

# Income Taxation with Frictional Labor Supply

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## Abstract

This paper characterizes the optimal labor income taxes in an environment where individual labor supply choices are subject to adjustment frictions. Agents incur a fixed cost of adjusting their hours of work in response to changes in their idiosyncratic wages or their tax rates. This fixed cost can be thought of as the cost of searching for a new job in an economy where hours are constrained within the firm. I derive a formula that characterizes the optimal long-run progressive tax schedule in this economy. Adjustment frictions generate endogenously an extensive margin of labor supply conditional on participation. In addition to the standard intensive margin disincentive effects of taxes, the optimal schedule takes into account their effects on the option value of adjusting hours of work, and therefore depends on several new elasticities and marginal social welfare weights. I then evaluate the quantitative magnitude of these novel theoretical effects and show that for a given intensive margin labor supply elasticity, the optimal long-run tax schedule is less progressive than a frictionless model would predict, because an increase in progressivity raises the dispersion of individual incomes around their desired values. The welfare miscalculations by wrongly assuming a frictionless economy can be large, and are decreasing in the size of the intensive margin labor income elasticity. The insights of this paper apply more broadly to models where fixed costs interact with non-linear policy instruments to yield long-run aggregate effects.

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# 1 Introduction

This paper characterizes optimal policy in models with fixed adjustment costs. Specifically, I analyze the long-run real effects generated by the interaction between fixed costs and non-linear policy. I study this question in an income taxation framework, where both non-linear (tax) policies and fixed (labor supply) adjustment costs arise naturally.

The theoretical models of optimal labor income taxation typically assume that labor supply can be adjusted costlessly and optimally in response to changes in wages or in the tax rates. The individual labor income elasticity, which measures the effects of a percentage change in the net-of-tax rate on the worker's taxable labor income, is the crucial parameter governing the behavioral impacts of taxes on government revenue. The optimal tax rate is determined by the trade-off between this revenue effect and the welfare effect (expressed in terms of public funds) of an increase in taxes, measured by the marginal utility of consumption, or marginal social welfare weights. Even models of taxation that incorporate explicitly an extensive margin of labor supply typically consider the binary decision of whether to participate in the labor force, keeping the assumption that conditional on participation hours are either exogenously fixed or fully flexible on the intensive margin. Unsurprisingly then, much theoretical and empirical research has been devoted to the analysis of the labor supply elasticity, or taxable income elasticity, with respect to the marginal tax rates.

A large and growing body of empirical evidence, however, shows that the adjustment of labor supply in response to productivity, wage or tax changes, is subject to substantial frictions. Specifically, workers in a given job face hour constraints set by firms and pay search costs to find new jobs. In other words, they must change jobs in order to adjust their hours of work, and this entails large fixed costs. The presence of such fixed adjustment costs generates endogenously an extensive margin of labor supply, conditional on participation, where the thresholds of adjustment are chosen optimally by the worker. There is little theoretical work that explicitly incorporates such adjustment frictions into models of labor income taxation and describes the revenue and welfare effects of taxes in this more realistic context. Several questions arise, some of which have been examined: the sluggish response of individuals to tax changes may generate small short-run (observed) labor income elasticities for large long-run elasticities, and non-trivial short-run welfare effects if taxes increase while individual hours are not maximizing their instantaneous utility. The question I address in this paper is whether, in the presence of labor supply frictions in a dynamic economy, *long-run* optimal taxes differ from those implied by the frictionless, intensive margin, models, and if so, what are their theoretical determinants in the frictional economy.

I set up an analytically tractable dynamic continuous-time model in which individuals choose their labor supply as a function of their stochastic idiosyncratic wage shocks and the non-linear tax schedule. The individual wage is exogenous and follows a random growth process with jumps, consistent with both micro and macro evidence, and the basis of a leading theory of income inequality. The tax schedule is restricted to have a constant rate of progressivity, a specific functional form that closely approximates the actual U.S. tax and transfer schedule. To keep the model tractable

I assume that individuals are born (or enter the labor force) and die (or retire) at an exogenous Poisson rate, and that they cannot save or borrow. In order to adjust their hours in response to wage or tax changes, individuals must pay a fixed cost. This fixed cost can be thought of as the cost of searching for a new job in an economy where hours are constrained within the firm, and is assumed proportional to the worker's foregone utility of consumption from the search activity. Once they decide to pay the fixed cost, i.e., to start searching for a new job, they receive a job offer, or a costless adjustment opportunity, at an exogenous Poisson rate, which captures in a reduced-form way the frictions on the demand side of the labor market. As a result, hours of work evolve in a lumpy manner at the individual level: workers remain inactive, that is keep the same job, until their wage is such that their optimal (frictionless) labor supply is far enough from their current, actual labor supply; at this point they pay the fixed cost and start searching for a new job. I show that the optimal range of inaction is an interval that I characterize analytically, and that the aggregate stationary income distributions have Pareto tails that can be written transparently in closed form as a function of the progressivity of the tax schedule.

I then derive formulas characterizing the optimal long-run progressive tax schedule in this economy, that is, the tax rates that maximize utilitarian social welfare subject to a government budget constraint. In the frictionless model, I show that the optimum is characterized by sufficient-statistic-type expressions extending the standard static formulas to the steady-state of a dynamic model. These formulas are written in terms of the individual intensive margin labor income elasticities and marginal social welfare weights that I define, capturing respectively the revenue and welfare effects of perturbing the tax rates. In the frictional model, I first show that the effect of a uniform increase in the marginal tax rates is given by the same formula as in the frictionless setting. In particular, the relevant labor income elasticity entering this formula is the individual elasticity of frictionless income, even though in the presence of frictions the individual's actual hours are generally different from her frictionless optimum. Intuitively, in the long run all individuals have had time to adjust their behavior to the new tax system and frictions wash out in the aggregate. I then turn to the effects of an increase in the progressivity of the tax schedule. There are several new effects that appear in the frictional economy that are not captured by the frictionless optimal taxation formula. First, in the frictional economy the individual's option value of adjusting labor supply is endogenous to taxes. This is because an increase in progressivity reduces the volatility of the income process, as higher incomes are taxed at a higher rate, which in turn reduces the option value and narrows the inaction region. I show that this induces non-zero effects on revenue and welfare unless these two effects exactly cancel out, i.e., unless the frequency of adjustment is exogenous to taxes. I define and characterize novel, "extensive margin" elasticities and marginal social welfare weights that summarize these effects in the optimal tax formula. Moreover, the presence of adjustment frictions implies that individuals who earn the same income differ in their utility, as the least productive of them (i.e., those with a lower wage) are working more hours to earn the same income, and this non-degenerate distribution is itself endogenous to the tax schedule. By treating the population earning a given income as a representative agent, the frictionless model thus miscalculates the welfare effects of perturbing taxes. This effect is captured by another novel marginal social welfare

weight in the optimal tax formula. Therefore, theoretically, the extensive margin of labor supply endogenously generated by hour requirements within jobs has non-trivial long-run effects on tax revenue and social welfare beyond the standard intensive margin effects.

Finally I calibrate the model and estimate numerically the magnitude of these novel theoretical effects. [**Note:** *Preliminary calculations for now.*] The first finding is that the extensive margin elasticities with respect to the rate of progressivity are non-negligible - of the same order of magnitude as the participation elasticities found in the literature. In the baseline calibration, these elasticities induce only a small behavioral effect on tax revenue: the revenue losses from raising progressivity calculated by wrongly assuming a frictionless economy are about 2% away from their true values. The reason is that the extensive margin elasticities are bounded, while an increase in progressivity induces unboundedly large changes in the marginal tax rates as income grows, and hence much larger intensive margin income responses to taxes. However, the *welfare* effects generated by the endogenous option value of adjusting labor supply and by the endogenous distribution of utilities within incomes are larger, and imply benefits of raising the progressivity of the tax schedule relative to the frictionless model: the frictionless values are more than 7% away from their true value. This translates in an optimal rate of progressivity equal to  $xx$  rather than  $xx$  [**Note:** *Computations are coming soon.*]. In contrast, assuming a larger intensive margin labor income elasticity ( $\varepsilon = 1$  rather than  $\varepsilon = 0.33$ ) dwarves these effects, as the standard intensive margin terms then dominate the new extensive margin terms; in this case, the frictionless model closely approximates the true long-run optimal tax rates. The reason why the extensive margin effects on welfare tend to reduce the gains of raising progressivity is the following. We saw that the standard marginal social welfare weights capture the true welfare effects of taxes if the decrease in the volatility of incomes due to an increase in progressivity is exactly compensated by the narrowing of the inaction region (option value effect). In general, however, the latter effect is dominated by the former, so that an increase in progressivity is equivalent to a higher dispersion of individual incomes around their desired values, which adversely affects welfare, relative to the frictionless benchmark. On the other hand, when the intensive margin labor income elasticity is large, the extensive margin welfare losses are dominated by the intensive margin losses.

Finally, the results I derive in this paper apply much more broadly than to this taxation theory framework. The key finding of this paper is that non-linear policies interact with fixed adjustment costs to yield real aggregate long-run effects. This is in contrast with most of the literature with fixed costs at the micro level, where typically the aggregate economy behaves in the long-run as a frictionless (representative agent) model. I show in this paper that this insight is correct only if we consider *linear* policies, which do not affect the option value of adjusting labor supply. On the other hand, the *curvature* of policy matters in the aggregate (for both efficiency and welfare) even in the long-run when adjustment at the individual level is frictional. This insight should apply generally to models with fixed costs and non-linear policy tools, e.g. to the investment literature.

