Statistical Externalities and the Labour Market in the Digital Age

Paul Seabright [*], [†] Ananya Sen [‡]

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Abstract

This paper examines whether a reduction in the cost of applying for jobs that leads to an increase in the number of candidates applying for jobs at a firm, may make the firm worse off. We build a model where there is worker heterogeneity and firms can choose to screen workers at a cost. In equilibrium, a reduction in application costs can lower firm payoffs by raising the number of applications from workers who, on average, are of lower quality than those who apply when application costs are high. An additional candidate can impose a negative externality on the firm by negatively affecting the statistical quality of its candidate pool. We discuss applications to the phenomenon of attention congestion through advances in digital technology.

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[*] Toulouse School of Economics, Institute for Advanced Studies at Toulouse and CEPR.

[†] Toulouse School of Economics.

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1 Introduction

Can too much choice make decision makers worse off? In this paper we address this question in the context of the labour market. In our model, we show that additional job applications by unemployed workers can impose a negative externality on firms. An equilibrium with sufficiently low application costs, and hence more applications than an equilibrium where costs are high, could reduce the average quality of the pool of applicants and lower the expected payoff of the firm. The intuition is that as the cost of making an application declines, the additional applications may come from unemployed workers of lower quality than those who previously applied. This may make the firm worse off if it does not screen workers, and may do so even if it does screen workers in equilibrium.

In standard matching models of the labour market, such as in Pissarides (1985) and Mortensen and Pissarides (1994), an increase in the number of unemployed workers increases the payoff of firms. This is because vacancies are costly, and an increase in unemployment reduces the length of time a vacancy is left unfilled. On the workers' side of the market, additional applications impose a negative externality since they reduce the probability of a given worker getting a job. These models developed the matching function which became a standard tool to analyse the labour market. Though convenient, the matching function was a reduced form way to capture various frictions, and hence was considered a 'black box'.

Several studies since then (Albrecht et al. (2004), Galenianos and Kircher (2009),Wolthoff (2012)) have tried to provide micro-foundations for an aggregate matching function. Our model falls in this area of the literature since we provide a story about how firms and workers form a match and highlight a friction involved in the process which has not been pointed out till now. We have a model of search with Nash bargaining over the surplus, where there is worker heterogeneity and where can screen at a cost. We describe a screening procedure which resembles the process of drawing balls from an urn without replacement.

In this setting, a fall in the costs of making applications can induce a rise in the number of applications that reduces the payoff of firms. This can happen if the additional applications reduce the average quality of the pool of applicants to a given firm. This can be true, most obviously, if the firm does not screen applicants, provided that the marginal applicant is of lower quality than the average applicant. But it may also be true even if the firm does screen applicants in equilibrium and therefore never hires an applicant of low quality. This is because the increase in applications may be what induces the firm to screens applicants and therefore to incur higher screening costs.

After looking at the benchmark case which involves one firm and two workers, we then look at a more general model with a large but finite number of firms and workers. We allow for ex-post competition for a worker's services between the firms, which may bid up wages in the event of a worker's applying successfully to more than one firm. We still find that for every symmetric equilibrium screening strategy, for some range of the parameters an increase in the number of applications can reduce the payoff of the firm.

The phenomenon described by the model is one with quite wide relevance to technological developments in the digital age. In spite of undeniably large benefits, it is widely believed that reductions in the cost of processing and transmitting information have created a new form of scarcity - the scarcity of attention. Many people feel themselves inundated with much more information than they can effectively process. Part of the difficulty consists in the simple costs of processing information, including costs embodied in the neurophysiological constraints inherent in the structure of our brains (see Klingberg, 2010). But an even more fundamental problem is statistical - if our limited information processing capacities were the only difficulty, an effective solution would consist in processing only a random subset of the information we receive. The reason this is not a real solution in practice is that as we receive more information, random subsets of that information are less valuable to us than the more limited information used to be. In short, there is declining marginal productivity of information, as there is of many inputs, but unlike with other inputs we face the difficulty of being unable to distinguish the marginal unit of information from the infra-marginal ones.

With the advent of the internet as a medium for job search, costs of applying for jobs or positions at a university have gone down drastically. There seems to be very little physical search involved with a many jobs offering online applications. It is easy to conceive that lower quality workers are more likely as a result to apply for positions in pure hope, just because it is now much easier to do so. Clearly the logic is relevant not only for workers applying to firms and students applying for places but also for sales people, hustlers, politicians, bloggers, media organizations, in short to everyone who something to sell, an idea to promote, a case to put. It's important therefore to understand that while the internet has lowered the costs of putting one's case for everyone, it has lowered them most, relative to the expected benefits, for those who have a relatively weak case to put. That, in a nutshell, is the origin of the scarce attention problem.

One obvious context is that of applications for colleges and universites. There is evidence that applications for college in the U.S have been on the increase over the past decade. Applications to only one institution declined as a proportion of the total from 22% in 1999 to 14% in 2009. Applications to six or more colleges increased from 19% in 1999 to 33% in 2009. The advent of the internet definitely plays a part in this phenomenon. This has led to some skepticism as Fred Hargadon, former dean of admissions at Princeton and Stanford, doubts that more and more applicants make for a stronger class. He says "I couldn't pick a better class out of 30,000 applicants than out of 15,000" [1]. This point of view is echoed by Karl M. Furstenberg, dean of admissions and financial aid at Dartmouth College from 1992 to 2007, who says "I don't think these larger applicant pools are materially improving the quality of their classes" [2]. In this paper we suggest reasons for fearing that large applicant pools may even be lowering the quality of the classes.

[1] Application Inflation: When Is Enough Enough?: New York Times (November, 2010).

[2] Application Inflation: When Is Enough Enough?: New York Times (November, 2010).

In the next section, we review the literature. In section 3, we analyse a simple model with one firm and two workers. In section 4, we allow for a general model with a large number of firms and workers. Section 5 looks at extensions to the general model while section 6 concludes.

2 A Look at the Literature

The literature on the search and matching approach to labour market outcomes was pioneered by Pissarides (1985), Mortensen and Pissarides (1994) and Diamond (1982). The authors introduced the labour market matching function as a convenient way of capturing different frictions in such a market. Since then, there has been a large body of work which has explicitly dealt with different frictions in search models of the labour market. Burdett et al. (2001) highlight the urn-ball matching friction in a model of directed search where there are multiple sellers and buyers but the latter can approach only one seller. Thus, in a large labour market, due to lack of co-ordination between the buyers there might be a situation where some sellers have more buyers than they can service while some may not receive enough. This is the urn-ball matching friction which is also looked at by Shimer (2005). Montgomery (1991) used a search theoretic framework to explain inter-industry wage differentials.

Albrecht et al. (2004) look at a model of directed search with wage posting where workers can make multiple applications to firms. The fact that a worker can send multiple applications reduces the chances of a firm's not getting any applications at all. The fact the same worker applies to different firms, on the other hand, reduces the chance that any given wage offer will be accepted. This is another co-ordination friction in addition to the urn-ball matching friction. Albrecht et al. (2006) look at a similar setting but allow for ex-post competition between firms for workers who receive multiple offers.

In contrast, our model is one where there is directed search but the wage setting mechanism is Nash bargaining. The friction that we highlight in the paper holds even in the case of a single firm and is thus distinct from the urn-ball matching friction and the friction arising from wage offers that are not accepted.

Villena-Roldan (2008) and Wolthoff (2012) focus more on the recruitment process, taking into account costs of screening and making applications while modeling the search and matching process. Wolthoff (2012) consists of the most general model of directed search with wage posting, a model that encompasses several special cases such as Burdett et al. (2001), Albrecht et al. (2004, 2006) and Galenianos and Kircher (2009)). Wolthoff (2012), like us, explicitly looks at costs of making an application for a worker as well the cost for a firm of interviewing applicants.

Eeckhout and Kircher (2010) look at a situation of sorting in a market with frictions within a competitive price setting. They find that under strong complementarities between a firm and worker's type, there is positive assortative matching between the firms and the workers. Galenianos and Kircher (2009) analyzes multiple job applications as portfolio choice since workers can apply to more than one firm. In equilibrium, there is wage dispersion even in the presence of homogeneous workers. Since high wage firms are likely to get more applications, this would lead an individual to have a small probability of getting that job. Thus, she would want to diversify and apply to certain 'safe', that is, low wage options as well.

Though there are several papers which focus on search frictions, we are unaware of any study which highlights the possibility of a firm being worse off when it gets more applications. To the best of our knowledge, the idea that as costs of making an application go down, the average quality of the pool of applicants might decline because marginal applicants are of lower quality than average applicants has not yet been explored.

3 A Simple Model

3.1 The General Setup

3.1.1 Workers and Firms

There are two risk neutral unemployed workers indexed by $i \in \{1, 2\}$. Each worker can be one of two types $\theta_i \in \{a, b\}$. Nature chooses the type of each worker according to the unconditional probabilities:

$$prob(\theta_i = a) = p_i \in (0, 1) \tag{1}$$

with:

$$p_1 > p_2$$

There is one risk neutral firm which has one vacancy and may hire either of the workers. The value of the match to the firm from hiring a particular type of worker is given by:

$$v(a) = V \tag{2}$$

$$v(b) = -X \tag{3}$$

where $V > X \geq 0$. Since $p_1 > p_2$, worker 1 always has a higher chance of being a good match for the firm [3].

The workers choose whether to apply or not to the firm. Each worker can make an application at a cost $t > 0$. We denote the action set for the worker i by $\{g_{i1},\ g_{i0}\}$, where g_{i1} corresponds to worker i applying and g_{i0} corresponds to worker i not applying.

[3] The specification of the value of a match to the firm is similar to that in Wolthoff (2012). In that paper, a firm can interview applicants and with some exogenous probability find an applicant to be 'qualified' or not.

The firm, upon receiving an application, needs to decide whether to incur a cost $c > 0$ per application to screen and identify the type θ_i of the worker (with probability 1) or not. Let $s_l^1, l \in \{0, 1\}$ be the screening action of the firm if it receives one application and $s_r^2, r \in \{0, 1, 2\}$ be the screening action of the firm if it receives two applications.

3.1.2 The Information Structure

Neither the firm nor the worker observes the intrinsic type of the worker, but the probability distribution over types for each worker is common knowledge. This means that worker i knows she is i but does not know whether she is a or b. However, there is an important source of informational asymmetry which develops between the firm and a worker who makes an application: workers observe their index but firms do not (though they may be able to infer it in equilibrium), and workers, unlike firms, therefore know whether their probability of being of type a is equal to p_1 or p_2.

Let $\lambda(f|n)$ denote the belief of the firm about the identity of the workers who have applied, with $f \in \{1, 2, (1, 2)\}$ when it gets $n \in \{0, 1, 2\}$ applications. When the firm gets two applications it obviously knows that both have applied but it cannot tell which is which.

3.1.3 Payoffs

Let $\pi : \{g_{11}, g_{10}\}$x $\{g_{21}, g_{20}\}$x $s_j^n \to R$ with $j \in \{1, 2\}$, denote the payoff function for the firm.

Let $U^i : \{g_{11}, g_{10}\}$x $\{g_{21}, g_{20}\}$x $s_j^n \to R$ with $i \in \{1, 2\}, j \in \{0, 1, 2\}$ denote the payoff function of the worker. To simplify notation for the sections below, we define the expected wage for each application and screening strategy as $u^i \left(g_{ik} | g_{-ih}, s_j^n \right)$ with a slight abuse of notation such that $i = 1(2)$ and $-i = 2(1)$.

