Dynamic adverse selection with learning

Saara Hämäläinen*

DRAFT: March 4, 2013

Abstract

We study the equilibrium quality distribution and the wage offer path in a labor market setup of dynamic adverse selection with learning. Firms and workers meet randomly and pairwise. During their contact, the workers make the firms repeated wage offers while the firms observe the workers completing projects. The time til the success number one is the key separation instrument. Conditional on no completion, the firm becomes more and more pessimistic about worker quality. The acceptable wage decreases until it is so low that the best workers return to market for another try. Since better workers both finish their tasks faster, and get a job, or walk away faster if luck turns against them, the equilibrium quality distribution can be dominated by the initial one. Better types are usually also better paid but there can also be wage variation between identical workers. Despite the lemons problem, the best workers need not idle for long and the market distribution can first order stochastically dominate the entry distribution. In addition to stationary equilibria, there can exist regular cycles with market freeze phases, or sunspot cycles with or without market freeze phases.

Keywords: Adverse selection; Search; Learning; Signaling; Perfect Sequential Equilibrium. **JEL-codes:** D82, D83.

*University of Helsinki and HECER. P.O. Box 17, FI-00014 University of Helsinki, FINLAND, Phone: +358503658192, Email: saara.hamalainen@helsinki.fi. I thank Hannu Vartiainen for his omnipatient supervision and Takuro Yamashita, Juuso Välimäki, Marko Terviö, Shouyong Shi, Lucas Maestri, Pauli Murto, Klaus Kultti, Aviad Heifetz and the audiences at FEA XXXIV meeting, HECER seminar, and SING 8 meeting for comments. I am grateful to Finnish Academy of Science and Letters, OP-Pohjola Group Research Foundation, and Yrjö Jahnsson Foundation for financial help. All errors are my own.

1 Introduction

It is a seemingly robust result in models of dynamic adverse selection that, even when it is impossible to clear the market immediately with prices only, (i) all qualities can be traded with time but (ii) higher qualities have to be traded more slowly or more rarely than lower qualities (see *Inderst and Muller* [2002]; *Moreno and Wooders* [2002], *Blouin* [2003], *Guerrieri et al.* [2010], *Moreno and Wooders* [2010], *Chang* [2011]; *Guerrieri and Shimer* [2012], and *Camargo and Lester* [2010]). If the maximal price the buyers would be willing to offer for average quality is below the minimal price the sellers would be willing to accept for high quality, the higher qualities would not be traded in static Walrasian equilibrium. This is the classic lemons problem (*Akerlof* [1970]). Yet, if the lower qualities were, indeed, sold the first, average quality would increase. The problem might become less severe, even vanish. Since trade is supported by (explicit or implicit) discounting, inefficiency is unavoidable, though. Agents dislike waiting but, to satisfy the incentive conditions that support trade, the best goods have to be left for last. As a result, liquidity necessarily decreases in quality.

That, however, appears to be at odds with the dynamics in canonical lemons markets. It does not seem to be the case that higher qualities would be left idle just to reveal their types, say, in the labor market, in the market for houses, the market for used cars, or in even in those for stocks or bonds. In fact, there are much more attractive separation mechanisms available once the deviation from the static Walrasian environment is made, i.e. when trading can take time and agents can affect the terms of trade (see *Hendel and Lizzeri* [1999, 2002]; *Hendel et al.* [2005]). Moreover, as the agents are waiting, they also tend to be learning. This appears quite natural, at least, when signals keep arriving over time and the learning cost is just the waiting cost. If the best workers are also the fastest ones, it may not take long to tell them apart. Information is rarely an endowment but, almost always, determined in equilibrium (see *Bergemann and Välimäki* [2006]). When there is asymmetric information which impedes trade, there are usually also ways to reveal or acquire information in order to improve the outcomes. The ignorant might have an incentive to learn, and the knowledgeable might have an incentive to allow them to learn. Abstracting from learning could, thus, somewhat exaggerate the inefficiency that originates from asymmetric information. If learning is possible in a lemons market, it probably occurs and matters.

We study the equilibrium quality distribution and the wage offer path in a labor market setup of dynamic adverse selection with learning. Firms and workers meet randomly and pairwise. During their contact, the workers make the firms repeated wage offers while the firms observe the workers completing projects. The time til the success number one is the key separation instrument. To avoid so called "nulls", types with zero success rate, or to reveal the lowest types, a firm never pays a higher wage before seeing, at least, the first project ready. Conditional on no completion, a firm becomes more and more pessimistic about worker quality. The acceptable wage decreases until it is so low that the best workers return to market for another try. After that dipping time, the firm beliefs and, hence, the wage offers decrease quickly as workers of first higher and then lower qualities resume their search. Since better workers both finish their tasks faster and walk away faster if their luck turns against them, the equilibrium quality distribution can be dominated by the initial one. Better types are typically also better paid but, given that the time til the first success is random, there is also wage variation between identical workers. Despite the lemons problem, the best workers need not idle for long. This is rare in models of dynamic adverse selection, yet, consistent with everyday empirics. Unemployed CEOs are rare.

