How to consult an expert? Opinion vs Evidence[1]

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Abstract

In this paper, two modes of non-binding communication between an expert and a decision-maker are compared. They are distinguished mainly by the nature of the information transmitted by the expert. In the first one, the expert reports only his opinion (soft information) concerning the desirability of a certain action, whereas in the second one, he is consulted to provide evidence (hard information) to convince the decision-maker. The expert's ability to provide evidence increases with the precision of his information. The paper shows that requiring evidence is always beneficial to the decision-maker whereas it is beneficial to the expert if and only if the preferences of both agents are different enough.

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1 Introduction

In many economic, political and social environments, decision-makers (such as corporate CEOs, investors, political leaders or jurors) do not have all the necessary knowledge to make the best decisions. That is why they resort to experts (such as marketing specialists, management consultants, stockbrokers, investment bankers, financial advisors, scientific committee or investigators). In these situations the sharing of information among agents is done strategically through communication processes. Conflicts of preferences between both parties can make the disclosure of information difficult. In this paper, we characterize all equilibria of a non-binding interaction between an expert and a decision-maker and analyze how these differ under two different forms of information-reporting.

The expert – in our case a privately informed agent who is interested in the final action chosen by the decision-maker – can be consulted in different ways. He can be consulted only to give an opinion or any other *soft information* on a specific subject. He can also be consulted to explain the basis of his opinion. In that case he has to provide facts, documents, proof or any other *hard information* justifying his beliefs. Providing irrefutable proof (as for example the existence of a given risk) makes him playing a more decisive role in the decision-making process. When sound proof is established, the decision-maker has to make a decision that is consistent with the truth revealed by the expert.

In this article, two games of communication are compared. In the first game, the expert is consulted by the decision-maker only to give an opinion concerning the desirability of a certain action. In contrast, in the second game, the decision-maker consults the expert to bring evidence – proof, arguments or facts – to be convincing. Providing evidence makes it possible for the expert to convince the decision-maker and the decision-maker can assess the accuracy or the precision of the information observed by the expert. We compare equilibria of these two games regarding the welfare of each agent to determine under what conditions on preferences to require evidence is beneficial.

The two games studied have a common structure. Each one is a sender-receiver game with costless and non-binding messages. Our two games only differ through the assumption made on the information that the expert can transmit (either soft or hard information). In the first game, whatever information he observes, the expert can lie about his own opinion as much as he wants. Formally, the set of available messages does not depend on the expert's private information (cheap-talk game). For instance, even if the information observed shows a strong probability that a specific risk occurs, the expert can provide the decision-maker with an opinion indicating the contrary. On the other hand, in the second game, the expert argues by certifying all or a part of his private information to the decision-maker. Formally, some messages are only available for some specific information (persuasion game). In this latter game, he can thus voluntarily prove some pieces of his information by transmitting a message which would not be available to him under other conditions (the proof of being in a state of the world is not accessible in another state). He can also voluntarily hide some information by only presenting some vague arguments. However he cannot lie by certifying some wrong information because the needed messages are not available given his information.

The ability of the expert to provide reliable information – evidence – increases with the precision of the information he has observed. We assume that the minimum level of a given precision is based on provable facts whereas the maximum level is not. For example, in the case of skills, a person can prove he is a good musician by performing well a difficult work, whereas there exists no analogous way to prove that one is not a good musician. We suppose the expert is able to certify all payoff-relevant information he has observed. However, as long as he does not know everything, he is unable to prove that he is not hiding any additional information. The provability is partial. Regarding the precision of some specific piece of information, the expert can cover up some of what he knows by providing documents he could have transmitted with less precise information. But he is unable to provide all the documents he could have given with some more precise information. The set of documents available to the expert increases with the precision of his information. In sum presenting specific evidence proves that the expert's precision of information is at least equal to a certain level. But it does not exclude the possibility that his information is more precise than this level.

We offer a model as simple as possible in order to characterize all equilibria of both interactions. We assume that the decision-maker has to choose an action that is part of a binary set such as undertaking a specific project or not. Moreover, depending on the information collected, the decision-maker and the expert may have potentially conflicting preferences. That is, for a given piece of information, either both agents share the same preferred action (both agents want the project to be either undertaken or not) or they do not (one agent wants the project to be undertaken while the other does not). Each agent would like the project to be undertaken only if the information is at least as favorable as a specific level, or threshold of indifference or reasonable doubt. Since both thresholds may differ a given information may be sufficiently favorable to carry out the project for one agent while it is not for the other.

Our main result shows that resorting to evidence is always beneficial to the decision-maker whereas it is beneficial to the expert if and only if the preferences of both agents are different enough. The idea that the provability of information (even if it is partial) is always beneficial to the decision-maker seems intuitive enough. However, the literature has a counter-example in a complete provability setting (see, Giovannoni and Seidmann (2007)). In our case, the provability in the evidence game often enables the decision-maker to extract all information using a skeptical strategy. This strategy consists in taking his most preferred action according to his prior as long as the expert does not report him convincing documents. But since provability is partial, this unraveling argument does not always apply. The unraveling argument fails when the expert's threshold of indifference is higher than the decision-maker's one. The expert has then an incentive to withhold any document that is not convincing him but would convince the decision-maker. In that case, equilibria may take a non trivial form but we show even so that the decision-maker is better off than when playing the opinion game. In particular, there is an equilibrium in which the expert still reveals evidences that are convincing both. While when agents' preferences are different enough no information is revealed at equilibrium of the opinion game. As for the expert, the idea that he prefers to provide evidence if and only if the preferences of both agents are different enough is explained as follows. In the opinion game, the equilibria characterize extreme economic situations since in a setting of binary actions (even with a continuum of types) cheap-talk is a very coarse instrument. Indeed either the preferences of both agents are close enough; so the decision-maker always applies the recommendations of the expert. Or the preferences are not close enough; so the decision-maker is really distrustful and does not listen to any recommendations. In this latter case, resorting to a more precise mode of communication as providing evidence sometimes still makes it possible for the expert to convince the decision-maker. Example 1 reflects the results expected.

Example 1 (Market attractiveness). Consider a company deciding whether or not to enter a new market. It uses consultant services to bring in information about the attractiveness of that market. The CEO prefers to enter the market only if the probability of success following entry is higher than his threshold of reasonable doubt. The work of the consultant consists in collecting and sorting out various information on that new market (e.g., companies' financial statements, fluctuations in profit margin, costs, revenues and potential demand). Due to parallel advice activity (as addressing key strategies), the consultant has preferences with respect to his own (maybe different) threshold of reasonable doubt. Any information collected is certified and cannot be forged. But the consultant can voluntary delete some of it from the file he is putting together. He is the only person to know the nature of his information – whether his file supports the attractiveness or not – and the precision of this information – to what extent the attractiveness is either sustained or ruled out. When he is consulted only to give his opinion (or transmit any soft information) he can say whatever he wants. Consequently it can be rational for the CEO to ignore any recommendation. So there is an equilibrium in which no information is transmitted. In addition, if the preferences of both parties are close enough, it can be rational for the CEO to trust the consultant. In this case, there is also an equilibrium in which the consultant can manipulate the final decision as he wants. To some extent, it would be as if the consultant took the final decision by himself. Conversely when he is consulted to argue his opinion with documents, he can totally reveal or hide his private information. He just has to provide the complete file or, on the contrary, to claim he has observed no relevant information (providing an empty file). He can also decrease the precision of the transmitted information by hiding some bits (by leaving out some document from his file). However he cannot increase this precision because he cannot add non-existing certified document. Since the information transmitted can now contain irrefutable evidence, the CEO's behavior that consists in never taking into account the information revealed no longer supports any equilibrium. When the CEO is more reluctant than the consultant to enter the new market, he can extract the most important pieces of the consultant's private information. To do so, he just has to be skeptical by deciding not to enter unless he is presented with sufficiently convincing documents.

Example 2 (Trial). In a jury environment, the model is interpreted as follows. A person on trial, who is either innocent or guilty, can be either acquitted or convicted. We suppose that to render his verdict, the decision-maker – a judge or jury – resorts to a investigator who is better informed. The decision-maker prefers to condemn the defendant only if the probability of his guilt is higher than his threshold of reasonable doubt. Consider that the information observed by the investigator either supports the guilt or innocence of the defendant. For instance, it can be a video on which one can see the defendant at a place far away from where the murder took place at the estimated time of the crime. Or on the contrary, we can see the defendant at the exact place of the crime. The investigator is the only person to know the nature of his information – the video supports the guilt or innocence of the defendant – and the precision of this information – either he has a video showing the defendant simply arguing with the victim or literally killing him. The investigator can continuously decrease the precision of the information transmitted by suppressing some images on the video. However he cannot increase this precision, he cannot add non-existing images. Results are as in the previous example. Here, the extraction of the consultant's private information (when asked to provide evidence) requires the judge to be more reluctant than the investigator to condemn the defendant.

Example 3 (Public policy). We could also consider the case of a scientific committee regarding an environmental or sanitary problem. The decision would be whether or not to authorize a product that is known to be potentially harmful. The private information of the committee would be collected in a scientific report. Only the committee would know the nature of his information – either the report claims that the product is harmful or harmless – and the precision of it – for instance, which steps of a scientific protocol defined by the public authorities beforehand have been validated. In the same way, we could compare the situation where the scientific committee gives only his opinion about the harm caused by the product to the situation in which he is obliged to provide documents – such as the elements contained in the scientific report – to convince the governing authority.

To assess whether resorting to evidence is beneficial to both agents, we compare two distinct games. This methodology can be criticized. An alternative method would have been to consider the whole as one game only. For instance, the problematic would be to determine under which conditions on both the set of available messages and the players' preferences full revelation occurs at equilibrium. This question has already been tackled in Mathis (2008).

We assume that the expert's private information is one-dimensional. We choose to proceed in that standard way (see Related works) because modeling a multi-dimensional information space would considerably complicate our study. In particular, as long as the beliefs are about a one-dimensional state of Nature, how to link updated beliefs with evidence is not obvious. Indeed, whenever the expert transmits an incomplete file sustaining a given state of Nature, whether the decision-maker should infer that the possibly missing documents are actually sustaining this state or the other is not clear.

