A Model of Financialization of Commodities*

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Abstract

A sharp increase in the popularity of commodity investing in the past decade has triggered an unprecedented inflow of institutional funds into commodity futures markets, referred to as the financialization of commodities. In this paper, we explore the effects of financialization in a model that features institutional investors alongside traditional futures markets participants. The institutional investors care about their performance relative to a commodity index. We find that in the presence of institutional investors prices and volatilities of all commodity futures go up, but more so for the index futures than for nonindex ones. The correlations amongst commodity futures as well as in equity-commodity correlations also increase, with higher increases for index commodities. Within a framework additionally incorporating storage, we show how financial markets transmit shocks not only to futures prices but also to commodity spot prices and inventories. Commodity spot prices and inventories go up with financialization. In the presence of institutional investors shocks to any index commodity spill over to all storable commodity prices.

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1. Introduction

In the past decade the behavior of commodity prices has become highly unusual. Commodity prices have experienced significant run-ups, and the nature of their fluctuations has changed considerably. An emerging literature on financialization of commodities attributes this behavior to the emergence of commodities as an asset class, which has become widely held by institutional investors seeking diversification benefits (Buyuksahin and Robe (2014), Singleton (2014)). Starting in 2004, institutional investors have been rapidly building their positions in commodity futures.[1] CFTC staff report (2008) estimates institutional holdings to have increased from $15 billion in 2003 to over $200 billion in 2008. Many of the institutional investors hold commodities through a commodity futures index, such as the Goldman Sachs Commodity Index (GSCI), the Dow Jones UBS Commodity Index (DJ-UBS) or the S&P Commodity Index (SPCI). Tang and Xiong (2012) document that after 2004 the behavior of index commodities has become increasingly different from those of nonindex, with the former becoming more correlated with oil, an important index constituent, and more correlated with the equity market. These intriguing facts could be attributed to the entry of institutional investors into commodity futures markets. The financialization theory has far-reaching implications for regulation: the 2004-2008 run-up in commodity prices has prompted many calls for curtailing positions of institutions who may have generated the run-up (see Masters' (2008) testimony).

While the empirical literature on financialization of commodities has been influential and has contributed to the policy debate, theoretical literature on the subject remains scarce. Our goal in this paper is to model the financialization of commodities and to disentangle the effects of institutional flows from the traditional demand and supply effects on commodity futures prices. We particularly focus on identifying the economic mechanisms through which institutions may influence commodity futures prices, volatilities, and their comovement, as well as how their presence may affect commodity spot prices and inventories.

We develop a multi-good, multi-asset dynamic model with institutional investors and standard futures markets participants. The institutional investors care about their performance

[1] Related empirical literature dates the start of the financialization of commodity futures around 2004 (Buyuksahin et al. (2008), Irwin and Sanders (2011), Tang and Xiong (2012), Hamilton and Wu (2014), Boons, de Roon, and Szymanowska (2014), among others), and some of these works explicitly test for and confirm a structural break around 2004.

relative to a commodity index. They do so because their investment mandate specifies a benchmark index for performance evaluation or because their mandate includes hedging against commodity price inflation. We capture such benchmarking through the institutional objective function. Consistent with the extant literature on benchmarking (originating from Brennan (1993)), we postulate that the marginal utility of institutional investors increases with the index. In particular, institutional investors dislike to perform poorly when their benchmark index does well and so have an additional incentive to do well when their benchmark does well.[2] All investors in our model invest in the commodity futures markets and the stock market. Prices in these markets fluctuate in response to three possible sources of shocks: (i) commodity supply shocks, (ii) commodity demand shocks, and (iii) (endogenous) changes in wealth shares of the two investor classes. We include in the index only a subset of the traded futures contracts. We can then compare a pair of otherwise identical commodities, one of which belongs to the index and the other does not. We capture the effects of financialization by comparing our economy with institutional investors to an otherwise identical benchmark economy with no institutions. The model is solved in closed form, and all our results below are derived analytically.

We first find that the prices of all commodity futures go up with financialization. However, the price rise is higher for futures belonging to the index than for nonindex ones. This happens because institutions strive to not fall behind when the index does well, thereby they value assets that pay off more in those high-index states. Hence, relative to the benchmark economy without institutions, futures whose returns are positively correlated with those of the index are valued higher. In our model, all futures are positively correlated because they are valued using the same discount factor, and so all futures prices go up with financialization. But, naturally, the comovement with the index is higher for futures included in the index. Therefore, prices of index futures rise more than those of nonindex. The larger the institutions, the more they affect pricing—or, more formally, the discount factor—making the above effects stronger.

The volatilities of both index and nonindex futures returns go up with financialization. This is because, absent institutions, there are only two sources of risk: supply and demand risks. With institutions present, some agents in the economy (institutional investors) face an additional risk of falling behind the index. This risk is reflected in the futures prices and it

[2] One may reasonably argue that there is also a category of institutional investors who want to perform well when the index does poorly (e.g., hedge funds).

raises the volatilities of futures returns. While the volatilities of all futures rise, those of index futures rise by more. Index futures are especially attractive to institutional investors because of their high comovement with the index. Hence, their volatilities rise enough to make them unattractive to the normal investors (standard market participants) so that they are willing to sell them to the institutions.

We also find that the correlations amongst commodity futures as well as in the equity-commodity correlations increase with financialization. The often-quoted intuition for this rise is that commodity futures markets were largely segmented before the inflow of institutional investors in mid-2000s, and institutions that entered these markets have linked them together, as well as with the stock market, through cross-holdings in their portfolios. We show that the argument does not need to rely on market segmentation, and that the rise in correlations may occur even under complete markets. Benchmarking institutional investors to a commodity index leads to the emergence of this index as a new (common) factor in commodity futures and stock returns. In equilibrium, all assets load positively on this factor, which increases their covariances and their correlations. We show that index commodity futures are more sensitive to this new factor, and so their covariances and correlations with each other rise more than those for otherwise identical nonindex commodities. Furthermore, we model explicitly demand shocks which allows us to disentangle the effects of institutional investors from the effects of demand and supply (fundamentals), and conclude that the effects of financialization are sizeable.

To address the question of how commodity spot prices and inventories are affected by financialization in our model, we follow the classical theory of storage (Deaton and Laroque (1992, 1996)) and introduce intermediate consumption and storage decisions. Our main departure from the extant storage literature is that cash flows from storing a commodity are discounted with a (stochastic) discount factor, which is influenced by all investors, including the institutions, and not at a constant riskless rate. We show that only storable commodity prices are affected by financialization. In the presence of institutions, storable commodity inventories and prices are higher than in the benchmark economy, and again, this effect is stronger for commodities included in the index. Storing a commodity is akin to buying an asset whose payoff is the commodity price in the future net of storage costs. In our model this payoff is positively related to the payoff of the commodity index and hence the same intuition developed in the context of futures prices also applies to spot prices of storable commodities. Because the discount factor is affected by the institutional investors (and depends on the index), outside shocks to index commodities spill over to prices and inventories of seemingly unrelated commodities. In contrast, there are no spillovers of shocks to nonindex commodities. Outside shocks here are broad and include shocks to a specific index commodity (related to its supply, supply volatility, or demand), stock market related (stock volatility and return), as well as inflows of institutions. These results challenge commonly held views that such shocks do not matter for commodity spot prices.

1.1. Related Literature

This paper is related to several strands of literature. The two papers that have motivated this work are Tang and Xiong (2012) and Singleton (2014). Singleton examines the 2008 boom/bust in oil prices and argues that flows from institutional investors have contributed significantly to that boom/bust. Tang and Xiong document that the comovement between oil and other commodities has risen dramatically following the inflow of institutional investors starting from 2004, and that the commodities belonging to popular indices have been affected disproportionately more. There was no difference in comovement patterns of index and nonindex commodities pre-2004. Using a proprietary dataset from the CFTC, Buyuksahin and Robe (2014) investigate the recent increase in the correlation between equity indices and commodities and argue that this phenomenon is due to the presence of hedge funds that are active in both equity and commodity futures markets. Recently, Henderson, Pearson, and Wang (2015) present new evidence on the financialization of commodity futures markets based on commodity-linked notes.

The impact of financialization on commodity futures and spot prices is the subject of much ongoing debate. Surveys by Irwin and Sanders (2011) and Fattouh, Kilian, and Mahadeva (2013) challenge the view that increased speculation in oil futures markets in post-financialization period was an important determinant of oil prices. Kilian and Murphy (2014) attribute the 2003-2008 oil price surge to global demand shocks rather than speculative demand shifts. Hamilton and Wu (2015) examine whether commodity index-fund investing had a measurable effect on commodity futures prices and find little evidence to support this hypothesis.

There still remains lack of agreement whether trades by institutional investors affect commodity futures prices. Our view is that, given the size of commodity index traders' (a proxy for institutional investors) futures holdings in the data, it is natural to expect that such traders affect prices. Furthermore, similar effects are reasonably well-established in other markets, especially equity markets. Starting from Harris and Gurel (1986) and Shleifer (1986), a large body of work documented that prices of stocks that are added to the S&P 500 and other indices increase following the announcement and prices of stocks that are deleted drop—a phenomenon widely attributed to the price pressure from institutional investors. Relatedly, a variety of studies document the so-called "asset class" effects: the "excessive" comovement of assets belonging to the same index or other visible category of stocks (e.g., Barberis, Shleifer, and Wurgler (2005) for the S&P500 vis-à-vis non-S&P500 stocks, Boyer (2011) for BARRA value and growth indices). These effects are attributed to the presence of institutional investors.

The closest theoretical work on the effects of institutions on asset prices is the Lucas-tree economy of Basak and Pavlova (2013). Basak and Pavlova focus on index and asset class effects in equity markets. Their model does not feature multiple commodities, nor is it designed to address some of the main issues in the debate on financialization; namely, whether institutional investors impact commodity futures and spot prices, as well as inventories. Another related theoretical study of an asset-class effect is by Barberis and Shleifer (2003), whose explanation for this phenomenon is behavioral. However, they also do not explicitly model commodities and so cannot address some questions specific to the current debate on the financialization of commodities.

Finally, there is a large and diverse literature going back to Keynes (1923) that studies the determination of commodity spot prices in production economies with storage and links the physical markets for commodities with the commodity futures markets.[3] In this literature, Baker (2013) studies the financialization of a storable commodity. His interpretation of financialization is the reduction in transaction costs of households for trading futures. He focuses on a single commodity, while we consider multiple commodities, distinguishing between index and nonindex ones. More generally, we contribute to this literature by modeling shareholders

[3]In this strand of literature, a recent paper by Sockin and Xiong (2015) shows that price pressure from investors operating in futures markets (even if driven by nonfundamental factors) can be transmitted to spot prices of underlying commodities. Acharya, Lochstoer, and Ramadorai (2013) stress the importance of capital constraints of futures' markets speculators and argue that frictions in financial (futures) markets can feedback into production decisions in the physical market. In a similar framework, Gorton, Hayashi, and Rouwenhorst (2013) derive endogenously the futures basis and the risk premium and relate them to inventory levels. Routledge, Seppi, and Spatt (2000) derive the term structure of forward prices for storable commodities, highlighting the importance of the non-negativity constraints on inventories.

of storage firms as risk-averse investors (some of which could be institutions), and highlight the influence of our discount factor channel and its role in generating cross-commodity spillovers.

Methodologically, this paper contributes to the asset pricing literature by providing a tractable multi-asset general equilibrium model with heterogeneous investors which is solved in closed form. Pavlova and Rigobon (2007) and Cochrane, Longstaff, and Santa-Clara (2008) highlight the complexities of multi-asset models and provide analytical solutions for the two-asset case. As Martin (2013) demonstrates, the general multi-asset case presents a formidable challenge. In contrast, our multi-asset model is surprisingly simple to solve. Our innovation is to replace Lucas trees considered in the above literature by zero-net-supply assets (futures) and model only the aggregate stock market as a Lucas tree. The model then becomes just as simple and tractable as a single-tree model.

The remainder of the paper is organized as follows. Sections 2 and 3 present our model and demonstrate how institutional investors affect commodity futures prices, volatilities, and their comovement. Section 4 examines the effect of institutions on commodity inventories and spot prices and Section 5 concludes. Appendix A provides all proofs and the online Appendix B presents the economy with demand shocks.

2. The Model

Our goal in this section is to develop a simple and tractable model of commodity futures markets. We consider a pure-exchange multi-good, multi-asset economy with a finite horizon T. Uncertainty is resolved continuously, driven by a K+1-dimensional standard Brownian motion $\omega \equiv (\omega_0, \ldots, \omega_K)^\top$. All consumption in the model occurs at the terminal date T, while trading takes place at all times $t \in [0, T]$.

Commodities. There are K commodities (goods), indexed by $k = 1, \ldots K$. The date-T supply of commodity k, D_{kT}, is the terminal value of the process D_{kt}, with dynamics

$$dD_{kt} = D_{kt}[\mu_k dt + \sigma_k d\omega_{kt}], \tag{1}$$

where μ_k and $\sigma_k > 0$ are constant. The process D_{kt} represents the arrival of news about D_{kT}. We refer to it as the commodity-k *supply news.* The price of good k at time t is denoted by p_{tk}. There is one further good in the economy, commodity 0, which we refer to as the generic good. This good subsumes all remaining goods consumed in the economy apart from the K commodities that we have explicitly specified above and it serves as the numeraire. The date-T supply of the generic good is D_T, which is the terminal value of the supply news process

$$dD_t = D_t[\mu dt + \sigma d\omega_{0t}], \tag{2}$$

where μ and $\sigma > 0$ are constant. Our specification implies that the supply news processes are uncorrelated across commodities ($dD_{kt}\, dD_{it} = 0,\ dD_{kt}\, dD_t = 0,\ \forall k,\ k \neq i$). This assumption is for expositional simplicity; it can be relaxed in future work.

Financial Markets. Available for trading are K standard futures contracts written on commodities $k = 1, \ldots, K$. A futures contract on commodity k matures at time $\tau < T$ and, upon maturity, gets rolled over to the next contract with maturity τ. This gets repeated till time T, at which time consumption takes place. The payoff of the contract is one unit of commodity k. Each contract is continuously resettled at the futures price f_{kt} and is in zero net supply. The gains/losses on each contract are posited to follow

$$df_{kt} = f_{kt}[\mu_{f_k t} dt + \sigma_{f_k t} d\omega_t], \tag{3}$$

where $\mu_{f_k t}$ and the $K+1$ vector of volatility components $\sigma_{f_k t}$ are determined endogenously in equilibrium (Section 3).

Our model makes a distinction between index and nonindex commodities because we seek to examine theoretically the asset class effect in commodity futures documented by Tang and Xiong (2012). A *commodity index* includes the first L commodities, $L \leq K$, and is defined as

$$I_t = \prod_{i=1}^{L} f_{it}^{1/L}. \tag{4}$$

This index represents a geometrically-weighted commodity index such as, for example, the S&P Commodity Index (SPCI). For expositional simplicity, our index weighs all commodities equally; this assumption is easy to relax.[4]

In addition to the futures markets, investors can trade in the stock market, S, and an instantaneously riskless bond. The stock market is a claim to the entire output of the economy

[4]To model other major commodity indices such as the Goldman Sachs Commodity Index and the Dow Jones UBS Commodity Index, it is more appropriate to define the index as $I_t = \sum_{i=1}^{L} w_i f_{it}$, where the weights w_i add up to one. Such a specification is less tractable but one can show numerically that most of the implications are in line with those in our analysis below.

at time T: $D_T + \sum_{k=1}^{K} p_{kT} D_{kT}$. It is in positive supply of one share and is posited to have price dynamics given by

$$dS_t = S_t[\mu_{St}dt + \sigma_{St}d\omega_t], \tag{5}$$

with μ_{St} and $\sigma_{St} > 0$ endogenously determined in equilibrium. The bond in zero net supply. It pays a riskless interest rate r, which we set to zero without loss of generality.[5]

We note that our formulation of asset cash flows is standard in the asset pricing literature. The main distinguishing characteristic of our model is that it avoids the complexities of multi-tree economies. This is because only the stock market is in positive net supply, while all other assets (futures) are in zero net supply. As we demonstrate in the ensuing analysis, this model is just as simple and tractable as a single-tree model.

Investors. The economy is populated by two types of market participants: normal investors, $\mathcal{N}$, and institutional investors, $\mathcal{I}$. The (representative) normal investor is a standard market participant, with logarithmic preferences over the terminal value of her portfolio:

$$u_{\mathcal{N}}(W_{\mathcal{N}T}) = \log(W_{\mathcal{N}T}), \tag{6}$$

where $W_{\mathcal{N}T}$ is wealth or consumption.

The institutional investor's objective function, defined over his terminal portfolio value (consumption) $W_{\mathcal{I}T}$, is given by

$$u_{\mathcal{I}}(W_{\mathcal{I}T}) = (a + bI_T)\log(W_{\mathcal{I}T}), \tag{7}$$

where a, $b > 0$. The institutional investor is modeled along the lines of Basak and Pavlova (2013), who study institutional investors in the stock market and also provide microfoundations for such an objective function, as well as a status-based interpretation.[6] The objective function has two key properties: (i) it depends on the index level I_T and (ii) the marginal utility of wealth is increasing in the benchmark index level I_T. This captures the notion of benchmarking: the

[5] This is a standard feature of models that do not have intermediate consumption. In other words, there is no intertemporal choice that would pin down the interest rate. Our normalization is commonly employed in models with no intermediate consumption (see e.g., Pastor and Veronesi (2012) for a recent reference).

[6] Direct empirical support for the status-based interpretation of our model is provided in Hong, Jiang, and Zhao (2015), who adopt the formulation in (7) in their analysis. Empirical work estimating objectives of institutional investors remains scarce, with a notable exception of Koijen (2014). There is also a broader interpretation of the specification in (7). Recently consumers have become more sensitive to commodity prices, and one way to capture this is via a formulation along the lines of (7). We thank an anonymous referee for this interpretation.

institutional investor is evaluated relative to his benchmark index and so he cares about the performance of the index. When the benchmark index is relatively high, the investor strives to catch up and so he values his marginal unit of performance highly (his marginal utility of wealth is high). When the index is relatively low, the investor is less concerned about his performance (his marginal utility of wealth is low). We use the commodity market index as the benchmark index because in this work we attempt to capture institutional investors with the mandate to invest in commodities, most of whom are evaluated relative to a commodity index. An alternative interpretation of the objective function is that the institutional investor has a mandate to hedge commodity price inflation; i.e., deliver higher returns in states in which the commodity price index is high.[7] We think of the terminal date T as the performance evaluation horizon of institutional asset managers, usually 3-5 years (BIS (2003)).

In this multi-good world, terminal wealth is defined as an aggregate over all goods, a consumption index (or consumption). We take the index to be Cobb-Douglas, i.e.,

$$W_n = C_{n_0}^{\alpha_0} C_{n_1}^{\alpha_1} \cdot \ldots \cdot C_{n_K}^{\alpha_K}, \qquad n \in \{\mathcal{N}, \mathcal{I}\}, \tag{8}$$

where $\alpha_k > 0$ for all k. For the case of $\sum_{k=0}^{K} \alpha_k = 1$, the parameter α_k represents the expenditure share on good k. Here we are considering a general Cobb-Douglas aggregator in which the weights do not necessarily add up to one, and hence we label α_k as the "commodity demand parameter."[8] We take the commodity demand parameters to be the same for all investors in the economy. Heterogeneity in demand for specific commodities is not the dimension we would like to focus on in this paper.

A change in α_k represents a demand shift towards commodity k. A change in the demand parameter α_k is the simplest and most direct way of modeling a demand shift, i.e., an outward movement in the entire demand schedule, as typical in classical demand theory (Varian (1992)).[9] In Section 3.3, we allow the demand parameters α_k to be stochastic, in order to capture a more realistic environment with demand shocks. Until then, we keep them constant so as to isolate

[7] One could reasonably argue that there is also a category of institutional investors whose marginal utility is decreasing in the index level, for example, hedge funds which may prefer higher payoffs when the index does poorly.

[8] In what follows, we are interested in comparative statics with respect to α_k. The expenditure share on commodity k, $\alpha_k / \sum_{k=0}^{K} \alpha_k$, is monotonically increasing in α_k. Hence all our comparative statics for α_k are equally valid for expenditure shares $\alpha_k / \sum_{k=0}^{K} \alpha_k$.

