BANK CREDIT AND UNEMPLOYMENT*

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Abstract

The worst employment slumps tend to follow the largest expansions of household debt. In this paper, we theoretically investigate an amplification mechanism by which debt on household balance sheets distorts labor market search behavior, leading to deeper employment slumps. Using a competitive search model, we find that levered households protected by limited liability engage in risk-shifting by searching for jobs with high wages but low employment probabilities. In general equilibrium, firms respond to this household preference distortion and post high wages but few vacancies. This *vacancy-posting effect* implies that high household debt leads to high unemployment, a result that casts light on why labor market recoveries are sluggish after financial crises. The equilibrium level of household debt is inefficiently high due to a *household-debt externality*—banks fail to internalize the effect that household leverage has on employment via the vacancy-posting effect. We analyze the role that a financial regulator can play in mitigating this externality. We find that loan-to-value caps for households and capital requirements for banks can elevate employment and improve efficiency, providing an alternative to monetary policy for labor market intervention.

Keywords: Household debt, employment, banking, financial regulation

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1 Introduction

The worst employment slumps tend to follow the largest expansions of household debt. For example, in the US, household debt as a percentage of GDP climbed from below fifty percent in 1980 to almost one hundred percent by 2006, and the Great Recession that accompanied the 2007–09 subprime crisis saw the US economy shed over eight million jobs. Ng and Wright (2013) note that the increase in leverage prior to the Great Recession was more pronounced for households than for firms. And Mian and Sufi (2014b) document that counties in the US that had the most highly levered households had the sharpest drops in employment. Furthermore, despite the economic recovery in terms of GDP growth, the labor market has lagged behind (see Daly and Hobijn (2015)). The connection between increases in household debt and spikes in unemployment followed by slow recovery in the labor market is *not* a unique feature of the last recession—it is rather commonplace in recessions associated with financial crises.[1] This raises the first set of questions we address in this paper: why do high levels of household debt amplify employment slumps and why does the labor market recover so slowly after financial crises?

Almost all household debt is created by consumer loans made by financial institutions. So the lending decisions of these institutions influence aggregate household debt, and those lending decisions, in turn, are affected by financial regulation. This raises the main policy question that we address in this paper: do financial regulatory policies have important effects on the labor market?

The central mechanism at work in our model relies on a two-way bridge between the labor and credit markets. Given that households are protected by limited liability, debt on their balance sheets distorts their preferences when they search for jobs. This distortion—analogous to the risk-shifting of levered firms in corporate finance—induces households to search inefficiently for high-wage jobs, even if the probability of finding employment is relatively low. Competitive firms respond to households' behavior by posting higher wages, but, as a result, they can only afford to post fewer job vacancies. This causes unemployment to go up. In other words, the debt-induced distortion of labor market search may be an important channel by which high levels of household debt amplify employment slumps. The higher unemployment resulting from household debt elevates default rates, but individual households and banks fail to internalize this negative labor-market-driven externality of increasing consumer credit, leading to excessive lending to households. We show that a central bank, in its role as a prudential regulator, can diminish this inefficiency and boost employment with a combination of capital requirements on banks and caps on household leverage.

[1] See, for example, Reinhart and Rogoff (2009) and Schularick and Taylor (2012).

We develop a two-date general equilibrium model of household borrowing and the labor market. Households live for two dates. At the early date, they borrow in a competitive credit market. At the late date, firms post wages competitively and households find jobs in a directed search market.[2] Employed households repay their debts, but unemployed households cannot repay their debts and so they default. Critically, households are protected by limited liability.

Our first main result is that increasing household debt raises the equilibrium unemployment rate. If a levered household finds a job, it must pay a large fraction of its wages to its creditors. On the other hand, if a household is unemployed, it is likely to default, in which case it is it is protected by limited liability. Thus, a levered household is relatively insensitive to the downside risk associated with unemployment. Like levered firms that shift risk to choose inefficiently risky projects in corporate finance, households in our model shift risk by searching for jobs with high wages, even if the associated probability of becoming employed is low. When competitive firms post vacancies, they recognize households' preferences, and that levered households will look for relatively high-paying jobs. Hence, an increase in household debt results in firms posting higher wages in order to attract workers. However, because they pay higher wages, firms must reduce the number of vacancies they post to maintain positive profits. This leads to a decrease in employment. We refer to this as the *vacancy-posting effect* of household debt.

The vacancy-posting effect suggests a connection between household debt and equilibrium wages. To analyze the implications of this connection over the business cycle, we include aggregate productivity shocks in an extension (Subsection 9.1). We demonstrate that wages are procyclical, consistent with the data. Further, our findings suggest that household debt may be a source of (well-documented) wage rigidity (Bewley (1999)). Note that other work on the connection between household debt and the labor market has assumed wage rigidity exogenously to generate employment fluctuations, whereas wage rigidity arises *endogenously* in our model. We also study an extension in which household debt is collateralized, so that when collateral values are high, unemployed households can liquidate their collateral to repay their debts. We find that low collateral prices exacerbate the vacancy-posting effect, consistent with the large employment slump that followed the fall in house prices in the Great Recession. This extension also suggests that wages should be rigid mainly when collateral prices are low, consistent with evidence that wages are especially slow to adjust downward.

Our second main result is that the level of household debt is *inefficiently high* in equilibrium. The result follows from the fact that when a bank extends credit to a household, it affects the likelihood that *other* households are employed, as implied

[2] In Subsection 8.2, we demonstrate that our results are not sensitive to the structure of the labor market.

by the vacancy-posting effect. Thus, since unemployed households do not repay their debts, the extension of bank credit to a household increases the probability that other households default on their bank loans. The interest rate a bank charges a borrower does not reflect this negative effect it has on other agents in the economy. In other words, there is a *household debt externality*, which operates through the vacancy posting effect. Since banks and households fail to internalize the negative effects of their debt on the labor market, the level of debt in the economy is higher than the socially-optimal level. Thus, there is scope for a financial regulator to intervene. It can cap debt at the socially efficient level to restore efficiency.[3]

To explore the connection between financial crises and labor market slumps associated with excessive household leverage, we expand the model to include an explicit role for a financial sector. To do this, we add asymmetric information, the most ubiquitous friction in the credit market. Specifically, we introduce heterogeneity in households, so that there are good households that want to work at the late date and bad households that never work. Banks have a screening technology that allows them to screen out bad households at a cost.

We demonstrate that better-capitalized banks screen households more intensely. The reason is that bank creditors bear the downside of banks making loans to bad households, so more leverage leads to a more severe agency problem and less screening.[4] More surprisingly, despite the agency frictions resulting from bank leverage, banks choose to raise all new capital via debt. This happens despite the fact that the environment lacks the usual reasons why banks favor high leverage such as taxes, deposit insurance, government safety nets and the like. In our model, high leverage serves as a commitment device for banks to *not* screen loans too intensely. Banks that are somewhat lax in screening are attractive to households who then face a lower risk of being screened out. In other words, banks use leverage to attract borrowers who do not want to be denied credit.

Our main result of this section is that a poorly capitalized financial sector amplifies the household debt externality and the vacancy posting effect. The reason is that decreasing bank equity decreases their screening. This implies that banks make more loans to bad households. Loans are thus riskier and interest rates are higher. As a result, there is more debt on household balance sheets; this exacerbates the household debt externality, moving the economy away from the socially efficient allocation.

From a policy point of view, this result implies that a financial regulator must be concerned not only with financial stability but also with stimulating employment,

[3]Such caps have been implemented in several countries in recent years, see, for example, Borio and Shim (2007), Crowe, Dell'Ariccia, Igan, and Rabanal (2011) and Ono, Uchida, Udell, and Uesugi (2014).

[4]There is empirical evidence that banks with higher capital levels screen more intensely. See Purnanandam (2011), for example.

an objective typically reserved for monetary policy. Our results suggest that capital requirements for banks have important implications for the labor market: increasing bank capital mitigates the household debt externality and the vacancy-posting effect, leading to higher employment in equilibrium.[5]

The rest of the Introduction includes a discussion of the realism of our assumptions and the consistency of our main predictions with empirical evidence; it also contains a review of the related literature. In Section 2 we develop the baseline model and in Section 3 we solve it. Here we show the vacancy-posting effect. In Section 4, we demonstrate that the equilibrium level of household debt exceeds the socially efficient level due to the household debt externality. In Section 5, we extend the model to include asymmetric information and bank screening. In Section 6 we find the equilibrium of the extended model and in Section 7 we demonstrate that a well-capitalized banking sector attenuates the vacancy-posting effect. In Section 8, we show that our results are robust to relaxing some of the main assumptions we make in the baseline model. In particular, (i) we generalize the form of households' preferences, studying risk-averse households, (ii) we consider a labor market characterized by random matching instead of directed search, (iii) we demonstrate that households prefer to borrow from banks than from investors directly, and (iv) we discuss the role of employment at the early date, which we do not include in the baseline model. In Section 9, we analyze the following three extensions to generate further results: (i) the addition of shocks to aggregate output and wage fluctuations, (ii) the addition of collateral securing household debt, and (iii) the addition of non-zero penalties for households default. Section 10 concludes. The Appendix contains all formal proofs and several omitted derivations as well as a glossary of symbols.

1.1 Realism of Assumptions and Predictions

The mechanism behind the vacancy posting effect relies on the following four ingredients: (i) households default when they are unemployed, (ii) households are protected by limited liability, (iii) unemployed households take into account their limited liability protection when searching for jobs, and (iv) firms internalize this household preference distortion when posting vacancies. Each of these ingredients has established empirical support in the literature, some of which we list here. (i) Geradi, Herkenoff, Ohanian, and Willen (2013) find that individual unemployment is the strongest predictor of default. Similarly, Herkenhoff (2012) finds that unemployment (and not negative equity) is the primary reason for household default; thus, households default mainly when they

[5]Note that we focus only on the effect of bank capital on screening loans and we abstract from other effects that bank capital requirements may have. For discussion and analysis of these other effects, see Opp, Opp, and Harris (2014) and Thakor (2014).

fail to find employment. (ii) Household limited liability in the event of default is salient in the US, where debtors can dissolve debt obligations by filing for personal bankruptcy (see Dobbie and Goldsmith-Pinkham (2015) and Mahoney (2015), for example). However, even in countries like Spain in which personal debt cannot be dissolved that way, the punishment for defaulting debtors is limited. (iii) Mahoney (2015) establishes that households do indeed take into account limited liability; they use the protection afforded by it as informal insurance. Further, Melzer (forthcoming) demonstrates that limited liability in the form of asset exemptions in mortgage default leads to distortions in households' investment decisions. Households with negative equity cut back substantially on home improvements, but continue to invest in durable assets that can be retained in the event of default. (iv) Work on the effects of unemployment provides evidence that firms do respond to distortions in households' labor market search behavior. Notably, Hagedorn, Manovskii, and Mitman (2015) exploit variation in unemployment insurance policies across US states to show that increasing unemployment insurance causes firms to post fewer vacancies. They estimate that cuts to unemployment insurance created about 1.8 million jobs in the US in 2014 due to increased job creation by firms. In our model, unemployment insurance has the same distortionary effect on household labor market search behavior as household debt does. This is because household debt is effectively a "tax" for finding employment—households repay their debts out of their wages—whereas unemployment insurance is a subsidy for *not* finding employment. The labor market distortions resulting from household leverage are likely to be even more important than those resulting from employment insurance. This is because personal bankruptcy results in more effective transfers than all state unemployment insurance programs combined (Lefgren, McIntyre, and Miller (2010)). Moreover, household limited liability is not limited to debt that is discharged in bankruptcy; in fact, bankruptcies constitute only about one-sixth of household defaults (Herkenhoff (2012)).

In addition to making reasonable assumptions about behavior at the microeconomic level, our model's predictions are consistent with empirical evidence about individual household behavior. In particular, Herkenhoff (2012) finds a spike in the employment rate of households when their debt expires, suggesting that when households discharge their debt, it mitigates the distortion of their labor market search, increasing the employment rate. Our results are also in line with the findings of Dobbie and Goldsmith-Pinkham (2015), who find that limited recourse for mortgage debt—i.e. household limited liability—leads to a decrease in the employment rate.[6]

[6]This paper finds seemingly contradictory evidence for homestead exemptions—it finds that these lead to increases in the employment rate. We think this may be because homestead exemptions, which essentially protect home equity from credit card and auto loans, are likely to cause households to discharge their debt sooner, thereby reducing household leverage and mitigating the vacancy-posting effect. This contrasts with

Brown and Matsa (forthcoming) find that an increase in an employer's distress results in fewer applicants, but applicants with more unemployment insurance are less sensitive to employment risk (i.e., to firm distress). Given that, in our model, an increase in unemployment insurance has the same effect as an increase in limited liability, this finding provides further support for our household risk-shifting channel: households with higher unemployment insurance look for relatively riskier jobs.

