Bank Capital and Credit Cycles

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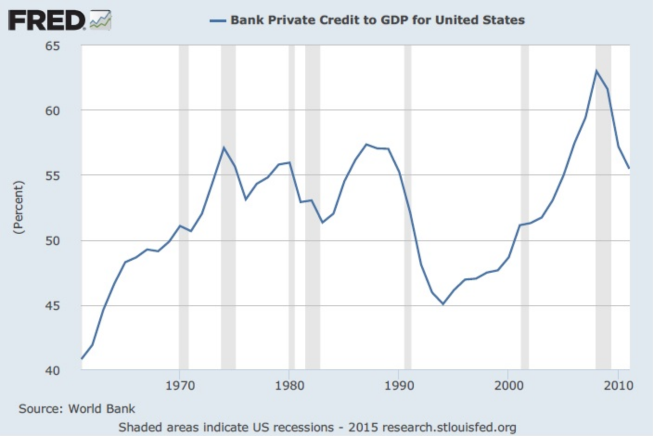
January 2015

Motivation

- Policymakers need macro-models that are specifically tailored to financial stability analysis (in the same way DSGE models were designed for guiding monetary policy).
- Necessary to guide macro-prudential policy decisions.
- For example, counter-cyclical banking regulations are implemented in some countries, without a clear understanding of the macroeconomic impact of bank capital requirements.
- In fact, very little is known about the determinants of aggregate bank lending, and the mechanism behind credit cycles.

Credit Cycles: Evidence



Schularik-Taylor (2012) data set: credit booms and leverage cycles in 12 countries over 1870-2008.

CREDIT CYCLES: THEORIES

- Fluctuations of collateral prices for constrained borrowers *(Bernanke-Gertler (1989,1990), Kiyotaki-Moore(1997))*
- Loans officers relax credit standards when things go well *(Ruckes 2004)*
- Strategic complementarities in risk taking between banks due to anticipated bail outs *(Farhi-Tirole 2012)*, relative performance evaluation of bank managers *(e.g., Rajan 1994, Aitken et al. 2012)*
- Pecuniary externalities *(Lorenzoni (2008), Bianchi (2011), Jeanne-Korinek (2011), Gersbach and Rochet (2013, 2014))*

OUR CONTRIBUTION

- General equilibrium model with financial frictions in the spirit of Brunnermeier & Sannikov (2014).
- Simpler than BS model and allows for closed-form solutions.
- Banks are explicitly modeled and bank capital dynamics plays a crucial role.
- Model lends itself to simple comparative statics and welfare analysis.

Related literature

1. **Bank capital channel:**
   - Meh & Moran (2010)
   - Blum & Hellwig (1995), Van den Heuvel (2008), Nguyen (2013)

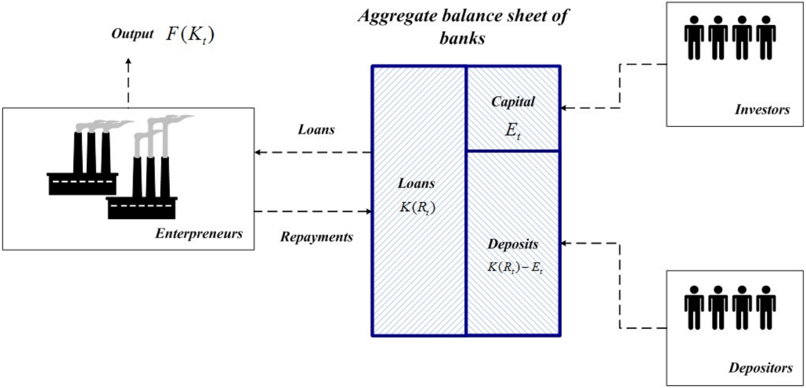
2. **Macro-finance in a continuous-time framework:**
   - Brunnermeier & Sannikov (2014), He & Krishnamurthy (2012, 2013, 2014), Adrian & Boyarchenko (2014)

3. **Corporate cash management**
   - Decamps et al. (2011) Isohätälä, Milne & Robertson (2014), Barth & Moreno (2014), Bolton et al (2011)

THE MODEL

- Dynamic general equilibrium model with a banking sector.
- One physical good, can be consumed or invested. Complete depreciation of capital at each period.
- Firms' investments can only be financed by bank loans (investment = credit K_t).
- Three types of agents: entrepreneurs and investors (both risk neutral) and depositors (infinitely risk-averse).
- Banks are financed by risky equity and riskless deposits.
- Depositors are less impatient (discount factor r) than investors (discount factor $\rho > r$).
- We focus on the case $r = 0$.

