Compatibility Choices, Switching Costs and Data Portability: On the Role of the Non-Negative Pricing Constraint *

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Abstract

We study mix-and-match compatibility choices of firms selling complementary products in a dynamic setting. In contrast to what happens in a static setting where symmetric firms choose compatibility (Matutes and Régibeau, 1988), when consumers face significant switching costs and firms can poach them by making behavior-based price discrimination, symmetric firms choose incompatibility to soften future competition. Even if this tends to harm consumers, incompatibility can increase welfare by reducing excessive switching. Data portability, by reducing switching costs, induces the firms to choose compatibility more often but, given a compatibility regime, benefits consumers only if the non-negative pricing constraint binds.

Key words: Compatibility, Switching Cost, Data Portability

JEL Codes: D43, L13, L41.

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1 Introduction

Will the future of the Internet be dominated by incompatible devices and applications? Back in the 90s' when the Internet was at its dawn, openness and compatibility seemed to be the rule. For instance, during the times, Microsoft was the dominant player in the personal computer market but decided to bring two of its most successful software, Internet Explorer and Microsoft Office, to Macs. However, after the turn of the 21st century, we seem to enter a new era in which platforms are becoming *"walled gardens"* trying to lock-in customers, either by making it hard to move data across platforms or by providing some benefits exclusively to those who use all from the same ecosystem. According to Larry Page, a cofounder of Google:

> "The Internet was made in universities and it was designed to interoperate. And as we've commercialized it, we've added more of an island-like approach to it, which I think is a somewhat a shame for users."[1]

We provide a theory which shows that competing firms selling complementary products embrace incompatibility in markets characterized by high switching costs and behavior-based price discrimination. Consider the market of Enterprise SaaS (Software as a Service).[2] In this market, switching from one vendor to another is very costly, for instance, in the cases of productivity software suites and file storage/hosting services. More precisely, switching from Microsoft Office to Google Docs/Sheets/Slides requires significant efforts to convert all existing files from one format to another and switching from Google Drive to Microsoft OneDrive requires moving all users' data from one service to another which can be very costly depending on the size of the client company. Other markets like cloud computing also exhibit high switching cost. As an executive of an Amazon Web Service (AWS) vendor partner put it, "data gravity makes lock-in worse with Amazon"[3], implying that as the data stocked in one platform grows, it becomes harder to move from the platform. This applies not only to enterprise markets but also to mass consumer markets as each consumer accumulates more and more data in one platform. For instance, 1.2 trillion photos were taken with smartphones in 2017[4]. Google and Apple offer their photo storage services, Google Photos and iCloud Photos. As a consumer accumulates more photos in one of the two platforms, it becomes harder to switch.

[1] http://fortune.com/2012/12/11/fortune-exclusive-larry-page-on-google/

[2] SaaS is one of the three models of cloud computing service and the other two are PaaS (Platform as a Service) and IaaS (Infrastructure as a Service). A recent communication from the European Commission (2020) proposing a European strategy for data points out market power in cloud computing service as a problem and discusses high concentration in the cloud computing industry and large platforms' data advantage.

[3] http://fortune.com/2015/10/08/aws-lock-in-worry/

[4] https://www.businessinsider.com/12-trillion-photos-to-be-taken-in-2017-thanks-to-smartphones-chart-2017-8

An increasingly common feature of the above-mentioned markets is behavior-based price discrimination: firms offer a discount to poach customers from rivals. For instance, in the Enterprise SaaS market, Microsoft tried to poach Google Drive users "by offering free OneDrive for Business for the remaining term of their existing contract with Google"[5]. At the same time, Google offered a similar incentive to Microsoft Office customers by "cover[ing] the fees of Google Apps until [their] contract runs out [... and] chip[ing] in on some of the deployment costs"[6]. In consumer markets, the rapid advance in information technology makes it easier for sellers to condition their price offers on consumers' prior purchase behavior. Firms can offer personalized discounts through targeted messages although list prices are publicly quoted (Acquisti and Varian, 2005). The behavior-based price discrimination can also take the form of trade-in, meaning that a vendor offers discounts to a rival's customers in exchange of their devices. For instance, Google offered up to $600 to iPhone user to switch to Google Pixel[7], Samsung offered the full price of a Google Pixel in exchange of a Samsung Galaxy[8], and Microsoft paid $650 to Apple MacBook users to trade-in their MacBook for a Microsoft Surface[9].

Our paper has three sets of results. First, we attempt to understand firms' mix-and-match compatibility choices from a dynamic perspective and find that symmetric firms make their products incompatible in order to soften future competition when customer lock-in arises due to high switching costs and they practice behavior-based price discrimination. Our result is opposite to what happens in a static model in which symmetric firms make their products compatible to soften competition (Matutes and Régibeau, 1988). In the Enterprise SaaS market, there is some incompatibility between one platform's file storage service and another's productivity software suit. For instance, it's not possible to store Google Docs in Microsoft OneDrive, or to use all functionalities of Microsoft Office, like real-time coauthoring, when stored in Google Drive. We also find a strong conflict between the compatibility regime chosen by the firms and the one maximizing consumer surplus. However, incompatibility can raise welfare relative to compatibility by reducing excessive switching.

Second, we study the interaction between compatibility choices and the non-negative pricing constraint (NPC). In our two-period model, the firms compete fiercely in period one to build a customer base, which can be exploited in period two due to the switching cost. This competition may lead to negative prices in period one, which may be impractical due to adverse selection and opportunistic behaviors of consumers (Farrell and Gallini,1988, Amelio and Jullien, 2012,

[5]https://www.microsoft.com/en-us/microsoft-365/blog/2018/02/06/make-the-switch-to-onedrive-for-improved-productivity-and-cost-savings/

[6]https://cloud.googleblog.com/2015/10/going-Google-just-got-easier.html

[7]https://bgr.com/2019/05/09/google-pixel-3a-deal-iphone-trade-in/

[8]https://lifehacker.com/samsung-will-pay-you-the-full-price-of-a-google-pixel-3-1836734167

[9]https://techcrunch.com/2016/10/28/microsoft-apple/

Choi and Jeon, forthcoming). Therefore, we study both the case in which the firms face the NPC and the case in which the constraint does not apply. When the NPC gets binding, it limits the dissipation of the second-period rent from locked-in consumers. As we find that the rent is always larger under incompatibility than under compatibility, the binding NPC expands the interval of switching costs under which incompatibility is chosen. We also find that the strong conflict between the compatibility regime chosen by the firms and the one maximizing consumer surplus exists regardless of whether the NPC binds. However, the binding NPC can mitigate the conflict between the compatibility regime chosen by the firms and the one maximizing welfare.

Third, as a policy remedy, we consider data portability, which is hotly discussed across the Atlantic.[10] We show that data portability, by lowering switching cost, induce the firms to embrace compatibility more often. Interestingly, we find that given a compatibility regime, whether data portability increases or reduces consumer surplus (and profits) completely depends on whether or not the NPC binds. If the constraint binds, data portability increases consumer surplus but reduces each firm's profit whereas the opposite holds when the constraint does not apply. At the end of the paper (in Section 7.1), we generalize the NPC and find an intermediate case in which data portability increases both consumer surplus and profits.

We extend the mix-and-match compatibility model of Matutes and Régibeau (1988) to two periods. They study compatibility choices made by two symmetric firms (A and B) which compete to sell a system of complementary products (x and y). Therefore, under compatibility, four systems are available $((A, A), (A, B), (B, A), (B, B))$ while under incompatibility, only two pure systems, (A, A) and (B, B), are available. They study a two-stage game in which the first stage of non-cooperative choice between compatibility and incompatibility is followed by the second stage of price competition. The first-period in our model is identical to the model of Matutes and Régibeau (1988). The firms make their compatibility choices in period one and the compatibility regime determined in period one is maintained in period two.[11] In period two, consumers incur switching costs when they consume products different from those consumed in the first period and each firm competes to poach consumers by offering prices dependent on their past purchase behavior (Chen, 1997b and Fudenberg and Tirole, 2000). Following Villas-Boas (2006) and Doganoglu (2010), we consider experience goods and assume that each consumer discovers the value that she obtains from a product after consuming it.

We assume for simplicity that all consumers incur the same switching cost per product $s > 0$. When all products are compatible, there are four submarkets in period two: the market i_j refers

[10] See the report commissioned by Vestager, the European Competition Commissioner, (Crémer et al., 2019), the report from UK digital competition expert panel (Furman et al., 2019), the Stigler report (2019), and the report on data from the Expert Group for the Observatory on Online Platform Economy (2020).

[11] In on-line appendix, we study an alternative scenario in which the firms make their compatibility choices in each period and find that the main results are similar (see Proposition 15 in the on-line appendix).

to the market composed of the consumers who bought product $j = x, y$ from firm $i = A, B$ in period one. If a consumer wants to switch from x of A to x of B, she should incur s and therefore firm A is dominant and firm B is dominated in market A_x. When products are incompatible, there are two submarkets (A, A) and (B, B) in period two, where the market (A, A) is composed of the consumers who bought the system (A, A) in period one. If a consumer wants to switch from (A, A) to (B, B), she should incur a switching cost of $2s$.

When we study how incompatibility affects the firms' second-period profits from the consumers who bought say (A, A) in period one, we find three thresholds of switching costs $(\bar{s}^1, \bar{s}^2, \bar{s}^3)$ with $\bar{s}^1 < \bar{s}^2 < \bar{s}^3$ such that the dominant firm's profit is higher under incompatibility than under compatibility for $s > \bar{s}^1$, the industry profit is higher under incompatibility for $s > \bar{s}^2$ and the dominated firm's profit is higher under incompatibility for $s > \bar{s}^3$. In other words, when the switching cost is high enough, incompatibility softens the second-period competition relative to compatibility. This result can be understood from Hahn and Kim (2012) and Hurkens, Jeon and Menicucci (2019), who extend Matutes and Régibeau (1988) to asymmetric firms (one is dominant and the other is dominated). They find that when the level of dominance is large enough, both the dominant firm and the dominated firm prefer incompatibility since incompatibility softens competition. This mechanism is in place in our model as the level of dominance increases with s (see Section 3.3).

When we study the first-period competition without the NPC, for each given regime of compatibility, each firm charges a price smaller than the one in Matutes and Régibeau (1988) by the difference equal to the second-period rent from a locked-in consumer. Hence, they completely dissipate this rent by competing aggressively in period one. This implies that each firm's total profit is the sum of the static profit in Matutes and Régibeau (1988) and the profit that a firm realizes in period two if it attracted no consumer in period one. The latter is equal to what we call the profit of the dominated firm. Therefore, for $s > \bar{s}^3$, if the relative weight of the second-period payoff is large enough, the firms choose incompatibility in period one in order to soften future competition. When the NPC binds, it limits the rent dissipation as the prices in period one are zero. Therefore, the firms choose incompatibility when it generates a higher industry profit in period two (i.e. when $s > \bar{s}^2$). Hence, the binding NPC expands the interval of switching costs under which incompatibility is chosen.

As the firms make compatibility choices mainly to soften competition, we find a strong conflict between the compatibility regime chosen by the firms and the one maximizing consumer surplus, regardless of whether the NPC binds. When the NPC does not apply, whenever the firms choose incompatibility, it generates a lower consumer surplus than under compatibility. The binding NPC induces the firms to choose incompatibility more often while inducing consumers to prefer compatibility more often, which preserves the conflict. We also find that incompatibility can increase welfare by reducing excessive switching for intermediate level of switching costs. However, when the NPC does not apply, the firms choose incompatibility only for very high switching costs such that whenever the firms choose incompatibility, it generates a lower welfare than under compatibility.

Finally, we analyze data portability policy in our setup. EU's General Data Protection Regulation (GDPR) provides consumers with the data portability right. According to Article 20 of GDPR,

"the data subject shall have the right to receive the personal data concerning him or her, which he or she has provided to a controller, in a structured, commonly used and machine-readable format and have the right to transmit those data to another controller without hindrance from the controller to which the personal data have been provided."

After discussing how the application of the article reduces switching cost in our context by distinguishing three categories of data, we analyze its impact. First, data portability, by reducing switching cost, can induce a change in the compatibility regime from incompatibility to compatibility. Second, we find that given a compatibility regime, whether data portability increases or reduces consumer surplus (and profits) completely depends on whether or not the NPC binds. If the constraint binds, data portability increases consumer surplus but reduces each firm's profit. By contrast, when the constraint does not apply, the opposite result holds. In Section 7.1, we generalize the NPC by allowing the firms to circumvent the NPC by offering "freebies" (Amelio and Jullien, 2012). As the freebies are likely to be less efficient than money in transferring utility from the firms to consumers, we introduce a pass-through rate and show that for intermediate pass-through rates, data portability increases both consumer surplus and profits.

1.1 Related literature

We merge two different strands of literature, the one on compatibility and the one on poaching. First, as incompatibility is equivalent to pure bundling, our paper is related to literature on bundling, in particular the one on competitive bundling which studies how bundling affects competition when entry or exit is not an issue (Matutes and Régibeau 1988, 1992, Economides, 1989, Carbajo, De Meza and Seidmann, 1990, Chen 1997a, Denicolo 2000, Nalebuff, 2000, Armstrong and Vickers, 2010, Carlton, Gans, and Waldman, 2010, Thanassoulis 2011).[12] Especially, our

[12] There are two other branches of the bundling literature. The first one includes the papers that view bundling as a price discrimination device for a monopolist (Stigler, 1968, Schmalensee, 1984, McAfee et al. 1989, Salinger 1995, Armstrong 1996, Bakos and Brynjolfsson, 1999, Fang and Norman, 2006, Chen and Riordan, 2013, Menicucci, Hurkens, and Jeon, 2015). The second is the leverage theory of bundling in which the main motive of bundling is to deter entry or induce the exit of rival firms in the competitive segment of the market (Whinston, 1990, Choi, 1996, Choi and Stefanadis 2001, Carlton and Waldman, 2002, Nalebuff, 2004, Peitz 2008, Jeon and Menicucci,

paper is related to the line of research on "mix and match" compatibility initiated by Matutes and Régibeau (1988), which has seen some recent development. While Matutes and Régibeau (1988) find that incompatibility intensifies competition in a symmetric duopoly, the extension to asymmetric duopoly by Hahn and Kim (2012) and Hurkens, Jeon and Menicucci (2019) shows that for large asymmetry, incompatibility softens competition. Kim and Choi (2015) and Zhou (2017) consider symmetric oligopoly of more than two firms and find that incompatibility can soften competition when the number of firms is above a threshold which can be small. We contribute to this literature by considering a dynamic setup and showing that even a symmetric duopoly can prefer incompatibility to soften competition. As we build on Matutes and Régibeau (1988), our model does not include network effects although network effects can be an important factor influencing firms' compatibility decisions. For instance, Katz and Shapiro (1985), Crémer, Rey and Tirole (2000) and Chen, Doraszelski and Harrington (2009) study compatibility choices in the presence of network effects. Note that in these papers, symmetric firms always adopt compatibility as they can only benefit from a larger network effect.

Second, our two-period model is similar to those considered in the literature on poaching in the presence switching costs (Chen, 1997b) or in their absence (Fudenberg and Tirole, 2000).[13] As our paper considers switching costs, it is closer to Chen (1997b) which studies a duopoly model with homogenous products and heterogenous switching costs. Both Chen (1997b) and Fudenberg and Tirole (2000) compare the allocation under poaching with the one without poaching. The main difference between our paper and theirs is that we consider multi-product firms and analyze their compatibility choices under poaching and how data portability affects the choices.

Our paper is also related to the large literature on switching costs.[14] Our model is very similar to that of Doganoglu (2010) which studies competition between two firms producing experience goods over an infinite horizon with overlapping generations of consumers. Utility of a consumer in our model is exactly the same as that of a consumer in Doganoglu (2010). However, Doganoglu (2010) considers neither poaching nor compatibility choices. To some extent, our model is similar to Somaini and Einav (2013), Rhodes (2014), Cabral (2016) and Lam (2017) which assume that consumers' valuations are independently and identically distributed over an Hotelling line across periods. Even if we do not formally make such assumption, a model with such assumption will generate exactly the same predictions as our current model.[15] Our contribution with respect to the literature on switching cost is twofold. First, we embed the mix-and-match compatibility

2006, 2012).

[13]The literature on behavior-based price discrimination is large. Aquisti and Varian (2005) is widely cited. See Fudenberg and Villas-Boas (2006) for a survey and Choe, King and Matsushima (2018) for a recent development.

[14]See Farrell and Klemperer (2007) for a survey.

[15]This is because we consider a Hotelling model for the first period and if we consider the same model for the second period as well, then the results from Hahn and Kim (2012) and Hurkens, Jeon and Menicucci (2019) apply as they consider either a Hotelling model or a log-concave family which includes the Hotelling model.

choices into a model of poaching under switching costs and study how switching costs (and data portability) affect the choices. Second, we show that whether a reduction in switching cost increases consumer surplus (and reduces profits) crucially depends on whether the NPC binds.

To our knowledge, Lam and Liu (2020) is the only economic article studying data portability. They consider an incumbent facing entry and find that data portability can hinder entry instead of facilitating it due to a demand-expansion effect: the possibility of porting data to an entrant induces consumers to provide more data to the incumbent. This can reduce switching as it increases the value of the incumbent's services based on big data analytics making use of inferred data, which is not subject to the portability obligation under EU's GDPR. Although their result is interesting, the forces generating the result are absent in our model. First, as we consider two symmetric incumbents and asymmetry in data analytic services does not exist.[16] Second, we do not consider consumers' active choices regarding how much data to provide because in reality most data is generated as a by-product of their consumption activities.[17] Therefore, we assume that data portability reduces switching costs. Our novelty consists in studying the interaction between data portability and compatibility choices, on the one hand, and the one between data portability and the NPC, on the other hand.

The paper is organized as follows. Section 2 describes the model. Section 3 (4) analyzes the second-period (the first-period) price competition given a compatibility regime. Section 5 analyzes compatibility choices. Section 6 provides the analysis of consumer surplus and welfare. Section 7 provides the analysis of data portability. All the proofs are gathered in Appendix. The on-line Appendix analyzes the scenario in which the firms make compatibility choices each period.

2 The model

There are two firms, $i = A, B$, which produce two perfectly complementary products, $j = x, y$.[18] Therefore, consumer demand is defined for the system composed of two products. When both firms' products are compatible, there are four systems available: (A, A), (A, B), (B, A), (B, B). When firm A's products are not compatible with those of firm B, only two systems are available:

[16] As both firms have significant market shares and hence have inferred data from their consumers, both can provide services based on big data analytics. In contrast, an entrant cannot provide such services as it initially has no stock of inferred data.

[17] See the report commissioned by Vestager, the European Competition Commissioner, (Crémer et al., 2019), the report from UK digital competition expert panel (Furman et al., 2019), the Stigler report (2019) and the report on data from the Expert Group for the Observatory on Online Platform Economy (2020).

[18] In fact, we obtain the same results even when the two products can be independently consumed instead of being perfect complements as long as (i) incompatibility is interpreted as pure bundling and (ii) each consumer obtains a high enough utility from each product x and y such that the market is fully covered for both products.

(A, A) and (B, B). We consider a two-period model in which consumers have switching costs in the second period.

We assume that each consumer obtains a high enough utility from the system that she buys one among the available systems in each period $t = 1, 2$. In addition, we assume that each consumer has a unit demand for the system: she buys only one among the available systems. Given these assumptions, we set the marginal cost of producing each product to zero for simplicity and interpret prices as margins.

Each firm simultaneously and non-cooperatively chooses between compatibility and incompatibility at the beginning of period one. Compatibility prevails only if both firms choose compatibility; otherwise, incompatibility prevails. The compatibility regime determined in period one is maintained in period two, which makes sense when compatibility choices are embedded into technical design of the products such that undoing the initial design is very costly.

In the first period, consumers have heterogeneous costs of learning to use different products as in Klemperer (1995). Precisely, each consumer is characterized by a pair of locations $(\theta_x, \theta_y) \in [0, 1]^2$ which determine her learning cost for each product: $t\theta_j$ $(t(1 - \theta_j))$ is the learning cost for product j of firm A (for product j of firm B) for $j = x, y$, for some $t > 0$.[19] Hence, a consumer located at $(\theta_x, \theta_y) \in [0, 1]^2$ incurs a total learning cost of $t\theta_x + t\theta_y$ to use system (A, A); the learning cost is $t\theta_x + t(1 - \theta_y)$ for system (A, B). The locations (θ_x, θ_y) are uniformly distributed over $[0, 1]^2$.

We consider experience goods as Villas-Boas (2006) and Doganoglu (2010) do. At the beginning of period one, every consumer has the same expected valuation $2v^e$ for each system. Therefore, depending on the compatibility regime, the first-period utility of a consumer located at (θ_x, θ_y) from purchasing (A, A) is given as follows. Under compatibility, it is

$$U_1(A, A) = 2v^e - p^A_{1,x} - p^A_{1,y} - t\theta_x - t\theta_y; \tag{1}$$

under incompatibility, it is

$$U_1(A, A) = 2v^e - P^A_1 - t\theta_x - t\theta_y,$$

where $p^i_{1,j}$ is the price for product j of firm i in period one under compatibility and P^i_1 is the price of system (i, i) in period one under incompatibility.[20] We assume that v^e is large enough to make the market fully covered. If there were no second period, then our model would be

[19]To make the model more reaslistic, we can add a common cost of learning k such that the total cost of learning becomes $k+ t\theta_j$ $(k + t(1 - \theta_j))$. k is distributed over $[\underline{k}, \overline{k}]$, independently of (θ_x, θ_y): some consumers have relatively high learning costs. As long as we assume that $v^e - \overline{k}$ is large enough to make the market fully covered, we obtain the same results.

[20]For prices, we use lower case letters under compatibility and upper case letters under incompatibility.

identical to that of Matutes and Régibeau (1988).

After a consumer uses product j $(= x, y)$ of firm i in period one, she discovers her own valuation v_j^i for the product, which she obtains only if the product is consumed together with a compatible product k ($k \neq j$ and $k = x, y$). v_j^i a random draw from a uniform distribution with support $[\underline{v}, \overline{v}]$ in which $\underline{v} > 0$. Hence, $v^e = (\underline{v} + \overline{v})/2$. We assume that the distribution of the valuation is independent across different products and different consumers.

Now we describe what happens in the second period. Consider a consumer who bought product x from A in period one. Then she has learnt her valuation v_x^A. Under compatibility, her choice in period two is either to consume the same product and obtain v_x^A, or to switch to product x of B. In the latter case her gross surplus is v^e minus the switching cost $s > 0$. We assume that each firm can engage in behaviour-based price discrimination to poach consumers: the price a firm charges to a consumer in period two can depend on the product she purchased in period one.

Regarding the switching cost per product s, for simplicity we assume that the switching cost is the same for all consumers and products.[21] s includes psychological and transactional cost of switching (Farrell and Klemperer, 2007). It also includes the cost of learning to use a different product in the second period, which is assumed to be much smaller than the one in the first period as she already learned to use a competing product.[22] Importantly, in the context of products providing data-based services, s captures the reduction in the quality of the service offered by the firm to which a consumer switches because it has no access to the data generated by the consumer in period one while she was using the rival's product. Therefore, data portability obligation can reduce the switching cost (see the detailed discussions in Section 7).

Suppose that the products are compatible and that a consumer who bought (A, A) in the first period switches to (A, B) in the second period. Then, her second-period utility is given by:

$$U_2(A, B)_{|(A,A)} = v_x^A + v^e - p_{2,x}^A(A) - p_{2,y}^B(A) - s, \tag{2}$$

where $p_{2,j}^i(h)$ is the second-period price charged by firm i for product j under compatibility to the consumers who bought product j of firm h in the first period, with $i, h \in \{A, B\}$.[23] Suppose now that the products are incompatible and that a consumer who bought (A, A) in the first

[21] Otherwise, each consumer has three-dimensional private information in the beginning of period two: the valution of each product she consumed in period one and her switching cost.

[22] In other words, given j $(= x, y)$, we assume that the cost of learning to use both product j of A and product j of B is much lower than the sum of the cost of learning to use product j of A only and the cost of learning to use product j of B only because of synergy in learning.

[23] We may allow $p_{2,j}^i(h)$ to depend not only on the firm h from which the consumer has bought product j in period one, but also on the firm from which the consumer has bought the other product in period one. Since each consumer's utility function is separable in the utility of the two products, such additional generality is irrelevant.

period switches to (B, B) in the second period. Then her second-period utility is given by:

$$U_2(B, B)_{|(A,A)} = 2v^e - P_2^B(A, A) - 2s,$$

where $P_2^i(h, h)$ is the second-period price charged by firm i for its system under incompatibility to the consumers who bought (h, h) in the first period with $i, h \in \{A, B\}$. Our model is similar to Beggs and Klemperer (1992), Klemperer (1995) and Doganoglu (2010) in that the Hotelling differentiation is assumed only in the first period.