Note, finally, that our model captures the following stylized facts at the macroeconomic level: (i) high household leverage causes severe employment slumps (Mian and Sufi (2010), Mian and Sufi (2014b)), (ii) unemployment is persistant (Hall (1975), Hagedorn, Manovskii, and Mitman (2015)), (iii) a weak financial sector can exacerbate labor market slumps (Reinhart and Rogoff (2009)), (iv) wages are rigid, especially downwardly[7] (Bewley (1999), Daly and Hobijn (2015)), (v) negative shocks to household collateral values (house prices) contribute to labor market slumps (Mian and Sufi (2014b)), and (vi) lax bank screening contributed to the last financial crisis (e.g. Keys, Mukherjee, Seru, and Vig (2010), Purnanandam (2011), and Thakor (forthcoming)).

One salient feature of the Great Recession emphasized by Mian and Sufi (2014b) is that the employment slump was concentrated in the non-tradable sector. In our baseline model, we do not include sectoral differences, and thus we do not address this fact explicitly. However, given that labor is not perfectly substitutable or perfectly mobile across sectors, the vacancy-posting effect would amplify any sector specific shocks. In other words, if a shock to aggregate demand, for example, causes a decline in employment in the non-tradable sector, our findings suggest that high levels of household debt will make this sectoral slump deeper and more persistent in that sector. Further, there is another reason that we expect our mechanism to amplify shocks to the non-tradable sector more than shocks to the tradable sector. Since our mechanism is driven by distortions to job seekers' search behavior, the vacancy-posting effect should be strongest in labor markets with high turnover and flexible contracts. Thus, our results should be stronger in service industries (non-tradable) than in manufacturing industries (tradable).

1.2 Related Literature

Our focus on the distortionary effect of household leverage on labor market search contrasts with previous research on the connection between household debt and unemployment, which has mainly focused on the transmission of credit market shocks to the labor

mortgage default, which is likely be be delayed, because it is typically associated with deadweight losses, perhaps due to foreclosure, relationship-specific investment, costs of relocation, or other personal difficulties.

[7]Note that in our model the wages of new hires are rigid. While some work has suggested that wages for new hires are relatively flexible (e.g. Pissarides (2009)), Gertler, Huckfeldt, and Trigari (2014) suggest that these findings are mainly due to compositional effects, and that the wages of new hires are indeed rigid.

market via the aggregate demand channel (see, e.g., Eggertsson and Krugman (2012), Guerrieri and Lorenzoni (2015), Mian and Sufi (2011, 2014b), Midrigan and Philippon (2011), and Mishkin (1978, 1978)). They key mechanism connecting household credit and labor markets in these papers is as follows. When credit tightens for households (for example due to falling house prices), highly levered households reduce their demand for goods, leading to falling prices for goods, which causes firms to cut production and hire fewer workers, increasing unemployment. This literature offers a compelling explanation of the connection between the expansion of household debt and job losses, which finds empirical support in Mian and Sufi's (2014b) analysis of job losses in the tradable and non-tradable sectors discussed above. We suggest a complementary mechanism that amplifies this aggregate demand channel, and we also explain why the labor market is slow to recover compared to the rest of the economy, why wages are rigid, and the connection between employment slumps and financial crises, which the literature on the aggregate demand channel in general does not explain.

Our paper incorporates household limited liability in a directed search framework (Moen (1997), Shimer (1996)). Other papers that embed household credit markets in search models of the labor market are Bethune, Rocheteau, and Rupert (2015), Kehoe, Midrigan, and Pastorino (2013), and Herkenhoff (2013). These papers focus on the expansion of lending in the boom period and the decrease in credit availability in the contraction, rather than the effect of debt already on household balance sheets at the beginning of a recession that we study. Further, this work does not include household limited liability and, as a result, does not analyze the distortionary effect of household debt on labor market search.

Our paper is also related to the literature on bank capital regulation. In the wake of the financial crisis of 2008–09, there has been active debate about the costs and benefits of high bank capital requirements. For example, Holmstrom and Tirole (1997) and Mehran and Thakor (2011) develop theories that highlight the benefits of higher bank capital, and Berger and Bouwman (2013) document that higher capital enhances bank performance during financial crises. See Thakor (2014) for a review. The papers in this literature that are most related to our work study the effects of increasing bank equity in a general equilibrium framework. For example, Opp, Opp, and Harris (2014) develop a model in which increasing bank equity has a non-monotonic effect on welfare. The reason is that banks that face competition from outside investors may react to higher capital requirements by taking on (socially) inefficient risk. In a structural model that lends itself to calibration, Nguyen (2014) shows that increasing capital requirements above present levels can produce welfare gains.

Various other papers have examined the consequences of the interaction between credit and labor markets. Acemoglu (2001) argues that failures of the credit market to channel funds to socially valuable projects can increase unemployment, and that the persistence of high unemployment in Europe, relative to in the US, may be explicable on this basis. Adams (2005) suggests that the ability of households to repay their loans depends on the likelihood of remaining employed, which means that lenders can predict default likelihood by looking at the labor market. Adams, Einav, and Levin (2009) document that automobile demand in the US increases sharply during the rebate season, and that household default rates rise with loan size, indicating the possible desirability of loan caps.[8] Buera, Fattal-Jaef, and Shin (2014) develop a theoretical model in which a credit crunch leads to a big drop in employment for small, young firms and a lesser drop for large, old firms. Boeri, Garibaldi, and Moen (2012) develop a model and provide evidence that more highly-leveraged sectors in the economy are associated with higher employment-to-output elasticities during banking crises. Kocherlakota (2012) develops an incomplete labor market model with an exogenous interest rate to show that a decline in the price of land can cause a reduction in employment if the real interest rate remains constant. Koskela and Stenbacka (2003) develop a model in which increased credit market competition leads to lower unemployment under certain conditions related to labor force mobility, whereas Gatti, Rault, and Vaubourg (2012) document that reduced banking concentration can lead to lower unemployment, but only under some labor market conditions.

2 Baseline Model: Setup

This section describes the model. There are two dates, Date 0 and Date 1 and three types of players: banks, firms, and households. Households become workers at Date 1 and provide labor to firms; households and firms meet in a directed search market at Date 1. Banks lend to households at Date 0. Households consume at both Date 0 and Date 1, whereas banks and firms maximize only expected Date 1 profits.

Note that "households" are also "borrowers" at Date 0 and "workers" at Date 1.

2.1 Preferences and Action Spaces of Players

Households. There is a unit continuum of impatient, risk-averse households. Households have a unit labor endowment at Date 1, but not at Date 0. Households consume c_0 at Date 0 and c_1 at Date 1. They have utility $u(c_0) + \delta u(c_1)$, where $\delta \in (0, 1)$ is the discount factor. Below we will assume that u is piecewise linear (Subsection 2.6), which will enable us to solve the model. Households want to smooth consumption, which

[8] The optimality of capping household loan-size is also an implication that arises in our analysis.

they can achieve by borrowing from banks at Date 0. Banks will step in to provide this credit to households.[9]

Banks. There is a unit continuum of risk-neutral banks with discount factor one. The credit market is competitive, so each bank earns an expected rate of return equal to the (zero) riskless interest rate.

Firms. There is a continuum of firms with measure significantly greater than one.[10] Firms pay search cost k to post wages w. Firms either find a worker/household or remain unmatched. Firms make revenue y if they find a worker in the labor market and zero if they remain unmatched. Firms are competitive and will receive zero expected profit in equilibrium.

2.2 Labor Market

We model the labor market with a one-shot version of a standard competitive search model. In our model, firms post wages w and households direct their search at a given wage. The *queue length* q_w for wage w is the ratio of the measure of households directing their search at w to the measure of firms posting w (this is the reciprocal of the so-called tightness of submarket w). We assume that for each wage w, households are matched with firms with intensity $\alpha(q_w)$ and firms are matched with households with intensity $q_w\alpha(q_w)$.

2.3 Contracts

There are two types of contracts in the model: the labor contract between households and firms and the borrowing contract between households and banks.

The labor contract between households and firms is defined entirely by a wage w. Each firm post a wage at the beginning of Date 1 and, if a household is matched with the firm, it receives w in exchange for devoting its unit of labor toward production.

The borrowing contract between a household and its bank is defined by the amount B borrowed by the household and its promised repayment F. We denote this debt contract by (B, F). We assume that contracts are enforceable but that households are protected by limited liability, so households repay whenever they are employed (and have sufficient income),[11] but households are not punished beyond the loss of

[9] Note that our main results are not sensitive to the households' motive for borrowing; for example, if households borrow to finance investment rather than consumption, our results go through as long as investment is risky.

[10] This restriction that the number of firms exceeds the number of workers is standard in models with free entry of firms. It ensures an interior solution in the sense that enough firms enter the market so they make zero profit in equilibrium.

[11] Note that even among households with negative home equity, unemployment is by far the dominant cause of household default (Gerardi, Herkenhoff, Ohanian, and Willen (2013)).

their income when they default.[12] Since there are only two outcomes ("employed" and "unemployed") and the cash flow to the household is zero when it is unemployed, our restriction to debt contracts is without loss of generality.

2.4 Summary of Timing

The sequence of moves is as follows. At Date 0, each bank posts a contract (B, F) competitively, is matched frictionlessly[13] with a household and lends B to the household in exchange for its promise to repay F. At Date 1, each firm observes the aggregate level of household debt[14] and posts a wage and each household directs its search at a given wage. Next, if firms are matched with households, they produce output y and pay wages w. Finally, employed households repay their debts. Unemployed households default. See Figure 1 for a timeline representation of the sequence of moves.

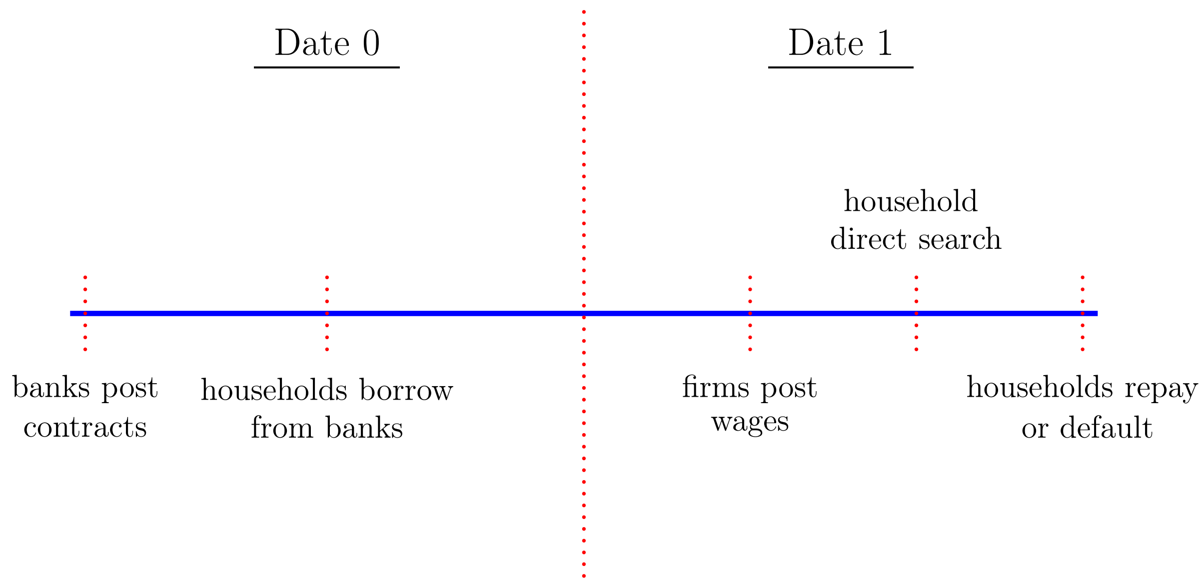
The markets for loans and workers are competitive. We capture competition via the free entry of banks and firms. Note that the only matching frictions are in the labor market.

[12]Our results are robust to more general contracts: in Subsection 9.3 we argue that our results are robust to the inclusion of default penalties and in Subsection 9.2 we argue that our results are robust to the inclusion of collateral.

[13]Formally, this corresponds to a Leontief matching technology.

[14]This assumption—that firms observe only the aggregate level of household debt, rather than each individual household's level of debt—simplifies the analysis by allowing us to abstract from the analysis of deviations by agents of zero measure. We also believe it is realistic. Further, in Section 8.2, we confirm that our results are insensitive to this assumption. There, we repeat the analysis with random matching without any restrictions on the observability of household debt and find the same equilibrium allocation.

Figure 1: A Timeline Representation of Sequence of Moves



2.5 Solution Concept

The solution concept is symmetric perfect Bayesian equilibrium.

2.6 Assumptions

In this section we impose several restrictions on functional forms and parameters. Specifically, we assume functional forms for the households' utility and the firm-household matching function that allow us to solve the model in closed form. We discuss the implications of relaxing some of these assumptions in Section 9.

