Dynamic efficiency and intergenerational altruism*

Emmanuel Thibault†

Toulouse School of Economics (GREMAQ)

Abstract: Can dynamic inefficiency that may occur in societies populated by non altruistic agents be removed by introducing intergenerational altruism ? Although the answer (see Abel, 1987, AER or Weil, 1987, JME) seems to be negative, this paper shows, by means of a simple example, that the presence of an arbitrarily low proportion of altruists can be sufficient to prevent a society from reaching a non Pareto optimal equilibrium. Intergenerational transfers from the old to the young can therefore provide an alternative —to public debt, fiat money or money bubbles which transfer goods from the young to the old— solution to the dynamic efficiency problem.

Keywords: OLG model, Dynamic Efficiency, Intergenerational Altruism.

J.E.L. classification: C 62 - D 91 - O 41.

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†TSE (GREMAQ) - Manufacture des Tabacs - 21, allée de Brienne - 31 000 Toulouse - France. Tel: (+33)-5-61-12-85-74 / Fax: (+33)-5-61-22-55-63 / E-mail: emmanuel.thibault@univ-tlse1.fr

1 Introduction

Dynamic efficiency is a key to understand a number of positive and normative questions raised by economic analysis. In particular it plays a critical role in the analysis of the effects of fiscal (debt) policies in growth models. This analysis is usually developed in the overlapping generations settings of Diamond (1965) and Barro (1974), both considering finitely lived agents. In Diamond (1965) people are pure life cyclers, dynamic inefficiency[1] can arise and public debt matters. In Barro (1974) agents are linked across generations by altruistic bequests. In such a setting public debt is neutral and the market equilibrium is dynamically efficient.

In view of such clear opposite conclusions one important question has been raised: *Can the dynamic inefficiency taking place in societies populated by non-altruistic agents be ruled out by introducing intergenerational altruism ?*

It is commonly assumed that the answer to this question is negative. Abel (1987) and Weil (1987) argue that the dynamic efficiency result of Diamond (1965) is a necessary condition for an altruistic bequest motive to matter and for the Ricardian equivalence theorem of Barro (1974) to hold.

In this paper, we show that this form of dynamic inefficiency can be ruled out by the introduction of Barro's (1974) intergenerational altruism. Indeed, an intergenerational transfer *from the old to the young* can provide a solution to the dynamic efficiency problem. Our solution therefore departs from the standard solution, public debt (Diamond, 1965), fiat money (Wallace, 1980) or money bubbles (Tirole, 1985), which transfer goods *from the young to the old.*

The paper starts by noting that Abel's (1987) and Weil's (1987) result relies on a set of assumptions placed on the model to insure existence, uniqueness and stability of the steady state of the underlying Diamond's (1965) model. In a first step, we discuss the relevance of these assumptions. In a second step, we relax most of them, just keeping standard concavity hypotheses on economic fundamentals (utility and production functions) and discuss the implications of our exercise. We finally provide a simple example, inspired by Galor and Ryder (1989) and Nourry and Venditti (2001), to es-

[1]Such an economy is said to be dynamically inefficient since a Pareto–improvement can be achieved by allowing the current generation to eat a portion of the capital stock and by leaving the consumption of all future generations intact.

tablish that the existence of an arbitrarily low proportion of altruistic agents can be sufficient to rule out dynamic inefficiency in Diamond's model.

The paper is organized as follows. Section 2 reviews both the seminal and the recent literature on the existence of a steady state with positive bequests in Barro's (1974) model. It then presents a graphical intuition of the paper and some methodological issues. In section 3, building on Galor and Ryder (1989), we develop a simple example in which Diamond's (1965) economy converges to a non Pareto optimal equilibrium whereas Barro's (1974) economy converges to a Pareto optimal steady state. Section 4 assesses the robustness and the relevance of the example.

2 An intuitive approach

The main idea of the seminal Robert Barro (1974) work is that a network of intergenerational transfers makes the typical person part of an infinitely lived family. Therefore, the economy converges to the so–called Modified Golden Rule (hereafter M.G.R.) equilibrium.[2] The main objection to Barro's result is that it requires positive bequests on top of the assumption of altruistic consumers to be established.

2.1 - Positive bequests and intergenerational altruism.

