Pareto Efficient Income Taxation when People are Inequality Averse**

Thomas Aronsson and Olof Johansson-Stenman+*

Abstract

This paper deals with tax policy responses to inequality aversion by examining the first-best Pareto efficient marginal tax structure when people are inequality averse. In doing so, we distinguish between five different widely used mesures of inequality. The results show that empirically and experimentally quantified degrees of inequality aversion have potentially very strong implications for Pareto efficient marginal income taxation. It also turns out that the exact type of inequality aversion (self-centered vs. non-self-centered), and measures of inequality used, matter a great deal for the structure of efficient income taxation. For example, based on simulation results the preferences suggested by Fehr and Schmidt (1999) imply monotonically increasing marginal income taxes, with large negative marginal tax rates for low-income individuals and large positive marginal tax rates for high-income individuals. In contrast, the often considered similar model by Bolton and Ockenfels (2000) implies close to zero marginal income tax rates for all.

Keywords: Pareto efficient taxation, Inequality aversion, Inequity aversion, Self-centered inequality aversion, Non-self-centered inequality aversion, Fehr and Schmidt preferences, Bolton and Ockenfels preferences.

JEL Classification: D03, D62, H23.

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* Address: Department of Economics, Umeå School of Business and Economics, Umeå University, SE – 901 87 Umeå, Sweden. E-mail: Thomas.Aronsson@econ.umu.se

+ Address: Department of Economics, School of Business, Economics and Law, University of Gothenburg, SE – 405 30 Gothenburg, Sweden. E-mail: Olof.Johansson@economics.gu.se

1. Introduction

There are several reasons for a government to tax its citizens, including reasons based on redistribution objectives and revenue collection to fund public expenditure. Most optimal tax models dealing with income redistribution assume that the government wants to redistribute from the well-off to the not so well-off, e.g., since low-income individuals are assumed to have higher marginal utility of consumption than high-income ones. We then often say that the government or the social planner is inequality averse.

At the same time, *individuals* are generally not assumed to care about inequality *per se*. That is, their utility is typically modelled to depend solely on their own private and public consumption, as well as on their own leisure time, and hence not on any measure of inequality. This is despite the fact that there is now ample evidence based on experimental research suggesting that people are inequality averse, in the sense that they prefer a more equitable allocation to an allocation which is in their narrow material self-interest; see, e.g., Fehr and Schmidt (1999) and Bolton and Ockenfels (2000).[1] In the present paper we will take this experimental evidence seriously, and assume that people do not only derive utility from their own consumption and leisure time (as in standard models of optimal taxation); they also prefer a more equal distribution of consumption to a less equal one, *ceteris paribus*. The purpose is to examine the optimal tax policy implications of inequality aversion and, more specifically, how inequality aversion affects the structure of marginal income taxation. In doing so, we also distinguish between self-centered inequality aversion (where each individual's aversion towards inequality is based on a comparison between his/her own consumption and other people's consumption) and non-self-centered inequality aversion.

As far as we know, our study is the first to derive efficient income tax policies based on models where people are inequality averse. This is in sharp contrast to the by now rich literature on different aspects of optimal taxation based on another kind of interdependent utility structure, where people instead of caring about inequality have preferences for their own relative consumption or relative income. That is, people prefer to have more than others

[1] There is of course also much other empirical evidence for other-regarding behavior, for example with respect to tax compliance (Pommerehne and Weck-Hannemann, 1996, Andreoni et al., 1998;), voting behavior and political preferences (Mueller, 1998; Fong, 2001; Carlsson et al., 2010), and charitable giving (List, 2011; Andreoni and Payne, 2013).

and dislike having less. This literature shows that relative consumption concerns have profound effects on the optimal tax structure by implying much higher marginal labor income and/or commodity tax rates than in standard models, as well as justifies capital income taxation both on efficiency grounds and for redistributive reasons.[2]

Although there are important similarities between preferences based on inequality aversion and preferences for relative consumption, since the individuals' consumption choices generate externalities in both cases, there are important differences too. In particular, when people care about their relative consumption compared to others, they typically impose negative externalities on one another. When people are inequality averse, on the other hand, the consumption externalities may be either positive or negative, depending on whether a consumption increase of a particular individual contributes to increase or decrease the inequality that other people care about. As we will see below, the latter also implies that the tax policy implications of different measures of inequality may differ quite much.

In the present paper, we focus on efficiency aspects of inequality aversion, i.e., the tax policy that these aspects motivate. This means that we assume that the government can observe individual ability and thus use ability-specific lump-sum taxes for purposes of redistribution. Our approach has two advantages. First, it allows for a simple characterization of the marginal tax policy incentives caused by inequality aversion *per se*, since all analyses below presuppose that inequality aversion is the only reason for distorting the labor-leisure choice. The insights gained from such a study are particularly useful in this case since there are no earlier studies dealing with the tax policy implications of consumer aversion against inequality. Second, since we aim at examining several different measures of inequality, it admits a straightforward comparison of social costs and corrective tax policies, which further emphasizes the need for a simple baseline model.

Section 2 present a continuous-type model and derives the choice rule for Pareto efficient marginal income taxation for a very general measure of consumption inequality. Based on the results in Section 2, we derive efficient marginal tax rates for two different versions of self-centered inequality aversion in Section 3, namely the ones proposed by Fehr and Schmidt

[2] This literature includes Boskin and Sheshinski (1978), Oswald (1983), Frank (1985a, b, 2005, 2008), Tuomala (1990), Persson (1995), Corneo and Jeanne (1997), Ljungqvist and Uhlig (2000), Ireland (2001), Dupor and Liu (2003), Abel (2005), Aronsson and Johansson-Stenman (2008, 2010, 2015), Wendner (2010, 2014), Alvarez-Cuadrado and Long (2011, 2012), Eckerstorfer and Wendner (2013), and Kanbur and Tuomala (2014).

(1999) and Bolton and Ockenfels (2000) respectively. As explained above, by “self-centered” we mean measures of inequality that are defined as relations between the individual’s own consumption and others’ consumption. Despite that the two models are quite similar, their policy implications are surprisingly different. Section 4 analyzes efficient taxation in economies with non-self-centered inequality aversion. We will focus on three variants of such inequality aversion, where the measures of inequality are given by the Gini coefficient, the coefficient of variation, and the ratio of the absolute mean deviation and mean consumption, respectively. Section 5 concludes that experimentally estimated parameters of inequality aversion, if generalized to the overall economy, motivate substantial marginal income taxes. It is also demonstrated that the exact nature of the inequality aversion measure has profound implications for the efficient marginal income tax structure. Proofs are presented in the Appendix.

2. Pareto Efficiency and Inequality Aversion

Suppose we have a continuous ability distribution without bunching and wholes. The government wants to maximize a Paretian social welfare function such that

$$W = \int_{w_{\min}}^{w_{\max}} \psi\left(u(w)\right) f(w)dw, \tag{1}$$

where $u(\cdot)$ denotes the utility function, which is common to all individuals, while $\psi(\cdot)$ constitutes an increasing transformation of individual utility. We do not assume that this transformation is necessarily concave (or quasi-concave). In fact, we will rather (implicitly) assume that it often gives a higher weight to the utility of high-ability individuals. As such, a natural interpretation is that it reflects the outcome of a political process where different people have different negotiation power. We also assume that each individual cares about the distribution of consumption, but not about the distribution of utility or leisure. Therefore, even though low-ability individuals may dislike the governmental objective function, all individuals will, conditional of this objective function, agree that there are good reasons to obtain a Pareto efficient allocation. Thus, for any distribution of negotiating power in the economy, all individuals agree that Pareto improvements should be made, and hence that the allocation should be Pareto efficient.[3]

Let us assume that the ability distribution results in a continuous equilibrium distribution for private consumption , such that higher ability will always result in higher consumption in equilibrium. We can then write the social objective function as

$$W = \int_{c_{\min}}^{c_{\max}} \psi\left(u(c,z,C)\right) f(c)dc\,, \tag{2}$$

where *c* is own consumption, *z* is own leisure, and C is a (possibly type-specific) measure of the overall consumption distribution (which we will specify further subsequently). Without loss of generality, we normalize the population size to unity such that

$$\int_{w_{\min}}^{w_{\max}} f(w)dw = \int_{c_{\min}}^{c_{\max}} f(c)dc = 1.$$

Individual Behavior

Each individual treats C as exogenous and chooses private consumption, c, and work hours, $l = 1 - z$, to maximize utility, $u(c,z,C)$, subject to his/her budget constraint. For an individual of ability *w*, the budget constraint is given by

$$c = wl - T(wl)\,, \tag{3}$$

in which $T(wl)$ denotes the individual's tax payment (positive or negative). The individual first-order conditions for consumption and work hours can then be combined as follows:

$$MRS_{cz} = \frac{1}{w\left(1 - T'(wl)\right)}\,, \tag{4}$$

where $T'(wl)$ denotes the marginal income tax rate, and the left hand side is the marginal rate of substitution between the individual's own private consumption and leisure, i.e.,

$$MRS_{cz} = \frac{\partial u(c,z,C)}{\partial c} \bigg/ \frac{\partial u(c,z,C)}{\partial z}\,.$$

Social Decision Problem

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[3] If we had instead restricted $\psi(\cdot)$ to be a concave transformation, a first-best optimization with respect to (1) would in general not imply that higher-ability individuals would have higher consumption in equilibrium.

