# The Economics of Collective Brands

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April 7, 2014

**Abstract**

We analyze the effect of a shared brand name, such as geographical names, on incentives of otherwise autonomous firms to establish a reputation for product quality. On the one hand, brand membership provides consumers with more information about past quality and therefore can motivate reputation building when the scale of production is too small to motivate reputation formation by stand alone firms. On the other hand, sharing a brand name may motivate free riding on the group's reputation, reducing investment in quality. We identify conditions under which collective branding delivers higher quality than is achievable by stand alone firms.

# 1 Introduction

There are many instances in which otherwise autonomous firms, which make independent business decisions and retain their own profits, market their products under a shared brand name. Often, the shared brand name is perceived as a badge of superior quality by consumers, who are willing to pay premium prices for them (e.g. Landon and Smith, 1998, and Loureiro and McCluskey, 2000, 2003). Examples include regional agricultural brands protected by designation of origin (PDO) and geographical indication (PGI) status in the EU such as champagne bubbly wine, Parma ham and cheese, Roquefort Cheese. In countries where such Protected Geographical Status laws are enforced, only products genuinely originating in that region are allowed to be identified as such in commerce. Similarly, the Jaffa label is shared by many independent Israeli orange growers and exporters.

Another important example is franchising which in 2007 accounted for 9.2 percent of total U.S. GDP (Kosova and Lafontaine, 2012) and which spans the range from fast food restaurants to accounting and law firms. In a typical business-format franchising arrangement, franchisees sell under the common franchise logo, but are otherwise independent businesses which retain their own profits after paying the chain the corresponding fees (typically based on the outlet's sales).

Some premium food products, though sold by individual producers, share a common logo. For example, many of Germany's top wine producers are members of the VDP wine association and carry the VDP logotype. VDP members must adhere to more stringent standards than those set down in the German wine law. Similarly, otherwise independent members of many prestigious professional organizations share a common logo (e.g., the German BFF association for professional photographers in which membership is determined through a jury selection process).

The fact that collective brand labels are associated with superior quality suggests that firms which are members of these brands invest more to maintain brand quality (or at least are perceived to do so by consumers) and earn higher profits than they would as stand alone firms. This seems surprising. If consumers' perception of the collective brand label's quality is jointly determined by their experience with the qualities provided by different individual members, and if the provision of high quality requires costly investment, it would seem that each member has an incentive to free ride on the investments of fellow members. If so, why are these brand labels perceived as badges of quality?

It is true that in some cases, the perception of superior quality may be partly attributable to exogenous advantages such as climate, soil quality, access to superior inputs, technology and so on. However, even when such natural advantages are present the achievement of superior quality presumably also requires the requisite investment of effort and other resources. The free riding problem might also be mitigated to some extent by monitoring the efforts and investments of individual members to maintain quality standards. However, monitoring is costly and imperfect and is therefore unlikely to eliminate free riding altogether. Thus it would seem that producers have less of an incentive to invest in quality as members of a collective brand than they would as stand alone firms.

The purpose of this paper is to show how collective branding may lead to higher quality in the market and increase welfare by incentivizing brand members to invest in quality, when they would not do so as stand alone firms.

The idea is the following. When product quality is difficult to observe before purchase and is revealed to consumers only after consuming the product ('experience goods'), their perception of quality and the amount they are willing to pay for the product is based on past experience with the product - its reputation. Thus the extent to which a firm is able to receive a good return on its investment in quality depends on how much information consumers have about its past performance. If firms are small, relative to the size of the market, consumers may not have much information about the past quality of any individual firm. In that case, an individual firm may be unable to effectively establish a robust reputation for quality on its own and consequently has little incentive to invest in quality. Here collective branding may come to the rescue and serve as a vehicle for reputation formation by increasing the relevant information available to consumers. Specifically, suppose small individual firms market their products under a collective brand name, sharing a collective reputation, while otherwise retaining full autonomy. Since the collective brand name covers a larger share of the market than any individual member firm, consumers are better able to assess the reputation of the brand than of individual members. This in turn increases the value of a good brand reputation for each member, and may thus incentivize members to invest in quality when they would otherwise not do so. This is the 'reputation effect' of collective branding.

But as noted above, branding may also have an opposing effect on investment incentives. Unless the brand is able to effectively monitor individual investment, sharing a collective reputation may encourage individual members to free ride on the efforts of other members. Therefore the full effect of collective branding on investment in quality is determined by the interaction of these two opposing factors - the fact that, on the one hand, a good collective reputation is more valuable than a stand alone reputation, against the incentive to free ride, on the other.

Accordingly, we analyze the effects of collective branding in two polar cases. In the first, called 'perfect monitoring', free riding on the brand's reputation is deterred because members which fail to invest are costlessly detected and excluded from using the brand name. Since then only the reputation effect is operative, a brand member's incentive to invest is always greater than that of a stand alone firm. Moreover, the incentive to invest increases with brand size (the number of firms which are members of the brand) - the larger the brand, the greater the incentive of each member to invest and therefore the more profitable membership is. Thus in this case "bigger is better". We show that this feature also applies if brand membership requires costly authentication of investment.

We find that, for appropriate parameters this pro - investment effect of collective branding also applies to the case of 'no-monitoring', in which failure to invest cannot result in exclusion from the brand. Specifically, collective branding can still facilitate investment if investment is a sufficiently important ingredient for the attainment of high quality - that is, if the difference between the expected product quality of a firm which invests in quality and one which doesn't is sufficiently large. However, in contrast to the case of perfect monitoring, here "bigger is better" only up to a point. Once the brand is sufficiently large, the marginal contribution of an individual member's investment to the brand's reputation becomes too small to override free riding, reducing the brand's incentive to invest relative to stand alone firms. Thus, in this case the brand size which maximizes firms' profits is large enough to enable successful reputation building but small enough to discourage individual free riding. Thus one might speculate that a regional brand like Champagne wine owes its success not only to unique soil and climatic conditions but also to fortuitous natural boundaries which encompass "just the right" number of producers under its brand label.

## 1.1 Empirical Evidence

Casual observation suggests that collective branding is often observed in situations where consumers are unlikely to have much information about individual producers. Thus, for example, the export of agricultural products is often managed by marketing boards and state trading enterprises rather than by the individual producers as foreign consumers are unlikely to recognize individual producers. Similarly, restaurants on highway stops, where there is little repeat business, almost always belong to well known chains. In the franchising context, Jin and Leslie (2009) provide evidence that chain restaurants - which share a collective brand name - maintain better hygiene than non-chain restaurants.

In an econometric study of the determinants of reputation in the Italian wine industry, Castriota and Delmastro (2008) show that brand reputation is increasing in the number of bottles produced by the brand and decreasing in the number of individual producers in the brand. This is consistent with our analysis. Keeping output fixed, an increase in the number of individual producers has no reputation effect since the number of units whose quality consumers observe is unchanged. However, it does increase the incentive for free riding (which increases with the number of members), and hence lowers investment incentives and reduces the brand's reputation.

