Competition, Reputation, and Sellers Truth-telling*

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Abstract

This paper studies the scope of honest trade in a competitive environment where sellers sell experience goods to consumers dynamically. Good sellers produce high-quality goods with some probability and bad sellers are inept. Both seller type and product quality are private information to a seller himself. We show that under this setup there exists a truth-telling equilibrium where good sellers establish a reputation by being honest about the quality and bad sellers are likely to be dishonest. Honest behavior is rewarded by higher future prices and larger customer base. Sellers who operate longer in the market face a larger customer base and charge higher prices.

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1 Introduction

Good sellers are usually more honest than bad sellers in dynamic competitive market. For instance, in Chinese sea food Dai Pai Dong market, where many restaurants operate at the same location, the restaurants with long queues usually tell customers when their sea food is not fresh. Also, a good diamond dealer is more likely to tell their buyers true product quality. In E-commerce, reputable sellers are less likely to make bold claims about product quality (Jin and Kato, 2006).

Under imperfect monitoring where a buyer cannot ascertain quality even after consumption, a seller with high reputation might have less incentive to report true quality than a seller with middle reputation, because a reputable seller's reputation is not sensitive to a bad result. Then why do good sellers or firms still adopt honest policy, even after they have developed a good reputation? This paper provides an explanation: market competition provides strong and credible punishment for dishonest sellers through the immediate loss of loyal consumers.

In the model, many sellers and buyers trade with each other repeatedly. Sellers are of two types, Good and Bad. Good sellers produce high quality products with some probability each period while bad sellers always produce low quality products. A high quality product is more likely to lead to a successful trade outcome than a low quality product does. Both seller type and product quality are a seller's private information. Each period before transaction takes place, sellers simultaneously post prices and quality information (which could be fake). Then buyers observe these prices and quality claims, and make their purchasing decisions. If buying, a consumer (she) pays up front. After that, each consumer learns her own trade outcome, and decides whether to be a loyal consumer. Switching incurs no cost.

I show in this setting, there exists a symmetric Perfect Bayesian equilibrium where good sellers always tell the truth and bad sellers randomize between truth-telling and lying. The idea is that, competition makes the threat of losing loyal consumers credible, which sustains truth-telling motives for good sellers.

Specifically, when a seller draws low quality products, he will not lie, if consumers can switch to other sellers immediately they get disappointed (i.e. being told quality, but experience bad outcome), and if these sellers would charge appropriate prices (so that consumers indeed would switch). Moreover, with a higher continuation value, a good seller would find it more costly to lie than a bad seller does. Hence if a bad seller is indifferent between lying and truth-telling, a good seller strictly prefers to tell the truth.

When a seller draws high quality products, he might have incentive to lie if high quality products do not lead to success very often. However, if claiming low quality is not as helpful in keeping loyal consumers as claiming high quality and meanwhile enjoying successful trade outcome is, he will choose to tell the truth.

As a seller remains longer and longer in the market, consumers are more and more sure that he is a good seller. In return, he can charge higher prices and enjoy larger consumer base. These results also carry over to a market where there is entry each period. Under some conditions, there exists an equilibrium where good sellers always tell the truth and bad sellers randomize. An age-old seller enjoys a higher reputation and larger consumer base, and can charge higher prices when reporting high quality.

This paper is closely related to Hörner (2002), who shows that competition helps sustain a high-effort equilibrium. In equilibrium, good firms always choose high effort, that leads to high quality service for sure. However, under uncertainty, that is, when good firms are not able to produce high quality for sure, another question arises naturally: would firms have incentive to report true quality? Our model tackles this question, and characterizes conditions under which a truth-telling (and high-effort) equilibrium exists. Jullien and Park (2012) studies the role of pre-trade communication in an (noncompetitive) experience good market where sellers differ in their ability to supply high quality items. They show under perfect monitoring, there exists a honest equilibrium where high-ability sellers always tell the truth; moreover, if exists, honest equilibrium is unique. Our paper differ from theirs in the sense that we study the environment of imperfect monitoring, where honest equilibrium does not exist under non-competitive market.

Section 2 provides the basic model where there is no entry except in the initial period, and characterizes the conditions for existence and equilibrium property. Section 3 extends the model to an environment where entry is allowed, and shows most results obtained in the basic model carry over to the model with entry. In section 4, we discuss the case of moral hazard, where it incurs a cost for good sellers to have some probability to draw high quality products. Section 5 concludes.

2 The Basic Model

This section we study a model where entry of sellers only occurs in the initial period, so as to focus on the role of competition. Section three relaxes this assumption and shows that most results carry over to the general model. Assume throughout that sellers and buyers are anonymous, and they cannot make contingent contracts.

In the market, a continuum of sellers repeatedly trade with a continuum of consumers in a infinite time horizon. The measure of consumers is 1, and the measure of sellers is λ_0 (to be determined). In each period t, a consumer(she) may buy one product from a seller, and in case of buying, she pays up front to the seller. Depending on the quality of a product traded, a transaction can lead to two outcomes: either a success($\bar{y}$), that brings utility 1 to a consumer; or a failure($\underline{y}$), that brings 0 to her. A product can be of high quality(g), or of low quality(b); high quality product yields a success with probability α; low quality yields a success with probability β. We assume $0 < \beta < \alpha \leq 1$.

Sellers are identical before entry, each with probability μ_0 of being a good seller(G) and $1 - \mu_0$ being a bad seller(B). Entry incurs cost C. After entry, each seller privately learns his own type, which is fixed over time. A good seller has probability P_G (exogenously determined) of selling high quality products each period. A bad seller is inept, selling low quality products all the time. Seller's outside option is 0. All agents share common prior: that a seller has probability μ_0 of being a good seller, and a good seller has probability P_G selling high quality products each period. After entry, a seller knows his own type and current-period product quality.

Consumers are homogeneous, long-lived agents. In each period, a consumer decides whether to buy (one product) and from which seller to buy. All consumers buying from the same seller experience the same result in that given period. Consumers are Bayesian, using Bayes' rule to update their beliefs about a seller's type and product quality whenever possible. If consumer j buys a product that has probability ρ_i of being high quality, she gets expected utility $u_j = \rho_i (\alpha - \beta) + \beta - p_{it}$, where p_{it} is the price charged by her seller i. After an outcome realized, a consumer decides whether to be a switching consumer next period. Switching incurs no cost. Each consumer is utility-maximizer, choosing a seller that maximizes her expected utility each time; if $u_j < 0$, she would not buy in period t.

Discount factor is δ, same for all consumers and sellers.

2.1 Timing

At time $t = 0$, sellers decide whether to enter the market. If a seller enters, he learns his own type, and the quality of his products that period, $q_{i0} \in \{g, b\}$. He then sends to the market a message about the quality of his products, $m_{i0} \in \{g, b\}$, which can be either truthful or not, and meanwhile charges a price p_{i0}. All sellers make these decisions simultaneously. After seeing the messages and prices posted by all sellers, consumers make their purchasing decisions: whether to buy and from whom to buy. If a consumer buys from seller i, she pays p_{i0}, and then experiences an outcome. After that, she updates her belief about seller's type and decides whether to be a loyal consumer of seller i, or to be a switching consumer. Sellers decide whether to exit.

At time $t \geq 1$, after having learned his product quality $q_{it} \in \{g, b\}$ and consumer base, seller i sends a message $m_{it} \in \{g, b\}$ to the market, charges price p_{it}. As in period $t = 0$, all sellers make these decisions simultaneously. A loyal consumer of seller i sees the message and price of seller i, and decides whether to trade with him. A switching consumer observes all sellers' messages, prices, and consumer base, and makes her purchasing decision. Then the timing goes the same as in period $t = 0$.

2.2 Markov strategies

The histories and strategies are described formally in the appendix. Here we only consider Markov strategies (after entry), which only depend on payoff relevant variables, here consumers belief, and current period actions. Consumers decide whether to buy or from whom to buy based on their belief about sellers types, the prices and messages set by sellers. They choose whether to be loyal based on their current period experience if buying. Sellers charge prices and send messages based on consumers belief and their consumer bases.

Let u_j be consumer j's belief about her current seller (if she has any), that is, the probability that her current seller is a Good seller; and θ_j be her belief about the proportion of good sellers serving the market. The state of the game for consumer j, is her belief ω_j about the distribution of $(\theta_j, u_j)_{j\in[0,1]}$, with $\omega_j \in \Omega_j$. The strategy of consumer j consists two mappings: σ_j as her purchasing strategy, and ϵ_j her switching strategy. The strategy of seller i is a mapping ξ_i, representing his message-sending and price-setting strategies.

