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A Pitfall with DSGE-Based, Estimated, Government Spending Multipliers*

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Abstract

This paper examines issues related to the estimation of the government spending multiplier (GSM) in a Dynamic Stochastic General Equilibrium context. We stress a potential source of bias in the GSM arising from the combination of *Edgeworth complementarity/substitutability* between private consumption and government expenditures and *endogenous government expenditures*. Due to cross-equation restrictions, omitting the endogenous component of government policy at the estimation stage would lead an econometrician to underestimate the degree of Edgeworth complementarity and, consequently, the long-run GSM. An estimated version of our model with US postwar data shows that this bias matters quantitatively. The results prove to be robust to a number of perturbations.

KEYWORDS: DSGE models, Edgeworth complementarity/substitutability, Government spending rules, Multiplier.

JEL CLASS.: C32, E32, E62.

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1 Introduction

In the current crisis context, there has been a renewed academic and policy interest in studying the effects of government activity.[1] A key quantity that has attracted considereable attention is the government spending multiplier (GSM), i.e. the increase in output consecutive to an increase in government spending.

In this paper, we study issues related to the estimation of this multiplier in a Dynamic Stochastic General Equilibrium (DSGE) context. We stress a potential source of bias in the GSM arising from the combination of (i) the transmission mechanism of government expenditures and (ii) the endogeneity of government expenditures. We find that the transmission mechanisms and the endogeneity of policy interact at the estimation stage through cross-equation restrictions, paving the way for potential biases.

To illustrate how these biases can arise, we focus on Edgeworth complementarity/substitutability between private consumption and government expenditures as an example of the transmission mechanism,[2] though alternative channels considered in the literature would yield similar results, as we discuss later in the paper. Depending on its, this transmission mechanism may mitigate the crowding-out effect of government spending shocks. The mechanics of the bias are then as follows.

Assume government spending policy is countercyclical, as suggested by several recent papers Cúrdia and Reis (2010), Jones (2002), McGrattan (1994).[3] Such an assumption would raise a severe challenge for Neoclassical models. In those setups, following any shock such that both output and consumption decline, countercyclical policy triggers an increase in public spending. This in turn increases output but reduces consumption even more (crowding-out effect), finally making private consumption even more negatively correlated with public expenditures than under an exogenous policy. This seems to be at odds with postwar US data. Typically, over the sample used in this paper, we observe a correlation between the growth rates of these aggregate quantities around 0.24. Allowing for Edgeworth complementarity helps mitigate this problem. With such a mechanism, a rise in public expenditures would make people want to consume more, thus counteracting the crowding-out effect. As a consequence, given a certain unconditional correlation between private consumption and government expenditures that we seek to

[1] See, among others, Christiano, Eichenbaum and Rebelo (2011), Cogan et al. (2010), Fernández-Villaverde (2010), Uhlig (2010). The common theme of these papers is to investigate under which circumstances the multiplier may or may not be large.

[2] Such a specification has now become standard, following the semnial work by Aschauer (1985), Bailey (1971), Barro (1981), Braun (1994), Christiano and Eichenbaum (1992), Finn (1998), McGrattan (1994).

[3] In Cúrdia and Reis (2010) and McGrattan (1994), the forcing variables are assumed to follow vector autoregressive processes, which can be interpreted as reduced-form policy rules when it comes to exogenous policy variables such as government spending or taxes. Importantly, in spite of specification or sample differences, these papers all find significant countercyclical policy rules that prove essential to the models fit.

match, allowing for a very countercyclical policy will require a high degree of Edgeworth complementarity. Since, as we show, the GSM increases with the degree of Edgeworth complementarity, this will mechanically translate into a large GSM. Conversely, omitting countercyclical policy will imply a small degree of complementarity, thus yielding a downward-biased GSM.

To establish these results formally, we first work out a simple model with only limited dynamic features. The model is simple enough that we can come up with an analytical characterization of the bias that would arise from omitting the countercyclical component to government spending policy. We use this framework to identify configurations in which this bias would be likely. We show that omitting the endogenous component of the policy rule at the estimation stage would always yield a downward-biased estimate of the GSM, provided shocks to government expenditures are not the only perturbations affecting the economy. Because countercyclical policy and Edgeworth complementarity work in opposite directions in terms of generating a certain pattern of correlation between consumption and government expenditures, we can reinterpret this bias as a simultaneous equation bias. As a matter of fact, the simple model allows us to derive a formula for the bias that closely resembles those appearing in standard econometrics textbooks in a demand-supply framework. By analogy with this celebrated framework, an econometrician omitting the countercyclical spending rule risks recovering the policy rule parameter when trying to estimate the private response to public spending. In all likelihood, this will happen when shocks to government spending account for a small portion of fluctuations and/or the feedback effect in the policy rule is strong. We show in an appendix that other transmission mechanisms considered in the literature would yield the same results. Hence, our conslusion does not hinge on Edgeworth complementarity/substitutability but holds more generally.

In a second step, using post-war US data, we estimate a quantitative model version via maximum likelihood techniques. We first find that government spending policy is indeed countercyclical. As a consequence, the same sort of bias is present when the econometrician omits the countercyclical component of government policy. This, in turn, translates into significant differences in the estimated long- and short-run government spending multipliers. In our benchmark specification with Edgeworth complementarity and countercyclical policy, the implied long-run multiplier amounts to 1.31. Using the same model and imposing an exogenous policy rule, we obtain a multiplier of 0.97, significantly smaller than our benchmark value by 0.34 point. Such a difference is clearly not neutral if the model is used to assess recovery plans of the same size as those recently enacted in the US. To illustrate this more concretely, we feed the American Recovery and Reinvestment Act (ARRA) fiscal stimulus package into our model. We obtain that omitting the countercyclical policy rule at the estimation stage would lead an analyst to understate the cumulated output effect of this package by 15% of the package itself or, equivalently, around 1.4% of US GDP prior to the shock.[4] Clearly, these are not negligible figures. Interestingly,

[4] In this paper, we insist on the long-run GSM. The main reason for this is that this number does not directly depend on the

simulating our preferred model while imposing the counterfactual hypothesis that policy is exogenous would not produce very different dynamics. This illustrates that, while countercyclical policy does not seem to play a major role when *simulating* the model, it turns out to be an essential feature when *estimating* the model.

To complement these results, we consider several robustness analyses. First, we investigate the robustness of policy countercyclicality. To do so, we reestimate our DSGE model under a large battery of alternative specifications for the endogenous component of government spending. The bottom line is that our benchmark specification is supported by the data. Second, to address potential issues of policy rules instability, we reestimate the model over two subsamples. Although the estimated parameters show moderate signs of time-variability, our main conclusion still holds. Finally, we investigate whether our results still obtain in a Smets-Wouters type model. Although this model yields different multipliers than in our benchmark specification, we still find that omitting the endogenous component of government expenditures would severely downward-bias the estimated multiplier.[5]

Our estimations yield a GSM which exceeds unity and a near zero multiplier for private consumption. These findings are broadly in the range of values reported by Hall (2009) - typically in between 0.5 and 1.7 for output. This range of values derives from various methods. These include, for example, GSM estimates from DSGE models (e.g. Christiano, Eichenbaum and Rebelo, 2011, Monacelli, Perotti and Trigari, 2010, Zubairy, 2010), single regressions on government purchases (Barro and Redlick, 2009, Hall, 2009), and Structural VectorAutoregressions (SVARs) (e.g. Blanchard and Perotti, 2002, Caldara and Kamps, 2008, Fisher and Peters, 2010). Interestingly, the DSGE literature itself generates such a range of values for the GSM: the lower bound obtains in typical calibrated neoclassical setups; the upper bound is obtained in Christiano, Eichenbaum and Rebelo (2011). A key contribution of the latter is to show under which circumstances the multiplier can be much larger than one, typically when the economy has reached the zero lower bound on the nominal interest rate. Our paper adds to this literature by spotting possible estimation biases that naturally emerge from such models. The strength of these models lies precisely in their cross-equation restrictions which significantly contribute to highlight identification problems for the GSM.[6]

The rest of the paper is organized as follows. In section 2, we expound the simple model and illustrate the trade-off between Edgeworth complementarity and countercyclical policy in terms of matching the

persistence of government policy shocks. As shown by Aiyagari, Christiano and Eichenbaum (1992), short-run multipliers can prove very sensitive to the persistence of these shocks, which would complicate the comparison between different model versions. Focussing on the long-run GSM allows us to sidestep this problem. However, all our results hold in an assessment of shorter-term multipliers.

[5]We also report several simulation exercises in appendix and obtain quantitative results that echo our analytical formula.

[6]Our analytical results also point out to potential sources of bias in SVARs, as emphasized by Caldara (2011).

observed correlation between output and government expenditures. We then characterize the bias that would result from omitting countercyclical policy. Section 3 develops a quantitative version of this model that we take to post-war US data. We then explore the quantitative implications of policy rule misspecification. In section 4, we investigate the robustness of our results. The last section briefly concludes.

2 A Simple Illustrative Example

In this section, we work out an equilibrium model simple enough to obtain closed-form formulas illustrating how short- and long-run government spending multipliers are biased when the econometrician omits the endogenous component of public policy. We focus on Edgeworth complementarity/substitutability between private consumption and public spending as the transmision mechanism of government expenditures. As claimed before, the literature has considered other mechanisms such as non-separable utility, externalites on preferences and technology, or deep habits. Whatever the mechanism considered, the log-linear equilibrium output takes the same form as that obtained below.[7]

2.1 The Model

Consider a discrete time economy populated with a large number of infinitely-lived, identical agents. The representative household seeks to maximize

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left\{ \log(c_{t+i} + \alpha_g g_{t+i}) - \frac{\eta}{1+\nu} n_{t+i}^{1+\nu} \right\} \tag{1}$$

subject to the sequence of budget constraints ($t \geq 0$)

$$c_t \leq w_t n_t - T_t, \tag{2}$$

where $\mathrm{E}_t\{\cdot\}$ is the expectation operator, conditioned on information available as of time t, $\beta \in (0,1)$ is the subjective discount factor, c_t is private consumption, g_t denotes public expenditures, n_t is the labor supply, w_t is the real wage rate, and T_t denotes lump-sum taxes. The Frisch elasticity of labor supply is $1/\nu$ and $\eta > 0$ is a scale parameter.

The parameter α_g, in turn, accounts for the complementarity/substitutability between private consumption c_t and public spending g_t.[8] If $\alpha_g \geq 0$, government spending substitutes for private consumption,

[7] See Appendix A for a review of alternative transmission mechanisms of government spending.

[8] Here we use the specification adopted by Christiano and Eichenbaum (1992), Finn (1998), McGrattan (1994), among others. Alternatively, CES specifications of utility have been considered (see Bouakez and Rebei, 2007, McGrattan, Rogerson and Wright, 1997). These yield the exact same log-linearized equilibrium conditions as our specification.

with perfect substitution if $\alpha_g = 1$, as in Christiano and Eichenbaum (1992). In this case, a permanent increase in government spending has no effect on output and hours but reduces private consumption, through a perfect crowding-out effect. In the special case $\alpha_g = 0$, we recover the standard business cycle model, with government spending operating through a negative income effect on labor supply (see Aiyagari, Christiano and Eichenbaum, 1992, Baxter and King, 1993). When the parameter $\alpha_g < 0$, government spending complements private consumption. Then, it can be the case (depending on the labor supply elasticity) that private consumption will react positively to an unexpected increase in government spending.

The representative firm produces a homogeneous final good y_t using labor as the sole input, according to the constant returns-to-scale technology

$$y_t = \mathrm{e}^{z_t} n_t.$$

Here, z_t is a shock to total factor productivity, assumed to be iid with $z_t \sim N(0, \sigma_z^2)$. Profit maximization implies that the marginal productivity of labor equals the real wage, i.e. $w_t = \mathrm{e}^{z_t}$.

Government purchases are entirely financed by taxes,

$$T_t = g_t.$$

As in the recent literature emphasizing the relevance of stabilizing government spending rules (see, among others, Cúrdia and Reis, 2010, Jones, 2002, Leeper, Plante and Traum, 2010, McGrattan, 1994) we specify a feedback rule of the following form

$$g_t = \bar{g} \left(\frac{y_t}{y_{t-1}} \right)^{-\varphi_g} \mathrm{e}^{u_t} \tag{3}$$

where $\bar{g}$ is a scale factor that pins down the deterministic steady-state level of government expenditures and φ_g governs the responsiveness of g_t to output growth. The random term u_t represents the discretionary part of policy and is assumed to be iid with $u_t \sim N(0, \sigma_u^2)$. A simpler rule would have government spending react to current output only. A problem with such a specification within our simplified model is that it would compromise identification of the policy parameter. Anticipating on the next section, we also notice that the dynamic rule (3) is favored by the data when we estimate a quantitative model version.

Finally, the market clearing condition on the goods market writes

$$y_t = c_t + g_t.$$

Combining the household's first order condition on labor, the profit maximization condition, and the resource constraint, one finally arrives at the equilibrium condition

$$\eta y_t^{\nu} = \frac{\mathrm{e}^{(1+\nu)z_t}}{y_t - (1-\alpha_g) g_t}. \tag{4}$$

Conditions (3) and (4) together constitute the equilibrium system governing the dynamics of the above economy. To ensure positiveness of the marginal utility of consumption, we henceforth impose the restriction $\alpha_g > (s_g - 1)/s_g$, where $s_g \equiv \bar{g}/\bar{y} \in [0, 1)$ is the steady-state public spending-output ratio.

In this economy, the long-run GSM is defined as follows.

Definition 1. *The long-run government spending multiplier, denoted by $\Delta y/\Delta g$, is the increase in steady-state output $\bar{y}$ consecutive to an increase in steady-state government spending expenditures $\bar{g}$, i.e. formally*

$$\frac{\Delta y}{\Delta g} \equiv \frac{\mathrm{d}\bar{y}}{\mathrm{d}\bar{g}}.$$

From this definition and the structure of the above model economy, the following proposition states key properties of the long-run GSM.

Proposition 1. *Under the preceding assumptions:*

1. *The long-run government spending multiplier $\Delta y/\Delta g$ is*
$$\frac{\Delta y}{\Delta g} = \frac{1 - \alpha_g}{1 + \nu[1 - s_g(1 - \alpha_g)]}.$$
2. *The multiplier is a decreasing function of α_g.*

Proof. See Appendix B. ■

This proposition establishes that the long-run GSM depends on the share of government spending in output (s_g), on the inverse Frisch elasticity of labor (ν), and on the parameter governing the degree of Edgeworth complementarity between private consumption and government expenditures (α_g). Importantly, $\Delta y/\Delta g$ does not depend directly on the degree of countercyclicality of the government spending rule (φ_g). The main thesis of this paper, though, is that φ_g can contaminate the long-run GSM indirectly.

To gain intuition as to how this can happen, we start by loglinearizing the system (3)-(4) in the neighborhood of the deterministic steady state. This yields

$$\hat{y}_t = \alpha \hat{g}_t + \zeta z_t \tag{5}$$

$$\hat{g}_t = -\varphi_g(\hat{y}_t - \hat{y}_{t-1}) + u_t \tag{6}$$

where a letter with a hat denotes the logdeviation (with respect to steady-state value) of the associated variable and the composite parameters α and ζ are defined as

$$\alpha \equiv \frac{s_g(1 - \alpha_g)}{1 + \nu[1 - s_g(1 - \alpha_g)]},$$
$$\zeta \equiv \frac{(1 + \nu)(1 - s_g(1 - \alpha_g))}{1 + \nu[1 - s_g(1 - \alpha_g)]}.$$

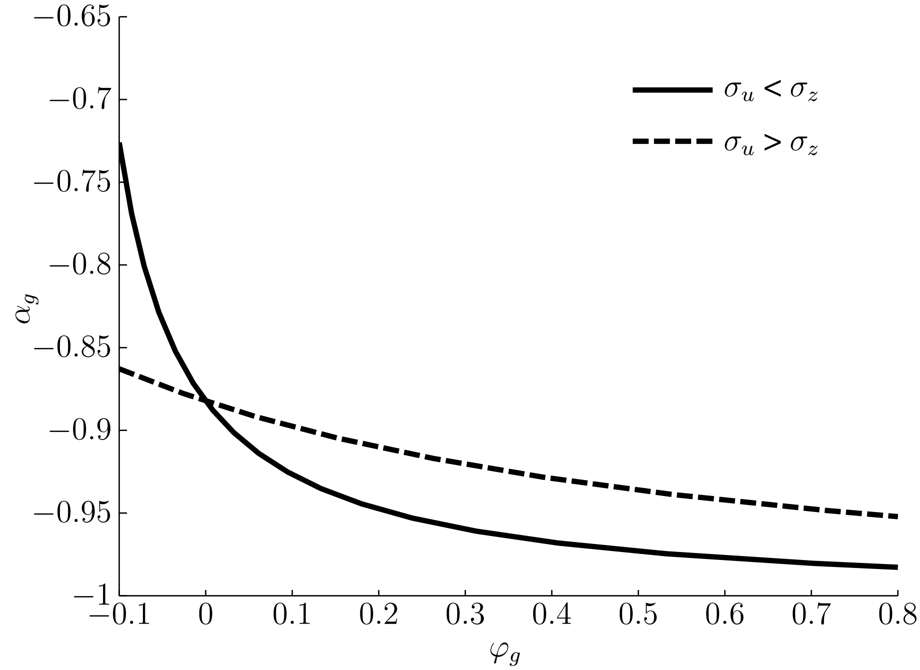

Figure 1: Iso-Correlation loci in the (φ_g, α_g) plane

Notes: The iso-correlation loci are computed assuming, for illustrative purposes, $\nu = 5$, $s_g = 0.2$, and $\mathrm{Corr}(\hat{y}, \hat{g}) = 0.3$. In the $\sigma_u < \sigma_z$ case, we impose $\sigma_u/\sigma_z = 0.5$. In the $\sigma_u > \sigma_z$ case, we impose $\sigma_u/\sigma_z = 2$.

In the remainder, to simplify the algebra, we drop the coefficient ζ from the dynamic system. This can always be done by rescaling appropriately the standard error of z_t.