Acemoglu and Shimer (1999) allow workers to be risk-averse in a labor model with directed search, but Rogerson, Shimer, and Wright (2005) note that this "...means it is no longer possible to solve the model explicitly" (p. 976). We avoid this difficulty by assuming that households' utility functions are piecewise linear. Households are risk averse globally, giving them the incentive to smooth consumption, but risk-neutral locally, allowing us to solve the Date 1 search model as if they have linear utility.[15]

Assumption 1.

$$u(c) = \begin{cases} c & \text{if } c \leq I, \\ I & \text{otherwise.} \end{cases} \tag{1}$$

[15]See Chassang (2013) and Dang, Gorton, Holmström, and Ordonez (2014) for a similar assumption on preferences.

Further, we assume a form for the matching probability α that enables us to solve the model in closed form.

ASSUMPTION 2. *The matching function is homogenous and the probability that a household is employed if it queues at a firm with queue length q is*

$$\alpha(q) = \frac{a}{\sqrt{q}}. \tag{2}$$

This probability satisfies the properties induced by standard matching functions in the literature, namely the probability α that a household matches with a firm is decreasing and convex in the queue length, whereas the probability $q\alpha$ that a firm matches with a household is increasing and concave in the queue length.

We must ensure that these matching probabilities are between zero and one in equilibrium, namely that for the equilibrium queue length $\alpha, q\alpha \in [0,1]$ or, equivalently,

$$a^2 < q < \frac{1}{a^2}. \tag{3}$$

a being sufficiently small suffices for this to hold in equilibrium. Rather than assuming this, we impose the following looser restriction on primitives.

ASSUMPTION 3.

$$a^2\left(y + \sqrt{y^2 - \frac{8kI}{a^2}}\right) < 4k < y + \sqrt{y^2 - \frac{8kI}{a^2}}. \tag{4}$$

Appendix A.6 demonstrates the sufficiency of these bounds to have $\alpha, q\alpha \in [0,1]$.

The household's utility function is flat whenever consumption is greater than the level I, at which point there is a kink. Therefore, in order to ensure that we have an interior solution for households' wages, it must be that households' consumption is less than I. Since employed households repay their debts before consuming, this condition corresponds to $w - F < I$. A sufficient condition for this to hold is given in terms of primitives in the next assumption, that firm output y is not too large relative to the kink parameter I.

ASSUMPTION 4.

$$2I > y. \tag{5}$$

Finally, we state one restriction on the endogenous variables w and F that we maintain throughout the paper. In Appendix A.7 we verify that the restriction is satisfied by these endogenous variables in equilibrium. We refer to this (subsequently verified) restriction as a "maintained hypothesis." This hypothesis ensures that the households are on the increasing part of their utility function at Date 1.

Maintained Hypothesis 1.

$$0 < w - F < I. \tag{6}$$

3 Baseline Model: Results

The analysis of the model is presented in this section. We look for a symmetric equilibrium in which all households' loans have the same face value F. We first solve the labor market in terms of F and then proceed by backward induction to find the equilibrium in the credit markets.

3.1 Labor Market

The solution to the directed search model in the labor market is mostly standard,[16] but the twist in our model is that households have debt with face value F. In this section we take the face value F as given; we solve for it in Subsection 3.2 below. Note that we are assuming for now that all households have the same level of debt. Later, we will verify that this is the case in equilibrium (see Proposition 2). The next proposition summarizes the equilibrium of the labor market.

Proposition 1. *The equilibrium queue length, wage, and employment rate are*

$$\sqrt{q} = \frac{2k}{a(y-F)}, \tag{7}$$

$$w = \frac{y+F}{2} \tag{8}$$

and

$$\alpha(q) = \frac{a^2(y-F)}{2k}. \tag{9}$$

The equilibrium queue length allows us to find the equilibrium matching probability α. Since unmatched households are unemployed, α also equals the employment rate. The next corollary says that the higher is the level of household debt, the lower is the employment rate. We call this result the *vacancy-posting effect.*

Corollary 1. *The employment rate α is decreasing in the level of household debt F.*

We explain the intuition for why increasing household debt decreases employment in two steps. The first step is that debt distorts household preferences, leading them to search for relatively high-wage jobs. The second step is that fewer firms post vacancies when households are indebted, because this household preference distortion leads to relatively expensive labor.

[16] See Rogerson, Shimer, and Wright (2005) for a primer.

To see why more indebted households search for only relatively high-paying jobs, consider a simple binary example with only two households and two jobs. One household has no debt and the other has debt F. They each must choose between a low-wage job that pays w_L and a high-wage job that pays $w_H > w_L$. For the purpose of this illustration we assume that the queue for the high-wage job is much longer than the queue for the low-wage job, so not only is the probability of being employed in the high-wage job less than the probability of being employed in the low-wage job, $\alpha_{w_H} < \alpha_{w_L}$, but further this difference in probabilities is large enough that the expected income from the low-wage job is higher than the expected income from the high-wage job, $\alpha_{w_L} w_L > \alpha_{w_H} w_H$. Thus, the unindebted household will always search for the low-wage job, because it maximizes its expected income, which coincides with its expected utility. But which job will the indebted household prefer? Since it repays its debt before it consumes, its payoff when it is employed is $w - F$, whereas its payoff when it is unemployed is zero, since it is protected by limited liability. Thus, its payoff when it is unemployed is not affected by its debt F. So, as long as F is sufficiently large, $\alpha_{w_H}(w_H - F) > \alpha_{w_L}(w_L - F)$, and it prefers the high-wage job.

This result is reminiscent of the result in corporate finance that debt distorts firms' project choices, leading them to engage in *risk-shifting*, taking on excessively risky projects. When households have limited liability, debt distorts their preferences in the same way, leading them to accept the risk of unemployment to look for high wages in the labor market.

To see this preference distortion outside of the context of the stylized binary example above, consider the sensitivities of household expected utility to employment probabilities and to wages. First, note that the household's expected utility at the beginning of Date 1, denoted by v, is

$$v = \alpha(w - F). \tag{10}$$

So we have that

$$\frac{\partial}{\partial F}\left(\frac{\partial v}{\partial \alpha}\right) = -1, \tag{11}$$

whereas

$$\frac{\partial}{\partial F}\left(\frac{\partial v}{\partial w}\right) = 0. \tag{12}$$

Note that increasing household debt decreases the marginal benefit of an increase in the employment probability. The reason is that when a household finds a job, it must pay a large fraction of its wage to its creditor. In contrast, increasing household debt does not affect the marginal benefit of an increase in wage, because a household keeps all of the upside from the extra wages it earns. Note that *household debt is analogous to unemployment insurance*. The reason is that unemployment insurance is high, the marginal benefit of finding a job is low. In contrast, the marginal benefit to higher wages conditional on employment is unchanged.

We now proceed to explain the second step in the argument, about firms' vacancy postings. Firms recognize these household preferences, and reason that they must therefore pay higher wages when they employ more-highly-indebted households. Consequently, firms' willingness to post vacancies decreases as household debt increases, and unemployment consequently increases. Hence, we refer to this as the *vacancy-posting effect* of household debt.

3.2 Credit Market

We now turn to the market in which households borrow from banks.

3.2.1 The Level of Household Debt

Before we turn to our main results of this section, we state a lemma that simplifies the analysis. The lemma pins down the amount households will borrow at Date 0. Specifically, households borrow exactly up to the kink in their utility functions, $B = I$. The reason that they do not borrow more is that their utility function is flat above the kink, so their marginal benefit from consuming above the kink at Date 0 is zero. The reason they do not borrow less is that they are impatient (their discount factor δ is less than one), so they have the incentive to move consumption forward. Thus, the amount of credit a household demands from his bank is exactly I.

LEMMA 1. *In equilibrium, households borrow $B = I$.*

3.2.2 The Face Value of Household Debt and the Household Debt Externality

We are now in a position to compute the face value of debt that a bank will post. Banks are competitive and therefore break even on average, i.e. the expected value of a bank's Date 1 repayment must equal its Date 0 loan amount I (recall that $B = I$ by Lemma 1). Since households are employed with probability α and repay their debt only if they are employed, a bank's expected repayment is given by αF and the bank's break-even condition is

$$\alpha F = I. \tag{13}$$

Recall that, as a result of the vacancy-posting effect, α depends on the face value of debt that households have when they enter the labor market (see equation (9)). To solve for the equilibrium, let $\hat{F}$ represent the face value of debt on a representative household balance sheet. Thus, for a face value F to be an equilibrium face value, it must satisfy two conditions. The first is the bank's zero-profit condition:

$$\frac{a^2}{2k}\big(y-\hat{F}\big)F = I, \tag{14}$$

where we have substituted in for $\alpha = \alpha(\hat{F})$ from equation (9) into equation (13). The second is the rational expectations condition

$$F = \hat{F}. \tag{15}$$

Combining equations (14) and (15) yields a quadratic equation in F:

$$(y-F)F - \frac{2kI}{a^2} = 0. \tag{16}$$

This quadratic equation has two solutions, corresponding to two equilibria of the model. There are multiple equilibria because *banks' beliefs about future employment are self-fulfilling:* when banks believe that the rate of employment will be high—and therefore that household default is unlikely—banks demand low face values and employment is indeed high. Likewise, when banks believe that the rate of employment will be low—and therefore that household default is likely—banks demand high face values and unemployment is indeed high. In other words, there is a high-debt, low-employment equilibrium and a low-debt, high-employment equilibrium. We refer to the face value of household debt in the high-debt equilibrium as F_+ and the face value of household debt in the low-debt equilibrium as F_-. These are summarized in the next proposition.

Proposition 2. *There are two equilibrium face values,*

$$F_- = \frac{1}{2}\left(y - \sqrt{y^2 - \frac{8kI}{a^2}}\right) \tag{17}$$

and

$$F_+ = \frac{1}{2}\left(y + \sqrt{y^2 - \frac{8kI}{a^2}}\right), \tag{18}$$

so long as the discriminant above is positive.

In the next section, we solve for the socially-optimal allocation and show formally that the equilibrium level of debt in the economy is inefficiently high (even in the low-debt equilibrium). This is because of a *household debt externality* that leads to excessive household leverage. The externality works as follows. When banks extend credit to households, they affect the employment rate α via the vacancy-posting effect. In other words, when a bank lends, it increases the aggregate level of debt in the economy, and thus decreases the employment rate. By decreasing the employment rate, it increases the probability that *other* households default on their debts to *other* banks, because unemployed households have no income with which to repay their debt. When a household and a bank agree to a loan, they fail to internalize this negative effect on other households and other banks.[17]

4 Baseline Model: Welfare and Policy Implications

In this section, we analyze social welfare and ask what is the socially-optimal level of household debt. We find that the socially-optimal debt level is always weakly below the equilibrium debt level in the low-debt equilibrium. In other words, as a result of the household-debt externality, households borrow too much even in the low-debt equilibrium. Thus, our main policy prescription in this subsection is that household borrowing should be limited. Note that despite the distortionary effect of household leverage on the labor market, *the social planner does not wish to forbid debt entirely*, because household debt has the positive effect of allowing households to smooth consumption.

The social planner's objective is to maximize household utility over both the face value of debt and the size of the loan, subject to banks' break-even condition. We state this formally in the next definition.

Definition 1. *The social planner's problem is to maximize*

$$u(B) + \alpha\delta\, u(w - F) \tag{19}$$

over B and F subject to

$$\alpha F = B. \tag{20}$$

We denote the social planner's solution for the optimal amount lent and face value of debt by B^{SP} and F^{SP}.

We are now in a position to present our main results of this section. We find that the socially-optimal level of household borrowing B^{SP} is less than the equilibrium level of household borrowing $B = I$. Thus, a financial regulator can intervene to implement the social optimum by capping household debt at B^{SP}. Note that this is a strict Pareto improvement, since banks always break even and households are strictly better off. The results below state the solution of the Social Planner's Problem and the optimal intervention formally.

[17] Note that the household debt externality is the only source of inefficiency in our model, since the equilibrium in the search market is constrained efficient, because the so-called Hosios condition is satisfied in our model (Hosios (1990)). The Hosios condition is that the fraction of surplus allocated to firms equals the elasticity of their matching probability in equilibrium; we compute this is Subsection 8.2.