Denote D the Borel measure on $\{g, b\} \times \mathbb{R} \times \mathbb{R}_+$(the Cartesian product of message space, price space, and consumer base space), with total mass equal to 1, and $\mathcal{D}$ the set of such measures. Let T and N represent trade and not trade respectively.

Before buying, a switching consumer (at t) observes the distribution of current period messages, prices, and consumer base in the market, and choose whether to buy or not. Hence her purchasing strategy is $\sigma_j(s) : \Omega_j \times \mathcal{D} \to \{N\} \bigcup \{g, b\} \times \mathbb{R} \times \mathbb{R}_+$, with $\sigma_j(s)(\omega_j \times D) \in \{N\} \bigcup SupportD$. After transaction, she observes her own transaction outcome, and decides whether to switch or not. Hence her switching strategy is $\epsilon_j : \Omega_j \times \{g, b\} \times \mathbb{R} \times \mathbb{R}_+ \times \{\bar{y}, \underline{y}\} \to [0, 1]$, where $\epsilon_j = a$ means to be loyal with probability a.

A loyal consumer can only observe the message, price, and consumer base of her own seller, and if trade occurs, her own transaction outcome. Hence her purchasing strategy $\sigma_j(l) : \Omega_j \times \{g, b\} \times \mathbb{R} \times \mathbb{R}_+ \rightarrow \{T, N\}$. Her switching strategy is the same as a switching consumer's.

Denote ω_i seller i's belief about the distribution of $(\theta_j, u_j)_{j \in [0,1]}$, with $\omega_i \in \Omega_i$. At the beginning of each period, a seller learns his quality and consumer base before charging a price and sending a message. If he is a good seller, his strategy is $\xi_i^G : \Omega_i \times \mathbb{R}_+ \times \{g, b\} \rightarrow \{g, b\} \times \mathbb{R}$; otherwise, if he is a bad seller, $\xi_i^B : \Omega_i \times \mathbb{R}_+ \rightarrow \triangle(\{g, b\} \times \mathbb{R})$(mixed strategy).

Consumers use Bayes' rule to update their beliefs whenever possible. The belief that a consumer holds upon seeing a seller with message m_t, price p_t, and consumer base n_t, is denoted by the mapping Ψ,

$$\Psi : [0,1] \times \{g, b\} \times \mathbb{R} \times \mathbb{R}_+ \rightarrow [0,1]$$

where $\Psi(\mu_t \mid m_t, p_t, n_t)$ denotes the posterior probability that a seller is of good type when the prior is μ_t , message m_t, price p_t, and consumer base n_t. The belief that a consumer holds upon seeing her own result, is denoted by the mapping Φ,

$$\Phi : [0,1] \times \{g, b\} \times \mathbb{R} \times \mathbb{R}_+ \times \{\bar{y}, \underline{y}\} \rightarrow [0,1]$$

where $\Phi(\mu_t \mid m_t, p_t, n_t, y_t)$ denotes the posterior probability that a seller is of good type when the belief in period t is μ_t, period t message m_t, price p_t, consumer base n_t, outcome is y_t.

2.3 Equilibrium

The goal of this paper is to characterize conditions under which truth-telling equilibria exist. Because non-revealing[1] equilibrium, provides the most severe punishment on misbehaved sellers (consumers leave a seller immediately they get disappointed), and hence strong impetus to sustain a truth-telling equilibrium, we focus on this set of equilibria.

Definition 1. *A* non-revealing truth-telling equilibrium *is an equilibrium where, on the equilibrium path, sellers who send the same message charge the same price in each period (prices could differ across time); and good sellers always report their true quality.*

Among all equilibria, we look for perfect Bayesian equilibria in symmetric, Markov strategies, that constitute non-revealing truth-telling equilibria.

[1] We follow the definition in Hörner (2002): in a non-revealing equilibrium, there is only one equilibrium price in each period (could be different over time). Our model considers message sending, hence non-revealing equilibrium allows prices differ for different messages in each period.

In the following of this section, if not mentioning, equilibrium refers to PBE in symmetric Markov strategies.

2.3.1 Seller behaviors

Suppose in equilibrium, a bad seller lies with probability $\gamma_{gt} \in [0,1]$, which of course could depend on his reputation. Denote p_{bt} (respectively, p_{gt}) the price for products that are claimed of low quality (respectively, high quality) at time t. On the equilibrium path, there are only two prices each period, p_{bt} and p_{gt}, hence the pair (m_t, p_{mt}) can be represented simply by m_t.

Let $\varphi(\mu_t \mid m_t)$ denote the posterior (on the equilibrium path) that a seller is of good type when the prior is μ_t , message m_t, price p_{mt}; and $\phi(\mu_t \mid m_t, y_t)$ the posterior probability that a seller is of good type when the belief in period t is μ_t, period t message m_t, price p_t, outcome is y_t, then applying Bayes rule, we have

$$\begin{aligned}
\varphi(\mu_t \mid g) &= \frac{\mu_t P_G}{\mu_t P_G + (1-\mu_t)\gamma_{gt}} \\
\varphi(\mu_t \mid b) &= \frac{\mu_t (1-P_G)}{\mu_t (1-P_G) + (1-\mu_t)(1-\gamma_{gt})}
\end{aligned}$$

and

$$\begin{aligned}
\phi(\mu_t \mid g, \bar{y}) &= \frac{\mu_t P_G \alpha}{\mu_t P_G \alpha + (1-\mu_t)\gamma_{gt}\beta} \\
\phi\left(\mu_t \mid g, \underline{y}\right) &= \frac{\mu_t P_G (1-\alpha)}{\mu_t P_G (1-\alpha) + (1-\mu_t)\gamma_{gt}(1-\beta)} \\
\phi(\mu_t \mid b, \bar{y}) &= \phi\left(\mu_t \mid b, \underline{y}\right) = \varphi(\mu_t \mid b)
\end{aligned}$$

Lemma 1 determines the probability that a bad seller randomizes on the equilibrium path.

Lemma 1. *In any non-revealing truth-telling equilibrium, sellers remaining in the market are those who never experience a failure in any period when they send message g . Moreover, by the end of a given period, sellers who send b in that given period should have the same reputation with those who send g and experience success. A bad seller lies with probability $\gamma_g = \frac{P_G \alpha}{P_G \alpha + (1-P_G)\beta}$ each period in equilibrium.*

Proof. (1) At the beginning of each period, sellers who have loyal consumers must possess the highest reputation. This is because (i) Facing price

$p_{gt}(> p_{bt})$, consumers get strictly higher payoff buying from seller with higher reputation; and they get the same level of payoff buying a product claimed of low quality (at price p_{bt}); (ii) At each period, each seller has strictly positive probability claiming high quality and charging a high price, sellers whose reputation is lower than average cannot keep any loyal consumer. Otherwise, the loyal consumer can simply switch and get an average utility (of all consumers). This means that sellers who are able to keep loyal consumers must have the highest reputation at the beginning of each period.

(2) Starting with the same prior at time t, the reputation of a seller who claimed high quality and experienced failure in that period, is surely lower than the reputation of a seller who claimed high quality and experienced success (since $\phi\left(\mu_t \mid g, \underline{y}\right) < \phi\left(\mu_t \mid g, \bar{y}\right)$). This implies consumers who bought from the former will choose to switch for sure.

In addition, since good sellers are willing to tell the truth when they draw low quality products, they must be able to reap some benefit in the future from sacrificing today. This implies that they can remain in the market and hence should have the same reputation as those who claimed high quality and enjoyed success in that period, that is

$$\phi\left(\mu_t \mid g, \bar{y}\right) = \varphi\left(\mu_t \mid b\right) \tag{1}$$

Equation (1) determines the probability that a bad seller randomize:

$$\gamma_{gt} = \gamma_g \equiv \frac{P_G\alpha}{P_G\alpha + (1 - P_G)\beta}$$

□

Because $\alpha > \beta$, we have $\gamma_g > P_G$, that is, a bad seller claims high quality more often than a good seller does. Since over time there are more good sellers left, in equilibrium, we should witness reputable sellers are more honest, which is consistent with Jin and Kato (2006) that reputable sellers are less likely to make bold claims.

We have shown that in any non-revealing truth-telling equilibrium, a good seller chooses action (g, p_{gt}) when drawing high quality products; and (b, p_{bt}) when drawing low quality products. A bad seller chooses (g, p_{gt}) with probability γ_g, and (b, p_{bt}) with probability $1 - \gamma_g$. It left to characterize consumer behavior.