After a worker is hired, production takes place, and the firm and the worker then bargain over the realized surplus. We use the Nash bargaining solution to capture the outcome, on the assumption that if bargaining breaks down the workers and the firm receive zero payoffs.

Let γ_1 and γ_2 be the bargaining power of the firm and the worker respectively with $\gamma_1 + \gamma_2 = 1$. Both workers have the same bargaining power. In the Nash bargaining stage, the screening costs incurred in the previous period do not matter since they are sunk. The wage $w(g_1, g_2, s_j^n)$ is set so that the logarithm of the joint Nash product is maximized.

We make two assumptions on parameter values to make the analysis tractable and interesting:

(1) $V > \frac{(1-p_2)}{p_1 p_2} X$.

(2) $c < p_2 \gamma_1 V$

First, (1) implies that a worker is always better off when she is the only applicant as opposed to when the firm gets two applications, for any screening strategy that it might adopt . This helps us to limits the number of cases we need to analyze without significant loss of generality. Then (2) implies that if the firm does decide to screen applications, it will be in its interest to screen both, at least ex ante (it may in the event need to screen only one to find a candidate of type a). This helps us limit the tedious algebra while the fundamental trade-offs remain exactly the same.

3.1.4 The Timing of the Game

The sequence of moves in this game is given by:

Period 0 : Nature chooses the type of the worker.

Period 1: Workers choose whether to apply for the job.

Period 2: The firm decides (a) whether to screen and (b) whether to hire.

Period 3: If the firm decides to hire, then production takes place and there is Nash bargaining over the division of the surplus- the size of which is observed by the firm and the hired worker.[4].

3.1.5 Equilibrium

We look for perfect Bayesian equilibria of the game in pure strategies, defined as follows:

Definition 1. A Perfect Bayesian Equilibrium outcome is a tuple given by:
$\left\{(s_j^{1*}, s_k^{2*}), \lambda^*(f|n)), (g_1^*, g_2^*), w^*\right\}$ such that:

(1) (s_j^{1*}, s_k^{2*}) is the firm's efficient screening strategy taking the application strategy (g_1^*, g_2^*) of the workers as given and taking into account the wage w^* paid out as the efficient split of the surplus. $\lambda^*(k|n)$ is a system of beliefs for the firm, which are updated using Bayes' rule along the equilibrium path, such that (s_j^{1*}, s_k^{2*}) is sequentially rational with respect to it.

(2) (g_1^*, g_2^*) is the workers' efficient strategy taking into account the efficient firm strategy (s_j^{1*}, s_k^{2*}) as well as the efficient wage w^* arising in the subsequent stages.

(3) (w^*) is the wage which is the efficient split of the surplus at the Nash Bargaining stage.

To characterize the equilibrium, we need to define the beliefs of the firm off the equilibrium path. This matters particularly when, in equilibrium, the firm gets applications from both workers (or from neither) and hence has to specify beliefs for the out of equilibrium event of getting one application[5]. This out of equilibrium event will have a non-singleton information set with $\lambda^{*'}(.)$ and $1 - \lambda^{*'}(.)$ capturing the distribution over the nodes in this set with The belief that the firm holds, off the equilibrium path, that it is worker 1 who has applied when it gets only one application is defined as $\lambda^{*'}(.)$.

[4]Thus, in this setup there is bargaining over the surplus ex-post. We avoid considering bargaining before the size of the surplus is known, which would add complications without increasing insight.

[5]When the firm gets only one application in equilibrium, off the equilibrium path the information set is a singletion and hence we will have $\lambda^{*'}(f = (1, 2)|n = 2) = 1$ in that case.

We now consider in detail the beliefs and actions of the players, solving backwards from the end. All proofs are in the Appendix .

3.2 Solving the Model

3.2.1 Period 3: Nash Bargaining Solution

In the spirit of backward induction, we start from period 3 which is the Nash bargaining stage.

Here, we look at the case when the firm gets only one application from worker 1 in equilibrium and decides not to screen. In this case, we have $\lambda(k = 1|n = 1) = 1$ using the bayes rule to update beliefs along the equilibrium path. The gross value of the match that the firm gets, in expectation, is $p_1V - (1 - p_1)X$. Since the firm and the hired worker bargain over the realized surplus and p_i is common knowledge, there is no asymmetric information at this stage. The wage w is set so that the logarithm of the joint Nash product is maximized: $max_w \; log\left[(p_1V - (1 - p_1)X - w - 0)^{\gamma_1}(w - 0)^{\gamma_2}\right]$ which gives a first order condition:

$$\frac{1 - \gamma_1}{w} - \frac{\gamma_1}{p_1V - (1 - p_1)X - w} = 0 \tag{4}$$

The second order condition is given by $\frac{\gamma-1}{w^2} - \frac{\gamma_1}{(p_1V-(1-p_1)X-w)^2} < 0$. The optimal wage is defined to be:

$$w_1^* = (1 - \gamma_1)\left(p_1V - (1 - p_1)X\right) \tag{5}$$

Similarly, we can carry out the same exercise for different informational situations depending on the screening strategy adopted. Since $p_1 > p_2$, we can establish the following lemma.

Lemma 1. We can establish an ordering on the wages generated by the split of the surplus such that $w_2^* \geq w_1^* \geq w_3^*$.

Screening for the type of the unemployed worker leads to a greater value of the match for the firm ex-post and hence the wage paid out to the hired worker[6].

3.2.2 Period 2: Firm's Screening Strategy

The payoff to the firm if it does not screen when it gets only one application from worker i[7] :

$$\pi(s_0^1|g_{i1}, g_{-i0}) = p_i\gamma_1V - (1 - p_i)\gamma_1X \tag{6}$$

The payoff to the firm if it screens in depth to find out θ_i is

[6]The different wages coming out of the Nash bargaining process are indexed by $w_d^*(.)$,$d \in \{1, 2, 3\}$

[7]Here, if $i = 1(2)$ then $-i = 2(1)$.

$$\pi(s_1^1|g_{11}, g_{20}) = p_i\gamma_1 V - c \tag{7}$$

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By screening for the type of the worker, the firm incurs a cost c but eliminates the possibility of hiring a worker with $\theta_i = b$. Thus, the firm will want to screen if $\pi(s_1^1|g_{i1}, g_{-i0}) - \pi(s_0^1|g_{i1}, g_{-i0}) \geq 0$, which holds when

$$p_i \leq p^* \equiv 1 - \frac{c}{\gamma_1 X}$$

If p_i is low enough then it is worth spending c to prevent a negative impact of $(1-p_i)X$. The fact that $p_1 > p_2$ implies that it is more likely that the firm will want to screen if it gets an application from only worker 2 in equilibrium.

If the firm gets two applications, it cannot distinguish which application corresponds to which worker. The firm can decide to evaluate both applications in detail to find out the θ_i or just randomly pick one out of the pile without incurring any cost. When the firm screens for the type of the worker, the evaluation procedure is such that the employer first picks out one application which is randomly chosen from the two. It evaluates the application for θ_i. If it does not turn out to be a type a worker then it will evaluate the next applicant. Thus, we allow for the firm to optimally choose the number of applications it would want to screen.

We define $p \equiv \frac{p_1+p_2}{2}$. The payoff to the firm under different scenarios is given by:

$$\pi(s_0^2|g_{11}, g_{21}) = p\gamma_1 V - (1-p)\,\gamma_1 X$$

$$\pi(s_2^2|g_{11}, g_{21}) = (2p - p_1 p_2)\gamma_1 V - (2-p)c$$

In the case of the event$\{s_2^2, g_{11}, g_{21}\}$, there is a $\frac{p_1}{2}$ chance of picking the application of the type 1 worker and finding her to be type a while there is $\frac{p_2}{2}$ chance of getting worker 2 who is of type a in the first draw out of the pile of applications. Thus, there is a $(1 - \frac{p_1+p_2}{2})$ chance of not finding $\theta_i = a$ which provides the continuation probability.

Conditional on getting two applications, it optimal for the firm to screen both for the worker type by incurring c per application if $\pi(s_2^2|g_{11}, g_{21}) \geq \pi(s_0^2|g_{11}, g_{21})$ which means:

$$c \leq c^+ \equiv \left(\frac{\gamma_1}{2-p}\right)((p - p_1 p_2)V + (1-p)X) \tag{8}$$

Thus, if the screening costs $c \leq c^+$are not too high, then it will be optimal for the firm to screen both applications.

3.2.3 Period 1: Workers' Application Strategy

In the first stage, the workers have to choose to either apply or not taking into account the screening strategy of the firm as well the wage they would get at the Nash bargaining stage.

The payoff to the worker i from applying (conditional on worker $-i$ not applying)is (a) if the firm does not screen: $U^i\left(g_{i1}|(g_{-i0}, s_0^1)\right) = (1-\gamma_1)(p_iV - (1-p_i)X) - t$ and (b) if the firm screens: $U^i\left(g_{i1}|(g_{-i0}, s_1^1)\right) = (1-\gamma_1)p_iV - t$ if the firm screens. If both workers apply and if the firm screens both applications in detail for θ_i then the payoff is:

$$U^i\left(g_{i1}|(g_{-i1}, s_2^2)\right) = (2-p_{-i})\frac{p_i}{2}(1-\gamma_1)V - t \tag{9}$$

There is $\frac{1}{2}$ probability that worker i gets chosen in the first draw and a probability p_i of turning out to be a type a individual. If a type a worker is not found in the first draw, the second application is evaluated with a continuation probability of $(1-p)$. If both workers apply and the firm does not screen, we have $U^1\left(g_{i1}|(g_{-i1}, s_0^2)\right) = \frac{1}{2}(1-\gamma_1)(p_iV - (1-p_i)X) - t$. The intuition behind these is based on the evaluation procedure as highlighted in the previous section.

We now define different thresholds to identify efficient application actions by the workers. We make a preliminary observation in the following lemma:

Lemma 2. If $u^1(g_{11}|g_{21}, s_k^2) \geq u^2(g_{21}|g_{10}, s_j^1), k \in \{0,1,2\}, j \in \{0,1\}$, then if the firm receives only one application it has to be from worker 1.

The condition $u^1(g_{11}|g_{21}, s_k^2) \geq u^2(g_{21}|g_{10}, s_j^1)$ implies that if the cost of making an application is low enough, such that worker 2 to applies then it has to be the case that worker 1 applies as well. This happens if the quality heterogeneity is large enough with $p_1 > p_2$. As long as $u^2(g_{21}|g_{11}, s_k^2) \equiv t_k^- \leq t \leq t_j^+ \equiv u^1(g_{11}|g_{20}, s_j^1)$, only worker 1 will apply. If $t \leq t_k^-$ then both workers would apply.

If $u^1(g_{11}|g_{21}, s_k^2) \leq t \leq u^2(g_{21}|g_{10}, s_1^1)$ with $c \in [\gamma_1(1-p_1)X, \gamma_1(1-p_2)X]$, only worker 2 will apply with the firm screening the application. Worker 1 does not apply because the firm does not have incentive to screen if it receives an application from her because c is too high. On the other hand, if $u^1(g_{11}|g_{21}, s_k^2) \leq t \leq u^2(g_{21}|g_{10}, s_j^1)$ with $c \notin [\gamma_1(1-p_1)X, \gamma_1(1-p_2)X]$, then there could potentially be multiple equilibria at the application stage. In this case the firm could get one application from either worker 1 or 2 since it might be in the interest of any one worker to apply but not both simultaneously.

