \lambda$*, female preferences are increasing in the number of wives for sufficiently high polygyny rates.*

While polygyny still is costly in the sense that the male's outputs get divided by a larger number of children, females now also derive some benefit from there being other wives as long as care is a somewhat public good. Hence, female preferences under household property are qualitatively similar to male preferences under private property (see Proposition 6). Importantly, however, here also the privateness of care (β) matters. If care is very private (i.e., if β is high), a female benefits only a little from other wives, and she then prefers monogamy over all polygyny rates even if protection is almost a fully public good (i.e., even for low valus of α), while under private property protection had to be fully private ($\alpha = 1/\lambda$) for males to prefer monogamy over all polygyny rates. But if both α and β are small, females no longer have a strict preference for monogamy. Thus, for intermediary degrees of "aggregate privateness" ($\lambda\alpha + \delta\beta$), female preferences are non-monotonic in the polygyny rate,[17] and for low levels of aggregate privateness, females prefer polygyny to monogamy.

Turning now to male preferences, let

$$\bar{\alpha} \equiv \hat{\alpha} + \frac{\delta (1 - \beta)}{\lambda}.$$

Then (as is shown in the appendix):

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[17] Lemma 5 in the proof of the proposition states the threshold values of k below which female reproductive success is decreasing and increasing, respectively. It should be noted that although $\hat{\alpha} \neq \tilde{\alpha}$ when $\delta (1 - \beta) < 1$, these threshold values are the same as in Lemma 4 ($\hat{k}_1$ and $\hat{k}_2$).

Proposition 9 *Under household property, among all the possible reproductive cultures* $\omega = (k, n) \in [1, +\infty)$*, conditional on mating successfully, males prefer the fertility rate* $n = n^*$ *(see (21)), and they:*
(i) prefer monogamy to any polygyny rate if $\alpha = [1 + \delta(1 - \beta)]/\lambda$ *(i.e., if* $\lambda = \alpha = \beta = 1$*), the preference being strict if* $y^*(1, \boldsymbol{\xi}) > 0$*;*
(ii) strictly prefer monogamy to low polygyny rates, and high polygyny rates to monogamy if $\alpha \in (\bar{\alpha}, [1 + \delta(1 - \beta)]/\lambda)$*;*
(iii) (weakly) prefer any polygyny rate $k > 1$ *to monogamy if* $\alpha \in [0, \bar{\alpha}]$*.*
For any $\alpha < [1 + \delta(1 - \beta)]/\lambda$*, male preferences are increasing in the number of wives for sufficiently high polygyny rates.*

These preferences are qualitatively similar to those under private property, although here the degree of publicness of care also matters: for monogamy to be preferred to all polygyny rates, both protection and care must be fully private goods ($\alpha = \beta = 1$); otherwise there always exists some polygyny rate that the male prefers to monogamy. For some parameter values, however, male reproductive success is non-monotonic in the polygyny rate, and males then prefer monogamy to low polygyny rates.

Finally, recall that under a balanced sex ratio at birth (Fisher, 1930) the reproductive culture that maximizes the reproductive success of males behind the veil of ignorance as to who will be lucky and who will be unlucky, coincides with the reproductive culture that maximizes female reproductive success. Now, is this reproductive culture really efficient? This is the question I address in the next section.

5 Efficiency

In a distant enough evolutionary past, there were no institutions. However, once institutions emerged, they opened the door to reflection about what is good for society as a whole. From the preceding sections, we know that, conditional on equilibrium time allocations being chosen, the reproductive culture that maximizes overall reproductive success is that which females prefer. But if the equilibrium time allocations are not efficient, in the sense that they fail to achieve the highest possible reproductive success given the reproductive culture at hand, then the reproductive culture preferred by females may fail to achieve the highest overall reproductive success. Here the goal is to investigate whether the equilibrium time allocations are efficient, and to draw conclusions based on this.

It is shown in the appendix that, for any reproductive culture, equilibrium time allocations *are* efficient.

Proposition 10 *Whether female outputs are private or household property, for any reproductive ecology* $(\boldsymbol{\xi}, \omega)$ *the (unique) equilibrium time allocation maximizes both female and male reproductive success.*

The intuition behind this result is that, for any given reproductive culture, the objectives of the male and the females are aligned: they all seek to maximize reproductive success. In particular, (by definition) there is no opportunity cost of time.[18]

In light of Proposition 10, a reproductive culture maximizes the expected reproductive success at birth of a female if and only if it maximizes the reproductive success that she achieves given the equilibrium time allocations, i.e., $\tilde{F}(n, k, \boldsymbol{\xi})$ under private property (see (19)), and $\tilde{F}^H(n, k, \boldsymbol{\xi})$ under household property (see (26)). Letting k^* denote an efficient polygyny rate, the following corollary follows immediately from Lemmas 1 and 2:

Corollary 1 *Suppose that a female's outputs are her private property. Then, in any ecology* $\boldsymbol{\xi}$*, it is efficient for each male to have a single wife,* $k^* = 1$*, and for each female to have* n^* *children. This is the unique efficient reproductive culture if, moreover,* $\alpha \cdot y^*(1, \boldsymbol{\xi}) > 0$.

Likewise, the following corollary follows immediately from Lemma 1 and Proposition 8. In this corollary, K denotes the total number of females that survive to sexual maturity:

Corollary 2 *Suppose that a female's outputs are household property. Then:*
(i) if $\alpha \geq \delta(1 - \beta)/\lambda$*, it is efficient for each male to have a single wife,* $k^* = 1$*, and for each female to have one child,* $n^* = 1$*; this is the unique efficient reproductive culture if, moreover,* $y^*(1, \boldsymbol{\xi}) > 0$*;*
(ii) if $\alpha < \delta(1 - \beta)/\lambda$*, the unique efficient reproductive culture is such that all the females share the same husband,* $k^* = K$*, and for each female to have* n^* *children.*

Under private property monogamy is more efficient than any polygyny rate, independent of the ecology $\boldsymbol{\xi}$. Monogamy is also efficient under household property in ecologies where the average privateness of protection and care is high enough ($\lambda\alpha + \delta\beta \geq \delta$). By contrast, under

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[18] The result would likely be different in a model that allows for unfaithfulness.

household property and a high average publicness of protection and care ($\lambda\alpha + \delta\beta < \delta$), efficiency requires that the number of females per male be maximized, which happens when all the K females share the same man.[19] The intuition behind this bang-bang result is reminiscent of the intuition behind the shape of male preferences under private property: either the negative effect of diluting male outputs over more than one female's children always outweighs the positive effect of sharing total female outputs among all the children in the household, or the former effect is small enough to be outweighed for high enough polygyny rates.

Interestingly, monogamy is efficient if it is also efficient for each woman to have a single child. More generally, the propensity for monogamy to be more efficient than some levels of polygyny[20] goes hand in hand with the propensity for low fertility rates to be efficient, since both are driven by a low average publicness of care.

6 Discussion

In this paper I propose a simple model to analyze how female and male preferences over reproductive cultures (number of children per female and number of females per male) may have been shaped by evolutionary forces. The originality of the model stems from its ability to link these preferences to the exogenously given ecology. In line with basic evolutionary logic the underlying assumption is that both females and males seek to maximize own reproductive success. While it is beyond the scope of this paper to analyze which reproductive culture in fact arises, I here discuss some potential implications for this of the results reported above.

Since female reproductive success dictates the expected reproductive success at birth of both females and males (Fisher, 1930), female preferences are of particular interest when it comes to understanding how resources may be utilized efficiently at the level of the group, and thus in which direction competition between groups may push the reproductive culture (see Henrich, Boyd, and Richerson, 2012, for a model of such group competition). In a nutshell, the analysis suggests that while group competition may be expected to favor monogamy

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[19] It goes without saying that this result is somewhat misleading to the extent that it does not take into account the frustration of unlucky males. But this issue is beyond the scope of this paper.

[20] While Corollary 2 does not report the non-monotonicity results, these should be clear from Proposition 8.

in many ecologies, allomothering sometimes makes polygyny attractive for the group, even absent the fact that polygyny would make some male labor available for fighting wars. More generally, four broad conclusions can be drawn from the analysis:

1. Under private property, females prefer monogamy over polygyny in all ecologies.
2. Under household property, females prefer either monogamy or full polygyny (one male marries all the available females). Furthermore, even when full polygyny is preferred, female preferences are non-monotonic in the polygyny rate if the average privateness of the care provided by males and females is high enough: monogamy then dominates low polygyny rates, but high polygyny rates dominate monogamy.
3. The ecology affects female preferences over polygyny rates indirectly by affecting time allocations. In particular, ecologies in which the male would devote some time to food production under monogamy tend to favor monogamy, at least over low polygyny rates. This is because food is a purely private good, so that when the male produces food the dilution effect of polygyny is more pronounced than when the male produces no food.
4. Female preferences over fertility rates are the same under private and household property. Furthermore, the preferred fertility rate tends to be lower in ecologies where monogamy is not dominated by all polygyny rates.