Related works

The analysis of cheap-talk game[4] was first explored by Crawford and Sobel (1982). They examine a sender-receiver game in which the sender has private information that the receiver, who must take a decision that affects them both, would like to know. In particular, they show that when agents have different preferences and when the report is unprovable, full revelation does not occur in equilibrium.

There exists a substantial literature considering strategic transmission of hard information between a self-interested expert and an uninformed decision-maker[5].

Some of this work consider a context of *complete provability* – i.e., assuming that the informed party has ability to prove both all his decision-relevant information and that he is not withholding information – (e.g., Giovannoni and Seidmann (2007), Grossman (1981), Grossman and Hart (1980), Koessler (2003), Matthews and Postlewaite (1985), Milgrom (1981), Milgrom and Roberts (1986)). The central result called as the *unraveling argument* (demonstrated in a general setting by Seidmann and Winter (1997)) is that at equilibrium, by using a skepticism strategy the decision-maker succeeds to fully extract the informed party's private information. Skepticism strategy consists in choosing an action relying on a worst case inference when the informed party does not reveal all his private information. The decision-maker has no difficulty to detect any withholding of information since the provability is complete.

Most of the literature that considers *partial provability* – *i.e.*, where the informed party has ability to certify something but not everything – focuses on an informed party with monotonic preferences – *i.e.*, willing the decision-maker to maximize (or minimize) the magnitude of his action. Okuno-Fujiwara *et al.* (1990) show that the unraveling argument extends to a situation where the decision-maker knows that the informed party wants to maximize the magnitude of his decision and is able to prove that the observed information is at least as favorable than a certain threshold[6]. Shin (1994a, 1994b) provides a model in which the expert is unable to prove the precision of his information. A skeptical inference then might be irrational and the author shows that there is no fully revealing equilibrium. Wolinsky (2003) considers a situation where the decision-maker does not know whether the expert is of a type that wants to maximize or minimize the magnitude of a certain action. In this setting, he characterizes a unique equilibrium outcome as a combination of the equilibria that would prevails in the certainty expert's preferences world.

Only very recent literature deals with situation where the expert's most preferred action depends on his decision-relevant private information in a setting of partial provability. Mathis (2008) generalizes Seidmann and Winter's (1997) results to the partial provability setting. He provides necessary and sufficient conditions on both players' preferences and information that can be certified for a Sender-Receiver game to possess a separating equilibrium, as well as sufficient conditions

[4] For a survey on cheap-talk games see Farrell and Rabin (1996) and Krishna and Morgan (2008).

[5] For a survey on Persuasion games see Forges and Koessler (2009).

[6] Although the authors study a situation with several experts, the full revelation of information does not rely on the experts' competition (contrary to Lipman and Seppi (1995)).

for any equilibrium of such a game to be separating. Lanzi and Mathis (2008) consider a situation where a decision-maker relies on the report of an expert prior to decide whether to undertake a certain project. Depending on the information collected, the two players may have conflicting preferences. Information contained in the report is partially verifiable in the sense that the expert can suppress favorable information sustaining the project but he cannot exaggerate it. They show that this setting favors the agent which is the less eager to undertake the project in that he always succeeds to induce his most preferred action.

The two closest papers to ours are Giovannoni and Seidmann (2007) and Lanzi and Mathis (2008).

Giovannoni and Seidmann (2007) consider a game with a continuum of actions and a finite number of types in which the verifiability is strong: the sender has the ability to prove any true event (formally, any subset of types containing the realized one is certifiable). They show that when the sender's preferences are such that there is no pair of types strictly preferring to be misidentified for one another (single-crossing property) a fully revealing equilibrium exists. This forced informativeness is then beneficial to the Receiver. They also give an example in which the decision-maker gets an *ex-ante* higher payoff in some cheap-talk equilibrium than in any equilibrium of the initial game.

Contrary to Giovannoni and Seidmann (2007), we consider a game with a binary set of actions and a continuum of types in which the provability is partial. The sender's preferences satisfy the Giovannoni-Seidmann's single-crossing property but due to partial provability, a fully revealing equilibrium may not exist, depending on how players' preferences are aligned. Our model also rules out their example. This is due to the absence of two types strictly preferring to be misidentified for one another. We then establish that resorting to evidence is beneficial to the decision-maker by characterizing and comparing all equilibria of our opinion and evidence games.

Although our paper does not address the same issue, our evidence game generalizes Lanzi and Mathis (2008) to situation where the decision-maker does not know which action is actually sustained by the expert's private information. In particular, we establish the robustness of their main result according to which the agent which is the less eager to undertake the project always succeeds to induce his most preferred action.

In addition, we address for the first time in the literature, whether or not the certification is beneficial to the *expert*.

The remainder of this article is organized as follows. Section 2 presents the basic sender-receiver game. In particular, we introduce the partial provability structure. Section 3 presents the equilibrium of each game and the main results of the paper. Section 4 concludes.

2 The Basic Framework

Communication game. Consider a simple communication game in which Nature draws a state from a binary set $\Omega \equiv \{X, Y\}$, whose typical element will be denoted ω. There is a prior equiprobable distribution on the states of Nature. The expert (S) observes a binary signal $\sigma \in \{\mathbf{x}, \mathbf{y}\}$ correlated with ω according to a given probability $p \equiv P(\sigma = \boldsymbol{\omega} \mid \omega)$ and then sends a message $m \in M$ to the decision-maker (R), who then must take an action $a \in \{x, y\}$ that determines the welfare of both.

Preferences. The vNM payoff of the player i, with $i = S, R$, is a function $u^i : \{x, y\} \times \Omega \mapsto \mathbb{R}$ where $u^i(a, \omega)$ denotes the i's utility level when action a is taken under the state of Nature ω. Let action x (*resp.* y) be appropriate in state X (*resp.* Y), *i.e.* $u^i(x, X) > u^i(y, X)$ (*resp.* $u^i(y, Y) > u^i(x, Y)$). Thus, by denoting p^i to be the i's posterior belief that $\omega = Y$, we have

$$\mathbb{E}_{p^i}[u^i(y, .)] > \mathbb{E}_{p^i}[u^i(x, .)]$$

$$\iff p^i > \frac{u^i(x, X) - u^i(y, X)}{u^i(x, X) - u^i(y, X) + u^i(y, Y) - u^i(x, Y)} \equiv q^i \in (0, 1)$$

The parameter q^i exactly characterizes the player i's *threshold of indifference* between the two decisions, *i.e.* the agent i will prefer the action y to the action x iff he believes the state of Nature is Y with probability p^i higher than q^i. We call the i's most preferred action according to his prior belief the i's *uninformed action*, *i.e.* the action y if $q^i \leq \frac{1}{2}$ and x if $q^i \geq \frac{1}{2}$. We say that player i is *the least eager* to undertake the project a if for $j \neq i$ ($\frac{1}{2} \leq q^j \leq q^i$ and $a = y$) or ($q^i \leq q^j \leq \frac{1}{2}$ and $a = x$). In words, player i is the least eager to undertake the project y if both players' uninformed action is x and preferring the action y requires for i a higher beliefs that the state of Nature is Y than does the other player. If there is no least eager agent *i.e.* if $q^i < \frac{1}{2} < q^j$, then the players are in conflict at the uninformed action. In the remainder, we assume that $q^S \leq q^R$, but given the symmetry of the model this is without loss of generality and could be reversed since state X (with respect to Y) is arbitrarily defined. Figure 1 depicts the full information benchmark for players' preferences.

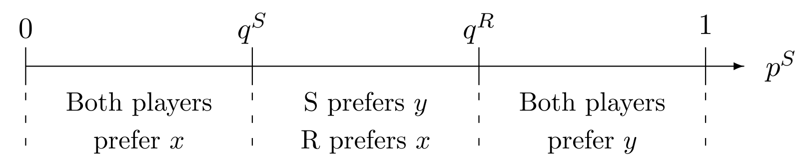

Figure 1: Full information benchmark

Expert's Information. We denote by $p \equiv P(\sigma = \boldsymbol{\omega} \mid \omega)$ the S's *signal quality* (information precision) which denotes the probability that S receives the 'correct' signal, *i.e.* the signal $\sigma = \boldsymbol{\omega}$

in state ω. We assume that the S's signal σ and signal quality p are both private information of the expert. We assume that p is uniformly distributed on $[\frac{1}{2}, 1]$[7].

Expert's Posterior Belief. Let $p^S = P(\omega = Y \mid \sigma, p) \in [0, 1]$ denotes the S's updated belief (or opinion) that the state of Nature is Y, considering the signal σ and the signal quality p are observed. Since the two states of Nature are equiprobable we have:

$$p^S \equiv P(\omega = Y \mid \sigma, p) = \begin{cases} p \text{ if } \sigma = \mathbf{y} \\ 1 - p \text{ if } \sigma = \mathbf{x} \end{cases}$$

Since R does not observe neither the S's signal σ nor the S's signal quality p all the S's payoff relevant private information is defined by the pair $(\sigma, p) \in \{\mathbf{x}, \mathbf{y}\} \times [\frac{1}{2}, 1]$ or equivalently by the S's updated beliefs $p^S \in [0, 1]$. We then consider the S-*type* – which denotes all the S's payoff relevant private information – as the S's updated beliefs p^S for which the prior distribution is uniform on the S-type set $[0, 1]$. We then define $p^S[\omega] = p^S$ if $\omega = Y$; and $p^S[\omega] = 1 - p^S$ if $\omega = X$.

Opinion vs Evidence. The two communication games studied have the previous structure and differ only from the set of messages available to the expert.

In the first game, we suppose that the expert reports only his opinion (formalized by his updated belief) concerning the desirability of the action a. His set of messages is then $[0, 1]$ and is not constrained by his private information. So, the set of messages is the same whatever the S-type. Formally, for any $p^S \in [0, 1]$, $M(p^S) = [0, 1]$.