For ν and s_g set at given values, the value of α summarizes the complementarity/substitutability between private and public consumption. This composite parameter and the long-run government spending multiplier are tightly linked since

$$\frac{\Delta y}{\Delta g} = \frac{\alpha}{s_g}$$

The system (5)-(6) makes clear how the degree of countercyclicality of the government spending rule and the degree of Edgeworth complementarity between private consumption and government spending work in opposite directions in terms of generating a positive correlation between $\hat{y}$ and $\hat{g}$. Intuitively, Edgeworth complementarity between private consumption and government spending (i.e. $\alpha_g < 0$) tends to increase the correlation between y and g, since under such a configuration an increase in government expenditures would induce people to consume more; at the same time, a countercyclical policy rule reduces this correlation.

This yields a trade-off: given an observed correlation between output and government spending, a highly countercyclical policy must be compensated by a high degree of Edgeworth complementarity. Conversely, if policy is exogenous, a lower degree of Edgeworth complementarity will suffice to match the observed pattern of correlation between output and government expenditures. This is illustrated in figure 1.

This figure shows two iso-correlation loci in the (φ_g, α_g) plane, depending on the relative sizes of the structural disturbances. Each point in these loci gives a particular (φ_g, α_g) combination resulting in the same correlation between output and government spending. When $\sigma_z > \sigma_u$, the iso-correlation locus is decreasing with φ_g, with a steep slope. This means that, as the degree of countercyclicality on policy increases, it takes more and more complementarity to match the observed correlation. When $\sigma_u > \sigma_z$, the iso-correlation curve is much flatter but the trade-off still exists.

This trade-off paves the way for a potential bias in the estimated degree of Edgeworth complementarity (and, by virtue of proposition 1, in the estimated multiplier). Suppose that an econometrician seeks to estimate α but uses a misspecified model in which φ_g is set to zero while actually $\varphi_g > 0$. The above reasoning suggests that this would result in a downward-biased estimate of α, immediately translating into a downward-biased estimated multiplier. The next section formally establishes this.

2.2 The Effect of Omitting Endogenous Policy

Direct calculations yield the model's reduced-form

$$\hat{y}_t = \frac{\alpha\varphi_g}{1+\alpha\varphi_g}\hat{y}_{t-1} + \frac{\alpha}{1+\alpha\varphi_g}u_t + \frac{1}{1+\alpha\varphi_g}z_t \tag{7}$$

$$\hat{g}_t = \frac{\varphi_g}{1+\alpha\varphi_g}\hat{y}_{t-1} + \frac{1}{1+\alpha\varphi_g}u_t - \frac{\varphi_g}{1+\alpha\varphi_g}z_t \tag{8}$$

From this reduced form, the structural parameters $(\alpha, \varphi_g, \sigma_u, \sigma_z)$ can be recovered using the $plim$ of the maximum likelihood estimation or an instrumental variable technique (with a relevant choice of instrumental variables). An easy way to obtain a consistent estimator of α relies on indirect estimation using the following representation of the reduced form

$$\hat{y}_t = \pi_1\hat{y}_{t-1} + \epsilon_{1,t} \tag{9}$$

$$\hat{g}_t = \pi_2\hat{y}_{t-1} + \epsilon_{2,t} \tag{10}$$

The $plim$ estimators of π_1 and π_2 are given by

$$\hat{\pi}_1 = \frac{\mathrm{E}\{\hat{y}_t\hat{y}_{t-1}\}}{\mathrm{E}\{\hat{y}_t^2\}} \quad \text{and} \quad \hat{\pi}_2 = \frac{\mathrm{E}\{\hat{g}_t\hat{y}_{t-1}\}}{\mathrm{E}\{\hat{y}_t^2\}}$$

from which we deduce

$$\hat{\alpha} = \frac{\hat{\pi}_1}{\hat{\pi}_2} = \frac{\mathrm{E}\{\hat{y}_t\hat{y}_{t-1}\}}{\mathrm{E}\{\hat{g}_t\hat{y}_{t-1}\}}$$

From (7)-(8), we obtain:

$$\mathrm{E}\{\hat{y}_t\hat{y}_{t-1}\} = \frac{\alpha\varphi_g}{1+\alpha\varphi_g}\mathrm{E}\{\hat{y}_t^2\} \quad \text{and} \quad \mathrm{E}\{\hat{g}_t\hat{y}_{t-1}\} = \frac{\varphi_g}{1+\alpha\varphi_g}\mathrm{E}\{\hat{y}_t^2\}$$

The indirect estimator $\hat{\alpha}$ of α is thus consistent. Similarly, $\hat{\varphi}_g$ is also a consistent estimator of φ_g.

Now, imagine the econometrician ignores the feedback rule and seeks to estimate the parameter α from data $(\hat{y}_t, \hat{g}_t)$ generated by the model (7)-(8). The model considered by this econometrician is thus of the form

$$\hat{y}_t = \tilde{\alpha}\tilde{u}_t + \tilde{z}_t \tag{11}$$

$$\hat{g}_t = \tilde{u}_t. \tag{12}$$

By ignoring the parameter φ_g, the econometrician is implicitly estimating the government spending effects on output through a single-equation approach in a simultaneous-equation setup. As is well known from standard econometrics textbooks, she potentially faces a severe *simultaneous-equation* bias (see Greene, 1997, Hamilton, 1994).

To see this clearly, the ML estimator $\hat{\tilde{\alpha}}$ of $\tilde{\alpha}$ would be

$$\hat{\tilde{\alpha}} = \frac{\mathrm{E}\{\hat{y}_t\hat{g}_t\}}{\mathrm{E}\{\hat{g}_t^2\}}$$

which simply corresponds to the OLS estimator. While φ_g exerts no influence on the long-run multiplier (see proposition 1), the next proposition establishes that this parameter corrupts the estimated composite parameter $\tilde{\alpha}$ when policy is assumed to be exogenous.

Proposition 2. *Under the previous hypotheses*

1. *The* plim *of the Maximum Likelihood estimator* $\hat{\tilde{\alpha}}$ *of* $\tilde{\alpha}$ *is*

$$\hat{\tilde{\alpha}} = \frac{\alpha(1+\alpha\varphi_g)\sigma_u^2 - \varphi_g\sigma_z^2}{(1+\alpha\varphi_g)\sigma_u^2 + 2\varphi_g^2\sigma_z^2}. \tag{13}$$

2. *Whenever* $\sigma_z > 0$ *and* $\varphi_g > 0$, $\hat{\tilde{\alpha}}$ *is downward-biased.*

3. $\forall\sigma_z > 0$ *and* $\forall\sigma_u > 0$, *we have*

   (a) *If* $\alpha \geq \sigma_z/\sigma_u$, *then*

$$\frac{\partial}{\partial\varphi_g}(\hat{\tilde{\alpha}} - \alpha) < 0, \quad \forall\varphi_g \geq 0.$$

(b) If $\alpha < \sigma_z/\sigma_u$, $\exists\ \bar{\varphi}_g(\alpha,\sigma_z,\sigma_u) > 0$ such that

$$\begin{aligned}
\frac{\partial}{\partial \varphi_g}(\hat{\tilde{\alpha}} - \alpha) &\leq 0, \quad \forall \varphi_g \in [0, \bar{\varphi}_g(\alpha,\sigma_u,\sigma_z)], \\
\frac{\partial}{\partial \varphi_g}(\hat{\tilde{\alpha}} - \alpha) &> 0, \quad \forall \varphi_g > \bar{\varphi}_g(\alpha,\sigma_u,\sigma_z).
\end{aligned}$$

Proof. See Appendix C. ■

Part 1 of proposition 2 establishes that the estimated value of $\hat{\tilde{\alpha}}$ is corrupted, in a non-linear way, by φ_g, σ_u and σ_z. The OLS regression does not pin down the effects of $\hat{g}$ on $\hat{y}$ but an average of private behavior and public policy, with weights that depend on the relative size of the shocks' variances. Notice that this is more or less the formula displayed in standard econometrics textbooks in a demand-supply setup (e.g., see Hamilton, 1994, chap. 9).

Part 2 of proposition 2 indicates under which circumstances omitting the endogenous component of government spending policy would result in a downward-biased estimate of α. Notice that under such circumstances, the long-run GSM is systematically downward-biased, by virtue of Proposition 1. Conversely, the only circumstances in which the bias vanishes are either $\varphi_g = 0$ or $\sigma_z = 0$. When $\varphi_g = 0$, the bias is obviously zero since, in this case, the model is well specified. If $\sigma_z = 0$, the bulk of fluctuations in $\hat{y}$ are accounted for by government spending shocks. In this case, the endogeneity bias vanishes. Endogenous public spending is positively related to the shock u and the (inverse) government policy shifts along the output equation.

When $\sigma_z > 0$ and $\sigma_u > 0$, the bias may increase or decrease with φ_g, depending on α, σ_z, and σ_u, as stated in part 3 of Proposition 2. If $\alpha \geq \sigma_z/\sigma_u$, the bias increases. If $\alpha < \sigma_z/\sigma_u$, the bias increases with φ_g, up to a threshold value above which it decreases. However, the bias never reverts back to zero since $\lim_{\varphi_g \to \infty} \hat{\tilde{\alpha}} = 0 < \alpha$.

In the special case $\sigma_z > 0$ and $\sigma_u \to 0$, government spending shocks do not contribute much to the variance of $\hat{y}$. In this case, we obtain $\hat{\tilde{\alpha}} \to -1/(2\varphi_g)$. Thus, omitting the endogeneity of government spending would lead us to estimate a negative value of $\tilde{\alpha}$. This is because endogenous public spending is negatively related to the shock z which shifts aggregate output. The covariance between $\hat{y}_t$ and $\hat{g}_t$ is negative and thus the estimated effect of public spending on output is negative. We are in the case when the output equation moves along the policy rule equation (which is truly downward slopping). In this case, the econometrician almost recovers the reverse government policy rule.

So far, we have focussed on bias for the long-run GSM. However, omitting the endogeneity of the policy rule also has critical consequences on the estimated short-run responses of output to a government spending shock. First, the estimated effect of government expenditures vanishes immediately one period after the shock. This is not the case in the model (7)-(8) where the effects of the government policy shock are long lasting, provided $\varphi_g > 0$. This is rather trivial and we do not make much of it. More importantly, the response of $\hat{y}_t$ on impact is biased provided $\varphi_g \neq 0$. However, the sign of the bias is less trivial than for the long–run GSM. Indeed, the parameter φ_g affects the response on impact of $\hat{y}_t$ in the model (7)-(8). This response is given by

$$\frac{\alpha}{1+\alpha\varphi_g},$$

and thus it is a decreasing function of φ_g for $\alpha > 0$. At the same time, an econometrician omitting the endogeneity of the government policy would obtain a response on impact equal to $\hat{\hat{\alpha}}$, which may be a decreasing function of φ_g, too (See Proposition 2 and the related discussion). It can be shown that if $\varphi_g > \bar{\varphi}_g$ and $\alpha < \sqrt{3}(\sigma_z/\sigma_u)$, with

$$\bar{\varphi}_g = \frac{\alpha^2(\sigma_u^2/\sigma_z^2) - 1}{\alpha\left(3 - \alpha^2(\sigma_u^2/\sigma_z^2)\right)},$$

then the estimated response under a misspecified policy rule will under-estimate the true response. The sign of $\bar{\varphi}_g$ depends on the value of α and on the relative variance of the two shocks σ_z/σ_u. When $\alpha > (\sigma_z/\sigma_u)$, $\bar{\varphi}_g$ is positive and the government policy must be sufficiently counter–cyclical to get a downward bias. This configuration corresponds to the situation where the government policy shock is the main driver of $\hat{y}$. Conversely, when $\alpha < (\sigma_z/\sigma_u)$, $\bar{\varphi}_g$ is negative and the downward bias under the misspecified policy rule will appear more likely, i.e. for a wider range of values for φ_g. This will happen when the shock to government policy accounts for a small portion of the the variance of $\hat{y}$. Notice that a counter-cyclical policy is a sufficient condition to under-estimate the true impact response of output.

To sum up, we have shown analytically in a tractable model that omitting the endogenous component of government spending can result in a downward-biased estimate of the long- or short-run government spending multiplier. In this simple setup, the downward bias is a mix of a simultaneous-equation bias and an omitted-variable bias. It is the result of two conflicting economic forces, one that magnifies the correlation between output and government spending (Edgeworth complementarity) and the other that reduces it (countercyclical government spending rule).

In the following section, we consider a quantitative DSGE model which we estimate on US data via maximum likelihood techniques. While the model is too complicated to get such a sharp bias characterization, it proves a useful tool to investigate whether omitting the endogenous component of government spending actually results in a quantitatively significant bias in estimated government spending multipliers.

3 A Quantitative Model

We now work out a quantitative extension of the previous model that we formally take to the data. We extend the previous setup by allowing for capital accumulation, habit formation in leisure decisions, and multiple shocks. While the model is arguably very stylized, it turns out to deliver a good fit.[9]

3.1 The Model

The representative household's intertemporal expected utility is

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left\{ \mathrm{e}^{a_{t+i}} \log(c_{t+i} + \alpha_g g_{t+i}) - \mathrm{e}^{b_{t+i}} \frac{\eta}{1+\nu} \left(\frac{n_{t+i}}{n_{t+i-1}^{\phi}} \right)^{1+\nu} \right\} \tag{14}$$

where $\mathrm{E}_t\{\cdot\}$ denotes the expectation operator conditional on the information set at period t and $\beta \in (0,1)$ is the subjective discount factor. As in the previous section, the parameter α_g governs the substitutability/complementarity between private consumption and public expenditures.

The parameter ϕ governs the habit persistence in labor supply and $\eta \geq 0$ is a scale parameter. When the parameter $\phi \neq 0$, labor supply decisions are subject to time non-separabilities. If $\phi < 0$, labor supply displays inter-temporal substitutability, whereas $\phi > 0$ implies inter-temporal complementarity. Eichenbaum, Hansen and Singleton (1988) showed that a specification with intertemporal complementarities is favored by the data. More recently, this specification has proven to be empirically relevant, as it translates habit persistence in leisure choices into aggregate output persistence (see Bouakez and Kano, 2006, Dupaigne, Fève and Matheron, 2007, Wen, 1998). While other specifications that allow to capture the persistence in hours have been considered in the literature (e.g. adjustment costs on labor input, as in Chang, Doh and Schorfheide 2007, or learning-by-doing, as in Chang, Gomes and Schorfheide 2002), it turns out that the implied reduced-form are almost identical to that resulting from our specification.

Utility derived from consumption is altered by a preference shock a_t, which obeys

$$a_t = \rho_a a_{t-1} + \sigma_a \zeta_{a,t}$$

where $|\rho_a| < 1$, $\sigma_a > 0$ and $\zeta_{a,t}$ is iid with $\zeta_{a,t} \sim N(0,1)$. Labor disutility is subject to a preference shock b_t, which obeys

$$b_t = \rho_b b_{t-1} + \sigma_b \zeta_{b,t}$$

[9] In Appendices D and E, we modify the benchmark specification allowing for (i) habits in consumption and dynamic adjustment costs and (ii) news shocks in the policy rule. Our results are robust to these perturbations.

where $|\rho_b| < 1$, $\sigma_b > 0$ and $\zeta_{b,t}$ is iid with $\zeta_{b,t} \sim N(0,1)$. As noted by Galí (2005), this shock accounts for a sizeable portion of aggregate fluctuations. Moreover, it allows us to capture various distortions on the labor market, labeled *labor wedge* in Chari, Kehoe and McGrattan (2007).

The representative household supplies hours n_t and capital k_t to firms, and pays a lump-sum tax T_t to the government. Accordingly, the representative household's budget constraint in every period t is

$$c_t + x_t \leq w_t n_t + r_t k_t - T_t \tag{15}$$

where w_t is the real wage, r_t is the rental rate of capital, and x_t denotes investment. The capital stock evolves according to

$$k_{t+1} = (1-\delta)k_t + x_t \tag{16}$$

where $\delta \in (0,1)$ is the constant depreciation rate. The representative household thus maximizes (14) subject to the sequence of constraints (15) and (16), $t \geq 0$.

The representative firm produces a homogeneous final good y_t through the constant returns-to-scale technology

$$y_t = k_t^{\theta} \left(\mathrm{e}^{z_t} n_t\right)^{1-\theta}$$

where k_t and n_t denote the inputs of capital and labor, respectively, $\theta \in (0,1)$ is the elasticity of output with respect to capital, and z_t is a shock to total factor productivity, which follows a random walk process with drift of the form

$$z_t = \log(\gamma_z) + z_{t-1} + \sigma_z \zeta_{z,t}$$

where $\sigma_z > 0$ and $\zeta_{z,t}$ is iid with $\zeta_{z,t} \sim N(0,1)$. The constant term $\gamma_z > 1$ is the drift term and accounts for the deterministic component of the growth process. Profit maximization implies $r_t = \theta y_t / k_t$ and $w_t = (1-\theta) y_t / n_t$.

Government expenditures are entirely financed by taxes $T_t = g_t$. Notice that Ricardian equivalence holds in our setup, so that introducing government debt is unnecessary. The stationary component of government spending is given by

$$g_t \mathrm{e}^{-z_t} = \bar{g}^s \tilde{g}_t \mathrm{e}^{g_t^\star},$$

where $\bar{g}^s$ denotes the deterministic steady-state value of $g_t \mathrm{e}^{-z_t}$. The endogenous policy component $\tilde{g}_t$ obeys

$$\log(\tilde{g}_t) = -\varphi_g \left(\Delta \log(y_t) - \log(\gamma_z)\right),$$

and the stochastic (discretionary) component is assumed to follow an autoregressive process of the form:

$$g_t^\star = \rho_g g_{t-1}^\star + \sigma_g \zeta_{g,t}$$

where $|\rho_g| < 1$, $\sigma_g > 0$ and $\zeta_{g,t}$ is iid with $\zeta_{g,t} \sim N(0,1)$. Here, Δ stands for the first-difference operator. The parameter φ_g is the policy rule parameter linking the stationary component of government policy to demeaned output growth. If $\varphi_g < 0$, the policy rule contains a procyclical component that triggers an increase in government expenditures whenever output growth is above its average value. In constrast, if $\varphi_g > 0$, the policy rule features a countercyclical component. In both cases however, assessing the degree of pro- or counter-cyclicality of the overall level of government spending requires taking the stochastic trend in productivity into account. For example, assuming that $\varphi_g = 0$, the growth rate of government expenditures would still be positively correlated with total factor productivity growth.[10]

The homogeneous good can be used for private consumption c_t, government consumption g_t, and investment x_t. The market clearing condition on the goods market accordingly writes

$$y_t = c_t + x_t + g_t.$$

In the context of this model featuring a stochastic trend in productivity, we must modify Definition 1. To this end, we start by defining the detrended variables $y_t^s \equiv y_t \mathrm{e}^{-z_t}$ and $g_t^s \equiv g_t \mathrm{e}^{-z_t}$.

