Structural Estimation of a Model of School Choices: the Boston Mechanism vs. Its Alternatives*

Caterina Calsamiglia[†], Chao Fu[‡] and Maia Güell[§]

October 28, 2014

Abstract

An important debate centers on what procedure should be used to allocate students across public schools. We contribute to this debate by developing and estimating a model of school choices by households under one of the most popular procedures known as the Boston mechanism (BM). We recover the joint distribution of household preferences and sophistication types using administrative data from Barcelona. Our counterfactual policy analyses show that a change from BM to the Gale-Shapley student deferred acceptance mechanism would create more losers than winners, while a change from BM to the top trading cycles mechanism has the opposite effect.

*We thank Yuseob Lee for excellent research assistance. We are grateful to the staff at IDESCAT, and in particular to Miquel Delgado, for their help in processing the data. We thank Steven Durlauf, John Kennan, Chris Taber and workshop participants at UCL, Royal Holloway and Virginia Tech for helpful comments. The authors acknowledge support from a grant from the Institute for New Economic Thinking (INET) for the Human Capital and Economic Opportunity Global Working Group (HCEO), an initiative of the Becker Friedman Institute for Research in Economics. The views expressed in this paper are those of the authors and not necessarily those of the funders or persons named here. Calsamiglia acknowledges financial support by the Spanish Plan Nacional I+D+I (SEJ2005-01481, SEJ2005-01690, SEJ2006-002789-E and FEDER), and the Grupo Consolidado de tipo C (ECO2008-04756), the Generalitat de Catalunya (SGR2005-00626), and the Severo Ochoa program. Güell acknowledges financial support from the Spanish Ministry of Science and Innovation (ECO2011-28965). All errors are ours.

[†]Universitat Autonoma de Barcelona and the Barcelona GSE

[‡]Corresponding author: Chao Fu, Department of Economics, University of Wisconsin, 1180 Observatory Dr., Madison, WI 53703, USA. Email: cfu@ssc.wisc.edu.

[§]The University of Edinburgh, CEP(LSE), CEPR, FEDEA, & IZA.

1 Introduction

In many countries, every child is guaranteed free access to education in some public school. However, not all public schools are of the same quality, nor are higher-quality schools distributed evenly across residential areas. Designed to broaden households' access to schools beyond their neighborhoods, public school choice systems have been increasingly adopted in many countries, including the U.S.[1] On the one hand, the quality of schools to which students are assigned can have significant long-term effects for individual families as well as important implications on the efficiency and equity for a society.[2] On the other hand, schools are endowed with certain capacities and not all choices can be satisfied. As a result, how to operationalize school choice, i.e., what procedure should be used to assign students to schools, becomes a non-trivial question that remains heatedly debated on among policy makers and researchers.

One important debate centers around a procedure known as the Boston mechanism (BM), which was used by Boston Public Schools (BPS) between 1999 and 2005 to assign K-12 pupils to city schools, and still is one of the most popular school choice systems in the world. In BM, a household submits its applications in the form of an ordered list of schools. All applicants are assigned to their first choices if there are enough seats in those schools. If a given school is over-demanded, applicants are accepted in the order of their priorities for that school.[3] Those rejected from their first choices face a dramatically decreased chance of being accepted to any other desirable schools since they can only opt for the seats that remain free after everyone's first choice has been considered. As a result, some parents may refrain from ranking schools truthfully, which makes BM vulnerable to manipulation (Abdulkadiroğlu and Sönmez (2003)). In 2005, the BPS replaced BM with the Gale-Shapley student deferred acceptance mechanism (GS), originally proposed by Gale and Shapley (1962), which

[1]Some studies have explored exogenous changes in families' school choice sets to study the impacts of school choice on students' achievement, e.g., Abdulkadiroğlu, Angrist, Dynarsky, Kane and Pathak (2010), Deming, Hastings, Kane and Staiger (2014), Hastings, Kane and Staiger (2009), Lavy (2010), Mehta (2013) and Walter (2013). Other studies focus on how the competition induced by student's school choices affects school performance, e.g., Hoxby (2003) and Rothstein (2006).

[2]See Heckman and Mosso (2014) for a comprehensive review of the literature on human development and social mobility.

[3]Priorities for a given school are often determined by whether or not one lives in the zone that contains that school, whether or not one has siblings enrolled in the same school, and some other socioeconomic characteristics, with a random lottery to break the tie.

provides incentives for households to reveal their true preferences.[4]

Although the vulnerability of BM to manipulation is widely agreed upon, it remains unclear whether or not it should be replaced in other cities as well.[5] In practice, the switch decision by the BPS was resisted by some parents.[6] In theory, the efficiency and equity comparison between BM and its alternatives remains controversial.[7] The welfare implications of various mechanisms thus become an empirical question, one that needs to be answered before a switch from BM to GS or some other mechanisms is recommended more widely.

To answer this question, one needs to quantify two essential but unobservable factors underlying households' choices, which is what we do in this paper. The first factor is household preferences, without which one could not compare welfare across mechanisms even if household choices were observed under each alternative mechanism. Moreover, as choices are often not observed under counterfactual scenarios, one needs to predict which households would change their behaviors and how their behaviors would change, were the current mechanism switched to a different one. The knowledge of household preferences alone is not enough for this purpose. Although BM gives incentives for households to act strategically, there may exist non-strategic households that simply rank schools according to their true preferences.[8] A switch from BM to GS, for example, will induce behavioral changes only among strategic households who hide their true preferences under BM. Therefore, the knowledge about the distribution of household types (strategic or non-strategic) becomes a second essential factor for one to assess the impacts of potential reforms in the school choice system.

We develop a model of school choices under BM by households who differ in both their preferences for schools and their strategic types. Households' preferences for schools depends on the school quality, the school-home distance and attendance fees, interacted with household characteristics and affected by household tastes. Non-

[4]See Abdulkadiroğlu, Pathak, Roth and Sonmez (2005) for a description of the Boston reform.

[5]Pathak and Sönmez (2013) document switches in Chicago and England from certain forms of the Boston mechanism to less-manipulable mechanisms, and argue that these switch decisions revealed government preferences against mechanisms that are (excessively) manipulable.

[6]See Abdulkadiroğlu, Che, and Yasuda (2011) for examples of the concerns parents had.

[7]See the literature review below.

[8]There is direct evidence that both strategic and non-strategic households exist. For example, Abdulkadiroğlu, Pathak, Roth and Sönmez (2006) show that some households in Boston obviously failed to strategize. Calsamiglia and Güell (2014) prove that some households obviously behave strategically. Estimation results in our paper are such that both types exist.

strategic households fill out their application forms according to their true preferences, while strategic households take admissions risks into account and may hide their true preferences.

We apply our model to a rich administrative data set from Barcelona, Spain, where a BM system has been used to assign students to public schools. The data contains information on applications, admissions and enrollment for all Barcelona families who applied for schools in the public school system in the years 2006 and 2007. We observe applicants' family addresses, hence home-school distances, and other family characteristics that allow us to better understand their decisions. Between 2006 and 2007, there was a drastic change in the official definition of school zones that significantly altered the set of schools a family had priorities for in the school assignment procedure. We estimate our model via simulated maximum likelihood using the 2006 pre-reform data. We conduct an out-of-sample validation of our estimated model using the 2007 post-reform data. The estimated model matches the data in both years well.

The results of the out-of-sample validation provide enough confidence in the model to use it to perform counterfactual policy experiments, where we assess the performance of two popular and truth-revealing alternatives to BM: GS and the top trading cycles mechanism (TTC) (Abdulkadiroğlu and Sönmez (2003)).[9] We find that a change from BM to GS benefits fewer than 10% of the households while hurting 28% of households. An average household loses by an amount equivalent to a 60-euro increase in school fees. In contrast, a change from BM to TTC benefits over 20% of households and hurts 19% of them. An average household benefits by an amount equivalent to a 88-euro decrease in school fees. Compared to TTC, BM and GS inefficiently assign households to closer-by but lower-quality schools. On the equity side, a switch from BM to GS is more likely to benefit those who live in higher-school-quality zones than those who live in lower-school-quality zones, hence enlarging the cross-zone inequality. In contrast, the quality of the school zone a household lives in does not impact its chance to win or to lose in a switch from BM to TTC. We also find that while TTC enables 76% of households whose favorite schools are out of their zones to attend their favorite schools, this fraction is only 59% under BM and 52% under GS.

Our paper contributes to the literature on school choices, in particular, the litera-

[9]TTC was inspired by top trading cycles introduced by Shapley and Scarf (1974) and adapted by Abdulkadiroğlu and Sönmez (2003). To our knowledge, only New Orleans has implemented TTC.

ture on the design of centralized choice systems initiated by Balinski and Sönmez (1999) for college admissions, and Abdulkadiroğlu and Sönmez (2003) for public school choice procedures.[10] Abdulkadiroğlu and Sönmez (2003) formulate the school choice problem as a mechanism design problem, and point out the flaws of BM, including manipulability. They also investigate the theoretical properties of two alternatives of BM: GS and TTC. Since then, researchers have been debating on the properties of BM. Some studies suggest that the fact that strategic ranking may be beneficial under BM creates a potential issue of equity since parents who act honestly (non-strategic parents) may be disadvantaged by those who are strategically sophisticated (e.g., Pathak and Sönmez (2008)). Using the pre-2005 data provided by the BPS, Abdulkadiroğlu, Pathak, Roth and Sönmez (2006) find that households that obviously failed to strategize were disproportionally unassigned. Calsamiglia and Miralles (2014) show that under certain conditions, the only equilibrium under BM is the one in which families apply for and are assigned to schools in their own school zones, which causes concerns about inequality across zones. Besides equity, BM has also been criticized on the basis of efficiency. Experimental evidence from Chen and Sönmez (2006) and theoretical results from Ergin and Sönmez (2006) show that GS is more efficient than BM in complete information environments. *However,* in a recent series of studies, Abdulkadiroğlu, Che, and Yasuda (2011); Featherstone and Niederle (2011); and Miralles (2008) all provide examples of specific environments where BM is more efficient than GS. Abdulkadiroğlu, Che, and Yasuda (2011) also point out that some non-strategic parents may actually be better off under BM than under GS.

Although there have been extensive discussions about the strength and weakness of alternative school choice mechanisms in the theoretical literature, empirical studies that are designed to quantify the differences between these alternatives have been sparse. Closely related to our paper, He (2012) estimates an equilibrium model of school choice under BM using data from one neighborhood in Beijing that contains four schools, for which households have equal priorities to attend. Under certain assumptions, he estimates household preference parameters by grouping household choices, without having to model the distribution of household sophistication types. Assuming that a given fraction of non-strategic households exists uniformly across all demographic groups, he calculated welfare changes implied by a switch from BM to

[10]While student priorities for a certain college depends on the college's own "preferences" over students; student priorities for public schools are defined by the central administration.

GS. On the one hand, the approach in He (2012) allows one to be agnostic about the distribution of household strategic types during the estimation, hence imposing fewer presumptions on the data. On the other, it restricts his model's ability to conduct cross-mechanism comparisons. Our paper complements He (2012) by estimating both households' preferences and the distribution of their strategic types. We apply our model to the administrative data that contain the application and assignment outcomes for the entire city of Barcelona, where households are given priorities to schools in their own school zones. With access to this richer data set, we are able to form a more comprehensive view of the alternative mechanisms in terms of the overall household welfare and the cross-neighborhood inequality. With a different focus, Abdulkadiroğlu, Agarwal and Pathak (2014) show the benefits of centralizing school choice procedures, using data from New York city where high school choices used to be decentralized.

Also related to our paper are studies that use out-of-sample fits to validate the estimated model.[11] Some studies do so by exploiting random social experiments, e.g., Wise (1985), Lise, Seitz and Smith (2005) and Todd and Wolpin (2006), or lab experiments, e.g., Bajari and Hortacsu (2005). Other studies do so using major regime shifts. McFadden and Talvitie (1977), for example, estimate a model of travel demand before the introduction of the BART system, forecast the level of patronage and then compare the forecast to actual usage after BART's introduction. Pathak and Shi (2014) aim at conducting a similar validation exercise on the data of school choices before and after a major change in households' choice sets of public schools, introduced in Boston in 2013.[12] Some studies, including our paper, deliberately hold out data to use for validation purposes. Lumsdaine, Stock and Wise (1992) estimate a model of worker retirement behavior of workers using data before the introduction of a temporary one-year pension window and compare the forecast of the impact of the pension window to the actual impact. Keane and Moffitt (1998) estimate a model of labor supply and welfare program participation using data after federal legislation that changed the program rules. They used the model to predict behavior prior to that policy change. Keane and Wolpin (2007) estimate a model of welfare participation, schooling, labor supply, marriage and fertility on a sample of women from five US states and validate the model based on a forecast of those behaviors on

[11] See Keane, Todd and Wolpin (2011) for a comprehensive review.

[12] The authors are waiting for the post-reform data to finish their project.

a sixth state.

The rest of the paper is organized as follows. The next section gives some background information about the public school system in Barcelona. Section 3 describes the model. Section 4 explains our estimation and identification strategy. Section 5 describes the data. Section 6 presents the estimation results. Section 7 conducts counterfactual experiments. The last section concludes. The appendix contains further details and additional tables.

2 Background

2.1 The Public School System in Spain

The public school system in Spain consists of two types of schools, public and semi-public.[13] Public schools are fully financed by the autonomous community government and are free to attend.[14] The operation of public schools follows rules that are defined both at the national and at the autonomous community level. Depending on the administrative level at which it is defined, a rule applies uniformly to all public schools nationally or autonomous-community-wise. This implies that all public schools in the same autonomous community are largely homogenous in terms of the assignment of teachers, school infrastructure, class size, curricula, and the level of (full) financial support per pupil.

Semi-public schools are run privately and funded via both public and private sources.[15] The level of public support per pupil for semi-public schools is defined at the autonomous community level, which is about 60% of that for public schools. Semi-public schools are allowed to charge enrollee families for complementary services. In Barcelona, the service fee per year charged by semi-public schools is $1,280$ euros on average with a standard deviation of 570 euros.[16] On average, of the total

[13]Semi-public schools were added into the system under a 1990 national educational reform in Spain (LOGSE).

[14]Spain is divided into 17 autonomous communities, which are further divided into provinces and municipalities. A large fraction of educational policies are run at the autonomous community (*Comunidades Autonomas)* and municipality levels (*municipios*) following policies determined both at the national and at the local levels. In particular, the Organic Laws (*Leyes Orgánicas*) establish basic rules to be applied nationally; while autonomous communities further develop these rules through what are called *Decretos*.