2.3.2 Consumer behavior

Consumers purchasing strategies. A switching consumer picks a seller who post messages and prices that maximize her utility (if trading). If there are many such sellers, we assume she picks one randomly, such that the probability that a seller trades with this consumer is proportional to the mass of his loyal consumers. A loyal consumer chooses to buy from her current seller if and only if she gets positive expected utility.

Consumers switching strategies. As is shown in Lemma 1, a consumer chooses to be loyal only if her current seller enjoys the highest reputation. Therefore, a consumer buying from a seller who claimed high quality and experienced a failure (hence $\left(g, \underline{y}\right)$), chooses to switch for sure. Denote $F_t^{g,\bar{y}}$ the probability that a consumer chooses to be loyal if a consumer bought from a seller who claimed high quality and enjoyed a success (hence $(g, \bar{y})$), on the equilibrium path; and F_t^b the probability that a consumer chooses to be loyal if she bought from a seller who claimed low quality, whatever the result is (hence$(b, \cdot)$). To simplify the analysis, we make the following assumption: $F_t^{g,\bar{y}} = 1$, and $F_t^b = F$, with $\alpha < F \leq 1$. That is, on the equilibrium path, a consumer does not leave if her seller was brave enough to claim high quality, and she enjoyed a good experience; she leaves with some probability if her seller claimed low quality. This probability will make sure that sellers do not lie when they sell high quality products. We make this assumption because it is simple, both for analysis and for consumers to follow; and because it captures the idea that, it matters that there is a difference between $F_t^{g,\bar{y}}$ and F_t^b; otherwise, even a seller drawing high quality products might want to claim low quality, which is unrealistic and uninteresting.

To sustain the equilibrium, we assume consumers hold such belief: *all consumers believe in each period, sellers who send the same message charge the same price; any prices different from equilibrium prices are taken as coming from a bad seller for sure.*

Finally, assume that a seller with zero loyal consumer chooses to exit, except at the beginning of period 0. The idea is that, because of consumers belief, a seller with 0 loyal consumer is believed to be bad for sure, and hence he can attract consumers only by charging a price lower than or equal to p_{bt} and claim low quality. However, in equilibrium, competition will be so fierce that p_{bt} will stay negative forever, hence such seller would never find it in his interest to charge p_{bt}. Therefore, he can either exit, or charge a price higher than p_{bt}, lingering in the market with 0 consumers. Hence it is without loss of generality to assume such seller chooses to exit.

2.3.3 Equilibrium Properties

We now derive the properties of equilibrium by first assuming the profile of strategies and belief updating rule above constitutes an equilibrium. Then we check that it is indeed an equilibrium.

1) Price difference

In equilibrium, at the beginning of any period, sellers operating in the market have the same reputation. For sellers who choose (b, p_{bt}) and those who choose (g, p_{gt}) to attract positive consumer base, the price difference in each period must make consumers indifferent between buying from either of them. That is,

$$\beta - p_{bt} = \varphi\left(\mu_t \mid g\right)(\alpha - \beta) + \beta - p_{gt} \tag{2}$$

Denote price difference at t by $\triangle_t \equiv p_{gt} - p_{bt}$, we have

$$\triangle_t \quad = \quad \frac{\mu_t P_G}{\mu_t P_G + (1-\mu_t)\,\gamma_g}\,(\alpha - \beta)$$

The higher a seller's reputation, the higher the price difference; same with a good seller's ability to draw high quality products, and the discrepancy between the success rates of two kinds of products.

2) consumer base

In equilibrium, a seller's consumer base grows proportionally with his loyal consumer base. Suppose seller i has loyal consumers n_i at the end of t, then at $t+1$, he will have consumers $n_i \cdot \frac{n_{t+1}}{n_t}$, where $\frac{n_{t+1}}{n_t}$ is the common growth rate of a seller's consumer base at $t+1$. Lemma 2 gives this growth rate.

Lemma 2. *In equilibrium, consumer base of each active seller grows at rate* $\frac{n_{t+1}}{n_t}$*, where*

$$\frac{n_{t+1}}{n_t}\left(\left[\mu_t P_G \alpha + (1-\mu_t)\,\gamma_g \beta\right] + \left[\mu_t\,(1-P_G) + (1-\mu_t)\,(1-\gamma_g)\right]F\right) = 1 \tag{3}$$

Proof. At time t, suppose there are mass λ_i sellers with consumer base n_{it}, and with same reputation μ_t, then by the end of t, mass $\lambda_i\left[\mu_t P_G \alpha + (1-\mu_t)\,\gamma_g \beta\right]$ sellers who claimed high quality remain, with loyal consumer base n_{it}; mass $\lambda_i\left[\mu_t\,(1-P_G) + (1-\mu_t)\,(1-\gamma_g)\right]$ sellers who claimed low quality remain, with loyal consumer base $F n_{it}$. Because consumer base grows at the same rate, $\frac{n_{t+1}}{n_t}$, we have

$$\begin{aligned}\sum_i \lambda_i n_{it} =& \sum_i \left[\mu_t P_G \alpha + (1-\mu_t)\,\gamma_g \beta\right]\lambda_i n_{it} \cdot \frac{n_{t+1}}{n_t} \\ &+ \left[\mu_t\,(1-P_G) + (1-\mu_t)\,(1-\gamma_g)\right]\lambda_i F n_{it} \cdot \frac{n_{t+1}}{n_t}\end{aligned}$$

or

$$\sum_i \lambda_i n_{it} = \frac{n_{t+1}}{n_t} \left(\left[\mu_t P_G \alpha + (1-\mu_t)\gamma_g \beta \right] \right.$$
$$\left. + \left[\mu_t (1-P_G) + (1-\mu_t)(1-\gamma_g) \right] F \right) \sum_i \lambda_i n_{it}$$

which is equation (3) $\square$

In equilibrium, a seller's consumer base increases with reputation, because $F \frac{n_{t+1}}{n_t} > 1$. The sellers who have the largest consumer base are those who always claim high quality and enjoy successful results $((g, \bar{y}))$. The more often a seller claims low quality, the smaller his size of consumer base. Sellers who claim low quality τ times, have consumer base $F^{\tau} n_t$, $\tau \leq t$. That is, while claiming low quality can protect a seller from being driven out of the market for that period, it is not a perfect substitute of $(g, \bar{y})$ if a seller's quality is indeed high. It only saves a seller when he unfortunately draws low quality.

3) Equilibrium prices

We show the sequence of equilibrium prices $\{p_{mt}\}_{m \in \{b,g\}, t \in \mathbb{N}}$ in Lemma 3.

Lemma 3. *Suppose the outside option for bad firms is 0, then in equilibrium,the price dynamics are*

$$p_{bt} = \frac{\triangle_{t-1}}{\delta (F-\beta)} \frac{n_{t-1}}{n_t} - \frac{\triangle_t F}{(F-\beta)} \tag{4}$$

$$p_{gt} = \frac{\triangle_{t-1}}{\delta (F-\beta)} \frac{n_{t-1}}{n_t} - \frac{\triangle_t \beta}{(F-\beta)} \tag{5}$$

$$p_{b0} = -\frac{\triangle_0 F}{(F-\beta)}$$

$$p_{g0} = -\frac{\triangle_0 \beta}{(F-\beta)}$$

$$p_{b\infty} = \frac{(\alpha-\beta)}{\delta (F-\beta)} \left(P_G \alpha + (1-P_G) F - \delta F \right)$$

$$p_{g\infty} = \frac{(\alpha-\beta)}{\delta (F-\beta)} \left(P_G \alpha + (1-P_G) F - \delta \beta \right)$$

Proof. The idea is to use sellers' indifference condition, and their outside option to pin down the sequence of prices (assuming μ_0 is known, will be determined later). In a non-revealing truth-telling equilibrium, a bad seller is indifferent between lying and truth-telling. Denote by V_t^B the equilibrium value (per consumer) of a bad seller in period t. By claiming high quality, he obtains high payoff p_{gt} today, but only has probability β remaining in the market; by claiming low quality, he obtains a low payoff p_{bt} today, but enjoys a higher continuation value. Because he is indifferent between the two options, we must have

$$\begin{aligned} V_t^B &= p_{bt} + \delta F \frac{n_{t+1}}{n_t} V_{t+1}^B \\ &= p_{gt} + \delta \beta \frac{n_{t+1}}{n_t} V_{t+1}^B \end{aligned} \tag{6}$$