The positiveness of bequests has been studied by several economists. Abel (1987) and Weil (1987) were the first to establish a formal condition for the existence of a steady state with positive bequests in Barro's (1974) model (hereafter $\mathcal{BM}$). Both of them assume that the underlying overlapping generations economy — the Diamond's (1965) model (hereafter $\mathcal{DM}$) — possesses a unique and (locally) stable steady state capital stock, k^D. This assumption implies that the function $\phi(k)$ that summarizes the dynamics of $\mathcal{DM}$ is locally concave (in a neighborhood of k^D). Under this restrictive assumption, Abel (1987, Proposition 1) and Weil (1987, Proposition 2) show that bequests x^M are positive in $\mathcal{BM}$ if and only if the M.G.R. capital stock, k^M, is larger than k^D. Since k^M is lower than the Golden Rule capital stock k^G, this condition implies that over-accumulation of capital[3] in $\mathcal{DM}$ rules out positive bequests in $\mathcal{BM}$. Then,

[2]The M.G.R. capital stock k^M is equal to $f^{'-1}[(1+n)/\beta]$ where $f(.)$ is the neoclassical production function, $n > -1$ the exogenous population growth rate and $\beta \in (0, 1]$ the degree of altruism.

[3]Over-accumulation of capital occurs when the Diamond equilibrium k^D is greater than the Golden Rule $k^G = f'^{-1}(1 + n)$ and thus also greater than the Modified Golden Rule: $k^D > k^G \geq k^M$.

the dynamic inefficiency that occurs in $\mathcal{DM}$ may not be removed by the introduction of intergenerational altruism.

It is worth noting that this result relies on the uniqueness of k^D and on the local concavity (around k^D) of $\phi(.)$. Recent researches indeed emphasize that these assumptions on $\mathcal{DM}$ are not fully appropriate to address $\mathcal{BM}$ for two main reasons.

First, Abel (1987, p. 1042) and Weil (1987, footnote 8) suggest that assuming uniqueness of k^D and local concavity (around k^D) of $\phi(.)$ in $\mathcal{DM}$ should be sufficient to avoid counterintuitive results in $\mathcal{BM}$. This conjecture, though, has not been formally proved. In a simple example, Thibault (2001) shows that an increase in the degree of altruism can result in a decrease in the steady state level of bequests even if k^D is unique and $\phi(.)$ is globally concave. To rule out this counterintuitive result, an assumption on the curvature of the production function $f(.)$ is needed.[4]

Second, Galor and Ryder (1989) show that, contrary to the common wisdom, existence (and, consequently, uniqueness) of k^D in $\mathcal{DM}$ is not guaranteed by standard assumptions on fundamentals (i.e., concavity and Inada conditions of production $f(.)$ and utility $U(.)$ functions). In fact, the concavity of $\phi(.)$ crucially depends on the third derivatives of $f(.)$ and $U(.)$, which can take a broad range of values without violating concavity of $f(.)$ and $U(.)$. As a result, even if k^D exists, its uniqueness requires stronger assumptions on the interaction of technology and preferences. Galor and Ryder (1989) exhibit examples which illustrate this result: multiple steady-states can arise in $\mathcal{DM}$ although both functions $f(.)$ and $U(.)$ are concave on their domains. Obviously, $\phi(.)$ is not locally concave around all the equilibria k^D in these examples.

Since assuming uniqueness of k^D and local concavity of $\phi(.)$ around this k^D is not relevant, Thibault (2000) extends the analysis of Abel (1987) and Weil (1987) by establishing a necessary and sufficient condition for the existence of a steady state with positive bequests in $\mathcal{BM}$ which holds whatever the number and stability properties of equilibria in $\mathcal{DM}$. When $f(.)$ and $U(.)$ are concave, Thibault (2000, Proposition 1) shows that bequests x^M are positive in $\mathcal{BM}$ if and only if the M.G.R. capital stock

[4]When $f''(k^M)$ is sufficiently low, a small increase in k^M induces an infinitesimal variation in the interest factor and an increase in the market wage. So, the labor income of children increases while saving income of parents does not vary. As a result, although more altruist, parents bequeath less.

k^M is larger than $\phi(k^M)$.[5] Importantly, this condition holds in a setting where agents *à la Diamond* and *à la Barro* coexist and have the same life-cycle utility $U(.)$. Indeed, according to Nourry and Venditti (2001, Proposition 1), the condition for altruists to leave a positive bequest does not depend on their proportion.

2.2 - Goals, intuition and graphical illustrations.

We now reinvestigate the impact of intergenerational altruism in the light of the existence results mentioned above. Can the dynamic inefficiency arising in $\mathcal{DM}$ be removed by the introduction of intergenerational altruism ?