All players have a common discount factor $\delta > 0$; δ can be larger than one since it represents the weight assigned to the second-period payoff. All firms have rational expectations. Whether consumers are myopic or forward-looking does not matter in our model. So we consider myopic consumers for our exposition and show in Section 4.3 that our results are unaffected when we consider forward-looking consumers.

As is typical in two-period models with switching costs, we find that the firms compete fiercely in period one to build a customer base, which can be exploited in period two due to the switching cost. This competition may lead to negative prices in period one, which may be impractical due to adverse selection and opportunistic behaviors by consumers (Farrell and Gallini, 1988, Amelio and Jullien, 2012 and Choi and Jeon, forthcoming). Then, the firms face a non-negative pricing constraint (NPC) in period one:[24]

$$\text{(NPC)} \qquad p_{1,j}^i \geq 0, \qquad P_1^i \geq 0 \qquad \text{for } j = x, y, \quad \text{for } i = A, B. \tag{3}$$

We analyze both the case in which the NPC does not need to be satisfied and the case in which it must be satisfied. The NPC is likely to be irrelevant in B2B markets while it is likely to matter in B2C markets. Section 7.1 provides an extension which generalizes the NPC such that it includes the two cases as extreme cases.

We introduce the following assumption to guarantee that a positive measure of consumers switch in each sub-market in period two:

Assumption 1: $s < \frac{3}{2}\Delta v$, where $\Delta v \equiv \overline{v} - \underline{v}$.

If this assumption is not satisfied, then no switching occurs in period two when the products are compatible.

The timing in period one is given by:

[24]In particular, Choi and Jeon (forthcoming) considers a two-period model with switching cost in which tying allows to circumvent the non-negative pricing constraint (see Section 2.B).

- Stage 1: Each firm simultaneously and non-cooperatively chooses between compatibility and incompatibility.
- Stage 2: After observing the compatibility regime determined by the choices, each firm simultaneously and non-cooperatively chooses its price(s).
- Stage 3: Consumers make purchase decisions.

In period two, only Stages 2 and 3 occur. Notice that there always exists an equilibrium in which both firms choose incompatibility. We assume that each firm plays its weakly dominant action and therefore compatibility arises if and only if both firms prefer compatibility.

In order to solve this two-period model, we first solve for the firms' second-period equilibrium behavior for any given first-period compatibility choice and market shares. We find that equilibrium prices and profits are linearly homogenous in Δv, therefore sometimes it is useful to normalize Δv to 1, as the model with $(\Delta v, s)$ is qualitatively equivalent to the one with $(1, s/\Delta v)$.

3 Second-period competition given a compatibility regime

In this section, we first study the second-period competition to poach consumers for *a given compatibility regime.* Second, we show how incompatibility affects the second-period profits relative to compatibility, which is important to understand the compatibility choices in period one.

3.1 Given compatibility

Suppose that the products are compatible and consider the market for product j composed of the consumers who bought this product from firm i in the first period. We call it market i_j. As all four markets A_x, A_y, B_x, B_y are alike, it is enough to analyze just one of them. We normalize the total mass of consumers in market i_j to one.

Consider market i_j. Because of the switching cost, firm i is the dominant firm and firm h ($\neq i$) is the dominated firm. Let p_2^+ (instead of $p_{2,j}^i(i)$) denote the price charged by the dominant firm and p_2^- (instead of $p_{2,j}^h(i)$) the price charged by the dominated firm. Likewise, let d_2^+ denote the demand for product j of the dominant firm and $\pi^+ = p_2^+ d_2^+$ its profit i; d_2^- and $\pi^- = p_2^- d_2^-$ are similarly defined.

A consumer with valuation v_j^i for product j of firm i is indifferent between buying again from firm i and switching to firm h if and only if

$$v_j^i - p_2^+ = v^e - p_2^- - s. \tag{4}$$

From (4) we obtain d_2^+ and d_2^- and the equilibrium prices, p_2^{+*} and p_2^{-*}, which maximize $p_2^+ d_2^+$ and $p_2^- d_2^-$, respectively.

Lemma 1. *Suppose that the products are compatible. Consider the second-period competition in market i_j composed of the consumers who bought product j from firm i in period one. We normalize the total mass of consumers in market i_j to one. Under Assumption 1, there exists a unique equilibrium. The equilibrium prices and profits are given by:*

$$p_2^{+*} = \frac{\Delta v}{2} + \frac{s}{3}, \qquad p_2^{-*} = \frac{\Delta v}{2} - \frac{s}{3} > 0;$$

$$\pi_2^{+*} = \frac{1}{\Delta v}\left(\frac{\Delta v}{2} + \frac{s}{3}\right)^2, \qquad \pi_2^{-*} = \frac{1}{\Delta v}\left(\frac{\Delta v}{2} - \frac{s}{3}\right)^2.$$

For the analysis of data portability in Section 7, it is useful to note that an increase in s reduces consumer surplus and increases the joint profit $\pi_2^{+*} + \pi_2^{-*}$. As s increases, the dominant firm has more market power and raises its price, which softens competition such that the sum of s and the dominated firm's price increases. Hence, both a consumer's payoff upon no switching and the one upon switching decrease with s whereas the competition-softening effect raises the joint profit.

Corollary 1. *In market i_j, as s increases, every consumer's payoff strictly decreases and the joint profit $\pi_2^{+*} + \pi_2^{-*}$ strictly increases.*

3.2 Given incompatibility

Suppose that the products are incompatible and consider the market (i, i) composed of the consumers who purchased both products from firm i in the first period. In this market, firm i is the dominant firm and firm h ($\neq i$) is the dominated firm. Let P_2^+ denote the price charged by the dominant firm and P_2^- the price charged by the dominated firm. Likewise, D_2^+, D_2^- and Π_2^+, Π_2^- denote the firms' demands and profits.

A consumer with valuations $\left(v_x^i, v_y^i\right)$ is indifferent between buying system (i, i) or system (h, h) if and only if

$$v_x^i + v_y^i - P_2^+ = 2v^e - P_2^- - 2s.$$

This allows to obtain D_2^+, D_2^- and the equilibrium prices, P_2^{+*}, P_2^{-*}.

Lemma 2. *Suppose that the products are incompatible. Consider the second-period competition in market (i, i) composed of the consumers who bought the system (i, i) in period one. We normalize the total mass of consumers in this market to one. There exists a unique equilibrium. The* *equilibrium prices and profits are given by:*

$$P_2^{+*} = \frac{1}{8}\left[3\sqrt{(2s-\Delta v)^2+8\Delta v^2}+5(2s-\Delta v)\right], \quad P_2^{-*} = \frac{1}{8}\left[\sqrt{(2s-\Delta v)^2+8\Delta v^2}-(2s-\Delta v)\right];$$

$$\Pi_2^{+*} = \left(1-\frac{2\left(P_2^{-*}\right)^2}{\Delta v^2}\right)P_2^{+*}, \quad \Pi_2^{-*} = \frac{2}{\Delta v^2}\left(P_2^{-*}\right)^3.$$

The effects of a higher switching cost on consumer surplus and the joint profit are qualitatively the same as under compatibility:

Corollary 2. *In market (i,i), as s increases, every consumer's payoff strictly decreases and the joint profit $\Pi_2^{+*}+\Pi_2^{-*}$ strictly increases.*

In addition, the following result is useful to understand the impact of compatibility choices on consumer surplus in Section 6.

Corollary 3. *We have*

$$\Pi_2^{+*} - \Pi_2^{-*} > 2\pi_2^{+*} - 2\pi_2^{-*} \text{ for any } s \in (0, \frac{3}{2}). \tag{5}$$

3.3 The effects of incompatibility: the demand size effect and the demand elasticity effect

In order to analyze how incompatibility affects the second-period profits relative to compatibility, we here apply the findings of Hurkens, Jeon and Menicucci (2019) to our model. They study pure bundling (which is equivalent to incompatibility) in a static model with two multi-product firms, assuming that one firm is dominant as it offers products with higher quality than the dominated rival firm. They decompose the effects of incompatibility into a demand size effect and a demand elasticity effect. We here apply their analysis to the second period of our model by considering the market composed of the consumers who bought both products from firm A (for instance) in period one. We normalize the mass of the consumers of this market to one. Let $F(\cdot)$ be the c.d.f. for the uniform distribution with support $[\underline{v}, \overline{v}]$, that is the distribution of v_j^A for $j = x, y$; $f(\cdot)$ is its density.[25]

Demand size effect. Suppose that under compatibility firm A charges p_2^+ and firm B charges p_2^- to each product. Let $\widetilde{v}_j^A$ be the valuation of the consumer who is indifferent between switching and no switching; hence the demand for A's product is $1-F(\widetilde{v}_j^A)$. Since $s > 0$, we expect $\widetilde{v}_j^A < v^e$ in equilibrium, hence $1-F(\widetilde{v}_j^A) > 1/2$. Suppose now that there is incompatibility and

[25] As Hurkens, Jeon and Menicucci (2019) consider a family of distributions which is log-concave and symmetric around the mean, the argument we describe here applies to the family.

that each firm sells its system at a price equal to the sum of the prices under compatibility, that is $P_2^+ = 2p_2^+$ and $P_2^- = 2p_2^-$. Then the indifferent consumer has the *average valuation* equal to $\widetilde{v}_j^A$ and the demand for A's system is $1 - \widehat{F}(\widetilde{v}_j^A)$, in which $\hat{F}$ is the distribution function for the average valuation. An important property of $\hat{F}$ is that $\hat{F}$ is more peaked around v^e than F: for each $\varepsilon \in (0, v^e - \underline{v})$, we have $\int_{v^e-\varepsilon}^{v^e+\varepsilon} f(s)ds < \int_{v^e-\varepsilon}^{v^e+\varepsilon} \hat{f}(s)ds$ where $\widehat{f}(\cdot)$ is the density of $\hat{F}$, which means that the distribution of the average valuation is more concentrated around the mean than the distribution of each individual valuation. This implies $1 - \widehat{F}(\widetilde{v}_j^A) > 1 - F(\widetilde{v}_j^A)$ for any $\widetilde{v}_j^A < v^e$. Hence, for given prices, incompatibility increases the demand for the dominant firm A and decreases the demand for the dominated firm B.

Demand elasticity effect. After the regime changes from compatibility to incompatibility, the firms will have incentives to choose prices different from $P_2^+ = 2p_2^+$ and $P_2^- = 2p_2^-$. Whether they want to charge higher or lower prices depends on how incompatibility affects demand elasticity, which in turn depends on the valuation of the indifferent consumer and hence on the level of the switching cost. For low levels of switching cost (that is, when $\widetilde{v}_j^A$ is not much smaller than v^e), incompatibility makes the demand more elastic: a given decrease in the average price of a system under incompatibility generates a higher boost in demand than the same decrease in the price of each product under compatibility because the distribution of the average valuation is more-peaked around v^e than the distribution of individual valuations. On the other hand, for high levels of switching cost (that is, when $\widetilde{v}_j^A$ is close to $\underline{v}$), incompatibility makes the demand less elastic. Precisely, $\widehat{f}(v)/f(v)$ converges to zero as v tends to $\underline{v}$. Hence, for $\widetilde{v}_j^A$ close enough to $\underline{v}$, we find that a given decrease in the average price of a system generates a smaller boost in demand than the same decrease in the price of each product. In summary, incompatibility changes the elasticity of demand such that firms compete more aggressively for low levels of switching costs but less aggressively for high levels of switching costs.

Second-period profit comparison. As a result of these two effects, we find the following profit comparison:

Corollary 4. *Let $\Delta v = 1$ without loss of generality. Then, there are three threshold values of switching cost, $\overline{s}^1$, $\overline{s}^2$, $\overline{s}^3$ with $\overline{s}^1 < \overline{s}^2 < \overline{s}^3$, such that*

$$2\pi_2^{+*} \gtreqless \Pi_2^{+*} \text{ if and only if } s \lesseqgtr \overline{s}^1 (= 0.701); \tag{6}$$

$$2\pi_2^{+*} + 2\pi_2^{-*} \gtreqless \Pi_2^{+*} + \Pi_2^{-*} \text{ if and only if } s \lesseqgtr \overline{s}^2 (= 0.825); \tag{7}$$

$$2\pi_2^{-*} \gtreqless \Pi_2^{-*} \text{ if and only if } s \lesseqgtr \overline{s}^3 (= 1.187). \tag{8}$$

Although each of π_2^{+*}, π_2^{-*}, Π_2^{+*}, Π_2^{-*} depends on s, in what follows we do not highlight this dependence unless it is necessary. Figure 1 shows $2\pi_2^{+*}$, $2\pi_2^{-*}$, Π_2^{+*} and Π_2^{-*} as a function

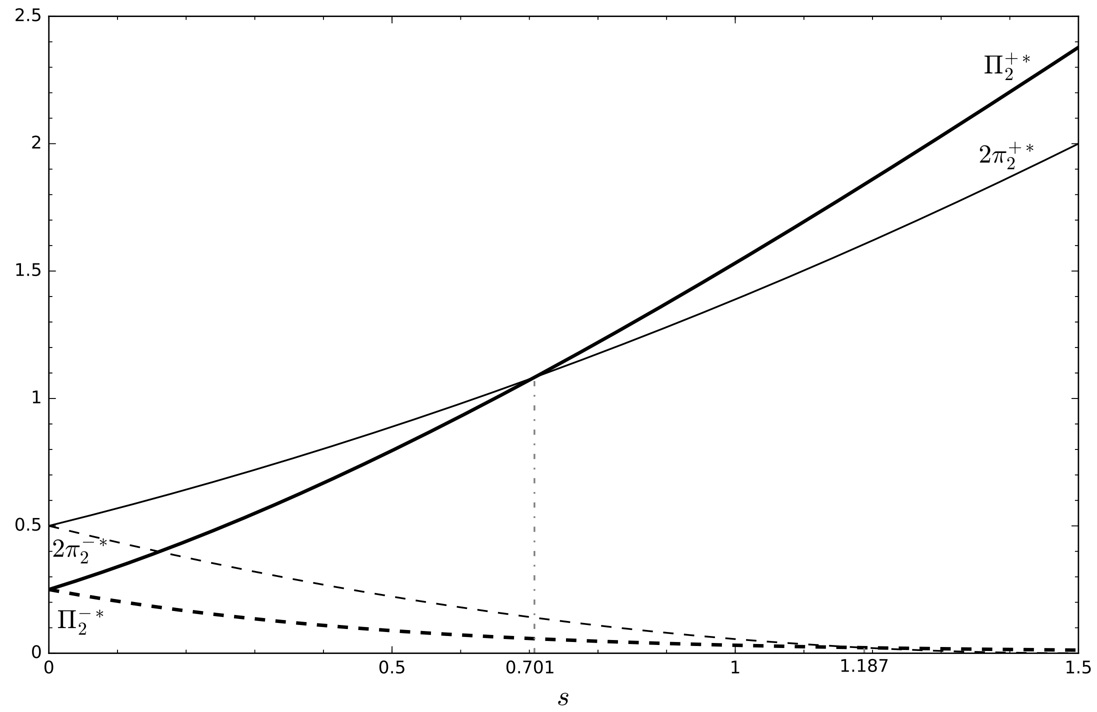

Figure 1: Second-period profits with or without compatibility

of $s \in (0, 3/2)$, given $\Delta v = 1$. There are three thresholds such that for $s > 0.701$ we have $\Pi_2^{+*} > 2\pi_2^{+*}$, for $s > 0.825$ we have $\Pi_2^{+*} + \Pi_2^{-*} > 2\pi_2^{+*} + 2\pi_2^{-*}$, and for $s > 1.187$ we have $\Pi_2^{-*} > 2\pi_2^{-*}$.

We can explain this result by using the two effects introduced above. For low switching costs (i.e., s close to zero), the demand size effect is negligible relative to the demand elasticity effect and the latter makes incompatibility intensify competition compared to compatibility. Therefore, incompatibility reduces both firms' profits. In particular, when $s = 0$, there is only the demand elasticity effect, which explains the result of Matutes and Régibeau (1988) that incompatibility intensifies competition for symmetric firms. For high switching costs (i.e., s close to 3/2), the demand size effect is again negligible relative to the demand elasticity effect, but now the latter makes incompatibility soften competition compared to compatibility. Therefore, incompatibility increases both firms' profits. For intermediate level of switching costs, the demand elasticity effect might be neutral but the demand size effect is positive for firm A and negative for firm B. Therefore, incompatibility increases A's profit but reduces B's profit.

4 First-period competition given a compatibility regime

We here study the competition in period one given a compatibility regime.

4.1 Given compatibility

Suppose first that compatibility was chosen in the beginning of the first period. Given a vector of the first-period prices $\left(p_{1,x}^A, p_{1,y}^A, p_{1,x}^B, p_{1,y}^B\right)$, firm i's total profit is given as follows:

$$\pi^i = d_{1,x}^i(p_{1,x}^i + \delta\pi_2^{+*}) + (1 - d_{1,x}^i)(\delta\pi_2^{-*}) + d_{1,y}^i(p_{1,y}^i + \delta\pi_2^{+*}) + (1 - d_{1,y}^i)(\delta\pi_2^{-*}), \quad \text{for } i = A, B, \tag{9}$$

where $d_{1,j}^i$ represents the demand for product j of firm i in period one (for $j = x, y$). From the indifference condition $p_{1,j}^A + t\theta_j = p_{1,j}^B + t(1 - \theta_j)$, we obtain $d_{1,j}^A$ ($d_{1,j}^B = 1 - d_{1,j}^A$):

$$d_{1,j}^A = \frac{1}{2} + \frac{1}{2t}(p_{1,j}^B - p_{1,j}^A). \tag{10}$$

Using (10) we can derive the equilibrium prices. In the next proposition, we distinguish the case in which the NPC (3) does not apply from the case in which it must be satisfied.

Proposition 1. *Under compatibility, there exists a unique equilibrium in which the first period equilibrium prices and each firm's total equilibrium profit (per product) are given as follows:*

- *(i) If the NPC (3) does not apply,*

$$p_{1j}^{i*} = t - \delta\left(\pi_2^{+*} - \pi_2^{-*}\right) \equiv p_1^*, \quad \textit{for } i = A, B \textit{ and } j = x, y. \tag{11}$$

$$\pi^{i*} = \frac{t}{2} + \delta\pi_2^{-*} \equiv \pi^*, \quad \textit{for } i = A, B. \tag{12}$$

- *(ii) If the NPC (3) must be satisfied,*

$$p_{1,j}^{i*} = \max\{t - \delta\left(\pi_2^{+*} - \pi_2^{-*}\right), 0\} \equiv p_1^*, \quad \textit{for } i = A, B \textit{ and } j = x, y. \tag{13}$$

$$\pi^{i*} = \max\{\frac{t}{2} + \delta\pi_2^{-*}, \delta(\frac{1}{2}\pi_2^{+*} + \frac{1}{2}\pi_2^{-*})\} \equiv \pi^*, \quad \textit{for } i = A, B. \tag{14}$$

Consider first case (i) in which the NPC (3) does not apply. Then the pricing in (11) is quite intuitive. For $\delta = 0$, each firm charges a price per product equal to t as in a standard Hotelling model. For $\delta > 0$, if firm i attracts a consumer from the rival in the first period, its expected profit from the customer in the second period is π_2^{+*}. But if the customer stays with the rival, then firm i's expected profit from that customer in the second period is π_2^{-*}. Therefore, each firm dissipates $\delta\left(\pi_2^{+*}-\pi_2^{-*}\right)$ the rent from a locked-in consumer and obtains a profit of $\frac{t}{2}+\delta\pi_2^{-*}$. What is interesting is that even if there is perfect competition in period one (i.e. $t=0$), each firm realizes a positive profit. The profit in (12) can be understood in a similar way. For $\delta=0$, each firm gets a profit of $t/2$ as in a standard Hotelling model. For $\delta>0$, each firm's profit is equal to $t/2+\delta\pi_2^{-*}$ as the rent $\delta\left(\pi_2^{+*}-\pi_2^{-*}\right)$ is dissipated away during the first-period competition. However, for a large δ, the term $\delta\left(\pi_2^{+*}-\pi_2^{-*}\right)$ is larger than t and hence p_1^* in (11) is negative. Then, if the NPC (3) must be satisfied, the equilibrium price is zero for both firms and each firm's profit coincides with its second period profit $\delta(\frac{1}{2}\pi_2^{+*}+\frac{1}{2}\pi_2^{-*})$.

4.2 Given incompatibility

Suppose now that incompatibility was chosen at the beginning of the first period. Given the period one prices $\left(P_1^A,P_1^B\right)$, firm i's total profit is given as follows:

$$\Pi^i=D_1^i(P_1^i+\delta\Pi_2^{+*})+(1-D_1^i)(\delta\Pi_2^{-*}),\qquad \text{for } i=A,B \tag{15}$$

where D_1^i is firm i's market share in period one. From the indifference condition $P_1^A+t\theta_x+t\theta_y=P_1^B+t(1-\theta_x)+t(1-\theta_y)$, we obtain the following demand

$$D_1^A=\frac{1}{2}\left(1+\frac{1}{2t}(P_1^B-P_1^A)\right)^2-\frac{1}{4t^2}(P_1^B-P_1^A)\max\{0,P_1^B-P_1^A\}. \tag{16}$$

and $D_1^B=1-D_1^A$.

Proposition 2. *Under incompatibility, there exists a unique equilibrium in which the first period equilibrium prices and each firm's total equilibrium profit are given as follows:*

- *(i) If the NPC (3) does not apply, then*

$$P_1^{i*}=t-\delta\left(\Pi_2^{+*}-\Pi_2^{-*}\right)\equiv P_1^*,\quad \textit{for } i=A,B. \tag{17}$$

$$\Pi^{i*}=\frac{t}{2}+\delta\Pi_2^{-*}\equiv\Pi^*,\quad \textit{for } i=A,B. \tag{18}$$

- *(ii) If the NPC (3) must be satisfied, then*

$$P_1^{i*}=\max\{t-\delta\left(\Pi_2^{+*}-\Pi_2^{-*}\right),0\}\equiv P_1^*,\quad \textit{for } i=A,B \tag{19}$$

$$\Pi^{i*}=\max\{\frac{t}{2}+\delta\Pi_2^{-*},\delta(\frac{1}{2}\Pi_2^{+*}+\frac{1}{2}\Pi_2^{-*})\}\equiv\Pi^*,\quad \textit{for } i=A,B. \tag{20}$$

Under incompatibility, if $\delta = 0$ it is well-known from Matutes and Régibeau (1988) that each firm charges a price for its system equal to t as in a two-dimensional Hotelling model. For $\delta > 0$, if firm i attracts a consumer from the rival in the first period, its expected profit from the customer in the second period is Π_2^{+*}. But if the customer stays with the rival, then firm i's expected profit from him in the second period is Π_2^{-*}. Therefore, each firm is ready to pay the rent from a locked-in consumer $\delta\left(\Pi_2^{+*} - \Pi_2^{-*}\right)$ to attract a consumer, which is dissipated away and hence each firm's equilibrium profit is $\frac{t}{2} + \delta\Pi_2^{-*}$. This holds as long as the NPC (3) does not need to be satisfied or if it must be satisfied and P_1^* in (17) is non-negative. Otherwise, the equilibrium price in period one is zero and each firm's total profit is equal to the second period profit $\delta(\frac{1}{2}\Pi_2^{+*} + \frac{1}{2}\Pi_2^{-*})$.

4.3 Forward-looking consumers

Up to now, we have assumed that consumers are myopic. We here show that if consumers were forward-looking, our results remain unaffected. As it is immediate that our analysis of the second period continues to apply regardless of whether consumers are myopic or forward-looking, we focus on the first period.

Consider compatibility. Then, a consumer's expected second-period utility does not depend on which products he buys in period one. Precisely, if he buys product j from firm A in period one, then his expected second-period utility is the expectation of $\max\{v_j^A - p_2^{+*}, v^e - p_2^{-*} - s\}$. If he buys product j from firm B in period one, then his utility is the expectation of $\max\{v_j^B - p_2^{+*}, v^e - p_2^{-*} - s\}$. Since v_j^A and v_j^B are identically distributed, the two expectations are equal and the consumer will choose in period one on the basis of his first-period utility, as a myopic consumer does. The same principle holds under incompatibility since the distribution of $v_x^A + v_y^A$ is the same as the distribution of $v_x^B + v_y^B$. This proves that a forward-looking consumer behaves exactly as a myopic consumer.

Proposition 3. *(forward-looking consumers) Forward-looking consumers behave in the same way as myopic consumers do.*

5 Compatibility choice

We here study the equilibrium compatibility regime, which depends on whether the NPC (3) must be satisfied or not. The next proposition distinguishes these two cases, relying on the firms' profits from the two compatibility regimes in (12), (18) and (14), (20). We recall that $\Pi_2^{-*} - 2\pi_2^{-*} > 0$ holds if and only if $s > \bar{s}^3$, $\Pi_2^{+*} + \Pi_2^{-*} > 2\pi_2^{+*} + 2\pi_2^{-*}$ holds if and only if $s > \bar{s}^2$ and $\bar{s}^2 < \bar{s}^3$.