Group competition may, however, not be expected to be the only force that shapes reproductive cultures. In particular, in ecologies where males would prefer to have more than one wife conditional on being successful at mating, they have an incentive to invest in activities that allow them to compete against other males, and marry a greater number of wives than the females would like. By showing how the preferred polygyny rate of a successful male depends on the ecology, the analysis thus sheds light on how the ecology affects the willingness of males to invest in such activities, which may be more or less wasteful. Furthermore, the analysis shows that if conditional on being successful at mating a male prefers polygyny to monogamy, then his wives tend to prefer a smaller polygyny rate than he does (except when they all prefer full polygyny). Hence, the model may deliver insights as to how the ecology affects the strength with which females would be opposed to the competition among males for wives.

Furthermore, the results suggest that history may matter when it comes to shaping a reproductive culture. In particular, consider a society where successful males would like to

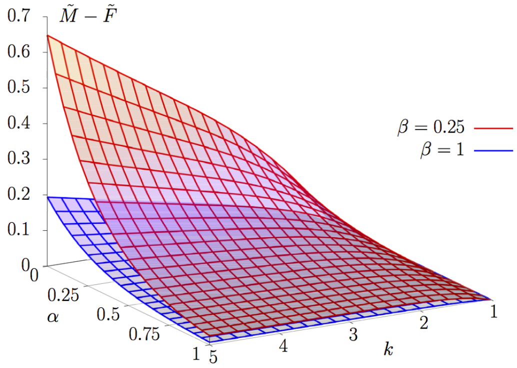

Figure 6: $\tilde{M}\left(n, k, \boldsymbol{\xi}\right) - \tilde{F}\left(n, k, \boldsymbol{\xi}\right)$ for $\lambda = \delta = 1$, $\tau = 0.5$, $\phi = 0.75$, $\rho = 1$, $\gamma = 0.1$, $\theta = 0.6$, and $n = 5$

have several wives but where monogamy arises at some point in history because of intense group competition, and suppose that group competition then becomes less intense. If the process by which the reproductive culture changes is gradual and males prefer monogamy to low polygyny rates, then monogamy may be preserved in the society even though males would prefer a high polygyny rate (conditional on mating successfully).

Finally, while the fact that females and males typically have different preferences over polygyny rates is not surprising, the model provides insights about how the magnitude of the disagreement between females and males depends on the ecology. This disagreement being literally about life and death, it may be expected to affect the quality of relations between men and women. As an illustration, Figure 6 shows the difference between the reproductive success of a lucky male and that of a female, as a function of the degree of publicness of male care and the polygyny rate. The graph suggests that for any α, the difference increases (from zero for $k = 1$) as k increases. Furthermore, for any k, the difference is smaller the higher is α: here, relations between males and females may be expected to be more pacific in ecologies where male care is a relatively private good.

7 Conclusion

The model proposed in this paper delivers novel insights about the trade-offs faced by females and males when it comes to reproductive strategies, and about how these trade-offs depend on the ecology in which the population evolves. In particular, it shows that it may not be innocuous to assume, as is often done, that male reproductive success automatically increases if he acquires more wives.

The model may be extended in many directions. First and foremost, the implications of the results for household formation should be explicitly analyzed. In particular, it is not clear how the non-monotonicity of male preferences over polygyny rates would affect household formation. It would also be interesting to embed households more explicitly in a market economy and allow for wealth accumulation over generations. Extending the model to allow for uncertainty in food production and risk sharing within the extended family might also be fruitful. The model may also be used to analyze the evolution of preferences of mothers and fathers for their children and for each other. These and other fascinating questions are left for future research.

8 Appendix

8.1 Proposition 1

A strategy profile $(x_1^*, x_2^*, ..., x_k^*, y^*) \equiv (\boldsymbol{x}^*, y^*)$ is a Nash equilibrium if and only if

$$\begin{cases} x_j^* \in \arg\max_{x_j \in [0,1]} a(n) \cdot n \cdot s(x_j, y^*) \qquad \forall j = 1, ..., k \\ y^* \in \arg\max_{y \in [0,1]} a(n) \cdot n \cdot \sum_{j=1}^{k} s\left(x_j^*, y\right), \end{cases} \tag{29}$$

where (from (14))

$$s(x_j, y) = \left(\frac{1-y}{N^\alpha}\right)^\lambda \cdot \left(\frac{1-\phi x_j}{n^\beta}\right)^\delta \cdot \left(\frac{\rho\gamma x_j}{n} + \frac{\theta y}{N}\right)^\tau. \tag{30}$$

To begin, note that $y = 1$ cannot be part of an equilibrium strategy profile. Suppose, to the contrary, that $(\boldsymbol{x}^*, y^*) = (\boldsymbol{x}^*, 1)$ for some $\boldsymbol{x}^* \in [0,1]^k$. Then $s\left(x_j^*, y^*\right) = 0$ for any $x_j^* \in [0,1]$, while for any $y \in (0,1)$, $s\left(x_j^*, y\right) > 0$. Similarly, $(\boldsymbol{x}^*, y^*) = (\mathbf{0}, 0)$, where $\mathbf{0} = (0, 0, ..., 0)$, cannot be an equilibrium strategy profile. Indeed, $s(\mathbf{0}, 0) = 0$, while, for any $y \in (0,1)$, $s(\mathbf{0}, y) > 0$.

Next, it is straightforward to verify that, for each reproductive ecology $(\boldsymbol{\xi}, R)$ and each $y \in [0,1)$, $s(x_j, y)$ is strictly concave in x_j. This implies that for each $y \in [0,1)$ there is a unique x_j that maximizes $s(x_j, y)$, implying that any equilibrium must be female-symmetric. Hence, it is sufficient to use a two-dimensional vector, (x^*, y^*), to describe any equilibrium strategy profile. Noting that the term $a(n) \cdot n$ is irrelevant, and that for $x_1^* = ... = x_k^* = x^*$, $\sum_{j=1}^{k} s\left(x_j^*, y\right) = k \cdot s(x^*, y)$, the system of $k+1$ equations (1) reduces to the following system of two equations:

$$\begin{cases} x^* \in \arg\max_{x \in [0,1]} s(x, y^*) \\ y^* \in \arg\max_{y \in [0,1]} s(x^*, y), \end{cases} \tag{31}$$

where

$$s(x, y) = \left(\frac{1-y}{N^\alpha}\right)^\lambda \cdot \left(\frac{1-\phi x}{n^\beta}\right)^\delta \cdot \left(\frac{\rho\gamma x}{n} + \frac{\theta y}{N}\right)^\tau. \tag{32}$$

Note that, in fact, then, the original $k+1$-player game may be viewed as a game between one male and one female; since the payoff functions are continuous and the strategy spaces are compact and convex, equilibrium existence is guaranteed.

It has already been noted that for each male time allocation $y \in [0,1)$, there exists a unique x that is a best response for the female to y. Likewise, for each female time allocation

$x \in [0,1]$, s is strictly concave in y, implying that for each $x \in [0,1]$, there exists a unique y that is a best response for the male to x. Moreover,

$$s(x,y) = \frac{1}{N^{\lambda\alpha} n^{\delta\beta} n^{\tau}} \cdot (1-y)^{\lambda} \cdot (1-\phi x)^{\delta} \cdot \left(\rho\gamma x + \frac{\theta y}{k}\right)^{\tau}.$$

Letting $\tilde{s}(x,y) = (1-y)^{\lambda} \cdot (1-\phi x)^{\delta} \cdot \left(\rho\gamma x + \frac{\theta y}{k}\right)^{\tau}$, it is clearly sufficient to study the first-order partial derivatives of $\tilde{s}(x,y)$ to determine the best response functions.