[9] For example, an increase in demand for soya beans due to the invention of biofuels and concerns about the environment.

the effects of supply shocks and the effects of financialization (fluctuations in institutional wealth invested in the market) on commodity futures prices.

The institutional and normal investors are initially endowed with fractions $\lambda \in [0, 1]$ and $(1-\lambda)$ of the stock market, providing them with initial assets worth $W_{\mathcal{I}0} = \lambda S_0$ and $W_{\mathcal{R}0} = (1-\lambda)S_0$, respectively. We will often refer to the parameter λ as the size of institutions.

Starting with initial wealth W_{n0}, each type of investor $n = \mathcal{N}, \mathcal{I}$, dynamically chooses a portfolio process $\phi_n = (\phi_{n_1}, \ldots, \phi_{n_K})^\top$, where ϕ_n and ϕ_{n_S} denote the fractions of the portfolio invested in the futures contracts 1 through K and the stock market, respectively. The wealth process of investor n, W_n, then follows the dynamics

$$dW_{nt} = W_{nt} \sum_{k=1}^{K} \phi_{n_k t}[\mu_{f_k t}dt + \sigma_{f_k t}d\omega_t] + W_{nt}\phi_{n_S t}[\mu_{St}dt + \sigma_{St}d\omega_t]. \tag{9}$$

3. Equilibrium Effects of Financialization of Commodities

We are now ready to explore how the financialization of commodities affects equilibrium prices, volatilities, and correlations. In order to understand the effects of financialization, we will often make comparisons with equilibrium in a *benchmark economy*, in which there are no institutional investors. We can specify such an economy by setting $b = 0$ in (7), in which case the institution in our model no longer resembles a commodity index trader and behaves just like the normal investor. Another way to capture the benchmark economy within our model is to set the fraction of institutions, λ, to zero.

Equilibrium in our economy is defined in a standard way: equilibrium portfolios, asset and time-T commodity prices are such that (i) both the normal and institutional investors choose their optimal portfolios, and (ii) futures, stock, bond and time-T commodity markets clear. Letting $M_{t,T}$ to denote the (stochastic) discount factor or the pricing kernel in our model, by no-arbitrage, the futures prices are given by

$$f_{kt} = E_t[M_{t,T}\, p_{kT}]. \tag{10}$$

The discount factor $M_{t,T}$ is the marginal rate of substitution of any investor, e.g., the normal investor, in equilibrium.

To develop intuitions for our results, it is useful to examine the time-T prices prevailing in our equilibrium. These are reported in the following lemma.

Lemma 1 (Time-T equilibrium quantities). *In equilibrium with institutional investors, we obtain the following characterizations for the terminal date quantities.*

$$\textit{Commodity prices:} \quad p_{kT} = \frac{\alpha_k}{\alpha_0}\frac{D_T}{D_{kT}}; \quad p_{kT} = \overline{p}_{kT}, \tag{11}$$

$$\textit{Commodity index:} \quad I_T = \frac{D_T}{\alpha_0}\prod_{i=1}^{L}\left(\frac{\alpha_i}{D_{iT}}\right)^{1/L}; \quad I_T = \overline{I}_T, \tag{12}$$

$$\textit{Stock market value:} \quad S_T = D_T\sum_{k=0}^{K}\frac{\alpha_k}{\alpha_0}; \quad S_T = \overline{S}_T, \tag{13}$$

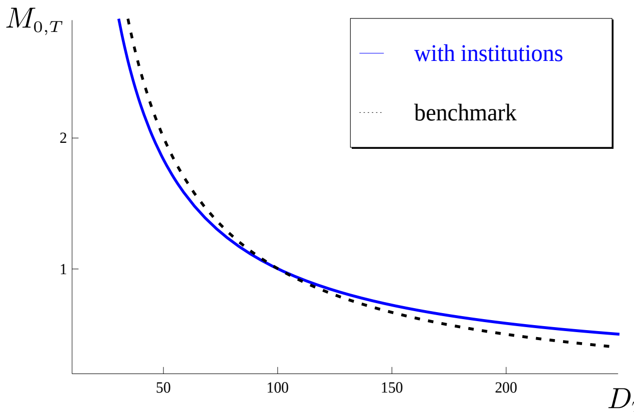
$$\textit{Discount factor:} \quad M_{0,T} = \overline{M}_{0,T}\left(1 + \frac{b\,\lambda(I_T - E[I_T])}{a + b\,E[I_T]}\right), \quad \overline{M}_{0,T} = \frac{e^{(\mu-\sigma^2)T}D_0}{D_T}, \tag{14}$$

where the expectation of the time-T index value, $E[I_T]$, is provided in the Appendix. The quantities with an upper bar denote the corresponding equilibrium quantities prevailing in the economy with no institutions.

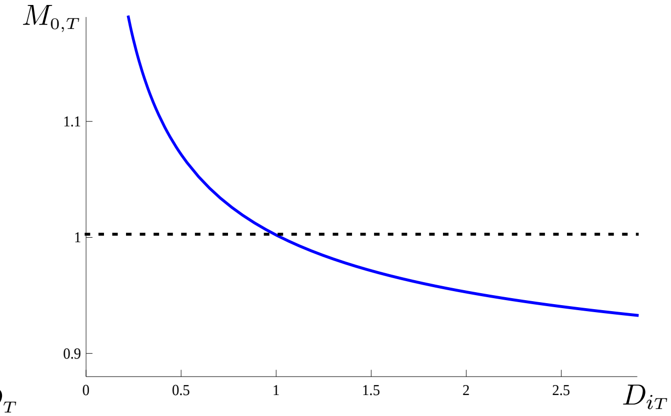
Lemma 1 reveals that the price of good k decreases with the supply of that good D_{kT}. As supply D_{kT} increases, good k becomes relatively more abundant. Hence, its price falls. A rise in the supply of the generic good D_T has the opposite effect. Now good k becomes more scarce relative to the generic good. Hence, its price rises. These are classical supply-side effects. A positive shift in α_k represents an increase in demand for good k. As a consequence, the price of good k goes up. This is a classical demand-side effect. Since the index is given by $I_T = \prod_{i=1}^{L} p_{iT}^{1/L}$, the terminal index value inherits the properties of the individual commodity prices. In particular, it declines when the supply of any index commodity i D_{iT} goes up, and rises when the supply of the generic good D_T rises.

We note that the time-T prices of commodities, and hence the commodity index coincide with their values in the benchmark economy with no institutions. We have intentionally set up our model in this way. By effectively abstracting away from the effects of financialization on underlying cash flows in (10), we are able to elucidate the effects of institutions in the futures markets coming via the discount factor channel. Financialization, however, can potentially affect time-t, $t < T$, commodity prices in our model. We explore this in Section 4.

The stock market is a claim against the aggregate output of all goods in the economy, $D_T + \sum_{k=1}^{K} p_{kT}D_{kT}$, which in this model turns out to be proportional to the aggregate supply of the generic good D_T, due to the investors' Cobb-Douglas consumption aggregators. So the aggregate wealth in the economy, the stock market value S_T, in equilibrium is simply a scaled supply of the generic good D_T. The quantity D is an important state variable in our model. In what follows, we will refer to it as aggregate wealth, or equivalently, aggregate output.



(a) Effect of aggregate output D_T



(b) Effect of index commodity supply D_{iT}

Figure 1: **Discount factor.** This figure plots the discount factor in the presence of institutions against aggregate output D_T and against an index commodity supply D_{iT}. The dotted lines correspond to the discount factor in the benchmark economy with no institutions. The plots are typical. The parameter values, when fixed, are: $L = 2$, $K = 5$, $a = 1$, $b = 1$, $T = 5$, $\lambda = 0.4$, $\alpha_0 = 0.7$, $D_T = D_0 = 100$, $D_{kT} = D_{k0} = 1$, $\mu = \mu_k = 0.05$, $\sigma = 0.15$, $\sigma_k = 0.25$, $\alpha_k = 0.06$, $k = 1, \ldots K$.[11]

In the benchmark economy, the discount factor depends only on aggregate output D_T. It bears the familiar inverse relationship with aggregate output (dotted line in Figure 1a), implying that assets with high payoffs in low-D_T (bad) states get valued higher. In the presence of institutions, the discount factor is also decreasing in aggregate output D_T, albeit at a slower rate. That is, the presence of institutions makes the discount factor less sensitive to news about aggregate output. Additionally, now the discount factor becomes dependent on the supply of each index commodity D_{iT} (Figure 1b). The channel through which institutions affect the discount factor is apparent from equation (14): the discount factor now becomes dependent on the performance of the index, pricing high-index states higher. This is the channel through which financialization affects asset prices in our model.

The new financialization channel works as follows. Institutional investors have an additional

[11]We discuss the parameter choices in the (online) Appendix B.

incentive to do well when the index does well. So relative to normal investors, they strive to align their performance with that of the index, performing better when the index does well in exchange for performing poorer when the index does poorly. As highlighted in our discussion of the equilibrium index value in (12), the index does well when aggregate output D_T is high and supply of index commodity D_{iT} is low. Because of the additional demand from institutions, these states become more "expensive" relative to the benchmark economy (higher Arrow-Debreu state prices or higher discount factor $M_{0,T}$). The financialization channel thus counteracts the benchmark economy inverse relation between the discount factor $M_{0,T}$ and aggregate output, making the discount factor less sensitive to aggregate output D_T (as evident from Figure 1a). Additionally, it also makes the discount factor dependent on and decreasing in each index commodity supply D_{iT}.

The graphs in Figure 1 are important because they underscore the mechanism for the valuation of assets in the presence of institutions. In particular, assets that pay off high in states in which the index does well (high D_T and low D_{iT}) are valued higher than in the benchmark economy with no institutions.

3.1. Equilibrium Commodity Futures Prices

Proposition 1 (Futures prices). *In the economy with institutions, the equilibrium futures price of commodity $k = 1, \ldots, K$ is given by*

$$f_{kt} = \overline{f}_{kt} \frac{a + b(1-\lambda) D_0 \prod_{i=1}^{L} \left(g_i(0)/D_{i0}\right)^{1/L} + b\,\lambda\, e^{1_{\{k \leq L\}} \sigma_k^2 (T-t)/L} D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}}{a + b(1-\lambda) D_0 \prod_{i=1}^{L} \left(g_i(0)/D_{i0}\right)^{1/L} + b\,\lambda\, e^{-\sigma^2 (T-t)} D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}}, \quad (15)$$

where the equilibrium futures price in the benchmark economy with no institutions $\overline{f}_{kt}$ and the quantity $g_i(t)$ are given by

$$\overline{f}_{kt} = \frac{\alpha_k}{\alpha_0} e^{(\mu - \mu_k - \sigma^2 + \sigma_k^2)(T-t)} \frac{D_t}{D_{kt}}, \qquad g_i(t) = \frac{\alpha_i}{\alpha_0} e^{(\mu - \mu_i + (1/L+1)\sigma_i^2/2)(T-t)} \quad (16)$$

Consequently, in the presence of institutions,

(i) The futures prices are higher than in the benchmark economy, $f_{kt} > \overline{f}_{kt}$, $k = 1, \ldots, K$.

(ii) The index futures prices rise more than nonindex ones for otherwise identical commodities, i.e., for commodities i and k with $D_{it} = D_{kt}$, $\forall t$, $\alpha_i = \alpha_k$, $i \leq L$, $L < k \leq K$.

Proposition 1 reveals that the commodity futures prices in the benchmark economy with no institutions $\overline{f}_{kt}$ inherit the features of time-T futures prices highlighted in Lemma 1. The benchmark economy futures prices rise in response to positive news about aggregate output D_t and fall in response to positive news about the supply of commodity k, D_{kt}. In contrast, in the economy with institutions the commodity futures prices f_{kt} depend not only on own supply news D_{kt} but also those of all *index* commodities D_{it}. Other characteristics of index commodities such as expected growth in their supply μ_i, volatility σ_i and their demand parameters α_i now also affect the prices of all futures traded in the market. Note that, just like in the benchmark economy, supply news D_k and other characteristics of *nonindex* commodities have no spillover effects on other commodity futures. Since there is one consumption date, however, we do not have a meaningful term structure of futures prices. All contracts get rolled over until time T, and so the maturity of a specific futures contract τ does not enter (15).

To understand why all futures prices go up (property (i) of Proposition 1), recall that the institutional investors desire high payoffs in states when the index does well. They therefore value assets that pay off highly in those states. All futures in the model are positively correlated with the index even in the benchmark economy because they are all priced using the common discount factor. Hence, all futures prices rise. However, the prices of index futures rise by more (property (ii)). The institutions specifically desire the futures that are included in the index because, naturally, the best way to achieve high payoffs in states when the index does well is to hold index futures. Therefore, index futures have higher prices than otherwise identical nonindex ones.[12]

Corollary 1. *The equilibrium commodity futures prices have the following additional properties.*

(i) All commodity futures prices f_{kt} are increasing in the size of institutions λ, $k = 1, \ldots, K$.

(ii) All commodity futures prices are more sensitive to aggregate output D_t than in the benchmark economy with no institutions; i.e., f_{kt} is increasing in D_t at a faster rate than does $\overline{f}_{kt}$, $k = 1, \ldots, K$. Moreover, index commodity futures are more sensitive to aggregate output that nonindex ones for otherwise identical commodities.

(iii) All commodity futures prices f_{kt}, $k = 1, \ldots, K$, react negatively to positive supply news of index commodities D_{it}, $i = 1, \ldots, L$, $k \neq i$, while in the benchmark economy such a price

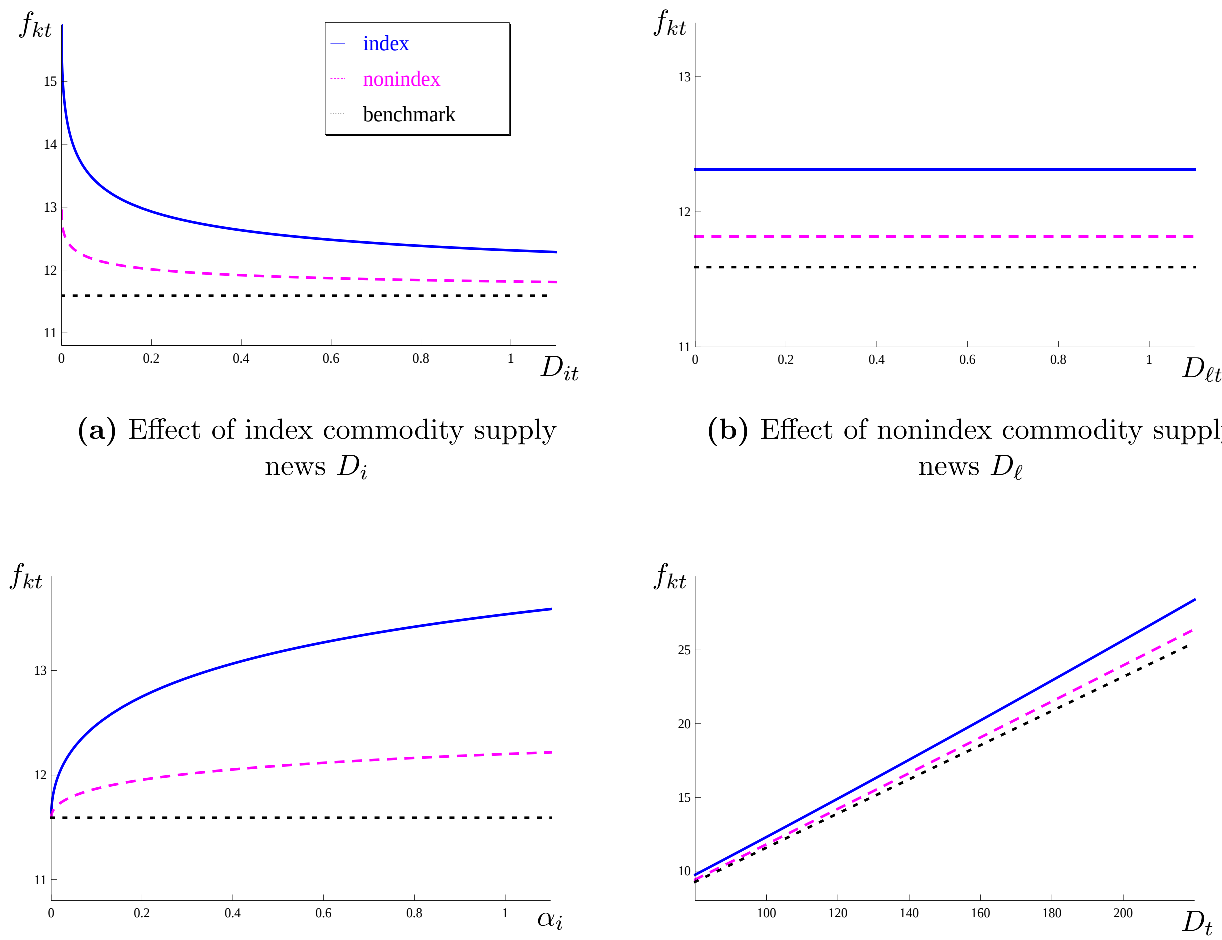
[12] One major difference of this model from the the one-good stock market economy of Basak and Pavlova (2013) is that in their analysis nonindex security prices are unaffected by the presence of institutions, although the institutions are modeled similarly. Consequently, in contrast to our findings, their nonindex assets have zero correlation among themselves and with index assets, and the nonindex asset prices and volatilities are not affected by institutional investors. The key reason for these differences is that in Basak and Pavlova, cashflows of nonindex securities are exogenous and they are uncorrelated with the index. Here, nonindex cashflows, which are endogenously determined commodity prices, end up being correlated with the index.

$\overline{f}_{kt}$ is independent of D_{it}. All prices f_{kt}, $k = 1, \ldots, K$, remain independent of nonindex commodities supply news $D_{\ell t}$, unless $k = \ell$.

(iv) *All commodity futures prices f_{kt}, $k = 1, \ldots, K$, react positively to a positive demand shift towards any index commodity α_i, $i = 1, \ldots, L$, $k \neq i$, while $\overline{f}_{kt}$ is independent of α_i. All prices f_{kt}, $k = 1, \ldots, K$, remain independent of nonindex commodities supply shifts α_ℓ, $\ell \neq k$.*

Figure 2 illustrates the results of the corollary. To elucidate the intuitions, we start from properties (iii) and (iv) of the corollary. Panel (a) shows that, unlike in the benchmark economy, futures prices decrease in response to positive index commodities' supply news D_{it}. Institutional investors strive to align their performance with the index, and as a result affect prices the most when the index is high. The index is high when D_{it} is low (supply of index commodity i is scarce) and low when D_{it} is high (supply is abundant). So the effects of the institutions on commodity futures prices f_{kt} are most pronounced for low D_{it} realizations and decline monotonically with D_{it}. These effects are absent in the benchmark economy in which agents are not directly concerned about the index. In contrast, futures prices f_{kt} do not react to news about supply of nonindex commodities (apart from that of own commodity k) because this news does not affect the performance of the index (panel (b) and Proposition 1). The demand-side effects on commodity futures prices are presented in panel (c). In contrast to the benchmark economy in which futures prices depend only on own commodity demand parameter α_k, in panel (c) it emerges that futures prices increase in demand parameters α_i for all commodities that are members of the index. An upward shift in demand for any index commodity leads to an increase in that commodity price (Lemma 1) and therefore leads to an increase in the index value. This is favorable to the institutions, and hence their impact on prices become increasingly more pronounced as α_k increases. In contrast, these effects are not present for nonindex commodities since a shift in demand for those commodities leaves the index unaffected (Proposition 1). A caveat to this discussion is that we are not formally modeling demand shifts in this section, but merely presenting comparative statics with respect to demand parameters α_k. In an economy with demand uncertainty, investors take into account of this uncertainty in their optimization (Section 3.3).

To illustrate property (ii), panel (d) demonstrates that aggregate output news D_t have stronger effects on futures prices f_{kt} than in the benchmark economy with no institutions. This is because good news about aggregate output not only increases the cashflows of all futures contracts (increases p_{kT}) but also increases the value of the index. This latter effect is responsible for the amplification of the effect of aggregate output news depicted in panel (d). The higher the aggregate output, the higher the index and hence the stronger the amplification effect. Property (i) shows that commodity futures prices rise when there are more institutions in the market. The more institutions there are, the stronger their effect on the discount factor and hence on all commodity futures prices. Finally, all panels in Figure 2 illustrate that in the presence of institutions, index futures rise more than nonindex.