Definition 2. *The long-run government spending multiplier, denoted by $\Delta y/\Delta g$, is the increase in steady-state, detrended output $\bar{y}^s$ after an increase in steady-state, detrended government spending expenditures $\bar{g}^s$, i.e. formally*

$$\frac{\Delta y}{\Delta g} \equiv \frac{\mathrm{d}\bar{y}^s}{\mathrm{d}\bar{g}^s}.$$

Using this definition, we can characterize how α_g and the long-run GSM are linked together. This relation is stated in the following proposition, which generalizes Proposition 1 to a setup with investment.

Proposition 3. *Under the preceding assumptions:*

1. *The long-run government spending multiplier $\Delta y/\Delta g$ is*

$$\frac{\Delta y}{\Delta g} = \frac{1-\alpha_g}{1-s_x+\mu[1-s_x-s_g(1-\alpha_g)]},$$

where

$$\mu \equiv (1+\nu)(1-\phi)-1, \quad s_x \equiv \frac{(\gamma_z-1+\delta)\theta\beta}{\gamma_z-\beta(1-\delta)}.$$

2. *The multiplier $\Delta y/\Delta g$ decreases with α_g.*

[10] In section 4, we explore the robustness of our results to alternative specifications of the feedback rule.

Proof. See Appendix F. ■

Notice that when α_g and ϕ are set to zero, the above formula collapses to those reported in Baxter and King (1993) and Aiyagari, Christiano and Eichenbaum (1992). As in the previous section, φ_g clearly does not show up in this formula. It remains to be seen whether omitting φ_g at the estimation stage can compromise the inference on α_g.

3.2 Data and Estimation

Before, taking the model to the data, we first induce stationarity by getting rid of the stochastic trend component z_t and we log-linearize the resulting system in the neighborhood of the deterministic steady state.[11] Then, let $\hat{s}_t$ denote the vector collecting the loglinear model variables. The log-linear solution is of the form

$$\hat{s}_t = \mathbf{F}(\psi)\hat{s}_{t-1} + \mathbf{G}(\psi)\begin{pmatrix} \zeta_{z,t} \\ \zeta_{a,t} \\ \zeta_{b,t} \\ \zeta_{g,t} \end{pmatrix}, \tag{17}$$

where ψ is the vector of model's parameters. The system matrices $\mathbf{F}(\psi)$ and $\mathbf{G}(\psi)$ are complicated functions of the model's parameters.

We use as observable variables in estimation the logs of output, consumption, hours worked, and government expenditures. The measurement equation is

$$\begin{pmatrix} \Delta\log(y_t) \\ \log(n_t) \\ \Delta\log(c_t) \\ \Delta\log(g_t) \end{pmatrix} = \begin{pmatrix} \gamma_z - 1 \\ m_n(\psi) \\ \gamma_z - 1 \\ \gamma_z - 1 \end{pmatrix} + \mathbf{H}\hat{s}_t. \tag{18}$$

Here, $m_n(\psi)$ is a function that gives average log hours as a function of ψ and $\mathbf{H}$ is a selection matrix. For a given ψ, using equations (17) and (18), the log-likelihood is evaluated via standard Kalman filter techniques. The estimated parameters are then obtained by maximizing the log-likelihood.[12]

The data used for estimation come from the Federal Reserve Bank of St. Louis' FRED II database and from the Bureau of Labor Statistics website. They consist of government consumption expenditures

[11] See Appendix F for further details on the procedure used to induce stationarity.

[12] We used different measurement equations, using the logged private consumption-output and logged government expenditures-output ratios instead of consumption growth and government expenditures growth. Estimation results were almost identical.

and gross investment (GCE), private investment and private consumption, all deflated by the implicit GDP deflator (GDPDEF). Private investment is defined as the sum of gross private domestic investment (GPDI) and personal consumption expenditures on durable goods (PCDG). Private consumption is measured as the sum of personal consumption expenditures on non-durable goods (PCND) and services (PCESV). Output is then defined as the sum of private investment, private consumption and government expenditures. Hours are borrowed from Francis and Ramey (2009). These hours data refer to the total economy and are adjusted for low-frequency movements due to changes in demographics, thus displaying less low-frequency behavior than unadjusted data. All the series are converted to per-capita terms by dividing them by the civilian population, age 16 and over (CNP16OV). All the series are seasonally adjusted except for population. Our sample runs from 1960:1 to 2007:4.[13]

The vector of parameters ψ is split in two subvectors ψ_1 and ψ_2. The first one, $\psi_1 = (\beta, \delta, \nu, \theta, s_g)$, contains parameters calibrated prior to estimation. Typically, these are parameters difficult to estimate in our framework. The subjective discount factor, β, is set to 0.9951, yielding a real annual interest rate of 3.75%. The depreciation rate, δ, is set to 0.0153, to match the average investment-output ratio. The parameter ν is set to 4 so that the long-run labor supply elasticity $\mu \equiv (1+\nu)(1-\phi) - 1$ is close to 2 in the benchmark model, in accordance with previous studies (see Smets and Wouters, 2007). Finally we set $\theta = 0.30$, so that the labor income share in output is 70%, and $s_g = 0.2$, so as to reproduce the average ratio of government expenditures to output in our sample.

The remaining parameters, contained in $\psi_2 = (\phi, \alpha_g, \gamma_z, \varphi_g, \rho_g, \rho_a, \rho_b, \sigma_z, \sigma_g, \sigma_a, \sigma_b)$, are estimated.[14] Estimation results are reported in table 1. We consider four model restrictions, according to whether α_g and φ_g are constrained. The table reports the log-likelihood $\mathscr{L}$ for each model specification, which we use naturally as our selection criterion. The restrictions are summarized below

- Model (1): $\alpha_g = 0$, $\varphi_g = 0$, so that g has no direct effect on the marginal utility of private consumption and is exogenous
- Model (2): $\alpha_g = 0$, $\varphi_g \neq 0$, so that g has no direct effect on the marginal utility of private consumption and is endogenous
- Model (3): $\alpha_g \neq 0$, $\varphi_g = 0$, so that g has a direct effect on the marginal utility of private consumption and is exogenous
- Model (4): $\alpha_g \neq 0$, $\varphi_g \neq 0$, so that g has a direct effect on the marginal utility of private consumption and is endogenous

[13] In section 4, we consider the robustness of our results to alternative estimation samples.

[14] The parameter vector ψ_2 also contains $\bar{n}$, which is the average level of log hours. In our setup, estimating $\bar{n}$ is equivalent to estimating η. This parameter does not play any role in the log-linearized dynamics and is not reported.

Overall, the model specifications yield precisely estimated parameters.[15] Notice that, except for ρ_g and σ_g, most parameters are pretty invariant to the restrictions imposed on α_g or φ_g.

The log-likelihood comparison suggests the following comments. First, comparing specifications (1) with (3) or (2) with (4), one can clearly see that the restrictions $\alpha_g = 0$ is strongly rejected by the data. The P-values of the associated likelihood ratio tests are almost zero. In specifications (3) and (4), the parameter α_g is negative and significantly different from zero, consistent with the above likelihood ratio tests. This result is not an artifact of allowing for an endogenous government policy. This suggests that private and public consumption are complements. This result echoes findings by Karras (1994). Bouakez and Rebei (2007) and Mazraani (2010) also reach similar conclusions with a specification of preferences in which private consumption and government expenditures interact through a CES subutility function. We also used such a model version. As discussed above, since both representations yield the same loglinear first-order conditions, this model version yields the exact same estimation results. However, the CES specification raises an identification problem that our specification eschews. It turns out that the government spending weight in utility is not separately identified in the CES case.

Second, the restriction $\varphi_g = 0$ is clearly rejected by the data. To see this, compare specifications (1) and (2) or (3) and (4). In both cases, the associated P-values are almost zero. These results strongly support the view that government policy comprises an endogenous component. To sum up, specification (4) is our preferred model. This specification thus features (i) a positive effect of government spending on the marginal utility of private consumption and (ii) a countercyclical feedback effect in government spending.[16]

Notice that the estimated value for φ_g is large, suggesting a strong countercyclical component to policy. However, this should not necessarily reflect an extremely countercyclical policy. To see this, recall that φ_g governs the cyclicality of government expenditures in deviation from the stochastic TFP trend. To the extent that innovations to TFP account for a significant portion of fluctuations, the overall level of government expenditures need not be overly countercyclical, even when φ_g is large.

Even though all second order moments are given the same weight in the likelihood, comparing unconditional moments from the models to their empirical counterparts is useful to get some intuition why model (4) is preferred. Results are reported in table 2. We consider moments documenting the volatility, persistence, and co-movement of key variables.

[15] We numerically checked the estimation convergence by shocking initial conditions on parameter values in the likelihood maximization step. Upon convergence, we also plotted slices of the likelihood function around each estimated parameter value to check local identification.

[16] In Appendix G, we use data simulated from specification (3) and (4) to establish that the negative empirical link found between α_g and φ_g is not idiosyncratic to our sample.

Table 1. Estimation Results

	Specification (1) $\alpha_g = 0, \varphi_g = 0$	Specification (2) $\alpha_g = 0, \varphi_g \neq 0$	Specification (3) $\alpha_g \neq 0, \varphi_g = 0$	Specification (4) $\alpha_g \neq 0, \varphi_g \neq 0$
α_g	–	–	-0.6340	-0.9452
			(0.1357)	(0.1410)
φ_g	–	0.5723	–	0.6117
		(0.0803)		(0.0660)
ϕ	0.4004	0.4138	0.3766	0.4110
	(0.0764)	(0.0730)	(0.0714)	(0.0638)
γ_z	1.0043	1.0043	1.0043	1.0044
	(0.0007)	(0.0007)	(0.0007)	(0.0007)
ρ_g	0.9469	0.9592	0.9535	0.9756
	(0.0087)	(0.0078)	(0.0079)	(0.0055)
ρ_a	0.9762	0.9775	0.9795	0.9834
	(0.0049)	(0.0047)	(0.0036)	(0.0031)
ρ_b	0.8346	0.8345	0.8366	0.8399
	(0.0424)	(0.0411)	(0.0366)	(0.0330)
σ_z	0.0107	0.0107	0.0107	0.0107
	(0.0005)	(0.0005)	(0.0005)	(0.0005)
σ_g	0.0130	0.0120	0.0129	0.0119
	(0.0007)	(0.0006)	(0.0007)	(0.0006)
σ_a	0.0103	0.0102	0.0123	0.0133
	(0.0009)	(0.0009)	(0.0011)	(0.0013)
σ_b	0.0270	0.0266	0.0273	0.0266
	(0.0014)	(0.0014)	(0.0015)	(0.0014)
$\mathscr{L}$	2655.3227	2679.4375	2665.9338	2701.3173

Notes: Sample period: 1960:1-2007:4. Standard errors in parentheses.

All the model versions perform equally well in terms of fitting standard errors of key aggregate variables. More interestingly, we see that whether or not $\alpha_g = 0$ is imposed, the correlation between changes in private consumption and changes in government spending is smaller when $\varphi_g > 0$. To see this, compare the results for specifications (1) against (2) or (3) against (4). This illustrates our claim that in a standard neoclassical growth model, allowing for a countercyclical government spending policy works toward reducing the correlation between consumption growth and government spending shocks. Similarly, comparing the same specifications, we see that relaxing the constraint $\varphi_g = 0$ decreases the correlation between output growth and government expenditures growth. Conversely, whether or not $\varphi_g = 0$,

Table 2. Moments Comparison

	Data	Specification (1) $\alpha_g = 0, \varphi_g = 0$	Specification (2) $\alpha_g = 0, \varphi_g \neq 0$	Specification (3) $\alpha_g \neq 0, \varphi_g = 0$	Specification (4) $\alpha_g \neq 0, \varphi_g \neq 0$
$\sigma(\Delta y)$	0.0093	0.0093	0.0093	0.0095	0.0095
$\sigma(\Delta c)$	0.0050	0.0071	0.0071	0.0074	0.0065
$\sigma(\Delta x)$	0.0327	0.0255	0.0281	0.0255	0.0308
$\sigma(\Delta g)$	0.0110	0.0170	0.0145	0.0168	0.0140
$\sigma(\Delta n)$	0.0066	0.0069	0.0068	0.0070	0.0070
$\rho(\Delta y)$	0.3200	0.1165	0.1226	0.1116	0.1304
$\rho(\Delta c)$	0.2482	0.0217	0.0232	0.0176	0.1980
$\rho(\Delta x)$	0.2057	0.1599	0.0413	0.1425	-0.0681
$\rho(\Delta g)$	0.0948	-0.0161	0.1076	-0.0139	0.1337
$\rho(\Delta n)$	0.3886	0.3299	0.3437	0.3100	0.3511
$\text{corr}(\Delta y, \Delta c)$	0.5115	0.6336	0.6461	0.7282	0.6541
$\text{corr}(\Delta y, \Delta x)$	0.9043	0.8025	0.8567	0.7315	0.8246
$\text{corr}(\Delta y, \Delta g)$	0.2913	0.5616	0.3354	0.5873	0.3697
$\text{corr}(\Delta c, \Delta g)$	0.2368	0.1999	0.0204	0.4976	0.4001
$\text{corr}(\Delta x, \Delta g)$	-0.0288	0.1719	0.0209	0.0213	-0.1001

Notes: Sample period: 1960:1-2007:4. $\sigma(\cdot)$, $\rho(\cdot)$, and $\text{corr}(\cdot,\cdot)$ stand for standard deviation, first-order autocorrelation coefficient, and correlation, respectively. Δ is the first difference operator, y is output, c is consumption, x is investment, g is government expenditures, n is hours worked.

relaxing the constraint $\alpha_g = 0$ increases the correlation between consumption growth and government spending growth. To see this, compare the results for specifications (1) against (3) or (2) against (4). This once again illustrates how Edgeworth complementarity and countercyclical policy interact in our model. Finally, we also see that, irrespective of constraints imposed on α_g, the restriction $\varphi_g = 0$ deteriorates the model's ability to capture the persistence of changes in government expenditures.

We complement the above results by performing specification tests for the innovation of each variables used for estimation in equation (18), *i.e.* $\Delta\log(y_t)$ output growth, $\log(n_t)$ the log of hours, $\Delta\log(c_t)$ private consumption growth, and $\Delta\log(g_t)$ government consumption growth. The innovations are obtained as the difference between the observed variables and their predicted value at convergence of the estimation stage. The specification tests, reported in table 3, are conducted for the four model's specifications. The first column reports the Shapiro and Wilk (1965) test statistic. The null hypothesis being tested is that the innovation of the variables listed on the left is normally distributed. A small value of the test statistic indicates a rejection of the null, whereas a value close to unity favors the normality assumptions. On the right, we report the P-value (in %) of the test statistic. Except for consumption growth, normality is rejected in all cases. However, rejection is essentially driven by a few outliers. Given the parametric parsimony of the model, such a rejection is hard to interpret.

Table 3. Specification Tests

Specification	Innovation in	Normality: Shapiro-Wilk Statistic	Normality: P-value (in %)	Serial Correlation: Coefficient	Serial Correlation: Confidence Interval (at 95%)
(1)	$\Delta\log(y_t)$	0.9657	0.0531	0.1883	$[0.0482\ ,\ 0.3284]$
	$\log(n_t)$	0.9737	0.3513	0.0055	$[-0.1372\ ,\ 0.1482]$
	$\Delta\log(c_t)$	0.9847	7.2771	0.3146	$[0.1783\ ,\ 0.4508]$
	$\Delta\log(g_t)$	0.9686	0.1019	0.1685	$[0.0267\ ,\ 0.3103]$
(2)	$\Delta\log(y_t)$	0.9668	0.0673	0.1821	$[0.0419\ ,\ 0.3224]$
	$\log(n_t)$	0.9738	0.3577	-0.0170	$[-0.1597\ ,\ 0.1256]$
	$\Delta\log(c_t)$	0.9850	7.8055	0.3125	$[0.1762\ ,\ 0.4489]$
	$\Delta\log(g_t)$	0.9672	0.0741	0.0393	$[-0.1045\ ,\ 0.1831]$
(3)	$\Delta\log(y_t)$	0.9655	0.0510	0.1948	$[0.0549\ ,\ 0.3348]$
	$\log(n_t)$	0.9746	0.4386	0.0354	$[-0.1071\ ,\ 0.1778]$
	$\Delta\log(c_t)$	0.9860	10.7253	0.2851	$[0.1476\ ,\ 0.4225]$
	$\Delta\log(g_t)$	0.9699	0.1405	0.1504	$[0.0081\ ,\ 0.2926]$
(4)	$\Delta\log(y_t)$	0.9681	0.0917	0.1787	$[0.0384\ ,\ 0.3191]$
	$\log(n_t)$	0.9744	0.4246	-0.0223	$[-0.1647\ ,\ 0.1201]$
	$\Delta\log(c_t)$	0.9865	12.3860	0.0924	$[-0.0505\ ,\ 0.2352]$
	$\Delta\log(g_t)$	0.9693	0.1211	-0.0008	$[-0.1447\ ,\ 0.1431]$

Notes: Sample period: 1960:1-2007:4. $\Delta\log(y_t)$ denotes output growth, $\log(n_t)$ hours, $\Delta\log(c_t)$ private consumption growth, and $\Delta\log(g_t)$ government consumption growth. Specification (1): $\alpha_g = \varphi_g = 0$, Specification (2): $\alpha_g = 0$, $\varphi_g \neq 0$, Specification (3): $\alpha_g \neq 0$, $\varphi_g = 0$, Specification (4): $\alpha_g \neq 0$, $\varphi_g \neq 0$. Coefficients are obtained by projecting each innovation on its own lag.