Thus, for the male:

$$\begin{aligned} \frac{\partial \tilde{s}(x,y)}{\partial y} &= \frac{\tau\theta}{k} \cdot (1-y)^{\lambda} \cdot (1-\phi x)^{\delta} \cdot \left(\rho\gamma x + \frac{\theta y}{k}\right)^{\tau-1} \\ &\quad -\lambda (1-y)^{\lambda-1} (1-\phi x)^{\delta} \cdot \left(\rho\gamma x + \frac{\theta y}{k}\right)^{\tau}. \end{aligned} \tag{33}$$

Dividing this by the strictly positive term $(1-y)^{\lambda-1} \cdot (1-\phi x)^{\delta} \cdot \left(\rho\gamma x + \frac{\theta y}{k}\right)^{\tau-1}$ and simplifying, one obtains that the sign of $\frac{\partial \tilde{s}(x_j,y)}{\partial y}$ is the same as the sign of

$$\theta\tau - k\lambda\rho\gamma x - \theta(\lambda+\tau) y. \tag{34}$$

This expression is strictly decreasing in y. It is non-negative for all $y \in [0,1]$ iff it is non-negative for $y = 1$, i.e., if $\theta\tau - k\lambda\rho\gamma x - \theta(\lambda+\tau) \geq 0$, which is false. The expression in (34) is non-positive for all $y \in [0,1]$ iff it is non-positive for $y = 0$, i.e., iff $\theta\tau - k\lambda\rho\gamma x \leq 0$, or

$$x \geq \theta\tau / (k\lambda\rho\gamma) \equiv x_1.$$

Thus, if $x \in (0, x_1)$, there exists a unique y such that the expression in (34) equals zero:

$$y = \frac{\tau}{\lambda+\tau} - \frac{k\lambda\rho\gamma}{\theta(\lambda+\tau)} x. \tag{35}$$

Hence, the male's best response to the female strategy x (where x is chosen by each of the k females) is:

$$\begin{cases} y^m = 0 \text{ if } x \geq x_1 \\ y^m = \frac{\tau}{\lambda+\tau} - \frac{k\lambda\rho\gamma}{\theta(\lambda+\tau)} x \text{ if } x \in [0, x_1]. \end{cases} \tag{36}$$

Turning now to the female:

$$\begin{aligned} \frac{\partial \tilde{s}(x,y)}{\partial x} &= \tau\rho\gamma \cdot (1-y)^{\lambda} \cdot (1-\phi x)^{\delta} \cdot \left(\rho\gamma x + \frac{\theta y}{k}\right)^{\tau-1} \\ &\quad - (1-y)^{\lambda} \cdot \phi\delta \cdot (1-\phi x)^{\delta-1} \cdot \left(\rho\gamma x + \frac{\theta y}{k}\right)^{\tau}. \end{aligned} \tag{37}$$

Dividing this by the strictly positive term $(1-y)^{\lambda} \cdot (1-\phi x)^{\delta-1} \cdot \left(\rho\gamma x + \frac{\theta y}{k}\right)^{\tau-1}$ and simplifying, one obtains that the sign of $\frac{\partial \tilde{s}(x,y)}{\partial x}$ is the same as the sign of

$$k\tau\rho\gamma - (\delta+\tau)\, k\rho\gamma\phi x - \delta\phi\theta y. \tag{38}$$

This expression is strictly decreasing in x. It is non-negative for all $x \in [0,1]$ iff it is non-negative for $x = 1$, i.e., if $k\tau\rho\gamma - (\delta+\tau)\, k\rho\gamma\phi - \delta\phi\theta y \geq 0$, i.e., iff

$$y \leq \frac{k\tau\rho\gamma}{\delta\phi\theta} - \frac{(\delta+\tau)\, k\rho\gamma}{\delta\theta} \equiv y_0.$$

Note that $y_0 \geq 0$ iff $\frac{\tau}{\tau+\delta} \geq \phi$. The expression in (38) is non-positive for all $x \in [0,1]$ iff it is non-positive for $x = 0$, i.e., iff $k\tau\rho\gamma - \delta\phi\theta y \leq 0$, or

$$y \geq k\tau\rho\gamma / (\delta\phi\theta) \equiv y_1.$$

Thus, if $y \in (\max\{0, y_0\}, y_1)$, there exists a unique x such that the expression in (38) equals zero:

$$x = \frac{\tau}{(\delta+\tau)\,\phi} - \frac{\delta\theta}{(\delta+\tau)\, k\rho\gamma} y. \tag{39}$$

Hence, a female's best response to the male strategy y is:

$$\begin{cases} x^f = 0 \text{ if } y \geq y_1 \\ x^f = \frac{\tau}{(\delta+\tau)\phi} - \frac{\delta\theta}{(\delta+\tau)k\rho\gamma} y \text{ if } y \in [\max\{0, y_0\}, y_1] \\ x^f = 1 \text{ if } y \leq \max\{0, y_0\}. \end{cases} \tag{40}$$

Noting that y^m is linear and strictly decreasing in x (whenever $x \in [0, x_1]$), and that x^f is linear and strictly decreasing in y (whenever $y \in [y_0, y_1]$), a necessary and sufficient condition for equilibrium to be unique is that the (absolute value of the) slope of the male's best response curve be strictly smaller than the (absolute value of the) slope of the female's best response curve, which is true: $\left|\frac{dy^m}{dx}\right| = \frac{k\lambda\rho\gamma}{\theta(\lambda+\tau)} < \frac{(\delta+\tau)k\rho\gamma}{\delta\theta} = \left|1/\left(\frac{dx^f}{dy}\right)\right|$.

Combining (36) and (40), two cases may be distinguished, depending on whether $y_0 \leq 0$ or $y_0 > 0$. First, if $\frac{\tau}{\delta+\tau} \leq \phi$ (i.e., if $y_0 \leq 0$),

$$\left(x^*(k, \boldsymbol{\xi}), y^*(k, \boldsymbol{\xi})\right) = \begin{cases} \left(\frac{\tau}{\phi(\delta+\tau)}, 0\right) \text{ if } \frac{\theta}{\rho\gamma} \leq \frac{k\lambda}{\phi(\delta+\tau)} \\ \left(\frac{(\lambda+\tau)\rho\gamma k - \delta\phi\theta}{\rho\gamma k\phi(\lambda+\delta+\tau)}, \frac{\theta\phi(\delta+\tau) - \rho\gamma\lambda k}{\theta\phi(\lambda+\delta+\tau)}\right) \text{ if } \frac{\theta}{\rho\gamma} \in \left[\frac{k\lambda}{\phi(\delta+\tau)}, \frac{k(\lambda+\tau)}{\delta\phi}\right] \\ \left(0, \frac{\tau}{\lambda+\tau}\right) \text{ if } \frac{\theta}{\rho\gamma} \geq \frac{k(\lambda+\tau)}{\delta\phi}. \end{cases} \tag{41}$$

Second, if $\frac{\tau}{\delta+\tau} > \phi$ (i.e., if $y_0 > 0$)

$$\left(x^*\left(k,\boldsymbol{\xi}\right), y^*\left(k,\boldsymbol{\xi}\right)\right) = \begin{cases} (1,0) \text{ if } \frac{\theta}{\rho\gamma} \leq \frac{k\lambda}{\tau} \\ \left(1, \frac{\tau\theta - k\lambda\rho\gamma}{\theta(\lambda+\tau)}\right) \text{ if } \frac{\theta}{\rho\gamma} \in \left[\frac{k\lambda}{\tau}, \frac{k[\lambda+\tau-\phi(\lambda+\delta+\tau)]}{\delta\phi}\right] \\ \left(\frac{(\lambda+\tau)\rho\gamma k - \delta\phi\theta}{\rho\gamma k\phi(\lambda+\delta+\tau)}, \frac{\theta\phi(\delta+\tau)-\rho\gamma\lambda k}{\theta\phi(\lambda+\delta+\tau)}\right) \text{ if } \frac{\theta}{\rho\gamma} \in \left[\frac{k[\lambda+\tau-\phi(\lambda+\delta+\tau)]}{\delta\phi}, \frac{k(\lambda+\tau)}{\delta\phi}\right] \\ \left(0, \frac{\tau}{\lambda+\tau}\right) \text{ if } \frac{\theta}{\rho\gamma} \geq \frac{k(\lambda+\tau)}{\delta\phi}. \end{cases} \tag{42}$$

8.2 Proposition 3

If $\frac{\tau}{\delta+\tau} \leq \phi$, the total amount of food that a child receives is

$$\frac{\rho\gamma x^*\left(k,\boldsymbol{\xi}\right)}{n} + \frac{\theta y^*\left(k,\boldsymbol{\xi}\right)}{kn} = \begin{cases} \frac{\rho\gamma\tau}{n\phi(\delta+\tau)} \text{ if } \frac{\theta}{\rho\gamma} \leq \frac{k\lambda}{\phi(\delta+\tau)} \\ \frac{\tau(\rho\gamma k+\theta\phi)}{nk\phi(\lambda+\delta+\tau)} \text{ if } \frac{\theta}{\rho\gamma} \in \left[\frac{k\lambda}{\phi(\delta+\tau)}, \frac{k(\lambda+\tau)}{\delta\phi}\right] \\ \frac{\theta\tau}{nk(\lambda+\tau)} \text{ if } \frac{\theta}{\rho\gamma} \geq \frac{k(\lambda+\tau)}{\delta\phi}. \end{cases} \tag{43}$$