If t is low enough such that $t \leq t_k^* \equiv u^2(g_{21}|g_{11}, s_k^2)$ then both workers will have the incentive to apply. If $u^2(g_{21}|g_{11}, s_k^2) \leq t \leq u^1(g_{11}|g_{21}, s_k^2)$, then it becomes a dominant strategy for worker 1 to always apply irrespective of what worker 2 is doing. If $u^2(g_{21}|g_{10}, s_j^1) \leq t \leq u^1(g_{11}|g_{20}, s_j^1)$, then only worker 1 would have an incentive to apply.

3.3 The Equilibria of the Basic Model

We establish a preliminary observation in the following lemma.

Lemma 3. The screening strategy (s_0^2, s_1^1) does not arise in equilibrium for any beliefs of the firm.

The restrictions on parameter values are such that such a screening strategy does not arise in equilibrium. This holds for any beliefs that the firm might hold in case it gets only one application off the equilibrium path.

We now characterize the different pure strategy perfect bayesian equilibrium outcomes of the model.

Proposition 1. The Perfect Bayesian Equilibrium outcomes for each set of parameter values is as follows:

(I) The firm gets two applications with both the workers applying:

If $t \leq t_k^-$ or $t \leq t_k^*$, then the equilibrium outcome is $\{s_2^{2*}, (g_{11}^*, g_{21}^*), w_2^*\}$, if $c \leq c^+$. If $c \geq c^+$ the outcome is $\{s_0^{2*}, (g_{11}^*, g_{21}^*), w_1^*\}$ if worker 1 is picked or $\{s_0^{2*}, (g_{11}^*, g_{21}^*), w_3^*\}$ if worker 2 is picked.

(II) The firm gets one application with only worker 1 applying :

If $t_k^- \leq t \leq t_j^+$ or if $u^1(g_{11}|g_{21}, s_k^2) \leq u^2(g_{21}|g_{10}, s_j^1)$ with $u^2(g_{21}|g_{11}, s_k^2) \leq t \leq u^1(g_{11}|g_{21}, s_k^2)$ with $p_1 \leq (\geq) p^*$, the equilibrium outcome is given by $\{s_{1(0)}^{1*}, (g_{11}^*, g_{20}^*), w_2^*\}$.

(III) The firm gets one application with only worker 2 applying :

If $t \in [(1-\gamma_1)(p_1 V - (1-p_1)X, (1-\gamma_1)p_1 V]$ with $c \in [\gamma_1(1-p_1)X, \gamma_1(1-p_2)X]$, then the equilibrium outcome will be $\{s_1^{1*}, (g_{10}^*, g_{21}^*), w_2^*\}$.

(IV) The firm receives one application either from worker 1 or worker 2:

If $u^1(g_{11}|g_{21}, s_k^2) \leq t \leq u^2(g_{21}|g_{10}, s_j^1) \leq u^1(g_{11}|g_{20}, s_f^1)$ with $c \notin [\gamma_1(1-p_1)X, \gamma_1(1-p_2)X]$, then there can be multiple equilibria. The outcome can be given by $\{s_{1(0)}^{1*}, (g_{11}^*, g_{20}^*), w_{2(1)}^*\}$ if $p_1 \leq (\geq) p^*$ or $\{s_{1(0)}^{1*}, (g_{10}^*, g_{21}^*), w_{2(3)}^*\}$ if $p_2 \leq (\geq) p^*$.

The proposition follows from the thresholds computed above. In equilibrium, for different parameter values, a firm may have different number of applicants ranging from no applicants to both workers applying. We characterize the equilibria in which the firm gets no applications in the Appendix. There can be an equilibrium where worker 2 is the only applicant as in (III) even though worker 1 is of a (ex-ante) higher quality. This is when the firm has incentive to screen if it is worker 2 making the application and not when it is worker 1. In (IV) there are multiple equilibria at the application stage in which either worker 1 applies or worker 2. It is never the case that, for these parameter values, both workers have the incentive to apply together.

In the next section, we establish a result which will capture the crux of the paper.

3.4 The Externality of the Marginal Applicant

The main idea we want to highlight is that the firm might be worse off getting two applications rather than one. We first provide a definition for what we mean by the marginal applicant:

Definition 2. Consider any equilibrium in which both workers apply, and increase t

while holding all other parameters constant. The marginal applicant is the first to switch from applying to not applying.

At the risk of some repetition, we establish the following lemma which identifies the marginal applicant under different parameter values. We assume that we start from an equilibrium in which both workers apply. We then identify the marginal applicant depending on the interval to which t belongs as it rises. Thus, we do not only focus on small changes in t but also identify the marginal applicant for large discontinuous changes as well[8].

Lemma 4. If lemma 1 holds or if $u^2(g_{21}|g_{11}, s_j^{2*}) \leq t \leq u^1(g_{11}|g_{21}, s_j^{2*}), j \in \{0, 2\}$ or if $u^2(g_{21}|g_{10}, s_f^1) \leq t \leq u^1(g_{11}|g_{20}, s_f^1), f \in \{0, 1\}$, then worker 2 is the marginal applicant. If $u^1(g_{11}|g_{21}, s_j^{2*}) \leq t \leq u^2(g_{21}|g_{10}, s_1^1)$ with $c \in [\gamma_1(1-p_1)X, \gamma_1(1-p_2)X]$, then worker 1 is the marginal applicant. In all other cases, there are two equilibria, in one of which the marginal applicant is worker 1 while in the other it is worker 2.

The lemma follows trivially from the analysis of the workers' application strategy. We now provide a formal definition of the externality imposed on the firm by an applicant:

Definition 3. The externality imposed by an applicant $k \in \{1, 2\}$ is defined by $\mathcal{E}(k) \equiv \pi(s_h^{2*}|g_{11}^*, g_{21}^*) - \pi(s_l^1|g_{-k1}, g_{k0})$, which is the difference in the payoff $\pi(s_h^{2*}|g_{11}^*, g_{21}^*)$, received by the firm in the equilibrium $\{(s_h^{2*}, s_l^{1*}), g_{11}^*, g_{21}^*\}, h \in \{0, 1, 2\}, l \in \{0, 1\}$ and the payoff $\pi(s_l^1|g_{-k1}, g_{k0})$ it would have got if it had received only one from applicant $-k$ such that if $k = 1(2)$ then $-k = 2(1)$.

Now, consider any equilibrium outcome in which the firm gets two applications. For any such equilibrium there is a range of parameter values such that , if t increases sufficiently to lead the firm to receive one application (with the marginal applicant switching from applying to not applying) instead of two, it will receive a higher payoff. Thus the marginal applicant, denoted by h, can have a negative externality, $\mathcal{E}(h) < 0$, on the firm. The result displays the interaction of screening costs, quality heterogeneity and the option value of having an additional application. We state this result more formally for each equilibrium with two applications in the following proposition.

Proposition 2. (I) If the equilibrium is $\{(s_0^{2*}, s_0^{1*}), g_{11}^*, g_{21}^*\}$, then $\mathcal{E}(h) < 0$ (> 0) if $h = 2(1)$.

(II) If the equilibrium is $\{(s_2^{2*}, s_1^{1*}), g_{11}^*, g_{21}^*\}$, then $\mathcal{E}(h) < 0$ (> 0) if $h = 2(1)$ and if $c \in [\frac{1}{1-p}\gamma_1 V(p_2 - p_1 p_2), c^+]$.

(III) If the equilibrium is $\{(s_2^{2*}, s_0^{1*}), g_{11}^*, g_{21}^*\}$, then $\mathcal{E}(h) < 0$ (> 0) if $h = 2(1)$ and if $c \in [\frac{1}{2-p}\gamma_1 V(p_2 - p_1 p_2), c^+]$.

The method to show this is quite straightforward. First, we need to compare the difference in the payoff that the firm receives when it gets two applications in equilibrium and with the payoff it would have got had it received only one application, keeping the screening strategy unchanged. This provides us with a condition on the parameters p_i or

[8]This is based on the idea that the internet might have reduced the cost of making an application in a large discontinuous way and not necessarily in a marginal fashion.

c. Using these conditions, we then need to verify whether the initial constraints on the firm's equilibrium strategies still hold ex-post.

Looking at $\{(s_0^{2*}, s_0^{1*}), g_{11}^*, g_{21}^*\}$, the firm will be trivially worse off if worker 2 is the marginal applicant because of the statistical decline in the average quality of the pool of applicants. Figure 3.1 shows how the firm's payoff evolves with the cost of making an application when lemma 1 holds.

If t is too high then no worker applies and if $t \leq t_0^+$, then only worker 1 applies to the firm. If $t \leq t_0^-$, there is a discontinuous fall in the expected payoff of the firm. If the marginal applicant is worker 1 then the firm will be better off because the statistical increase in the quality of applicants.

This can be seen in figure 3.2, where the firm does not screen and lemma 1 does not hold. In this case there can be multiple equilibria with the marginal application coming from either worker 1 or worker 2. Thus, there is a non-monotonic relationship beteween the firm's payoff and the cost of making an application.

Next, looking at $\{(s_2^{2*}, s_1^{1*}), g_{11}^*, g_{21}^*\}$, the firm will be worse off if the marginal applicant is worker 2 and if the quality effect outweighs the option value effect. . Additionally, $\frac{\partial}{\partial p_2}\left(\frac{1}{1-p}\gamma_1 V\left(2p - p^2 - p_1\right)\right) > 0$ which means that if the intrinsic quality of worker 2 increases, the firm is less likely to be worse off getting an additional application.

If the firm screens applications and it gets the marginal application from worker 1 then it is always better off. The additional application provides an increase in the average quality of the pool of applicants. It also provides an additional option value in case when it screens a worker and she turns out to be type b. The graphical intuition of this case is very similar to figures 3.1 and 3.2.

3.5 Robustness

In this section, we briefly look at a couple of assumptions made in the basic model and examine whether the results are robust even if we relax them.

3.5.1 Limited Liability

In the model, we assume that there are no limited liability concerns for the worker. Suppose worker 1 gets hired by the firm when it has not screened. The ex-ante payoff to the worker is $(1-\gamma_1)(p_1 V - (1-p_1)X)$ while ex-post the worker might end up with $(1-\gamma_1)V$ or $-(1-\gamma_1)X$. We assume that the worker has the capacity to share the cost $-X$ if it turns out that $\theta_1 = b$. Suppose the worker was protected by limited liability and this was not the case. Then the remuneration profile for the worker would be $(1-\gamma_1)V$ if $\theta_i = a$ and 0 if $\theta_i = b$ and thus the payoff to the firm becomes $\gamma_1 V$ if $\theta_i = a$ and $-X$ if $\theta_i = b$. We can show that with this profile of payoffs for the workers and the firm, the results will remain qualitatively the same. As an illustrative example, consider the situation where the firm does not screen irrespective of the number of applications it receives. The payoff to the firm if receives two applications becomes $\frac{1}{2}(p_1\gamma_1 V-(1-p_1)X)+\frac{1}{2}(p_2\gamma_1 V-(1-p_2)X)$. The firm gets a share $0<\gamma_1<1$ of V if the worker turns out to be type a while it gets $\gamma_1=1$ share of $-X$. If in addition, when the firm gets only one application it is from worker 1 its payoff becomes $p_1\gamma_1 V-(1-p_1)X$. We can easily see from these payoffs that the trade-offs at play in terms of the externality are exactly the same. The firm will be trivially worse off if the marginal applicant is worker 2. The results are driven by the heterogeneity in worker quality with $p_1>p_2$. We can check that the results for different screening strategies are qualitatively the same.