In the second game, the expert has to report evidence put forth as proofs or facts to convince the decision-maker. In this game, we suppose that the set of messages depends on the information observed by the expert. We consider an evidence as a list of documents which the expert can present. For a given signal $\mathbf{x}$ or $\mathbf{y}$, each document the expert can provide is a fact exclusively supporting the signaled state x or y. The more exact the precision is, the more the list of presentable documents is long and convincing. Consequently to each pair observed – (information, precision of information) – is matched a fixed list of presentable documents.

We suppose the expert can present either the entire list of documents he has or only a short version of it according to his will. Besides we suppose that the expert is unable to create or add any documents he does not have. Thus the expert can delete some information which is favorable to the signaled state, but he cannot exaggerate it. An evidence – incomplete or not – is then considered as a message which proves the inclusion of the observed couple (σ, p) to a given subset.

Formally, we assume that $M(p^S) = [p^S, \frac{1}{2}]$ if $p^S \leq \frac{1}{2}$ and $M(p^S) = [\frac{1}{2}, p^S]$ if $p^S \geq \frac{1}{2}$. If the expert receives no relevant information (his updated belief is equal to his prior), then he cannot report any document or logical proof that substantiates a particular conclusion. We model this limited possibility for argumentation by considering a singleton for his set of messages – formally, for any σ with $p = \frac{1}{2}$, we have $p^S = \frac{1}{2}$ and $M(p^S = \frac{1}{2}) = \{\frac{1}{2}\} \subset M(p^{S\prime})$ for any $p^{S\prime} \in [0, 1]$. Alternatively,

[7]Since the expert observes σ and p, assuming $p < \frac{1}{2}$ would be irrelevant.

if S receives an information which constitutes an irrefutable proof that Y is the state of Nature, then he can report this proof, as he can partially or completely withhold it, but he cannot report any document substantiating that X is the state of Nature – formally, if $\sigma = \mathbf{y}$ with $p = 1$, we have $p^S = 1$ and $M(p^S = 1) = [\frac{1}{2}, 1]$. While, if S receives an information which constitutes a partial proof substantiating that X may be the state of Nature, then he can partially or completely report this information but not exaggerate it – formally, if $\sigma = \mathbf{x}$ with $p \in (\frac{1}{2}, 1)$, we have $p^S = 1 - p$ and $M(p^S = 1 - p) = [1 - p, \frac{1}{2}]$. In both games the whole set of messages will be noted as M, with $M \equiv \cup_{p^S \in [0,1]} M(p^S) = [0, 1]$.

The timing of both games is represented by Figure 2.

Information phase	Talking phase	Action phase
Expert learns signal σ correlated with state ω w.r.t. p Both σ and p are S's private information S updates his belief $p^S \equiv \mathrm{P}[\omega = Y \vert \sigma, p]$	Expert reports $m \in M(p^S)$ Opinion game: $M(p^S) = [0, 1]$ Evidence game: $M(p^S) = [\frac{1}{2}, p^S]$ if $p^S > \frac{1}{2}$; $M(p^S) = [p^S, \frac{1}{2}]$ else	DM updates his belief w.r.t. m and chooses action $a \in \{x, y\}$

Figure 2: Timing of the games

Players' Strategies. A *(mixed) strategy* for player R is a map $\alpha : M \mapsto [0, 1]$. This has the interpretation that when R receives the message m, he selects the action y with probability $\alpha(m)$. Here, whatever the played game and the used strategies, we assume that a message m has the intrinsic meaning that the higher it is, the stronger it recommends playing action y. So, we confine our attention to R's strategies which are *monotonic and increasing*[8], that is to the map $\alpha : M \mapsto [0, 1]$ that are increasing in m. Recall that S-type is the S's posterior belief $p^S \in [0, 1]$. A *pure strategy* for player S is a map $\mu : [0, 1] \mapsto M$ such that $\mu(p^S) \in M(p^S)$. This has the interpretation that when S learns that his type is p^S, he selects the message $\mu(p^S)$ in his set of available messages $M(p^S)$. We shall not consider S's mixed strategies[9].

In this paper, we are interesting in the S-type's payoff (or equivalently the interim S's payoff) and the *ex-ante* R's payoff. Let (μ, α, p^R) denote the triple of a S's message strategy μ, a R's action strategy α and a R's posterior belief p^R – which specifies the R's conditional beliefs that $\omega = Y$ for each S's sent message m.

Equilibrium. Our equilibrium concept is *perfect Bayesian equilibrium* (PBE) that is sustained by R's increasing strategy and S's pure strategy. For the two games, such equilibrium is described

[8] Intrinsic meaning of the messages is not assumed in Lanzi-Mathis (2004) and non-monotonic equilibria outcome are characterized.

[9] S's mixed (distributional) strategies are considered in Lanzi-Mathis (2004).

by a triple $(\mu^*, \alpha^*, p^{R*})$ which satisfies the following. First, S's message strategy μ^* maximizes his expected utility for each S-type p^S, taking R's action strategy α^* as given, that is for any $p^S \in [0,1]$,

$$\mu^*(p^S) \in \arg\max_{\mu(p^S)} \mathbb{E}_{(\mu(p^S),\alpha^*,p^S)}[u^S(.,.)] \tag{1}$$

Second, R's mixed action strategy α^* maximizes his expected utility, taking his beliefs p^{R*} as induced by the given S's message strategy μ^*, that is

$$\forall m \in M,\ \alpha^*(m) \begin{cases} = 0 \text{ if } p^{R*}(m) < q^R \\ \in [0,1] \text{ if } p^{R*}(m) = q^R \\ = 1 \text{ if } p^{R*}(m) > q^R \end{cases} \tag{2}$$

Third, for any received message m, R updates his beliefs p^{R*} in a *consistency*[10] manner. That is, if m is *on the equilibrium path i.e.* $m \in Range(\mu^*) \equiv \{m \in M | \exists p^S \in [0,1] \text{ such that } \mu^*(p^S) = m\}$, and according to μ^*, m is chosen by a nonzero measure set of S-types, then R's posterior beliefs is formed using Bayes' rule. Formally, whenever $\int_{\{p^S|\mu^*(p^S)=m\}} dp^S > 0$, we write:

$$p^{R*}(m) = \frac{\int_{\{p^S|\mu^*(p^S)=m\}} p^S dp^S}{\int_{\{p^S|\mu^*(p^S)=m\}} dp^S}$$

If $m \in Range(\mu^*)$ but m is a message that is chosen by a measure-zero set of S-types, then $p^{R*}(m)$ is an expectation value of S-types according to any probability distribution that has full support on the set of S-types that actually send m under the strategy μ^*. Say differently, $p^{R*}(m)$ belongs to the convex hull of the set of S-types that send m under the equilibrium strategy, *i.e.* $p^{R*}(m) \in co\{p^S|\mu^*(p^S) = m\}$. Finally, if m is *off the equilibrium path i.e.* for every $p^S \in [0,1]$, $\mu^*(p^S) \neq m$, then the only constraint is that $p^{R*}(m)$ belongs to the set of S-types for whom message m is available (*information set*).

Information Set. In the opinion game, for any $p^S \in [0,1]$, we have $M(p^S) = [0,1]$. Thus for any message the information set is $[0,1]$. While in the evidence game, $M(p^S) = [p^S, \frac{1}{2}]$ if $p^S \leq \frac{1}{2}$ and $M(p^S) = [\frac{1}{2}, p^S]$ if $p^S \geq \frac{1}{2}$. Thus, for a given message $m \in M$ the information set is $[0,m]$ if $m < \frac{1}{2}$, $[m,1]$ if $m > \frac{1}{2}$ and $[0,1]$ if $m = \frac{1}{2}$.

Equilibria Distinction. A *fully revealing equilibrium* (*FRE*) is an equilibrium (μ, α, p^R) in which R always learns the true S-type p^S. Formally, for any message m received there is a unique S-type $p^S \in [0,1]$ such that $\mu(p^S) = m$, and so from consistency we obtain $p^R(m) = p^S$. A *pooling* (or *uninformative*) *equilibrium* is an equilibrium (μ, α, p^R) in which R never learns any information on the S-type distribution, and then his updated belief p^R is equal to his prior whatever the message he receives. A *semi-pooling* (or *partially informative*) *equilibrium* is an equilibrium (μ, α, p^R) which is neither FRE nor pooling.

[10]see Ramey (1996) for the definition of consistency with a continuum of types.

Since the goal of this paper is to study whether the use of evidence is beneficial we shall use both the individual players' payoffs and the Pareto efficiency concept.

Definition 1. *Equilibrium* $(\mu^*, \alpha^*, p^{R*})$ *is* ***Pareto-efficient*** *if there is no triple* (μ, α, p^R) *that R's payoff dominates* $(\mu^*, \alpha^*, p^{R*})$ *i.e.*

$$\mathbb{E}_{(\mu,\alpha,p^R)}[u^R(.,.)] \geq \mathbb{E}_{(\mu^*,\alpha^*,p^{R*})}[u^R(.,.)];$$

and that S-type's payoff dominates $(\mu^*, \alpha^*, p^{R*})$ *for every S-type i.e.*

$$\mathbb{E}_{(\mu,\alpha,p^R),p^S}[u^S(.,.)] \geq \mathbb{E}_{(\mu^*,\alpha^*,p^{R*}),p^S}[u^S(.,.)] \text{ for every } p^S \in [0,1],$$

with at least one strict inequality either for R or for a subset of S-type to which the prior distribution assigns positive probability[11].

Every equilibrium (μ, α, p^R) induces for each S-type $p^S \in [0,1]$ a conditional distribution over actions. Usually, this conditional distribution is called the "equilibrium outcome". Here, with a slight abuse of language, we shall call *equilibrium outcome* such a conditional distribution for every S-type p^S, except for $p^S = q^S$[12]. Observe that messages are costless in these two games. So, to compare the equilibria players' payoffs it shall be sufficient to compare their outcomes. We state this as a property that shall be useful.