Lemma 2. *The solution to the social planner's problem is given by*

$$F^{\mathrm{SP}} = \frac{1-\delta}{2-\delta}\, y \tag{21}$$

and

$$B^{\mathrm{SP}} = \frac{a^2}{2k} \frac{1-\delta}{(2-\delta)^2} y^2. \tag{22}$$

Proposition 3. *The socially-optimal amount of household debt is less than the equilibrium amount of household debt (even in the low-debt equilibrium), i.e.*

$$I > B^{\mathrm{SP}} \tag{23}$$

and

$$F_+ > F_- > F^{\mathrm{SP}}. \tag{24}$$

Corollary 2. *The social planner can implement the social optimum by capping household debt at B^{SP}.*

5 Extended Model: Heterogeneous Households and Bank Screening

Since we want to understand the connection between the health of the financial system and labor market slumps, we extend the model to include a more active role for banks. To do so, we introduce asymmetric information at the household level and we give the bank the ability to screen borrowers. Further, to close the model, we introduce a fourth type of player, investors, who can lend to banks either via debt or via equity. In Subsection 8.3 we show that even if investors could lend to households directly, households would prefer to borrow from banks.

5.1 Preferences and Action Spaces of Players

Households. Households have the same preferences as those specified in Section 2.1. The only difference is that now they can be either good $\tau = g$ or bad $\tau = b$; g-type households have a unit labor endowment at Date 1, whereas b-type households have no labor endowment. Households do not know their types initially, when they apply for loans, and they learn them at the end of Date 0, before they are matched with firms in the labor market at Date 1. Let $\theta \in [0,1]$ be the prior probability that the household is $\tau = g$.

Banks. Banks have the same preferences as those specified in Subsection 2.1. However, now each bank has initial equity e and it raises new capital by issuing debt and new (outside) equity at Date 0. Specifically, it raises an amount D in exchange for the promise to repay RD and it raises an amount Δ in exchange for a fraction $1-\beta$ of the bank's equity. Therefore, the Date-0 asset value of the bank is $e+\Delta+D=E+D$, where $E:=e+\Delta$ denotes the total value of bank equity. At Date 0, the bank can lend an amount B to a household in exchange for the household's promise to repay face value F at Date 1. The credit market is competitive, so each bank earns an expected rate of return equal to the (zero) riskless interest rate.

Banks have a noisy screening technology that enables them to screen out b-type households. Each bank observes a signal $s \in \{s_g, s_b\}$ about the household it will potentially lend to. Both the bank's choice of screening precision and the signal realization are public information.[18] For a signal precision of $\sigma \in [0,1]$, the type-conditional signal distribution is as follows:

$$\mathbb{P}\left[s = s_\tau \,|\, \tau, \sigma\right] = \begin{cases} 1 & \text{if } \tau = g \\ \sigma & \text{if } \tau = b. \end{cases} \tag{25}$$

This specification implies that $\sigma = 0$ yields a completely uninformative signal—the bank can anticipate observing $s = s_g$ regardless of the borrower's type. Thus, the bank's posterior belief about the borrower's type is its prior belief in this case. Precision $\sigma = 1$ yields a perfect signal that reveals the borrower's type with no error.[19] See Figure 2 for a pictorial representation. Increasing the screening precision σ allows the bank to reduce the probability of lending to b-type households, but increasing σ is costly for the bank. Specifically, the bank can pay cost $c(s) = \gamma\sigma^2/2$ to achieve screening precision σ.

Investors. There is a unit continuum of risk-neutral investors with discount factor one, each with wealth $I-e$. They divide their capital between bank debt, bank equity, and consumption at Date 0. The deposit market is competitive, so investors' expected return is equal to one for their investments in bank debt and bank equity.

Firms. Firms are as specified in Subsection 2.1.

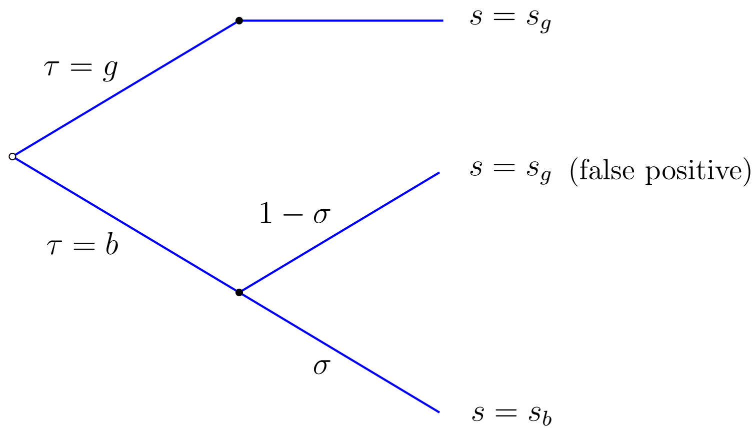
5.2 Contracts

Two types of contracts were introduced in Subsection 2.3—the labor contract between the firm and the household and the borrowing contract between the bank and the household. We now introduce an additional contract—that between the bank and its

[18]Note that this assumption is without loss of generality: since no bad types enter the market at Date 1, this does not affect any inference problem at Date 1.

[19]This asymmetric signal structure implies that banks' screening errors can lead them to grant credit to b-type borrowers but never to deny credit to g-type borrowers. This will imply that all households searching in the labor market at Date 1 will have the same level of debt. This provides a substantial simplification since we can avoid solving the labor market problem with heterogeneous households, i.e. households with different levels of debt.

Figure 2: A Pictorial Representation of the Signal Structure



investors. Banks raise funds via an optimal mix of debt and equity. The bank raises an amount D via debt and promises to repay RD to debtholders. We do not distinguish here between deposits and other forms of bank debt. The bank raises an amount Δ of equity in exchange for an equity stake that promises a proportion $1 - \beta$ of the bank's cash flows net of debt repayments. We denote the debt claim by (D, R) and the equity claim by (Δ, β).

5.3 Summary of Timing and Information Structure

The sequence of moves is as follows. At Date 0, each bank raises capital, is matched with a household, invests in a screening technology, and observes a signal about the household's type. We assume, without loss of generality, that each bank's screening precision and the signal are public information. Each bank funds itself via an optimal mix of debt and equity, which it raises from competitive investors. Then, each bank proceeds to lend to a household. At the end of Date 0, households learn their types. At Date 1, each firm posts a wage and each g-type household directs its search at a given wage. Next, if firms are matched with households, they produce output and pay wages. Finally, employed households repay their debts and banks repay their investors (shareholders and debtholders).

The markets for deposits, loans, and households are competitive. We capture competition via the free entry of banks, investors, and firms. Note that the only matching frictions are in the labor market.

See Figure 3 for a timeline representation of the sequence of moves.

Figure 3: A Timeline Representation of Sequence of Moves

Date 0

Each bank determines optimal capital structure and raises capital

Banks post contracts $\big((D, R), (\Delta, \beta)\big)$

Households search for loans

Banks chooses screening precision σ and observe signal s

Each banks agrees to lend B to a household or not lend

Investors and households consume

Households learn their types

Date 1

Firms pays k to post wage w or stay out

g-type households direct search for jobs and matches take place

Each indebted household repays F or defaults

Each bank repays RD or defaults

Investors, households, banks, and firms consume

5.4 Assumptions

We maintain Assumption 1, Assumption 2, and Assumption 4 from Subsection 2.6, but, to adapt to the extended environment with bank screening, we replace Assumption 3 from the baseline model with the Assumption 5 below.

Assumption 5.

$$a^2\left(y + \sqrt{y^2 - 4\Gamma_{\text{min}}}\right) < 4k < y + \sqrt{y^2 - 4\Gamma_{\text{min}}} \tag{26}$$

where

$$\Gamma_{\text{min}} := \frac{2k}{a^2\theta}\left(1 - \frac{(1-\theta)^2 I^2}{2\gamma}\right). \tag{27}$$

Appendix A.14 demonstrates the sufficiency of these bounds for the matching probabilities to be well-defined, $\alpha, q\alpha \in [0, 1]$.

One additional assumption on parameters ensures that I is not too small. This assumption is useful to ensure that the solution of the bank's funding program is well-behaved (we use it in the proof of Proposition 5).

Assumption 6.

$$(1-\theta)^2 I^2 > 4\gamma(I-e). \tag{28}$$

Finally, we state another maintained hypothesis on endogenous variables. In Appendix A.15 we verify that it is indeed satisfied in equilibrium. This maintained hypothesis simply says that banks' Date-0 book equity $I - RD$ is positive.

Maintained Hypothesis 2.

$$I > RD. \tag{29}$$

6 Extended Model: Results

We solve the model backwards. All the labor market results in Section 3.1 remain unchanged, with the only caveat being that the households looking for a job at Date 1 are the g-households. So we can proceed directly to solving for the equilibrium in the credit markets.

6.1 Credit Markets

We now turn to the two credit markets in the model, namely the market in which households borrow from banks, and the market in which banks borrow from investors.

6.1.1 *The Level of Household Debt*

As in Subsection 3.2.1 households borrow exactly up to the kink in their utility functions, $B = I$. Further, since each bank is matched with one household, and the only alternative to lending is riskless storage, a bank has no incentive to hold assets in excess of I at Date 0. Therefore, a bank borrows $D = I - E$ so that its entire cash holdings at Date 0 are I. The result of this argument is stated formally in the next lemma.

Lemma 3. *In equilibrium, households borrow $B = I$ and banks raise capital $D + \Delta = I - e$.*

6.1.2 *Bank Screening for a Given Bank Capital Structure*

We now solve for the equilibrium level of bank screening. Each bank chooses its screening precision σ to maximize its expected Date-1 equity value net of screening costs. Note that in equilibrium banks extend credit if they observe signal s_g and deny credit if they observe signal s_b. The proposition below states banks' equilibrium level of screening.

Proposition 4. *Banks screen with precision*

$$\sigma = \begin{cases} \frac{1}{\gamma}(1-\theta)(I-RD) & \text{if } \ 0 < I - RD < \frac{\gamma}{1-\theta}, \\ 1 & \text{if } \ I - RD > \frac{\gamma}{1-\theta}. \end{cases} \tag{30}$$

The expression for σ in Proposition 4 above allows us to perform comparative statics on the screening precision as a function of bank leverage. In particular, we see that more highly levered banks screen less. The reason is that screening reduces the probability of lending to bad borrowers: an increase in bank leverage leads bank equity holders to bear a smaller proportion of the cost of these bad loans, so they have less incentive to screen them out. Since the screening precision choice is made to maximize the value of the bank's equity, screening precision declines with bank leverage. In the next corollary we state how changes in D affect σ.

Corollary 3. *Screening precision is decreasing in bank leverage. In particular,*

$$\frac{\partial \sigma}{\partial D} = -\frac{(1-\theta)R}{\gamma} < 0$$

whenever $0 < I - RD < \gamma/(1-\theta)$*, and otherwise the derivatives is zero or undefined.*

Note that the corollary above takes into account only the direct effects of D on σ. This corresponds to studying what would happen if bank capital structure were exogenous. In the full model, capital structure is endogenous. In Subsection 6.1.3 below, we will analyze a bank's optimal mix of debt and equity funding. To find the optimum we will have to consider not only the direct effect of an increase in debt D on screening precision σ that we calculate above, but also its indirect effect on screening precision through a change in the deposit interest rate. Note that Corollary 3 suggests that the direct effects of increasing D on σ are negative. In the proof of Proposition 5 below, we show that this intuition carries through to this total derivative, i.e. $d\sigma/dD$ is indeed negative.

6.1.3 Bank Capital Structure

In this section, we find the optimal capital structure for banks. The proposition below states that banks raise all their new capital via debt.

Proposition 5. *In equilibrium, banks raise capital only via debt, i.e.* $\Delta = 0$ *and* $D = I - e$*. That is, banks have minimum leverage.*

The intuition is as follows. Increasing bank leverage decreases the bank's incentives to screen, i.e. decreases σ, as we discussed earlier (see Corollary 3). This increases the probability that the household is granted a loan at Date 0 and, therefore, the probability that the household consumes early. Since households are impatient, they value this early consumption and are willing to repay more tomorrow in order to be able to borrow today—they are willing to compensate banks for lending to bad borrowers.[20] Since the only mechanism banks have at their disposal to commit not to screen is their own leverage, a highly levered bank can appeal more to households in search of loans.

6.1.4 The Face Value of Household Debt

We are now in a position to compute the equilibrium face value F. As in Subsection 3.2.2, we compute it from the bank's break-even condition which says that the expected payoff from lending is equal to the cost of lending. First, since only a proportion θ of households are the g-type that will search for employment and thus repay, the expected repayment to banks is $\theta\alpha(\hat{F})F$. Banks screen out bad households with probability σ, thus extend credit I to a proportion $1-(1-\theta)\sigma$ of households. Finally, banks bear the cost of screening $c(\sigma)$. This gives the analog of the break-even condition in equation (13),

$$\theta\alpha(\hat{F})F = \big(1-(1-\theta)\sigma\big)I + c(\sigma). \tag{31}$$

This condition together with the belief consistency equation $F = \hat{F}$ again yields a quadratic equation for F, the solutions of which are summarized in the proposition below.