In an experimental study, Huck and Lűncer (2009) find that more sellers invest in quality when buyers are informed about the average past quality of all sellers - which corresponds to a collective brand in our model - than when they only know the record of the seller from whom they actually buy. However, consistent with our analysis, when the number of sellers increases, the average quality declines.

Online hiring markets also provide evidence for reputational effects of collective branding. Stanton and Thomas (2010) find that employers are willing to pay more to inexperienced online workers (which have yet to establish individual reputation) affiliated with outsourcing agencies than to inexperienced independent contractors and that this advantage dissipates over time as employers learn about individual productivity.

## 1.2 Relationship to the Literature

The centrality of *individual* firms' reputation for quality for their success is the theme of a very large literature (see the survey article of Bar Issac and Tadelis (2008)). By contrast our concern is to understand how autonomous firms can form a collective reputation. Tirole (1996) analyzes how group behavior affects individual incentives to invest (behave honestly) when the group size is fixed exogenously. By contrast, our focus is precisely on the role of the group size on individual investment incentives.

Our analysis is closely related to a substantial literature on brand extension or umbrella branding, which refers to the practice of multiproduct firms to use the same brand name on otherwise unrelated products in order to signal quality of experience goods to consumers.[1] A seminal paper by Wernerfelt (1988) considers a model of adverse selection in which a firm sells an old and new product of exogenous quality and establishes conditions under which an umbrella brand sells only high quality products. Choi (1998)

[1] Relatedly Rob and Fishman (2005) show that a firm's investment in quality increases with size and Yacouel (2005) and Guttman and Yacouel (2006) show that larger firms benefit more from a good reputation.

considers an infinite horizon signalling model with adverse selection in which a firm discovers a new product of given quality in every period, and shows that it is less costly (in terms of price distortion) to signal high quality to consumers if old products' brand name is extended to new products than if products are separately branded. Cabral (2000) analyzes the role of brand extension in a setting with ongoing learning about exogenous quality when qualities of old and new products are correlated and shows that higher quality sellers have stronger incentives to extend their brands. Miklos-Thal (2012) shows that if the quality of an existing product is determined by past decisions, then brand extension strengthens the incentives to invest in the quality of a new product only if the existing product is high quality.

More directly related papers in this literature focus on the role of brand extension to incentivize investment in all of the firm's products. Andersson (2002), and Cabral (2009) consider a repeated game with moral hazard in which a multi product firm must repeatedly invest in the quality of each of its products and show that brand extension can support high quality equilibria which are not feasible if products are sold as separate brands[2]. Cai and Obara (2009) and Hakenes and Peitz (2008) consider moral hazard settings when the firm makes a once and for all investment in the quality of both of its products. In a related vein, Dana and Spier (2009) consider a repeated game with imperfect observability of quality in which bundling different products together can incentivize investment because consumers who buy more products from a firm collect more information about its quality and can better monitor its behavior. Hakenes and Peitz (2009) explore whether umbrella branding can partially or fully substitute for external certification of quality and Rasmusen (2011) shows how a firm with a monopoly on one product may use umbrella branding to capture the market for a competitive product[3].

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[2] Andersson (2002) assumes asymmetric products, while Cabral (2009) assumes symmetric products but imperfect observability.

[3] Johnson (2013) and Choi and Jeon (2007) also consider contexts in which products of different firms share reputations but the issue of brand size effects and free riding do not arise. Johnson analyzes the relationship between upstream and downstream firms in the presence of asymmetric information about final product quality and, specifically, considers whether quality is better assured if consumers look to

Both collective branding and umbrella branding provide firms with greater incentives to invest in quality than if products are branded separately. The main difference is that in an umbrella brand a central authority makes investment decisions for each of the brand's products and internalizes the effect of each individual product's quality on the reputation of the entire brand. By contrast, in a collective brand, individual members are concerned only with the effect of their investment decisions on the value of their own product. Therefore, umbrella branding incentivizes investment more than collective branding, but the latter can nevertheless support higher quality than stand alone firms.

Another context which address related issues is the literature on reputation in teams (e.g., Che and Yoo, 2001) in which the payoff of each team member depends on both her own efforts as well as those of other team members. Our analysis can also contribute to understanding the role of cooperatives. While the conventional approach (e.g., Sexton and Sexton, 1987) views cooperatives as a means of joint integration allowing for the exploitation of scale economies, market power and risk pooling, our analysis suggests an additional important function of cooperatives— joint signaling of information. Our approach is also related to the common trait literature (e.g., Benabou and Gertner, 1993, Fishman 1996), in which an agent's behavior reveals information about a common trait that she shares with other agents in the group.

# 2 The Model: Stand Alone Firms

We consider a market for an experience good - consumers observe quality only after buying, but not at the time of purchase. There are two periods, $N$ risk neutral firms and we normalize the number of consumers per firm to be 1. There are two possible product qualities, low ($l$) and high ($h$). Firms are of two types, $H$ and $L$, which are distinguished by their technological ability to produce high quality. An $L$ firm produces high quality with probability $b$ at each period whether or not it invests. An $H$ firm produces high

the upstream manufacturer or the downstream retailer for final product quality assurance. Choi and Jeon consider when a new firm (with no reputation) can facilitate its ability to signal product quality by employing components produced by a firm with an established reputation (an instance of co - branding).

quality with probability $b$ if it does not invest but if it invests, it produces high quality with probability $g$ at each period, where $0 < b < g \leq 1$. In either case the realized quality at period 2 is independent of its realization at period 1. The cost of investment is fixed at $e$ and investment is "once and for all": Prior to period 1, each firm decides whether or not to invest and that investment determines the probability with which it produces high quality at periods 1 and 2. We denote by $N_H$ and $N_L$ the total number of $H$ and $L$ firms respectively, $N_L \geq N_H$, and by $r = \frac{N_H}{N_H+N_L}$ the proportion of $H$ firms in the market.

Each consumer is in the market for one period, demands at most one (discrete) unit, and exits the market at the end of the period. Her utility from a low quality unit is zero, from one high quality unit is 1 and her utility from any additional unit is zero. A consumer buys if her expected utility from a unit is greater or equal to the price she pays. We assume that $g - b > e$, so that investment is efficient.