The social optimization problem means choosing private consumption and leisure time (or work hours) for each individual to maximize the social welfare function given in equation (2) subject to a resource constraint for the economy as a whole. In doing so, the social planner also recognizes the relationship between each individual's consumption, *c*, and the measure of the overall distribution of consumption, *C*. This implies the following Lagrangean:

$$\int_{c_{\min}}^{c_{\max}} \psi\left(u(c,z,C)\right) f(c)dc + \lambda\left(\int_{w_{\min}}^{w_{\max}} w(1-z(w))\, f(w)dw - \int_{c_{\min}}^{c_{\max}} c\ f(c)dc\right). \quad (5)$$

Consider the social first-order conditions with respect to consumption and leisure, respectively, for individuals with consumption level $\tilde{c}$, for whom the associated level of leisure and type-specific measure of the overall consumption distribution are $\tilde{z}$ and $\tilde{C}$, respectively. These social first-order conditions can be written as

$$\int_{c_{\min}}^{c_{\max}} \psi'\frac{\partial u(c,z,C)}{\partial C}\frac{\partial C}{\partial \tilde{c}}\, f(c)dc + \tilde{\psi}'\frac{\partial u(\tilde{c},\tilde{z},\tilde{C})}{\partial \tilde{c}}\, f(\tilde{c}) = \lambda f(\tilde{c}), \quad (6)$$

$$\tilde{\psi}'\frac{\partial u(\tilde{c},\tilde{z},\tilde{C})}{\partial \tilde{z}}\, f(\tilde{c}) = \lambda \tilde{w} f(\tilde{c}). \quad (7)$$

where $\tilde{\psi}' = \dfrac{d\psi}{du(\tilde{c},\tilde{z},\tilde{C})}$.

We are now ready to characterize the optimal marginal tax policy for the model set out above, in which we have made no assumption about the preferences for inequality aversion (other than that *C* might be type specific). This general characterization will thus be very useful in later parts of the paper, where the tax policy implications of more specific forms of inequality aversion are addressed. Let

$$MWTP(\tilde{c}) = -\frac{1}{f(\tilde{c})}\frac{\dfrac{\partial u(c,z,C)}{\partial C}\dfrac{\partial C}{\partial \tilde{c}}}{\dfrac{\partial u(c,z,C)}{\partial c}} \quad (8)$$

denote the marginal willingness to pay of an individual with consumption level c for individuals with consumption level $\tilde{c}$ to reduce their consumption.[4] We can then derive the

[4] The normalization choice of consumption for one individual is made for convenience, and is harmless and could be replaced by any number, despite the fact that it may seem a bit strange since the overall population is normalized to one.

following result by combining the private and social first-order conditions in equations (4), (6), and (7):

Lemma 1. *The Pareto efficient marginal income tax rate implemented for individuals with gross income* $\tilde{w}\tilde{l}$ *and consumption* $\tilde{c}$ *is given by*

$$T'(\tilde{w}\tilde{l}) = \int_{c_{\min}}^{c_{\max}} \frac{1-T'(\tilde{w}\tilde{l})}{1-T'(wl)} MWTP(\tilde{c}) f(c) dc . \qquad (9)$$

This tax formula looks almost like a conventional Pigouvian tax, i.e. the sum of all other peoples marginal willingness to pay for avoiding one additional consumption unit from individuals with gross income $\tilde{w}\tilde{l}$ and consumption $\tilde{c}$. The only difference is the weight factor $\left(1-T'(\tilde{w}\tilde{l})\right)/\left(1-T'(wl)\right)$ attached to the measure of marginal willingness to pay on the right hand side.

To see the rationale behind this weight factor, consider first the logic behind a conventional first-best tax of an externality-generating good. In that case, the discrepancy between the social and private marginal value, as reflected by the externality-correcting tax, simply consists of the sum of other people's marginal willingness to pay for the individual not to consume one additional unit of the good. This would have been the case here as well had the first term on the left hand side of equation (6) been the same for everybody, i.e., if the externality were atmospheric.[5] In general, however, the externality examined here is *non-atmospheric*, meaning that the externality generated by consuming one additional unit will typically differ depending on who consumes the additional unit. In this case, the social first-order condition does not imply equalization of the social marginal utility of consumption among consumers, i.e. that $\psi' \partial u(c,z,C)/\partial c$ should be the same for all consumption (and hence ability) levels. Instead, as revealed from (6), what should be equalized is $\psi' \partial u(c,z,C)/\partial c$ plus a term that reflects the value of the marginal externality that the individual's consumption imposes on other people. This, in turn, means that the social

[5] A similar result would follow if we were to introduce a numeraire good that does not generate externalities. The reason is that a government that maximizes a social welfare function, and is able to redistribute without any social cost, will equalize the social marginal utility of consumption of the numeraire good among individuals.

marginal utility of consumption is larger at the optimum for individuals whose consumption generates large negative externalities, and vice versa, which explains the weight factor.

Note that equation (9) can alternatively be written as

$$\frac{T'(\tilde{w}\tilde{l})}{1-T'(\tilde{w}\tilde{l})} = \int_{c_{\min}}^{c_{\max}} \frac{MWTP(\tilde{c})}{1-T'(wl)} f(c)dc .$$

Hence, the ratio of the marginal tax rate and the part of the additional income that is not taxed away equals the sum (measured over all individuals) of the ratio of the marginal willingness to pay and the fraction of the marginal income that is not taxed away. The marginal income tax rate faced by individuals with before-tax wage rate $\tilde{w}$ and associated consumption $\tilde{c}$ is thus interpretable to depend on other people's marginal willingness to pay measured in terms of gross income.[6]

An analytically useful special case of equation (9) arises when all marginal income tax rates are small enough, yet not necessarily similar, such that

$$\frac{1-T'(\tilde{w}\tilde{l})}{1-T'(wl)} \approx 1$$

in which it is possible to obtain an algebraic closed-form solution.[7] In this case, the marginal tax rate faced by an individual of ability $\tilde{w}$ and consumption $\tilde{c}$ can be approximated as

$$T'(\tilde{w}\tilde{l}) = \int_{c_{\min}}^{c_{\max}} MWTP(\tilde{c}) f(c)dc . \qquad (10)$$

The Pareto efficient marginal income tax rate implemented for any individual would in this case simply equal the sum of all people's marginal willingness to pay for this particular individual to reduce his/her consumption. Note also that the assumption that all marginal tax rates are small does not mean that their relative size is similar (only that the net of marginal tax rates are similar in size). Instead, since the externalities are generally non-atmospheric,

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[6] Yet another way to write the tax formula is in terms of a marginal rate of substitution between C and z, i.e.,

$$\frac{T'(\tilde{w}\tilde{l})}{1-T'(\tilde{w}\tilde{l})} = -\int_{c_{\min}}^{c_{\max}} w \frac{\partial u(c,z,C)/\partial C}{\partial u(c,z,C)/\partial z} \frac{\partial C/\partial \tilde{c}}{f(\tilde{c})} f(c)dc .$$

The integrand is interpretable as the value of leisure that an individual of ability w is willing to sacrifice for an individual of ability $\tilde{w}$ to decrease his/her consumption marginally.

[7] Note that equation (10) is not a reduced form, since $\tilde{c}$ depends on $T'(\tilde{w}\tilde{l})$.

their relative size may be very different and some optimal marginal tax rates may be negative while other are positive.

3. Pareto Efficient Taxation under Self-Centered Inequality Aversion

In the previous section, we derived general expressions for Pareto efficient marginal taxation when people are inequality averse, or more generally when the utility of each individual depends on the consumption of all individuals. Yet, we have not further explored the determination of the marginal willingness to pay (WTP) measures *per se*. This is the task of the present section, where we will explore the marginal WTPs on the right hand side of equations (9) and (10) based on the two most famous models of self-centered inequality aversion, namely those suggested by Fehr and Schmidt (1999) and Bolton and Ockenfels (2000), and then illustrate how the Pareto efficient marginal income taxes will vary with the gross income based on a realistic distribution of consumption. That is, in our numerical analysis we will not start from a given before-tax wage distribution, or ability distribution, and then derive the resulting consumption distribution based on the Pareto efficient income taxes. The main reason is, of course, that this information is not sufficient for calculating the after-tax consumption distribution. Recall that we are dealing with the efficiency problem in a first-best world where the government can use type-specific lump-sum taxes for redistribution. This means that the consumption distribution will depend crucially on the distribution of lump-sum taxes.

Instead, we will take as our point of departure a given, and fairly realistic, consumption distribution. We assume that this distribution is the result of a Pareto efficient income tax policy, including an efficient set of type-specific lump-sum taxes. We can then calculate what the marginal income tax rates must be, for a continuum of consumption levels under different assumptions about the structure and magnitude of the inequality aversion.

3.1 The Fehr-Schmidt Model

Let us start with the model suggested by Fehr and Schmidt (1999), which has become something like the industry standard in the context of self-centered inequality aversion. This is presumably due to a combination of a high degree of parsimony, since the model is based on only two parameters, and the model's ability to rather well explain the outcomes of many experimental games.

While the Fehr and Schmidt (1999) model is most often used in either two or few individual settings, it is straightforward to generalize it to a continuous distribution of individuals. The utility of an individual with consumption c can then be written as

$$\begin{aligned} u(c,z,C) &= u\left(c, z, \int_{c_{\min}}^{c} \beta(c-\hat{c}) f(\hat{c}) d\hat{c} + \int_{c}^{c_{\max}} \alpha(\hat{c}-c) f(\hat{c}) d\hat{c}\right) \\ &= v\left(c-C, z\right) = v\left(c - \int_{c_{\min}}^{c} \beta(c-\hat{c}) f(\hat{c}) d\hat{c} - \int_{c}^{c_{\max}} \alpha(\hat{c}-c) f(\hat{c}) d\hat{c}, z\right) \end{aligned}. \quad (11)$$

The parameters $\beta \geq 0$ and $\alpha \geq 0$ are interpretable to reflect the strengths of the aversion against inequality that is to the individual's material advantage and disadvantage, respectively.