Proposition 4. *(i) Suppose that the NPC (3) does not need to be satisfied. If* $\delta\left(\Pi_2^{-*} - 2\pi_2^{-*}\right) > \frac{t}{2}$*, then in the unique equilibrium both firms choose incompatibility. Otherwise, there exists an equilibrium in which both firms choose compatibility and this equilibrium weakly Pareto-dominates the incompatibility equilibrium.*

(ii) Suppose that the NPC (3) must be satisfied. If $\Pi_2^{+*} + \Pi_2^{-*} > 2\pi_2^{+*} + 2\pi_2^{-*}$ *and* $\frac{\delta}{t} > \frac{1}{\frac{1}{2}\Pi_2^{+*}+\frac{1}{2}\Pi_2^{-*}-2\pi_2^{-}}$*, then in the unique equilibrium both firms choose incompatibility. Otherwise, there exists an equilibrium in which both firms choose compatibility and this equilibrium weakly Pareto-dominates the incompatibility equilibrium.*

(iii) The presence of the NPC expands the range of parameter values for which the firms choose incompatibility.

Consider first the case in which the NPC (3) does not need to be satisfied. Then incompatibility intensifies competition between symmetric firms in a one-period game. Precisely, when $\delta = 0$, incompatibility reduces each firm's profit from t to $t/2$, a well-known result from Matutes and Régibeau (1988). Proposition 4(i) generalizes this result for δ/t small enough and/or s small. First, as long as δ/t is small, the compatibility equilibrium emerges because compatibility softens competition in period one and the second-period profits are relatively unimportant. Second, when s is small (i.e. $s < \bar{s}^3$), in period two, the dominated firm's profit is larger under compatibility than under incompatibility. As the total profit of each firm is a constant (which is greater under compatibility) plus δ times the second-period profit of the dominated firm (see (12) and (18)), both firms choose compatibility.

However, the finding of Matutes and Régibeau (1988) is reversed if both the switching cost and the weight of the second period are large enough. Precisely, if $s > \bar{s}^3$, then $\Pi_2^{-*} > 2\pi_2^{-*}$ holds and incompatibility emerges for high δ/t. In this case, both firms choose incompatibility as it softens competition in period two. Even if part of the increased second-period profit is dissipated away, each firm retains $\delta(\Pi_2^{-*} - 2\pi_2^{-*})$ in terms of increased profit, which more than compensates the reduction by $t/2$ in the first-period profit if δ/t is large.

The NPC (3) limits the dissipation of rent from locked-in consumers. Corollary 3 shows that the rent from a locked-in consumer is larger under incompatibility than under compatibility for any level of switching cost. Therefore, when the NPC binds regardless of the compatibility regime (which occurs for δ/t large), the rent dissipation is constrained more under incompatibility than under compatibility, which induces the firms to choose incompatibility more often. More precisely, when the NPC binds regardless of regime, the firms make profits only from the second period and hence choose incompatibility if and only if $\left(\Pi_2^{+*} + \Pi_2^{-*}\right)/2 > \pi_2^{+*} + \pi_2^{-*}$, which is equivalent to $s > \bar{s}^2$.[26]

[26]For simplicity, in our explanation, we focused on the case in which the NPC binds no matter the compatibility

6 Consumer surplus and welfare

In this section we compare consumer surplus and welfare under compatibility with those under incompatibility. Without loss of generality, we normalize Δv to one.

6.1 Consumer surplus

Consider first the case in which the NPC (3) does not need to be satisfied. In period one, consumer surplus is greater under incompatibility than under compatibility for any s and δ/t for two reasons. The first reason is known from Matutes and Régibeau (1988): for symmetric firms, incompatibility intensifies the first-period competition. As we explained in Section 3.3, this has to do with the demand elasticity effect of incompatibility. Even if incompatibility increases the transportation costs incurred by consumers, this effect is dominated by the effect of more intensive competition. The second reason is that from Corollary 3, the rent from a locked-in consumer is larger under incompatibility than under compatibility, implying that the firms dissipate more rent under incompatibility than under compatibility: see (11) and (17).

Consumer surplus in period two depends on s. For instance, consider a consumer who bought both products from firm A in period one. Then, for s large, incompatibility softens competition in period two with respect to compatibility such that we have $P_2^{+*} > 2p_2^{+*}$ and $P_2^{-*} > 2p_2^{-*}$. The inequalities imply that the consumer is better off under compatibility than under incompatibility for any possible realization of $\left(v_x^A, v_y^A\right)$. Conversely, for s close to zero, the opposite inequalities hold. As a consequence, the second-period consumer surplus is higher under compatibility if and only if $s > 0.876$.

Let CS^C (CS^I) denote the total consumer surplus under compatibility (incompatibility). The previous arguments seem to suggest that $CS^C > CS^I$ when $s > 0.876$ and δ/t is large. But we need to take into account the fact that the higher rent from the locked-in consumers under incompatibility from (5) is transferred to consumers through a more intense first-period competition: an increase in δ reduces P_1^* more than $2p_1^*$. Hence, we find $CS^C > CS^I$ if $s > 1.168$ and δ/t is above a suitable threshold $r_{CS}(s)$ specified in Proposition 5. However, from Proposition 4(i), the firms choose incompatibility when $s > \bar{s}^3 (= 1.187)$ and $\frac{\delta}{t} > \frac{1}{2\left(\Pi_2^{-*} - 2\pi_2^{-*}\right)}$. Since $r_{CS}(s) < \frac{1}{2\left(\Pi_2^{-*} - 2\pi_2^{-*}\right)}$ holds for $s > \bar{s}^3 (= 1.187)$, whenever the firms choose incompatibility, $CS^C > CS^I$ holds. A similar property holds with respect to compatibility: when it arises in equilibrium, it is often the case that $CS^I > CS^C$. Next proposition summarizes our results.

regime. But an incompatibility equilibrium can also exist when the NPC binds under incompatibility but not under compatibility. Then, the firms choose incompatibility if and only if $\delta(\frac{1}{2}\Pi_2^{+*} + \frac{1}{2}\Pi_2^{-*}) > t + 2\delta\pi_2^{-}$, which is equivalent to $s > \bar{s}^2$ and $\frac{\delta}{t} > \frac{1}{\frac{1}{2}\Pi_2^{+*} + \frac{1}{2}\Pi_2^{-*} - 2\pi_2^{-}}$ in Proposition 4(ii). Proposition 4(iii) follows from $\bar{s}^2 < \bar{s}^3$ and $\frac{1}{\frac{1}{2}\Pi_2^{+*} + \frac{1}{2}\Pi_2^{-*} - 2\pi_2^{-}} < \frac{t}{2(\Pi_2^{-*} - 2\pi_2^{-*})}$.

Proposition 5. *(consumer surplus: no NPC) Suppose that the NPC (3) does not need to be satisfied. Then*
(i) Consumer surplus under compatibility is

$$CS^C = 2v^e - 2p_1^* - \frac{1}{2}t + \delta(2v^e - 2p_2^{+*} + \pi_2^{-*}) \tag{21}$$

(ii) Consumer surplus under incompatibility is

$$CS^I = 2v^e - P_1^* - \frac{2}{3}t + \delta(2v^e - P_2^{+*} + \frac{2}{3}\Pi_2^{-*}) \tag{22}$$

(iii) The inequality $CS^C > CS^I$ holds if and only if $s > 1.168$ and

$$\frac{\delta}{t} > r_{CS}(s) = \frac{5}{12\pi_2^{+*} - 6\pi_2^{-*} - 12p_2^{+*} + 6P_2^{+*} - 6\Pi_2^{+*} + 2\Pi_2^{-*}}$$

Whenever incompatibility arises in equilibrium, $CS^C > CS^I$ holds. If compatibility arises in equilibrium, then $CS^I > CS^C$ for $s < 1.168$.

What is remarkable is the conflict between the compatibility regime chosen by the firms and the one maximizing consumer surplus, which arises except for a small range of parameters. This is because the firms choose (in)compatibility in order to soften competition.

When the NPC (3) must be satisfied, we focus on the case in which (3) binds regardless of the compatibility regime. As the rent is higher under incompatibility than under compatibility from (5), the binding NPC constrains more the rent dissipation under incompatibility than under compatibility, which makes consumers prefer compatibility more often. By contrast, the binding constraint makes the firms choose incompatibility more often (Proposition 4(iii)). Therefore, the conflict between the compatibility regime chosen by the firms and the one maximizing consumer surplus is preserved when the NPC binds.

Proposition 6. *(consumer surplus: NPC) Suppose that the NPC (3) must be satisfied and is binding in both compatibility regimes. Then*
(i) CS^C is given by (21) with $p_1^* = 0$; CS^I is given by (22) with $P_1^* = 0$.
(ii) The inequality $CS^C > CS^I$ holds if and only if $s \geq 0.876$, or if $s < 0.876$ and $\frac{\delta}{t} < \frac{1}{4\Pi_2^{-*}+12p_2^{+*}-6\pi_2^{-*}-6P_2^{+*}}$. The binding NPC expands the range of parameter values for which the consumers prefer compatibility.
(iii) The compatibility regime chosen by the firms leads to the lower consumer surplus for a set of parameters that includes at least all $(s, \frac{\delta}{t})$ such that $s < 0.749$ or $s \geq 0.876$.

6.2 Social welfare

We note first that given a compatibility regime, the NPC (3) has no effect on social welfare as it only affects distribution of surplus between consumers and the firms.

We start by describing the first-best allocation. In period one, the first-best requires a consumer with location θ_j for good j ($j = x, y$) to buy product j of firm A if and only if $\theta_j \leq \frac{1}{2}$. In period two, the first-best requires a consumer who purchased product j of A (for instance) in period one and observes v_j^A to keep buying it from A if $v_j^A \geq v^e - s$, but to switch to B if $v_j^A < v^e - s$. In particular, no switching occurs in the first-best if $s \geq \frac{1}{2}$.

Under compatibility, the first-period allocation coincides with the first-best one, but some inefficiency emerges in the second period since $p_2^{+*} > p_2^{-*}$ implies that excessive switching occurs. However, this efficiency loss is small if s is close to zero or if s is close to $\frac{3}{2}$. In the former case, p_2^{+*} is close to p_2^{-*} and hence the switching is only slightly excessive. In the latter case, the proportion of switching consumers tends to 0 as s tends to $\frac{3}{2}$.

Incompatibility generates efficiency loss in each period. In period one, consumers cannot mix and match, which increases the transportation costs they incur. In period two, likewise, incompatibility forces consumers to make switching decisions only at a system level. However, for some intermediate values of s (i.e., for $s \in (0.535, 1.152)$), the *second-period* social welfare is higher under incompatibility. This occurs because for these values of s, social welfare is maximal if there is no switching, and the market share of the dominant firm is significantly larger under incompatibility than under compatibility precisely because of the demand size effect explained in Section 3.3. As a consequence, incompatibility generates a higher *total* welfare for $s \in (0.535, 1.152)$ if δ/t is large (see Proposition 7(iii)).

The NPC (3) matters when we compare the compatibility regime chosen by the firms with the one maximizing welfare. When (3) is not imposed, from Proposition 4(i), we know that $s > \bar{s}^3$ is a necessary condition for incompatibility to emerge in equilibrium. Since $\bar{s}^3 > 1.152$, we conclude that whenever the firms choose incompatibility, welfare is higher with compatibility. When instead the NPC (3) must be satisfied and is binding in both regimes, there is no clear contrast between the compatibility regime chosen by the firms and the one maximizing social welfare. For instance, if incompatibility emerges (see Proposition 4(ii)), it is socially optimal if $s < 1.152$ and $\frac{\delta}{t}$ is sufficiently large.

Proposition 7. *(welfare) (i) Social welfare under compatibility is*

$$SW^C = 2(1+\delta)v^e - \frac{1}{2}t + \delta(2\pi_2^{+*} + 3\pi_2^{-*} - 2p_2^{+*}).$$

(ii) Social welfare under incompatibility is

$$SW^I = 2(1+\delta)v^e - \frac{2}{3}t + \delta(\Pi_2^{+*} + \frac{5}{3}\Pi_2^{-*} - P_2^{+*}).$$

(iii) The inequality $SW^I > SW^C$ *holds if and only if* $s \in (0.535, 1.152)$ *and* $\frac{\delta}{t}$ *is larger than* $\frac{1}{6\Pi_2^{+*}+10\Pi_2^{-*}-6P_2^{+*}-12\pi_2^{+*}-18\pi_2^{-*}+12p_2^{+*}}$. *If the NPC (3) does not need to be satisfied, then* $SW^I < SW^C$ *whenever incompatibility arises in equilibrium. If the NPC (3) binds regardless of the compatibility regime, the incompatibility chosen by the firms generates the higher welfare if* $s \in (\bar{s}^2, 1.152)$ *and* $\frac{\delta}{t}$ *is large.*

7 Policy intervention: data portability

We here study how data portability policy affects consumer surplus, profits and social welfare. Data portability is expected to lower switching costs and thereby to enhance competition among Internet firms providing data-based services.[27] How much switching cost can be lowered by data portability depends on the categories of portable data. World Economic Forum (2014) distinguishes personal data into three categories: volunteered data, observed data and inferred data.[28] Data portability applies to volunteered data and is likely to extend to observed data but not to inferred data (Crémer et al. (2019), p.81). Inferred data matter for the services based on big data analytics. As we consider two incumbents with similar market shares, no portability of inferred data is less a concern relative to an asymmetric situation in which an incumbent faces an entry. An entrant has no stock of inferred data and should convince consumers to switch to (or multi-home) its product/service in order to generate data. In contrast, in the second-period of our model, each incumbent already has inferred data from its own consumer base and therefore it can use the volunteered and observed data of a switching consumer in order to identify the doppelgängers whose profiles closely match the switching consumer and use the inferred data of the identified doppelgängers to provide big data analytic service (Stephens-Davidowitz, 2017). Therefore, in our context, data portability could significantly reduce the switching cost.

Our first result is that data portability, by lowering switching cost, induces the firms to choose compatibility more often regardless of whether or not the NPC binds, which is straightforward

[27] "Being able to port one's data directly lowers the cost of moving from one service to another, which in turn causes businesses to compete harder to keep those customers." (the Stigler report, 2019, p.88).

[28] "Volunteered data" refer to data which is intentionally contributed by a user such as name, image, review, post etc. "Observed data" refers to more behavioral data obtained automatically from a user's activity such as location data and web browsing data. "Inferred data" is obtained by transforming in a non-trivial manner volunteered and/or observed data while still related to a specific individual. This includes a shopper's profiles resulting from clustering algorithms or predictions about a person's propensity to buy a product. See also the report on data from the Expert Group for the Observatory on Online Platform Economy (2020).

from Proposition 4.[29]

The next lemma presents our second result:

Lemma 3. *(i) Suppose that the NPC does not need to be satisfied. Then, given each regime of compatibility, each firm's profit decreases with s and consumer surplus increases with s.*

(ii) Suppose that the NPC binds regardless of the compatibilty regime. Then, given each regime of compatibility, each firm's profit increases with s and consumer surplus decreases with s.

The lemma shows that how the switching cost affects profits and consumer surplus completely differs depending on whether or not the NPC binds. Suppose that the constraint binds regardless of the compatibility regime. Then, each firm's profit is $\left(\Pi_2^{+*} + \Pi_2^{-*}\right)/2$ or $\pi_2^{+*} + \pi_2^{-*}$ depending on the regime and each of these profits increases with s according to Corollary 1 and 2. This is because a higher switching cost relaxes the second-period competition. For the same reason, consumer surplus decreases with s under each compatibility regime because s affects consumer surplus only through the second-period surplus, which decreases with s according to Corollary 1 and 2. By contrast, when the NPC does not need to be satisfied, because of the full dissipation of the rent from locked-in consumers, each firm's profit is $t/2 + \delta\Pi_2^{-*}$ or $t + \delta 2\pi_2^{-*}$ depending on the compatibility regime and each of these profits decreases with s as the dominated firm's second-profit profit (Π_2^{-*} or π_2^{-*}) decreases with s. As a higher s intensifies the first-period competition to attract consumers and this effect dominates the second-period competition-softening effect, consumer surplus increases with s under each compatibility regime. In what follows, we focus on the case in which the NPC binds regardless of the incompatibility regime and very briefly discuss what happens when the constraint does not apply. This is because full rent dissipation requires negative prices for δ high, which may be implausible, and firms are likely to benefit from high switching costs instead of suffering from them (Farrell and Klemperer, 2007).

Suppose that data portability reduces s from s' to s'' and that the NPC always binds. If the reduction in the switching cost does not affect the compatibility regime, it reduces the profits but increases consumer surplus. Therefore, we consider the case in which data portability changes the compatibility regime from incompatibility to compatibility. Recall that the firms choose incompatibility if and only if $s > \bar{s}^2$ and $\frac{\delta}{t}$ not small. This implies $s' > \bar{s}^2 > s''$. Clearly, the

[29] However, there is a caveat. When the NPC does not apply, the statement is correct except when the initial switching cost is larger than 1.45 and the reduction in s is small since $\Pi_2^{-*} - 2\pi_2^{-*}$ is decreasing (although positive) for s in $(1.45, 1.5)$.

reduction in the switching cost lowers the firms' profits as we have

$$\begin{aligned}\frac{\Pi_2^{+*}(s')+\Pi_2^{-*}(s')}{2} &> \frac{\Pi_2^{+*}(\bar{s}^2)+\Pi_2^{-*}(\bar{s}^2)}{2}\\ &= \pi_2^{+*}(\bar{s}^2)+\pi_2^{-*}(\bar{s}^2) > \pi_2^{+*}(s'')+\pi_2^{-*}(s'').\end{aligned}$$

Regarding consumer surplus, we fix $t = 1$ without loss of generality. We have $CS^I(s', \delta) < CS^I(\bar{s}^2, \delta)$ and $CS^C(\bar{s}^2, \delta) < CS^C(s'', \delta)$ as consumer surplus decreases with s under each compatibility regime. Therefore, if $CS^C(\bar{s}^2, \delta) \geq CS^I(\bar{s}^2, \delta)$, then we can conclude that $CS^C(s'', \delta) > CS^I(s', \delta)$. Indeed, $CS^C(\bar{s}^2, \delta) \geq CS^I(\bar{s}^2, \delta)$ holds if and only if δ is between 1.818 and 4.822.[30] For $\delta > 4.822$, we have $CS^C(\bar{s}^2, \delta) < CS^I(\bar{s}^2, \delta)$ and therefore if s', s'' are close to $\bar{s}^2$, $CS^C(s'', \delta) < CS^I(s', \delta)$ holds and data potability reduces consumer surplus. However, if the reduction in switching cost is large enough to satisfy $s'' \leq 0.783$, then $CS^C(s'', \delta) > CS^I(s', \delta)$ and data portability increases consumer surplus.

Regarding welfare, for $s < \bar{s}^2$, the firms choose compatibility and welfare decreases in s whereas for $s > \bar{s}^2$, the firms choose incompatibility and welfare decreases in s for $s \in [\bar{s}^2, 1]$ and increases in s for $s \in [1, 1.5]$: SW^I is non-monotonic in s and reaches the maximum at $s = 1.5$. Hence, if data portability does not change the compatibility regime, it increases welfare for $s \leq 1$. Suppose now that data portability reduces s from $s' \left(> \bar{s}^2\right)$ to $s'' \left(< \bar{s}^2\right)$ such that the regime changes from incompatibility to compatibility. We have that $SW^C(\bar{s}^2, \delta) > SW^I(1.5, \delta)$ for each $\delta \in (1.8181, 3.58]$, thus in this case data portability increases welfare as we have $SW^C(s'', \delta) > SW^C(\bar{s}^2, \delta) > SW^I(1.5, \delta) \geq SW^I(s', \delta)$. For $\delta > 3.58$, we may have $SW^C(s'', \delta) < SW^I(s', \delta)$ if s'' is just a bit smaller than $\bar{s}^2$, but $SW^C(s'', \delta) > SW^I(s', \delta)$ holds if $s'' < 0.655$.

Finally, we very briefly discuss the effects of the data portability when the NPC does not need to be satisfied. Then, data portability always increases the firms' profits. It reduces consumer surplus if it does not induce any change in compatibility regime. If the regime change from incompatibility to compatibility occurs, it is very likely that this change reduces consumer surplus even more than when there is no regime change because of the conflict between the compatibility regime chosen by the firms and the one maximizing consumer surplus. Note that welfare analysis given a compatibility regime does not depend on whether the NPC binds or not. As the cut-off switching cost triggering the regime change is larger than $\bar{s}^3 (> 1.152)$ for δ large, from Proposition 7(iii), we conclude that a small reduction in switching cost that induces the regime change improves welfare in contrast to what happens when the NPC binds.

Summarizing, we have:

Proposition 8. *The effects of the data portability policy are as follows.*

[30] $\delta = 1.818$ is the lower bound for δ in order to have the NPC bind for both regimes given $s = \bar{s}^2$.

(i) The policy induces the firms to choose compatibility more often instead of incompatibility.[31]

(ii) The policy reduces the firms' profits if the NPC binds but increases the profits if the constraint does not apply or is slack.

(iii) When the NPC does not apply or is slack, then the policy reduces consumer surplus, except for a "small" set of parameters where the change in the compatibility regime occurs. When the NPC binds, it increases consumer surplus if it does not induce any change in compatibility regime; if the regime change from incompatibility to compatibility occurs, the policy increases consumer surplus unless the reduction in switching cost is very small.

7.1 Extension: Incomplete pass-through by tying freebies

Lemma 3 shows that under each compatibility regime, there is a conflict between profits and consumer surplus in the sense that if the data portability increases profits, it reduces consumer surplus and vice versa. We here extend our model by allowing for incomplete pass-through of the rent from locked-in consumers when the NPC binds and show that for an intermediate range of pass-through rates, data portability can increase both profits and consumer surplus.

Suppose that the NPC binds regardless of the regime. Then, each firm wants to charge a negative first-period price but cannot because a direct implementation of negative prices is not possible due to opportunistic behavior and adverse selection. However, the firms can circumvent the NPC to some extent by tying another (complementary) product (called "freebies") together with the original product(s) (Amelio and Jullien, 2012). In order to avoid attracting undesirable consumers, the tied product should generate more value when it is used together with the original one such as free parking provided by shopping malls.

We consider that the tied free good is competitively supplied and divisible like data storage capacity. As the freebies are likely to be less efficient than money in transferring utility from the firms to consumers, we introduce a parameter $\lambda \in [0,1]$ which represents the pass-through rate in the sense that one dollar spent on freebies translates as λ utility to consumers in dollar terms. The case of $\lambda = 1$ represents the complete pass-through and is equivalent to the case when the NPC does not apply. By contrast, the case of $\lambda = 0$ represents no pass-through and is equivalent to the case in which the NPC binds and the firms cannot offer freebies. In what follows, we study how λ affects the impact of a reduction in the switching cost on profits and consumer surplus.

Under compatibility, let $\left(f_j^A, f_j^B\right) \in R_+^2$ be the amount of freebies offered by the firms for product j: under incompatibility, let $\left(F^A, F^B\right) \in R_+^2$ be the amount of freebies offered by the

[31] An exception occurs when the NPC does not apply, the initial s is largeer than 1.45 and a small reduction in s occurs.

firms. We have

Lemma 4. *Suppose that the NPC binds regardless of compatibility regime and that the firms can tie freebies to circumvent the NPC..*

(i) Under compatibility, there exists $\hat{\lambda}^C = \frac{t}{\delta(\pi_2^{+*} - \pi_2^{-*})} \in (0,1)$ *such that both firms offer no freebies for* $\lambda \leq \hat{\lambda}^C$ *and offer* $f_j^A = f_j^B = \delta(\pi_2^{+*} - \pi_2^{-*}) - \frac{t}{\lambda} > 0$ *for* $\lambda > \hat{\lambda}^C$*. Each firm's total profit is* $\delta\pi_2^{+*} + \delta\pi_2^{-*}$ *for* $\lambda \leq \hat{\lambda}^C$ *and* $\frac{t}{\lambda} + 2\delta\pi_2^{-*}$ *for* $\lambda > \hat{\lambda}^C$*. Consumer surplus is* $2v^e - \frac{1}{2}t + \delta(2v^e - 2p_2^{+*} + \pi_2^{-*})$ *for* $\lambda \leq \hat{\lambda}^C$ *and* $2(1+\delta)v^e + \delta(\lambda(2\pi_2^{+*} - 2\pi_2^{-*}) + \pi_2^{-*} - 2p_2^{+*}) - \frac{5}{2}t$ *for* $\lambda > \hat{\lambda}^C$*.*

(ii) Under incompatibility, there exists $\hat{\lambda}^I = \frac{t}{\delta(\Pi_2^{+*} - \Pi_2^{-*})} \in (0,1)$ *such that both firms offer no freebies for* $\lambda \leq \hat{\lambda}^I$ *and offer* $F^A = F^B = \delta(\Pi_2^{+*} - \Pi_2^{-*}) - \frac{t}{\lambda}$ *for* $\lambda > \hat{\lambda}^I$*. Each firm's total profit is* $\frac{1}{2}(\delta\Pi_2^{+*} + \delta\Pi_2^{-*})$ *for* $\lambda \leq \hat{\lambda}^I$ *and* $\frac{t}{2\lambda} + \delta\Pi_2^{-*}$ *for* $\lambda > \hat{\lambda}^I$*. Consumer surplus is* $2v^e - \frac{2}{3}t + \delta(2v^e - P_2^{+*} + \frac{2}{3}\Pi_2^{-*})$ *for* $\lambda \leq \hat{\lambda}^I$ *and* $2(1+\delta)v^e - \frac{5}{3}t + \delta\left(\lambda(\Pi_2^{+*} - \Pi_2^{-*}) - P_2^{+*} + \frac{2}{3}\Pi_2^{-*}\right)$ *for* $\lambda > \hat{\lambda}^I$*.*

Consider compatibility. When the pass-through rate is smaller than $\hat{\lambda}^C$, freebies are not cost-effective means to attract consumers and hence no firm offers freebies. Once the pass-through rate is above $\hat{\lambda}^C$, both firms offer freebies. As λ increases, they offer more freebies and hence each firm's profit decreases with λ and consumer surplus increases with λ. We note that the profit increases with s for $\lambda \leq \hat{\lambda}^C$ and decreases with s for for $\lambda > \hat{\lambda}^C$. Consumer surplus decreases with s for $\lambda \leq \hat{\lambda}^C$. However, when $\lambda > \hat{\lambda}^C$, consumer surplus can be non-monotonic in s for intermediate levels of λ. The same remarks apply to the case of incompatibility.