3.5.2 Different Reservation Values

In the basic model, we have assumed that both worker 1 and worker 2 have the same reservation values which were, for simplicity, normalized to zero. It could be argued that worker 1 may have a higher reservation value than worker 2. Keeping this in mind, we could allow for a reservation value for worker 1 to be given by $\bar{w}>0$. Thus, worker 1 would earn a premium over the wage it would have received with a 0 reservation value exactly because $\bar{w}>0$. As long as $\bar{w}$ is not too large, the firm would still prefer to hire worker 1 rather than worker 2. As an example, suppose the firm found it efficient to not screen applications. Then, the firm would prefer to pick out worker 1 rather than worker 2 if $\gamma_1(p_1V-(1-p_1)X)-\gamma_1\bar{w}>\gamma_1(p_2V-(1-p_2)X)$. This inequality will hold if $\bar{w}<(p_1-p_2)(V-X)$. In general, we can find a reservation value which is not too high such that the firm still prefers worker 1 to worker 2. This will diminish the surplus earned by the firm and would result in a transfer to worker 1 but will not change the results if the value is still low enough.

4 The General Model

We now setup a general model with a large but finite number of heterogenous workers and firms. Similar to the basic setup, each worker can be a potentially good match with any of the firms depending on their intrinsic abilities. Ex-post, a worker will be a good match with only one of the firms. The intrinsic ability, defined over a partition of an interval, is revealed if the firm screens applicants. It is because of this that workers get multiple offers only if the firms do not screen. We allow for firms to compete for a worker's services if a worker gets multiple offers. The firms compete in the Bertrand sense for a worker by offering a higher wage. We characterize the symmetric equilibrium of the general model and analyse the existence of a negative externality of the marginal applicant.

4.1 The Setting

4.1.1 Preferences of Workers and Firms

We denote workers by $j\in\{1,2,....,L\}$ and the firms by $m\in\{1,2,...,K\}$ with $L>K$.

The intrinsic ability of a worker j is given by $\theta^j \in \Theta = [0,1]$. We consider a partition of $\Theta = \cup_{m=1}^{K}\Theta_m$ such that $\Theta_k \cap \Theta_h = \phi$ if $k \neq h$. Let $\Theta_1^j = [0, \theta_1^j]$ and $\Theta_K^j = [\theta_{K-1}^j, 1]$. Let the measure of Θ_m^j be $\mu(\Theta_m^j) > 0, \forall j, m$. This means that each worker has a positive probability to be a good match (generate a positive surplus) with any of the firms.

The intrinsic ability (or type) of the worker j has a distribution function $F(\theta^j)$ which admits a density $f(\theta^j) = F'(\theta^j)$. The value of the match to firm m by hiring a worker of type j is:

$$v(\theta^j \in \Theta_m^j) = V(\theta^j)$$
$$v(\theta^j \notin \Theta_m^j) = -X(\theta^j)$$

Thus, firm m would like to hire worker j if she is of type $\theta^j \in \Theta_m^j$ and not if she is of any other type. $V(\theta^j)$ and $X(\theta^j)$ are piece-wise continuous (and hence integrable) with $\frac{\partial V(\theta)}{\partial \theta^j}|\theta^j \in \Theta^j, \frac{\partial X(\theta)}{\partial \theta^j}|\theta^j \notin \Theta^j > 0$.

The information structure remains the same as in the basic model with the probability distribution over types being common knowledge and firms forming a belief, $\lambda(f|n)$ over the identity of the workers who have applied. Thus, a particular distribution over types for each worker will define an initial ranking of firms over workers and vice-versa in terms of the gross surplus generated due to the match. This ranking defines an ordering for the workers over firms which they would prefer if they faced no competition from any other applicant. Their eventual application strategy will, of course, depend on the number of other applicants applying to a particular firm.

A worker , as before, can apply to a firm at a cost $t > 0$. We allow the workers to send an arbitrary number of applications with a maximum of one per firm. The cost of evaluating applications is again captured by $c > 0$.

We focus on symmetric screening and application strategies where the workers choose to send the same number of applications and each firm adopts the same screening strategy in equilibrium. We denote a profile screening actions by the firms as a K dimensional vector $\mathbf{s_N} = (s_1, ..., s_K)$ with $s_m = s_m(n_m)$ where $n_m \in \{0, 1, 2....., L\}$ is the number of applications received by firm m. With a slight abuse of notation, we denote $\mathbf{s_N}$ by $(s_m(n_m), \mathbf{s}_{-m})$ while focussing on firm m's screening action while the other $-m$ firms screening strategy is summarized by $\mathbf{s}_{-m}$. Similarly, we can denote a symmetric application profile as a L dimensional vector $\mathbf{g_h} = (g_h^1, g_h^2, ..., g_h^L)$ with $h \in \{0, 1, 2, ..., K\}$. We denote $\mathbf{g_h}$ by $(g_j^k, \mathbf{g}_{-j}^h)$ while focussing on worker j's application action while all the other $-j$ workers make h applications.

4.1.2 Payoffs

Let $\pi : \prod_{j=1}^{L}\{g_0^j, g_1^j, ..., g_K^j\} \times \prod_{m=0}^{L}\{s_m(n_1), ..., s_m(n_K)\} \to R$, denote the payoff function for the firm. Let $U : \prod_{j=1}^{L}\{g_0^j, g_1^j, ..., g_K^j\} \times \prod_{m=0}^{L}\{s_m(n_1), ..., s_m(n_K)\} \to R$, denote the payoff function of the worker. As an illustration, in terms of notation, $\pi((0, L)_m|\mathbf{g_K}, (\mathbf{L}, \mathbf{L})_{-\mathbf{m}})$

is the payoff that firm m gets when it receives L applications and it does not screen $((0, L)_m)$ while the other $-m$ firms screen all the applications $((\mathbf{L}, \mathbf{L})_{-\mathbf{m}})$ when each worker makes K applications each. For the workers $U\left(g_K^j|\mathbf{g}_{\mathbf{K}}^{-\mathbf{j}}, (\mathbf{L}, \mathbf{L})_{\mathbf{N}}\right)$ denotes the payoff to worker j when it makes K applications and so do the other workers while all the firms screen all the applications $((\mathbf{L}, \mathbf{L})_{\mathbf{N}})$.

Define $E(M_m^j) \equiv \int_{\Theta_m^j} V(\theta^j)dF(\theta^j) - \sum_{k\neq m}\int_{\Theta_k^j} X(\theta^j)dF(\theta^j)$. Assume that $E(M_m^g) > 0$ where worker g ranks the lowest on firm m's list . This ensures that the expected match value if the firm does not screen is positive. Also, c being constant implies that it will be in the interest of a firm to screen all applications if it does decide to screen.

The timing of the game remains similar except whether the firm is able to hire a worker or not depends also on the outcome of the competition if the worker gets multiple offers. In this setting, since we consider a partition over Θ, a worker can get multiple offers only if the firms do not screen. Thus, if a worker gets multiple offers then firms compete in a Bertrand manner over the wage offered. We assume that a firm knows the identity (but not the type) of the worker if it does compete for her[9]. If a worker only gets one offer then there is bilateral bargaining over the realized surplus as in the basic model with γ being the firm's share.

We still focus on perfect Bayesian equilibria in pure strategies. We consider players actions, beliefs and payoffs in detail solving from the back.

4.2 Backward Induction

4.2.1 Period 3: Wage Setting Process

The way the wage is determined now differs from the basic model if a worker gets multiple offers. If a worker gets multiple offers, then firms engage in Bertrand competition for her services by offering a higher wage. In this competition, what matters is the expected value of the surplus of its highest and second highest offers. Consider firm m which provides the highest value amongst the offers made to worker j. Let firm s be the second highest option available. Since each firm knows the index of the worker being competed for, firm m will be able to hire at a wage $w^* = E(M_s^j)$ with its own payoff being $E(M_m^j) - E(M_s^j)$.

If the firm screens and decides to hire or if the worker does not get multiple offers when the firm does not screen then, as in the basic model, there is bilateral bargaining over the realized value of the surplus. In that case, the firm just pays out a share γ of the gross surplus created.

[9]This is to get rid of any informational asymmetry between the firms and the worker. Also, this gives information to the firms about which wage level to compete till against the other firm.

4.2.2 Period 2: Screening Actions of the Firms

We characterize the payoffs to the firms when the cost of applying is low enough such that each worker makes k applications.

The expected payoff to firm m if each firm gets l applications and they decide not to screen is: $\pi((0,l)_m|\mathbf{g_k},(\mathbf{0},\mathbf{l})_{-\mathbf{m}}) =$

$$\sum_{j=1}^{l}\{\frac{1}{l}\left(1-\frac{1}{l}\right)^{k-1}\gamma E(M_m^j)+\sum_{i=1}^{k-1}\mathbf{1}_{\{m\succ s\}_j}$$
$$[\frac{1}{l}\binom{l}{i}\frac{1}{l^i}\left(1-\frac{1}{l}\right)^{k-i}(E(M_m^j)-E(M_s^j))]\}$$

This expression follows from the discussion above. If no other firm picks out worker j, then firm m gets a share γ of the surplus created. This happens with probability $\frac{1}{l}\left(1-\frac{1}{l}\right)^{k-1}$. If worker j does get multiple offers, firm m will only be able to hire her if it provides the highest offer. This is denoted by $\{m \succ s\}_j$, which is equivalent to $E(M_m^j) > E(M_s^j)$, where s makes the second highest offer to the worker. There is a $\binom{l}{i}\frac{1}{l^i}\left(1-\frac{1}{l}\right)^{k-i}$ probability of exactly i other firms picking out the same applicant j.

Now suppose each firm gets l applications and they decide to screen for θ^j. As mentioned before, since we consider a partition of Θ, a worker can only be a good match with only one particular firm and hence there will be no ex-post competition for the worker. The screening procedure resembles the process of drawing balls from an urn without replacement. We have already noted that conditional on screening, the firm finds it efficient to always screen all l applications.

Since firm m gets l applications, there can be $l!$ sequences in which applications are picked out which define the expected payoff for the firm: $\pi((l,l)_m|\mathbf{g_k},(\mathbf{l},\mathbf{l})_{-\mathbf{m}})$=

$$\frac{1}{l!}\sum_{l!}\left\{\sum_{h=0}^{l-1}\frac{1}{l-h}(1-\int_{\Theta_m^i}dF(\theta))^h(\gamma\int\limits_{\Theta_m^k}V(\theta)dF(\theta)-c)\right\}$$

In the first draw of applications, if worker i is picked out then the expected payoff is $\frac{1}{l}(\gamma\int_{\Theta_m^k}V(\theta)dF(\theta)-c)$. Thus, there is a probability $\frac{1}{l}(1-\int_{\Theta_m^i}dF(\theta))$, that the firm does find say worker i with $\theta^i\in\Theta_m^i$ in the first draw of applications. The firms keeps evaluating applications at a cost c till it does not find a $\theta^j\in\Theta_m^j$. The above expression is with a slight abuse of notation because the product $(1-\int_{\Theta_1^i}dF(\theta))^h$ as well as the worker picked out at any draw will depend on the exact identity of applicants picked out in previous draws.