Global Picture



ENTREPRENEURS

- have individual projects of duration Δt.
- if successful: return $1 + x\Delta t$, repay $1 + R\Delta t$.
- x is heterogenous (density $g(x)$) and private information.
- if not successful: default (limited liability) repay nothing.
- entrepreneurs have no funds: borrow iff $x > R$. Demand for credit

$$K(R) = \int_R^\infty g(x)dx$$

- ϵ_t: aggregate shock ± 1 with probability $\frac{1}{2}$.
- probability of default: $p_t = p\Delta t + \sigma_0\sqrt{t}\epsilon_t$.
- return on loans for banks (continuous time limit):

$$(R_t - p)dt - \sigma_0 dZ_t,$$

where Z_t is Brownian motion. First best allocation $R_t \equiv p$.

Banks

- k_t denotes lending volume of a bank (aggregate K_t) at time t
- Equity of the bank follows:

$$de_t = k_t[(R_t - p)dt \quad - \underbrace{\sigma_0 dZ_t}_{\text{shocks}}] - \underbrace{d\delta_t}_{\text{dividends}} + \underbrace{di_t}_{\text{recapitalizations}}$$

- Recapitalizations involve proportional costs γ (MAIN FRICTION: if $\gamma = 0$ equilibrium: first best).
- Focus on Markov equilibria: loan rate R_t follows:

$$dR_t = \mu(R_t)dt + \sigma(R_t)dZ_t,$$

where $\mu(R_t)$ and $\sigma(R_t)$ are to be determined.

Roadmap

1. Optimal decisions of individual banks.
2. Equilibrium.
3. Credit cycles and financial (in)stability.
4. Welfare analysis.

The optimal decisions of a bank

Maximization problem of an individual bank:

$$v(e_0, R_0) = \max_{k_t, d\delta_t, di_t} \underbrace{\mathbb{E}\left[\int_0^{+\infty} e^{-\rho t} d\delta_t\right]}_{\text{dividends}} - \underbrace{(1+\gamma)\mathbb{E}\left[\int_0^{+\infty} e^{-\rho t} di_t\right]}_{\text{recapitalizations}}$$

$$de_t = k_t[(R_t - p)dt - \sigma_0 dZ_t] - d\delta_t + di_t$$

$$dR_t = \mu(R_t)dt + \sigma(R_t)dZ_t$$

The optimal decisions of a bank

- Value function of the bank satisfies:

$$\rho v = \max_{k, d\delta, di} \Big\{ d\delta(1 - v_e) - di(1 + \gamma - v_e) + \\ + k\left[(R - p)v_e + \frac{k\sigma_0^2}{2} v_{ee} - \sigma_0 \sigma(R) v_{eR}\right] + \\ + \mu(R) v_R + \frac{\sigma^2(R)}{2} v_{RR} \Big\}$$

- Homogeneity of $v(e, R)$ in e implies:

$$v(e, R) = eu(R),$$

where $u(R)$: market-to-book ratio (same for all banks).

THE OPTIMAL DECISIONS OF A BANK

Homogeneity of $v(e, R)$ leads to:

$$\rho u(R) = \max_{k, d\delta, di} \Big\{ \frac{d\delta}{e}[1 - u(R)] - \frac{di}{e}[1 + \gamma - u(R)] + \\ + \frac{k(e,R)}{e}[(R - p)u(R) - \sigma_0 \sigma(R) u'(R)] + \\ + \mu(R) u'(R) + \frac{\sigma^2(R)}{2} u''(R) \Big\}$$

The optimal decisions of a bank

- FOC for k implies:

$$\frac{u'(R)}{u(R)} = \frac{R - p}{\sigma_0 \sigma(R)},$$

- FOC for $d\delta$ implies that dividends are paid whenever R_t reaches barrier R_{min} such that

$$u(R_{min}) = 1$$

- Similarly, FOC for di implies a recapitalization barrier R_{max} such that

$$u(R_{max}) = 1 + \gamma$$

THE OPTIMAL DECISIONS OF A BANK

Proposition 1

Let the loan rate process R_t be defined by $dR_t = \mu(R_t)dt + \sigma(R_t)dZ_t$ on $[R_{min}, R_{max}]$ (reflected at both ends). Decision problem of individual banks has a solution iff market-to-book value

$$u(R) = \int_{R_{min}}^{R} \frac{(s-p)}{\sigma_0 \sigma(s)} ds$$

satisfies $u(R_{max}) = 1 + \gamma$ and

$$\rho u(R) = \mu(R)u'(R) + \frac{\sigma^2(R)}{2}u''(R).$$

This determines $\mu(R)$:

$$\mu(R) = \frac{\sigma(R)}{2}\left[\sigma'(R) - \frac{\sigma(R) - 2\rho\sigma_0}{R-p} - \frac{R-p}{\sigma_0}\right]$$

Competitive equilibrium

- In the region $R \in (R_{min}, R_{max})$, **aggregate bank capital** evolves according to

$$dE_t = K(R_t)[(R_t - p)dt - \sigma_0 dZ_t]$$

- **Equilibrium** is characterized by two processes : loan rate R_t and aggregate bank capital E_t such that lending market clears.

