How Venture Capitalist Compensation Affects Investment Decisions
Preliminary and incomplete

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Abstract

Value-of-distribution rules of venture capital limited partnership agreements determine when venture capitalists receive their share of profits and often generate an interest-free loan between limited partners and venture capitalists. This paper explores how value-of-distribution rules affect venture capitalist's incentives on the timing of starting and exiting decisions, hence the duration, of investments. When the roles of capital provider and decision maker coincide, the investor wants to start the investment as soon as possible if there is only one project under consideration; when there are two projects, due to the convex monitoring cost, the investor intends to keep the overlapping period short by decreasing the duration of the first project and delaying the starting date of the second one. When the roles of capital provider and decision maker separate as in reality, if there is only one project, the Return First contract can restore optimal outcomes given certain level of carried interest for the venture capitalist. The Escrow contract can achieve optimal duration with smaller share of carries for the venture capitalist, but cannot achieve the optimal starting date. The duration under the Payback contract is always shorter than the optimal. When there are two projects, Under the Payback contract, the duration of the second project is shorter than optimal; the difference in project's duration between the Payback and the other two contracts is larger when two projects are considered. Under the Return First contract, the second project has a shorter duration than the first one. Under the Escrow and the Payback contract, the duration of the second project doesn't depend on its starting date.

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1 Introduction

A venture capital fund is typically formed under the limited partnership agreement. The agreement rules the behavior of investors and venture capitalists (henceforth VCs) in the relationship over the entire life of fund. Individual and institutional investors serve as limited partners (henceforth LPs), VCs run the fund as general partners. The partnership agreement explicitly specifies the terms of VC compensation. The visible components of VC compensation are management fee and carried interest. The invisible component is value-of-distribution rules which determine when the carries are paid (see Litvak, 2009). The complexity of VC compensation lies in the form the distribution rules. A venture fund makes several investments at different times throughout its life, and each investment has its own exit date and that is when profits are realized. When the VCs receive their carries becomes an important issue: early distribution of the carries reduces the costs of outside borrowing and creates correct incentives for the VCs who control investment decisions. If the VCs receive their carried interest at the exit dates of investments, it is equivalent to receive a interest-free loan from the LPs through early distributions. Therefore, the timing of distribution because it is highly-valuable to the VCs[1]. The fact that VC compensation terms are relatively standardized (see Axelrad and Wright 1997, Litvak 2009), at least in its basic structure, makes it possible to compare the impact of different distribution rules across funds.

This paper theoretically explores how value-of-distribution rules affect VCs' incentives on the timing of starting and exiting decisions, hence the duration, of investments. The distribution rules we look into are Escrow contract, Return First contract and Payback contract. Under the Escrow contract, the VCs' share goes to an escrow account when each investment is realized, and the VCs only receive payments at the fund liquidation date. Under the Return First contract, the VCs receive no distributions until the invested capital has been fully paid back to the LPs for each investment. After this threshold, the VCs can receive their share of the profits at each exit date. Under the Payback contract, the VCs receive their share of the revenues at the investment exit date, and pay back the invested capital they receive back to the LPs when the fund liquidates.

First we provide the optimal outcomes when the roles of capital provider and decision makers coincide, then we compare the investment decisions under different value-of-distribution rules to the optimal outcome. The results we find are the following. As for the optimal outcomes, we find that the investor wants to start the investment as soon as possible if there is only one project under consideration; when there are two projects, fixing the first one to be normal, due to the convex management cost, the investor intends to keep the overlapping period short by starting the first project at the beginning date and decreasing the duration of it, and delaying the starting date of the second one. When the roles of capital provider and decision makers separate as in reality, if there is only one

[1] Litvak (2009) shows that a shift from the most popular distribution rule to the second most popular rule can effect the VC compensation as much or more than common variations in management fee or carried interest.

project, Return First contract can restore optimal outcomes given certain level of carried interest. Escrow contract can achieve optimal duration with smaller carries for the venture capitalist, but postpone the starting date. The duration under Payback contract is always shorter than the optimal. When there are two projects, also fixing the first one to be normal, under the Payback contract, the duration of the second project is shorter than optimal; the difference in project's duration between the Payback and the other two contracts is larger when two projects are considered. Under the Return First contract, the second project has a shorter duration than the first one. Under the Escrow and the Payback contract, the duration of the second project doesn't depend on the starting date.

To the best of our knowledge, this paper is the first attempt to analyze the impact of value-of-distribution rules on investment decisions. The main contribution of the paper lies in highlighting the impact of free-interest loans between VCs and LPs generated under different distribution rules. It is the crucial element to understand the opaqueness and complexity behind the venture capital agreement. The fact that the VCs can only keep their share of profits instead of revenues makes monitoring costs relatively larger pushes for an early exit of single projects, while the VCs' convex monitoring cost function pushes for a shorter overlapping period when there are two projects, with either an early exit or a later starting date; the interest free loan from the LPs to the VCs pushes for an early exit while the interest free loan from the VCs to the LPs has a contradicting effect. With these incentives, we study the efficiency of different value-of-distribution rules. We show that the Return First contract can provide optimal outcomes when there is only one project. In the sample of Litvak (2009), the Return First contract is the most popular one, used by almost half of the funds in her sample. We provide some theoretical explanation to the question why the Return First contract is more commonly used than others. Gompers and Lerner (1996) find empirical evidence suggest that the price of venture capital services shift if the demand for venture funds changes while the supply of fund managers remains fixed in the short-run, this may help to explain why our theoretical results don't fit the empirical evidence perfectly when there are two projects.