To answer this question, we only assume that the production $f(.)$ and life-cycle utility $U(.)$ functions are concave and satisfy Inada conditions. Relaxing the restrictive assumption of Abel (1987) and Weil (1987) implies that $\mathcal{DM}$ may exhibit multiple non-trivial steady states. Our objective is then to construct an example where the dynamics of capital in $\mathcal{DM}$ take the form depicted in Figure 1. Under this configuration, $\mathcal{DM}$ has three non-trivial steady states. Note that if the Golden Rule capital stock k^G corresponds to k_2^D, then k_1^D and k_2^D are dynamically efficient and k_3^D is not Pareto optimal. Starting from $k_0 > k_2^D$, $\mathcal{DM}$ exhibits monotone convergence towards k_3^D. The multiplicity of steady states in $\mathcal{DM}$ (see the LHS of Figure 1) is necessary to get at the same time positive bequests in $\mathcal{BM}$ (see the RHS of Figure 1) and over-accumulation of capital in $\mathcal{DM}$.

According to Figure 1, we have $\phi(k^M) < k^M$. Then, the M.G.R. equilibrium, k^M, is a steady state of $\mathcal{BM}$ with positive bequests, x^M. This equilibrium is stable if, starting from any k_0 in a neighborhood of k^M, $\mathcal{BM}$ converges toward k^M. Indeed, for each k_0 close to k^M, there exists a unique implicit price of bequest $q_0^\star$ such that $(k_0, q_0^\star)$ is on the stable manifold and the equilibrium path converges to the M.G.R. Then, Figure 1 identifies a situation where the introduction of altruistic individuals prevents a society from converging to a non Pareto optimal equilibrium. Indeed, starting from k_0, a selfish economy converges to a non Pareto optimal equilibrium k_3^D, whereas an economy with intergenerational altruism converges to a Pareto optimal steady state, i.e., the M.G.R. k^M. The configuration depicted in Figure 1 therefore

[5] Obviously, when k^D is unique and $\phi(.)$ is locally concave (around k^D), $k^M > \phi(k^M)$ if and only if $k^M > k^D$. We recognize in this special case the condition derived by Abel (1987) and Weil (1987).

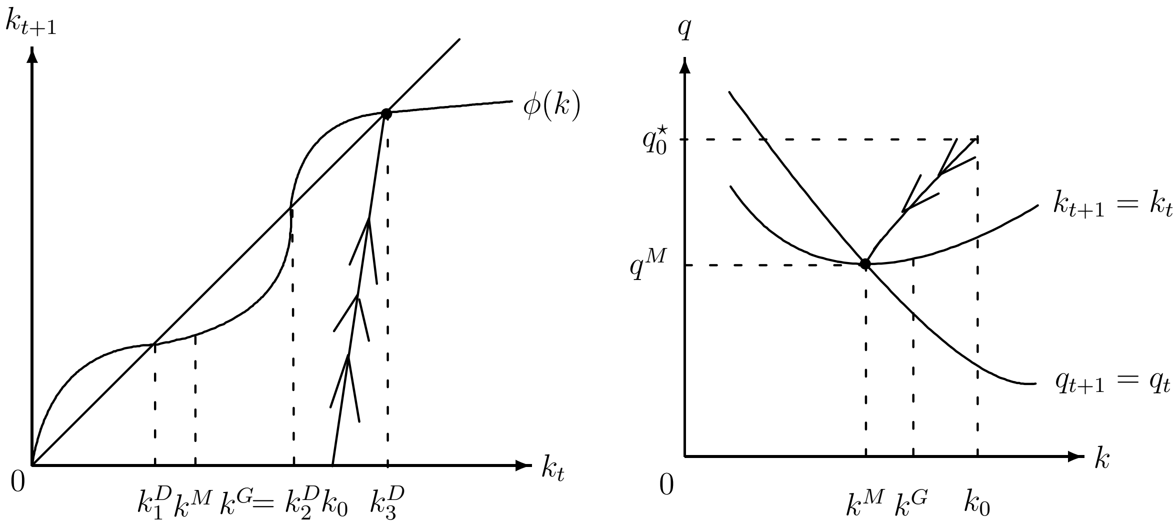

Figure 1: The intergenerational altruism rules out the dynamic efficiency.

enlightens the positive influence of intergenerational altruism on dynamic inefficiency. Nevertheless the following puzzle remains to be solved: **is it possible to achieve this outcome under reasonable assumptions?** The next section provides the reader with an example in this direction.