In order to focus on the reputational effects of collective branding on investment incentives in the most direct way, it is convenient to assume that firms have monopolistic market power and can make take it or leave it offers to consumers. Specifically, if consumers' expected utility from a unit of firm $i$ is $v_i$, firm $i'$s price is assumed to be $v_i$. Thus branding has no effect on firm pricing power or market share, and can only affect firm investment incentives via reputational considerations.[4]

Firms can distinguish each others' type. In contrast, consumers face both adverse selection and moral hazard; they cannot directly observe a firm's type ($H$ or $L$) and also do not observe if it has invested. Let $s_i \in \{l, h\}$ denote the realized quality of firm $i$ at period 1 and let $S = (s_1, s_2, ..., s_N)$ be the industry profile of realized qualities at period 1. We assume that at the beginning of period 2, consumers are perfectly informed (e.g.,

[4] This could be because consumers have high transportation costs which effectively endows firms with local monopoly pricing power. Alternatively, consider a standard search model: A consumers knows only the price distribution but not which firm charges what price, is randomly and costlessly matched with one firm and can either buy from that firm or sequentially search for other firms, incurring a positive search cost at each search. As is well known, these assumptions imply that firms have monopoly pricing power (Diamond, 1971).

by interacting with consumers of the previous generation) about $S$ and update their beliefs about firms.

A firm's profit is the sum of its revenues at periods 1 and 2 less the investment cost, $e$, if it invests. Let $f$ denote a firm's strategy:[5]

$$f : \{H, L\} \longrightarrow \{I, NI\}$$

where $I$ and $NI$ denote investing and not investing, respectively.

A consumer's belief about firm $i$ is the probability with which she believes that the firm is an $H$ firm which has invested.[6] Consumers form beliefs about *each* firm at each period, where at period 2, beliefs possibly depend on $S$. [7] Let $B_1$ denote consumers' belief at period 1, $B_1 \in [0, 1]^N$, and $B_2(S)$ be consumers' belief at period 2, where

$$B_2 : S \longrightarrow [0, 1]^N.$$

An *equilibrium* is a strategy $f$ for each firm and consumer beliefs $B_1$ and $B_2(S)$ such that:

- Each firm's strategy $f$ maximizes its profit, given the strategies of all other firms and consumer beliefs.
- $B_1$ and $B_2(S)$ are consistent with firms' strategies.
- Consumers maximize their expected utility (i.e., they buy if and only if the price is less or equal to the expected value of the good according to their beliefs).

Obviously $L$ firms don't invest in any equilibrium since consumers don't observe investment and investment has no effect on their quality. Trivially, there always exists an

[5] We do not formally include a firm's price as part of its strategy since we assume that its price always equals consumers' expected utility.

[6] As far as a consumer is concerned, an $H$ firm which has not invested is equivalent to an $L$ firm since both produce high quality with the same probability.

[7] We implicitly assume that, since all consumers have the same information, they also form the same beliefs about each firm.

equilibrium in which no firm invests.[8] The more interesting possibility is the existence of an 'investment equilibrium' (*IE*) in which $H$ firms invest.

Suppose there is an equilibrium in which all $H$ firms invest. Since at period 1 firms have no history and since $H$ firms invest, consumers believe that any firm is an $H$ firm which invests with probability $r$. Therefore at period 1 the expected utility from any firm - and hence its price - is $rg + (1-r)b$.

At period 2, consumers are informed about $S$ and update their beliefs. Let $\Pr(H \mid s_i, S_{-i})$ be the posterior probability - and hence consumers' belief[9] - at period 2 that a randomly selected firm $i$ is type $H$ when its realized quality at period 1 is $s_i$ and those of the other firms is $S_{-i} \equiv (S \backslash s_i)$. Then the actual price of firm $i$ at period 2 is $g \Pr(H \mid s_i, S_{-i}) + b(1 - \Pr(H \mid s_i, S_{-i}))$. However, since $S$ is of course unknown at the time of investment, what is relevant for firms' investment strategy is the *expected* price, as evaluated at the time of investment. This is calculated as follows. Let $E_{S_{-i}} \Pr(H \mid s_i, S_{-i})$ be the *expected* (with respect to $S_{-i}$) consumer belief at period 2 - *as evaluated by firm i at the time of investment* - that firm $i$ is type $H$, given that its realized quality will be $s_i$. Thus:

$$\begin{aligned} E_{S_{-i}} \Pr(H \quad \mid \quad s_i, S_{-i}) &= \sum_{S_{-i}} \Pr(H \mid s_i, S_{-i}) \Pr(S_{-i} \mid s_i) = \sum_{S_{-i}} \frac{\Pr(H, s_i, S_{-i})}{\Pr(s_i, S_{-i})} \frac{\Pr(s_i, S_{-i})}{\Pr(s_i)} \\ &= \sum_{S_{-i}} \frac{\Pr(H, s_i, S_{-i})}{\Pr(s_i)} = \frac{\Pr(H, s_i)}{\Pr(s_i)} = \Pr(H \mid s_i). \end{aligned} \tag{1}$$

That is, while consumers' actual belief at period 2 will depend on the realization of $S_{-i}$, their *expected* belief at the time of investment does not.

Thus if $p(s_i)$ is a firm's expected - as evaluated at the time of investment - second

[8] In this equilibrium consumers believe that no firm invests, which makes it optimal for firms not to invest.

[9] For any realization of $s_i, S_{-i}$ consistent with firms' strategy, consumers' equilibrium beliefs must be consistent with Bayesian updating.

period price, conditional on its realized quality being $s_i$,

$$\begin{aligned} p(s_i) &= gE_{S_{-1}} \Pr(H \mid s_i, S_{-i}) + b(1 - E_{S_{-1}} \Pr(H \mid s_i, S_{-i})) \\ &= g \Pr(H \mid s_i) + b(1 - \Pr(H \mid s_i)) \end{aligned}$$

Since an $H$ firm which invests produces high quality with probability $g$ and an $L$ firm produces high quality with probability $b$, Bayes' rule gives (henceforth we omit subscript $i$):

$$\Pr(H \mid h) = \frac{gr}{gr + b(1-r)}$$

$$\Pr(H \mid l) = \frac{(1-g)r}{(1-g)r + (1-b)(1-r)}$$

and thus:

$$\begin{aligned} p(h) &= g \Pr(H \mid h) + b(1 - \Pr(H \mid h)) \\ &= b + (g-b) \Pr(H \mid h) = b + \frac{(g-b)gr}{gr + b(1-r)} \end{aligned} \tag{2}$$

and similarly:

$$\begin{aligned} p(l) &= g \Pr(H \mid l) + b(1 - \Pr(H \mid l)) \\ &= b + (g-b) \Pr(H \mid l) = b + \frac{(g-b)(1-g)r}{(1-g)r + (1-b)(1-r)}. \end{aligned} \tag{3}$$

Denoting by $R$ and $R_{-1}$ the expected second period revenues of an $H$ firm that invests and doesn't invest, respectively, we have:

$$R = gp(h) + (1-g)p(l) \tag{4}$$

and

$$R_{-} = bp(h) + (1-b)p(l) \tag{5}$$

Thus an $H$ firm's expected gain from investment is $e^* \equiv R - R_{-}$ and thus by (2) - (5):

$$e^* = (g-b)^2 \left[ \frac{gr}{gr + b(1-r)} - \frac{(1-g)r}{(1-g)r + (1-b)(1-r)} \right].$$

Thus:

**Proposition 1** *When firms stand alone an IE exists if and only if* $e \leq e^*$.

In the 'stand alone' setting, firms have only a limited opportunity to establish a reputation for quality, since consumers' information is limited to one observation per firm. Hence if $e > e^*$, an *IE* does not exist because the cost of investment exceeds the individual firm's expected return from acquiring a good reputation.