Property 1. *All equilibria with same outcome induce the same S-type's payoff for every S-type and the same (ex-ante) decision-maker's payoff.*

In every game where there are less actions than types, two equilibria which do not reveal the same information may have same outcome. For instance, FRE outcome is the outcome of any equilibrium for which the decision is the full-information decision. However, due to Property 1, outcome equivalent equilibrium keeps Pareto-(in)efficiency property.

[11] Here, considering strict inequality for any non-empty subset of S-type is not appropriate. For instance, increasing the payoff of a particular S-type has no impact on the R's payoff. (The R's payoff is the same because a particular S-type has a prior measure null.)

[12] The reason is that when $p^S = q^S$, firstly S is indifferent between both actions, and secondly such a prior measure zero event $\{p^S = q^S\}$ has no impact on the *ex-ante* R's payoff.

3 Opinion vs Evidence Games

In this section, we exhibit and compare equilibria from each game.

3.1 Equilibria of the Opinion Game

We now establish the opinion game main result. A well-known result in cheap-talk literature is that there always exists a pooling equilibrium[13]. The following proposition establishes that our opinion game may contain – depending on the agent's preferences – either one or two equilibria outcomes.

Proposition 1 *(Outcomes equilibria of the opinion game). In the opinion game there are at most two outcomes equilibria:*
i) A Pareto-inefficient one in which no information is revealed (pooling). It exists whatever the agents' preferences; and
ii) A Pareto-efficient one, denoted as $\mathcal{E}(q^S)$, in which the decision-maker learns whether p^S is lower or higher than q^S and the expert always succeeds to induce his most preferred action (i.e. the action x when $p^S < q^S$, and y when $p^S > q^S$). It exists if and only if $q^R \leq E[p^S|p^S > q^S]$.

Figure 3 depicts the outcomes equilibria of the opinion game such as described by Proposition 1.

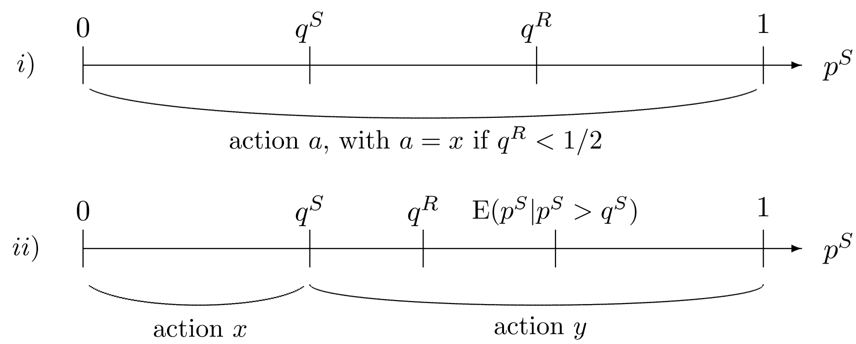

Figure 3: Outcomes equilibria of the opinion game

Proposition 1 states in particular that our opinion game has an equilibrium in which some information is (partially) revealed iff players' preferences are sufficiently similar *i.e.* $q^R \leq E[p^S|p^S > q^S]$,.or equivalently $(q^R - q^S) \leq \frac{1-q^S}{2}$[14]. Such a result is not surprising and such a condition was already highlighted by Crawford and Sobel (1982).

[13] Indeed, consider the R's strategy in which R plays his uninformed action whatever the message received. So the S's payoff will not depend on his strategy and the set of S's best response is then his whole strategy set. Thus in particular, the S's strategy which consists in sending the same message regardless his S-type sustains this R's strategy.

[14] Without the assumption that $q^S \leq q^R$ this condition would be $E[p^S|p^S < q^S] \leq q^R \leq E[p^S|p^S > q^S]$, or equivalently $|q^R - q^S| \leq \max\{\frac{q^S}{2}, \frac{1-q^S}{2}\}$.

In the semi-pooling equilibrium $\mathcal{E}(q^S)$, the expert always succeeds to induce his most preferred action. This is due to the lack of provability constraint and the possibility that the two players could share the same preferred action for some belief. To see the intuition, suppose that the decision-maker's strategy is such that there are two messages which respectively induce the action x and y. Since there is no provability constraint, the expert's best response is then to send a message which induces his most preferred action. That is the action x when $p^S < q^S$ and the action y when $p^S > q^S$. In response, the decision-maker follows the expert's messages if and only if the two players' preferences are not too distant. This is the equilibrium $\mathcal{E}(q^S)$. Otherwise, the decision-maker never follows the expert's messages and takes, by only comparing his threshold of reasonable doubt q^R to his prior, his uninformed action (pooling strategy). This is the pooling equilibrium (existing whatever the players' preferences) as represented in the hatched area in Figure 4.

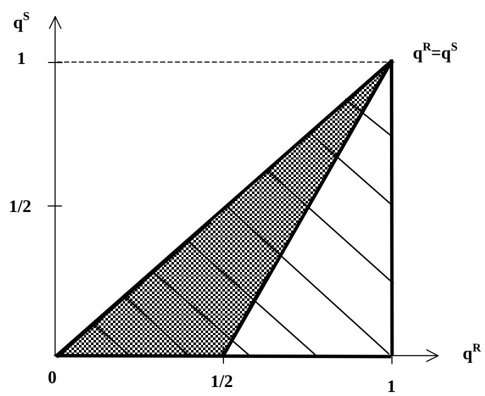

Figure 4: Condition on players' preferences (with $q^S \leq q^R$) for the existence of opinion game equilibrium. Hatched area: pooling equilibrium. Shadowed area: $\mathcal{E}(q^S)$.

Figure 5 illustrates the case where $\frac{1}{2} < q^S < q^R \leq \frac{1+q^S}{2}$ when $0 = u^R(x, X) > u^R(y, Y) > u^R(x, Y) > u^R(y, X)$. Proposition 1 states that there are both a pooling equilibrium (in which the decision-maker always takes his uninformed action x) and a semi-pooling equilibrium in which the expert succeeds to induce his most preferred action. By comparing to the case where the decision-maker would be truthfully informed on the expert type (the decision-maker takes the action x if $p^S < q^R$ and the action y if $p^S > q^R$), the hatched area represents the decision-maker's expected loss induced by the pooling equilibrium, whereas the shadowed area represents it for the semi-pooling one. By comparing both areas, we can see that the semi-pooling equilibrium induces a higher *ex-ante* decision-maker's payoff than does the pooling one (the shadowed area is smaller than the hatched one). Suppose that you can switch q^S to the left, the semi-pooling equilibrium remains *ex-ante* decision-maker's payoff superior (the shadowed area remains smaller than the hatched one) as long as $q^R \leq E[p^S | p^S > q^S]$.

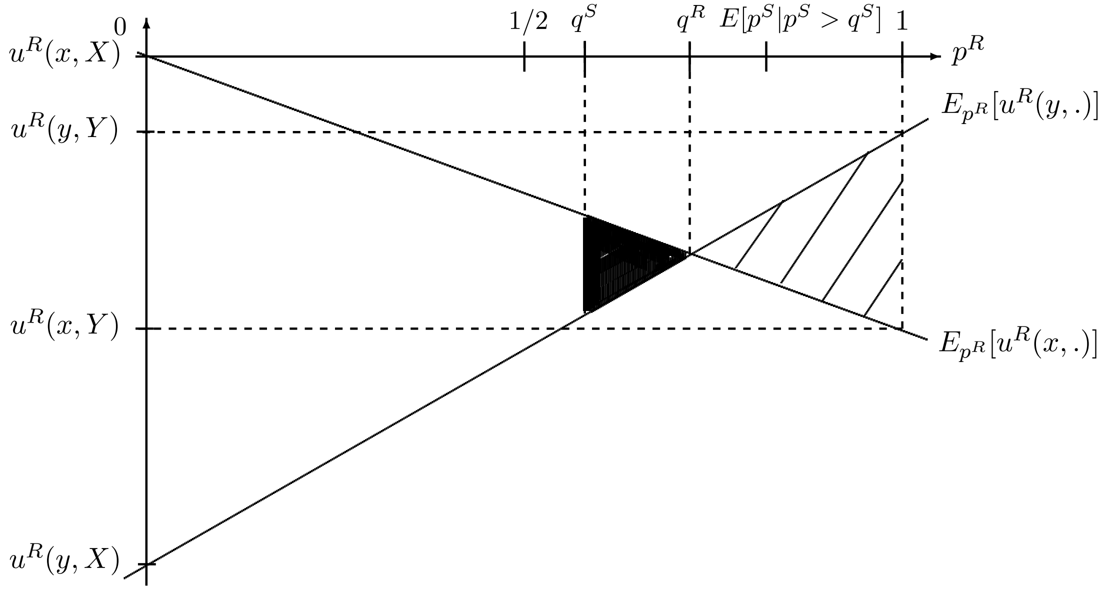

Figure 5: R's expected loss (w.r.t. full information) induced by pooling equilibrium (hatched area) and $E(q^S)$ (shadowed area).

3.2 Equilibria of the Evidence Game

We now consider the equilibria of the evidence game. Since the expert has to prove some facts by arguing, there is no pooling equilibrium. This is due to the fact that when $p^S < \min\{q^S, q^R, \frac{1}{2}\}$ (resp. $p^S > \max\{q^S, q^R, \frac{1}{2}\}$), the expert can convince the decision-maker to take the action x (resp. action y) by simply reporting p^S. More generally, since we suppose that $q^S \leq q^R$ we have the following property:

Property 2. *Any outcome equilibrium of the evidence game induces action x for every $p^S < \min\{q^S, \frac{1}{2}\}$, and action y for every $p^S > \max\{q^R, \frac{1}{2}\}$.*

Proposition 2 exhibits the Pareto-efficient equilibria of the evidence game.