This paper proceeds as follows. Section 2 Section 3 introduces the model. The first best solutions are presented in Section 4 as the benchmark cases. The impact of different VC partnership agreements on the VCs' investment and exit decisions is presented in Section 5. The last section concludes.

2 Related literature

Compared to the extensive literature on the contracts between VCs and their portfolio enterprises, little light has been shed on the contracts between VCs and the LPs of venture funds. The dearth of literature that focuses on limited partnership has been due to the difficulty in obtaining the partnership agreements. To the best of our knowledge, there are only few research papers of VC compensation.

Gompers and Lerner (1999) empirically examine VC compensation with a dataset containing 419 partnerships dated from 1978 to 1992. They observe that the VC compensation of new and smaller funds displays less sensitivity to performance, has less variation and higher fixed base component. Their evidence shows no relation between incentive compensation and performance, and it is consistent with a learning model, in which the pay of new VCs is less sensitive to performance because reputational concerns induce them to work hard. However, the agreements in their dataset are relatively old. In their study, they only consider the visible components of VC compensation (management fee and carried interest), the value-of-distribution rules are ignored.

Metrick and Yasuda (2009) estimate the expected revenue to managers as a function of their investor contracts, with detailed records on 238 funds raised between 1993 and 2006. But their dataset consists of both venture capital funds and buyout funds[2]. In their sample, buyout funds are of larger size than venture capital funds; buyout funds usually has a hurdle rate of 8%, while venture capital funds have no hurdle rate but a 80-20% split in the carries. They test how this estimated revenue varies across the characteristics. They find that, among their sample of funds, about two-thirds of expected revenue comes from fixed revenue components (management fee, transaction fee etc.) that are not sensitive to performance. They find sharp differences between venture capital and buyout funds. Buyout managers build on their prior experience by increasing the size of their funds faster than the VCs do. This leads to significantly higher revenue per partner and per professional in later buyout funds.

Our study is most inspired by Litvak (2009). It is the first study that pays attention to value-of-distribution rules. Litvak (2009) suggests that the value-of-distribution rules should be the third element of VC compensation, along with management fee and carried interest. Through the interest-free loan generated by value-of-distribution rules, the VCs almost always capture a higher fraction of funds' profits than the nominal carry percentage. Compared to Gompers and Lerner (1999), Metrick and Yasuda (2009), Litvak (2009) uses a hand-collected dataset which is of relatively small size[3]. She shows that more complex management fee provisions predict lower total compensation while common proxies for VC quality predict higher levels of the more transparent forms of VC compensation (carried interest and management fee). However, there is no prediction of the levels of opaque compensation (the value of capital distribution rules). VC compensation is less performance-sensitive than commonly believed. On average, a 1% increase in fund returns predicts a 0.47% increase in total VC compensation, which is similar to the performance-compensation elasticity of executive payment of public listed company of the same time period.

[2] 94 are VC funds and 144 are BO funds.

[3] Litvak's (2009) sample consists of partnership agreements of 68 venture capital funds, raised by 28 venture capital firms.

3 Model

There are projects that each needs an investment F. The value of the project depends on the duration of the investment d in two ways. Firstly: longer duration increases the value of a normal project $V(d)$ in a decreasing rate, i.e., $V'(d) > 0$ and $V''(d) < 0$. Secondly: the project may either remain normal or fail. The event of a failed project follows a Poisson process with intensity parameter λ_F. A failed project pays 0. Time is continuous. We also consider that if a project has shown to be a failure, then uncertainty is realized[4]. It means that a project can only change state if it is normal. Therefore the probability that a project fails at any given moment t is $\lambda_F e^{-\lambda_F t}$. The revenue of a normal project can either be sufficient to compensate the invested capital, i.e. $V(d) > F$, or not, i.e. $V(d) < F$. If it cannot compensate the invested capital, i.e. $V(d) < F$, the VCs will terminate the project immediately. So, we only consider the case $V(d) > F$.

The VCs are risk neutral, cash poor and care about their monetary returns. Time is discounted at rate r. The VCs have to monitor the project, due to their limited time and energy, we consider that there is a convex cost of doing so: when there are two active investments at the same time, the funds may need more VCs and administrative personnel to monitor these investments just to keep the monitoring intensity at the same level as that of one project. The monitoring cost incurred by the VCs up to time t is

$$\begin{aligned} c(d_a, d_b, t) &= \delta \int_a^t e^{r(T-t)} \left(\Phi_a(d_a, t) + \Phi_b(d_b, t)\right)^2 d\tau, \\ 0 &\leq a \leq b \leq t, \end{aligned}$$

where $\Phi_i(d_i, t) = 1$ if project i is active at time t and zero otherwise. The subscripts a, b indicate the starting date of each project. The VCs decide when to start the project and when to exit, hence its duration. We present all the expressions in future value evaluated at T, which is the fund liquidation date, it allows a nature comparison between these contracts that the LPs and the VCs sign.