# 3 Collective Branding

In this section we show that, in contrast to the stand alone setting, investment equilibria may exist even when $e > e^*$ if otherwise autonomous firms market their products under a shared brand name. The idea is that when the products of two or more firms share a common brand name, consumers may condition their beliefs about a specific firms' type based on the past performance of all the brand's members rather than on the firm's individual performance. Thus, branding may provide consumers with better information which may in turn increase the incentive of $H$ firms to invest.

In order to facilitate the comparison of collective brands with stand alone firms, it is convenient to assume that consumers become aware of firms' brand affiliation only after the first period, so that first period revenue is the same in both settings. Any effect of branding on investment incentives can now only be due to its effect on second period revenues.

Let $\wp$ be the set of all the possible partitions of the $N$ firms and let $P \in \wp$. $P$ is determined exogenously. Each element $Q \in P$ is called a collective brand and each firm $i \in Q$ assigned to $Q$ by $P$ is called a member of brand $Q$. Let $\pi_i(Q)$ denote firm $i's$ profit as a member of brand $Q$ and let $\pi_i$ be its profit if it stands alone.

In this setting firms' strategies and consumers' beliefs at period 2 may depend not only on $S$ but also on $P$. That is,

$$f : \wp \times \{H, L\} \longrightarrow \{I, NI\}$$

$$B_2 : \wp \times S \longrightarrow [0,1]^N$$

We then define a $BE$ (Brand Equilibrium) by $P \in \wp, f, B_1$ and $B_2$ such that:

- Each firm's strategy $f$ maximizes its profit, given the strategies of all other firms and consumer beliefs.
- $B_1$ and $B_2(\wp, S)$ are consistent with firms' strategies.
- (individual rationality) For each $Q \in P$ and $i \in Q$, $\pi_i(Q) \geq \pi_i$. That is, if a firm is assigned to brand $Q$ by $P$, membership in $Q$ must be at least as profitable as standing alone.
- $\nexists i, Q \in P$ s.t. $: \forall j \in Q,\ i \notin Q,\ i \in Q' \in P,\ \pi_j(Q \cup \{i\}) \geq \pi_j(Q), \pi_i(Q \cup \{i\}) \geq \pi_i(Q')$, with the inequality strict for at least one $j$ or $i$. That is, adding an additional member to brand $Q \in P$ can not increase both its profit and the profit of existing (assigned) members of $Q$.

For any $m \in \{1, ..., N_H\}$, let $n_H^m$ and $n_L^m$ be the largest integer $\leq \frac{N_H}{m}$, and the largest integer $\leq \frac{N_L}{m}$, respectively. Define an $m$ *partition* as a partition consisting of $n_H^m$ brands, each of which has exactly $m$ type $H$ members - henceforth called $H$ brands - and $n_L^m$ brands each of which has exactly $m$ type $L$ members - henceforth called $L$ brands - and $N - m(n_H^m + n_L^m)$ stand alone firms.

We shall refer to the number of firms which are members of a brand as the *brand size* and define a $BIE$ as a $BE$ in which each member of each $H$ brand invests. Let $q = \frac{n_H^m}{n_H^m + n_L^m}$ be the proportion of $m$ size brands which are $H$.

We analyze branding equilibria under two alternative regimes. Under *perfect monitoring*, membership in an $H$ brand constitutes a binding commitment to invest, such that a firm which is assigned by $P$ to an $H$ brand and does not invest is precluded from using the brand name.[10] The interpretation is that the brand can costlessly detect a member which doesn't invest and exclude it from the brand. By contrast, in the *no-monitoring* regime, membership in an $H$ brand cannot be conditioned on investment. The interpretation is that failure to invest is undetectable and cannot jeopardize brand membership.

## 3.1 Perfect Monitoring

In this section we analyze collective branding under perfect monitoring. Let $e_m$ be the largest value of $e$ for which a $BIE$ exists for an $m$ partition under perfect monitoring.

**Proposition 2** *Corresponding to every* $m \in \{2, ..., N_H\}$

*(i)* $e_m > e^*$.

*(ii)* $e_m$ *is strictly increasing in* $m$.

**Proof of proposition**: We construct a $BIE$ for any $m$ partition.

Suppose all members of all the $H$ brands invest. Let a brand's *record* be the total number of high quality units produced by all the members of the brand at period 1. Denote the record of brand $i$ of size $m$ as $s_i^m \in \{0, 1, ...., m\}$, let

$S^m = (s_1^m, ..., s^m_{n_H^m + n_L^m}, s^1_{n_H^m + n_L^m + 1}, ..., s^1_{N-(m-1)(n_H^m + n_L^m)})$, and let $S_{-i}^m \equiv (S^m \backslash s_i^m)$.

Let consumer beliefs at period 2 be: A stand alone firm or a firm which is a member of a brand of size $\neq m$ is either type $L$ or type $H$ which has not invested.[11] Let $\Pr(H^m \mid s_i^m, S_{-i}^m)$ be the posterior probability - and therefore consumers' belief[12] that, given $S_{-i}^m$, brand $i$ of size $m$ and record $s_i^m$ is an $H$ brand. To simplify notation, in the

[10] In that case, we assume that one of the stand alone firms (if there are any under $P$) , or one of the firms assigned to an $L$ brand is randomly chosen to be reassigned to replace the non-investor. Thus excluding a non-investing $H$ firm from the brand leaves the brand size and the expected number of high quality units produced by the brand unchanged, and hence cannot decrease the expected profits of the remaining brand members which do invest, while it can only increase the profit of the replacement firm.

[11] With respect to brands of size $\neq m$ this is an out of equilibrium belief.