Proposition 6. *There are two equilibrium face values,*

$$F_- = \frac{1}{2}\Big(y - \sqrt{y^2 - 4\Gamma_D}\,\Big) \tag{32}$$

and

$$F_+ = \frac{1}{2}\Big(y + \sqrt{y^2 - 4\Gamma_D}\,\Big), \tag{33}$$

so long as the discriminant above is positive, where Γ_D is defined in equation (102) in Appendix A.11.

7 Extended Model: Welfare Analysis and Policy Implications

In this section we analyze welfare in the extended model. As in our analysis of welfare in the baseline model (Subsection 4), the analysis here revolves around the utility of

[20]The assumption that households do not know their own types when they apply for bank loans is important for this result. Although somewhat different, this result is reminiscent of the Hirshleifer (1971) effect that risk-averse agents would prefer not to have information revealed before they trade. However, here it is not risk aversion *per se* that is driving this result, but rather the impatience to consume and the need to guarantee that consumption.

households, since banks, investors, and firms, all break even. The social planner's problem in the extended model is complicated, so we cannot solve for the global social optimum as we did in the baseline model. Rather, we show that implementing caps on household borrowing and capital requirements for banks raises welfare above the *laissez-faire* equilibrium level. We can now state our first result of this section.

PROPOSITION 7. *Employment and output are higher in the equilibrium associated with* F_- *than in the equilibrium associated with* F_+.

This result that output and employment are higher in the low-debt equilibrium than in the high-debt equilibrium follows from the observation that lower household debt means a diminished externality of debt on employment. Thus, the equilibria are efficiency-ranked from the point of view of GDP. Simply the common belief that banks will offer low interest rates can arrest a drop in employment and prevent a recession. Thus, there is scope for financial regulation to restore efficiency in the labor market, something typically left to monetary policy. In fact, an immediate corollary of this result is that a financial regulator can ensure the economy is in the high-employment equilibrium with an appropriate cap on household debt.

COROLLARY 4. *If banks are restricted to offer households debt with face values below a cap* $\bar{F} \in (F_-, F_+)$, *then the unique equilibrium of the model is the high-employment equilibrium of the model without the cap.*

We now turn to the effect of regulating bank equity. We perform comparative statics on banks' equity E to represent regulatory capital requirements. Our next main result is that increasing bank equity increases employment, provided that the face value of household debt is capped to ensure the economy is in the high-employment equilibrium.[21]

PROPOSITION 8. *As long as the banks's initial equity* e *is not too small, if banks are restricted to offer households debt with face values below a cap* $\bar{F} \in (F_-, F_+)$, *then increasing each bank's equity* E *increases the rate of employment* α, *i.e.,*

$$\frac{d\alpha}{dE} > 0. \tag{34}$$

The intuition behind this result is as follows. Increasing bank capital requirements leads banks to increase their equity, since absent such regulatory intervention they raise all new capital via debt (Proposition 6.1.3). This increases bank screening (Corollary 3), leading them to grant fewer loans to b-type households. This reduces the cross-subsidy from g-type households to b-type households, and therefore lowers the face

[21] This shows an additional and previously unexplored beneficial effect of bank capital that goes beyond its role in promoting financial stability, as discussed, for example in Thakor (2014).

value of debt that g-type households have on their balance sheets when they search in the labor market at Date 1. This weakens the distortion of debt on labor market search and thereby mitigates the vacancy-posting effect, increasing employment.

8 Robustness

In this section, we show that our results are robust to relaxing the following four main assumptions we have made so far: (i) that households' utility is piecewise linear, (ii) that households and firms are matched in a directed search market, (iii) that investors cannot lend directly to households, and (iv) that households are unemployed at Date 0.

8.1 Robustness to the Functional Form of Utility

The assumption that households' utility is piecewise linear facilitates tractability. The *local* risk neutrality implied by this assumption allows us to solve the directed search model of the labor market in closed form (see the discussion in Rogerson, Shimer, and Wright (2005)). But the *global* risk aversion of households implied by this assumption gives them the incentive to borrow.

Here we demonstrate that the main mechanism behind the vacancy-posting effect would only be amplified if we were to use a more traditional utility function that was strictly increasing and concave everywhere. Consider the utility of a household with debt F searching in the labor market at Date 1. Its expected utility is given by

$$v = \alpha u(w - F) + (1 - \alpha)u(0), \tag{35}$$

where, as in the main model, the household receives $w - F$ if employed (which occurs with probability α) and zero otherwise (which occurs with probability $1 - \alpha$). To show that households have incentive to shift risk with a more general utility function, we compute the analogs of equations (11) and (12) below:

$$\frac{\partial^2 v}{\partial F \partial w} = -\alpha u''(w - F) > 0, \tag{36}$$

and

$$\frac{\partial^2 v}{\partial F \partial \alpha} = -u'(w - F) < 0. \tag{37}$$

Thus, increasing F increases the marginal value of higher wages, but decreases the marginal value of a higher probability of employment. This is exactly the mechanism behind the vacancy-posting effect induced by household leverage. As a result, we expect that *the vacancy-posting effect of household debt is robust to more general utility specifications.*

8.2 The Effect of Directed Search rather than Random Matching

There are two modeling frameworks in labor search theory: the directed search model that we use in our analysis and the Diamond–Mortensen–Pissarides random-matching model. In this section we demonstrate that the random matching framework generates the same equilibrium unemployment and wages as the directed search framework we use.

Consider a variant of the model with random search, in which firms and households are randomly matched at Date 1. As above, α denotes the probability that a household is matched with a firm and $q\alpha$ denotes the probability that a firm is matched with a household. Once a household and a firm are matched, they set wages by Nash bargaining, to split the surplus fifty-fifty.[22] All outside options are zero, since the model is one-shot. The total surplus to be shared between the household and the firm is $y - F$, which is the total output generated by the match minus the amount that must be paid to a third party, namely the household's creditor. Solving this model gives the same equilibrium unemployment and wages as the baseline model above, as formalized in the next proposition.

PROPOSITION 9. *The equilibrium employment and wages in the random matching model coincide with those in the baseline model as specified in Proposition 1.*

8.3 Investors Do Not Lend Directly to Households: Why We Need Banks

In the extended model we have assumed that investors do not lend to households directly. We now show that if we allowed investors to lend directly to households, they would still prefer to invest in banks, eschewing the direct-lending option. This provides the *raison d'être* for banks.

To do this, we change the model slightly and assume that investors have endowment I and have the same screening technology that banks have, thereby "levelling the playing field" and allowing us to establish a reason for banks to exist even if they are not endowed

[22]Note that this case of the fifty-fifty split is the case in which the Hosios condition is satisfied (Hosios (1990)); namely that the equilibrium elasticity of the firm matching probability coincides with the firm's bargaining power, computed as follows:

$$\varepsilon \equiv \frac{q(q\alpha)'}{q\alpha} = \frac{\left(a\sqrt{q}\right)'}{a/\sqrt{q}} = \frac{1}{2}. \tag{38}$$

with any special advantage in screening. We will argue that the result that households always prefer to borrow from banks is effectively a corollary of the result that banks are always maximally levered (Proposition 5).

The reason is as follows. An investor who posts a contract to lend directly to households is exactly an all-equity funded bank. But Proposition 5 implies that such an all-equity bank cannot exist in equilibrium. To see this, recall the intuition for the proposition, which goes as follows. Banks' screening precision is decreasing in bank leverage. Thus, the probability that the household is granted a loan at Date 0—and, therefore, the probability that the household can consume early—is also decreasing in bank leverage. Since households value early consumption, they prefer to borrow from a levered bank than from an (otherwise identical) unlevered bank. An investor who lends directly to households is effectively an unlevered bank, so households always prefer to borrow from banks than from investors directly. Thus, the leverage that banks find privately optimal to put on their balance sheets acts as a commitment device to not screen borrowers and becomes the *raison d'être* for banks. We state this result formally below:

Proposition 10. *In equilibrium, investors are better off investing via banks than they would be offering loans directly to households. Further, there cannot be an equilibrium in which investors lend directly to households rather than investing in banks.*

8.4 Date-0 Employment

So far, we have assumed that labor and production occur only at Date 1, so all households are unemployed when they borrow at Date 0. In this section, we argue briefly that this is just a simplifying assumption and that our results are robust to households' being employed at Date 0.

Suppose that households are all employed at Date 0, but still have an incentive to borrow.[23] As above, they promise to repay F to borrow B. Then, as is standard in dynamic labor models, a random fraction μ of households lose their jobs at the end of Date 0. This fraction μ of households are the unemployed households searching in the labor market at Date 1. They have debt on their balance sheets as above; hence, they have incentive to shift-risk in their job search and the vacancy-posting effect is at work. Specifically, the probability α that an unemployed household finds a job at Date 1 is decreasing in the level of household debt and, thus, so is the employment rate.[24]

[23]In our setup, in which households borrow to smooth consumption, households will have the incentive to borrow so long as their Date-0 wage w_0 is below the kink in their utility function, $w_0 < I$. However, other motives for household borrowing, notably investment in housing, also generate the same results.

[24]In this extension, the employment rate is given by the proportion of households that do not lose their jobs at Date 0, plus the proportion of those that are reemployed at Date 0 after having lost their jobs at Date 0 or $1 - (1 - \alpha)\mu$.

9 Extensions

In this section we analyze the following three extensions: (i) the addition of shocks to aggregate output and wage fluctuations, (ii) the addition of collateral securing household debt, and (iii) the addition of non-zero penalties for household default.

9.1 Aggregate Shocks and Wage Dynamics

In this section, we discuss the effects of changes in firm output y on employment and wages. We argue that household debt may be a source of sticky wages, and discuss the complementarities between our household debt externality channel of unemployment and the aggregate demand channel that is well-established in the literature.

We now include two possible aggregate states, a boom in which firm output is y_H and a recession in which firm output is $y_L < y_H$. Thus, given household debt with face value F, the calculations in Subsection 3.1 give the labor market outcomes in the boom and recession states. In particular, we have the equations for the wages

$$w_H = \frac{y_H + F}{2} \quad \text{and} \quad w_L = \frac{y_L + F}{2}. \tag{39}$$

The following proposition says that the fluctuation of wages across macroeconomic states decreases as household debt increases, suggesting that high levels of household debt represent a potential source of wage rigidity (see Bewley (1999)).

PROPOSITION 11. *The percentage change of wages across macroeconomic states,*

$$\frac{w_H - w_L}{w_H} = \frac{y_H - y_L}{y_H + F} \tag{40}$$

is decreasing in the level of household debt F.

Now turn to the employment rates. We see that

$$\alpha_H = \frac{a}{2k}\big(y_H - F\big) \quad \text{and} \quad \alpha_L = \frac{a}{2k}\big(y_L - F\big), \tag{41}$$

suggesting that high levels of household debt may retard employment expansion during booms and more importantly, amplify employment slumps in recessions. Thus, while our channel of unemployment, based on the impact of household debt on the labor market, is novel, it is complementary to channels based on varying aggregate output. In particular, when aggregate demand decreases, firm revenues decrease. In our model, this corresponds to a decrease in y. This shock to y has a more severe effect on the labor market when households are more highly levered (F is higher). This is consistent with evidence in studies of the aggregate demand channel, notably Mian and Sufi (2014a).

9.2 The Inclusion of Collateral

In this section we show that our results are robust to the inclusion of collateral. We now suppose that households have collateral in place. Call the household's value of the collateral h and call the liquidation value of the collateral λh, where $\lambda \in [0, 1]$ is a parameter that represents the inefficiency of liquidation.[25]

PROPOSITION 12. *Whenever $\lambda h < F$, households' preferences are distorted, leading to risk shifting in the labor market search. Thus, the vacancy positing effect is robust to the inclusion of collateral.*

This extension not only confirms the robustness of our model, but also yields the additional empirical prediction that the vacancy posting effect should be strongest when $\lambda h < F$, namely when collateral values h are low or liquidation discounts $1 - \lambda$ are high. This explains why the connection between household debt and unemployment is strongest in economic downturns, when asset values are depressed and asset values are illiquid. This was the case for housing during the Great Recession when household collateral values were low due to the fall in house prices. This is consistent with evidence in Mian and Sufi (2014b).

9.3 Allowing for a Default Penalties

In this section we show that our results are robust to the inclusion of default penalties. Note that this may provide cross-sectional variation to test our model as there is large cross-sectional variation in default penalties in the US.[26]

We now assume that a household that defaults on its debt suffers a penalty $-d$. Thus, if a household has debt F before searching in the labor market, its expected Date-1 utility is

$$v = \alpha(w - F) + (1 - \alpha)(-d) \tag{42}$$

$$= \alpha\big(w - (F - d)\big) - d. \tag{43}$$

[25] This can be interpreted as a loss of value due to the forced sale, because the buyer of the collateral in liquidation is unlikely to be the party that can employ the collateral most efficiently (as, for example, in the fire sales analysis of Shleifer and Vishny (1992)). See Besanko and Thakor (1987) for a similar assumption in an analysis of how collateral can be used as a sorting device with a priori unobservable borrower heterogeneity.

