Stagnation Traps

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Abstract

We provide a Keynesian growth theory in which pessimistic expectations can lead to permanent, or very persistent, slumps characterized by unemployment and weak growth. We refer to these episodes as *stagnation traps*, because they consist in the joint occurrence of a liquidity and a growth trap. In a stagnation trap, the central bank is unable to restore full employment because weak growth pushes the interest rate against the zero lower bound, while growth is weak because low aggregate demand results in low profits, limiting firms' investment in innovation. Policies aiming at restoring growth can successfully lead the economy out of a stagnation trap, thus rationalizing the notion of job creating growth.

1 Introduction

Can insufficient aggregate demand lead to economic stagnation, i.e. a protracted period of low growth and high unemployment? Economists have been concerned with this question at least since the Great Depression, but recently interest in this topic has reemerged motivated by the two decades-long slump affecting Japan since the early 1990s, as well as by the slow recoveries characterizing the US and the Euro area in the aftermath of the 2008 financial crisis. Indeed, all these episodes have been characterized by long-lasting slumps in the context of policy rates at, or close to, their zero lower bound, leaving little room for conventional monetary policy to stimulate demand. Moreover, during these episodes potential output growth has been weak, resulting in large deviations of output from pre-slump trends.[1]

In this paper we present a theory in which permanent, or very persistent, slumps characterized by unemployment and weak growth are possible. Our idea is that the connection between depressed demand, low interest rates and weak growth, far from being casual, might be the result of a two-way interaction. On the one hand, unemployment and weak aggregate demand might have a negative impact on firms' investment in innovation, and result in low growth. On the other hand, low growth might depress the real interest rates and push nominal rates close to their zero lower bound, thus undermining the central bank's ability to maintain full employment by cutting policy rates.

To formalize this insight, and explore its policy implications, we propose a *Keynesian growth* framework that sheds lights on the interactions between endogenous growth and liquidity traps.[2] The backbone of our framework is a standard model of vertical innovation, in the spirit of Aghion and Howitt (1992) and Grossman and Helpman (1991). We modify this classic endogenous growth framework in two directions. First, we introduce nominal wage rigidities, which create the possibility of involuntary unemployment, and give rise to a channel through which monetary policy can affect the real economy.[3] Second, we take into account the zero lower bound on the nominal interest rate, which limits the central bank's ability to stabilize the economy with conventional monetary policy. Our theory thus combines the Keynesian insight that unemployment might arise due to weak aggregate demand, with the notion, developed by the endogenous growth literature, that productivity growth is the result of investment in innovation by profit-maximizing agents. We

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[1]Ball (2014) estimates the long-run consequences of the 2008 global financial crisis in several countries and documents significant losses in terms of potential output. Christiano et al. (2015) find that the US Great Recession has been characterized by a very persistent fall in total factor productivity below its pre-recession trend. Cerra and Saxena (2008) analyse the long-run impact of deep crises, and find, using a large sample of countries, that crises are often followed by permanent negative deviations from pre-crisis trends. A similar conclusion is reached by Blanchard et al. (2015), who also find that recessions are in many cases followed by a slowdown in the growth rate of the economy.

[2]We refer to our model as a Keynesian growth framework because it combines a model of long-run endogenous growth with short run Keynesian frictions. In this model the level of output and employment over a long time period are not taken as given as in the current New Classical and New Keynesian synthesis, but they are endogenously determined.

[3]A growing body of evidence emphasizes how nominal wage rigidities represent an important transmission channel through which monetary policy affects the real economy. For instance, this conclusion is reached by Christiano et al. (2005) using an estimated medium-scale DSGE model of the US economy, and by Olivei and Tenreyro (2007), who show that monetary policy shocks in the US have a bigger impact on output in the aftermath of the season in which wages are adjusted. Eichengreen and Sachs (1985) and Bernanke and Carey (1996) describe the role of nominal wage rigidities in exacerbating the downturn during the Great Depression. Similarly, Schmitt-Grohé and Uribe (2011) document the importance of nominal wage rigidities for the 2001 Argentine crisis and for the 2008-2009 recession in the Eurozone periphery. Micro-level evidence on the importance of nominal wage rigidities is provided by Fehr and Goette (2005), Gottschalk (2005), Barattieri et al. (2010) and Fabiani et al. (2010).

show that the interaction between these two forces can give rise to prolonged periods of low growth and high unemployment. We refer to these episodes as *stagnation traps*, because they consist in the joint occurrence of a liquidity and a growth trap.

In our economy there are two types of agents: firms and households. Firms' investment in innovation determines endogenously the growth rate of productivity and potential output of our economy. As in the standard models of vertical innovation, firms invest in innovation to gain a monopoly position, and so their investment in innovation is positively related to profits. Through this channel, a slowdown in aggregate demand that leads to a fall in profits, also reduces investment in innovation and the growth rate of the economy. Households supply labor and consume, and their intertemporal consumption pattern is characterized by the traditional Euler equation. The key aspect is that households' current consumption is affected by the growth rate of potential output, because productivity growth is one of the determinants of households' future income. Hence, a low growth rate of potential output is associated with lower future income and a reduction in current aggregate demand.

This two-way interaction between productivity growth and aggregate demand results in two steady states. First, there is a full employment steady state, in which the economy operates at potential and productivity growth is robust. However, our economy can also find itself in an unemployment steady state. In the unemployment steady state aggregate demand and firms' profits are low, resulting in low investment in innovation and weak productivity growth. Moreover, monetary policy is not able to bring the economy at full employment, because the low growth of potential output pushes the interest rate against its zero lower bound. Hence, the unemployment steady state can be thought of as a stagnation trap.

Expectations, or animal spirits, are crucial in determining which equilibrium will be selected. For instance, when agents expect growth to be low, expectations of low future income reduce aggregate demand, lowering firms' profits and their investment, thus validating the low growth expectations. Through this mechanism, pessimistic expectations can generate a permanent liquidity trap with involuntary unemployment and stagnation. We also show that, aside from permanent liquidity traps, pessimistic expectations can give rise to liquidity traps of finite, but arbitrarily long, duration.

We then examine the policy implications of our framework by focusing on the role of growth-enhancing policies. While these policies have been studied extensively in the context of the endogenous growth literature, here we show that they operate not only through the supply side of the economy, but also by stimulating aggregate demand during a liquidity trap. In fact, we show that an appropriately designed subsidy to innovation can push the economy out of a stagnation trap and restore full employment, thus capturing the notion of job creating growth. However, our framework suggests that, in order to be effective, the subsidy to innovation has to be sufficiently aggressive, so as to provide a "big push" to the economy.

This paper is related to several strands of the literature. First, the paper is related to Hansen's secular stagnation hypothesis (Hansen, 1939), that is the idea that a drop in the real natural interest rate might push the economy in a long-lasting liquidity trap, characterized by the absence of any self-correcting force to restore full employment. Hansen formulated this concept inspired by the US Great Depression, but recently some researchers, most notably Summers (2013) and Krugman

(2013), have revived the idea of secular stagnation to rationalize the long duration of the Japanese liquidity trap and the slow recoveries characterizing the US and the Euro area after the 2008 financial crisis. To the best of our knowledge, the only existing framework in which permanent liquidity traps are possible due to a fall in the real natural interest rate has been provided by Eggertsson and Mehrotra (2014).[4] However, the source of their liquidity trap is very different from ours. In their framework, liquidity traps are generated by shocks that alter households' lifecycle saving decisions. Instead, in our framework the drop in the real natural interest rate that generates a permanent liquidity trap originates from an endogenous drop in investment in innovation and productivity growth.[5]

Second, our paper is related to the literature on poverty and growth traps. This literature discusses several mechanisms through which a country can find itself permanently stuck with inefficiently low growth. Examples of this literature are Murphy et al. (1989), Matsuyama (1991), Galor and Zeira (1993) and Azariadis (1996).[6] Different from these contributions, we show that a liquidity trap can be the driver of a growth trap. Indeed, the intimate connection between the two traps lead us to put forward the notion of stagnation traps.

As in the seminal frameworks presented by Aghion and Howitt (1992), Grossman and Helpman (1991) and Romer (1990), long-run growth in our model is the result of investment in innovation by profit-maximizing agents. A small, but growing, literature has considered the interactions between short-run fluctuation and long run growth in this class of models (Fatas, 2000; Barlevy, 2004; Comin and Gertler, 2006; Aghion et al., 2010; Nuño, 2011; Queraltó, 2013), as well as some of the implications for fiscal or monetary policy (Aghion et al., 2009, 2014; Chu and Cozzi, 2014). However, to the best of our knowledge, we are the first ones to study monetary policy in an endogenous growth model featuring a zero lower bound constraint on the policy rate, and to show that the interaction between endogenous growth and monetary policy creates the possibility of long periods of stagnation.

Finally, our paper is linked to the literature on fluctuations driven by confidence shocks and sunspots. Some examples of this vast literature are Kiyotaki (1988), Benhabib and Farmer (1994, 1996), Francois and Lloyd-Ellis (2003), Farmer (2012) and Bacchetta and Van Wincoop (2013). We contribute to this literature by describing a new channel through which pessimistic expectations can give rise to economic stagnation.

The rest of the paper is composed of four sections. Section 2 describes the model. Section 3 shows that pessimistic expectations can generate arbitrarily long lasting stagnation traps. Section 4 studies the role of growth policies as a tool to stimulate aggregate demand and pull the economy out of a stagnation trap. Section 5 concludes.