This gives us the exact value of V_{t+1}^B :

$$V_{t+1}^B = \frac{\triangle_t}{\delta (F - \beta)} \frac{n_t}{n_{t+1}} \tag{7}$$

Apply it in equation (6) to get equilibrium price

$$\begin{aligned} p_{bt} &= V_t^B - \delta F \frac{n_{t+1}}{n_t} V_{t+1}^B \\ &= \frac{\triangle_{t-1}}{\delta (F - \beta)} \frac{n_{t-1}}{n_t} - \frac{\triangle_t F}{(F - \beta)} \end{aligned}$$

Then using free entry condition of bad sellers: $V_0^B = 0$, we get the price in the initial period

$$\begin{aligned} p_{b0} &= V_0^B - \delta F \frac{n_1}{n_0} V_1^B \\ &= -\frac{\triangle_0 F}{(F - \beta)} \end{aligned}$$

In this way, the whole sequence of prices are determined. □

The price for products claimed of high quality increases over time, implying that reputable sellers enjoy a price premium. The reason for price premium is that, products sold by reputable sellers are more likely to be of high quality because there are more good sellers among reputable sellers. This drives up prices. Initially, price is negative, so that no seller can make a profit by hit and run. Then it rises and finally converges. The larger the initial fraction of good sellers, μ_0, the shorter time it takes for sellers beginning to make profits. However, because of free entry, the initial price is also lower when μ_0 is high, making sure a bad seller gets 0 upon entry.

Notice that if agents are patient enough, or, if $\delta > \underline{\delta} \equiv \frac{(1-P_G)F + P_G \alpha}{F}$, then the prices for products claimed of low quality stays negative, deterring any incentive to cut price. Because if a seller tries to cut price and get the whole market, he will obtain strictly negative payoff in current period, and at most outside option next period (since he will be taken as a bad seller for sure by charging non-equilibrium price).

4) The profile of strategies is an equilibrium.

(i) Given the system of belief, and other agents' strategies, a consumer's strategies maximize her utility.

(ii) No seller has incentive to charge out-of-the-equilibrium price, provided that agents are patient enough: $\delta > \frac{(1-P_G)F+P_G\alpha}{F}$.

(iii) A bad seller has no incentive to deviate from the randomizing strategy since he is indifferent between lying and truth-telling (equation (6)).

(iv) Now we study the truth-telling incentives for a good seller.

Let V_t^{Gb} (respectively, V_t^{Gg}) be the equilibrium value (*per consumer*) of a good seller in period t, after he learns his products are of low quality (respectively, high quality); and V_t^G be the value (*per consumer*) of a good seller at t before he learns his products quality.

For type Gb, a good seller drawing low quality products, incentive compatibility requires that:

$$\begin{aligned} V_t^{Gb} &= p_{bt} + \delta F \frac{n_{t+1}}{n_t} V_{t+1}^G \\ &\geq p_{gt} + \delta \beta \frac{n_{t+1}}{n_t} V_{t+1}^G \end{aligned} \tag{8}$$

By using a bad seller's indifference condition $\triangle_t = \delta (F-\beta) \frac{n_{t+1}}{n_t} V_{t+1}^B$, this inequality is satisfied trivially, since $V_{t+1}^G - V_{t+1}^B > 0$ for all t. The intuition is that, a good seller has a higher continuation value than a bad seller does, hence lying is more costly for the former. If a bad seller is indifferent between lying and being honest, a good seller strictly prefers to be honest.

For type Gg, a good seller drawing high quality products, incentive compatibility requires that

$$\begin{aligned} V_t^{Gg} &= p_{gt} + \delta \alpha \frac{n_{t+1}}{n_t} V_{t+1}^G \\ &\geq p_{bt} + \delta F \frac{n_{t+1}}{n_t} V_{t+1}^G \end{aligned} \tag{9}$$

If the difference between being honest and lying: $\triangle_t - \delta (F-\alpha) \frac{n_{t+1}}{n_t} V_{t+1}^G > 0$ for all t, then $\triangle_t - \delta (F-\alpha) \frac{n_{t+1}}{n_t} V_{t+1}^G$ is increasing over time (see appendix); by setting F such that $p_{g0} + \delta \alpha \frac{n_1}{n_0} V_1^G = p_{b0} + \delta F \frac{n_1}{n_0} V_1^G$, all incentive compatibility constraints are satisfied (set $F = 1$ if $p_{g0} + \delta \alpha \frac{n_1}{n_0} V_1^G > p_{b0} + \delta \frac{n_1}{n_0} V_1^G$).

That is, a reputable seller benefits more from truth-telling (when quality is high) than a new one because of monotonicity ; if a new seller is brave enough to tell the truth and report true quality, reputable sellers will also do so.

5) Existence

Proposition 1 characterize the sufficient conditions under which, non-revealing truth-telling equilibrium exists.

Proposition 1. *There exists a $\underline{\delta}$, such that if $\delta > \underline{\delta}$, then non-revealing truth-telling equilibria exists.*

Proof. See appendix. □

The idea is that, if people are patient enough, in order for bad seller to make 0 profit, p_{bt} has to be negative (as is shown above). Then we impose the free entry condition to get the initial mass of sellers entering the market $\lambda_0 \equiv \frac{1}{n_0}$:

$$n_0 \left((1 - \mu_0) V_0^B + \mu_0 V_0^G \right) = C$$

If the equilibrium value of a good seller at time 0 is finite, then there exists such a λ_0.

Competition in the basic model is, in some sense, inappropriate, since as time goes on, there are fewer and fewer sellers operating in the market, making the threat of consumer switching unrealistic. However, if we relax the no-entry condition, as in section 3, it turns out that the results obtained here are robust. That is, under some condition, there exists an equilibrium where good sellers are honest and bad sellers randomize.

3 Extension: Model with Entry

Now we consider the model with entry and exit each period. Suppose time horizon goes from $-\infty$ to $+\infty$. At the beginning of each period, sellers can choose whether to enter or not, and at the end of each period, whether to exit or not. As before, the total mass of consumers is 1. Because a non-stationary model would be too formidable to get some feature, we only focus on stationary Markov equilibrium, where an equal mass (λ_0) of sellers enter and exit the market each period, so that the mass of sellers with different ages remains constant over time.

3.1 Timing

At time t, sellers outside the market, choose whether to enter the market. Then new entrants learn their own type. Meanwhile, each seller learns the quality of his products, $q_i \in \{g, b\}$, a seller of age i sends a message $m_i \in \{g, b\}$, posts a price $p_i \in \mathbb{R}$ to the market. As before, sellers make these decisions simultaneously. Then a switching consumer observes the distribution of messages, prices, and consumer bases, of all sellers; a loyal consumer only observes her own seller's message and price. Based on the information obtained, a switching consumer decides whether to buy and from whom to buy; a loyal consumer decides whether to trade with her current seller. If a consumer trades with a seller of age i, she pays p_i up front, and then experiences an outcome. By the end of t, she updates her belief about her seller's type and decides whether to be a loyal consumer of seller i, or to be a switching consumer. At the end of t, sellers learn the distribution of messages and prices posted at t, and consumer bases, then choose whether to exit the market.

3.2 Equilibrium

In the following, we focus on stationary non-revealing truth-telling equilibria in symmetric, Markov strategies.

Definition 2. *A stationary non-revealing truth-telling equilibrium is an equilibrium where, on the equilibrium path, among sellers of the same age, those who send the same message charge the same price; good sellers always report their true quality; the mass of sellers at each age remains the same in equilibrium.*

In equilibrium, there are prices (at most) $(p_{mi})_{m=g,b;i=1,2,\dots}$ each period, where p_{mi} refers to the price of products claimed to be quality $m \in \{g, b\}$ and sold by seller of age i. Applying the same reasoning, we obtain a modification of Lemma 1: sellers with the same age, must possess the same reputation; the randomizing probability on the equilibrium path for a bad seller is γ_g, same for all bad sellers.

In a *stationary non-revealing truth-telling equilibrium*, a good seller of age i chooses action (g, p_{gi}) when drawing high quality products; and (b, p_{bi}) when drawing low quality products. A bad seller of age i chooses (g, p_{gi}) with probability γ_g, and (b, p_{bi}) with probability $1 - \gamma_g$. A seller who loses all his loyal consumers will exit the market. Consumers choose to leave for sure if $\big(g, \underline{y}\big)$; otherwise, they adopt mix strategies, such that for seller with the same reputation, if they choose to stay with probability a when $(g, \bar{y})$ occurs, they will choose to stay with probability $F \cdot a$ in case of $(b, \cdot)$. The belief sustaining the equilibrium is: all consumers believe that in any given period, among sellers of the same age, those who send the same message charge the same price; any prices different from equilibrium prices are taken as coming from a bad seller for sure.