- In a Markov equilibrium, one must have $E_t = E(R_t)$.

Competitive equilibrium

- By Itô's lemma, we have:

$$dE_t = \left(\mu(R_t)E'(R_t) + \frac{\sigma^2(R_t)}{2} E''(R_t) \right) dt + \sigma(R_t)E'(R_t)dZ_t$$

- Matching the drift and volatility terms of E_t yields:

$$(R - p)K(R) = \mu(R)E'(R) + \frac{\sigma^2(R)}{2} E''(R),$$

$$-\sigma_0 K(R) = \sigma(R)E'(R)$$

- We obtain a second relation between μ and σ:

$$\mu(R) = \frac{\sigma(R)}{2} \left[\sigma'(R) - \frac{\sigma(R)K'(r)}{K(R)} - \frac{2(R-p)}{\sigma_0} \right]$$

Competitive equilibrium

Proposition 2

There exists a unique Markov competitive equilibrium characterized by

$$\sigma(R) = \frac{(R-p)^2 + 2\rho\sigma_0^2}{\sigma_0\Big[1 + (R-p)\Big(-\frac{K'(R)}{K(R)}\Big)\Big]}$$

$R_{min} = p$, and $log(1+\gamma) = \int_p^{R_{max}} \frac{(R-p)}{\sigma_0\sigma(R)} dR.$

Aggregate bank capital satisfies

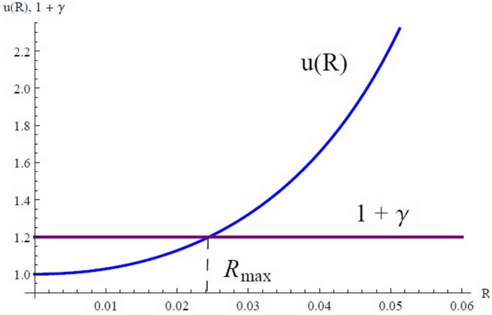
$$E'(R) = -\frac{\sigma_0 K(R)}{\sigma(R)} < 0,$$

$$E(R_{max}) = 0.$$

COMPETITIVE EQUILIBRIUM

- R_{max} is determined by:

$$u(R_{max}) = \exp\left(\int_p^{R_{max}} \frac{(s-p)}{\sigma_0\sigma(s)} ds\right) = 1 + \gamma$$



General properties of equilibrium

- γ only impacts R_{max} but not μ, σ, u.
- $\sigma(p) = 2\rho\sigma_0 > 0$. Moreover $\sigma'(p) < 0$.
- $\mu(p) = 0$. Moreover $\mu'(p) < 0 \Leftrightarrow K'' < 0$.
- For most specifications, the drift μ is very small compared with the volatility σ.
- When γ is small, σ is almost constant and μ is almost zero on $[p, R_{max}]$. Thus R_t behaves like a Brownian motion without drift that is reflected at both ends of an interval.

Capital transmission channel

Bank capital transmission channel:

- An adverse shock depletes bank capital.
- In order to reduce the probability of costly recapitalizations and to avoid delays in dividend payments, banks reduce their risk exposure, by cutting down credit supply.
- Credit supply $\downarrow \quad \Rightarrow \quad R_t \quad \uparrow \quad \Rightarrow \quad K(R_t) \quad \downarrow \quad \Rightarrow$ output $\downarrow$
- When γ is high, banks cannot quickly rebuild capital and the economy slides into a long phase of credit crunch.