[26] In particular, asset exemption laws, which specify the types and levels of assets that can be seized in bankruptcy, vary across states. According to Mahoney (2015), "Kansas, for example, allows households to exempt an unlimited amount of home equity and up to \$40,000 in vehicle equity. Neighboring Nebraska allows households to keep no more than \$12,500 in home equity or take a \$5,000 wildcard exemption that can be used for any type of asset." Chapter 7 is the most popular form of bankruptcy which accounts, according to White (2007), for 70 percent of filings.

The last term $-d$ is an additive constant and therefore does not affect household behavior. Comparison with equation (10) reveals that a household with debt F that will suffer a penalty $-d$ in the event of default has equivalent preferences to a household with debt $F' := F - d$ that will suffer no penalty in the event of default. Thus, *the vacancy-posting effect is robust to the inclusion of default penalties*, and we view the zero default penalty in the model as just a normalization.

Higher default penalties do, however, attenuate the vacancy-posting effect. The reason is that the preference distortion due the risk-shifting problem is mitigated by the default penalty—the effect of $-d$ on preferences exactly offsets the effect of F on preferences.

10 Conclusion

Paper summary. This paper examines the effect of household credit and financial regulation on the labor market. We find that debt on household balance sheets distorts labor market search, leading households to search for high-wage jobs despite the potentially low associated probabilities of finding employment. This labor market search behavior is a result of household limited liability, which leads to a form of risk-shifting behavior that is similar to that of limited liability corporations in corporate finance. Firms in our model respond to households' distorted search behavior by posting high wages but few vacancies. This *vacancy-posting effect* explains why high levels of household debt precede unemployment slumps, and offers insight into why those slumps are persistent.

Because borrowing allows households to smooth consumption, some amount of household borrowing is beneficial for social welfare, despite the labor market distortion it induces. However, households and banks fail to internalize the negative effect that they have on *other* households and *other* banks via the vacancy-posting effect. This *household debt externality* leads to excessive household debt in equilibrium. A financial regulator can intervene to mitigate this externality by capping debt.

To explore the connection between financial crises and labor market slumps, we enrich the model to include asymmetric information and bank screening. We find that banks with low levels of equity screen too little, exacerbating the vacancy-posting effect. This connects financial crises with employment slumps—a poorly capitalized financial system amplifies labor market slumps. A financial regulator can intervene and increase welfare by imposing capital requirements on banks.

This role of bank capital requirements in promoting employment is novel. It shows that a well-capitalized banking system can not only contribute to a reduction in financial fragility, as is well recognized, but it can also foster a reduction in unemployment.

This role of prudential bank regulation seems significant in light of persistently undercapitalized banks and stubbornly-high unemployment in some parts of the world today, notably in several European countries.

Future research. There is an active body of research on the connection between household debt and the labor market. One key contribution of this paper is to focus on the role of household limited liability in distorting labor market search. We hope that future research will investigate its importance quantitatively. We also think that there is scope for further theoretical work extending our analysis, for example by adding housing and mortgage markets to our model or by incorporating household limited liability and default in a dynamic business cycle model.

Future research should also explore the connection between bank lending and employment in greater detail. In particular, what are the effects of interbank competition and competition between markets and banks on unemployment in the real sector? We know from the theories of relationship banking that these factors affect both the nature and level of relationship lending (e.g., Boot and Thakor (2000)) and there is empirical evidence that banking concentration can affect unemployment (e.g., Gatti, Rault, and Vaubourg (2012)). These insights may be joined to explore a host of additional issues that have potentially rich regulatory implications.

A Appendix

A.1 Proof of Proposition 1

This proposition states the solution of a one-shot labor market with directed search in which households have debt on their balance sheets and limited liability.

Firms' free entry for each wage w implies that

$$\Pi = q_w \alpha(q_w)(y - w) - k = 0 \tag{44}$$

for each wage w. Since $\alpha(q) = a/\sqrt{q}$, this implies

$$\sqrt{q_w} = \frac{k}{a(y - w)} \tag{45}$$

Each household then directs its search at the wage that maximizes its expected utility v. Thus, w maximizes

$$v = \alpha(q_w)\big(w - F\big) \tag{46}$$

$$= \frac{a}{\sqrt{q_w}}\big(w - F\big) \tag{47}$$

where q_w is given by equation (45) above. Substituting in for q_w gives the concave objective function

$$\frac{a^2}{k}(y - w)(w - F). \tag{48}$$

Maximizing this over w gives the equilibrium wage

$$w = \frac{y + F}{2}. \tag{49}$$

The expressions in the proposition follow immediately from the expression for $q = q_w$ above and the functional of the matching probability α. □

A.2 Proof of Corollary 1

Immediate from the expression for α in equation (9). □

A.3 Proof of Lemma 1

First, note that households do not have incentive to borrow more than I even if the funds are available, since households do not value consumption above I given their utility function $u(c_0) = \min\{I, c_0\}$. So, we are left to show that households do not want to consume below the kink in their utility function I. Below the kink, households are risk-neutral. And, because households are impatient, there are gains from trade between households and banks, so long as households are still below the kink in their utility functions. Households and banks will always exploit these gains from trade in equilibrium; to do this banks lend to households I. We show this formally by conjecturing an equilibrium in which households borrow less than I and showing that there is a profitable deviation to another contract in which households borrow I.

Suppose (in anticipation of a contradiction) that in equilibrium the bank posts a lending contract (B, F) with $B < I$ in equilibrium. Now consider the deviation (B', F') with $I \geq B' > B$. For the primed contract to be a profitable deviation both the household and the bank must be better off under such contract than under (B, F). The household is better off under the primed contract if

$$B' + \delta\alpha\left(w - F'\right) \geq B + \delta\alpha\left(w - F\right), \tag{50}$$

and the bank is better off if

$$-B' + \alpha F' > -B + \alpha F. \tag{51}$$

These two inequalities can be respectively rewritten as

$$\delta\alpha\left(F' - F\right) \leq B' - B \tag{52}$$

and

$$B' - B < \alpha\left(F' - F\right). \tag{53}$$

There exists a deviation (B', F') whenever $\delta \in (0, 1)$. $\square$

A.4 Proof of Proposition 2

Immediate from applying the quadratic formula to equation (16). $\square$

A.5 Proofs of Lemma 2 and Proposition 3

We present the proofs of Lemma 2 and Proposition 3 together. The reason we prove the two of them together is that we prove both results under a maintained hypothesis that we use the second result to verify.

We solve the social planner's problem under the maintained hypothesis that $B^{\mathrm{SP}} < I$ and $w - F^{\mathrm{SP}} < I$, so that households are on the linear increasing part of their utility functions. Under these assumptions, we derive the expressions in Lemma 2. At the end of the proof, we go back and verify that this is indeed the case given the optimal intervention.

We begin the proof by substituting the constraint of the problem in Definition 1 into the objective function. This gives the following new objective function

$$\alpha F + \alpha\delta\,(w - F). \tag{54}$$

We now insert the equilibrium values of α and w from Proposition 1 and we have that the social planner maximizes the function

$$\frac{a^2}{2k}\left((y-F)F + \frac{\delta}{2}(y-F)^2\right) \tag{55}$$

over F. This is a negative quadratic; we solve for the global maximum via the first-order approach, which gives the following equation for the socially-optimal level of debt

$$F^{\mathrm{SP}} = \frac{1-\delta}{2-\delta}\,y. \tag{56}$$

This proves the first part of Lemma 2.

From the equilibrium expression for α, we now find that at the social optimum the rate of employment is

$$\begin{aligned} \alpha &= \frac{a^2}{2k}(y-F) && (57)\\ &= \frac{a^2}{2k}\,\frac{y}{2-\delta}. && (58) \end{aligned}$$

From the constraint in Definition 1, we find that the socially-optimal level of debt is given by

$$\begin{aligned} B^{\mathrm{SP}} &= \alpha F^{\mathrm{SP}} && (59)\\ &= \frac{\theta a^2}{2k}\,\frac{1-\delta}{(2-\delta)^2}y^2. && (60) \end{aligned}$$

This proves the second part of Lemma 2.

We now turn to the proof of Proposition 3. First, we show that this socially-optimal amount borrowed is less than the equilibrium amount borrowed I. To do this we make use of Assumption 4, which says that $y < 2I$, and Assumption 3, which implies that $y < 4k/a^2$. Together these say that

$$y^2 < \frac{8kI}{a^2}. \tag{61}$$

We use this to express a bound on the socially optimal amount borrowed B^{SP} as follows:

$$
\begin{aligned}
B^{\mathrm{SP}} &= \frac{a^2}{2k}\,\frac{1-\delta}{(2-\delta)^2}y^2 && (62)\\
&< \frac{a^2}{2k}\,\frac{1-\delta}{(2-\delta)^2}\,\frac{8kI}{a^2} && (63)\\
&= \frac{4(1-\delta)I}{(2-\delta)^2}. && (64)
\end{aligned}
$$

A sufficient condition for this last expression to be less than I is

$$
\frac{4(1-\delta)}{(2-\delta)^2} < 1 \tag{65}
$$

which holds for all $\delta \in (0,1)$. Thus, we have proved that the socially-optimal amount borrowed is less than the equilibrium amount borrowed, $B^{\mathrm{SP}} < I$.

Now we show further that the socially-optimal face value F^{SP} is less than the lower equilibrium face value F_{-}. Replacing F^{SP} with the expression in equation (56) above and F_{-} with the expression in equation (17), we see that $F^{\mathrm{SP}} < F_{-}$ if and only if

$$
\frac{1-\delta}{2-\delta}\,y < \frac{1}{2}\left(y - \sqrt{y^2 - \frac{8kI}{a^2}}\right) \tag{66}
$$

which can be rewritten as

$$
\frac{1-\delta}{(2-\delta)^2}y^2 < \frac{2kI}{a^2}. \tag{67}
$$

Observe that this last inequality is equivalent to the inequality $B^{\mathrm{SP}} < I$ derived above (compare with the expression for B^{SP} in equation (62)). Therefore, it is satisfied and we can conclude that $F^{\mathrm{SP}} < F_{-}$.

It remains to verify the maintained assumptions that $B^{\mathrm{SP}} < I$ and $w - F^{\mathrm{SP}} < I$. That $B^{\mathrm{SP}} < I$ is immediate from the findings above, since the kink is at I and the socially-optimal consumption at Date 0 equals the amount borrowed, which is less than or equal to I, i.e. $B^{\mathrm{SP}} < I$. To complete the proof, we show that $w - F^{\mathrm{SP}} < I$ by computation. Recalling that in equilibrium $w = (y + F)/2$, we have that at the social optimum

$$
\begin{aligned}
w - F^{\mathrm{SP}} &= \frac{y + F^{\mathrm{SP}}}{2} - F^{\mathrm{SP}} && (68)\\
&= \frac{y}{2(2-\delta)}. && (69)
\end{aligned}
$$

Thus, the household is on the increasing part of its utility function whenever

$$
\frac{y}{2(2-\delta)} < I, \tag{70}
$$

which always holds by Assumption 4 since $\delta < 1$. $\square$

A.6 Sufficiency of Bounds in Assumption 3

Here we show the sufficiency of the bounds stated in Assumption 3 for the matching probabilities to be well-defined. Substituting the equilibrium q from Proposition 1 we can rewrite the condition in equation (3) as

$$a^2(y - F) < 2k < (y - F).$$

Plugging in for for the smallest F from Proposition 2, i.e., F_-, in the left-hand side of the equation and for the largest F from Proposition 2, i.e., F_+, in the right-hand side of the equation we obtain sufficient conditions for the inequality above to hold, namely

$$a^2\left(y + \sqrt{y^2 - \frac{8kI}{a^2}}\right) < 4k < y - \sqrt{y^2 - \frac{8kI}{a^2}}.$$

$\square$

A.7 Verification that $0 < w - F < I$

In Subsection 3.1 we solved the model under the hypothesis that $0 < w - F < I$. Here we show that given the equilibrium values of w and F, Assumption 4 suffices for the hypothesis to hold. Substituting in for w from Proposition 1 gives the necessary and sufficient condition

$$F < y < 2I + F. \tag{71}$$

The expressions for F in Proposition 2 show that $0 < F < y$, so Assumption 4 that $y < 2I$ suffices. $\square$

A.8 Proof of Lemma 3

The proof is analogous to that of Lemma 1. It is more tedious because we now find a deviation that satisfies the incentive constraints of more players than there are in the baseline model. Further, we circumvent the effects of changes in screening σ by focusing on deviations that do not affect σ. Note that it is without loss of generality to focus on the case in which $E + D = B$. This follows from Proposition 5.[27]

Suppose (in anticipation of a contradiction) that in equilibrium the bank posts a

[27] Proposition 5 implies that $E + D - B < RD$. This says that if a bank holds cash in excess of the amount B it lends, it will all go to debt holders if its borrower does not find a job.

lending contract (B, F) and funding contract $\big((D, R), (\Delta, \beta)\big)$ with $B < I$ in equilibrium. Now consider the deviation (B', F'), $\big((D', R'), (\Delta', \beta')\big)$ with $B' \leq I$.