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[4]The literature studying liquidity traps in micro-founded models has traditionally focused on slumps generated by ad-hoc preference shocks, as in Krugman (1998), Eggertsson and Woodford (2003), Eggertsson (2008) and Werning (2011), or by financial shocks leading to tighter access to credit, as in Eggertsson and Krugman (2012) and Guerrieri and Lorenzoni (2011). In all these frameworks liquidity traps are driven by a temporary fall in the natural interest rate, and permanent liquidity traps are not possible. Benhabib et al. (2001) show that, when monetary policy is conducted through a Taylor rule, changes in inflation expectations can give rise to permanent liquidity traps. However, in their framework the real natural interest rate during a permanent liquidity trap is equal to the one prevailing in the full employment equilibrium.

[5]On a technical note, Eggertsson and Mehrotra (2014) rely on an overlapping generation model to generate liquidity traps, while our mechanism is also at work in economies in which agents are infinitely lived.

[6]See Azariadis and Stachurski (2005) for an excellent survey of this literature.

2   Model

Consider an infinite-horizon closed economy. Time is discrete and indexed by $t \in \{0, 1, 2, ...\}$. The economy produces a continuum of goods indexed by $j \in [0, 1]$, which are used for consumption and as inputs in research. The economy is inhabited by households, firms, and by a central bank that sets monetary policy.[7]

2.1   Households

There is a continuum of measure one of identical households. The lifetime utility of the representative household is:

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1 - \sigma} \right) \right],$$

where $0 < \beta < 1$ is the subjective discount factor, σ denotes the inverse of the elasticity of intertemporal substitution, and $E_t[\cdot]$ is the expectation operator conditional on information available at time t. C is a quality-adjusted consumption index defined as:

$$C_t = \exp \left(\int_0^1 \ln q_{jt} c_{jt} dj \right),$$

where c_j denotes consumption of good j with associated quality q_j.[8]

Define P_j as the nominal price of good j, and $X_t \equiv \int_0^1 P_{jt} c_{jt} dj$ as the household's expenditure in consumption at time t. Each period the household allocates X_t to maximize C_t given prices. The optimal allocation of expenditure implies:

$$c_{jt} = \frac{X_t}{P_{jt}},$$

so that the household allots identical expenditure shares to all consumption goods. Hence, we can write:

$$X_t = \frac{P_t C_t}{Q_t},$$

where $Q_t \equiv \exp(\int_0^1 \ln q_{jt} dj)$ captures the average quality of the consumption basket, while $P_t \equiv \exp(\int_0^1 \ln P_{jt} dj)$ is the consumer price index.

Each household is endowed with one unit of labor and there is no disutility from working. However, due to the presence of nominal wage rigidities to be described below, a household might be able to sell only $L_t < 1$ units of labor on the market. Hence, when $L_t = 1$ the economy operates at full employment, while when $L_t < 1$ there is involuntary unemployment, and the economy operates below capacity.

Households can trade in one period, non-state contingent bonds b. Bonds are denominated in units of currency and pay the nominal interest rate i. Moreover, households own all the firms and each period they receive dividends d from them.

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[7]To ease exposition, in the main text we consider a cashless economy. In appendix B we show that introducing money does not affect our results.

[8]More precisely, for every good j q_j represents the highest quality available. In principle, households could consume a lower quality of good j. However, as in Aghion and Howitt (1992) and Grossman and Helpman (1991), the structure of the economy is such that in equilibrium only the highest quality version of each good is consumed.

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The intertemporal problem of the representative household consists in choosing C_t and b_{t+1} to maximize expected utility, subject to the budget constraint:

$$\frac{P_t C_t}{Q_t} + \frac{b_{t+1}}{1 + i_t} = W_t L_t + b_t + d_t,$$

where b_{t+1} is the stock of bonds purchased by the household in period t, and b_t is the payment received from its past investment in bonds. W_t is the nominal wage, so that $W_t L_t$ is the household's labor income.

The optimality conditions are:

$$\lambda_t = C_t^{-\sigma} \frac{Q_t}{P_t} \tag{1}$$

$$\lambda_t = \beta(1 + i_t) E_t \left[\lambda_{t+1} \right], \tag{2}$$

where λ denotes the Lagrange multiplier on the budget constraint.

2.2 Firms and innovation

In every industry j producers compete as price-setting oligopolists. One unit of labor is needed to manufacture one unit of consumption good, regardless of quality, and hence every producer faces the same marginal cost W_t. Our assumptions about the innovation process will ensure that in every industry there is a single leader able to produce good j of quality q_{jt}, and a fringe of competitors which are able to produce a version of good j of quality q_{jt}/γ, where $\gamma > 1$ captures the distance in quality between the leader and the followers. It is then optimal for the leader to capture the whole market for good j by charging the price:[9]

$$P_{jt} = \gamma W_t.$$

This expression implies that every good j is charged the same price, and so:

$$P_t = \gamma W_t. \tag{3}$$

Moreover, as we will verify later, every good faces the same demand y_t, so that the profits of the leader are:

$$y_t W_t(\gamma - 1), \tag{4}$$

implying that leaders make the same profits independently of the sector in which they operate.

Research and innovation. There is a large number of entrepreneurs that can attempt to innovate upon the existing products. A successful entrepreneur researching in sector j discovers a new version of good j of quality γ times greater than the best existing version. The successful entrepreneur becomes the leader in the production of good j, and maintains the leadership until a

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[9]Intuitively, the lowest price that competitors can charge without incurring losses is equal to the marginal cost W_t. Since one unit of the leading quality version of the good gives the same utility as γ units of the quality provided by the competitors, the leader can capture all the market by charging a price epsilon below γW_t. Indeed, it is optimal for the leader to charge this price. In fact, charging a higher price would result in loosing the market to the competitors, while charging a lower price would not result in an increase in the revenue from sales, while leading to a reduction in profits.

new version of good j is discovered.[10]

In order to discover a new product an entrepreneur needs to make an investment in terms of the differentiated goods.[11] In particular, the probability that an entrepreneur innovates is:

$$\frac{\chi I_t}{Q_t},$$

where $\chi > 0$ is a parameter capturing the productivity of research, I is an aggregate of the differentiated goods defined as:

$$I_t \equiv \exp\left(\int_0^1 \ln q_{jt}\iota_{jt}dj\right),$$

and ι_j denotes the quantity of good j invested in research. This formulation implies that goods of higher quality are more productive in research. The presence of the term Q captures the idea that as the economy grows and becomes more complex, a higher investment is required in order to make a new discovery.[12] This assumption is needed to ensure stationarity in the growth process.

It is optimal for an entrepreneur to allocate her research expenditure equally across all the goods j, and so we can drop the j subscripts and write $I_t = Q_t\iota_t$. Hence, the innovation probability is:

$$\chi\iota_t. \tag{5}$$

Since profits are the same for all industries j, entrepreneurs are indifferent with respect to which good they target their research efforts. We focus on symmetric equilibria in which all products are targeted with the same intensity. ι_t can then be interpreted as a measure of aggregate investment in innovation activities, and $\chi\iota_t$ is the probability that an innovation occurs in any sector.

We now turn to the reward from research. Denote by V_t the value of becoming a leader. Assuming that gaining a leadership in period t allows a firm to start producing in period $t + 1$, V_t is given by:

$$V_t = \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\left(y_{t+1}W_{t+1}(\gamma - 1) + (1 - \chi\iota_{t+1})V_{t+1}\right)\right]. \tag{6}$$

The value of becoming a leader at time t is equal to the expected profits to be gained in period $t + 1$, $y_{t+1}W_{t+1}(\gamma - 1)$, plus the value of being a leader in period $t + 1$, V_{t+1}, times the probability that the entrepreneur remains the leader in period $t + 1$, $1 - \chi\iota_{t+1}$. The entrepreneur acts in the interest of the households, and so discounts future payoffs using the households' discount factor $\beta\lambda_{t+1}/\lambda_t$.

We focus on equilibria in which some research is conducted in every period. Then, free entry

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[10]As discussed by Aghion and Howitt (1992) and Grossman and Helpman (1991), in this setting incumbents do not perform any research, because the value of improving over their own product is smaller than the profits that they would get from developing a leadership position in a second market.

[11]The assumption that goods are used as inputs into research follows chapter 7 of Barro and Sala-i Martin (2004) and Howitt and Aghion (1998). Alternatively, one could assume, as in Grossman and Helpman (1991), that labor is used as input into research. We chose the first formulation because it simplifies the exposition.

[12]Similar assumptions are also present in chapter 7 of Barro and Sala-i Martin (2004) and Howitt and Aghion (1998).

into research implies that expected profits from researching are zero, and so:[13]

$$P_t = \chi V_t.$$

Combining this condition with expression (6) gives:

$$\frac{P_t}{\chi} = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left(y_{t+1} W_{t+1}(\gamma - 1) + (1 - \chi \iota_{t+1}) \frac{P_{t+1}}{\chi} \right) \right]. \tag{7}$$

This condition determines the optimal investment in research.

2.3 Aggregation and market clearing

In equilibrium, production is equal across all the goods j. Hence, we can drop the j subscripts and write the goods market clearing condition as:[14]

$$y_t = c_t + \iota_t, \tag{8}$$

which states that all the production has to be consumed or invested in research. Since labor is the only factor of production:

$$y_t = L_t \leq 1, \tag{9}$$

where the inequality derives from the assumption of a unitary endowment of labor. Since labor is supplied inelastically by the households, this expression implies that when $y_t = 1$ the economy operates at full employment, while when $y_t < 1$ there is involuntary unemployment. Hence, we can interpret y_t as a measure of the output gap.

Long run growth in this economy takes place through increases in the quality of the consumption goods, captured by increases in the quality index Q. Since a higher Q increases the utility that households obtain from consumption, as well as the productivity of investment in research, we will refer to the growth rate of Q as the productivity growth rate of the economy.[15] Recalling that $\chi \iota_t$ is the probability that an innovation occurs in any sector, and using the law of large numbers, the

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[13]To see this, consider that an entrepreneur that invests ι_t in research has a probability $\chi \iota_t$ of becoming a leader which carries value V_t. Hence, the expected return from this investment is $\chi \iota_t V_t$. On the other hand, the investment costs $P_t \iota_t$. The zero expected profits condition in the research sector then implies:

$$P_t \iota_t = \chi \iota_t V_t.$$

Simplifying we obtain the expression in the main text.