Because of the non-montonicity of consumer surplus in s, in the next proposition, we focus on a marginal reduction in s.

Proposition 9. *Suppose that the NPC binds regardless of compatibility regime and that the firms can tie freebies to circumvent the NPC.*

(i) Consider compatibility and a given s.

(a) If $\hat{\lambda}^C(s) < \frac{1}{2}$*, then for* λ *between* $\hat{\lambda}^C(s)$ *and 1/2, both consumer surplus and profits are locally decreasing in s.*

(b) There exists $(\delta/t)^C(s) > 0$ *such that for any* $\delta/t > (\delta/t)^C(s)$*,* $\hat{\lambda}^C(s) < 1/2$ *holds and hence a marginal reduction in s increases both profits and consumer surplus for* $\lambda \in (\hat{\lambda}^C(s), \bar{\lambda}^C(s))$*.*

(ii) Consider incompatibility and a given s.

(a) If $\hat{\lambda}^I(s) < 9/10$*, then for* λ *between* $\hat{\lambda}^I(s)$ *and 9/10, both consumer surplus and profits are locally decreasing in s.*

(b)There exists $(\delta/t)^I(s) > 0$ *such that for any* $\delta/t > (\delta/t)^I(s)$*,* $\hat{\lambda}^I(s) < 9/10$ *holds and hence a marginal reduction in s increases both profits and consumer surplus for* $\lambda \in (\hat{\lambda}^I(s), \frac{9}{10})$*.*

The proposition shows that when δ/t is high enough, data portability policy can increase both profits and consumer surplus for an intermediate range of pass-through rates.

8 Conclusion

Our theory captures well Larry Page's claim of an "island-like" Internet. When moving data across Internet firms is hard, consumer lock-in arises very naturally. Then, our theory predicts that firms embrace incompatibility today in order to soften future competition. In general, we find a conflict between firms' compatibility choice and the one maximizing consumer surplus, regardless of whether the NPC binds. We also find a conflict between firms' compatibility choice and the one maximizing welfare when the NPC does not apply. However, the binding NPC mitigates this conflict. On the one hand, for intermediate levels of switching cost, incompatibility generates a higher welfare than compatibility by reducing excessive switching. On the other hand, the binding NPC expands the regime of incompatibility to include some intermediate levels of switching cost. Data portability reduces switching cost and thereby induces firms to choose compatibility more often. However, given a compatibility regime, data portability benefits consumers only when the NPC binds.

9 References

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10 Appendix

Proof of Lemma 1

The demand for product j of firm i in market i_j is given by:

$$d_2^+ = \begin{cases} 1 & \text{if } \quad p_2^+ < s - \frac{\Delta v}{2} + p_2^- \\ \frac{1}{2} + \frac{1}{\Delta v}(s + p_2^- - p_2^+) & \text{if } s - \frac{\Delta v}{2} + p_2^- \leq p_2^+ \leq s + \frac{\Delta v}{2} + p_2^- \\ 0 & \text{if } s + \frac{\Delta v}{2} + p_2^- < p_2^+ \end{cases} \quad (23)$$

and the demand for product j of firm h is $d_2^- = 1 - d_2^+$.

The F.O.C. for the maximization of $p_2^+ d_2^+$ with respect to p_2^+ and of $p_2^- d_2^-$ with respect to p_2^- are as follows (they are sufficient as $p_2^+ d_2^+$ is concave in p_2^+, and $p_2^- d_2^-$ is concave in p_2^-):

$$\begin{aligned} -\frac{p_2^+}{\Delta v} + d_2^+ &= 0 \Leftrightarrow \frac{\Delta v}{2} + s + p_2^- - 2p_2^+ = 0 \\ -\frac{p_2^-}{\Delta v} + d_2^- &= 0 \Leftrightarrow \frac{\Delta v}{2} - s - 2p_2^- + p_2^+ = 0 \end{aligned}$$

From this we obtain p_2^{+*}, p_2^{-*} and then π_2^{+*}, π_2^{-*} in Lemma 1.

Proof of Lemma 2

In market (i, i), the demand function for firm h's system is given by (for $-\Delta v < P_2^+ - P_2^- - 2s < \Delta v$)

$$D_2^- = \frac{1}{2(\Delta v)^2}\left[\Delta v - 2s - P_2^- + P_2^+\right]^2 - \frac{1}{(\Delta v)^2}(P_2^+ - P_2^- - 2s)\max\{0, P_2^+ - P_2^- - 2s\}, \quad (24)$$

and $D_2^+ = 1 - D_2^-$ is the demand for the system of firm i. The F.O.C. for the maximization of $D_2^+ P_2^+$ with respect to P_2^+ and of $D_2^- P_2^-$ with respect to P_2^- are as follows:[32]

$$P_2^+ = 2s - \Delta v + 3P_2^- \quad \text{and} \quad -8(P_2^-)^2 - 2(2s - \Delta v)P_2^- + \Delta v^2 = 0$$

[32]Since D_2^- is such that $\frac{\partial D_2^- / \partial P_2^-}{D_2^-}$ is negative and decreasing in P_2^- (i.e., D_2^- is log-concave in P_2^-), satisfying the first order condition relative to $P_2^- D_2^-$ suffices to maximize $P_2^- D_2^-$ with respect to P_2^-. A similar remark applies to the maximization of $P_2^+ D_2^+$ with respect to P_2^+.

This yields P_2^{+*}, P_2^{-*} and then Π_2^{+*}, Π_2^{-*} in Lemma 2.

Proof of Proposition 1

Given compatibility, the total profit of firm A from product j is:

$$d_{1,j}^A p_{1,j}^A + \delta \left(d_{1,j}^A \pi_2^{+*} + (1 - d_{1,j}^A) \pi_2^{-*} \right) \tag{25}$$

where $d_{1,j}^A$ in (10) is the first-period demand (for $p_{1,j}^A \in [p_{1,j}^B - t, p_{1,j}^B + t]$). From (25) we obtain the first order condition

$$d_{1,j}^A - \frac{1}{2t}(p_{1,j}^A + \delta\pi_2^{+*} - \delta\pi_2^{-*}) = 0$$

A similar first order condition, $d_{1,j}^B - \frac{1}{2t}(p_{1,j}^B + \delta\pi_2^{+*} - \delta\pi_2^{-*}) = 0$, needs to hold for firm B.[33] Solving then, we find the first-period equilibrium prices and then the profits in Proposition 1(i): see (11), (12). However, this is correct as long as (3) does not need to be satisfied, or (3) applies but $t - \delta\left(\pi_2^{+*} - \pi_2^{-*}\right)$ in (11) is positive or zero. In case that (3) must be satisfied and $t - \delta\left(\pi_2^{+*} - \pi_2^{-*}\right) < 0$, then we show that the equilibrium price for each firm in period one is zero. In order to see this, consider firm A and suppose that $p_{1,j}^B = 0$. Then $d_{1,j}^A = 0$ for each $p_{1,j}^A \geq t$, and the derivative of firm A's profit is $\frac{1}{2} - \frac{1}{2t}p_{1,j}^A - \frac{1}{2t}(p_{1,j}^A + \delta\pi_2^{+*} - \delta\pi_2^{-*})$, which is negative for each $p_{1,j}^A < t$.

Proof of Proposition 2

Given incompatibility, the total profit of firm A is given by $\delta\Pi_2^{-*} + D_1^A(P_1^A + \delta\Pi_2^{+*} - \delta\Pi_2^{-*})$, with D_1^A in (16) (for $P_A \in [P_1^B - 2t, P_1^B + 2t]$). Assume without loss of generality that the equilibrium first-period prices, P_1^{A*}, P_1^{B*}, satisfy $P_1^{A*} \geq P_1^{B*}$. Then , P_1^{A*}, P_1^{B*} solve the F.O.C. given by[34]

$$\begin{aligned} -\frac{1}{2t}\left[1 + \frac{1}{2t}(P_1^B - P_1^A)\right](P_1^A + \delta\Pi_2^{+*} - \delta\Pi_2^{-*}) + D_1^A &= 0 \\ -\frac{1}{2t}\left[1 + \frac{1}{2t}(P_1^B - P_1^A)\right](P_1^B + \delta\Pi_2^{+*} - \delta\Pi_2^{-*}) + D_1^B &= 0 \end{aligned}$$

The unique solution is $P_1^{A*} = P_1^{B*} = P_1^*$ in (17),[35] and from (15) we obtain that equilibrium profit for each firm is Π^* in (18). However, this is correct as long as (3) does not need to be

[33] As in Lemma 1, these first order conditions are sufficient since the profit functions are concave.

[34] Since D_1^A (D_1^B) is log-concave in P_1^A (P_1^B), the same argument given in footnote 32 applies here to establish that F.O.C. are sufficient.

[35] If we consider $P_1^A > P_1^B$, then $D_1^A < D_1^B$, which implies that not both equalities can be satisfied.

satisfied, or (3) applies but P_1^* in (17) is positive or zero. In case that (3) must be satisfied and $t-\delta\left(\Pi_2^{+*}-\Pi_2^{-*}\right)<0$, then we show that the equilibrium price for each firm in period one is zero. In order to see this, consider firm A and suppose that $P_1^B=0$. Then $D_1^A=0$ for each $P_1^A\geq 2t$, and the derivative of firm A's profit is $-\frac{1}{2t}(1-\frac{1}{2t}P_1^A)(P_1^A+\delta\Pi_2^{+*}-\delta\Pi_2^{-*})+\frac{1}{2}(1-\frac{1}{2t}P_1^A)^2=\frac{1}{2}(1-\frac{1}{2t}P_1^A)\left(1-\frac{1}{t}(P_1^A+\delta\Pi_2^{+*}-\delta\Pi_2^{-*})-\frac{1}{2t}P_1^A\right)$, which is negative for each $P_1^A<2t$.

Proof of Proposition 4

(i) The result is obtained by comparing each firm's profit under compatibility, $t+2\delta\pi_2^{-*}$, with each firm's profit under incompatibility, $\frac{t}{2}+\delta\Pi^{-*}$.

(ii) First notice that under compatibility, the NPC binds if and only if $\frac{\delta}{t}>\frac{1}{\pi_2^{+*}-\pi_2^{-*}}$; under incompatibility, NPC binds if and only if $\frac{\delta}{t}>\frac{1}{\Pi_2^{+*}-\Pi_2^{-*}}$. Furthermore, $\frac{1}{\Pi_2^{+*}-\Pi_2^{-*}}<\frac{1}{\pi_2^{+*}-\pi_2^{-*}}$.

If $s\leq\bar{s}^2$, then we know from Proposition 4(i) that each firm prefers compatibility if the NPC does not bind in either regime, that is if $\frac{\delta}{t}\leq\frac{1}{\Pi_2^{+*}-\Pi_2^{-*}}$. If $\frac{\delta}{t}$ is between $\frac{1}{\Pi_2^{+*}-\Pi_2^{-*}}$ and $\frac{1}{\pi_2^{+*}-\pi_2^{-*}}$, then the NPC binds under incompatibility and each firm's profit is $\delta(\frac{1}{2}\Pi_2^{+*}-\frac{1}{2}\Pi_2^{-*})$, whereas under compatibility it is still $t+2\delta\pi_2^{+*}$. The inequality $t+2\delta\pi_2^{+*}>\delta(\frac{1}{2}\Pi_2^{+*}-\frac{1}{2}\Pi_2^{-*})$ is equivalent to $\frac{1}{\frac{1}{2}\Pi_2^{+*}-\frac{1}{2}\Pi_2^{-*}-2\pi_2^{+*}}>\frac{\delta}{t}$ and is satisfied since $\frac{\delta}{t}<\frac{1}{\pi_2^{+*}-\pi_2^{-*}}$. Finally, if $\frac{\delta}{t}>\frac{1}{\pi_2^{+*}-\pi_2^{-*}}$, then the NPC binds in both regimes and $\delta(\pi_2^{+*}+\pi_2^{-*})\geq\delta(\frac{1}{2}\Pi_2^{+*}+\frac{1}{2}\Pi_2^{-*})$ since $s\leq\bar{s}^2$.

If $s>\bar{s}^2$, then $\frac{1}{\Pi_2^{+*}-\Pi_2^{-*}}<\frac{1}{\frac{1}{2}\Pi_2^{+*}+\frac{1}{2}\Pi_2^{-*}-2\pi_2^{-*}}<\frac{1}{\pi_2^{+*}-\pi_2^{-*}}$. When $\frac{\delta}{t}\leq\frac{1}{\Pi_2^{+*}-\Pi_2^{-*}}$, the NPC does not bind in either regime and each firm prefers compatibility because $\frac{\delta}{t}\leq\frac{1}{\Pi_2^{+*}-\Pi_2^{-*}}$ implies $t+2\delta\pi_2^{+*}<\frac{t}{2}+\delta\Pi_2^{-*}$. When $\frac{1}{\Pi_2^{+*}-\Pi_2^{-*}}<\delta\leq\frac{1}{\frac{1}{2}\Pi_2^{+*}+\frac{1}{2}\Pi_2^{-*}-2\pi_2^{-*}}$, the NPC binds in under incompatibility, and the profit under incompatibility is greater than under compatibility if and only if $\frac{\delta}{t}>\frac{1}{\frac{1}{2}\Pi_2^{+*}-\frac{1}{2}\Pi_2^{-*}-2\pi_2^{+*}}$, which is the condition state in Proposition 4(ii). Finally, if $\delta>\frac{1}{\pi_2^{+*}-\pi_2^{-*}}$ then the NPC binds under each regime and each firm prefers incompatibility since $s>\bar{s}^2$ implies $\delta(\pi_2^{+*}+\pi_2^{-*})<\delta(\frac{1}{2}\Pi_2^{+*}+\frac{1}{2}\Pi_2^{-*})$.

Proof of Propositions 5-6

(i) Under compatibility, the second-period consumer surplus in market j is given by

$$\frac{1}{\Delta v}\int_{\underline{v}}^{v^e+p_2^{+*}-p_2^{-*}-s}(v^e-p_2^{-*}-s)dv_j^i+\frac{1}{\Delta v}\int_{v^e+p_2^{+*}-p_2^{-*}-s}^{\bar{v}}(v_j^i-p_2^{+*})dv_j^i=v^e-p_2^{+*}+\frac{1}{2}\pi_2^{-*}.$$

The first-period consumer surplus in market j is $2\int_0^{\frac{1}{2}}(v^e-p_1^*-tx)dx=v^e-p_1^*-\frac{t}{4}$. Hence, the total consumer surplus in market j is $v^e-p_1^*-\frac{t}{4}+\delta(v^e-p_2^{+*}+\frac{1}{2}\pi_2^{-*})$ and the total consumer surplus is given by (21).

(ii) Under incompatibility, the second-period consumer surplus is given by

$$
\begin{aligned}
& \frac{1}{(\Delta v)^2}\int_{\underline{v}}^{\bar{v}}\int_{\underline{v}}^{\bar{v}}(v_x^i+v_y^i-P_2^{+*})dv_x^i dv_y^i \\
- & \frac{1}{(\Delta v)^2}\int_{\underline{v}}^{2v^e-2s-P_2^{-*}+P_2^{+*}-\underline{v}}\int_{\underline{v}}^{2v^e-2s-P_2^{-*}+P_2^{+*}-v_x^i}(v_x^i+v_y^i-P_2^{+*})dv_x^i dv_y^i \\
+ & \frac{1}{(\Delta v)^2}\int_{\underline{v}}^{2v^e-2s-P_2^{-*}+P_2^{+*}-\underline{v}}\int_{\underline{v}}^{2v^e-2s-P_2^{-*}+P_2^{+*}-v_x^i}(2v^e-P_2^{-*}-2s)dv_x^i dv_y^i \\
= & \ 2v^e-P_2^{+*}+\frac{2}{3}\Pi_2^{-*}
\end{aligned}
$$

The first-period consumer surplus is $2\int_0^1\int_0^{1-x}(2v^e-P_1^*-tx-ty)dydx = 2v^e-P_1^*-\frac{2}{3}t$. Hence, the total consumer surplus is given by (22).

(iii) From Propositions 1(i) and 2(i) we insert p_1^* and P_1^* into CS^C and CS^I and find that $CS^C > CS^I$ if and only if $\delta\left(2\pi_2^{+*}-\pi_2^{-*}-2p_2^{+*}+P_2^{+*}-\Pi_2^{+*}+\frac{1}{3}\Pi_2^{-*}\right) > \frac{5}{6}t$. The left hand side is negative or zero if $s \le 1.168$, thus $CS^C < CS^I$ in this case. If $s > 1.168$, then the left hand side is positive and $CS^C > CS^I$ if and only if $\frac{\delta}{t} > \frac{5}{6\left(2\pi_2^{+*}-\pi_2^{-*}-2p_2^{+*}+P_2^{+*}-\Pi_2^{+*}+\frac{1}{3}\Pi_2^{-*}\right)}$. Incompatibility arises if and only if $s > \bar{s}^3 = 1.187$ and $\frac{\delta}{t} > \frac{1}{\Pi_2^{-*}-2\pi_2^{-*}}$, and such inequality implies $\frac{\delta}{t} > \frac{5}{6\left(2\pi_2^{+*}-\pi_2^{-*}-2p_2^{+*}+P_2^{+*}-\Pi_2^{+*}+\frac{1}{3}\Pi_2^{-*}\right)}$.

From Propositions 1(ii) and 2(ii) we insert p_1^* and P_1^* into CS^C and CS^I and find that $CS^C > CS^I$ if and only if $\frac{1}{6}t > \delta\left(2p_2^{+*}-\pi_2^{-*}-P_2^{+*}+\frac{2}{3}\Pi_2^{-*}\right)$. The right hand side is negative or zero if $s \ge 0.876$, thus $CS^C > CS^I$ in this case. If $s < 0.876$, then the right hand side is positive and $CS^C > CS^I$ if and only if $\frac{1}{6\left(2p_2^{+*}-\pi_2^{-*}-P_2^{+*}+\frac{2}{3}\Pi_2^{-*}\right)} > \frac{\delta}{t}$. Incompatibility arises if and only if $s > \bar{s}^2$ (given that we are considering the case in which the NPC binds in both regimes), hence for each $s > 0.876$ we have that $CS^C > CS^I$ but incompatibility emerges. Conversely, if $s < 0.749$, then compatibility emerges and $\frac{\delta}{t} > \frac{1}{\pi_2^{+*}-\pi_2^{-*}}$ (the inequality that implies that the NPC binds in both regimes) implies $\frac{\delta}{t} > \frac{1}{6\left(2p_2^{+*}-\pi_2^{-*}-P_2^{+*}+\frac{2}{3}\Pi_2^{-*}\right)}$, that is $CS^C < CS^I$.

Proof of Proposition 7

(i-ii) The welfare is simply obtained by adding consumer surplus to profits, which we presented in the previous propositions.

(iii) It is immediate to see that $SW^I - SW^C = -\frac{1}{6}t+\delta(\Pi_2^{+*}+\frac{5}{3}\Pi_2^{-*}-P_2^{+*}-2\pi_2^{+*}-3\pi_2^{-*}+2p_2^{+*})$. It turns out that $\Pi_2^{+*}+\frac{5}{3}\Pi_2^{-*}-P_2^{+*}-2\pi_2^{+*}-3\pi_2^{-*}+2p_2^{+*} \le 0$ if $s \le 0.535$ or $s \ge 1.152$, hence in this case $SW^I < SW^C$. If $s \in (0.535, 1.152)$, then $\Pi_2^{+*}+\frac{5}{3}\Pi_2^{-*}-P_2^{+*}-2\pi_2^{+*}-3\pi_2^{-*}+2p_2^{+*} > 0$ hence $SW^I > SW^C$ if $\frac{\delta}{t} > \frac{1}{6(\Pi_2^{+*}+\frac{5}{3}\Pi_2^{-*}-P_2^{+*}-2\pi_2^{+*}-3\pi_2^{-*}+2p_2^{+*})}$. If the NPC does not apply, then incompatibility requires $s > \bar{s}^3 = 1.187$; hence $SW^I < SW^C$ when incompatibility emerges. If the NPC binds, then incompatibility emerges if and only if $s \geq \bar{s}^2$, and then $SW^I > SW^C$ if and only if $s < 1.152$ and $\frac{\delta}{t} > \frac{1}{6(\Pi_2^{+*} + \frac{5}{3}\Pi_2^{-*} - P_2^{+*} - 2\pi_2^{+*} - 3\pi_2^{-*} + 2p_2^{+*})}$.

Proof of Lemma 3

(i) Under compatibility, each firm's profit is $t + 2\delta\pi_2^{-*}$, which is decreasing in s because π_2^{-*} is decreasing in s. Consumer surplus is given by Proposition 5(i). After inserting the equilibrium values from Lemma 1 and Proposition 1 into the consumer surplus expression, we find an increasing function of s. Under incompatibility, each firm's profit is $\frac{t}{2} + \delta\Pi_2^{-*}$, which is decreasing in s because Π_2^{-*} is decreasing in s. Consumer surplus is given by Proposition 5(ii). After inserting the equilibrium values from Lemma 2 and Proposition 2 into the consumer surplus expression, we find an increasing function of s.

(ii) Under compatibility, each firm's profit is $\delta(\frac{1}{2}\pi_2^{+*} + \frac{1}{2}\pi_2^{-*})$, which is increasing in s because $\pi_2^{+*} + \pi_2^{-*}$ is increasing in s (see Corollary 1). Consumer surplus is given by Proposition 5(i). After inserting the equilibrium values from Lemma 1 and Proposition 1 into the consumer surplus expression, we find a decreasing function of s. Under incompatibility, each firm's profit is $\delta(\frac{1}{2}\Pi_2^{+*} + \frac{1}{2}\Pi_2^{-*})$, which is increasing in s because $\Pi_2^{+*} + \Pi_2^{-*}$ is increasing in s (see Corollary 2). Consumer surplus is given by Proposition 5(ii). After inserting the equilibrium values from Lemma 2 and Proposition 2 into the consumer surplus expression, we find a decreasing function of s.

Proof of Lemma 4

(i) Under compatibility, suppose that $\frac{\delta}{t} > \frac{1}{\pi_2^{+*} - \pi_2^{-*}}$, which implies that the NPC binds. Then the indifference condition for consumers in market j is $-\lambda f_j^A + t\theta_j = -\lambda f_j^B + t(1-\theta_j)$. Therefore the demand for firm A is $d_j^A = \frac{1}{2} + \frac{\lambda}{2t}\left(f_j^A - f_j^B\right)$; the demand for firm B is $1 - d_J^A$. The profit for firm A in market j is $d_j^A(\delta\pi_2^{+*} - f_j^A) + (1 - d_j^A)\delta\pi_2^{-*}$ and the first order condition for a symmetric equilibrium, such that $f_j^A = f_j^B = f_j$, is

$$\frac{\lambda}{2t}(\delta\pi_2^{+*} - f_j) - \frac{1}{2} - \frac{\lambda}{2t}\delta\pi_2^{-*} = 0.$$

This equation has no positive solution with respect to f_j if $\lambda \leq \hat{\lambda}^C$, whereas if $\lambda > \hat{\lambda}^C$ then $f_j = \delta(\pi_2^{+*} - \pi_2^{-*}) - \frac{t}{\lambda} > 0$ is the unique solution to the first order condition. Proving that no profitable deviation exists is standard. Notice that $\frac{\delta}{t} > \frac{1}{\pi_2^{+*} - \pi_2^{-*}}$ implies $\hat{\lambda}^C < 1$.