A sufficient condition for the primed contract to be a profitable deviation for the bank is that it make equity holders strictly better off without making households or debt holders worse off. The corresponding incentive constraints for these conditions are as follows:

$$
\begin{aligned}
&\theta\alpha(F' - R'D') + (1-\theta)\sigma'(B' - R'D') - c(\sigma') - E' > \theta\alpha(F - RD) + (1-\theta)\sigma(B - RD) - c(\sigma) - E \\
&\big(\theta + (1-\theta)(1-\sigma')\big)B' + \delta\theta\alpha(w - F') \geq \big(\theta + (1-\theta)(1-\sigma)\big)B + \delta\theta\alpha(w - F), \\
&\theta\alpha R'D' + (1-\theta)\sigma' R'D' - D' \geq \theta\alpha RD + (1-\theta)\sigma RD - D,
\end{aligned}
$$

where σ is the level of screening that the bank chooses optimally given the initial contract and, likewise, σ' is the level of screening that the bank chooses optimally given the primed contract. Note that we have used the notation $E = e + \Delta$ and $E' = e + \Delta'$ and, further, that we have aggregated the incentive constraints of inside and outside equityholders into a single constraint. Since we will show a deviation that strictly increases the total value of equity, we will always be able to find a division of the surplus β' that makes all equity holders better off. Thus, we write the aggregate constraint down immediately for simplicity.

We will look at a deviation (primed contract) such that $B' - R'D' = B - RD$. This is a sufficient condition for $\sigma' = \sigma$ (this follows from Proposition 4). Substituting for $\sigma' = \sigma$ and reducing these inequalities gives

$$
\begin{aligned}
&\theta\alpha(F' - F) + (1-\theta)\sigma(B' - B) - (E' - E) > \big(\theta\alpha + (1-\theta)\sigma\big)\big(R'D' - RD\big) && (\mathrm{IC_E}) \\
&\big(1 - (1-\theta)\sigma\big)\big(B' - B\big) \geq \delta\theta\alpha(F' - F) && (\mathrm{IC_w}) \\
&\big(\theta\alpha + (1-\theta)\sigma\big)\big(R'D' - RD\big) \geq D' - D. && (\mathrm{IC_D})
\end{aligned}
$$

Note that the right-hand side of $\mathrm{IC_E}$ coincides with the left-hand side of $\mathrm{IC_D}$ in the system above. Thus, a sufficient condition for there to exist R' such that the incentive constraints $\mathrm{IC_E}$ and $\mathrm{IC_D}$ are satisfied is the following condition:

$$\theta\alpha(F' - F) + (1-\theta)\sigma(B' - B) - (E' - E) > D' - D.$$

Since $E' + D' = B'$ and $E + D = B$, the last inequality can be rewritten as

$$\theta\alpha(F' - F) > \big(1 - (1-\theta)\sigma\big)\big(B' - B\big). \tag{72}$$

This constraint and $\mathrm{IC_w}$ are the only two constraints that remain to be satisfied. Note that the left-hand side of $\mathrm{IC_w}$ coincides with the right-hand side of equation (72). Thus, a sufficient condition for there to exist B' such that the constraints are satisfied is

$$\theta\alpha(F' - F) > \delta\theta\alpha(F' - F).$$

This is always satisfied when $\delta < 1$. Thus, if $B < I$ there exists a profitable deviation for the bank, hence $B \geq I$ in equilibrium. Combined with the arguments above, this says that $B = I$ and it completes the proof. □

A.9 Proof of Proposition 4

Since the bank lends if and only if it observes signal s_g its objective function is

$$\begin{aligned} &\mathbb{P}\left[s = s_g\right]\mathbb{P}\left[\tau = g \,|\, s = s_g\right]\alpha\left(F - RD\right) + \mathbb{P}\left[s = s_b\right]\left(I - RD\right) - c(\sigma) \\ &\qquad = \theta\alpha\left(F - RD\right) + (1-\theta)\sigma(I - RD) - \frac{\gamma\sigma^2}{2}. \end{aligned} \tag{73}$$

This objective function is a negative quadratic polynomial in σ, so the first-order condition determines the global maximum, i.e., whenever there is an interior solution $\sigma \in (0, 1)$, the optimal screening precision solves

$$(1-\theta)(I - RD) - \gamma\sigma = 0. \tag{74}$$

When the global maximizer is to the right of the boundary at $\sigma = 1$, $\sigma = 1$ maximizes the quadratic on the domain $[0, 1]$. In contrast, we can see immediately from the first-order condition for $\sigma > 0$ whenever $I > RD$—there is never a corner solution at zero. □

A.10 Proof of Proposition 5

First, we write down the bank's problem to set its borrowing and lending contracts as a constrained maximization program. The constraints are determined by competition. In particular, banks and investors are competitive, so they break even in expectation. The objective function in the program is the expected utility of the household. The reason is that in order to be able to make a loan, a bank must appeal to households who want to borrow. Only banks whose contracts maximize the households' expected utility receive any loan applications, because these are the only banks at which households direct their search. Precisely, the bank must maximize the expected utility of a household subject to four constraints. They are as follows: (i) old shareholders break even; (ii) new shareholders break even; (iii) investors break even; and (iv) amount of deposits available satisfies the investor's wealth constraint. The program is thus

$$\text{Maximize } \big(\theta + (1-\theta)(1-\sigma)\big)I + \delta\theta\alpha(w - F) \tag{75}$$

subject to

$$\beta\Big(\theta\alpha(F - RD) + (1-\theta)\sigma(I - RD) - c(\sigma)\Big) = e, \tag{76}$$

$$(1-\beta)\Big(\theta\alpha(F - RD) + (1-\theta)\sigma(I - RD) - c(\sigma)\Big) = \Delta, \tag{77}$$

$$\Big(\theta\alpha + (1-\theta)\sigma\Big)RD = D, \tag{78}$$

$$e + \Delta + D = I \tag{79}$$

over F, β, Δ, R, and D, where σ is as in Proposition 4.

To prove that $\Delta = 0$ in equilibrium, we suppose an interior solution $\Delta \in (0, I - e)$ and then show that changing D (equivalent to changing Δ) is a profitable deviation for the bank. As a result, it must be that the program has a corner solution. There are two possible corner solutions, $D = 0$ and $D = I - e$. We compare these directly and show that $D = I - e$ corresponds to a higher value of the objective than $D = 0$, so leverage is maximal in equilibrium.

We divide the proof into five steps. In Step 1, we eliminate the variables that appear linearly in the constraints to simplify the program. In Step 2, we consider a marginal change in leverage. We do this by differentiating the objective along the surface defined by the binding constraints. In Step 3, we show that increasing debt either increases the objective function or first increases the objective and then decreases it. In Step 4, we eliminate the possibility that the objective function is first increasing and then decreasing in debt. In Step 5, we conclude that debt is maximal.

Step 1: Reducing the program. We begin the proof by rewriting the bank's program from equations (75)–(79), having substituted for the equilibrium value of screening intensity σ, so the program is to maximize

$$\left(1 - \frac{(1-\theta)^2}{\gamma}\big(I - RD\big)\right) I + \delta\theta\alpha(w - F) \tag{80}$$

subject to

$$\beta\left(\theta\alpha(F-RD)+\frac{(1-\theta)^2}{2\gamma}\big(I-RD\big)^2\right)=e, \tag{81}$$

$$(1-\beta)\left(\theta\alpha(F-RD)+\frac{(1-\theta)^2}{2\gamma}\big(I-RD\big)^2\right)=\Delta, \tag{82}$$

$$\theta\alpha RD+\frac{(1-\theta)^2}{\gamma}\big(I-RD\big)RD=D, \tag{83}$$

$$e+\Delta+D=I \tag{84}$$

over F, β, Δ, R, and D. Now, observe that the system is linear in the variables β, e, and Δ that appear in the constraints. Thus, we can collapse the three equations (81), (82), and (84) into a single equation, which we then add to equation (83), to eliminate these three variables to rewrite the program again, so it is to maximize

$$\left(1-\frac{(1-\theta)^2}{\gamma}\big(I-RD\big)\right)I+\delta\theta\alpha(w-F) \tag{85}$$

subject to

$$\theta\alpha F+\frac{(1-\theta)^2}{2\gamma}\big(I^2-R^2D^2\big)=I, \tag{86}$$

$$\theta\alpha R+\frac{(1-\theta)^2}{\gamma}\big(I-RD\big)R=1. \tag{87}$$

Step 2: The effect of an incremental change in D on the objective. Now suppose that $\Delta > 0$ or, equivalently, $D < I - e$ and consider a marginal increase in D. First, we solve for $\partial F/\partial D$ by differentiating the first constraint (as expressed in equation (86)). This gives

$$\theta\alpha\frac{\partial F}{\partial D}-\frac{(1-\theta)^2 RD}{\gamma}\frac{\partial}{\partial D}(RD)=0. \tag{88}$$

We now differentiate the objective and substitute for $\partial F/\partial D$ from the equation above:

$$\frac{(1-\theta)^2 I}{\gamma}\frac{\partial}{\partial D}(RD)-\delta\theta\alpha\frac{\partial F}{\partial D}=\frac{(1-\theta)^2}{\gamma}\big(I-\delta RD\big)\frac{\partial}{\partial D}(RD). \tag{89}$$

Since $\delta < 1$, $I - \delta RD > I - RD$, which is positive by Maintained Hypothesis 2. Thus, a necessary and sufficient condition for the objective to be increasing in D is for RD to be increasing in D. We will now use the second constraint (equation (87)) to show that this condition is satisfied.

Step 3: Showing the program has a corner solution. Differentiating the second constraint (as expressed in equation (87)) with respect to D gives

$$\left(\theta\alpha + \frac{(1-\theta)^2}{\gamma}\big(I - RD\big)\right)\frac{\partial R}{\partial D} - \frac{(1-\theta)^2}{\gamma}R\frac{\partial}{\partial D}(RD) = 0. \tag{90}$$

We now simplify this equation using the fact that

$$\frac{\partial}{\partial D}(RD) = R + D\frac{\partial R}{\partial D}, \tag{91}$$

to recover

$$\frac{\partial R}{\partial D} = \frac{(1-\theta)^2 R^2}{\theta\alpha\gamma + (1-\theta)^2(I - 2RD)}. \tag{92}$$

Now turn to the condition established above—i.e. that RD is increasing in D—and substitute for $\partial R/\partial D$ from the last equation above (equation (92)) to see exactly when it is satisfied:

$$\frac{\partial}{\partial D}(RD) = R + D\frac{\partial R}{\partial D} \tag{93}$$

$$= \left(\frac{\theta\alpha\gamma + (1-\theta)^2(I - RD)}{\theta\alpha\gamma + (1-\theta)^2(I - 2RD)}\right)R. \tag{94}$$

Maintained Hypothesis 2 (that $I > RD$) implies that the numerator is always positive and the derivative above is never zero. Further, at $D = 0$ the denominator is strictly positive, $\theta\alpha\gamma + (1-\theta)^2 I > 0$. Thus, at $D = 0$, RD is increasing in D and therefore so is the objective function. Thus there are two cases:

Case 1: RD is increasing for $D \in [0, I - e]$

Case 2: There is a point $D^* \in (0, I - e)$ at which the denominator in equation (94) is zero; so RD is increasing on $[0, D^*)$ and then decreasing on $(D^*, I - e]$.

Step 4: Showing that RD must be always increasing in equilibrium.

We now show that Case 2 above cannot obtain in equilibrium. We state this result as a lemma, because it will be useful also in subsequent proofs.

LEMMA 4. *RD is increasing in D on* $[0, I - e]$.