In addition to the stationary pooling equilibrium, which arises naturally in a market with enough nulls, there can exist quite efficient stationary semi separating equilibria, or non stationary equilibria in which market freeze, pooling and semi separating phases alternate in intricate ways. The pooling equilibrium can fail to exist, especially, if the market quality is so high that the firm prefers expoiting a new worker to employing the old worker even after a success. As semi separation can either increase or decrease market quality while pooling can only decrease it, semi separation can pave way for pooling in cases where the initial market quality is too high for pooling after a success, yet, too low for pooling before a success. Under regular cycles, there might need to be, in addition, a market freeze phase in between the two phases as higher or lower types are waiting for better times. The market freeze can be regarded as capturing the idea how incomplete markets can arise endogenously. Under sunspot cycles, the market freeze might not be needed, however, if the pooling and semi separation phase probabilities adjust appropriately to market changes.

Our findings, hence, resonate with existing work on cyclic equilibria presented previously, for example, by *Moreno and Wooders* [2002, 2010]; *Janssen and Roy* [2004], in settings quite similar to ours, and by *Gu and Wright* [2011], with credit cycles that originate merely from belief fluctuations. However, as learning can overturn market dynamics, pooling and semi separation can play upset roles in our work in contrast to the previous work. In a sense, the time in the meeting replaces the time on the market (as in *Taylor* [1999]) as signaling and learning takes place in the pairwise meetings (as in*Bar-Isaac* [2003]; *Kremer and Skrzypacz* [2007] with a single seller) and is not visible to the market as whole. Still, no work to our knowledge has analyzed the effects on private learning and signaling on equilibrium quality distribution and the wage offer path. The case of public learning and signaling and the connections between the Akerlofian (1970) and the Spencian (1973) approaches are considered in the seminal paper by *Daley and Green* [2011]. That also results in improving quality composition; our setup is a unique one to predict a decline in that. The effect of learning on unemployment is covered by *Gonzalez and Shi* [2010].

The focus of the paper is on Perfect Sequential equilibria (see *Grossman and Perry* [1986]), which refines Perfect Bayesian equilibria by requiring that the out of equilibrium path beliefs are derived, whenever possible, by updating the equilibrium path beliefs in a Bayesian fashion. In particular, the support of the updated (out of equilibrium path) distribution must be contained in the support of the original (equilibrium path) distribution. In continuous time the refinement is consistent only with semi separating and not with fully separating equilibrium patterns as a revealing move on the path cannot be supported by type uncertainty out of path. We also impose a mild technical continuity requirement on the out of equilibrium path beliefs, i.e. we assume that slight deviations from equilibrium offers do not give rise to drastic changes in beliefs. That removes the ambiguity about firm values as the equilibrium wage offers have to keep the firms at their continuation values. Interestingly, the firm continuation values are positive even in stationary equilibria as firms benefit from "free work" (the first success if it occurs) during "probation" (the time before the success or the return to the market). The setup, hence, escapes the *Diamond* [1971] paradox that arises very often in stationary search equilibria like the ones analyzed here. Our mild requirement does not, in particular, imply that profits have to be zero although the workers have the entire bargaining power.

The presentation is organized in the following way. In Section 2, the model is set up. In Section 3, the stationary pooling equilibrium is constructed step by step. Parts of the analysis remains to be written, yet, most results stem directly form our earlier analysis conducted under discrete time and types. Section 4 analyzes the equilibria of the game more generally, in particular, the existence and properties of other different classes of equilibria such as stationary semi separating equilibria and non-stationary equilibria where pooling and separation vary either deterministically or stochastically, and extensions with types on both sides. Section 5 relates the main findings to past work and future research avenues.

2 Model

Entry, meetings, types Consider a large continuum market: firms and workers of continuous types meeting one another randomly and pairwise in continuous time. The firms are indexed by $i \in \mathcal{I} = [0,1]$ and the workers by $j \in \mathcal{J} = [0,1]$. The instants are indexed by $t \in \mathcal{T} = [0,\infty)$, in the market, and by $s \in \mathcal{S} = [0,\infty)$, in a meeting between a firm and a worker. A unit mass of new firms and new workers enters the market each moment. Both live for ever, and share discount factor r. Their arrivals, meetings and success in projects are each governed by a Poisson process. At any infinitesimal time interval $dt > 0$, or any simultaneous time interval $ds > 0$, the firms and workers enter the market for probability $\epsilon > 0$ (arrival rate), find a pair for probability $\mu > 0$ (meeting rate), and get a project completed for probability $\theta > 0$ (success rate). The entering firms are identical but the entering workers have private types $\theta = F^{-1}(j) \in \Theta = [\underline{\theta}, \overline{\theta}]$, for $0 < \underline{\theta} < \overline{\theta}$, drawn independently from a continuous distribution F with a density f. The type is, thus, worker's success rate. High types are fast, low types are slow.[1]

Payoffs, gains from work, terms of work, lemons problem The firm value for a success is one while the worker value for the success is zero. The flow cost of work is $c(\theta)$. It is a smooth function of type. In consequence, during any interval $dt = ds > 0$, the utility of working is θ for the firm while the disutility of working is $c(\theta)$ for the worker. There is asymmetric information. The worker knows his type θ but the firm does not see it. Neither can affect the cost $c(\theta)$. In other words, the problem is that of adverse selection but not of moral hazard.