More interestingly, table 3 also includes serial correlation tests. We report the least-squares coefficient obtained by projecting each innovation on its own lag. For each coefficient, we report the associated 95% confidence interval. We find that omitting the feedback rule deteriorate the results. Indeed, comparing specification (3) and (4) shows that consumption and government spending innovations display less serial correlation when the policy rule coefficient is not constrained to zero.

Having explored the empirical properties of model (4), we now use this version to investigate the quantitative effects of omitting the government feedback rule.

3.3 Quantitative Implications for the Multiplier

In this section, we assess the quantitative impact of omitting the countercyclical component of government expenditures policy. We proceed in two steps. First, guided by the analytical results obtained in the previous section, we investigate the consequences for the long-run GSM of misspecifiying the policy rule. Second, we illustrate that these effects are also present in an assessment of shorter-term multipliers.

3.3.1 Long-Run Government Spending Multiplier

Upon inspecting models (3) and (4), we see that imposing $\varphi_g = 0$ strongly affects the estimated value of α_g. When φ_g is freely estimated, we obtain $\alpha_g = -0.95$ whereas we get $\alpha_g = -0.63$ when we impose $\varphi_g = 0$. Importantly, these parameter estimates are significantly different from each other, according to a standard Wald test.

What does this imply for long-run government spending multiplier? To answer this question, we use the formula in Proposition 3 to estimate the long-run GSM. The estimated multipliers are reported in table 4, together with their standard errors. When α_g is restricted to zero, the estimated long-run multiplier is typically less than one, as obtains in standard models (see Aiyagari, Christiano and Eichenbaum, 1992, Baxter and King, 1993). Concretely, depending on the restrictions imposed on φ_g, we obtain values roughly comprised between 0.53 and 0.55. Importantly, comparing specifications (1) and (2) in table 4, one can clearly see that φ_g has almost no discernible effect on the long-run multiplier. The reason why is simple: the parameters β, δ, and ν are restricted prior to estimation. In addition, the estimation results show that ϕ is relatively insensitive to the different specifications (see table 1).

In our framework, omitting the feedback effect in the policy rule can impact on the long-run output multiplier only when the parameter α_g is freely estimated. This is the novel feature of our model, because the link between the policy feedback parameter (φ_g) and the degree of Edgeworth complementarity between public and private consumptions (α_g) could obviously not be studied in frameworks imposing $\alpha_g = 0$. The columns associated with specifications (3) and (4) in table 4 thus give new results. More precisely, in model (3), the output multiplier is 0.97 while it reaches 1.31 in model (4). These values are significantly different from each other at conventional levels. As explained above, the higher multiplier in (4) derives from a smaller α_g than in (3).[17]

[17] Similar results obtain for the multiplier on private investment. This is not surprising since this multiplier is strictly

Table 4. Estimated Multipliers

	Specification (1)	Specification (2)	Specification (3)	Specification (4)
	$\alpha_g = 0, \varphi_g = 0$	$\alpha_g = 0, \varphi_g \neq 0$	$\alpha_g \neq 0, \varphi_g = 0$	$\alpha_g \neq 0, \varphi_g \neq 0$
$\Delta y/\Delta g$	0.5319	0.5429	0.9738	1.3128
	(0.0606)	(0.0603)	(0.1410)	(0.1803)
$\Delta c/\Delta g$	-0.5955	-0.5872	-0.2596	-0.0020
	(0.0461)	(0.0458)	(0.1072)	(0.1370)
$\Delta x/\Delta g$	0.1274	0.1301	0.2333	0.3148
	(0.0146)	(0.0145)	(0.0339)	(0.0433)

Notes: Sample period: 1960:1-2007:4. Standard errors in parentheses.

To complement on these results, we consider the following exercise. We set the feedback parameters φ_g to values on a grid between $\varphi_g = 0$ and the estimated value obtained in specification (4), i.e. $\varphi_g = 0.6117$. For each value, all the remaining parameters in ψ_2 are re-estimated. The results are reported on figure 2.

The upper left panel reports the log-likelihood as a function of φ_g. The grey area corresponds to restrictions on φ_g that are not rejected at the 5% level according to a likelihood ratio test. This grey area is also reported in each of the other panels in figure 2. The upper right panel reports the estimated value of α_g as a function of φ_g. The bottom panels report the long-run multipliers on output and consumption. The figure makes clear that even loose restrictions on φ_g (i.e. restrictions not too far from the estimated value) are easily rejected and rapidly translate into higher α_g and much lower multipliers. Importantly, the continuous and decreasing mapping from φ_g to α_g (and thus on long-run multipliers) echoes the analytical findings obtained in the simple model explored in the first section.

To sum up, there exists a strong interaction between the estimated values of φ_g and α_g that have potentially dramatic implications for the quantitative assessment of the long-run government spending multiplier. We argued previously that this bias could also corrupt shorter-term multipliers. The next section provides a quantitative illustration of this.

proportional to $\Delta y/\Delta g$ and the proportionality factor does not depend on α_g. The multiplier on private consumption is not significantly different from zero.

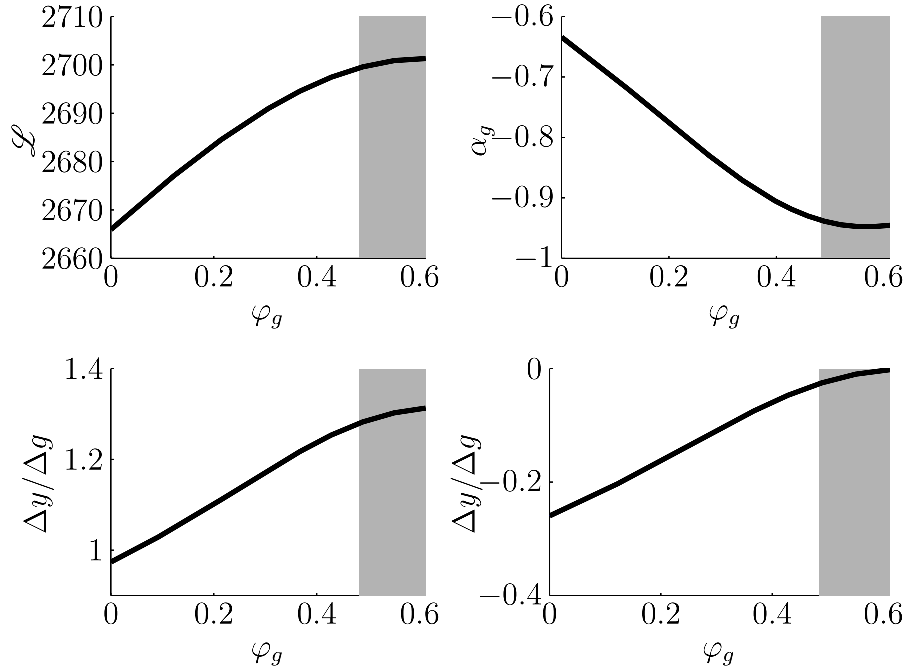

Figure 2: Sensitivity to constraints on policy rule parameter φ_g

Notes: Sample period: 1960:1-2007:4. The feedback parameters φ_g takes values on a grid between 0 and 0.6117. For each value, all the remaining parameters in ψ_2 are re-estimated. The upper left panel reports the log-likelihood as a function of φ_g. The grey area corresponds to restrictions on φ_g that are not rejected at the 5% level according to a likelihood ratio test. This grey area is also reported in each of the other panels. The upper right panel reports the estimated value of α_g as a function of φ_g. The bottom panels report the long-run multipliers on output and consumption.

3.3.2 Implications for Short-Run Multipliers

So far, we have insisted on the long-run GSM. The main reason for this is that this number does not directly depend on the serial correlation of government policy. Focussing on this multiplier thus illustrates in an unambiguous way how misspecifying the policy rule can result in a downward biased multiplier. However, all our results should in principle hold equally in an assessment of shorter-term multipliers.

To illustrate this, we follow Uhlig (2010) and specify an autoregressive process for the exogenous component of government spending designed to approximate the time profile of a recovery plan. More precisely, we assume that the discretionary component to policy follows the process

$$g_t^\star = 1.653 g_{t-1}^\star - 0.672 g_{t-2}^\star,$$

which is initialized by setting $g_0^\star = 0$ and $g_1^\star = 0.32$, assuming that date $t = 1$ corresponds to 2009. As argued by Uhlig (2010), this process closely approximates the government spending path reported in Cogan et al. (2010) (see, in particular, Uhlig, 2010, figure 1).

This process is fed into three model versions which we use to compute different versions of the short-run multiplier. First, we use specification (3), which imposes an exogenous policy at the estimation stage.

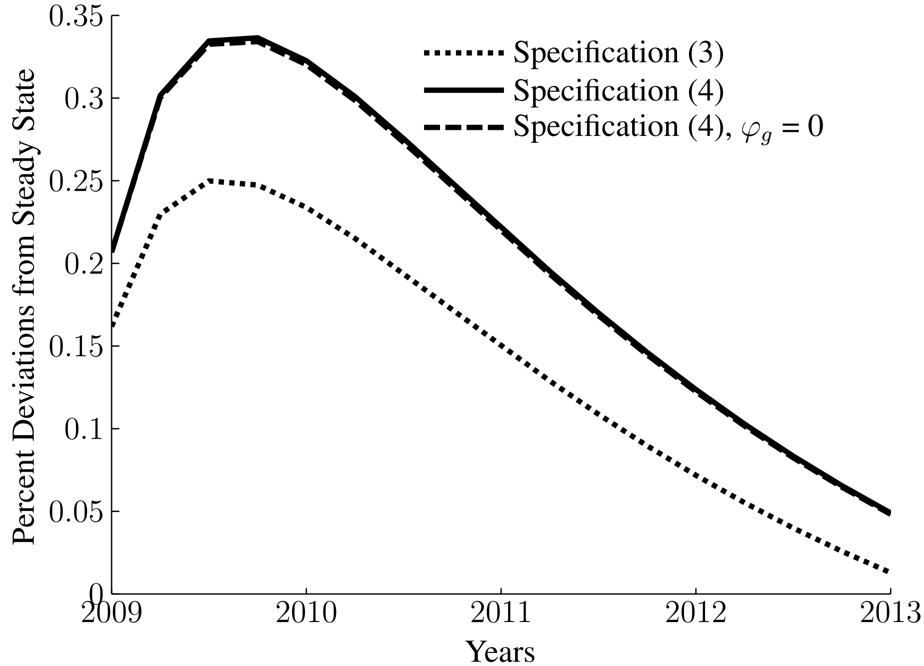

Figure 3: Stimulus Effect on Output,

Notes: Estimated dynamic effects of government purchases in the February 2009 stimulus legislation on output, in relative deviation from steady state. Specification (3): $\alpha_g \neq 0$ and $\varphi_g = 0$; specification (4): $\alpha_g \neq 0$ and $\varphi_g \neq 0$; in specification (4) with $\varphi_g = 0$, we use the estimated value of α_g in (4) and impose the counterfactual $\varphi_g = 0$.

Second, we use our preferred specification (4), in which policy parameters are freely estimated. Finally, we freeze all non-policy parameters obtained in this specification and impose an exogenous policy rule (i.e. $\varphi_g = 0$). We do this to make sure that allowing for a countercyclical policy rule does not change the resulting short-run multiplier too much.

The results are reported in figure 3. The thick dark line corresponds to the percent deviation of GDP from its steady state after the discretionary shock in specification (4). The dashed, dark line is the same impulse response obtained under an exogenous policy in specification (4). Finally, the dotted line is the GDP response obtained under specification (3). As the figure makes clear, with or without systematic policy, the GDP response is always much higher in specification (4) than in specification (3).

Over the selected horizon, the cumulated difference between the response obtained under specification (4) and that obtained under specification (3) is approximately 1.4% of steady-state US GDP. Now, to make things concrete, the extra government expenditures of the ARRA package amount to 8.8% of US, steady-state output (see Cogan et al., 2010). Thus, an econometrician using specification (3) would understate the cumulated output effect of the ARRA package by approximately 15.5% of the package's size. Interestingly, the impact response of output under specification (3) is much smaller than that obtained under specification (4). This downward bias echoes analytical results on the impact response of output derived in our simple model.

Notice finally that the exact value of φ_g used in simulating specification (4) does not affect quantitatively the multiplier. This is consistent with our previous long-run findings.[18] As argued before, this parameter plays a key role only at the estimation stage. Omitting it would lead an econometrician to downward bias the degree of Edgeworth complementarity, resulting in a seriously flawed assessment of the ARRA impact.

4 Robustness

In this section, we investigate the robustness of our results to (i) alternative specifications for the government spending rule, (ii) alternative estimation subsamples, and (iii) an alternative model specification very close to Smets and Wouters (2007).

4.1 Alternative Specifications of the Feedback Rule

In this paper, the assumed fiscal rule lacks strong theoretical underpinnings, contrary to the celebrated Taylor rule for monetary policy. It is thus central, at the very least, to establish that our specification can be defended on empirical grounds. To this end, we reestimate the model under a large battery of alternative government spending rules. We then use the likelihood to compare these specifications.

The alternative rules considered in this section are specified as follows

$$
\begin{array}{lrcl}
\text{(A)} & \log(\tilde{g}_t) & = & -\varphi_g(\log(y_t) - z_t - \log(\bar{y}^s)) \\
\text{(B)} & \log(\tilde{g}_t) & = & -\varphi_g(\log(y_{t-1}) - z_{t-1} - \log(\bar{y}^s)) \\
\text{(C)} & \log(\tilde{g}_t) & = & -\varphi_z(\Delta z_t - \log(\gamma_z)) \\
\text{(D)} & \log(\tilde{g}_t) & = & -\varphi_z(\Delta z_t - \log(\gamma_z)) - \varphi_a \Delta a_t - \varphi_b \Delta b_t \\
\text{(E)} & \log(\tilde{g}_t) & = & -\varphi_g(\Delta \log(y_t) - \log(\gamma_z)) - \varphi_z(\Delta z_t - \log(\gamma_z)) \\
\text{(F)} & \log(\tilde{g}_t) & = & -\varphi_g(\Delta \log(y_t) - \log(\gamma_z)) - \varphi_z(\Delta z_t - \log(\gamma_z)) - \varphi_a \Delta a_t - \varphi_b \Delta b_t \\
\text{(G)} & \log(\tilde{g}_t) & = & -\varphi_g(\Delta \log(y_{t-1}) - \log(\gamma_z)) \\
\text{(H)} & \log(\tilde{g}_t) & = & -\varphi_g(\lambda(\Delta \log(y_t) - \log(\gamma_z)) + (1-\lambda)(\Delta \log(y_{t-1}) - \log(\gamma_z)))
\end{array}
$$

where, as before, $\bar{y}^s$ denotes the steady-state value of detrended output.

[18] Notice however that if we were to impose $\alpha_g = \hat{\alpha}_g$ and $\varphi_g = 0$ in model (4), the fit would be dramatically deteriorated. The associated log-likelihood would now equal to 2663.3995. With such a restriction, the model would generate too much pro-cyclical movements between public spending and both output and private consumption.

Results are reported in table 5. For comparison purpose, we reproduce the results obtained with our preferred specification (4), referred to here as the benchmark specification. The other specifications are: (A) the stationary component of government spending reacts to current deviations of output from its stochastic trend; (B) the stationary component of government spending reacts to once-lagged deviations of output from its stochastic trend; (C) the stationary component of government spending reacts to changes in total factor productivity, resembling the specification used by Smets and Wouters (2007); (D) the stationary component of government spending reacts to changes in all the structural shocks; (E) combines our benchmark specification with (C); (F) combines the benchmark specification with (D). We also consider an alternative rule (G) to our benchmark specification where we assess the effect of $\log(\tilde{g}_t)$ responding to lagged output growth. With this specification, automatic stabilizers work with some delay. Finally, specification (H) nests our benchmark and (G), where the parameter λ put a weight on contemporaneous output growth.[19]

Table 5 reports the estimated values of α_g and the policy rule parameters, together with the implied long-run multipliers and the log-likelihood. To ease comparison, we also report the estimation results obtained under an exogenous policy (i.e. specification (3)).

As is clear from table 5, our benchmark specification dominates the alternative specifications from (A) to (D). Notice that these specifications are not nested with our benchmark. Since our benchmark implies a higher log-likelihood, the alternative rules are not better descriptions of the data. Moreover, the specification test results deteriorate (not reported), especially so in terms of serial correlation of innovations.

The specifications that yield the lowest fit are (A) and (B). Specification (A) implies a procyclical government spending policy. In this case, the implied α_g is higher than under an exogenous policy (-0.58 in specification (A) and -0.63 under an exogenous policy). This mechanically translates into a smaller multiplier in (A), as expected from our previous analysis. In specification (B), the policy rule is almost acyclical and we obtain roughly similar multipliers than under an exogenous policy.[20]

Specification (C) implies a countercyclical policy rule in response to technology shocks, since the estimated values of φ_z is positive. Once again, as expected, the estimated α_g is slightly lower than under an exogenous policy ($\alpha_g = -0.71$), resulting in a higher multiplier.

[19] We also considered versions of the fiscal rule with hours growth $\Delta \log(n_t)$ instead of output growth. We basically obtain the same results as in our benchmark specification.

[20] In Appendix G, we use our benchmark model as a DGP and estimate on simulated data a mispecified model assuming that the government spending rule takes the form (A). While government policy is indeed countercyclical in the DGP, estimation outocmes under the mispecified model result on average in a procyclical policy rule. Interestingly, the modal estimate of φ_g in this experiment is very close to that obtained when estimating the model with rule (A) on actual data. We use this result to set up an encompassing test which leads us to reject specification (A) in favor of specification (4).