The equilibrium profit of each firm per market is $\frac{1}{2}(\delta\pi_2^+ + \delta\pi_2^- - f_j)$, which is $\frac{1}{2}(\delta\pi_2^{+*} + \delta\pi_2^{-*})$ if $\lambda < \hat{\lambda}^C$, is $\frac{t}{2\lambda} + \delta\pi_2^{-*}$ if $\lambda > \hat{\lambda}^C$. In the first case, it is increasing in s; in the second case, it is decreasing in s. Consumer surplus is equal to $2v^e + 2\lambda f_j - \frac{1}{2}t + \delta(2v^e - 2p_2^{+*} + \pi_2^{-*})$ (see Proposition 6). Hence if $\lambda < \hat{\lambda}^C$, it is $2v^e - \frac{1}{2}t + \delta(2v^e - 2p_2^{+*} + \pi_2^{-*})$; if $\lambda > \hat{\lambda}^C$, it is $2(1+\delta)v^e + \delta(\lambda(2\pi_2^{+*} - 2\pi_2^{-*}) + \pi_2^{-*} - 2p_2^{+*}) - \frac{5}{2}t$.

(ii) Under incompatibility, suppose that $\frac{\delta}{t} > \frac{1}{\Pi_2^{+*} - \Pi_2^{-*}}$, which implies that the NPC binds. Then the indifference condition is $-\lambda F^A + t\theta_x + t\theta_y = -\lambda F^B + t(1-\theta_x) + t(1-\theta_y)$ and the demand for firm A is $D^A = \frac{1}{2}\left(1 + \frac{\lambda}{2t}\left(F^A - F^B\right)\right)^2$ (if $F^A \geq F^B$); the demand for firm B is $1 - D^A$. The profit for firm A is $D^A(\delta\Pi_2^{+*} - F^A) + (1 - D^A)\delta\Pi_2^{-*}$ and the first order condition for a symmetric equilibrium, such that $F^A = F^B = F$, is

$$\frac{\lambda}{2t}(\delta\Pi_2^{+*} - F) - \frac{1}{2} - \frac{\lambda}{2t}\delta\Pi_2^{-*} = 0.$$

This equation has no positive solution with respect to F if $\lambda \leq \hat{\lambda}^I$, whereas if $\lambda > \hat{\lambda}^I$, then $F = \delta(\Pi_2^{+*} - \Pi_2^{-*}) - \frac{t}{\lambda}$ is the unique solution to the first order condition. Proving that no profitable deviation exists is standard. Notice that $\frac{\delta}{t} > \frac{1}{\Pi_2^{+*} - \Pi_2^{-*}}$ implies $\hat{\lambda}^I < 1$.

The equilibrium profit of each firm is $\frac{1}{2}(\delta\Pi_2^+ + \delta\Pi_2^- - F)$, that is $\frac{1}{2}(\delta\Pi_2^{+*} + \delta\Pi_2^{-*})$ if $\lambda < \hat{\lambda}^I$, is $\frac{t}{2\lambda} + \delta\Pi_2^{-*}$ if $\lambda > \hat{\lambda}^I$. In the first case it is increasing in s; in the second case, it is decreasing in s.

Consumer surplus is equal to $2v^e + \lambda F^* - \frac{2}{3}t + \delta(2v^e - P_2^{+*} + \frac{2}{3}\Pi_2^{-*})$ (see Proposition 6). Hence if $\lambda < \hat{\lambda}^I$, it is $2v^e - \frac{2}{3}t + \delta(2v^e - P_2^{+*} + \frac{2}{3}\Pi_2^{-*})$; if $\lambda > \hat{\lambda}^I$, it is $2(1+\delta)v^e - \frac{5}{3}t + \delta\left(\lambda(\Pi_2^{+*} - \Pi_2^{-*}) - P_2^{+*} + \frac{2}{3}\Pi_2^{-*}\right)$.

Proof of Proposition 9

(i) When $\lambda > \hat{\lambda}^C$, consumer surplus given by $2(1+\delta)v^e + \delta(\lambda(2\pi_2^{+*} - 2\pi_2^{-*}) + \pi_2^{-*} - 2p_2^{+*}) - \frac{5}{2}t$ is decreasing in s if $\lambda < \frac{1}{2}$. If $\frac{\delta}{t}$ is sufficiently large, then $\hat{\lambda}^C(s) < \frac{1}{2}$ and for λ between $\hat{\lambda}^C(s)$ and $\frac{1}{2}$, both profits and consumer surplus are decreasing in s. Hence a marginal reduction in s increases both profits and consumer surplus.

(ii) When $\lambda > \hat{\lambda}^I$, consumer surplus given by $2(1+\delta)v^e - \frac{5}{3}t + \delta\left(\lambda(\Pi_2^{+*} - \Pi_2^{-*}) - P_2^{+*} + \frac{2}{3}\Pi_2^{-*}\right)$ is decreasing in s if $\lambda < \frac{9}{10}$. If $\frac{\delta}{t}$ is sufficiently large, then $\hat{\lambda}^I(s) < \frac{9}{10}$ and for λ between $\hat{\lambda}^I$ and $\frac{9}{10}$, both profits and consumer surplus are decreasing in s. Hence a marginal reduction in s increases both profits and consumer surplus.

11 On-line Appendix (not for publication): Dynamic Compatibility Choices

We here provide the analysis of dynamic compatibility choices: each firm makes a non-cooperative compatibility choice in the beginning of each period. In other words, Stages 1-3 are repeated in each period. We still consider two periods. We assume that consumers are myopic.

We study this alternative model for two different reasons. First, we want to check the robustness of the prediction generated by the model we analyzed previously (i.e., Proposition 4). We find that the prediction is robust. Second, this alternative model reveals some interesting dynamics of compatibility choices that arises because of the interaction between compatibility choices and poaching. This result is clearly seen when we study the second-period compatibility choice for a given first-period market outcome.

We gather all the proofs at the end of this on-line Appendix.

11.1 Second-period price competition and compatibility choice

11.1.1 Second-period price competition

Lemma 1 can be applied to the second-period price competition in each market i_j as long as there is compatibility in period two, independently of the first-period compatibility regime. Similarly, Lemma 2 can be applied to each market (i, i) as long as there is incompatibility in period two, independently of the first-period compatibility regime. Therefore, what remains to be studied is the competition under incompatibility in the market (i, h) composed of the consumers who have bought a hybrid system in period one. This competition arises only if the firms chose compatibility in period one and incompatibility in period two.

We normalize the total mass of consumers in market (i, h) to one. A consumer with valuations $\left(v_x^i, v_y^h\right)$ is indifferent between buying (i, i) and (h, h) if and only if

$$v_x^i + v^e - P^i - s = v^e + v_y^h - P^h - s. \tag{26}$$

It turns out that the demand for firm i's system (for $P^h - \Delta v < P^i < P^h + \Delta v$) is $D_2^i(i, h) = \frac{1}{2(\Delta v)^2}\left(\Delta v - P_2^i + P_2^h\right)^2 - \frac{1}{(\Delta v)^2}\left(P_2^h - P_2^i\right)\max\{0, P_2^h - P_2^i\}$ (and $D_2^h(i, h) = 1 - D_2^i(i, h)$) and coincides with the demand for its system in market (i, i) when $s = 0$ (see (24)). Therefore, Lemma 5 below is a special case of Lemma 2 and identifies a symmetric equilibrium.

Lemma 5. *Suppose that compatibility was chosen in period one while incompatibility was chosen in period two. Consider the period-two competition in the market composed of the consumers who bought the hybrid system (A, B) (or (B, A)) in period one. We normalize to one the total mass* *of consumers in this market. There exists a unique equilibrium, which is symmetric and such that*

(i) the equilibrium price for each firm is $P_2^{0*} = \frac{\Delta v}{2}$;

(ii) each firm's second-period profit from this market is $\Pi_2^{0*} = \frac{\Delta v}{4}$.

Notice that Π_2^{0*} coincides with Π_2^{+*} and Π_2^{-*} when $s = 0$. As $2\pi_2^{+*} + 2\pi_2^{-*} > \Pi_2^{+*} + \Pi_2^{-*}$ when $s = 0$ and both $2\pi_2^{+*} + 2\pi_2^{-*}$ and $\Pi_2^{+*} + \Pi_2^{-*}$ are strictly increasing in s, it follows that the second-period competition in market (A, B) or (B, A) under incompatibility is more intensive than the competition in market (A, A) or (B, B) under incompatibility and the competition in any product market under compatibility.

11.1.2 Second-period compatibility choice

The detailed analysis of the second-period compatibility choice can be found in Section 11.5, where Lemma 6 (Lemma 7) describes our findings given the first-period compatibility (incompatibility). We below provide a summary of the main findings (see also Figure 2). Basically, we can distinguish two cases depending on whether or not the first-period market shares affect the second-period compatibility choice.

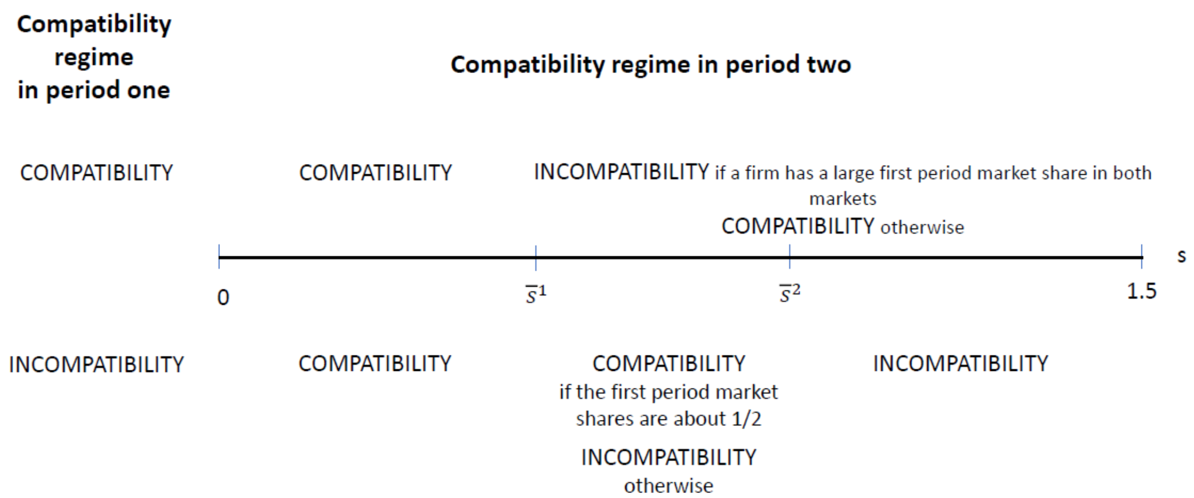

Figure 2: Compatibility choice in period 2 given a compatibility regime in period 1

- Case 1: the second-period compatibility choice is determined independently of the first-period market shares

- For low switching cost (i.e., $s < \overline{s}^1$), both firms always choose compatibility in period two independently of the compatibility regime of the first-period.
- Given the first-period incompatibility, for high switching cost (i.e., $s > \overline{s}^2$), at least one firm chooses incompatibility in period two.

• Case 2: the second-period compatibility choice depends on the first-period market shares

- Given the first-period incompatibility, for intermediate switching cost (i.e., $\overline{s}^1 \leq s < \overline{s}^2$), compatibility emerges in period two if the firms' first-period market shares are not too different; otherwise, incompatibility emerges in period two.
- Given the first-period compatibility, for high switching cost (i.e., $s > \overline{s}^1$), incompatibility emerges in period two if and only if one firm's first-period market share in both products is close to one.

The first finding of Case 1 results simply from the fact that $2\pi_2^{+*} > \Pi_2^{+*}$, $2\pi_2^{-*} > \Pi_2^{-*}$ for $s < \overline{s}^1$, that is both firms prefer compatibility in the second period. The second finding of Case 1 is obtained because given the first-period incompatibility and $s > \overline{s}^2$, the second period industry profit is higher with incompatibility. Therefore, at least the firm with a weakly larger (first-period) market share prefers incompatibility in the second period.

Consider now the first finding of Case 2, the first-period incompatibility with $\overline{s}^1 \leq s < \overline{s}^2$. If the firms have similar market shares, then $s < \overline{s}^2$ implies that both prefer compatibility. But if one firm has a much larger market share, then $\overline{s}^1 \leq s$ implies that the firm prefers incompatibility. Consider now the case of the first-period compatibility with $s > \overline{s}^1$. As the second-period incompatibility intensifies competition in the market for the consumers who bought a hybrid system in period one, the firms choose incompatibility only if the size of these markets is small enough, which happens if one firm has a large (first-period) market share in both markets. Conversely, if the first-period market shares are more or less symmetric, the first-period compatibility leads to the second-period compatibility.

In summary, the interaction between compatibility choices and poaching generates an interesting asymmetry in the dynamics of compatibility choices. For a low switching cost, there is no path dependency in that the firms end up choosing compatibility in period two no matter the compatibility regime in period one. In contrast, for high switching cost, we have path dependency if the shares of the hybrid markets are significant enough under the first-period compatibility. Then, compatibility (incompatibility) in period one leads to compatibility (incompatibility) in period two. This path dependency arises even if we make the first-period market shares endogenous: namely, according to Proposition 10(ii) and Proposition 11, we have such path dependency for $s > \overline{s}^2$ as long as δ is small enough.

11.2 First-period price competition

In this subsection, we study the first-period price competition given a first-period compatibility regime. The analysis is relatively straightforward in the case in which the second-period compatibility choice is determined independently of the first-period market shares, because then first-period price competition has no dynamic effect. The analysis is more involved in the case in which the first-period market shares has an impact on the second-period compatibility choices. For this reason, we start by analyzing the regime of the first-period incompatibility.

11.2.1 Given incompatibility in the first period

We below present a proposition that covers $s < \bar{s}^1$ and $s \geq \bar{s}^2$; in both cases, the second-period compatibility choice does not depend on the first-period market shares.

Proposition 10. *Suppose that incompatibility was chosen in the first period. Let* $\Delta v = 1$ *without loss of generality.*

(i) If $\Pi_2^{+*} < 2\pi_2^{+*}$ *(i.e., if* $s < \bar{s}^1$*) and the NPC does not apply, then there exists a unique equilibrium, which is symmetric and such that*

$$P_1^{i*} = t - \delta\left(2\pi_2^{+*} - 2\pi_2^{-*}\right) \quad \textit{for } i = A, B.$$

Both firms choose compatibility in the second period. Each firm's total profit is

$$\Pi^{i*} = \frac{t}{2} + \delta 2\pi_2^{-*} \quad \textit{for } i = A, B.$$

If instead the NPC must be satisfied, then

$$P_1^{i*} = \max\{t - \delta\left(2\pi_2^{+*} - 2\pi_2^{-*}\right), 0\} \quad \textit{for } i = A, B.$$

Both firms choose compatibility in the second period. Each firm's total profit is

$$\Pi^{i*} = \max\{\frac{t}{2} + \delta 2\pi_2^{-*}, \delta(\pi_2^{+*} + \pi_2^{-*})\} \quad \textit{for } i = A, B.$$

(ii) If $2\pi_2^{+*} + 2\pi_2^{-*} \leq \Pi_2^{+*} + \Pi_2^{-*}$ *(i.e., if* $s \geq \bar{s}^2$*), there exists a unique equilibrium, which coincides with the equilibrium described by Proposition 2. Both firms choose incompatibility in the second period and each firm's total profit is* Π^* *as described in Proposition 2.*

Consider first Proposition 10(ii) which considers $s \geq \bar{s}^2$. It is similar to Proposition 2, as both yield the same equilibrium prices and profits, although a major difference is that the second-period compatibility choice is endogenous in Proposition 10(ii) while it is exogenous in Proposition 2.

Proposition 10(i) considers $s < \overline{s}^1$ and describes a change in the compatibility regime over the two periods. However, the equilibrium prices and profits it identifies can be explained by arguments very similar to those provided after Proposition 1 and 2.

Finally, if s is between $\overline{s}^1$ and $\overline{s}^2$, then the compatibility regime in period two depends on the first-period market shares: compatibility emerges if the first-period market shares are not too different. Proposition 16 in Section 11.5 establishes that if a pure strategy equilibrium exists, then the prices are the same as in Proposition 10(i) and compatibility prevails in the second period. When the NPC does not need to hold, the equilibrium exists if $\delta/t \leq r_{IC}(s)$ for a suitable function r_{IC} defined in Section 11.5; when the NPC must be satisfied, the equilibrium exists except for a tiny set of parameters which is included in the set of $(s, \frac{\delta}{t})$ satisfying $s \in [0.819, \bar{s}^2]$ and $r_{IC}(s) \leq \delta/t \leq 1$.

11.2.2 Given compatibility in the first period

If compatibility was chosen in period one, then for $s \leq \overline{s}^1$, compatibility arises also in period two independently of the first-period market shares. Hence, there exists an equilibrium which is outcome-equivalent to the equilibrium described by Proposition 1. Next proposition establishes that such equilibrium exists also if $s > \overline{s}^1$ and δ/t is small.

Proposition 11. *Suppose that compatibility was chosen in the first period.*

(i) When the NPC must be satisfied, there exists an equilibrium like the one described in Proposition 1(ii)

(ii) When the NPC does not apply, the exists an equilibrium like the one described in Proposition 1(i) if and only if $\Pi_2^{+*} - 2\pi_2^{+*} \leq \frac{t}{\delta}$. *In either case, each firm chooses compatibility in the second period and each firm's total profit is given as in the equilibrium of Proposition 1.*

When the NPC is not imposed, the equilibrium described in Proposition 1(i) may not exists if $s > \overline{s}^1$ because for a firm it may be profitable to induce incompatibility in the second period by choosing low prices and cornering both markets in the first period. Indeed, for δ large, this is the best deviation as there are economies of scale under the second-period incompatibility. In order to corner both markets, the deviating firm should charge $p_1^* - t$ in period one; then it obtains a profit of Π_2^{+*} in period two and a total profit of $\delta\left[\Pi_2^{+*} - 2\left(\pi_2^{+*} - \pi_2^{-*}\right)\right]$. Therefore, the cornering deviation is not profitable if the deviation profit is smaller than $t + \delta 2\pi_2^{-*}$, which is equivalent to $\Pi_2^{+*} - 2\pi_2^{+*} \leq \frac{t}{\delta}$. This inequality is obviously satisfied for $s \leq \overline{s}^1$, but for $s > \overline{s}^1$ it fails to hold if $\frac{\delta}{t}$ is high and then the equilibrium described does not exist. However, this deviation is infeasible when the NPC must be satisfied, thus existence is not subject to any restriction.

When the NPC does not apply and $s > \bar{s}^3$, there exists a cornering equilibrium if δ/t is large. In this equilibrium, one firm charges a price $\bar{p} < 0$ (i.e., below the marginal cost) in each market and the other firm charges $\bar{p} + t$. Hence, the first firm has full market share in both markets in period one and chooses incompatibility in period two, with a total profit of $2\bar{p} + \delta\Pi_2^{+*}$. The other firm has zero market share in both markets in period one and its total profit is $\delta\Pi_2^{-*}$. The value of $\bar{p}$ must be small enough in order to discourage the cornered firm from earning a positive market share in period one, but high enough to deter the cornering firm from reducing its period-one market share in order to reduce its period-one losses. Since the possible deviations are somewhat intricate to describe,[36] stating precisely the conditions under which this equilibrium exists is complicated, except that for each $s > \bar{s}^3$ there exists $r_{CI}(s)$ such that a cornering equilibrium exists if and only if $\frac{\delta}{t} \geq r_{CI}(s)$. A $\frac{\delta}{t}$ sufficiently high is needed because each firm makes zero or negative profit in period one, and a large $\frac{\delta}{t}$ discourages deviations in period one that yield a positive profit in period one but reduce the profit in period two.[37] Moreover, we find that $r_{CI}(s)$ is between $\frac{3/2}{\Pi_2^{+*} - 2\pi_2^{+*}}$ and $\frac{16/9}{\Pi_2^{+*} - 2\pi_2^{+*}}$.

Proposition 12. *Suppose that compatibility was chosen in the first period and that the NPC does not apply. Assume $\Pi_2^{-*} - 2\pi_2^{-*} > 0$. Then there exists a cornering equilibrium if and only if $\frac{\delta}{t} \geq r_{CI}(s)$. Precisely,*

(i) in the first period, in both markets one firm charges price $\bar{p} = \gamma - \frac{\delta}{2}(\Pi_2^{+*} - \Pi_2^{-*})$ (for a suitable γ in $(0, t)$) and the other firm charges price $\bar{p} + t$; hence the first firm corners both markets;

(ii) in the second period, incompatibility is chosen at least by the firm who cornered the markets, which earns a total profit of $2\gamma + \delta\Pi_2^{-*}$; the other firm's total profit is $\delta\Pi_2^{-*}$.

11.3 First-period compatibility choice

In the previous subsection we have analyzed the first-period price competition given a first-period compatibility regime. We now study the firms' first-period compatibility choices. Let us normalize Δv to one without loss of generality and let us consider the case in which the NPC does not apply. If the switching cost is low enough (i.e., $s < \bar{s}^1$), then by Proposition 10(i) and Proposition 11, the firms choose compatibility in period two no matter the compatibility

[36] They might include the choice of the deviating firm to include compatibility in period two.

[37] Such deviations lead to compatibility in period two, thus reducing each firm's profit in period two as $s > \bar{s}^3$.

regime in period one. As compatibility in period one generates a higher first-period profit than incompatibility (see Propositions 10(i) and 11), the firms will choose compatibility also in period one.

Suppose now that the switching cost is high enough (i.e., $s > \bar{s}^3$). Then, incompatibility in period one leads to incompatibility in period two, with the profit of $\frac{t}{2} + \delta\Pi_2^{-*}$ for each firm (see Proposition 10(ii)). Regarding the case of compatibility in period one, we have to distinguish δ/t small from δ/t large. If δ/t is small (i.e., $\frac{\delta}{t} \leq \frac{1}{\Pi_2^{+*}-2\pi_2^{+*}}$), compatibility in period one leads to compatibility in period two by Proposition 11. In this case, each firm's profit is $t + \delta 2\pi_2^{-*}$. Since $\frac{1}{\Pi_2^{+*}-2\pi_2^{+*}} < \frac{1}{2(\Pi_2^{-*}-2\pi_2^{-*})}$ for each $s \in (\bar{s}^3, \frac{3}{2})$, it follows that $\frac{\delta}{t} \leq \frac{1}{\Pi_2^{+*}-2\pi_2^{+*}}$ implies $\frac{\delta}{t} < \frac{1}{2(\Pi_2^{-*}-2\pi_2^{-*})}$, that is $t + \delta 2\pi_2^{-*} > \frac{t}{2} + \delta\Pi_2^{-*}$ and therefore the firms choose compatibility for δ/t small. If δ/t is large enough (i.e., larger than $r_{CI}(s)$), then compatibility in period one leads to incompatibility in period two through the cornering equilibrium described in Proposition 12. In such an equilibrium, the cornered firm earns $\delta\Pi_2^{-*}$, which is less then $\frac{t}{2} + \delta\Pi_2^{-*}$. Therefore, such a firm will choose incompatibility in period one.

Proposition 13 below covers values of parameters such that a pure strategy equilibrium exists given either compatibility regime in period one. We have

Proposition 13. *(first-period compatibility choice without the NPC) Consider the model in which each firm makes a compatibility choice in every period and the NPC does not apply.*

(i) If either the switching cost is low enough to satisfy $2\pi_2^{+*} \geq \Pi_2^{+*}$ (i.e., $s \leq \bar{s}^1$), or if δ/t is small enough to satisfy $\delta/t \leq \frac{1}{\Pi_2^{+*}-2\pi_2^{+*}}$ (and, furthermore, $\delta/t \leq r_{IC}(s)$ for $s \in (\bar{s}^1, \bar{s}^2)$) then there is a unique equilibrium, which is symmetric. In the equilibrium, the firms choose compatibility in both periods.

(ii) If both the switching cost and δ/t are high enough to satisfy $\Pi_2^{-*} > 2\pi_2^{-*}$ and $\delta/t \geq r_{CI}(s)$, then there is a unique equilibrium, which is symmetric and such that the firms choose incompatibility in both periods.

For the case in which the NPC must be satisfied, matters are simpler because the NPC makes more frequent the existence of an equilibrium and leads to an outcome which essentially coincides with the outcome of Proposition 4(ii).