There is a lemons problem. The surplus of work is positive for any given type[2]

$$\begin{array}{cc} & r\int_0^\infty e^{-rt}\theta dt - r\int_0^\infty e^{-rt}c(\theta)dt > 0, \\ \iff & \theta - c(\theta) > 0, \end{array}$$

yet, the expected discounted utility of working with an entrant type $E(\theta|F)$ is below the expected discounted disutility of working with the entrant type $\overline{\theta}$

$$\begin{array}{cc} & r\int_{\underline{\theta}}^{\overline{\theta}} \left(\int_0^\infty e^{-rt}\theta dt\right) dF(\theta) - r\int_0^\infty e^{-rt}c(\overline{\theta})dt < 0, \\ \iff & E(\theta|F) - c(\overline{\theta}) < 0. \end{array}$$

The life time payoffs u are linear in the lump sum wage w:

$$\begin{array}{c} u^i(\theta, w) = \theta - w, \\ u^j(\theta, w) = w - c(\theta). \end{array}$$

This implies that, if the equilibrium market distribution $G(\theta)$ equals the entry distribution $F(\theta)$, there is no single, fixed wage w, or wage path $w(t)$ from now on, that all firms and all workers could accept without any revelation mechanism.

$$\begin{array}{c} E(\theta|F) - w \leq c(\overline{\theta}) + w, \\ \forall w := r\int_0^\infty e^{-rt}w(t)dt \geq 0. \end{array}$$

There are gains from trade for all types θ, and that is common knowledge, but to overcome the lemons problem, the terms of trade cannot be the same for all types θ. To support working, more elaborate labor contracts are called for.

[1] We use masculine pronouns for workers and feminine pronouns for firms.

[2] To ease the exposition, the multiplier r is used to normalize the total payoffs to the same scale as the flow payoffs.

Contracts, histories, strategies Once a firm and a worker meet, the latter sets to work while the former supervises the tasks and their completion times. The *success records* are captured by $(n, \boldsymbol{s}) = (n; s_1, ..., s_n, s_{n+1}) \in \mathbb{Z}_+ \times \mathbb{R}_+^{n+1}$, where n is the number of projects finished and $\boldsymbol{s}$ is the vector of times spent so far on doing tasks $1, .., n$ (completed) and task $n+1$ (not yet completed) totaling $s = s_1 + ... + s_n + s_{n+1}$. Concurrently with project work, the worker keeps on offering wages $w \in [0, \infty) \cup \{\infty\} = \mathcal{W}^j$ and the firm either keeps on rejecting the offers or, finally, accepts an offer $\alpha(w) \in \{0, 1\} = \mathcal{A}^i$. If the firm accepts a wage, the pair signs a perpetual labor contract, exits together form the market, and the firm either immediately pays the worker the wage w or commits to a wage path $w(t)$ that induces the wage w. If the firm rejects a wage, the meeting just continues.

The *meeting histories* $h = (t, n, \boldsymbol{s}, w(s)) \in \mathcal{H}$, or h^s, at moment s, $h^{si} \in \mathcal{H}^i$ before the firm's move and $h^{sj} \in \mathcal{H}^j$ before the worker's move depending on emphasis, consist of the meet time t, the success records $(n, \boldsymbol{s})$ and the offer curve $w(s)$ from the meet time until now. The histories are always, by definition, cut between the meetings: The same firm and the same worker almost never meet again. The actions in the pairwise meetings are not visible to the others. As new higher or lower offers can be made any time and as infinite offers are always one option, the worker's (mixed) strategy is representable by the *offer curve* $w(h) \in \triangle\mathcal{W}^j$ and the firm's (mixed) strategy by the *acceptance probability* $\alpha(w|h) \in \triangle\mathcal{A}^i$. Both can also choose to resume their search any time. This is denoted by $\mathcal{O}$, the option to search.

The timing withing an instant is: first, the worker offers a wage (or chooses $\mathcal{O}$) and the firm either accepts or rejects it (or chooses $\mathcal{O}$), then, there either is a success or no success. If there is one, the firm profits from it right away. When in a meeting with a firm, the worker is constantly subjected to the cost of work.

Game, solution concept We consider Perfect Bayesian equilibria (PBE) $(w, \alpha; \pi)$ in the dynamic game of incomplete information $(\mathcal{H}, \mathcal{A}, \mathcal{W}, \Theta, F, u)_{ijt \in \mathcal{I} \times \mathcal{J} \times \mathcal{T}}$. A PBE is a strategy profile (w, α) and a belief system π such that (i) the strategies are sequentially rational given the beliefs and (ii) the beliefs are consistent with the strategies.

Beliefs The game may feature both Bayesian and non-Bayesian learning. If offers are not revealing, as in pooling equilibria, the firm's beliefs π about the worker's type θ, and the underlying type distribution P with the density p, have to satisfy the Bayes' law on the equilibrium path and, in addition, the Intuitive Criterion (Cho and Kreps) out of equilibrium path.[3] If offers are revealing, as in separating or semi-separating

[3] Which is enough to reject most deviations, though, other refinements would typically be satisfied also. For convenience, and to avoid extremely harsh punishments, it is assumed that beliefs are continous in offers, i.e. if at $h = (t, n, \boldsymbol{s}, w(s))$ and $h' = (t, n, \boldsymbol{s}, w'(s))$ the wage paths are "close" $\|w(s) - w'(s)\| < \delta$ then the beliefs are also "close" $\|\pi(h) - \pi(h')\| < \epsilon$, where the norm $\|\cdot\|$ could be, for intance, the sup norm $\|f\| = sup_x |f(x)|$.