Proof. Suppose (in anticipation of a contradiction) that RD is not increasing in D on $[0, I - e]$. Thus, Case 2 above must obtain. Namely, there is a point D^* after which RD is decreasing in D. At D^* the denominator of equation (94) must be zero. So,

$$RD^* = \frac{1}{2}\left(I + \frac{\theta\alpha\gamma}{(1-\theta)^2}\right). \tag{95}$$

Now, observe that D^* must maximize RD and, therefore, maximize the objective function. Thus, D^* must be the equilibrium level of debt. Hence, we can solve for R by subsisting for RD^* in equation (87):

$$\theta\alpha R + \frac{(1-\theta)^2}{\gamma}\left[I - \frac{1}{2}\left(I + \frac{\theta\alpha\gamma}{(1-\theta)^2}\right)\right]R = 1$$

or

$$R = \frac{2\gamma}{\theta\alpha\gamma + (1-\theta)^2 I}. \tag{96}$$

Solving for D^* from equation (95) now gives

$$\begin{aligned} D^* &= \frac{\left(\theta\alpha\gamma + (1-\theta)^2 I\right)^2}{4\gamma(1-\theta)^2} && (97) \\ &> \frac{(1-\theta)^2 I^2}{4\gamma} && (98) \\ &> I - e, && (99) \end{aligned}$$

where the last line above follows from Assumption 6. But this conclusion that $D^* > I-e$ violates the hypothesis that $D^* \in (0, I-e)$. This is a contraction. Thus, we conclude that Case 1 must obtain, or that RD is increasing in D on $[0, I-e]$. □

Step 5: Concluding the in equilibrium $D = I - e$. We have shown that RD is increasing in D on $[0, I-e]$. From equation (88), this implies that the objective is maximized at the corner $D = I - e$. Thus, in equilibrium, banks maximize leverage, setting $D = I - e$ and $\Delta = 0$. □

A.11 Proof of Proposition 6

To compute the face value of household debt, we use the bank's break-even condition

$$\theta\alpha F + (1-\theta)\sigma I - c(\sigma) = I, \tag{100}$$

analogous to equation (13) in the baseline model.[28]

Combining this with the expression for α from equation (9) and the rational expectations condition $F = \hat{F}$ gives the following quadratic equation for F

$$(y - F)F - \Gamma_D = 0, \tag{101}$$

[28]Note that we work with the break-even condition of all bank claimants for now. We separate the break-even conditions of debt- and equity-holders in Subsection A.13, when we discuss the equilibrium deposit rate.

where the function Γ_D does not depend explicitly on F and is defined as follows:

$$\Gamma_D := \frac{2k}{a^2\theta}\Big(\big(1-(1-\theta)\sigma\big)I + c(\sigma)\Big)$$
$$= \begin{cases} \dfrac{2k}{a^2\theta}\left(I - \dfrac{\big(1-\theta\big)^2\big(I^2-R^2D^2\big)}{2\gamma}\right) & \text{if } 0 < I - RD < \dfrac{\gamma}{1-\theta}, \\ \dfrac{2k}{a^2\theta}\left(\theta I + \dfrac{\gamma}{2}\right) & \text{otherwise.} \end{cases} \tag{102}$$

The quadratic in equation (101) has two solutions. The expressions for these solutions, which we call F_+ and F_-, are given in the statement of the proposition. □

A.12 Proof of Proposition 8

Since, by hypothesis, the equilibrium face value $F < \bar{F} < F_+$, Corollary 4 implies that the equilibrium face value is F_-. Thus, we compute the total derivative $d\alpha/d\Delta$ directly given the face value of debt F_- as

$$\frac{d\alpha}{dD} = -\frac{a^2}{2k\sqrt{y^2-4\Gamma_D}}\left(\frac{\partial \Gamma_D}{\partial D} + \frac{\partial \Gamma_D}{\partial R}\frac{\partial R}{\partial D}\right). \tag{103}$$

Now we consider two cases.

$$\text{Case 1} : I - RD > \frac{\gamma}{1-\theta} \tag{104}$$
$$\text{Case 2} : I - RD \leq \frac{\gamma}{1-\theta}. \tag{105}$$

In Case 1 Γ_D is constant, so

$$\frac{\partial \Gamma_D}{\partial D} = \frac{\partial \Gamma_D}{\partial R} = 0$$

and, thus, $d\alpha/d\Delta = 0$, as desired.

In Case 2 we have

$$\frac{\partial \Gamma_D}{\partial D} = \frac{2kI(1-\theta)^2R^2D}{a^2\theta\gamma} > 0,$$

and

$$\frac{\partial \Gamma_D}{\partial R} = \frac{2kI(1-\theta)^2RD^2}{a^2\theta\gamma} > 0.$$

As a result, a sufficient condition for $d\alpha/dD$ to be negative is that $\partial R/\partial D$ is positive.

Now, R is defined implicitly by the debt holders' break-even condition

$$\frac{\theta a^2}{4k}\left(y + \sqrt{y^2-4\Gamma_D}\right)R + \frac{1}{\gamma}(1-\theta)^2(I-RD)R = 1. \tag{106}$$

So we use the implicit function theorem to determine the sign of $\partial R/\partial D$. Below, we refer to $\partial R/\partial D$ as R'. Implicitly differentiating the equation above we find

$$\frac{\theta a^2}{4k}\left(y+\sqrt{y^2-4\Gamma_D}\right)R' - \frac{\theta a^2 R}{2k\sqrt{y^2-4\Gamma_D}}\left(\frac{\partial \Gamma_D}{\partial R}R' + \frac{\partial \Gamma_D}{\partial D}\right) + \\ + \frac{1}{\gamma}(1-\theta)^2\left[IR' - R^2 - 2RDR'\right] = 0$$

or

$$\left[\frac{\theta a^2}{4k}\left(y+\sqrt{y^2-4\Gamma_D}\right) + \frac{1}{\gamma}(1-\theta)^2(I-RD) - \frac{(1-\theta)^2 RD}{\gamma}\left(\frac{IRD}{\sqrt{y^2-4\Gamma_D}}+1\right)\right]R' = \\ = \frac{(1-\theta)^2R^2}{\gamma}\left(\frac{IRD}{\sqrt{y^2-4\Gamma_D}}+1\right).$$

Thus, R' is positive as long as

$$\frac{\theta a^2}{4k}\left(y+\sqrt{y^2-4\Gamma_D}\right) + \frac{1}{\gamma}(1-\theta)^2(I-RD) - \frac{(1-\theta)^2 RD}{\gamma}\left(\frac{IRD}{\sqrt{y^2-4\Gamma_D}}+1\right) > 0,$$

which holds as long as D is not too large. Since $D = I - e$ in equilibrium, this holds as long as e is not too small. Thus, whenever the bank's initial equity e is not too small, $d\alpha/dD < 0$ or $d\alpha/dE > 0$ as desired. □

A.13 The Equilibrium Deposit Rate

In this section we briefly discuss the equilibrium deposit rate R. Here we derive the equation that defines R implicitly. We do this more to close the model than to derive further results. The following lemma summarizes the result.

LEMMA 5. *If the economy is in an equilibrium associated with F_-, then the deposit rate R_- solves*

$$\frac{\theta a^2}{4k}\left(y+\sqrt{y^2-4\Gamma_D}\right)R + \frac{1}{\gamma}(1-\theta)^2(I-RD)R = 1 \tag{107}$$

If the economy is in an equilibrium associated with F_+, then the deposit rate R_+ solves

$$\frac{\theta a^2}{4k}\left(y-\sqrt{y^2-4\Gamma_D}\right)R + \frac{1}{\gamma}(1-\theta)^2(I-RD)R = 1. \tag{108}$$

The equilibrium deposit rate is the one that makes the investors' break-even condition bind. By Maintained Hypothesis 2, $I > RD$, the bank defaults either when it lends to a b-type household, which occurs with probability $(1-\theta)(1-\sigma)$, or when it lends to a g-type household who remains unemployed, which occurs with probability

$\theta(1-\alpha)$. The investors' break-even condition thus reads

$$\theta\alpha RD + (1-\theta)\sigma RD = D. \tag{109}$$

Recall that α is both the employment rate and repayment rate. Replacing α in the equation above with the expression in terms of the conjectured face value of debt $\hat{F}$, we have

$$\frac{\theta a^2}{2k}\big(y-\hat{F}\big) + (1-\theta)\sigma = \frac{1}{R}. \tag{110}$$

This reveals immediately that the equilibrium deposit rate also depends on banks' belief about the equilibrium they will be in. □

A.14 Sufficiency of Bounds in Assumption 5

Here we show the sufficiency of the bounds stated in Assumption 5 for the matching probabilities to be well-defined. Substituting the equilibrium q from Proposition 1 we can rewrite the condition in equation (3) as

$$a^2(y-F) < 2k < (y-F).$$

Plugging in for for the smallest F from Proposition 2, i.e., F_-, in the left-hand-side of the equation and for the largest F from Proposition 2, i.e., F_+, in the right hand side of the equation we obtain sufficient conditions for the inequality above to hold, namely

$$a^2\left(y+\sqrt{y^2-4\Gamma_D}\right) < 4k < y-\sqrt{y^2-4\Gamma_D}.$$

These bounds are tightest when Γ_D is smallest. Thus, we minimize Γ_D over all possible σ. We do this by minimizing the expression[29]

$$\Gamma_D = \frac{2k}{a^2\theta}\Big(\big(1-(1-\theta)\sigma\big)I + c(\sigma)\Big) \tag{111}$$

and replacing σ with the minimizer

$$\sigma = \frac{(1-\theta)I}{\gamma} \tag{112}$$

to find the expression for $\Gamma_{\min}$ in the statement of the assumption. □

[29] The expression is a negative quadratic, so the first-order condition suffices to find the global minimizer.

A.15 Verification of Maintained Hypothesis $I - RD > 0$

We use the equation

$$\theta\alpha RD + \frac{(1-\theta)^2}{\gamma}(I - RD)RD = D \tag{113}$$

which follows from plugging in for the equilibrium σ in equation (109) to show that the hypothesis $I > RD$ (Maintained Hypothesis 2) holds in equilibrium. This equation says that

$$I - RD = \frac{\gamma(1-\theta\alpha R)}{(1-\theta)^2 R}. \tag{114}$$

Therefore $I > RD$ exactly when

$$1 > \theta\alpha R. \tag{115}$$

This holds by equation (113) since $I - RD > 0$ by hypothesis. $\square$

A.16 Proof of Proposition 9

Since households receive only their wages, and Nash bargaining implies that firms' revenue less wages $y - w$ equal to half the surplus $y - F$, we have

$$y - w = \frac{y - F}{2} \tag{116}$$

or $w = (y + F)/2$ as in Proposition 1.

We now find the employment rate α from the firms' zero-profit condition,

$$\Pi = q\alpha(y - w) - k = 0. \tag{117}$$

Now, just substituting $\alpha(q) = a/\sqrt{q}$ and $w = (y + F)/2$ gives

$$\sqrt{q} = \frac{2k}{a(y - F)} \quad \text{and} \quad \alpha(q) = \frac{a^2(y - F)}{2k} \tag{118}$$

as in Proposition 1. $\square$

A.17 Proof of Proposition 12

We should distinguish between the following two cases:

Case 1: $\lambda h \geq F$,

Case 2: $\lambda h < F$.

In Case 1, the liquidation value of the collateral exceeds the face value of debt, or $\lambda h \geq F$. Here the household never defaults, even if $w = 0$. As a result, we can write household's expected Date 1 utility as

$$v = \alpha(w + h - F) + (1 - \alpha)(\lambda h - F) \tag{119}$$

$$= \alpha(w + h) + (1 - \alpha)\lambda h - F. \tag{120}$$

Here, there is *no interaction between F and α*. Thus, we do not expect household leverage to distort household preferences, inducing risk shifting. As such, the vacancy posting effect is not at work when collateral liquidation values are sufficiently high, or $\lambda h \geq F$.

Now turn to Case 2, in which the liquidation value of collateral is less than the face value of debt, or $\lambda h < F$. In this case, the household defaults when it is unemployed, because without wages it cannot repay its debt even if it liquidates its collateral. As a result, we can write the household's expected Date 1 utility as

$$v = \alpha(w + h - F). \tag{121}$$

Except for the constant h, this expression coincides with the expression in the baseline model without collateral, as can be seen from a comparison with equation (10). Specifically, a household with collateral value h in this model has equivalent preferences to a household with debt $F' := F - h$ in the baseline model. Thus, *whenever liquidation values of collateral are low, the vacancy-posting effect is robust to the inclusion of collateral* and we view the baseline model as just a normalization of collateral values to zero. □

A.18 Table of Notations

	Labor Market
q_w	the queue length in the submarket associated with wage w
q	the equilibrium queue length
α	probability that a household is matched with a firm
$q\alpha$	probability that a firm is matched with a household
	Households
v	households' Date 1 expected utility
u	households' single date utility ("felicity") function
I	households' utility parameter—i.e. the point of the kink in households' utility
c_t	households' consumption at Date t
δ	households' discount factor
$\tau \in \{g, b\}$	household type
	Banks
$s \in \{s_g, s_b\}$	signal about household type
σ	banks' screening precision
$c(\sigma)$	banks' cost of screening
γ	cost function parameter
e	banks' initial equity
Γ_D	shorthand notation defined in equation (102)
	Firms
y	firms' output
k	firms' search cost
Π	firms' profit
	Contracts
B	the amount households' borrow from banks
F	the face value of household debt
F_-, F_+	the two possible equilibrium face values of household debt
D	the amount banks borrow from investors via debt
R	the interest rate on bank deposits
Δ	the amount banks raise from investors via equity
β	the proportion of equity retained by initial equity holders
E	bank's total equity $e + \Delta$
w	the wage firms pay households

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