Based on this type of inequality aversion, we can evaluate the marginal WTP-measures in the general policy rule for Pareto efficient marginal income taxation presented in Lemma 1, and immediately obtain the following result:

Proposition 1. *Based on Fehr and Schmidt preferences for inequality aversion, the Pareto efficient marginal income tax rate implemented for individuals with gross income* $\tilde{w}\tilde{l}$ *and consumption* $\tilde{c}$ *is given by*

$$T'(\tilde{w}\tilde{l}) = \alpha \int_{c_{\min}}^{\tilde{c}} \frac{1-T'(\tilde{w}\tilde{l})}{1-T'(wl)} f(c) dc - \beta \int_{\tilde{c}}^{c_{\max}} \frac{1-T'(\tilde{w}\tilde{l})}{1-T'(wl)} f(c) dc. \quad (12)$$

Equation (12) is clearly an implicit formulation since the Pareto efficient marginal income tax implemented for gross income $\tilde{w}\tilde{l}$ is expressed in terms of the Pareto efficient marginal income taxes for all consumption levels. As a consequence, it is not straightforward to interpret the policy rule. In particular, it is not apparent how the marginal income tax rate varies in the consumption distribution, or even how it relates to the consumption rank. We will deal with this limitation in two ways. First we will present the results of the special case given in equation (10), where all marginal tax rates are small enough to imply that the weight factor $\left(1-T'(\tilde{w}\tilde{l})\right)/\left(1-T'(wl)\right)$ is negligible. Then we will present simulation results based on the general case.

The special case where all marginal tax rates are small result in a much simpler efficiency marginal tax rule, as follows:

Corollary 1. *If all marginal income tax rates are small, then based on Fehr and Schmidt preferences for inequality aversion the Pareto efficient marginal income tax rate implemented for individuals with gross income* $\tilde{w}\tilde{l}$ *and consumption* $\tilde{c}$ *is given by*

$$T'(\tilde{w}\tilde{l}) = -\beta + (\alpha+\beta)\mathrm{Rank}(\tilde{c})\,. \tag{13}$$

Equation (13) implies that the Pareto efficient marginal income tax rate increases in the consumption rank, and that this is moreover an affine relationship. The Pareto efficient marginal tax for the lowest consumption level (where $\mathrm{Rank}(\tilde{c}) = 0\%$) is given by $T'(w_{\min}l) = -\beta$, whereas the Pareto efficient marginal tax for the highest consumption level (where $\mathrm{Rank}(\tilde{c}) = 100\%$) is given by $T'(w_{\max}l) = \alpha$. Thus, the Pareto efficient marginal tax rate increases monotonically from $-\beta$ for the individual with the lowest consumption to α for the individual with the highest consumption. While the efficient marginal tax rate increases linearly in the consumption rank, it typically increases nonlinearly with the consumption level, where the specific pattern depends on the resulting consumption distribution in the population.

To illustrate how the Pareto efficient marginal tax rates vary with consumption in the general case when the marginal tax rates are not necessarily small, we will make use of numerical simulations, for which we have to make some further assumptions. In particular, the results will depend on the resulting consumption distribution. Let us take as a point of departure the disposable income in the US 2013. According to the Luxemburg Income Study (LIS), the mean disposable income per (equivalence-scale adjusted) capita was then 39322 USD, and the corresponding Gini coefficient was 0.377. We will here for convenience approximate the actual distribution with a log-normal one, such that mean income and the Gini coefficient equal the above values,[8] and moreover assume that the consumption distribution equals the disposable income distribution. Although the results naturally depend on these distributional assumptions, most qualitative insights remain the same for other realistic distributions. We will use the same distributional assumption throughout in this paper, i.e., also for other measures of inequality.

We must also make parametric assumptions within the Fehr and Schmidt model of inequality aversion. In accordance with Fehr and Schmidt (1999, p. 844), who based their own judgment on ample experimental evidence, we first assume that $\alpha = 0.85$ and $\beta = 0.315$. These parameter values clearly imply substantial marginal tax rates, suggesting that we cannot rely on equation (12) as a good approximation of the Pareto efficient marginal tax policy. Indeed, whereas the distribution based on the simplified equation (12) implies a marginal tax range from -0.315 to 0.85, the efficient marginal tax distribution according to equation (13) ranges from approximately -0.6 to 0.5, as can be seen in Figure 1 below.

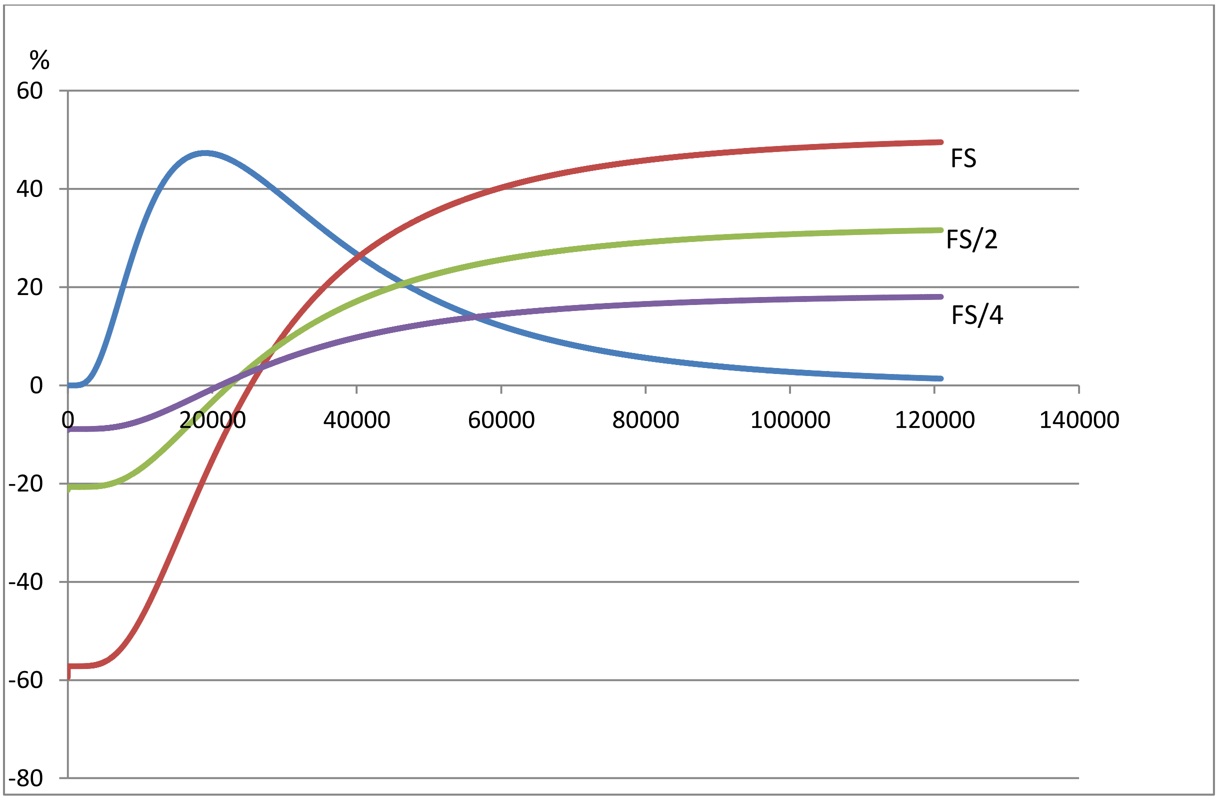

Figure 1. Pareto efficient marginal income tax rates as a function of the consumption levels (for the log-normal consumption distribution discussed above) in equilibrium, based on the

[8] We also have data on the 10-percentile, the median and the 90-percentile. Our lognormal approximation is reasonably good (for our purposes) also for these values.

Fehr and Schmidt model of inequity aversion. FS: $\alpha = 0.85$, $\beta = 0.315$; FS/2: $\alpha = 0.425$, $\beta = 0.15075$; FS/4: $\alpha = 0.2125$, $\beta = 0.075375$.

Overall, the Pareto efficient marginal income tax rates are substantial. Furthermore, the marginal tax rate increases (quite sharply) up to a certain consumption (and thus income) level and then remains fairly constant. Low levels of income should be subsidized in response to inequality aversion. Note also that this qualitative pattern remains the same even if we assume half or a quarter of the values of α and β suggested by Fehr and Schmidt (1999), as can be seen in the figure.

3.2 The Bolton-Ockenfels Model

The second most often referred to model of self-centered inequality aversion is the one suggested by Bolton and Ockenfels (2000). While also this model is typically used in either two or few individual settings (as the Fehr and Schmidt model), the utility function can be written in the same way in a continuous-type framework

$$u(c,z,C) = u\left(c, z, \frac{c}{\overline{c}}\right), \tag{14}$$

where $\frac{\partial u}{\partial (c/\overline{c})} > 0$ for $c < \overline{c}$, $\frac{\partial u}{\partial (c/\overline{c})} = 0$ for $c = \overline{c}$ and $\frac{\partial u}{\partial (c/\overline{c})} < 0$ for $c > \overline{c}$.