3.2.1 Consumer base

Denote n_i the consumer base of the age-i sellers, who always reported high quality and enjoyed success (that is, $\big(g, \underline{y}\big)$) in the past, and $\frac{n_i}{n_{i-1}}$ the common growth rate of age-i seller's consumer base. In a stationary equilibrium the mass of sellers of any age is strictly positive and remains constant, hence equilibrium prices must make consumers indifferent among buying from any of them, that is,

$$\beta - p_{bi} = \beta - p_{b0} \tag{10}$$

which gives us the dynamic of seller-i's consumer base.

$$\frac{n_i}{n_{i-1}} = \frac{\triangle_{i-1}}{\delta F \left(\triangle_i - \triangle_0\right)} \tag{11}$$

3.2.2 Equilibrium prices

Using the conditions that a bad seller is indifferent between lying and truth-telling (equation (6)), that price difference should make consumers indifferent between buying products claimed of low quality or products claimed of high quality (equation (2)), and that the outside option of a bad seller is 0, we get the sequence of equilibrium prices as before.

$$\begin{aligned} p_{bi} &= \frac{\triangle_{i-1}}{\delta \left(F - \beta\right)} \frac{n_{i-1}}{n_i} - \frac{\triangle_i F}{\left(F - \beta\right)} \\ p_{gi} &= \frac{\triangle_{i-1}}{\delta \left(F - \beta\right)} \frac{n_{i-1}}{n_i} - \frac{\triangle_i \beta}{\left(F - \beta\right)} \end{aligned}$$

Applying equation (10), we have

$$\begin{aligned} p_{bi} &= -\frac{\triangle_0 F}{\left(F - \beta\right)} \\ p_{gi} &= \triangle_i - \frac{\triangle_0 F}{\left(F - \beta\right)} \end{aligned} \tag{12}$$

As in the basic model, price for products claimed of high quality is initially negative, then rises over time, and finally converges. The higher the initial fraction of good sellers, the lower the price in the infinite future, show the intensiveness of competition. On the whole, consumer base grows over time. If starting with a low $\triangle_0$, then consumer base firstly grows swiftly at the beginning, then the growth rate slows down, (and can shrink for some period); but in the limit, it keeps growing, although at a low speed. That a seller's consumer base grows quickly at young age, and slows down when getting old, is consistent with the stylized fact of firm growth.

3.2.3 Existence

In equilibrium, there are mass λ_i of age-i sellers[2], with λ_0 to be determined. Initially, a new seller serves mass n_0 of consumers, then his consumer base increases with his age. Specifically, if a seller of age i has chosen message b for τ times in his consecutive active $i+1$ periods, his consumer base is $F^l n_i$. The total mass of consumers served by age-i sellers are $\eta_i n_i$, where[3]

$$\begin{aligned}\eta_i &= \left(\mu_{i-1}P_G\alpha + (1-\mu_{i-1})\gamma_g\beta + (\mu_{i-1}(1-P_G) + (1-\mu_{i-1})(1-\gamma_g))F\right)\eta_{i-1} \\ \eta_0 &= 1\end{aligned}$$

Therefore, the total mass of consumers served by all firms is $\sum_{i=0}^{\infty}\eta_i n_i$, which should equal to 1 in equilibrium. If $\sum_{i=0}^{\infty}\eta_i n_i < \infty$, then for given n_0, we can find $\eta_0 \equiv \lambda_0$ such that such that this is satisfied. Proposition 2 establishes the conditions under which a stationary equilibrium exists.

Proposition 2. *There exist $\underline{\delta}$ and $\tilde{\mu}_0$, such that, if $\delta > \underline{\delta}$, and $\mu_0 < \tilde{\mu}_0$, there exists a stationary non-revealing truth-telling equilibrium in symmetric and Markov strategies.*

---

[2]

$$\begin{aligned}\lambda_i &= \left(\mu_{i-1}P_G\alpha + (1-\mu_{i-1})\gamma_g\beta + (\mu_{i-1}(1-P_G) + (1-\mu_{i-1})(1-\gamma_g))\right)\lambda_{i-1} \\ &= \mu_{i-1}(P_G\alpha + (1-P_G))\left(1 + \left(\frac{1}{\mu_{i-1}} - 1\right)\frac{\gamma_g\beta}{P_G\alpha}\right)\lambda_{i-1}\end{aligned}$$

[3]Thus η_i is the mass of age-i sellers, had consumers adopted the same switching strategy in this section, but that consumers of the same sellers actually choose the same action. That is, with probability $(1-F)$, a seller who chooses (b, p_{bi}) will be driven out of the market, and with probability F, the seller will stay, with loyal consumer base η_i. Since this randomization (of consumers strategy) yields the same effect on the mass of consumers served by the whole group of the same age, we can say that firms of age i serve a mass $\eta_i n_i$ of consumers.

Proof. IC constraints can be shown to be satisfied in the same way as in the basic model. Here we only establish the existence.

If $\lim_{i\to\infty}\frac{\eta_{i+1}n_{i+1}}{\eta_i n_i}<1$, then we have $\sum_{i=0}^{\infty}\eta_i n_i<\infty$, thus for given n_0, we can find $\eta_0\equiv\lambda_0$ such that $\sum_{i=0}^{\infty}\eta_i n_i=1$. Then use the free entry condition to find the mass of new sellers entering the market each period.

The condition $\lim_{i\to\infty}\frac{\eta_{i+1}n_{i+1}}{\eta_i n_i}=\frac{(1-P_G)F+P_G\alpha}{\delta F\left(1-\frac{\triangle_0}{\alpha-\beta}\right)}<1$ requires $\triangle_0<\tilde{\triangle}_0$, where $\tilde{\triangle}_0\equiv(\alpha-\beta)\left[1-\frac{(1-P_G)F+P_G\alpha}{\delta F}\right]$. It is possible iff $\delta>\frac{(1-P_G)F+P_G\alpha}{F}$, and $\mu_0<\tilde{\mu}_0$, where $\tilde{\mu}_0$ is such that $\tilde{\triangle}_0=\varphi\left(\tilde{\mu}_0\mid g\right)(\alpha-\beta)$.

As in proposition 1, if $V_0^G<\infty$ (proved in the appendix), then applying the free entry condition to get the mass of new sellers ($\lambda_0\equiv\frac{1}{n_0}$) entering the market each period. □

Most results obtained in the basic model carry over here: A reputable seller enjoys a larger consumer base (generally speaking), and a higher price premium, and hence obtains a higher payoff. That this is a no-mercy world: as seller gets older, consumers are more and more sure that he is a good seller; but if a reputable seller disappoints his consumers, they will leave for sure because they believe there are plenty of other sellers who will charge attractive prices. It is this potential punishment that enforces a good seller to report quality truthfully, whatever the quality, whatever the age.

4 Discussion: moral hazard

In this section, we consider the problem that, when a good seller has to work hard to obtain probability P_G drawing high quality products each period, whether there exists an non-revealing equilibrium, where good sellers always choose to work hard, and be honest about the quality of his products all the time.

To tackle this problem, we modify the model with entry as follows: at the beginning of each period t, with cost c, a good seller chooses whether to spend a unit effort, to obtain probability P_G drawing high quality products. A bad seller is inept, always selling low quality products. The outside option of a seller C is assume to be 0. The timing after effort choice is the same as in section 3.