(a) Effect of index commodity supply news D_i

(b) Effect of nonindex commodity supply news D_ℓ

(c) Effect of index commodity demand parameter α_i

(d) Effect of aggregate output D_t

Figure 2: **Futures prices.** This figure plots the equilibrium futures prices against several key quantities. The plots are typical. We set $t = 0.1$, $D_t = 100$, $D_{kt} = 1$, $k = 1, \ldots K$. The solid blue line is for index futures, the magenta dashed line is for nonindex futures, and the black dotted line is for the benchmark economy. The remaining parameter values (when fixed) are as in Figure 1.

3.2. Futures Volatilities and Correlations

The past decade in commodity futures markets has been characterized by an increase in volatility, attracting attention of policymakers and commentators. We explore commodity futures volatilities in this section in order to highlight the sources of this increased volatility. Our objective is to demonstrate how standard demand and supply risks can be amplified in the presence of institutions. Propositions 2 reports the futures return volatilities in closed form.[13]

Proposition 2 (Volatilities of commodity futures). *In the economy with institutions, the volatility vector of loadings of index commodity futures k returns on the Brownian motions are given by*

$$\sigma_{f_k t} = \overline{\sigma}_{f_k} + h_{kt}\,\sigma_{It}, \quad h_{kt} > 0, \quad k = 1, \ldots, L, \tag{17}$$

and nonindex by

$$\sigma_{f_k t} = \overline{\sigma}_{f_k} + h_t\,\sigma_{It}, \quad h_t > 0, \quad k = L+1, \ldots, K, \tag{18}$$

where $\overline{\sigma}_{f_k}$ is the corresponding volatility vector in the benchmark economy with no institutions and σ_{It} is the volatility vector for the conditional expectation of the index $E_t[I_T]$, given by

$$\overline{\sigma}_{f_k} = (\sigma,\, 0, \ldots, -\sigma_k, 0, \ldots, 0),\ \sigma_{It} = (\sigma,\, -\tfrac{1}{L}\sigma_1, \ldots, -\tfrac{1}{L}\sigma_L,\, 0, \ldots, 0), \tag{19}$$

and where h_t and h_{kt} are strictly positive stochastic processes provided in the Appendix with the property $h_{kt} > h_t$.

Consequently, in the presence of institutions,

(i) The volatilities of all futures prices, $\|\sigma_{f_k t}\|$, are higher than in the benchmark economy, $k = 1, \ldots K$.

(ii) The volatilities of index futures rise more than those of nonindex for otherwise identical commodities, i.e., for commodities i and k with $D_{it} = D_{kt}$, $\forall t$, $\alpha_i = \alpha_k$, $i \leq L$, $L < k \leq K$.

The general formulae presented in Proposition 2 can be decomposed into individual loadings of futures returns on the primitive sources of risk in our model, the Brownian motions $\omega_0,\ \omega_1, \ldots, \omega_K$. Table 1 presents this decomposition and illustrates the role of each individual source of risk. Recall that in our model the supply news of individual commodities D_{kt} are independent of each other and of the generic good supply news D_t. Each of these processes is driven by own Brownian motion. Since in the benchmark economy the futures price depends only on own D_{kt} and aggregate output D_t, it is exposed to only two primitive sources of risk:

[13] The notation $||z||$ denotes the square root of the dot product $z \cdot z$.

Brownian motions ω_k and ω_0. In the presence of institutions, futures prices become additionally dependent on supply news of all index commodities and therefore exposed to sources of uncertainty $\omega_1, \ldots \omega_L$. (The dependence is negative, as revealed by Corollary 1.) Additionally, as argued in Corollary 1, shocks to D_t are amplified in the presence of institutions. Proposition 2 formalizes these intuitions by explicitly reporting the loadings on $\omega_0, \omega_1, \ldots, \omega_K$, the driving forces behind $D, D_1, \ldots, D_K$, respectively. Hence, commodity futures become more volatile for two reasons: (i) their volatilities are amplified because futures prices react stronger to news about aggregate output D_t and (ii) the prices now depend on additional shocks driving index commodity supply news $D_1, \ldots, D_L$. Our model delivers increased sensitivity to the underlying shocks in the presence of institutions because the shocks affect the value of the index.

	Sources of risk associated with								
	Generic	Index commodities					Nonindex commodities		
	ω_0	ω_1	$\ldots$	ω_k	$\ldots$	ω_L	ω_{L+1}	$\ldots$	ω_K
Loadings									
Benchmark $\overline{\sigma}_{f_k}$	σ	0	$\ldots$	-σ_k	$\ldots$	0	0	$\ldots$	0
Index σ_{f_k}	$\sigma(1+h_{kt})$	-$\sigma_1 \frac{1}{L} h_{kt}$	$\ldots$	-$\sigma_k(1+\frac{1}{L}h_{kt})$	$\ldots$	-$\frac{1}{L}\sigma_L h_{kt}$	0	$\ldots$	0

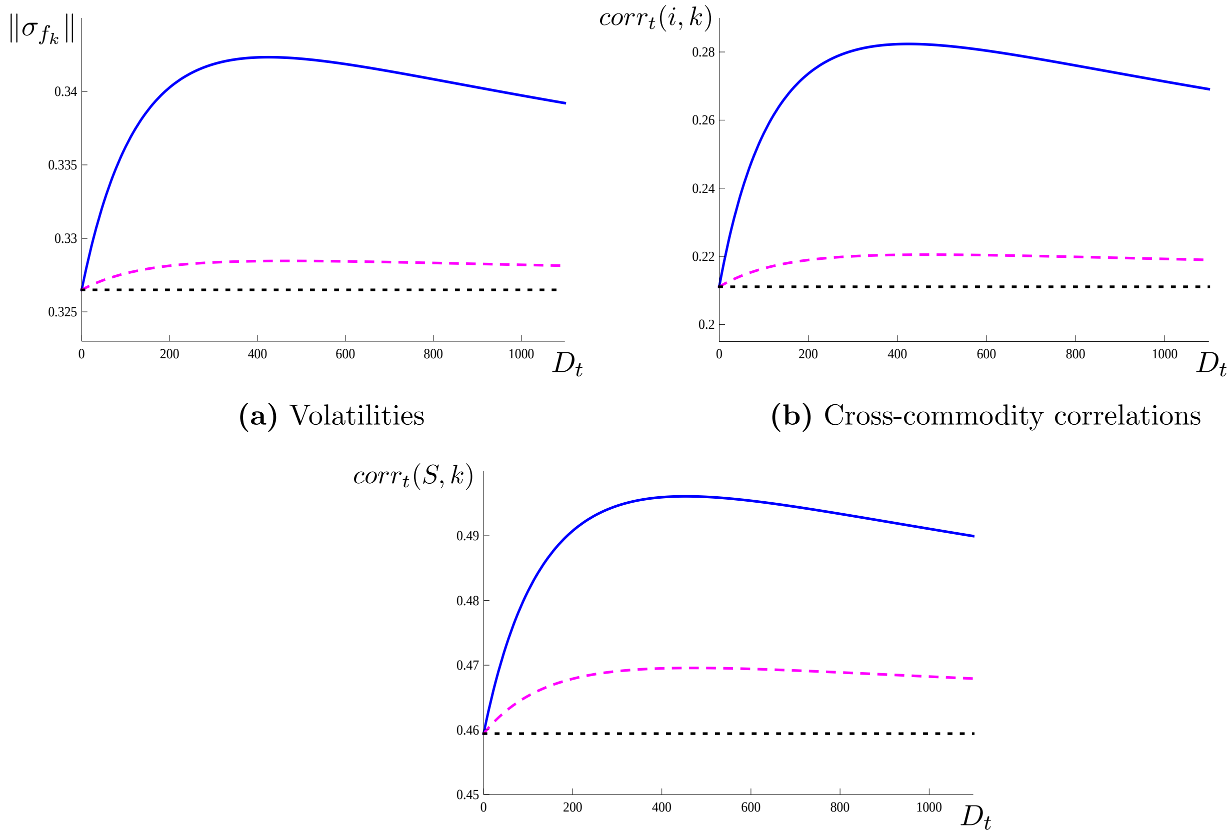
(a) Index commodity futures $k = 1, \ldots, L$

	Sources of risk associated with								
	Generic	Index commodities			Nonindex commodities				
	ω_0	ω_1	$\ldots$	ω_L	ω_{L+1}	$\ldots$	ω_k	$\ldots$	ω_K
Loadings									
Benchmark $\overline{\sigma}_{f_k}$	σ	0	$\ldots$	0	0	$\ldots$	-σ_k	$\ldots$	0
Nonindex σ_{f_k}	$\sigma(1+h_t)$	-$\sigma_1 \frac{1}{L} h_t$	$\ldots$	-$\frac{1}{L}\sigma_L h_t$	0	$\ldots$	-σ_k	$\ldots$	0

(b) Nonindex commodity futures $k = L+1, \ldots, K$

Table 1: **Individual volatility components of futures prices.**

Figure 3 provides an illustration. All plots in the figure are against an index commodity supply news D_i, which is a new state variable identified by our model, but the plots against aggregate output D look similar. Figure 3a also reveals that the volatilities of index and nonindex futures are differentially affected by the presence of institutions. Tang and Xiong



(a) Volatilities

(b) Cross-commodity correlations

(c) Equity-commodity correlations

Figure 3: **Commodity futures volatilities, cross-commodity correlations, and equity-commodity correlations.** This figure plots the commodity futures volatility $||\sigma_{f_k t}||$, cross-commodity correlations $\sigma_{f_i t} \cdot \sigma_{f_k t}/(\|\sigma_{f_i t}\| \, \|\sigma_{f_k t}\|)$, and equity-commodity correlations $\sigma_{f_S t} \cdot \sigma_{f_k t}/(\|\sigma_{St}\| \, \|\sigma_{f_k t}\|)$ in the presence of institutions against aggregate output D_t. The solid blue line is for index futures, the magenta dashed line is for nonindex futures, and the black dotted line is for the benchmark economy. The parameter values are as in Figure 2 .

(2012) document that since 2004, and especially during 2008, index commodities have exhibited higher volatility increases than nonindex ones. Our results are consistent with these findings.[14] The volatilities of index futures are higher than those of nonindex because index futures, by construction, pay off more when the index does well. The volatilities of index futures become high enough to make them unattractive to the normal investors (standard market participants) so that they are willing to sell the index futures to the institutions.

[14]In Figure 3 we do not attempt to generate realistic magnitudes of volatility increases; we simply illustrate our comparative statics results in Proposition 2. For more realistic magnitudes of the volatilities, see our richer model in Section 3.3 (Figure B2).

We next turn to examining the (instantaneous) correlations of futures returns, defined as $corr_t(i,k) = \sigma_{f_i t} \cdot \sigma_{f_k t} / (\|\sigma_{f_i t}\| \, \|\sigma_{f_k t}\|)$. Recent evidence indicates that financialization of commodities markets has coincided with a sharp increase in the correlations across a wide range of commodity futures returns (Tang and Xiong (2012)). The increase in correlations is especially pronounced for index futures returns. Tang and Xiong hypothesize that the commodity markets were largely segmented before 2000, and the inflow of institutional investors who hold multiple commodities in the same portfolio has linked together the commodity futures markets and increased the correlations among commodities, and especially the index ones. Our model shows that one does not need to rely on market segmentation to produce these effects. Arguably, commodity market speculators investing across commodity markets were present before 2004. Our model produces both the increase in the correlations amongst commodities and the higher increase in the correlations of index commodities under complete markets.[15] Our key mechanism is that in the presence of institutional investors benchmarked to a commodity index, this index (more precisely, $E_t[I_T] = D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}$) emerges as a common factor in returns of all commodities, raising their correlations. However, the sensitivity to this new factor is higher for index commodity futures (Proposition 2), making their returns more correlated than those of nonindex futures. The above intuition is precise for covariances. However, it carries through also to the correlations because the effect of rising volatilities is smaller than the effect of rising covariances. Figure 3b plots the correlations occurring in our model.

Moreover, the new factor $E_t[I_T]$ can be mapped in our model into the performance to date of institutional investors relative to that of normal investors or the time-t wealth distribution in the economy.[16] Intuitively, because institutional investors have an additional reason to hold index futures, they end up with a long position in these futures, while the normal investors take the other side. So the higher the expected level of the index, the higher the portfolio return of institutions relative to that of normal investors. As institutional investors get wealthier relative to normal investors, they comprise a higher proportion of the market. This effect is similar to that of fund inflows, with inflows following good relative performance to date. The resulting volatilities and correlations become time-varying and they depend, in principle, on all state vari-

[15] This result can be shown analytically when the volatilities of commodity supply news are the same, i.e., $\sigma_k = \sigma_j$, $\forall k, j = 1, \ldots, K$. For different volatility supply news parameters, all cross correlations (including the stock) can be analytically shown to increase for $L = 1$.

[16] Indeed, it can be shown that the time-t wealth distribution is $W_{\mathcal{I}t}/W_{\mathcal{N}t} = (a + bE_t[I_T])/(a + bE[I_T])$.

ables and parameters of the model (see Proposition 2). We have attempted to identify the ones to which the volatilities and correlations react the most. We find that they go up significantly in the presence of a common demand shock (Section 3.3, online Appendix B). Additionally, they are quite sensitive to the underlying supply news and aggregate output volatilities (σ_i and σ). The latter can be mapped within our model into the VIX (see Proposition A1): the higher the VIX, the higher are the commodity return volatilities and correlations. Finally, they are sensitive to the size of institutional investors λ and their relative performance to date (as captured by $E_t[I_T]$), although the sensitivity to both of these is lower.

Since investors in our model invest in both the futures and stock markets, one may expect that the effects we find are also present in equity-commodity correlations. Our main focus is on commodity markets, however, and so we do not incorporate all driving forces pertinent in stock markets.[17] The quantities $corr_t(S,k) = \sigma_{f_S t} \cdot \sigma_{f_k t}/(\|\sigma_{S t}\| \, \|\sigma_{f_k t}\|)$, for all k, are the (instantaneous) equity-futures correlations in our model, where the stock market level and volatility vector are presented in the Appendix (Proposition A1). These correlations always rise in the presence of institutions. In other words, we do get a theoretical confirmation within our model to support the assertion that the recent rise in the equity-commodity correlations can be attributed to financialization.[18] Figure 3c depicts the equity-commodity correlations in our model. The correlations of the stock market and the commodity futures returns go up because both the stock market and the commodities returns depend positively on the new common factor: the commodity index. The correlations of the stock market and the index commodities is higher than that with the nonindex because the index commodity futures have a higher loading on the new factor.

3.3. Economy with Demand Shocks

Our setting so far has been missing demand shocks, and such shocks have been argued to be critical in understanding the behavior of prices of oil and other commodities (Fattouh, Kilian, and Mahadeva (2013)). Commentators often link the increase in commodity prices and cross-

[17]For example, investors who are benchmarked to a stock market index (e.g., S&P 500) would have a confounding effect on the stock market valuation. Their index would also appear in the equilibrium stock market level. Our model can be extended to incorporate such investors.

[18]Tan and Xiong document that the correlation between GSCI commodity index and the S&P500 rose after 2004, and have been especially high in 2008. Relatedly, Buyuksahin and Robe (2014) find that the GSCI-S&P500 correlation rose since the 2008 financial crisis, but not before.

commodity correlations to China, whose high growth has led to an increase in demand for a number of key commodities. In the online Appendix B, we introduce common demand shocks, affecting a group of commodities. In particular, in the consumption index (8) of the investors,

$$W_n = C_{n_0}^{\alpha_0} C_{n_1}^{\alpha_1} \cdot \ldots \cdot C_{n_K}^{\alpha_K}, \qquad n \in \{\mathcal{N}, \mathcal{I}\},$$

we allow two demand parameters, α_1 and α_2, to be strictly positive stochastic processes. These processes are modeled so that the demand for a commodity is increasing with aggregate output (as in the model of oil prices of Dvir and Rogoff (2009)). We may think of commodities 1 and 2 as representing energy commodities, both in the commodity index.

Within this richer setting, we demonstrate the validity of earlier results that all futures prices and their volatilities are higher in the presence of institutions, with those of index futures exceeding nonindex ones (Proposition B1 in Appendix B). However, these effects are stronger than those in the baseline model. In the presence of demand shocks, the index becomes more volatile and so the institutional investors' incentive to not fall behind the index strengthens further, amplifying our earlier results. Furthermore, the cross-commodity futures return correlations go up sizably, reaching the levels documented in post-financialization period in the data (Tang and Xiong (2012)). Within this setup we can disentangle the effects of financialization from the effects of fundamentals (demand and supply) by comparing economies with and without institutional investors. Within a plausible numerical illustration, presented in Appendix B, we quantify the fraction of futures price rise that is due to financialization, and find it to be sizeable. Our results support the view advocated in Kilian and Murphy (2014) that fundamentals, and especially demand shocks, are important in explaining commodity prices, but there is also a contribution of financialization, and moreover, the presence of institutions amplifies the effects of a rising demand.[19]

4. Commodity Prices and Inventories

Commodity spot prices are important determinants of the cost of living worldwide. Spiralling food and energy prices observed in recent years have sparked an intense debate whether the

[19]This amplification effect suggests that the specifications used in structural econometric models of commodity prices, such as in Kilian and Murphy, may not be time-invariant, and in particular the sensitivity of commodity prices to structural shocks may have changed since the inflow of institutional investors from 2004 onwards. This is a testable implication that we leave for future empirical work.

inflow of institutional investors into the futures markets may be hurting millions of households. In his congressional testimony, Masters (2008) argues that the price spiral is unequivocally due to the inflow of institutional commodity investors. In a formal study, Singleton (2014) presents evidence in favor of this view.

The framework we have developed so far does not carry direct implications for intermediate commodity spot prices p_{kt}, $t < T$. To determine prices p_{kt} one needs to extend our model and incorporate additional features. In this section we introduce intermediate consumption and storage. Towards that we adopt the classical optimal storage framework for commodity pricing which follows Deaton and Laroque (1992, 1996). The main departure from Deaton and Laroque is that the cash flows of storage firms are priced by a discount factor that reflects the risk aversion of their shareholders and that is influenced by institutional investors (because institutional investors can hold shares of storage firms). This departure elucidates how financial markets transmit outside shocks not only to futures prices (Section 3) but also to commodity spot prices and inventories, via the discount factor channel.

4.1. Incorporating Storage

We introduce additional economic agents, consumers and firms operating within a competitive storage sector. These new agents exist alongside our normal and institutional investors. The new agents are present over two dates, t and $t+1$. The firms make a one-time decision at time t to store a commodity until time $t+1$. To close the model, we incorporate consumers who consume at times t and $t+1$.[20] The choice of a one-time storage decision is for tractability. The complexity and intractability of the dynamic storage framework is well-acknowledged in the literature (Deaton and Laroque (1992), Dvir and Rogoff (2009)). Our setting has the added complexities of having risk-averse investors and elaborate financial markets. Our formulation is the simplest possible set-up that is sufficient to illustrate our main economic mechanism.[21]

[20]We remark that we can easily recast our baseline model (Section 2) in discrete time so that, like the storage decision, asset allocation decisions are made over discrete intervals. The only changes needed to discretize the model would be to take the supply news to be the discrete-time analogues of processes in equations (1)-(2) (each $D_t, D_{kt}, k = 1, \ldots, K$ is conditionally lognormal) and to complete the financial markets with enough zero net supply securities to compensate for the loss of spanning owing to the removal of continuous re-trading. With these two changes, our key insights, and in particular all our expressions and results in Proposition 1 and Corollary 1, remain as they are. Hence, we can frame our model with storage in discrete time.

[21]Two related recent works in finance which employ commodity storage models in the presence of financial markets, Acharya, Lochstoer, and Ramadorai (2013) and Gorton, Hayashi, and Rouwenhorst (2013), also resort

Commodities. As before, we have $K+1$ commodities, where now the economy is additionally endowed with output D_t, D_{kt}, units of commodities $k=1,\ldots,K$ at time t and $D_{t+1}, D_{k\,t+1}$ at time $t+1$. One of these commodities, x, is storable in the sense that putting aside X_t units of the commodity at time t yields $(1-\delta)X_t$ units of this commodity at time $t+1$, where δ is the storage cost. The inventory X_t is optimally chosen by the storage firms. The remaining commodities are non-storable. We here make a distinction between storable and non-storable commodities because we intend to demonstrate that the effects of financialization on storable vs. non-storable commodities are very different.