3 A parameterized example

We first define the fundamentals (production and utility functions) of the economy and then distinguish different cases depending on whether individuals are altruistic or not.

3.1 - Fundamentals.

Consider a perfectly competitive economy which evolves over an infinite horizon. Time is discrete; population is assumed constant ($n = 0$) and consists of agents who live for two periods. Young agents born in t supply a fixed amount of labor, receive the market wage w_t, consume c_t and save s_t. When old, they earn and consume d_{t+1}. Preferences are represented by the following life-cycle utility $U(.)$ function:

$$U(c_t, d_{t+1}) = \ln c_t + \ln d_{t+1}$$

Firms produce a homogenous good using physical capital, K_t, and labor, L_t, according to a constant returns to scale technology represented by the production function $F(.)$. Homogeneity of degree one allows us to write output *per* young as a function $f(.)$ of the capital stock *per*-young, i.e., $f(k_t) = F(k_t, 1) + (1-\delta)k_t$, where $k_t = K_t/L_t$ is the capital stock *per*-young. We let $\delta \in [0,1]$ be the constant depreciation rate of capital. Since markets are perfectly competitive, each factor is paid at its marginal product, i.e., $w_t = F_L(k_t, 1) = f(k_t) - k_t f'(k_t)$ and $R_t = F_K(k_t, 1) - \delta = f'(k_t)$, where R_t is the interest factor at time t.

Following Galor and Ryder (1989), we consider a piecewise-defined production function f, depicted in Figure 2, which satisfies standard properties (it is taken to be twice continuously differentiable, positive, increasing and strictly concave):

$$f(k_t) = \begin{cases} (2 + 3\ln 1.5 + \ln 5)\ k_t - k_t \ln k_t + 1 & \text{if } \ 0 < k_t \leq 4 \\ (5 + \ln 5 + 3\ln 6)\ k_t - 4\ k_t \ln k_t - 11 & \text{if } \ 4 < k_t \leq 5 \\ (4 + 3\ln 6)\ k_t - 3\ k_t \ln k_t - 6 & \text{if } \ 5 < k_t \leq 7 \\ (2.5 + 3\ln 6 - 1.5\ln 7)\ k_t - 1.5\ k_t \ln k_t + 4.5 & \text{if } \ 7 < k_t \leq 10 \end{cases}$$

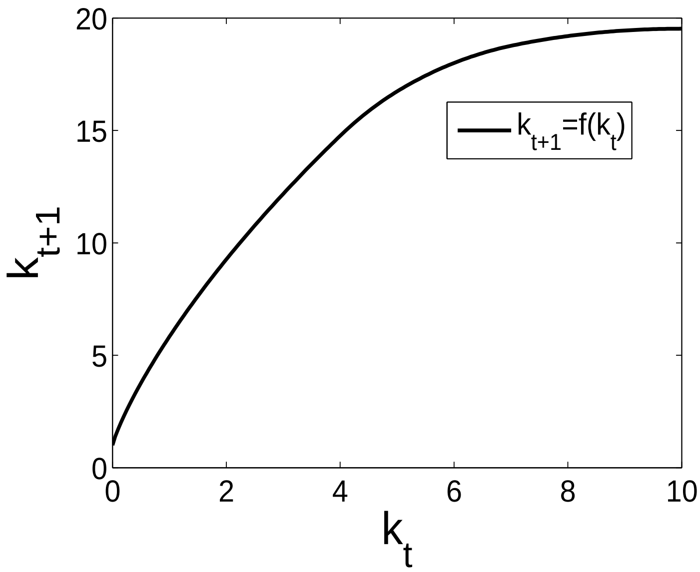

Figure 2: The production function $f(.)$

Using $f(.)$, it is straightforward to show that w_t is a continuous, positive, increasing function of k_t and that R_t is a continuous, positive, decreasing function of k_t:

$$w_t = \begin{cases} k_t + 1 & \text{if } \ 0 < k_t \leq 4 \\ 4k_t - 11 & \text{if } \ 4 < k_t \leq 5 \\ 3k_t - 6 & \text{if } \ 5 < k_t \leq 7 \\ 1.5k_t + 4.5 & \text{if } \ 7 < k_t \leq 10 \end{cases} \quad R_t = \begin{cases} 1 + 3\ln 1.5 + \ln 5 - \ln k_t & \text{if } \ 0 < k_t \leq 4 \\ 1 + \ln 5 + 3\ln 6 - 4\ln k_t & \text{if } \ 4 < k_t \leq 5 \\ 1 + 3\ln 6 - 3\ln k_t & \text{if } \ 5 < k_t \leq 7 \\ 1 + 3\ln 6 - 1.5\ln 7 - 1.5\ln k_t & \text{if } \ 7 < k_t \leq 10 \end{cases}$$

The capital stock in period $t+1$ is financed by the savings of the generation born in t. Hence, using an intensive form representation, we get: $k_{t+1} = s_t$. We can now distinguish different cases depending on whether individuals are altruistic or not.