Proposition 14. *(first-period compatibility choice given the NPC) Consider the model in which each firm makes a compatibility choice in every period and the NPC must be satisfied. If $\Pi_2^{+*} + \Pi_2^{-*} > 2\pi_2^{+*} + 2\pi_2^{-*}$ and $\frac{\delta}{t} > \frac{1}{\frac{1}{2}\Pi_2^{+*}+\frac{1}{2}\Pi_2^{-*}-2\pi_2^{-}}$, then in the unique equilibrium both firms choose incompatibility in both periods. Otherwise (with the exception of the parameter set mentioned at the of Section 11.2.1) there exists an equilibrium in which both firms choose compatibility in both periods.*

11.4 Comparison of the two models

We now compare Proposition 4 from the model in which compatibility choices are made in period 1 only (called, Model 1) with Propositions 13 and 14 from the model in which compatibility choices are made in each period (called, Model 2). First notice, for the case in which the NPC must be satisfied, that Proposition 4(ii) is essentially equivalent to Proposition 14.

When the NPC does not need to be satisfied, we note first that the two models still generate quite similar predictions. If $\frac{\delta}{t}$ and the switching cost are large enough (i.e., if $s > \bar{s}^3$ and $\frac{\delta}{t} > \frac{1}{2(\Pi_2^{-*} - 2\pi_2^{-*})}$ holds), then the incompatibility equilibrium arises in both models since $r_{CI}(s) < \frac{1}{2(\Pi_2^{-*} - 2\pi_2^{-*})}$. Similarly, if either the switching cost is low enough (i.e., $s \leq \bar{s}^1$) to satisfy $2\pi_2^{+*} \geq \Pi_2^{+*}$, or $s > \bar{s}^1$ and δ/t is small enough to satisfy $\delta/t \leq \frac{1}{\Pi_2^{+*} - 2\pi_2^{+*}}$ (plus $\frac{\delta}{t} \leq \bar{r}_{IC}(s)$ if $s \in (\bar{s}^1, \bar{s}^2)$), then the compatibility equilibrium arises in both models.

When we focus on the differences between the two propositions, we find that the incompatibility equilibrium arises in Model 2 more frequently than in Model 1. In order to see why, notice that in Model 1 the incompatibility equilibrium emerges if and only if each firm's profit under incompatibility, $\frac{t}{2} + \delta\Pi_2^{-*}$, is larger than the one under compatibility, $t + \delta 2\pi_2^{-*}$. In Model 2, the incompatibility equilibrium arises as long as the cornering equilibrium exists given compatibility in period one, since this induces at least one firm to choose incompatibility in period one. This occurs if and only if $\frac{\delta}{t} \geq r_{CI}(s)$, and since $r_{CI}(s) < \frac{1}{2(\Pi_2^{-*} - 2\pi_2^{-*})}$, the incompatibility equilibrium is found more frequently in Model 2 than in Model 1.

Summarizing, we have:

Proposition 15. *(i) Suppose that the NPC must be satisfied. Then, both models generate essentially the same predictions.*
(ii) Suppose that the NPC does not need to be satisfied.
(a)In both models, the compatibility equilibrium arises if the switching cost is low enough (i.e., $s \leq \bar{s}^1$) or if δ/t is small enough to satisfy $\delta/t \leq \frac{1}{\Pi_2^{+*} - 2\pi_2^{+*}}$ for $s \in (\bar{s}^1, \frac{3}{2})$ (plus $\frac{\delta}{t} \leq \bar{r}_{IC}(s)$ if $s \in (\bar{s}^1, \bar{s}^2)$). In both models, the incompatibility equilibrium arises if $\frac{\delta}{t}$ and the switching cost are large enough (i.e., if $s > \bar{s}^3$ and $\Pi_2^{-*} - 2\pi_2^{-*} > \frac{t}{2\delta}$ holds).
(b) If the incompatibility equilibrium arises in Model 1, it arises also in Model 2. But there are parameter values under which the incompatibility equilibrium arises in the latter while it does not in the former.

11.5 Proofs

Second-period compatibility choice
Case 1: when compatibility was chosen in period one.

Suppose that both firms chose compatibility in the first period. Let $\Delta v = 1$ without loss of generality. If both firms choose compatibility in the second period, then firm i's second-period profit is:

$$\left(d_{1,x}^i + d_{1,y}^i\right) \pi_2^{+*} + \left(d_{1,x}^h + d_{1,y}^h\right) \pi_2^{-*}, \tag{27}$$

where $d_{1,j}^i$ is the first-period market share of firm i for product j, and $d_{1,j}^i + d_{1,j}^h = 1$. If any of the two firms chooses incompatibility in the second period, then i's profit is

$$d_{1,x}^i d_{1,y}^i \Pi_2^{+*} + \left(d_{1,x}^i d_{1,y}^h + d_{1,x}^h d_{1,y}^i\right) \Pi_2^{0*} + d_{1,x}^h d_{1,y}^h \Pi_2^{-*}. \tag{28}$$

Therefore, firm i chooses compatibility in the second period as long as (27) is at least as large as (28).We have the following result:

Lemma 6. *Suppose that compatibility was chosen in period one (and let $\Delta v = 1$ w.l.o.g.). Then compatibility choices in the second period are as follows:*

(i) If s is such that $2\pi_2^{+*} - \Pi_2^{+*} \geq 0$ (i.e., $s \leq \bar{s}^1$), then both firms choose compatibility for any $(d_{1,x}^i, d_{1,y}^i)$ in $[0, 1]^2$.

(ii) If s is such that $2\pi_2^{+*} - \Pi_2^{+*} < 0$ (i.e., $s > \bar{s}^1$), then at least one firm chooses incompatibility if and only if $d_{1,x}^i$ and $d_{1,y}^i$ are both close to 1 or both close to 0.

Proof. (i) Using (27) and (28), we see that firm i prefers compatibility if and only if

$$2\pi_2^{-*} - \Pi_2^{-*} \geq d_{1,x}^i d_{1,y}^i (\Pi_2^{+*} + \Pi_2^{-*} - \frac{1}{2}) + (d_{1,x}^i + d_{1,y}^i)(\frac{1}{4} - \Pi_2^{-*} - \pi_2^{+*} + \pi_2^{-*}) \tag{29}$$

Likewise, firm h prefers compatibility if and only if

$$2\pi_2^{+*} - \Pi_2^{+*} \geq d_{1,x}^i d_{1,y}^i (\Pi_2^{+*} + \Pi_2^{-*} - \frac{1}{2}) + (d_{1,x}^i + d_{1,y}^i)(\frac{1}{4} - \Pi_2^{+*} + \pi_2^{+*} - \pi_2^{-*}) \tag{30}$$

If $s \leq \bar{s}^1$, then we prove that (29) holds for each $(d_{1,x}^i, d_{1,y}^i) \in [0, 1]^2$ by showing that the maximum of the right hand side, considered as a function of $(d_{1,x}^i d_{1,y}^i) \in [0, 1]^2$, is not larger than $2\pi_2^{-*} - \Pi_2^{-*}$, the left hand side. To this purpose, notice that the Hessian matrix of the right hand side is indefinite, hence no maximum point exists in the interior of $[0, 1]^2$, and we examine the four edges of the square $[0, 1]^2$. For instance, if we consider the edge such that $d_{1,x}^i = 1$ then the right hand side of (29) is $d_{1,y}^i(\Pi_2^{+*} + \Pi_2^{-*} - \frac{1}{2}) + (1 + d_{1,y}^i)(\frac{1}{4} - \Pi_2^{-*} - \pi_2^{+*} + \pi_2^{-*})$, a linear function of $d_{1,y}^i$ which is smaller than $2\pi_2^{-*} - \Pi_2^{-*}$ both at $d_{1,y}^i = 0$ and at $d_{1,y}^i = 1$. The other three edges of the square are dealt with similarly. The proof for (30) follows the same lines.

(ii) The proof follows from immediate manipulations of (29)-(30).■

For $s > \overline{s}^1$, firm i prefers incompatibility if

$$d^i_{1,x} > \frac{\pi_2^{+*} + \pi_2^{-*} - \Pi_2^0}{\Pi_2^{+*} + \pi_2^{-*} - \pi_2^{+*} - \Pi_2^0} \text{ and } d^i_{1,y} > \frac{2\pi_2^{-*} - \Pi_2^{-*} - (\Pi_2^0 - \Pi_2^{-*} - \pi_2^{+*} + \pi_2^{-*})d^i_{1,x}}{(\Pi_2^0 - \Pi_2^{-*} - \pi_2^{+*} + \pi_2^{-*}) + (\Pi_2^{+*} + \Pi_2^{-*} - 2\Pi_2^0)d^i_{1,x}},$$

that is if $d^i_{1,x}$ and $d^i_{1,y}$ are close enough to one. This is intuitive, as when $d^i_{1,x}$ and $d^i_{1,y}$ are both close to 1, the comparison between (27) and (28) approximately reduces to $2\pi_2^{+*}$ vs Π_2^{+*}, and $s > \overline{s}^1$ implies $2\pi_2^{+*} < \Pi_2^{+*}$. Likewise, firm h prefers incompatibility if $d^h_{1,x}$ and $d^h_{1,y}$ are close to 1 (i.e., when $d^i_{1,x}$ and $d^i_{1,y}$ are close to zero). For instance, when $s = 1.1$, Figure 3 shows that firm i prefers incompatibility if $(d^i_{1,x}, d^i_{1,y})$ is above the thin curve; firm h prefers incompatibility if $(d^i_{1,x}, d^i_{1,y})$ is below the thick curve.

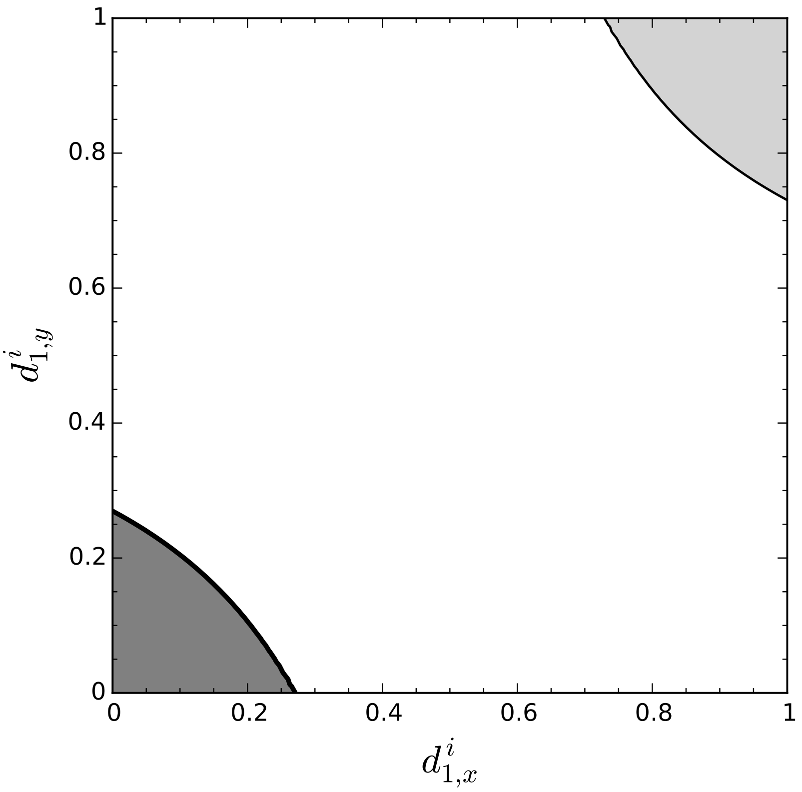

Figure 3: Second-period compatibility choice when compatibility was chosen in period one for $s = 1.1$

As s increases, the set of $(d^i_{1,x}, d^i_{1,y})$ such that at least one firm prefers incompatibility becomes wider, but for no s it includes the point such that $d^i_{1,x} = d^i_{1,y} = \frac{1}{2}$, as the following corollary states.

Corollary 5. *Suppose that the firms chose compatibility in the first period (and let $\Delta v = 1$ w.l.o.g.). In the case of symmetric first-period market shares, that is $d^A_{1,x} = d^B_{1,x} = d^A_{1,y} = d^B_{1,y} = \frac{1}{2}$, both firms prefer compatibility to incompatibility for any $s \in (0, 3/2)$.*

Given symmetric first-period market shares, from comparing (27) with (28) we find, for any $s \geq 0$

$$\pi_2^{+*} + \pi_2^{-*} > \frac{1}{4}(\Pi_2^{+*} + \Pi_2^{-*}) + \frac{1}{2}\Pi_2^{0*},$$

where the L.H.S. (the R.H.S) is each firm's second-period profit under compatibility (under incompatibility). As we wrote after Lemma 5, competition in market (i, h) under incompatibility is more intensive than competition in (i, i) under incompatibility or under compatibility. More precisely, $\Pi_2^{0*} < \min\left\{\pi_2^{+*} + \pi_2^{-*}, \frac{1}{2}(\Pi_2^{+*} + \Pi_2^{-*})\right\}$ for any $s > 0$. This force induces the firms to embrace compatibility no matter what the level of switching cost even if $\pi_2^{+*} + \pi_2^{-*} < \left(\Pi_2^{+*} + \Pi_2^{-*}\right)/2$ holds for $s > \overline{s}^2$.

Case 2: when incompatibility was chosen in period one.

We now consider the case in which firms have chosen incompatibility period one, thus no consumer buys a hybrid system in period one. Let D_1^i denote i's first-period market share.

Suppose first that incompatibility prevails in period two. Competition in each market (i, i) is described by Lemma 2. Hence firm i's second-period profit is

$$D_1^i \Pi_2^{+*} + D_1^h \Pi_2^{-*}. \tag{31}$$

Suppose now that the first-period incompatibility is followed by the second-period compatibility. Then the firms' second-period profits in each market i_j do not depend on the first-period compatibility regime. Therefore, Lemma 1 applies and firm i's second-period profit is

$$2\left(D_1^i \pi_2^{+*} + D_1^h \pi_2^{-*}\right). \tag{32}$$

In order to compare (31) and (32), we use $D_1^h = 1 - D_1^i$ and see that firm i prefers second-period compatibility if and only if

$$D_1^i 2\pi_2^{+*} + (1 - D_1^i)2\pi_2^{-*} > D_1^i \Pi_2^{+*} + (1 - D_1^i)\Pi_2^{-*},$$

which is equivalent to $D_1^i < \frac{2\pi_2^{-*} - \Pi_2^{-*}}{\Pi_2^{+*} - \Pi_2^{-*} - (2\pi_2^{+*} - 2\pi_2^{-*})} \equiv \overline{D}$.[38] Likewise, firm h with $h \neq i$ prefers

[38] By (5), the denominator denominator of $\bar{D}$ is positive for each $s \in (0, \frac{3}{2})$.

second-period compatibility if and only if

$$D_1^i 2\pi_2^{-*} + (1 - D_1^i)2\pi_2^{+*} > D_1^i \Pi_2^{-*} + (1 - D_1^i)\Pi_2^{+*},$$

which is equivalent to $D_1^i > \frac{\Pi_2^{+*} - 2\pi_2^{+*}}{\Pi_2^{+*} - \Pi_2^{-*} - (2\pi_2^{+*} - 2\pi_2^{-*})} \equiv \underline{D}$. Then the following lemma is immediate.

Lemma 7. *Suppose that the firms chose incompatibility in the first period (and let $\Delta v = 1$ w.l.o.g.). Then compatibility choices in the second period are as follows:*

(i) If $\Pi_2^{+*} < 2\pi_2^{+*}$ (that is, if $s < \bar{s}^1$), then $\overline{D} > 1$ and $\underline{D} < 0$. Hence compatibility emerges in the second period for any $D_1^i \in [0, 1]$.

(ii) If $\Pi_2^{+*} + \Pi_2^{-*} \geq 2\pi_2^{+*} + 2\pi_2^{-*}$ (that is, if $s \geq \bar{s}^2$), then $\overline{D} \leq \underline{D}$. Hence incompatibility emerges in the second period for any $D_1^i \in [0, 1]$.

(iii) If $\Pi_2^{+*} \geq 2\pi_2^{+*}$ and $\Pi_2^{+*} + \Pi_2^{-*} < 2\pi_2^{+*} + 2\pi_2^{-*}$ (that is, if $\bar{s}^1 \leq s < \bar{s}^2$), then $0 \leq \underline{D} < \frac{1}{2} < \overline{D} \leq 1$. Hence compatibility (incompatibility) emerges in the second period if $\underline{D} < D_1^i < \overline{D}$ (if $D_1^i \leq \underline{D}$, or $D_1^i \geq \overline{D}$).

Proof. It is omitted as it is straightforward.■

We know from Corollary 4 that if the dominant firm prefers compatibility (i.e., if $\Pi_2^{+*} < 2\pi_2^{+*}$), then also the dominated firm prefers compatibility (i.e., $\Pi_2^{-*} < 2\pi_2^{-*}$). Therefore, as long as the dominant firm prefers compatibility (i.e., if $s < \bar{s}^1$), (31) is greater than (32) for any D_1^i and both firms choose compatibility in the second period. As the switching cost becomes larger than $\bar{s}^2$, the inequality $\Pi_2^{+*} + \Pi_2^{-*} > 2\pi^{+*} + 2\pi_2^{-*}$ holds. Then incompatibility emerges since this inequality means that the sum of the second-period profits is greater under incompatibility than under compatibility. Therefore at least one firm prefers incompatibility. Lemma 6(i) and Lemma 7 show that if $s < \bar{s}^1$, the firms always end up choosing compatibility in period two regardless of the first-period compatibility regime and market shares. But such strong prediction cannot be made for s between $\bar{s}^1$ and $\bar{s}^2$.

Proof of Proposition 10

(i) Given $s < \bar{s}^1$, we will have the second period compatibility regardless of the first period market shares, by Lemma 7(i). As a consequence, given a pair of the first-period prices under incompatibility $\left(P_1^i, P_1^h\right)$, we consider the following profit functions:

$$\Pi^i = D_1^i(P_1^i + \delta 2\pi_2^{+*}) + (1 - D_1^i)(\delta 2\pi_2^{-*}), \qquad \Pi^h = (1 - D_1^i)(P_1^h + \delta 2\pi_2^{+*}) + D_1^i(\delta 2\pi_2^{-*}), \quad (33)$$

where D_1^i is firm i's market share in period one and D_1^A is given by (16). Then, we can argue as in the proof of Proposition 2 to find the first period equilibrium prices and the equilibrium profit in Proposition 10(i).

(ii) For $s \in [\bar{s}^2, \frac{3}{2})$, Lemma 7 implies that for any $\left(P_1^i, P_1^h\right)$, the second-period compatibility regime will be incompatibility. Then the proof of Proposition 2 applies.

Statement and Proof of Proposition 16

Proposition 16. *Suppose that incompatibility was chosen in the first period. Let $\Delta v = 1$ without loss of generality. Suppose that $\Pi_2^{+*} \geq 2\pi_2^{+*}$ and $2\pi_2^{+*} + 2\pi_2^{-*} > \Pi_2^{+*} + \Pi_2^{-*}$ (i.e., s is between $\bar{s}^1$ and $\bar{s}^2$).*
(i) Suppose the NPC does not apply. If a pure strategy equilibrium exists, then $P_1^{A*} = P_1^{B*} = t - \delta\left(2\pi_2^{+*} - 2\pi_2^{-*}\right)$, as in Proposition 10(i), and that is an equilibrium if $\delta/t \leq \bar{r}_{IC}(s) \equiv \frac{\sqrt{2(1-\bar{D})}+2\cdot(3\bar{D}-2)}{\left(\Pi_2^{+*}-\Pi_2^{-*}-(2\pi_2^{+*}-2\pi_2^{-*})\right)\sqrt{2(1-\bar{D})}}$.
(ii) Suppose the NPC must be satisfied. If a pure strategy equilibrium exists, then $P_1^{A*} = P_1^{B*} = \max\{t - \delta\left(2\pi_2^{+*} - 2\pi_2^{-*}\right), 0\}$, and that is an equilibrium if $\delta/t \leq \bar{r}_{IC}(s)$, or if $P_1^{A*} = P_1^{B*} = 0$.

Proof of (i) Suppose that $s \in (\bar{s}^1, \bar{s}^2)$. We prove three claims: (a) no equilibrium is such that incompatibility arises at stage two; (b) the only candidate for equilibrium is such that $P_1^{i*} = P_1^{h*} = t - \delta\left(2\pi_2^{+*} - 2\pi_2^{-*}\right)$; (c) no profitable deviation exists if and only if $\frac{\delta}{t} \leq \bar{r}_{IC}(s)$.

Proof of claim (a) There exists no equilibrium such that $\underline{D} < D^i < \bar{D}$ for some firm i

We first prove that in no equilibrium we have $D^i \geq \bar{D}$.

Suppose that P_1^{i*}, P_1^{h*} is an equilibrium with $D_1^i = \overline{D}$. Then the profit of firm i if $P^i < P_1^{i*}$ is Π^i in (15), and its derivative with respect to P^i needs to be non-negative at $P^i = P_1^{i*}$, that is $\frac{dD_1^i}{dP_1^i}(P_1^{i*} + \delta(\Pi_2^{+*} - \Pi_2^{-*})) + \overline{D} \geq 0$. The profit for firm i if P^i is slightly higher than P_1^{i*} is Π^i in (33) and its derivative with respect to P^i needs to be non-positive at $P^i = P_1^{i*}$, that is $\frac{dD_1^i}{dP_1^i}(P_1^{i*} + \delta(2\pi_2^{+*} - 2\pi_2^{-*})) + \overline{D} \leq 0$. However, these inequalities cannot both hold since $\frac{dD_1^i}{dP_1^i} < 0$ and $\Pi_2^{+*} - \Pi_2^{-*} > 2\pi_2^{+*} - 2\pi_2^{-*}$.

Suppose that P_1^{i*}, P_1^{h*} is an equilibrium such that $\overline{D} < D_1^i < 1$, which implies $P_1^{i*} < P_1^{h*}$. Then the profit functions are (15) in a neighborhood of (P_1^{i*}, P_1^{h*}), and from these profit functions we obtain first order conditions as in the proof of Proposition 2, which rule out $P_1^{i*} < P_1^{h*}$.

Suppose that P_1^{i*}, P_1^{h*} is an equilibrium with $D_1^i = 1$. Then the derivative of the profit function of firm i with respect to P^i is equal to one at $P^i = P^{i*}$ because $\frac{\partial D_1^i}{\partial P_1^i} = 0$. Therefore it is profitable for firm i to increase slightly P^i above P^{i*}.

Now consider the case of $D^i \leq \underline{D}$. If there exists an equilibrium such that $D^i \leq \underline{D}$, then $D^h \geq \bar{D}$ and we can rule out this case as we have ruled out above that $D^i \geq \bar{D}$.

Proof of claim (b) The only possible equilibrium is such that $P_1^{i*} = P_1^{h*} = t - \delta\left(2\pi_2^{+*} - 2\pi_2^{-*}\right)$

We know that if P^{*i}, P^{*h} is an equilibrium, then $\underline{D} < D^i < \bar{D}$. Hence the profit functions are given by (33) in a neighborhood of (P^{*i}, P^{*h}), and from these profit functions we obtain first order conditions as in the proof of Proposition (10)(i), which imply $P_1^{i*} = P_1^{h*} = t - \delta\left(2\pi_2^{+*} - 2\pi_2^{-*}\right)$.

Proof of claim (c) The prices $P_1^{i*} = P_1^{h*} = t - \delta\left(2\pi_2^{+*} - 2\pi_2^{-*}\right)$ constitute an equilibrium if and only if δ/t is sufficiently close to 0

Given $P_1^{i*} = P_1^{h*} = t - \delta\left(2\pi_2^{+*} - 2\pi_2^{-*}\right)$, it is immediate that for no firm there exists a profitable deviation which leads to compatibility in period two, hence we consider here deviations which lead to incompatibility in period 2, that is such that $D^i \leq \underline{D}$, or $D^i \geq \bar{D}$.

Deviations with large P Given that firms are symmetric, it suffices to consider the point of view of firm i at stage one, given incompatibility at stage one. Given $P_1^{h*} = P_1^* = t - \delta(2\pi_2^{+*} - 2\pi_2^{-*})$, let $\underline{P}$ be the price of firm i such that $D_1^i = \underline{D}$ (recall that for $s \in (\bar{s}^1, \bar{s}^2)$, we have $0 < \underline{D} < \frac{1}{2}$). Precisely, $\underline{P}$ belongs to $(P_1^*, P_1^* + 2t)$ and is such that $\frac{1}{2}(1 + \frac{P_1^* - \underline{P}}{2t})^2 = \underline{D}$, that is $\underline{P} = P_1^* + 2t - 2t\sqrt{2\underline{D}}$. Since $P_1^i = \underline{P}$ induces incompatibility in period two, the profit of firm i from playing $P_1^i = \underline{P}$ is $\underline{D}\left(t - \delta(2\pi_2^{+*} - 2\pi_2^{-*}) + 2t - 2t\sqrt{2\underline{D}} + \delta\Pi_2^{+*}\right) + (1 - \underline{D})\delta\Pi_2^{-*}$, that is

$$(3 - 2\sqrt{2\underline{D}})\underline{D}t + \delta\left(\Pi_2^{+*} - 2\pi_2^{+*} + \Pi_2^{-*}\right) \tag{34}$$

after using $\underline{D} = \frac{\Pi_2^{+*} - 2\pi_2^{+*}}{\Pi_2^{+*} - \Pi_2^{-*} - (2\pi_2^{+*} - 2\pi_2^{-*})}$. The difference between the equilibrium profit $\frac{t}{2} + \delta 2\pi_2^{-*}$ and (34) is

$$\left(\frac{1}{2} - (3 - 2\sqrt{2\underline{D}})\underline{D}\right)t + \delta(2\pi_2^{+*} + 2\pi_2^{-*} - \Pi_2^{+*} - \Pi_2^{-*})$$

which is positive for each $\underline{D} \in (0, \frac{1}{2})$ since $\frac{1}{2} - (3 - 2\sqrt{2\underline{D}})\underline{D} = (1 + 2\sqrt{2\underline{D}})(\frac{1}{2}\sqrt{2} - \sqrt{\underline{D}})^2 > 0$ and $2\pi_2^{+*} + 2\pi_2^{-*} - \Pi_2^{+*} - \Pi_2^{-*} > 0$ as $s < \bar{s}^2$.