The total amount of the storable commodity at times t and $t+1$ is, respectively, $D_{xt}-X_t$ and $D_{x\,t+1}+(1-\delta)X_t$. A governing entity distributes these quantities of commodity x and all of the output of the other commodities to the consumers in the form of endowment (or labor income).

Competitive storage sector. The firms in the competitive storage sector at time t buy X_t units of good x at the equilibrium price p_{xt} and carry an inventory over to the next period, liquidating it at time $t+1$. The shares in these firms are traded by investors in our economy, both the normal and institutional investors. The firm shares are in zero net supply. A firm's objective is to choose an optimal inventory level X_t so as to maximize its value given by

$$-\,p_{xt}X_t + E_t\left[M_{t,t+1}\,p_{x\,t+1}(1-\delta)X_t\right] \tag{20}$$

For tractability, we abstract away from inventory stockouts and do not impose an explicit non-negativity constraint on inventories.[22]

Firms in the storage sector are perfectly competitive, which ensures that the equilibrium prices p_{xt}, $p_{x\,t+1}$ must satisfy

$$p_{xt} = (1-\delta)E_t\left[M_{t,t+1}\,p_{x\,t+1}\right]. \tag{21}$$

Otherwise, if for example, p_{xt} in equation (21) were less than the quantity on the right-hand side, the firms would have an incentive to store more at time t and sell the commodity at time $t+1$. The relationship (21) underpins the classical theory of storage (Deaton and Laroque

to one-time storage decisions. Notably, Baker (2013) considers dynamic storage decisions.

[22]Our focus here is on the interaction of financial markets and commodity prices and inventories, and we chose to highlight it in the simplest possible way. It is possible to impose non-negativity constraints on inventories, as in the storage literature, but our analysis here does not have much to add regarding the effects of inventory stockouts beyond what is already in the literature.

(1992) and others). Our main departure from this literature is that we do not assume that the shareholders of the storage firms are risk-neutral whereby the discounting of future cash flows is at a constant riskless interest rate. Instead, our firm shareholders discount future cash flows using a stochastic discount factor. Since the shareholders are the normal and institutional investors in our economy, the relevant stochastic discount factor is that determined from the equilibrium conditions in the financial markets where our investors trade (presented in Section 3) and it reflects their attitudes towards risk. The importance of replacing the riskless interest rate by a stochastic discount factor in the pricing of storable commodities has recently been emphasized by Singleton (2014) (Cassasus and Collin-Dufresne (2005) also employ such discounting, and it implicitly appears in earlier works in finance such as Gibson and Schwartz (1990)). Note that equation (21) holds with equality because we have abstracted away from the non-negativity constraints on inventories. We further note that, as evident from equations (20)–(21), the value of the firm's profits is zero. The firm shares can be viewed as redundant assets in our economy, and so they are priced as redundant securities under complete markets.

Investors. The investors are the same as in Section 2, normal and institutional investors.

Consumers. The storage literature typically specifies the demand functions for commodities in reduced form. We here opt to microfound these functions by explicitly modeling the end consumers of commodities. We model the consumers $\mathcal{C}$ as consumer-workers who live hand-to-mouth from time t until $t+1$. At the two dates t and $t+1$ they receive an endowment (labor income), which they fully consume. These consumers neither save nor invest and they are distinct from the investors in our model. Such hand-to-mouth consumers were introduced by Campbell and Mankiw (1989) and first applied in an asset pricing context in Weil (1992).[23] The values of the representative consumer's endowments at times t and $t+1$ are, respectively,

$$W_{\mathcal{C}t} = D_t + p_{1t}D_{1t} + \ldots + p_{xt}(D_{xt} - X_t) + \ldots + p_{Kt}D_{Kt},$$

$$W_{\mathcal{C}t+1} = D_{t+1} + p_{1t+1}D_{1t+1} + \ldots + p_{xt+1}(D_{xt+1} + (1-\delta)X_t) + \ldots + p_{Kt+1}D_{Kt+1}.$$

Since the consumer consumes all his endowment every period, at each $s = t,\ t+1$ the consumer

[23] This way of modeling consumers has been employed extensively in macroeconomics; it has been argued that accounting for hand-to-mouth consumers helps rationalize aggregate consumption data and is important for policy experiments (e.g., impact of a fiscal stimulus). In a recent paper, Kaplan and Violante (2014) document that in the US between 18% and 37% of households live hand-to-mouth (consume all of their paycheck).

maximizes his utility given by

$$\log C_{0s}^{\alpha_0} C_{1s}^{\alpha_1} \cdot \ldots \cdot C_{Ks}^{\alpha_K}$$

subject to the budget constraint

$$C_{0s} + p_{1s} C_{1s} + \ldots + p_{ks} C_{ks} = W_{\mathcal{C}s}.$$

The demand for commodities resulting from this optimization is closely related to reduced-form demand functions adopted in the storage literature.

Equilibrium now involves the following additional market clearing conditions for each good.

$$C_{0s} = D_s,\ C_{ks} = D_{ks},\ k \neq x,\ s = t, t+1, \tag{22}$$

$$C_{xt} = D_{xt} - X_t,\ C_{xt+1} = D_{xt+1} + (1-\delta) X_t. \tag{23}$$

4.2. Equilibrium Commodity Prices and Inventories

Proposition 3 reveals how the discount factor is affected by institutional investors and summarizes the equilibrium commodity prices and inventories in our economy with storage.

Proposition 3 (Commodity prices and inventories). *In the economy with institutions, the equilibrium inventories X_t of the storable commodity satisfy*

$$\frac{D_t}{D_{xt} - X_t} = (1-\delta) E_t \left[M_{t,t+1} \frac{D_{t+1}}{D_{xt+1} + (1-\delta) X_t} \right], \tag{24}$$

where the stochastic discount factor, $M_{t,t+1}$ is given by

$$M_{t,t+1} = e^{\left(\mu - \sigma^2\right)} \frac{D_t}{D_{t+1}} \frac{A + b\lambda D_{t+1} \prod_{i=1}^{L} \left(g_i\left(t+1\right)/D_{it+1}\right)^{1/L}}{A + b\lambda e^{-\sigma^2} D_t \prod_{i=1}^{L} \left(g_i\left(t\right)/D_{it}\right)^{1/L}}, \tag{25}$$

the deterministic function $g_i(t)$ is as in (16) and the constant A is reported in Appendix A.

The equilibrium commodity prices are given by

$$p_{xt} = \frac{\alpha_x}{\alpha_0} \frac{D_t}{D_{xt} - X_t}, \qquad \textit{(storable commodity)} \tag{26}$$

$$p_{x\,t+1} = \frac{\alpha_k}{\alpha_0} \frac{D_{t+1}}{D_{x\,t+1} + (1-\delta) X_t}, \qquad \textit{(storable commodity)} \tag{27}$$

$$p_{ks} = \frac{\alpha_k}{\alpha_0} \frac{D_s}{D_{ks}},\ s = t, t+1, k \neq x. \qquad \textit{(non-storable commodities)} \tag{28}$$

In the benchmark economy with no institutions, the equilibrium inventories $\overline{X}_t$ satisfy (24) with $M_{t,t+1}$ replaced by its corresponding benchmark value $\overline{M}_{t,t+1} = e^{(\mu-\sigma^2)} D_t / D_{t+1}$. The benchmark economy commodity prices are given by (26)–(28) with $\overline{X}_t$ replacing X_t.

Consequently, in the presence of institutions

(i) The storable commodity time-t price is higher than in the benchmark economy, $p_{xt} > \overline{p}_{xt}$.

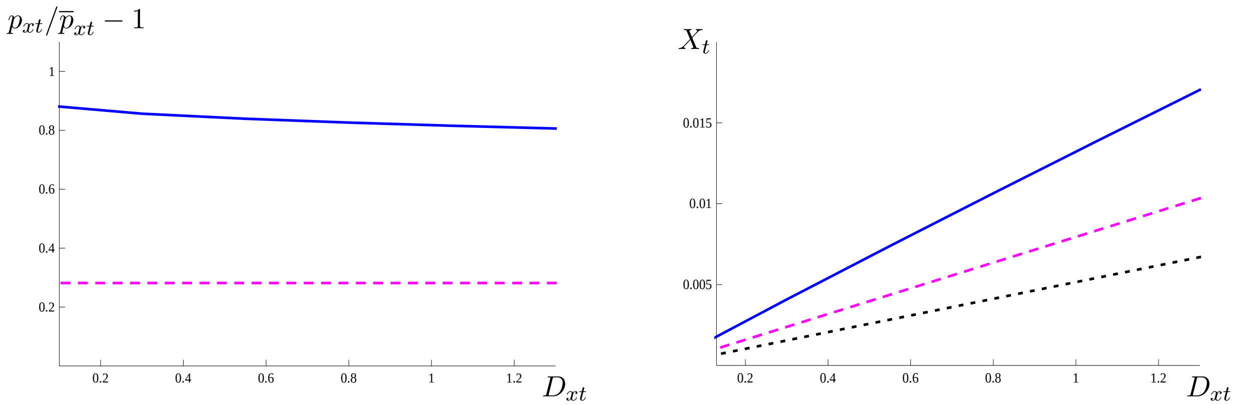
(ii) The inventories of the storable commodity is higher than in the benchmark economy, $X_t > \overline{X}_t$.

(iii) An index commodity price and inventory rise more than those of a nonindex one for an otherwise identical storable commodity.

Proposition 3 reveals that financialization affects commodity inventories and prices, but only for those commodities that can be stored. The prices of non-storable commodities at each time are determined by the supply and demand at that time. Such commodities cannot be viewed as investable assets, and hence the relationship (21) does not apply to them. Since most of the commodities for which futures are traded are storable, albeit at some cost, we now focus on the storable commodity in our model, as do Deaton and Laroque (1992) and subsequent works.

Proposition 3 reports, in closed form, the discount factor prevailing in our economy. Even in the benchmark economy with no institutions, this discount factor is stochastic, because it depends on the aggregate output. Therefore, in the benchmark economy, the discounting of cash flows of storage firms is not at a constant (riskless) rate, as in much of the extant storage literature. This implies that the expected return on storing a commodity in our model, just like that of any other asset, depends on the covariance of the return from holding that commodity with the discount factor. Since the discount factor is determined in financial markets, the financial markets and commodity spot markets are therefore intertwined. In particular, outside shocks affecting financial markets may also affect commodity spot prices.

The discount factor in the economy with institutions, presented in Proposition 3, depends additionally on the characteristics of index commodities (e.g., D_i, α_i, σ_i, $i = 1, \ldots L$). This is due to the presence of institutional investors, whose marginal utilities are represented in the discount factor. As a consequence, prices of storable commodities are affected by the presence of institutions. In particular, as Proposition 3 demonstrates, commodity prices and inventories are higher than in the benchmark economy with no institutions. The intuition is as follows. As the demand for commodities and hence the commodity prices in our model are increasing with



(a) Price difference relative to benchmark (%)

(b) Commodity inventory

Figure 4: **Commodity prices and inventories.** Panel (a) plots the difference between the storable commodity price in the economy with institutions relative to that in the benchmark economy (in percent). Panel (b) plots the inventory of the storable commodity x. The plots are typical. The solid blue line is for index commodities, the magenta dashed line is for nonindex commodities, and the black dotted line is for the benchmark economy. We set the storage cost $\delta = 0.02$. The remaining parameters are from Figure 2.

aggregate output (as also discussed in Dvir and Rogoff (2009)), all commodity prices comove positively with each other and with the commodity index. Since institutional investors strive to not fall behind when the commodity index does well, they particularly value such assets—i.e., assets that pay off more in the states when the index is expected to do well.[24] Storing a unit of a commodity from time t until $t+1$ can be viewed as investing in an asset whose future (time-$t+1$) return is positively correlated with that of the commodity index. Hence, the price of such assets, p_{xt} in this case, is higher in the presence of institutions. Because there is a one-to-one mapping in our model between the inventory of commodity x and its time-t price p_{xt} (equation (26)), the inventory of commodity x has to increase at the same time. Moreover, if commodity x is included in the index, its price, naturally, comoves more with the index. Therefore storing such a commodity is especially attractive to institutional investors and therefore the effects we have described are stronger, and in particular the commodity price and inventories increase by more. Finally, at the end of the storage period $t+1$, inventories have to be liquidated and all of the available commodities consumed. Since inventories are higher in the presence of institutions, mechanically the time-$t+1$ commodity price has to be lower in the presence

[24] If a commodity is scarce at time $t+1$ and hence its price is high, it is expected to remain scarce over the horizon of the investors because our driving processes are persistent.

of institutions. This effect of the last storage date will be there even in an extension of our framework to multiple storage decisions, provided that firms live for a finite number of periods. This suggests that an infinite horizon model with storage is perhaps more appropriate, but it is beyond the scope of this work. Figure 4 illustrates the results of Proposition 3.

4.3. Cross-Commodity Spillovers and Impact of Income Shocks

We now turn to describing cross-commodity and other spillovers in our model. These spillovers act through the discount factor channel and are novel to our model.

Proposition 4 (Spillovers). *Consider an index commodity i.*

(i) In the economy with institutions, the following spillovers from commodity i to the storable commodity x occur.

	Increase in		
Spillover to	*Supply of index commodity D_{it}*	*Demand of index commodity α_i*	*Mean growth μ_i or volatility σ_i of supply*
Price of storable commodity x, p_{xt}	$-$	$+$	$\neq 0$
Inventory of storable commodity x, X_t	$-$	$+$	$\neq 0$

(ii) From a nonindex commodity ℓ, there are no spillovers to any commodity—i.e., all entries in the table above are zero. Furthermore, there are no such spillovers in the benchmark economy with no institutions.

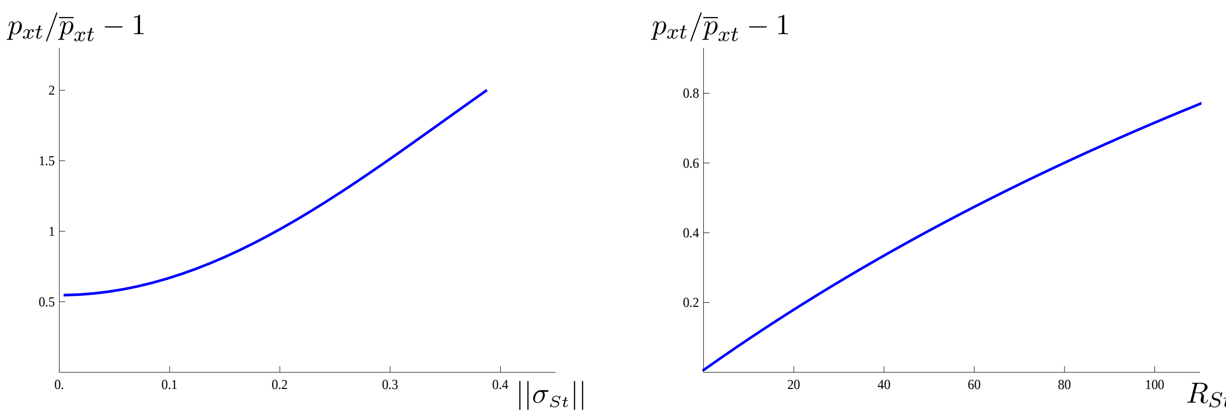
(iii) In the economy with institutions, an inflow of institutions (an increase in λ) increases the storable commodity's price p_{xt} and inventory X_t.

Proposition 4(i) shows how outside shocks affecting an index commodity spill over to all other storable commodities' spot prices and inventories. The main driver behind such spillovers is that the discount factor in (21) is affected by institutions and in particular, it becomes dependent on the characteristics of index commodities. The intuition is as follows. Recall that a *negative* supply shock to an index commodity implies that the commodity price, and hence the index value, is expected to be higher. This induces more storage in the economy as institutions seek to not fall behind the index. Since all storable commodities are positively correlated with the index, the inventories of all commodities, and especially of index commodities, increase. The equilibrium storable commodity prices then have to rise in response to this increased demand for storage (equation (26)). The opposite is true for a positive supply shock. Now consider a positive demand shift for an index commodity. Following such a shift, the affected commodity's price is higher and therefore the index value is expected to be higher. This increases the demand of institutions for all assets that are positively correlated with the index and in particular increases their demand for commodity storage. Hence prices and inventories of all storable commodities rise. Finally, consider a shift in the mean growth rate or volatility of any index commodity's supply. Such a shift affects the discount factor in the economy with institutions (see (25)), and hence it has an impact on every storable commodity's price and inventories. We do not report the sign of the effect, as it can be shown analytically that it can be positive or negative, depending on parameter values. In contrast, a shock to a nonindex commodity does not affect the discount factor and hence does not spill over to other commodities (provided, of course, that the shock does not have a common component with shocks to index commodities) because such shocks do not alter the discount factor (Proposition 4(ii)). There are also no such spillovers in the benchmark economy with no institutions because in that economy the discount factor depends only on the aggregate output and is not directly affected by index commodities.

As evident from Proposition 4(iii), an inflow of institutional investors, which in our model is captured by an increase in λ, increases prices and inventories of storable commodities. This occurs because the incentives of the institutions to do well relative to the commodity index are impounded into the discount factor, and the more institutions there are, the bigger their influence on the discount factor. The resulting increase in the storable commodity's price and inventory, revealed in Proposition 3, therefore becomes more pronounced.

We note that such spillovers of outside shocks through financial markets may present challenges for identification strategies commonly adopted in empirical work. Since supply and demand shocks are not directly observable, they have to be inferred from commodity prices, inventories and other observable variables. An econometrician may then interpret an increase in prices and inventories of a commodity as, for example, a result of a demand shift for that commodity. Proposition 4 shows that such an increase may have nothing to do with demand or supply for that particular commodity but may come from a shock to another commodity transmitted via financial markets.

In our framework there are also spillovers from the stock market to the spot prices and inventory of the storable commodity. Identifying these spillovers, however, requires a more nuanced approach because the stock and commodity prices comove even in the benchmark economy with no institutions as they load on a common factor, aggregate output D. So in the economy with institutions one needs to separate the interdependence from genuine spillovers occurring only in the presence of institutions. We can do so within our model by comparing the economies with and without institutions and focusing on the difference, which we call a spillover. Figure 5 plots the spillovers from the stock market to commodity prices. In particular, in the presence of institutions higher stock market returns and volatility spills over to commodity prices, pushing them up. We have not been able to sign analytically the spillover, but for reasonable parameter values it is positive, as in Figure 5. The spillovers occur via our discount factor channel. Recent empirical evidence documents spillovers akin to the ones occurring in Figure 5. Diebold and Yilmaz (2012) develop a new methodology to identify spillovers and document volatility spillovers from US stock to commodity markets in recent data.



(a) Spillover of stock volatility

(b) Spillover of stock return

Figure 5: **Spillovers from stock market.** This figure plots commodity prices (difference relative to benchmark (%)) against stock volatility $||\sigma_S||$ and stock return (%) $R_{St} \equiv S_t/S_0 - 1$. We set $\delta = 0.02$, $\rho = 0.99$, $\mu_x = 0$. The remaining parameters are as in Figure 2.

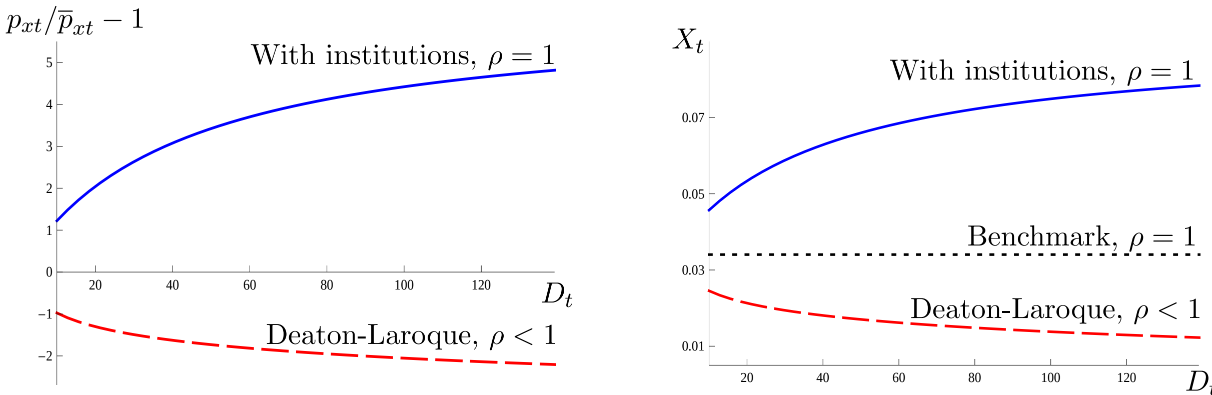
Our final goal in this section is to examine the effects of income shocks on commodity inventories and prices.[25] In classical storage models (e.g., Deaton and Laroque (1996)), storage always has stabilizing effects on prices. This is because a positive income shock (temporarily)

[25]The shocks we are considering here are shocks to the income of the consumers. Such shocks induce shifts in the demand schedule for each commodity, and therefore they are commonly referred to as *demand* shocks in models that specify consumer demand exogenously.

increases a commodity price, but at the same time firms have a reduced incentive to store the commodity because they expect lower prices in the future. This makes the commodity more abundant today. So storage "leans against the wind" and mitigates the effects of the shock on commodity prices. Dvir and Rogoff (2009) challenge this conclusion on empirical grounds because the above mechanism is unable to deliver sufficient persistence of commodity prices. They point out that income in Deaton and Laroque and related literature is assumed to follow an AR(1) process in levels, and this assumption leads to a drop in storage following a positive income shock. Dvir and Rogoff propose to consider instead *permanent* income (demand) shocks, which end up implying that firms actually store more following such shocks (because prices are expected to remain high in the future).