Proposition 2 (*Outcomes Pareto-efficient equilibria of the evidence game). Outcomes Pareto-efficient equilibria of the evidence game are such that:*
i) if $q^R \leq E[p^S|p^S > q^S]$ then there is an outcome equilibrium, denoted as $\mathcal{E}(q^S)$, in which the expert succeeds to induce his most preferred action;
ii) if $q^R > \frac{1}{2}$, then there is a fully revealing outcome equilibrium (FRE). More generally for any $q \in [q^S, q^R]$ such that $q > \frac{1}{2}$ and $q^R \leq E[p^S|p^S > q]$, there is an outcome equilibrium which is partially (or fully when $q = q^R$) revealing, denoted as $\mathcal{E}(q)$, which induces the action x when $p^S < q$ and the action y when $p^S \geq q$;
iii) if $q^R = \frac{1}{2}$, then there is a fully revealing outcome equilibrium (FRE).
There is no other outcome Pareto-efficient equilibrium.

Figure 6 depicts the outcomes Pareto-efficient equilibria of the evidence game such as described by Proposition 2.

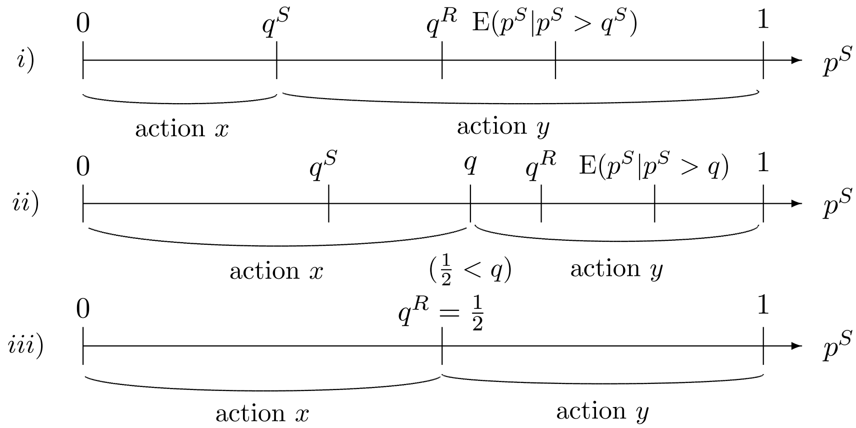

Figure 6: Outcomes Pareto-efficient equilibria of the evidence game

The first kind of equilibrium, $\mathcal{E}(q^S)$ corresponds to the semi-pooling equilibrium outcome of the opinion game. The expert always succeeds to induce his most preferred action.

The second kind of equilibrium, $\mathcal{E}(q)$ – which in fact is a continuum of equilibrium parameterized by q – corresponds either to the case where the two agents do not have the same uninformed action ($q^S < \frac{1}{2} < q^R$) or to the case where the decision-maker is the less eager agent ($\frac{1}{2} \leq q^S$). In both cases, the decision-maker uses a skepticism strategy to constraint the expert to partially reveal his type p^S. Here this skepticism strategy consists in threatening to take his uninformed action unless the expert convinces him by reporting sufficient evidence. That is, when $q^R < \frac{1}{2}$ (resp. $q^R > \frac{1}{2}$), the decision-maker takes the action y (resp. x) unless he receives a message $m \leq q^R$ (resp. $m \geq q^R$).

If $q = q^R$, then the $\mathcal{E}(q = q^R)$ equilibrium outcome contains the FRE and also all semi-pooling equilibria in which the decision-maker induces his most preferred action but the expert withholds piece of information (*e.g.*, by sending the same message $m = q^R$ for every S-type $p^S > q^R$). But all of these equilibria have the following property: the decision-maker uses skepticism strategy in which, since $q^R > \frac{1}{2}$, he takes the uninformed action x as long as, by reporting some evidence the expert does not prove that his S-type p^S is greater than the decision-maker's threshold of reasonable doubt q^R.

If $q < q^R$, then the $\mathcal{E}(q)$ equilibrium outcome can only be constituted by semi-pooling equilibria. In this continuum of equilibrium parameterized by $q \in [q^S, q^R)$, the decision-maker uses a more subtle strategy which we call *weakened skepticism* strategy. The weakened skepticism strategy consists of taking the action x as long as the expert does not provide evidence that his S-type p^S is greater than a weakened threshold of reasonable doubt $q \in [q^S, q^R)$. On the interval $[q^S, q)$, the decision-maker succeeds to induce his most preferred action x which is not the most preferred action of the expert. On the contrary on the interval $[q, q^R]$, the expert succeeds to induce his most preferred action y which is not the most preferred action of the decision-maker. The lower is q, the less is the skepticism of the decision-maker. The case where $q = q^S > \frac{1}{2}$, corresponds to $\mathcal{E}(q^S)$.

The third kind of equilibrium is trivial. Since $q^R = \frac{1}{2}$, any strategy of the decision-maker which satisfy (2) and a rational updating must satisfy $\alpha(m) = 0$ for any $m < \frac{1}{2}$, and $\alpha(m) = 1$ for any $m > \frac{1}{2}$. Thus, such strategy α either sustain the first kind of equilibrium $\mathcal{E}(q^S)$ provided that $q^R \leq E[p^S | p^S > q^S]$ or a fully revealing equilibrium.

In a game of persuasion where there is uncertainty about what the informed party exactly knows – *i.e.* the decision-maker does not know whether the expert has perfectly observed the state of Nature – Shin (1994a,b) states that there is no fully revealing equilibrium. In our evidence game, when the expert is not the least eager agent such equilibrium does exist. This is although the precision of the information coming from the expert is also unknown to the decision-maker. In these two models of persuasion games, when the expert sends a message which does not perfectly reveals the state of Nature the decision-maker cannot detect whether the expert is withholding information because the expert privately observes the precision of his information. In the models of Shin (1994a,b), there are no uncertainty about the expert's preferences, so the decision-maker has ability to identify the expert's least favorable action. However, in his models to any distinct state of nature corresponds a distinct appropriate action. Consequently, when the decision-maker receives a message which does not perfectly reveal the state of nature, the expert's least favorable action might not correspond to the action which is appropriate to the actual state of Nature. Thus skepticisms may be costly and a non-credible threat. In our evidence game, the situation is different. Either the expert is not the least eager agent so that a skepticisms strategy is costless for the decision-maker as he threat consists in selecting his own preferred action; hence a fully revealing equilibrium exists. Or the expert is the least eager agent which means that a skepticisms strategy is not a threat as the decision-maker is selecting the expert's preferred action; hence there is no fully revealing equilibrium.

Our evidence game generalizes the situation studied by Lanzi and Mathis (2008). They consider a situation where a decision-maker relies on the report of an expert prior to decide whether to undertake a certain project. Depending on the information collected, the two agents may have conflicting preferences. Information contained in the report is partially verifiable in the sense that the expert can suppress favorable information sustaining the project but he cannot exaggerate it. They show that this setting favors the agent which is the less eager to undertake the project in that he always succeeds to induce his most preferred action. Here, their set-up is extended to situation where the decision-maker does not know which action is actually sustained by the expert's private information. (Formally, he does not know whether p^S is higher or lower than $\frac{1}{2}$; and information sustaining state X as well as state Y can be suppressed.) Our Proposition 2 establishes the robustness of their main result according to which the agent which is the less eager to undertake the project always succeeds to induce his most preferred action.

All these equilibria have a common structure in that the induced action is x when $p^S < q^S$ and y when $p^S > q^R$. It seems to be natural since both players are in conflict at the full information decision only when $p^S \in [q^S, q^R]$. However, the following lemma establishes the existence of other equilibrium in the evidence game. These are supported by a more aggressive R's skepticism strategy. In these equilibria, the decision-maker threaten to take the more unfavorable action (for both players) as long as the expert transmits an information which is potentially transmissible by every S's types, *i.e.* if $m = \frac{1}{2}$. Indeed, by sending such a message $m = \frac{1}{2}$, the expert presents no element of proof to the decision-maker. The following definition formally defines this strategy.

Definition 3. *If both agents have the same unique uninformed action, a decision-maker's strategy α is an* ***aggressive-skepticism strategy*** *if the uninformed action is not chosen surely when the expert brings no information i.e.* $[q^S > \frac{1}{2}$ *and* $\alpha(\frac{1}{2}) > 0]$ *or* $[q^R < \frac{1}{2}$ *and* $\alpha(\frac{1}{2}) < 1]$.

Lemma 1 states that any Pareto-inefficient equilibrium of the evidence game is supported by a decision-maker's aggressive-skepticism strategy.

Lemma 1 *(Pareto-inefficient equilibria of the evidence game). An equilibrium of the evidence game is Pareto-inefficient iff it is supported by a decision-maker's aggressive-skepticism strategy. Moreover, if preferences are sufficiently distant, that is if* $q^R > E[p^S|p^S > q^S]$*, then there is no Pareto-inefficient equilibrium.*

All inefficient equilibria (see Definition 1) are supported by a strategy in which the decision-maker threatens to select the most unfavorable action for both agents when the expert brings no evidence. This result highlights a 'perverse' effect by which the players coordinate each other on equilibria inducing a payoff which is inferior to the one reached at equilibrium of the opinion game. This only occurs if agents' preferences are similar enough. On the contrary, when the agents' preferences are distant enough the evidence game has no equilibrium supported by a decision-maker's aggressive-skepticism strategy.

3.3 Equilibria Comparison

When preferences are sufficiently distant, Proposition 1 establishes that the unique equilibrium of the opinion game is the Pareto-inefficient pooling one. Whereas Lemma 1 establishes that there is no Pareto-inefficient equilibrium in the evidence game. This establishes a sense in which, when preferences are sufficiently distant, requiring evidence is beneficial to both players.

The equilibria comparison between both games is summarized in the following result.