Thus, an individual prefers that the average consumption level is as close as possible to his/her own consumption level, *ceteris paribus*. Based on equation (14) we can immediately derive the following measure of marginal WTP by using equation (8):

$$MWTP(\tilde{c}) = MWTP = \frac{c}{\overline{c}^2} \frac{\frac{\partial u(c,z,C)}{\partial C}}{\frac{\partial u(c,z,C)}{\partial c}}, \tag{15}$$

which is clearly independent of $\tilde{c}$. This marginal WTP thus reflects how much an individual with consumption level c is willing to pay for a decrease in any individual's consumption. While an individual's marginal willingness to pay is positive if the average income is larger than the individual's own income, and vice versa, it is thus independent of which individual the potential consumption change refers to. In other words, the consumption externality that inequality aversion gives rise to is atmospheric in this case. The reason is that each individual only cares about the average consumption, in addition his/her own consumption and leisure. We can then derive a closed-form solution to the Pareto efficient tax problem also in the general case, when the marginal tax rates are not small. Lemma 1 and equation (15) implies the following result:

Proposition 2. *Based on Bolton and Ockenfels's preferences for inequality aversion, the Pareto efficient marginal income tax rate implemented for individuals with gross income* $\tilde{w}\tilde{l}$ *and consumption* $\tilde{c}$ *is given by*

$$T'(\tilde{w}\tilde{l}) = \frac{1}{\bar{c}^2}\int_{c_{\min}}^{c_{\max}} c\left(\frac{\partial u(c,z,C)}{\partial C}\Big/\frac{\partial u(c,z,C)}{\partial c}\right) f(c)dc = \int_{c_{\min}}^{c_{\max}} MWTP\ f(c)dc\,. \quad (16)$$

Equation (16) clearly implies that the Pareto efficient marginal tax income tax rate is the same for all, irrespective of consumption level. The intuition is as follows: Each individual derives disutility if his/her consumption deviates from the average consumption in the economy as a whole, *ceteris paribus*. This means that an individual with a consumption level below the mean will prefer that others reduce their consumption. Yet, this individual is indifferent regarding which of the other individuals that reduces his/her consumption. Hence, their marginal WTP is the same for a reduction by the rich as for an equally large reduction by the poor. Similarly, an individual above the mean would prefer that others increase their consumption, and he/she would be willing to pay the same amount to a rich and a poor individual for a given consumption increase. The resulting Pareto efficient marginal tax rate will then reflect the net effect of such positive and negative marginal WTPs. While this Pareto efficient marginal tax rate is not in general strictly equal to zero, it will presumably be very close to zero in most cases.

In order to shed some more light on the order of magnitudes for estimate the level of the Pareto efficient marginal tax rate, let us consider a more specific version of the Bolton and Ockenfels preferences as follows:

$$u(c,z,C) = v\left(c - \phi\left(\frac{1}{C}-1\right)^2, z\right) = v\left(c - \phi\left(\frac{\bar{c}}{c}-1\right)^2, z\right). \quad (17)$$

Using this utility function in equation (16) one can show that

$$T'(\tilde{w}\tilde{l}) = 2\phi\int_{c_{\min}}^{c_{\max}} \frac{1}{c}\left(\frac{\bar{c}}{c}-1\right) f(c)dc\,. \quad (18)$$

Equation (18) implies that the marginal tax rate is the same for all individuals (the intuition for which we discussed above), and also that it is proportional to a parameter measuring the strength of the aversion against inequality, ϕ. By using simulations based on the same consumption distribution as in the Fehr and Schmidt model examined above , the resulting Pareto efficient marginal tax rate turns out to be very close to zero for all reasonable values of ϕ. Overall, the policy implications in terms of Pareto efficient taxation turn out to be strikingly different for the Fehr-Schmidt and the Bolton-Ockenfels models, respectively.

4. Pareto Efficient Taxation under Non-Self-Centered Inequality Aversion

Although much work on social preferences has focused on self-centered inequality aversion, one may question such a point of departure in a many-individual society. In particular, an individual may prefer a more equal consumption distribution to a less equal one regardless of the relationship between his/her own consumption and the consumption of others. For example, with respect to the Fehr and Schmidt (1999) model, an individual may prefer a society with fewer super rich and fewer super poor persons, for a given own consumption level, consumption rank, and mean consumption levels among those who have higher as well as lower consumption than the individual himself/herself. In this section we explore the marginal *WTP*s in equations (9) and (10) based on different models of *Non*-Self-Centered, or general, inequality aversion. This means that the inequality measure is the same for all individuals such that $\partial\tilde{C}/\partial\tilde{c} = \partial C/\partial\tilde{c}$ for all *C*.

We consider three such measures of inequality: the Gini coefficient, the coefficient of variation, and the ratio between the mean absolute deviation and mean consumption, respectively, as the basis for studying the optimal tax policy responses to inequality aversion. We will for ease of comparability in each case consider a Cobb-Douglas specification of the relationship between the individual's own consumption and the measure of inequality, following Carlsson et al. (2005). For each of these three measures of inequality we can then write the utility function as follows:

$$u(c,z,C) = v\left(c(C)^{-\gamma}, z\right), \qquad (19)$$

where $\gamma > 0$ is a parameter reflecting the degree of inequality aversion. Thus, an individual always prefers less to more general inequality, regardless of the relation between the individual's own consumption and the consumption of others.

4.1 Gini Coefficient

Let us start with the most commonly used inequality measure on the social level, namely the Gini coefficient, *G*, such that $C = G$ in (19). The Gini coefficient is half of the relative mean absolute consumption difference which, in turn, is defined as the ratio of the mean absolute consumption difference, *D*, and the mean consumption. Therefore, $G = 0.5D/\overline{c}$, where

$$D = \int_0^\infty f(\hat{c}) \int_0^{\hat{c}} f(\breve{c})(\hat{c} - \breve{c}) d\breve{c} d\hat{c} + \int_0^\infty f(\hat{c}) \int_{\hat{c}}^\infty f(\breve{c})(\breve{c} - \hat{c}) d\breve{c} d\hat{c} .$$

Based on this measure of inequality, we can derive the marginal WTP measures used to form the marginal tax policy rules in equations (9) and (10).

Let us start with the general case where the marginal taxes are not necessarily small, implying the following result:

Proposition 3. *For the model of inequality aversion based on the Gini coefficient, the Pareto efficient marginal income tax rate implemented for individuals with gross income* $\tilde{w}\tilde{l}$ *and consumption* $\tilde{c}$ *is given by*

$$T'(\tilde{w}\tilde{l}) = \frac{\gamma}{D}\left(2\text{Rank}(\tilde{c}) - 1 - G\right) \int_{c_{\min}}^{c_{\max}} c \frac{1 - T'(\tilde{w}\tilde{l})}{1 - T'(wl)} f(c) dc . \qquad (20)$$

Again there is no closed-form algebraic solution in the general case. However, we can easily calculate the critical levels for when the marginal income tax is positive and when it is negative. From (21) it clearly follows that $T'(\tilde{w}\tilde{l}) = 0$ for $\text{Rank}(\tilde{c}) = 0.5 + D/2\overline{c}$ and also that $T'(\tilde{w}\tilde{l}) < 0$ for $\text{Rank}(\tilde{c}) < 0.5 + D/2\overline{c}$ and $T'(\tilde{w}\tilde{l}) > 0$ for $\text{Rank}(\tilde{c}) > 0.5 + D/2\overline{c}$.

Let us next turn to the simplified case where all marginal income tax rates are small, as given in equation (10), where we instead obtain a closed-form solution as follows:

Corollary 2. *If all marginal income tax rates are small, and if the inequality aversion is based on the Gini coefficient, the Pareto efficient marginal income tax rate implemented for individuals with gross income* $\tilde{w}\tilde{l}$ *and consumption* $\tilde{c}$ *is given by*

$$T'(\tilde{w}\tilde{l}) = \frac{\gamma}{G}\left(2\mathrm{Rank}(\tilde{c}) - G - 1\right). \qquad (21)$$

Equation (21) is reminiscent of equation (13), i.e., the corresponding marginal tax policy derived under the Fehr and Schmidt preferences for inequality aversion, and we can observe a monotonically increasing affine relationship between marginal tax rates and consumption (and hence also a monotonic relation with gross income, by assumption). The marginal tax rate will start from $-\gamma(G+1)/G<0$ for the individual with the lowest consumption rank and end with $\gamma(1-G)/G>0$ for the individual with the highest consumption rank. The intuition is that all individuals would benefit from a more equal consumption distribution, *ceteris paribus*, which can be accomplished through increased consumption in the lower end of the distribution and decreased consumption in the upper end. Since marginal taxation affects the before-tax income via the labor supply decision, the tendency to supply too much labor in the upper end of the distribution and too little labor in the lower end is counteracted through the policy illustrated in the figure.

Returning to the general case, where we do not assume small marginal tax rates, consider next simulations based on the same consumption distribution as before, with Gini coefficient equal to 0.33, and hence a relative mean absolute consumption difference equal to 0.66. The results are presented in Figure 2. As expected from the qualitative analysis above, the marginal Pareto efficient marginal tax rates vary with the (optimal) consumption level in the same general way as for the Fehr and Schmidt preferences for self-centered inequality aversion. This means that the non-self-centered inequality aversion discussed here may have tax policy implications qualitatively similar to those associated with self-centered inequality aversion, even if the levels of marginal taxation differ between Figures 1 and 2.