Our setting naturally lends itself to an exploration of permanent income shocks because all our driving processes are persistent (geometric Brownian motions). Figure 6 depicts the effects of an income shock on prices and inventories in our model adopted to the Deaton and Laroque setting (dashed red line), in our benchmark economy without institutional investors (dotted black line), and in the economy with institutions (solid blue line). The Deaton-Laroque line corresponds to an economy with discounting at the constant riskless rate and output D_t following an AR(1) process ($\log D_{t+1} = \rho \log D_t + \varepsilon_{t+1}$, $\rho \in (0, 1)$, $\varepsilon_{t+1} \sim \mathcal{N}(0, \sigma^2)$).

Consistent with the classical storage literature, in the Deaton-Laroque case inventories indeed decrease in response to a positive income shock (a positive change in D_t). On the other hand, in our benchmark economy with no institutions we find that inventories *do not respond* to income shocks. There are now two counteracting forces. As before, a positive income shock increases the price p_{xt} (e.g., see equation (26)). In the Deaton-Laroque case, this reduces incentives to store. But now the shock is permanent, and income and prices are expected to remain high in the future, which acts to increase storage. In Dvir and Rogoff the latter force dominates but in our case the two forces exactly offset each other. This suggests that simply replacing an AR(1) output processes by any I(1) process is not sufficient to make inventories increasing with an income shock; one needs a more nuanced specification, as e.g., the one suggested by Dvir and Rogoff. On the the other hand, in the presence of institutions we see that inventories go up with a shock, magnifying the effects of an income shock on commodity prices. Firms have an incentive to store more because higher output today implies higher levels of the commodity price index in the future. Anticipating that, institutional investors increase their demand for all assets whose payoffs are correlated with the index, and in particular increase their demand for storage. This result complements the findings of Dvir and Rogoff but here we offer an alternative channel and a more nuanced view on the connection between permanent income shocks and commodity inventories.



(a) on commodity price (difference relative to benchmark with $\rho = 1$ (%))

(b) on commodity inventory

Figure 6: **Effects of an income shock.** This figure plots commodity prices and inventories against consumers' income (D_t). We set the gross interest rate for the Deaton-Laroque case $R = 1.02$. The remaining parameters are as in Figure 5.

5. Conclusion and Discussion

In this paper we explore theoretically how the presence of institutional investors may affect commodity futures prices and their dynamics. We find that in the presence of institutions futures prices of all commodities rise, with futures prices of index commodities increasing by more. We also find that in the presence of institutional investors shocks to fundamentals (demand and supply) of index commodities get transmitted to prices of all other commodities. Furthermore, the volatilities of all commodity futures rise in the presence of institutions, with those of index commodities increasing by more. Moreover, the presence of institutions leads to an increase in the cross-commodity and equity-commodity correlations, with those for index commodity futures increasing by more. Finally, we find that storable commodity spot prices and inventories go up in the presence of institutions. The financial markets serve as a conduit in transmitting outside shocks to commodity spot prices.

To keep our focus, we have not explored the implications of our model for the commodity futures risk premia. The risk premium is defined as the difference between the expected spot price of a commodity and its futures price. This quantity should be positive according to the hedging pressure theory (Keynes (1930), Hicks (1939), Hirshleifer (1988)). If producers of a commodity want to hedge their price risk by selling futures contracts, then arbitrageurs who take the other side of the contract should receive the risk premium in compensation for taking that risk. According to our model, the buying pressure from institutional investors exerts a similar effect in the opposite direction, which should reduce the risk premium. Consistent with this prediction, Hamilton and Wu (2014) document that the risk premium in crude oil futures on average decreased and became more volatile since 2005. Moreover, it would be interesting to explore the effects of institutions on the futures curve. In our model, the (very simplistic) futures curve for any commodity exhibits a contango in the classical sense (as defined by Keynes) in that the futures price exceeds its corresponding expected spot price, but only in the presence of institutions. Owing to its one-consumption date nature, however, our model is not immediately suitable for a proper analysis of futures curves, but its extended version could be.

Our model also has implications for the open interest in the futures markets. Cheng, Kirilenko, and Xiong (2015) show that the positions of commodity index traders fall in response to an increase in the overall economic uncertainty, as captured by the VIX Volatility Index. We anticipate that, qualitatively, our model delivers this implication. In a recent paper, Hong and Yogo (2012) document that open interest predicts asset prices and macroeconomic variables. It would be interesting to examine whether our model delivers this intriguing finding.

In our model information is symmetric and investors have the same beliefs. Sockin and Xiong (2015) develop a model with asymmetric information, in which producers learn about the state of the economy from futures prices, a channel absent in our framework. In our model, trade between investors occurs because their interim relative performance fluctuates. Another realistic motive for trade is expectations-based speculation, driven by investors' differences of opinion. It is desirable but not straightforward to extend our model to include asymmetric information, expectations-based speculation, and inefficient risk sharing. Finally, our analysis of financialization is based on comparing economies with and without institutional investors. But it would be desirable to address the deeper issue of why the institutions entered the commodity futures markets in the first place. We leave these important extensions for future research.

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Appendix A: Proofs

Proof of Lemma 1. We first determine the institutional and normal investors' optimal demands in each commodity. Since the securities market is dynamically complete in our setup with $K+1$ risky securities and $K+1$ sources of risk ω, there exists a state price density process, ξ, such that the time-t value of a payoff Q_T at time T is given by $E_t\big[\xi_T Q_T\big]/\xi_t$. In our setting, the state price density is a martingale. Accordingly, investor n's, $n = \mathcal{N}, \mathcal{I}$, dynamic budget constraint (9) can be restated as

$$E_t\Big[\xi_T \sum_{k=0}^{K} p_{kT} C_{n_k T}\Big] = \xi_t W_{nt}. \tag{A1}$$

Maximizing the institutional investor's expected objective function (7), with the Cobb-Douglas aggregator (8) substituted in, subject to (A1) evaluated at time $t = 0$ leads to the institution's optimal demand in commodity $k = 1, \ldots, K$ and generic good, respectively, as

$$C_{\mathcal{I}_k T} = \frac{\alpha_k \left(a + bI_T\right)}{y_I p_{kT} \xi_T}, \quad C_{\mathcal{I}_0 T} = \frac{\alpha_0 \left(a + bI_T\right)}{y_I \xi_T}, \tag{A2}$$

where $1/y_I$ solves (A1) evaluated at $t = 0$. Substituting (A2) into (A1) at $t = 0$, we obtain

$$\frac{1}{y_I} = \frac{\lambda \xi_0 S_0}{\sum_{j=0}^{K} \alpha_j \left(a + bE\left[I_T\right]\right)}.$$

Consequently, the institution's optimal commodity demands are given by

$$C_{\mathcal{I}_k T} = \frac{\alpha_k}{\sum_{j=0}^{K} \alpha_j} \frac{\lambda \xi_0 S_0}{p_{kT} \xi_T} \frac{a + bI_T}{a + bE\left[I_T\right]}, \qquad k = 1, \ldots, K, \tag{A3}$$

$$C_{\mathcal{I}_0 T} = \frac{\alpha_0}{\sum_{j=0}^{K} \alpha_j} \frac{\lambda \xi_0 S_0}{\xi_T} \frac{a + bI_T}{a + bE\left[I_T\right]}. \tag{A4}$$

Similarly, we obtain the normal investor's optimal commodity demands at time T as

$$C_{\mathcal{N}_k T} = \frac{\alpha_k}{\sum_{j=0}^{K} \alpha_j} \frac{(1-\lambda)\, \xi_0 S_0}{p_{kT} \xi_T}, \qquad k = 1, \ldots, K, \tag{A5}$$

$$C_{\mathcal{N}_0 T} = \frac{\alpha_0}{\sum_{j=0}^{K} \alpha_j} \frac{(1-\lambda)\, \xi_0 S_0}{\xi_T}. \tag{A6}$$

We now proceed to determine the equilibrium prices at time T. To obtain the equilibrium state price density, we impose the market clearing condition for the generic good, $C_{\mathcal{N}_0 T} + C_{\mathcal{I}_0 T} = D_T$, and substitute (A4) and (A6) to obtain

$$\frac{\alpha_0}{\sum_{j=0}^{K} \alpha_j} \frac{\xi_0 S_0}{\xi_T} \left(1 - \lambda + \lambda \frac{a + bI_T}{a + bE\left[I_T\right]}\right) = D_T,$$

which after rearranging leads to the equilibrium terminal state price density:

$$\xi_T = \frac{\alpha_0}{\sum_{j=0}^{K} \alpha_j} \frac{\xi_0 S_0}{D_T} \left(1 + \frac{\lambda b \left(I_T - E\left[I_T\right]\right)}{a + bE\left[I_T\right]}\right). \tag{A7}$$

The equilibrium state price density in the benchmark economy with no institutions is obtained by considering the special case of $b = 0$ in (A7). The time-T discount factor is defined as $M_{0,T} = \xi_T/\xi_0$, which after substituting (A7) leads to the expression (14) reported in Lemma 1.

To determine the equilibrium commodity prices at T, we impose the market clearing condition $C_{\mathcal{N}_k T} + C_{\mathcal{I}_k T} = D_{kT}$ for each commodity $k = 1, \ldots, K$, and substitute (A3) and (A5) to obtain

$$\frac{\alpha_k}{\sum_{j=0}^{K} \alpha_j} \frac{\xi_0 S_0}{p_{kT}\xi_T} \left(1 - \lambda + \lambda \frac{a + bI_T}{a + bE\left[I_T\right]}\right) = D_{kT},$$

which after substituting the equilibrium state price density (A7) and rearranging leads to the equilibrium commodity price expressions (11) in Lemma 1. Substituting the equilibrium commodity prices (11) that are in the commodity into the definition of the index (4) leads to the equilibrium commodity index value (12). Moreover, substituting the equilibrium commodity prices (11) into the stock market terminal value $S_T = D_T + \sum_{k=1}^{K} p_{kT} D_{kT}$ leads to the expression (13) in Lemma 1. To determine the unconditional expectation of the index, we make use of the fact that D_T, D_{iT}, $i = 1, \ldots, L$, are lognormally distributed and hence obtain

$$E\left[I_T\right] = E\left[\frac{D_T}{\alpha_0} \prod_{i=1}^{L} \left(\frac{\alpha_i}{D_{iT}}\right)^{1/L}\right] = e^{\left(\mu - \frac{1}{L}\sum_{i=1}^{L}\left(\mu_i - \frac{1}{2}\left(\frac{1}{L}+1\right)\sigma_i^2\right)\right)T} \frac{D_0}{\alpha_0} \prod_{i=1}^{L} \left(\frac{\alpha_i}{D_{i0}}\right)^{1/L}. \tag{A8}$$

Finally, we note that the equilibrium commodity and stock prices at time T are as in the benchmark economy with no institutions (the special case of $b = 0$, $a = 1$).

Q.E.D.

Proof of Proposition 1. By no arbitrage, the time-t futures price of a futures contract with maturity τ on commodity $k = 1, \ldots, K$ is given by $f_{kt} = E_t\left[\xi_{t+\tau} p_{k\,t+\tau}\right]/\xi_t$. Iteratively substituting the next rollover price upon maturity, $f_{k\,t+\tau}$, till T, we obtain

$$f_{kt} = \frac{E_t\left[\xi_T p_{kT}\right]}{\xi_t}. \tag{A9}$$

We proceed by first determining the equilibrium state price density process ξ. Since the state price density process is a martingale, its time-t value is given by

$$\begin{aligned} \xi_t &= E_t\left[\xi_T\right] \\ &= \bar{\xi} E_t\left[1/D_T\right] \left(a + b\left(1 - \lambda\right) E\left[I_T\right] + \lambda b \frac{E_t\left[I_T/D_T\right]}{E_t\left[1/D_T\right]}\right), \end{aligned} \tag{A10}$$

where the second equality follows by substituting ξ_T from (A7) and rearranging, and

$$\bar{\xi} = \frac{\alpha_0}{\sum_{j=0}^{K} \alpha_j} \frac{\xi_0 S_0}{a + bE\left[I_T\right]}. \tag{A11}$$

Substituting (12) and using the fact that D_T, D_{iT}, $i = 1, \ldots, L$, are lognormally distributed, we obtain

$$\begin{aligned} E_t\left[I_T/D_T\right] &= \frac{1}{\alpha_0} E_t\left[\prod_{i=1}^{L} \left(\alpha_i/D_{iT}\right)^{1/L}\right] \\ &= \frac{1}{\alpha_0} e^{-\frac{1}{L}\sum_{i=1}^{L}\left(\mu_i - \frac{1}{2}\left(\frac{1}{L}+1\right)\sigma_i^2\right)(T-t)} \prod_{i=1}^{L} \left(\alpha_i/D_{it}\right)^{1/L}. \end{aligned} \tag{A12}$$

Substituting (A8), (A12) and $E_t\left[1/D_T\right] = e^{\left(\sigma^2-\mu\right)(T-t)}/D_t$ into (A10), we obtain

$$\xi_t = \bar{\xi}\frac{e^{\left(\sigma^2-\mu\right)(T-t)}}{D_t}\left(a + b\left(1-\lambda\right) D_0 \prod_{i=1}^{L}\left(g_i\left(0\right)/D_{i0}\right)^{1/L} + b\lambda e^{-\sigma^2(T-t)} D_t \prod_{i=1}^{L}\left(g_i\left(t\right)/D_{it}\right)^{1/L}\right), \tag{A13}$$

where $g_i\left(t\right)$ is as given in (16).

To compute the expected deflated futures payoff of commodity $k = 1, \ldots, K$, we substitute (A7) and (11), and rearrange to obtain

$$E_t\left[\xi_T p_{kT}\right] = \bar{\xi}\frac{\alpha_k}{\alpha_0} E_t\left[1/D_{kT}\right]\left(a + b\left(1-\lambda\right) E\left[I_T\right] + b\,\lambda\,\frac{E_t\left[I_T/D_{kT}\right]}{E_t\left[1/D_{kT}\right]}\right), \tag{A14}$$

where $\bar{\xi}$ is as in (A11).

For nonindex futures contracts $k = L+1, \ldots, K$, we proceed by considering

$$\begin{aligned} E_t\left[I_T/D_{kT}\right] &= \frac{1}{\alpha_0} E_t\left[D_T/D_{kT}\prod_{i=1}^{L}\left(\alpha_i/D_{iT}\right)^{1/L}\right] \\ &= \frac{1}{\alpha_0} E_t\left[D_T\prod_{i=1}^{L}\left(\alpha_i/D_{iT}\right)^{1/L}\right] E_t\left[1/D_{kT}\right], \end{aligned}$$

where in the first equality we have substituted (12) and in the second we have made use of the fact that D_{kT} is independent of D_T, D_{iT}, $i = 1, \ldots, L$. Consequently, using the fact that D_T, D_{iT}, $i = 1, \ldots, L$, are lognormally distributed, we obtain

$$\frac{E_t\left[I_T/D_{kT}\right]}{E_t\left[1/D_{kT}\right]} = D_t\prod_{i=1}^{L}\left(g_i\left(t\right)/D_{it}\right)^{1/L}, \tag{A15}$$

where $g_i\left(t\right)$ is as in (16). Substituting (A13)–(A15), (A8) and $E_t\left[1/D_{kT}\right] = e^{\left(\sigma_k^2-\mu_k\right)(T-t)}/D_{kt}$ into (A9), and rearranging, we arrive at the equilibrium nonindex futures price expression reported in (15) for $k = L+1, \ldots, K$. The equilibrium futures price $\bar{f}_k$ in the benchmark economy with no institutions (16) follows by considering the special case of $a = 1$, $b = 0$ in (15).

For index futures contracts $k = 1, \ldots, L$, we substitute (12) and again compute

$$E_t\left[I_T/D_{kT}\right] = \frac{1}{\alpha_0} E_t\left[D_T/D_{kT} \prod_{i=1}^{L} \left(\alpha_i/D_{iT}\right)^{1/L}\right]$$
$$= \frac{1}{\alpha_0} e^{\left(-\mu+\mu_k+\left(\frac{1}{L}+1\right)\sigma_k^2 - \frac{1}{L}\sum_{i=1}^{L}\left(\mu_i - \frac{1}{2}\left(\frac{1}{L}+1\right)\sigma_i^2\right)\right)(T-t)} \frac{D_t}{D_{kt}} \prod_{i=1}^{L} \left(\alpha_i/D_{it}\right)^{1/L}.$$

So, using $E_t\left[1/D_{kT}\right] = e^{\left(\sigma_k^2-\mu_k\right)(T-t)}/D_{kt}$ we obtain

$$\frac{E_t\left[I_T/D_{kT}\right]}{E_t\left[1/D_{kT}\right]} = e^{\frac{1}{L}\sigma_k^2(T-t)} D_t \prod_{i=1}^{L} \left(g_i\left(t\right)/D_{it}\right)^{1/L}, \tag{A16}$$

where $g_i\left(t\right)$ is as in (16). Substituting (A13), (A14), (A16), and (A8) into (A9) and rearranging leads to the equilibrium index futures price expression reported in (15) for $k = 1, \ldots, L$. The property (i) that the futures prices are higher than in the benchmark economy follows by observing that the factor multiplying $\bar{f}_{kt}$ in expression (15) is strictly greater than one. Similarly, the property (ii) that the index futures price rise is higher than that of nonindex futures follows by observing that the factor multiplying $\bar{f}_{kt}$ in expression (15) is higher for an otherwise identical index futures.

Q.E.D.

Proof of Corollary 1. The stated properties follow by taking the appropriate partial derivatives of the expressions (15)–(16), and comparing the relevant magnitudes of the partial derivatives of interest.

Q.E.D.

Proof of Proposition 2. We write the equilibrium index futures price in (15) for $k = 1, \ldots, L$ as

$$f_{kt} = \bar{f}_{kt} \frac{Z_t}{Y_t}, \tag{A17}$$

where

$$\bar{f}_{kt} = \frac{\alpha_k}{\alpha_0} e^{(\mu-\mu_k-\sigma^2+\sigma_k^2)(T-t)} \frac{D_t}{D_{kt}},$$

$$Z_t = a + b(1-\lambda)D_0 \prod_{i=1}^{L} \left(g_i(0)/D_{i0}\right)^{1/L} + b\,\lambda\, e^{\sigma_k^2(T-t)/L} D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L},$$

$$Y_t = a + b(1-\lambda)D_0 \prod_{i=1}^{L} \left(g_i(0)/D_{i0}\right)^{1/L} + b\,\lambda\, e^{-\sigma^2(T-t)} D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}.$$

where $g_i(t)$ is as in (16).