[12] For a brand of size $m$, any record $s^m$ is on the equilibrium path and consumer beliefs about such a brand must be consistent with Bayesian updating.

remainder of the proof we omit the subscript and superscript of $s_i^m$ when this does not lead to any ambiguity. A completely analogous argument to (1) implies that consumers' *expected* (with respect to $S_{-i}^m$) belief - *evaluated at the time of investment* - that a brand with record $s$ is an $H$ brand is given by:

$$\Pr(H^m \mid s) = \frac{qg^s(1-g)^{m-s}}{qg^s(1-g)^{m-s} + (1-q)b^s(1-b)^{m-s}} \tag{6}$$

Thus at the time of investment, the expected revenue (price) of each member of an $H$ brand at period 2, conditional on the brand's realized record being $s$, is given by $p^m(s)$ :

$$\begin{aligned} p^m(s) &= g \Pr(H^m \mid s) + b(1 - \Pr(H^m \mid s)) \\ &= b + (g-b) \Pr(H^m \mid s) \end{aligned} \tag{7}$$

and thus its unconditional expected revenue at period 2 is:

$$R^m = \sum_{s=0}^{m} \binom{m}{s} g^s(1-g)^{m-s} p^m(s) \tag{8}$$

Analogously, the expected revenues of each member of an $L$ brand of size $m$ at period 2 is:

$$R_L^m = \sum_{s=0}^{m} \binom{m}{s} b^s(1-b)^{m-s} p^m(s). \tag{9}$$

Since consumers believe that stand alone firms are either type $L$ or type $H$ which don't invest, the second period revenue of any stand alone firm is $b$. Consider an $L$ firm which is a member of an $L$ brand. As a brand member its expected profit is $R_L^m$ and as a stand alone firm its profit is $b < R_L^m$, where the inequality follows from (7) and (9). Thus brand membership is more profitable for $L$ firms than standing alone.

Consider an $H$ firm which is a member of an $H$ brand. As a brand member, it invests and its expected profit is $R^m - e$. If it doesn't invest, it must stand alone (since membership in the $H$ brand is contingent on investment) and its profit is $b$. Thus brand membership is more profitable than standing alone if $R^m - b \geq e$.

Consider a stand alone firm. If it joins one of the $L$ brands, the brand size will increase to $m+1$, and the profit of every existing member will be $b$ (since consumers will not pay more than $b$ to a firm which is a member of a brand of size $\neq m$), while otherwise its profit is $R_L^m > b$. By the same reasoning, if it joins one of the $H$ brands, the revenue of every existing member will decrease from $R^m$ to $b$.

Thus the equilibrium conditions are satisfied if $R^m - b \geq e$. To prove the proposition it thus remains to characterize $e$ which satisfies the condition $R^m - b \geq e$. This is achieved by using the following lemma, proved in the appendix.

**Lemma 1** *For every $m \geq 1$, $R^m$ is increasing and $R_L^m$ is decreasing with $m$ .*

Let $\varepsilon_m \equiv R^m - R_L^m$. By equations (4) - (9), $R^1 = R$ and $R_L^1 = R_-$. Hence by Lemma 1, and the definition of $e^*$ it follows that for every $m \geq 2$:

$$\varepsilon_m = R^m - R_L^m > R^1 - R_L^1 = R - R_- = e^*.$$

Let $e_m = \varepsilon_m$. Thus, $R^m - b > e$ for $e \leq e_m$. This completes the proof of part $(i)$ of the proposition. Part $(ii)$ then follows immediately from Lemma 1.

■

**Remark 1**: Although we do not formally require, as an equilibrium condition, that members of H brands earn greater profits when the brand collectively invests than if no members invest, it follows immediately from the definition of $\varepsilon_m$ in the preceding proof that our equilibrium constructions do indeed satisfy this sensible criterion.

Thus under collective branding with perfect monitoring, there are multiple brand sizes which can support investment in equilibrium when $e > e^*$. As the preceding proposition establishes, these may be ranked in terms of their effect on investment: The larger is the brand size, $m$, the greater is the range of investment costs for which investment is sustainable in equilibrium. In particular, the largest investment cost under which investment is sustainable corresponds to the brand size $N_H$ where all $H$ firms are in the same brand. In this equilibrium, "bigger is better" in the sense that the larger is $N_H$, the larger the range of investment costs which can support equilibrium investment. The same observation applies to the relationship between brand size and firm profits: As shown in the proof of the proposition, the larger the equilibrium brand size, the greater the $H$ firms' profit and the lower the $L$ firms' profit.

This suggests that the equilibrium brand size $m = N_H$ is supported by more plausible consumer beliefs than $m < N_H$. Specifically, as is shown in the proof of the proposition, equilibria in which $m < N_H$ require that consumers believe that a brand of size larger than $m$ is either type $L$ or type $H$ which doesn't invest. But, it is precisely the $H$ firms which would profit, while $L$ firms would lose, if the brand size increased, as long as consumers believed that a brand size $> m$ with a record greater or equal to that of a brand of size $m$ is at least as likely to invest. Thus, consumer beliefs which associate larger brand size with lower quality seem somewhat unpalatable. By contrast, consumers appropriately associate a brand size larger than $N_H$ with lower quality because such a brand must include at least some $L$ firms.

The same reasoning suggests that equilibria in which brands are of different sizes would similarly be based on less plausible consumer beliefs. Specifically, in an equilibrium in which $H$ brands are of different sizes, the smaller $H$ brands would be less profitable and could profitably increase in size unless consumers implausibly associate larger brand size with lower quality.

Although the preceding argument suggests a theory of 'mega' brands, this conclusion must be tempered once the assumption of perfect monitoring is relaxed, as the following section shows.

## 3.2 No-Monitoring

We now turn to examine the extent to which the analysis of the previous section applies in the case of no-monitoring. In this setting failure to invest cannot prevent a firm from using the brand label and thus firms have less of an incentive to invest than in the perfect monitoring regime. Nevertheless, the following proposition establishes that if $g$

is sufficiently large, collective branding can still incentivize investment when stand alone firms will not invest. Let $\widetilde{e}_m$ be the largest value of $e$ for which a $BIE$ exists for an $m$ partition under no-monitoring.

**Proposition 3** *Under no-monitoring, for every $m \in \{2, ..., N_H\}$ there is $g(m) < 1$ such that if $g \geq g(m)$, $\widetilde{e}_m > e^*$.*

**Proof:** For any $m$ partition, let the firms' strategies and consumer beliefs be the same as described in the proof of proposition 2. Then exactly the same arguments as in the proof of proposition 2 imply that the equilibrium conditions are satisfied for all stand alone firms and members of $L$ brands. Consider a member of an $H$ brand. If it invests its revenue (given that the other $m-1$ members invest) is $R^m$ given by (8). However, in contrast to the case of perfect monitoring, here it has the option of remaining in the $H$ brand without investing. If it doesn't invest, then its revenue is:[13]

$$R_{-1}^m = \sum_{s=0}^{m-1} \binom{m-1}{s} g^s (1-g)^{m-1-s} \left[(1-b)p^m(s) + bp^m(s+1)\right] \tag{10}$$

If it stands alone, its revenue is $b < R_{-1}^m$ where the inequality follows from (7) and (10). Let $\widetilde{\varepsilon}_m \equiv R^m - R_{-1}^m$. Thus the equilibrium conditions are satisfied if $e \leq \widetilde{\varepsilon}_m$.