**Related literature.** The empirical literature estimating the (Hicksian) labor income elasticities is vast: see, e.g., Saez, Slemrod, and Giertz (2012) and Keane and Rogerson (2012) for recent

surveys. Rogerson and Wallenius (2009, 2013) and Ljungqvist and Sargent (2011) argue that the (small) micro and (large) macro elasticities typically found in the literature can be reconciled if the primary margin of adjustment of labor supply is the choice of career length rather than hours conditional on participation. Chetty, Guren, Manoli, and Weber (2012) and Chetty (2012) argue instead that adjustment frictions can fully explain the difference of steady-state elasticities, and that the extensive margin (participation) elasticities are smaller than the intensive margin (hours) elasticities. Holmlund and Söderström (2008) argue that the short-run and the long-run elasticities may differ. More generally there is a large empirical literature that points to the presence of frictions in the adjustment of labor supply. Altonji and Paxson (1992) show that changes in labor supply preferences have a much larger effect on hours of work when individual change jobs, suggesting that adjusting behavior entails substantial fixed costs. Other papers have similarly argued that labor supply is constrained by adjustment costs and hours requirements set by firms, e.g., Cogan (1981), Altonji and Paxson (1988), Dickens and Lundberg (1993), Chetty, Friedman, Olsen, and Pistaferri (2011), Gelber, Jones, and Sacks (2013). My contribution is to incorporate explicitly these fixed costs into a dynamic taxation framework and derive the consequences for long-run optimal income taxes. These fixed costs generate endogenous extensive margin responses for employed individuals and long-run elasticities that differ from the short-run elasticities due to the sluggish adjustment of hours.

This paper relates to several strands in the theoretical optimal taxation literature. My frictionless model is related to that of Heathcote, Storesletten, and Violante (2014), who also restrict the set of available tax instruments to two-parameter schedules, and analyze the effects of progressivity on social welfare with imperfect private insurance and investment in skills. My frictionless model is simpler than theirs, which allows me to introduce frictional labor supply and keep the tractability of the model. The literature on the sufficient statistic approach to taxation, see e.g. Saez (2001) in the static setting, Golosov, Tsyvinski, and Werquin (2014) in the dynamic setting, and Chetty (2009) for a general exposition, derives optimal tax formulas for a very large class of models, irrespective of the underlying functional forms for the utility functions, the sources of heterogeneity, etc. However, these models generally assume that labor supply can always be set optimally at no cost. In this paper I show that these sufficient statistic formulas do not generally hold, even in the long run, in the presence of simple adjustment frictions. There is a theoretical taxation literature with labor supply responses on the extensive margin. Saez (2002), Choné and Laroque (2011), Jacquet, Lehmann, and Van der Linden (2013), Shourideh and Troshkin (2014), Lehmann, Kroft, Kucko and Schmieder (2015) study optimal taxation problems where individuals face a fixed cost of working, leading to binary participation decisions. I extend these papers' insights by generating and studying more general sources of extensive margin responses of labor supply to taxes, namely, conditional on participation. It would be straightforward to include an explicit participation margin in my setting (similar to Alvarez, Borovickova and Shimer (2015)), although empirically much of the difference in labor supply across countries with different tax regimes is driven by hours worked conditional on employment (see Davis and Henrekson (2005), and Chetty, Guren, Manoli and Weber (2011)). Chetty, Looney, and Kroft (2009) propose a model of bounded rationality (i.e., where the

fixed adjustment cost is interpreted as a cognitive cost) where individuals' responses to taxes are affected by tax salience, and show that this feature affects the calculation of the impact of taxes on social welfare. Chetty, Friedman, Olsen, and Pistaferri (2011) study a model in which labor supply is subject to search costs and jobs are characterized by hours requirements. These models are primarily static, and do not capture the dynamic decisions of individuals based on their option value of waiting, nor the long-run effects of taxes on social welfare. A paper related to mine is that of Alvarez, Borovickova and Shimer (2015) who also model labor supply adjustment decisions as a stopping time problem. They only consider the transitions between employment and unemployment, however, and do not focus on the implications of this class of models for optimal taxation.

Finally, the technical tools I use to analyze my model of individual behavior are those developed in the impulse control literature originally developed for operations research questions. Dixit and Pindyck (1994) and Stokey (2008) summarize many references and applications of these models to economics, primarily on monetary and investment topics. Scarf (1959), Harrison, Sellke and Taylor (1985), Bertola and Caballero (1994), Grossman and Laroque (1990), Caballero and Engel (1999), and more recently Alvarez and Lippi (2013) and Alvarez, Le Bihan, and Lippi (2014), to cite only a few, have made important theoretical contributions to this literature, on which this paper builds. In public finance, there is a rich literature on investment in the presence of adjustment costs: Hall and Jorgensen (1967), Summers (1981), Abel (1983), Auerbach and Hines (1987), Auerbach (1989), Auerbach Hassett (1992). I bring this literature to the study of labor supply, now that we know that labor adjustment costs and not just capital adjustment costs can be important, and moreover I study optimal policy in this class of models.

The structure of the paper is as follows. I set up the environment and describe the individual and government problems in Section 2. I then analyze the optimal individual behavior in Section 3, and characterize the aggregate steady-state of the economy in Section 4. I define the new elasticities and marginal social welfare weights, and derive formulas for optimal taxes in Section 5. The numerical exercises are in Section 6, and Section 7 concludes. The proofs of all the results are gathered in the Appendix.

## 2 Environment

There is a continuum of mass one of individuals in the economy. Time is continuous.

**Preferences.** Individuals have the following Greenwood, Hercowitz and Huffman (1988) utility function of consumption  $c$  and hours of work  $h$  with isoelastic disutility of labor supply:

$$U(c, h) = \frac{1}{1-\gamma} \left( c - \frac{1}{1+1/\varepsilon} h^{1+1/\varepsilon} \right)^{1-\gamma}, \quad (1)$$

with  $\gamma \in [0, 1)$ . They discount the future at rate  $\rho_1$ . They are born (or enter the labor force) and die (or retire) at an exogenous and constant Poisson rate  $\rho_2$ . I denote the function  $g(x) =$

$$(1 - \gamma)^{-1} x^{1-\gamma}.$$

**Technology.** Individual productivity  $\theta$  is exogenous. The production function is linear in the labor input, so that in equilibrium workers' wages  $w_t$  are always equal to their marginal productivity  $\theta_t$ , and they can freely choose to be employed at any labor supply  $h(\theta_t)$ . Therefore, in the sequel, I substitute a worker's exogenous productivity  $\theta$  with her wage  $w$ .

An individual's wage (i.e., productivity) at birth,  $w_0$ , is drawn from a log-normal distribution with mean  $m_w$  and variance  $s_w^2$ , i.e.,  $f_{w_0}(\cdot) \sim \log \mathcal{N}(m_w, s_w^2)$ . The idiosyncratic wage  $w_t$  then evolves stochastically over time  $t \geq 0$  according to a geometric Brownian motion with jumps, that is,

$$d \ln w_t = \mu_w dt + \sigma_w d\mathcal{W}_t + \nu_{w,t} d\mathcal{J}_t, \quad (2)$$

where  $\mathcal{W}_t$  is a Wiener process and  $d\mathcal{J}_t$  is a jump process with intensity  $\iota$ . Thus, in the absence of jumps, the wage follows a random growth process with expected growth rate  $\mu_w + \frac{1}{2}\sigma_w^2$  and volatility  $\sigma_w$ . There is a jump in  $[t, t + dt)$  (i.e.,  $d\mathcal{J}_t = 1$ ) with probability  $\iota dt$  and no jump (i.e.,  $d\mathcal{J}_t = 0$ ) with probability  $1 - \iota dt$ . The innovations  $\nu_{w,t}$  are drawn from a distribution  $f_\nu$ . Individuals know their wage process and observe its realization at every instant  $t$ .

The reduced-form equation (2) for the exogenous wage process can be microfounded, see e.g. Gabaix, Lasry, Lions and Moll (2015), and the references therein. This random growth formulation is a leading theory of income inequality as it naturally generates Pareto tails for the wage distributions (see, e.g., Gabaix (2009) and Section 4 below), a stylized fact that plays an important role in the optimal taxation literature (Saez (2001)). The empirical literature (see Meghir and Pistaferri (2011) for a survey) estimates wage specifications of this form (without the jumps) and its findings are consistent with the presence of a unit root in the wage process  $w_t$ , that is, permanent wage shocks. Assuming a double-Pareto distribution  $f_\nu$ , the jump process furthermore implies that the distribution of income *growth rates*  $d \ln w_t$  itself also has Pareto tails, consistent with the evidence found in Guvenen, Karahan, Ozkan and Song (2014).

**Budget constraint and taxes.** An individual with wage  $w$  who works  $h$  hours earns *taxable labor income*  $y = wh$  and pays taxes  $T(y)$  to the government. I assume that she cannot save or borrow, so that she consumes her net income at every instant:

$$c = y - T(y).$$

The tax-and-transfer system is restricted within a class of two-parameter schedules (see, e.g., Benabou (2002), Heathcote, Storesletten, and Violante (2014)), defined as

$$T(y) = y - \frac{1 - \tau}{1 - p} y^{1-p}, \quad (3)$$

with  $\tau \in \mathbb{R}$  and  $p < 1$ . I denote the tax schedule interchangeably by  $T(\cdot)$  or  $\{\tau, p\}$ . The parameter  $p$  is the coefficient of marginal rate progression (see Musgrave and Thin (1948)), or *progressivity* of