4 First Best

We provide the first best case where the roles of capital provider and investment decision makers coincide as the benchmark case. The projects have the same characteristics as we describe before.

[4]Venture funds invest in portfolio enterprises that have a novel technology or business model in some high technology industries, such as biotechnology, IT, software, etc; they offer high potential but high risk (see Sahlman, 1990). Once the portfolio enterprises fail, venture funds usually just write them down or off. The case of Apple is rather rare.

4.1 One Project Only

We start with the case where there is only one project which starts at date 0, the starting date of the venture fund. The investor must evaluate whether to exit from or remain in the investment at each point in time. If the project fails, the investor will terminate the investment immediately. There will be no revenue, but further monitoring cost can be avoided by exiting.

At date 0 the expected profits of having a project with duration d, as long as it remains normal, are

$$\begin{aligned} E\left(\Pi\left(d_N\right)\right) &= e^{-\lambda_F d}\left(e^{r(T-d)}V\left(d\right)-\int_0^d \delta e^{r(T-\tau)}d\tau\right) \\ &\quad +\int_0^d \lambda_F e^{-\lambda_F t}\left(-\int_0^t \delta e^{r(T-\tau)}d\tau\right)dt - e^{rT}F. \end{aligned}$$

The first part of the first two terms concerns the probability of being at each specific state after duration d and the second part (in brackets) concerns the future value of the payoff of each possible state, minus the invested capital of the project, evaluated at date T.

The investor solves

$$\max_d E\left(\Pi\left(d\right)\right)$$

The investor's strategy is determined by a first order condition that depends on whether the optimal exit date of a normal project has been passed or not. If the project remains normal then the exit condition is

$$rV\left(d_N\right)+\delta = V'\left(d_N\right)-\lambda_F V\left(d_N\right).$$

On the left hand side we have the marginal cost of not exiting, it consists of interest opportunity cost and the increase in monitoring cost. On the right hand side lies the marginal benefit of staying. It includes not only the growth of a regular project, but also the possibility that a project may turn out to be a failure.

We then ask when the investor wants to start the project. If the starting date is not fixed to be date 0 which date will the investor choose? The invetor's expected profits are

$$\begin{aligned} E\left(\Pi\left(d,a\right)\right) &= e^{-\lambda_F d}\left(e^{r(T-d-a)}V\left(d\right)-\int_a^{a+d} \delta e^{r(T-\tau)}d\tau\right) \\ &\quad +\int_0^d \lambda_F e^{-\lambda_F t}\left(-\int_a^{a+t} \delta e^{r(T-\tau)}d\tau\right)dt - e^{r(T-a)}F. \end{aligned}$$

The expected profits can be rewritten as

$$E\left(\Pi\left(d,a\right)\right)=e^{r(-a)}E\left(\Pi\left(d\right)\right).$$

The derivative with respect to a is

$$\frac{\partial E\left(\Pi\left(d,a\right)\right)}{\partial a}=-re^{r(-a)}E\left(\Pi\left(d\right)\right)<0.$$

Therefore, if there is only one project, the investor always wants to start financing at date 0.

4.2 Two Projects

Now we consider the case of two projects. The investor will have to decide when to start financing and when to exit from the investments. We assume that the first project starts at date a and the second one starts at date b. For simplicity, we assume the first project stays normal for now[5]. The second project can either stay normal or fail. Let d_a and d_b be the durations of two projects, respectively.

The existence of a second project, which may eventually overlap with the first, changes the VCs' effort cost.

The expected profits at date 0 are

$$\begin{aligned}
&E\left(\Pi\left(d_a,d_b,a,b\right)\right)= \\
&e^{-\lambda_F d_b}\left(\begin{array}{c} e^{r(T-d_a-a)}V\left(d_a\right)+e^{r(T-d_b-b)}V\left(d_b\right) \\ -\delta\left(\int_a^b e^{r(T-\tau)}d\tau+\int_b^{d1N}4e^{r(T-\tau)}d\tau+\int_{d1N}^{d2N}e^{r(T-\tau)}d\tau\right)\end{array}\right)+ \\
&\int_a^{d_b}\lambda_F e^{-\lambda_F t}\left(\begin{array}{c} e^{r(T-d_a-a)}V\left(d_a\right) \\ -\delta\left(\int_a^b e^{r(T-\tau)}d\tau+\int_b^{d1F}4e^{r(T-\tau)}d\tau+\int_{d1F}^{d2F}e^{r(T-\tau)}d\tau\right)\end{array}\right)dt+ \\
&-e^{r(T-a)}F-e^{r(T-b)}F,
\end{aligned}$$

$$0\leq a\leq b\leq t,$$

where the number 1 or 2 of the state designation indicates the project ends first or later, respectively; the letters N and F of the state designation refer to the states of the second project. $d1N=\min\left\{a+d_a,b+d_b\right\}$ denotes the duration of the project that ends first and $d2N=\max\left\{a+d_a,b+d_b\right\}$ denotes the duration of the project that ends second in state N. When the second project fails, the duration of the project that ends first is $d1F=\min\left\{b+t,a+d_a\right\}$ and that of the project which ends second is $d2F=\max\left\{b+t,a+d_a\right\}$.