The following lemma shows that a similar result to Lemma 1 applies under no monitoring in the special case $g = 1$.

**Lemma 2** *Under no- monitoring, if $g = 1$, $\widetilde{\varepsilon}_m$ is strictly increasing in $m$ for $m \geq 1$.*

By equations (4) - (10), $R^1 = R$ and $R_{-1}^1 = R_-$. Hence $\widetilde{\varepsilon}_1 = R^1 - R_{-1}^1 = e^*$. Thus it follows from the lemma that if $g = 1$, then $\widetilde{\varepsilon}_m > e^*$ for all $m > 1$. By equations (6) - (8) and (10), $\widetilde{\varepsilon}_m$ is continuous in $g$, implying that there is $g(m) < 1$, such that for $g \geq g(m)$, $\widetilde{\varepsilon}_m > e^*$. Finally, let $\widetilde{e}_m = \widetilde{\varepsilon}_m$. This completes the proof. ■

[13] $\binom{m-1}{s} g^s (1-g)^{m-1-s}$ is the probability that the other, $m-1$ investing firms, produce $s$ high quality units. With probability $1-b$ the firm which doesn't invest produces low quality in which case the brand produces $s$ high quality units and each member receives the price $p^m(s)$. With probability $b$ the $m$-th firm produces high quality and the price is $p^m(s+1)$.

The intuition behind Proposition 3 is that the incentive to free ride on the investment of other brand members reflects the adverse effect of a single low quality observation on the brand's reputation. If $g$ is sufficiently large, even a single low quality unit has enough of a negative effect on consumer beliefs to deter free riding.

However, this is true only as long as the brand size is not "too large". Once the brand size is sufficiently large, the effect of a single low observation on the brand's reputation is too small to deter free riding. Therefore, in contrast to the case of perfect monitoring, under no-monitoring it is not generally true that 'bigger is better'. In fact, the following proposition shows that under no-monitoring, for sufficiently large $m$, a *BIE* for an $m$ partition does not exists for $e \geq e^*$ .

**Proposition 4** *Under no-monitoring, for every $g < 1$, there is $m(g)$ such that for $m \geq m(g)$, $\widetilde{e}_m \leq e^*$.*

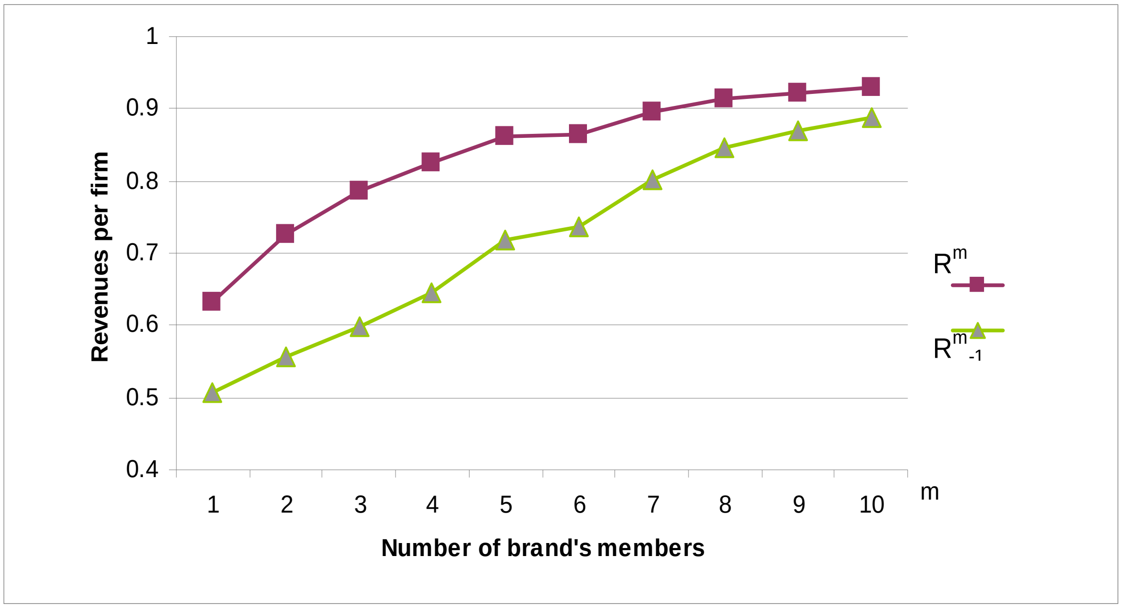
**Proof:** In the Appendix.

Thus, if $\widetilde{m}$ denotes the brand size which maximizes $\widetilde{e}_m$ - the brand size for which a $BIE$ exists for the largest range of investment costs - then, $\widetilde{m} < N_H$ if $N_H$ is sufficiently large, in contrast to the case of perfect monitoring. Also, in contrast to perfect monitoring, if $\widehat{m}$ is the equilibrium size which is most profitable for $H$ firms, then $\widehat{m} > \widetilde{m}$ if $N_H$ is sufficiently large, as the example directly below illustrates.[14] Recall that in the case of perfect monitoring it was argued that $N_H$ is a more plausible equilibrium brand size than smaller brand sizes. Thus by a completely analogous argument, under no-monitoring the same holds true for $\widehat{m}$.

**Example**

In figure 1, $R^m$ and $R^m_{-1}$ are sketched for the parameters $b = 0.4$, $g = 0.95$, $N_H = 10$, $N_L = 30$ and $e = 0.14$. $\widetilde{e}_m$ is represented by the distance between $R^m$ and $R^m_{-1}$. In this

[14] This is because $\widetilde{m}$ is the $m$ which maximizes $R^m - R^m_{-1}$ while the most profitable brand size for a given $e$ is the largest $m$ for which $R^m - R^m_{-1}$ is still $\geq e$.




example[15], $e^* = 0.12$, $BIE$ exist for $2 \leq m \leq 5$ and $\widetilde{m} = 3$ (where $\tilde{e}_3 = 0.19 > e^* = 0.12$).

The equilibrium brand size which maximizes $H$ firms' profit is $m = 5$. By contrast, under perfect monitoring, as we know from the previous section, both $e_m$ and $H$ firms' profit is maximized when $m = N_H = 10$.