[15]See http://www.idescat.cat/cat/idescat/publicacions/cataleg/pdfdocs/dossier13.pdf for details.

[16]The median annual housesehold income is $25,094$ euros in Spain and $26,418$ euros in Catalunya.

financial resources for semi-public schools, government funding accounts for 63%, service fees account for 34%, and private funding accounts for 3%. Semi-public schools have much higher level of autonomy than public schools. They can freely choose their infrastructure facilities, pedagogical preferences and procedures. Subject to the government-imposed teacher credential requirement, semi-public schools have controls over teacher recruiting and dismissal. However, there are some important regulations semi-public schools are subject to. In particular, all schools in the public school system, public or semi-public, have to unconditionally accept all the students that are assigned to them via the centralized school choice procedure that we describe in the next subsection; and no student can be admitted to the public school system without going through the centralized procedure. In addition, all schools have the same national limit on class sizes.

Outside of the public school system, there are a small number of private schools, accounting for only 4% of all schools in Barcelona. Private schools receive no public funding and charge very high tuition, ranging from 5,000 to 16,000 euros per year in Barcelona. Private schools are subject to very few restrictions on their operation; and they do not participate in the centralized school choice program.[17]

2.2 School Choice within the Public School System

The Organic Law 8/1985 establishes the right for families to choose schools in the public school system for their children. The national reform in 1990 (LOGSE) extended families' right to guarantee the universal access for a child 3 years or older to a seat in the public school system, by requiring that preschool education (ages 3-5) be offered in the same facilities that offer primary education (ages 6-12). Although a child is guaranteed a seat in the public school system, individual schools can be over-demanded. The Organic Law from 2006 (LOE) specifies broad criteria that autonomous communities shall use to resolve the overdemand for schools. Catalunya, the autonomous community for the city of Barcelona, has its own *Decretos* in which it specifies, under the guideline of LOE, how overdemand for given schools shall be resolved. In particular, it describes broad categories over which applicants may be ranked and prioritized, known as the priority rules.

[17]For this reason, information on private schools is very limited. Given the lack of information on private schools and the small fraction of schools they account for, we treat private schools as part of the (exogenous) outside option in the model.

Families get access to schools in the public school system via a centralized school choice procedure run at the city or municipality level, in which almost all families participate.[18] Every April, participating families with a child who turns three in that calendar year are asked to submit a ranked list of up to 10 schools. Households who submit their applications after the deadline (typically between April 10th and April 20th) can only be considered after all on-time applicants have been assigned.[19] All applications are typed into a centralized system, which assigns students to schools via a Boston mechanism.[20] The final assignment is made public and finalized between April and May, and enrollment happens at the beginning of September, when school starts. In the assignment procedure, all applicants are assigned to their first choice if there are enough seats. If there is overdemand for a school, applicants are prioritized according to the government-specified priority rules. In Catalunya, a student's priority score is a sum of various priority points: the presence of a sibling in the same school (40 points), living in the zone that contains that school (30 points), and some other characteristics of the family or the child (e.g., disability (10 points)). Ties in total priority scores are broken through a fair lottery. The assignment in every round of the procedure is final, which implies that an applicant rejected from her first-ranked school can get into her second-ranked school only if this school still has a free seat after the first round. The same rule holds for all later rounds.

In principle, a family can change schools within the public school system after the assignment. This is feasible only if the receiving school has a free seat, which is a near-zero-probability event in popular schools. The same difficulty of transferring schools persists onto the preschool-to-primary-school transition because a student has the priority to continue her primary-school education in the same school she enrolled for preschool education, and because school capacities remain the same in preschools and primary schools (which are offered in the same facilities). A family's initial school choice continues to affect the path into secondary schools as students are given priorities to attend specific secondary schools depending on the schools they enrolled for primary-school education. On the one hand, besides the direct effect of quality of the preschool on their children's development, families' school choice for their 3-year-old

[18]For example, in 2007, over 95% of families with a 3-year old child in Barcelona participated in the application procedure.

[19]See Calsamiglia and Güell (2014) for more details on the application forms and the laws underlying this procedure.

[20]We will describe the exact procedure in the model section.

children have long-term effects on their children's educational path due to institutional constraints. On the other hand, the highly centralized management of public schools in Barcelona reduces the stakes families take by narrowing the differences across schools.

2.3 Changes in the Definition of Zones (2007)

Before 2007, the city of Barcelona was divided into fixed zones; families living in a given zone had priorities for *all* the schools in that zone.[21] Depending on their specific locations within a zone, families could have priorities for in-zone schools that were far away from their residence while no priority for schools that were close-by but belonged to a different zone. This is particularly true for families living around the corner of different zones. In 2007, a family's school zone is redefined as the smallest area around its residence that covered the closest 3 public and the closest 3 semi-public schools, for which the family was given residence-based priorities.[22] The 2007 reform was announced abruptly on March 27th, 2007, before which there had been no public discussions about it. Families were informed via mail by March 30th, who had to submit their lists by April 20th.

3 Model

3.1 Primitives

There are J public schools distributed across various school zones in the city. In the following, schools refer to non-private (public, semi-public) schools unless specified otherwise. There is a continuum of households/applicants/parents of measure 1 (we use the words household, applicant and parent interchangeably). Each household submits an ordered list of schools before the official deadline, after which a centralized procedure is used to assign students according to their applications, the available

[21]Before 2007, zones were defined differently for public and semi-public schools. A family living at a given location had priorities for a set of public schools defined by its public-school zone, and a set of semi-public schools defined by its semi-public-school zone. Throughout the paper, in-zone schools refer to the union of these two sets of schools; and two families are said to live in the same zone if they have the same set of in-zone schools.

[22]There were over 5,300 zones under this new definition. See Calsamiglia and Güell (2014) for a detailed description of the 2007 reform.

capacity of each school and a priority structure.[23] A student can either choose to attend the school she is assigned to or opt for the outside option.

3.1.1 Schools

Each school j is endowed with a location l_j and a vector w_j of characteristics consisting of school quality, capacity, tuition and an indicator of semi-public school. School characteristics are public information.[24] A school's capacity lie between $(0,1)$ hence no school can accommodate all students. The total capacity of all schools is at least 1, hence each student is guaranteed a seat in the public school system.

3.1.2 Households

A household i is endowed with characteristics x_i, a home location l_i, tastes $\epsilon_i = \{\epsilon_{ij}\}_j$ for schools $j = 1, .., J$, and a type $T \in \{0, 1\}$ (non-strategic or strategic).[25] Household tastes and types, known to households themselves, are unobservable to the researcher. The fraction of strategic households varies with household characteristics and home locations, given by $\lambda(x_i, l_i)$. Types differ only in their behaviors, which will become clear when we specify a household's problem, but all households share the same preference parameters.

As is common in discrete choice models, the absolute level of utility is not identified, we normalize the ex-ante value of the outside option to zero for all households. That is, a household's evaluation of each school is relative to its evaluation of the outside option, which may differ across households. Let $d_{ij} = d(l_i, l_j)$ be the distance between household i and school j, and $d_i = \{d_{ij}\}_j$ be the vector of distances to all schools for i. Household i's utility from attending school j, regardless of its type, is

[23] As mentioned in the background section, almost all families participate in the application procedure. For this reason, we assume that the cost of application is zero and that all families participate. This is in contrast with the case of college application, which can involve significant monetary and non-monetary application costs, e.g., Fu (2014).

[24] We assume that households have full information about school characteristics. Our data do not allow us to separate preferences from information frictions. Some studies have taken a natural or field experiment approach to shed lights on how information affects schooling choices, e.g., Hastings and Weinstein (2008) and Jensen (2010).

[25] Our model is flexible enough to accomodate but does not impose any restriction on the existence of either strategic and non-strategic types. The distribution of the two types is an empirical question. With a parsimonious two-point distribution of sophistication types, the model fits the data well. We leave, as a future extension, more general specifications of the type distribution with more than two levels of sophistication.

given by

$$u_{ij} = U\left(w_j, x_i, d_{ij}\right) + \epsilon_{ij}.$$

where $U\left(w_j, x_i, d_{ij}\right)$ is a function of the school and household characteristics and home-school distance, and ϵ_{ij} is i's idiosyncratic tastes for school j.[26] We assume the vector $\epsilon_i \sim i.i.d.F_\epsilon\left(\epsilon\right)$.[27]

Between application and enrollment (about 6 months), the value of the outside option is subject to a shock $\eta_i \sim i.i.d.N(0, \sigma_\eta^2)$. A parent knows the distribution of η_i's before submitting the application, and observes it afterwards. For example, a parent may experience a wage shock that changes her ability to pay for the private school. This post-application shock rationalizes the fact that some households in the data chose the outside option even after being assigned to the schools of their first choice.

3.2 Priority and Assignment

In this subsection, we describe the official rules on priority scores and the assignment procedure.

3.2.1 The Priority Structure

A household i is given a priority score s_{ij} for each of the schools $j = 1, ..., J$, determined by household characteristics, its home location and the location of the school. Locations matter only up to whether or not the household locates within the school zone a school belongs to. Let z_l be the school zone that contains location l. Household characteristics x_i consists of two parts: demographics x_i^0 and the vector of length J $\left\{sib_{ij}\right\}_j$, where $sib_{ij} = 1$ ($sib_{ij} = 0$) if student i has some (no) sibling enrolled in

[26] Our initial estimation allows a function of zone characteristics to also enter household utility function in order to capture some common preference factors that exist among households living in the same zone. In a likelihood ratio test, we cannot reject that the simpler specification presented here explains the data just as well as the more complicated version. As most studies on the school choice mechanisms, we abstract from peer effects and social interactions from our model. The major complication is the potential multiple equilibria problem embedded in the presence peer effects and social interactions, which implies that truth-telling may no longer hold even under mechanisms such as GS and TTC. See Epple and Romano (2011) and Blume, Brock, Durlauf and Ioannides (2011) for comprehensive reviews on peer effects in education and on social interactions, respectively.

[27] In particular, we assume $\epsilon_{ij} \sim i.i.d.N(0, \sigma_\epsilon^2)$.

school j. Priority score s_{ij} is given by

$$s_{ij} = x_i^0 a + b_1 I\left(l_i \in z_{l_j}\right) + b_2 sib_{ij}, \tag{1}$$

where a is the vector of official bonus points that applies to household demographics, $b_1 > 0$ is the bonus point for schools within one's zone, $I\left(l_i \in z_{l_j}\right)$ indicates whether or not household i lives in school j's zone, and b_2 is the bonus point for the school one's sibling is enrolled in.[28] To reduce its own computational burden, the administration stipulates that a student's priority score of her first choice carries over for all schools on her application list.[29]

3.2.2 The Assignment Procedure: BM

Schools are gradually filled up over rounds. There are $R < J$ rounds, where R is also the official limit on the length of an application list.

Round 1: Only the first choices of the students are considered. For each school, consider the students who have listed it as their first choice and assign seats of the school to these students one at a time following their priority scores from high to low (with random lotteries as tie-breakers) until either there are no seats left or there is no student left who has listed it as her first choice.

Round $r \in \{2, 3, ..., R\}$: Only the r^{th} choices of the students *not previously assigned* are considered. For each school with still available seats, assign the remaining seats to these students one at a time following their priority scores from high to low (with random lotteries as tie-breakers) until either there are no seats left or there is no student left who has listed it as her r^{th} choice.

The procedure terminates after any step $r \leq R$ when every student is assigned a seat at a school, or if the only students who remain unassigned listed no more than r choices. Let $p_j^r(s_{ij})$ be the probability of being admitted to school j in Round r for a student with priority score s_{ij} for school j, who listed j as the r^{th} application. The assignment procedure implies that the admissions probability is (weakly) decreasing in priority scores within each round, and is (weakly) decreasing over rounds for all

[28]It follows from the formula that a student can have 2, 3 or 4 levels of priority scores, depending on whether or not the school is in-zone or out-of-zone, whether or not one has sibling(s) in some in-zone and/or out-of-zone schools. See the appendix for details.

[29]For example, if a student lists an in-zone (out-of-zone) sibling school as her first choice, she carries $x_i^0 a + b_1 + b_2$ $(x_i^0 a + b_2)$ for all the other schools she listed regardless of whether or not they are within her zone and whether or not she has a sibling in those schools.

priority scores. A student who remains unassigned after the procedure ends can propose a school that still has empty seat and be assigned to it.

3.3 Household Problem

We start with a household's enrollment problem. After seeing the post-application shock η_i to its outside option and the assignment result, a household chooses the better between the school it is assigned to and the outside option. Let the expected value of being assigned to school j be v_{ij}, such that

$$v_{ij} = E_{\eta_i} \max \{u_{ij}, \eta_i\}.$$

As seen from the assignment procedure, if rejected by all schools on its list, a household can opt for a school that it prefers the most within the set of schools that still have empty seats after everyone's applications have been considered. Label these schools as "leftovers," and i's favorite "leftover" school as i's backup. The value (v_{i0}) of being assigned to its backup school for household i is given by

$$v_{i0} = \max \{v_{ij}\}_{j \in leftovers}.$$

In the following, we describe a household's application problem, in which it chooses an ordered list of up to R schools. We do this separately for non-strategic and strategic households.

3.3.1 Non-Strategic Households

A non-strategic household lists schools on its application form according to its true preferences $\{v_{ij}\}_j$. Without further assumptions, any list of length n $(1 \leq n \leq R)$ that consists of the ordered top n schools according to $\{v_{ij}\}_j$ is consistent with non-strategic behavior. We impose the following extra structure: suppose household i ranks its backup school as its n_i^*-th favorite, then the length of i's application list n_i is such that

$$n_i \geq \min \{n_i^*, R\}. \tag{2}$$

That is, when there are still slots left on its application form, a non-strategic household will list at least up to its backup school.

Let $A_i^0 = \left\{a_1^0, a_2^0, ..a_{n_i}^0\right\}$ be an application list for non-strategic ($T = 0$) household i, where a_r^0 is the ID of the r^{th}-listed school and n_i satisfies (2). The elements in A_i^0 are given by

$$\begin{aligned} a_1^0 &= \arg\max \{v_{ij}\}_j \\ a_r^0 &= \arg\max \{v_{ij} | j \neq a_{r'<r}\}_j, \text{ for } 1 < r \leq \min\{n_i^*, n_i\}. \end{aligned} \tag{3}$$

Define $\mathbf{A}^0(x_i, \epsilon_i, l_i)$ as the set of lists that satisfy (2) and (3) for a non-strategic household with characteristics x_i, location l_i and tastes ϵ_i. If $n_i^* \geq R$, the set $\mathbf{A}^0(\cdot)$ is a singleton, and the length of the application list $n_i = R$. If $n_i^* < R$, all lists in the set $\mathbf{A}^0(\cdot)$ are identical up to the first n_i^* elements, and they all imply the same allocation outcome for household i.