In this expression we include the payoff and the investment of both projects. We also take into account the new monitoring cost. Here we allow for any ending dates of the projects. It may be the case that the first project ends before the second, or the other way around.

[5] We will get rid of this assumption in the near future.

4.2.1 Exit decisions

As we mention before, if a project fails, the investor will terminate it immediately. There will be no revenue, but further monitoring cost can be avoided. So the investor only intentionally decides the exiting timing for a normal project. The presence of the other project increases the monitoring cost and consequently changes the exit decision of the current one, depending on whether the other one is active by the time of the exit decision.

If the second project has started and does not fail, and therefore is still active by d_0^* the the exit condition is

$$rV\left(d_a^*\right) + 3\delta = V'\left(d_a^*\right).$$

In case the second project ends before d_0^* the exit condition of the first project is

$$rV\left(d_a\right) + \delta = V'\left(d_a\right),$$

and coincides with the exit decision of one single project. Obviously $d_a^* < d_a$.

We can conclude that the introduction of a second investment decreases the investment duration of the first project in case the second project is active. The second project creates an externality on the first one, through the increase of the monitoring cost. This externality is only present while the second project is active, hence we can also conclude that, conditional on having a failure on the second project, it is better to have it before the first project ends than after it ends, as this will enable the investor of making a better decision on the first project.

The exit decision of the second project resembles the exit decision when there is one project only. The only possible difference is whether the first one is still active by the exiting date of the second project . In that case the marginal monitoring cost would increase to 3δ.

4.2.2 Starting decisions

Then we look into when the investor starts the projects. The investor solves

$$\max_{a,b} E\left(\Pi\left(d_a, d_b, a, b\right)\right).$$

The first derivative for starting date a is

$$\frac{\partial E\left(\Pi\left(d_a, d_b, a, b\right)\right)}{\partial a} = -re^{r(-a)} E\left(\Pi\left(d, b\right)\right) < 0,$$

which is very simply to to that of the single project case.

The first derivative for starting date b is

$$\frac{\partial E\left(\Pi\left(d_a, d_b, a, b\right)\right)}{\partial b} = -r\left(E\left(\Pi\left(d, b\right)\right) - 2e^{r(T-b)}\right).$$

The only difference is the change in the monitoring cost. When there are two projects, overlapping has to be taken into account. When there are two projects it is optimal to start the second as late as possible. It will be the case when $E\left(\Pi\left(d,a\right)\right) < 2e^{r(T-a)}$, i.e. when the individual profits are not sufficient to compensate for the increase in the monitoring cost.

5 Contracts

In the market the VCs usually are not the owners of capital. Wealthy institutional and individual investors put up a fund and hire VCs to run it. The partnership agreement consists of the committed capital, K, the fund liquidation date, T, the management fee, the size of the carried interest for the VCs, α, and the rules of capital distribution (β, γ).

The committed capital K is the total amount that the LPs make available for investment. Therefore, the capital is owned and retained by the LPs, but is available to the VCs. For simplicity, we assume the committed capital K is available from date 0. The fund liquidation T date is the date when the fund ends, and where all payments are finalized. The fund typically has a contractually limited life of 10 years with a provision for an extension of 1 to 3 years. The VCs earn their payments through two channels: firstly, through manegement fees, which usually is set as a percentage of the total committed capital[6]; secondly, through the carried interest α, the VCs receive α percentage of the profits of the fund. The final feature of the contracts is the value-of-distribution rules describing when each party receives the payment they are entitled to.

Our interest is to understand the impact of value-of-distribution rules on the VCs' incentives on the timing of starting and exiting decisions, hence the duration, of investments. In order to capture the impact of the distribution rules, we keep the committed capital K, the fund liquidation date T and the carried interest α constant to see how the VCs' decisions change with different distribution rules.

There are some typical distribution rules of contracts that can be fully described by using the following parameters (β, γ). β indicates whether the VC is entitled to some interest of the investment revenues if the exit occurs before the liquidation date T, it takes value of 0 or 1. Therefore, $\beta = 1$ means that the VC receives some capital before the fund liquidates. γ is the share of the invested capital F that the VC has to return to the LPs by the time when investments exit before the liquidation date T. γ may either be constant or a function of investment revenues.

We consider three possible contracts. They generate an interest-free loan either from the VCs to the LPs (Escrow contract), or from the LPs to the VCs (Payback Contract), or no interest-free loan (Return First contract).