equilibria, the sellers learn from the offers also. As the firm value depends only on the mean of the type distribution, not on the other moments of the type distribution, is it sufficient to represent the beliefs for market conditions G and meeting histories h by

$$\pi_s(\theta|h) = E_s(\theta|G, h).$$

Conditional on no success nor other revealing signals in an interval $ds > 0$, the density slopes incrementally to

$$p_{s+ds}(\theta) = \frac{(1-\theta ds)\, p_s(\phi)}{\int (1-\phi ds)\, p_s(\phi) d\phi} = \frac{1-\theta ds}{1-E_s(\theta)ds} p_s(\phi),$$

where the update factor

$$\frac{1-\theta ds}{1-E_s(\theta)ds}$$

is smaller than one for above average types and greater than one for below average types. However, if there is a success in an interval, the density jumps immediately to

$$p_{s+}(\theta) = lim_{ds\to 0+} \frac{\left(1-e^{-\theta ds}\right) p_{s-}(\theta)}{\int \left(1-e^{-\phi ds}\right) p_{s-}(\phi) d\phi},$$

from the left hand limit $lim_{ds\to 0-} p_{s+ds}(\theta) = p_{s-}(\theta)$, and simplifies by l'Hotelling rule to

$$= \frac{\theta p_{s-}(\theta)}{\int \phi p_{s-}(\phi) d\phi} = \frac{\theta}{E_{s-}(\theta)} p_{s-}(\theta),$$

where the update factor

$$\frac{\theta}{E_{s-}(\theta)}$$

is, in contrast to the previous one, smaller than one for below average types and greater than one for above average types.

A history h is suffixed by a plus sign $h+$ if there is a success after the history and by a minus sign $h-$ if there is no success after the history. As a result, around a success, beliefs slope downwards to

$$\pi_{s+ds}(\theta|h) = \int \phi \frac{1-\phi ds}{1-E_s(\theta)ds} p_s(\phi) d\phi$$

$$= \frac{1}{1-E_s(\theta)ds} \int (1-\phi ds)\, \phi p_s(\phi) d\phi = \frac{E_s(\theta) - E_s(\theta^2)ds}{1-E_s(\theta)ds}$$

$$= \frac{\pi_s(\theta|h) - \pi_s(\theta^2|h)ds}{1-\pi_s(\theta|h)ds} =: \pi_s(\theta|h-).$$

while, at a success, beliefs jump upwards to

$$\pi_{s+ds}(\theta|h) = \int \phi \frac{\phi}{E_{s-}(\theta)} p_{s-}(\phi) d\phi$$

$$= \frac{1}{E_{s-}(\theta)} \int \phi^2 p_{s-}(\phi) d\phi = \frac{E_{s-}(\theta^2)}{E_{s-}(\theta)}$$

$$= \frac{\pi_{s-}(\theta^2|h)}{\pi_{s-}(\theta|h)} =: \pi_s(\theta|h+).$$

Interestingly, due to the asymmetric information, the firm beliefs and the worker beliefs about the firm beliefs can evolve entirely differently. As beliefs have to be a martingale, the changes in firm beliefs have to be unanticipated. However, knowing the true success rate θ, the workers of high types $\theta > \pi(\theta|h)$ (low types $\theta < \pi(\theta|h)$) can still expect low (high) enough firm beliefs to drift upwards (downwards) over time.

Values, flows Any strategy profile (w,α) induces a profile of continuation values $V^i = (V_t^i, V_s^i(h))_{t,s}$ and $V^j(\theta) = \left(V_t^j(\theta), V_s^j(\theta,h)\right)_{t,s}$ for the firms and the workers respectively. The meeting values depend on type θ and history h which includes also market time t. The market values are, in addition, indexed by market time t and the meeting values by meeting time s.

Sequential rationality and the *meeting values* are captured by the following Bellman equations. Firms choose between searching, accepting and rejecting

$$\begin{array}{rcc} V_s^i(h) & = & max_{\alpha,\mathcal{O}}\left\{V_t^i, \pi(\theta|h) - w(h),\right. \\ & & \pi(\theta|h)ds\left(r + (1-rds)V_{s+ds}^i(h+)\right) + \\ & & \left.(1-\pi(\theta|h)ds)\,(1-rds)V_{s+ds}^i(h-)\right\} \end{array}$$

and workers choose between searching and offering a wage associated with an acceptance probability

$$\begin{array}{rcc} V_s^j(h) & = & max_{w,\mathcal{O}}\left\{V_t^j, \alpha(w|h)\,(w - c(\theta)) + \right. \\ & & (1-\alpha(w|h))\left(-rc(\theta)ds + \theta ds\,(1-rds)\,V_{s+ds}^j(h+) + \right. \\ & & \left.\left.(1-\theta ds)\,(1-rds)\,V_{s+ds}^j(h-)\right)\right\} \end{array}$$

where $1 - rds$ is an approximation for e^{-rds}.

The *market values* are determined by

$$V_t^i = \mu dt\,(1-rdt)\,V_0^i(h) + (1-\mu dt)\,(1-rdt)\,V_{t+dt}^i(h),$$

and

$$V_t^j(\theta) = \mu dt\,(1-rdt)\,V_0^j(h,\theta) + (1-\mu dt)\,(1-rdt)\,V_{t+dt}^j(h,\theta).$$

for the firms and the workers respectively.