Remark 1 *Notice that Condition* (2) *requires that, instead of being totally naive, a non-strategic household know which schools will be leftovers. We have imposed this extra condition for the following reasons. First of all, to know the set of leftover schools involves far less sophistication than to know all admissions probabilities by school and by round. It is reasonable to believe that even the non-strategic households may have this (minimal) level of sophistication. Second, in order to calculate welfare, we need to predict the content of an application list at least up to the point beyond which listing any additional schools will not affect the allocation outcome. Condition* (2)*, together with Condition* (3)*, gives the model such a predictive power without assuming too much sophistication for non-strategic households.*

3.3.2 Strategic Households

Strategic households are fully aware of the admissions probabilities in all rounds and take them into account when applying for schools. A household's expected payoffs depend not only on which schools it includes on its application list, but also on how these schools are listed.[30] In other words, a strategic household has to choose one particular permutation from the set of all schools. When the total number of schools J is relatively big, solving for a fully optimal *ordered* list of length $n \in \{1, 2, ...R\}$ out of all J schools will soon become a daunting task for any household as R goes beyond

[30]Because admissions probabilities are household-school-round-specific, for a given household, the admissions probabilities to a given school vary with where the household put it on the application list.

1. In the case of Barcelona, J is over 300 and R is 10. We assume that a strategic household uses the following less-demanding decision-making process. First, from *all J schools*, a strategic household i *chooses* a smaller set of candidate schools $J_i^* \subset \{1, ..., J\}$. Then, household i makes an application list out of its candidate set J_i^* to maximize its expected utility, taking into account the admissions probabilities in all rounds.

Candidate Schools We assume that in the first step of its optimization problem, a strategic household i narrows down all schools into its own candidate set (J_i^*) of size N, composed of three non-overlapping groups of schools.[31]

The first group (the favorite) consists of N_1 schools that the household prefers the most out of the ones to which it has some positive probability of being admitted.

The second group (the middle ground) consists N_2 schools that are not in the first group and that generate the highest one-shot expected values, $p_j^1(s_{ij})\, v_{ij}$, where the expectation is based on the first round admissions probability only.

The third group (the insurance) consists N_3 schools that the household prefers the most among those that are not already included in the first two groups and that remain available after the first round and the backup school.[32]

Remark 2 *Notice that all households face the same choice set, i.e., the set of all J schools. From this choice set, a household* ***chooses*** *its set of candidate schools, which depends on their preferences and therefore on parameter values. As J^* approaches J, the decision-making process converges to the the fully optimal process.*

Optimal Lists The second step of household i's decision procedure involve choosing the optimal application list out of *any ordered subset* of i's candidate set (J_i^*) chosen in the first step. Recall that a student's priority score is kept constant over all rounds in Barcelona. Define the remaining value of list $A = \{a_1, ..., a_R\}$ from round $r \leq R$

[31] Given household preference parameters, a household i may have a favorite school j that is also a middle-ground and/or an insurance school. We require that the three groups of schools be non-overlapping not only to ensure that school j be included in i's candidate set but also to avoid shrinking i's candidate set.

[32] In practice, we set $N_1 = 10$, $N_2 = 10$, $N_3 = 3$. Our estimates are robust to the expansion of the third group (safe schools) because over 95% of households were assigned within the first two rounds. Given the estimated parameter values, our model predictions are robust to the expansion of all three groups.

onwards for household i with some priority score s as

$$V^{r}\left(A,s,x_i,l_i,\epsilon_i\right)=p^{r}_{a_r}\left(s\right)v_{ia_r}+(1-p^{r}_{a_r}\left(s\right))V^{r+1}\left(A,s,x_i,l_i,\epsilon_i\right),$$

and

$$V^{R+1}\left(A,s,x_i,l_i,\epsilon_i\right)=v_{i0}.$$

An optimal list out of J_i^* for a strategic household i, denoted as $A_i^1=\{a_{i1}^1,...a_{iR}^1\}$, solves the following problem

$$\max_{A\subset J_i^*} V^1\left(A,s,x_i,l_i,\epsilon_i\right) \qquad (4)$$
$$s.t.\ s=s_{ia_1},$$

where the constraint reflects the fact that a student's priority score of her first choice carries over for all schools on her application list.

There can be multiple optimal application lists yielding the same value. Let $\mathbf{A}^1\left(x_i,l_i,\epsilon_i\right)$ be the set of optimal lists for a strategic household. All lists in the optimal set are identical up to the payoff-relevant part of the lists and imply the same allocation outcome. For example, consider a list $A^1=\{a_1^1,...,a_r^1,...a_R^1\}$, by the specification of $\{u_{ij}\}$, each a_r^1 is generically unique if there is no school listed before it has a 100% admissions rate for the household. However, if for some $r<R$, the admissions rate for the r^{th} listed school is one, then any list that shares the same first r ordered elements is also optimal. See the appendix for other cases.

Remark 3 *It is worth noting that we do not take a stand on why some households are strategic while some are not. This is an important research question, especially if households can choose whether or not to be strategic, and if a policy change may affect such choices. This is less of a concern in our case, because the major goal of this paper is to investigate the impacts of switching BM to some other mechanisms that are truth-revealing. Under a truth-revealing mechanism, all households, strategic or not, will rank schools according to their true preferences, i.e., types no longer matter. Once we recover household preferences and the (current) distribution of strategic versus non-strategic types in the data, we can compare the current regime with its alternatives without knowing how household types will change in the new environment.*

4 Estimation

4.1 Additional Structure

Conditions (2) and (3) for the non-strategic and Condition (4) for the strategic are sufficient for the model to predict student welfare and allocation results. This implies that if an observed household listed a leftover school, all schools listed after it will not be informative about the household's preferences because these later schools would not affect the household's payoff. In order to use the data to its full potential, we assume that the schools listed after a leftover school are ranked *within* themselves. Notice, however, we do not impose any additional structure, beyond Conditions (2)-(4) on how these later-listed schools compare with other schools, nor do we require that they belong to a strategic household's candidate set J_i^*. Let $\mathbf{\Lambda}^T(x_i, l_i, \epsilon_i)$ the subset of the optimal application set $\left(\mathbf{A}^T(x_i, l_i, \epsilon_i)\right)$ for type T household that satisfies this additional structure.[33]

4.2 Likelihood

The model is estimated via the simulated maximum likelihood estimation method. The estimates of the model parameters should maximize the probability of the observed application and enrollment outcomes conditional on household observables (x_i, l_i), school characteristics and location (w_j, l_j), and student-school assignments.[34] Denote the vector of model parameters as $\Theta \equiv [\Theta_u, \Theta_T]$, where Θ_u is the vector of parameters that govern household preferences, and Θ_T is the vector of parameters that govern the distribution of household types. In particular, Θ_u is composed of 1) the parameters that govern the net benefit function $U(\cdot)$ of attending schools, 2) the dispersion of household tastes for schools σ_ϵ, and 3) the dispersion of post-application shocks to the value of the outside option σ_η.

Let $O_i \equiv \left[\widetilde{A}_i, \widetilde{e}_i | \widetilde{j}_i\right]$ be the observed outcomes for household i, where $\widetilde{A}_i$ is the observed application list, $\widetilde{j}_i$ is the school student i was assigned to, and $\widetilde{e}_i$ is the observed enrollment decision given one being assigned to school $\widetilde{j}_i$. Recall that a

[33]See Appendix D formal details. He (2012) takes a similar approach and assumes that if a household includes on its list some schools that are worse than its outside option, the ranking of these schools reveals the household's true preference.

[34]Notice that given applications, student assignment is a mechanical procedure that does not depend on parameters of the model, so it does not contribute to the likelihood per se.

household can either enroll in the assigned school or choose the outside option, hence $\widetilde{e}_i = I(\text{enroll})$, where $I(\cdot)$ is the indicator function.

Conditional on being type T, the probability of observing O_i is given by

$$L_i^T(\Theta_u) = \int \left\{ \begin{array}{l} I\left(\widetilde{A}_i \in \mathbf{\Lambda}^T(x_i, l_i, \epsilon_i; \Theta_u)\right) \times \\ \left[\widetilde{e}_i \Phi\left(\frac{U\left(w_{\widetilde{j}_i}, x_i, d_{i\widetilde{j}_i}; \Theta_u\right) + \epsilon_{i\widetilde{j}_i}}{\sigma_\eta}\right) + (1 - \widetilde{e}_i)\left(1 - \Phi\left(\frac{U(\cdot) + \epsilon_{i\widetilde{j}_i}}{\sigma_\eta}\right)\right)\right] \end{array} \right\} dF_\epsilon(\epsilon; \sigma_\epsilon),$$

where $\mathbf{\Lambda}^T(x_i, l_i, \epsilon_i; \Theta_u)$ is the subset of model-predicted optimal application lists for a type-T household with (x_i, l_i, ϵ_i) as described in the previous subsection. $\Phi\left(\frac{U(\cdot)+\epsilon_{i\widetilde{j}_i}}{\sigma_\eta}\right)$ is the model-predicted probability that this household will enroll in school $\widetilde{j}_i$, which happens if only if the post-application shock to the outside option is lower than the utility of attending $\widetilde{j}_i$.

To obtain household i's contribution to the likelihood, we integrate over the type distribution

$$L_i(\Theta) = \lambda(x_i, l_i; \Theta_T) L_i^1(\Theta_u) + (1 - \lambda(x_i, l_i; \Theta_T)) L_i^0(\Theta_u).$$

Finally, the total log likelihood of the whole sample is given by

$$\pounds(\Theta) = \sum_i \ln(L_i(\Theta)).$$

4.3 Identification

We give an overview of the identification in this subsection and leave the formal proof in Appendix B2. Besides the regular conditions such as utility functions be continuous, the identification relies on the following assumptions.

IA1: There does not exist a vector of household observable x and a school j, such that all households with x has zero admissions probabilities to school j.

IA2: Household tastes ϵ are drawn from an i.i.d. unimodal distribution, with mean normalized to zero. Tastes are independent of school observables w, household observables (x, l) and household type (T).

IA3: At least one continuous variable in the utility function is excluded from the type distribution. Conditional on variables that enter the type distribution function, the excluded variable is independent of household type T.

To illustrate the identification challenge, consider a situation where each household only applies to one school, which is a less favorable situation for identification because we would have less information, and suppose there is no post-application shock.[35] If all households are non-strategic, the model boils down to a multinomial discrete choice model with a household choosing the highest $U(w_j, x_i) + \epsilon_{ij}$. The identification of such models is well-established under very general conditions (e.g., Matzkin (1993)). If all households are strategic, the model is modified only in that a household considers the admissions probabilities $\{p_{ij}\}_j$ and chooses the option with the highest expected value. With $\{p_{ij}\}$ observed from the data, this model is identified with the additional condition IA1, which requires that for any x, the expected value of applying for school j is not degenerate. The challenge exists because we allow for a mixture of both types of households. In the following, we first explain IA2-IA3, then give the intuition underlying the identification proof.

We observe application lists with different distance-quality-risk combinations with different frequencies in the data. The model predicts that households of the same type tend to make similar application lists. Given IA2, the distributions of type-related variables will differ around the modes of the observed choices, which informs us of the correlation between type T and these variables. IA3 guarantees that different behaviors can arise from exogenous variations *within a type*. To satisfy IA3, we need to make some restrictions on how household observables (x_i, l_i) enter type distribution and utility. Conditional on distance, a non-strategic household may not care too much about living to the left or the right of a school, but a strategic household may be more likely to have chosen a particular side so as to take advantage of the priority zone structure.[36] However, given that households, strategic or not, share the same preferences about school characteristics and distances, there is no particular reason to believe that everything else being equal, the strategic type will live closer to a particular school than the non-strategic type do only for the sake of being close. In other words, because the only difference between a strategic type and a non-strategic type is whether or not one considers the admissions probabilities, which are affected by one's home location only via the zone to which it belongs to, we assume that home

[35] The post-application shock is identified from the observed allocation and enrollment outcomes.

[36] Without directly modeling households' location choices, we allow household types to be correlated with the characteristics of the school zones they live in. We leave the incorporation of household location choices for future extensions.

location l_i enters the type distribution only via the school zone z_{l_i}, i.e.,

$$\lambda(x_i, l_i) = \lambda(x_i, z_{l_i}).$$

In contrast, household utility depends directly on the home-school distance vector d_i, where $d_{ij} = d(l_i, l_j)$. Conditional on being in the same school zone, households with similar characteristics x but different home addresses still face different home-school distance vectors d, as required in IA3.

Conditional on (x, z_l), the variation in d induces different behaviors within the same type; and conditional on (x, z_l, d), different types behave differently. In particular, although households share the same preference parameters, different types of households will behave as if they have different sensitivities to distance. For example, consider households with the same (x, z_l) and a good school j out of their zone z_l, as the distance to j decreases along household addresses, more and more non-strategic households will apply to j because of the decreasing distance cost. However, the reactions will be much less obvious among the strategic households, because they take into account the risk of being rejected, which remains unchanged no matter how close j is as long as it is out of z_l. In fact, as the home address moves closer and closer to the border of the school zones, strategic households may appear to "prefer" schools that are further away. The different distance-elasticity among households therefore inform us of the type distribution within (x, z_l).[37] This identification argument does not depend on specific parametric assumptions. For example, Lewbel (2000) shows that similar models are semiparametrically identified when an IA3-like excluded variable with a large support exists. However, to make the exercise feasible, we have assumed specific functional forms. Appendix B2 shows a formal proof of identification given these additional specifications.