The first is the "Escrow, All Interest To Fund" contract (Escrow contract). The profits are distributed to investors throughout the fund's life, the VCs' share goes to an escrow account, and the interest of this accont belongs to the

[6] The typical management fee is for a fund to charge an annual fee of about 2% of total committed capital (see Kaplan 1999, Gompers and Lerner 1999, Litvak 2009).

LPs. The VCs only receive payments when the fund liquidates. In this case, the amount of the invested capital that is paid back to the LPs at the exit date of each investment is irrelevant because all revenues are held by the LPs in any case. In practical terms, the VCs are making an interest-free loan of the carried interest to the LPs during the time period between the investment exit date and the fund liquidation date. In this model, the Escrow contract implies $\beta = 0$ and $\gamma = 1$.

The second is the "Payback with No Interest Note" contract (Payback contract). The VCs receive α share of the revenues at the exit date of each investment, and transfers between the LPs and the VCs happen to adjust final payoffs at the liquidation date. This is because the VCs are only entitled to α share of realized profits, they should simultaneously pay back the α share of invested capital to the LPs. The repayment usually is made at liquidation date (see Litvak, 2009). The VCs are entitled to the interest of the amount they receive at the investment exit date. Actually, the LPs make an interest-free loan of the amount equivalent to α share of the invested capital between the exit date and fund liquidation date. In this model, the Payback contract implies $\beta = 1$ and $\gamma = 0$.

The third is the "Return All Capital Contributions First" contract (Return First contract). For each investment, the VCs receive no distributions until total invested capital (F) has been paid back to the LPs. After this threshold, the VCs can receive their carry of the profits. In the model, it implies $\beta = 1$ because the VCs are entitled to their carry of profits as long as the revenues exceed the invested capital. It also implies $\gamma = \min\left\{\frac{V(d)}{F}, 1\right\}$, if the revenues are not enough to cover the invested capital, they're all spent compensating the LPs.

5.1 One Project Only

Assume there is only one project and it starts at date a with the same uncertainty we describe before. We consider each decision the VCs make under different contracts, and compare them with the optimal decisions.

We first define the payoff function of the VCs. If the investment is profitable, then the VCs will get paid according to the contract. If the investment is not profitable, the VCs will deliver all project revenues to the LPs[7]. Therefore the compensation scheme for the VCs is

$$\pi = e^{\beta r(T-d)}\alpha\left(V\left(d\right) - \gamma F\right) - \alpha F\left(1 - \gamma\right)$$

[7] Under the Escrow and the Return First contract all the revenues will be delivered to the LPs on the exit date; under the Payback contract, the VCs can still receive a share of revenue on the exit date. If one of the projects fail, the VCs can receive nothing under the Escrow and the Return First contract. But under the Payback contract, there is effectively an interest-free loan for the VCs from the LPs, this may change the VCs' decision on exiting.

In case $V(d) > F$, the expected payoff of the VCs is

$$E[\Pi(d,a)] =$$
$$e^{-\lambda_F d}\left(e^{\beta r(T-a-d)}\alpha\left(V(d)-\gamma F\right)-\delta\int_a^{a+d}e^{r(T-\tau)}d\tau\right)+$$
$$\int_a^{a+d}\lambda_F e^{-\lambda_F(t-a)}\left(-\delta\int_a^t e^{r(T-\tau)}d\tau\right)dt-\alpha F(1-\gamma),$$

where d is the duration of a project if it remains normal and a is the starting date of the project. The first part of each term concerns the probability that either of the possible states is achieved, and the second part concerns the correspondent payoff. For instance, the probability that the project fails if the VC remains in an investment that starts at a for the duration d is $\int_a^{a+d}\lambda_F e^{-\lambda_F(t-a)}dt$. In this case the VC will receive nothing, also he has a monitoring cost, he would exit from the investment immediately.

The VCs solve

$$\max_{d,a} E[\Pi(d,a)].$$

5.1.1 Exit Decision

For $V(d) > F$, the exit condition for an interior solution of a normal project is

$$\begin{aligned}\beta r e^{\beta r(T-a-d_N)}\alpha\left(V(d_N)-\gamma F\right)&\\ +\delta e^{r(T-a-d_N)} &= e^{\beta r(T-a-d_N)}\alpha V'(d_N)-e^{\beta r(T-a-d_N)}\lambda_F\alpha V(d_N)\end{aligned}$$

On the left hand side of the condition again we have the marginal cost of staying in the investment which comprises the interest opportunity cost and the marginal monitoring cost. On the right hand side we have the growth of a normal project, the possibility that the project fails and all revenue is lost.

For all contracts, the fact that the VCs only keep a share α of the profits makes the monitoring cost relatively higher. It is the effort incentive distortion that pushes for an early exit of the project.

Under the Escrow contract the exit condition is

$$\delta e^{r\left(T-a-d_N^E\right)}=\alpha V'\left(d_N^E\right)-\lambda_F\alpha V\left(d_N^E\right)$$

Since the VCs will not receive anything until the fund liquidate, they face no interest opportunity cost, it pushes for a later exit. But there are the effort incentive distortion and the timing distortion both pushing for an early exit. The timing distortion arises from the fact that the VCs only get paid at T increases the marginal cost of staying longer because $e^{r\left(T-a-d_N^E\right)} > 1$.