[14]The goods market clearing condition can also be derived combining the households' budget constraint, with the expression for firms' profits:

$$d_t = P_t y_t - W_t L_t - P_t \iota_t,$$

where profits are net of research expenditure, and the equilibrium condition $b_{t+1} = 0$, deriving from the assumption of identical households.

[15]To strengthen the parallel between the growth rate of Q and productivity growth, one could assume that households consume a unique consumption good, produced by competitive firms using the j goods as intermediate inputs. Under this interpretation, growth in the quality of the intermediate goods would allow competitive firms to increase the quantity of the final good produced, and growth in the quality index Q would capture the productivity growth of intermediate inputs. Grossman and Helpman (1991) show that this model is isomorphic to the one presented in the main text.

growth rate of Q can be written as:[16]

$$g_{t+1} \equiv \frac{Q_{t+1}}{Q_t} = \exp\left(\chi \iota_t \ln \gamma\right). \tag{10}$$

Hence, higher investment in research in period t is associated with faster growth between periods t and $t+1$.

2.4 Nominal rigidities

To introduce the possibility of involuntary unemployment, and a role for monetary policy as a stabilization tool, we consider an economy with nominal wage rigidities. For simplicity, we start by considering an economy in which nominal wage inflation is constant and equal to $\bar{\pi}$:

$$W_t = \bar{\pi} W_{t-1}. \tag{11}$$

For instance, this expression could be derived from the presence of large menu costs from deviating from the constant wage inflation path. Since, by equation (3), prices are proportional to wages, it follows that also CPI inflation is constant and equal to $\bar{\pi}$.

Considering an economy with constant inflation simplifies the analysis, and allows us to characterize transparently the key economic forces at the heart of our model.[17] However, in section 3.3 we generalize our results to an economy featuring downward nominal wage rigidities, giving rise to a Phillips curve.

2.5 Monetary policy

The central bank implements its monetary policy stance by setting the nominal interest rate i_t. We consider a central bank that sets monetary policy according to the interest rate rule:

$$1 + i_t = \max\left(\left(1 + \bar{i}\right) y_t^{\phi}, 1\right), \tag{12}$$

where $\bar{i} > 0$ and $\phi > 0$. Under this rule, the central bank responds to a fall in the output gap, or equivalently to a rise in unemployment, by lowering the policy rate to stimulate aggregate demand. However, by standard arbitrage between money and bonds, the nominal interest rate cannot be

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[16]To derive this expression, consider that:

$$Q_{t+1} = \exp\left(\int_{[0,1]} \ln q_{jt+1} dj\right) = \exp\left(\int_{\mathcal{I}_t} \ln \gamma q_{jt} dj + \int_{[0,1]\setminus\mathcal{I}_t} \ln q_{jt} dj\right) = \exp\left(\int_{\mathcal{I}_t} \ln \gamma dj + \int_{[0,1]} \ln q_{jt} dj\right),$$

where $\mathcal{I}_t \in [0,1]$ is the mass of entrepreneurs who successfully innovate at time t. The probability of successful innovation $\chi \iota_t$ is the same and independent across entrepreneurs, hence using the law of large numbers the last expression simplifies to:

$$\exp\left(\chi \iota_t \ln \gamma\right) \exp\left(\int_{[0,1]} \ln q_{jt} dj\right) = \exp\left(\chi \iota_t \ln \gamma\right) Q_t.$$

[17]In particular, focusing on an economy with constant inflation makes clear that the possibility of permanent liquidity traps in our model does not rely on self-fulfilling drops in expected inflation, of the type described by Benhabib et al. (2001).

negative, $i_t \geq 0$.[18] Hence, there is a zero lower bound constraint on the nominal interest rate, which might interfere with the central bank's ability to stabilize employment.

2.6 Equilibrium

The equilibrium of our economy can be described by four simple equations. The first one is the *Euler* equation, which captures households' consumption decisions. Combining households' optimality conditions (1) and (2) gives the Euler equation:

$$C_t^{-\sigma} \frac{Q_t}{P_t} = \beta(1+i_t)E_t\left[\frac{C_{t+1}^{-\sigma}Q_{t+1}}{P_{t+1}}\right].$$

Since households consume the same amount of every good $C_t = Q_t c_t$, while constant inflation implies $P_{t+1}/P_t = \bar{\pi}$. Combining these two conditions with the previous expression gives:

$$c_t^\sigma = \frac{\bar{\pi} g_{t+1}^{\sigma-1}}{\beta(1+i_t)E_t\left[c_{t+1}^{-\sigma}\right]}. \tag{13}$$

The Euler equation relates demand for consumption with the nominal interest rate. As it is standard in models with price rigidities, a fall in the nominal interest rate stimulates present consumption. Similarly, a rise in expected future consumption stimulates present consumption.

The only non-standard feature of this Euler equation is the presence of the growth rate of productivity, captured by the term g_{t+1}. The impact of productivity growth on present demand for consumption depends on the elasticity of intertemporal substitution, $1/\sigma$. Intuitively, there are two effects. On the one hand, faster growth generates higher lifetime utility from consumption. This income effect leads households to increase their demand for current consumption after a rise in the growth rate of the economy. On the other hand, faster growth is associated with a rise in the quality of future consumption goods compared to the quality of present consumption goods. This substitution effect points toward a negative relationship between growth and current demand for consumption. For low levels of intertemporal substitution, i.e. for $\sigma > 1$, the income effect dominates and the relationship between growth and demand for consumption is positive. Instead, for high levels of intertemporal substitution, i.e. for $\sigma < 1$, the substitution effect dominates and the relationship between growth and demand for consumption is negative. Finally, for the special case of log utility, $\sigma = 1$, the two effects cancel out and growth does not affect present demand for consumption.

Empirical estimates based on aggregate consumption data point toward a low elasticity of intertemporal substitution (Hall, 1988).[19] Hence, in the main text we will focus attention on the case $\sigma > 1$, while we provide an analysis of the cases $\sigma < 1$ and $\sigma = 1$ in the appendix.

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[18]In appendix B we introduce money in the model and derive explicitly a zero lower bound on the nominal interest rate.

[19]Similar results are reached by Ogaki and Reinhart (1998) and Basu and Kimball (2002). Using estimates based on micro data, Vissing-Jørgensen (2002) finds higher values of the elasticity of intertemporal substitution, but they still tend to be lower than 1.

Assumption 1 *The parameter σ satisfies:*

$$\sigma > 1.$$

Under this assumption, the Euler equation implies a positive relationship between the pace of innovation and demand for present consumption.

The second key relationship in our model is the *growth equation*, which describes the supply side of the economy. To derive the growth equation, plug equations (1) and (10) in the optimality condition for investment in research (7), and use $W_t/P_t = 1/\gamma$, to obtain:

$$1 = \beta E_t \left[\left(\frac{c_t}{c_{t+1}} \right)^\sigma g_{t+1}^{1-\sigma} \left(\chi \frac{\gamma - 1}{\gamma} y_{t+1} + 1 - \frac{\ln g_{t+2}}{\ln \gamma} \right) \right]. \tag{14}$$

This equation captures the optimal investment in research by entrepreneurs. It implies a positive relationship between growth and the output gap, because a rise in the output gap is associated with higher monopoly profits. In turn, higher profits induce entrepreneurs to invest more in research, leading to a positive impact on the growth rate of the economy.

The third equation combines the goods market clearing condition (8) with the equation relating growth to investment in innovation (10):

$$c_t = y_t - \frac{\ln g_{t+1}}{\chi \ln \gamma}. \tag{15}$$

Keeping output constant, this equation implies a negative relationship between consumption and growth, because to generate faster growth the economy has to devote a larger fraction of the output to innovation activities, reducing the resources available for consumption.

Finally, the fourth equation is the monetary policy rule:

$$1 + i_t = \max \left((1 + \bar{i}) \, y_t^\phi, 1 \right). \tag{16}$$

We are now ready to define an equilibrium as a set of processes $\{y_t, c_t, g_{t+1}, i_t\}_{t=0}^{+\infty}$ satisfying equations $(13) - (16)$.

3   Confidence shocks and stagnation traps

In this section we show that our economy can get stuck in prolonged periods of stagnation. We start by considering non-stochastic steady states, and show that our model features two steady states: one characterized by full employment, and one by involuntary unemployment.

3.1   Non-stochastic steady states

In steady state growth g, the output gap y, consumption c and the nominal interest rate i are constant. Hence, the steady state equilibrium is described by the system:

$$g^{\sigma-1} = \frac{\beta(1+i)}{\bar{\pi}} \tag{17}$$

$$\frac{g^{\sigma-1}}{\beta} + \frac{\ln g}{\ln \gamma} = \chi \frac{\gamma - 1}{\gamma} y + 1 \tag{18}$$

$$c = y - \frac{\ln g}{\chi \ln \gamma} \tag{19}$$

$$1 + i = \max\left((1 + \bar{i}) y^\phi, 1\right), \tag{20}$$

where the absence of a time subscript denotes the value of a variable in a non-stochastic steady state.