We will particularly focus on the case when the cost of applying is low enough such that all workers apply to all firms. If $n_m = L, \forall m$, firm m will want to not screen any applicants given that other firms do not screen as well if :

$$\pi((0,L)_m|\mathbf{g_K},(\mathbf{0},\mathbf{L})_{-\mathbf{m}}) \geq \pi((L,L)_m|\mathbf{g_K},(\mathbf{0},\mathbf{L})_{-\mathbf{m}})$$

This inequality gives a threshold on $c \geq \hat{c}_m \equiv \frac{1}{Q}(\sum_h I_h \int_{\Theta_m^h} V(\theta)dF(\theta) + \sum_{k\neq m}\sum_h G_h \int_{\Theta_k^h} X(\theta)dF(\theta))$. The exact form that the coefficients I_h and G_h take are in the appendix.

Similarly, firm m will want to screen L applications when all the other firms are as well if $\pi((L,L)_m|\mathbf{g_K},(\mathbf{L},\mathbf{L})_{-\mathbf{m}}) \geq \pi((0,L)_m|\mathbf{g_K},(\mathbf{L},\mathbf{L})_{-\mathbf{m}})$. This gives us another threshold given by $c \leq c_m^+ \equiv \frac{1}{Q}(\sum_h S_h \int_{\Theta_m^h} V(\theta)dF(\theta) + \sum_{k\neq m}\sum_h T_h \int_{\Theta_k^h} X(\theta)dF(\theta))$ below which firm m will find it efficient to screen given that the other firms are screening for θ^j as well. The coefficients S_h and T_h will be defined in same way as I_h and G_h above.

We can define cost thresholds for the firms if each worker sends $K-k$ applications (for an arbitrary $K > k > 0$). The cost threshold for the firms to not screen is given by $c \geq \hat{c}_m^k, \forall m$. If the firms screen then the thresholds are given by $\hat{c}_m^{k'}$. To define a symmetric equilibrium, we just need that each worker sends the same number of applications which does not imply that these applications are distributed uniformly over the firms. In this case, each firm might not end up getting the same number of applications but the fundamental trade-offs involved are exactly the same as highlighted above.

4.2.3 Payoffs of the Workers

We now look at the payoffs that worker j would get from making k applications and the conditions under which it would be efficient to do so.

Suppose the firms were not screening for θ^j, then if all the other workers were applying to k firms, worker j's payoff would be: $U\left(g_k^j|\mathbf{g_k^{-j}},(\mathbf{0},\mathbf{l})_{\mathbf{N}}\right) =$

$$\frac{1}{l}\left(1-\frac{1}{l}\right)^{k-1}\gamma\sum_{h=1}^{k}E(M_h^j) + \sum_{i=2}^{k}\binom{l}{i}\frac{1}{l^i}\left(1-\frac{1}{l}\right)^{k-i}E(M_s^j) - kt$$

The payoffs follow the screening procedure. There are $\binom{l}{i}\frac{1}{l^i}\left(1-\frac{1}{l}\right)^{k-i}$ ways in which worker j can be picked out by i firms. Worker j chooses the offer from the firm which wins the ex-post competition. This firm pays out as a wage the surplus generated with the firm which provides the second highest offer which by convention is given by s. The worker will pay a total cost of kt for making k applications.

Again, we focus on the case where each worker applies to all firms. Given that firms do not screen and worker j will make K applications rather than $K-1$ if:

$$U\left(g_K^j|\mathbf{g_K^{-j}},(\mathbf{0},\mathbf{L})_{\mathbf{N}}\right) \geq U\left(g_{K-1}^j|\mathbf{g_K^{-j}},(\mathbf{L},\mathbf{L})_{\mathbf{N}}\right)$$

Let applying to firm g give the lowest ex-ante payoff to worker j, then this will provide a lower bound on the cost of making an application such that it is optimal to make K applications. This lower bound is given by $t \leq \hat{t}_g \equiv \frac{1}{L}\left(1-\frac{1}{L}\right)^{K-1}(E(M_g^j))$ since j will accept firm g's offer only if she gets no other offer.

Next, we look at the payoff to the worker when the firm does screen for intrinsic ability. If all the other workers were applying to k firms, worker j's payoff would be:$U\left(g_k^j|\mathbf{g_k^{-j}},(\mathbf{l},\mathbf{l})_\mathbf{N}\right) =$

$$(1-\gamma)\sum_{m=1}^{k}\frac{1}{l!}\sum_{l!}\left(\sum_{h=0}^{l-1}\frac{1}{l-h}(1-\int_{\Theta_m^i}dF(\theta))^h(\int_{\Theta_m^j}V(\theta)dF(\theta))\right)-kt$$

This expression follows the evaluation procedure adopted by the firms. Worker j could, from an ex-ante perspective, be a good match for any of the k firms. The worker could be picked in the first draw or in the second draw and so on. This can happen for each of the firms that the worker has applied to which leads to the expression above.

Worker j will make K applications when the firms screen if $U\left(g_K^j|\mathbf{g_K^{-j}},(\mathbf{L},\mathbf{L})_\mathbf{N}\right) \geq U\left(g_{K-1}^j|\mathbf{g_K^{-j}},(\mathbf{L},\mathbf{L})_\mathbf{N}\right)$. This implies that $t \leq t_g^{++} \equiv (1-\gamma)(\sum_{h=0}^{L-1}\frac{1}{L-h}(1-\int_{\Theta_g^i}dF(\theta))^h(\int_{\Theta_g^j}V(\theta)dF(\theta)))$. This again, provides a lower bound on the willingness of worker j to make its Kth application. We can define such thresholds for each of the workers $t_j^{++}, \forall j$.

If the cost of making an application $t \in [\ t_j^{k^*+}, t_j^{k+}], \forall j$ with the firms not screening then each worker sends $K-k$ applications. If the firms screen then the interval is given by $t \in [t_j^{k'^*+}, t_j^{k'+}]$.

4.3 The Equilibrium of the General Model

We focus first on equilibria where all workers apply to all the firms and then characterize a symmetric equilibrium where workers send applications which are striclty less than K.

Proposition 3. The symmetric pure strategy equilibrium outcomes of the game are as follows:

(I) All workers apply to all the firms-

(a) Each worker applies to all firms and the firms screen if $t \leq t_j^{++}, \forall j$ and $c \leq$min $\{c_m^+, \frac{\gamma}{L}\sum_{j=1}^{L}\int_{\Theta_m^j}V(\theta^j)dF(\theta^j)\}\ \forall m$.

(b) Each worker applies to all firms and the firms do not screen if $t \leq \hat{t}_j, \forall j$ and $c \geq$min $\{\hat{c}_m, \frac{\gamma}{L}\sum_{j=1}^{L}\int_{\Theta_m^j}V(\theta^j)dF(\theta^j)\}\ \forall m$.

(II) Workers apply to a subset of firms-

(c) All worker will make $0 < K-h < K$ applications and the firms screen if $t \in [t_j^{h'^*+}, t_j^{h'+}]$ and $c\leq$min $\{\hat{c}_m^{h'}, \frac{\gamma}{L}\sum_{j=1}^{L}\int_{\Theta_m^j}V(\theta^j)dF(\theta^j)\}$.

(d) All worker will make $0 < K-h < K$ applications and the firms do not screen if $t \in [t_j^{h'^*+}, t_j^{h+}]$ and $c\geq$min $\{\hat{c}_m^{h}, \frac{\gamma}{L}\sum_{j=1}^{L}\int_{\Theta_m^j}V(\theta^j)dF(\theta^j)\}$.

The decisions to apply and screen depend on the different parameter values. In part (I), for example, the cost of making an application is low enough such that all workers apply to all firms taking into account their screening strategy. The firms screen or not depending on the value of c as well as the measure of the intrinsic ability of individuals who have applied. For the firm to screen, it must be that $c \leq \frac{\gamma}{L}\sum_{j=1}^{L}\int_{\Theta_m^j}V(\theta^j)dF(\theta^j)$ such that it is low enough which gives the firm an incentive to do so. In part (II), the decision by firms and workers depend on a very similar logic. Though, as pointed out earlier, each firm might not get the same number of applications in equilibrium due to different rankings of workers over firms. The equilibrium in which no (or some) firm gets any applications can be characterized exactly as in the basic model. We can isolate unique out of equilibrium beliefs to deal with issues of multiplicity of equilibria.

4.4 The Externality in the General Model

We consider a symmetric equilibrium in which each firm m gets l applications. We define the marginal applicant and the externality in the same way as in the basic model.

Definition 4. Consider any equilibrium in which each firm gets l applications, and increase t while holding all other parameters constant. The marginal applicant, for firm m, is the first to switch from applying to not applying.

Definition 5. The externality imposed by an applicant c is defined by

$$\mathcal{E}(c) \equiv \pi((q,l)_m|\mathbf{g_k^*}, \mathbf{s^*_{-m}}(\mathbf{l})) - \pi((h,l-1)_m|(g_{j,k-1}, \mathbf{g_{-j,k}}), \mathbf{s_{-m}}(\mathbf{l}))$$

, which is the difference in the payoff $\pi((q,l)_m|\mathbf{g_k^*}, \mathbf{s^*_{-m}}(\mathbf{l}))$, received by the firm in the equilibrium $\{((q,l)^*_m, (h,l-1)^*_m), \mathbf{g_k^*}, \mathbf{s^*_{-m}}(\mathbf{l})\}$, with $h=0(l-1)$ if $q=0(l)$and the payoff $\pi((h,l-1)_m|(g_{j,k-1}, \mathbf{g_{-j,k}}), \mathbf{s_{-m}}(\mathbf{l})$ it would have got if it had received only $l-1$ aplications[10].

Consider any equilibrium outcome in which each firm gets l applications. For any such equilibrium there is a range of parameter values such that , if t increases sufficiently to lead firm m to receive $l-1$ applications (with the marginal applicant switching from applying to not applying) instead of l, it will receive a higher payoff. Thus the marginal applicant, denoted by y, can have a negative externality, $\mathcal{E}(y)< 0$, on the firm. Denote by $\frac{l}{2}$ the index of the average applicant from the existing pool based on firm m's initial ranking. Define

$$\nabla_{l-1} \equiv \sum_{j=1}^{l-1} \frac{1}{l-1}\left(1-\frac{1}{l}\right)^{k-1} \gamma E(M_m^j) - \sum_{j=1}^{l}[\frac{1}{l}\left(1-\frac{1}{l}\right)^{k-1} E(M_m^j)]$$

which will capture what will be called the quality effect and

$$\triangle_{l-1} \equiv \sum_{j=1}^{l-1}\sum_{i=1}^{k-1} \mathbf{1}_{\{m\succ s\}_j}[\frac{1}{l-1}\binom{l}{i}\frac{1}{l^i}\left(1-\frac{1}{l}\right)^{k-i}(E(M_m^j)-E(M_s^j))]$$

$$-\sum_{j=1}^{l}\sum_{i=1}^{k-1} \mathbf{1}_{\{m\succ s\}_j}[\frac{1}{l}\binom{l}{i}\frac{1}{l^i}\left(1-\frac{1}{l}\right)^{k-i}(E(M_m^j)-E(M_s^j))]$$

[10]Note that $\{((q,l)^*_m, (h,l-1)^*_m), \mathbf{g_k^*}, \mathbf{s^*_{-m}}(\mathbf{l})\}$ does not completely define the equilibrium since we would need to define how many applications each firm would screen for any number that it might receive.

which will capture the competition effect in the proposition which follows:

Proposition 4. (I) If the equilibrium is $\{((0,l)^*_m,(0,l-1)^*_m),\mathbf{g}^*_{\mathbf{k}},(\mathbf{0},\mathbf{l})^*_{-\mathbf{m}}\}$, then $\mathcal{E}(y)<0$ if $\nabla_{l-1}+\triangle_{l-1}<0$.