Under the Payback contract the exit condition is

$$r\alpha V\left(d_N^P\right)+\delta=\alpha V'\left(d_N^P\right)-\lambda_F\alpha V\left(d_N^P\right)$$

In this case, the only distortion is the effort incentive distortion that pushes for an early exit.

Finally, under the Return First contract the exit condition is

$$r\alpha\left(V\left(d_N^R\right)-F\right)+\delta=\alpha V'\left(d_N^R\right)-\lambda_F\alpha V\left(d_N^R\right)$$

Now there are two distortions leading towards opposite directions. On the one hand we still observe the effort incentive distortion, which leads to an early exit; on the other hand the interest opportunity cost of not exiting is now smaller, pushing for a later exit. This new effect is not present under the Payback contract and makes the VCs delay the exit decision. The intuition behind this is that the VCs are still not be able to grasp all the interest of the carry, and therefore will rather allow the project to grow more.

Proposition 1 *If $V(d)>F$ then the duration of a normal project under the Payback contract is shorter than that of the first best case.*
There exists an $\underline{\alpha}^R\equiv\frac{V'(d_N)-\lambda_F V(d_N)-rV(d_N)}{V'(d_N)-\lambda_F V(d_N)-rV(d_N)+rF}$ such that the duration under the Return First contract coincides with the first best case. If $\alpha<\underline{\alpha}^R$ then the duration under the Return First contract is shorter than that of the first best case.
There exists an $\underline{\alpha}^E\equiv\frac{e^{r(T-a-d_N)}(V'(d_N)-\lambda_F V(d_N)-rV(d_N))}{V'(d_N)-\lambda_F V(d_N)}$ such that the duration under the Escrow contract coincides with the first best case. If $\alpha<\underline{\alpha}^E$ then the duration under the Escrow contract is shorter than that of the first best case.

From the previous proposition we can state that if the LPs are seeking to achieve the efficient project duration, then they cannot rely on the Payback contract. The Escrow contract and the Return First contract are able to achieve optimal duration.

Proposition 2 *For a same share α the duration under the Payback contract is shorter than that of the Return First contract.*

This proposition follows directly from the fact that under the Return First contract the interest oppotunity cost is smaller than that of the Payback contract, which makes the VCs stay longer.

5.1.2 The starting date

Concerning the starting date of the project the derivative with respect to a is

$$(-r)\,e^{-\lambda_F d_N}\left(-\delta\int_a^{a+d_N}e^{r(T-\tau)}d\tau\right)+$$
$$(-r)\int_a^{a+d_N}\lambda_F e^{-\lambda_F(t-a)}\left(-\delta\int_a^t e^{r(T-\tau)}d\tau\right)dt.$$

Under the escrow contract, the derivative is

$$(-r)\,e^{-\lambda_F d_N^E}\left(-\delta\int_a^{a+d_N^E}e^{r(T-\tau)}d\tau\right)+$$
$$(-r)\int_a^{a+d_N^E}\lambda_F e^{-\lambda_F(t-a)}\left(-\delta\int_a^t e^{r(T-\tau)}d\tau\right)dt>0.$$

Under the Payback contract the first derivative coincides with

$$(-r)\,E\left[\Pi\left(d_N^P, a\right) + \alpha F\right] < 0.$$

Under the Return First contract

$$(-r)\,E\left[\Pi\left(d_N^R, a\right)\right] < 0.$$

Proposition 3 *Under both the Payback contract and the Return First contract, the VCs always want to start the project at date* 0*. Under the Escrow contract, the VCs decide to start as late as possible.*

The intuition is that under the Escrow contract the VCs will not receive any profits until T, hence they will rather postpone the project to decrease the monitoring cost.

5.2 Two Projects

Same as before, we assume the first project starts at date a and stays normal, the second project starts at date b, it can stay normal or fail. If both projects remain normal until their optimal exit decisions then there will be enough revenue for the fund to have profits, one project cannot generate enough revenue to cover the invested capital for two projects, i.e. $F < V(d) < 2F$ [8]. Let d_a and d_b be the duration of two projects respectively.

Since now we have two projects under consideration, we need to define how much of the invested capital is returned to the LPs at each exit date. The Escrow contract implies $\beta = 0$, $\gamma_a + \gamma_b = 1$, which means that the VCs will only receive their share of carry at the fund liquidation date T. In this case it does not matter how much of the invested capital is paid by each project. The payback contract implies $\beta = 1$, $\gamma_a = \gamma_b = 0$, the VCs only return the invested capital when the fund liquidates at date T. Finally, the Return First contract suggests $\beta = 1$, which means the VCs can receive some profits before T as long as the invested capital is returned to the LPs. But the revenue of one project cannot cover the invested capital for two projects, under the Return First contract, the revenue of the project that ends first is all returned to the LPs, the revenue of the project that ends later is partly returned to the LPs and the rest, if there is any, $\gamma_a = \frac{V_a}{2F}$, $\gamma_b = \frac{2F - V_a}{2F}$ if the first project ends first and the second one ends later; $\gamma_a = \frac{2F - V_b}{2F}$, $\gamma_b = \frac{V_b}{2F}$ if the first project ends after the second one.