Applying Itô's Lemma to both sides of (A13), we obtain

$$\sigma_{f_k t} = \overline{\sigma}_{f_k} + \sigma_{Zt} - \sigma_{Yt}, \tag{A18}$$

where

$$\begin{aligned}
\overline{\sigma}_{f_k} &= (\sigma,\, 0, \ldots, -\sigma_k, 0, \ldots, 0) \\
\sigma_{Zt} &= \frac{b\,\lambda\, e^{\sigma_k^2 (T-t)/L} D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}}{a + b(1-\lambda) D_0 \prod_{i=1}^{L} \left(g_i(0)/D_{i0}\right)^{1/L} + b\,\lambda\, e^{\sigma_k^2 (T-t)/L} D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}} \sigma_{It} \\
\sigma_{Yt} &= \frac{b\,\lambda\, e^{-\sigma^2 (T-t)} D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}}{a + b(1-\lambda) D_0 \prod_{i=1}^{L} \left(g_i(0)/D_{i0}\right)^{1/L} + b\,\lambda\, e^{-\sigma^2 (T-t)} D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}} \sigma_{It},
\end{aligned}$$

and σ_{It} is the volatility vector of $D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L} = E_t\left[I_T\right]$ given by

$$\sigma_{It} = \left(\sigma,\, -\tfrac{1}{L}\sigma_1, \ldots, -\tfrac{1}{L}\sigma_L,\, 0, \ldots, 0\right).$$

We note that $Y_t \sigma_{Yt} = Z_t \sigma_{Zt} e^{-\left(\sigma^2 + \sigma_k^2/L\right)(T-t)}$. Hence, we have

$$\begin{aligned}
Z_t \sigma_{Zt} Y_t - Y_t \sigma_{Yt} Z_t &= Z_t \sigma_{Zt} \left(Y_t - e^{-\left(\sigma^2 + \sigma_k^2/L\right)(T-t)} Z_t\right) \\
&= Z_t \sigma_{Zt} \left(1 - e^{-\left(\sigma^2 + \sigma_k^2/L\right)(T-t)} Z_t\right) \left(a + b(1-\lambda) D_0 \prod_{i=1}^{L} \left(g_i(0)/D_{i0}\right)^{1/L}\right),
\end{aligned} \tag{A19}$$

where the second equality follows by substituting Z_t and Y_t and manipulating terms. Substituting (A19) into the expression $\sigma_{Zt} - \sigma_{Yt} = \left(Z_t \sigma_{Zt} Y_t - Y_t \sigma_{Yt} Z_t\right)/Y_t Z_t$, and then into (A18) leads to the equilibrium volatility vector of loadings of index commodity futures in (17) where

$$\begin{aligned}
h_{kt} = &\frac{b\,\lambda\, e^{\sigma_k^2 (T-t)/L} \left(1 - e^{-\left(\sigma^2 + \sigma_k^2/L\right)(T-t)}\right) \left(a + b(1-\lambda) D_0 \prod_{i=1}^{L} \left(g_i(0)/D_{i0}\right)^{1/L}\right)}{a + b(1-\lambda) D_0 \prod_{i=1}^{L} \left(g_i(0)/D_{i0}\right)^{1/L} + b\,\lambda\, e^{\sigma_k^2 (T-t)/L} D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}} \\
&\times \frac{D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}}{a + b(1-\lambda) D_0 \prod_{i=1}^{L} \left(g_i(0)/D_{i0}\right)^{1/L} + b\,\lambda\, e^{-\sigma^2 (T-t)} D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}} > 0,
\end{aligned} \tag{A20}$$

where $g_i(t)$ is as in (16).

To determine the volatility vector of loadings of nonindex futures $k = L+1, \ldots, K$, as reported in (18), we follow the same steps as above for index futures, and obtain the stochastic process h_t as

$$\begin{aligned}
h_t = &\frac{b\,\lambda \left(1 - e^{-\sigma^2 (T-t)}\right) \left(a + b(1-\lambda) D_0 \prod_{i=1}^{L} \left(g_i(0)/D_{i0}\right)^{1/L}\right)}{a + b(1-\lambda) D_0 \prod_{i=1}^{L} \left(g_i(0)/D_{i0}\right)^{1/L} + b\,\lambda\, D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}} \\
&\times \frac{D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}}{a + b(1-\lambda) D_0 \prod_{i=1}^{L} \left(g_i(0)/D_{i0}\right)^{1/L} + b\,\lambda\, e^{-\sigma^2 (T-t)} D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}} > 0,
\end{aligned} \tag{A21}$$

where $g_i(t)$ is as in (16).

The property that volatilities of all futures prices are higher than in the benchmark economy follow immediately from (17)–(18). To prove property (ii), we note that for commodities i and k with $D_{it} = D_{kt}$, $\alpha_i = \alpha_k$, we have $h_{kt} > h_t$ from (A20)–(A21), and hence the volatility increase for an index futures is higher than that for an otherwise identical nonindex futures.

Q.E.D.

Proposition A1 (Stock market level and volatility). In the economy with institutions, the equilibrium stock market level and volatility vector are given by

$$S_t = \overline{S}_t \frac{a + b(1-\lambda)D_0 \prod_{i=1}^{L} (g_i(0)/D_{i0})^{1/L} + b\,\lambda\, D_t \prod_{i=1}^{L} (g_i(t)/D_{it})^{1/L}}{a + b(1-\lambda)D_0 \prod_{i=1}^{L} (g_i(0)/D_{i0})^{1/L} + b\,\lambda\, e^{-\sigma^2(T-t)} D_t \prod_{i=1}^{L} (g_i(t)/D_{it})^{1/L}}, \tag{A22}$$

$$\sigma_{St} = \overline{\sigma}_S + h_{St}\sigma_{It}, \quad h_{St} > 0, \tag{A23}$$

where $\overline{S}_t$ and $\overline{\sigma}_S$ are the corresponding quantities in the benchmark economy with no institutions, given by

$$\overline{S}_t = \frac{\sum_{k=0}^{K} \alpha_k}{\alpha_0} e^{(\mu-\sigma^2)(T-t)} D_t, \quad \overline{\sigma}_S = \sigma, \tag{A24}$$

and h_{St} is a strictly positive stochastic process given by (A21), and σ_{It} is as in Proposition 2. Consequently, in equilibrium, the stock market level and its volatility $\|\sigma_{St}\|$ are increased in the presence of institutions.

Proof. By no arbitrage, the stock market level is given by

$$S_t = \frac{E_t[\xi_T D_T]}{\xi_t}. \tag{A25}$$

To compute the expected deflated stock market payoff, we substitute (A7) and (12) to obtain

$$E_t[\xi_T D_T] = \bar{\xi} \sum_{k=0}^{K} \frac{\alpha_k}{\alpha_0} \left(a + b(1-\lambda)D_0 \prod_{i=1}^{L} (g_i(0)/D_{i0})^{1/L} + b\,\lambda\, D_t \prod_{i=1}^{L} (g_i(t)/D_{it})^{1/L} \right), \tag{A26}$$

where we have used the fact that D_T, D_{iT}, $i = 1, \dots, L$ are lognormally distributed, and $\bar{\xi}$ is as in (A11) and $g_i(t)$ is as in (16). Substituting (A26) and (A13) into (A25), and manipulating, leads to the reported equilibrium stock market level in (A22). The equilibrium stock market level $\bar{S}_t$ in the benchmark economy (A24) follows by considering the special case of $a = 1$, $b = 0$ in (13).

To derive the stock market volatility vector (A23), we follow the same steps for the index futures in the proof of Proposition 2, and obtain the stochastic process h_{St} to be as in (A21). The property that the stock market level and its volatility are higher than those in the benchmark follow straightforwardly from the expressions (A22)–(A24).

Q.E.D.

Proof of Proposition 3. Maximizing the consumer's objective function leads to his optimal commodity demands as

$$C_{0s} = \frac{\alpha_0}{\sum_{j=0}^{K} \alpha_j} W_{c_s}, \; C_{ks} = \frac{\alpha_k}{\sum_{j=0}^{K} \alpha_j} \frac{W_{c_s}}{p_{ks}}, \quad s = t, \, t+1, \tag{A27}$$

implying

$$p_{ks} = \frac{\alpha_k}{\alpha_0} \frac{C_{0s}}{C_{ks}}, \quad s = t, \, t+1. \tag{A28}$$

Substituting the market clearing conditions (22)–(23) for each good into (A28) leads to the equilibrium storable and non-storable commodity prices reported in (26)–(28). Substituting the storable commodity equilibrium prices (26)–(27) into (21) leads to the equilibrium inventories satisfying (24). The stochastic discount factor in (24) is given by $M_{t,t+1} = \xi_{t+1}/\xi_t$ and is determined by substituting the equilibrium state price density ξ_t in (A13), leading to the expression in (25) where

$$A = a + b\,(1-\lambda)\, D_0 \prod_{i=1}^{L} \left(g_i\,(0)\,/D_{i0}\right)^{1/L}. \tag{A29}$$

To prove the stated properties, we first note that the equilibrium inventories in the benchmark economy with no institutions $\overline{X}_t$ satisfy

$$\begin{aligned}
\frac{D_t}{D_{xt} - \overline{X}_t} &= (1-\delta) E_t \left[\frac{\bar{\xi}_{t+1}}{\bar{\xi}_t} \frac{D_{t+1}}{D_{xt+1} + (1-\delta)\overline{X}_t} \right] \\
&= (1-\delta) E_t \left[\frac{\bar{\xi}_{t+1}}{\bar{\xi}_t} D_{t+1} \right] E_t \left[\frac{1}{D_{xt+1} + (1-\delta)\overline{X}_t} \right] \\
&< (1-\delta) E_t \left[\frac{\xi_{t+1}}{\xi_t} D_{t+1} \right] E_t \left[\frac{1}{D_{xt+1} + (1-\delta)\overline{X}_t} \right],
\end{aligned} \tag{A30}$$

where the second equality uses the independence of D_{t+1} and D_{xt+1}, and $\bar{\xi}_t$ is given by (A13) with $b = 0$, and the inequality follows from the stock market implication in Proposition A1 and the law of iterated expectations applied to (A30). On the other hand, the equilibrium inventories with institutions satisfy

$$\frac{D_t}{D_{xt}-X_t} = (1-\delta)E_t\left[\frac{\xi_{t+1}}{\xi_t}\frac{D_{t+1}}{D_{xt+1}+(1-\delta)X_t}\right] \tag{A31}$$

$$\begin{aligned} &= (1-\delta)E_t\left[\frac{\xi_{t+1}}{\xi_t}D_{t+1}\right]E_t\left[\frac{1}{D_{xt+1}+(1-\delta)X_t}\right] \\ &+ \frac{1-\delta}{\xi_t}\mathrm{Cov}_t\left(\xi_{t+1}D_{t+1}, \frac{1}{D_{xt+1}+(1-\delta)X_t}\right). \end{aligned} \tag{A32}$$

Substituting ξ_t from (A13) and using the independence of D_t, D_{xt}, D_{it}, we see that both arguments of the covariance term are decreasing in D_{xt+1}, implying that the covariance term is positive if commodity x is in the index, and is zero if commodity x is not in the index, and hence

$$\frac{D_t}{D_{xt}-X_t} \geq (1-\delta)E_t\left[\frac{\xi_{t+1}}{\xi_t}D_{t+1}\right]E_t\left[\frac{1}{D_{xt+1}+(1-\delta)X_t}\right]. \tag{A33}$$

The inequalities (A30) and (A33) imply the inventory property result (ii), which then implies the commodity price property (i). The last property (iii) follows from the fact that the weak inequality in (A33) holds with a strict inequality for an index commodity x, and holds with an equality for a nonindex commodity. *Q.E.D.*

Proof of Proposition 4. We first prove the properties in (i). Rearranging (A31) gives

$$\frac{D_t}{(1-\delta)} = E_t\left[\frac{\xi_{t+1}}{\xi_t}D_{t+1}\frac{D_{xt}-X_t}{D_{xt+1}+(1-\delta)X_t}\right]. \tag{A34}$$

Taking the derivative with respect to the supply of index commodity D_{it} of both sides of (A34) yields

$$0 = E_t\left[\frac{\partial}{\partial D_{it}}\left(\frac{\xi_{t+1}}{\xi_t}D_{t+1}\right)\frac{D_{xt}-X_t}{D_{xt+1}+(1-\delta)X_t} + \frac{\xi_{t+1}}{\xi_t}D_{t+1}\frac{\partial}{\partial D_{it}}\left(\frac{D_{xt}-X_t}{D_{xt+1}+(1-\delta)X_t}\right)\right]. \tag{A35}$$

Substituting ξ_t from (A13), we have

$$\frac{\xi_{t+1}}{\xi_t}D_{t+1} = D_t e^{\left(\mu-\sigma^2\right)}\frac{A+b\lambda D_{t+1}\prod_{i=1}^{L}\left(g_i\left(t+1\right)/D_{it+1}\right)^{1/L}}{A+b\lambda e^{-\sigma^2}D_t\prod_{i=1}^{L}\left(g_i\left(t\right)/D_{it}\right)^{1/L}}, \tag{A36}$$

where $g_i\left(t\right)$ is as in (16) and A as in (A29). Hence we have

$$\frac{\partial}{\partial D_{it}}\frac{\xi_{t+1}}{\xi_t}D_{t+1} = -\frac{A}{LD_{it}}\frac{1}{A+b\lambda e^{-\sigma^2}D_t\prod_{i=1}^{L}\left(g_i\left(t\right)/D_{it}\right)^{1/L}}\left[\frac{\xi_{t+1}}{\xi_t}D_{t+1}-D_t e^{\left(\mu-\sigma^2\right)}\right], \tag{A37}$$

and

$$E_t\left[\frac{A+b\lambda D_{t+1}\prod_{i=1}^{L}\left(g_i\left(t+1\right)/D_{it+1}\right)^{1/L}}{A+b\lambda e^{-\sigma^2}D_t\prod_{i=1}^{L}\left(g_i\left(t\right)/D_{it}\right)^{1/L}}\right] = \frac{A+b\lambda D_t\prod_{i=1}^{L}\left(g_i\left(t\right)/D_{it}\right)^{1/L}}{A+b\lambda e^{-\sigma^2}D_t\prod_{i=1}^{L}\left(g_i\left(t\right)/D_{it}\right)^{1/L}} > 1, \tag{A38}$$

also implying from (A36) that

$$E_t\left[\frac{\xi_{t+1}}{\xi_t}D_{t+1}\right] > D_t e^{\left(\mu-\sigma^2\right)}. \tag{A39}$$

Substituting (A37) into the first expectation of (A35) gives

$$\begin{aligned}
&E_t\left[\frac{\partial}{\partial D_{it}}\left(\frac{\xi_{t+1}}{\xi_t}D_{t+1}\right)\frac{D_{xt}-X_t}{D_{xt+1}+(1-\delta)X_t}\right]\\
&=-\frac{A}{LD_{it}}\frac{D_{xt}-X_t}{A+b\lambda e^{-\sigma^2}D_t\prod_{i=1}^{L}\left(g_i(t)/D_{it}\right)^{1/L}}E_t\left[\left(\frac{\xi_{t+1}}{\xi_t}D_{t+1}-D_te^{\left(\mu-\sigma^2\right)}\right)\frac{1}{D_{xt+1}+(1-\delta)X_t}\right]\\
&=-\frac{A}{LD_{it}}\frac{D_{xt}-X_t}{A+b\lambda e^{-\sigma^2}D_t\prod_{i=1}^{L}\left(g_i(t)/D_{it}\right)^{1/L}}\\
&\quad\times\left(E_t\left[\frac{\xi_{t+1}}{\xi_t}D_{t+1}-D_te^{\left(\mu-\sigma^2\right)}\right]E_t\left[\frac{1}{D_{xt+1}+(1-\delta)X_t}\right]+\mathrm{Cov}_t\left[\frac{\xi_{t+1}}{\xi_t}D_{t+1},\frac{1}{D_{xt+1}+(1-\delta)X_t}\right]\right)<0,
\end{aligned}$$

where the inequality follows from (A39) and the fact that the covariance term is positive as in the proof of Proposition 3 (see equation (A32)). Since the first expectation of (A35) is negative, the second expectation term must be positive, i.e.,

$$\begin{aligned}
0 &< E_t\left[\frac{\xi_{t+1}}{\xi_t}D_{t+1}\frac{\partial}{\partial D_{it}}\left(\frac{D_{xt}-X_t}{D_{xt+1}+(1-\delta)X_t}\right)\right]\\
&= -E_t\left[\frac{\xi_{t+1}}{\xi_t}D_{t+1}\frac{D_{xt+1}+(1-\delta)D_{xt}}{\left(D_{xt+1}+(1-\delta)X_t\right)^2}\right]\frac{\partial X_t}{\partial D_{it}},
\end{aligned}$$

implying $\partial X_t/\partial D_{it}<0$. Therefore from (26), we deduce

$$\frac{\partial p_{xt}}{\partial D_{it}}=\frac{\alpha_x}{\alpha_0}\frac{D_t}{\left(D_{xt}-X_t\right)^2}\frac{\partial X_t}{\partial D_{it}}<0.$$

To prove the spillover property from the demand of the index commodity, we take the derivative of both sides of (A34) with respect to α_i:

$$0=E_t\left[\frac{\partial}{\partial\alpha_i}\left(\frac{\xi_{t+1}}{\xi_t}D_{t+1}\right)\frac{D_{xt}-X_t}{D_{xt+1}+(1-\delta)X_t}+\frac{\xi_{t+1}}{\xi_t}D_{t+1}\frac{\partial}{\partial\alpha_i}\left(\frac{D_{xt}-X_t}{D_{xt+1}+(1-\delta)X_t}\right)\right]. \tag{A40}$$

Taking the derivative of (A36), substituting $\partial g_i(t)/\partial\alpha_i=g_i(t)/\alpha_i$, $\partial A/\partial\alpha_i=(A-a)/L\alpha_i$ and manipulating, we obtain

$$\frac{\partial}{\partial\alpha_i}\frac{\xi_{t+1}}{\xi_t}D_{t+1}=\frac{a}{\alpha_iL}\frac{1}{A+b\lambda e^{-\sigma^2}D_t\prod_{i=1}^{L}\left(g_i(t)/D_{it}\right)^{1/L}}\left[\frac{\xi_{t+1}}{\xi_t}D_{t+1}-D_te^{\left(\mu-\sigma^2\right)}\right]. \tag{A41}$$

Substituting (A41) into the first expectation of (A40) gives

$$
\begin{aligned}
&E_t\left[\frac{\partial}{\partial\alpha_i}\left(\frac{\xi_{t+1}}{\xi_t}D_{t+1}\right)\frac{D_{xt}-X_t}{D_{xt+1}+(1-\delta)\,X_t}\right]\\
=&\frac{a}{\alpha_i L}\frac{D_{xt}-X_t}{A+b\lambda e^{-\sigma^2}D_t\prod_{i=1}^{L}\left(g_i\left(t\right)/D_{it}\right)^{1/L}}E_t\left[\left(\frac{\xi_{t+1}}{\xi_t}D_{t+1}-D_t e^{\left(\mu-\sigma^2\right)}\right)\frac{1}{D_{xt+1}+(1-\delta)\,X_t}\right]\\
=&\frac{a}{\alpha_i L}\frac{D_{xt}-X_t}{A+b\lambda e^{-\sigma^2}D_t\prod_{i=1}^{L}\left(g_i\left(t\right)/D_{it}\right)^{1/L}}\\
&\times\left(E_t\left[\frac{\xi_{t+1}}{\xi_t}D_{t+1}-D_t e^{\left(\mu-\sigma^2\right)}\right]E_t\left[\frac{1}{D_{xt+1}+(1-\delta)\,X_t}\right]+\mathrm{Cov}_t\left[\frac{\xi_{t+1}}{\xi_t}D_{t+1},\frac{1}{D_{xt+1}+(1-\delta)\,X_t}\right]\right)>0,
\end{aligned}
$$

where the inequality follows from (A39) and the covariance term being positive as in equation (A32) of the proof of Proposition 3. Since the first expectation of (A40) is positive, the second expectation term must be negative, i.e.,

$$
\begin{aligned}
0&>E_t\left[\frac{\xi_{t+1}}{\xi_t}D_{t+1}\frac{\partial}{\partial\alpha_i}\left(\frac{D_{xt}-X_t}{D_{xt+1}+(1-\delta)\,X_t}\right)\right]\\
&=-E_t\left[\frac{\xi_{t+1}}{\xi_t}D_{t+1}\frac{D_{xt+1}+(1-\delta)\,D_{xt}}{\left(D_{xt+1}+(1-\delta)\,X_t\right)^2}\right]\frac{\partial X_t}{\partial\alpha_i},
\end{aligned}
$$

implying $\partial X_t/\partial\alpha_i>0$. Therefore from (26), we deduce

$$
\frac{\partial p_{xt}}{\partial\alpha_i}=\frac{\alpha_x}{\alpha_0}\frac{D_t}{\left(D_{xt}-X_t\right)^2}\frac{\partial X_t}{\partial\alpha_i}>0.
$$

To demonstrate the spillover from the mean growth μ_i and volatility σ_i of the index commodity supply, we note that the equilibrium inventories X_t must satisfy (A31), which is driven by the state price density ξ_t. The equilibrium ξ_t as given in (A13) is itself driven by both μ_i and σ_i, leading to the stated dependence.