If $\frac{\tau}{\delta+\tau} > \phi$, it is

$$\frac{\rho\gamma x^*\left(k,\boldsymbol{\xi}\right)}{n} + \frac{\theta y^*\left(k,\boldsymbol{\xi}\right)}{kn} = \begin{cases} \frac{\rho\gamma}{n} \text{ if } \frac{\theta}{\rho\gamma} \leq \frac{k\lambda}{\tau} \\ \frac{\tau(\rho\gamma k+\theta)}{nk(\lambda+\tau)} \text{ if } \frac{\theta}{\rho\gamma} \in \left[\frac{k\lambda}{\tau}, \frac{k[\lambda+\tau-\phi(\lambda+\delta+\tau)]}{\delta\phi}\right] \\ \frac{\tau(\rho\gamma k+\theta\phi)}{nk\phi(\lambda+\delta+\tau)}\text{if } \frac{\theta}{\rho\gamma} \in \left[\frac{k[\lambda+\tau-\phi(\lambda+\delta+\tau)]}{\delta\phi}, \frac{k(\lambda+\tau)}{\delta\phi}\right] \\ \frac{\theta\tau}{nk(\lambda+\tau)} \text{ if } \frac{\theta}{\rho\gamma} \geq \frac{k(\lambda+\tau)}{\delta\phi}. \end{cases} \tag{44}$$

In either case, the total amount of food is:

1. strictly increasing in τ, except if $\frac{\tau}{\delta+\tau} > \phi$ and $\frac{\theta}{\rho\gamma} \leq \frac{k\lambda}{\tau}$, in which case it is constant in τ;
2. strictly decreasing in δ if $\frac{\tau}{\delta+\tau} > \phi$ and $\frac{\theta}{\rho\gamma} \leq \frac{k(\lambda+\tau)}{\delta\phi}$, or if $\frac{\tau}{\delta+\tau} > \phi$ and $\frac{\theta}{\rho\gamma} \in \left[\frac{k[\lambda+\tau-\phi(\lambda+\delta+\tau)]}{\delta\phi}, \frac{k(\lambda+\tau)}{\delta\phi}\right]$; otherwise it is constant in δ;
3. strictly decreasing in λ, except if $\frac{\tau}{\delta+\tau} > \phi$ and $\frac{\theta}{\rho\gamma} > \frac{k\lambda}{\phi(\delta+\tau)}$, or if $\frac{\tau}{\delta+\tau} > \phi$ and $\frac{\theta}{\rho\gamma} > \frac{k\lambda}{\tau}$, in which case it is constant in λ;
4. strictly increasing in θ, except if $\frac{\tau}{\delta+\tau} \leq \phi$ and $\frac{\theta}{\rho\gamma} \leq \frac{k\lambda}{\phi(\delta+\tau)}$, or if $\frac{\tau}{\delta+\tau} > \phi$ and $\frac{\theta}{\rho\gamma} \leq \frac{k\lambda}{\tau}$, in which case it is constant in θ;
5. strictly increasing in ρ, except if $\frac{\theta}{\rho\gamma} \geq \frac{k(\lambda+\tau)}{\delta\phi}$, in which case it is constant in ρ;
6. strictly increasing in γ, except if $\frac{\theta}{\rho\gamma} \geq \frac{k(\lambda+\tau)}{\delta\phi}$, in which case it is constant in γ;

7. strictly decreasing in ϕ if $\frac{\tau}{\delta+\tau} \leq \phi$ and $\frac{\theta}{\rho\gamma} < \frac{k(\lambda+\tau)}{\delta\phi}$, or if $\frac{\tau}{\delta+\tau} > \phi$ and $\frac{\theta}{\rho\gamma} \in \left[\frac{k[\lambda+\tau-\phi(\lambda+\delta+\tau)]}{\delta\phi}, \frac{k(\lambda+\tau)}{\delta\phi}\right]$; otherwise, it is constant in ϕ;
8. strictly decreasing in k, except if $\frac{\tau}{\delta+\tau} \leq \phi$ and $\frac{\theta}{\rho\gamma} \leq \frac{k\lambda}{\phi(\delta+\tau)}$, or if $\frac{\tau}{\delta+\tau} > \phi$ and $\frac{\theta}{\rho\gamma} \leq \frac{k\lambda}{\tau}$, in which case it is constant in k;
9. strictly decreasing in n.

8.3 Lemma 1

Combining (13), (15), and (18), recalling that $a(n) = \max\{0, 1-bn\}$, and noting that the equilibrium time allocations are functions of k and $\boldsymbol{\xi}$, female and male reproductive success may be written as functions of the reproductive culture and the ecology. Letting $\tilde{F}$ and $\tilde{M}$ denote these functions:

$$\begin{aligned}\tilde{F}(n,k,\boldsymbol{\xi}) &\equiv F(x^*(k,\boldsymbol{\xi}), y^*(k,\boldsymbol{\xi}), n, k, \boldsymbol{\xi}) \\ &= \max\{0,1-bn\} \cdot n^{1-\lambda\alpha-\delta\beta-\tau} \cdot \left(\frac{1-y^*(k,\boldsymbol{\xi})}{k^{\alpha}}\right)^{\lambda} \cdot \\ &\quad [1-\phi x^*(k,\boldsymbol{\xi})]^{\delta} \cdot \left[\rho\gamma x^*(k,\boldsymbol{\xi}) + \frac{\theta y^*(k,\boldsymbol{\xi})}{k}\right]^{\tau}\end{aligned} \tag{45}$$

and

$$\begin{aligned}\tilde{M}(n,k,\boldsymbol{\xi}) &\equiv M(x^*(k,\boldsymbol{\xi}), y^*(k,\boldsymbol{\xi}), n, k, \boldsymbol{\xi}) \\ &= k \cdot F(x^*(k,\boldsymbol{\xi}), y^*(k,\boldsymbol{\xi}), n, k, \boldsymbol{\xi}) \\ &= k^{1-\lambda\alpha} \cdot \max\{0,1-bn\} \cdot n^{1-\lambda\alpha-\delta\beta-\tau} \cdot [1-y^*(k,\boldsymbol{\xi})]^{\lambda} \cdot \\ &\quad [1-\phi x^*(k,\boldsymbol{\xi})]^{\delta} \cdot \left[\rho\gamma x^*(k,\boldsymbol{\xi}) + \frac{\theta y^*(k,\boldsymbol{\xi})}{k}\right]^{\tau}.\end{aligned} \tag{46}$$

Since the expression in (45) may be written

$$\begin{aligned}\tilde{F}(n,k,\boldsymbol{\xi}) &= g(n) \cdot \left(\frac{1-y^*(k,\boldsymbol{\xi})}{k^{\alpha}}\right)^{\lambda} \cdot [1-\phi x^*(k,\boldsymbol{\xi})]^{\delta} \cdot \\ &\quad \left[\rho\gamma x^*(k,\boldsymbol{\xi}) + \frac{\theta y^*(k,\boldsymbol{\xi})}{k}\right]^{\tau},\end{aligned} \tag{47}$$

where $g(n) = \max\{0, 1-bn\} \cdot n^{1-\lambda\alpha-\delta\beta-\tau}$, and that in (46) may be written

$$\begin{aligned}\tilde{M}(n,k,\boldsymbol{\xi}) &= g(n) \cdot k^{1-\lambda\alpha} \cdot [1-y^*(k,\boldsymbol{\xi})]^{\lambda} \cdot [1-\phi x^*(k,\boldsymbol{\xi})]^{\delta} \cdot \\ &\quad \left[\rho\gamma x^*(k,\boldsymbol{\xi}) + \frac{\theta y^*(k,\boldsymbol{\xi})}{k}\right]^{\tau},\end{aligned} \tag{48}$$

the maximization problems

$$\max_{n\in[1,+\infty)} \tilde{F}\left(n,k,\boldsymbol{\xi}\right)$$

and

$$\max_{n\in[1,+\infty)} \tilde{M}\left(n,k,\boldsymbol{\xi}\right)$$

are both equivalent to

$$\max_{n\in[1,+\infty)} g\left(n\right). \tag{49}$$

To solve this maximization problem, note first that $g\left(0\right)=0$, while $g\left(1\right)>0$ (since $b\in(0,1)$), so that the preferred $n\geq 1$. Now, since

$$g'\left(n\right)=\left(1-\lambda\alpha-\delta\beta-\tau\right)\left(1-bn\right)\cdot n^{-\lambda\alpha-\delta\beta-\tau}-bn^{1-\lambda\alpha-\delta\beta-\tau},$$

two cases immediately arise. First, if $1-\lambda\alpha-\delta\beta-\tau\leq 0$, $g'\left(n\right)<0$ for all $n\in[1,+\infty)$, which implies that $n=1$ is the unique solution to (49). Second, if $1-\lambda\alpha-\delta\beta-\tau>0$,

$$g''\left(n\right)=\left(1-\lambda\alpha-\delta\beta-\tau\right)\left[-\left(\lambda\alpha+\delta\beta+\tau\right)\left(1-bn\right)n^{-\lambda\alpha-\delta\beta-\tau-1}-2bn^{-\lambda\alpha-\delta\beta-\tau}\right]$$

is strictly negative, so that either $n=1$ solves (49) (if $g'\left(1\right)\leq 0$), or there exists $n>1$ that satisfies the necessary first-order condition for an interior solution, $g'\left(n\right)=0$. Since $g'\left(n\right)$ iff $n=\frac{(1-\lambda\alpha-\delta\beta-\tau)}{(2-\lambda\alpha-\delta\beta-\tau)b}$. In sum, then, the solution writes:

$$n^{*}=\max\left\{1,\frac{\left(1-\lambda\alpha-\delta\beta-\tau\right)}{\left(2-\lambda\alpha-\delta\beta-\tau\right)b}\right\}.$$