The identification of our model is further facilitated by the fact that we can partly observe household type directly from the data: there is one particular type of "mistakes" that a strategic household will never make, which is a sufficient (but not necessary) condition to spot a non-strategic household. Intuitively, if a household's admissions status is still uncertain for all schools listed so far, and there is another

[37] Although our identification does not rely on the following extreme case, one can also take the argument to the case of households along the border of two zones. Were all households non-strategic, the applications should be very similar by households across the border. In contrast, were all households strategic, drastically different applications could occur.

school j it desires, it never pays to waste the current slot listing a zero-probability school instead of j because the admissions probabilities decrease over rounds.[38] The idea is formalized in the following claim and proved in the appendix.[39]

Claim 1 *An application list with the following features is sufficient but not necessary evidence that the household must be non-strategic: 1) for some r^{th} element a_r on the list, the household faces zero admissions probability at the r^{th} round $\left(p^r_{a_r}(s_i) = 0\right)$, and 2) it faces admissions probabilities lower than 1 for all schools listed in previous rounds $\left(p^{r'}_{a_{r'}}(s_i) < 1 \text{ for all } r' < r\right)$, and 3) it faces a positive but lower than 100% admissions probability for the school listed in a later slot $r" \geq r+1$ $\left(0 < p^{r"}_{a_{r"}}(s_i) < 1\right)$ and no school listed between a_r and $a_{r"}$ admits the household with probability 1.*

5 Data

Our analysis focuses on the applications among families with children that turned 3 years old in 2006 or 2007 and lived in Barcelona. For each applicant, we observe the list of schools applied for, the assignment and enrollment outcomes. We also have information on the applicant's home address, family background, and the ID of the school(s) her siblings were enrolled in the year of her application. For each school in the public school system, we observe its type (public or semi-public), a measure of school quality, school capacity and the level of service fees. The final data set consists of merged data sets from five different administrative units: the *Consorci d'Educacio de Barcelona* (local authority handling the choice procedure in Barcelona), *Department d'Ensenyament de Catalunya* (Department of Education of Catalunya), the *Consell d'Avaluacio de Catalunya* (public agency in charge of evaluating the Catalunya educational system), the *Instituto Nacional de Estadistica* (national institute of statistics) and the *Institut Catala d'Estadistica* (statistics institute of Catalunya).[40]

[38] Abdulkadiroğlu, Pathak, Roth and Sonmez (2006) use a mistake similar to Feature 1) in Claim 1 to spot non-strategic households, which is to list a school over-demanded in the first round as one's second choice.

[39] If the support of household characteristics is full conditional on being obviously non-strategic, household preferences can be identified using this subset of households without IA1, since ϵ is independent of (x, l). However, our identification is only faciliated by, not dependent on the existence of obviously non-strategic households.

[40] These five different data sources were merged and anonimized by the Institut Catala d'Estadistica (IDESCAT).

5.1 Data Sources

From the *Consorci d'Educacio de Barcelona,* we obtain access to every applicant's application form, as well as the information on the school assignment and enrollment outcomes. An application form contains the entire list of ranked schools a family submitted. In addition, it records family information that was used to determine the priority the family had for various schools (e.g., family address, the existence of a sibling in the first-ranked school and other relevant family and child characteristics). The geocode in this data set allows us to compute a family's distance to each school in the city.

From the Census and local register data, we obtain information on the applicant's family background, including parental education and whether or not both parents were registered in the applicant's household. Since information on siblings who were not enrolled in the school the family ranked first is irrelevant in the school assignment procedure, it is not available from the application data. However, such information is relevant for family's application decisions. From the Department of Education, we obtained the enrollment data for children aged 3 to 18 in Catalunya. This data set is then merged with the local register, which provides us with the ID of the schools enrolled by each of the applicant's siblings at the time of the application.

To measure the quality of schools, we use the external evaluation of students conducted by *Consell d'Avaluacio de Catalunya*. Since 2009, such external evaluations have been imposed on all schools in Catalunya, in which students enrolled in the last year of primary school are tested on math and language subjects.[41] From the 2009 test results that we obtained, we calculated the average test score across subjects for each student, then use the average across students in each school as a measure of the school's quality.[42] Finally, to obtain information on the fees charged by semi-public schools (public schools are free to attend), we use the survey data collected by the *Instituto Nacional de Estadistica.*[43]

[41]As mentioned in the background section, a student has the priority to continue her primary-school education in the same school (with the same capacity) she enrolled for preschool education, which makes it very unlikely that one can transfer to a better school between preschool-primary school transition. For example, at least 94% of the 2010 preschool cohort were still enrolled in the same school for primary school education in 2013.

[42]Following the same rule used in Spanish college admissions, we use unweighted average of scores across subjects for each student.

[43]See http://www.idescat.cat/cat/idescat/publicacions/cataleg/pdfdocs/dossier13.pdf for a summary of the survey data.

5.2 Admissions Probabilities and Sample Selection

It is well-known that BM can give rise to multiple equilibria, which can greatly complicate the estimation of an equilibrium model.[44] However, assuming each household is a small player that takes the admissions probabilities as given, we can recover all the model parameters by estimating an individual decision model. This is possible because the assignment procedure is mechanical and because we observe the applications and assignment results for *all* participating families, which we use to calculate the admissions probabilities. Due to the fact that the assignment to over-demanded schools depends on random draws to break the tie between applicants of the same priority, we obtain the admissions probabilities as follows. Taking the observed applications as given, we take random draws for all applicants and simulate the assignment results, which yields the round-school-priority-score-specific admissions probabilities $\left\{p_j^r(s)\right\}$ for the given set of random draws. We obtain the admissions probabilities by repeating the process 1,000 times and then integrating over the results. The simulated admissions probabilities are treated as the ones that the households expected when they apply, i.e., before the realization of the tie-breaking random draws.

In 2006, 11,871 Barcelona households participated in the application for schools in the Barcelona public school system. After we calculated the admissions probabilities using the entire sample, we conduct sample selection for the estimation as follows. We drop 3,152 observations whose home location information cannot be consistently matched with the GIS (geographic information system) data, for example, due to typos. We exclude 191 families whose children have special (physical or mental) needs or who submitted applications after the deadline, the latter were ineligible for assignment in the regular procedure. We drop 31 households whose applications, assignment and/or enrollment outcomes are inconsistent with the official rule, e.g., students being assigned to over-demanded schools they did not apply for. Finally, we delete observations missing critical information such as parental education and the enrollment information of the applicant's older sibling(s).[45] The final sample size for

[44] He (2012) did not detect multiple equilibria in his simulations and hence estimated the equilibrium model assuming uniqueness.

[45] Our model distinguishes between high-school education and college eduation. Therefore, the observations excluded from the estimation sample include 748 parents who reported their education levels as "high school or above." In policy simulations, however, we do include this subsample and simulate their application behaviors in order to be able to conduct the city-wise assignment under alternative mechanisms. We interpolate the probability of each of these 748 households as being high school or college educated by comparing them with those who reported exactly high-school

estimation is 6,836.

5.3 Summary Statistics

There were 158 public schools and 159 semi-public schools in our sample period. Table 1 summarizes school characteristics separately for the two groups of schools. The first row summarizes school quality as measured by the average test scores of students in each school.[46] The average quality of public schools is 7.4 with a standard deviation of 0.8. Semi-public schools have higher average quality of 8.0 and a smaller dispersion of 0.5. Although public schools are free to attend, semi-public schools charge on average 1,280 euros per year with a standard deviation of 570 euros. The average capacity for the incoming 3-year-old students in public schools is 1.4 classes, as compared to 1.8 in semi-public schools.

Table 1 School Characteristics

	Public	Semi-Public	All
Quality	7.4 (0.8)	8.0 (0.5)	7.7 (0.7)
Fees (100 Euros)	0	12.8 (5.7)	6.4 (7.5)
# Classes	1.4 (0.5)	1.8 (1.0)	1.6 (0.8)
Observations	158	159	317

Table 2 summarizes the household characteristics of the 2006 estimation sample. Among all households, about 30% parents had less than high school education and about 40% had college education.[47] For about 15.8% of the sample, only one parent was registered in the applicant's household. We refer such households as "single parent" households throughout the paper. Over 42% of applicants had at least one older sibling enrolled in some preschool or primary school in 2006, almost all of these older siblings were enrolled in the Barcelona public school system (40.7% out of 42.2%). Depending on their home locations, the numbers of schools for which households had priorities were different, so was the average quality of these schools. On average, a household had priority for 22 schools in 2006 with a standard deviation

education or college education. We estimate the probabilities via a flexible function of all the other observable characteristics, such as gender, residential area, age, number of children etc. The model fit for this subsample is as good as that for the estimation sample, available on request.

[46]We measure test scores on a scale from 0 to 10, distance in 100 meters and tuition in 100 euros.

[47]Following the literature on child development, we use mother's education as the definition of parental education if the mother is present in the household, otherwise, we use the father's education.

of almost 8 schools. The average quality of schools within one's priority zone was 7.8 and the cross zone dispersion was 0.3.

Table 2 Household Characteristics

Parental Edu[a] < HS	29.8%
Parental Edu = HS	30.4%
Parental Edu ≥College	39.8%
Single Parent	15.8%
Have school-age older sibling(s)	42.2%
# Schools in Zone	22.3 (7.9)
Average school quality in zone	7.8 (0.3)
Observations	6,836

[a]Parental Edu: mother's edu if she is present, o/w father's edu.

Table 3 shows the number of schools households listed on their application forms. Households were allowed to list up to 10 schools, but most of households listed no more than 3 schools, with 47% of households listing only one school. Across different educational groups, parents with lower-than-high-school education were more likely to have a shorter list, while parents with exact high school education tended to list more schools than the others. Single parents also tended to list more schools compared to both-parent households.

Table 3 Number of Schools Listed (%)

	1	2	3	4 or more
All	46.9	12.4	16.9	23.8
Parental Edu < HS	49.8	15.0	19.5	15.7
Parental Edu = HS	43.4	12.1	18.4	26.2
Parental Edu ≥College	47.4	10.6	13.9	20.1
Single-Parent	43.3	14.7	16.8	25.2

Table 4 shows the round at which households were assigned. By definition, a household was assigned to its r^{th} listed school if it was assigned in round r; and remained unassigned if it failed to get in any of its listed schools. Ninety three percent of households were assigned in the first round; 2.8% were assigned in the second round and 2.7% were unassigned. Across educational groups, college-educated parents were most likely to be assigned to their first choices (93.7%), followed by the lowest educational group. High-school educated parents were the least likely to be assigned to their first choice and to be assigned at all. Single parents were more likely to be assigned to their first choice compared to their counterpart.

Table 4 Assignment Round (%)

	1st	2nd	3rd-10th	Unassigned
All	93.0	2.8	1.5	2.7
Parental Edu $<$ HS	93.2	2.7	1.6	2.5
Parental Edu $=$ HS	92.0	3.5	1.5	3.0
Parental Edu $\geq$College	93.7	2.3	1.3	2.7
Single-Parent	94.0	1.9	1.4	2.7

Given that most households were assigned to their first choices, Table 5 summarizes the characteristics of the top-listed schools. For all students, the average quality of the top-listed schools was 7.9. The home-school distance was about 710 meters. The distance-quality trade-offs seem to differ across educational groups: as parental education goes up, the quality of top-listed schools increases while the distance decreases. Single parents were more likely to top-list a school with higher quality yet longer distance, compared to an average household.

Table 5 Top-Listed Schools

	Quality	Distance (100m)	Fees (100Euros)
All	7.9 (0.6)	7.1 (8.7)	8.1 (7.7)
Parental Edu $<$ HS	7.6 (0.7)	5.2 (6.2)	5.4 (6.6)
Parental Edu $=$ HS	7.9 (0.5)	7.0 (8.6)	8.1 (7.5)
Parental Edu $\geq$College	8.2 (0.4)	8.7 (9.9)	9.9 (8.1)
Single-Parent	8.0 (0.6)	8.1 (9.9)	8.6 (8.4)

Finally, Table 6 lists the fraction of all students, assigned or unassigned, who were enrolled in the public school system (recall that a household can propose a school with an available seat and be assigned to it after the regular admissions if the household remains unassigned to any of its listed schools). Overall, 97% of applicants were enrolled in the public school system. Applicants with college-educated parents and/or single parents were less likely to enroll. The last row shows that 2.2% of households chose not to enroll even though they had been assigned to their first choice. The ex-post shocks introduced in the model are meant to rationalize such behaviors.

Table 6 Enrollment in Public System (%)

All	96.7
Parental Edu < HS	97.0
Parental Edu = HS	97.1
Parental Edu ≥College	96.3
Single-Parent	96.1
Assigned in Round 1	97.8

6 Results

6.1 Parameter Estimates

Table 7 presents the estimated parameters governing household preferences, with standard errors shown in parentheses. The utility function is linear with interactions between household characteristics and school characteristics. We treat high-school educated both-parent households as the base group. High-school educated parents value schools within the public school system more than other educational groups, especially the college-educated group. One explanation is that college-educated parents are more likely to be able to afford a costly outside option (a private school). Single parents also tend to value public schools less than their counterpart. The next row shows that it is especially attractive for a household to send the child to the same school where her older sibling was enrolled in. These parameter estimates are consistent with the observed behaviors. For example, Table 5 and Table 6 show that although college-educated parents and single parents were more likely to be assigned to their top choices, they were less likely to enroll their children in the public school system. It is also observed that most households with more than one child sent the younger child to her sibling's school.

The sixth row on the left panel of Table 7 shows that holding everything else constant, semi-public schools are more preferable to public schools, which may reflect the fact parents value the more flexible management and curriculum in semi-public schools. The next four rows show the effect of school fees. Price sensitivities decrease with education levels; and the cost of fees is concave. In particular, the cost of fees peaks around 1,500 (1,600) euros per year for the college (high-school) educated parents, which is around the 75^{th} percentile of the distribution of fees charged by semi-public schools. For the lowest-education group, the cost peaks around 2,100 euros, which is the 95^{th} percentile of of the distribution of fees charged by semi-public schools. The finding that tuition cost is concave reflects the fact that a large part of the fees are charged for additional services provided by semi-public schools, which apparently are valued by parents. Our preference parameters on fees capture the effects of monetary costs net of the benefits associated with these fees.[48] The last three rows on the left panel show households' additional preferences for schools with capacities of more than one first-year class. Consistent with the fact that larger schools tend to have more resources and lower closing-down probabilities, our parameter estimates show that households prefer schools with larger capacity.