To prove property (ii), we again note that the equilibrium inventories X_t must satisfy (A31). Consider a decrease in D_{it} while keeping all other state variables and the inventory X_{kt} fixed. If the commodity i is not in the index, ξ_s, $s=t,t+1$ and D_{kt} are independent of D_{it} (see (1)–(2) and (A13)), and D_{it} does not directly enter (A32). Hence X_{kt} is independent of D_{it}. From (26), the price p_{kt} is also unchanged. The same goes for α_i. Furthermore, in the benchmark economy with no institutions the state price density $\overline{\xi}_t$ is given by (A13) with $b=0$, which is not dependent on D_{it} or α_i. Hence, in the benchmark economy the storable good inventory and price do not depend on D_{it} or α_i.

To prove property (iii), the spillover from an inflow of institutions, we take the partial derivative of both sides of (A34) with respect to λ:

$$
0=E_t\left[\frac{\partial}{\partial\lambda}\left(\frac{\xi_{t+1}}{\xi_t}D_{t+1}\right)\frac{D_{xt}-X_t}{D_{xt+1}+(1-\delta)\,X_t}+\frac{\xi_{t+1}}{\xi_t}D_{t+1}\frac{\partial}{\partial\lambda}\left(\frac{D_{xt}-X_t}{D_{xt+1}+(1-\delta)\,X_t}\right)\right]. \tag{A42}
$$

Taking the derivative of (A36), we obtain

$$\frac{\partial}{\partial\lambda}\frac{\xi_{t+1}}{\xi_t}D_{t+1} = D_t e^{\left(\mu-\sigma^2\right)}\frac{\partial A/\partial\lambda + bD_{t+1}\prod_{i=1}^{L}\left(g_i\left(t+1\right)/D_{it+1}\right)^{1/L}}{A + b\lambda e^{-\sigma^2}D_t\prod_{i=1}^{L}\left(g_i\left(t\right)/D_{it}\right)^{1/L}}$$
$$-\frac{\xi_{t+1}}{\xi_t}D_{t+1}\frac{\partial A/\partial\lambda + be^{-\sigma^2}D_t\prod_{i=1}^{L}\left(g_i\left(t\right)/D_{it}\right)^{1/L}}{A + b\lambda e^{-\sigma^2}D_t\prod_{i=1}^{L}\left(g_i\left(t\right)/D_{it}\right)^{1/L}}, \tag{A43}$$

where $\partial A/\partial\lambda = -bD_0\prod_{i=1}^{L}\left(g_i\left(0\right)/D_{i0}\right)^{1/L}$. Substituting (A43) into the first expectation of (A42) yields

$$E_t\left[\frac{\partial}{\partial\lambda}\left(\frac{\xi_{t+1}}{\xi_t}D_{t+1}\right)\frac{D_{xt}-X_t}{D_{xt+1}+\left(1-\delta\right)X_t}\right]$$
$$=E_t\left[D_t e^{\left(\mu-\sigma^2\right)}\frac{\partial A/\partial\lambda + bD_{t+1}\prod_{i=1}^{L}\left(g_i\left(t+1\right)/D_{it+1}\right)^{1/L}}{A + b\lambda e^{-\sigma^2}D_t\prod_{i=1}^{L}\left(g_i\left(t\right)/D_{it}\right)^{1/L}}\frac{D_{xt}-X_t}{D_{xt+1}+\left(1-\delta\right)X_t}\right]$$
$$-E_t\left[\frac{\xi_{t+1}}{\xi_t}D_{t+1}\frac{\partial A/\partial\lambda + be^{-\sigma^2}D_t\prod_{i=1}^{L}\left(g_i\left(t\right)/D_{it}\right)^{1/L}}{A + b\lambda e^{-\sigma^2}D_t\prod_{i=1}^{L}\left(g_i\left(t\right)/D_{it}\right)^{1/L}}\frac{D_{xt}-X_t}{D_{xt+1}+\left(1-\delta\right)X_t}\right].$$

Following similar steps to those in the proof of property (i), after some algebra, we deduce that the above expression is positive. Since the first expectation of (A42) is positive, the second expectation term must be negative, i.e.,

$$0 > E_t\left[\frac{\xi_{t+1}}{\xi_t}D_{t+1}\frac{\partial}{\partial\lambda}\left(\frac{D_{xt}-X_t}{D_{xt+1}+\left(1-\delta\right)X_t}\right)\right]$$
$$= -E_t\left[\frac{\xi_{t+1}}{\xi_t}D_{t+1}\frac{D_{xt+1}+\left(1-\delta\right)D_{xt}}{\left(D_{xt+1}+\left(1-\delta\right)X_t\right)^2}\right]\frac{\partial X_t}{\partial\lambda},$$

implying $\partial X_t/\partial\lambda > 0$. Hence, from (26) we obtain

$$\frac{\partial p_{xt}}{\partial\lambda} = \frac{\alpha_x}{\alpha_0}\frac{D_t}{\left(D_{xt}-X_t\right)^2}\frac{\partial X_t}{\partial\lambda} > 0.$$

Q.E.D.

Online Appendix B: Economy with Demand Shocks

In this section we introduce commodity demand shocks to our baseline model. While our setup with supply-side-only uncertainty is capable of delivering our main insights, we need a richer model to explore the quantitative effects of financialization. It has been argued in the literature that demand shocks are very important in explaining the behavior of prices of oil and other commodities (see Fattouh, Kilian, and Mahadeva (2013) for a survey). For example, Kilian and Murphy (2014) argue that the 2004-2008 surge in oil prices can be attributed to demand shocks. Furthermore, commentators frequently attribute commodity price rises to an increased demand from China, as its industry grows and population becomes wealthier and so its consumption basket becomes more commodity-intensive. They also link the increase in the cross-commodity correlations to China, whose high growth has led to a simultaneous increase in demand for a number of key commodities. In this section, we consider correlated demand shocks and show that such shocks alone can generate a sizeable increase in futures prices and their comovement. But, importantly, demand shocks also magnify the effects of financialization. Within our model we can disentangle how much of a rise in futures prices and their comovement can be attributed to positive demand shocks alone and how much to financialization.

To model demand shocks, we make the following modification to our model. In the consumption index (8) of the investors, repeated here for expositional clarity,

$$W_n = C_{n0}^{\alpha_0} C_{n1}^{\alpha_1} \cdot \ldots \cdot C_{nK}^{\alpha_K}, \qquad n \in \{\mathcal{N}, \mathcal{I}\}, \tag{B1}$$

we allow two demand parameters, α_1 and α_2, to be stochastic. Shocks to α_1 and α_2 then represent shifts in demand for goods 1 and 2 in the commodity index; we hereafter refer to them as demand shocks. We do not consider shocks to demand for other goods, but our model can be extended to incorporate such shocks. We assume that α_1 and α_2 are strictly positive processes with dynamics

$$d\alpha_{1t} = \alpha_{1t}\sigma_{\alpha_1} dw_{0t} \quad \text{and} \quad d\alpha_{2t} = \alpha_{2t}\sigma_{\alpha_2} dw_{0t}, \tag{B2}$$

where $\sigma_{\alpha_1}, \sigma_{\alpha_2} > 0$ are constant. Implicit in this assumption is that α_1 and α_2 are driven by the same source of risk. This source of risk, Brownian motion w_0, is the one driving aggregate output D—i.e., α_1 and α_2 have one-to-one mappings with aggregate output.[1] Now an investor's time-T demand for goods 1 is not simply a (decreasing) function of its price p_{1T}, but also an (increasing) function of the aggregate output D_T (through α_{1T}). (The demand for good 2 is determined analogously.) The latter assumption has been advocated by Dvir and Rogoff (2009) in their model of oil prices. In the numerical illustration that follows, we associate commodities 1 and 2 with energy and refer to them as energy 1 and energy 2. We include futures on both energy 1 and energy 2 in the index, consistently with energy futures being included in all popular commodity indices.

Proposition B1 reports the equilibrium futures prices and their return volatilities in the economy with demand shocks in closed form.

[1] Using the dynamics in (2) and (B2), one can establish the following mapping between α_{it} and D_t: $\alpha_{it} = \alpha_{i0}\, e^{-\sigma_{\alpha_i}^2/2\, t + \sigma_{\alpha_i}/\sigma[\log Y_t/Y_0 - (\mu - \sigma^2/2)t]}$, $i = 1, 2$.

Proposition B1 (Futures prices and volatilities with demand shocks). *In the economy with institutions and demand shocks, the equilibrium futures price of commodity $k = 1, \ldots, K$ and its associated volatility vector of loadings are given by*

$$f_{kt} = \overline{f}_{kt} \frac{B + b\,\lambda e^{\left(1_{\{k\leq L\}}\sigma_k^2/L + 1_{\{k\leq 2\}}(L\sigma + \sigma_{\alpha 1} + \sigma_{\alpha 2})\sigma_{\alpha k}/L\right)(T-t)} \alpha_{1t}^{1/L} \alpha_{2t}^{1/L} D_t \prod_{i=1}^{L} \left(\hat{g}_i(t)/D_{it}\right)^{1/L}}{B + b\lambda e^{-(\sigma^2 + (\sigma\sigma_{\alpha 1} + \sigma\sigma_{\alpha 2})/L)(T-t)} \alpha_{1t}^{1/L} \alpha_{2t}^{1/L} D_t \prod_{i=1}^{L} \left(\hat{g}_i(t)/D_{it}\right)^{1/L}}, \tag{B3}$$

$$\sigma_{f_k t} = \overline{\sigma}_{f_k} + \hat{h}_{kt}\sigma_{It}, \quad \hat{h}_{kt} > 0, \tag{B4}$$

where $B, \hat{g}_i(t) > 0$, and $\overline{f}_{kt}$ is the equilibrium futures price in the benchmark economy with no institutions, $\overline{\sigma}_{f_k}$ its corresponding volatility vector, and σ_{It} is the volatility vector of the conditional expected index $E_t[I_T]$, given by

$$\overline{f}_{kt} = \frac{\left(\alpha_k 1_{\{k>2\}} + \alpha_{kt} 1_{\{k\leq 2\}}\right) e^{(\mu - \mu_k - \sigma^2 + \sigma_k^2)(T-t)}}{\alpha_0} \frac{D_t}{D_{kt}}, \tag{B5}$$

$$\overline{\sigma}_{f_k} = (\sigma + \sigma_{\alpha 1} 1_{\{k=1\}} + \sigma_{\alpha 2} 1_{\{k=2\}}, 0, \ldots, -\sigma_k, 0, \ldots, 0), \tag{B6}$$

$$\sigma_{It} = (\sigma + \tfrac{1}{L}(\sigma_{\alpha 1} + \sigma_{\alpha 2}), -\tfrac{1}{L}\sigma_1, \ldots, -\tfrac{1}{L}\sigma_L, 0, \ldots, 0), \tag{B7}$$

and the constant B, the deterministic quantity $\hat{g}_i(t)$ and the stochastic process $\hat{h}_{kt}$ are explicitly provided in the proof, located at the end of this appendix.

Consequently, in equilibrium, all futures prices and their volatilities $\|\sigma_{f_k}\|$ are higher than in the benchmark economy.

Proposition B1 confirms our earlier result that all futures prices are higher in the presence of institutions, with prices of index futures exceeding those of nonindex ones. The distinguishing feature of our economy with demand shocks is that these effects become stronger than in the economy without demand shocks and the cross-commodity futures return correlations go up sizably, reaching the levels documented in post-financialization period in the data (Tang and Xiong (2012)). Below we identify plausible parameter values to assess the quantitative importance of our results.

Since we have taken commodities 1 and 2 to represent energy, we infer the demand parameters α_{1t} and α_{2t} from the energy expenditure share in total consumption. The total expenditure share in our model is given by $(\alpha_{1t} + \alpha_{2t})/(\sum_{k=0,k\neq 1,2}^{K} \alpha_k + \alpha_{1t} + \alpha_{2t})$. For convenience, we set our baseline parameters such that $\sum_{k=0,k\neq 1,2}^{K} \alpha_k + \alpha_{1t} + \alpha_{2t} = 1$ at time t. We obtain data on the energy expenditure share in the US from BEA Table 2.3.5U from 1959:M1 through 2012:M12.[2] The average expenditure share in the sample is about 6%, and so we set $\alpha_{1t} = \alpha_{2t} = 0.03$. As also noted by Hamilton (2013), the energy expenditure share series is very volatile. We set $\sigma_{\alpha_1} = \sigma_{\alpha_2} = 9.8\%$ to match the volatility estimate obtained from the series. Finally, the series does not have a deterministic trend, and we cannot reject the null that it has a unit root, which supports our specification in (B2). The expenditure share of the generic good is taken to be 70%, and the remaining expenditure is spread equally across the remaining commodities (other

[2] Hamilton (2013) uses the same data source in his detailed analysis of the energy expenditure share.

Parameter or State Variable	Symbol	Value
Mean growth rate of generic good's supply news	μ	0.05
Volatility of generic good's supply news	σ	0.15
Mean growth rate of commodity k supply news, $k = 1, \ldots, K$	μ_k	0.05
Volatility of commodities 1 and 2 (energy) supply news	σ_1, σ_2	0.29
Volatility of commodity $k \neq 1, 2$ (non-energy) supply news	σ_k	0.24
Volatility of commodities 1 and 2 demand shocks	$\sigma_{\alpha_1}, \sigma_{\alpha_2}$	0.098
Demand parameter, generic good	α_0	0.7
Time-0 and time-t demand parameter for energy	α_{10}, α_{20}, α_{1t}, α_{2t}	0.03
Demand parameter, commodity $k = 1, \ldots K$	α_k	0.08
Number of commodities	K	5
Number of commodities in the index	L	2
Terminal date	T	5 years
Current date	t	0.1 years
Size of institutions	λ	0.3
Objective function parameters	a, b	1
Time-0 and time-t supply of generic good	D_0, D_t	100
Time-0 and time-t supply of commodity k, $k = 1, \ldots, K$	D_{k0}, D_{kt}	1

Table B1: **Parameter values and state variables.**

than energy). We set the volatility of the process for the generic good supply news D_t to be consistent with the stock market volatility expression (B25) below in the Supplement to Table B1 (using the value of 16% for the aggregate US stock market volatility in the data).[3] The parameter value for the volatility of generic good supply news, σ, is then around 15%, which is consistent with the aggregate dividend volatility in the data). The model-implied volatility parameters of the processes for D_1, D_2 and D_k, $k = 3, \ldots, K$ are obtained from equation (B7) using data on the average volatilities of the energy-sector and non-energy sector futures from Gorton, Hayashi, and Rouwenhorst (2013). We set the mean growth rates $\mu = \mu_k = 0.05$ for all k; our results do not vary much for a wide range of alternative values for these parameters.

The horizon T is set in line with the typical performance evaluation horizons of fund managers, usually 3-5 years (BIS (2003)). We interpret our parameter λ as the fraction of investors in the commodity futures market who are benchmarked to a commodity index. The closest measure for this in the data is the fraction of commodity index traders in this market. Using proprietary data, Aulerich, Irwin, and Garcia (2010) and Cheng, Kirilenko, and Xiong (2015) provide suggestive evidence for what the value of λ could be, based on the percent of total open interest held by commodity index traders (net long positions). The magnitudes vary across commodities, averaging around 0.32 post-2004 (Aulerich et al.'s data). Since this measure is just a rough proxy, to be conservative, we set $\lambda = 0.2$ and report our results for a range of values

[3]We recognize that equilibrium models with logarithmic preferences, such as ours, are unable to deliver a realistic equity premium if one imposes that aggregate consumption is equal to aggregate dividend and uses consumption data to infer model parameters. In order to match the equity premium, one would need to employ more sophisticated preferences or to break the relationship that aggregate consumption equals to aggregate dividend (e.g., by assuming that the stock market is a levered claim on aggregate consumption).

around $\lambda = 0.2$.[4] These and the remaining parameter values are summarized in Table B1.

Figure B1 illustrates the results of Proposition B1 and disentangles the contribution of financialization over and above that of fundamentals (demand and supply). We vary the demand parameter α_{1t} for energy commodity 1 to highlight the contribution of a rising/decreasing demand. As one can see from the figure, increasing demand for energy pushes up its futures price even in the benchmark economy with no institutions (the dashed lines).[5] But in the presence of institutions, the futures price increases even more, and especially so in the presence of demand shocks (solid blue line). This is because there is now an additional risk in the economy—shifts in demand for energy—that affects the value of the index. Therefore assets whose payoffs are positively correlated with these demand shocks—the energy futures—become even more valuable than in the economy without demand shocks. Proposition B1 also uncovers that there is a spillover of demand shocks to energy onto the other futures prices. The mechanism for these spillovers echoes the one in Section 3; the spillovers to other futures occur because demand shocks affect the value of the index.

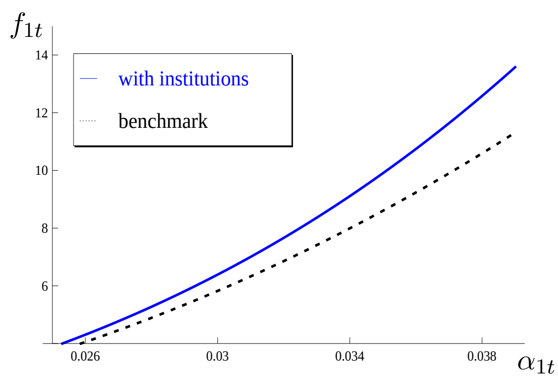


Figure B1: **Futures prices.** This figure plots index futures 1 price in the economy with demand shocks (solid blue line) against the demand parameter α_{1t} for energy commodity 1. The dotted black line is for the corresponding prices in the benchmark economy with no institutions. The parameter values are as in Table B1.

[4]Our leading interpretation of λ is the fraction of (long-only) institutional investors in the commodity futures market. Much of the financialization literature has been motivated by the fact that there has been a very significant inflow of such investors into this market. However, because our model is a general equilibrium model, our institutional investors also participate in the overall stock market clearing, and hence could be interpreted as the percentage of commodity-oriented institutional investors in all capital markets. Under the second interpretation, λ should be smaller. One possible way to address the problem with the multiple interpretations of λ is to have several separate classes of institutional investors: the ones that are benchmarked to other popular indexes, and most notably, the stock market index, and our commodity-oriented institutional investors. Each class of investors will then have its own λ, to be interpreted as the fraction of institutional investors in the corresponding market. It would then be much easier to assign a value to the total and each individual λ.

[5]Equation (B5) reveals that the benchmark price of energy futures is linear in the energy 1 demand parameter α_{1t}. However, there is also an indirect dependence of benchmark futures prices of all commodities on α_{1t} through the aggregate output D_t. This is because α_{1t} and D_t are driven by the same source of risk, the Brownian motion ω_0, and a rise in α_{1t} always coincides with a rise in D_t. That is why the plot of the futures price against α_{1t} is convex, even in the benchmark economy without institutions.

		$\lambda = 0.1$, $\alpha_{1t} = 0.03$	$\lambda = 0.2$, $\alpha_{1t} = 0.03$	$\lambda = 0.3$, $\alpha_{1t} = 0.03$	$\lambda = 0.1$, $\alpha_{1t} = 0.04$	$\lambda = 0.2$, $\alpha_{1t} = 0.04$	$\lambda = 0.3$, $\alpha_{1t} = 0.04$
	0.24	4.00%	8.13%	12.38%	9.81%	17.60%	24.56%
σ_1	0.29	4.78%	**9.71%**	14.80%	11.71%	21.31%	29.31%
	0.34	5.79%	11.76%	17.91%	14.16%	25.74%	35.40%

Table B2: **Increase in energy futures prices with financialization,** $(f_{1t} - \overline{f}_{1t})/\overline{f}_{1t}$**.** The parameter values are as in Table B1, unless specified otherwise. The figure in bold is for our baseline parameter values (in Table B1).