NUMERICAL ILLUSTRATION

- Consider the following specification of loan demand:

$$K(R) = (\overline{R} - R)^{\beta} \qquad \text{where} \qquad \beta > 0, \quad \overline{R} > p$$

- We have

$$\frac{-K'(R)}{K(R)} = -\frac{\beta}{\overline{R} - R}$$

- Volatility of the loan rate:

$$\boldsymbol{\sigma(R)} = \frac{(\overline{R} - R)\left(2\rho\sigma^2 + (R-p)^2\right)}{\sigma_0(\overline{R} + (\beta - 1)R - \beta p)}$$

- Drift of the loan rate:

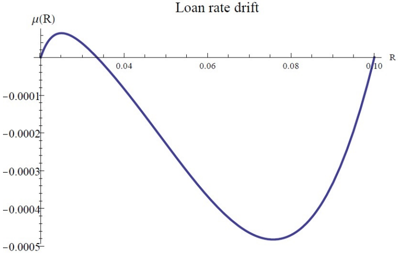
$$\boldsymbol{\mu(R)} = \sigma(R)\frac{\beta(R-p)\left[(1-\beta)((R-p)^2 - 2\rho\sigma_0^2) - 2(R-p)(\overline{R}-p)\right]}{2\sigma_0(\overline{R} + (\beta-1)R - \beta p)^2}$$

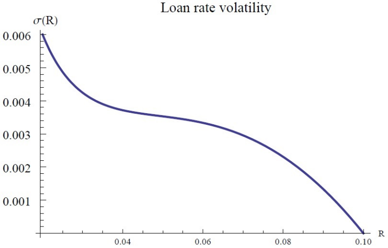
NUMERICAL ILLUSTRATION

- When $\beta < 1$, $\mu(R) < 0$ for all $R \in (p, R_{max})$.
- When $\beta > 1$, $\mu(R)$ is first positive then negative. It vanishes for $R = R^*$, the positive root of

$$Q(R) = (1-\beta)((R-p)^2 - 2\rho\sigma_0^2) - 2(R-p)(\overline{R}-p)$$

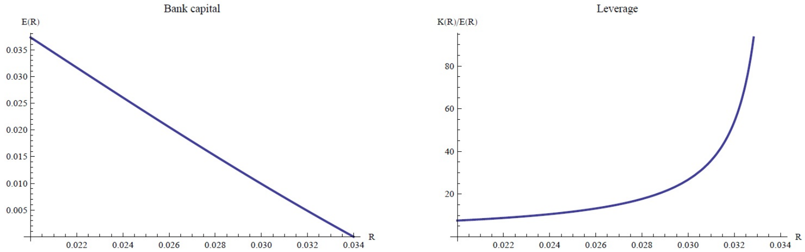
NUMERICAL ILLUSTRATION





Remark: $\mu(R)$ is typically very small w.r.t. $\sigma(R)$

NUMERICAL ILLUSTRATION



Aggregate bank capital and leverage
($\rho = 0.05$, $p = 0.02$, $\overline{R} = 0.1$, $\sigma_0 = 0.1$, $\beta = 2$, $\gamma = 0.1$)

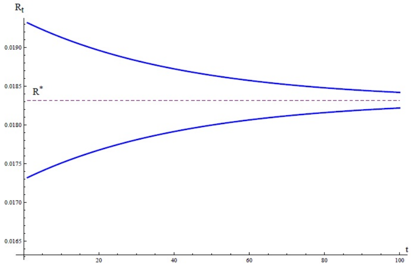
Impulse response analysis

- classical methodology in DSGE models: start at the deterministic steady state $R = p$
- assume $dZ_0 > 0$ but $dZ_t \equiv 0$ for $\forall t > 0$.
- $dZ_0 > 0$ induces a drop in bank capital $\Rightarrow$ loan rate rises to R_0. Then R_t evolves according to ODE:

$$dR_t = \mu(R_t)dt$$

- If $\beta < 1$, $\mu < 0$ and DSS $R = p$ is "stable".
- If $\beta > 1$, μ is first positive then negative. There are two DSS: $R = p$ (instable) and $R = R^*$ (locally stable).
- However we have seen that μ is typically very small w.r.t. σ. Thus this impulse response analysis can be very misleading!

Impulse response analysis



Dynamics of loan rate after a single unexpected shock
($\rho = 0.05$, $p = 0$, $\overline{R} = 0.1$, $\sigma_0 = 0.2$, $\beta = 2$)

STATIONARY DISTRIBUTION

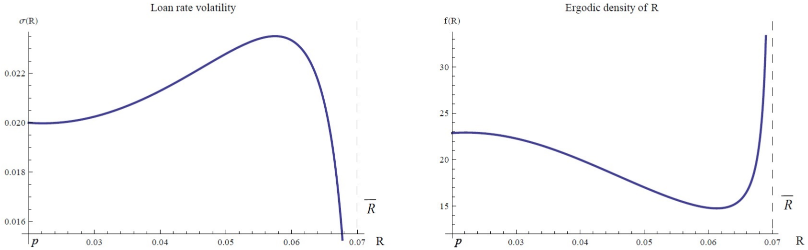
Proposition 3

- There is a unique stationary distribution, which is ergodic.
- Its density f solves the ODE

$$\frac{f'(R)}{f(R)} = \frac{2\mu(R)}{\sigma^2(R)} - \frac{2\sigma'(R)}{\sigma(R)}$$

- γ only impacts the support of the ergodic distribution.