Table 7 Preference Parameters

Constant	3478.9 (122.4)	q*I(Edu < HS)	0.01 (1.87)
Single Parent	-261.9 (172.6)	q*I(Edu = HS)	6.4 (0.4)
Education < HS	-57.7 (471.3)	q*I(Edu > HS)	1.6 (0.2)
Education > HS	-405.6 (150.7)	$I\left(q > q^g\right)*\left(q - q^g\right)^2$*I(Edu < HS)	7.8 (3.1)
Sibling School	1618.7 (194.5)	$I\left(q > q^g\right)*\left(q - q^g\right)^2$*I(Edu = HS)	20.6 (1.3)
Semi-Public School	37.6 (0.3)	$I\left(q > q^g\right)*\left(q - q^g\right)^2$*I(Edu > HS)	41.6 (0.2)
Fee*I(Edu < HS)	-5.1 (0.04)	$I\left(q < q^b\right)*\left(q^b - q\right)^2$	-10.6 (0.2)
Fee*I(Edu = HS)	-3.8 (0.01)	Distance (100m)	-1 (n/a)
Fee*I(Edu > HS)	-3.6 (0.01)	Distance^2	-0.05 (0.001)
Fee^2	0.12 (0.001)	Distance>5 (100m)	-46.3 (0.1)
# Classes= 2	22.0 (0.1)	Distance>10 (100m)	-23.5 (0.3)
# Classes= 3	40.7 (0.4)	σ_ϵ(taste dispersion)	53.8 (0.1)
# Classes> 3	54.2 (0.4)	σ_η(post-application shock)	2131.7 (40.6)

$q^g\left(q^b\right)$ is the quality of the school at the 75th (25th) percentile.

The right panel of Table 7 shows the trade-offs between quality (q) and distance. There are three sets of quality parameters: 1) education-group-specific linear impacts of quality, 2) education-group-specific square terms on school quality beyond the 75^{th} percentile of the quality distribution (q^g), and 3) a square term on school quality below the 25^{th} percentile $\left(q^b\right)$. Except for the high-school-educated parents, the linear effects of school quality are small, especially for the lowest-educational group, for

[48]It will be interesting to disentangle the cost from the benefits associated with service fees, which requires information that is unavailable in our data sets.

whom the linear term is almost zero. Households do, however, value the top schools. As shown in the 4th to 6th row, the preferences for schools that are ranked at the higher end of the quality distribution are strongly convex, especially for the higher educated groups. The next row shows that households also have a strong aversion against schools at the lower end of the quality distribution. In sum, although households may not care too much about the quality differences across schools in the middle of the quality distribution, they do care much more about the very good schools and the very bad ones. The next four rows show preferences on distances. The linear preference parameter on distance is normalized to -1. The cost of distance is convex with the square term being 0.05. In addition, we allow two jumps in the cost of distances. The first jump is set at 500 meters, which is meant to capture an easy-to-walk distance even for the 3-year old. Another jump is at the 1 kilometer threshold, which is a long yet perhaps still manageable walking distance. As households may have to rely on some other transportation methods when a school is beyond walking distances, it is not surprising to see that the cost of distance jumps significantly at the thresholds, by about 4.6 kilometers at the first threshold and by another 2.3 kilometers at the second. Our findings that parents of different education levels differ in their views of the trade-offs among quality, fees and distance are consistent with those in the literature.[49] Finally, the last two rows on the right panel show, respectively, the dispersion of household preferences across schools and that of post-application shocks. The latter is necessary to rationalize the fact that some households gave up their first choices after being assigned. In addition, households take expectations over these ex-post shocks when applying, thus application provides another benefit, i.e., an option value.

Table 8 presents the estimated parameters governing the probability that a household is strategic, which takes a logistic functional form. Single parents are slightly less likely to be strategic. Compared to high-school-educated parents, those with lower or higher education levels are more likely to be strategic. We do not find that strategic households are more likely to live in zones with more schools, and in fact, the coefficient is slightly negative. However, we do find that strategic households are more likely to live in zones with higher average school quality. Although not precisely estimated, households with older children are more likely to be strategic.

[49] For example, Burgess et al (2009), Hastings, Kane and Staiger (2008), He (2012) and Abdulkadiroğlu, Agarwal and Pathak (2014).

Table 8 Type Distribution

Constant	-17.7 (1.5)
Single Parent	-0.2 (0.2)
Education < HS	1.1 (0.2)
Education > HS	0.4 (0.2)
No. schools in zone	-0.1 (0.01)
Average school quality in zone	3.0 (0.2)
Have an older sibling	38.0 (42.6)

Based on the estimates in Table 8, Table 9 shows the simulated type distribution in our sample. The left panel shows that 96% of all households were strategic, i.e., very few households applied without considering the odds of being admitted.[50] The next 5 rows shows the fraction of strategic households for each of the subgroups of the sample. Across educational groups, the high-school educated households were the least likely to be strategic. Households with both parents and those with older children were both more likely to be strategic. The upper-right panel of Table 9 shows the average characteristics of the zones lived in by different types of households. On average, strategic (non-strategic) households lived in zones with 22.3 (23.1) schools and the average quality of these schools was 7.9 (7.8).

Table 9 Strategic vs. Non-Strategic Type: Simulation

	Strategic (%)		Strategic	Non-Strategic
All	96.0	Schools in zone		
Parental Edu < HS	97.3	No. Schools	22.3	23.1
Parental Edu = HS	94.8	Ave. quality	7.9	7.8
Parental Edu ≥College	96.5			
Single-Parent	95.6			
Have an older sibling	97.0			

6.2 Model Fits and Out-of-Sample Validation

The 2007 re-definition of priority zones abruptly changed the school-household-specific priorities. For example, the number of schools to which a household had priority became 7.0 on average with a standard deviation of 1.5 in 2007, as compared to the 22.3

[50] We find a much smaller fraction of non-strategic households than Abdulkadiroğlu, Pathak, Roth and Sönmez (2006) did. By incorporating the outside option and the leftover schools into the framework, our model rationalizes the choices by a substantial fraction of households.

(7.9) figure in 2006. More importantly, the priority schools became those that surrounded each home location, which also changed the risk-quality-distance trade-offs faced by households. In this section, we show the model fits for both the 2006 and the 2007 samples.[51] To simulate the 2007 outcomes, we first calculate the admissions probabilities in 2007 via the same procedure as we do for the year 2006, using the entire 2007 sample. Then we use the selected 2007 sample of 7,437 households to conduct an out-of-sample validation, where the sample selection rule is the one used for the 2006 sample.[52] To the extent that the change came as a surprise to households, it is reasonable to believe households had been unable to reallocate before submitting their applications in 2007. As such, we simulate the distribution of 2007 household types using the characteristics of their residential zones according to the 2006 definition.[53]

Considered as the most informative test of the model, the first two rows of Table 10 explore the changes in the definition of priority zones.[54] The 2007 reform led to situations where some schools were in the priority zone for a household in one year but not in the other, which would affect the behavior of a strategic household. As shown in the first row of Table 10, in 2006, 24% of the households in our sample top-listed a school that was in their priority zone by the 2006 definition but not by the 2007 definition. In 2007, the fraction of households that top listed these schools dropped to 12%. On the other hand, the second row of Table 10 shows that the fraction of households that top-listed schools in their priority zone only by the 2007 definition but not by the 2006 definition increased from 3% to 12% over the two years. The model is able to replicate such behaviors and predicts the changes as being from 22% to 15% for the first case, and from 5.7% to 11.2% for the second case. The next 3 rows of Table 10 show that the model fits the data well in terms of the characteristics of the top-listed schools, including average quality, distances and fees. In particular, the distance of the top-listed school went down from 7.1 to 6.6 in the data between

[51]Appendix Table A1-A6 show the fits for subgroups of households conditional on demographics.

[52]In 2007, 12,335 Barcelona households participated. We follow the same sample selection rule as that for the 2006 sample. In particular, the 7,437 households in 2007 do not include the 998 parents who reported "high school or above" as their education levels. We interpolate the probability of being college-educated for these parents and include them in the counterfactual policy experiments.

[53]As shown below, we can fit the data in both years, suggesting that our assumption about household type distribution in 2007 is a reasonable one.

[54]Calsamiglia and Güell (2014) use this change to show that the observed application behavior was driven largely by admissions priorities.

the two years, as priority schools became those surrounding one's home location. The model-predicted change in distance is from 7.4 to 6.9.

Table 10 Top-Listed Schools

	2006		2007	
	Data	Model	Data	Model
In Zone 06 Only (%)	24.1	22.0	12.0	15.1
In Zone 07 Only (%)	3.0	5.7	12.0	11.2
Quality	7.9	7.9	7.9	7.9
Distance (100m)	7.1	7.4	6.6	6.9
Fee (100 Euros)	8.1	8.1	7.9	8.0

As mentioned in the model section, there can be multiple lists that are payoff-equivalent and imply the same allocation results. All these lists have identical ordered elements that are allocation-relevant, which is what our model can explain. For example, consider a list of length 4, the third element of which was a leftover school. Our model is designed to replicate the first three elements of that list, not how many schools to be listed beyond that point. Table 11 presents the model fit for the length of the allocation-relevant part of household application lists. In both years, about 86% of households' lists contained only one allocation-relevant school and fewer than 3% of households had more than 2 relevant schools on their lists, which is not surprising given that most households were assigned in the first round. The model-predicted distribution of the list length lies slightly to the right of the data distribution for 2006. The model under-predicts the fraction of households listing 2 schools in 2007.

Table 11 Relevant List Length (%)

	2006		2007	
	Data	Model	Data	Model
1	85.8	83.2	86.1	86.9
2	11.5	12.3	11.7	10.1
≥ 3	2.7	4.5	2.2	3.0

Table 12 shows the rounds at which households were assigned. The model slightly over-predicts the fraction of households assigned in the first round. Table 13 shows that the model closely replicates the enrollment rate within the public school system. In particular, with the ex-post shocks, the model replicates the non-enrollment behavior by households who were assigned to their first choice.

Table 12 Assignment Round (%)

	2006		2007	
	Data	Model	Data	Model
1	93.0	94.3	92.0	94.5
2	2.8	2.3	3.1	2.0
≥ 3	1.5	0.2	1.8	0.4
Unassigned	2.7	3.2	3.1	3.1

Table 13 Enrollment in the Public System (%)

	2006		2007	
	Data	Model	Data	Model
All	96.7	96.5	97.6	96.5
Assigned in Round 1	97.8	96.7	98.3	97.1

7 BM vs. GS vs. TTC

Using the estimated model, we are ready to answer the question we posed at the beginning of the paper. How does the current Boston mechanism (BM) compare with two of its alternatives, the Gale-Shapley student deferred acceptance mechanism (GS) and the top trading cycles mechanism (TTC)? In a different experiment, presented in the appendix, we assess the impacts of the 2007 reform, which changed the student-school priority structure by redefining priority zones. In both experiments, households' welfare refers to their evaluations of their assignment outcomes relative to their outside options, i.e., v_{ij}.[55]

7.1 Theoretical Background

This subsection briefly discusses the properties of the three alternative mechanisms; Appendix E contains detailed descriptions.[56] The GS procedure is similar to BM with the key differences that students are only temporarily assigned to schools in each round and that one's chance of being finally admitted to a school does not depend on the ranking of the school on her application list. The TTC algorithm

[55] All simulations include the interpolated sample, as mentioned in footnotes 45 and 52. All simulations use the school-household-specific priority scores given by (1) , as they were defined by the official rules in the relevant year.

[56] See Abdulkadiroğlu and Sönmez (2003) for further theoretical discussions.

has a very different structure. Intuitively, in each round TTC creates cycles of trade between individuals. Each individual in a cycle trades off a seat in her highest-priority school for a seat in her announced most preferred school among those that still have seats. Whenever such a cycle is formed the allocation is final.

Three properties are considered as desirable for a mechanism, i.e., Pareto efficiency, truth revealing and the elimination of justified envy (also known as stability).[57] Unfortunately, all three properties may not hold simultaneously. BM satisfies none of the properties. GS and TTC are both truth revealing. Between the other two conflicting properties, GS eliminates justified envy at the cost of Pareto efficiency, while TTC achieves Pareto efficiency at the cost of stability.

Despite the fact that BM does not satisfy any of the three desirable properties, the welfare comparison between BM and its alternatives is ambiguous.[58] The ambiguity arises from the coexistence of two competing forces. On the one hand, BM can lead to potential misallocations because households hide their true preferences. This source of misallocation is absent in the truth-revealing GS and TTC. On the other hand, BM may better "respect" households' cardinal preferences than GS and TTC (Abdulkadiroğlu, Che and Yasuda (2011)). BM-induced household behaviors increase the chance that the "right match" is formed, where a school being matched to households that value it more.[59] Under a truth-revealing mechanism, households who share the same ordinal preferences will rank schools the same way in their applications and be given the same chance of being allocated to various schools, regardless of who will gain the most from each school. Given that it is theoretically inconclusive, the welfare comparison between BM and GS or TTC becomes an empirical question, one that we answer below.

[57] Stability requires that there be no unmatched student-school pair (i, s) where student i prefers school s to her assignment and she has higher priority than some other student who is assigned a seat at school s.

[58] The ambiguity has been reflected in the conflicting findings in the theoretical and lab experimental studies that compare BM with GS. Pareto efficiency is an ordinal concept, which does not necessarily imply the highest level of total household welfare.

[59] The intuition can be explained by the following simple example with equal priorities. Consider three schools and a set of households who share the same ordinal but different cardinal preferences for these schools, where the schools are ranked from high to low as Schools 1, 2 and 3. Under BM, the strategic decision is whether to take the high risk and top-list School 1 or to play it safe and top-list School 2. Given the same evaluation for School 1, a household whose evaluations for Schools 2 and 3 are similar is more likely to choose the risky strategy because it has less to lose from the gamble. Given the same evaluation for School 3, a household that values School 1 much higher than School 2 is more likely to choose the risky strategy because it has more to gain from the gamble.

7.2 Results

Under both GS and TTC, all households, strategic or not, will list schools according to their true preferences. As such, we simulate each household's application list according to their true preferences and assign them using GS and then using TTC.[60] We compare the results from these two counterfactual mechanisms with those from the baseline.[61] We present our results under the more recent, i.e., the 2007, priority zone structure.[62]

Remark 4 *In the following, we will present consequences of the reforms from BM to GS and TTC on the total household welfare, the distribution of winners and losers among different subgroups of households, as well as the assignment outcomes. Notice that the level of total household welfare is not necessarily the criterion for social welfare, which may involve different weights across households. Given that we can calculate the welfare changes at the household level, our results can be used to calculate any weighted social welfare. Moreover, household welfare may not be the only factor that policy makers consider. For example, policy makers may put high value on truth-revealing and the elimination of justified envy, which will make GS particularly attractive. Therefore, we do not necessarily recommend one mechanism over another in this paper. However, given a social objective, our results can easily be used for policy-making purposes.*

7.2.1 Household Welfare Comparison

To form the basis for comparison, the first column of Table 14 shows the average welfare and the standard deviations among the population (in parentheses) under BM. The second column of Table 14 shows changes in welfare (Δutils) when BM is replaced by GS. The average household welfare is decreased by 2.7 from the BM level of 4,146. Moreover, the average welfare decreases among all subgroups of households, as defined

[60] Notice that all allocation mechanisms we consider use random lotteries to rank students with the same priority score. As such, for each experiment we simulate the overall allocation procedure and obtain the outcomes for all students for a given set of random lotteries. We repeat this process many times to obtain the expected (average) outcomes for each simulated student.