Proposition 3 *(Equilibria Comparison). i) If preferences are sufficiently distant, that is if* $q^R > E[p^S|p^S > q^S]$*, then requiring evidence is beneficial for both agents in the sense that all equilibria of the evidence game players' payoffs dominates the unique equilibrium of the opinion game.*
ii) If preferences are sufficiently similar, that is if $q^R \leq E[p^S|p^S > q^S]$*, then requiring evidence is:*

- *beneficial for the decision-maker in the sense that all Pareto-efficient (resp. inefficient) equilibria of the evidence game the decision-maker's payoff dominates all (resp. the inefficient) equilibria of the opinion game;*

- *not beneficial for the expert in the sense that, for every S-type, the Pareto-efficient equilibrium of the opinion game S-type's payoff dominates all equilibria of the evidence game.*

When preferences are sufficiently similar, the opinion game contains a unique Pareto-efficient equilibrium, denoted as $\mathcal{E}(q^S)$ where the expert always succeeds to induce his most preferred action. In the Pareto-efficient equilibria of the evidence game the expert is potentially compelled by the decision-maker's (weakened) skepticism strategies and for all $q \in [q^S, q^R]$ the equilibrium $\mathcal{E}(q)$ the decision-maker's payoff dominates the unique Pareto-inefficient (pooling) equilibrium of the opinion game (see Claim in the Appendix).

4 Conclusion

In this article, we are interested in the problem of the transmission of the information between a better informed and self-interested expert and a decision-maker. We have compared two modes of communication. In the first one, the expert only reports his opinion (soft information) concerning the desirability of a certain action. In the second one, he is consulted to report documents (hard information) supporting his opinion. We have assumed that the ability of the expert to provide evidence depended on the precision of his information. More specifically, the more his information is precise or substantial the more he is able to provide evidence supporting his opinion. Concerning the provability through providing evidence, we have supposed that the expert is able to prove everything he knows. However, since he does not potentially know everything, he is unable to prove that he does not hide additional information. The question raised has been to know who would benefit from the resorting to evidence, knowing that it modifies the manipulability of the information. In order to answer this question, we have compared the equilibria of both games ruled by these two modes of communication based on the concept of Pareto-efficiency and the players' payoff. We have demonstrated that resorting to evidence is always beneficial to the decision-maker. This first result can seem intuitive but the literature has a counter-example in which the expert can prove everything he knows and also he does not hide any information (Giovannoni and Seidmann (2007)). We have also demonstrated that resorting to evidence is beneficial to the expert if and only if both players have preferences which are distant enough. The intuition is simple. When the agents' preferences are distant enough, in the absence of convincing proofs the decision-maker does not trust the expert and chooses an action according to his prior belief. Only resorting to evidence can then enable the expert to convince the decision-maker (whenever the information he has observed is determining enough). On the contrary, if the preferences are not distant enough, the decision-maker always completely trusts the expert when consulted on the basis of his opinion. However requiring evidence makes it possible for the decision-maker to behave skeptically and then extract all or some pieces of the information held by the expert.

In this study, we have chosen a specific form of communication. The expert's ability to provide evidence is objective and directly depends on the quality of the information he has observed. For instance, if we suppose there is a positive correlation between the skills of an expert and the quality of his expertise regarding the acquired information, our modeling of the information mentions he will be more likely to provide the decision-maker with irrefutable proofs. This model does not tackle the more subjective aspect of the communication in which the semantic contents given the set of messages would be different. We think for instance to the rhetoric qualities of a salesman who, even with some non-precise information on the quality of the proposed product, would be able to convince many consumers to buy it. An extension of this study would be to model this more subjective form of the argumentation.

Also our model is quite simple since we have considered two states of Nature – but a continuum of types –, two decisions and an interaction between two players only. Another extension of this article would be to consider the evidence game with N experts. Thus we will get a quite relevant debate model in which the ability of the agents to provide evidence would depend on their private information. This is little studied in the literature – cf. Glazer and Rubinstein (2001, 2004) – for debates models between two experts with provable information. Moreover this extension would be the direct extension of Lipman and Seppi (1995) to the case where experts may observe different information.

Appendix

The appendix is organized as follows. Since Property 1 is trivial, we do not expose its proof. Appendix A states the proof of Proposition 1. Appendix B starts with the proof of Property 2 and Proposition 2. Then we states Proposition 4 that exhibits the Pareto-inefficient equilibria of the evidence game. As in the proof of Proposition 2, the proof of Proposition 4 requires the Lemma 2 next introduced. The proof of Lemma 1 is then established. Finally, in Appendix C we demonstrate Proposition 3.

In the following, remember that when reasoning on the outcome equilibrium equivalence, it shall not be required to specify any action distribution at the point $p^S = q^S$.

Proof of Property 1. See Lemma 1 in Lanzi and Mathis (2004).

Appendix A: Equilibria of the Opinion Game

Proof of Proposition 1. We proceed in three steps. First, to prove existence of each considered outcome equilibrium, we exhibit an appropriate triple (μ, α, p^R). Second, we demonstrate that any equilibrium generates one of the two considered outcomes. Third, we discuss the Pareto-efficiency.

1. Straightforward to (1), (2) and the consistency hypothesis, the two following triple are equilibria.

i) Pooling equilibrium. (μ, α, p^R) such that $\mu(p^S)$ does not depend on the S-type p^S; for each message received m: the R's belief is equal to the prior *i.e.* $p^R(m) = \frac{1}{2}$ for any $m \in M$, and R takes the action x if $q^R > \frac{1}{2}$, the action y if $q^R < \frac{1}{2}$, and chooses the action y with any probability if $q^R = \frac{1}{2}$.

ii) $\mathcal{E}(q^S)$. (μ, α, p^R) such that:

$$\mu(p^S) = \begin{cases} \{m_x\} \text{ if } p^S \leq q^S \\ \{m_y\} \text{ if } p^S > q^S \end{cases}, \text{ with } m_x, m_y \in M,\ m_x \neq m_y,$$

$$\alpha(m) = \begin{cases} 0 \text{ if } m = m_x \\ 1 \text{ if } m = m_y \end{cases} \text{ and } p^R(m) = \begin{cases} \frac{q^S}{2} \text{ if } m = m_x \\ \frac{q^S+1}{2} \text{ if } m = m_y \end{cases},$$

and else (m is off the equilibrium path): $\alpha(m)$ is any constant in $[0, 1]$ and $p^R(m) = q^R$.

2. Assume (μ, α, p^R) is an equilibrium. There are two distinct cases:

(i) R adopts the same strategy whatever the message received *i.e.* for any $m_1, m_2 \in M$, we have $\alpha(m_1) = \alpha(m_2)$. Therefore (μ, α, p^R) is a pooling equilibrium.

(ii) There are two messages for which the R's strategy is different *i.e.* there are m_1 and m_2 such that $\alpha(m_1) < \alpha(m_2)$. Due to the increase of α on M, we have $0 \leq \alpha(0) \leq \alpha(m_1) < \alpha(m_2) \leq \alpha(1) \leq 1$. By (2), we then have $p^R(0) \leq q^R \leq p^R(1)$. And by (1), we must have for any $p^S < q^S$, $\mu(p^S) \in \{m \in M | \alpha(m) = \alpha(0)\}$ and for any $p^S > q^S$, $\mu(p^S) \in \{m \in M | \alpha(m) = \alpha(1)\}$. This satisfy (2) and the consistency hypothesis only if $\frac{q^S}{2} \leq q^R \leq \frac{1+q^S}{2}$. From $q^S \leq q^R$, we obtain that the triple (μ, α, p^R) necessarily generates the equilibrium outcome $\mathcal{E}(q^S)$.

3. The pooling equilibrium is not Pareto-efficient since $q^i \notin \{0, 1\}$, $i = R, S$. The partially revealing equilibrium $\mathcal{E}(q^S)$ is Pareto-efficient since each S-type succeeds to induce his most preferred action while it is not possible to increase the R's payoff without decreasing the payoff of a subset of S-type to which the prior distribution assigns positive probability. ■

Appendix B: Equilibria of the Evidence Game

Proof of Property 2. Assume that triple (μ, α, p^R) is an equilibrium. From (2) and consistency hypothesis, p^R and α are such that:

$$\begin{aligned} \forall m &< \min\{q^R, \frac{1}{2}\},\ p^R(m) < q^R \text{ and } \alpha(m) = 0 \\ \forall m &> \max\{q^R, \frac{1}{2}\},\ p^R(m) > q^R \text{ and } \alpha(m) = 1 \end{aligned}$$

and from (1), μ must then satisfies:

$$\begin{aligned}\forall p^S &< \min\{q^S, \frac{1}{2}\},\ \mu(p^S) \in \{m \in M(p^S) | \alpha(m) = 0\} \\ \forall p^S &> \max\{q^R, \frac{1}{2}\},\ \mu(p^S) \in \{m \in M(p^S) | \alpha(m) = 1\}\end{aligned}$$

■

Proof of Proposition 2. As in proof of Proposition 1, we proceed in three steps. First, to prove existence of each considered outcome equilibrium, we exhibit an appropriate triple (μ, α, p^R). Second, we show that these equilibria are Pareto-efficient. Third, we demonstrate that all equilibria which are Pareto-efficient necessarily generate one of the considered outcomes.

1. Straightforward to (1), (2) and the consistency hypothesis the three following triples are equilibria. For all these triples, if m is off the equilibrium path it suffices to consider that $\alpha(m)$ satisfies (2) with $p^R(m) = m$.

i) $\mathcal{E}(q^S)$. If $q^S \le \frac{1}{2}$, (μ, α, p^R) such that:

$$\mu(p^S) = \begin{cases} p^S \text{ if } p^S < q^S \\ \frac{1}{2} \text{ if } p^S \ge q^S \end{cases},$$

$$\alpha(m) = \begin{cases} 0 \text{ if } m < q^S \\ 1 \text{ if } m = \frac{1}{2} \end{cases} \text{ and } p^R(m) = \begin{cases} m \text{ if } m < q^S \\ \frac{1+q^S}{2} \text{ if } m = \frac{1}{2} \end{cases}.$$

If $q^S > \frac{1}{2}$ then consider $\mathcal{E}(q)$ with $q = q^S$.

ii) $\mathcal{E}(q)$. (μ, α, p^R) such that:

$$\mu(p^S) = \begin{cases} \frac{1}{2} \text{ if } p^S < q \\ q \text{ if } p^S \ge q \end{cases},$$

$$\alpha(m) = \begin{cases} 0 \text{ if } m = \frac{1}{2} \\ 1 \text{ if } m = q \end{cases} \text{ and } p^R(m) = \begin{cases} \frac{q}{2} \text{ if } m = \frac{1}{2} \\ \frac{1+q}{2} \text{ if } m = q \end{cases}.$$

iii) FRE. (μ, α, p^R) such that: $p^R(m) = m$ for any $m \in M$; $\alpha(m) = 0$ if $m \le \frac{1}{2}$ and 1 else; $\mu(p^S) = p^S$.