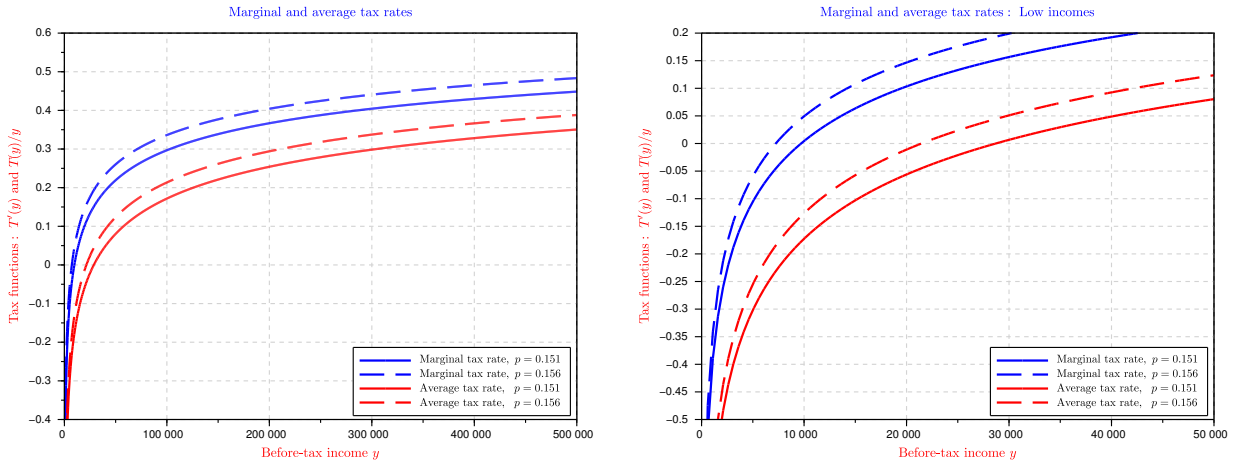
the tax schedule. It is equal to the elasticity of the net-of-tax rate with respect to taxable income,

$$p = -\frac{d \ln(1 - T'(y))}{d \ln y}. \quad (4)$$

If  $p = 0$ , the income tax schedule is linear with constant marginal tax rate  $\tau$ . If  $p \in (0, 1)$ , the ratio of the marginal tax rate to the average tax rate is  $T'(y) / \{T(y)/y\} > 1$ , so that the tax schedule is progressive. If  $p < 0$ , the tax schedule is regressive. Note that the marginal and average tax rates are monotone in earnings, and that average tax rates are negative for incomes  $y$  below  $\left(\frac{1-\tau}{1-p}\right)^{1/p}$ .

This functional form for the tax schedule provides a close approximation of the actual tax system in the U.S. (see Heathcote, Storesletten, and Violante (2014)). The first panel of Figure 1 shows the marginal and average tax rates for two values of the progressivity parameter:  $p = 0.151$ , which is calibrated to the rate of progressivity of the US tax code (see Section 6) and  $p = 0.156$ . The second panel graphs these tax schedules at the bottom of the income distribution.

Figure 1: Tax schedule



**Individual problem.** Individuals choose their labor supply  $h_t$  endogenously as a function of their information at time  $t$ . Consider a frictionless environment first, that is, where they can adjust their labor supply optimally at every instant. A worker with current wage  $w_0$  then solves the following problem:

$$\begin{aligned} \max_{\{h_t\}_{t \geq 0}} \quad & \mathbb{E} \left[ \int_0^\infty e^{-(\rho_1 + \rho_2)t} U(c_t, h_t) dt \mid w_0 \right], \\ \text{s.t. } \quad & \forall t \geq 0, \quad c_t = w_t h_t - T(w_t h_t), \\ & d \ln w_t = \mu_w dt + \sigma_w d\mathcal{W}_t + \nu_{w,t} d\mathcal{J}_t. \end{aligned} \quad (5)$$

The solution to this problem gives the agent's *frictionless*, or *desired*, labor supply  $\{h_t^*\}_{t \geq 0}$  and consumption  $\{c_t^*\}_{t \geq 0}$ . I denote by  $\mathcal{V}^*(w)$  (or, equivalently,  $\mathcal{V}^*(y)$ ) the value function of an individual with current wage  $w$  (or current income  $y$ ) in this frictionless environment.



I now suppose that in order to adjust her labor supply (which I also refer to as a “job”) from  $h$  to  $h'$ , the individual must pay a *fixed utility cost*  $\kappa \geq 0$ . I interpret this fixed cost as the search cost of finding a new job (see the empirical evidence provided in Altonji and Paxson (1992), Chetty, Friedman, Olsen and Pistaferri (2011), Gelber, Jones and Sacks (2013)). Formally, I assume that  $\kappa$  is proportional to the utility  $g(c^*)$  from the foregone (frictionless) disposable income due to the search activity.<sup>1</sup> Thus we have

$$\kappa = \kappa \times g(c^*), \tag{6}$$

with  $\kappa \geq 0$ .

After paying the fixed cost, the individual waits until she receives a job offer. Offers arrive at an exogenous Poisson rate  $q$ . Whenever she receives one, she can adjust her hours  $h$  optimally and costlessly given her current productivity (i.e., wage)  $w$ . As long as she does not receive the offer, she keeps the same labor supply.<sup>2</sup> Intuitively,  $q$  captures in a reduced-form way the frictions on the demand side of the labor market: the larger the  $q$ , the faster an individual searching for a job finds one.<sup>3</sup> As  $q \rightarrow \infty$  with  $\kappa > 0$ , the model converges to a two-sided  $(S, s)$  model similar to those studied in the operations research, monetary or investment literatures (see, e.g., Harrison, Sellke and Taylor (1985), Dixit and Pindyck (1994), Stokey (2004), and the references therein). In this case the adjustments in hours are entirely driven by labor supply considerations, i.e., productivity, taxes, search costs. As  $\kappa \rightarrow 0$  with  $q < \infty$ , on the other hand, the model converges to an environment similar to the Calvo (1983) model; in this case adjustments are driven purely by labor demand.

Individuals decide when and by how much to adjust their labor supply as their wage evolves. They can choose their hours optimally and costlessly at birth. Let  $\{\mathcal{F}_t\}_{t \geq 0}$  denote the filtration generated by  $\mathcal{W}_t$ . An *impulse control policy* is defined as a sequence of stopping times  $0 = t_0 < t_1 < \dots < t_i < \dots$  adapted to  $\{\mathcal{F}_t\}$ , and a sequence of random variables  $h_{t_0}, h_{t_1}, \dots, h_{t_i}, \dots$  that are measurable with respect to the minimum  $\sigma$ -algebra  $\{\mathcal{F}_{t_i}\}$  of events up to time  $t_i$ . These represent respectively the timing of adjustments and the optimal choice of labor supply upon adjustment.

I say that an individual is *inactive* if she is not currently searching for a job, that is, if she has not yet paid the fixed adjustment cost since she started working at her current job. I say that the individual is *searching* if she has paid the fixed cost but has not yet received an adjustment opportunity. Consider an individual who has just received an adjustment opportunity at time 0 when her current wage is  $w_0$ , and call  $\hat{V}(w_0)$  her value function at that instant. Her new labor supply  $h_0$  and her next adjustment stopping time  $t_1 > 0$  are the solution to the following recursive

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<sup>1</sup>Assuming  $\gamma < 1$  in (1) implies that this fixed cost is strictly positive and increasing in income.

<sup>2</sup>In a previous version of this paper (Chapter 1 in Werquin (2015)), I assumed instead that income  $y$ , rather than hours  $h$ , is subject to the adjustment cost. In this case,  $w$  and  $h$  are interpreted respectively as productivity and effort, and their product  $y$  is the agent’s effective labor supply, or income. As the individual becomes more productive and stays in her current job ( $w$  gets higher and  $y$  remains constant), she needs to provide less effort ( $h$  gets lower) to produce the required amount  $y$ . She adjusts her income upwards (resp., downwards) when she becomes so productive (resp., unproductive) that she spends most of her time idle (resp., when she provides too much effort) to produce  $y$ . The main results of the paper are unaffected by this alternative specification.

<sup>3</sup>In this paper I focus on the labor supply effects of taxation, and thus assume that the labor demand frictions, summarized by the parameter  $q$ , are exogenous to taxes.

problem:

$$\begin{aligned}
\hat{V}(w_0) &= \max_{\{h_0, t_1\}} \mathbb{E} \left[ \int_0^{t_1} e^{-(\rho_1 + \rho_2)t} U(c_t, h_t) dt + e^{-(\rho_1 + \rho_2)t_1} \left\{ \hat{V}(w_{t_1}) - \kappa_{t_1} \right\} | w_0 \right], \\
\text{s.t. } \forall t \in [0, t_1), \quad & c_t = w_t h_t - T(w_t h_t), \\
& h_t = h_0, \\
& d \ln w_t = \mu_w dt + \sigma_w d\mathcal{W}_t + \nu_{w,t} d\mathcal{J}_t.
\end{aligned} \tag{7}$$

Let  $\mathcal{V}_i(w, h)$ , respectively  $\mathcal{V}_s(w, h)$ , denote the value function, i.e., the expected present discounted value of lifetime utility net of the adjustment costs, of an inactive, respectively searching, worker with wage  $w$  and labor supply  $h$ . In Section 3 I set up the recursive formulation of the individual's problem (7) and formally define and characterize the value functions  $\mathcal{V}_i(w, h)$  and  $\mathcal{V}_s(w, h)$ .

**Government's problem.** The government chooses the tax schedule  $T(\cdot) = \{\tau, p\}$  and evaluates social welfare according to a utilitarian objective over all living individuals. Due to the assumption of exponential deaths, individuals receive equal weights independently of their age. The government maximizes the long-run (steady-state) social welfare subject to a budget balance constraint that imposes that the total tax revenue net of transfer payments must be at least as large as an exogenous revenue requirement  $\bar{R}$ . Assuming for now their existence (see Section 4), let  $f_{w,h}^i, f_{w,h}^s$  denote the stationary joint density of wages  $w$  and hours  $h$  for inactive and searching individuals, respectively. The government solves:

$$\max_{T(\cdot)} \int_0^\infty \int_0^\infty \sum_{x \in \{i, s\}} \mathcal{V}_x(w, h) f_{w,h}^x(w, h) dw dh \tag{8}$$

subject to

$$\int_0^\infty \int_0^\infty T(wh) \sum_{x \in \{i, s\}} f_{w,h}^x(w, h) dw dh \geq \bar{R}. \tag{9}$$

Let  $\lambda$  denote the marginal value of public funds, i.e., the Lagrange multiplier associated with the budget constraint of the maximization problem (8,9). Let  $\mathcal{R}(T)$  denote the long-run tax revenue given the tax schedule  $T(\cdot)$ , i.e., the left-hand side of the constraint (9). Finally let  $\mathcal{W}(T)$  denote social welfare, equal to the sum of individual indirect utilities (the maximand in (8)), normalized by the shadow value  $\lambda$  so as to obtain a money metric for welfare.