[61] Notice that to simulate GS and TTC, it is sufficient to know household preferences. However, to compare GS or TTC with the baseline (Boston) mechanism, one needs to know the distribution of household strategic types.

[62] The 2006 results are similar, available on request.

by strategic sophistication or education levels.[63] To translate the welfare changes into more intuitive measures, we ask the following question:"Holding everything else constant at the baseline level, what is the adjustment in attendance fees that will bring a household's welfare from its level under BM to its level under the counterfactual (new) system?". Let Δ_i^{fees} denote the welfare-equalizing fee adjustment for household i.[64] Notice that a negative Δ_i^{fees} implies an increase in welfare.

Table 14 Household Welfare: BM vs. GS vs. TTC

%	BM	GS-BM		TTC-BM	
		Δutils	Δ^{fees}	Δutils	Δ^{fees}
All	4,146 (752)	-2.7 (23.5)	59.9 (543.0)	1.4 (33.2)	-88.0 (581.3)
Strategic	4,145 (753)	-2.6 (23.4)	57.4 (540.1)	1.4 (32.8)	-88.6 (580.9)
Non-strategic	4,173 (752)	-5.5 (27.4)	116.8 (603.7)	0.1 (40.4)	-75.5 (591.4)
Edu<HS	4,141 (722)	-4.5 (22.4)	152.6 (510.0)	1.9 (24.9)	-7.4 (478.0)
Edu= HS	4,308 (736)	-2.6 (26.2)	53.3 (576.6)	1.4 (36.3)	-106.6 (634.4)
Edu≥College	4,022 (760)	-1.6 (22.0)	3.2 (528.5)	1.1 (35.4)	-127.1 (594.7)

Column 3 of Table 14 reports the average welfare-equalizing fee adjustments. Averaging over all households, the change from BM to GS causes a welfare loss that is equivalent to a 60-euro increase in school fees, with a 543-euro standard deviation across the population. Average non-strategic households suffer more, with a loss equivalent to a 117-euro increase in fees. The average loss among the lowest education group is equivalent to a 153-euro increase in fees, while the highest education group suffers the least.

The last two columns of Table 14 compares BM with TTC. For an average household, the change from BM into TTC increases the welfare by an amount equivalent

[63]Our finding that GS decreases welfare for both strategic and non-strategic households is consistent with some recent theoretical work, e.g., Abdulkadiroğlu, Che, and Yasuda (2011).

[64]Formally, let j be the school household i is assigned to under BM and $v_{ij} = \pi(\widetilde{w}_j, fee_j, x_i, d_{ij}, \epsilon_{ij})$ be i's value of being assigned to j, where $\widetilde{w}_j$ is the vector of school j's characteristics excluding the annual fees (fee_j); let $v_{ij^{new}}$ be household i's welfare under the new (GS or TTC) system which assigns i to school j^{new}. For each household, define the welfare-equalizing fee adjustment Δ_i^{fees} as the solution to the following equation

$$\pi\left(\widetilde{w}_j, fee_j + \Delta_i^{fees}, x_i, d_{ij}, \epsilon_{ij}\right) = v_{ij^{new}}.$$

Notice that for some households, the adjustment may make the fee negative, leading to positive cash transfers to households.

to decreasing school fees by 88 euros. As such, TTC leads to the highest total household welfare among all three alternatives. Moreover, the average welfare increases within each subgroup of households, with strategic households gaining more than non-strategic households and higher-education groups gaining more than lower-education groups.

Result 1: In terms of the level of total household welfare, the three mechanisms are ranked as TTC > BM > GS.

Winners and Losers We next present the effects by distinguishing between winners and losers. Table 15 focuses on fractions of winners and losers under each change. The first two columns show the case for a change from BM to GS, which benefits fewer than 10% of households while hurting over 28% of them. Moreover, the fact that there are more losers than winners holds for all subgroups of households. In contrast, as shown in Columns 3 and 4, a change from BM to TTC generates more winners (20.4%) than losers (18.9%), which again persists across all subgroups of households. Table 16 shows the welfare changes among winners and among losers.[65] Under both counterfactual scenarios, the magnitude of gains among winners is larger than that of losses among losers. From BM to GS (from BM to TTC), an average winner gains by an amount equivalent to a 872-euro (804-euro) decrease in school fees, while the average loser loses by a 502-euro (408-euro) increase in fees.

Result 2: There are more losers than winners from a change of BM into GS, and more winners than losers from a change of BM into TTC.

Table 15 Winners and Losers (%)

	BM to GS		BM to TTC	
	Winner	Loser	Winner	Loser
All	9.5	28.5	20.4	18.9
Strategic	9.6	28.2	20.5	18.8
Non-strategic	8.6	35.4	19.7	22.0
Edu < HS	8.5	31.3	20.9	18.1
Edu = HS	10.9	31.2	22.5	21.0
Edu ≥College	9.2	24.4	18.5	17.8

[65] Appendix Tables A7-A8 show the same statistics for subgroups of households.

Table 16 Gains and Losses

	BM to GS		BM to TTC	
	Δutils	Δ^{fees}	Δutils	Δ^{fees}
Winner	35.3 (45.0)	-872.2 (775.0)	30.7 (40.8)	-804.0 (743.8)
Loser	-21.3 (20.5)	502.1 (578.7)	-25.7 (48.6)	408.4 (597.5)

7.2.2 Cross-Zone Inequality

A household's welfare can be significantly affected by the school quality within its zone not only because of the quality-distance trade-off, but also because of the quality-risk trade-off created by the priority structure. For equity concerns, a replacement of BM will be more desirable if it is more likely to benefit those living in poor-quality zones. Table 17 tests whether or not each of the counterfactual reforms meets this goal by showing the zone quality among winners and losers from each reform. The first two columns show the case for the change from BM to GS. The winners are those who live in better zones than the losers: the average zone quality among all winners is 7.75 while that among losers is 7.65. As shown in the next three rows, this pattern persists across all educational groups. Therefore, a change from BM to GS increases the dependence of welfare on zone quality, which is against the goal of equity across zones. The next two columns show that changing from BM to TTC, the average zone quality is similar across winners and losers, which implies that such a reform is unlikely to reduce or to enlarge the cross-zone inequality as compared to BM.

Result 3: Welfare dependence on zone quality increases with a change from BM to GS, and remains unaffected by a change from BM to TTC.

Table 17 Zone Quality: Winners vs. Losers

	BM to GS		BM to TTC	
	Winner	Loser	Winner	Loser
All	7.75	7.65	7.70	7.71
Edu $<$ HS	7.57	7.48	7.52	7.51
Edu $=$ HS	7.72	7.65	7.69	7.69
Edu $\geq$College	7.88	7.80	7.84	7.85

The Cost of the Elimination of Justified Envy Underlying the results in Table 17 is the residence-based priority and the high respect GS has for priorities, the latter enabling GS to eliminate justified envy but not without cost.[66] The first three columns of Table 18 show the fractions of households assigned to schools in their own school zones under alternative mechanisms. The first row shows that about 70% of all households are assigned to schools in zone under GS, followed by the case of BM (65%) and finally TTC (58%). The second row shows this fraction among households whose favorite schools are in their school zones. While over 95% of these households are assigned to in-zone schools under BM and GS, this figure is only 90% under TTC. Most illustrative of the point, the third row shows this fraction among households whose favorite schools are out of their school zones. Even though a household would like to attend a school out of its zone, due to its high respect for priorities, GS assigns 30% of them within their zone, as compared to 20% under BM and 10% under TTC.

Table 18 The Cost of the Elimination of Justified Envy

	Assigned in Zone (%)			Assigned to Favorite (%)		
	BM	GS	TTC	BM	GS	TTC
All Households	65.5	69.7	58.5	79.4	75.0	80.7
Favorite is in Zone	95.3	95.4	89.9	92.6	89.8	83.8
Favorite is out of Zone	19.5	30.0	10.0	58.9	52.0	76.0

The last three columns of Table 18 shows the extent to which the respect for priorities hampers households' chances of being assigned to their favorite schools. Row 1 shows that over all households, the chance of being assigned to their favorite schools is the highest under TTC (81%) and lowest under GS (75%). Row 2 shows that BM is the best at accommodating households preferences if their favorite schools are in their school zones, followed by GS and then TTC. Finally, while TTC enables 76% of households whose favorite schools are out of their zones to attend their favorite schools, this fraction decreases to 59% under BM and 52% under GS.

[66]Under BM, a risk-taking poor-zone household only needs to compete with other households who top-listed the same school, since the assignment is final at each round. Under GS, the same poor-zone household has to compete not only with those who have the same favorite school but also with those who are unable to get their favorite schools, because the assignment in each round is only temporary. This can make it harder for a poor-zone household to get into a better school out of its zone under GS than under BM, which in turn can make such a household worse off under GS. Under TTC, having high priority to a better school increases one's chance to form a trading cycle. However, conditional on forming a cycle, the assignment does not depend one's priority for the receiving school. Therefore, who wins and who loses from the change of BM into TTC depends much less on the quality of one's own zone. See the appendix for the details of the GS and the TTC algorithms.

Result 4: GS assigns the largest fraction of households to in-zone schools, followed by BM and then TTC. In terms of enabling households to get out of their zones to attend their desired schools, the three mechanisms are ranked as TTC > BM > GS.

Remark 5 *As is common in the literature on school choice mechanisms, our cross-mechanism comparisons takes student-school priority structures as given. These structures differ across cities; and they play an essential role in the allocation of students. We leave it for future research, with data from multiple cities, to understand the trade-offs and social objectives underlying these different priority structures.*

7.2.3 School Assignment

Table 19 compares the assignment outcomes across mechanisms. The first three columns show the changes in the characteristics of schools households are assigned to when BM is replaced by GS.[67] School quality increases slightly for all groups of households by similar amount of around 0.006.[68] School-home distance reduces by about 30 meters for an average household, which is assigned to a less costly school with average fees lowered by 20 euros. The changes are heterogenous across educational groups. The least-educated group sees the smallest deduction in distance, while an increase in fees by 50 euros on average. In contrast, the highest-educated group sees the smallest increase in quality and biggest deduction in fees. Our parameter estimates suggest that on average higher educated households values quality more in the trade-offs between quality, distance and fees. The assignment outcomes under GS goes against these preferences.

Table 19 School Assignment

	GS-BM			TTC-BM		
	Quality	Distance	Fees	Quality	Distance	Fees
All	0.0068 (0.3)	-0.3 (3.6)	-0.2 (2.9)	0.008 (0.3)	0.6 (4.2)	0.07 (3.8)
Edu $<$ HS	0.006 (0.3)	-0.2 (3.6)	0.5 (2.6)	0.03 (0.3)	0.5 (3.9)	0.05 (3.2)
Edu $=$ HS	0.006 (0.3)	-0.4 (3.8)	-0.4 (3.2)	0.002 (0.3)	0.8 (4.6)	0.10 (4.1)
Edu $\geq$College	0.005 (0.2)	-0.3 (3.4)	-0.6 (3.0)	0.005 (0.3)	0.5 (4.1)	0.05 (3.9)

[67] The baseline case is presented in the appendix Table A9.

[68] A non-zero average change in quality is possible because there are more school seats than students city-wise.

The last three columns of Table 19 show the changes when BM is replaced by TTC. School quality increases by 0.008 on average, with the lowest-educational group experiencing the biggest increase of 0.03. School-home distance increases by about 60 meters for an average household. As an illustration of the misallocation arising from people hiding their true preferences under BM, Table 19 shows that the current BM makes households inefficiently apply for close-by schools that they have higher priority for, while giving up higher-quality schools with longer distance that they have lower priority for.

Result 5: Compared to TTC, both BM and GS inefficiently assign students to schools that are of shorter distance but lower quality.

8 Conclusion

We have developed and estimated a model of school choices by households under the Boston mechanism. We have recovered the joint distribution of household preferences and their strategic vs. non-strategic types, using administrative data from Barcelona before a drastic change in the definition of households' priority school zones. The estimated model has been validated using data after this drastic change.

We contribute to the on-going debates on school choice mechanism designs by quantifying the welfare impacts of replacing the Boston mechanism with its two alternatives, GS and TTC. A change from the Boston mechanism to GS creates more losers than winners. This change also increases the dependency of a household's welfare on the quality of its school zones, leading to further inequality concerns across residential zones. In contrast, a change from the Boston mechanism to TTC creates more winners than losers. However, the change of BM to TTC is unlikely to reduce or to enlarge the cross-zone inequality.

The methods developed in this paper and the main empirical findings are promising for future research. One particularly interesting extension is to incorporate household's residential choices into the framework of this paper. Individual households may relocate in order to take advantage of changes in school choice mechanisms and/or in residence-based priority structures. Such individual incentives will in turn affect the housing market. There is a large literature on the capitalization of school quality for housing prices, as reviewed by Black and Machin (2010) and Gibbons and Machin

(2008).[69] An important yet challenging research project involves combining this literature and the framework proposed in our paper, in order to form a more comprehensive view of the equilibrium impacts of school choice mechanisms on households' choices of schools and residential areas, and on the housing market.

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[69] Ries and Somerville (2010) exploit changes in the catchment areas of public schools in Vancouver and find significant effects of school performance on housing prices. Epple and Romano (2003) conjecture that school choice systems can eliminate the capitalization of school quality on the housing market. Machin and Salvanes (2010) exploit policy reforms in Oslo that allowed students to attend schools without having to live in the school's catchment area, and find a significant decrease in the correlation between a school's quality and housing prices.

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Appendix

A1. Properties of the optimal list for a strategic household.

Consider an optimal list $A_i^1 = \{a_1^1, ..., a_r^1, ...a_{R'}^1\}$ derived by the backward induction as in Section 3.3.2, if the student does not face 100% admissions rate for any of the first $r-1$ listed schools, and she does for the r^{th} listed school $\left(p^r_{a_r^1}(s_i) = 1\right)$, then the following lists all generate the same value for the household as A_i^1 does, and hence are all optimal:

1) a list that shares the same first r elements of A_i^1.