We define the payoff function of the VCs. If these investments are profitable, then the VCs will get paid according to the types of contract. If these investments are not profitable, they will to deliver all project revenues to the LPs[9]. So, the payoff function for the VCs when the second project remains normal is

$$\pi = e^{\beta r(T-d_a-a)}\alpha\left(V\left(d_a\right) - \gamma_a F\right) + e^{\beta r(T-b-d_b)}\alpha\left(V\left(d_b\right) - \gamma_b F\right) - \alpha 2F\left(1 - \gamma_a - \gamma_b\right)$$

[8] Later we would relax this assumption, we would assume one project could generate enough revenue to cover the total invested capital, which we believe would be a better fit to industrial facts. We would also assume the first project can also fail.

[9] Under the Escrow and the Return First contract all the revenues will be delivered to

5.2.1 Exit decisions

The VCs maximize their expected payoff by deciding over the starting and exit dates of projects. They solve

$$\max_{d_a, d_b, a, b} \quad E\left[\Pi\left(d_{aN}, d_{bN}, a, b\right)\right] = -\alpha 2F\left(1-\gamma_a-\gamma_b\right)$$

$$e^{-\lambda_F d2N}\left(\begin{array}{c} e^{\beta r\left(T-d_{aN}-a\right)}\alpha\left(V\left(d_{aN}\right)-\gamma_a F\right)+e^{\beta r\left(T-b-d_{bN}\right)}\alpha\left(V\left(d_{bN}\right)-\gamma_b F\right) \\ -\delta\left(\int_a^b e^{r(T-\tau)}d\tau+\int_b^{d1N} 4e^{r(T-\tau)}d\tau+\int_{d1N}^{d2N} e^{r(T-\tau)}d\tau\right) \end{array}\right)+$$

$$\int_b^{d1N} \lambda_F e^{-\lambda_F(t-a)}\left(-\delta\left(\int_a^b e^{r(T-\tau)}d\tau+\int_b^t 4e^{r(T-\tau)}d\tau\right)\right)dt+$$

$$\int_{d1N}^{b+d2N} \lambda_F e^{-\lambda_F(t-a)}\left(-\delta\left(\int_a^b e^{r(T-\tau)}d\tau+\int_b^{d1N} 4e^{r(T-\tau)}d\tau+\int_{d1N}^t e^{r(T-\tau)}d\tau\right)\right)dt,$$

where the number 1 or 2 of the state designation indicates the project ends first or later, respectively; the letters N and F of the state designation refer to the states of the second project. $d1N=\min\{a+d_a, b+d_b\}$ denotes the duration of the project that ends first and $d2N=\max\{a+d_a, b+d_b\}$ denotes the duration of the project that ends second in state N.

Exit decision of the first project When deciding whether to exit or to stay longer in the first project, the VCs have to consider the possible states of the second project. The first order condition of the exit decision of the first project, in case the second project is normal by d_{aN} is quite cumbersome, so we only present it for each specific contract. In case the second project fails before d_{aN} then the VCs exit immediately as all the revenue off this project will belongs to the LPs.

If the first project is normal then the exit condition under the escrow contract is

$$3\delta e^{r\left(T-d_{aN}^E\right)}=\alpha V'\left(d_{aN}^E\right)e^{-\lambda_F\left(a+d_{bN}^E-d_{aN}^E\right)}$$

where $e^{-\lambda_F\left(a+d_{bN}^E-d_{aN}^E\right)}$ is the probability that the second project remains normal, evaluated at d_{aN}^E and conditional on b and d_{bN}^E.

For the escrow contract there is no interest opportunity cost. The growth is multiplied by the probability that the second project does not fail because if it does fail then the VCs won't be entitled to any payments.

If the second project is normal then the exit condition under the Payback contract is

$$e^{-\lambda_F\left(a+d_{bN}^P-d_{aN}^P\right)}+3\delta=\alpha V'\left(d_{aN}^P\right)e^{-\lambda_F\left(a+d_{bN}^P-d_{aN}^P\right)}.$$

the LPs on each exit date; under the Payback contract, the VCs can still receive a share of revenues on each exit date. If one of the projects fail, the VCs can receive nothing under the Escrow and the Return First contract. But under the Payback contract, there is effectively an interest-free loan for the VCs from the LPs. If the benefits are large enough to cover the monitoring cost, the VCs will still keep the projects active even they fail.

The first term on the left hand side is the expected value of the interest that the VCs retain. It is in expectation because the VCs will only have access to it if the second project doesn't fail. The second term is the marginal monitoring cost. The term on the right hand side is the growth of the first project, which the VCs will be able to benefit from if the second project does not fail.