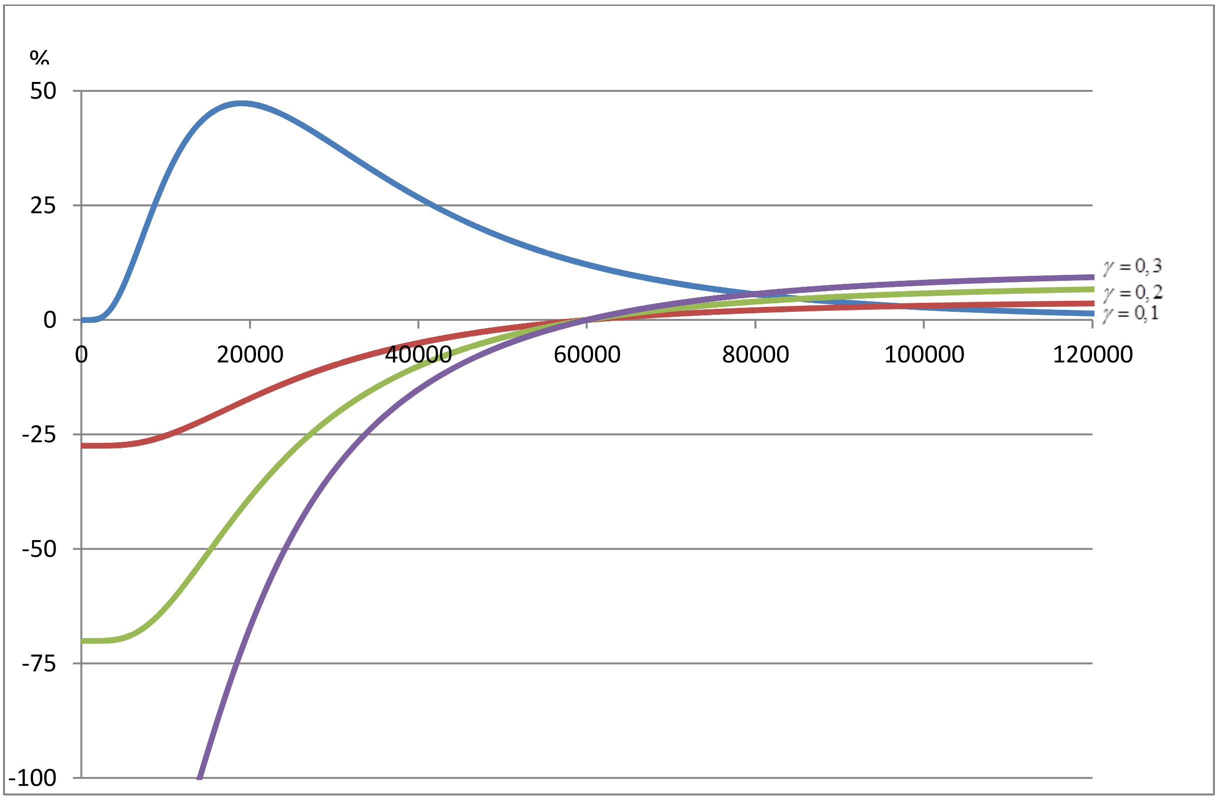

Figure 2. Pareto efficient marginal income tax rates as a function of the consumption levels (for a log-normal consumption distribution) in equilibrium, based on non-self-centered inequality aversion where inequality is measured as the Gini coefficient.

4.2 Coefficient of Variation

Consider next what is presumably the second most often used general inequality measure, namely the coefficient of variation, *V*, defined as the ratio between the standard deviation of the consumption distribution in the population, σ, and mean consumption, $\bar{c}$, such that $C = V = \sigma / \bar{c}$. Carlsson et al. (2005) analyze and parameterize this measure of inequality, based on a questionnaire-experimental approach. They conclude that the mean degree of inequality aversion is such that $\gamma \approx 0.2$ in equation (19).

In the general case, when the marginal taxes are not necessarily small, the utility function of equation (19) and Lemma 1 imply the following result:

Proposition 4. *For the model of inequality aversion based on the coefficient of variation, the Pareto efficient marginal income tax rate implemented for individuals with gross income* $\tilde{w}\tilde{l}$ *and consumption* $\tilde{c}$ *is given by*

$$T'(\tilde{w}\tilde{l}) = \gamma\left(\frac{\tilde{c}-\bar{c}}{\sigma^2} - \frac{1}{\bar{c}}\right)\int_{c_{\min}}^{c_{\max}} c\frac{1-T'(\tilde{w}\tilde{l})}{1-T'(wl)}f(c)dc\,. \tag{22}$$

As expected, there is no closed-form algebraic solution here either. However, before turning to simulations we can easily derive the marginal income tax rates for the lowest and highest possible consumption levels. When the consumption of the taxed individual approaches zero we obtain from (22):

$$\lim_{\tilde{c}\to 0} T'(\tilde{w}\tilde{l}) = -\gamma\left(\frac{\bar{c}}{\sigma^2} + \frac{1}{\bar{c}}\right)\int_{c_{\min}}^{c_{\max}} c\frac{1-T'(\tilde{w}\tilde{l})}{1-T'(wl)}f(c)dc < 0\,. \tag{23}$$

For a sufficiently large γ, this means

$$\lim_{\tilde{c}\to 0} \frac{T'(\tilde{w}\tilde{l})}{1-T'(\tilde{w}\tilde{l})} = -\gamma\left(\frac{\bar{c}}{\sigma^2} + \frac{1}{\bar{c}}\right)\int_{c_{\min}}^{c_{\max}} \frac{c}{1-T'(wl)}f(c)dc < -1\,.$$

Therefore, by the mean value theorem, there exist a $c^* > 0$ such that

$$\lim_{c\to c^*} T'(\tilde{w}\tilde{l}) = -\infty\,.$$

In other words, there is a positive consumption level at which the marginal income tax rate approaches minus infinity. The intuition is that, based on this measure of inequality aversion, it is simply not possible to obtain consumption levels below c^* as a part of a Pareto efficient allocation, regardless of the social welfare function (as long at is Paretian). The reason is that, at this consumption level, the social value of increased consumption is positive even if society puts no weight whatsoever to the utility of individuals with consumption c^*. Similarly, when consumption (and hence gross income) approaches infinity we obtain:

$$\begin{aligned}\lim_{\tilde{c}\to\infty} \frac{T'(\tilde{w}\tilde{l})}{1-T'(\tilde{w}\tilde{l})} &= \tilde{c}\left(\frac{1}{\sigma^2} - \frac{\bar{c}}{\tilde{c}\sigma^2} - \frac{1}{\tilde{c}\bar{c}}\right)\gamma\int_{c_{\min}}^{c_{\max}} \frac{c}{1-T'(wl)}f(c)dc \\ &= \tilde{c}\frac{\gamma}{\sigma^2}\int_{c_{\min}}^{c_{\max}} \frac{c}{1-T'(wl)}f(c)dc = \infty\end{aligned}\,. \tag{24}$$

This, in turn, clearly implies that

$$\lim_{c \to \infty} T'(\tilde{w}\tilde{l}) = 1,$$

i.e. that the marginal income tax rate approaches 100% when consumption goes to infinity.

Moreover, it is straightforward to obtain the critical levels for when the marginal income tax is positive and when it is negative. From (23) it clearly follows that $T'(\tilde{w}\tilde{l}) = 0$ for $\tilde{c} = \bar{c}\left(\rho^2 + 1\right)$ and also that $T'(\tilde{w}\tilde{l}) < 0$ for $\tilde{c} < \bar{c}\left(\rho^2 + 1\right)$ and $T'(\tilde{w}\tilde{l}) > 0$ for $\tilde{c} > \bar{c}\left(\rho^2 + 1\right)$.

In the simplified case where all marginal income tax rates are small, we obtain instead a closed-form solution summarized as follows:

Corollary 3. *If all marginal income tax rates are small, and if the inequality aversion is based on the coefficient of variation, the Pareto efficient marginal income tax rate implemented for individuals with gross income $\tilde{w}\tilde{l}$ and consumption $\tilde{c}$ is given by*

$$T'(\tilde{w}\tilde{l}) = \gamma\left(\frac{\tilde{c} - \bar{c}}{\bar{c}}\frac{1}{V^2} - 1\right). \tag{25}$$

Again we can observe a monotonic positive relationship between the marginal tax rates and consumption, starting from $-\gamma\left(1/V^2 + 1\right)$ for $\tilde{c} = 0$. The intuition is, of course, the same as for the marginal policy implied by equation (21), where the inequality aversion is based on the Gini coefficient.

The simulation in Figure 3 below shows the efficient marginal tax rates for different inequality parameters γ, based on the same distributional assumptions as before for the general case (without assuming small marginal tax rates). We can observe that the Pareto efficient marginal tax rates vary strongly with the consumption level; that it may become very large at non-extreme consumption levels; and that it can take extreme negative values for low levels of consumption. Indeed, it can be shown that the case with $\gamma = 0.3$ implies that the Pareto efficient marginal tax approaches minus infinity for a positive consumption level. Despite level differences, however, the general patter in Figure 3 resembles that in Figure 2.

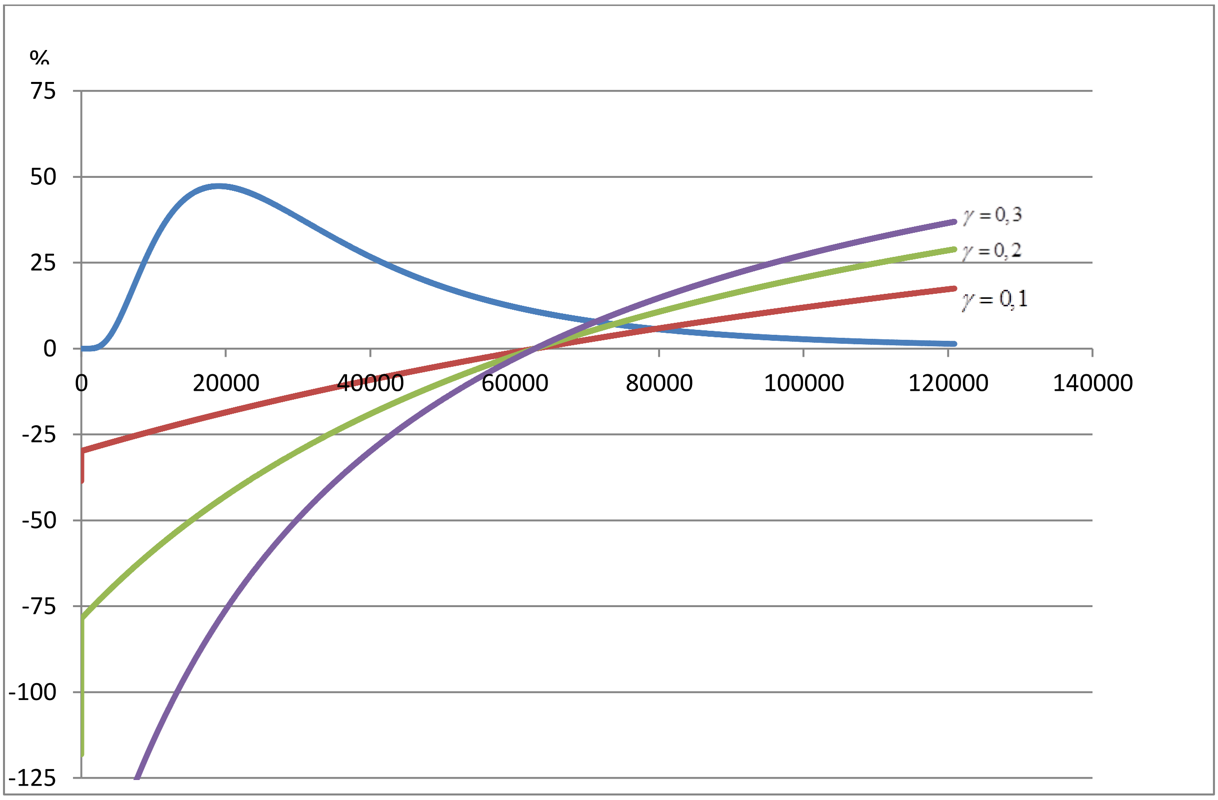

Figure 3. Pareto efficient marginal income tax rates as a function of the consumption levels (for a log-normal consumption distribution) in equilibrium, based on non-self-centered inequality aversion where inequality is measured as the coefficient of variation.