2) a list of length n $(r < n \leq R)$, which shares the same first $r-1$ elements of A_i and the last (n^{th}) element is a_r^1, and for all elements $r' \in \{r, ..., n-1\}$, the household faces 0 admissions probability.

3) Furthermore, if this r^{th} listed school is one's backup school with $p_{a_r}(:) = 1$, then

any list of length n $(r-1 \leq n \leq R)$ is also optimal if it has the same first $r-1$ elements of A_i^1 and the admissions probabilities to the other elements are all 0.
For components on a list that do not affect the value of the list, we do not impose that they be in the set J_i^* chosen in the first part of a strategic household's decision procedure.

A2. Proof for Claim 1

An application list with the following features reveals that the household must be non-strategic: 1) for some r^{th} $(r > 1)$ element a_r on the list $p_{a_r}^r(s_i) = 0$, and 2) $p_{a_{r'}}^{r'}(s_i) < 1$ for all $r' < r$, and 3) for some $r^{"} \geq r+1$, $0 < p_{a_{r"}}^{r"}(s_i) < 1$ and $p_{a_{r'''}}^{r'''}(s_i) < 1$ for any $r < r''' < r''$.

Without Feature 2), the list can still be strategically optimal due to Remark 1. Without Feature 3) a household may still be strategic if it prefers some sure-to-get-in school listed later over any of the schools listed after a_r, including a_r. All three features guarantee that the household is non-strategic.

Proof. Take a given list that satisfies all three features in Claim 1: $A = \{a_1, ..., a_r, ..., a_{r"}, ...\}$, where a_r is the *first* school that satisfies feature 3). Let $W_i^r(A)$ be the residual value of this list starting from the r^{th} element.

$$\begin{aligned} W_i^r(A) &= p_{a_r}^r(s_i)\, v_{ia_r} + (1 - p_{a_r}^r(s_i))W_i^{r+1}(A) \\ &= W_i^{r+1}(A) \\ &= p_{a_{r+1}}^{r+1}(s_i)\, v_{ia_r} + (1 - p_{a_{r+1}}^{r+1}(s_i))W_i^{r+2}(A) \\ &= ... \\ &= p_{a_{r"}}^{r"}(s_i)\, v_{ia_r} + (1 - p_{a_{r"}}^{r"}(s_i))W_i^{r"}(A) \\ &= W_i^{r"}(A). \end{aligned}$$

The equalities follow from the fact that any school listed between a_r and $a_{r"}$ must have admissions probability of zero for household i.
Consider an alternative (not necessarily optimal) application list $B = \{a_1, ..., a_{r"}, ..., a_{r"}, ...\}$, which differs from A only in that it replace a_r with $a_{r"}$. The residual value of this list

at its r^{th} element (now $a_{r"}$) is given by

$$\begin{aligned} W_i^r(B) &= p_{a_{r"}}^r(s_i)\, v_{ia_{r"}} + (1 - p_{a_{r"}}^r(s_i))W_i^{r+1}(B) \\ &= p_{a_{r"}}^r(s_i)\, v_{ia_{r"}} + (1 - p_{a_{r"}}^r(s_i))W_i^{r+1}(A) \\ &= p_{a_{r"}}^r(s_i)\, v_{ia_{r"}} + (1 - p_{a_{r"}}^r(s_i))W_i^{r"}(A) \\ &> p_{a_{r"}}^{r"}(s_i)\, v_{ia_{r"}} + (1 - p_{a_{r"}}^{r"}(s_i))W_i^{r"}(A) \\ &= W_i^{r"}(A) \end{aligned}$$

The second and third line holds because the rest of list B is the same as A, and the value of $W_i^{k+1}(\cdot)$ is independent of what one chooses in slot k, for any $k > 1$. The third line follows because $p_{a_{r"}}^r(s_i) > p_{a_{r"}}^{r"}(s_i)$ (admissions probabilities decrease over rounds) and

$$v_{ia_{r"}} = E \max\{u_{ia_{r"}}, \eta\} > E(\eta) = 0.$$

Given that the first $r-1$ elements are also unchanged, it is immediate that the value of the whole list $W_i^1(B) > W_i^1(A)$. ■

B1. Detailed Functional Forms

Household Characteristics: $x_i = [x_{i1}, ..., x_{i5}]$, where $x_{i1} = I(\text{edu}_i < \text{high school})$, $x_{i2} = I(\text{edu}_i = \text{high school})$, $x_{i3} = I(\text{edu}_i \geq College)$, $x_{i4} = I(\text{single parent}_i = 1)$, $x_{i5} =$ sibling's school ($x_{i5} = 0$ if outside school, $\in \{1, ...J\}$ if non-private school, -9 if no sibling).

School Characteristics: $w_j = [w_{j1}, w_{j2}, w_{j3}, w_{j4}]$, where w_{j1} is school quality, w_{j2} is tuition level, w_{j3} is capacity, and $w_{j4} = I(\text{semi-public})$. Let q^g (q^b) be the 75^{th} (25^{th}) percentile of school quality.

Home-school distance: d_{ij}, measured in 100 meters.

Zone Characteristics: Let N_z be the number of schools in zone z, and $\overline{q}_z$ be the average school quality in zone z.

B1.1 Utility functions

Household utility is given by $u_{ij} = U(w_j, x_i, d_{ij}) + \epsilon_{ij}$. Define $g(\cdot)$ and $C(\cdot)$ such that $U(w_j, x_i, d_{ij}) = g(w_j, x_i) - C(d_{ij})$.

$$g(w_j, x_i) = \alpha_0 + \sum_{m=1}^{4} \alpha_m x_{im} + w_{j1}\left(\sum_{m=1}^{3} \alpha_{4+m} x_{im}\right) + (w_{j1} - q^g)^2 I(w_{j1} > q^g)\left(\sum_{m=1}^{3} \alpha_{7+m} x_{im}\right)$$
$$+ \alpha_{11}\left(w_{j1} - q^b\right)^2 I\left(w_{j1} > q^b\right) + \alpha_{12}\left[I(x_{i5} = j) - I(x_{i5} = 0)\right]$$
$$+ \alpha_{13} I(w_{j3} = 2) + \alpha_{14} I(w_{j3} = 3) + \alpha_{15} I(w_{j3} > 3) + \alpha_{16} w_{j4}$$
$$+ w_{j2}\left(\sum_{m=1}^{3} \alpha_{16+m} x_{im}\right) + \alpha_{20}(w_{j2})^2,$$

and

$$C(d_{ij}) = \left[d_{ij} + c_1 d_{ij}^2 + c_2 I(d_{ij} > 5) + c_3 I(d_{ij} > 10)\right].$$

B1.2 Type distribution

$$\lambda(x_i, l_i) = \lambda(x_i, z_{l_i}) = \frac{\exp(\beta_0 + \sum_{m=1}^{4} \beta_m x_{im} + \beta_5 I(x_{i5} \geq 0) + \beta_6 N_{z_{l_i}} + \beta_7 \overline{q}_{z_{l_i}})}{1 + \exp(\beta_0 + \sum_{m=1}^{4} \beta_m x_{im} + \beta_5 I(x_{i5} \geq 0) + \beta_6 N_{z_{l_i}} + \beta_7 \overline{q}_{z_{l_i}})}.$$

B2. Identification

Since the dispersion of post-application shocks is mainly identified from the enrollment decisions, to ease the illustration, we show the identification of the model without post-application shocks. A household has observables (x_i, l_i) and can be one of two types $T = 0, 1$. Home-school distance is given by $d_{ji} = d(l_i, l_j)$ and z_{l_i} is the zone that l_i belongs. Let the taste for school be $\epsilon_{ij} \sim i.i.d.N(0, 1)$.[70] In line with (IA2) and (IA3) in the paper, assume that d is independent of T conditional on (x, z_l) and ϵ is independent of (x, l, T). To give the idea, consider the case where a household can apply only to one school from the choice set of schools 1 and 2, and where all households face the same admissions probabilities. Household-specific admissions probabilities provide more variations, which will provide more identification power.

Let $\overline{u}_{ij}$ be the utility net of individual taste, $\overline{u}_{ij} = g(w_j, x_i) - C(d_{ij})$. Let p_j be the probability of admission to school j and $p_1 \neq p_2$, and $p_j > 0$ (IA1). Let y be the

[70] Given that the linear distance enters the utility function with coefficient of minus one, the standard deviation of ϵ is identified from the variation in distance within (x, z_l) group. To simplify the notation, we will present the case where σ_ϵ is normalized to 1.

decision to list 1 (regardless of whether or not 2 is listed). y is related to the latent variable y^* in the following way

$$y\left(x_i, l_i, \epsilon_i, T\right) = 1 \text{ if only if}$$

$$y^*\left(x_i, l_i, \epsilon_i, T\right) = T\left(p_{i1}u_{i1} - p_{i2}u_{i2}\right) + (1-T)(u_{i1} - u_{i2}) > 0.$$

Hence the probability of observing the decision to list 1 by someone with (x_i, l_i) is

$$H\left(x_i, l_i\right) = \lambda\left(x_i, z_{l_i}\right)\Phi\left(\frac{p_1\overline{u}_{i1} - p_2\overline{u}_{i2}}{\sqrt{p_1^2 + p_2^2}}\right) + \left(1 - \lambda\left(x_i, z_{l_i}\right)\right)\Phi\left(\frac{\overline{u}_{i1} - \overline{u}_{i2}}{\sqrt{2}}\right).$$

Fix (x, z_l), $H(\cdot)$ only varies with d, so we can suppress the dependence on (x, z_l). Within (x, z_l), let $g(w_j, x) = g_j$ such that

$$\begin{aligned} H(d) = {} & \lambda\Phi\left(\frac{(p_1g_1 - p_2g_2) - (p_1C(d_1) - p_2C(d_2))}{\sqrt{p_1^2 + p_2^2}}\right) \\ & + (1-\lambda)\Phi\left(\frac{(g_1 - g_2) - (C(d_1) - C(d_2))}{\sqrt{2}}\right). \end{aligned} \tag{5}$$

B2.1 Identification of $g(\cdot)$ and $\lambda(\cdot)$

The following theorem shows that fix any (x, z_l), $g(w_j, x)$ and $\lambda(x, z_l)$ are identified.

Theorem 1 *Assume that 1) $\lambda \in (0,1)$, 2) there exists an open set $D^* \subseteq D$ such that for $d_{ij} \in D^*$, $C'(d_{ij}) \neq 0$. Then the parameters $\theta = [g_1, g_2, \lambda]'$ in* (5) *are locally identified from the observed application decisions.*

Proof. The proof draws on the well-known equivalence of local identification with positive definiteness of the information matrix. In the following, I will show the positive definiteness of the information matrix for model (5).

Step 1. Claim: *The information matrix $I(\theta)$ is positive definite if and only if there exist no $\omega \neq 0$, such that $\omega' \frac{\partial H(d)}{\partial \theta} = 0$ for all d.*

The log likelihood of an observation (y, d) is

$$L(\theta) = y\ln(H(d)) + (1-y)\ln(1 - H(d)).$$

The score function is given by

$$\frac{\partial L}{\partial \theta} = \frac{y - H(d)}{H(d)(1 - H(d))} \frac{\partial H(d)}{\partial \theta}.$$

Hence, the information matrix is

$$I(\theta|d) = E\left[\frac{\partial L}{\partial \theta}\frac{\partial L}{\partial \theta'}|d\right] = \frac{1}{H(d)(1 - H(d))} \frac{\partial H(d)}{\partial \theta}\frac{\partial H(d)}{\partial \theta'}.$$

Given $H(d) \in (0, 1)$, it is easy to show that the claim holds.
Step 2. Show $\omega' \frac{\partial H(d)}{\partial \theta} = 0$ for all $d \Longrightarrow \omega = 0$.
Define $p_j^* = \frac{p_j}{\sqrt{p_1^2 + p_2^2}}$, $B_1(d) = (p_1^* g_1 - p_2^* g_2) - (p_1^* C(d_1) - p_2^* C(d_2))$, and $B_0(d) = \left(\frac{(g_1 - g_2) - (C(d_1) - C(d_2))}{\sqrt{2}}\right)$, $\frac{\partial H(d)}{\partial \theta}$ is given by:

$$\begin{aligned}
\frac{\partial H(d)}{\partial \lambda} &= \Phi(B_1(d)) - \Phi(B_0(d)) \\
\frac{\partial H(d)}{\partial g_1} &= \lambda \phi(B_1(d)) p_1^* + (1 - \lambda)\phi(B_0(d)) \frac{1}{\sqrt{2}} \\
\frac{\partial H(d)}{\partial g_2} &= -\lambda \phi(B_1) p_2^* - (1 - \lambda)\phi(B_0) \frac{1}{\sqrt{2}}.
\end{aligned}$$

Suppose for some ω, $\omega' \frac{\partial H(d)}{\partial \theta} = 0$ for all d :

$$\begin{aligned}
&\omega_1[\Phi(B_1) - \Phi(B_0)] + \omega_2\left(\lambda\phi(B_1)p_1^* + (1 - \lambda)\phi(B_0)\frac{1}{\sqrt{2}}\right) \\
&-\omega_3\left(\lambda\phi(B_1)p_2^* + (1 - \lambda)\phi(B_0)\frac{1}{\sqrt{2}}\right) = 0
\end{aligned}$$

Take derivative with respect to d_2 evaluated at some $d_2 \in D^*$

$$\begin{aligned}
&\omega_1[\phi(B_1)p_2^* - \frac{\phi(B_0)}{\sqrt{2}}]C'(d_2) + \omega_2\left(\lambda\phi'(B_1)p_1^*p_2^* + (1 - \lambda)\phi'(B_0)\frac{1}{2}\right)C'(d_2) \qquad (6) \\
&- \omega_3\left(\lambda\phi'(B_1)(p_2^*)^2 + (1 - \lambda)\phi'(B_0)\frac{1}{2}\right)C'(d_2) = 0.
\end{aligned}$$

Define $\gamma(d) = \frac{\phi(B_1)}{\phi(B_0)}$, divide (6) by $\phi(B_0)$:

$$\omega_1[\gamma(d)\, p_2^* - \frac{1}{\sqrt{2}}] - \omega_2 \left(\lambda B_1 \gamma(d)\, p_1^* p_2^* + (1-\lambda)\, B_0 \frac{1}{2} \right)$$
$$+\omega_3 \left(\lambda B_1 \gamma(d) \left(p_2^*\right)^2 + (1-\lambda)\, B_0 \frac{1}{2} \right) = 0$$

$$\gamma(d)\,[\omega_1 p_2^* - \lambda B_1 p_2^* (\omega_2 p_1^* - \omega_3 p_2^*)] - [\frac{\omega_1}{\sqrt{2}} + (\omega_2 - \omega_3)(1-\lambda)\, B_0 \frac{1}{2}] = 0 \quad (7)$$

Since $\gamma(d)$ is a nontrivial exponential function of d, (7) hold for all $d \in D^*$ only if both terms in brackets are zero for each $d \in D^*$, i.e.

$$\omega_1 p_2^* - \lambda B_1(d)\, p_2^* (\omega_2 p_1^* - \omega_3 p_2^*) = 0 \quad (8)$$
$$\frac{\omega_1}{\sqrt{2}} + (\omega_2 - \omega_3)(1-\lambda)\, B_0(d)\, \frac{1}{2} = 0.$$

Take derivative of (8) again with respect to d_2, evaluated at $d_2 \in D^*$:

$$-\lambda C'(d_2) \left(p_2^*\right)^2 (\omega_2 p_1^* - \omega_3 p_2^*) = 0$$
$$(\omega_2 - \omega_3)(1-\lambda)\, C'(d_2)\, \frac{1}{2\sqrt{2}} = 0.$$

Since $\lambda \in (0,1)$, $p_j > 0$ (hence $p_2^{*2} > 0$) and $C'(d_2) \neq 0$ for some d, we have

$$\omega_2 p_1^* - \omega_3 p_2^* = 0$$
$$\omega_2 - \omega_3 = 0.$$

Given $p_1 \neq p_2$ (hence $p_1^* \neq p_2^*$), follows that $\omega = 0$. ■

B2.2 Identification of $C(d_{ij})$.