Table 5. Alternative Government Spending Rules

Specification	α_g	φ_g	λ	φ_z	φ_a	φ_b	ϕ	$\Delta y/\Delta g$	$\mathscr{L}$
Benchmark	-0.9452	0.6117	—	—	—	—	0.4110	1.3128	2701.3173
			—						
(A)	-0.5774	-0.4866	—	—	—	—	0.4072	0.9652	2669.6339
(B)	-0.6287	0.1029	—	—	—	—	0.3710	0.9623	2666.1522
(C)	-0.7091	—	—	0.3952	—	—	0.3995	1.0693	2691.5604
(D)	-0.7893	—	—	0.4254	0.0133	-0.0500	0.3769	1.1105	2694.4357
(E)	-1.0398	0.8176	—	-0.1660	—	—	0.4181	1.4244	2702.0942
(F)	-1.2220	2.5102	—	-1.4928	-0.1132	0.2264	0.3700	1.5455	2714.2522
(G)	-0.6342	0.0104	—	—	—	—	0.3769	0.9736	2665.9424
(H)	-1.0730	1.0802	0.7023	—	—	—	0.4137	1.4514	2712.7980
			—						
Exogenous Policy	-0.6340	—	—	—	—	—	0.3766	0.9738	2665.9338

Notes: Sample period: 1960:1-2007:4. Benchmark is specification (4): $\alpha_g \neq 0$ and $\varphi_g \neq 0$. Exogenous policy is specification (3): $\alpha_g \neq 0$ and $\varphi_g = 0$.

Specification (D) adds to the former case by allowing policy to respond to all the shocks. This specification implies a countercyclical policy rule, in the sense that the estimated values of φ_z and φ_a are positive while the estimated value of φ_b is negative. As expected, the Edgeworth complementarity parameter turns out to be lower than under an exogenous policy ($\alpha_g = -0.79$).

Specifications (E) and (F), which nest our benchmark, imply higher log-likelihoods, by construction. A likelihood ratio test would not reject our benchmark when compared to specification (E). In contrast, the log-likelihood is much higher in specification (F). However, specification tests outcomes do not improve much when compared to our benchmark.

It is not obvious a priori to tell whether or not specifications (E) and (F) feature countercyclicality of government spending policy. However, if our claim holds in this context, one can interpret the low value obtained for α_g as suggestive of a high degree of countercyclicality.

Estimation results show that specification (G) is clearly dominated by our benchmark representation. The log-likelihood is close to that obtained under an exogenous policy. Specification (H) confirms this finding. When contemporaneous and once-lagged output growth appear both in the rule, our estimation results clearly favor a contemporaneous response of government spending (the weight λ is equal to 0.7023). Notice that in this case, the degree of Edgeworth complementarity is more pronounced and, accordingly, the model yields a higher multiplier.

4.2 Subsample Analysis

Perotti (2005) showed that empirical measures of government spending multipliers can prove sensitive to the particular sample selected. Importantly, he argues that multipliers over the sample 1980:1-2007:4 are smaller than those found over the sample 1960:1-1979:4. We here investigate whether our results still hold if we re-estimate our model over the same subsamples.

Results are reported in table 6. Our previous conclusions are broadly confirmed. First, the restriction $\varphi_g = 0$ is rejected, suggesting that government spending policy is endogenous, irrespective of the selected sample. Second, when this restriction is imposed, we obtain a higher α_g, resulting in a smaller multiplier. This holds over both subsamples. We also obtain a smaller long-run GSM over the second subsample, confirming results in Perotti (2005).

Table 6. Subsample Robustness

Specification	α_g	φ_g	ϕ	$\Delta y/\Delta g$	$\mathscr{L}$
1960:1-1979:4					
(4)	-1.0167	0.6033	0.3755	1.3263	1101.3585
(3)	-0.7910	–	0.3896	1.1276	1088.1722
1980:1-2007:4					
(4)	-0.8068	0.5597	0.4146	1.1856	1614.9772
(3)	-0.6100	–	0.3777	0.9580	1599.7944

Notes: Specification (4): $\alpha_g \neq 0$ and $\varphi_g \neq 0$; Specification (3): $\alpha_g \neq 0$ and $\varphi_g = 0$.

4.3 A Smets-Wouters type model

It has become common in macroeconomics to estimate richer models with many more shocks and frictions. As a final robustness check, we thus investigate whether our results hold within a medium-scale DSGE model along the lines of Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007). The model combines a neoclassical growth core with several shocks and frictions. It includes features such as habit formation, investment adjustment costs, variable capital utilization, monopolistic competition in goods and labor markets, and nominal price and wage rigidities, which help to replicate the data. The economy is populated by five classes of agents: producers of a final good, intermediate goods producers, households, employment agencies and a governement. We adopt the specification investigated by Justiniano, Primiceri and Tambalotti (2010), except that we allow for (i) Edgeworth complementarity/substitutability and (ii) endogeneity of public spending. On top of these refinements,

Table 7. Estimation Results for the Smets-Wouters Type Model

Specification	ω	α_g	φ_g	$\Delta y/\Delta g$	Marginal Likelihood
(4)	–	-0.626	0.752	0.869	-1277.683
		[-0.914,-0.332]	[0.545,0.964]	[0.598,1.139]	
(3)	–	-0.199	–	0.524	-1298.946
		[-0.454,0.040]		[0.364,0.721]	
(4')	0.149	-1.133	0.728	1.168	-1275.890
	[0.085,0.211]	[-1.443,-0.819]	[0.528,0.919]	[0.883,1.486]	
(3')	0.165	-0.870	–	0.893	-1296.944
	[0.095,0.231]	[-1.201,-0.552]		[0.675,1.160]	

Notes: Specification (3): $\omega = 0$, $\alpha_g \neq 0$ and $\varphi_g = 0$; Specification (4): $\omega = 0$, $\alpha_g \neq 0$ and $\varphi_g \neq 0$; Specification (3'): $\omega \neq 0$, $\alpha_g \neq 0$ and $\varphi_g = 0$; Specification (4'): $\omega \neq 0$, $\alpha_g \neq 0$ and $\varphi_g \neq 0$. Figures in brackets are the cutoff values for the 5th and 95th percentile of the posterior distributions.

following Cogan et al. (2010), we consider two model versions depending on whether we allow for non-Ricardian agents. Indeed, as shown by (Galí, López-Salido and Vallés, 2007), allowing for liquidity-constrained agents has proven to be a useful modeling device for reproducing the aggregate effect of government spending shocks. In the remainder, we let ω denote the fraction of non-Ricardian agents.

We follow the Bayesian approach to estimate the log-linearized model. The dynamic system is cast in a state-space representation for the set of observable variables. The Kalman filter is then used (i) to measure the likelihood of the observed variables and (ii) to form the posterior distribution of the structural parameters by combining the likelihood function with a joint density characterizing some prior beliefs. Given the specification of the model, the posterior distribution cannot be recovered analytically but may be computed numerically, using a Monte-Carlo Markov Chain (MCMC) sampling approach. More specifically, we rely on the Metropolis-Hastings algorithm to obtain a random draw of size 400,000 from the posterior distribution of the parameters. We estimate the model using seven series of US quarterly data: consumption growth, investment growth, government expenditures growth, the logarithm of hours worked, the inflation rate, and the nominal interest rate. As before, real variables are divided by the civilian population, age 16 and over. Once again, all the data come from the Federal Reserve Bank of St. Louis' FRED II database and from the Bureau of Labor Statistics website, except hours worked that are borrowed from Francis and Ramey (2009). Recall that the sample runs from 1960:1 to 2007:4.

We consider four model versions, labelled (3), (4), (3'), and (4'). As before, in (3), we impose $\varphi_g = 0$

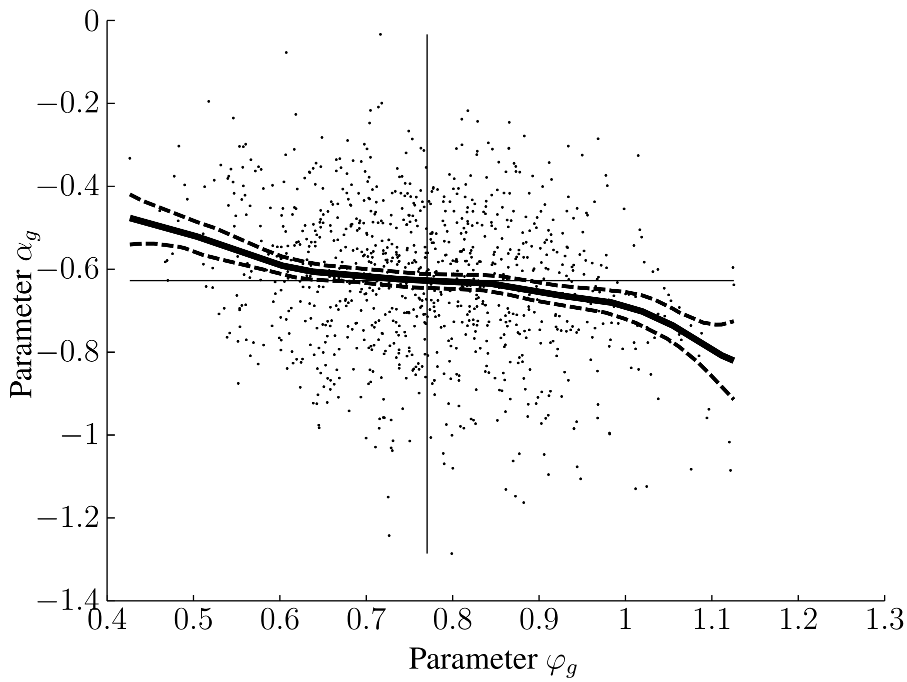

Figure 4: Relationship between φ_g and α_g

Notes: The thick plain line is the non-parametric regression and the thick dashed lines delineate the 95% confidence interval obtained by standard bootstrap techniques. The cross indicates the average parameter values for α_g and φ_g.

while in (4) this parameter is freely estimated. In both specifications, we also impose that the mass of non-Ricardian agents be zero, i.e. $\omega = 0$. In specifications (3') and (4'), we freely estimate ω, while considering $\varphi_g = 0$ in (3') and $\varphi_g \neq 0$ in (4'). To save space, we only report estimation results for ω, φ_g and α_g.[21] Importantly, we specify for φ_g and α_g uniform priors centered on zero and with a standard deviation of 0.65 and 1.30, respectively, to reflect our agnostic view concerning these key parameters.

Table 7 reports the posterior means and the associated 90% confidence interval for the parameters discussed above. Upon inspecting model (3) and (4), we see again that imposing $\varphi_g = 0$ strongly affects the estimated value of α_g: when φ_g is freely estimated, we obtain $\alpha_g = -0.63$ whereas we get $\alpha_g = -0.20$ when we set $\varphi_g = 0$. The table makes clear that ignoring the feedback rule would lower the degree of Edgeworth complementarity. As a consequence, this would also lead to a lower long-run GSM. It is worth noting that the implied difference in multipliers is very close to that reported in table 4. All these findings confirm the results obtained from our benchmark model. In addition, the marginal likelihood strongly favors the model version with endogenous policy.

When considering specifications (3') and (4'), we obtain a small fraction of non-Ricardian agents. Notice that the presence of these agents does not change our basic message: the prior mean of α_g is smaller in absolute value when φ_g is constrained to zero. Once again, the marginal likelihood points to specifi-

[21]Model's details and full estimation results are relegated in Appendix H.

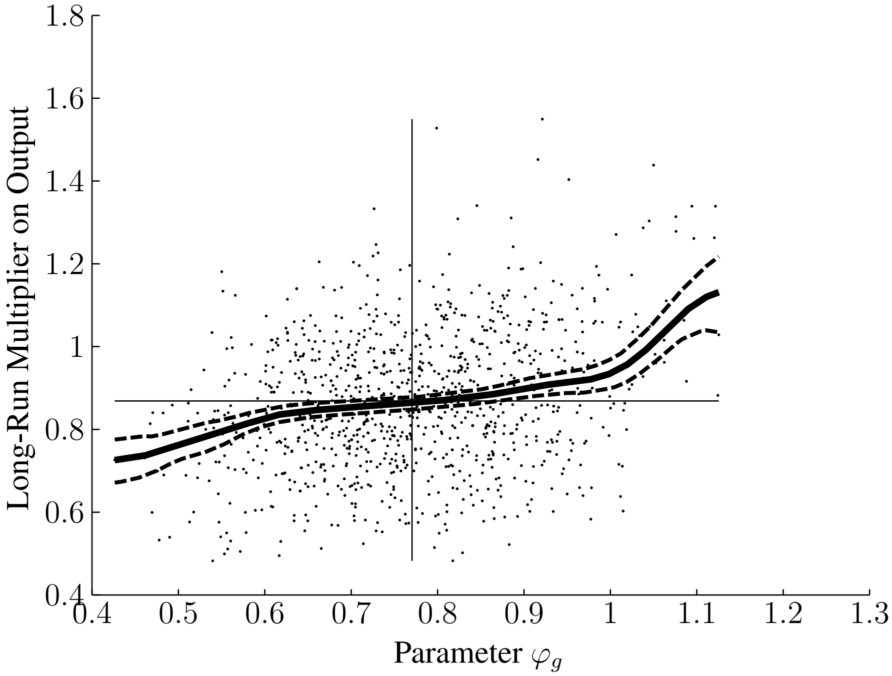

Figure 5: Relationship between φ_g and $\Delta y/\Delta g$

Notes: The thick plain line is the non-parametric regression and the thick dashed lines delineate the 95% confidence interval obtained by standard bootstrap techniques. The cross indicates the average parameter values for α_g and φ_g.

cation (4') as the preferred model version.

Finally, we plot draws from the posterior distribution of (i) φ_g and α_g, and (ii) φ_g and $\Delta y/\Delta g$ for specification (4). Figures 4 and 5 report the outcome of this exercice. The thick plain line is the non-parametric regression and the thick dashed lines delineate the 90% confidence interval obtained by standard bootstrap techniques. The scatter diagram corresponds to the estimation of model (4). The cross indicates the average parameter values for φ_g and α_g in figure 4, and the average parameter values for φ_g and $\Delta y/\Delta g$, in figure 5. The figures clearly reveal -in the neighborhood of the posterior means- the existence of (i) a negatively slopped relation between φ_g and α_g and (ii) a positively slopped relation between φ_g and $\Delta y/\Delta g$. Once again, these additional quantitative exercices corroborate findings from the simple analytical model and those from the benchmark setup.

5 Conclusion

This paper has proposed to assess quantitatively the consequences of misspecifying the government spending rule on the estimated government spending multiplier within a DSGE framework. We first considered a simplified model version to show analytically that omitting the feedback rule at the estimation stage yields a downward-biased estimate of the long-run government spending multiplier. To establish this, we first showed that the multiplier is an increasing function of the degree of Edgeworth complementarity. In turn, complementarity and countercyclical policy interact through cross-equation restrictions, paving the way for a potential bias. We also showed in appendix that this result holds for other commonly considered transmission mechanisms. We then estimated on postwar US data a quantitative model version and obtained that omitting the endogeneity of government spending exerts a severe, downward impact on the estimated long-run multiplier. Such a bias also characterizes shorter-term multipliers and thus can seriously affect the quantitative assessment of fiscal packages such as the ARRA stimulus. Our results appear to be very robust to a series of perturbations to the benchmark specification.

In our framework, we have deliberately abstracted from relevant details in order to highlight, as transparently as possible, the empirical link between policy rule parameters and the degree of Edgeworth complementarity between private and public consumption. However, the recent literature insists on other modeling issues that might potentially affect our results. We mention two of them. First, as put forth by Leeper, Plante and Traum (2010), a more general specification of government spending rule, lump-sum transfers, and distortionary taxation is needed to properly fit US data. This richer specification includes in addition to the automatic stabilizer component, a response to government debt and co-movement between tax rates. An important quantitative issue may be to assess which type of stabilization (automatic stabilization and/or debt stabilization) interacts with the estimated degree of Edgeworth complementarity. Second, Fiorito and Kollintzas (2004) have suggested that the degree of complementarity/substitutability between government and private consumptions is not homogeneous over types of public expenditures. This suggests to disaggregate government spending and inspect how feedback rules affect the estimated degree of Edgeworth complementarity in this more general setup. These three issues will constitute the object of future researches.

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Appendix Not Intended for Publication

A Alternative Transmission Mechanisms

In the body of the paper, we considered Edgeworth complementarity/substitutability as the transmission mechanism of government spending. In this section, we propose other mechanisms that yield the exact same reduced form. We still consider that aggregate output is equal to the the sum of private and public consumptions ($y_t = c_t + g_t$). It follows that a multiplier exceeding one is equivalent to a positive multiplier on private consumption. In our simple setup, the multiplier is a decreasing function of α_g and it exceeds unity when $\alpha_g < \nu(1 - s_g)/(1 + \nu s_g)$.

In all the following alternative specifications, we show that the log-linear approximation of the economy takes the form

$$\hat{y}_t = \alpha \hat{g}_t + \zeta z_t, \tag{A.1}$$

as in the simple model of section 2. In what follow, we focus exclusively on the parameter α, since the parameter ζ can be normalized to unity by rescaling appropriately the standard error of the technology shock. Moreover, we discuss conditions on deep parameters representing preferences or technology for the long-run GSM to exceed unity. We also show that the value of this multiplier is unambiguously related to a single parameter summarizing the transmission mechanism of public spending.

A.1 Non-Separable Preferences

Following Linnemann (2006) and Bilbiie (2009), we consider a more general utility function. The utility function in equation (1) rewrites more generally as $u(c_t, l_t)$, where leisure l_t satisfies $l_t = 1 - n_t$. Combining the optimality conditions of the households' problem yields $u_l(c_t, l_t) = u_c(c_t, l_t)w_t$.

Firms produce a homogenous good y_t using the same constant returns-to-scale technology as in 2, so that profit maximization yields $w_t = \mathrm{e}^{z_t}$. It immediately comes that $u_l(c_t, l_t) = u_c(c_t, l_t)\mathrm{e}^{z_t}$. After log-linearizing this equation, we get

$$\varphi \hat{n}_t = z_t - \psi \hat{c}_t,$$

where $\varphi \equiv (u_{cl}n/u_c) - (u_{ll}n/u_l)$ and $\psi \equiv -(u_{cc}c/u_c) + (u_{cl}c/u_l)$. Using the other equilibrium conditions, we exactly obtain equation (A.1), where the parameter α is now defined by

$$\alpha \equiv \frac{\psi s_g}{\psi + \varphi(1 - s_g)}$$

The long-run government spending multiplier is then

$$\frac{\Delta y}{\Delta g} = \frac{\psi}{\psi + \varphi(1 - s_g)}.$$

The multiplier exceeds unity when $\psi > 0$ and $\varphi < 0$. As stated in Bilbiie (2009), this imposes that consumption be an inferior good (see also Bilbiie, 2011). The multiplier is a decreasing function of φ. Assuming all the other parameters constant (in particular ψ), the long-run multiplier is then directly linked to the key parameter φ summarizing households' preferences.