# 4 Umbrella brands

Umbrella branding refers to the practice of multiproduct firms to market otherwise unrelated products under the same brand name in order to signal quality. How do the incentives of collective brands to invest in reputation compare with those of umbrella brands? To address this question in our setting, consider an $m$ partition each element of which is now a multiproduct firm which makes investment decisions for, bears the investment costs of and owns the the profits of each 'member' (product). Thus, if the

[15] $(n_H, n_L, q)$ equals (10,30,0.25), (5,15,0.25), (3,10,0.23), (2,7,0.22), (2,6,0.25), (1,5,0.17), (1,4,0.2), (1,3,0.25), (1,3,0.25), (1,3,0.25), for $m = 1, 2, ..., 10$, respectively. Note that $R^m$ is almost flat between $m = 5$ and $m = 6$ (increasing from 0.862 to 0.865) because of the sharp decline in $q$ between those two values.

umbrella brand is size $m$, and the price of each of its members (products) is $p$, the brand's revenue is $pm$. We compare the umbrella brand's investment incentives with those of the collective brand under no-monitoring[16].

In the case of collective brands under no-monitoring, the highest investment cost for which a BIE exists for an $m$ partition is $\widetilde{e}_m = R^m - R^m_{-1}$. If the umbrella brand of size $m$ invests in all its members, then its second period expected profit is $m(R^m - e)$. For the same reason, if it invests in only $m-1$ of its products, its profit is $m(R^m_{-1} - e) + e$. Thus a $BIE$ exists for umbrella brands of size $m$ if:

$$m(R^m - e) - m(R^m_{-1} - e) - e = m(R^m - R^m_{-1}) - e = m\widetilde{e}_m - e \geq 0$$

Thus, while a $BIE$ exists for collective brands only if $e \leq \widetilde{e}_m$, in the case of umbrella brands it exists for $e \leq m\widetilde{e}_m$. Thus umbrella branding incentivizes investment more than collective branding.

The intuition for this is straightforward. In the cases of both collective brands and umbrella brands, a low quality realization of one member reduces the reputation of the entire brand. In the case of the collective brand, individual members are only concerned about how this affects the value of their own product. By contrast, the umbrella brand internalizes the effect of its investment in each individual member (product) on the reputational value of the brand's entire product line.

# 5 Monitoring costs

We have considered two polar cases of collective brands; perfect monitoring, in which members of $H$ brands are committed to invest, and imperfect monitoring, in which brand members invest only if investment is individually optimal. Consider an intermediate case in which the brand cannot costlessly detect failure to invest and, accordingly, membership

[16] The appropriate comparison is to no-monitoring because under perfect monitoring brand members have no discretion with respect to investment decisions while the owner of the umbrella brand can decide in which products to invest.

in an $H$ brand requires a firm to incur a fixed monitoring cost $c$ to verify that it invests - for example by hiring a reliable external auditor to certify its investment[17]. Then, a brand member's profit is $R^m - e - c$ while the profit from standing alone is $b$. Thus, a $BIE$ exists for the $m$ partition if $R^m - (e + c) > b$ . Thus, since $R^m$ increases with $m$, investment incentives and $H$ firms' profit increase with $m$, just as in the case of perfect monitoring without monitoring costs. This also suggests that under monitoring costs there is a minimal brand size - the brand must be large enough for reputational gains associated with increased size to cover monitoring costs in addition to investment costs.

# 6 Franchising

Franchising shares some features of umbrella brands and some features of collective brands. The franchisor collects a share of each franchisee's revenues - and thus benefits from the investment of each outlet - but franchisees bear investment costs. In practice, franchisors tend to monitor franchisees quite closely, by contractually requiring that the service be in accordance with the pattern determined by the franchisor, through field support, external service audits, peer review and consumer feedback (Spinelli Jr, Rosenberg, Birley, 2004), all of which suggests that quality assurance is costly to the franchisor. Thus, if the franchisor incurs a monitoring cost $c$ for each franchisee that it monitors and gets a fraction $\alpha$ of franchisees' revenue, its profit is $m(\alpha R^m - c)$ which, since $R^m$ is increasing, increases with $m$. This suggests that, as in the case of collective brands with perfect or costly monitoring, the franchisor's profit increases with the number of franchisees and that the number of franchisees must be large enough for reputational gains to cover monitoring costs.

Indeed, leading franchise chains are huge and seem to strive for unlimited growth. For example, in the US alone, there are over 20,000 Subway, 14,000 McDonalds , 7000 Pizza Hut, 11000 Starbucks and 13000 H&R Block tax preparation locations. However, it should be noted that the number of chain outlets or locations can greatly exaggerate

[17] Alternatively and equivalently, the cost $c$ is shared by all brand members.

the number of "brand members" since franchisees often own multiple units. Indeed, the policy of many large chains is to actively encourage franchisees to take on multiple outlets. For example, Domino's Pizza and Subway offer reduced fees for franchisees that acquire further units (see https://www.businessfranchise.com/special-features/multiple). According to NatWest/BFA Franchise Survey 2008, one fifth of franchisees own multiple units, with an average of seven units each. This policy might be designed to reduce monitoring costs. First, owners of multiple units have more of an incentive to internalize the effects of their investment on the brand reputation than owners of single units (particularly if the outlets owned are in close geographical proximity). Second, it may be cheaper for the franchisor to monitor owners of multiple units than single unit owners. For example, the former can be effectively monitored by retaliating against all its units if the quality of one randomly sampled unit is defective, while in the case of single unit owners, it is necessary to monitor each unit individually.[18]

# 7 Concluding Remark

It has been shown that collective branding can lead to higher quality than is attainable by stand alone firms. Institutions such as marketing boards and state trading enterprises are often viewed as means to foster collusion and obstacles to efficient markets and on these grounds have been targeted by free market advocates in WTO negotiations. Our analysis suggests that to the contrary, by enhancing reputational incentives, such institutionalized collective brands may increase efficiency and welfare by enabling higher product quality than would be attainable in their absence. [19]

[18] Moreover, there is some evidence that monitoring by franchisors is less than perfect, possibly to save on monitoring costs. For example, Jin and Leslie (2008) show that within a chain, company owned restaurants tend to have better hygiene than franchisee owned restaurants, suggesting at least some free riding by franchisees on the chain reputation. Relatedly, Ater and Rigby (2012) show that chain outlets at locations in which repeat business is infrequent tend to be company owned, possibly to save on monitoring costs at locations in which individual incentives to free ride are particularly strong.

[19] An alternative position expressed in defense of STE's is that they provide economies of scale in production and promotion.