Under the Return First contract if the second project is normal then the exit condition is of the first project is

$$3\delta = \alpha V' \left(d_{aN}^{R}\right) e^{-\lambda_F \left(a + d_{bN}^{R} - d_{aN}^{R}\right)} e^{r\left(d_{aN}^{R} - b - d_{bN}^{R}\right)}$$

In the Return First contract the VC evaluates the marginal monitoring cost against the growth of the project. We have the probabilities with different associated weights.

Exit decision of the second project If the second project is normal at d_{bN} then the first order condition for any contract, of an interior solution is

$$\begin{aligned}
&r\beta e^{\beta r(T-b-d_{bN})}\alpha\left(V\left(d_{bN}\right)-\gamma_b F\right)+e^{r(T-b-d_{bN})}\delta = \\
&e^{\beta r(T-b-d_{bN})}\alpha V'\left(d_{bN}\right) - \\
&\lambda_F\left(e^{\beta r(T-a-d_{bN})}\alpha\left(V\left(d_{bN}\right)-\gamma_b F\right)+e^{\beta r(T-d_{aN}-a)}\alpha\left(V\left(d_{aN}\right)-\gamma_a F\right)\right).
\end{aligned}$$

Under the Escrow contract the exit condition is

$$e^{r\left(T-a-d_{bN}^{E}\right)}\delta = \alpha V'\left(d_{bN}^{E}\right) - \lambda_F \alpha \left(V\left(d_{bN}^{E}\right) + V\left(d_{0N}^{E}\right) - 2F\right).$$

Under the Payback contract the exit condition for a normal project is

$$r\alpha V\left(d_{bN}^{P}\right) + \delta = \alpha V'\left(d_{bN}^{P}\right) - \lambda_F\left(\alpha V\left(d_{bN}^{P}\right) + e^{r\left(a + d_{bN}^{P} - d_{aN}^{P}\right)}\alpha V\left(d_{aN}^{P}\right)\right).$$

Under the Return First contract the exit condition of a normal project is

$$r\alpha\left(V\left(d_{bN}^{R}\right)+V\left(d_{aN}^{R}\right)-2F\right)+\delta = \alpha V'\left(d_{bN}^{R}\right)-\lambda_F\alpha\left(V\left(d_{bN}^{R}\right)+V\left(d_{aN}^{R}\right)-2F\right).$$

For the three contracts the distortions towards the optimal are just as in the case of the exit decision when there is one project only except for one. In the first best the consequence of the failing of the project is the project's lost revenue. This was also true when there was one project only that could pay for all the committed capital. When there are two projects the evaluated loss is contract dependant. Under the Escrow and the Return First contracts the VCs lose the total profits which are smaller than the total revenues. This increases the benefit of staying longer in the project and therefore this distortion pushes for a later exit. In case of the Payback contract the VCs lose the present value of the total revenue generated by the two projects. This is obviously larger than the revenue generated just by the second project. In this case the fear of failure has a larger value than in the first best and therefore the distortion pushes for an early exit.

Corollary 4 *The difference in project's duration between the Payback and the other two contracts is larger when two projects are considered.*

One more important point about the comparison of the three contracts is that the result first contract is the only contract for which the exit decision of the second project depends on the generated revenue in the first project.

Corollary 5 *Under the Return First contract, in a fund with two projects, the second project has a shorter duration.*

It is more likely that the revenue of projects that start later does not contribute to pay for the invested capital. Therefore these projects have a larger interest opportunity cost, and hence a smaller duration.

Corollary 6 *Under the Escrow and Payback contracts, in a fund with two projects, the projects' duration does not depend on their starting date.*

6 Conclusion

This paper serves two main objectives. The first is to theoretically investigate how value-of-distribution rules of venture capital limited partnership agreements affect the VCs' incentives on the timing of starting and exiting decisions, hence the duration, of investments. This is the first attempt to analyze the impact of value-of-distribution rules on investment decisions. As we model the failing rate of projects to be exogenous, which may not fit to the industrial facts, our results need to be viewed merely as an explanation on the VCs' incentives on timing. Nevertheless, we believe that our results are useful and can shed some light on relationship between the investors and the VCs.

The second objective of the paper is to provide some theoretical explanation to the question why a certain type of contract, namely the Return First contract, is more commonly used than others. In this respect, our model successfully shows that under the Return First contract, the VCs make optimal starting and exiting investment decisions if there is only one project under consideration. However, Litvak's (2009) hand-collect dataset may suffer from selection bias issues[10]. Venture capital limited partnership agreements are usually confidential documents, due to the difficulty in obtaining the partnership agreements, we can not compare our theoretical results to other samples of agreements for now.

The main focus of this paper is the VCs' incentives on timing of investment decisions. For future research it would be interesting to move our focus from the VCs' incentives on timing to the incentives on monitoring projects. The VCs monitor the investments and provide the portfolio enterprises with managerial consulting etc. It would be ideal if we could modify our model to capture the

[10] Litvak's (2009) main dataset is relatively small; it contains partnership agreements of 68 venture capital funds, raised by 28 venture capital firms. Compare to other major database (VentureXpert), the funds in her sample are better than average in terms of fund size and success rate in their portfolios.

effect of monitoring on the failing rate of projects. This interesting idea is left for being explored soon.

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