Internet Regulation, Two-Sided Pricing, and Sponsored Data: Online Supplementary Appendix

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Optimal price cap

We focused in the paper on standard regulation methods with little informational requirement. In this supplementary section, we want to discuss the optimal price cap on the content price assuming that the regulator is uncertain about the other parameters. To make the analysis as general as possible, we keep the assumption that the content provider's cost is heterogenous.

Following the reasoning of the proof of proposition 2, faced to a price cap $\sigma \geq s_\ell$, the network will choose $b_\ell = s_\ell$, $q_h \geq q_\ell$ and $s_h = \inf\{\sigma, b_h - (b_h - b_\ell)\frac{q_\ell}{q_h}\}$. The reduced program for the network then writes as

$$\begin{aligned}\max_{q_h, q_\ell} \quad & \lambda \left[U(q_h) - \theta_h q_h + \inf\{\sigma q_h, b_h q_h - (b_h - b_\ell) q_\ell\}\theta_h\right] \\ & +(1-\lambda)\left[U(q_\ell) + (b_\ell - 1)\theta_\ell q_\ell\right]\end{aligned}$$

The choice of the price cap σ should balance the potential inefficiencies on the HB content with higher consumption of the LB content. Inefficiencies arise because the network will react to a tightening of the price cap by rebalancing its revenue between content providers and consumers and raising the consumers price. In our model with inelastic demand, rebalancing is not an issue for welfare as long as it entails only an increase in the fixed fee. Thus only consumption distortions matter.

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As shown in the proof of proposition 2, as long as the price cap is above cost, the consumption of HB content is efficient and the consumption of LB content decreases with σ. Thus an optimal price cap is below the cost. The question is whether it is strictly below or equal to the cost. We first derive the network's choice of prices (and therefore the induced consumption levels) for any price cap σ.

Lemma 1 *For $\sigma \leq 1$, let $q_\ell^+ = D\left((1-b_\ell)\theta_\ell + \frac{1-\sigma}{b_h-\sigma}\frac{\lambda}{1-\lambda}(b_h-b_\ell)\theta_h\right)$ and $q_h^{\sigma *} = D((1-\sigma)\theta_h)$. A price cap $\sigma \geq b_\ell$ leads to $\bar{s}_\ell = b_\ell,\ \bar{s}_h = \sigma$ and*

$$\begin{cases} (i) \quad \bar{q}_h = D(0) \ \text{ and } \bar{q}_\ell = q_\ell^* \ \text{ if } \frac{q_\ell^*}{D(0)} \geq \frac{b_h-\sigma}{b_h-b_\ell} \\ (ii) \quad \bar{q}_h = D(0) \ \text{ and } \bar{q}_\ell = D(0)\frac{b_h-\sigma}{b_h-b_\ell} \ \text{ if } \frac{q_\ell^+}{D(0)} \geq \frac{b_h-\sigma}{b_h-b_\ell} \geq \frac{q_\ell^*}{D(0)} \\ (iii) \quad D(0) > \bar{q}_h > q_h^{\sigma *} \ \text{ and } \bar{q}_\ell = \bar{q}_h\frac{b_h-\sigma}{b_h-b_\ell} \ \text{ if } \frac{q_\ell^{FB}}{q_h^{\sigma *}} > \frac{b_h-\sigma}{b_h-b_\ell} > \frac{q_\ell^+}{D(0)} \\ (iv) \quad \bar{q}_h = q_h^{\sigma *} \ \text{ and } \ \bar{q}_\ell = q_\ell^{FB} \ \text{ if } \frac{b_h-\sigma}{b_h-b_\ell} \geq \frac{q_\ell^{FB}}{q_h^{\sigma *}} \end{cases}$$

A price cap $\sigma < b_\ell$ induces uniform pricing $\bar{s} = \sigma$ and $\bar{q} = D((1-\sigma)\mathbb{E}(\theta))$.