In fact, firm i can achieve incompatibility in the second period by choosing any $P^i \in (\underline{P}, P_i^* + 2t]$, and in such a case the profit of firm i is $D_1^i[P^i + \delta(\Pi_2^{+*} - \Pi_2^{-*})] + \delta\Pi_2^{-*}$, with derivative

$$\frac{dD_1^i}{dP^i}(P^i + \delta(\Pi_2^{+*} - \Pi_2^{-*})) + D_1^i \tag{35}$$

Now consider the profit under compatibility in the second period, which is $D_1^i[P^i + \delta(2\pi_2^{+*} - 2\pi_2^{-*})] + \delta(2\pi_2^{-*})$, and its derivative with respect to P^i, $\frac{dD_1^i}{dP^i}[P^i + \delta(2\pi_2^{+*} - 2\pi_2^{-*})] + D_1^i$, which is larger than (35) because of (5). Since $\frac{dD_1^i}{dP^i}[P^i + \delta(2\pi_2^{+*} - 2\pi_2^{-*})] + D_1^i < 0$ for each $P^i > P_1^*$,[39] in particular for $P^i > \underline{P}$, it follows that (35) is negative.

[39] This follows from the log-concavity of D_1^i: see footnote 34.

Deviations with small P Given $P_1^{h*}=P_1^*=t-\delta(2\pi_2^{+*}-2\pi_2^{-*})$, let $\bar{P}$ be the price of firm i such that $D_1^i=\bar{D}$ (recall that for $s\in(\bar{s}^1,\bar{s}^2)$, we have $\frac{1}{2}<\bar{D}<1$). Precisely, $\bar{P}$ belongs to (P^*-2t,P^*) and is such that $1-\frac{1}{2}(1-\frac{P_1^*-\bar{P}}{2t})^2=\bar{D}$, hence $\bar{P}=P_1^*-2t+2t\sqrt{2(1-\bar{D})}$. Since $P_1^i=\bar{P}$ induces incompatibility in period two, the profit of firm i from playing $P_1^i=\bar{P}$ is $\bar{D}\left(t-\delta(2\pi_2^{+*}-2\pi_2^{-*})-2t+2t\sqrt{2(1-\bar{D})}+\delta\Pi_2^{+*}\right)+(1-\bar{D})\delta\Pi_2^{-*}$, that is

$$\left(2\sqrt{2(1-\bar{D})}-1\right)\bar{D}t+\delta 2\pi_2^{-*} \tag{36}$$

after using $\bar{D}=\frac{2\pi_2^{-*}-\Pi_2^{-*}}{\Pi_2^{+*}-\Pi_2^{-*}-(2\pi_2^{+*}-2\pi_2^{-*})}$. The difference between the equilibrium profit $\frac{t}{2}+\delta 2\pi_2^{-*}$ and (36) is

$$\left(\frac{1}{2}-2\bar{D}\sqrt{2(1-\bar{D})}+\bar{D}\right)t=\frac{\left(2\bar{D}-1\right)^2\left(8\bar{D}+1\right)}{2(1+2\bar{D}+4\bar{D}\sqrt{2(1-\bar{D})})}t$$

which is positive for each $\bar{D}>\frac{1}{2}$.

In fact, i can achieve incompatibility in the second period by choosing any $P^i\in[P^*-2t,\bar{P})$, and in such a case the profit of firm i is $D_1^i(P^i+\delta(\Pi_2^{+*}-\Pi_2^{-*})]+\delta\Pi_2^{-*}$, with derivative (35) which is equal to

$$\frac{1}{4t^2}\left(-\frac{3}{2}(P^i)^2-(2t+\Delta\Pi\delta+4\Delta\pi\delta)P^i-\frac{4(\Delta\pi)^2\delta^2-7t^2+2\Delta\Pi t\delta+4\Delta\pi t\delta+4\Delta\Pi\Delta\pi\delta^2}{2}\right) \tag{37}$$

with $\Delta\Pi=\Pi_2^{+*}-\Pi_2^{-*}$, $\Delta\pi=2\pi_2^{+*}-2\pi_2^{-*}$. Notice that if (37) is positive or zero at $P^i=\bar{P}$, then we can conclude that the profit from the deviation is increasing with respect to P^i for $P^i\in[P^*-2t,\bar{P})$, hence no profitable deviation exists, since we have proved that deviating with $P^i=\bar{P}$ is unprofitable. Precisely, at $P^i=\bar{P}$ we find that (37) is equal to $-2+3\bar{D}+\frac{t+2\Delta\pi\delta-\Delta\Pi\delta}{2t}\sqrt{2(1-\bar{D})}$, which is non-negative since $\delta/t\leq\bar{r}_{IC}(s)$.

Proof of (ii) Suppose that the NPC must hold. Then $P_1^{A*}=P_1^{B*}=t-\delta\left(2\pi_2^{+*}-2\pi_2^{-*}\right)$ if and only if $\frac{\delta}{t}<\frac{1}{2\pi_2^{+*}-2\pi_2^{-*}}$. In case that $\frac{\delta}{t}\leq r_{IC}(s)$, the arguments in the proof of part (i) apply to establish that no profitable deviation exists. The inequality $\frac{1}{2\pi_2^{+*}-2\pi_2^{-*}}\leq r_{IC}(s)$ holds for each $s\in(\bar{s}^1,0.819]$, hence a profitable deviation may exist only if $s\in(0.819,\bar{s}^2)$ and $r_{IC}(s)<\frac{\delta}{t}<\frac{1}{2\pi_2^{+*}-2\pi_2^{-*}}$. For $s\in(0.819,\bar{s}^2)$ we have that $\frac{1}{2\pi_2^{+*}-2\pi_2^{-*}}<1$ as mentioned at the end of Section 11.2.1.

If $\frac{\delta}{t}\geq\frac{1}{2\pi_2^{+*}-2\pi_2^{-*}}$, then $P_1^{A*}=P_1^{B*}=0$ and each firm's total profit is $\delta(\pi_2^{+*}+\pi_2^{-*})$. Firm i cannot deviate by lowering P^i below 0. However, it can deviate by increasing P^i in such a way that incompatibility emerges in period two. In case that $D^i=\underline{D}$, then $P^i=\underline{P}=2t-2t\sqrt{2\underline{D}}$ and firm i's profit is $\underline{D}(2t-2t\sqrt{2\underline{D}}+\delta\Pi_2^{+*})+(1-\underline{D})\delta\Pi_2^{-*}$ and $\delta(\pi_2^{+*}+\pi_2^{-*})-\underline{D}(2t-2t\sqrt{2\underline{D}}+$

$\delta\Pi_2^{+*}) - (1-\underline{D})\delta\Pi_2^{-*} = \frac{(\Pi_2^{+*}-\Pi_2^{-*}-\pi_2^{+*}+\pi_2^{-*})(2\pi_2^{+*}+2\pi_2^{-*}-\Pi_2^{+*}-\Pi_2^{-*})}{\Pi_2^{+*}-\Pi_2^{-*}-2\pi_2^{+*}+2\pi_2^{-*}}\delta - \frac{\Pi_2^{+*}-2\pi_2^{+*}}{\Pi_2^{+*}-\Pi_2^{-*}-2\pi_2^{+*}+2\pi_2^{-*}}(2t - 2t\sqrt{2\underline{D}})$, which is positive since $\frac{\delta}{t} \geq \frac{1}{2\pi_2^{+*}-2\pi_2^{-*}}$. In fact, firm i can achieve incompatibility in the second period by choosing any $P^i \in (\underline{P}, 2t]$, and in such a case the profit of firm i is $D_1^i[P^i + \delta(\Pi_2^{+*} - \Pi_2^{-*})] + \delta\Pi_2^{-*}$, with derivative $\frac{1}{2}(1-\frac{1}{2t}P^i)\left(1-\frac{3}{2t}P^i - \frac{\delta}{t}(\Pi_2^{+*}-\Pi_2^{-*})\right)$, which is negative since $\frac{\delta}{t} \geq \frac{1}{2\pi_2^{+*}-2\pi_2^{-*}}$.

Proof of Proposition 11

(ii) Consider first the case in which the NPC does not apply. Consider an equilibrium such that for each firm i, the profit in (27) is strictly greater than the profit in (28) at the equilibrium values for $d_{1,x}^i, d_{1,y}^i$. Then compatibility arises also in period two and the profit functions are given by (9) in a neighborhood of first period equilibrium prices. Then we can argue as in the proof of Proposition 1 and find first period equilibrium prices and profits p_1^*, π^* in (11)-(12). Moreover, thanks to Corollary 5 we know that given $d_{1,x}^i = d_{1,y}^i = \frac{1}{2}$, both firm choose compatibility in the second period.

Given the equilibrium candidate we mentioned above, now we show that no profitable deviation exists if and only if $\Pi_2^{+*} - 2\pi_2^{+*} \leq \frac{t}{\delta}$.

(a) First, Proposition 1 implies that no profitable deviation exists which induces compatibility in period two. Therefore, no profitable deviation exists if $\Pi_2^{+*} \leq 2\pi_2^{+*}$ (that is, if $s \leq \bar{s}^1$) because then Lemma 4(i) reveals that compatibility necessarily emerges in the second period.
(b) Now consider the case in which $\Pi_2^{+*} > 2\pi_2^{+*}$. Then, by Lemma 4(ii) there exist deviations of firm i which lead to incompatibility in the second period such that $d_{1,x}^i, d_{1,y}^i$ are both close to 1, or both close to 0. We show that no profitable deviation exists for firm i if and only if $\Pi_2^{+*} - 2\pi_2^{+*} \leq \frac{t}{\delta}$. Given incompatibility in period two, the total profit of firm i is

$$\begin{aligned} \Pi^i &= d_{1,x}^i p_{1,x}^i + d_{1,y}^i p_{1,y}^i \\ &+ \delta\left[d_{1,x}^i d_{1,y}^i \Pi_2^{+*} + d_{1,x}^i d_{1,y}^h \Pi_2^{0*} + d_{1,y}^i d_{1,x}^h \Pi_2^{0*} + d_{1,x}^h d_{1,y}^h \Pi_2^{-*}\right] \\ &= d_{1,x}^i p_{1,x}^i + d_{1,y}^i p_{1,y}^i + \delta B d_{1,x}^i d_{1,y}^i + \delta(\Pi_2^{0*} - \Pi_2^{-*})(d_{1,x}^i + d_{1,y}^i) + \delta\Pi_2^{-*} \qquad (38) \end{aligned}$$

where $d_{1,j}^i = \frac{1}{2} + \frac{1}{2t}(p_1^* - p_{1,j}^i)$ for $j = x, y$, and $B = \Pi_2^{+*} + \Pi_2^{-*} - \frac{1}{2} > 0$. We now prove that to maximize (38) with respect to $p_{1,x}^i, p_{1,y}^i$, we can restrict to considering $p_{1,x}^i = p_{1,y}^i$:

Lemma 8. *Suppose that the firms chose compatibility in the first period and that a deviating firm expects that the incompatibility prevails in the second period. Then, without loss of generality, we can restrict attention to deviations in the first period such that $p_{1,x}^i = p_{1,y}^i$.*

Proof of the lemma. Suppose that firm i deviates with $p_{1,x}^i \neq p_{1,y}^i$. Then consider $\hat{p}_{1,x}^i, \hat{p}_{1,y}^i$ such that $\hat{p}_{1,x}^i = \hat{p}_{1,y}^i = \frac{1}{2}(p_{1,x}^i + p_{1,y}^i) \equiv \hat{p}_1^i$. Then the demand for the product of firm i is

$\hat{d}_1^i = \frac{1}{2}(d_{1,x}^i + d_{1,y}^i)$ in each market and i's profit is[40]

$$2\hat{d}_1^i\hat{p}_1^i + \delta B(\hat{d}_1^i)^2 + \delta\left[\Pi_2^{0*} - \Pi_2^{-*}\right]2\hat{d}_1^i + \delta\Pi_2^{-*}$$

which is larger than (38) because (i) $2\hat{d}_1^i\hat{p}_1^i = 2\frac{1}{2}(d_{1,x}^i + d_{1,y}^i)\frac{1}{2}(p_{1,x}^i + p_{1,y}^i) = d_{1,x}^i p_{1,x}^i + d_{1,y}^i p_{1,y}^i + \frac{1}{2}(p_{1,x}^i - p_{1,y}^i)(d_{1,y}^i - d_{1,x}^i) = d_{1,x}^i p_{1,x}^i + d_{1,y}^i p_{1,y}^i + \frac{1}{4t}(p_{1,x}^i - p_{1,y}^i)^2 > d_{1,x}^i p_{1,x}^i + d_{1,y}^i p_{1,y}^i$; (ii) $\delta B(\hat{d}_1^i)^2 > \delta B d_{1,x}^i d_{1,y}^i$ because $\delta B > 0$ and $(\hat{d}_1^i)^2 > d_{1,x}^i d_{1,y}^i$, given that $\hat{d}_1^i = \frac{1}{2}(d_{1,x}^i + d_{1,y}^i)$; (iii) $\delta\left[\Pi_2^{0*} - \Pi_2^{-*}\right]2\hat{d}_1^i = \delta\left[\Pi_2^{0*} - \Pi_2^{-*}\right](d_{1,x}^i + d_{1,y}^i)$. ■

Given a symmetric deviation such that $p_{1,x}^i = p_{1,y}^i \equiv p$, we have $d_{1,x}^i = d_{1,y}^i = 1 - \frac{\delta}{2t}\left(\pi_2^{+*} - \pi_2^{-*}\right) - \frac{1}{2t}p \equiv d$ and

$$\begin{aligned}\Pi^i &= 2dp + \delta d^2 B + 2\delta d(\Pi_2^{0*} - \Pi_2^{-*}) + \delta\Pi_2^{-*} \\ &= (B\delta - 4t)d^2 + (4t - 2A\delta)d + \delta\Pi_2^{-*},\end{aligned} \tag{39}$$

where $A = -\Pi_2^{0*} + \Pi_2^{-*} + \pi_2^{+*} - \pi_2^{-*}$. Then we need to consider a few cases, as a function of A and B.

Suppose that $1 - \frac{\delta}{4t}B \leq 0$, which implies that Π^i is convex in d. Then Π^i is maximized at $d = 0$ or at $d = 1$. Precisely, the profit at $d = 0$ is $\delta\Pi_2^{-*}$, which is smaller than the profit at $d = 1$, equal to $\delta\left(\Pi_2^{+*} - 2\pi_2^{+*} + 2\pi_2^{-*}\right)$. The latter is not larger than the candidate equilibrium profit $t + 2\delta\pi_2^{-*}$ if and only if $\Pi_2^{+*} - 2\pi_2^{+*} \leq \frac{t}{\delta}$.

Now suppose that $1 - \frac{\delta}{4t}B > 0$, which implies that Π^i is concave in d Then the maximum point for Π^i is either $d = 0$, or $d = 1$, or $d = \frac{\frac{1}{2} - \frac{\delta}{4t}A}{1 - \frac{\delta}{4t}B}$ if $0 < \frac{1}{2} - \frac{\delta}{4t}A < 1 - \frac{\delta}{4t}B$.[41] We have already dealt with the cases of $d = 0$ and $d = 1$. When $0 < \frac{1}{2} - \frac{\delta}{4t}A < 1 - \frac{\delta}{4t}B$, the deviation profit is:

$$\Pi^i = 4t\frac{\left(\frac{1}{2} - \frac{\delta}{4t}A\right)^2}{1 - \frac{\delta}{4t}B} + \delta\Pi_2^{-*}.$$

[40] Notice that if incompatibility emerges in the second period given $d_{1,x}^i, d_{1,y}^i$, then it also emerges given the market shares $\hat{d}_1^i, \hat{d}_1^i$ in each market, given the shape of the set described in Lemma 4(ii), and immediately after the lemma.

[41] Notice that $d = \frac{\frac{1}{2} - \frac{\delta}{4t}A}{1 - \frac{\delta}{4t}B}$ is the point where the derivative of Π^i in (39) vanishes. In fact, we should verify whether $d = \frac{\frac{1}{2} - \frac{\delta}{4t}A}{1 - \frac{\delta}{4t}B}$ induces incompatibility in period two, but we prove below that Π^i in (39) at $d = \frac{\frac{1}{2} - \frac{\delta}{4t}A}{1 - \frac{\delta}{4t}B}$ is smaller than the candidate equilibrium profit.

The difference between the candidate equilibrium profit $t + 2\delta\pi_2^{-*}$ and the deviation profit is

$$
\begin{aligned}
t + 2\delta\pi_2^{-*} - \Pi^i &= \frac{1}{1-\frac{\delta}{4t}B}\left[t(1-\frac{\delta}{4t}B) + \delta(1-\frac{\delta}{4t}B)C - 4t\left[\frac{1}{2} - \frac{\delta}{4t}A\right]^2\right] \\
&= \frac{\delta}{1-\frac{\delta}{4t}B}\left[A - \frac{B}{4} + C - \frac{\delta}{4t}(A^2 + BC)\right],
\end{aligned}
$$

where $C = 2\pi_2^{-*} - \Pi_2^{-*}$. Since $s > \bar{s}^1$, the following inequalities hold: $\frac{1}{2} - \frac{\delta}{4t}A \leq 1 - \frac{\delta}{4t}B$, $0 < B - A$, and they imply $\frac{\delta}{4t} \leq \frac{1}{2(B-A)}$. Since $BC + A^2 > 0$, we find that

$$
A - \frac{B}{4} + C - \frac{\delta}{4t}(A^2 + BC) \geq A - \frac{B}{4} + C - \frac{A^2 + BC}{2(B-A)} \tag{40}
$$

Since the right hand side in (40) is positive for each $s \in (0, \frac{3}{2})$, it follows that $t + 2\delta\pi_2^{-*} - \Pi^i > 0$.

(i) Now we consider the case in which the NPC must be satisfied. Suppose first that $\frac{\delta}{t} < \frac{1}{\pi_2^{+*} - \pi_2^{-*}}$, so that $p_1^* > 0$. In this case, when we consider a deviation of firm i, the term $B\delta - 4t$ in (39) is negative, hence the profit in (39) is concave but $\frac{\delta}{t} < \frac{1}{\pi_2^{+*} - \pi_2^{-*}}$ implies that $d = 0$, or $d = 1$ (in fact, $d = 1$ is actually infeasible as firm i cannot reduce its price below zero to increase its market share) or $d = \frac{\frac{1}{2} - \frac{\delta}{4t}A}{1 - \frac{\delta}{4t}B}$ are all non-profitable deviations. Now suppose that $\frac{\delta}{t} \geq \frac{1}{\pi_2^{+*} - \pi_2^{-*}}$, so that $p_1^* = 0$ and each firm's profit is $\delta(\pi_2^{+*} + \pi_2^{-*})$. Firm i can only deviate by setting $p^i > 0$, which implies that the demand for firm i is smaller than $\frac{1}{2}$. We only need to consider the case in which the regime for period two switches to incompatibility; in such case, the deviation profit is $(B\delta - 4t)d^2 + t\left(2 + (\frac{1}{2} - 2\Pi_2^{-*})\frac{\delta}{t}\right)d + \delta\Pi_2^{-*}$. If $\frac{\delta}{t} > \frac{4}{B}$, then the profit from the deviation is a convex function of d, hence we can restrict to the cases of $d = 0$, $d = \frac{1}{2}$ (even though we know that $d = \frac{1}{2}$ does not lead to period two incompatibility). In case of $d = 0$, the profit is $\delta\Pi_2^{-*}$, which is smaller than $\delta(\pi_2^{+*} + \pi_2^{-*})$. In case of $d = \frac{1}{2}$, the deviation profit is $\left(\frac{1}{16}\Pi_2^{+*} + \frac{1}{16}\Pi_2^{-*} + \frac{7}{32}\right)\delta + \frac{3}{4}t$, and $\delta(\pi_2^{+*} + \pi_2^{-*}) - \left(\frac{1}{16}\Pi_2^{+*} + \frac{1}{16}\Pi_2^{-*} + \frac{7}{32}\right)\delta - \frac{3}{4}t = t\left(\left(\pi_2^{+*} + \pi_2^{-*} - \frac{1}{16}\Pi_2^{+*} + \frac{1}{16}\Pi_2^{-*} - \frac{7}{32}\right)\frac{\delta}{t} - \frac{3}{4}\right)$ is positive since $\frac{\delta}{t} > \frac{4}{B}$. The final possibility is such that $\frac{1}{\pi_2^{+*} - \pi_2^{-*}} \leq \frac{\delta}{t} \leq \frac{4}{B}$. Then the deviation profit is concave, with maximum point at $d = \frac{(4 + \frac{\delta}{t} - 4\Pi_2^{-*}\frac{\delta}{t})}{(16 + 2\frac{\delta}{t} - 4\Pi_2^{-*}\frac{\delta}{t} - 4\Pi_2^{-*}\frac{\delta}{t})}$ which is larger than the maximal d that leads to incompatibility in period two, given below, from (30)

$$
\frac{1}{Y}\left(\begin{array}{c} 4\Pi_2^{+*} - 4\pi_2^{+*} + 4\pi_2^{-*} - 1 \\ -\sqrt{-8\pi_2^{+*} - 8\pi_2^{-*} - 16\Pi_2^{+*}\Pi_2^{-*} + 32\Pi_2^{+*}\pi_2^{-*} + 32\Pi_2^{-*}\pi_2^{+*} - 32\pi_2^{+*}\pi_2^{-*} + 16(\pi_2^{+*})^2 + 16(\pi_2^{-*})^2 + 1} \end{array}\right)
$$

where $Y = 4\Pi_2^{+*} + 4\Pi_2^{-*} - 2$. At this d, the deviation profit is smaller than $\delta(\pi_2^{+*} + \pi_2^{-*})$ since $\frac{1}{\pi_2^{+*} - \pi_2^{-*}} \leq \frac{\delta}{t}$, thus the same inequality holds for each lower d.

Proof of Proposition 12

We are considering the equilibrium candidate in which a firm i corners the other firm, h, in the first period in both markets. Precisely, firm i plays a price $\bar{p}$ in each market and firm h plays price $\bar{p}+t$ in each market, with $\bar{p}$ to be determined. As a consequence, the profit of firm i is $2\bar{p}+\delta\Pi_2^{+*}$, the profit of firm h is $\delta\Pi_2^{-*}$.

Step 1 No deviation which induces incompatibility is profitable for any firm if and only if

$$\bar{p} = \eta t - \frac{\delta}{2}(\Pi_2^{+*} - \Pi_2^{-*}) \qquad \text{for} \quad \text{some} \quad \eta \in [0,1] \tag{41}$$

Consider a deviation of firm i such that it plays price $p^i \in (\bar{p}, \bar{p}+2t]$ in both markets; this implies that the quantity sold by firm i in each market is $d^i = \frac{1}{2}+\frac{1}{2t}(\bar{p}+t-p^i)$. Second-period incompatibility arises if the market share of firm i is either close to 1 in both markets, or close to zero in both markets. Then the profit of firm i is $f^i(d^i) = 2d^ip^i+\delta\left(\Pi_2^{+*}(d^i)^2 + \frac{1}{2}d^i(1-d^i) + \Pi_2^{-*}(1-d^i)^2\right)$, with $B = \Pi_2^{+*}+\Pi_2^{-*}-\frac{1}{2}$ and $C = \frac{1}{4}-\Pi_2^{-*}$. Using $p^i = 2t+\bar{p}-2d^it$ we obtain $f^i(d^i) = (B\delta-4t)(d^i)^2 + (4t+2\bar{p}+2C\delta)\,d^i + \delta\Pi_2^{-*}$. In equilibrium, firm i is supposed to play $p^i = \bar{p}$, such that $d^i = 1$ and $f^i(1) = 2\bar{p}+\delta\Pi_2^{+*}$. We are considering $\frac{\delta}{t} > \frac{4}{B}$, hence f^i is convex and for firm i no deviation which induces incompatibility in the second period is profitable if and only if $f^i(1) \geq f^i(0)$, that is if and only if $2\bar{p}+\delta\Pi_2^{+*} \geq \delta\Pi_2^{-*}$, or $-\frac{\delta}{2}(\Pi_2^{+*}-\Pi_2^{-*}) \leq \bar{p}$.