A strategy profile (w,α) generates also a pattern of *outflows* $\varpi = \left(\varpi_t^i, \varpi_t^j(\theta)\right)_t$ from the market to the meetings and *backflows* $\beta = \left(\beta_t^i, \beta_t^j(\theta)\right)_t$ from the meetings to the market. As a result, the mass of workers of type θ in the market evolves as

$$m_{t+dt}^j(\theta) = \epsilon dt f(\theta) + \left(1 - \mu\varpi_t^j(\theta)dt\right) m_t^j(\theta) + \beta_t^j(\theta)dtM_t^j(\theta) - \mu\varpi_t^j(\theta)dtm_t^j(\theta),$$

as new workers enter and old ones either remain in the market, meet a firm, or return to the market. The mass of workers of type θ in the meetings fluctuates, in parallel, as

$$M_{t+dt}^{j}(\theta) = \mu\varpi_t^j(\theta)dtm_t^j(\theta) - \beta_t^j(\theta)dtM_t^j(\theta),$$

when pairs meet and separate. As the market sides are of equal length, the mass of firms in the market and in the meetings is just the sum of workers in the market and in the meetings

$$m_t^i = \int_0^1 m_t^j(\theta)d\theta,$$

$$M_t^i = \int_0^1 M_t^j(\theta)d\theta.$$

3 Stationary pooling equilibrium

Three types To get a feel of the equilibria, we begin by discussing the key properties and providing a parametric example of the first equilibrium class: the stationary pooling equilibrium. To keep the analysis as simple as possible, we focus on two basic worker types, the high type $\overline{\theta}$, the low type $\underline{\theta}$, and one new type 0. More precisely, we modify the model by adding to the market a mass λ of workers of type $\theta = 0$, so called nulls. To maintain the market sides equally long, we also insert to the market a mass λ of firms.

There are no gains from working with the nulls. They almost never succeed. The firms try to avoid the nulls and the other worker types attempt to distinguish themselves from the nulls. Their presence serves multiple purposes, though.

Most importantly, it facilitates the construction of the stationary pooling equilibrium, that is presented the first, by stopping the low types from separating from the high types as early as $s = 0$. The other equilibrium classes can be presented in a sequence as its extensions. Moreover, the mass $\lambda > 0$ is also an important parameter for other equilibria where the firms benefit from the free work before hiring, as it helps to keep the firm market value low enough so that the firms, in the end, prefer employing the current type to exploiting a novel type.

Besides, the introduction of the null type does seem to change the model into yet more realistic direction as an application to markets for experts. Some applicants might, indeed, have zero success rate for the job. Getting them sorted out before communicating with the others would then, obviously, represent a major concern to recruiters. If the other, better types $\theta > 0$ would be relatively fast, that might be most conveniently accomplished by never hiring a worker without a positive success record $n > 0$. In this sense, the presence of nulls would support learning.

Equilibrium construction We construct an equilibrium where wages are accepted, not after, nor before, but always at the very moment of the first success. To convey the main idea as efficiently as possible, two simple modeling tricks are utilized so as to pin down the equilibrium in the easiest possible fashion here. As shown later on, the equilibrium can arise under weaker conditions also. The first "trick" is the resortation to harsh out of equilibrium path beliefs (yet, continuous and consistent with the Intuitive Criterion): the high types do not want to separate late, at or after the success, as the deviations are attributed primarily to the low type. The second "trick" is related to the introduction of the nulls: low types do not want to separate early, before the success, as the deviations are attributed primarily to the null type. Both "tricks" are, of course, clearly legitimate modeling choices. Although they are not necessary for the existence of the stationary pooling equilibrium, they do facilitate its construction significantly.

The *duration of probation* d, i.e. the supremum over meeting time s before the acceptance, turns out to be the key endogenous variable. As will be explained later, the worker meeting value given success record will fluctuate, with slight abuse of notation, between a floor $\lfloor \theta \rfloor$ (prompting the worker to return to the market) and a ceiling $\lceil \theta \rceil$ (prompting the worker to make an acceptable wage offer). In a stationary pooling equilibrium, the high type floor $\lfloor \overline{\theta} \rfloor$ is, in general, so close to the high type ceiling $\lceil \underline{\theta} \rceil$ that a single success at the floor suffices to hit the ceiling. In this example the effect is amplified by the adverse out of equilibrium path beliefs, that also make sure that the low type floor and ceiling coincide with the high type floor and ceiling. The floor determines the probation duration $d = \lfloor \overline{\theta} \rfloor = \lfloor \underline{\theta} \rfloor$. In other words, by construction, if there is a success at $s < d$, both types make an acceptable offer determined below and, it if there is no success by d, both types return to the market for search.

As a result, the stationary pooling equilibrium features only Bayesian, not non-Bayesian, learning. The time til the first success is the key separation instrument. The firms update their beliefs upon that wait time. If the entry distribution is given by (f^H, f^L) and the market distribution by (g^H, g^L), the firm beliefs at the moment of success s would jump right up to

$$\pi_s(\theta) = \frac{(\theta^H)^2 g^H e^{-\theta^H s} + (\theta^L)^2 g^L e^{-\theta^L s}}{\theta^H g^H e^{-\theta^H s} + \theta^L g^L e^{-\theta^L s}}.$$

As the analysis in conducted under smooth beliefs and the workers have the entire bargaining power, the equilibrium wage offers have to keep the firms just indifferent between accepting and rejecting them. The offer curve is, in consequence, the difference between the firm beliefs and the firm market value

$$w_s = \pi_s(\theta) - V^i.$$

This is with no loss of generality since offers are accepted at and only at the moment of success. Interestingly, the random success times generate natural variation to the wage. Moreover, as better workers are faster than worse workers, better types are typically also better paid. They are more likely to succeed during any interval. Both properties are consistent with data.