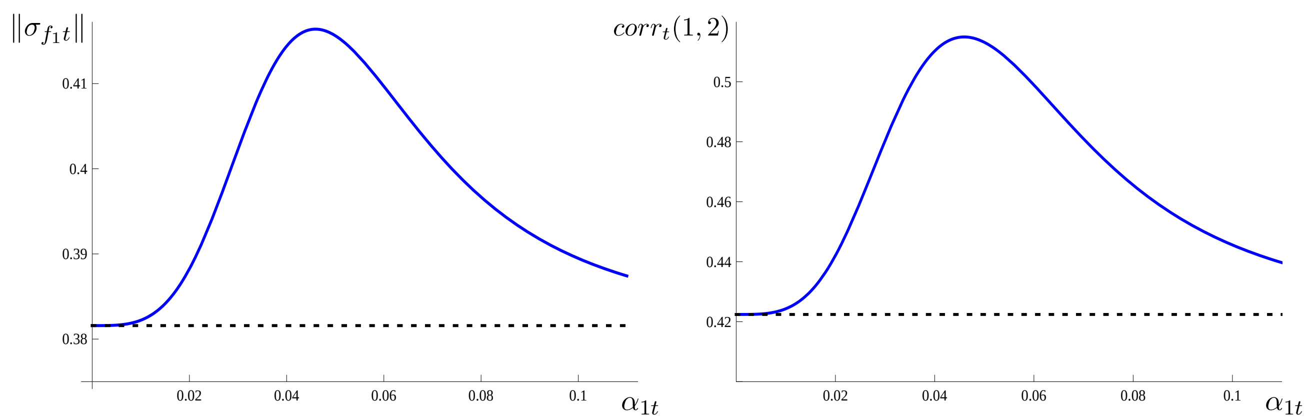
Above effects are quantitatively important. As revealed by Table B2, for our baseline parameterization, we find that the increase in energy futures price with financialization is 9.71%. This result is sensitive to the parameter λ, with our estimates for the price increase ranging from 4.71% for $\lambda = 0.1$ to 14.80% for $\lambda = 0.3$. We also perform a sensitivity analysis around our parameter values for the energy supply news volatility and report the resulting values in Table B2. Our results are not out of line with the findings of Kilian and Murphy (2014) that fluctuations in fundamentals are important in explaining the fluctuations in commodity prices, but we also stress a significant contribution of financialization. To this discussion, we should add a caveat that our model abstracts away from several important influences on commodity futures prices and so our quantitative results should be taken with caution.

Table B2 also highlights that the magnitudes of the impact of financialization on futures prices are rather sensitive to the volatility of the supply news; the more volatile the energy commodity supply news are, the bigger the increase of the commodity futures prices that is attributable to financialization. In unreported analysis, we find that the effects of financialization are also stronger the larger the aggregate output news volatility σ and the larger the demand uncertainly σ_{α_i}. These comparative statics may shed light on the time-variation in futures prices. During periods of high uncertainty the effects of financialization are amplified, pushing prices much higher than what could have been justified by fundamentals (supply and demand) alone.

Kilian and Murthy argue further that most of the 2003-2008 increase in energy prices (specifically, oil prices) was due to global demand shocks. Our model delivers this result, but also uncovers an important interaction: the effects of financialization become stronger with higher global demand. To illustrate this implication, we explore within our model the effects of an upward demand shift for energy from the baseline value of $\alpha_{1t} = 0.03$ to 0.04—a 33% increase. The last three columns of Table B2 present the (recomputed) increases in futures prices that are attributable to financialization. As one can see clearly, financialization becomes significantly more important. For our baseline parameter values, the energy futures price increase attributable to financialization rises from 9.71% to 21.31%.[6]

[6]Table B2 demonstrates that our results are quite sensitive to the energy supply news volatility σ_1. For robustness, we re-examine our results parameter values based on a study of Vassilev (2010). Vassilev's study includes fewer commodities, and his data implies parameter values $\sigma_1 = 0.33$ and $\sigma_k = 0.25$, $k > 2$. (The remaining parameter values remain as in Table B1). In this new exercise, we find that the increase in energy futures prices with financialization rises from 9.71% to 11.37%. For an upward demand shift for energy from

Proposition B1 also confirms that our remaining results of Section 3 continue to hold in the presence of demand shocks. In particular, futures return volatilities are higher in the presence of institutions. Moreover, we find in our numerical illustration that they become much higher in the presence of demand shocks. To understand the intuition behind this new result, it is useful to note that the energy futures is exposed to an additional source of risk, demand shocks, and so it is more volatile even in the benchmark economy with no institutions. Additionally, the membership of energy futures in the index makes the index riskier than in the economy without demand shocks. Since falling behind the index is a source of risk for institutional investors, all futures prices depend on the expected index, as we have highlighted before. The (expected) index appears as a new risk factor in the futures prices, and this factor is now more volatile (higher $||\sigma_I||$). Consequently, all futures prices are more volatile as well. In terms of magnitudes, for our baseline parameter values, the volatility of energy futures prices in our model rises from around 0.33 in the economy without demand shocks (Section 3, Figure 3) to 0.41 in the economy with demand shocks. Figure B2(a) presents the sensitivity of these magnitudes to the energy demand parameter α_{1t}.



(a) Volatility of energy futures

(b) Energy futures return correlation

Figure B2: **Volatilities and correlations.** Panel (a) plots return volatility of energy futures (solid blue line). Panel (b) plots return correlations of energy futures, $corr_t(1, 2)$ (solid blue line). Both plots are against the energy demand parameter α_{1t}. The dotted lines are for the corresponding quantities in the benchmark economy with no institutions. The parameters are from Table B1.

The commodity futures return correlations also rise with financialization. As before, this is because the expected index emerges as a common factor affecting all assets in the economy, and hence the covariances of all assets with each other increase more than in the economy without demand shocks. The same ends up being true for the corresponding correlations. To illustrate the effects of financialization on the correlations quantitatively, in Figure B2(b) we plot energy futures return correlations in the economies with and without institutions. Our model with demand shocks delivers large correlation increases, part of which is due to fundamentals (the

$\alpha_{1t} = 0.03$ to 0.04, this value becomes 24.88%.

common demand shock) and part of which to the presence of institutions. For example, in the baseline model in the benchmark economy (Section 3, Figure 4), the correlation between two index futures returns is 0.21, while in the economy with the common demand shock and institutions it rises to 0.5 (of this rise, the rise from 0.21 to 0.42 is attributed to fundamentals). These magnitudes are roughly consistent with the evidence in Tang and Xiong (2012), who document that the average correlation of index commodities with oil rose from 0.1 pre-2004 to about 0.5 in 2009. Furthermore, the correlation of nonindex commodities in this numerical exercise is only 0.28—much lower than that of index (energy) commodities, which is roughly in line with the data (Tan and Xiong). This is because in our numerical illustration nonindex commodities are not affected by the common demand shock, which we believe is a reasonable assumption. That could be part of an explanation behind the large difference in the index vs. nonindex commodities returns correlations in the data.

Proof of Proposition B1. We first consider the investors' optimal demands in each commodity. Maximizing the institutional investor's expected objective function (7), subject to (A1) evaluated at $t = 0$ leads to the institution's optimal demand in commodity $k = 3, \ldots, K$ and generic good as in (A2) of Section 3, and demand in commodities 1 and 2 as

$$C_{\mathcal{I}_1 T} = \frac{\alpha_{1T}\left(a + bI_T\right)}{y_{\mathcal{I}} p_{1T} \xi_T}, \quad C_{\mathcal{I}_2 T} = \frac{\alpha_{2T}\left(a + bI_T\right)}{y_{\mathcal{I}} p_{2T} \xi_T}. \tag{B8}$$

Here, $1/y_{\mathcal{I}}$ solves (A1) evaluated at $t = 0$, and using the lognormal distribution property of $D_T, D_{kT}, \alpha_{1T}, \alpha_{2T}$, is given by

$$\frac{1}{y_{\mathcal{I}}} = \frac{\lambda\, \xi_0 S_0}{a\left(\sum_j \alpha_j + \alpha_1 + \alpha_2\right) + b\left(\sum_j \alpha_j E\left[I_T\right] + E\left[\alpha_{1T} I_T\right] + E\left[\alpha_{2T} I_T\right]\right)}, \tag{B9}$$

where henceforth the summation $\sum_j$ denotes the summation over all commodities but the first two, i.e., $j = 0, 3, \ldots, K$, and $\alpha_1 \equiv \alpha_{10}$, $\alpha_2 \equiv \alpha_{20}$ denote the initial values of the processes α_{1t} and α_{2t}, respectively. Similarly, we obtain the normal investor's optimal commodity demands at time T for $k = 0, 3, \ldots, K$ to be as previously in (A5)–(A6), and for commodities 1 and 2 as

$$C_{\mathcal{N}_1 T} = \frac{\alpha_{1T}\left(1 - \lambda\right) \xi_0 S_0}{(\sum_j \alpha_j + \alpha_1 + \alpha_2) p_{1T} \xi_T}, \quad C_{\mathcal{N}_2 T} = \frac{\alpha_{2T}\left(1 - \lambda\right) \xi_0 S_0}{(\sum_j \alpha_j + \alpha_1 + \alpha_2) p_{2T} \xi_T}. \tag{B10}$$

To determine the equilibrium state price density, we impose market clearing for the generic good, $C_{\mathcal{N}_0 T} + C_{\mathcal{I}_0 T}$, substitute (A4) and (A6), and rearrange to obtain at T

$$\xi_T = \bar{\xi} \frac{1}{D_T}\left(B + b\lambda I_T\right), \tag{B11}$$

where

$$\bar{\xi} = \frac{\alpha_0 \xi_0 S_0}{a\left(\sum_j \alpha_j + \alpha_1 + \alpha_2\right) + b\left(\sum_j \alpha_j E\left[I_T\right] + E\left[\alpha_{1T} I_T\right] + E\left[\alpha_{2T} I_T\right]\right)}, \tag{B12}$$

$$B = a + b\left(1 - \lambda\right) \frac{\sum_j \alpha_j E\left[I_T\right] + E\left[\alpha_{1T} I_T\right] + E\left[\alpha_{2T} I_T\right]}{\sum_j \alpha_j + \alpha_1 + \alpha_2}. \tag{B13}$$

To determine the equilibrium commodity prices at T, we impose the market clearing condition $C_{\mathcal{N}_kT} + C_{\mathcal{I}_kT} = D_{kT}$ for each commodity $k = 1, \ldots, K$, and substitute (A3), (A5), (B8) and (B10) and the equilibrium state price density (B11) to obtain the same commodity prices (11) as in Lemma 1 for $k = 3, \ldots, K$, and for commodities 1 and 2 we obtain

$$p_{1T} = \frac{\alpha_{1T}}{\alpha_0}\frac{D_T}{D_{1T}}, \quad p_{2T} = \frac{\alpha_{2T}}{\alpha_0}\frac{D_T}{D_{2T}}. \tag{B14}$$

Substituting the equilibrium commodity prices (11), (B14), into the definition of the index (4) leads to the time-T equilibrium commodity index value

$$I_T = \frac{\alpha_{1T}^{1/L}\alpha_{2T}^{1/L}}{\alpha_1^{1/L}\alpha_2^{1/L}}\frac{D_T}{\alpha_0}\prod_{i=1}^{L}\left(\alpha_i/D_{iT}\right)^{1/L}. \tag{B15}$$

Hence, making use of the lognormal property of D_T, D_{iT}, α_{1T}, α_{2T}, we deduce the unconditional expected index value to be

$$E\left[I_T\right] = e^{\left(\mu+\frac{1}{L}\left(\sigma\sigma_{\alpha 1}+\sigma\sigma_{\alpha 2}+\frac{1}{2}\left(\frac{1}{L}-1\right)\left(\sigma_{\alpha 1}^2+\sigma_{\alpha 2}^2\right)+\frac{1}{L}\sigma_{\alpha 1}\sigma_{\alpha 2}\right)-\frac{1}{L}\sum_{i=1}^{L}\left(\mu_i-\frac{1}{2}\left(\frac{1}{L}+1\right)\sigma_i^2\right)\right)T}\frac{D_0}{\alpha_0}\prod_{i=1}^{L}\left(\frac{\alpha_i}{D_{i0}}\right)^{1/L}, \tag{B16}$$

and the unconditional expectations in (B12)–(B13) as

$$E\left[\alpha_{1T}I_T\right] = \alpha_1 e^{\left(\sigma\sigma_{\alpha 1}+\left(\sigma_{\alpha 1}+\sigma_{\alpha 2}\right)\sigma_{\alpha 1}/L\right)T}E\left[I_T\right],$$

$$E\left[\alpha_{2T}I_T\right] = \alpha_2 e^{\left(\sigma\sigma_{\alpha 2}+\left(\sigma_{\alpha 1}+\sigma_{\alpha 2}\right)\sigma_{\alpha 2}/L\right)T}E\left[I_T\right].$$

We now determine the equilibrium futures prices. First, the equilibrium time-t state price density follows by taking the conditional expectation of (B11), substituting (B15), using the lognormality of D_T, D_{iT}, α_{1T}, α_{2T}, and after some algebra we get

$$\xi_t = \bar{\xi}e^{\left(\sigma^2-\mu\right)(T-t)}\frac{1}{D_t}\left(B + b\lambda e^{-\left(\sigma^2+\left(\sigma\sigma_{\alpha 1}+\sigma\sigma_{\alpha 2}\right)/L\right)(T-t)}\alpha_{1t}^{1/L}\alpha_{2t}^{1/L}D_t\prod_{i=1}^{L}\left(\hat{g}_i\left(t\right)/D_{it}\right)^{1/L}\right), \tag{B17}$$

where $\bar{\xi}$ and B are as in (B12)–(B13), and

$$\hat{g}_i\left(t\right) = \frac{\alpha_i}{\alpha_0\alpha_1^{1/L}\alpha_2^{1/L}}e^{\left(\mu+\frac{1}{L}\left(\sigma\sigma_{\alpha 1}+\sigma\sigma_{\alpha 2}+\frac{1}{2}\left(\frac{1}{L}-1\right)\left(\sigma_{\alpha 1}^2+\sigma_{\alpha 2}^2\right)+\frac{1}{L}\sigma_{\alpha 1}\sigma_{\alpha 2}\right)-\mu_i+\frac{1}{2}\left(\frac{1}{L}+1\right)\sigma_i^2\right)(T-t)}. \tag{B18}$$

To compute the expected deflated futures payoff of commodity $k = 3, \ldots, K$, we substitute (B11) and (11), and rearrange to obtain

$$E_t\left[\xi_T p_{kT}\right] = \frac{\bar{\xi}}{\alpha_0}\alpha_k\frac{e^{\left(\sigma_k^2-\mu_k\right)(T-t)}}{D_{kt}}\left(B + b\lambda\frac{E_t\left[I_T/D_{kT}\right]}{E_t\left[1/D_{kT}\right]}\right). \tag{B19}$$

For nonindex futures contracts $k = L+1, \ldots, K$, using the lognormality of D_T, D_{iT}, α_{1T}, α_{2T}, and substituting (B15) we obtain

$$\frac{E_t\left[I_T/D_{kT}\right]}{E_t\left[1/D_{kT}\right]} = \alpha_{1t}^{1/L}\alpha_{2t}^{1/L}D_t\prod_{i=1}^{L}\left(\hat{g}_i\left(t\right)/D_{it}\right)^{1/L}, \tag{B20}$$

where $\hat{g}_i(t)$ is as in (B18). For index futures contracts except for the first two commodity futures, $k = 3, \ldots, L$, we get

$$\frac{E_t\left[I_T/D_{kT}\right]}{E_t\left[1/D_{kT}\right]} = e^{\frac{1}{L}\sigma_k^2(T-t)}\alpha_{1t}^{1/L}\alpha_{2t}^{1/L}D_t\prod_{i=1}^{L}\left(\hat{g}_i(t)/D_{it}\right)^{1/L}. \tag{B21}$$

Finally, for the first two index futures contracts $k = 1, 2$, we substitute (B11) and (B14), and rearrange to obtain

$$E_t\left[\xi_T p_{kT}\right] = \frac{\bar{\xi}}{\alpha_0}\alpha_{kt}\frac{e^{\left(\sigma_k^2-\mu_k\right)(T-t)}}{D_{kt}}\left(B + b\lambda\frac{E_t\left[\alpha_{kT}I_T/D_{kT}\right]}{E_t\left[\alpha_{kT}/D_{kT}\right]}\right),$$

using the lognormality of D_T, D_{iT}, α_{1T}, α_{2T}, and substituting (B15) we obtain

$$\frac{E_t\left[\alpha_{kT}I_T/D_{kT}\right]}{E_t\left[\alpha_{kT}/D_{kT}\right]} = e^{\left(\frac{1}{L}(L\sigma+\sigma_{\alpha 1}+\sigma_{\alpha 2})\sigma_{\alpha k}+\frac{1}{L}\sigma_k^2\right)(T-t)}\alpha_{1t}^{1/L}\alpha_{2t}^{1/L}D_t\prod_{i=1}^{L}\left(\hat{g}_i(t)/D_{it}\right)^{1/L}. \tag{B22}$$

Substituting (B19)–(B22) and (B17) into (A9) and rearranging leads to the equilibrium index futures price expression reported in (B3). The equilibrium futures price $\bar{f}_k$ in the benchmark economy with no institutions (B5) follows by considering the special case of $a = 1$, $b = 0$ in (B3). The property that the futures prices are higher than in the benchmark economy follows by observing that the factor multiplying $\bar{f}_{kt}$ in expression (B3) is strictly greater than one.

To derive the equilibrium volatility vector of loadings, we apply Itô's Lemma to the futures price expression (B3), and follow similar steps to those in the proof of Proposition 2 to deduce (B4) in Proposition B1, where

$$\begin{aligned}\hat{h}_{kt} = &\frac{b\,\lambda\,B\left(e^{\left(1_{\{k\le L\}}\sigma_k^2/L+1_{\{k\le 2\}}(L\sigma+\sigma_{\alpha 1}+\sigma_{\alpha 2})\sigma_{\alpha k}/L\right)(T-t)} - e^{-\left(\sigma^2+(\sigma\sigma_{\alpha 1}+\sigma\sigma_{\alpha 2})/L\right)(T-t)}\right)}{B + b\,\lambda e^{\left(1_{\{k\le L\}}\sigma_k^2/L+1_{\{k\le 2\}}(L\sigma+\sigma_{\alpha 1}+\sigma_{\alpha 2})\sigma_{\alpha k}/L\right)(T-t)}\alpha_{1t}^{1/L}\alpha_{2t}^{1/L}D_t\prod_{i=1}^{L}\left(\hat{g}_i(t)/D_{it}\right)^{1/L}} \\ &\times\frac{\alpha_{1t}^{1/L}\alpha_{2t}^{1/L}D_t\prod_{i=1}^{L}\left(\hat{g}_i(t)/D_{it}\right)^{1/L}}{B + b\,\lambda e^{-(\sigma^2+(\sigma\sigma_{\alpha 1}+\sigma\sigma_{\alpha 2})/L)(T-t)}\alpha_{1t}^{1/L}\alpha_{2t}^{1/L}D_t\prod_{i=1}^{L}\left(\hat{g}_i(t)/D_{it}\right)^{1/L}} > 0,\end{aligned} \tag{B23}$$

where B and $\hat{g}_i(t)$ are as in (B13) and (B18), respectively. The property that volatilities of all futures price returns are higher than in the benchmark economy follows immediately from (B4) since $h_{kt} > 0$.

Q.E.D.

Supplement to Table B1: Stock volatility. To determine the equilibrium stock market volatility for our numerical illustration we note that in the benchmark economy with no institutions the stock market terminal value is given by

$$\overline{S}_T = \frac{\sum_j \alpha_j + \alpha_{1T} + \alpha_{2T}}{\alpha_0}D_T,$$

where we have substituted (11) and (B8). Following similar steps in the determination of equilibrium futures prices above, we arrive at the following equilibrium stock market level and its associated vector of loadings in the benchmark economy with demand shocks:

$$\bar{S}_t = \frac{\sum_j \alpha_j + \alpha_{1t} + \alpha_{2t}}{\alpha_0} e^{\left(\mu - \sigma^2\right)(T-t)} D_t, \tag{B24}$$

$$\bar{\sigma}_{St} = \left(\sigma + \frac{\alpha_{1t}\sigma_{\alpha_1} + \alpha_{2t}\sigma_{\alpha_2}}{\sum_j \alpha_j + \alpha_{1t} + \alpha_{2t}}, 0, \ldots, 0 \right). \tag{B25}$$

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