I characterize the optimal tax schedule  $T(\cdot) = \{\tau, p\}$  in Section 5 by deriving the first-order effects on social welfare of perturbations  $(d\tau, dp)$  of the tax schedule as  $d\tau, dp \rightarrow 0$ .<sup>4</sup> The marginal value of public funds  $\lambda$ , respectively the optimal rate of progressivity  $p$ , is such that the perturbation  $d\tau$ , respectively  $dp$ , has no first-order effects on social welfare. The resulting two equations (along with the budget constraint) fully characterize the optimum tax system.

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<sup>4</sup>More generally, we can use this method to characterize the first-order welfare effects of revenue-neutral *tax reforms* away from the optimum tax schedule; see Golosov, Tsyvinski and Werquin (2014) for a general exposition.

### 3 Individual behavior

In this section I characterize the optimal individual behavior in the model. [*Note:* from here on, I solve the model without jumps (i.e.,  $d\mathcal{J}_t = 0$ ). The theoretical results are not affected by the presence of jumps (only some specific derivations are, e.g. equations (19) and (35)). The text will be updated to include the jumps.]

**Frictionless model.** The solution to the frictionless problem (5) is as follows. At each instant  $t$ , the individual's optimal labor supply  $h_t^*$  is an increasing function of her current wage  $w_t$  and her net-of-tax rate  $(1 - T'(w_t h_t^*))$ . The frictionless taxable income  $y_t^* = w_t h_t^*$  and disposable income  $c_t^*$  are given by:

$$\begin{aligned} y_t^* &= (1 - T'(y_t^*))^\varepsilon w_t^{1+\varepsilon} = (1 - \tau)^{\frac{\varepsilon}{1+p\varepsilon}} w_t^{\frac{1+\varepsilon}{1+p\varepsilon}}, \\ c_t^* &= y_t^* - T(y_t^*) = \frac{1}{1-p} (1 - \tau)^{\frac{1+\varepsilon}{1+p\varepsilon}} w_t^{\frac{(1-p)(1+\varepsilon)}{1+p\varepsilon}}. \end{aligned} \quad (10)$$

Equation (10) shows that the structural elasticity parameter  $\varepsilon$  and the parameters of the tax schedule  $(\tau, p)$  govern the relationship between an individual's wage and her corresponding choice of labor supply or taxable income. In particular, the magnitude by which higher wages (or lower marginal tax rates) translate into higher incomes is increasing in the elasticity  $\varepsilon$ , which therefore measures the disincentive effects of taxation (see below).

Using equation (10), we find that the laws of motion of the taxable and disposable incomes are given by the following random growth processes:

$$\begin{aligned} d \ln y_t^* &= \mu_y dt + \sigma_y d\mathcal{W}_t, \quad \text{with} \quad \{\mu_y, \sigma_y\} = \frac{1 + \varepsilon}{1 + p\varepsilon} \{\mu_w, \sigma_w\}, \\ d \ln c_t^* &= \mu_c dt + \sigma_c d\mathcal{W}_t, \quad \text{with} \quad \{\mu_c, \sigma_c\} = (1 - p) \frac{1 + \varepsilon}{1 + p\varepsilon} \{\mu_w, \sigma_w\}. \end{aligned} \quad (11)$$

Equation (11) shows that the income processes are given endogenously from the wage process as functions of the labor supply elasticity  $\varepsilon$  and the progressivity  $p$  of the tax schedule. In particular, a higher elasticity  $\varepsilon$  and a lower rate of progressivity  $p$  lead to a higher volatility  $\sigma_y$  of the income process.

I assume that

$$\rho \equiv \rho_1 + \rho_2 - (1 - \gamma) \mu_c - \frac{1}{2} (1 - \gamma)^2 \sigma_c^2 > 0,$$

which will ensure that the individual indirect lifetime utility is finite.

**Frictional model.** I now analyze the frictional problem (7) with  $\kappa > 0$ . This problem has two state variables: the current wage  $w$ , and the current labor supply  $h$ . The crucial variable for the analysis is the *labor supply deviation*  $\delta$ , which is defined as the log-difference between the *actual* and the *desired* (or frictionless) hours of work  $h$  and  $h^*$ , that is,

$$\delta_t \equiv \ln(h_t) - \ln(h_t^*) = \ln(y_t) - \ln(y_t^*), \quad (12)$$

where  $y_t^*$  is given by (10). While the individual does not adjust her labor supply, her deviation  $\delta_t$  evolves according to the following process:

$$d\delta_t = -d \ln h_t^* = \mu_\delta dt + \sigma_\delta d\mathcal{W}_t, \quad \text{with} \quad \{\mu_\delta, \sigma_\delta\} = -\frac{(1-p)\varepsilon}{1+p\varepsilon} \{\mu_w, \sigma_w\}. \quad (13)$$

In the sequel I change variables and use either  $(y^*, \delta)$  or  $(y, \delta)$  (rather than  $(w, h)$ ) as the two state variables of the individual's problem, that is, the optimal frictionless or actual taxable income, and the deviation of her labor supply away from the frictionless optimum. There are one-to-one correspondences between these pairs of variables, given by the relationships (10) and (12). Accordingly, with a slight abuse of notation, from now on I denote the individual's utility function by  $U(y^*, \delta)$  and her value functions by  $\mathcal{V}_i(y^*, \delta)$  and  $\mathcal{V}_s(y^*, \delta)$ .

We can easily show that the flow utility  $U(y^*, \delta)$  is homogeneous in the utility of frictionless disposable income  $c^*$ ,

$$U(y^*, \delta) = \frac{1}{1-\gamma} \left( \frac{1-\tau}{1-p} y^{*(1-p)} \right)^{1-\gamma} \left[ e^{(1-p)\delta} - \frac{1-p}{1+1/\varepsilon} e^{(1+1/\varepsilon)\delta} \right]^{1-\gamma} \equiv g(c^*) \times u(\delta). \quad (14)$$

A second-order Taylor approximation of the function  $u(\delta)$  around the frictionless optimum  $\delta = 0$  shows that the utility loss from failing to optimize is locally quadratic around the frictionless optimum  $h^*$ ,

$$u(\delta) \underset{\delta \rightarrow 0}{\sim} \left( \frac{1+p\varepsilon}{1+\varepsilon} \right)^{1-\gamma} \left[ 1 - \frac{1}{2}(1-\gamma)(1-p) \left( 1 + \frac{1}{\varepsilon} \right) \delta^2 \right]. \quad (15)$$

Since the function  $u(\delta)$  in (14) is not well defined for  $\delta$  far away from 0, I assume for simplicity that the utility of deviation is given by its quadratic approximation (15) for any  $\delta \in \mathbb{R}$ .<sup>5</sup> Equation (14) together with the homogeneity of the fixed adjustment cost (6) and the random growth property of the law of motion of frictionless disposable income (11) allow us to crucially reduce the dimensionality of the state space. Specifically, I show the following proposition:

**Proposition 1.** *The policy functions and the value functions  $\mathcal{V}_i(y^*, \delta)$  and  $\mathcal{V}_s(y^*, \delta)$  of inactive and searching individuals with frictionless taxable income  $y^*$  and labor supply deviation  $\delta$  are homogeneous of degree one in the utility of desired consumption  $g(c^*)$ . The value functions can thus be written as*

$$\mathcal{V}_i(y^*, \delta) = g \left( \frac{1-\tau}{1-p} y^{*(1-p)} \right) v_i(\delta), \quad \text{and} \quad \mathcal{V}_s(y^*, \delta) = g \left( \frac{1-\tau}{1-p} y^{*(1-p)} \right) v_s(\delta). \quad (16)$$

*Proof.* See Appendix. □

I now analyze the individual's optimal adjustment behavior, given by the solution to the impulse control problem (7). Proposition 2 below shows that for any level of labor supply  $h$  (i.e., for any given job), the optimal impulse control policy can be characterized by an interval of inaction  $(\underline{\delta}, \bar{\delta})$

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<sup>5</sup>Alternatively we can keep the exact expression if  $\gamma = 0$  (we should then add curvature to the social welfare function in the government's problem); none of the qualitative results would be affected.

and a return point  $\delta^*$ , with  $\underline{\delta} < \delta^* < \bar{\delta}$ . No control is exerted as long as the state process  $\delta$  is in  $(\underline{\delta}, \bar{\delta})$ . When the state process strikes or is below  $\underline{\delta}$  or above  $\bar{\delta}$ , the individual pays the fixed cost and waits (on average a duration  $q^{-1}$ ) until she receives an adjustment opportunity. At this time she adjusts the state to  $\delta^*$ , i.e., hours jump from  $h$  to  $h' = he^{\delta^* - \delta}$ , where  $\delta$  is the labor supply deviation at the time the signal is received.

Consider a searching individual first, i.e., who has paid the fixed cost but not yet received the adjustment offer. Define, for any  $x \in \{w, y, c, \delta\}$  and  $\rho > 0$ ,

$$r_{1,x}^\rho \equiv \frac{\mu_x}{\sigma_x^2} - \sqrt{\frac{\mu_x^2}{\sigma_x^4} + \frac{2\rho}{\sigma_x^2}}, \quad \text{and} \quad r_{2,x}^\rho \equiv \frac{\mu_x}{\sigma_x^2} + \sqrt{\frac{\mu_x^2}{\sigma_x^4} + \frac{2\rho}{\sigma_x^2}}. \quad (17)$$

As long as the individual is searching, her value function  $v_s(\delta)$  satisfies the following Hamilton-Jacobi-Bellman (HJB) equation: for all  $\delta \in \mathbb{R}$ ,

$$\frac{1}{2}\sigma_\delta^2 v_s''(\delta) + [\mu_\delta + (1-\gamma)\sigma_c\sigma_\delta]v_s'(\delta) - \left[ \rho_1 + \rho_2 + q - (1-\gamma)\mu_c - \frac{1}{2}(1-\gamma)^2\sigma_c^2 \right] v_s(\delta) = -u(\delta). \quad (18)$$

The intuition for this equation is as follows. Interpreting the entitlement to the flow of disposable incomes and deviations as an asset, and  $\mathcal{V}_s(y^*, \delta)$  as its value, we can write:

$$(\rho_1 + \rho_2)\mathcal{V}_s(y^*, \delta) = U(y^*, \delta) + \frac{\mathbb{E}_t[d\mathcal{V}_s(y^*, \delta)]}{dt} + q[\mathcal{V}_i(y^*, \delta^*) - \mathcal{V}_s(y^*, \delta)].$$

The left hand side gives the normal return per unit time that an individual, using  $(\rho_1 + \rho_2)$  as the discount rate, would require for holding this asset. The right hand side is the expected total return per unit time from holding the asset. The first term is the immediate payout or dividend from the asset. The second term is its expected rate of capital gain or loss. The third term is the change in the value of the asset in case a job opportunity is received (so that the agent goes from searching to inaction), which occurs at rate  $q$  per unit time. The equality is a no-arbitrage condition, expressing the investor's willingness to hold the asset. Using Itô's formula, we can express the second term in the right hand side as a function of the first and second partial derivatives of the value function  $\mathcal{V}_s$  and the drifts and volatilities of the income and deviation processes. We then obtain the HJB equation (18) for  $v_s(\delta)$  using the homogeneity of the value function shown in Proposition 1.