4.3 The ratio between the mean absolute deviation and mean consumption

We now turn to our final inequality measure. This measure is analogous to the coefficient of variation with the only difference that we will here use the mean absolute deviation, *M*, instead of the standard deviation, such that $C = M / \bar{c}$. Thus, our measure of inequality is the ratio between the mean absolute consumption deviation and the mean consumption.

In this case it turns out that each individual will have a different marginal willingness to pay to reduce another individual's consumption depending on whether that individual's consumption is smaller or larger than the average consumption. Yet, the marginal willingness to pay will be the same regardless of *how much* higher or lower the individual's consumption is compared to the mean consumption. As a consequence, there will only be two different Pareto efficient marginal income tax rates; one for individuals below the mean consumption level, and another for those about that level. Let us use the short notation $T_l{}'$ for the Pareto efficient marginal tax for individuals below the average consumption, and T_h' for the marginal tax rate above this level. Also, let n_l and n_h denote the number of persons who consume less and more than the average, respectively, and similarly $\bar{c}_l$ and $\bar{c}_h$ denote the mean consumption levels among those who consume less and more than the average. Based on equation (9), we can then derive the following result:

Proposition 5. *For the model of inequality aversion based on the ratio between mean absolute deviation and mean consumption, the Pareto efficient marginal income tax rate implemented for individuals with gross income* $\tilde{w}\tilde{l}$ *and consumption* $\tilde{c}$ *is given by*

$$T_l' = T'(\tilde{w}\tilde{l}\,|\tilde{c} < \bar{c}) = -\gamma \frac{\bar{c}_l + \bar{c}_h}{\bar{c}} \frac{M + 2n_h\bar{c}}{M - 2\gamma\bar{c}_h}\,. \tag{26a}$$

$$T_h' = T'(\tilde{w}\tilde{l}\,|\tilde{c} > \bar{c}) = \gamma \frac{\bar{c}_l + \bar{c}_h}{\bar{c}} \frac{2n_l\bar{c} - M}{M + 2\gamma\bar{c}_l} \tag{26b}$$

From equation (26b), we find that the efficient marginal tax on those above the mean consumption is strictly positive, since we always have $2n_l\bar{c} > M$ (in the extreme case with a symmetric distribution with two atoms in each end we get that $2n_l\bar{c} = \bar{c} = M$). Equation (26a) we implies that the efficient marginal income tax converges to minus infinity for γ approaching $\frac{M}{2\bar{c}_h}$ (from below). In a way similar to the case where the inequality aversion is based on the coefficient of variation, we also find that for sufficient large γ a particular consumption distribution is simply not consistent with a Pareto efficient allocation, regardless of the social welfare function (as long as it is Paretian). At this value of γ individuals above mean consumption level would simply be indifferent between giving one dollar to those below the mean consumption level and not doing so. Based on the consumption distribution used for simulations in this paper, this critical value of γ is as low as 0.145. When $\gamma = 0.1$ we get $T_l' = -1.89, T_h' = 0.29$.

5. Conclusion

As far as we know, this is the first paper to analyze Pareto efficient marginal income taxation in economies where people are inequality averse. We started by examining a general model, in which we made no other assumption about the inequality aversion other than that people prefer a more equal distribution of consumption (or disposable income) to a less equal one, *ceteris paribus*. Based on the policy rules for marginal income taxation derived in the context of this general model, we examined the implications of five more specific types of inequality aversion; two of them self-centered and three non-self-centered. The basic purposes were to understand how and why inequality aversion motives marginal tax wedges in the labor market, and how the Pareto efficient marginal tax rate varies along the distribution of consumption.

The take home message of the paper is twofold. First, empirically and experimentally quantified degrees of inequality aversion have potentially very important implications for Pareto efficient income taxation. More specifically, several of the models show that the Pareto efficient marginal tax rates required to internalize the externalities caused by inequality aversion are substantial, and that these marginal tax rates may vary quite much among consumers depending on their consumption level in equilibrium. Several of the models imply a progressive tax structure, where low income levels are subsidized at a diminishing marginal rate and high levels of income are tax at an increasing marginal rate.

Second, the exact nature of the inequality aversion, and measures of inequality used, matter a great deal for the structure of efficient marginal income taxation. The most striking result comes from comparing the two models of self-centered inequality aversion, with seemingly similar consumer preferences. Whereas the Fehr and Schmidt (1999) type of preference for inequality aversion imply monotonically increasing marginal income tax rates, with large negative marginal tax rates for low-income individuals and large positive tax rates for high-income individuals, the often considered similar inequality aversion model by Bolton and Ockenfels (2000) implies close to zero marginal income tax rates for all. The intuition is that the consumption externality caused by inequality aversion is non-atmospheric in the former case and atmospheric in the latter.

Future research may take several direction and we shall briefly mention three of them here. One would be to extend the analysis to a second-best framework of asymmetric information, preferably in a two-type self-selection model. Another is to include a broader set of policy instruments. For instance, since inequality aversion leads to (private) consumption externalities, it is likely to have implications also for the efficient provision of public goods and the public provision of private goods. A third avenue would be to allow for a broader spectrum of social interaction, where the policy implications of inequality aversion are examined alongside the implications of other types of (empirically established) forms of interaction such as relative consumption concerns and/or social norms. We hope to address these questions in future research.

Appendix

Proof of Lemma 1

Start by rewriting equation (6) to read

$$\tilde{\psi}'\frac{\partial u(\tilde{c},\tilde{z},\tilde{C})}{\partial \tilde{c}} f(\tilde{c}) = \lambda f(\tilde{c}) - \int_{c_{\min}}^{c_{\max}} \psi' \frac{\partial u(c,z,C)}{\partial C}\frac{\partial C}{\partial \tilde{c}} f(c)dc . \qquad \text{(A1)}$$

By combining equations (A1) and (7) we get

$$\tilde{w}\tilde{MRS}_{cz} = 1 - \frac{\int_{c_{\min}}^{c_{\max}} \psi' \frac{\partial u(c,z,C)}{\partial C}\frac{\partial C}{\partial \tilde{c}} f(c)dc}{\lambda f(\tilde{c})} . \qquad \text{(A2)}$$

Next, substitute the private first-order condition for work hours in equation (4) into equation (A2) to obtain

$$T'(\tilde{w}\tilde{l}) = -\left(1 - T'(\tilde{w}\tilde{l})\right)\frac{\int_{c_{\min}}^{c_{\max}} \psi'() \frac{\partial u(c,z,C)}{\partial C}\frac{\partial C}{\partial \tilde{c}} f(c)dc}{\lambda f(\tilde{c})} . \qquad \text{(A3)}$$

Substituting (8) into (A3) and using the MRS definition gives

$$T'(\tilde{w}\tilde{l}) = \left(1 - T'(\tilde{w}\tilde{l})\right)\int_{c_{\min}}^{c_{\max}} \psi' WTP(\tilde{c})\frac{MRS_{cz}}{\lambda}\frac{\partial u(c,z,C)}{\partial z} f(c)dc . \qquad \text{(A4)}$$

Finally, using

$$\frac{\partial u(c,z,C)}{\partial z} = \lambda w / \psi'$$

from equation (7), $wMRS_{cz} = 1/(1-T'(wl))$ from equation (4), and then substituting into equation (A4), we obtain (9). QED Equation (10) follows as the special case where $\left(1-T'(\tilde{w}\tilde{l})\right)/\left(1-T'(wl)\right) \approx 1$.

Proof of Proposition 1

It follows from (11) that $\frac{\partial u(c,z,C)}{\partial C} = -\frac{\partial u(c,z,C)}{\partial c}$ such that

$$MWTP(\tilde{c}) = \frac{1}{f(\tilde{c})}\frac{\partial C}{\partial \tilde{c}}. \tag{A5}$$

For $c < \tilde{c}$ it follows that $\partial C / \partial \tilde{c} = \alpha f(\tilde{c})$ and

$$MWTP(\tilde{c}) = \alpha \tag{A6}$$

Thus, all individuals with a consumption level smaller than $\tilde{c}$ will on the margin be willing to pay the same amount, α, per consumption reduction unit of an individual with consumption $\tilde{c}$. Similarly when $c > \tilde{c}$ it follows that $\partial C / \partial \tilde{c} = -\beta f(\tilde{c})$ and

$$MWTP(\tilde{c}) = -\beta . \tag{A7}$$

Therefore, all individuals with a consumption level larger than $\tilde{c}$ will instead be willing to pay β per unit consumption increase of an individual with consumption $\tilde{c}$.