2. According to the following remarks, all of these equilibria are Pareto-efficient.

i) $\mathcal{E}(q^S)$. As in the proof of Proposition 1. Each S-type succeeds to induce his most preferred action while it is not possible to increase the R's payoff without decreasing the payoff for a subset of S-type to which the prior distribution assigns positive probability.

ii) $\mathcal{E}(q)$. Each S-type $p^S \in [0, q^S] \cup [q, 1]$ succeeds to induce his most preferred action while on the open interval (q^S, q) the players do not share the same preferred action and the R's *ex-ante* payoff is maximal.

iii) FRE. R is fully informed and succeeds to induce his most preferred action while it is not possible to increase the payoff for a subset of S-type to which the prior distribution assigns positive probability without decreasing the R's payoff.

3. Straightforward from Lemma 2 *(see below)*. $\blacksquare$

Before establishing the Proof of Lemma 1, we need the two following results.

Proposition 4 *(Evidence game Pareto-inefficient equilibria). Outcomes Pareto-inefficient equilibria of the evidence game are such that:*
i) if $\frac{1}{4} \leq q^R < \frac{1}{2}$ *then there is an equilibrium, denoted as* $\mathcal{E}(\frac{1}{2})$*, which induces the action* x *when* $p^S \leq \frac{1}{2}$ *and* y *when* $p^S > \frac{1}{2}$*;*
ii) if $q^R = \frac{q^S + \frac{1}{2}}{2} < \frac{1}{2}$ *then there is an equilibrium, denoted as* $\mathcal{E}(\bar{\alpha})$*, which induces the action* x *when* $p^S < q^S$ *the action* y *when* $p^S > \frac{1}{2}$ *and the action* y *with probability* $\bar{\alpha} \in (0,1)$ *if* $p^S \in [q^S, \frac{1}{2}]$*;*
iii) if $\frac{1}{2} < q^S$ *and* $q^R < \frac{3}{4}$*, then there is an equilibrium, denoted as* $\mathcal{E}(\frac{1}{2}')$*, which induces the action* x *when* $p^S < \frac{1}{2}$ *and the action* y *when* $p^S \geq \frac{1}{2}$*.*

Proof of Proposition 4. We proceed in three steps. First, to prove existence of each considered outcome, we exhibit an appropriate triple (μ, α, p^R). Second, we show that these equilibria are Pareto-inefficient. Third, we demonstrate that all Pareto-inefficient equilibria necessarily generate one of the considered outcomes.

1. Straightforward to (1), (2) and the consistency hypothesis the three following triples support equilibria.

i) $\mathcal{E}(\frac{1}{2})$. (μ, α, p^R) such that:

$$\mu(p^S) = \begin{cases} \frac{1}{2} \text{ if } p^S \leq \frac{1}{2} \\ p^S \text{ if } p^S > \frac{1}{2} \end{cases},$$

$$\alpha(m) = \begin{cases} 0 \text{ if } m = \frac{1}{2} \\ 1 \text{ if } m > \frac{1}{2} \end{cases} \text{ and } p^R(m) = \begin{cases} \frac{1}{4} \text{ if } m = \frac{1}{2} \\ m \text{ if } m > \frac{1}{2} \end{cases},$$

if m is off the equilibrium path (*i.e.* $m < \frac{1}{2}$) then $\alpha(m) = 0$ and $p^R(m) \leq q^R$.

ii) $\mathcal{E}(\bar{\alpha})$. (μ, α, p^R) such that:

$$\mu(p^S) = \begin{cases} p^S \text{ if } p^S < q^S \\ \frac{1}{2} \text{ if } p^S \in [q^S, \frac{1}{2}] \\ p^S \text{ if } p^S > \frac{1}{2} \end{cases},$$

$$\alpha(m) = \begin{cases} 0 \text{ if } m < q^S \\ \bar{\alpha} \text{ if } m = \frac{1}{2} \\ 1 \text{ if } m > \frac{1}{2} \end{cases} \text{ and } p^R(m) = \begin{cases} m \text{ if } m < q^S \\ \frac{q^S + \frac{1}{2}}{2} \text{ if } m = \frac{1}{2} \\ m \text{ if } m > \frac{1}{2} \end{cases},$$

if m is off the equilibrium path (*i.e.* $m \in [q^S, \frac{1}{2})$) then $\alpha(m) = 0$ and $p^R(m) \leq q^R$.

iii) $\mathcal{E}(\frac{1}{2}')$. (μ, α, p^R) such that:

$$\mu(p^S) = \begin{cases} \frac{1}{2} \text{ if } p^S \geq \frac{1}{2} \\ p^S \text{ if } p^S < \frac{1}{2} \end{cases},$$

$$\alpha(m) = \begin{cases} 0 \text{ if } m < \frac{1}{2} \\ 1 \text{ if } m = \frac{1}{2} \end{cases} \text{ and } p^R(m) = \begin{cases} m \text{ if } m < \frac{1}{2} \\ \frac{3}{4} \text{ if } m = \frac{1}{2} \end{cases},$$

if m is off the equilibrium path (*i.e.* $m > \frac{1}{2}$) then $\alpha(m) = 1$ and $p^R(m) \geq q^R$.

2. For each considered equilibrium, we exhibit an outcome which Pareto-dominates it.

i) $\mathcal{E}(\frac{1}{2})$. Since $q^R < \frac{1}{2}$, $\mathcal{E}(\frac{1}{2})$ is Pareto-dominated by the outcome which induces the action x if $p^S \leq q^R$ and y if $p^S > q^R$.

ii) $\mathcal{E}(\bar{\alpha})$. Since $\alpha > 0$ and $q^S < \frac{1}{2}$, $\mathcal{E}(\bar{\alpha})$ is Pareto-dominated by $\mathcal{E}(q^S)$.

iii) $\mathcal{E}(\frac{1}{2}')$. Since $q^S > \frac{1}{2}$, $\mathcal{E}(\frac{1}{2}')$ is Pareto-dominated by $\mathcal{E}(q^S)$.

3. Straightforward from Lemma 2 *(see below)*. ■

Lemma 2. *There is no other outcome equilibrium supported by an R's monotone increasing strategy and an S's pure strategy than those exhibited in Proposition 2 and Proposition 4.*

Proof of Lemma 2. We proceed in two steps. First we prove the result in the case $q^R \leq \frac{1}{2}$, second in the case $q^R > \frac{1}{2}$. Assume (μ, α, p^R) is an equilibrium where μ is an S's pure strategy and α is an R's monotone increasing strategy.

1. Suppose $q^R \leq \frac{1}{2}$. By Property 2, for any $p^S < q^S$, $\mu(p^S) \in \{m \in M(p^S) | \alpha(m) = 0\} \neq \emptyset$ and for any $p^S > \frac{1}{2}$, $\mu(p^S) \in \{m \in M(p^S) | \alpha(m) = 1\} \neq \emptyset$. So (μ, α, p^R) induces the action x if $p^S < q^S$ and y if $p^S > \frac{1}{2}$. Now, to determine the induced action when $p^S \in [q^S, \frac{1}{2}]$ we distinguish different cases. By (2) and the consistency hypothesis, for any $m < q^R \leq \frac{1}{2}$, we have $p^R\,(m) < q^R$ and $\alpha(m) = 0$.

(i) Suppose $\alpha(\frac{1}{2}) \in (0, 1)$. Then by (2) we have $p^R\,(\frac{1}{2}) = q^R$. By increase of the R's strategy, for all $m \leq \frac{1}{2}$, we have $\alpha(m) \leq \alpha(\frac{1}{2})$, and by (1), for any $p^S \in (q^S, \frac{1}{2}]$, $\mu(p^S) \in \{m \in M(p^S) | \alpha(m) = \alpha(\frac{1}{2})\}$.

$(i)(a)$ If $q^R = \frac{1}{2}$ then by the consistency hypothesis, we must have $q^S = q^R$. Property 2 implies that the equilibrium outcome (μ, α, p^R) must be the one generated by the FRE.

$(i)(b)$ If $q^R < \frac{1}{2}$ since by definition $\mu(\frac{1}{2}) = \frac{1}{2}$, then by the consistency hypothesis, there is $\bar{p}^S \in [q^S, q^R)$ such that $\mu(\bar{p}^S) = \frac{1}{2}$. For all $p^S \in (q^S, \frac{1}{2}]$, since for any $m \in \mu(p^S)$, $p^R\,(m) = q^R$, we then have $q^R = \frac{q^S + \frac{1}{2}}{2}$. Moreover, equilibrium (μ, α, p^R) must induces the action x if $p^S < q^S$, the action y if $p^S > \frac{1}{2}$ and the action x with probability $\alpha(\frac{1}{2}) \in (0, 1)$ if $p^S \in (q^S, \frac{1}{2}]$. Therefore, (μ, α, p^R) generates the outcome equilibrium $\mathcal{E}(\overline{\alpha} = \alpha(\frac{1}{2}))$.

(ii) Suppose $\alpha(\frac{1}{2}) = 0$. By increase of the R's strategy, and by (2), for all $m \leq \frac{1}{2}$ we have $\alpha(m) = 0$ and $p^R(m) \leq q^R$.

$(ii)(a)$ If $q^R = \frac{1}{2}$ then the outcome equilibrium (μ, α, p^R) must be the one generated by the FRE.