Proof. We consider first the case of a price cap at $\sigma \in [b_\ell, 1]$. The constraints can be written as

$$\begin{aligned} & b_\ell \geq s_\ell \quad \text{and } \sigma \geq s_h \\ & (b_\ell - s_\ell)\, q_\ell \geq (b_\ell - s_h)\, q_h \\ & (b_h - s_h)\, q_h \geq (b_h - s_\ell)\, q_\ell \end{aligned}$$

As the network value is decreasing with the prices paid by content providers, we have

$$\begin{aligned} s_\ell &= \inf\{b_\ell, b_\ell - (b_\ell - s_h)\frac{q_h}{q_\ell}\} \\ s_h &= \inf\{\sigma, b_h - (b_h - s_\ell)\frac{q_\ell}{q_h}\} \end{aligned}$$

The reasoning of the proof of proposition 2 shows that $s_\ell = b_\ell$ and $q_\ell \leq q_h$. The program then writes as

$$\max_{q_h \geq q_\ell} \lambda\left[U(q_h) - \theta_h q_h + \inf\{\sigma q_h, b_h q_h - (b_h - b_\ell)q_\ell\}\theta_h\right] + (1-\lambda)\left[U(q_\ell) + (b_\ell - 1)\theta_\ell q_\ell\right]$$

- There is a solution $q_h = D(0)$ if $\sigma q_h > b_h q_h - (b_h - b_\ell)q_\ell$: then the price cap is not biding and $q_\ell = q_\ell^*$ which requires that

$$q_\ell^* > \frac{b_h - \sigma}{b_h - b_\ell} D(0).$$

- There is a solution $q_h = D((1-\sigma)\theta_h)$ if $\sigma q_h < b_h q_h - (b_h - b_\ell)q_\ell$: then the price cap is strictly biding but not the incentive compatibility condition and $q_\ell = q_\ell^{FB}$ which requires that

$$q_\ell^{FB} < \frac{b_h - \sigma}{b_h - b_\ell} D((1-\sigma)\theta_h).$$

Suppose now that $q_\ell^{FB} \geq \frac{b_h-\sigma}{b_h-b_\ell} D((1-\sigma)\theta_h)$ and $q_\ell^* \leq \frac{b_h-\sigma}{b_h-b_\ell} D(0)$. Then the solution verifies $\sigma q_h = b_h q_h - (b_h - b_\ell)q_\ell$ or $q_\ell = \frac{b_h-\sigma}{b_h-b_\ell} q_h$. The choice of q_h solves

$$\max_{q_h} \lambda [U(q_h) + (\sigma - 1)\theta_h q_h] + (1-\lambda)\left[U(\frac{b_h - \sigma}{b_h - b_\ell} q_h) + (b_\ell - 1)\theta_\ell \frac{b_h - \sigma}{b_h - b_\ell} q_h\right]$$

The slope is

$$\lambda [U'(q_h) + (\sigma - 1)\theta_h] + (1-\lambda)\left[U'(\frac{b_h - \sigma}{b_h - b_\ell} q_h) + (b_\ell - 1)\theta_\ell\right] \frac{b_h - \sigma}{b_h - b_\ell}$$

Notice that the slope at $q_h = D((1-\sigma)\theta_h)$ is nonnegative because $q_\ell^{FB} \geq \frac{b_h-\sigma}{b_h-b_\ell} D((1-\sigma)\theta_h)$, so $q_h \geq D((1-\sigma)\theta_h)$.

The slope at $q_h = D(0)$ is positive if

$$U'(\frac{b_h - \sigma}{b_h - b_\ell} D(0)) > (1 - b_\ell)\theta_\ell + \frac{1-\sigma}{b_h - \sigma} \frac{\lambda}{1-\lambda} (b_h - b_\ell)\theta_h.$$

which writes as $\frac{b_h-\sigma}{b_h-b_\ell} D(0) < q_\ell^+$. This does not hold if $q_\ell^{FB} = \frac{b_h-\sigma}{b_h-b_\ell} D((1-\sigma)\theta_h)$ because then $\frac{b_h-\sigma}{b_h-b_\ell} D(0) > q_\ell^{FB} > q_\ell^+$. However this holds if $q_\ell^* = \frac{b_h-\sigma}{b_h-b_\ell} D(0)$ because $q_\ell^+ > q_\ell^*$. Thus we have two regions:

- If $\frac{b_h-\sigma}{b_h-b_\ell} D(0) \geq q_\ell^+$, then

$$\lambda [U'(\bar{q}_h) + (\sigma - 1)\theta_h] + (1-\lambda)\left[U'(\frac{b_h - \sigma}{b_h - b_\ell} \bar{q}_h) + (b_\ell - 1)\theta_\ell\right] \frac{b_h - \sigma}{b_h - b_\ell} = 0.$$

- If $q_\ell^+ \geq \frac{b_h-\sigma}{b_h-b_\ell} D(0)$ then $\bar{q}_h = D(0)$.

We consider here the case of a price cap at $\sigma \leq b_\ell$. The constraints can be written as $q_h \geq q_\ell$ and $s_\ell = \inf\{\sigma, b_\ell - (b_\ell - s_h)\frac{q_h}{q_\ell}\}$; $s_h = \inf\{\sigma, b_h - (b_h - s_\ell)\frac{q_\ell}{q_h}\}$

Let us show that $s_h = s_\ell = \sigma$. Suppose that $s_h = b_h - (b_h - s_\ell)\frac{q_\ell}{q_h}$ and $s_\ell = b_\ell - (b_\ell - s_h)\frac{q_h}{q_\ell}$. It means that

$$s_h = b_h - (b_h - s_\ell)\frac{q_\ell}{q_h} = (b_h - b_\ell)\left(1 - \frac{q_\ell}{q_h}\right) + s_h,$$

which is only possible if $q_\ell = q_h$ and thus $s_h = s_\ell$.

Suppose that $s_h = \sigma$ and $s_\ell = b_\ell - (b_\ell - \sigma)\frac{q_h}{q_\ell} \leq \sigma$. It means that

$$\sigma \geq b_h - (b_h - s_\ell)\frac{q_\ell}{q_h} = (b_h - b_\ell)\left(1 - \frac{q_\ell}{q_h}\right) + \sigma,$$

which is only possible if $q_\ell = q_h$ and thus $\sigma = s_\ell$.

Suppose that $s_h = b_h - (b_h - s_\ell)\frac{q_\ell}{q_h} \leq \sigma$ and $s_\ell = \sigma$. It means that

$$s_h = b_h - (b_h - \sigma)\frac{q_\ell}{q_h} = (b_h - \sigma)\left(1 - \frac{q_\ell}{q_h}\right) + \sigma,$$

which is only possible if $q_\ell = q_h$ and thus $\sigma = s_h$.

Thus we have $s_h = s_\ell = \sigma$ which requires $q_\ell = q_h = q$. The program then writes as

$$\max_q U(q) - (1-\sigma)\,\mathbb{E}(\theta)\, q$$

which yields $q = D\left((1-\sigma)\,\mathbb{E}(\theta)\right)$. ∎

We can interpret the results as follows. In case (i), the price cap is not binding, i.e. $s_h^* < \sigma$. Starting from this case, let us reduce σ sightly below s_h^*. Then the network sets $s_h = \sigma$. Given this price, the network would like to induce the quantity $q_h^{\sigma*}$ that maximizes the network value $U(q) + (\sigma - 1)\,\theta_h q$ for HB content at $s = \sigma$. Thus, it would like to raise the consumer price r_h above zero. But to prevent the HB content providers from opting for the LB content tariff, the network must reduce q_ℓ by $\frac{b_h-\sigma}{b_h-b_\ell}$ for each unit of reduction of q_h. As q_ℓ is distorted downward to reduce the HB content rent, the network faces a tradeoff between excessive consumption of HB content and insufficient consumption of LB content. When the cost of reducing q_ℓ outweighs the benefit of reducing q_h, the network chooses to keep the HB content price for consumers at 0. This is the case when $q_\ell^+ \geq q_\ell$ so that the distortion on the LB content is large (case (ii)). The consumption of HB content is only distorted when the efficient consumption $D(0)$ would lead to a consumption of LB content above q_ℓ^+. In this case, the network increases the consumer prices and reduces consumptions below $D(0)$ and $D(0)\frac{b_h-\sigma}{b_h-b_\ell}$ for the HB and LB contents respectively. In case (iii), the internally optimal price $r_h = (1-\sigma)\,\theta_h$ would require a suboptimal consumption of LB content, so that the network prefers to set $r_h < (1-\sigma)\,\theta_h$. The quantity $q_h = q_\ell\,(b_h - b_\ell)\,/\,(b_h - \sigma)$ is given by the first-order condition