I show in the Appendix that this differential equation can be integrated using the appropriate boundary conditions at  $\pm\infty$ , yielding the following expression for the value function  $v_s(\delta)$  of a searching individual: for all  $\delta \in \mathbb{R}$ ,

$$v_s(\delta) = \frac{\int_{-\infty}^{\infty} \left[ e^{r_{2,\delta}^{\rho_1+\rho_2+q}x} \mathbb{I}_{\{x \leq 0\}} + e^{r_{1,\delta}^{\rho_1+\rho_2+q}x} \mathbb{I}_{\{x > 0\}} \right] u(x+\delta) dx}{\frac{\sigma_\delta^2}{2} e^{(1-\gamma)(1+\frac{1}{\varepsilon})} \left( r_{2,\delta}^{\rho_1+\rho_2+q} - r_{1,\delta}^{\rho_1+\rho_2+q} \right)} + \frac{q}{\rho_1 + \rho_2} v_i(\delta^*). \quad (19)$$

The first term in the right hand side of (19) is the flow utility from the time at which the cost is paid until the adjustment occurs, i.e.,  $\mathbb{E} \left[ \int_0^T e^{-(\rho_1+\rho_2)t} U(y_t^*, \delta_t) dt \right]$ , where the stopping time  $T$  is

exponentially distributed (Poisson shocks). The second term is the expected value of returning to  $(y^*, \delta^*)$ , i.e.,  $\mathbb{E} [e^{-(\rho_1 + \rho_2)T} \mathcal{V}_i(y_T^*, \delta_T^*)]$ .

Next consider an inactive individual, i.e., who has not yet paid the fixed cost since she started her current job. As long as she remains inactive, her value function  $v_i(\delta)$  satisfies the following Hamilton-Jacobi-Bellman (HJB) equation: for all  $\delta \in (\underline{\delta}, \bar{\delta})$ ,

$$\frac{1}{2} \sigma_\delta^2 v_i''(\delta) + [\mu_\delta + (1 - \gamma) \sigma_c \sigma_\delta] v_i'(\delta) - \left[ \rho_1 + \rho_2 - (1 - \gamma) \mu_c - \frac{1}{2} (1 - \gamma)^2 \sigma_c^2 \right] v_i(\delta) = -u(\delta). \quad (20)$$

The intuition for this equation is similar to that of equation (18).

The boundary conditions at  $\underline{\delta}, \delta^*, \bar{\delta}$  that complete the characterization of the individual's optimal policy are the following. First, at the time the agent decides to pay the cost and search for a new job, she must be indifferent between doing so and continuing with her current job; thus the following *value-matching conditions* hold at the thresholds  $\underline{\delta}, \bar{\delta}$ :

$$v_i(\bar{\delta}) = v_s(\bar{\delta}) - \kappa, \quad \text{and} \quad v_i(\underline{\delta}) = v_s(\underline{\delta}) - \kappa. \quad (21)$$

Second, the marginal value and the marginal cost of starting to search must be equal, i.e., the two value functions  $v_i(\delta)$  and  $v_s(\delta)$  meet tangentially at  $\underline{\delta}, \bar{\delta}$ ; thus the following *smooth-pasting conditions* hold:

$$v_i'(\bar{\delta}^+) = v_i'(\bar{\delta}^-) = v_s'(\bar{\delta}), \quad \text{and} \quad v_i'(\underline{\delta}^+) = v_i'(\underline{\delta}^-) = v_s'(\underline{\delta}). \quad (22)$$

Third, the optimal return point upon adjustment is the maximum of the value function  $v_i(\delta)$  of the newly inactive individual, so that the following *optimality condition* holds at  $\delta^*$ :

$$v_i'(\delta^*) = 0. \quad (23)$$

Equations (20), (19), (21), (22), (23) fully characterize the optimal individual policy.

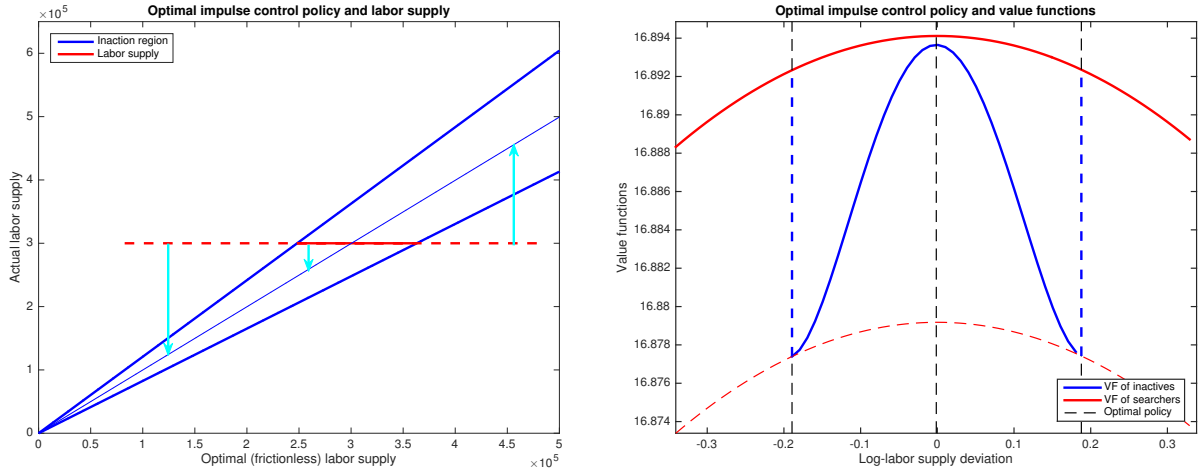
**Proposition 2.** *Suppose that there exist  $\{\underline{\delta}, \delta^*, \bar{\delta}\}$  such that the functions  $v_i \in \mathcal{C}^1(\mathbb{R}) \cap \mathcal{C}^2(\mathbb{R} \setminus \{\underline{\delta}, \bar{\delta}\})$  and  $v_s \in \mathcal{C}^2(\mathbb{R})$  solve the differential equation problem (20), (19), (21), and that the conditions (22) and (23) hold. Then the control band  $\{\underline{\delta}, \delta^*, \bar{\delta}\}$  is the optimal policy and  $v_i(\delta), v_s(\delta)$  are the value functions of inactive and searching individuals respectively.*

*Proof.* See Appendix. □

Figure 2 shows graphically the optimal individual behavior (left panel) and the corresponding value functions (right panel). In the left panel, an individual's labor supply moves along the horizontal red line as long as she remains inactive; that is, her actual labor supply  $h$  is constant while her frictionless labor supply  $h^*$  tracks the evolution of her productivity. When she reaches the boundaries of the inaction region (the thick blue lines  $h^* = e^{-\bar{\delta}} h$  or  $h^* = e^{-\underline{\delta}} h$ ), she starts searching and, as soon as she receives an offer, adjusts up or down to a new labor supply level on the central

blue line  $h' = e^{\delta^*} h^*$ . The right panel shows the value functions  $v_i(\delta)$  and  $v_s(\delta)$  of inactives (in blue) and searchers (in red) as well as the optimal control band. The value of inactives is bell-shaped and reaches its maximum at  $\delta^*$ . When reaching the boundaries  $\underline{\delta}$  and  $\bar{\delta}$  of the inaction region, the individual becomes a searcher and her value function jumps up along the corresponding dashed blue line. The size of the jump, i.e. the difference between the two functions at these values, is exactly equal to the fixed cost  $\kappa$ . Finally, the dashed red curve is the function  $v_s(\delta) - \kappa$ , which illustrates that the two value functions meet tangentially at the boundaries of the inaction region (smooth-pasting).