3.2 - Diamond's (1965) model ($\mathcal{DM}$).

In this context, agents are selfish and solve:

$$\begin{array}{cl} \max\limits_{c_t, s_t, d_{t+1}} & U(c_t, d_{t+1}) \\ s.t. & w_t = c_t + s_t \quad \text{and} \quad R_{t+1} s_t = d_{t+1}. \end{array}$$

Since $U(.) = \ln c_t + \ln d_{t+1}$, we obtain $s_t = w_t/2$. Then, the dynamics of capital stock $k_{t+1} = s_t = \phi(k_t)$ of $\mathcal{DM}$, which is represented Figure 3, take the following form:

$$k_{t+1} = \phi(k_t) = \frac{w_t}{2} = \begin{cases} 0.5\ k_t + 0.5 & \text{if } 0 < k_t \leq 4 \\ 2\ k_t - 5.5 & \text{if } 4 < k_t \leq 5 \\ 1.5\ k_t - 3 & \text{if } 5 < k_t \leq 7 \\ 0.75\ k_t + 2.25 & \text{if } 7 < k_t \leq 10 \end{cases}$$

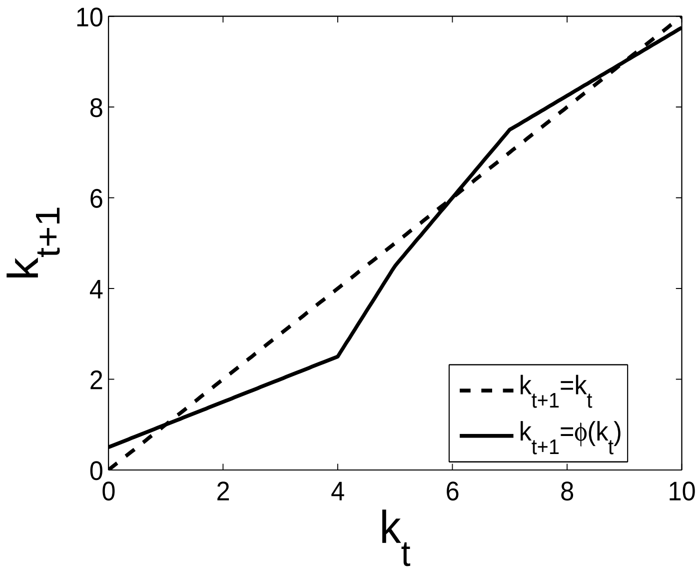

Figure 3: Dynamics in the Diamond's (1965) model.

As $k^G = f'^{-1}(1) = 6$, $\mathcal{DM}$ possesses two steady states, $k_1^D = 1$ and $k_2^D = 6$, which are dynamically efficient, and one, $k_3^D = 9$, which is non Pareto optimal. Importantly, as in Figure 1, k_2^D is unstable but k_1^D and k_3^D are locally stable. Then:

Result 1.

Starting from $k_0 > k_2^D$, $\mathcal{DM}$ exhibits monotone convergence towards k_3^D, i.e., towards a non Pareto optimal equilibrium.

Note that, even if the stability of k_3^D is local, k_0 can be as low as one wants. Indeed, our example is a generic case rather than a special case since it was obtained solving a system of 10 equations in 12 unknowns. So, one can exhibit an infinite number of economies satisfying the LHS of Figure 1. Then, one can always choose constant parameters of $f(.)$ to obtain given values of k_1^D, k_2^D and k_3^D.

3.3 - Barro's (1974) model ($\mathcal{BM}$).