Full employment steady state. Let us start by describing the full employment steady state, which we denote by the superscripts f. In the full employment steady state the economy operates at full capacity ($y^f = 1$), and so, by equation (18), productivity growth solves:

$$\frac{\left(g^f\right)^{\sigma-1}}{\beta} + \frac{\ln g^f}{\ln \gamma} = \chi \frac{\gamma - 1}{\gamma} + 1. \tag{21}$$

The nominal interest rate that supports this steady state can be obtained by rearranging equation (17):

$$i^f = \frac{\bar{\pi} \left(g^f\right)^{\sigma-1}}{\beta} - 1.$$

We now make some assumptions about the parameters governing the monetary policy rule, to ensure that the full employment equilibrium exists and is unique.

Assumption 2 *The parameters $\bar{i}$, $\bar{\pi}$, χ, γ, σ, β and ϕ satisfy:*

$$\bar{i} = \frac{\bar{\pi} \left(g^f\right)^{\sigma-1}}{\beta} - 1 \tag{22}$$

$$\bar{i} \geq 0 \tag{23}$$

$$\phi \geq 1, \tag{24}$$

where g^f solves:

$$\frac{\left(g^f\right)^{\sigma-1}}{\beta} + \frac{\ln g^f}{\ln \gamma} = \chi \frac{\gamma - 1}{\gamma} + 1.$$

Proposition 1 *Suppose assumption 2 holds. Then, there exists a unique full employment steady state.*[20]

Intuitively, assumptions (22) and (23) guarantee that inflation and trend growth in the full employment steady state are sufficiently high so that the zero lower bound constraint on the nominal interest rate is not binding. In this case there exists a unique steady state in which a positive nominal interest rate is consistent with full employment. Instead, assumption (24) ensures that, in absence of the zero lower bound, there are no steady states other than the full employment steady state.

Unemployment steady state. Aside from the full employment steady state, the economy can find itself in a permanent liquidity trap with low growth and involuntary unemployment. We

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[20] All the proofs can be found in appendix A.

denote this unemployment steady state with superscripts u. To derive the unemployment steady state, consider that with $i = 0$ equation (17) implies:

$$g^u = \left(\frac{\beta}{\bar{\pi}}\right)^{\frac{1}{\sigma-1}} < g^f,$$

where the inequality holds because $\sigma > 1$. To see that the liquidity trap steady state is characterized by unemployment, rewrite equation (18) as:

$$y^u = \left(\frac{(g^u)^{\sigma-1}}{\beta} + \frac{\ln g^u}{\ln \gamma} - 1\right)\frac{\gamma}{\chi(\gamma-1)} < 1,$$

where the inequality follows from $g^u < g^f$ and from the fact that equation (18) gives a monotonically increasing relationship between g and y. Since $\iota \geq 0$, this equilibrium exists if $g^u \geq 1$, i.e. if $\beta/\bar{\pi} \geq 1$.[21] If this is the case, assumption 2 guarantees that when the output gap is equal to y^u the central bank sets the nominal interest rate to zero. The following proposition summarizes our results about the unemployment steady state.

Assumption 3 *The parameters β and $\bar{\pi}$ satisfy:*

$$\frac{\beta}{\bar{\pi}} \geq 1.$$

Proposition 2 *Suppose assumptions 1, 2 and 3 hold. Then, there exists a unique unemployment steady state. At the unemployment steady state the economy is in a liquidity trap $(i^u = 0)$, there is involuntary unemployment $(y^u < 1)$, and growth is lower than in the full employment steady state $(g^u < g^f)$.*

To understand the economics behind assumption 3, consider that when $i = 0$, to be consistent with households' Euler equation, the economy must grow at rate $(\beta/\bar{\pi})^{1/(\sigma-1)}$. In turn, the growth rate has a lower bound equal to 1, because knowledge does not depreciate.[22] Hence, assumption 3 guarantees that there exists a level of output gap $y^u > 0$ consistent with the economy growing at rate $(\beta/\bar{\pi})^{1/(\sigma-1)}$.

We think of this second steady state as a *stagnation trap*, that is the combination of a liquidity and a growth trap. In a liquidity trap the economy operates below capacity because the central bank is constrained by the zero lower bound on the nominal interest rate. In a growth trap, lack of demand for firms' products depresses investment in innovation and prevents the economy from developing its full growth potential. In a stagnation trap these two events are tightly connected. We illustrate this point with the help of a graphical analysis.

Figure 1 depicts the two key relationships that characterize the steady states of our model in the $y - g$ space. The first one is the growth equation (18), which corresponds to the upward-sloped

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[21]Since $\beta < 1$, in our baseline model an unemployment steady state exists only if inflation is non-positive, and so if $\bar{\pi} \leq 1$. However, as we show in section 3.3, this is not a strict implication of our framework, and it is not hard to modify the model to allow for positive inflation in the unemployment steady state, for instance by introducing precautionary savings due to idiosyncratic shocks.

[22]More generally, in an economy in which knowledge depreciates, an unemployment steady state exists if $(\beta/\bar{\pi})^{1/(\sigma-1)}$ is smaller than the rate of knowledge depreciation.
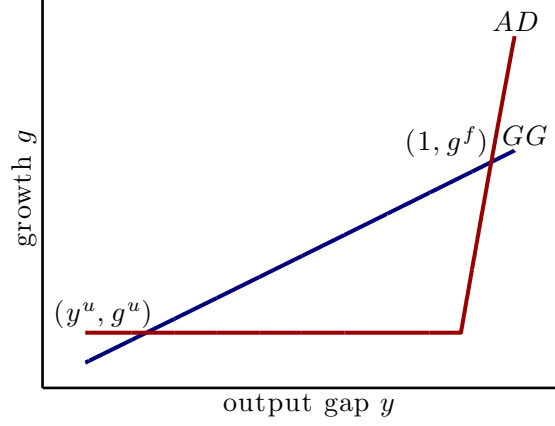
Figure 1: Non-stochastic steady states.

GG schedule. Intuitively, the output gap is positively related with growth because an increase in production is associated with a rise in firms' profits. Since firms invest in innovation to appropriate monopoly profits, higher profits generate an increase in investment in innovation and productivity growth, giving rise to a positive relationship between y and g.

The second key relationship combines the Euler equation (17) and the policy rule (20):

$$g^{\sigma-1} = \frac{\beta}{\bar{\pi}} \max \left((1+\bar{i})y^{\phi}, 1 \right).$$

Graphically, this relationship is captured by the *AD*, i.e. aggregate demand, curve. The upward-sloped portion of the *AD* curve corresponds to cases in which the zero lower bound constraint on the nominal interest rate is not binding.[23] In this part of the state space, the central bank responds to a rise in the output gap y by increasing the nominal rate. Since inflation is constant, the increase in the nominal rate directly translates into a rise in the real rate. In turn, according to the private aggregate demand equation, the real interest rate is increasing in the growth rate of the economy.[24] Hence, when monetary policy is active the *AD* curve generates a positive relationship between y and g. Instead, the horizontal portion of the *AD* curve corresponds to a situation in which the zero lower bound constraint binds. In this case, the central bank sets $i = 0$ and steady state growth is independent of y and equal to $(\beta/\bar{\pi})^{1/(\sigma-1)}$. As long as assumptions 1, 2 and 3 hold, the two curves cross twice and two steady states are possible.

Importantly, both the presence of the zero lower bound and the procyclicality of investment in innovation are needed to generate steady state multiplicity. Suppose that the central bank is not constrained by the zero lower bound, and hence that liquidity traps are not possible. As illustrated by the left panel of figure 2, in this case the *AD* curve reduces to an upward sloped curve, steeper than the *GG* curve, and the unemployment steady state disappears. Intuitively, in absence of the zero lower bound, the central bank's reaction to unemployment is always sufficiently strong to ensure that the only possible steady state is the full employment one.

Now suppose instead that productivity growth is constant and equal to g^f. In this case the *GG* curve reduces to a horizontal line at $g = g^f$, and again the full employment steady state is the only

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[23]Precisely, the zero lower bound constraint does not bind when $y \geq (1+\bar{i})^{-1/\phi}$.
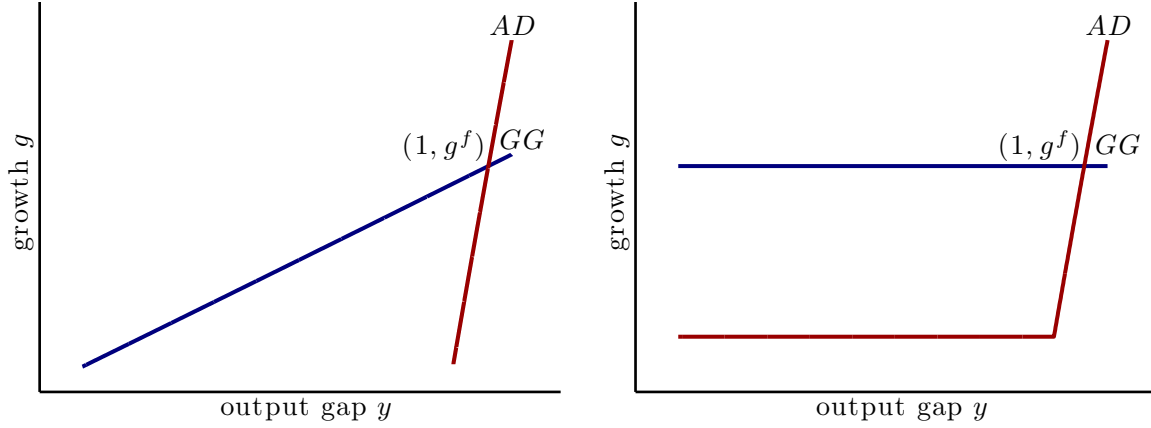[24]Recall that we are focusing on the case $\sigma > 1$.

Figure 2: Understanding stagnation traps. Left panel: economy without zero lower bound. Right panel: economy with exogenous growth.

possible one. Intuitively, if growth is not affected by variations in the output gap, then aggregate demand is always sufficiently strong so that in steady state the zero lower bound constraint on the nominal interest rate does not bind, ensuring that the economy operates at full employment. We refer to the unemployment steady state as a stagnation trap to capture the tight link between liquidity and growth traps suggested by our model.