STATIONARY DISTRIBUTION



Remark: the economy spends most of the time at the states with the lowest endogenous volatility

Welfare analysis

- Take aggregate bank capital E as the state variable
- Assume that loan demand is linear ($\beta = 1$):

$$K(E) = \overline{R} - R(E)$$

- Aggregate welfare in the economy:

$$W(E) = \underbrace{\mathbb{E}\left[\int_0^{+\infty} e^{-\rho t}\Pi(E_t)dt\right]}_{\text{Firms' value}} + \underbrace{\mathbb{E}\left[\int_0^{+\infty} e^{-\rho t}(d\Delta_t - (1+\gamma)dI_t)\right]}_{\text{Banks' value}},$$

where firms' expected profits at time t are given by:

$$\Pi(E_t) = F(K(E_t)) - K(E_t)F'(K(E_t)) = \frac{[K(E_t)]^2}{2}$$

WELFARE ANALYSIS

- Equity value at the competitive equilibrium satisfies:

$$E'(R) = -\frac{\sigma_0 K(R)}{\sigma(R)} = -\frac{(\overline{R}-p)\sigma_0^2}{(R-p)^2 + 2\rho\sigma_0^2},$$
$$E(R_{max}) = 0.$$

- Then,

$$E(R) = -\frac{1}{\sqrt{2\rho}}(\overline{R}-p)\sigma_0 \arctan\left(\frac{R-p}{\sqrt{2\rho}\sigma_0}\right) + E_0$$

- Credit volume at the competitive equilibrium:

$$K(E) = \overline{R} - R(E) = \overline{R} - p - \sqrt{2\rho}\sigma_0 \tan\left(\frac{\sqrt{2\rho}}{(\overline{R}-p)\sigma_0}(E_0 - E)\right)$$

Welfare analysis

- Welfare function satisfies:

$$\rho W(E) = \frac{K^2(E)}{2} + K(E)(\overline{R} - p - K(E))W'(E) + \frac{\sigma_0^2}{2}K^2(E)W''(E),$$

where $K(E)$ is credit volume at the competitive equilibrium

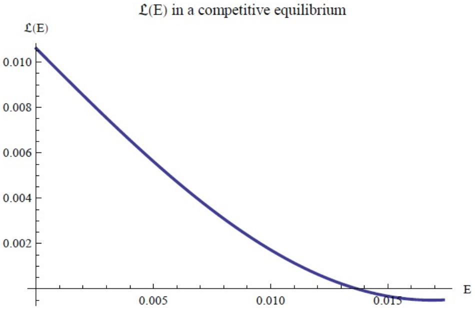
- Boundary conditions:

$$W'(E_{max}) = 1$$
$$W'(0) = 1 + \gamma$$

- Differentiating this equation with respect to $K(E)$ yields:

$$\mathcal{L}(E) := K(E)[1 - 2W'(E) + \sigma_0^2 W''(E)] + (\overline{R} - p)W'(E)$$

Welfare analysis



Remarks:

- $\mathcal{L}(E) > 0$: social welfare can be increased by expanding lending
- $\mathcal{L}(E) < 0$: social welfare can be increased by reducing lending

Welfare analysis

- Private cost of lending:

$$R_c(E) - p = \sigma_0^2 K_c(E)\Big[-\frac{\hat{u}'(E)}{\hat{u}(E)}\Big],$$

where $\hat{u}(E) \equiv u[R(E)]$.

- Social costs of lending:

$$R(E) - p = \sigma_0^2 K_c(E)\Big[-\frac{W''(E)}{W'(E)}\Big] + \underbrace{R'(K_c(E))K_c(E)\Big[\frac{1}{W'(E)} - 1\Big]}_{>0, \text{ since } R'(K)<0 \text{ and } W'(E)>0}$$

Conclusion

- Simple dynamic macro model where bank capital impacts credit volume.
- Closed form solutions.
- Asymptotic behavior given by ergodic distribution (also explicit).
- Impact of bank crises can be fully analyzed without having to linearize around the DSS.
- Financing frictions + elastic loan demand may give rise to credit cycles
- Competitive equilibrium leads to too much lending when things go well, and too little when they go badly.
- Model can be extended in many directions.

Thank you!