$$\lambda U'(q_h) + (1-\lambda)\left(\frac{b_h-\sigma}{b_h-b_\ell}\right)U'(q_\ell) = \lambda(1-\sigma)\theta_h + (1-\lambda)\left(\frac{b_h-\sigma}{b_h-b_\ell}\right)(1-b_\ell)\theta_\ell$$

as long as the incentive compatibility of the HB content providers is binding. In case (iv), the quantity $q_h^{\sigma *}$ is large enough and the price cap is enough so that the "optimal tariff" $\{(\sigma,(1-\sigma)\theta_h);(b_\ell,(1-b_\ell)\theta_\ell)\}$ satisfies the incentive compatibility conditions.

We remark that q_ℓ^+ is increasing in σ. As the price cap σ decreases, the solution moves continuously from case (i) to case (ii) and then to case (iii).[1] Finally a price cap strictly below b_ℓ reduces consumption uniformly.

It is interesting to see what happens for σ close to b_ℓ. We may distinguish two cases. Whenever $\theta_h \geq \theta_\ell$, $(b_h-\sigma)/(b_h-b_\ell) < q_\ell^{FB}/q_h^{\sigma *}$ so case (iii) prevails. In this case, the consumption levels $\bar{q}_h$ and $\bar{q}_\ell$ converge to $D((1-b_\ell)\mathbb{E}(\theta))$ and thus quantities evolve continuously with σ. However when $\theta_h < \theta_\ell$, then $(b_h-\sigma)/(b_h-b_\ell) > q_\ell^{FB}/q_h^{\sigma *}$, case (iv) prevails and the consumptions $\bar{q}_h$ and $\bar{q}_\ell$ converge to $D((1-b_\ell)\theta_h)$ and $D((1-b_\ell)\theta_\ell)$ respectively. Thus, quantities are discontinuous at $\sigma = b_\ell$. The reason is the following. Suppose that $\sigma = b_\ell$, then the network sets $s_h = s_\ell = b_\ell$. The providers of LB content are indifferent between any quantity because they receive no profit, while the providers of HB content prefer higher quantities. The network can then implement q_ℓ^{FB} for the LB content and the internally efficient quantity $D((1-b_\ell)\theta_h)$ for the HB content if it is larger than q_ℓ^{FB}, hence if $\theta_h \leq \theta_\ell$. But if σ is reduced below b_ℓ, this becomes unfeasible because then the LB content providers have a margin $b_\ell - \sigma > 0$ and they also prefer higher quantities. Thus as σ falls below b_ℓ, the network has no other choice but to implement uniform prices. From this discussion it appears that:

Proposition 1 *The optimal price cap belongs to the interval* $[b_\ell, 1]$.

Proof. If $D(0)\frac{b_h-1}{b_h-b_\ell} \geq q_\ell^{FB}$ then $\sigma = 1$ is optimal because it yields an efficient allocation. Otherwise lowering the price cap raises the consumption of LB content without affecting the consumption of HB content as long as $q_\ell^+ \geq D(0)\frac{b_h-\sigma}{b_h-b_\ell}$. It is then optimal to set $\sigma < 1$. Clearly b_ℓ dominates any price cap below b_ℓ as it yields more efficient consumption levels. ■

Thus, an optimal price cap is always positive and that may or may not be at cost.

[1] As $D((1-\sigma)\theta_h)\frac{b_h-\sigma}{b_h-b_\ell}$ is non monotonic, the solution may alternate between (iii) and (iv).