Figure 2: Optimal control policy and value functions



**Effects of taxes on individual behavior.** I first describe how tax policy affects the frictionless income variables. Equation (10) implies that the effects of perturbing the parameters  $\tau, p$  of the tax schedule on the individual frictionless taxable income  $y^*$  are given by

$$\frac{d \ln y^*}{d \ln (1 - \tau)} = \frac{\varepsilon}{1 + p\varepsilon}, \quad \text{and} \quad \frac{d \ln y^*}{dp} = -\frac{\varepsilon}{1 + p\varepsilon} \ln y^*. \quad (24)$$

The interpretation of equation (24) is as follows. The behavioral change in income following a tax increase (both in  $\tau$  and in  $p$ ) is determined by the structural elasticity parameter  $\varepsilon$ . If the baseline tax system is linear, i.e.  $p = 0$ , (24) implies immediately that the elasticity of labor income  $y^*$  with respect to the net-of-tax rate  $1 - \tau$  is equal to  $\varepsilon$ . Suppose now that the baseline tax system is progressive or regressive, i.e.  $p \neq 0$ . Then a change in the marginal tax rate  $T'(y^*)$  that an individual faces induces a *direct* reduction of his labor income  $y^*$  by the amount  $\varepsilon$ , by definition of the labor income elasticity. This direct adjustment generates in turn an *indirect* change in the marginal tax rate that the individual faces, due to the non-linearity of the baseline tax schedule. The amount of this change is equal to  $d(T'(y)) = T''(y) dy$ , and it induces a further labor income adjustment given by the elasticity  $\varepsilon$  and the curvature  $p$  of the baseline tax schedule. Thus the total change in income following a perturbation of the net-of-tax rate  $(1 - T'(y^*))$  of an individual with

income  $y^*$  is given by

$$\frac{d \ln y^*}{d \ln (1 - T'(y^*))} = \frac{\varepsilon}{1 + T''(y^*) \frac{y^* \varepsilon}{1 - T'(y^*)}} = \frac{\varepsilon}{1 + p\varepsilon}. \quad (25)$$

Equations (24) and (25) thus show that, from the point of view of individuals, the effect on income of a percent perturbation of the parameter  $(1 - \tau)$  is equivalent to a percent perturbation of the net-of-tax rate  $(1 - T'(y))$  at every income level. Similarly, the effect of a perturbation of the parameter  $p$  is equivalent to perturbing the marginal tax rates faced by all individuals by an amount proportional to their log-income, so that the magnitude of the tax increase raises with income.

Taxes also affect the laws of motion of the frictionless income variables. Equations (11) imply that the effects of perturbing the parameters  $\tau, p$  on the drift and volatility of the frictionless taxable and disposable income processes are given by:

$$\begin{aligned} \frac{d \ln \{|\mu_y|, |\sigma_y|\}}{d \ln (1 - \tau)} = 0, \quad \text{and} \quad \frac{d \ln \{|\mu_y|, |\sigma_y|\}}{dp} = -\frac{\varepsilon}{1 + p\varepsilon} < 0, \\ \frac{d \ln \{|\mu_c|, |\sigma_c|\}}{d \ln (1 - \tau)} = 0, \quad \text{and} \quad \frac{d \ln \{|\mu_c|, |\sigma_c|\}}{dp} = -\frac{1}{1 - p} \frac{1 + \varepsilon}{1 + p\varepsilon} < 0, \end{aligned} \quad (26)$$

where the absolute values allow for the possibility that the drift and volatility parameters are negative. Thus a higher rate of progressivity of the tax schedule lowers the drift and the volatility of both the taxable and disposable income processes. Intuitively, individual income responses following an increase in productivity are attenuated by the fact that higher incomes pay higher marginal tax rates if the tax schedule is progressive. Note that a uniform change in the marginal tax rates (i.e., a change in  $\tau$ ), on the other hand, does not affect the drift or the volatility of income since all incomes are shifted by a proportional amount.

Next, I turn to the effects of taxes on the optimal individual adjustment policy in the frictional model. The parameter  $\tau$  has no effect on the optimal behavior  $\{\underline{\delta}, \delta^*, \bar{\delta}\}$ . Equation (26) shows that a decrease in  $p$ , however, increases the volatilities  $\sigma_y, \sigma_\delta$  of the income and the deviation processes, which raises the option value of waiting to adjust labor supply, and therefore widens the optimal inaction region:

$$\frac{d \ln \{|\underline{\delta}|, \bar{\delta}\}}{d \ln (1 - \tau)} = 0, \quad \text{and} \quad \frac{d \ln \{|\underline{\delta}|, \bar{\delta}\}}{dp} < 0. \quad (27)$$

Intuitively, this is because a less progressive tax schedule magnifies the unexpected shocks to the wage. This raises the incentives for the individual to keep her current job and “wait and see” the evolution of her productivity before carrying out the adjustment, in order to save on new search costs.<sup>6</sup>

The drift and volatility of the labor supply deviation (or of frictionless hours) is also affected by

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<sup>6</sup>The same option value effect is at play in the Mortensen and Pissarides (1994) search model with endogenous job destruction.



taxes. Specifically,

$$\frac{d \ln \{|\mu_\delta|, |\sigma_\delta|\}}{d \ln (1 - \tau)} = 0, \quad \text{and} \quad \frac{d \ln \{|\mu_\delta|, |\sigma_\delta|\}}{dp} = -\frac{1}{1-p} \frac{1 + \varepsilon}{1 + p\varepsilon} < 0, \quad (28)$$

Importantly, note that a lower rate of progressivity has an ambiguous effect on the frequency of adjustment: on the one hand, the higher volatility of the deviation process makes individuals reach the boundaries of their inaction region faster; on the other hand, the inaction region is wider, which tends to make them adjust less often. In practice, however, the volatility effect typically dominates the size-of-the-bands effect, so that a less progressive tax schedule increases the frequency of adjustment. Intuitively, the desired (frictionless) income moves away from the current income faster, so that it is optimal for the individual to carry out the costly adjustment more often.

**Individual welfare.** In the frictionless model, the value function  $\mathcal{V}^*(y)$  of an individual with current income  $y$  is given by

$$\mathcal{V}^*(y) = \frac{1}{\rho_1 + \rho_2 - (1 - \gamma) \mu_c - \frac{1}{2} (1 - \gamma)^2 \sigma_c^2} g \left( \frac{1 + p\varepsilon}{1 + \varepsilon} \frac{1 - \tau}{1 - p} y^{1-p} \right). \quad (29)$$

Note that in this expression the relevant discount rate  $\rho$  used to compute the present value of utility (the denominator of (29)) depends on the growth rate of the future utility of consumption, and is therefore endogenous to taxes.

In the frictional model, consider an inactive or searching individual with *actual* (as opposed to frictionless) income  $y$  and deviation  $\delta$ , and let  $\tilde{\mathcal{V}}_x(y, \delta) \equiv \mathcal{V}_x(ye^{-\delta}, \delta)$  for  $x \in \{i, s\}$  denote her value functions. Equation (16) implies that this value function is equal to the value  $\mathcal{V}^*(y)$  that the planner would compute for an individual with income  $y$  *assuming wrongly* that the world is frictionless, times a correction factor  $\tilde{v}_x(\delta)$  which depends on the deviation  $\delta$  of her income away from its desired level. That is, we have

$$\tilde{\mathcal{V}}_x(y, \delta) = \mathcal{V}^*(y) \times \tilde{v}_x(\delta), \quad (30)$$

where the values of deviation  $\tilde{v}_i(\delta), \tilde{v}_s(\delta)$  conditional on an *actual* income level are defined, for  $x \in \{i, s\}$ , by

$$\tilde{v}_x(\delta) = \left( \rho_1 + \rho_2 - (1 - \gamma) \mu_c - \frac{1}{2} (1 - \gamma)^2 \sigma_c^2 \right) \left( \frac{1 + p\varepsilon}{1 + \varepsilon} \right)^{\gamma-1} e^{-(1-p)(1-\gamma)\delta} v_x(\delta). \quad (31)$$

These correction terms  $\tilde{v}_x(\delta)$  are strictly decreasing in the deviation  $\delta$ , as individuals who earn a given income  $y$  but work fewer hours (i.e., have a higher wage) get a higher utility than those who earn the same income but work more hours at a lower wage. I finally denote by  $\tilde{\mathcal{V}}(y, \delta)$  and  $\tilde{v}(y, \delta)$  the weighted averages of the value functions of inactive and searching individuals, that is,

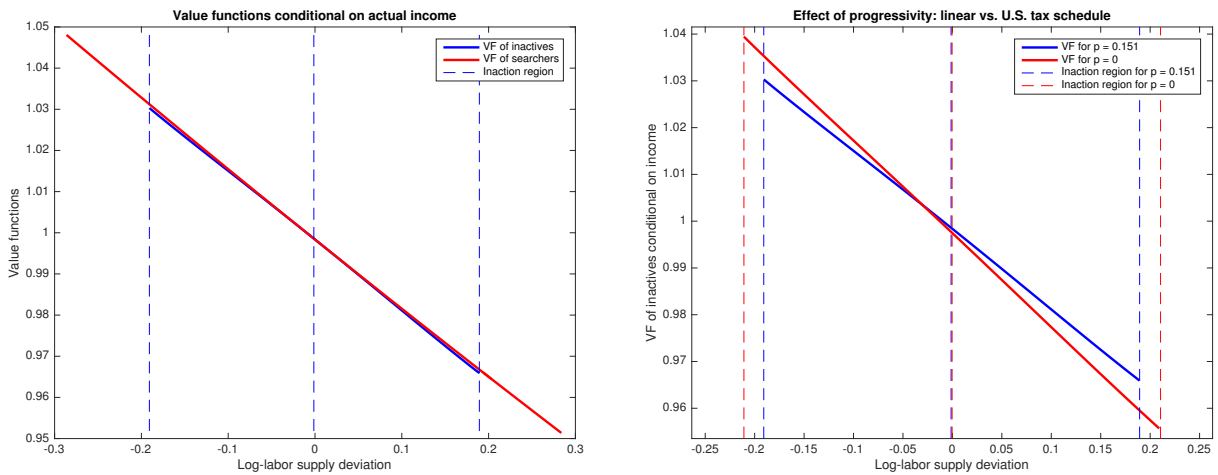
$$\tilde{v}(y, \delta) = \tilde{v}_i(\delta) \frac{f_{y,\delta}^i(y, \delta)}{f_{y,\delta}(y, \delta)} + \tilde{v}_s(\delta) \frac{f_{y,\delta}^s(y, \delta)}{f_{y,\delta}(y, \delta)}, \quad \text{and} \quad \tilde{\mathcal{V}}(y, \delta) = \mathcal{V}^*(y) \tilde{v}(y, \delta), \quad (32)$$

where  $f_{y,\delta}^i, f_{y,\delta}^s$  denote the stationary joint densities of inactive and searching individuals at income and deviation  $(y, \delta)$  (see Section 4), and  $f_{y,\delta} = f_{y,\delta}^i + f_{y,\delta}^s$  is the total stationary density of individuals at  $(y, \delta)$ .