(II) If the equilibrium is $\{((l,l)^*_m,(l-1,l-1)^*_m),\mathbf{g}^*_{\mathbf{k}},(\mathbf{l},\mathbf{l})^*_{\mathbf{m}}\}$, then $\mathcal{E}(y)<0$ if $h\geq\frac{l}{2}$ and if $c\geq\frac{1}{\triangle}\sum_h B_h\int_{\Theta^h_m}V(\theta)dF(\theta)$ otherwise $\mathcal{E}(y)>0$.

The method to prove this proposition is very similar to what we saw in the basic model. The intuition behind (I) is different because of the ex-post competition. If the firm does not screen in equilibrium, then there two effects of the marginal applicant. As in the basic model, there is a quality effect. A marginal applicant of greater (less) than average quality increases (decreases) the quality of the pool of applicants. This quality effect is captured by ∇_{l-1}. With ex-post competition, this quality effect interacts with the competition effect-captured by $\triangle_{l-1}$. If firm m does not rank 'high' or if the marginal applicant has outside options which will lead to a high wage being paid as a result of the competition, then the firm might still be worse off even with higher than average applicant. Additionally, firm m might be better off with a marginal applicant whose ability is lower than the average quality of the existing pool. This happens if the decline in quality is outweighed by weak competition from other firms for this particular candidate.

The trade-off in (II) is the same as in the basic model. If in equilibrium all the firms screen, firm m can still be worse off with the additional application if the option value provided by the marginal applicant is outweighed by the increased screening cost and the statistical decline in average quality. If the marginal applicant has an ex-ante ability higher than the average ability of the existing pool of applicants then it always imposes a positive externality if the firm screens.

This exercise carried out above can be done for any arbitrary number of applications made by the workers and screened by the firms.

5 Extensions

In this section, we will look at certain assumptions that were made in the model and try to relax them one at a time to see how the results would change.

5.1 Pricing Applications (Very Preliminary)

First, we look to endogenize the cost of making an application such that the firm has control over it. In reality we see examples of firms/institutions controlling this cost. This can be a monetary cost like an application fee or can come from making the application a very long and time consuming process. This is one of the main reasons why the University of Chicago for a lot of years got significantly less applications for its undergraduate class compared to other similarly ranked institutions. They had what was called the "Uncommon Application," in contrast to the Common Application, the standardized form that allows students to apply to any of hundreds of participating colleges (New York Times, November

2010). Its application forced applicants, apart from other things, to write long and creative essays on topics such as "If you could balance on a tightrope, over what landscape would you walk? (No net)". Such exercises were used to weed out candidates who would not be a good match for the institution.

We allow firm j to set a cost e_j on top of t such that $t + e_j$, with $e_j \geq 0$ is the total cost of making an application. To keep things simple, we assume that a firm can impose this on the applicants costlessly. Now, before the worker makes the applications, the firms simultaneously decide on e_j . Upon observing the total cost $t + e_j$, workers decide to apply or not (and how many to apply to). The firms then decide to either not screen or screen (and decide how many to screen).

We can solve the model working backwards and compute the externality for each of firms. If the firm faces a negative externality it has the option of raising the price paid by the workers for the applications made which would restrict the number of applications it gets. Now, the firms have access to another screening device which was earlier not in their control.

We consider the preference profile highlighted in figure 4.1 and assume that t was low enough such that each firm was getting a large number of applications in a symmetric equilibrium. We can show that there is an outcome where each firm sets its price so high that it gets only one application from its most preferred worker.

Proposition 5. If preferences are as in figure 4.1, then there is a set of prices $\{t + e_1, t + e_2, ..., t + e_K\}$ such that worker j applies to only firm j,$\forall j \in J, M$ in a symmetric equilibrium.

To arrive at this result we need, first, that the applicant should have the incentive to apply in the first place and thus, e_j has to ensure that she gets a weakly positive payoff in equilibrium. Then, applicant j should apply only to firm j and not to any other firm. Similarly, no other worker $h \neq j$ should want to apply to firm j. We show that these inequalities can be satisfied together.

If preferences are as in figure 4.4, even then we can find a set of prices (with a bit more algebra) such that worker j applies to only firm j,$\forall j \in J, M$ in a symmetric equilibrium. This means that firm 1 gets the best worker, firm 2 gets the second best worker and so on.

These implications discussed in the context of the labour market are similar in spirit to policies debated or adopted in other areas such as taxing e-mails to reduce spam. In the same way, Ayres and Nalebuff (2003) have suggested individual consumers should be allowed to set their own personal prices to be contacted by telemarketers.

6 Conclusion

In this paper we address the question of whether a potential employer can be worse off with more applications than less. We show that, contrary to conventional wisdom, this can happen in a variety of cases. The basic mechanism which drives the results is that as application costs decline if the additional applications are made by a lower quality applicant then the firm can be worse off. We provide a flexible framework with an endogenous screening procedure in which a particular firm can be better or worse off with an additional application. We show that even when the firm has access to a screening technology, it can be worse off. If worker heterogeneity is high enough, then the additional application to the firm always comes from a worker who has a lower ex-ante quality which makes the possibility of a negative externality greater.

In the basic model, we show that there only one particuler range of parameter values such that a lower ex-ante quality worker will be the only applicant to a firm with probability 1. The only other ocassion when a firm gets a low quality worker as the only applicant is in a situation where there is multiple equilibria and either worker could have made the application to that firm. Thus, the occasions when a negative externality of an additional application can arise are far greater than when a positive externality occurs. We then generalize the model to allow for a large number of firms and workers to show that the basic insights still go through. In the general model, we also allow for competition for a worker's service between the firms if she receives more than one offer.

This question has yet to be addressed in the literature and we feel that it is extremely pertinent especially when looked at in the context of the digital age. With communication costs declining rapidly, individuals are often swamped with much more information than they would want. This phenomenon has also been noticed in the context of firms and other institutions receiving many more applications than they would ideally want as highlighted above.

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Appendix

Lemma 1: The event (g_{11}, g_{20}, s_0^1) leads to the value of the surplus being $p_1V - (1 - p_1)X$ which when plugged into the Nash bargaining problem yields $w_1^*.(g_{11}, g_{20}, s_1^1)$ and (g_{11}, g_{21}, s_2^2) lead to a surplus V such that $w_2^* = argmax_w\ \gamma_1 log(V - w) + (1 - \gamma_1)log(w)$. The cost c is treated as sunk. $w_2^* = (1 - \gamma_1)V \geq w_1^* = (1 - \gamma_1)(p_1V - (1 - p_1)X)$ since $X \geq 0$. The event (g_{11}, g_{21}, s_0^1) leads to a wage $w_1^* = (1 - \gamma_1)(p_1V - (1 - p_1)X)$ for worker 1 and $w_3^* = (1 - \gamma_1)(p_2V - (1 - p_2)X)$ for worker 2 since the values are realized in period 3. Since $X \geq 0$ and $p_1 \geq p_2$ we can establish the result as stated in the lemma. QED

Lemma 2: To establish the lemma, we look at the application strategies for different parameter values.

If $u^1\left(g_{11}|g_{21}, s_k^2\right) > t > u^2(g_{21}|g_{10}, s_j^1), \forall j, k \in \{0, 1, 2\}$, then it is a dominant strategy for worker 1 to always apply. The sufficient condition for this to hold is $(p_1 - p_2)V \geq (1 - p_1)X$ which requires $p_1 - p_2 > 0$ to be large enough. As the cost goes down and is low enough, $t < u^2(g_{21}|g_{11}, s_k^2)$ worker 2 will apply. Since the match surplus is determined ex-post, the beliefs of the firm influence the screening strategy of the firm and hence the worker needs to take this into account. One can notice, additionally, that if t is such that $t < u^1\left(g_{11}|g_{21}, s_k^2\right) < u^2(g_{21}|g_{10}, s_j^1)$, then it becomes a dominant strategy for worker 1 to always apply. QED

Lemma 3. The screening strategy (s_0^2, s_1^1) implies that $c \leq (1-p_i)\gamma_1 X$ and that $c \geq c^+ \equiv \left(\frac{\gamma_1}{2-p}\right)((p - p_1p_2)V + (1-p)X)$ must hold in equilibrium. To be satisfied simultaneously we need $c \in [c^+, (1-p_2)\gamma_1 X] \neq \{\phi\}$. We focus on $p_i = p_2$ to look at the mildest restriction possible. One can check that it is never the case that $c^+ \leq c \leq (1-p_2)\gamma_1 X$ with $\frac{p_1p_2}{1-p_2}V > X$- which is the intial assumption made on parameter values. Thus, there will be no belief of the firm which will allow such a strategy to hold in equilibrium. QED

Proposition 1: We will establish the result by examining the different cases one at a time.

(I) We consider cases depending on whether $u^1(g_{11}|g_{21}, s_k^2) \gtreqless u^2(g_{21}|g_{10}, s_j^1)$. Consider $u^1(g_{11}|g_{21}, s_k^2) \geq u^2(g_{21}|g_{10}, s_j^1)$. If $t \leq t_k^-$ then it is in the interest of both workers to apply. If either worker deviates, it will get a payoff $\equiv 0$. Conditional on getting two applications the equilibrium strategy for the firm is to screen for θ_i if $\pi(s_2^2|g_{11}, g_{21}) \geq \pi(s_0^2|g_{11}, g_{21})$, that is if $c \leq c^+$. When $u^1(g_{11}|g_{21}, s_k^2) \leq u^2(g_{21}|g_{10}, s_j^1)$ and if $t \leq t_k^*$, the same logic applies for the workers again. The firm, again, depending on c would decide to screen or not. The belief on the equilibrium path is $\lambda^*(f = (1,2)|n = 2) = 1$ since this information set is a singleton. Off the equilibrium path, there is a non singleton information set for which we need to specify beliefs. The belief that the firm holds off the equilibrium path, that it is worker 1 who has applied when it gets only one application is defined as $\lambda^{*'}(.)$. If the worker deviates she receives zero, we can have $\lambda^{*'}(.) \in (0,1)$ which will sustain this equilibrium outcome for the given parameter values due to equilibrium domination . To have an equilibrium screening strategy of (s_2^{2*}, s_1^{1*}) it has be that:

$$\lambda^{*'}(p_1\gamma_1 V - (1-p_1)\gamma_1 X) + (1-\lambda^{*'})(p_2\gamma_1 V - (1-p_2)\gamma_1 X) \leq$$

$$\lambda^{*'}(p_1\gamma_1 V - c) + (1-\lambda^{*'})(p_2\gamma_1 V - c)$$

$$\Rightarrow \lambda^{*'} \leq \lambda^+ \equiv \frac{(1-p_2)}{p_1 - p_2} - \frac{c}{\gamma_1 X(p_1 - p_2)}$$

(II) We again consider two cases depending on $u^1(g_{11}|g_{21}, s_k^2) \gtreqless u^2(g_{21}|g_{10}, s_j^1)$. When $u^1(g_{11}|g_{21}, s_k^2) \geq u^2(g_{21}|g_{10}, s_j^1)$ with $t_k^- \leq t \leq t_j^+$, it is a dominant strategy for worker 1 to always apply and hence deviating (by not applying) would give her a lower payoff $\equiv 0$. In this case worker 2 will not apply because $U^2\left(g_{21}|(g_{11}, s_k^2)\right) \leq 0$. Any deviation would imply that worker 2 gets a negative payoff while on the equilibrium path in this case she gets 0. Using Bayes' rule along the equilibrium path implies that $\lambda^*(f = 1|n = 1) = 1$. The firm's efficient action will be s_1^{1*} if $p_1 \leq p^*$ and s_0^{1*} if $p_1 \geq p^*$. Belief off the equilibrium path $\lambda^{*'}(k = (1,2)|n = 2) = 1$ is unique since the information set is a singleton. If

$u^1(g_{11}|g_{21}, s_k^2) \leq u^2(g_{21}|g_{10}, s_j^1)$ with $u^2(g_{21}|g_{11}, s_k^2) \leq t \leq u^1(g_{11}|g_{21}, s_k^2)$ then worker 1 will apply even though it is not a dominant strategy to do so. Worker 2 will still not apply since even if she was the only applicant, the cost of applying would outweigh the benefit. The firm's efficient action and beliefs would remain the same as described above.