In order for a good seller to choose effort, the expected payment of high effort must exceeds the payoff of shirking, that is

$$\begin{aligned}&p_{bi}+\triangle_i P_G-\frac{c}{n_i}+\delta\left(P_G\alpha+(1-P_G)F\right)\frac{n_{i+1}}{n_i}V_{i+1}^G\\ \geq\ &p_{bi}+\delta F\frac{n_{i+1}}{n_i}V_{i+1}^G\end{aligned}$$

or, by rearranging,

$$P_G\left(\triangle_i-\delta\frac{n_{i+1}}{n_i}\left(F-\alpha\right)V^G_{i+1}\right)-\frac{c}{n_i}\geq 0,\forall i\geq 0$$

Applying the recursive form of V^G_{i+1}, the inequality becomes

$$\begin{array}{cc} & \frac{\alpha-\beta}{F-\beta}\triangle_i-\delta\left(F-\alpha\right)\frac{\tilde{\alpha}-\tilde{\beta}}{F-\beta}\sum_{\tau=1}^{\infty}\left(\frac{\tilde{\alpha}}{F}\right)^{\tau}\prod_{s=1}^{\tau}\frac{\triangle_{i+s}}{\left(\triangle_{i+s}-\triangle_0\right)}\triangle_{i+1} \\ \geq & \frac{c}{n_i}\frac{1-\delta F}{1-\delta\tilde{\alpha}} \end{array}$$

where $\tilde{\alpha}=\left(1-P_G\right)F+P_G\alpha$, $\tilde{\beta}=\left(1-P_G\right)F+P_G\beta$. The LHS increases with i, as is shown in section 3, the RHS decreases with i, hence if the condition holds for period 0, they are going to be satisfied for any periods later on. Now choose F so that the condition holds at $i=0$, or,

$$P_G\left(\triangle_i-\delta\frac{n_1}{n_0}\left(F-\alpha\right)V^G_{i+1}\right)-\frac{c}{n_0}=0$$

and n_0 such that

$$n_0P_G\left(p_{g0}+\delta\frac{n_1}{n_0}\alpha V^G_{i+1}\right)=c$$

Notice that type Gg's truth-telling constraints are satisfied trivially if high-effort is incentive compatible for a good seller at any time.

Under similar conditions as in proposition 2, we have stationary non-revealing truth-telling high-effort equilibrium, where good sellers choose high effort as long as they have loyal consumers, and they report their true product quality each period. Bad sellers randomize between truth-telling and lying.

5 Conclusion

We show that competition might provide impetus for honest trade under imperfect monitoring. Considers an environment where good sellers have some probability but not always drawing high quality products, then "lemon problem" might come from both types of sellers. However, if, because of competition, consumers can switch to other better sellers whenever disappointed, sellers might find it in their interest to be honest. Specifically, because a good seller enjoys a high future payoff, if a bad seller is indifferent between lying and truth-telling, a good seller strictly prefers to do so.

We construct an equilibrium where good sellers always tell the truth, and bad sellers randomize. In equilibrium, reputable sellers enjoy larger consumer base, charge higher prices, and hence obtain higher payoff. The market weeds out more disappointing sellers, either unlucky good sellers or bad sellers, in the beginning, then less as time goes on.

Although consumers are more and more sure that reputable sellers are good sellers, they do not forgive a reputable seller even if this seller screwed up just once. This severe punishment enforces good sellers to be honest whenever they get an unlucky draw.

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A Histories and strategies

A.1 Histories

Because entry decision is trivial, we only define history and strategy after sellers enter, that is, after they learn their types.

History of a seller. We assume that in each period t, seller i (if still in the market) observes the messages and prices of all sellers operating in the market, the measure of consumers buying from him, the realization of outcomes, and the measure of his loyal consumers. Denote D the Borel measure on $\{g, b\} \times \mathbb{R} \times \mathbb{R}_+$(the Cartesian product of message space, price space, and consumer base space), with total mass equal to 1, and $\mathcal{D}$ the set of such measures. thus the set of history (of seller i) is denoted by:

$$\mathscr{H}_i^0 = \emptyset$$

for $t > 0$,

$$\mathscr{H}_i^t = \left[\{g, b\} \times \bigcup_{D \in \mathcal{D}} D \times SupportD \times \mathbb{R}_+ \times \left\{\bar{y}, \underline{y}\right\} \times \mathbb{R}_+ \right]^t$$

$$\mathscr{H}_i = \bigcup_{t \geq 0} \mathscr{H}_i^t$$

Note that if seller i is a bad seller, his set of history at time t is

$$\mathscr{H}_i^t = \left[\{b\} \times \bigcup_{D \in \mathcal{D}} D \times SupportD \times \mathbb{R}_+ \times \left\{\bar{y}, \underline{y}\right\} \times \mathbb{R}_+ \right]^t$$

History of a consumer. Till time t, there are three kinds of information that consumer j can gather from previous trading: the information she gathered when she was a one-period switching consumer: chose to switch by the end of both $s-1$ and s; that she gathered when she once was a loyal consumer of some seller, which means that she interacted with the seller for at least 2 consecutive periods but finally ended it; that she gathers when she is a current loyal consumer of some seller, which means she still chooses to be loyal for some seller by the end of the current period. We first describe the sets of such information, and characterize the history set of a consumer.

If consumer j is a one-period switching consumer, she gets information about the distribution of messages, prices, consumer bases at t, her own purchasing decisions, her trading outcome if she trades, hence the set of information she gets is

$$I_s = \bigcup_{D \in \mathcal{D}} \left[D \times \{N\} \bigcup SupportD \times \left\{ \bar{y}, \underline{y} \right\} \times \{s\} \right]$$

where the set$\{T, N\}$ denotes her decision about whether to trade or not, hence $\{N\} \bigcup SupportD$ refers to the set of her purchasing decisions: not buy, or buy from a seller sending message $m \in \{g, b\}$, posting price $p \in \mathbb{R}$, with consumer base $n \in \mathbb{R}_+$, where (m, p, n) is in the support of D; s (l) in the set$\{s, l\}$ refers to switch (and respectively loyal).

If consumer j is a loyal consumer of a seller from $t+1$ to $t+\tau$, which means she begins to trade with the seller at time t and ends the relationship at the end of time $t+\tau+1$ and hence the length of this relationship is $\tau+2$, the set of information she gathers from t to $t+2$ is

$$\begin{aligned} I_{ls}(\tau) &= \left[\bigcup_{D \in \mathcal{D}} \left[D \times (SupportD) \times \left\{ \bar{y}, \underline{y} \right\} \times \{l\} \right] \right] \\ &\quad \times \left[\left[\{g, b\} \times \mathbb{R} \times \{T\} \times \left\{ \bar{y}, \underline{y} \right\} \times \{l\} \right]^{\tau} \right] \\ &\quad \times \left[\left[\{g, b\} \times \mathbb{R} \times \{N\} \cup \left(\{T\} \times \left\{ \bar{y}, \underline{y} \right\} \times \{s\} \right) \right] \right] \end{aligned}$$

$$I_{ls} = \{ I_{ls}(\tau) : \tau \geq 0 \}$$

If consumer j is a loyal consumer of a seller from $t+1$ till the current period$t+\tau+1$and still chooses to be a loyal consumer by the end of time $t+\tau+1$, the set of information she gathers is

$$\begin{aligned} I_l(\tau) &= \left[\bigcup_{D \in \mathcal{D}} \left[D \times (SupportD) \times \left\{ \bar{y}, \underline{y} \right\} \times \{l\} \right] \right] \\ &\quad \times \left[\left[\{g, b\} \times \mathbb{R} \times \{T\} \times \left\{ \bar{y}, \underline{y} \right\} \times \{l\} \right]^{\tau} \right] \end{aligned}$$

$$I_l = \{ I_l(\tau) : \tau \geq 0 \}$$

Denote by $l\,()$the number of periods that a consumer interacts consecutively with a seller. Hence we have

$$l(i_s) = 1, \ \forall i_s \in I_s$$

$$l\left(i_{ls}\right)=\tau+2,\ \forall i_{ls}\in I_{ls}\left(\tau\right)$$

$$l\left(i_{l}\right)=\tau+1,\ \forall i_{l}\in I_{l}\left(\tau\right)$$

Suppose at the end of time t, a consumer has interacted with n different relationships with some sellers. Denote R_k the information she got from each relationship k, then the history of a consumer j is thus defined by

$$\mathscr{H}_j^0=\emptyset$$

for $t>0$,

$$\mathscr{H}_j^t\left(n\right)=\left\{\prod_{k=1}^{n}R_k:\ R_k\in I_s\cup I_{ls},\forall k<n,\ and\, R_n\in I_s\cup I_{ls}\cup I_l.\ \sum_{k=1}^{n}l\left(R_k\right)=t\right\}$$

$$\mathscr{H}_j^t=\bigcup_{t\geq n}\mathscr{H}_j^t\left(n\right),\ \forall t>0$$

$$\mathscr{H}_j=\bigcup_{t\geq 0}\mathscr{H}_j^t$$

Denote$\mathscr{H}_{j,s}^t\equiv\mathscr{H}_j^t\left(n\right)$, if $R_n\in I_s\times I_{ls}$, as the history set of a consumer who chooses to switch by the end of t; and$\mathscr{H}_{j,l}^t\equiv\mathscr{H}_j^t\left(n\right)$, if $R_n\in I_s\times I_{ls}\times I_l$, as the history set of a consumer who chooses to be loyal to her seller by the end of t.