We now adopt Barro's (1974) specification of the bequest motive: parents (born at time t) assign a positive weight to their children's utility in their own utility function, and possibly leave them a bequest x_{t+1}. Importantly, bequests x_t are restricted to be non-negative at all date t. Then, an altruist solves:

$$\begin{array}{rcl} V_t(x_t) & = & \max\limits_{c_t, d_{t+1}, x_{t+1}} \quad U(c_t, d_{t+1}) + \beta V_{t+1}(x_{t+1}) \\ s.t. & & w_t + x_t = c_t + s_t, \; R_{t+1}s_t = d_{t+1} + (1+n)x_{t+1} \quad \text{and} \quad x_{t+1} \geq 0. \end{array}$$

where $V_{t+1}(x_{t+1})$ denotes the utility of a representative descendant who inherits x_{t+1} and $\beta \in (0,1]$ the intergenerational degree of altruism of the dynasty.

According to Section 2.2, altruists choose to leave positive bequests in the long run if and only if $\phi(k^M) < k^M$. Since $\beta \in (0,1]$, we have $k^M = f'^{-1}(1/\beta) \in (0, k^G]$. Therefore, as indicated in the LHS of Figure 1, bequests are non-negative if and only if $k^M \in (k_1^D, k^G]$; the area where $\phi(k^M) \leq k^M$. Then, the M.G.R. equilibrium is a steady state equilibrium of $\mathcal{BM}$ with positive bequest if and only if: $\beta > \beta^\star = 1/f'(k_1^D) = 1/f'(1) = 1/[1 + 3\ln 1.5 + \ln 5]$.

Denoting by q_t the implicit price of bequests x_t, it is easy to obtain the following two dimensional dynamical system which describes the equilibrium paths in a neighborhood of the M.G.R. steady state (k^M, q^M):

$$k_{t+1} = f(k_t) - \frac{(1+\beta)}{\beta q_t} \equiv \psi(k_t, q_t) \quad \text{and} \quad q_{t+1} = \frac{q_t}{\beta f'(\psi(k_t, q_t))}$$

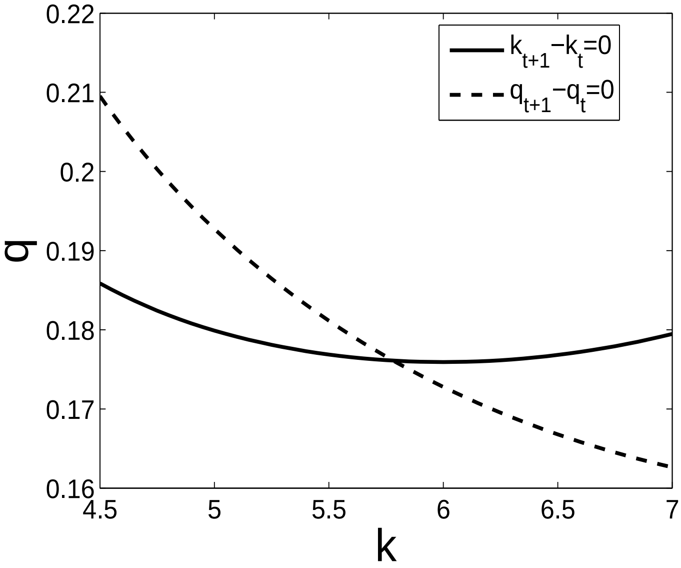

Figure 4: Dynamics in the Barro's (1974) model.

Figure 4 depicts the phase diagram of this dynamical system for a given degree of altruism ($\beta = 0.9$). For each degree of altruism β compatible with positive bequests, our model is formally equivalent to a model with infinitely lived agents and (k^M, q^M) is a regular saddle point. Thus, the form of the dynamics of the $\mathcal{BM}$ is equivalent to the one described in the RHS of Figure 1 and we have:

Result 2.

For every $\beta \in (\beta^\star, 1]$, there exists $\varepsilon > 0$ such that starting from $k_0 \in (k^M - \varepsilon, k^M + \varepsilon)$, $\mathcal{BM}$ exhibits saddle point convergence towards k^M, i.e, towards a Pareto optimal equilibrium.

Therefore, our example identifies a situation where altruistic individuals prevent a society from converging to a non Pareto optimal equilibrium. Indeed, when the degree of altruism is sufficiently large, local methods are sufficient to guarantee that there exist initial values of the capital stock k_0 (and of x_0) for which the economy without altruism converges to a non-Pareto-efficient steady state while the economy with altruism does converge to a Pareto efficient steady state from the same k_0.