Substituting equations (A6) and (A7) into equation (9) directly yields (13). QED

Proof of Corollary1

Substituting equations (A6) and (A7) into equation (10) yields

$$\begin{aligned} T'(\tilde{w}\tilde{l}) &= \alpha \int_{c_{\min}}^{\tilde{c}} f(c)dc - \beta \int_{\tilde{c}}^{c_{\max}} f(c)dc = \alpha \mathrm{Rank}(\tilde{c}) - \beta(1-\mathrm{Rank}(\tilde{c})) \\ &= -\beta + (\alpha+\beta)\mathrm{Rank}(\tilde{c}) \end{aligned}. \tag{A8}$$

QED

Proof of Proposition 2

When the marginal tax rates are small, it follows from equation (10) that

$$T'(\tilde{w}\tilde{l}) = \frac{1}{\bar{c}^2}\int_{c_{\min}}^{c_{\max}} c \frac{\frac{\partial u(c,z,C)}{\partial C}}{\frac{\partial u(c,z,C)}{\partial c}} f(c)dc . \tag{A9}$$

Instead, if based on the more general equation (10) we obtain

$$\frac{T'(\tilde{w}\tilde{l})}{1-T'(\tilde{w}\tilde{l})}=\frac{1}{\bar{c}^2}\int_{c_{\min}}^{c_{\max}} c\,\frac{\dfrac{\partial u(c,z,C)}{\partial C}}{\dfrac{\partial u(c,z,C)}{\partial c}}\frac{1}{1-T'(wl)}f(c)dc \tag{A10}$$

which is also the same for all, implying that (A10) reduces to (A9), and hence to equation (16). QED

Derivation of equation (18)

From equation (17) follows that

$$\frac{\partial u(c,z,C)}{\partial c}=\frac{\partial v}{\partial\left(c-\phi\left(\dfrac{1}{C}-1\right)^2\right)}, \tag{A11}$$

and

$$\frac{\partial u(c,z,C)}{\partial C}=\frac{\partial v}{\partial\left(c-\phi\left(\dfrac{1}{C}-1\right)^2\right)}\frac{2\phi}{C^2}\left(\frac{1}{C}-1\right). \tag{A12}$$

Equations (A11) and (A12) imply

$$\frac{\dfrac{\partial u(c,z,C)}{\partial C}}{\dfrac{\partial u(c,z,C)}{\partial c}}=2\phi\left(\frac{\bar{c}}{c}\right)^2\left(\frac{\bar{c}}{c}-1\right). \tag{A13}$$

Substituting equation (A13) into equation (17) gives equation (18). QED

Proof of Proposition 3

From equation (19) we obtain

$$\frac{\dfrac{\partial u(c,z,C)}{\partial C}}{\dfrac{\partial u(c,z,C)}{\partial c}}=-\gamma\frac{c}{G}. \tag{A14}$$

We know that the Gini coefficient is half of the relative mean absolute consumption difference, which in turn is defined as the ratio of the mean absolute consumption difference, *D*, and the mean consumption, such that $G=0.5D/\bar{c}$. By definition we have

$$D=\int_0^\infty f(\hat{c})\int_0^{\hat{c}} f(\breve{c})(\hat{c}-\breve{c})d\breve{c}d\hat{c}+\int_0^\infty f(\hat{c})\int_{\hat{c}}^\infty f(\breve{c})(\breve{c}-\hat{c})d\breve{c}d\hat{c} \tag{A15}$$

implying

$$\begin{aligned}\frac{\partial D}{\partial \tilde{c}} &= \frac{\partial\left(\int_0^{\infty} f(\hat{c})\int_0^{\hat{c}} f(\breve{c})(\hat{c}-\breve{c})d\breve{c}d\hat{c} + \int_0^{\infty} f(\hat{c})\int_{\hat{c}}^{\infty} f(\breve{c})(\breve{c}-\hat{c})d\breve{c}d\hat{c}\right)}{\partial \tilde{c}} \\ &= f(\tilde{c})\left(\int_0^{\tilde{c}} f(\hat{c})d\hat{c} - \int_{\tilde{c}}^{\infty} f(\hat{c})d\hat{c}\right) = f(\tilde{c})\left(2\text{Rank}(\tilde{c})-1\right)\end{aligned}. \qquad \text{(A16)}$$

We can then derive

$$\begin{aligned}\frac{\partial C}{\partial \tilde{c}} &= \frac{\partial G}{\partial \tilde{c}} = 0.5\frac{1}{\bar{c}}\frac{\partial D}{\partial \tilde{c}} - 0.5\frac{D}{\bar{c}^2}\frac{\partial \bar{c}}{\partial \tilde{c}} = 0.5\frac{f(\tilde{c})}{\bar{c}}\left(2\text{Rank}(\tilde{c})-1-\frac{D}{\bar{c}}\right) \\ &= 0.5\frac{f(\tilde{c})}{\bar{c}}\left(2\text{Rank}(\tilde{c})-1-G\right)\end{aligned} \qquad \text{(A17)}$$

and

$$\begin{aligned}MWTP(\tilde{c}) &= -0.5\frac{\frac{\partial u(c,z,C)}{\partial C}}{\frac{\partial u(c,z,C)}{\partial c}}\frac{1}{\bar{c}}\left(2\text{Rank}(\tilde{c})-1-G\right) \\ &= \frac{\gamma c}{D}\left(\text{Rank}(\tilde{c})-\frac{1+G}{2}\right)\end{aligned}. \qquad \text{(A18)}$$

Substituting equation (A18) into equation (9) yields equation (20). QED

Proof of Corollary 2

If the marginal income tax rates are small, equation (20) implies

$$\begin{aligned}T'(\tilde{w}\tilde{l}) &= \frac{\gamma}{D}\left(\text{Rank}(\tilde{c})-\frac{1+G}{2}\right)\bar{c} \\ &= \frac{\gamma}{0.5G}\left(\text{Rank}(\tilde{c})-\frac{1+G}{2}\right). \\ &= \frac{\gamma}{G}\left(2\text{Rank}(\tilde{c})-1-G\right)\end{aligned}$$

QED

Proof of Proposition 4

It follows from equation (19) that

$$\frac{\partial C}{\partial \tilde{c}} = \frac{1}{\bar{c}}\frac{\partial \sigma}{\partial \tilde{c}} - \frac{\sigma}{\bar{c}^2}\frac{\partial \bar{c}}{\partial \tilde{c}}$$

$$= \frac{1}{\bar{c}} \frac{\partial \left(\int_{c_{\min}}^{c_{\max}} (\hat{c} - \bar{c})^2 f(\hat{c}) d\hat{c} \right)^{0.5}}{\partial \tilde{c}} - \frac{\sigma}{\bar{c}^2} f(\tilde{c})\,. \tag{A20}$$

$$= \frac{f(\tilde{c})}{\bar{c}} \left(\frac{\tilde{c} - \bar{c}}{\sigma} - \frac{\sigma}{\bar{c}} \right)$$

It also follows from equation (10) that

$$MWTP(\tilde{c}) = -\frac{\dfrac{\partial u(c,z,C)}{\partial C}}{\dfrac{\partial u(c,z,C)}{\partial c}} \frac{1}{\bar{c}} \left(\frac{\tilde{c} - \bar{c}}{\sigma} - \frac{\sigma}{\bar{c}} \right). \tag{A21}$$

By using equation (19), the marginal utilities of c and C become (with the measure of inequality given by the coefficient of variation)

$$\frac{\partial u(c,z,C)}{\partial c} = -\frac{\partial v}{\partial \left(c \left(\frac{\sigma}{\bar{c}} \right)^{\gamma} \right)} \left(\frac{\sigma}{\bar{c}} \right)^{-\gamma},$$

and

$$\frac{\partial u(c,z,C)}{\partial C} = -\frac{\partial v}{\partial \left(c \left(\frac{\sigma}{\bar{c}} \right)^{\gamma} \right)} \gamma c \left(\frac{\sigma}{\bar{c}} \right)^{-\gamma-1},$$

and the following marginal rate of substitution:

$$\frac{\dfrac{\partial u(c,z,C)}{\partial C}}{\dfrac{\partial u(c,z,C)}{\partial c}} = -\gamma c \left(\frac{\sigma}{\bar{c}} \right)^{-1}. \tag{A22}$$

Substituting equation (A21) into equation (A22) gives

$$MWTP(\tilde{c}) = \gamma c \left(\frac{\tilde{c} - \bar{c}}{\sigma^2} - \frac{1}{\bar{c}} \right). \tag{A23}$$

Substituting equation (A23) into equation (9) yields equation (22). QED

Proof of Corollary 3

When all marginal tax rates are small it then follows from equation (10) that

$$T'(\tilde{w}\tilde{l}) = \int_{c_{\min}}^{c_{\max}} \gamma c \left(\frac{\tilde{c} - \overline{c}}{\sigma^2} - \frac{1}{\overline{c}} \right) f(c)dc = \gamma \left(\overline{c}\, \frac{\tilde{c} - \overline{c}}{\sigma^2} - 1 \right), \tag{A24}$$

which can be rewritten as equation (25). QED

Proof of Proposition 5

From equation (19) follows that

$$\frac{\frac{\partial u(c,z,C)}{\partial C}}{\frac{\partial u(c,z,C)}{\partial c}} = -\gamma c \left(\frac{M}{\overline{c}} \right)^{-1} < 0, \tag{A25}$$