A.2 Strategies

Consumer strategy. A strategy of consumer j consists two mappings: σ_j as her purchasing strategy, and ϵ_j her switching strategy.

If consumer j is a switching consumer, her purchasing strategy in period t is denoted by $\sigma_j^t\left(s\right):\mathscr{H}_{j,s}^t\times\mathcal{D}\rightarrow\{N\}\bigcup\{g,b\}\times\mathbb{R}\times\mathbb{R}_+$, where $\sigma_j^t\left(s\right)\left(h_{j,s}^t\times D\right)\in\{N\}\bigcup SupportD$. Her switching strategy in period t is $\epsilon_j^t\left(s\right):\mathscr{H}_{j,s}^t\times\{g,b\}\times\mathbb{R}\times\mathbb{R}_+\times\left\{\bar{y},\underline{y}\right\}\rightarrow\{s,l\}$.

If consumer j is a loyal consumer, her purchasing strategy in period t is denoted by $\sigma_j^t\left(l\right):\mathscr{H}_{j,l}^t\times\{g,b\}\times\mathbb{R}\times\mathbb{R}_+\rightarrow\{T,N\}$. If she buys in period t, her switching strategy in period t is $\epsilon_j^t\left(l\right):\mathscr{H}_{j,l}^t\left\{g,b\right\}\times\mathbb{R}\times\mathbb{R}_+\times\left\{\bar{y},\underline{y}\right\}\rightarrow\{s,l\}$

Seller strategy. The strategy of seller i is a mapping ξ_i, representing his message-sending and price-setting strategies. If he is a good seller, $\xi_i^t:\mathscr{H}_i^t\times\{g,b\}\times\mathbb{R}_+\rightarrow\{g,b\}\times\mathbb{R}$; otherwise, his strategy is $\xi_i^t:\mathscr{H}_i^t\times\mathbb{R}_+\rightarrow\{g,b\}\times\mathbb{R}$.

B Proofs

B.1 Monotonicity of $\triangle_t - \delta(F-\alpha)\frac{n_{t+1}}{n_t}V_{t+1}^G$

Denote $\tilde{\alpha} = (1-P_G)F + P_G\alpha$, $\tilde{\beta} = (1-P_G)F + P_G\beta$.

The value per consumer for a good seller at t+1 is

$$\begin{aligned}
V_{t+1}^G &= \sum_{\tau=0}^{\infty}\delta^\tau \frac{n_{t+\tau+1}}{n_{t+1}}\left\{[(1-P_G)F+P_G\alpha]^\tau\right\}(p_{bt+\tau+1}+\triangle_{t+\tau+1}P_G) \\
&= \frac{\triangle_t}{\delta(F-\beta)}\frac{n_t}{n_{t+1}} + \frac{\tilde{\alpha}-\tilde{\beta}}{F-\beta}\sum_{\tau=0}^{\infty}\delta^\tau\frac{n_{t+\tau+1}}{n_{t+1}}\triangle_{t+\tau+1}\left\{[(1-P_G)F+P_G\alpha]^\tau\right\} \\
&= \frac{\triangle_t}{\delta(F-\beta)}\frac{n_t}{n_{t+1}} + \frac{\tilde{\alpha}-\tilde{\beta}}{F-\beta}\sum_{\tau=0}^{\infty}\delta^\tau\left[\prod_{s=1}^{\tau}\frac{1}{\mu_{t+s}+(1-\mu_{t+s})\frac{\gamma_g\beta}{P_G\alpha}}\right]\triangle_{t+\tau+1}
\end{aligned}$$

Hence

$$\begin{aligned}
&\triangle_t - \delta(F-\alpha)\tfrac{n_{t+1}}{n_t}V_{t+1}^G \\
= \; &\triangle_t\left\{\tfrac{\alpha-\beta}{F-\beta} - \delta(F-\alpha)\tfrac{\tilde{\alpha}-\tilde{\beta}}{F-\beta}\textstyle\sum_{\tau=1}^{\infty}\left(\tfrac{\tilde{\alpha}}{F}\right)^\tau\prod_{s=1}^{\tau}\delta^\tau\left[\prod_{s=1}^{\tau}\tfrac{1}{\mu_{t+s-1}+(1-\mu_{t+s-1})\frac{\gamma_g\beta}{P_G\alpha}}\right]\tfrac{\triangle_{t+\tau+1}}{\triangle_t}\right\}
\end{aligned}$$

Suppose we have $\triangle_t - \delta(F-\alpha)\frac{n_{t+1}}{n_t}V_{t+1}^G > 0$ for all t. We know$\triangle_t$ increases with t; $\frac{1}{\mu_{t+s-1}+(1-\mu_{t+s-1})\frac{\gamma_g\beta}{P_G\alpha}}$ and $\frac{\triangle_{t+\tau+1}}{\triangle_t}$ decrease with t, hence the part in {} increases with t also (and positive). Therefore, $\triangle_t - \delta(F-\alpha)\frac{n_{t+1}}{n_t}V_{t+1}^G$ increases with t.

B.2 Proof of proposition 1

Free entry requires $n_0\left((1-\mu_0)V_0^B + \mu_0 V_0^G\right) = c$, or

$$\lambda_0 \equiv \frac{1}{n_0} = \frac{\mu_0 V_0^G}{c}$$

This condition would be satisfied if $V_0^G < \infty$.

The value per consumer for a good seller at time 0 is

$$
\begin{aligned}
V_0^G &= \sum_{\tau=0}^{\infty} \delta^{\tau} \frac{n_\tau}{n_0} \left\{ \left[\left(1-P_G\right) F + P_G \alpha \right]^{\tau} \right\} \left(p_{b\tau} + \triangle_\tau P_G \right) \\
&= \frac{\tilde{\alpha}-\tilde{\beta}}{F-\beta} \sum_{\tau=0}^{\infty} \delta^{\tau} \frac{n_\tau}{n_0} \triangle_\tau \left\{ \left[\left(1-P_G\right) \bar{F} + P_G \alpha \right]^{\tau} \right\} \\
&= \frac{\tilde{\alpha}-\tilde{\beta}}{F-\beta} \sum_{\tau=0}^{\infty} \delta^{\tau} \left[\prod_{s=1}^{\tau} \frac{1}{\mu_{s-1} + \left(1-\mu_{s-1}\right) \frac{\gamma_g \beta}{P_G \alpha}} \right] \triangle_\tau \\
&= \frac{\tilde{\alpha}-\tilde{\beta}}{F-\beta} \sum_{\tau=0}^{\infty} \delta^{\tau} \left[\prod_{s=1}^{\tau} \frac{1}{1 + \left(\frac{1}{\mu_0} - 1\right) \left(\frac{\gamma_g \beta}{P_G \alpha}\right)^{s}} \left[1 + \left(\frac{1}{\mu_0} - 1\right) \left(\frac{\gamma_g \beta}{P_G \alpha}\right)^{s-1} \right] \right] \triangle_\tau \\
&= \frac{\tilde{\alpha}-\tilde{\beta}}{F-\beta} \sum_{\tau=0}^{\infty} \delta^{\tau} \left[\frac{\left[1 + \left(\frac{1}{\mu_0} - 1\right) \right]}{1 + \left(\frac{1}{\mu_0} - 1\right) \left(\frac{\gamma_g \beta}{P_G \alpha}\right)^{\tau}} \right] \frac{1}{1 + \left(\frac{1}{\mu_0} - 1\right) \left(\frac{\gamma_g \beta}{P_G \alpha}\right)^{\tau} \frac{\gamma_g}{P_G}} \\
&= \frac{\left(\alpha-\beta\right)\left(\tilde{\alpha}-\tilde{\beta}\right)}{F-\beta} \sum_{\tau=0}^{\infty} \delta^{\tau} f\left(\tau\right)
\end{aligned}
$$

where $f\left(\tau\right) = \left[\frac{\left[1+\left(\frac{1}{\mu_0}-1\right)\right]}{1+\left(\frac{1}{\mu_0}-1\right)\left(\frac{\gamma_g \beta}{P_G \alpha}\right)^{\tau}} \right] \frac{1}{1+\left(\frac{1}{\mu_0}-1\right)\left(\frac{\gamma_g \beta}{P_G \alpha}\right)^{\tau} \frac{\gamma_g}{P_G}}$.