(III) If $t \in [(1-\gamma_1)p_2V, (1-\gamma_1)p_1V]$ with $c \in [\gamma_1(1-p_1)V, \gamma_1(1-p_2)V]$, then it is in the interest of only worker 2 to apply since the firm will be willing to screen it. Worker 1 would want to apply if the firm would screen her as well, but the cost of screening is high enough such that it does not compensate the firm for the negative impact of the worker 1 turning out to be a type b worker. The belief on the equilibrium path is $\lambda^*(f = 2|n = 1) = 1$ with the outcome being $\{s_1^{1*}, (g_{10}^*, g_{21}^*), w_2^*\}$.

(IV) If $(u^1(g_{11}|g_{21}, s_k^2) \leq u^2(g_{21}|g_{10}, s_j^1)$ with $u^1(g_{11}|g_{21}, s_k^2) \leq t \leq u^2(g_{21}|g_{10}, s_j^1))$ then worker 1 would find it profitable to apply if worker 2 does not and vice-versa. Thus, there could be two equilibria at the application stage in pure strategies. One is in which worker 1 applies (since $t \leq u^2(g_{21}|g_{10}, s_j^1) \leq u^1(g_{11}|g_{20}, s_j^1))$ and worker 2 does not (since $u^1(g_{11}|g_{21}, s_k^2) \leq t$). The other one is in which worker 2 applies and worker 1 does not. The belief on the equilibrium path has to be updated according to bayes' rule with $\lambda^*(f = 1(2)|n = 1) = 1$ if worker 1 (2) applies in equilibrium. The firm's screening strategy will be to screen for θ_i if $p_i \geq p^*$. The belief off the equilibrium path is again unique $(\lambda^{*'}(f = (1,2)|n = 2) = 1)$ since the particular information set is a singleton as above. QED

Characterization of the No Application Equilibria:

Here, we briefly characterize the equilibria in which the firm will get no applications. This can happen in a variety of cases. If $t \geq (1-\gamma_1)p_1V$, then no worker would apply irrespective of the screening strategy adopted by the firm. If $(1-\gamma_1)p_2V \geq t \geq (1-\gamma_1)(p_1V - (1-p_1)X)$ with $c \geq \gamma_1(1-p_2)X$, then the workers would only apply if the firm was willing to screen but the cost of screening is too high for the firm such that not screening becomes a dominant strategy for the firm.

An interesting case arises when $(1-\gamma_1)p_1V \geq t \geq (1-\gamma_1)p_2V$ with $(1-p_2)X \geq c \geq \gamma_1(1-p_1)X$. Here, worker 1 would like to apply if the firm screens but the firm would only screen if worker 2 applies. Thus, we can have an application profile such that neither worker applies. To completely characterize the equilibrium we need to define the beliefs off the equilibrium path for the firm. If $\lambda^{*'} \in (0,1)$, then there could be potential multiplicity of equilibria because worker 1 might apply if $\lambda^{*'}(.)$ is low enough. Using equilibrium dominance, we can isolate a unique out of equilibrium belief. The firm knows that the equilibrium value dominates for worker 2 which means that no deviation by the worker would fetch her a higher payoff. Thus, the firm should have a belief $\lambda^{*'}(f = 1|n = 1) = 1$ off the equilibrium path which would result in no applications being made.

Proposition 2: We establish the result taking the different scenarios into account.

(a) Consider the two application-equilibrium given by $\{(s_0^{2*}, s_0^{1*}), g_{11}^*, g_{21}^*\}$. There is a negative externality of the marginal applicant h if

$$\pi(s_0^{2*}|g_{11}^*, g_{21}^*) - \pi(s_0^1|g_{1e}, g_{2-e}) \leq 0$$

, which implies that:

$$p_i\gamma_1 V - (1-p_i)\gamma_1 X - p\gamma_1 V + (1-p)\gamma_1 X \geq 0 \tag{10}$$

This will hold if $e = 1$ which means $i = 1$. This holds by assumption since $p_1 > p_2$ and hence $p_1 > \frac{p_1+p_2}{2}$. This is the case for sure if $u^1(g_{11}|g_{21}, s_k^2) \geq u^2(g_{21}|g_{10}, s_j^1)$. If this inequality does not hold, then i could be either 1 or 2. If $i = 2$, then there would be a positive externality since $p_2 < p$ with $\pi(s_0^{2*}|g_{11}^*, g_{21}^*) - \pi(s_0^1|g_{10}, g_{21}) \geq 0$.

(b) Consider the equilibrium given by $\{(s_2^{2*}, s_1^{1*}), g_{11}^*, g_{21}^*\}$. The firm is worse off if $\mathcal{E}(h) \equiv \pi(s_1^{2*}|g_{11}^*, g_{21}^*) - \pi(s_1^1|g_{11}, g_{20}) \leq 0$, which means

$$p_i\gamma_1 V - c \geq (2p - p_1p_2)\gamma_1 V - (2-p)c \tag{11}$$

This gives us a threshold on c :

$$c \geq \frac{1}{1-p}\gamma_1 V\,(2p - p_1p_2 - p_i)$$

This can hold if $i = 1$. If the marginal applicant is worker 2 then the inequality is less likely to hold as p_2 increases because $\frac{\partial}{\partial p_2}\left(\frac{1}{1-p}\gamma_1 V\,(p_2 - p_1p_2)\right) > 0$. We can further show that $c \in [\frac{1}{1-p}\gamma_1 V\,(2p - p_1p_2 - p_1)\,, c^+] \neq \{\phi\}$. If $u^1(g_{11}|g_{21}, s_k^2) \leq u^2(g_{21}|g_{10}, s_j^1)$, then i can be 2 as well. In that case, it is always that $\pi(s_1^{2*}|g_{11}^*, g_{21}^*) - \pi(s_1^1|g_{10}, g_{21}) \geq 0$ with $c \leq \frac{1}{1-p}\gamma_1 V\,(p_1 - p_1p_2)$. This imposes a restriction on c which is less tight than the equilibrium restriction of c^+.

(c) Finally, consider the equilibrium given by $\{(s_2^{2*}, s_0^{1*}), g_{11}^*, g_{21}^*\}$.

For the negative externality to exist, we need to show that $\pi(s_2^{2*}|g_{11}^*, g_{21}^*) - \pi(s_0^1|g_{11}, g_{20}) \leq 0$, which means:

$$(2p - p_1p_2)\gamma_1 V - (2-p)c \leq p_i\gamma_1 V - (1-p_i)\gamma_1 X$$

which holds if the cost of screening is high enough:

$$c \geq \frac{1}{2-p}\left[\gamma_1 V(2p - p_1p_2 - p_i) + (1-p_i)X\right]$$

This inequality will only hold if $i = 1$. In that case, we can further show that $c \in [\frac{1}{2-p}\left[\gamma_1 V(2p - p_1p_2 - p_1) + (1-p_1)X\right], c^+] \neq \{\phi\}$ such that these inequalities hold simultaneously. As in part (b), we have $\frac{\partial}{\partial p_2}\left(\frac{1}{2-p}\left[\gamma_1 V(p_2 - p_1p_2) + (1-p_1)X\right]\right) > 0$ which is in line with the intuition of the model. If $p_i = p_2$ then there will be a positive externality with $c \leq \frac{1}{2-p}\left[\gamma_1 V(p_1 - p_1p_2) + (1-p_2)X\right] \leq c^+$.QED

Proposition 3. We will characterize the equilibrium looking at one case at a time.

(I) Let us first look at the response of the firms. Given that all workers apply to all firms, firm m will screen all the L applications if the cost of screening $c \leq c_m^+$ otherwise will screen none. Taking into account, workers apply to all K firms if $t \leq t_j^{++}, \forall j$. The Kth application would bring the lowest ex-ante payoff and hence would provide the tightest restriction on the cost of making an application. On the equilibrium path, we have $\lambda^{m*}(f = (1, 2, ..., L)|n = L) = 1$ for each of the m firms. As in a previous proof, there is equilibrium domination and hence this equilibrium can be sustained by any beliefs $\lambda^{m'*}(.) \in (0, 1)$ off the equilibrium path. In a similar way, we can see that if $c \geq \hat{c}_m, \forall m$ and $t \leq \hat{t}_j, \forall j$, then in equilibrium all workers make K applications and no firm screens any of the applications.

(II) If $t_j^{h'^*+} \leq t \leq t_j^{h'+}$, workers apply to $K - h$ firms since the cost of applying is low enough. The firms screen if $c \leq \hat{c}_m^{h'}$, which means the cost of screening is low enough. The beliefs of firm m , for example, on the equilibrium path is given by $\lambda^{m*}(f|n) = 1$. As before, this equilibrium can be sustained with any beliefs $\lambda^{m'*}(.) \in (0, 1)$ off the equilibrium path. In this case, even though all workers make $K - h$ applications, each firm might end receiving a different number of applications since the initial ranking of firms by workers can be different. This, however, does not change the fundamental trade-offs involved in the analysis.