A.2 Externalities

We consider two types of externalities yielding equation (A.1) exactly. One relates to the specification of preferences, whereas the second relates to the production technology.

A.2.1 Externality in Labor Supply

We adapt Benhabib and Farmer (2000) to our simple setup. Let us rewrite the instantaneous utility function in equation (1) as

$$\log(c_t) - \frac{\eta}{1+\nu}\left(\frac{n_t}{\bar{n}_t^{\vartheta}}\right)^{1+\nu},$$

where $\bar{n}_t$ represents the average labor supply in the economy. The parameter ϑ measures the external effect of other households' labor on individual utility. For example, when $\vartheta > 0$, individual and aggregate labor supplies are complement. Using the firm's first-order condition and the aggregate resources constraint, we obtain $\eta(y_t/z_t)^{\nu-\vartheta(1+\nu)} = \mathrm{e}^{z_t}/(y_t - g_t)$. Loglinearizing this yields

$$\alpha = \frac{s_g}{1 + (1-s_g)(\nu - \vartheta(1+\nu))}$$

The long-run GSM is then

$$\frac{\Delta y}{\Delta g} = \frac{1}{1 + (1-s_g)(\nu - \vartheta(1+\nu))}$$

The multiplier exceeds unity whenever $\vartheta > \nu/(1+\nu)$. Notice that for these values of ϑ, the constant-consumption, aggregate labor supply is downward slopping. The multiplier is an increasing function of ϑ. For s_g and ν set to given values, the multiplier is unambiguously linked to the labor supply externality.

A.2.2 Externality in production

We now adapt Benhabib and Farmer (1994) and Devereux, Head and Lapham (1996) to our simple setup. Let us assume that the technology takes the form

$$y_t = \mathrm{e}^{z_t} n_t s_t.$$

Here s_t is an externality on production specified as $s_t = \bar{n}_t^{\theta_n}$. As before, $\bar{n}_t$ represents the average level of labor. The parameter θ_n governs the productive externality. Notice that the technology displays constant returns-to-scale at the private level, but increasing returns at the social level when $\theta_n > 0$. In equilibrium, the real wage obeys $w_t = \mathrm{e}^{z_t} n_t^{\theta_n}$. Plugging this equation into the households' optimality condition and using the aggregate resources constraint, we obtain finally $\eta (y_t/z_t)^{(\nu-\theta_n)/(1+\theta_n)} = \mathrm{e}^{z_t}/(y_t - g_t)$. Loglinearizing this yields equation (A.1), where the parameter α is now given by

$$\alpha = \frac{s_g(1+\theta_n)}{1+\theta_n(1-s_g)(\nu-\theta_n)}$$

The long-run GSM is

$$\frac{\Delta y}{\Delta g} = \frac{1+\theta_n}{1+\theta_n(1-s_g)(\nu-\theta_n)}$$

The multiplier exceeds unity whenever $\theta_n > \nu$. Under this restriction, the aggregate labor demand is more upward sloping than the constant-consumption labor supply (see Bilbiie, 2011). The multiplier is an increasing function of θ_n. Keeping all the other parameters constant, the multiplier is unambiguously related to the size of the production externality.

A.3 Deep Subsistence Point

We now adapt Ravn, Schmitt-Grohé and Uribe (2006) and Ravn, Schmitt-Grohé and Uribe (2008) to our simple setup. The instantaneous utility rewrites as

$$\log(x_t^c) - \frac{\eta}{1+\nu} n_t^{1+\nu},$$

where x_t^c represents a composite consumption good, composed of a continuum of differentiated goods indexed by $j \in [0, 1]$. The composite good x_t^c is given by

$$x_t^c = \left[\int_0^1 \left(c_{j,t} - c_j^\star \right)^{1-1/\rho} \mathrm{d}j \right]^{1/(1-1/\rho)}, \tag{A.2}$$

where $c_j^\star$ is the subsistence level of consumption of good j. The parameter $\rho > 0$ is the elasticity of substitution across varieties. Minimizing total consumption expenditures $\int_0^1 P_{j,t} c_{j,t} \mathrm{d}j$, where $P_{j,t}$ denotes the price of good j, subject to the aggregation constraint (A.2) yields the demand for each good j

$$c_{j,t} = (P_{j,t}/P_t)^{-\rho} x_t^c + c_j^\star,$$

where $P_t = [\int_0^1 P_{j,t}^{1-\rho} \mathrm{d}j]^{1/(1-\rho)}$ is the price index. Notice that the price elasticity is not constant as soon as $c_j^\star > 0$. The optimality condition of households' problem is given by $\eta n_t^\nu = w_t / x_t^c$.

Symmetrically with the households'problem, the government allocates spending among individual varieties of goods, $g_{j,t}$, so as to maximize the quantity x_t^g of a composite good

$$x_t^g = \left[\int_0^1 \left(g_{j,t} - g_j^\star \right)^{1-1/\rho} \mathrm{d}j \right]^{1/(1-1/\rho)},$$

where $g_j^\star$ denotes the subsistence level of consumption of public good j. Given the budget constraint $\int_0^1 P_{j,t} g_{j,t} \mathrm{d}j \leq P_t g_t$, the government demand for each good is given by $g_{j,t} = (P_{j,t}/P_t)^{-\rho} x_t^g + g_j^\star$.

Finally, each good j is produced by a monopolist using the technology $y_{j,t} = \mathrm{e}^{z_t} n_{j,t}$. Each firm sets its price and satisfies demand, $\mathrm{e}^{z_t} n_{j,t} \geq c_{j,t} + g_{j,t}$. After substituting for the expressions for the demands $c_{j,t}$ and $g_{j,t}$ into the previous constraint, we obtain the first order conditions $\mathrm{mc}_{j,t} = w_t/\mathrm{e}^{z_t}$ and $P_{j,t}/P_t = \mathrm{mku}_{j,t} \times \mathrm{mc}_{j,t}$, where $\mathrm{mc}_{j,t}$ denotes the marginal cost of labor and $\mathrm{mku}_{j,t}$ the associated markup. This markup is given by

$$\mathrm{mku}_{j,t} = \left[1 - \frac{1}{\rho\left(1 - \frac{c_j^\star + g_j^\star}{y_{j,t}}\right)}\right]^{-1}$$

For simplicity, we impose $c_j^\star = c^\star$ and $g_j^\star = g^\star$. In a symmetric equilibrium, $P_{j,t}/P_t = 1\ , \forall j \in [0,1]$, so that $w_t = \mathrm{e}^{z_t}/\mathrm{mku}_t$. The equilibrium of this economy is then given by

$$(y_t/\mathrm{e}^{z_t})^\nu = \left(1 - \frac{1}{\rho\left(1 - \frac{c^\star + g^\star}{y_t}\right)}\right) \frac{\mathrm{e}^{z_t}}{y_t - g_t - c^\star}$$

Denoting $c^\star = \omega c$ and $g^\star = \omega g$, the steady-state share of subsistence consumption $c^\star + g^\star$ in output, we exactly obtain equation (A.1), where the parameter α is defined by

$$\alpha = \frac{s_g}{(\nu(1-s_g)(1-\omega)+1) - (1-s_g)\omega(\mathrm{mku}-1)},$$

where $\mathrm{mku} > 1$ is the steady-state markup, provided $\rho(1-\omega) - 1 \equiv (\mu-1)^{-1} > 0$. The long-run GSM is

$$\frac{\Delta y}{\Delta g} = \frac{1}{(\nu(1-s_g)(1-\omega)+1) - (1-s_g)\omega(\mathrm{mku}-1)}$$

The multiplier exceeds unity when $\omega > \nu/(\nu + \mu - 1)$. The multiplier is an increasing function of ω. Keeping all the other parameters constant (for example, mku is fixed), the multiplier unambiguously depends on the relative size of the deep subsistence point.

B Proof of Proposition 1

To prove the first part of Proposition 1, evaluate equation (4) in the deterministic steady state, which implies

$$\eta \bar{y}^\nu = \frac{1}{\bar{y} - (1-\alpha_g)\bar{g}}, \tag{B.3}$$

where $\bar{y}$ and $\bar{g}$ are the steady-state values of y and g, respectively.

Total differentiation of the above equation then yields

$$\nu \mathrm{d}\bar{y} = -\frac{\bar{y}}{\bar{y} - (1-\alpha_g)\bar{g}} \mathrm{d}\bar{y} + \frac{(1-\alpha_g)\bar{y}}{\bar{y} - (1-\alpha_g)\bar{g}} \mathrm{d}\bar{g}. \tag{B.4}$$

Rearranging this expression and using Definition 1, one obtains finally

$$\frac{\mathrm{d}\bar{y}}{\mathrm{d}\bar{g}} = \frac{1-\alpha_g}{1+\nu[1-s_g(1-\alpha_g)]}, \tag{B.5}$$

as was to be shown.

To establish the second part of Proposition 1, differentiate the long-run GSM with respect to α_g

$$\frac{\partial}{\partial \alpha_g}\left(\frac{\Delta y}{\Delta g}\right) = -\frac{1+\nu}{(1+\nu[1-s_g(1-\alpha_g)])^2} < 0.$$

Thus $\Delta y/\Delta g$ decreases with α_g, as was to be shown.

C Proof of Proposition 2

C.1 Proof of Part 1

The proof of part 1 of Proposition 2 proceeds as follows. The reduced-form equations (7) and (8) rewrite

$$\begin{aligned} \hat{y}_t &= \rho \hat{y}_{t-1} + \frac{\alpha}{1+\alpha\varphi_g} u_t + \frac{1}{1+\alpha\varphi_g} z_t, && \text{(C.6)} \\ \hat{g}_t &= \rho \hat{g}_{t-1} + \frac{1}{1+\alpha\varphi_g} u_t - \frac{\varphi_g}{1+\alpha\varphi_g} \Delta z_t, && \text{(C.7)} \end{aligned}$$

where $\Delta z_t = z_t - z_{t-1}$ and

$$\rho \equiv \frac{\alpha\varphi_g}{1+\alpha\varphi_g}.$$

Provided $\alpha_g < 1$ and $\varphi_g \geq 0$, we have $\rho \in [0, 1)$, so that (C.6) and (C.7) display second-order stationarity. Accordingly, they can be restated as

$$\begin{aligned} \hat{y}_t &= \frac{1}{1+\alpha\varphi_g} \sum_{i=0}^{\infty} \rho^i L^i (\alpha u_t + z_t), && \text{(C.8)} \\ \hat{g}_t &= \frac{1}{1+\alpha\varphi_g} \sum_{i=0}^{\infty} \rho^i L^i (u_t - \varphi_g \Delta z_t), && \text{(C.9)} \end{aligned}$$

where L is the backshift operator. Equation (C.9) can be reformulated as

$$\hat{g}_t = \frac{1}{1+\alpha\varphi_g}\left[\left(\sum_{i=0}^{\infty}\rho^i L^i\right) u_t - \varphi_g\left(1 + \frac{\rho-1}{\rho}\sum_{i=1}^{\infty}\rho^i L^i\right) z_t\right] \tag{C.10}$$

Combining (C.8) and (C.10), one obtains

$$\mathrm{E}[\hat{y}\hat{g}] = \frac{1}{(1+\alpha\varphi_g)^2}\left(\frac{\alpha\sigma_u^2}{1-\rho^2} - \frac{\varphi_g\sigma_z^2}{1+\rho}\right). \tag{C.11}$$

Then using the definition of ρ, (C.11) rewrites

$$\mathrm{E}[\hat{y}\hat{g}] = \frac{(1+\alpha\varphi_g)\alpha\sigma_u^2 - \varphi_g\sigma_z^2}{(1+\alpha\varphi_g)(1+2\alpha\varphi_g)}. \tag{C.12}$$

From (C.10), one also obtains

$$\mathrm{E}[\hat{g}^2] = \frac{1}{(1+\alpha\varphi_g)^2}\left(\frac{\sigma_u^2}{1-\rho^2} + \frac{2\varphi_g^2\sigma_z^2}{1+\rho}\right). \tag{C.13}$$

Then using the definition of ρ, (C.13) rewrites

$$\mathrm{E}[\hat{g}^2] = \frac{(1+\alpha\varphi_g)\sigma_u^2 + 2\varphi_g^2\sigma_z^2}{(1+\alpha\varphi_g)(1+2\alpha\varphi_g)}. \tag{C.14}$$

Now, combining (C.12), (C.14), and the definition of $\hat{\hat{\alpha}}$ yields

$$\hat{\hat{\alpha}} \equiv \frac{\mathrm{E}[\hat{y}\hat{g}]}{\mathrm{E}[\hat{g}^2]} = \frac{(1+\alpha\varphi_g)\alpha\sigma_u^2 - \varphi_g\sigma_z^2}{(1+\alpha\varphi_g)\sigma_u^2 + 2\varphi_g^2\sigma_z^2},$$

as was to be shown.

C.2 Proof of Part 2

The proof of part 2 of Proposition 2 is straightforward. Form the difference $\hat{\hat{\alpha}} - \alpha$, which yields

$$\hat{\hat{\alpha}} - \alpha = -\frac{\varphi_g\sigma_z^2(1+2\alpha\varphi_g)}{(1+\alpha\varphi_g)\sigma_u^2 + 2\varphi_g^2\sigma_z^2}. \tag{C.15}$$

This expression is strictly negative whenever $\varphi_g > 0$ and $\sigma_z > 0$, as was to be shown.

C.3 Proof of Part 3

To prove the third part of proposition 2, differentiate (C.15) with respect to φ_g. Assuming $\sigma_z > 0$, this yields

$$\frac{\partial}{\partial\varphi_g}(\hat{\hat{\alpha}} - \alpha) = -\left(\frac{1}{\omega(1+\alpha\varphi_g) + 2\varphi_g^2}\right)^2 P(\varphi_g), \tag{C.16}$$

where we defined $\omega \equiv (\sigma_u/\sigma_z)^2$ and

$$P(\varphi_g) \equiv 2(\alpha^2\omega - 1)\varphi_g^2 + 4\alpha\omega\varphi_g + \omega. \tag{C.17}$$

The expression in equation (C.16) is defined for $\varphi_g = 0$ whenever $\omega > 0$, i.e. whenever $\sigma_u > 0$. To simplify the analysis, we henceforth impose this condition. The sign of $\partial(\hat{\hat{\alpha}} - \alpha)/\partial\varphi_g$ depends on the sign of $P(\varphi_g)$. In turn, the sign of $P(\varphi_g)$ depends on the sign of $\alpha^2\omega - 1$, which calls for the following discussion:

- If $\alpha = \sigma_z/\sigma_u$, then $\alpha^2\omega - 1 = 0$, and thus $P(\varphi_g) = 4\alpha\omega\varphi_g + \omega > 0$ for $\varphi_g \geq 0$. Hence

$$\forall \varphi_g \geq 0,\ \frac{\partial}{\partial \varphi_g}(\hat{\hat{\alpha}} - \alpha) < 0.$$

- Now, if $\alpha > \sigma_z/\sigma_u$, then $\alpha^2\omega - 1 > 0$, and thus $P(\varphi_g)$ admits two negative roots (as can be read from the coefficients). Thus, for $\varphi_g \geq 0$, $P(\varphi_g) > 0$. Hence

$$\forall \varphi_g \geq 0,\ \frac{\partial}{\partial \varphi_g}(\hat{\hat{\alpha}} - \alpha) < 0.$$

- Finally, if $\alpha < \sigma_z/\sigma_u$, then $\alpha^2\omega - 1 < 0$, and thus $P(\varphi_g)$ admits roots of opposite signs (as can be read from the coefficients). Let $\bar{\varphi}_g(\alpha, \sigma_z, \sigma_u)$ define the positive root, i.e.

$$\bar{\varphi}_g(\alpha, \sigma_z, \sigma_u) \equiv \frac{2\alpha\omega + \sqrt{2\omega(\alpha^2\omega + 1)}}{2(1 - \alpha^2\omega)} > 0.$$

  Since the parabola opens downward, $P(\varphi_g) \geq 0$ for $\varphi_g \in [0, \bar{\varphi}_g(\alpha, \sigma_z, \sigma_u)]$ and $P(\varphi_g) < 0$ for $\varphi_g > \bar{\varphi}_g(\alpha, \sigma_z, \sigma_u)$. Thus

$$\begin{aligned} \frac{\partial}{\partial \varphi_g}(\hat{\hat{\alpha}} - \alpha) &\leq 0, \quad \forall \varphi_g \in [0, \bar{\varphi}_g(\alpha, \sigma_u, \sigma_z)], \\ \frac{\partial}{\partial \varphi_g}(\hat{\hat{\alpha}} - \alpha) &> 0, \quad \forall \varphi_g > \bar{\varphi}_g(\alpha, \sigma_u, \sigma_z). \end{aligned}$$

This completes the proof.

D Additional Real Frictions

A central ingredient of our preferred specification is the presence of dynamic complementarities in labor supply. Importantly, output dynamics inherit the built-in persistence of hours worked generated by this mechanism.

The recent DSGE literature, however, has emphasized alternative real frictions capable of generating very strong aggregate persistence. Important such mechanisms are habits in consumption and dynamic investment adjustment costs, see Christiano, Eichenbaum and Evans (2005). We considered versions of our preferred model augmented with these additional mechanisms.