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# 8 Appendix

## 8.1 Proof of Lemma 1

By equations (6) - (8)

$$\begin{aligned} R^m &= b+(g-b)\sum_{s=0}^{m}\binom{m}{s}g^s(1-g)^{m-s}\frac{g^s(1-g)^{m-s}q}{g^s(1-g)^{m-s}q+b^s(1-b)^{m-s}(1-q)} \\ &= b+(g-b)\sum_{s=0}^{m}\binom{m}{s}g^s(1-g)^{m-s}\frac{q}{q+(1-q)x_s^m} \\ &= b+(g-b)\sum_{s=0}^{m}\binom{m}{s}g^s(1-g)^{m-s}k_s^m \end{aligned}$$

where

$$x_s^m \equiv \frac{b^s(1-b)^{m-s}}{g^s(1-g)^{m-s}} \quad \text{and } k_s^m \equiv \frac{q}{q+(1-q)x_s^m}$$

Let $S$ be a binomial random variable with the parameters $(m, g)$. Let

$$X^m \equiv \frac{b^S(1-b)^{m-S}}{g^S(1-g)^{m-S}} \quad \text{and } K^m \equiv \frac{q}{q+(1-q)X^m}$$

Note that

$$E(X^{m+1} \mid X^m) = g\frac{b^{S+1}(1-b)^{m-S}}{g^{S+1}(1-g)^{m-S}} + (1-g)\frac{b^S(1-b)^{m+1-S}}{g^S(1-g)^{m+1-S}} = bX^m + (1-b)X^m = X^m$$

implying that $X^1, X^2, X^3, ...$ is a martingale. Since $X^m \geq 0$, $K^m$ is a strictly convex function of $X^m$, then by Jensen's Inequality, $EK^{m+1} > EK^m$. Hence,

$$R^{m+1} = b+(g-b)\sum_{s=0}^{m+1}\binom{m+1}{s}g^s(1-g)^{m+1-s}k_s^{m+1} > b+(g-b)\sum_{s=0}^{m}\binom{m}{s}g^s(1-g)^{m-s}k_s^m = R^m$$

which proves that $R^m$ is increasing with $m$.

Substitute equations (6) and (7) into (9) yielding

$$
\begin{aligned}
R_L^m &= b + (g-b)\sum_{s=0}^{m}\binom{m}{s} b^s(1-b)^{m-s}\frac{g^s(1-g)^{m-s}q}{g^s(1-g)^{m-s}q + b^s(1-b)^{m-s}(1-q)} \\
&= b + (g-b)\sum_{s=0}^{m}\binom{m}{s} g^s(1-g)^{m-s}\frac{qx_s^m}{qx_s^m + (1-q)}
\end{aligned}
$$

Since $\frac{qX^m}{qX^m+1-q}$ is a concave function of $X^m$, by Jensen's Inequality

$$E\frac{qX^{m+1}}{qX^{m+1}+1-q} < E\frac{qX^m}{qX^m+1-q}$$

implying

$$
\begin{aligned}
R_L^{m+1} &= b + (g-b)\sum_{s=0}^{m+1}\binom{m+1}{s} g^s(1-g)^{m+1-s}\frac{qx_s^{m+1}}{qx_s^{m+1}+1-q} \\
&< b + (g-b)\sum_{s=0}^{m}\binom{m}{s} g^s(1-g)^{m-s}\frac{qx_s^m}{qx_s^m+1-q} = R_L^m
\end{aligned}
$$

which proves that $R_L^m$ is decreasing with $m$. Thus completing the proof . ■

## 8.2 Proof of Lemma 2

**Proof:** When $g = 1$, the $m-1$ investing firms produce high quality with certainty. If the $m$th firm doesn't invest it produces high quality with probability $b$, in which case its revenues (and that of every other member of the brand) are $R^m$. With probability $1-b$ it produces low quality in which case $s = m-1$ and, by equations (6) and (7) $\Pr(H^m \mid m-1) = 0$ and $p^m(s) = b$. Hence,

$$R_{-1}^m = bR^m + (1-b)b.$$

It follows that

$$\widetilde{\varepsilon}_m = R^m - R_{-1}^m = (1-b)(R^m - b).$$

Since by Lemma 1 $R^m$ is increasing with $m$, it follows that $\widetilde{\varepsilon}_m$ is increasing with $m$. ■

## 8.3 Proof of Proposition 4

The proof is using the following Claim.

**Claim**

$$R^m = \sum_{s=0}^{m-1} \binom{m-1}{s} g^s (1-g)^{m-1-s} \left[ g p^m(s+1) + (1-g) p^m(s) \right] \tag{11}$$

**Proof of the Claim**: Let $s'$ be the number of high quality units produced by any given group of $m-1$ members of an $H$ brand of size $m$. Since the $m$th firm invests, it produces high quality with probability $g$ and low quality with probability $1-g$. Hence, the brand produces $s'+1$ high quality units and receives a price of $p^m(s'+1)$ with probability $g$ and produces $s'$ high quality units and receives a price of $p^m(s')$ with probability $1-g$. Since the probability that $m-1$ members produce $s'$ high quality units is $\binom{m-1}{s'} g^{s'} (1-g)^{m-1-s'}$ it follows that

$$R^m = \sum_{s=0}^{m-1} \binom{m-1}{s} g^s (1-g)^{m-1-s} \left[ g p^m(s+1) + (1-g) p^m(s) \right]$$

which proves the Claim.

Using equations (10) and (11)

$$\widetilde{\varepsilon}_m = R^m - R^m_{-1} = (g-b) \sum_{s=0}^{m-1} \binom{m-1}{s} g^s (1-g)^{m-1-s} \left[ p^m(s+1) - p^m(s) \right]$$

Substituting for $p^m(s)$ from equations (6) and (7) and recalling from the proof of Lemma 1 that $x^m_s \equiv \frac{b^s(1-b)^{m-s}}{g^s(1-g)^{m-s}}$:

$$\widetilde{\varepsilon}_m = (g-b)^2 \sum_{s=0}^{m-1} \binom{m-1}{s} g^s (1-g)^{m-1-s} \frac{q(1-q)(x^m_s - x^m_{s+1})}{\left[q + (1-q)x^m_{s+1}\right]\left[q + (1-q)x^m_s\right]}.$$

Substituting

$$x^m_s - x^m_{s+1} = \frac{b^s(1-b)^{m-s}}{g^s(1-g)^{m-s}} - \frac{b^{s+1}(1-b)^{m-s-1}}{g^{s+1}(1-g)^{m-s-1}} = \frac{b^s(1-b)^{m-s-1}}{g^s(1-g)^{m-s-1}} \left( \frac{1-b}{1-g} - \frac{b}{g} \right)$$

yields

$$\widetilde{\varepsilon}_m = (g-b)^2 \sum_{s=0}^{m-1} \binom{m-1}{s} b^s (1-b)^{m-s-1} \frac{q(1-q)\left(\frac{1-b}{1-g} - \frac{b}{g}\right)}{\left[q + (1-q)x^m_{s+1}\right]\left[q + (1-q)x^m_s\right]}.$$

Hence, and since $\lim_{m\to\infty} x_s^m = \infty$ and $\sum_{s=0}^{m-1} \binom{m-1}{s} b^s (1-b)^{m-s-1} = 1$ it follows that

$$\lim_{m\to\infty} \widetilde{\varepsilon}_m = 0$$

and the lemma follows immediately. ∎