8.4 Lemma 2

Based on the expression in (47),

$$\begin{aligned}\lim_{k\to+\infty}\tilde{F}\left(n,k,\boldsymbol{\xi}\right) &= \lim_{k\to+\infty}\frac{1}{k^{\lambda\alpha}}\cdot\left[g\left(n^{*}\right)\cdot\left[1-\phi\max\left\{1,\frac{\tau}{\phi\left(\delta+\tau\right)}\right\}\right]^{\delta}\cdot\right.\\ &\qquad\left.\left[\rho\gamma\max\left\{1,\frac{\tau}{\phi\left(1+\tau\right)}\right\}\right]^{\tau}\right].\end{aligned}$$

Hence,

$$\lim_{k\to+\infty}\tilde{F}\left(n,k,\boldsymbol{\xi}\right)=0$$

for all $\alpha > 0$, and

$$\begin{aligned}\lim_{k\rightarrow+\infty}\tilde{F}(n,k,\boldsymbol{\xi}) &= g(n^{*})\cdot\left[1-\phi\max\left\{1,\frac{\tau}{\phi(\delta+\tau)}\right\}\right]^{\delta}\cdot\\ &\left[\rho\gamma\max\left\{1,\frac{\tau}{\phi(1+\tau)}\right\}\right]^{\tau}\end{aligned}$$

for $\alpha = 0$.

To determine the polygyny rate $k \geq 1$ preferred by females, if any, the sign of $\frac{d\tilde{F}(n,k,\boldsymbol{\xi})}{dk}$ is studied. Since $g(n)$ does not depend on k and $g(n) > 0$, it is sufficient to determine the sign of $\frac{ds(x^{*}(k,\boldsymbol{\xi}),y^{*}(k,\boldsymbol{\xi}),n,k,\boldsymbol{\xi})}{dk}$.

$$\begin{aligned}\frac{ds(x^{*}(k,\boldsymbol{\xi}),y^{*}(k,\boldsymbol{\xi}),n,k,\boldsymbol{\xi})}{dk} &= \left.\frac{\partial s(x,y,n,k,\boldsymbol{\xi})}{\partial x}\right|_{(x,y)=(x^{*},y^{*})}\cdot\frac{\partial x^{*}(k,\boldsymbol{\xi})}{\partial k}\\ &+\left.\frac{\partial s(x,y,n,k,\boldsymbol{\xi})}{\partial y}\right|_{(x,y)=(x^{*},y^{*})}\cdot\frac{\partial y^{*}(k,\boldsymbol{\xi})}{\partial k}\\ &+\left.\frac{\partial s(x,y,n,k,\boldsymbol{\xi})}{\partial k}\right|_{(x,y)=(x^{*},y^{*})}.\end{aligned}$$

From the proof of Proposition 1, for all (x^{*},y^{*})

$$\left.\frac{\partial s(x,y,n,k,\boldsymbol{\xi})}{\partial x}\right|_{(x,y)=(x^{*},y^{*})}\cdot\frac{\partial x^{*}(k,\boldsymbol{\xi})}{\partial k}=\left.\frac{\partial s(x,y,n,k,\boldsymbol{\xi})}{\partial y}\right|_{(x,y)=(x^{*},y^{*})}\cdot\frac{\partial y^{*}(k,\boldsymbol{\xi})}{\partial k}=0.$$

Hence,

$$\begin{aligned}&k^{-(\lambda\alpha)}(1-y^{*}(k,\boldsymbol{\xi}))^{\lambda}\cdot[1-\phi x^{*}(k,\boldsymbol{\xi})]^{\delta}\cdot\\ &\left[\rho\gamma x^{*}(k,\boldsymbol{\xi})+\frac{\theta y^{*}(k,\boldsymbol{\xi})}{k}\right]^{\tau},\end{aligned}\tag{50}$$

$$\begin{aligned}\frac{ds(x^{*}(k,\boldsymbol{\xi}),y^{*}(k,\boldsymbol{\xi}),n,k,\boldsymbol{\xi})}{dk} &= \left.\frac{\partial s(x,y,n,k,\boldsymbol{\xi})}{\partial k}\right|_{(x,y)=(x^{*},y^{*})}\\ &= -\frac{\lambda\alpha[1-y^{*}(k,\boldsymbol{\xi})]^{\lambda}}{k^{\lambda\alpha+1}n^{\lambda\alpha}}\cdot\left(\frac{1-\phi x^{*}(k,\boldsymbol{\xi})}{n^{\beta}}\right)^{\delta}\cdot\\ &\left(\frac{\rho\gamma x^{*}(k,\boldsymbol{\xi})}{n}+\frac{\theta y^{*}(k,\boldsymbol{\xi})}{kn}\right)^{\tau}\\ &-\frac{\tau\theta y^{*}(k,\boldsymbol{\xi})}{k^{2}n}\cdot\left(\frac{1-y^{*}(k,\boldsymbol{\xi})}{k^{\alpha}n^{\alpha}}\right)^{\lambda}\cdot\left(\frac{1-\phi x^{*}(k,\boldsymbol{\xi})}{n^{\beta}}\right)^{\delta}\cdot\\ &\left(\frac{\rho\gamma x^{*}(k,\boldsymbol{\xi})}{n}+\frac{\theta y^{*}(k,\boldsymbol{\xi})}{kn}\right)^{\tau-1}.\end{aligned}\tag{51}$$

Three cases arise: *(i)* $\alpha = 0$ and $y^*(1, \boldsymbol{\xi}) = 0$: since $y^*(k, \boldsymbol{\xi})$ is non-increasing in k, then $\frac{ds(x^*(k,\boldsymbol{\xi}),y^*(k,\boldsymbol{\xi}),n,k,\boldsymbol{\xi})}{dk} = 0$ for all $k \geq 1$; *(ii)* $\alpha = 0$ and $y^*(1, \boldsymbol{\xi}) > 0$: since $y^*(k, \boldsymbol{\xi})$ is non-increasing in k, and since there exists $\tilde{k}$ such $y^*(k, \boldsymbol{\xi}) = 0$ for all $k \geq \tilde{k}$, then $\frac{ds(x^*(k,\boldsymbol{\xi}),y^*(k,\boldsymbol{\xi}),n,k,\boldsymbol{\xi})}{dk} < 0$ for all $k < \tilde{k}$ and $\frac{ds(x^*(k,\boldsymbol{\xi}),y^*(k,\boldsymbol{\xi}),n,k,\boldsymbol{\xi})}{dk} = 0$ for all $k \geq \tilde{k}$; *(iii)* $\alpha > 0$: then $\frac{ds(x^*(k,\boldsymbol{\xi}),y^*(k,\boldsymbol{\xi}),n,k,\boldsymbol{\xi})}{dk} < 0$ for all $k \geq 1$.

8.5 Lemmas 3 and 4

Based on the expression in (48),

$$\begin{aligned}\lim_{k\to+\infty} \tilde{M}(n,k,\boldsymbol{\xi}) &= \lim_{k\to+\infty} k^{1-\lambda\alpha} \cdot \left[g(n^*) \cdot \left[1 - \phi \max\left\{1, \frac{\tau}{\phi(1+\tau)}\right\}\right] \cdot \right. \\ &\qquad \left. \left[\rho\gamma \max\left\{1, \frac{\tau}{\phi(1+\tau)}\right\}\right]^\tau \right],\end{aligned}$$

so that

$$\begin{aligned}\lim_{k\to+\infty} \tilde{M}(n,k,\boldsymbol{\xi}) &= g(n^*) \cdot \left[1 - \phi \max\left\{1, \frac{\tau}{\phi(1+\tau)}\right\}\right] \cdot \\ &\qquad \left[\rho\gamma \max\left\{1, \frac{\tau}{\phi(1+\tau)}\right\}\right]^\tau\end{aligned}$$

for $\lambda\alpha = 1$, and

$$\lim_{k\to+\infty} \tilde{M}(n,k,\boldsymbol{\xi}) = +\infty$$

for all $\alpha \in [0, 1)$.