When either of these are included, they do not significantly contribute to the model's fit. A standard likelihood ratio test would not reject the restriction of no habits in consumption and/or no dynamic adjustment costs. For example, in the case of specification (4) augmented with habits in consumption and dynamic adjustment costs, the log-likelihood is equal to 2702.22, to be compared to our reference specification (4) where the log-likelihood is equal to 2701.32. The habits in consumption parameter is equal to 0.11 (not significantly different from zero at conventional levels) and the adjustment cost parameter is almost zero. In addition, we redo the specification tests (normality and serial correlation). Including habits in consumption and dynamic adjustment costs does not improve upon the model performance: the normality test statistic is almost the same for each innovation and the serial correlation coefficients are very similar.

More importantly for our purpose, the empirical interaction between α_g and φ_g still holds under this more complete framework. In particular, when φ_g is constrained to zero, we obtain $\alpha_g = -0.21$, yielding a multiplier $\Delta y/\Delta g = 0.85$. In contrast, when φ_g is freely estimated, government policy turns out to be countercyclical ($\varphi_g = 0.60$) and the parameter $\alpha_g = -0.79$, implying a multiplier equal to 1.19. This confirms our main result.

E News Shocks in the Government Spending Rule

As emphasized by Ramey (2009) and Schmitt-Grohé and Uribe (2008), the expected component in public expenditures constitutes an important element of government policy. We accordingly modify our benchmark specification to allow for news shocks in the government spending rule, according to

$$g_t^\star = \rho_g g_{t-1}^\star + \sum_{i=0}^{q} \sigma_{g,i}\zeta_{g,t-i}, \;\; \forall i \in \{0, \dots, q\}, \sigma_{g,i} \geq 0,$$

where the $\zeta_{g,t}$ is iid with $\zeta_{g,t} \sim N(0,1)$.

We first imposed $q = 4$. According to our estimation results, we obtain that lags $i = 1, 2, 3$ are not significant. This specification delivers a significantly better fit to the data than our preferred model (4), according to the likelihood ratio test (in this case, the log-likelihood is equal to 2717.39). However, the parameter estimates do not change too much compared to specification (4). In particular, the parameter α_g is now equal to -0.86, whereas the feedback rule parameter is equal to 0.59. In addition, allowing for news shocks in government spending does not improve upon the specification tests of our reference model (4).

Importantly, adding news shocks does not modify our main conclusion. When policy is exogenous ($\varphi_g = 0$), we obtain $\alpha_g = -0.21$, with an associated multiplier equal to 0.56. In contrast, when φ_g is freely estimated, we obtain $\alpha_g = -0.86$, resulting in a higher multiplier $\Delta y/\Delta g = 1.11$.

F Proof of Proposition 3

To prove the first part of Proposition 3, we start by characterizing the deterministic steady state. The latter is defined in terms of detrended variables. To be more specific, we induce stationarity according to the following formulas

$$c_t^s \equiv c_t \mathrm{e}^{-z_t}, \;\; x_t^s \equiv x_t \mathrm{e}^{-z_t}, \;\; g_t^s \equiv g_t \mathrm{e}^{-z_t}, \;\; y_t^s \equiv y_t \mathrm{e}^{-z_t}, \;\; k_{t+1}^s \equiv k_{t+1} \mathrm{e}^{-z_t}.$$

Using these definitions and the equilibrium conditions, the model steady state is solution to the system of equations

$$\bar{c}^s + \bar{x}^s + \bar{g}^s = \bar{y}^s, \tag{F.18}$$

$$\bar{y}^s = \left(\frac{\bar{k}^s}{\gamma_z}\right)^\theta \bar{n}^{1-\theta}, \tag{F.19}$$

$$\frac{1}{\bar{c}^s + \alpha_g \bar{g}^s}(1-\theta)\bar{y}^s = \eta(1-\beta\phi)\bar{n}^{(1+\nu)(1-\phi)}, \tag{F.20}$$

$$1 = \frac{\beta}{\gamma_z}\left(1 - \delta + \frac{\theta\gamma_z\bar{y}^s}{\bar{k}^s}\right), \tag{F.21}$$

$$\bar{x}^s = \left(1 - \frac{1-\delta}{\gamma_z}\right)\bar{k}^s. \tag{F.22}$$

From (F.21) and (F.22), one can solve for the capital-output and investment-output ratios

$$\frac{\bar{y}^s}{\bar{k}^s} = \frac{\gamma_z - \beta(1-\delta)}{\beta\theta\gamma_z},$$

$$s_x \equiv \frac{\bar{x}^s}{\bar{y}^s} = \frac{\beta\theta(\gamma_z - 1 + \delta)}{\gamma_z - \beta(1-\delta)}.$$

It follows from these relations that s_x and $\bar{y}^s/\bar{k}^s$ do not depend on $\bar{g}^s$. Thus, the ratio $\bar{y}^s/\bar{n}$ does not either depend on $\bar{g}^s$.

Now, differentiating equation (F.18) with respect to $\bar{g}^s$ yields

$$\frac{\mathrm{d}\bar{c}^s}{\mathrm{d}\bar{g}^s} = (1 - s_x)\frac{\mathrm{d}\bar{y}^s}{\mathrm{d}\bar{g}^s} - 1. \tag{F.23}$$

Differentiating equation (F.20), one obtains

$$(s_c + \alpha_g s_g)\frac{\mathrm{d}\bar{y}^s}{\mathrm{d}\bar{g}^s} = (1+\nu)(1-\phi)(s_c + \alpha_g s_g)\frac{\bar{y}^s}{\bar{n}}\frac{\mathrm{d}\bar{n}}{\mathrm{d}\bar{g}^s} + \frac{\partial\bar{c}^s}{\partial\bar{g}^s} + \alpha_g,$$

where

$$s_c \equiv \frac{\bar{c}^s}{\bar{y}^s}.$$

Now, since the ratio $\bar{y}^s/\bar{n}$ does not depend on $\bar{g}^s$, it must be the case that

$$\frac{1}{\bar{n}}\frac{\mathrm{d}\bar{n}}{\mathrm{d}\bar{g}^s} = \frac{1}{\bar{y}^s}\frac{\mathrm{d}\bar{y}^s}{\mathrm{d}\bar{g}^s},$$

so that the previous equation rewrites

$$(s_c + \alpha_g s_g)[1 - (1+\nu)(1-\phi)]\frac{\mathrm{d}\bar{y}^s}{\mathrm{d}\bar{g}^s} = \frac{\mathrm{d}\bar{c}^s}{\mathrm{d}\bar{g}^s} + \alpha_g.$$

Using equation (F.23) and Definition 1, one arrives at

$$\frac{\Delta y}{\Delta g} = \frac{1-\alpha_g}{1 - s_x + \mu[1 - s_x - s_g(1-\alpha_g)]},$$

where we made use of the identity $s_c = 1 - s_x - s_g$.

To prove the second part of Proposition 3, differentiate the long-run GSM with respect to α_g, which implies

$$\frac{\partial}{\partial \alpha_g}\left(\frac{\Delta y}{\Delta g}\right) = -\frac{(1-s_x)(1+\mu)}{(1-s_x+\mu[1-s_x-s_g(1-\alpha_g)])^2} < 0.$$

This completes the proof.

G Results from Simulated Data

In this section, we expound additional results based on simulated data.

G.1 Simulation results on the estimation bias for the degree of Edgeworth complementarity

Based on our previous results, one can suspect that the greater α_g obtained under model (3) is the outcome of a misspecification bias. Indeed, we previously saw that omitting φ_g always increases the estimated value of α_g. In the simple model considered in the first section, we were able to formally show the existence of such a bias. In our DSGE framework, no such analytical results is available, though the same economic forces seem to be at play when we estimate our model on actual data.

In this appendix, we resort to simulation techniques in order to investigate whether the negative link between α_g and φ_g is idiosyncratic to our sample. In addition, resorting to simulation enables us to investigate whether φ_g can be estimated to non-zero values even in a world where no such mechanism exists (an exercise that we can hardly perform on actual data).

To investigate these points, we develop a controlled experiment in which we use our model as our DGP, using the estimated values reported in table 1. More specifically, using model (4) as our DGP we first want to make sure that (i) estimating specification (4) on simulated data delivers consistent parameter estimates and (ii) estimating specification (3) on the exact same simulated data yields severely biased estimates of α_g. To complement on this, we also run the symmetric estimations in which we use model (3) as our DGP and successively estimate specifications (3) and (4) on simulated data. In this case, the crucial point is to check whether our estimation procedure is able to properly reject a policy feedback rule when no such rule exists in simulated data. Our Monte Carlo simulations are run as follows: using either model (4) or model (3), we generate 1000 samples of observables $(\Delta \log(y_t), \log(n_t), \Delta \log(c_t), \Delta \log(g_t)$, with the same sample size as actual data, after having eliminated 800 initial observations, thus ensuring that initial conditions do not contaminate our estimation results. To do so, the four structural shocks innovations are drawn from independent Gaussian distributions with zero mean and unit variance. On each simulated sample, we estimate specifications (3) and (4) and thus generate a population of estimated parameters.

Table 8. Simulation Results

	DGP: Specification (4) $\alpha_g \neq 0, \varphi_g = 0$			DGP: Specification (3) $\alpha_g \neq 0, \varphi_g \neq 0$		
		Estimated Models			Estimated Models	
Parameters	True Value	(4)	(3)	True Value	(4)	(3)
α_g	-0.9452	-0.9095	-0.5945	-0.6340	-0.6100	-0.6134
φ_g	0.6117	0.6140	—	—	-0.0075	—
ϕ	0.4110	0.4052	0.3848	0.3766	0.3845	0.3845
γ_z	1.0044	1.0045	1.0043	1.0043	1.0044	1.0044
ρ_g	0.9756	0.9729	0.9181	0.9535	0.9455	0.9459
ρ_a	0.9834	0.9736	0.9572	0.9795	0.9698	0.9699
ρ_b	0.8399	0.8302	0.8093	0.8366	0.8213	0.8215
σ_z	0.0107	0.0107	0.0107	0.0107	0.0107	0.0107
σ_g	0.0119	0.0118	0.0137	0.0129	0.0128	0.0129
σ_a	0.0133	0.0123	0.0096	0.0123	0.0113	0.0113
σ_b	0.0266	0.0265	0.0277	0.0273	0.0272	0.0272

Notes: Simulation results obtained from 1000 replications. Model (4): $\alpha_g \neq 0$ and $\varphi_g \neq 0$; Model (3): $\alpha_g \neq 0$ and $\varphi_g = 0$. In each case, we report the average value of parameters across simulations.

Table 8 reports the simulation results when using either specifications (4) or (3) as DGP and/or estimated model. We first check whether estimating model (4) on data simulated from model (4) yields consistent parameter estimates. It turns out that this is the case. Indeed, we see from table 8 that the average parameters estimates almost coincide with the true ones. Now, consider what happens when estimating model (3) on data simulated from model (4). In this case, all the parameters linked to government policy (α_g, ρ_g, σ_g) turn out to be biased. This is particularly striking when it comes to α_g, the average value of which is almost twice as small (in absolute term) as the true one. Interestingly, the average estimated value of α_g from our simulation experiment is very similar to what obtains from actual data when estimating model (3).

Consider now what happens when using specification (3) as our DGP. The results are reported in table 8. We first check whether estimating specification (3) on data simulated from model (3) yields consistent estimates. This again turns out to be the case. Now, consider what happens when estimating model (4) on data simulated from model (3). Basically, this procedure is able to recover the true parameters on average. This is particularly striking when it comes to the feedback parameter φ_g, which is zero on average. Recall that the latter does not exist in model (3), the DGP used for this simulation experiment, and appears only in model (4). This implies that a significant φ_g on actual data does not seem to be an artifact of our particular sample.

G.2 Simulation results on countercyclical policy

In our robustness analysis, we considered several alternative government spending rules and used simple likelihood comparisons to motivate why we selected our benchmark specification. In particular, we used this approach to discriminate between our benchmark specification and an alternative rule in which the endogenous component of detrended government expenditures react to detrended output (in relative deviation from its steady-state value). This alternative rule is widespread in the literature and has a very important implication: using it, we would most definitely conclude that governement spending is procyclical, in stark contrast with what obtains under our preferred specification. In the remainder, our benchmark is labelled (4) and the alternative specification is labelled (A), as in our analysis of alternative fiscal rules.

In this section, we consider an alternative empirical validation exercise which consists in resorting to an encompassing criterion.[22] Under this criterion, a specification must not just be judged based on the associated likelihood. It must also be able to predict the results based on an opposing model (specification (A) in our case). If one of the two views fails this encompassing test, the one that passes should be preferred.

Table 9. Simulation Results – Encompassing test

	DGP: Specification (4)			DGP: Specification (4')		
		Estimated Models			Estimated Models	
Parameters	True Value	(4)	(A)	True Value	(4)	(A)
α_g	-0.9452	-0.9095	-0.5498	-0.5773	-0.8950	-0.5676
φ_g	0.6117	0.6140	-0.3791	-0.4866	0.0681	-0.4854
ϕ	0.4110	0.4052	0.4051	0.4072	0.3702	0.4149
γ_z	1.0044	1.0045	1.0043	1.0043	1.0043	1.0043
ρ_g	0.9756	0.9729	0.9116	0.9547	0.9730	0.9478
ρ_a	0.9834	0.9736	0.9564	0.9776	0.9703	0.9686
ρ_b	0.8399	0.8302	0.8024	0.8396	0.8425	0.8158
σ_z	0.0107	0.0107	0.0107	0.0107	0.0107	0.0107
σ_g	0.0119	0.0118	0.0134	0.0123	0.0128	0.0122
σ_a	0.0133	0.0123	0.0094	0.0119	0.0131	0.0112
σ_b	0.0266	0.0265	0.0275	0.0273	0.0268	0.0271

Notes: Simulation results obtained from 1000 replications. Fiscal rule under model (4): $\log(\tilde{g}_t) = -\varphi_g(\Delta\log(y_t) - \log(\gamma_z))$; Fiscal rule under model (A): $\log(\tilde{g}_t) = -\varphi_g(\log(y_t^s) - \log(\bar{y}^s))$, where, $y_t^s = y_t \mathrm{e}^{-z_t}$ and $\bar{y}^s$ is the steady-state value of y_t^s. In each case, we report the average value of parameters across simulations.

We proceed as before. Using either (4) or (A) as our DGP, we estimate specifications (4) and (A) on simulated data. Our Monte Carlo simulations are conducted in the exact same way as before.

[22] See Christiano, Eichenbaum and Vigfusson (2003) for an early example of this approach.

These simulation results are reported in table 9. For each DGP, the table reports the "true value", i.e. the parameter values obtained when estimating the model on actual data and used to simulate the model. Consider first what happens when specification (4) is our DGP. As before, we see that when estimating (4) on data simulated from (4), our procedure recovers parameter values close to their true values (albeit with a small sample bias). More interestingly, when we estimate specification (A) on data simulated from (4), we obtain parameter values close to what obtains when estimating (A) on actual data. Thus, specification (4) encompasses specification (A). Now, let us proceed the other way around and use (A) as our DGP. Clearly, if (A) were to be estimated on data simulated from (A), we would on average recover the true parameter values (once again, with a small sample bias). However, if (4) were to be estimated on data simulated from (A), we would not recover the parameter values obtained when estimating (4) on actual data. Thus, (A) fails to encompass (4).

To sum up, specification (4) encompasses specification (A) while the converse is not true. This is yet another confirmation that our benchmark specification is a good description of the data. It also serves the purpose of reinforcing our empirical results on the countercyclicality of the government spending rule.

H The Smets-Wouters model

This section describes our medium-scale DSGE model, which is similar to Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007). The model combines a neoclassical growth core with several shocks and frictions. It includes features such as habit formation, investment adjustment costs, variable capital utilization, monopolistic competition in goods and labour markets, and nominal price and wage rigidities.

The economy is populated by five classes of agents: producers of a final good, intermediate goods producers, households, employment agencies, and the government.

H.1 Household sector

H.1.1 Employment agencies

As in Erceg, Henderson and Levin (2000), each household, indexed by $j \in [0,1]$, is a monopolistic supplier of specialized labor $n_t(j)$. At every point in time t, a large number of competitive "employment agencies" combine households' labor into a homogenous labor input n_t sold to intermediate firms, according to

$$n_t = \left(\int_0^1 n_t(j)^{\frac{1}{\lambda_{w,t}}} \mathrm{d}j \right)^{\lambda_{w,t}}, \tag{H.24}$$

where $\lambda_{w,t}=\lambda_w \mathrm{e}^{\varepsilon_{w,t}}$, λ_w is the desired steady-state wage markup over the marginal rate of substitution between consumption and leisure, and the markup shock $\varepsilon_{w,t}$ is assumed to evolve according to

$$\varepsilon_{w,t}=\rho_w\varepsilon_{w,t-1}+\sigma_w\zeta_{w,t},\quad \zeta_{w,t}\sim Niid(0,1).$$

Profit maximization by the perfectly competitive employment agencies implies the labor demand function

$$n_t(j)=\left(\frac{W_t(j)}{W_t}\right)^{-\frac{\lambda_{w,t}}{\lambda_{w,t}-1}} n_t, \tag{H.25}$$

where $W_t(j)$ is the wage paid by the employment agencies to the household supplying labor variety j, while

$$W_t\equiv\left(\int_0^1 W_t(j)^{\frac{1}{\lambda_{w,t}-1}}\mathrm{d}j\right)^{\lambda_{w,t}-1} \tag{H.26}$$

is the wage paid to homogenous labor.

H.1.2 Household's preferences

Household j has preferences given by

$$\mathrm{E}_t\sum_{s=0}^{\infty}\beta^s \mathrm{e}^{\varepsilon_{c,t+s}}\left[\log(\tilde{c}_{t+s}-h\tilde{c}_{t+s-1})-\frac{\eta}{1+\nu}n_{t+s}(j)^{1+\nu}\right], \tag{H.27}$$

where E_t denotes the mathematical expectation operator conditional upon information available at t, $\tilde{c}_t\equiv c_t+\alpha_g g_t$, c_t denotes consumption, g_t is government expenditures, $n_t(j)$ is labor of type j, β is the subjective discount factor, $h\in[0,1]$ denotes the degree of habit formation, $\nu>0$ is the elasticity of labor supply with respect to the real wage, η is a scale parameter, and $\varepsilon_{c,t}$ is a preference shock evolving according to

$$\varepsilon_{c,t}=\rho_c\varepsilon_{c,t-1}+\sigma_c\zeta_{c,t},\quad \zeta_{c,t}\sim Niid(0,1).$$

As we explain below, households are subject to idosyncratic shocks about whether they are able to re–optimize their wage. Hence, the above described problem makes the choices of wealth accumulation contingent upon a particular history of wage rate decisions, thus leading to households heterogeneity. For the sake of tractability, we assume that the momentary utility function is separable across consumption and leisure. Combining this with the assumption of a complete set of contingent claims market, all the households will make the same choices regarding consumption and will only differ by their wage rate and supply of labor. This is directly reflected in our notations.