We are left with determining what makes the economy settle in one of the two steady states. This role is fulfilled by expectations. Suppose that agents expect that the economy will permanently fluctuate around the full employment steady state. Then, their expectations of high future growth sustain aggregate demand, so that a positive nominal interest rate is consistent with full employment. In turn, if the economy operates at full employment then firms' profits are high, inducing high investment in innovation and productivity growth. Conversely, suppose that agents expect that the economy will permanently remain in a liquidity trap. In this case, low expectations about growth and future income depress aggregate demand, making it impossible for the central bank to sustain full employment due to the zero lower bound constraint on the interest rate. As a result the economy operates below capacity and firms' profits are low, so that investment in innovation is also low, justifying the initial expectations of weak growth. Hence, in our model expectations can be self-fulfilling, and sunspots, that is confidence shocks unrelated to fundamentals, can determine real outcomes.

Summarizing, the combination of growth driven by investment in innovation from profit-maximizing firms and constraints on monetary policy can produce stagnation traps, that is permanent, or very long lasting, liquidity traps characterized by unemployment and low growth. All it takes is a sunspot that coordinates agents' expectations on the unemployment steady state.

3.2 Sunspots and temporary liquidity traps

Though our model can allow for economies which are permanently in a liquidity trap, it is not difficult to construct equilibria in which the expected duration of a trap is finite.

To construct an equilibrium featuring a temporary liquidity trap we have to put some structure on the sunspot process. Let us start by denoting a sunspot by ξ_t. In a sunspot equilibrium agents form their expectations about the future after observing ξ, so that the sunspot acts as
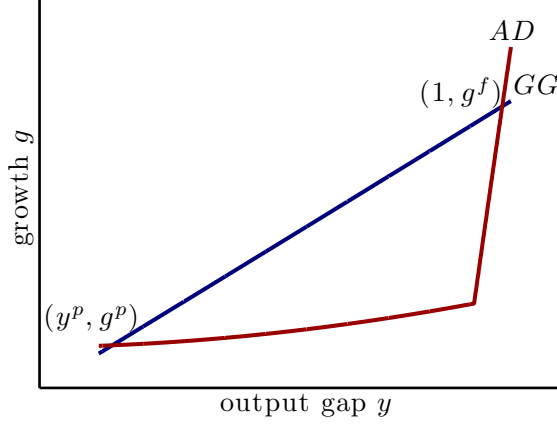
Figure 3: Temporary liquidity trap.

a coordination device for agents' expectations. To be concrete, let us consider a two-state discrete Markov process, $\xi_t \in (\xi_o, \xi_p)$, with transition probabilities $Pr\left(\xi_{t+1} = \xi_o | \xi_t = \xi_o\right) = 1$ and $Pr\left(\xi_{t+1} = \xi_p | \xi_t = \xi_p\right) = q_p < 1$. The first state is an absorbing optimistic equilibrium, in which agents expect to remain forever around the full employment steady state. Hence, once $\xi_t = \xi_o$ the economy settles on the full employment steady state, characterized by $y = 1$ and $g = g^f$. The second state ξ_p is a pessimistic equilibrium with finite expected duration $1/(1 - q_p)$. In this state the economy is in a liquidity trap with unemployment. We consider an economy that starts in the pessimistic equilibrium.

Under these assumptions, as long as the pessimistic sunspot shock persists the equilibrium is described by equations (13), (14) and (15), which, using the fact that in the pessimistic state $i = 0$, can be written as:

$$(g^p)^{\sigma-1} = \frac{\beta}{\pi}\left(q_p + (1 - q_p)\left(\frac{c^p}{c^f}\right)^\sigma \right) \tag{25}$$

$$\frac{(g^p)^{\sigma-1}}{\beta} = q_p\left(\chi\frac{\gamma-1}{\gamma}y^p + 1 - \frac{\ln g^p}{\ln\gamma} \right) + (1 - q_p)\left(\frac{c^p}{c^f}\right)^\sigma\left(\chi\frac{\gamma-1}{\gamma} + 1 - \frac{\ln g^f}{\ln\gamma} \right) \tag{26}$$

$$\frac{c^p}{c^f} = \frac{y^p - \frac{\ln g^p}{\chi\ln\gamma}}{1 - \frac{\ln g^f}{\chi\ln\gamma}}, \tag{27}$$

where the superscripts p denote the equilibrium while pessimistic expectations prevail. Similar to the case of the unemployment steady state, in the pessimistic equilibrium the zero lower bound constraint on the interest rate binds, there is involuntary unemployment and growth is lower than in the optimistic state.

While characterizing analytically the equilibrium is challenging, from equation (25) it is possible to see that temporary liquidity traps are characterized by slower growth than permanent ones. In fact, the term c^p/c^f is smaller than one, because switching to the optimistic steady state entails an increase in consumption. Intuitively, if the liquidity trap is temporary agents' consumption is expected to rise. However, when $i = 0$ the real interest rate is constant and equal to $1/\bar{\pi}$. Hence, by households' Euler equation the expected growth rate of households' quality-adjusted consumption is also constant, and independent of the expected duration of the trap. It follows that, for the Euler equation to hold, the expected increase in the quantity of goods consumed has to be compensated
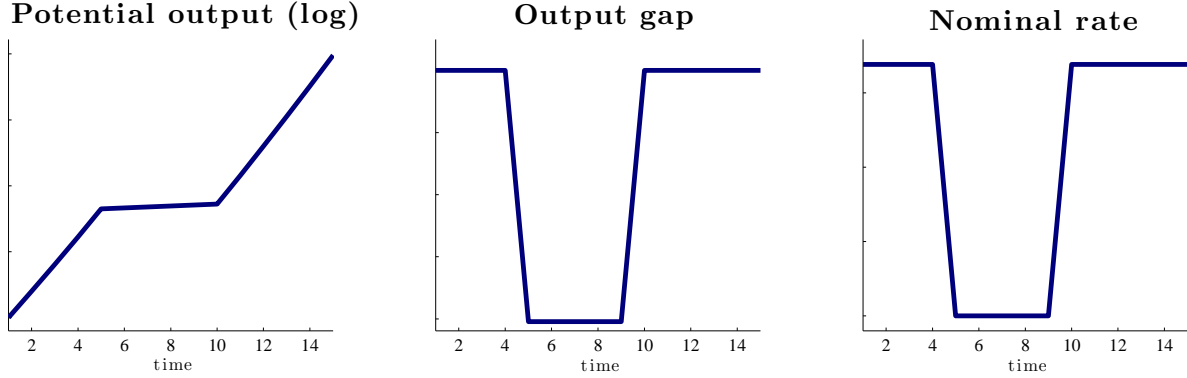
Figure 4: Dynamics around a temporary liquidity trap.

by a fall in the growth rate of quality. The result is that growth is slower in a temporary liquidity trap compared to a permanent one.

Figure 3 displays the equilibrium determination in terms of the AD and GG curves. The key change with respect to the case of non-stochastic steady states is that the AD curve is upward sloped for values of y such that $i = 0$.[25] Moreover, the steepness of the AD curve is decreasing in the expected duration of the trap, so that traps of shorter expected duration are characterized by lower growth and higher unemployment.

Figure 4 displays the dynamics around a liquidity trap of expected finite duration. The economy starts at the full employment steady state. In period 5 pessimistic expectations materialize, due to an unexpected confidence shock. The economy falls into a liquidity trap of finite expected duration, characterized by weak potential output growth and negative output gap, which lasts as long as pessimistic expectations prevail.[26] In the example of the figure expectations turn optimistic in period 10, and the economy exits the trap. However, the post-trap increase in the growth rate is not sufficiently strong to make up for the low growth during the trap, and so the economy experiences a permanent loss in output.

This example shows that pessimistic expectations can plunge the economy into a temporary liquidity trap with unemployment and low growth. Eventually the economy will recover, but the liquidity trap lasts as long as pessimistic beliefs persist. Hence, long lasting liquidity trap driven by pessimistic expectations can coexist with the possibility of a future recovery.

3.3 Two extensions to the basic framework

In this section we consider two extensions to the basic model: precautionary savings and variable inflation. We first show that the introduction of precautionary savings can give rise to stagnation traps characterized by positive growth and positive inflation. Then, we show that our key results do not rely on the assumption of a constant inflation rate.

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[25]Algebraically, this can be seen by combining equations (25) and (27) to obtain:

$$(g^p)^{\sigma-1} = \frac{\beta}{\pi} \left(q_p + (1 - q_p) \left(\frac{y^p - \frac{\ln g^p}{\chi \ln \gamma}}{1 - \frac{\ln g^f}{\chi \ln \gamma}} \right)^{\sigma} \right).$$

[26]Define quality-adjusted output as $Q_t y_t$. We then refer to potential output as Q_t, which is the quality-adjusted production prevailing when all the labor is employed.

Precautionary savings. One of the implications of assumption 3 is that in our basic framework positive growth and positive inflation cannot coexist during a permanent liquidity trap. Intuitively, if the economy is at the zero lower bound with positive inflation, then the real interest rate must be negative. But then, to satisfy households' Euler equation, the steady state growth rate of the economy must also be negative. Conversely, to be consistent with positive steady state growth the real interest rate must be positive, and when the nominal interest rate is equal to zero this requires deflation.

However, it is not hard to think about mechanisms that could make positive growth and positive steady state inflation coexist in an unemployment steady state. One possibility is to introduce precautionary savings. In appendix C, we lay down a simple model in which every period a household faces a probability p of becoming unemployed. An unemployed household receives an unemployment benefit, such that its income is equal to a fraction $b < 1$ of the income of an employed household. Unemployment benefits are financed with taxes on the employed households. We also assume that unemployed households cannot borrow and that trade in firms' share is not possible.