Turning now to determining the preferred polygyny rate of the male, if any, note first that since $\tilde{M}(n,k,\boldsymbol{\xi}) = k \cdot \tilde{F}(n,k,\boldsymbol{\xi})$,

$$\frac{d\tilde{M}(n,k,\boldsymbol{\xi})}{dk} = \tilde{F}(n,k,\boldsymbol{\xi}) + k \cdot \frac{d\tilde{F}(n,k,\boldsymbol{\xi})}{dk}.$$

Hence, upon division by $n \cdot a(n) > 0$, $\frac{d\tilde{M}(n,k,\boldsymbol{\xi})}{dk}$ has the same sign as

$$s(x^*(k,\boldsymbol{\xi}), y^*(k,\boldsymbol{\xi}), n, k, \boldsymbol{\xi}) + k \cdot \frac{ds(x^*(k,\boldsymbol{\xi}), y^*(k,\boldsymbol{\xi}), n, k, \boldsymbol{\xi})}{dk},$$

which, using (51), writes:

$$\begin{aligned}
&\left(\frac{1-y^*(k,\boldsymbol{\xi})}{k^\alpha n^\alpha}\right)^\lambda\cdot\left(\frac{1-\phi x^*(k,\boldsymbol{\xi})}{n^\beta}\right)^\delta\cdot\left(\frac{\rho\gamma x^*(k,\boldsymbol{\xi})}{n}+\frac{\theta y^*(k,\boldsymbol{\xi})}{kn}\right)^\tau\\
&-k\cdot\frac{\lambda\alpha\left[1-y^*(k,\boldsymbol{\xi})\right]^\lambda}{k^{\lambda\alpha+1}n^{\lambda\alpha}}\cdot\left(\frac{1-\phi x^*(k,\boldsymbol{\xi})}{n^\beta}\right)^\delta\cdot\left(\frac{\rho\gamma x^*(k,\boldsymbol{\xi})}{n}+\frac{\theta y^*(k,\boldsymbol{\xi})}{kn}\right)^\tau\\
&-k\cdot\frac{\tau\theta y^*(k,\boldsymbol{\xi})}{k^2 n}\cdot\left(\frac{1-y^*(k,\boldsymbol{\xi})}{k^\alpha n^\alpha}\right)^\lambda\cdot\left(\frac{1-\phi x^*(k,\boldsymbol{\xi})}{n^\beta}\right)^\delta\cdot\left(\frac{\rho\gamma x^*(k,\boldsymbol{\xi})}{n}+\frac{\theta y^*(k,\boldsymbol{\xi})}{kn}\right)^{\tau-1}\\
=\;&\frac{1}{n}\cdot\left(\frac{1-y^*(k,\boldsymbol{\xi})}{k^\alpha n^\alpha}\right)^\lambda\cdot\left(\frac{1-\phi x^*(k,\boldsymbol{\xi})}{n^\beta}\right)^\delta\cdot\left(\frac{\rho\gamma x^*(k,\boldsymbol{\xi})}{n}+\frac{\theta y^*(k,\boldsymbol{\xi})}{kn}\right)^{\tau-1}\cdot\\
&\left[(1-\lambda\alpha)\left(\rho\gamma x^*(k,\boldsymbol{\xi})+\frac{\theta y^*(k,\boldsymbol{\xi})}{k}\right)-\frac{\tau\theta y^*(k,\boldsymbol{\xi})}{k}\right].
\end{aligned}$$

Dividing by the strictly positive term $\frac{1}{n}\cdot\left(\frac{1-y^*(k,\boldsymbol{\xi})}{k^\alpha n^\alpha}\right)^\lambda\cdot\left(\frac{1-\phi x^*(k,\boldsymbol{\xi})}{n^\beta}\right)^\delta\cdot\left(\frac{\rho\gamma x^*(k,\boldsymbol{\xi})}{n}+\frac{\theta y^*(k,\boldsymbol{\xi})}{kn}\right)^{\tau-1}$, and rearranging terms, leads to the conclusion that $\frac{d\tilde{M}(n,k,\boldsymbol{\xi})}{dk}$ has the same sign as

$$(1-\lambda\alpha)\,\rho\gamma x^*(k,\boldsymbol{\xi})+(1-\lambda\alpha-\tau)\,\theta\frac{y^*(k,\boldsymbol{\xi})}{k}. \tag{52}$$

Recalling the expressions for $x^*(k,\boldsymbol{\xi})$ and $y^*(k,\boldsymbol{\xi})$, the following observations and ensuing results are immediate:

1. If $\lambda\alpha=1$, then $\frac{d\tilde{M}(n,k,\boldsymbol{\xi})}{dk}<0$ if $y^*(k,\boldsymbol{\xi})>0$, and $\frac{d\tilde{M}(n,k,\boldsymbol{\xi})}{dk}=0$ if $y^*(k,\boldsymbol{\xi})=0$. Since $\frac{\partial y^*(k|\boldsymbol{\xi})}{\partial k}\leq 0$, and since there exists $\tilde{k}>1$ such that $\frac{\partial y^*(k|\boldsymbol{\xi})}{\partial k}<0$ for any $k\in[1,\tilde{k})$, and $\frac{\partial y^*(k|\boldsymbol{\xi})}{\partial k}=0$ for any $k\geq\tilde{k}$, this means that if $\lambda\alpha=1$ there are two cases: *(i)* $y^*(1,\boldsymbol{\xi})=0$, in which case $\frac{d\tilde{M}(n,k,\boldsymbol{\xi})}{dk}=0$ for all $k\geq 1$; *(ii)* $y^*(1,\boldsymbol{\xi})>0$, in which case $\frac{d\tilde{M}(n,k,\boldsymbol{\xi})}{dk}<0$ for any $k\in[1,\tilde{k})$, and $\frac{d\tilde{M}(n,k,\boldsymbol{\xi})}{dk}=0$ for any $k\geq\tilde{k}$.

2. For any λ,α such that $\lambda\alpha\in[0,1)$, there exists $\bar{k}\geq 1$ such that for all $k\geq\bar{k}$, $x^*(k,\boldsymbol{\xi})>0$ while $y^*(k,\boldsymbol{\xi})=0$, which implies that for all $k\geq\bar{k}$, $\frac{d\tilde{M}(n,k,\boldsymbol{\xi})}{dk}>0$. Combined with the remark made above that, for any $\alpha\in[0,1)$, $\lim_{k\to+\infty}\tilde{M}(n,k,\boldsymbol{\xi})=+\infty$, this implies that there then exists no $k\geq 1$ that maximizes $\tilde{M}(n,k,\boldsymbol{\xi})$. Nonetheless, $\tilde{M}(n,k,\boldsymbol{\xi})$ need not be monotonically increasing in k. A sufficient condition for $\tilde{M}(n,k,\boldsymbol{\xi})$ to be non-monotonic in k is that $\left.\frac{d\tilde{M}(n,k,\boldsymbol{\xi})}{dk}\right|_{k=1}<0$, which is true if and only if

$$(1-\lambda\alpha)\left[\rho\gamma x^*(1,\boldsymbol{\xi})+\theta y^*(1,\boldsymbol{\xi})\right]<\tau\theta y^*(1,\boldsymbol{\xi}). \tag{53}$$

   Because $x^*(1,\boldsymbol{\xi})$ and $y^*(1,\boldsymbol{\xi})$ do not depend on α, the left-hand side can be viewed as an affine and strictly decreasing function of α, which takes the value 0 for $\alpha=1/\lambda$

and the value $\rho\gamma x^*(1,\boldsymbol{\xi}) + \theta y^*(1,\boldsymbol{\xi})$ for $\alpha = 0$; and since $\rho\gamma x^*(1,\boldsymbol{\xi}) + \theta y^*(1,\boldsymbol{\xi}) > 0$, (53) is equivalent to

$$\alpha > \frac{\rho\gamma x^*(1,\boldsymbol{\xi}) + \theta(1-\tau)\,y^*(1,\boldsymbol{\xi})}{\lambda\left[\rho\gamma x^*(1,\boldsymbol{\xi}) + \theta y^*(1,\boldsymbol{\xi})\right]} \equiv \hat{\alpha}.$$

In sum, if $\alpha > \hat{\alpha}$, there exists $\hat{k}_1$ and $\hat{k}_2 \geq \hat{k}_1$ such that for any $k \in [1, \hat{k}_1)$, $\tilde{M}(n,1,\boldsymbol{\xi}) > \tilde{M}(n,k,\boldsymbol{\xi})$, while for any $k > \hat{k}_2$, $\tilde{M}(n,k,\boldsymbol{\xi}) > \tilde{M}(n,1,\boldsymbol{\xi})$.