Because $\lim_{\tau\to\infty} \frac{\delta^{\tau+1} f(\tau+1)}{\delta^{\tau} f(\tau)} = \delta < 1$, hence $\sum_{\tau=0}^{\infty} \delta^{\tau} f\left(\tau\right) < \infty$, so we have $V_0^G < \infty$.

B.3 Proof of $V_0^G < \infty$ in proposition 2

Manipulating the equilibrium values, we have

$$
\begin{aligned}
V_i^G &= \frac{\triangle_{i-1}}{\delta\left(F-\beta\right)} \frac{n_{i-1}}{n_i} + \frac{\tilde{\alpha}-\tilde{\beta}}{F-\beta} \sum_{\tau=0}^{\infty} \delta^{\tau} \tilde{\alpha}^{\tau} \frac{n_{i+\tau}}{n_i} \triangle_{i+\tau} \\
V_i^B &= \frac{\triangle_{i-1}}{\delta\left(F-\beta\right)} \frac{n_{i-1}}{n_i}
\end{aligned}
$$

$$
V_0^G = \frac{\tilde{\alpha}-\tilde{\beta}}{F-\beta} \left[\sum_{\tau=1}^{\infty} \left(\frac{\tilde{\alpha}}{F}\right)^{\tau} \prod_{s=1}^{\tau} \frac{\triangle_s}{\left(\triangle_s - \triangle_0\right)} \triangle_0 \right]
$$

Denote $g\left(\tau\right) = \left(\frac{\tilde{\alpha}}{F}\right)^{\tau} \prod_{s=1}^{\tau} \frac{\triangle_s}{\left(\triangle_s - \triangle_0\right)} \triangle_0$, then under the conditions in Proposition 2,

$$
\lim_{\tau\to\infty} \frac{g\left(\tau+1\right)}{g\left(\tau\right)} = \frac{\tilde{\alpha}}{F} \frac{\triangle_\infty}{\left(\triangle_\infty - \triangle_0\right)} < \delta < 1
$$

Therefore, $V_0^G < \infty$.

C Equilibrium construction

C.1 A profile of strategies that constitute an equilibrium in the basic model

Now construct strategies and beliefs $\left((\Psi, \Phi, \sigma, \epsilon), \left(\omega, \xi^G, \xi^B\right)\right)$ that constitute an equilibrium we discussed above.

a) Consumer belief. Upon seeing message m, price p, consumer base n from a seller, a consumer updates her belief about this seller according to $\Psi(m, p, n)$:

$$
\begin{aligned}
\Psi(b, p_{bt}, Y) &= \varphi(\mu_t \mid b), \ \forall t \geq 0 \\
\Psi(g, p_{gt}, Y) &= \varphi(\mu_t \mid g), \ \forall t \geq 0 \\
\Psi(z) &= 0, \ \forall z \in \{\{b, g\} \times \mathbb{R} \times \{Y, N\}\} \setminus \left\{(b, p_{bt})_{t\geq 0}, (g, p_{gt})_{t\geq 0}\right\}
\end{aligned}
$$

where Y refers to $n > 0$, N refers to $n = 0$.

Upon an outcome realized, a consumer updates her belief about her trading seller according to $\Phi(m, p, n, y)$

$$
\begin{aligned}
\Phi(b, p_{bt}, Y, \bar{y}) &= \varphi(\mu_t \mid b), \ t \geq 0 \\
\Phi(g, p_{gt}, Y, \bar{y}) &= \phi(\mu_t \mid g, \bar{y}), \ t \geq 0 \\
\Phi\left(g, p_{gt}, Y, \underline{y}\right) &= \phi\left(\mu_t \mid g, \underline{y}\right), \ t \geq 0
\end{aligned}
$$

Purchasing strategy: suppose at time t, there are mass λ_i sellers with consumer base n_{it}, with $\sum_i \lambda_i = 1$. Let $\sigma(s)(m, p_{it}, B)$ denote a switching consumer's probability of selecting the groups of sellers who have consumer base n_{it} and charge price p_{it+1} at $t+1$. Set $\sigma(s)(m, p_{it+1}, B) = \frac{\lambda_i n_t}{\sum_i \lambda_i n_t} sign(\Psi(m, p_{t+1}))$, and then select seller in group i with equal probability. Loyal consumer:$\sigma(l)(m, p_{it}, \cdot) = sign(\Psi(m, p_{it}))$, where i refers to her trading seller.

Switching strategy: $\epsilon(b, p_{bt}) = F$, $\epsilon(g, p_{gt}, \bar{y}) = 1$, otherwise, $\epsilon(z) = 0$.

b) Seller strategy. Ω_i is the belief that seller of age-i holds about consumers' beliefs.

Good seller (i) :$\xi_i^G(\omega_i, n_i > 0, q_t) = (m = q_t, p = p_{qt})$, where ω_i is the true consumer beliefs, $q_t \in \{b, g\}$ is the quality of products that a good seller sells at time t. Bad seller (i) :$\xi_i^B(\omega_i, n_i > 0) = \gamma_g(g, p = p_{gt}) + (1 - \gamma_g)(b, p = p_{bt})$ is a bad seller's randomizing strategy. If $n_i = 0$, a seller chooses to exit.

C.2 A profile of strategies that constitutes an equilibrium in the basic model

a) Consumer belief. Upon seeing message m, price p from a seller, and the seller has consumer base$\{Y, N\}$, where Y and N refer to strictly positive consumer base and 0 consumer base respectively, a consumer updates her belief about this seller according to $\Psi(m, p,)$:

$$
\begin{aligned}
\Psi(b, p_{bi}, Y) &= \varphi(\mu_i \mid b), \ \forall i > 0 \\
\Psi(g, p_{gi}, Y) &= \varphi(\mu_i \mid g), \ \forall i > 0 \\
\Psi(b, p_{b0}, N) &= \varphi(\mu_0 \mid b) \\
\Psi(g, p_{g0}, N) &= \varphi(\mu_0 \mid g) \\
\Psi(z) &= 0, \ \forall z \in \{\{b, g\} \times \mathbb{R} \times \{Y, N\}\} \setminus \\
& \quad \left\{(b, p_{bi}, Y)_{i>0}, (g, p_{gi}, Y)_{i>0}, (b, p_{b0}, N), (g, p_{g0}, N)\right\}
\end{aligned}
$$

Upon an outcome realized, a consumer updates her belief about her trading seller according to $\Phi(p, y)$

$$
\begin{aligned}
\Phi(b, p_{bi}, Y, \bar{y}) &= \varphi(\mu_i \mid b), \ i \geq 0 \\
\Phi(g, p_{gi}, \bar{y}) &= \phi(\mu_t \mid g, \bar{y}), \ i \geq 0 \\
\Phi\left(g, p_{gi}, \underline{y}\right) &= \phi\left(\mu_t \mid g, \underline{y}\right), \ i \geq 0
\end{aligned}
$$

Purchasing strategy:

Switching consumer: $\sigma(s)(m, p_i, \centerdot) = \max\left\{0, \frac{\eta_i(n_i - n_{i-1})}{\sum_{i \in I} \eta_i(n_i - n_{i-1})} sign(\Psi(m, p_i, \centerdot))\right\}$ assigns the probability that a switching consumer chooses the group of age-i sellers in the market. If a switching consumer decides to choose sellers of age i, she randomizes among all sellers in this group, giving weight proportional to their consumer base.

Loyal consumer: $\sigma(s)(m, p_i, \centerdot) = sign(\Psi(m, p_i, \centerdot))$, where i refers to her trading seller.

Switching strategy: $\epsilon(b, p_{bi}) = F \cdot \min\left\{1, \frac{n_{i+1}}{n_i}\right\}$, $\epsilon(g, p_{gt}, \bar{y}) = \min\left\{1, \frac{n_{i+1}}{n_i}\right\}$, otherwise, $\epsilon(z) = 0$, where i refers to her trading seller.

b) Seller strategy. Ω_i is the belief that seller of age-i holds about consumers' beliefs.

Good seller of age i:$\xi_i^G(\omega_i, n_i > 0, q) = (m = q, p = p_{qi})$, where ω_i is the true consumer beliefs, $q \in \{b, g\}$ is the quality of products that a good seller sells at age-i. Bad seller of age i:$\xi_i^B(\omega_i, n_i > 0) = \gamma_g(g, p = p_{gi}) + (1 - \gamma_g)(b, p = p_{bi})$ is a bad seller's randomizing strategy. If $n_i = 0$, a seller chooses to exit.