and also

$$\begin{aligned}
\frac{\partial C}{\partial \tilde{c}} &= \frac{1}{\overline{c}} \frac{\partial M}{\partial \tilde{c}} - \frac{M}{\overline{c}^2} \frac{\partial \overline{c}}{\partial \tilde{c}} = \frac{1}{\overline{c}} \frac{\partial \left(\int_{c_{\min}}^{\overline{c}} (\overline{c} - \hat{c}) f(\hat{c}) d\hat{c} + \int_{\overline{c}}^{c_{\max}} (\hat{c} - \overline{c}) f(\hat{c}) d\hat{c} \right)}{\partial \tilde{c}} - \frac{M}{\overline{c}^2} f(\tilde{c}) \\
&= \frac{1}{\overline{c}} \frac{\partial \left(\int_{c_{\min}}^{\overline{c}} \overline{c} f(\hat{c}) d\hat{c} - \int_{\overline{c}}^{c_{\max}} \overline{c} f(\hat{c}) d\hat{c} \right)}{\partial \tilde{c}} - \frac{1}{\overline{c}} \frac{\partial \left(\int_{c_{\min}}^{\overline{c}} \hat{c} f(\hat{c}) d\hat{c} - \int_{\overline{c}}^{c_{\max}} \hat{c} f(\hat{c}) d\hat{c} \right)}{\partial \tilde{c}} - \frac{M}{\overline{c}^2} f(\tilde{c}) \\
&= \frac{1}{\overline{c}} \left(\int_{c_{\min}}^{\overline{c}} f(\hat{c}) d\hat{c} - \int_{\overline{c}}^{c_{\max}} f(\hat{c}) d\hat{c} \right) \frac{\partial \overline{c}}{\partial \tilde{c}} - \frac{1}{\overline{c}} \frac{\partial \left(\int_{c_{\min}}^{\overline{c}} \hat{c} f(\hat{c}) d\hat{c} - \int_{\overline{c}}^{c_{\max}} \hat{c} f(\hat{c}) d\hat{c} \right)}{\partial \tilde{c}} - \frac{M}{\overline{c}^2} f(\tilde{c}) \\
&= \frac{1}{\overline{c}} (n_l - n_h) f(\tilde{c}) - \frac{1}{\overline{c}} \frac{\partial \left(\int_{c_{\min}}^{\overline{c}} \hat{c} f(\hat{c}) d\hat{c} - \int_{\overline{c}}^{c_{\max}} \hat{c} f(\hat{c}) d\hat{c} \right)}{\partial \tilde{c}} - \frac{M}{\overline{c}^2} f(\tilde{c})
\end{aligned} \tag{A26}$$

For $\tilde{c} < \overline{c}$, we can derive

$$\begin{aligned}
\frac{\partial C}{\partial \tilde{c}} &= \frac{1}{\overline{c}} (n_l - n_h) f(\tilde{c}) - \frac{1}{\overline{c}} \frac{\partial \left(\int_{c_{\min}}^{\overline{c}} \hat{c} f(\hat{c}) d\hat{c} \right)}{\partial \tilde{c}} - \frac{M}{\overline{c}^2} f(\tilde{c}) \\
&\frac{1}{\overline{c}} (n_l - n_h) f(\tilde{c}) - \frac{1}{\overline{c}} f(\tilde{c}) - \frac{M}{\overline{c}^2} f(\tilde{c}) \\
&= \frac{1}{\overline{c}} f(\tilde{c}) \left(n_l - n_h - 1 - \frac{M}{\overline{c}} \right) \\
&= -\frac{1}{\overline{c}} f(\tilde{c}) \left(2n_h + \frac{M}{\overline{c}} \right) < 0
\end{aligned} \tag{A27}$$

and then

$$MWTP(\tilde{c}) = \frac{\dfrac{\partial u(c,z,C)}{\partial C}}{\dfrac{\partial u(c,z,C)}{\partial c}} \frac{1}{\overline{c}}\left(2n_h + \frac{M}{\overline{c}}\right) = -\frac{\gamma c}{M}\left(2n_h + \frac{M}{\overline{c}}\right) = c\Psi_l < 0 \text{ ,(A28)}$$

where we have used (A25) and introduced the short notation $\Psi_l < 0$ for the proportionality factor that determines $MWTP(\tilde{c})$. Similarly, for $\tilde{c} > \overline{c}$ it follows that

$$\begin{aligned}
\frac{\partial C}{\partial \tilde{c}} &= \frac{1}{\overline{c}}\left(n_l - n_h\right) f(\tilde{c}) + \frac{1}{\overline{c}} \frac{\partial\left(\int_{\overline{c}}^{c_{\max}} \hat{c} f(\hat{c}) d\hat{c}\right)}{\partial \tilde{c}} - \frac{M}{\overline{c}^2} f(\tilde{c}) \\
&= \frac{1}{\overline{c}}\left(n_l - n_h\right) f(\tilde{c}) + \frac{1}{\overline{c}} f(\tilde{c}) - \frac{M}{\overline{c}^2} f(\tilde{c}) \\
&= \frac{1}{\overline{c}} f(\tilde{c})\left(n_l - n_h + 1 - \frac{M}{\overline{c}}\right) \\
&= \frac{1}{\overline{c}} f(\tilde{c})\left(2n_l - \frac{M}{\overline{c}}\right)
\end{aligned} \quad , \qquad \text{(A29)}$$

and then

$$MWTP(\tilde{c}) = -\frac{\dfrac{\partial u(c,z,C)}{\partial C}}{\dfrac{\partial u(c,z,C)}{\partial c}} \frac{1}{\overline{c}}\left(2n_l - \frac{M}{\overline{c}}\right) = \frac{\gamma c}{M}\left(2n_l - \frac{M}{\overline{c}}\right) = c\Psi_h > 0 \text{ .(A30)}$$

Substituting equation (A30) into equation (9) gives

$$\begin{aligned}
T_l' &= \Psi_l\left(\int_{c_{\min}}^{\overline{c}} c\frac{1-T_l'}{1-T_l'} f(c)dc + \int_{\overline{c}}^{c_{\max}} c\frac{1-T_l'}{1-T_h'} f(c)dc\right) \\
&= \Psi_l\left(\int_{c_{\min}}^{\overline{c}} cf(c)dc + \frac{1-T_l'}{1-T_h'}\int_{\overline{c}}^{c_{\max}} cf(c)dc\right) \\
&= \Psi_l \overline{c}_l + \Psi_l \frac{1-T_l'}{1-T_h'}\overline{c}_h
\end{aligned} \quad . \qquad \text{(A31)}$$

Similarly, for $\tilde{c} > \overline{c}$ we obtain

$$\begin{aligned}
T_h' &= \Psi_h\left(\int_{c_{\min}}^{\overline{c}} c\frac{1-T_h'}{1-T_l'} f(c)dc + \int_{\overline{c}}^{c_{\max}} c\frac{1-T_h'}{1-T_h'} f(c)dc\right) \\
&= \Psi_h\left(\frac{1-T_h'}{1-T_l'}\int_{c_{\min}}^{\overline{c}} cf(c)dc + \int_{\overline{c}}^{c_{\max}} cf(c)dc\right) \\
&= \Psi_h \frac{1-T_h'}{1-T_l'}\overline{c}_l + \Psi_h \overline{c}_h
\end{aligned} \quad . \qquad \text{(A32)}$$

From equation (A31) we get

$$1 - T_l' = 1 - \Psi_l \bar{c}_l - \Psi_l \frac{1 - T_l'}{1 - T_h'} \bar{c}_h$$

$$= \frac{1 - \Psi_l \bar{c}_l}{1 + \dfrac{\Psi_l}{1 - T_h'} \bar{c}_h} \qquad \text{(A33)}$$

$$= \frac{1 - \Psi_l \bar{c}_l}{1 - T_h' + \Psi_l \bar{c}_h} (1 - T_h')$$

Substituting equation (A33) into equation (A32) yields

$$T_h' = \Psi_h \bar{c}_l \frac{1 - T_h' + \Psi_l \bar{c}_h}{1 - \Psi_l \bar{c}_l} + \Psi_h \bar{c}_h$$

$$= \frac{\Psi_h \bar{c}_l \dfrac{1 + \Psi_l \bar{c}_h}{1 - \Psi_l \bar{c}_l} + \Psi_h \bar{c}_h}{1 + \dfrac{\Psi_h \bar{c}_l}{1 - \Psi_l \bar{c}_l}} \quad . \qquad \text{(A34)}$$

$$= \frac{\Psi_h \bar{c}_l \left(1 + \Psi_l \bar{c}_h\right) + \Psi_h \bar{c}_h \left(1 - \Psi_l \bar{c}_l\right)}{1 - \Psi_l \bar{c}_l + \Psi_h \bar{c}_l}$$

$$= \frac{\bar{c}_l + \bar{c}_h}{1 + (\Psi_h - \Psi_l) \bar{c}_l} \Psi_h$$

Substituting next for Ψ_l and Ψ_h gives

$$T_h' = \frac{\bar{c}_l + \bar{c}_h}{1 + \dfrac{\gamma}{M} \left(2n_l + 2n_h\right) \bar{c}_l} \frac{\gamma}{M} \left(2n_l - \frac{M}{\bar{c}}\right)$$

$$= \frac{\bar{c}_l + \bar{c}_h}{M + 2\gamma \bar{c}_l} \left(2n_l - \frac{M}{\bar{c}}\right) \gamma \quad . \qquad \text{(A35)}$$

$$= \gamma \frac{\bar{c}_l + \bar{c}_h}{\bar{c}} \frac{2n_l \bar{c} - M}{M + 2\gamma \bar{c}_l}$$

By symmetry,

$$T_l' = \frac{\bar{c}_l + \bar{c}_h}{1 - (\Psi_h - \Psi_l) \bar{c}_h} \Psi_l \, . \qquad \text{(A36)}$$

Substituting again for Ψ_l and Ψ_h we get

$$
\begin{aligned}
T_l' &= -\frac{\bar{c}_l + \bar{c}_h}{M - \gamma\left(2n_l - \frac{M}{\bar{c}}\right)\bar{c}_h - \gamma\left(2n_h + \frac{M}{\bar{c}}\right)\bar{c}_h}\gamma\left(2n_h + \frac{M}{\bar{c}}\right) \\
&= -\gamma\frac{\bar{c}_l + \bar{c}_h}{M - 2\gamma\bar{c}_h}\left(2n_h + \frac{M}{\bar{c}}\right) \\
&= -\gamma\frac{\bar{c}_l + \bar{c}_h}{\bar{c}}\frac{M + 2n_h\bar{c}}{M - 2\gamma\bar{c}_h}
\end{aligned}
\quad . \qquad \text{(A37)}
$$

QED

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