8.6 Proposition 7

For the purpose of this proof some additional notation is introduced. From equation (24) it is clear that, for any given female it is only the sum of the efforts of the other females that matter, and not how these efforts are distributed among these other females. As a result, when it is necessary to single out how s depends on the effort of some female under consideration, say female j, one can write $s(x_j, \boldsymbol{x}_{-j}, y)$, where x_j is the effort of the female at hand, and $\boldsymbol{x}_{-j}$ is the vector of the other females' efforts.

A strategy profile $(x_1^e, x_2^e, ..., x_k^e, y^e)$ is a Nash equilibrium if and only if

$$\begin{cases} x_j^e \in \arg\max_{x_j \in [0,1]} n \cdot a(n) \cdot s\left(x_j, \boldsymbol{x}_{-j}^e, y^e\right) & \forall j = 1, ..., k \\ y^e \in \arg\max_{y \in [0,1]} k \cdot n \cdot a(n) \cdot s(\boldsymbol{x}^e, y), & \end{cases} \tag{54}$$

where (from (24))

$$\begin{aligned} s(x_j, \boldsymbol{x}_{-j}, y) &= \left(\frac{1-y}{(kn)^{\alpha}}\right)^{\lambda} \cdot \left(\frac{1-\phi x_j + \sum_{i\neq j}(1-\phi x_i)}{(kn)^{\beta}}\right)^{\delta} \\ &\quad \cdot \left[\frac{\rho\gamma\left[x_j + \sum_{i\neq j} x_i\right]}{kn} + \frac{\theta y}{kn}\right]^{\tau} \end{aligned}$$

and

$$s(\boldsymbol{x}, y) = \left(\frac{1-y}{(kn)^{\alpha}}\right)^{\lambda} \cdot \left(\frac{\sum_{j=1}^{k}(1-\phi x_j)}{(kn)^{\beta}}\right)^{\delta} \cdot \left(\frac{\rho\gamma\sum_{j=1}^{k} x_j}{kn} + \frac{\theta y}{kn}\right)^{\tau}.$$

Note first that $y = 1$ cannot be part of an equilibrium strategy profile. Suppose, to the contrary, that $(\boldsymbol{x}^e, y^e) = (\boldsymbol{x}^e, 1)$ for some $\boldsymbol{x}^e \in [0,1]^k$. Then $s(\boldsymbol{x}^e, y^s) = 0$, while, for any $y \in (0,1)$, $s(\boldsymbol{x}^e, y) > 0$. Similarly, $(\boldsymbol{x}^e, y^e) = (\mathbf{0}, 0)$, where $\mathbf{0} = (0,0,...,0)$, cannot be an equilibrium strategy profile. Indeed, $s(\mathbf{0}, 0) = 0$, while, for any $y \in (0,1)$, $s(\mathbf{0}, y) > 0$.

Next, it is straightforward to verify that, for each reproductive ecology $(\boldsymbol{\xi}, R)$, and for each $\left(\boldsymbol{x}^e_{-j}, y^e\right) \in [0,1]^{k-1} \times [0,1)$, $s\left(x_j, \boldsymbol{x}^e_{-j}, y^e\right)$ is strictly concave in x_j. This implies that any equilibrium must be female-symmetric. To see this, suppose to the contrary that there exists some equilibrium in which $k' \geq 1$ females choose x' while k'' females choose $x'' > x'$ (and where $k' + k'' = k$). Then, it must be that each female j is indifferent between x' and x''. But strict concavity of s in x_j implies that for any $x''' \in (x', x'')$, a female playing x''' rather than x' achieves a strictly higher payoff.

Finally, note that for each $\boldsymbol{x}^e \in [0,1]^k$, $s\left(\boldsymbol{x}^e, y\right)$ is strictly concave in y.

Because any equilibrium is female-symmetric, any equilibrium srategy profile may be fully described by a two-dimensional vector, denoted (x^e, y^e). Furthermore, since $s\left(x_j, \boldsymbol{x}_{-j}, y\right)$ is strictly concave in x_j for each $\left(\boldsymbol{x}_{-j}, y\right) \in [0,1]^{k-1} \times [0,1)$, and $s\left(\boldsymbol{x}, y\right)$ is strictly concave in y for each $\boldsymbol{x} \in [0,1]^k$ it is sufficient to analyze the first-order partial derivatives $\partial s\left(x_j, \boldsymbol{x}_{-j}, y\right)/\partial x_j$ and $\partial s\left(\boldsymbol{x}, y\right)/\partial y$ to determine the best response functions of the females and the male.

Writing $\mathbf{x}^e_{-j}$ to denote the $(k-1)$-dimensional vector whose components all equal x^e, and $\mathbf{x}^e$ to denote the k-dimensional vector whose components all equal x^e, and noting that the terms $n \cdot a(n)$, $k \cdot n \cdot a(n)$, and $\frac{1}{(kn)^{\lambda\alpha+\delta\beta+\tau}}$ are irrelevant, female-symmetry implies that the system of $k+1$ equations (54) reduces to the following system of two equations:

$$\begin{cases} x^e \in \arg\max_{x_j \in [0,1]} (kn)^{\lambda\alpha+\delta\beta+\tau} \cdot s\left(x_j, \mathbf{x}^e_{-j}, y^e\right) \\ y^e \in \arg\max_{y \in [0,1]} (kn)^{\lambda\alpha+\delta\beta+\tau} \cdot s\left(\mathbf{x}^e, y\right), \end{cases} \tag{55}$$

where

$$\begin{aligned} (kn)^{\lambda\alpha+\delta\beta+\tau} \cdot s\left(x_j, \mathbf{x}^e_{-j}, y^e\right) &= (1-y^e)^\lambda \cdot \left[1 - \phi x_j + (k-1)(1-\phi x^e)\right]^\delta \\ &\quad \cdot \left[\rho\gamma\left[x_j + (k-1)x^e\right] + \theta y^e\right]^\tau \end{aligned} \tag{56}$$

and

$$(kn)^{\lambda\alpha+\delta\beta+\tau} \cdot s\left(\mathbf{x}^e, y\right) = (1-y)^\lambda \cdot \left[k(1-\phi x^e)\right]^\delta \cdot (\rho\gamma k x^e + \theta y)^\tau. \tag{57}$$

Next, disregarding the strictly positive term $(kn)^{\lambda\alpha+\delta\beta+\tau}$, the sign of $\partial s\left(x_j, \mathbf{x}^e_{-j}, y^e\right)/\partial x_j$ is studied in order to determine a female's best response to $\left(\mathbf{x}^e_{-j}, y^e\right)$. It is straightforward to verify that upon division by $(1-y^e)^\lambda > 0$ and simplification, the sign of $\left.\frac{\partial s\left(x_j, \mathbf{x}^e_{-j}, y^e\right)}{\partial x_j}\right|_{x_j = x^e}$ is the same as the sign of

$$\begin{aligned} & k^{\tau+\delta-1} \cdot \tau\rho\gamma \cdot (1-\phi x^e)^\delta \cdot \left(\rho\gamma x^e + \frac{\theta y^e}{k}\right)^{\tau-1} \\ & -k^{\tau+\delta-1} \cdot \phi\delta \cdot (1-\phi x^e)^{\delta-1} \cdot \left(\rho\gamma x^e + \frac{\theta y^e}{k}\right)^\tau, \end{aligned} \tag{58}$$

which, since $k^{\tau+\delta-1} > 0$, is proportional to and has the same sign as the expression in (37), which was used to determine the equilibrium time allocation for a female under private property. Hence, $x^e = x^*$. It is straightforward to see that it follows directly that $y^e = y^*$ as well.

8.7 Proposition 8

To begin, note that the reproductive success achieved by a female living in a household with a total of k females under household property, and with protection and care publicness parameters α and β, respectively,

$$\begin{aligned} \tilde{F}^H(n,k,\boldsymbol{\xi}) &= k^{\delta-\delta\beta} \cdot \tilde{F}(n,k,\boldsymbol{\xi}) = k^{\delta-\delta\beta-\lambda\alpha} \cdot g(n) \cdot [1-y^*(k,\boldsymbol{\xi})]^{\lambda} \cdot [1-\phi x^*(k,\boldsymbol{\xi})]^{\delta} \cdot \\ & \left[\rho\gamma x^*(k,\boldsymbol{\xi}) + \frac{\theta y^*(k,\boldsymbol{\xi})}{k}\right]^{\tau}, \end{aligned} \tag{59}$$

is the same as that achieved by a male living in a household with a total of k females, private property, and in an ecology where $1-\lambda\alpha' = \delta-\delta\beta-\lambda\alpha$, as can be seen by replacing $1-\lambda\alpha'$ by $\delta-\delta\beta-\lambda\alpha$ in

$$\begin{aligned} \tilde{M}(n,k,\boldsymbol{\xi}) &= g(n) \cdot k \cdot \left(\frac{1-y^*(k,\boldsymbol{\xi})}{k^{\alpha'}}\right)^{\lambda} \cdot [1-\phi x^*(k,\boldsymbol{\xi})]^{\delta} \cdot \\ & \left[\rho\gamma x^*(k,\boldsymbol{\xi}) + \frac{\theta y^*(k,\boldsymbol{\xi})}{k}\right]^{\tau}. \end{aligned} \tag{60}$$