$(ii)(b)$ If $q^R < \frac{1}{2}$ then $[q^R, \frac{1}{2}) \neq \emptyset$. Since the prior is uniform, a necessary condition for the equilibrium existence is that $q^R \geq \frac{1}{4}$. Moreover, equilibrium (μ, α, p^R) must induces the action x if $p^S \leq \frac{1}{2}$ and the action y if $p^S > \frac{1}{2}$. Therefore, (μ, α, p^R) generates the outcome equilibrium $\mathcal{E}(\frac{1}{2})$.

(iii) Suppose $\alpha(\frac{1}{2}) = 1$.
So for any $p^S > q^S$, $\mu(p^S) \in \{m \in M(p^S) | \alpha(m) = 1\} \neq \emptyset$. A necessary condition for the equilibrium existence is that $q^R \leq \frac{1+q^S}{2}$. Thus, the equilibrium (μ, α, p^R) must induces the action x if $p^S < q^S$ and the action y if $p^S > q^S$. Therefore, (μ, α, p^R) generates the outcome equilibrium $\mathcal{E}(q^S)$.

2. Suppose $q^R > \frac{1}{2}$. By Property 2, for any $p^S < \min\{\frac{1}{2}, q^S\}$, $\mu(p^S) \in \{m \in M(p^S) | \alpha(m) = 0\} \neq \emptyset$ and for any $p^S > q^R$, $\mu(p^S) \in \{m \in M(p^S) | \alpha(m) = 1\} \neq \emptyset$.

(i) Suppose $\alpha(\frac{1}{2}) = 0$. Then by (1), for any $p^S < q^S$, $\mu(p^S) \in \{m \in M(p^S) | \alpha(m) = 0\} \neq \emptyset$.

$(i)(a)$ Suppose there is a message $m' \in (\frac{1}{2}, q^R)$ such that $\alpha(m') > 0$. Then by (2) we have $p^R\ (m') \geq q^R$. Thus, there is $p^S > q^R$ such that $m' = \mu(p^S)$ and by (1), we must have $\alpha(m') = 1$. Let $M^{\alpha=1} \equiv \{m \in (\frac{1}{2}, q^R) | \alpha(m) = 1\}$. For all $p^S \in M^{\alpha=1} \backslash [0, q^S]$, we have $\mu(p^S) \in \{m \in M(p^S) | \alpha(m) = 1\} \neq \emptyset$. Let $m' = \inf M^{\alpha=1}$. For such a $m' \in (\frac{1}{2}, q^R)$ such that $q^R \leq \frac{m'+1}{2}$ if $m' \geq q^S$ and such that $q^R \leq \frac{q^S+1}{2}$ else, equilibrium (μ, α, p^R) must induces the action x if $p^S < \max\{m', q^S\}$, the action y if $p^S > \max\{m', q^S\}$ and the action x (resp. y) if $p^S = m'$ and $m' \notin \inf M^{\alpha=1}$ (resp. $m' \in \inf M^{\alpha=1}$). So, (μ, α, p^R) generates the outcome equilibrium $\mathcal{E}(q = \max\{m', q^S\})$.

$(i)(b)$ Suppose for any $m \in (\frac{1}{2}, q^R)$, $\alpha(m) = 0$. Hence, equilibrium (μ, α, p^R) must induces the action x if $p^S \leq q^R$ and the action y if $p^S > q^R$. So, (μ, α, p^R) generates the outcome equilibrium $\mathcal{E}(q = q^R)$.

(ii) Suppose $\alpha(\frac{1}{2}) = 1$. By increase of the R's strategy, we have $\alpha(m) = 1$ for all $m \geq \frac{1}{2}$.

$(ii)(a)$ If $q^S \leq \frac{1}{2} < q^R$, trivially a necessary condition on the existence of the equilibrium is that $q^R \leq \frac{1+q^S}{2}$. So, the equilibrium (μ, α, p^R) must induces the action x if $p^S < q^S$ and the action y if $p^S > q^S$. Thus, (μ, α, p^R) generates the outcome equilibrium $\mathcal{E}(q = q^S)$.

$(ii)(b)$ If $\frac{1}{2} < q^S \leq q^R$, then by (1), for any $p^S \in (q^S, 1]$, $\mu(p^S) \in \{m \in M(p^S) | \alpha(m) = 1\} \neq \emptyset$. Since the prior distribution is uniform, a necessary condition on the existence of an equilibrium is that $q^R \leq \frac{3}{4}$. So, equilibrium (μ, α, p^R) must induces the action x if $p^S < \frac{1}{2}$ and the action y if $p^S \geq \frac{1}{2}$. Thus, (μ, α, p^R) generates the outcome equilibrium $\mathcal{E}(\frac{1}{2}')$.

(iii) Suppose $\alpha(\frac{1}{2}) \in (0, 1)$. Then by (2) we have $p^R\ (\frac{1}{2}) = q^R$ and by (1) we have for any $p^S > q^R$,

$\frac{1}{2} \neq \mu(p^S) \in \{m \in M(p^S) | \alpha(m) = 1\} \neq \emptyset$ whereas $\mu(\frac{1}{2}|\frac{1}{2}) = 1$, contrary to the consistency hypothesis. ■

Proof of Lemma 1. Firstly, let us prove that none of the Pareto-efficient equilibrium is supported by an R's aggressive skepticism strategy while this is the case for Pareto-inefficient equilibrium. From Lemma 2, all equilibria of the evidence game generate the outcome being either exhibited in Proposition 2 or Proposition 4. We need to prove that the Pareto-inefficient equilibria are exactly those considered in Proposition 4. From the proof of Lemma 2, the following outcome equilibria are supported by an R's aggressive skepticism strategy: $\mathcal{E}(\bar{\alpha})$ case $1(i)(b)$ where $q^R < \frac{1}{2}$ and $\alpha(\frac{1}{2}) \in (0,1)$; $\mathcal{E}(\frac{1}{2})$ case $1(ii)(b)$ where $q^R < \frac{1}{2}$ and $\alpha(\frac{1}{2}) = 0$; $\mathcal{E}(\frac{1}{2}')$ case $2(ii)(b)$ where $\frac{1}{2} < q^S$ and $\alpha(\frac{1}{2}) = 1$. All these outcomes are those exhibited in Proposition 4.
From the proof of Lemma 2, none of the following outcome equilibrium is supported by an R's aggressive skepticism strategy. *FRE:* case $1(i)(a)$ and $1(ii)(a)$ where $q^R = \frac{1}{2}$; $\mathcal{E}(q^S)$: case $1(iii)$ where $q^R \leq \frac{1}{2}$ and $\alpha(\frac{1}{2}) = 1$ and case $2(ii)(a)$ where $q^S \leq \frac{1}{2} < q^R$; $\mathcal{E}(q)$: case $2(i)(a)$ where $\frac{1}{2} < q^R$ and $\alpha(\frac{1}{2}) = 0$ and case $2(i)(b)$ where $q = q^R$. All these outcomes equilibria are those exhibited in Proposition 2.

Secondly, let us prove that if preferences are sufficiently distant then there is no equilibrium supported by an R's aggressive skepticism strategy. If preferences are sufficiently distant, that is $(q^R - q^S) > \frac{1-q^S}{2}$, then $q^R > \frac{1+q^S}{2} \geq \frac{1}{2}$. So, $q^R > \frac{1}{2}$ and if $q^S > \frac{1}{2}$ then $q^R > \frac{3}{4}$. Hence, from Proposition 4, there is no Pareto-inefficient equilibrium (and hence supported by an R's aggressive skepticism strategy). ■

Appendix C: Equilibria Comparison

Remember that by Property 1, to study the equilibria players' payoff, we shall only need to compare the outcomes equilibria.

Proof of Proposition 3. Firstly, we prove the result for the R's payoff and secondly for the S's payoff.

1. *a*) Let us show that whatever the preferences, all equilibria of the evidence game *ex-ante* R's payoff dominates the pooling equilibrium of the opinion game. For this, consider the following Claim. Trivially, any signaling game where messages are costless possesses a pooling equilibrium.

Claim. For any signaling game where messages are costless, the lower *ex-ante* R's equilibrium payoff is that of the pooling equilibrium.

Proof of the Claim. Let (μ, α, p^R) and $(\mu', \alpha', p^{R\prime})$ be two equilibria. If $(\mu', \alpha', p^{R\prime})$ is pooling then whatever the S's strategy $\tilde{\mu}$, the R's payoff is the same under $(\mu', \alpha', p^{R\prime})$ and $(\tilde{\mu}, \alpha', p^{R\prime})$ since messages are costless. In particular for $\tilde{\mu} = \mu$. Now, since (μ, α, p^R) is an equilibrium, the pair (α, p^R) must be a R's best response to μ. Then (μ, α, p^R) R's payoff dominates $(\mu', \alpha', p^{R\prime})$. ■

b) Let us show that when the preferences are not sufficiently distant, all Pareto-efficient equilibria of the evidence game *ex-ante* R's payoff dominates the Pareto-efficient equilibrium of the opinion game. From Proposition 2, the Pareto-efficient equilibria of the evidence game induces the action x for every $p^S < q^S$ and the action y for every $p^S > q^R$. The partially revealing equilibrium of the opinion game has also this outcome distribution while it induces the R's least preferred action for every $p^S \in (q^S, q^R)$.

2. a) Let us show that when the preferences are sufficiently distant, all equilibria of the evidence game S-type's payoff dominates the pooling equilibrium of the opinion game. Since $q^R > \frac{1+q^S}{2} \geq \frac{1}{2}$, the pooling equilibrium of the opinion game induce the action x for every p^S. From Lemma 1, all equilibria of the evidence game are Pareto-efficient and induces the action x for every $p^S < q^S$ while they induce the action y for every $p^S > q^R$.

b) Since in $\mathcal{E}(q^S)$, each S-type succeeds to induces his most preferred action, this establishes that when the preferences are not sufficiently distant, the Pareto-efficient equilibrium of the opinion game S-type's payoff dominates all equilibria of the evidence game. ■

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