In a stationary equilibrium, market and meeting inflows have to agree with outflows. The outflows ϖ from the market are given by

$$\mu m^H, \mu m^L$$

and the inflows ϵf and the backflows β by

$$\epsilon f^H + \mu m^H e^{-\theta^H d}, \epsilon f^H + \mu m^L e^{-\theta^L d}.$$

These equal for all types H, L, and 0, for any short a while $dt > 0$ iff

$$m^H = \frac{\epsilon f^H}{\mu(1 - e^{-\theta^H d})}, m^L = \frac{\epsilon f^L}{\mu(1 - e^{-\theta^L d})}.$$

The market distribution g^θ can, thus, be derived from the proportion of the entry probabilities f^θ and the success probabilities $1 - e^{-d\theta}$

$$\frac{g^H}{g^L} = \left(\frac{f^H}{f^L}\right)\left(\frac{1 - e^{-\theta^L d}}{1 - e^{-\theta^H d}}\right).$$

The market values V^j for workers are determined by

$$V^j = \mu dt\,(1 + rdt)\,V^j(h^0) + (1 - \mu dt)\,(1 + rdt)\,V^j$$

which ignoring the terms of order $(dt)^2$ results in the following

$$V^j = \frac{\mu dt\,(1+rdt)}{1-(1-\mu dt)\,(1+rdt)} V^j(h^0) = \frac{\mu}{r+\mu} V^j(h^0).$$

The market values for firms are otherwise the same but they have to be adjusted for the presence of the nulls

$$V^i = \frac{\left(\frac{m^L+m^H}{m^L+m^H+\lambda}\right)\mu dt\,(1+rdt)}{1-\left(1-\left(\frac{m^L+m^H}{m^L+m^H+\lambda}\right)\mu dt\right)(1+rdt)} V^j(h^0) = \frac{\left(\frac{m^L+m^H}{m^L+m^H+\lambda}\right)\mu}{r+\left(\frac{m^L+m^H}{m^L+m^H+\lambda}\right)\mu} V^j(h^0).$$

The market values depend only on the meeting values $V(h^0)$, the level of meeting frictions $1/\mu$ and impatience r (both decreasing them).

The firm meeting values incorporate, in essence, only the value of "free project work" the firms enjoy during the possibly multiple subsequent learning processes. Apart from that, the firms are never paid more than their market values V^i. The firm meeting values are, therefore, captured by

$$V^i(h^0) = r\int_0^d e^{-rs}\left(1+V^i\right)\pi_0(\theta)ds + e^{-\pi_0(\theta)d}e^{-rd}V^i,$$

$$V^i(h^0) = \pi_0(\theta)\left(1-e^{-rd}\right)\left(1+V^i\right) + e^{-\pi_0(\theta)d}e^{-rd}V^i.$$

If there is a success during the interval $s \in [0,d]$, the firm gets $1+V^i$ (in the 1st term on the RHS), otherwise, the firm gets only V^i (in the 2nd term on the RHS). It is obvious that the firms would never return to the market before the workers would: in the meeting, they have a chance to obtain V^i+1 whereas, in the market, they get no more than V^i.

The beliefs $\pi_0(\theta)$, as featured in the meeting value $V^i(h^0)$, are pinned down by the probation duration d both directly and through the market distribution $g(d)$. The duration of probation is determined by the worker Bellman equations, which have to be such that the low type prefers to return to the market if and only if the high type prefers. In the Bellman equations, the market values $V^j(\theta)$ depend on the first meeting values $V^j(h^0,\theta)$ through $V^j(\theta) = \frac{\mu}{r+\mu}V^j(h^0,\theta)$. The first meeting values are, in turn, given by

$$V^j(h^0,\theta) = -c(\theta)(1-e^{-rd})$$

$$+r\int_0^d e^{-rs}\left(\pi_s(\theta) - V^i\right)\theta ds$$

$$-\left(1-e^{-\theta d}\right)e^{-rd}c(\theta) + e^{-\theta d}e^{-rd}V^j(\theta).$$

Whether there is a success or no success, the worker always works through the full probation and endures the related cost (the 1st term on the RHS). If there is a success, the worker gets a wage, the firm beliefs net of the firm market value (the 2nd term on the RHS), but has to bear the cost of work after probation (the 3rd term on the RHS). If there is no success, the worker continues searching after probation (the 4th term on the RHS).