Given the identification result from B2.1, and given that $C(d_{ij})$ is common across (x, z_l)'s, the parameters in $C(d_{ij})$ solves for the system of equations (5), where one equation corresponds to one (x, z_l).

C. Priority Score Structure

Case 1: Those who does not have a sibling in school have two levels: $x_i a$ $(x_i a + b_1)$ for out-of-zone (in-zone) schools.

Case 2: Those whose sibling(s) is (are) in in-zone schools has 3 levels: $x_i a$ $(x_i a + b_1)$

for out-of-zone (in-zone) non-sibling schools, and $x_i a + b_1 + b_2$ for sibling schools.
Case 3: Those whose sibling(s) is (are) in out-of-zone schools has 3 levels: $x_i a$ $(x_i a + b_1)$ for out-of-zone (in-zone) non-sibling schools, and $x_i a + b_2$ for sibling schools.
Case 4: Those with sibling(s) in some in-zone school and sibling(s) in some out-of-zone school has 4 levels: $x_i a$ $(x_i a + b_1)$ for out-of-zone (in-zone) non-sibling schools, and $x_i a + b_2$ $(x_i a + b_1 + b_2)$ for out-of-zone (in-zone) sibling schools.

D. Additional Assumptions to Use the Data to its Full Potential

Consider an observed application list by household i $\left(\widetilde{A}_i\right)$ of length n, in which the r^{th} element is the first school that has admissions probability of 1 in the relevant round $\left(p_j^r(s_i) = 1\right)$, and the t^{th} element is the first leftover school listed $(r \leq t)$. We impose the following structure.

A1. If i is strategic, then the schools listed after r^{th} element are ranked, i.e., $v_{ia_i^{r+1}} > ... > v_{ia_i^n}$. However, we do not impose structure on how these schools are ranked with respect to other schools, nor do we require that they belong to the candidate set J_i^*.

A2. If i is non-strategic, then the schools listed after the t^{th} element are ranked, i.e., $v_{ia_i^{t+1}} > ... > v_{ia_i^n}$. By Condition (3), the schools listed after a_i^t are ranked below any schools listed before them, but we do not make assumptions on how they compare with non-listed schools.

Notice that whether or not a household is strategic is probabilistic from the researcher's point of view. In our simulation, each household in the data will be replicated by a large number of simulated households with identical observable characteristics but different unobserved tastes for schools and different types.

E. The GS and TTC Algorithms

E1. The GS algorithm assigns students as follows.

Round 1: Each school j tentatively assigns its seats to students who top-listed it, one at a time following their priority order. If school j is over-demanded, lower-ranked applicants are rejected.

In general, at Round r: Each school j considers the students it has been holding, *together with* students who were rejected in the previous round but listed j as their r^{th} choice. Seats in school j are tentatively assigned to these students, one at a time following their priority order. If school j is over-demanded, lower-ranked applicants are rejected.

The algorithm terminates when no student is rejected and each student is assigned her final tentative assignment.

The key differences between GS and BM are 1) in each round, students are only temporarily assigned to a school until the whole procedure ends; and 2) temporarily held students are considered based only on priorities along with students who were rejected from their choices in previous rounds and added into a school's student pool in the current round. As such, a previously held student can be crowded out by a newly-added student who has higher priority. That is, top-listing a school does not improve one's chance of being finally admitted to this school, which makes truth-telling a (weakly) dominant strategy for households under GS. Moreover, GS eliminates justified envy. The appealing properties of GS, however, may conflict with Pareto efficiency, as shown by Abdulkadiroğlu and Sönmez (2003).

E2. The TTC algorithm assigns students as follows.

Round 1: Assign a counter for each school which keeps track of how many seats are still available at the school, initially set to equal the school capacity. Each school points to the student who has the highest priority for the school. Each student points to her favorite school *under her announced preferences*.[71] This will create ordered lists of distinct schools (s) and distinct students (i) : ($s_1, i_1, s_2, i_2, ...$), where s_1 points at i_1, i_1 points at s_2, and s_2 points at i_2, etc. Because there are finite number of schools, at least one cycle will be formed, where i_k ($k \geq 1$) points at s_1. Although there may be multiple cycles formed in a round, each school can be part of at most one cycle and each student can be part of at most one cycle. Every student in a cycle is assigned a seat at the school she points to and is removed. The counter of each school in a cycle is reduced by one and if it reduces to zero, the school is also removed. Counters of all schools that are not in any cycle stay put.

In general, at Round r: Each remaining school points to the student with highest priority among the remaining students and each remaining student points to her favorite school among the remaining schools. Every student in a cycle is assigned a seat at the school that she points to and is removed. The counter of each school in a cycle is reduced by one and if it reduces to zero the school is also removed. Counters of all other schools stay put.

The algorithm terminates when all students are assigned a seat.

Intuitively, in each round TTC creates cycles of trade between individuals. Each

[71] A student announces her entire list of schools before the assignment starts. As such, the "pointing" by a student is mechanically following her announced list.

individual in a cycle trades off a seat in her highest-priority school for a seat in her most preferred school among those that still have seats. Whenever such a cycle is formed the allocation is final. Hence, the only way for an individual to improve her allocation is through "stealing" another individual's school assignment, which will in turn make this other individual worse off. As such, TTC is Pareto efficient as shown by Abdulkadiroğlu and Sönmez (2003), who also prove that TTC is truth-revealing. However, TTC does not eliminate justified envy because student-school priorities are ignored in the TTC trade between individuals.

F. Policy Evaluation: The 2007 Reform

The 2007 reform gives priorities for households to access schools that are closest to their home locations. Depending on households' home locations and strategic types, the reform may have affected them differently. In order to assess these impacts, we simulate the counterfactual outcomes for the 2007 applicants had they lived under the 2006 regime, taking as given the 2006 admissions probabilities. The results generated from this experiment can be interpreted in two ways: 1) the results are at the individual level, i.e., "what would have happened to a 2007 applicant had she applied in 2006?"; 2) assuming that the 2006 and 2007 cohorts are two i.i.d. random samples drawn from the same distribution, the results tell us "what would have happened to all 2007 households if the reform had not happened and if they had played the same equilibrium as the 2006 cohort?".

The first two columns of Table A10 present the fractions of winning and losing households due to the 2007 reform. About 18% of households gained and 14% of households lost from the reform. More non-strategic households were affected, with 27% winners and 22% losers. Across educational groups, the high-school educated group was the most likely to win (20%) and also the most likely to lose (15%) from the reform. The last row shows the distribution within a particular group of households who lived at the corner of school zones under the 2006 regime. In particular, we define corner households as those at least half of whose 2007 priority schools were not in their pre-reform school zones. They accounted for over 15% of all 2007 households. Not surprisingly, these households were more likely to be affected by the reform than an average household: 21% of them gained and 15% of them lost from the reform. The last two columns of Table A9 show the changes in welfare, as measured in utils and in welfare-equalizing fees. Overall, the gain from the 2007 reform was worth a 23-euro decrease in school fees. The average welfare impacts were small for strategic households, but much more significant for non-strategic households. Across educational groups, households with higher educated parents gained more.

G. Additional Tables

G1. Model Fit

Table A1 Model Fit: Relevant List Length 2006 (%)

	Edu < HS		Edu = HS		Edu ≥College		Single Parents	
	Data	Model	Data	Model	Data	Model	Data	Model
1	87.1	85.3	83.7	82.1	86.8	84.5	85.8	81.8
2	11.2	11.7	13.0	13.0	10.6	11.2	11.4	13.7
≥ 3	1.7	3.0	3.3	3.9	2.6	4.3	2.8	4.5

Table A2 Model Fit: Assignment Round 2006 (%)

	Edu < HS		Edu = HS		Edu ≥College		Single Parents	
	Data	Model	Data	Model	Data	Model	Data	Model
1	93.2	95.1	92.0	93.7	93.7	94.1	94.0	93.4
2	2.7	1.7	3.5	2.3	2.3	2.6	1.9	2.7
≥ 3	1.6	0.1	1.5	0.5	1.3	0.2	1.4	0.2
Unassigned	2.5	3.1	3.0	3.5	2.7	3.1	2.7	3.7

Table A3 Model Fit: Top-Listed Schools 2006

	Quality		Distance (100m)		Tuition (100 Euros)	
	Data	Model	Data	Model	Data	Model
Parental Edu < HS	7.6	7.6	5.2	6.3	5.4	5.4
Parental Edu = HS	7.9	7.9	7.0	7.3	8.1	8.3
Parental Edu ≥College	8.2	8.1	8.7	8.3	9.9	10.0
Single-Parent	8.0	8.0	8.1	7.8	8.6	8.8

Table A4 Model Fit: Relevant List Length 2007 (%)

	Edu < HS		Edu = HS		Edu ≥College		Single Parents	
	Data	Model	Data	Model	Data	Model	Data	Model
1	85.2	86.7	84.8	85.2	87.6	88.4	87.0	85.6
2	12.4	10.3	12.4	11.0	10.7	8.8	11.2	10.4
≥ 3	2.4	3.0	2.8	3.8	1.7	2.8	1.8	4.0

Table A5 Model Fit: Assignment Round 2007 (%)

	Edu < HS		Edu = HS		Edu ≥College		Single Parents	
	Data	Model	Data	Model	Data	Model	Data	Model
1	91.0	94.8	90.6	94.3	93.7	95.0	91.3	94.4
2	3.7	1.7	3.2	2.3	2.2	1.7	2.9	2.0
≥ 3	1.6	0.2	2.2	0.2	1.6	0.7	1.4	0.5
Unassigned	3.5	3.3	4.0	3.5	2.5	2.6	3.3	3.1

Table A6 Model Fit: Top-Listed Schools 2007

	Quality		Distance (100m)		Tuition (100 Euros)	
	Data	Model	Data	Model	Data	Model
Parental Edu < HS	7.5	7.5	5.2	5.9	5.3	5.2
Parental Edu = HS	8.0	7.9	6.3	6.5	8.2	8.2
Parental Edu ≥College	8.2	8.1	7.8	8.0	9.7	9.9
Single-Parent	8.0	7.9	6.8	7.0	8.2	8.5

G2. Counterfactual Experiments

Table A7 Gains and Losses: BM to GS

	Winner		Loser	
	Δutils	Δ^{fees}	Δutils	Δ^{fees}
Strategic	35.0 (44.9)	-868.7 (774.5)	-21.0 (20.3)	498.5 (577.5)
Non-strategic	43.2 (46.6)	-987.1 (778.3)	-26.0 (23.4)	567.7 (596.4)
Edu < HS	31.0 (39.0)	-556.2 (559.9)	-22.9 (21.8)	637.2 (574.0)
Edu = HS	38.4 (48.2)	-937.2 (793.9)	-21.7 (20.6)	496.7 (547.5)
Edu ≥College	35.1 (45.2)	-1003.8 (816.0)	-19.6 (19.2)	392.5 (589.3)

Table A8 Winners and Losers: BM to TTC

	Winner		Loser	
	Δutils	Δ^{fees}	Δutils	Δ^{fees}
Strategic	30.5 (40.7)	-802.7 (743.6)	-25.5 (47.9)	409.2 (598.2)
Non-strategic	34.5 (43.1)	-836.2 (747.1)	-30.4 (61.1)	392.6 (583.1)
Edu < HS	27.4 (35.5)	-513.2 (528.0)	-21.3 (25.7)	551.0 (575.8)
Edu = HS	33.0 (43.6)	-873.7 (763.5)	-28.8 (47.4)	432.4 (582.9)
Edu ≥College	30.9 (41.4)	-954.2 (799.5)	-25.8 (59.9)	289.1 (601.5)

Table A9 School Assignment: BM

	Quality	Distance (100m)	Fees (100Euros)
All	7.8 (0.7)	7.2 (7.7)	7.3 (7.6)
Edu $<$ HS	7.5 (0.8)	6.4 (6.7)	4.6 (6.1)
Edu $=$ HS	7.8 (0.6)	7.0 (7.3)	7.2 (7.2)
Edu $\geq$College	8.0 (0.5)	8.0 (8.1)	9.2 (8.1)

Table A10 Winners and Losers: the 2007 Reform

	Winner(%)	Loser(%)	Δutils	Δ^{fees}
All	18.4	13.6	1.5 (85.7)	-22.6 (539.7)
Strategic	18.0	13.2	1.0 (12.0)	-6.9 (358.0)
Non-strategic	27.4	21.5	11.6 (417.1)	-385.9 (1981.1)
Edu $<$ HS	16.2	14.3	0.4 (53.6)	-17.8 (393.7)
Edu $=$ HS	20.4	15.3	0.9 (96.5)	-35.6 (621.2)
Edu $\geq$College	18.4	11.7	2.6 (93.5)	-39.3 (552.7)
Corner	21.1	14.9	2.3 (84.4)	-23.9 (558.1)