Under these assumptions, the Euler equation can be derived from the Euler equation of employed households and, as showed in the appendix, can be written as:

$$c_t^\sigma = \frac{\bar{\pi} g_{t+1}^{\sigma-1}}{\beta(1+i_t)\rho E_t\left[c_{t+1}^{-\sigma}\right]},$$

where:

$$\rho \equiv 1 - p + p/b^\sigma > 1.$$

The unemployment steady state is now characterized by:

$$g^u = \left(\frac{\rho\beta}{\bar{\pi}}\right)^{\frac{1}{\sigma-1}}.$$

Since $\rho > 1$, an unemployment steady state in which both inflation and growth are positive is now possible.

The key intuition behind this result is that the presence of uninsurable idiosyncratic risk depresses the natural interest rate.[27] Intuitively, the presence of uninsurable idiosyncratic risk drives up the demand for precautionary savings. Since the supply of saving instruments is fixed, higher demand for precautionary savings leads to a lower equilibrium interest rate. This is the reason why an economy with uninsurable unemployment risk can reconcile positive steady state growth with a negative real interest rate. Hence, once the possibility of uninsurable unemployment risk is taken into account, it is not hard to imagine a permanent liquidity trap with positive growth and positive inflation.

Introducing a Phillips curve. Our basic model features a constant inflation rate. Here we show that our results are consistent with the introduction of a Phillips curve, which creates a positive link between inflation and the output gap.

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[27]See Bewley (1977), Huggett (1993) and Aiyagari (1994).

To make things simple, let us assume that the nominal wage is downwardly rigid:

$$W_t \geq \psi(y_t) W_{t-1},$$

with $\psi' > 0$ and $\psi(1) = \bar{\pi}$.[28] This formulation, which follows Schmitt-Grohé and Uribe (2012), allows wages to fall at a rate which depends on unemployment. Capturing some nonmonetary costs from adjusting wages downward, here wages are more downwardly flexible the more output is below potential. Since prices are proportional to wages, this simple form of wage setting gives rise to a nonlinear Phillips curve. For levels of inflation greater than $\bar{\pi}$ output is at potential. Instead, if inflation is below $\bar{\pi}$ there is a positive relationship between inflation and the output gap.

A steady state of the economy is now described by (17)-(20) and:

$$\pi \geq \psi(y), \tag{28}$$

where π denotes the steady state gross inflation rate.

It is easy to check that the presence of the Phillips curve (28) does not have any impact on the full employment steady state, which is characterized by $y = 1$, $g = g^f$, $i = i^f$ and $\pi = \bar{\pi} \equiv \pi^f$.

Let us now turn to the unemployment steady state. Combining equations (17) and (28), and using $i = 0$, gives:

$$g^u = \left(\frac{\beta}{\psi(y^u)} \right)^{\frac{1}{\sigma - 1}}. \tag{29}$$

This expression implies a negative relationship between growth and the output gap. To understand this relationship, consider that in a liquidity trap the real interest rate is just the inverse of expected inflation. Due to the Phillips curve (28), as the output gap increases wages rise and firms increase their prices generating higher inflation. Hence, in a liquidity trap a higher output gap is associated with a lower real interest rate. The consequence is that during a permanent liquidity trap a rise in the output gap must be associated with lower productivity growth, to be consistent with the lower real interest rate. As illustrated by figure 5, graphically this is captured by the fact that the AD curve, obtained by combining equations (17) and (28), is downward sloped for values of y low enough so that the zero lower bound constraint binds.

To solve for the equilibrium unemployment steady state, combine equations (18) and (29) to obtain:

$$y^u = \left(\frac{1}{\psi(y^u)} + \frac{(\sigma - 1)(\ln \beta - \ln \psi(y^u))}{\ln \gamma} - 1 \right) \frac{\gamma}{\chi(\gamma - 1)} < 1.$$

Since the left-hand side of this expression is increasing in y^u, while the right hand-side is decreasing in y^u, there is a unique y^u that characterizes the unemployment steady state.[29] Moreover, since $y^u < 1$, the presence of a Phillips curve implies that the unemployment steady state is now characterized by lower inflation than the full employment steady state. In sum, the presence of a

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[28]To ensure the existence of a unique steady state in which the zero lower bound constraint does not bind we have to make sure that the Phillips curve is not too steep. This requires $(\phi - 1)\psi(y)/(y\psi'(y)) > 1$ for every $(1 + \bar{i})^{-1/\phi} < y < 1$.

[29]Notice that, since $\psi(y^u) < \bar{\pi}$, keeping everything else constant the unemployment steady state features a higher output gap and growth in the variable inflation case, compared to the model with fixed inflation. Hence, while the introduction of a Phillips curve does not alter any of our qualitative results, in quantitative applications it might be important to take into account the possibility that firms will respond to weak demand by cutting prices.
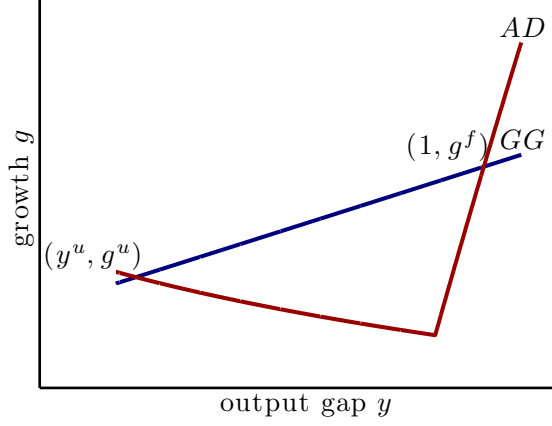
Figure 5: Steady states with variable inflation.

Phillips curve does not alter the key properties of the unemployment steady state, while adding the realistic feature that in the unemployment steady state the central bank undershoots its inflation target.

4   Growth Policy

We now turn to the policy implications of our model. One of the root causes of a stagnation trap is the weak growth performance of the economy, which is in turn due to entrepreneurs' limited incentives to innovate due to weak demand for their products. This suggests that subsidies to investment in innovation or to firms' profits might be a helpful tool in the management of twin traps. In fact, these policies have been extensively studied in the context of endogenous growth models as a tool to overcome inefficiencies in the innovation process. However, here we show how policies that foster productivity growth can also play a role in stimulating aggregate demand and employment during a liquidity trap.

The most promising form of growth policies to exit a stagnation trap are those that loosen the link between profits and investment in innovation. For instance, suppose that the government gives a fixed subsidy s to entrepreneurs' investment in innovation.[30] Let us also assume that the subsidy is financed with taxes on households.[31] Under these assumptions, the zero profit condition in the research sector is:[32]

$$V_t = \frac{P_t}{\chi}\left(1 - \frac{s}{\iota_t}\right).$$

The presence of the term s/ι_t is due to the fact that entrepreneurs have to finance only a fraction $1 - s/\iota_t$ of the investment in research, while the rest is financed by the government. This expression implies that entrepreneurs will be willing to invest in innovation even when the value of becoming

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[30]More precisely, we assume that the government devotes an aggregate amount of resources s to sustain innovation. These resources are equally divided among all the entrepreneurs operating in innovation.

[31]Since there is no disutility from working, it does not matter whether the subsidy is financed with lump sum taxes on households, or with taxes on labor income.

[32]With the subsidy, the cost of investing ι_t in research is $P_t(\iota_t - s)$, which gives an expected gain of $\chi\iota_t V_t$. The zero expected profits condition in the research sector then implies:

$$P_t\left(\iota_t - s\right) = \chi\iota_t V_t.$$

Rearranging this expression we obtain the expression in the main text.

a leader is zero, because the expression above implies that if $V_t = 0$ then $\iota_t = s$. Assuming that the government can ensure that entrepreneurs cannot divert the subsidy away from innovation activities, investment in innovation will be always at least equal to the subsidy s, so $\iota_t \geq s$.

Let us now consider the macroeconomic implications of the subsidy. For simplicity, we will focus on steady states. In this case, growth in the full employment steady state is now characterized by:

$$\left(\frac{\left(g^f\right)^{\sigma-1}}{\beta} - \left(1 - \frac{\ln g^f}{\ln \gamma}\right) \right) \left(1 - \frac{s\chi \ln \gamma}{\ln g^f}\right) = \chi \frac{\gamma - 1}{\gamma}, \tag{30}$$

and a higher subsidy is associated with faster growth in the full employment equilibrium.[33]

Now turn to the unemployment steady state. In an unemployment steady state, to satisfy the aggregate demand equation, productivity growth must be equal to $g^u = (\beta/\bar{\pi})^{(1/(\sigma-1))}$. As discussed in section 3.1, this condition implies that an unemployment steady state exists only if there exists a nonnegative value for the output gap such that productivity growth is equal to $g^u = (\beta/\bar{\pi})^{(1/(\sigma-1))}$. However, a sufficiently high subsidy can guarantee that the growth rate of the economy will always be higher than $(\beta/\bar{\pi})^{(1/(\sigma-1))}$. It follows that by setting a sufficiently high subsidy the government can rule out the possibility that the economy will fall in a stagnation trap.

Proposition 3 *Suppose that there is a subsidy to innovation satisfying:*

$$s > \frac{(\sigma - 1)(\ln \beta - \ln \bar{\pi})}{\chi \ln \gamma}, \tag{31}$$

and that the central bank sets $\bar{i}$ according to:

$$\bar{i} = \frac{\bar{\pi} \left(g^f\right)^{\sigma-1}}{\beta} - 1 \geq 1,$$

where g^f solves equation (30), and $\phi \geq 1$. Then there exists a unique steady state characterized by full employment.