The 2nd term on the RHS is the key term. It can be simplified followingly

$$r\int_0^d e^{-rs}\frac{(\theta^H)^2 g^H e^{-\theta^H s} + (\theta^L)^2 g^L e^{-\theta^L s}}{\theta^H g^H e^{-\theta^H s} + \theta^L g^L e^{-\theta^L s}}\theta^j ds =$$

$$r\int_0^d e^{-rs}\frac{(\theta^H)^2\theta^j + be^{ars}}{\theta^H + ce^{ars}}ds =$$

where $\theta^H - \theta^L = \Delta\theta = ar$ for some $a > 0$, $b(\theta, d) = \theta^j(\theta^L)^2\frac{f^L}{f^H}\frac{1-e^{-d\theta^H}}{1-e^{-d\theta^L}}$, and $c(\theta, d) = \theta^L\frac{f^L}{f^H}\frac{1-e^{-d\theta^H}}{1-e^{-d\theta^L}}$, $c'(\theta, d) = \frac{c}{\theta^H} = \frac{\theta^L}{\theta^H}\frac{f^L}{f^H}\frac{1-e^{-d\theta^H}}{1-e^{-d\theta^L}}$ and, thus, $\frac{b}{c} = \theta^j\theta^L$. The term can be simplified even more as

$$r\int_0^d \exp(-rs)\frac{\frac{b}{c}\left(\theta^H + c\exp(ars)\right) - \frac{b}{c}\theta^H + (\theta^H)^2\theta^j}{\theta^H + c\exp(ars)}ds =$$

$$\theta^L\theta^j\left(1-\exp(-rd)\right)-\left(\Delta\theta\right)\theta^j r\int_0^d \exp(-rs)\frac{1}{1+c'\exp(ars)}ds$$

What remains is, therefore, the integral in the last term on the RHS. Though, it is not obvious to us how to solve this in general in closed form, it is possible in many particular, parametric examples. To show how the model works, we proceed by focusing on the cases where $a=1$. By use of, first, substitutions $t=\exp(rs)$ and $dt=rtds$

$$\int_0^d \exp(-rs)\frac{1}{1+c'\exp(rs)}ds=\frac{1}{r}\int_{\exp(0)}^{\exp(rd)}\frac{1}{t^2}\frac{1}{1+dt}dt=$$

and, then, partial fraction decomposition

$$\frac{1}{t^2}\frac{1}{1+c't}=\frac{1}{t^2}+\frac{\left(c'\right)^2}{1+c't}-\frac{c'}{t}$$

the integration can be conducted as

$$\frac{1}{r}\int_{\exp(0)}^{\exp(rd)}\left(\frac{1}{t^2}+\frac{\left(c'\right)^2}{1+c't}-\frac{c'}{t}\right)dt=$$

$$\frac{1}{r}\left(1-exp(-rd)+c'rd+c'ln(1+c'exp(rd))-c'ln(1+c')\right).$$

In the stationary pooling equilibrium under consideration, high type workers return to the market for another try once the firm beliefs have decreased below a threshold value $\pi_d(\theta)$. There exists, hence, a wage $w_s=\pi_s(\theta)-V^i$ such that the worker is indifferent between returning to the market today after no success and returning to the market tomorrow after no success. That wage is determined by

$$V^j(\theta^H)=-rc(\theta^H)ds+\theta^H ds\left(1+rds\right)\left(w_{s+ds}-c(\theta^H)\right)+\left(1-\theta^H ds\right)\left(1+rds\right)V^j(\theta^H)$$

which, ignoring the terms of order $(dt)^2$ and defining the lowest wage that would be offered as $w_d := w_{s+ds}$, results in the following

$$w_d = rc(\theta^H) + \frac{c(\theta^H)ds}{\theta^H ds\,(1+rds)} + \frac{1-\left(1-\theta^H ds\right)(1+rds)}{\theta^H ds\,(1+rds)} V^j(\theta^H)$$

$$= rc(\theta^H) + \frac{c(\theta^H)}{\theta^H} + \frac{\theta^H - r}{\theta^H} V^j(\theta^H) = rc(\theta^H) + \frac{c(\theta^H)}{\theta^H} + \frac{\theta^H - r}{\theta^H}\frac{\mu}{\mu+r} V^j(h^0, \theta^H).$$

Notice that this is feasible only if $\left(\theta^H\right)^2 > c(\theta^H)\left(r\theta^H + 1\right)$ since, otherwise, the wage w_d would have to exceed θ^H which is obviously impossible.

After the high type workers return to the market, the low type workers face essentially a stationary optimal stopping problem. If they succeed, they are recognized as low types $\pi(\theta) = \theta^L$, if they do not succeed, they are regarded as null types $\pi(\theta) = 0$. In the equilibrium we are constructing now, they prefer searching to waiting for a successful separation from the nulls.

$$V^j(\theta^L) \geq V^j(h^d, \theta^L) = -rc(\theta^L)ds + \theta^L ds\,(1+rds)\left(\theta^L - V^i\right) +$$

$$\left(1-\theta^L ds\right)(1+rds)\,V^j(h^d, \theta^L) = -\frac{rc(\theta^L)}{\theta^L - r} + \frac{\theta^L}{\theta^L - r}\left(\theta^L - V^i\right).$$

It is also necessary to ascertain that the low types and the null types would not pool together. This is ruled out as long as there are so many nulls that the firms are not willing to pay enough to an average unsuccessful worker to make the low types accept the same deal as the null types

$$\frac{g^L}{\lambda + g^L}\theta^L < c(\theta^L).$$

Simple parametric example Consider an economy where $c(\theta^L) = 0 < \theta^L = 1 < c(\theta^H) = 3 < \theta^H = 5$. Notice that the gains from trade are larger for high types than for low types. Suppose that both types enter the market at the same rate $f^H = f^L = \frac{1}{2}$ and $\epsilon = 1$. In addition, the market is populated by some positive mass of nulls $\lambda > 0$, that is determined later on.