Result 2 guarantees that there exists a positive number $\varepsilon > 0$ such that starting from $k_0 \in (0, k^M + \varepsilon)$, the Barro economy converges to the M.G.R. equilibrium when

$\beta = 1$.[6] Since $k^M = k^G = k_2^D$, starting from $k_0 \in \mathcal{I}_1 = (k_2^D, k_2^D + \varepsilon) \neq \emptyset$, the dynamic inefficiency that occurs in the Diamond economy is removed by the introduction of Barro's (1974) intergenerational altruism. Regarding the robustness of the non-empty property of $\mathcal{I}_1$, the local analysis is sufficient to show (by continuity) that $\mathcal{I}_1$ is still non-empty for values of β different from one. To resume: **when β is large enough the dynamic inefficiency that occurs in the Diamond (1965) economy is removed by the introduction of Barro's (1974) intergenerational altruism.**

4 The efficiency of altruism

Our parameterized example suggests that the introduction of altruistic agents can prevent a society from converging to a non Pareto optimal equilibrium. However, the example only focuses on two polar cases (agents are either life cyclers or altruists). Considering a society in which these two types of individual coexist is a major issue, as it provides a way to gauge the actual positive influence of intergenerational altruism. **How much altruism is necessary to rule out the dynamic inefficiency ?**

In order to address this question, we consider, following Michel and Pestieau (1998) and Nourry and Venditti (2001), an OLG model that embeds Diamond's (1965) and Barro's (1974) framework. The population now consists of a proportion $p > 0$ of altruistic agents (which behaviors have been examined in Section 3.2) and of a proportion $1 - p$ of non-altruistic agents (described in Section 3.1).

When the long run optimal bequest x^M is positive, the steady state capital stock k^M satisfies the M.G.R., and does not depend on the proportion of altruists p. An arbitrarily small proportion of altruistic agents, whose degree of altruism is sufficiently large, is sufficient to obtain the results mentioned in Section 3.3 (see Michel and Pestieau, 1998 or Nourry and Venditti, 2001). The proportion p of altruists has an impact on the amount x^M of long-run bequests but neither the existence nor the stability properties of the M.G.R are affected by p. However, the dimension of the dynamical system which describes the equilibrium paths in a neighborhood of the M.G.R. steady state depends on whether $p = 1$ or $p < 1$. The previous section has shown that the case

[6] Recently Le Van and Morhaim (2006) fill the gap between growth models when the discount factor β is close to one and when it equals one. They show that the optimal growth problem is continuous with respect to the discount factor β under standard assumptions. Such a global turnpike result guarantees the fact that the analysis of this section are well defined when β is equal to one.

$p = 1$ induces a two-period dynamical system. If $p < 1$, the dimension of the corresponding dynamical system which describes the equilibrium paths in a neighborhood of the M.G.R. is three. We hence show (see Appendix A) that, when β is close to one, the M.G.R. equilibrium k^M is a regular saddle point whatever $p > 0$. Thus, we do not need population to consist exclusively of altruists to rule out the dynamic inefficiency.

RESULT 3.

The existence of an arbitrarily low proportion of altruists can be sufficient to prevent a society from converging to a non Pareto optimal equilibrium.

To sum up, the main contribution of this paper is to show that there exist economies which the intertemporal equilibria converge toward a steady state characterized by an over-accumulation of capital when there are only non-altruistic agents ($p = 0$), leading thus to dynamic efficiency, but converge toward the M.G.R. as soon as a positive proportion ($p > 0$) of altruistic agents is introduced. As a consequence, even if the proportion of altruists remains close to zero, dynamic efficiency is removed. It could be interesting to analyze how this main result can be extended to framework with uncertain lifetimes (e.g., Fuster, 2000), political process (e.g., Boldrin and Rustichini, 2000), endogenous labor supply (e.g., Cazzavillan and Pintus, 2004) or endogenous intergenerational altruism (e.g., Rapoport and Vidal, 2007).

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Appendix A. Stability of the Modified Golden Rule equilibrium

We study the (local) stability of the M.G.R. equilibrium in an economy where there is a proportion $0 < p < 1$ of agents *à la Barro* and a proportion $1 - p$ of agents *à la Diamond.* Denoting by x_t the level of bequests and by q_t the implicit price of bequests, it is easy to obtain the following three dimensional dynamical system:

$$k_{t+1} = \frac{(1+p)[f(k_t) - k_t f'(k_t)]}{2} + p\, x_t - \frac{p}{q_t} \equiv \phi(k_t, q_t, x_t)$$

$$q_{t+1} = \frac{q_t}{\beta f'(\phi(k_t, q_t, x_t))}$$

$$x_{t+1} = f'(\phi(k_t, q_t, x_t))\left[f(k_t) - k_t f'(k_t) + x_t - \frac{2}{q_t}\right]$$