Intuitively, the subsidy to innovation guarantees that even if firms' profits were to fall substantially, investment in innovation would still be relatively high. In turn, a high investment in innovation stimulates growth and aggregate demand, since a high future income is associated with a high present demand for consumption. By implementing a sufficiently high subsidy, the government can eliminate the possibility that aggregate demand will be low enough to make the zero lower bound constraint on the nominal interest rate bind. It is in this sense that growth policies can be thought as a tool to manage aggregate demand in our framework. Importantly, to be effective a subsidy to innovation has to be large enough, otherwise it might not have any positive impact on the economy.

The left panel of figure 6 illustrates these points graphically. The solid lines correspond to the benchmark economy, while the dashed lines represent an economy with a subsidy to investment in innovation. The subsidy makes the GG curve shift up, because for a given level of output gap the

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[33]To ensure the existence of a full employment equilibrium, the central bank has to adjust the intercept of the interest rate rule to take into account the impact of the subsidy on growth.
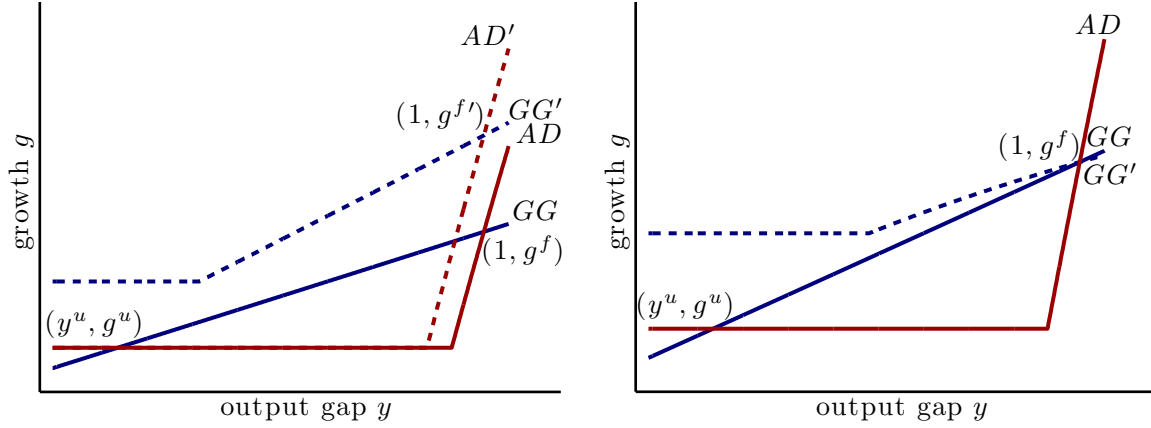
Figure 6: Subsidy to innovation.

subsidy increases the growth rate of the economy.[34] Moreover, the GG curve has now a horizontal portion, because the subsidy places a floor in the investment in research made by entrepreneurs. If the subsidy is sufficiently high, as it is the case in the figure, the unemployment steady state disappears and the only possible steady state is the full employment one.

One potential issue is that ruling out stagnation traps with a constant subsidy to innovation could come at the cost of an inefficiently high investment in innovation in the full employment steady state. However, it is not difficult to design a subsidy scheme that rules out the unemployment steady state, while maintaining the full employment steady state unchanged. For example, consider a countercyclical subsidy to innovation such that $s_t = s(1 - y_t)$.

Under this policy, the subsidy is decreasing in the output gap. In particular, the subsidy is equal to zero when the economy operates at full employment, so that the full employment steady state is not affected by the subsidy. However, if the subsidy is sufficiently large so that condition (31) holds, this countercyclical policy rules out the unemployment steady state.

The impact of a countercyclical subsidy is illustrated by the right panel of figure 6. The subsidy makes the GG curve rotate up. If s is sufficiently high, the movement of the curve is large enough to eliminate the unemployment steady state, while still leaving the full employment steady state unchanged. Hence, it is possible to design subsidy schemes that rule out the unemployment steady state without distorting the full employment one.

Summarizing, there is a role for subsidies to growth-enhancing investment, a typical supply side policy, in stimulating aggregate demand so as to rule out liquidity traps driven by expectations of weak future growth. In turn, the stimulus to aggregate demand has a positive impact on employment. In this sense, our model helps rationalizing the notion of job creating growth.

5   Conclusion

We develop a Keynesian growth model in which endogenous growth interacts with the possibility of involuntary unemployment due to weak aggregate demand. The model features two steady states. One is a stagnation trap, that is a permanent liquidity trap characterized by unemployment and weak growth. All it takes for the economy to fall into a stagnation trap is a shift toward

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[34]The subsidy also induces a shift left of the AD curve, because it has an impact on $\bar{i}$.

pessimistic expectations about future growth. Aside form permanent liquidity traps, the model can also generate liquidity traps of arbitrarily long expected duration. We show that large policy interventions to support growth can lead the economy out of a stagnation trap, thus shedding light on the role of growth policies in stimulating aggregate demand and employment.

Appendix

A Proofs

A.1 Proof of proposition 1 (existence and uniqueness of full employment steady state)

We start by proving existence. A steady state is described by the system:

$$g^{\sigma-1} = \frac{\beta(1+i)}{\bar{\pi}} \tag{A.1}$$

$$\frac{g^{\sigma-1}}{\beta} + \frac{\ln g}{\ln \gamma} = \chi \frac{\gamma-1}{\gamma} y + 1 \tag{A.2}$$

$$c = y - \frac{\ln g}{\chi \ln \gamma} \tag{A.3}$$

$$1 + i = \max\left((1+\bar{i})y^{\phi}, 1\right), \tag{A.4}$$

Setting $y = 1$, equation $(A.2)$ implies:

$$\frac{\left(g^f\right)^{\sigma-1}}{\beta} + \frac{\ln g^f}{\ln \gamma} = \chi \frac{\gamma-1}{\gamma} + 1 \tag{A.5}$$

Then equation $(A.1)$ implies:

$$1 + i^f = \frac{\bar{\pi}\left(g^f\right)^{\sigma-1}}{\beta}.$$

Assumption 2 guarantees that $i^f \geq 0$, and that $\bar{i} = i^f$, so that the interest rate rule $(A.4)$ is compatible with the existence of a full employment steady state. We are left to prove that $c^f > 0$. Combining equations $(A.3)$ and $(A.5)$ gives the $c^f > 0$ if:

$$\frac{\chi}{\gamma} + \frac{\left(g^f\right)^{\sigma-1}}{\beta} > 1,$$

which is always the case, since $g^f > 1$ and $\beta < 1$. Hence, the full employment steady state exists.

To prove uniqueness, consider that equation $(A.5)$ implies that there is only one value of g consistent with the full employment steady state, while equation $(A.1)$ establishes that there is a unique value of i consistent with $g = g^f$. Hence, the full employment steady state is unique. ∎

A.2 Proof of proposition 2 (existence and uniqueness of unemployment steady state)

To prove existence, consider that when $i = 0$ equation $(A.1)$ implies:

$$g^u = \left(\frac{\beta}{\bar{\pi}}\right)^{\frac{1}{\sigma-1}} < g^f. \tag{A.6}$$

Then equation $(A.2)$ implies:

$$y^u = \left(\frac{(g^u)^{\sigma-1}}{\beta} + \frac{\ln g^u}{\ln \gamma} - 1\right) \frac{\gamma}{\chi(\gamma-1)}. \tag{A.7}$$

Assumption 3 guarantees that $g^u \geq 1$. Since $1 \leq g^u < g^f$, then equation $(A.7)$ implies $0 < y^u < 1$, while it is easy to check that equation $(A.3)$ implies $c^u > 0$. To see that when $y = y^u$ then the interest rate rule implies $i = 0$, suppose that this is not the case, so that:

$$(1 + \bar{i})(y^u)^\phi > 1.$$

Substituting for $\bar{i}$ using equation (22), we can write the condition above as:

$$\frac{\bar{\pi}}{\beta}\left(g^f\right)^{\sigma-1}(y^u)^\phi > 1.$$

Combining this expression with equations $(A.5)$, $(A.6)$ and $(A.7)$, and rearranging gives:

$$\frac{\chi\frac{\gamma-1}{\gamma} + 1 - \frac{\ln g^f}{\ln \gamma}}{\chi\frac{\gamma-1}{\gamma}y^u + 1 - \frac{\ln g^u}{\ln \gamma}}(y^u)^\phi > 1.$$

Since $y^u < 1$ and, by assumption 2, $\phi > 1$ the condition above cannot hold.[35] Hence, we have found a contradiction and so at the unemployment steady state the interest rate rule implies $i = 0$. This proves the existence of the unemployment steady state.

To prove uniqueness, consider that when $i = 0$, then equation $(A.6)$ pins down a unique value of g^u. Then equation $(A.7)$ implies a unique value for y^u, so that there exists a unique unemployment steady state in which $i = 0$.