This implies that, but for the nulls, there really would not be a lemons problem

$$E(\theta|F, \lambda = 0) = f^H\theta^H + f^L\theta^L = 3 = c(\theta^H),$$

but, with them,

$$E(\theta|F, \lambda > 0) = \frac{f^H}{f^H + f^L + \lambda}\theta^H + \frac{f^L}{f^H + f^L + \lambda}\theta^L < 3 = c(\theta^H),$$

and it is impossible to empty the market immediately by a single price $p: \; c(\theta^H) \leq p \leq E(\theta|F, \lambda > 0)$.

In other words, any positive mass of nulls can help to initiate the lemons problem. However, in a stationary pooling equilibrium, a lemons problem could also emerge in itself,

$$E(\theta|G, \lambda = 0) = g^H\theta^H + g^L\theta^L < 3 = c(\theta^H),$$

as the low types return to the market more frequently than the high types and, therefore, the market distribution g is first order stochastically dominated by the entry distribution f.

To proceed, let us set the meeting rate to $\mu = 1$ and the discount factor to $r = 1$. The search values for the high type can be obtained as a function of the duration d.

$$\frac{r+\mu}{\mu}V^j = -c(\theta^H)(1 - e^{-rd}) - \left(1 - e^{-\theta^H d}\right)e^{-rd}c(\theta^H)$$

$$+\theta^L\theta^H\left(1 - \exp(-rd)\right)$$

$$-\left(\Delta\theta\right)\theta^H\left(1 - \exp(-rd) + c'rd + c'\ln(1 + c'\exp(rd)) - c'\ln(1 + c')\right)$$

$$- \left(1 - e^{-\theta d}\right) e^{-rd} c(\theta) + e^{-\theta d} e^{-rd} V^j(\theta). \tag{1}$$

The search values for the firms can also be obtained as a function of the duration d.

$$\frac{r + \left(\frac{1+\frac{f^H(1-e^{-\theta^L d})}{f^L(1-e^{-\theta^H d})}}{1+\frac{f^H(1-e^{-\theta^L d})}{f^L(1-e^{-\theta^H d})}+\mu\lambda\frac{(1-e^{-\theta^L d})}{f^L}}\right)\mu}{\left(\frac{1+\frac{f^H(1-e^{-\theta^L d})}{f^L(1-e^{-\theta^H d})}}{1+\frac{f^H(1-e^{-\theta^L d})}{f^L(1-e^{-\theta^H d})}+\mu\lambda\frac{(1-e^{-\theta^L d})}{f^L}}\right)\mu} V^i =$$

$$\pi_0(\theta)\left(1 - e^{-rd}\right)\left(1 + V^i\right) + e^{-\pi_0(\theta)d} e^{-rd} V^i. \tag{2}$$

The lowest wage the high workers accept is pinned down by the high worker type search value

$$w_d = c(\theta^H) + \frac{c(\theta^H)}{\theta^H} + \frac{\theta^H - r}{\theta^H} V^j(\theta^H),$$

while the firm beliefs that correspond with the wage are pinned down by the firm search value

$$\pi_d(\theta) = \frac{4\left(\frac{1-e^{-d}}{1-e^{-2d}}\right) e^{-2d} + e^{-d}}{2\left(\frac{1-e^{-d}}{1-e^{-2d}}\right) e^{-2d} + e^{-d}} = w_d + V^i,$$

which leads to

$$c(\theta^H) + \frac{c(\theta^H)}{\theta^H} + \frac{\theta^H - r}{\theta^H} V^j(\theta^H) + V^i = \frac{4\left(\frac{1-e^{-d}}{1-e^{-2d}}\right) e^{-2d} + e^{-d}}{2\left(\frac{1-e^{-d}}{1-e^{-2d}}\right) e^{-2d} + e^{-d}}. \tag{3}$$

So we have a system of three equations 1, 2, and 3 for three unknowns $V^j(\theta^H)$, V^i and $-d = \ln x < 0$ for some $0 < x < 1$. Solving it suffices to solving all other unknowns of interest. Just plugging into the equations all fixed parameter values and rearranging, the system takes the form (cont'd).

Proposition 1. In a stationary equilibrium, there exist a floor $\lfloor \theta \rfloor$ and a ceiling $\lceil \theta \rceil$ such that, if the beliefs $\pi(\theta)$ get below the floor, type θ returns to the market and, if the beliefs get above the floor, the type θ offers the wage that agrees with the beliefs $w = \pi(\theta) - V^i$.

4 Conclusion

In this paper we develop a model of adverse selection in a dynamic matching setup which takes into account the incentives for learning that arise naturally under quality uncertainty. We show that allowing for learning can reverse the dynamic trading patterns in a market for lemons, namely, the finding that higher quality has to be less liquid than lower quality. This implies that, in contrast to the earlier results, it may not be the case that the lemons problem would be automatically cured over time as the higher quality accumulates to the market and the mean of the type distribution rises. If learning is possible, as it is in most conceivable applications, like the market for used cars (test drives, inspections), labor market (CVs, interviews, selection tests, internships), or credit market (regular ratings and reports on performance), it might not be reasonable to expect that the highest qualities would remain idle in the market for long periods just to reveal their types. Other more subtle mechanisms might be at work as well, some maybe more efficient.

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