Unlike the frictionless environment, in which there is a representative agent at each income level  $y$ , the labor supply adjustment frictions imply that there is a *heterogeneous* population of individuals who earn the same income but reach different welfare levels – both because their wage-hours bundles vary, and because their employment status (inactive or searching) differ. Importantly, this distribution of utilities within income levels, summarized by the function  $\tilde{v}$ , depends on the progressivity parameter  $p$  and is thus *endogenous* to tax policy. I describe the implications of these observations for optimal taxation in Section 5.

Figure 3 shows graphically the distributions of utilities within income groups  $\tilde{v}_i(\delta), \tilde{v}_s(\delta)$  (left panel) and the effects of perturbing the progressivity of the tax schedule on the optimal inaction region and on this distribution (right panel). The left panel shows that (both inactive and searching) individuals with higher deviation but the same income are worse off. As in Figure 2, within the inaction region the value of searching is always strictly higher than, although very close to, the value of inactivity. The right panel thus shows only the value of inactives  $\tilde{v}_i(\delta)$ . As the progressivity decreases (linear versus U.S. tax schedule), the inaction region widens, as discussed above, and the distribution of utilities adjusts endogenously within the new bands. [Note: more on this: explain the direction of change.]

Figure 3: Value function conditional on income  $\tilde{v}(\delta)$  and effects of progressivity  $p$



## 4 Aggregation

In this section I analyze the properties of the long-run wage and income distributions obtained by aggregating the optimal individual policies described in Section 3.

## 4.1 Stationary wage distribution

We say that the variable  $x$  has a double-Pareto-lognormal distribution (DPLN), or that  $\ln x$  has a Normal-Laplace distribution, with parameters  $(r_1, r_2, m, s^2)$  if its density is given by

$$f_x(x) = \frac{|r_1| r_2}{|r_1| + r_2} \left\{ e^{\frac{1}{2} r_1^2 s^2 - r_1 m} x^{r_1 - 1} \Phi\left(\frac{\ln x - m}{s} + r_1 s\right) + e^{\frac{1}{2} r_2^2 s^2 - r_2 m} x^{r_2 - 1} \Phi^c\left(\frac{\ln x - m}{s} + r_2 s\right) \right\}. \quad (33)$$

The double-Pareto-lognormal distribution closely approximates the actual wage and income distributions observed empirically (see, e.g., Reed (2003), Reed and Jorgensen (2004), Toda (2012)). The following proposition shows that the wage distribution converges to a DPLN stationary distribution:

**Proposition 3.** *The distribution of wages  $w$  converges towards a unique stationary distribution  $f_w(\cdot)$  which is double-Pareto-lognormal with parameters  $(r_{1,w}^{\rho_2}, r_{2,w}^{\rho_2}, m_w, s_w^2)$ , where  $r_{1,w}^{\rho_2}, r_{2,w}^{\rho_2}$  are defined in (17) and  $m_w, s_w^2$  are the mean and variance of the wage distribution at birth  $f_{w_0}(\cdot)$ . In particular, the stationary wage distribution  $f_w(\cdot)$  exhibits power-law behavior in both tails, with Pareto coefficients on the right and left tail respectively given by  $(r_{1,w}^{\rho_2}, r_{2,w}^{\rho_2})$ , that is,*

$$f_w(w) \underset{w \rightarrow 0}{\sim} w^{r_{2,w}^{\rho_2} - 1}, \quad \text{and} \quad f_w(w) \underset{w \rightarrow \infty}{\sim} w^{r_{1,w}^{\rho_2} - 1}. \quad (34)$$

*Proof.* See Appendix. □

The lognormal “bulk” of the wage distribution is inherited from the lognormal density of productivities at birth (or entry into the labor force)  $f_{w_0}(\cdot)$ . The aggregation of the random growth individual wage processes naturally generates the distribution’s Pareto tails (see, e.g., Nirei and Souma (2007), Gabaix (2009)), which form one of the most well-known empirical regularities (discovered by Pareto (1896)). The wage process (2) therefore fits both the microeconomic empirical evidence (see Section 2) and the macroeconomic evidence. The higher the Pareto coefficient (in absolute value), the thinner the tail, the more equal the distribution. A higher drift  $\mu_w$ , a higher volatility  $\sigma_w$  of individual income, and a lower death (or retirement) rate  $\rho_2$ , lead to a more unequal distribution, i.e., a smaller value of  $|r_{1,w}^{\rho_2}|$ .

Note that the frictionless taxable and disposable incomes  $y^*, c^*$  are also log-normally distributed at birth with respective mean and variance  $(m_y, s_y)$  and  $(m_c, s_c)$  (see Appendix) and follow random growth processes (11) from then on. Hence their corresponding stationary distributions  $f_{y^*}, f_{c^*}$  are also double-Pareto lognormal with respective parameters  $(r_{1,y}^{\rho_2}, r_{2,y}^{\rho_2}, m_y, s_y^2)$  and  $(r_{1,c}^{\rho_2}, r_{2,c}^{\rho_2}, m_c, s_c^2)$ .

## 4.2 Stationary income distributions

I now characterize the stationary joint distributions  $f_{\ln y^*, \delta}^i$  and  $f_{\ln y^*, \delta}^s$  of frictionless taxable incomes  $\ln y^*$  and labor supply deviations  $\delta$  for inactive and searching individuals, respectively. Denote by  $f_1, f_2$  their partial derivatives with respect to the first and second variables, and  $f_{11}, f_{12}, f_{22}$  their

second partial derivatives. We have  $f^i = 0$  for all  $\delta < \underline{\delta}$  and  $\delta > \bar{\delta}$ . Moreover, for all  $\ln y^* \in \mathbb{R}$ , all  $\delta \in (\underline{\delta}, \delta^*) \cup (\delta^*, \bar{\delta})$  if  $f = f^i$ , and all  $\delta \in \mathbb{R} \setminus \{\underline{\delta}, \bar{\delta}\}$  if  $f = f^s$ , these distributions are the solutions to the following Kolmogorov-forward (or Fokker-Planck) equations:

$$0 = -(\rho_2 + q\mathbb{I}_{f^s})f - \mu_y f_1 + \mu_\delta f_2 + \frac{1}{2}\sigma_y^2 f_{11} + \frac{1}{2}\sigma_\delta^2 f_{22} - \sigma_y \sigma_\delta f_{12}, \quad (35)$$

where  $\mathbb{I}_{f^s}$  is equal to one if  $f = f_{\ln y^*, \delta}^s$  and zero if  $f = f_{\ln y^*, \delta}^i$ .

The Kolmogorov forward equations (35) have the following interpretation. At a given frictionless income level  $\ln y^*$ , the density of individuals with deviation  $\delta \notin \{\underline{\delta}, \delta^*, \bar{\delta}\}$  is reduced by those who move away from there, and is increased by those who move to  $\delta$  from a former deviation  $\delta' \neq \delta$ , following either an increase in their wage if  $\delta' > \delta$  (so that they would now like to work more), or a decrease in their wage if  $\delta' < \delta$ . These flows occur both because of the drift  $\mu_y$  (second and third terms of (35)) and the volatility  $\sigma_y$  (fourth to sixth terms of (35)) of individual incomes and deviations. Moreover, at any point  $(\ln y^*, \delta)$ , the distribution loses mass at rate  $\rho_2$  (due to the movements out of the labor force) plus  $q$  for the individuals who are searching (due to the exogenous adjustment opportunities they receive). In the steady-state, these flows in and out of  $(\ln y^*, \delta)$  must balance on net and are thus equal to zero. Note that these equations do not hold at  $\delta^*$  for  $f_{\ln y^*, \delta}^i$ , and at  $\{\underline{\delta}, \bar{\delta}\}$  for  $f_{\ln y^*, \delta}^s$ , where the inflows from births and from endogenous adjustments produce kinks in the densities.

The boundary conditions of the partial differential equations (35) are the following. First, the density functions  $f_{\ln y^*, \delta}^i$  and  $f_{\ln y^*, \delta}^s$  are continuous in  $\delta^*$  and  $\{\underline{\delta}, \bar{\delta}\}$  respectively: for all  $u \in \mathbb{R}$ ,

$$f_{\ln y^*, \delta}^{i,s}(u, \delta^-) = f_{\ln y^*, \delta}^{i,s}(u, \delta^+), \text{ for } \delta \in \{\underline{\delta}, \delta^*, \bar{\delta}\}. \quad (36)$$

Second, the boundaries  $\underline{\delta}$  and  $\bar{\delta}$  are absorbing for  $f_{\ln y^*, \delta}^i$ , so that there is no mass of inactive individuals at the edges of the inaction region: for all  $u \in \mathbb{R}$ ,

$$f_{\ln y^*, \delta}^i(u, \underline{\delta}) = f_{\ln y^*, \delta}^i(u, \bar{\delta}) = 0. \quad (37)$$

Intuitively, this is because individuals who reach a boundary of their inaction region immediately start searching and leave the inaction state. Third, the density of searchers in a given job converges to zero as  $\delta \rightarrow \pm\infty$ : for all  $u \in \mathbb{R}$ ,

$$\lim_{\delta \rightarrow \pm\infty} f_{\ln y^*, \delta}^s \left( u - \frac{\sigma_y}{\sigma_\delta} \delta, \delta \right) = 0. \quad (38)$$

Fourth, total flows in and out of  $\underline{\delta}, \delta^*, \bar{\delta}$  must balance, which yields three functional equations linking the density functions  $f_{\ln y^*, \delta}^i$  and  $f_{\ln y^*, \delta}^s$ . Letting  $\hat{f}^x$  denote the function  $\left( \frac{\sigma_y}{\sigma_\delta} f_1^x + f_2^x \right)$  for  $x \in \{i, s\}$ ,

these conditions write: for all  $u \in \mathbb{R}$ ,

$$\begin{aligned} \hat{f}^i(u, \delta^{*+}) - \hat{f}^i(u, \delta^{*-}) &= \frac{2\rho_2}{\sigma_\delta^2} f_{\ln y_0^*}(u) + \frac{2q}{\sigma_\delta^2} f_{\ln y^*}^s(u), \\ \hat{f}^i(u, \underline{\delta}^+) + \hat{f}^s(u, \underline{\delta}^+) - \hat{f}^s(u, \underline{\delta}^-) &= 0, \\ \hat{f}^i(u, \bar{\delta}^-) + \hat{f}^s(u, \bar{\delta}^-) - \hat{f}^s(u, \bar{\delta}^+) &= 0. \end{aligned} \tag{39}$$

These equations equate the inflows and outflows of individuals going from one state (inaction, search, non-participation) into another, following a change in their wage and hence desired hours, the reception of a new job opportunity, or a “birth”. Finally, a normalizing condition imposing that the total mass of individuals in the population is equal to one completes the full characterization of the economy’s steady-state:

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} [f_{\ln y^*, \delta}^i(u, \delta) + f_{\ln y^*, \delta}^s(u, \delta)] dud\delta = 1. \tag{40}$$

In the Appendix I show that the densities of inactive and searching individuals can be expressed analytically in terms of a single function  $\tilde{f}(\cdot)$  which satisfies a simple integral equation, allowing us to easily compute numerically the steady-state of the economy.