We are left to prove that it is not possible to have a steady state with unemployment if $i > 0$. Suppose that this is not the case, and that there exists a steady state with $i = \tilde{i} > 0$ and $y = \tilde{y} < 1$. Then equations $(A.1)$ and $(A.2)$ imply:

$$\tilde{g}^{\sigma-1} = \frac{(1 + \tilde{i})\beta}{\bar{\pi}}$$

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[35]To see this, consider that:

$$\chi\frac{\gamma-1}{\gamma}(y^u)^{1-\phi} \geq \chi\frac{\gamma-1}{\gamma},$$

since $y^u < 1$ and $\phi \geq 1$, and that:

$$\left(1 - \frac{\ln g^u}{\ln \gamma}\right)(y^u)^{-\phi} > \left(1 - \frac{\ln g^f}{\ln \gamma}\right),$$

since $g^u < g^f$.

$$\frac{\tilde{g}^{\sigma-1}}{\beta} + \frac{\ln \tilde{g}}{\ln \gamma} = \chi \frac{\gamma - 1}{\gamma} \tilde{y} + 1.$$

Combining the two equations above and using $1 + \tilde{\tilde{i}} = (1 + \bar{i})\tilde{y}^\phi$ gives:

$$\frac{(1 + \bar{i})}{\bar{\pi}} \tilde{y}^\phi = \chi \frac{\gamma - 1}{\gamma} \tilde{y} + 1 - \frac{\ln \tilde{g}}{\ln \gamma}.$$

Substituting for $\bar{i}$ using equations (22) and $(A.2)$ and rearranging, we can write the equation above as:

$$\chi \frac{\gamma - 1}{\gamma} \left(\tilde{y}^\phi - \tilde{y} \right) + \tilde{y}^\phi \left(1 - \frac{\ln g^f}{\ln \gamma} \right) - \left(1 - \frac{\ln \tilde{g}}{\ln \gamma} \right) = 0.$$

Since $\tilde{y} < 1$ it follows that $\tilde{g} < g^f$. Moreover, by assumption 2, $\phi \geq 1$. Hence, the left hand side of this expression is negative, and we have found a contradiction. This implies that it is not possible to have an unemployment steady state with a positive nominal interest rate. We have thus proved uniqueness of the unemployment steady state. ∎

A.3 Proof of proposition 3 (steady state with subsidy to innovation)

The proof that a full employment steady state exists and is unique follows the steps of the proof to proposition 1.

We now prove that there is no steady state with unemployment. Following the proof to proposition 2, one can check that if another steady state exists, it must be characterized by $i = 0$. Equation $(A.2)$ implies that in this steady state growth must be equal to $(\beta/\bar{\pi})^{(1/(\sigma-1))}$. But with the subsidy in place, the lowest growth rate possible is $\exp(\chi s \ln \gamma)$. Then, condition (31) implies $\exp(\chi s \ln \gamma) > (\beta/\bar{\pi})^{(1/(\sigma-1))}$, so that an unemployment steady state is not possible. ∎

B Model with money

In this appendix we explicitly introduce money in the model. The presence of money rationalizes the zero lower bound constraint on the nominal interest rate, but does not alter the equilibrium conditions of the model.

Following Krugman (1998) and Eggertsson (2008) we assume that households need to hold at least a fraction $\nu > 0$ of production in money balances M:

$$M_t \geq \nu P_t y_t. \tag{B.1}$$

The household's budget constraint is now:

$$\frac{P_t C_t}{Q_t} + \frac{b_{t+1}}{1 + i_t} + M_t = W_t L_t + b_t + M_{t-1} + d_t - T_t, \tag{B.2}$$

where M_t denotes money holdings at time t to be carried over at time $t + 1$, and T are lump-sum taxes paid to the government.

The optimality condition with respect to M_t is:

$$\lambda_t = \beta E_t [\lambda_{t+1}] + \mu_t, \tag{B.3}$$

where $\mu_t > 0$ is the Lagrange multiplier on constraint $(B.1)$. Combining optimality conditions (2) and $(B.3)$ it is easy to see that the presence of money implies $i_t \geq 0$. Intuitively, households will always prefer to hold money, rather than an asset which pays a non-contingent negative nominal return. It is also easy to see that constraint $(B.1)$ binds if $i_t > 0$, but it is slack if $i_t = 0$. Hence, households' money demand is captured by the complementary slackness condition:

$$i_t \left(M_t - \nu P_t y_t \right) = 0, \tag{B.4}$$

with $i_t \geq 0$ and $M_t \geq \nu P_t y_t$.

To close the model, we assume that the government runs a balanced budget:

$$T_t = M_t - M_{t-1},$$

so that seignorage revenue is rebated to households via lump sum taxes.

We can now define an equilibrium as a set of processes $\{y_t, c_t, g_{t+1}, i_t, M_t, P_t\}_{t=0}^{+\infty}$ satisfying equations $(13) - (16)$, $(B.4)$ and $P_t = \bar{\pi} P_{t-1}$, given an initial value P_{-1}.[36]

Notice that to solve for the path of y_t, c_t, g_{t+1} and i_t only equations $(13) - (16)$ are needed. Given values for y_t and P_t, the only use of the money demand equation $(B.4)$ is to define the money supply M_t consistent with the central bank's interest rate rule. Specifically, when $i_t > 0$ the equilibrium money supply is $M_t = \nu P_t y_t$, while when $i_t = 0$ any money supply $M_t \geq \nu P_t y_t$ is consistent with equilibrium.

These results do not rest on the specific source of money demand assumed. For instance, similar results can be derived in a setting in which households derive utility from holding real money balances, as long as a cashless limit is considered (Eggertsson and Woodford, 2003).

C   Model with unemployment risk

In this appendix, we lay down the model with idiosyncratic unemployment risk described in section 3.3. In this model, each household faces in every period a constant probability p of being unemployed. The employment status is revealed to the household at the start of the period. An unemployed household receives an unemployment benefit, such that its income is equal to a fraction $b < 1$ of the income received by employed households. Unemployment benefits are financed with taxes on employed households.

The budget constraint of a household now becomes:

$$c_t + \frac{b_{t+1}}{1 + r_t} = \nu_t W_t L_t + b_t + d_t + T_t.$$

The only change with respect to the benchmark model is the presence of the variables ν and T, which summarize the impact of the employment status on a household's budget. ν is an indicator variable that takes value 1 if the household is employed, and 0 if the household is unemployed. T represents a lump-sum transfer for unemployed households, and a tax for employed households. T

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[36]To derive the law of motion for P_t we have used the equilibrium condition $P_t = \gamma W_t$ and the law of motion for W_t.

is set such that the income of an unemployed household is equal to a fraction b of the income of an employed household.[37]

Moreover, here we assume that households cannot borrow:

$$b_{t+1} \geq 0,$$

and that trade in firms' shares is not possible, so that every household receives the same dividends d.

The Euler equation is now:

$$c_t^{-\sigma} = \beta \frac{1 + i_t}{\bar{\pi}} E_t \left[c_{t+1}^{-\sigma} \right] + \mu_t,$$

where μ is the Lagrange multiplier on the borrowing constraint.

We start by showing that the borrowing constraint binds only for unemployed households. Since neither households nor firms can borrow, in equilibrium every period every household consumes her entire income. Denoting, by c^e and c^u the consumption of respectively an employed and an unemployed household, we have $c_t^u = bc_t^e < c_t^e$. Moreover, due to the assumption of i.i.d. idiosyncratic shocks, $E_t \left[c_{t+1}^{-\sigma} \right]$ is independent of the employment status. Hence, from the Euler equation it follows that $\mu > 0$ only for the unemployed, and so the borrowing constraint does not bind for employed households.

The Euler equation of the employed households is:

$$(c_t^e)^{\sigma} = \frac{\bar{\pi} g_{t+1}^{\sigma-1}}{\beta(1 + i_t)\rho E_t \left[\left(c_{t+1}^e \right)^{-\sigma} \right]},$$

where $\rho \equiv 1 - p + p/b^{\sigma} > 1$, and we have used the fact that the probability of becoming unemployed is independent of aggregate shocks. Moreover, using $c_t = pc_t^u + (1 - p)c_t^e = c_t^e(bp + 1 - p)$, we can derive the Euler equation:

$$c_t^{\sigma} = \frac{\bar{\pi} g_{t+1}^{\sigma-1}}{\beta(1 + i_t)\rho E_t \left[c_{t+1}^{-\sigma} \right]},$$

which determines the demand for consumption in the model with idiosyncratic unemployment risk.

D   The cases of $\sigma = 1$ and $\sigma < 1$

In the main text we have focused attention on the empirically relevant case of low elasticity of intertemporal substitution, by assuming that $\sigma > 1$. In this appendix we consider the alternative

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[37]More precisely, an unemployed household receives a transfer:

$$T = \frac{bW_t L_t + (b - 1)d_t}{1 + \frac{bp}{1-p}},$$

while an employed household pays a tax

$$T = -\frac{p}{1 - p} \frac{bW_t L_t + (b - 1)d_t}{1 + \frac{bp}{1-p}}.$$

cases $\sigma = 1$ and $\sigma < 1$. The key result is that, in general, under these cases the steady state is unique.

We start by analyzing the case of $\sigma = 1$. In steady state, equation (17) can be written as:

$$1 = \frac{\beta(1+i)}{\bar{\pi}}.$$

Intuitively, under this case changes in the growth rate of the economy have no impact on the equilibrium nominal interest rate. Hence, if there exists a full employment equilibrium featuring a positive nominal interest rate, it is easy to check that no unemployment equilibrium can exist.

We now turn to the case $\sigma < 1$. Under this case, equation (17) implies a negative relationship between growth and the nominal interest rate. Supposing that a full employment equilibrium featuring a positive nominal interest rate exists, if a liquidity trap equilibrium exists, it must feature a higher growth rate than the full employment one, i.e. $g^u > g^f$. Since $y^f = 1$, it must be the case that $y^u \leq y^f$. Now suppose that the left-hand side of equation (18) is increasing in g. Then we cannot have a steady state in which $g^u > g^f$ and $y^u \leq y^f$, so that an unemployment steady state cannot exist. The left-hand side of equation (18) is increasing in g if:

$$g > \left(\frac{(1-\sigma)\ln\gamma}{\beta} \right)^{\frac{1}{1-\sigma}}.$$

For plausible values of γ and β we have $\ln\gamma < \beta$, implying that the inequality above always hold and that there is a unique steady state characterized by full employment. If the inequality above does not hold there might be multiple steady states, and the unemployment steady state will be characterized by faster growth than the full employment one.

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