We can notice that different application profiles can arise in equilibrium for different parameter values depending on the competition faced at each firm. Consider an application profile in which each worker makes only one application (because t is very high) to its least preferred firm in terms of its initial ranking and suppose the firm does not screen. For concreteness, let the distribution over types lead to an initial ranking over firms for worker j given by $e \succ h \succ ..... \succ g$. Worker j can apply to firm e and get $\frac{1}{2}\left((1-\gamma)E(M_e^j)\right) - t$ since there is a $\frac{1}{2}$ probability of getting picked. By applying to firm h, the worker can get $\left((1-\gamma)E(M_h^j)\right) - t$ for sure. Thus, if $\int_{\Theta_e^j} V(\theta)dF(\theta) \leq 2\int_{\Theta_h^j} V(\theta)dF(\theta)$ then a profile where each worker makes her only application to the least preferred firm can be sustained in equilibrium. We can see that this intuition extends to other cases easily and different application profiles can hold in equilibrium. QED

Proposition 4. We will prove this proposition by looking at firm m . We look at the different cases as follows:

(a) Let us look at the equilibrium outcome $\{((0, l)_m^*, (0, l-1)_m^*), \mathbf{g}_\mathbf{k}^*, (\mathbf{0}, \mathbf{l})_{-\mathbf{m}}^*\}$. For there to be a negative externality from the marginal applicant y, it must be the case that:

$$\pi(0, l)_m|\mathbf{g}_\mathbf{k}^*, (\mathbf{0}, \mathbf{l})_{-\mathbf{m}}^*) - \pi((0, l-1)_m|(g_{j,k-1}, \mathbf{g}_{-\mathbf{j},\mathbf{k}}), (\mathbf{0}, \mathbf{l})_{-\mathbf{m}}) \leq 0$$

If the firm gets only $l - 1$ applications, its payoff is $\sum_{j=1}^{l-1}\{\frac{1}{l-1}\left(1-\frac{1}{l}\right)^{k-1}\gamma E(M_m^j) + \sum_{i=2}^{k}\mathbf{1}_{\{m\succ s\}_j}[\frac{1}{l-1}\binom{l-1}{i}\frac{1}{l^i}\left(1-\frac{1}{l}\right)^{k-i}(E(M_m^j) - (E(M_s^j)]\}$. An additional application reduces the probability of the initial $l - 1$ being the only applicants to be picked. We know that

$$\nabla_{l-1} \equiv \sum_{j=1}^{l-1} \frac{1}{l-1} \left(1 - \frac{1}{l}\right)^{k-1} \gamma E(M_m^j) - \sum_{j=1}^{l} [\frac{1}{l} \left(1 - \frac{1}{l}\right)^{k-1} E(M_m^j)]$$

. $\nabla_{l-1} < 0 (> 0)$ if the marginal applicant y is below (above) the average quality of the pool of applicants. We can have $\triangle_{l-1} \lesseqgtr 0$ depending on the competition from the other firms for the candidate picked out. To see this, re-write $\triangle_{l-1}$ as:

$$\triangle_{l-1} \equiv \sum_{j=1}^{l-1} \sum_{i=1}^{k-1} \mathbf{1}_{\{m \succ s\}_j} [\frac{1}{l-1} \binom{l}{i} \frac{1}{l^i} \left(1 - \frac{1}{l}\right)^{k-i} (E(M_m^j) - E(M_s^j))]$$
$$- \sum_{j=1}^{l-1} \sum_{i=1}^{k-1} \mathbf{1}_{\{m \succ s\}_j} [\frac{1}{l} \binom{l}{i} \frac{1}{l^i} \left(1 - \frac{1}{l}\right)^{k-i} (E(M_m^j) - E(M_s^j))]$$
$$- \sum_{i=2}^{k-1} \mathbf{1}_{\{m \succ s\}_y} [\frac{1}{l} \binom{l}{i} \frac{1}{l^i} \left(1 - \frac{1}{l}\right)^{k-i} (E(M_m^j) - E(M_s^y))]$$

We can see that the sum of the first two terms is always positive. Thus, whether $\triangle_{l-1}$ is positive or negative depends on the magnitude of the first two terms and how it compares with the third. For $\triangle_{l-1} < 0$ marginal applicant y can have an ability which is greater or less than the average applicant for firm m.

For $\mathcal{E}(y) < 0$, it must be that $\nabla_{l-1} + \triangle_{l-1} < 0$. There are two effects at play. As in the basic model, there is a quality effect. A marginal applicant of greater (less) than average quality increases (decreases) the quality of the pool of applicants. With ex-post competition, this quality effect might be outweighed by the competition effect. If firm m does not rank 'high' on the marginal applicant's list or has outside options which give her a strong bargaining stance, then the firm might still be worse off even with higher than average applicant.

(b) We next look at the equilibrium outcome $\{((l,l)_m^*, (l-1, l-1)_m^*), \mathbf{g}_\mathbf{k}^*, (\mathbf{l}, \mathbf{l})_{-\mathbf{m}}^*\}$. For a negative externality from the marginal applicant y, it must be the case that:

$$\pi(l,l)_m | \mathbf{g}_\mathbf{k}^*, (\mathbf{l}, \mathbf{l})_{-\mathbf{m}}^*) - \pi((l-1, l-1)_m | (g_{j,k-1}, \mathbf{g}_{-\mathbf{j},\mathbf{k}}), (\mathbf{l}, \mathbf{l})_{-\mathbf{m}}) \leq 0$$

This inequality can be written as
$\sum_{l!} \frac{1}{l!} \left\{ \sum_{h=0}^{l-1} \frac{1}{l-h} (1 - \int_{\Theta_1^i} dF(\theta))^h (\gamma \int_{\Theta_1^k} V(\theta) dF(\theta) - c) \right\} \leq$
$\sum_{l-1!} \frac{1}{l-1!} \left\{ \sum_{h=0}^{l-1} \frac{1}{L-h} (1 - \int_{\Theta_1^i} dF(\theta))^h (\gamma \int_{\Theta_1^k} V(\theta) dF(\theta) - c) \right\}$
which gives the following condition:

$$c \geq \frac{1}{\triangle} \left(\sum_h B_h \int_{\Theta_m^h} V(\theta) dF(\theta) \right)$$

where $\Delta \equiv \sum_{l!} \frac{1}{l!} \left(\sum_{h=0}^{l-1} \frac{1}{L-h} (1 - \int_{\Theta_m^i} dF(\theta))^h \right)$

$- \sum_{l-1!} \frac{1}{l-1!} \left(\sum_{h=0}^{l-2} \frac{1}{l-h} (1 - \int_{\Theta_m^i} dF(\theta))^h \right)$ is the coefficent on c. We define B_h as $\left(\frac{\gamma}{l} + \frac{1}{l} \sum_s (1 - \int_{\Theta_m^s} dF(\theta))(\frac{\gamma}{l-1}) + ... \right) - \left(\frac{\gamma}{l-1} + \frac{1}{l-1} \sum_s (1 - \int_{\Theta_m^s} dF(\theta))(\frac{\gamma_1}{l-2}) + ... \right)$. If the inequality holds, the additional cost imposed on the firm from screening outweighs the option value provided by the marginal applicant. Also, it must be that $c \in [\frac{1}{\triangle} \sum_h B_h \int_{\Theta_m^h} V(\theta) dF(\theta), c_m^+] \neq \{\phi\}, \forall m$. We can show that this set is non-empty so that the initial equilbrium restrictions are met simultaneously. The above inequality holds if the rank of $y \geq \frac{l}{2}$ for firm m, i.e the marginal applicant is above the average quality otherwise $\mathcal{E}(y) > 0$. QED

Expressions for I_h,G_h: To be written out more completely..

The expression for I_1, for example, which is the coefficent on $\int_{\Theta_m^1} V(\theta) dF(\theta)$ takes the form $\frac{1}{L} \left(1 - \frac{1}{L}\right)^{K-1} \gamma + \sum_{i=1}^{K-1} \mathbf{1}_{\{1 \succ s\}_j} \frac{1}{L} \binom{L}{i} \frac{1}{L^i} \left(1 - \frac{1}{L}\right)^{K-i} - \sum_{L!} \frac{1}{L!} \left(\sum_{h=0}^{L-1} \frac{1}{L-h} (1 - \int_{\Theta_m^1} dF(\theta))^h \right) \gamma$. Similarly, $Q \equiv \sum_{L!} \frac{1}{L!} \left(\sum_{h=0}^{L-1} \frac{1}{L-h} (1 - \int_{\Theta_m^i} dF(\theta))^h \right)$is the coefficient on c. We can, in the same way, define I_h and G_h,$\forall h$ by gathering different terms in the payoffs highlighted above.

Proposition 5. We focus on firm 1 without loss of generality and look at the case, first, when firms do not screen. First, we need worker 1 to have the incentive to apply to firm 1 in the first place which gives us $(1-\gamma_1)(\int_{\Theta_1^1} V(\theta) dF(\theta) - \sum_{h \neq 1} \int_{\Theta_h^1} X(\theta) dF(\theta)) \geq t + e_1$. We additionally need an e_1 such that it is in the interest of worker 1 to be the only applicant to firm 1 rather than being the second applicant to any other firm. This gives us a sequence of inequalities involving e_1. We denote $(1 - \gamma_1)(\int_{\Theta_j^1} V(\theta) dF(\theta) - \sum_{h \neq j} \int_{\Theta_h^1} X(\theta) dF(\theta)) \equiv E(M_j^1)$. The following $(K-1)$ inequalities involving e_1 should hold simultaneously:

$$E(M_1^1) - \frac{1}{2} E(M_j^1) \geq e_1 - e_j, \forall j \in M, j \neq 1$$

Additionally, we have another $K-1$ inequalities such that worker h should apply to her most preferred (ex-ante) firm than firm 1:

$$E(M_h^h) - \frac{1}{2} E(M_1^k) \geq e_k - e_1, \forall k \in M, k \neq 1$$

We can add the $2K-2$ inequalities to get a single inequality:

$$(K-1)E(M_1^1) + \sum_{h=2}^{K} E(M_h^h) - \frac{1}{2}(\sum_{j=2}^{K} E(M_j^1) - \sum_{h=2}^{K} E(M_1^h)) \geq 0$$

This inequality holds by assumption since preferences in figure 5.1 imply that $E(M_j^j) > E(M_h^j), \forall j, h$. In addition to these inequalities, we need restrictions such that workers do not have incentive to apply to two firms rather than 1. In the case that firms do not screen, it will never be in the interest of worker 1 to deviate from just applying to firm 1. If she applies to j as well, she pays an additional $t + e_j$ but still gets the same payoff since she gets an offer from firm 1 with probability 1.

Next, look at the case where the firms screen in equilibrium. If worker 1 applies only to firm 1 in equilibrium if $(1-\gamma_1)\int_{\Theta_1^1} V(\theta)dF(\theta) \geq t + e_1$. If the worker applies to firm j instead, she gets $\frac{1}{2}(1-\gamma_1)\int_{\Theta_j^1} V(\theta)dF(\theta)(2 - \int_{\Theta_j^j} dF(\theta))\ -t - e_j$. $E(M_1^1) > E(M_j^1)$ in this case as well and hence there is an e_1 such that the resulting $2K-2$ inequalities can be satisfied simultaneously. In this case, we will also need to take into account the restrictions which ensure that firm 1 does not get more than one application because each worker has a positive probability of not being employed by the firm they apply to. This gives another set of constraints such that $\forall j \in M$:

$$e_1 \geq \frac{(1-\gamma_1)}{2}(1 - \int_{\Theta_j^j} dF(\theta))(2 - \int_{\Theta_1^1} dF(\theta)) \int_{\Theta_1^j} V(\theta)dF(\theta) - t$$

Combining this set of restrictions with the $2K-2$ inequalities above gives us an upper and a lower bound on e_1:

$$(1-\gamma_1)(\int_{\Theta_1^1} V(\theta)dF(\theta) - \frac{1}{2}\int_{\Theta_K^1} V(\theta)dF(\theta)(2 - \int_{\Theta_K^K} dF(\theta)) + e_K \geq e_1 \geq$$

$$max\{\frac{(1-\gamma_1)}{2}(1 - \int_{\Theta_K^K} dF(\theta))(2 - \int_{\Theta_1^1} dF(\theta)) \int_{\Theta_1^K} V(\theta)dF(\theta)) - t,$$

$$e_2 - [\int_{\Theta_2^2} V(\theta)dF(\theta) - \frac{1}{2}(2 - \int_{\Theta_1^1} dF(\theta)) \int_{\Theta_1^2} V(\theta)dF(\theta)]\}$$

We can define such inequalities for each of the K firms and we can show that they hold simultaneously after a bit of algebra.