Now consider firm h and a deviation such that firm h plays price $p^h \in [\bar{p}-t, \bar{p}+t)$ in both markets; this implies that the quantity sold by firm h in each market is $d^h = \frac{1}{2}+\frac{1}{2t}(\bar{p}-p^h)$, and its profit is $f^h(d^h) = 2d^hp^h+\delta\left(\Pi_2^{+*}(d^h)^2+\frac{1}{2}d^h(1-d^h)+\Pi_2^{-*}(1-d^h)^2\right)$. Using $p^h = t+\bar{p}-2d^ht$ we obtain $f^h(d^h) = (B\delta-4t)(d^h)^2+2(t+\bar{p}+C\delta)\,d^h+\delta\Pi_2^{-*}$. In equilibrium, firm h is supposed to play $p^h = \bar{p}+t$ such that $d^h = 0$ and $f^h(0) = \delta\Pi_2^{-*}$. Since f^h is convex (as $\frac{\delta}{t} > \frac{4}{B}$), it follows that for firm h no deviation which induces incompatibility in the second period is profitable if and only if $f^h(0) \geq f^h(1)$ that is if $\bar{p} \leq t - \frac{\delta}{2}(\Pi_2^{+*}-\Pi_2^{-*})$.

Step 2 A lower bound on δ/t

We need some preliminaries in order to state the result precisely. In addition to $B = \Pi_2^{+*} + \Pi_2^{-*} - \frac{1}{2}$, we define $E = \Pi_2^{+*}+\pi_2^{-*}-\frac{1}{4}-\pi_2^{+*} > 0$, $F = \Pi_2^{-*}+\pi_2^{+*}-\frac{1}{4}-\pi_2^{-*} > 0$, $\Delta^+ = \Pi_2^{+*}-2\pi_2^{+*} > 0$, $\Delta^- = \Pi_2^{-*}-2\pi_2^{-*} > 0$, $\Delta^{\pm} = \Delta^+ - \Delta^- > 0$.

Compatibility in period two arises if and only if the inequalities (29)-(30) in the proof of Lemma 4 are satisfied, and notice that they can be indifferently written as a function of d_x^i, d_y^i, or of d_x^h, d_y^h. Here we use the latter variables:

$$F(d_x^h+d_y^h) - Bd_x^hd_y^h \geq \Delta^- \qquad E(d_x^h+d_y^h) - Bd_x^hd_y^h \geq \Delta^+ \tag{42}$$

The set of d_x^h, d_y^h which satisfies (42) is represented in Figure 3 (after changing the labels of the axes to d_x^h, d_y^h). Notice that

- in case that firm h plays the same price in both markets, we have $d_x^h = d_y^h \equiv d^h$, and then both inequalities in (42) are satisfied if and only if d^h is included between a lower bound d_ℓ and an upper bound d_u, that is $d_\ell \le d^h \le d_u$ with $d_\ell = \frac{E-\sqrt{E^2-B\Delta^+}}{B}$ and $d_u = \frac{F+\sqrt{F^2-B\Delta^-}}{B}$;
- the southwest border of the feasible set is the graph of the curve

$$d_y^h = q(d_x^h), \quad \text{with} \quad q(d_x^h) = \frac{\Delta^+ - E d_x^h}{E - B d_x^h} \quad \text{for} \quad d_x^h \in [0, \frac{\Delta^+}{E}] \quad \text{and } q(d_\ell) = d_\ell \tag{43}$$

We now define several values of $r = \frac{\delta}{t}$, which we use to identify a lower bound for $\frac{\delta}{t}$:

$$\tilde{r}(s,\eta) \equiv \frac{(4+2\eta)d_\ell - 2\eta - 4d_\ell^2}{\Delta^- + \Delta^\pm d_\ell}, \qquad \bar{r}(s,\eta) \equiv \frac{2(2+\eta)\Delta^\pm + 8\Delta^- - 4\sqrt{(2\eta\Delta^\pm + 4\Delta^-)\Delta^+}}{(\Delta^\pm)^2} \tag{44}$$

$$r_i(s,\eta) = \max\{\tilde{r}(s,\eta), \bar{r}(s,\eta)\} \qquad r_h(s,\eta) \equiv \frac{2d_\ell + 2\eta d_\ell - 4d_\ell^2}{\Delta^- + \Delta^\pm d_\ell} \tag{45}$$

Finally, for each $s \in (\bar{s}^3, \frac{3}{2})$, let $\eta_m(s)$ denote the unique $\eta \in (0,1)$ such that $r_i(s,\eta) = r_h(s,\eta)$ (existence and uniqueness of $\eta_m(s)$ are proved in Step 2.1.3 below). The function $r_{CI}(s)$ we mention in Proposition 12 is defined as $r_{CI}(s) = r_h(s, \eta_m(s))$. Now we can state Proposition 12 in detail:

Proposition 12: Suppose that compatibility was chosen in the first period, and suppose that $s \in (\bar{s}^3, \frac{3}{2})$. Then a cornering equilibrium exists if and only if $\frac{\delta}{t} \ge r_{CI}(s)$. Precisely, in the first period one firm charges price $\bar{p} = \eta_m(s)t - \frac{\delta}{2}(\Pi_2^{+*} - \Pi_2^{-*})$ in both markets, the other firm charges price $\bar{p} + t$ in both markets, and the first firm corners both markets. In the second period, incompatibility is chosen at least by the firm who cornered the markets, which earns a total profit of $2\eta_m(s)t + \delta\Pi_2^{-*}$; the other firm's total profit is $\delta\Pi_2^{-*}$. For each $s \in (\bar{s}^3, \frac{3}{2})$, we have $\frac{3/2}{\Pi_2^{+*} - 2\pi_2^{+*}} < r_{CI}(s) < \frac{16/9}{\Pi_2^{+*} - 2\pi_2^{+*}}$.

In the proof we give below we consider first (in Step 2.1) deviations such that each firm plays the same price in both markets. In Step 2.2 we show that no profitable deviation exists even if the deviating firm can charge different prices in different markets.

Step 2.1: For each $s \in (\bar{s}^3, \frac{3}{2})$, for no firm there exists a profitable deviation which induces compatibility in period two and such that the firm chooses the same price in both markets if and only if $\frac{\delta}{t} \ge r_{CI}(s)$

Step 2.1.1: No profitable deviation exists for firm i if and only if $\frac{\delta}{t} \geq r_i(s,\eta)$; moreover, $r_i(s,\eta) = \bar{r}(s,\eta)$ if $\eta > 0.059234$.

Given the price p^i played by firm i in both markets, the demand for product i_j is $d^i = \frac{1}{2} + \frac{1}{2t}(\bar{p} + t - p^i)$ (for $j = x, y$), and inverting this relation we obtain $p^i = 2t + \bar{p} - 2td^i$. Hence, the profit of firm i is $g^i(d^i) = 2d^ip^i + 2\delta[d^i\pi_2^{+*} + (1-d^i)\pi_2^{-*}] = -4t(d^i)^2 + (4t + 2\eta t - \delta\Delta^{\pm})d^i + 2\delta\pi_2^{-*}$ for $d^i \in [d_\ell, d_u]$. A necessary condition for no profitable deviation to exist for firm i is $g^i(d_\ell) \leq 2\eta t + \delta\Pi_2^{-*}$ ($2\eta t + \delta\Pi_2^{-*}$ is the equilibrium profit of firm i), which is equivalent to $\frac{\delta}{t} \geq \tilde{r}(s,\eta)$. We need to distinguish two cases, depending on the value of η.

$$-4(d)^2 + (4 + 2\eta)d - 2\eta < \frac{\delta}{t}(\Delta^- + \Delta^{\pm}d)$$

- Case A: $0.059234 < \eta \leq 1$. The derivative of g^i is $g^{i\prime}(d^i) = -8td^i + 4t + 2\eta t - \delta\Delta^{\pm}$ and $g^{i\prime}(d_\ell) \leq 0$ if and only if $\frac{\delta}{t} \geq \frac{4+2\eta-8d_\ell}{\Delta^{\pm}} \equiv \hat{r}(s,\eta)$. It turns out that $\tilde{r}(s,\eta) < \hat{r}(s,\eta)$ for each $s \in (\bar{s}^3, \frac{3}{2})$ since $\eta > 0.059234$.[42] Therefore, no profitable deviation exists for firm i when $\frac{\delta}{t} \geq \hat{r}(s,\eta)$.

- Now consider $\frac{\delta}{t}$ between $\tilde{r}(s,\eta)$ and $\hat{r}(s,\eta)$. Then the profit maximizing d^i is $\frac{4+2\eta-\frac{\delta}{t}\Delta^{\pm}}{8}$, which is larger than d_ℓ but smaller than d_u.[43] We have that $g^i(\frac{4+2\eta-\frac{\delta}{t}\Delta^{\pm}}{8}) = \frac{1}{16t}(4t + 2\eta t - \delta\Delta^{\pm})^2 + 2\delta\pi_2^{-*}$ is smaller than $2\eta t + \delta\Pi_2^{-*}$ if and only if

$$-(\Delta^{\pm})^2(\frac{\delta}{t})^2 + 4\left((2+\eta)\Delta^{\pm} + 4\Delta^-\right)\frac{\delta}{t} - 4(2-\eta)^2 \geq 0 \qquad (46)$$

  At $\frac{\delta}{t} = \tilde{r}(s,\eta)$, the left hand side of (46) is negative since $g^i(d_\ell) = 2\eta t + t\tilde{r}(s,\eta)\Pi_2^{-*}$ and $g^{i\prime}(d_\ell) > 0$. At $\frac{\delta}{t} = \hat{r}(s,\eta)$, the left hand side in (46) is positive since $\frac{4+2\eta-\hat{r}(s,\eta)\Delta^{\pm}}{8} = d_\ell$ and $g^i(d_\ell) < 2\eta t + t\hat{r}(s,\eta)\Pi_2^{-*}$. Therefore, in case A no profitable deviation exists for firm i if and only if $\frac{\delta}{t}$ is at least as large as the smallest solution to (46), which is $\bar{r}(s,\eta)$, a number between $\tilde{r}(s,\eta)$ and $\hat{r}(s,\eta)$.

- Case B: $0 \leq \eta \leq 0.059234$. In this case $\hat{r}(s,\eta) \leq \tilde{r}(s,\eta)$ for some $s \in (\bar{s}^3, \frac{3}{2})$, and then no profitable deviation exists for firm i if and only if $\frac{\delta}{t} \geq \tilde{r}(s,\eta)$ (in this case $\bar{r}(s,\eta) < \tilde{r}(s,\eta)$). If the inequality $\tilde{r}(s,\eta) < \hat{r}(s,\eta)$ holds for some $s \in (\bar{s}^3, \frac{3}{2})$, then we can argue as for case A above that no profitable deviation exists for firm i if and only if $\frac{\delta}{t} \geq \bar{r}(s,\eta)$. Hence, in case B no profitable deviation exists for firm A if and only if $\frac{\delta}{t} \geq \max\{\tilde{r}(s,\eta), \bar{r}(s,\eta)\}$.

Joining the two cases A and B, we conclude that no profitable deviation exists for firm i if and only if $\frac{\delta}{t} \geq \max\{\tilde{r}(s,\eta), \bar{r}(s,\eta)\}$, which is defined as $r_i(s,\eta)$ in (45).

[42] Precisely, $\tilde{r}(s,\eta) < \hat{r}(s,\eta)$ is equivalent to $2d_\ell^2\Delta^{\pm} \leq 2(1 - 2d_\ell)\Delta^- + \eta\Delta^+$.

[43] For each $s \in (\bar{s}^3, \frac{3}{2})$, we have that $\frac{3}{4} < d_u$ and $g^{i\prime}(\frac{3}{4}) = 2\eta t - 2t - \delta\Delta^{\pm} < 0$.

Step 2.1.2: No profitable deviation for firm h exists if and only if $\frac{\delta}{t} \geq r_h(s,\eta)$.

Given the price p^h played by firm h in both markets, the demand for product h_j is $d^h = \frac{1}{2} + \frac{1}{2t}(\bar{p} - p^h)$ (for $j = x, y$), and inverting this relation we obtain $p^h = t + \bar{p} - 2td^h$. Hence the profit of firm h is $g^h(d^h) = -4t(d^h)^2 + (2t + 2\eta t - \delta\Delta^{\pm})d^h + 2\delta\pi_2^{-*}$ for $d^h \in [d_\ell, d_u]$. Notice that a necessary condition for no profitable deviation to exist for firm h is $g^h(d_\ell) \leq \delta\Pi_2^{-*}$ ($\delta\Pi_2^{-*}$ is the equilibrium profit of firm h), which is equivalent to $\frac{\delta}{t} \geq r_h(s,\eta)$. The derivative of g^h is $g^{h\prime}(d^h) = -8td^h + 2t + 2\eta t - \delta\Delta^{\pm}$, and $g^{h\prime}(d_\ell) \leq 0$ if and only if $\frac{\delta}{t} \geq \frac{2+2\eta-8d_\ell}{\Delta^{\pm}}$. It turns out that $r_h(s,\eta) > \frac{2+2\eta-8d_\ell}{\Delta^{\pm}}$ for each $s \in (\bar{s}^3, \frac{3}{2})$ and each $\eta \in [0,1]$, hence no profitable deviation for firm h exists if and only if $\frac{\delta}{t} \geq r_h(s,\eta)$.

Step 2.1.3: For each $s \in (\bar{s}^3, \frac{3}{2})$*, there exists a unique* η *which minimizes* $\max\{r_i(s,\eta), r_h(s,\eta)\}$

For given s and η, from Steps 2.1 and 2.2 it follows that no profitable deviation exists for firm i and firm h if and only if $\frac{\delta}{t} \geq \max\{r_i(s,\eta), r_h(s,\eta)\}$. Then, for a given s, we choose η to minimize $\max\{r_i(s,\eta), r_h(s,\eta)\}$. It turns out that (i) $r_i(s,0) > r_h(s,0)$ and $r_i(s,1) < r_h(s,1)$ for each $s \in (\bar{s}^3, \frac{3}{2})$; (ii) $r_i(s,\eta)$ is decreasing in η and $r_h(s,\eta)$ is increasing in η. Hence, for each $s \in (\bar{s}^3, \frac{3}{2})$ we conclude that $\max\{r_i(s,\eta), r_h(s,\eta)\}$ is minimized by the unique $\eta \in (0,1)$ such that $r_i(s,\eta) = r_h(s,\eta)$. We denote this value with $\eta_m(s)$. Suppose that for some s, $\eta_m(s)$ is such that $r_i(s,\eta_m(s))$, that is $\max\{\tilde{r}(s,\eta_m(s)), \bar{r}(s,\eta_m(s))\}$, is equal to $\tilde{r}(s,\eta_m(s))$. Then from the equation $\tilde{r}(s,\eta_m(s)) = r_h(s,\eta)$ we find that $\eta_m(s) = d_\ell$, but d_ℓ turns out to be greater than 0.17 for each $s \in (\bar{s}^3, \frac{3}{2})$. Since $0.17 > 0.059234$, from case A we know that $\tilde{r}(s,\eta_m(s)) < \bar{r}(s,\eta_m(s))$ and this contradicts that $\max\{\tilde{r}(s,\eta_m(s)), \bar{r}(s,\eta_m(s))\} = \tilde{r}(s,\eta_m(s))$. Therefore, for each s, $\eta_m(s)$ is such that $r_i(s,\eta_m(s)) = \bar{r}(s,\eta_m(s))$. From the equation $\bar{r}(s,\eta) = r_h(s,\eta)$ we find that $\eta_m(s) \in (\frac{2}{9}t, \frac{3}{11}t)$, which implies that $r_{CI}(s) = r_h(s,\eta_m(s))$ satisfies $\frac{3/2}{\Pi_2^{+*} - 2\pi_2^{+*}} < r_{CI}(s) < \frac{16/9}{\Pi_2^{+*} - 2\pi_2^{+*}}$.

Step 2.2: For each $s \in (\bar{s}^3, \frac{3}{2})$**, let** η **be equal to** $\eta_m(s)$ **and** $\frac{\delta}{t} \geq r_{CI}(s)$**. Then for no firm there exists a profitable deviation which induces compatibility in period two, even though each firm can charge different prices in different markets**

Step 2.2.1: Proof for firm i.

Let p_x^i, p_y^i denote the prices charged by firm i in period one and d_x^i, d_y^i the resulting demands for the products of firm i. Arguing as in Step 1.1 we find $p_x^i = 2t + \bar{p} - 2td_x^i$ and $p_y^i = 2t + \bar{p} - 2td_y^i$, hence the profit of firm i under compatibility is

$$G^i(d_x^i, d_y^i) = -2t(d_x^i)^2 + (2t + t\eta_m(s) - \frac{\delta}{2}\Delta^{\pm})d_x^i - 2t(d_y^i)^2 + (2t + t\eta_m(s) - \frac{\delta}{2}\Delta^{\pm})d_y^i + 2\delta\pi_2^{-*}$$

Notice that G^i is a concave function to maximize in the feasible set we denote with $\mathbf{F}$: see

Figure 3. Hence, if there exists a critical point (d_x^{i*}, d_y^{i*}) of G^i in $\mathbf{F}$ (a point where both partial derivatives of G^i vanish), then (d_x^{i*}, d_y^{i*}) is a global max point for G^i. The proof of Step 2.1 reveals that $(d_x^{i*}, d_y^{i*}) = (\frac{4+2\eta_m(s)-\frac{\delta}{t}\Delta^\pm}{8}, \frac{4+2\eta_m(s)-\frac{\delta}{t}\Delta^\pm}{8})$ is a critical point for G^i in $\mathbf{F}$ if $r_{CI}(s) < \frac{\delta}{t} < \hat{r}(s, \eta_m(s))$ (recall that $r_{CI}(s) = r_i(s, \eta_m(s)) = \bar{r}(s, \eta_m(s)))$ but since $d_x^{i*} = d_y^{i*}$, we know from Step 2.1 that it does not constitute a profitable deviation for firm i.

Conversely, for $\frac{\delta}{t} > \hat{r}(s, \eta_m(s))$ there exists no critical point for G^i in $\mathbf{F}$. This suggests to inspect the boundaries of $\mathbf{F}$, but it is immediate that the north-east boundary does not contain any maximum point, by the virtue of the remark in footnote 43. About the southwest boundary, consider any point (d_x^i, d_y^i) which belongs to this boundary for which we know that $G(d_x^i, d_y^i) < 2\eta_m(s)t + \delta\Pi_2^{-*}$ when $\frac{\delta}{t} = \hat{r}(s, \eta_m(s))$. For $\frac{\delta}{t} > \hat{r}(s, \eta_m(s))$, the same inequality holds because the derivative of the right hand side with respect to δ, Π_2^{-*}, is greater than the derivative of the left hand side with respect to δ, $2\pi_2^{-*} - \frac{1}{2}(d_x^i + d_y^i)\Delta^\pm$. Therefore, for each $\frac{\delta}{t} \geq r_{CI}(s)$ no profitable deviation exists for firm i even though firm i can charge different prices in different markets.

Step 2.2.2: Proof for firm h.

Let p_x^h, p_y^h denote the prices charged by firm h in period one and d_x^h, d_y^h the resulting demands for the products of firm h. Arguing as in Step 2.1 we find $p_x^h = t + \bar{p} - 2td_x^h$ and $p_y^h = t + \bar{p} - 2td_y^h$, hence the profit of firm h under compatibility is

$$G^h(d_x^h, d_y^h) = -2t(d_x^h)^2 + (t + \eta_m(s)t - \frac{\delta}{2}\Delta^\pm)d_x^h - 2t(d_y^h)^2 + (t + \eta_m(s)t - \frac{\delta}{2}\Delta^\pm)d_y^h + 2\delta\pi_2^{-*}$$

From the proof of Step 2.1 we know that there exists no critical point of G^h in $\mathbf{F}$, given that $\frac{\delta}{t} \geq r_{CI}(s)$. Hence each global maximum point of G^h belongs to the southwest boundary of $\mathbf{F}$,[44] which is the curve $d_y^h = q(d_x^h)$ described in (43) Notice that $q'(d_x^h) = \frac{B\cdot\Delta^+ - E^2}{(E - Bd_x^h)^2} < 0$ with $q'(d_\ell) = -1$, and $q''(d_x^h) = \frac{2B(B\cdot\Delta^+ - E^2)}{(E - Bd_x^h)^3} < 0$. Hence firm h's profit along the southwest border is given by

$$g_q(d_x^h) = G^h(d_x^h, q(d_x^h)) = -2t(d_x^h)^2 + (t + \eta_m(s)t - \frac{\delta}{2}\Delta^\pm)d_x^h - 2t(q(d_x^h))^2 + (t + \eta_m(s)t - \frac{\delta}{2}\Delta^\pm)q(d_x^h) + 2\delta\pi_2^{-*}$$

for $d_x^h \in [0, \frac{\Delta^+}{E}]$.

Now we prove that no profitable deviation exists for firm h when $\frac{\delta}{t} = r_h(s, \eta_m(s))$, because in this case g_q is maximized at $d_x^h = d_\ell$ as $g_q'(d_\ell) = 0$ and g_q is concave; thus no profitable deviation

[44] The northeast boundary cannot include any global optimum point because $\frac{\partial G^h}{\partial d_x^h} = -4td_x^h + t + t\eta_m(s) - \frac{\delta}{2}\Delta^\pm$, which is negative for each $d_x^h \geq \frac{1}{2}$. A similar property holds for $\frac{\partial G^h}{\partial d_y^h}$.

exists for firm h since $g_q(d_\ell) = \delta\Pi_2^{-*}$ is equal to the equilibrium profit. Precisely, we find that

$$g_q'(d_x^h) = -4td_x^h + t + \eta_m(s)t - \frac{\delta}{2}\Delta^{\pm} - 4tq(d_x^h)q'(d_x^h) + (t + \eta_m(s)t - \frac{\delta}{2}\Delta^{\pm})q'(d_x^h)$$

and $g_q'(d_\ell) = 0$ since $q(d_\ell) = d_\ell$ and $q'(d_\ell) = -1$. Moreover,

$$\begin{aligned} g_q''(d_x^h) &= -4t - 4t(q'(d_x^h))^2 - 4tq(d_x^h)q''(d_x^h) + (t + \eta_m(s)t - \frac{\delta}{2}\Delta^{\pm})q''(d_x^h) \\ &= -4t - 4t\frac{(B\cdot\Delta^+ - E^2)^2}{(E - Bd_x^h)^4} - 8t\frac{B(\Delta^+ - Ed_x^h)(B\cdot\Delta^+ - E^2)}{(E - Bd_x^h)^4} + (t + \eta_m(s)t - \frac{\delta}{2}\Delta^{\pm})q''(d_x^h) \end{aligned}$$

and if $\frac{\delta}{t} = r_h(s, \eta_m(s))$ we have $(t + \eta_m(s)t - \frac{\delta}{2}\Delta^{\pm})q''(d_x^h) < 0$ because $t[1 + \eta_m(s) - \frac{\delta}{2t}\Delta^{\pm}] = t\frac{(1+\eta_m(s))\Delta^- + 2\Delta^{\pm}d_\ell^2}{\Delta^- + \Delta^{\pm}d_\ell} > 0$ and $q''(d_x^h) < 0$. The other terms in $g_q''(d_x^h)$ have the same sign as $-(E - Bd_x^h)^4 - (B\Delta^+ - E^2)^2 + 2B(E^2 - B\Delta^+)(\Delta^+ - Ed_x^h)$. For any fixed s, this is a concave function of d_x^h which is maximized $d_x^h = \frac{E}{B} - \frac{1}{B}\left(\frac{E^3 - B\cdot\Delta^+\cdot E}{2}\right)^{1/3}$ and the maximum value of the function is negative. Therefore $g_q''(d_x^h) < 0$.

We have established above that $g_q(d_x^h) \leq \delta\Pi_2^{-*}$ for each $d_x^h \in [0, \frac{\Delta^+}{E}]$ when $\frac{\delta}{t} = r_h(s, \eta_m(s))$. When instead $\frac{\delta}{t} > r_h(s, \eta_m(s))$, the inequality $g_q(d_x^h) < \delta\Pi_2^{-*}$ still holds for each $d_x^h \in [0, \frac{\Delta^+}{E}]$ because the derivative of the right hand side with respect to δ, Π_2^{-*}, is greater than the derivative of the left hand side with respect to δ, which is $2\pi_2^{-*} - \frac{1}{2}\Delta^{\pm}d_x^h - \frac{1}{2}\Delta^{\pm}q(d_x^h)$.