Household j's period budget constraint is given by

$$P_t(c_t+x_t)+T_t+B_t\leq B_{t+1}/R_t+Q_t(j)+D_t+W_t(j)n_t(j)+P_t r_t^k u_t\bar{k}_{t-1}-P_t\vartheta(u_t)\bar{k}_{t-1}, \tag{H.28}$$

where x_t is investment, T_t denotes nominal lump–sum taxes (transfers if negative), B_t is the quantity one-period riskless nominal bond acquired at t and maturing at $t+1$, R_t is the nominal interest rate on bonds, $Q_t(j)$ is the net cash flow from household j's portfolio of state contingent securities, D_t is the equity payout received from the ownership of firms, and r_t^k is the real rental rate of capital. The capital utilization rate u_t transforms physical capital $\bar{k}_t$ into the service flow of effective capital k_t according to

$$k_t = u_t \bar{k}_{t-1}, \tag{H.29}$$

and the effective capital is rented to intermediate firms at the real rental rate r_t^k. The costs of capital utilization per unit of capital is given by the convex function $\vartheta(u_t)$. We assume that $u = 1$, $\vartheta(1) = 0$, and we define

$$\eta_u \equiv \frac{\vartheta''(1)/\vartheta'(1)}{1 + \vartheta''(1)/\vartheta'(1)}.$$

Later, we estimate η_u rather than the elasticity $\vartheta''(1)/\vartheta'(1)$ to avoid convergence issues.

The physical capital accumulates according to

$$\bar{k}_t = (1-\delta)\,\bar{k}_{t-1} + \mathrm{e}^{\varepsilon_{x,t}}\left(1 - S\left(\frac{x_t}{x_{t-1}}\right)\right)x_t \tag{H.30}$$

where δ is the depreciation rate of capital, $S(\cdot)$ is an adjustment cost function which satisfies $S(\gamma_z) = S'(\gamma_z) = 0$ and $S''(\gamma_z) = \eta_k > 0$, γ_z is the steady-state growth rate of technology, and $\varepsilon_{x,t}$ is an investment shock, evolving according to

$$\varepsilon_{x,t} = \rho_x \varepsilon_{x,t-1} + \sigma_x \zeta_{x,t}, \quad \zeta_{x,t} \sim Niid(0,1).$$

Households set nominal wages in a staggered fashion, following Calvo (1983). In each period, a fraction α_w of households cannot choose their wage optimally, but adjust it to keep up with the increase in the general wage level in the previous period according to the indexation rule

$$W_t(j) = \pi^{1-\gamma_w}\pi_{t-1}^{\gamma_w} W_{t-1}(j), \tag{H.31}$$

where $\pi_t \equiv P_t/P_{t-1}$ represents the gross inflation rate, π is steady–state (or trend) inflation and the coefficient $\gamma_w \in [0,1]$ is the degree of indexation to past wages. The remaining fraction of households chooses instead an optimal wage, to maximize (H.27), subject to the labor demand function (H.25).

H.2 Business sector

H.2.1 Final good producers

At every point in time t, perfectly competitive firms produce a final good y_t by combining a continuum of intermediate goods $y_t(i)$, $i \in [0,1]$, according to the technology

$$y_t = \left(\int_0^1 y_t(i)^{\frac{1}{\lambda_{p,t}}}\mathrm{d}i\right)^{\lambda_{p,t}}, \tag{H.32}$$

where $\lambda_{p,t} = \lambda_p \mathrm{e}^{\varepsilon_{p,t}}$, λ_p is the desired steady–state price markup over the marginal cost of intermediate firms, and the markup shock $\varepsilon_{p,t}$ is assumed to evolve according to

$$\varepsilon_{p,t} = \rho_p \varepsilon_{p,t-1} + \sigma_p \zeta_{p,t}, \quad \zeta_{p,t} \sim Niid(0,1).$$

Final good producing firms take their output price, P_t, and their input prices, $P_t(i)$, as given and beyond their control. Profit maximization implies the following first-order condition

$$y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\lambda_{p,t}}{\lambda_{p,t}-1}} y_t. \tag{H.33}$$

Integrating (H.33) and imposing (H.32), we obtain the following relationship between the final good and the prices of the intermediate goods

$$P_t \equiv \left(\int_0^1 P_t(i)^{\frac{1}{\lambda_{p,t}-1}} \mathrm{d}i\right)^{\lambda_{p,t}-1}. \tag{H.34}$$

H.2.2 Intermediate-goods firms

Intermediate good $i \in [0,1]$ is produced by a monopolist firm using the following production function

$$y_t(i) = k_t(i)^\theta \left(\mathrm{e}^{z_t} h_t(i)\right)^{1-\theta} - \mathrm{e}^{z_t} F, \tag{H.35}$$

where θ denotes the capital share, $k_t(i)$ and $h_t(i)$ denote the amounts of capital and effective labor used by firm i, F is a fixed cost of production that ensures that profits are zero in steady state, and z_t is an exogenous labor–augmenting productivity, evolving according to

$$z_t = \log(\gamma_z) + z_{t-1} + \varepsilon_{z,t}$$

where $\varepsilon_{z,t}$ is a permanent productivity shock

$$\varepsilon_{z,t} = \rho_z \varepsilon_{z,t-1} + \sigma_z \zeta_{z,t}, \quad \zeta_{z,t} \sim Niid(0,1).$$

In addition, we assume that intermediate firms rent capital and labor in perfectly competitive factor markets.

Intermediate firms set prices in a staggered fashion, following Calvo (1983). In each period, a fraction α_p of firms cannot choose their price optimally, but adjust it to keep up with the increase in the general price level in the previous period according to the indexation rule

$$P_t(i) = \pi^{1-\gamma_p} \pi_{t-1}^{\gamma_p} P_{t-1}(i), \tag{H.36}$$

where the coefficient $\gamma_p \in [0,1]$ indicates the degree of indexation to past prices. The remaining fraction of firms chooses its price $P_t^\star(i)$ optimally, by maximizing the present discounted value of future profits

$$\mathrm{E}_t \sum_{s=0}^{\infty} (\beta\alpha_p)^s \frac{\Lambda_{t+s}}{\Lambda_t} \left\{\Pi_{t,t+s}^p P_t^\star(i) y_{t+s}(i) - \left[W_{t+s} h_{t+s}(i) - r_{t+s}^k k_{t+s}(i)\right]\right\} \tag{H.37}$$

where Λ_t denotes the Lagrange multiplier on household j's nominal budget constraint and

$$\Pi^p_{t,t+s} \equiv \begin{cases} \prod_{\nu=1}^{s} \pi^{1-\gamma_p} \pi_{t+v-1}^{\gamma_p} & s > 0 \\ 1 & s = 0, \end{cases} \tag{H.38}$$

subject to the demand from final goods firms given by equation (H.33) and the production function (H.35).

H.3 Public sector

The government faces the budget constraint

$$P_t g_t + B_t = T_t + \frac{B_{t+1}}{R_t}.$$

The government spending rule is, as before, given by

$$g_t \mathrm{e}^{-z_t} = \bar{g}^s \tilde{g}_t \mathrm{e}^{\varepsilon_{g,t}},$$

where $\bar{g}^s$ denotes the deterministic steady–state value of $g_t \mathrm{e}^{-z_t}$. The endogenous policy component $\tilde{g}_t$ obeys

$$\log(\tilde{g}_t) = -\varphi_g \left(\Delta \log(y_t) - \log(\gamma_z)\right), \tag{H.39}$$

and the exogenous component evolves according to

$$\varepsilon_{g,t} = \rho_g \varepsilon_{g,t-1} + \sigma_g \zeta_{g,t}, \quad \zeta_{g,t} \sim Niid(0,1).$$

The short–run nominal interest rate R_t is set according to

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\phi_R} \left[\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi} \left(\frac{y_t}{y_{p,t}}\right)^{\phi_y}\right]^{(1-\phi_R)} \left(\frac{y_t y_{p,t-1}}{y_{p,t} y_{t-1}}\right)^{\phi_{\Delta y}} \mathrm{e}^{\sigma_R \zeta_{R,t}} \tag{H.40}$$

where R is the steady state of the gross nominal interest rate and $\zeta_{R,t}$ is $Niid\,(0,1)$. The monetary authorities follow a generalized Taylor rule by gradually adjusting the nominal rate in response to inflation and the output gap, defined as the ratio of actual to potential output (*i.e.* the level of output that would prevail under flexible prices and wages and constant elasticity of substitution among intermediate goods and labor types). In addition, there is a short-run feedback from the change in the output gap.

H.4 Market clearing

Market clearing condition on final goods market is given by

$$y_t = c_t + x_t + g_t + \vartheta\,(u_t)\,\bar{k}_{t-1}. \tag{H.41}$$

$$\Delta_{p,t} y_t = \left(u_t \bar{k}_{t-1}\right)^{\theta} \left(\mathrm{e}^{z_t} n_t\right)^{1-\theta} - \mathrm{e}^{z_t} F \tag{H.42}$$

where $\Delta_{p,t} = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\lambda_p \varepsilon_{p,t}}{\lambda_p \varepsilon_{p,t-1}}} \mathrm{d}i$ is a measure of the price dispersion.

H.5 Non-Ricardian Agents

As in Cogan et al. (2010), we also consider a version of the SW model featuring a fraction ω of non-Ricardian agents. These agents do not have access to financial markets and simply consume their disposable income in each and every period. Disposable income, in turn, equals wage receipts net of lump-sum taxes.

H.6 Detailed Estimation Results

Four parameters are calibrated before estimation. These are: the discount factor $\beta = 0.99$, the depreciation rate $\delta = 0.025$, the steady-state wage markup $\lambda_w = 0.15$, and the steady-state share of government spending in output $s_g = 0.20$.

In table 10, we report our estimation results in the SW model. As before, we consider analogs to specifications (3) and (4). In (3), we impose $\varphi_g = 0$ while in (4) this parameter is freely estimated. In both specifications, we also impose that the mass of non-Ricardian agents be zero, i.e. $\omega = 0$. In specifications (3') and (4'), we freely estimate ω, while considering $\varphi_g = 0$ in (3') and $\varphi_g \neq 0$ in (4').

Table 10. Estimated parameters in the SW model

	Prior	Posterior			
		SW Specification		SW with NR agents	
		Model 3	Model 4	Model 3'	Model 4'
ω	$\mathscr{B}$ [0.50,0.2]	–	–	0.165 [0.095,0.231]	0.149 [0.085,0.211]
φ_g	$\mathscr{U}$ [0,0.65]	–	0.752 [0.545,0.964]	–	0.628 [0.528,0.919]
α_g	$\mathscr{U}$ [0,1.30]	-0.199 [-0.454,0.040]	-0.626 [-0.914,-0.332]	-0.870 [-1.201,-0.552]	-1.133 [-1.443,-0.819]
ν	$\mathscr{N}$ [2,0.75]	2.829 [1.882,3.782]	2.952 [1.991,3.895]	3.054 [2.062,4.030]	3.356 [2.184,4.094]
h	$\mathscr{B}$ [0.6,0.1]	0.642 [0.579,0.709]	0.634 [0.557,0.709]	0.571 [0.484,0.666]	0.585 [0.480,0.693]
η_u	$\mathscr{B}$ [0.5,0.1]	0.544 [0.395,0.699]	0.665 [0.487,0.852]	0.587 [0.440,0.736]	0.608 [0.444,0.770]
η_k	$\mathscr{N}$ [4,1]	6.066 [4.698,7.438]	5.450 [3.886,6.979]	5.747 [4.392,7.081]	5.624 [4.260,6.973]
θ	$\mathscr{N}$ [0.33,0.05]	0.174 [0.144,0.204]	0.204 [0.173,0.234]	0.179 [0.154,0.205]	0.200 [0.172,0.229]
$\log(\gamma_z)$	$\mathscr{N}$ [0.5,0.1]	0.485 [0.399,0.574]	0.458 [0.360,0.548]	0.476 [0.392,0.559]	0.461 [0.373,0.548]
α_p	$\mathscr{B}$ [0.66,0.1]	0.910 [0.884,0.941]	0.876 [0.834,0.925]	0.906 [0.865,0.947]	0.884 [0.844,0927]
α_w	$\mathscr{B}$ [0.66,0.1]	0.906 [0.867,0.945]	0.889 [0.852,0.929]	0.921 [0.884,0.959]	0.907 [0.872,0.944]
γ_p	$\mathscr{B}$ [0.5,0.15]	0.083 [0.028,0.135]	0.089 [0.032,0.144]	0.074 [0.026,0.121]	0.083 [0.029,0.135]
γ_w	$\mathscr{B}$ [0.5,0.15]	0.703 [0.566,0.842]	0.725 [0.592,0.864]	0.712 [0.583,0.851]	0.724 [0.595,0.854]
λ_p	$\mathscr{N}$ [0.15,0.1]	0.117 [0.031,0.204]	0.226 [0.136,0.322]	0.107 [0.021,0.192]	0.196 [0.109,0.283]

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	Prior	Posterior			
		SW Specification		SW with NR agents	
		Model 3	Model 4	Model 3'	Model 4'
ϕ_R	$\mathscr{B}$ [0.6,0.2]	0.805 [0.769,0.841]	0.800 [0.767,0.835]	0.820 [0.784,0.856]	0.819 [0.788,0.852]
ϕ_π	$\mathscr{N}$ [1.7,0.3]	1.589 [1.364,1.805]	1.528 [1.338,1.720]	1.657 [1.421,1.902]	1.597 [1.380,1.788]
ϕ_y	$\mathscr{N}$ [0.125,0.05]	0.067 [0.026,0.113]	0.018 [−0.011,0.056]	0.063 [0.003,0.108]	0.018 [−0.012,0.051]
$\phi_{\Delta y}$	$\mathscr{N}$ [0.125,0.05]	0.287 [0.239,0.338]	0.252 [0.199,0.302]	0.281 [0.232,0.331]	0.252 [0.201,0.301]
ρ_w	$\mathscr{B}$ [0.6,0.2]	0.253 [0.144,0.366]	0.244 [0.133,0.352]	0.277 [0.159,0.387]	0.272 [0.159,0.384]
ρ_b	$\mathscr{B}$ [0.6,0.2]	0.759 [0.689,0.832]	0.788 [0.710,0.872]	0.824 [0.746,0.899]	0.842 [0.757,0.931]
ρ_x	$\mathscr{B}$ [0.6,0.2]	0.758 [0.662,0.839]	0.888 [0.754,0.997]	0.780 [0.683,0.873]	0.873 [0.764,0.977]
ρ_p	$\mathscr{B}$ [0.6,0.2]	0.858 [0.781,0.934]	0.871 [0.783,0.964]	0.898 [0.830,0.987]	0.873 [0.788,0.958]
ρ_z	$\mathscr{B}$ [0.3,0.2]	0.251 [0.129,0.377]	0.180 [0.062,0.298]	0.184 [0.059,0.297]	0.132 [0.023,0.228]
ρ_g	$\mathscr{B}$ [0.6,0.2]	0.987 [0.977,0.998]	0.978 [0.958,0.998]	0.989 [0.980,0.999]	0.984 [0.970,0.999]
σ_w	$\mathscr{IG}$ [0.1,2]	0.231 [0.195,0.267]	0.239 [0.202,0.275]	0.223 [0.186,0.260]	0.230 [0.193,0.266]
σ_b	$\mathscr{IG}$ [0.1,2]	0.041 [0.029,0.052]	0.041 [0.026,0.057]	0.046 [0.032,0.060]	0.046 [0.027,0.064]
σ_x	$\mathscr{IG}$ [0.1,2]	0.429 [0.353,0.507]	0.424 [0.310,0.537]	0.437 [0.359,0.512]	0.407 [0.325,0.482]
σ_p	$\mathscr{IG}$ [0.1,2]	0.049 [0.033,0.065]	0.055 [0.036,0.073]	0.042 [0.027,0.056]	0.053 [0.034,0.070]
σ_z	$\mathscr{IG}$ [0.1,2]	0.860 [0.772,0.951]	0.933 [0.824,1.041]	0.848 [0.762,0.935]	0.905 [0.804,1.004]
σ_g	$\mathscr{IG}$ [0.1,2]	1.116 [1.012,1.214]	1.207 [1.078,1.330]	1.114 [1.013,1.213]	1.187 [1.069,1.307]
σ_R	$\mathscr{IG}$ [0.1,2]	0.229 [0.208,0.249]	0.222 [0.202,0.243]	0.227 [0.207,0.247]	0.222 [0.202,0.241]
$\mathscr{L}$		−1298.946	−1277.683	−1296.944	−1275.890

Notes: Sample period: 1960:1-2007:4. Notice that all the data have been multiplied by 100. In the column labelled Prior, $\mathscr{B}$, $\mathscr{N}$, $\mathscr{U}$, $\mathscr{IG}$ denote Beta, Normal, Uniform, and Inverse Gamma prior densities, respectively. The figures in brackets are the prior mean and the prior standard deviation. In the columns labelled Posterior, the figures correspond to the posterior mean and the figures in brackets below the posterior mean indicate the posterior 90% interval. The model specification are as follows. In (3), we impose $\omega = 0$, $\varphi_g = 0$, and $\alpha_g \neq 0$; in (4), we impose $\omega = 0$, $\varphi_g \neq 0$, and $\alpha_g \neq 0$; in (3') we impose $\omega \neq 0$, $\varphi_g = 0$, and $\alpha_g \neq 0$; in (4'), we impose $\omega \neq 0$, $\varphi_g \neq 0$, and $\alpha_g \neq 0$. Finally, $\mathscr{L}$ denotes the marginal log-likelihood.