The logic followed to determine male preferences over the polygyny rates under private property (see the proof of Lemmas 3, and 4) can therefore be applied here to determine female preferences. Upon replacing $1-\lambda\alpha$ by $\delta-\delta\beta-\lambda\alpha$ in (52), the following Lemma is therefore immediate:

Lemma 5 *For each ecology $\boldsymbol{\xi}$ and each fertility rate n, $\tilde{F}^H(n,1,\boldsymbol{\xi}) \geq \tilde{F}^H(n,k,\boldsymbol{\xi})$ for all $k \geq 1$ if and only if $\alpha \geq \delta(1-\beta)/\lambda$, with a strict inequality for all $k > 1$ if, moreover, $y^*(1,\boldsymbol{\xi}) > 0$. Furthermore, if $\alpha \in (\tilde{\alpha}, \delta(1-\beta)/\lambda)$ there exists $\hat{k}_1 > 1$ and $\hat{k}_2 \geq \hat{k}_1$ such that $\tilde{F}^H(n,1,\boldsymbol{\xi}) > \tilde{F}^H(n,k,\boldsymbol{\xi})$ for any $k \in [1,\hat{k}_1)$, while $\tilde{F}^H(n,k,\boldsymbol{\xi}) > \tilde{F}^H(n,1,\boldsymbol{\xi})$ for any $k > \hat{k}_2$. Finally, if $\alpha \in [0,\tilde{\alpha}]$, then $\tilde{F}^H(n,k,\boldsymbol{\xi}) \geq \tilde{F}^H(n,1,\boldsymbol{\xi})$ for all $k \geq 1$, and there exists $\hat{k} \geq 1$ such that $\tilde{F}^H(n,k,\boldsymbol{\xi}) > \tilde{F}^H(n,1,\boldsymbol{\xi})$ for all $k > \hat{k}$.*

This lemma, together with the fact that preferred fertility does not depend on k, implies the statement in the proposition.

8.8 Proposition 9

To begin, note that the reproductive success achieved by a male living in a household with a total of k females under household property, and with protection and care publicness parameters α and β, respectively,

$$\begin{aligned} \tilde{M}^{H}(n,k,\boldsymbol{\xi}) &= k^{\delta-\delta\beta}\cdot\tilde{M}(n,k,\boldsymbol{\xi}) = k^{1+\delta-\delta\beta-\lambda\alpha}\cdot g(n)\cdot\left[1-y^{*}(k,\boldsymbol{\xi})\right]^{\lambda}\cdot\left[1-\phi x^{*}(k,\boldsymbol{\xi})\right]^{\delta}\cdot \\ &\quad \left[\rho\gamma x^{*}(k,\boldsymbol{\xi})+\frac{\theta y^{*}(k,\boldsymbol{\xi})}{k}\right]^{\tau}, \end{aligned} \tag{61}$$

is the same as that achieved by a male living in a household with a total of k females, private property, and in an ecology where $\lambda\alpha'$ would take the value $\lambda\alpha+\delta\beta-\delta$, as can be seen by replacing $\lambda\alpha'$ by $\lambda\alpha+\delta\beta-\delta$ in

$$\begin{aligned} \tilde{M}(n,k,\boldsymbol{\xi}) &= g(n)\cdot k\cdot\left(\frac{1-y^{*}(k,\boldsymbol{\xi})}{k^{\alpha'}}\right)^{\lambda}\cdot\left[1-\phi x^{*}(k,\boldsymbol{\xi})\right]^{\delta}\cdot \\ &\quad \left[\rho\gamma x^{*}(k,\boldsymbol{\xi})+\frac{\theta y^{*}(k,\boldsymbol{\xi})}{k}\right]^{\tau}. \end{aligned} \tag{62}$$

The logic followed to determine male preferences over the polygyny rates under private property (see the proof of Lemmas 3, and 4) can therefore be applied here to determine female preferences. Upon replacing $1-\lambda\alpha$ by $1+\delta-\delta\beta-\lambda\alpha$ in (52), the following Lemma is therefore immediate:

Lemma 6 *For each ecology $\boldsymbol{\xi}$ and each fertility rate n, $\tilde{M}^{H}(n,1,\boldsymbol{\xi})\geq\tilde{M}^{H}(n,k,\boldsymbol{\xi})$ for all $k\geq 1$ if and only if $\alpha=(1+\delta-\delta\beta)/\lambda$ (i.e., iff $\lambda\alpha=\beta=1$) and the inequality holds strictly for all $k>1$ if $y^{*}(1,\boldsymbol{\xi})>0$. Furthermore, if $\alpha\in(\bar{\alpha},(1+\delta-\delta\beta)/\lambda)$ there exists $\tilde{k}_1>1$ and $\tilde{k}_2\geq\tilde{k}_1$ such that $\tilde{M}^{H}(n,1,\boldsymbol{\xi})>\tilde{M}^{H}(n,k,\boldsymbol{\xi})$ for any $k\in[1,\tilde{k}_1)$, while $\tilde{M}^{H}(n,k,\boldsymbol{\xi})>\tilde{M}^{H}(n,1,\boldsymbol{\xi})$ for any $k>\tilde{k}_2$. Finally, if $\alpha\in[0,\bar{\alpha}]$, then $\tilde{M}^{H}(n,k,\boldsymbol{\xi})\geq\tilde{M}^{H}(n,1,\boldsymbol{\xi})$ for all $k\geq 1$, and there exists $\tilde{k}\geq 1$ such that $\tilde{M}^{H}(n,k,\boldsymbol{\xi})>\tilde{M}^{H}(n,1,\boldsymbol{\xi})$ for all $k>\tilde{k}$.*

This lemma, together with the fact that preferred fertility does not depend on k, implies the statement in the proposition.

8.9 Proposition 10

Under private property and under household property, a strategy profile $(\hat{x}_1,\hat{x}_2,...,\hat{x}_k,\hat{y})\equiv(\hat{\boldsymbol{x}},\hat{y})$ is efficient if and only if

$$(\hat{x}_1,\hat{x}_2,...,\hat{x}_k,\hat{y})\in\arg\max_{(x_1,x_2,...,x_k,y)\in[0,1]^{k+1}} n\cdot a(n)\cdot s(\boldsymbol{x},y), \tag{63}$$

where

$$s\left(\boldsymbol{x},y\right)=\left(\frac{1-y}{\left(kn\right)^{\alpha}}\right)^{\lambda}\cdot\left(\frac{\sum_{j=1}^{k}\left(1-\phi x_{j}\right)}{\left(kn\right)^{\beta}}\right)^{\delta}\cdot\left(\frac{\rho\gamma\sum_{j=1}^{k}x_{j}}{kn}+\frac{\theta y}{kn}\right)^{\tau}.$$

To begin, note that $\hat{y}\neq 1$, since $s\left(\boldsymbol{x},1\right)=0$ for any $\boldsymbol{x}\in\left[0,1\right]^{k}$, while for any $y\in\left(0,1\right)$ and any $\boldsymbol{x}\in\left[0,1\right]^{k}$, $s\left(\boldsymbol{x},1\right)>0$. Similarly, $\left(\hat{\boldsymbol{x}},\hat{y}\right)\neq\left(\mathbf{0},0\right)$, where $\mathbf{0}=\left(0,0,...,0\right)$, since $s\left(\mathbf{0},0\right)=0$, while, for any $y\in\left(0,1\right)$, $s\left(\mathbf{0},y\right)>0$.

Next, it is clear that while the sum $\sum_{j=1}^{k}x_{j}$ matters, the individual x_{j}'s don't. Hence, without loss of generality, let $x_{j}=x$ for all j, so that

$$s\left(\boldsymbol{x},y\right)=s\left(x,y\right)=k^{\delta-\delta\beta}\left(\frac{1-y}{\left(kn\right)^{\alpha}}\right)^{\lambda}\cdot\left(\frac{\left(1-\phi x\right)}{n^{\beta}}\right)^{\delta}\cdot\left(\frac{\rho\gamma x}{n}+\frac{\theta y}{kn}\right)^{\tau}.$$

This is proportional to the expression for s under private property (see (30)). From the proof of Proposition 1 we know that there exists a unique $\left(x^{*},y^{*}\right)$ such that

$$\begin{cases} x^{*}\in\arg\max_{x\in[0,1]}s\left(x,y^{*}\right)\\ y^{*}\in\arg\max_{y\in[0,1]}s\left(x^{*},y\right).\end{cases}\tag{64}$$

Hence, $\left(x^{*},y^{*}\right)$ is efficient.

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