After some tedious computations[7] we obtain the characteristic polynomial $\mathcal{P}(\lambda) =$

[7]Our dynamical system corresponds to eqs. (18) to (20) of Nourry and Venditti (2001), when

$\lambda^3 - \mathcal{T}\lambda^2 + \mathcal{J}\lambda - \mathcal{D} = 0$ with $\mathcal{T} = 1 + 1/\beta - \beta f''(k^M)[(1+p)f(k^M) - 2k^M]/2$, $\mathcal{J} = 1/\beta - f''(k^M)(1-p)[f(k^M) - k^M]/2$ and $\mathcal{D} = -(1-p)k^M f''(k^M)/(2\beta)$. Since we have two predetermined variables, k_t, x_t, and one forward variable, q_t, the M.G.R. steady state (k^M, q^M, x^M) is said to be saddle point stable if (and only if) two eigenvalues of $\mathcal{P}(.)$ are inside the unit circle and one outside.

To stress the relevance and the robustness of our parameterized example, we now establish that the M.G.R. can be a regular saddle point whatever the proportion p of altruists, i.e., even with an arbitrarily small proportion of altruists. This result can be established focusing on economies in which the degree of altruism β is close enough to one. When $\beta = 1$, we have $k^M = 6$, $f(k^M) = 18$ and $f''(k^M) = -1/2$. Hence, the characteristic polynomial $\mathcal{P}(\lambda)$ which describes the equilibrium paths in the neighborhood of (k^M, x^M, q^M) becomes:

$$\mathcal{P}(\lambda) = \lambda^3 - \Big(\frac{7}{2} + \frac{9}{2}p\Big)\lambda^2 + \Big(4 - 3p\Big)\lambda - \frac{3}{2} + \frac{3}{2}p = 0$$

Then, $\mathcal{P}'(\lambda) = 3\lambda^2 - \Big(7 + 9p\Big)\lambda + 4 - 3p$ is positive for all negative λ. Moreover $\mathcal{P}'(1) = -12p$ is negative. Then, the characteristic polynomial $\mathcal{P}(\lambda)$ varies as follows:

λ	$-\infty$	0	λ'	1	λ''		$+\infty$
$\mathcal{P}'(\lambda)$			0	$-$	0	$+$	
$\mathcal{P}(\lambda)$	$-\infty$	$\nearrow$		$\searrow$		$\nearrow$	$+\infty$

Let $\kappa = 3(1-p)/2$ the product of the roots of $\mathcal{P}(.)$. Since $\mathcal{P}(0) = 3(p-1)/2$, $\mathcal{P}(1) = -6p$ and $\mathcal{P}(\kappa) = -27p(1-p)^2/2$ are negative for all positive p, $\mathcal{P}(\lambda)$ has a real root $\tilde{\lambda}_1$ outside the unit circle ($\tilde{\lambda}_1 > \max\{\kappa, 1\}$). The other two roots $\tilde{\lambda}_2$ and $\tilde{\lambda}_3$ are either both complex and conjugate (i.e, $|\tilde{\lambda}_2| = |\tilde{\lambda}_3| = m$) or both positive, real and lower than one (i.e., $0 < \tilde{\lambda}_2 < \tilde{\lambda}_3 < 1$). Assume that $\tilde{\lambda}_2$ and $\tilde{\lambda}_3$ are complex roots (of modulus m) of $\mathcal{P}(.)$. Since the product of the roots of $\mathcal{P}(.)$ is equal to κ, we have $m^2\tilde{\lambda}_1 = \kappa$. Since $\tilde{\lambda}_1 > \kappa$ we have necessarily $m < 1$. Consequently, whatever $p > 0$, we have (at most) two cases. Either $\mathcal{P}(.)$ experiences two complex roots $\tilde{\lambda}_2$, $\tilde{\lambda}_3$ inside the unit circle and one positive real root $\tilde{\lambda}_1$ outside the unit circle. Either $\mathcal{P}(.)$ experiences three real positive roots such that: $0 < \tilde{\lambda}_2 < \tilde{\lambda}_3 < 1 < \tilde{\lambda}_1$. Hence, as two eigenvalues of $\mathcal{P}(.)$ are inside the unit circle and one outside, the M.G.R. steady state (k^M, q^M, x^M) is saddle point stable whatever $p > 0$. □

$U^a(.) = U^e(.) = \ln c_t + \ln d_{t+1}$.