**Proposition 4.** *The unique stationary distributions  $f_{\ln y^*, \delta}^i$  and  $f_{\ln y^*, \delta}^s$  of inactive and searching individuals are fully characterized by equations (36), (37), (38), (39), and (40). The stationary distributions of taxable and disposable incomes  $f_y, f_c$  have Pareto right and left tails with respective Pareto coefficients  $(r_{1,y}^{\rho_2}, r_{2,y}^{\rho_2})$  and  $(r_{1,c}^{\rho_2}, r_{2,c}^{\rho_2})$ .*

*Proof.* See Appendix. □

The Pareto coefficients of the tails of the taxable and disposable income distributions are given in closed form, as a function of the exogenous Pareto coefficients of the wage distribution, by

$$\left\{ r_{1,y}^{\rho_2}, r_{2,y}^{\rho_2} \right\} = \frac{1+p\varepsilon}{1+\varepsilon} \left\{ r_{1,w}^{\rho_2}, r_{2,w}^{\rho_2} \right\}, \quad \text{and} \quad \left\{ r_{1,c}^{\rho_2}, r_{2,c}^{\rho_2} \right\} = \frac{1}{1-p} \frac{1+p\varepsilon}{1+\varepsilon} \left\{ r_{1,w}^{\rho_2}, r_{2,w}^{\rho_2} \right\}. \tag{41}$$

These expressions show that the elasticity of labor income  $\varepsilon$  and the parameter  $p$  of the tax system determine the amount by which inequality in exogenous productivity or wage translates into income inequality. In particular, the distribution of desired taxable income is more unequal (thicker tail) than the wage distribution, because wage differences are amplified by the positive labor supply elasticity  $\varepsilon$ . This is consistent with the findings of Krueger, Perri, Pistaferri and Violante (2009). Importantly, the Pareto coefficients (41) of the income distributions are endogenous to tax policy and obtained in closed-form as a function of the progressivity  $p$  of the tax schedule.

An increase in the parameter  $\tau$  of the tax schedule does not affect the Pareto coefficients. This is because these coefficients depend on the ratio between the average income above a threshold  $\bar{y}$ , and the threshold  $\bar{y}$  (as  $\bar{y} \rightarrow \infty$ ). Changing the scaling parameter  $\tau$  has the same multiplicative effect on the incomes of each individual, so that the ratio is unaffected. Note also that the Pareto

coefficients satisfy  $r_{1,y}^{\rho_2} \leq r_{1,c}^{\rho_2} < r_{1,w}^{\rho_2}$  if  $p \geq 0$  and  $r_{1,c}^{\rho_2} \leq r_{1,y}^{\rho_2} < r_{1,w}^{\rho_2}$  if  $p \leq 0$ , where the inequalities are strict if  $p \neq 0$ . The distribution of frictionless disposable income is less unequal (thinner tail) than the distribution of desired taxable income if and only if the tax schedule is progressive,  $p > 0$ .

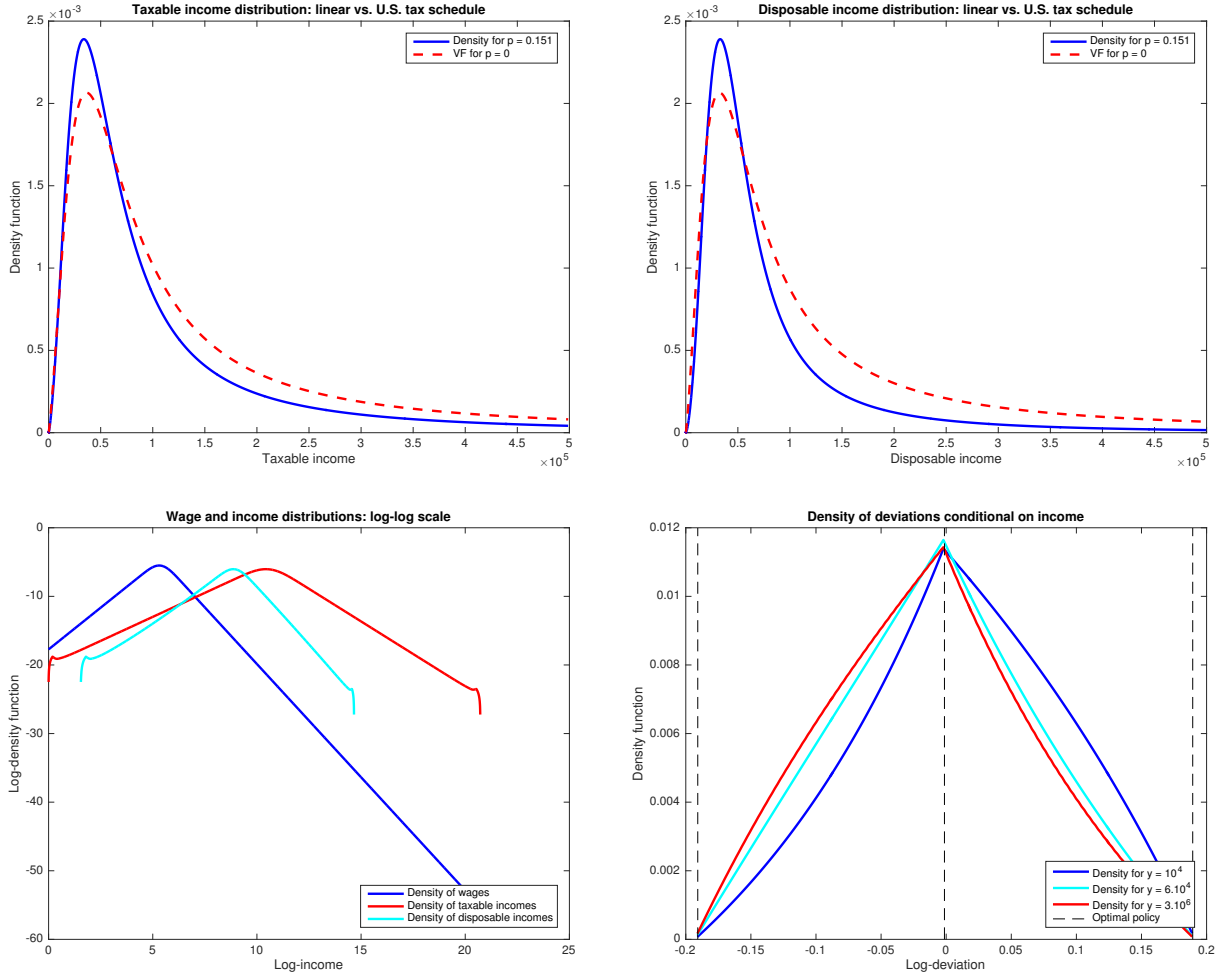
The Pareto coefficients of the income distributions are increasing in the progressivity  $p$ , i.e., both the before-tax and the after-tax income distributions have a thinner tail (are less unequal) when the tax schedule is more progressive. Specifically, we find:

$$\frac{d \ln \left\{ \left| r_{1,y}^{\rho_2} \right|, r_{2,y}^{\rho_2} \right\}}{dp} = \frac{\varepsilon}{1 + p\varepsilon}, \quad \text{and} \quad \frac{d \ln \left\{ \left| r_{1,c}^{\rho_2} \right|, r_{2,c}^{\rho_2} \right\}}{dp} = \frac{1}{1 - p} \frac{1 + \varepsilon}{1 + p\varepsilon}. \quad (42)$$

Thus the effect of progressivity on the tails of the after-tax income distribution is stronger than on those of the pre-tax income distribution. That is, a more progressive tax schedule reduces inequality in after-tax incomes more than it reduces inequality in pre-tax incomes.

Figure 4 summarizes these results graphically and show the stationary distributions of wages, incomes and deviations in the economy, as well as the effects of taxes on the income distributions. The top two graphs of Figure 4 show the distribution of taxable incomes (left panel) and disposable incomes (right panel), and their change when the tax schedule goes from the U.S. rate of progressivity to a linear tax rate. The mean and variance of both distributions are lower when  $p$  is higher; the tails are thinner, and to a much larger extent in the case of the disposable income distribution than the taxable income distribution. The bottom left panel of Figure 4 shows the wage, taxable income and disposable income distributions in log-log scale. This representation illustrates clearly the fact that those distributions all have left and right Pareto tails, corresponding to the asymptotic straight lines whose slopes are equal to the Pareto coefficients. The smaller the slopes in absolute value, the more unequal the distribution: the wage (or productivity) distribution is the most equal, the taxable income distribution is the most unequal (due to the positive labor supply elasticity); the inequality of disposable incomes is smaller than that of taxable incomes and closer to that of wages due to the positive rate of progressivity. Finally, the bottom right panel of Figure 4 shows the stationary distributions of deviations  $\delta$  conditional on several given income levels  $y$  for inactive individuals, along with the boundaries of the optimal inaction region  $\{\underline{\delta}, \delta^*, \bar{\delta}\}$ .

Figure 4: Stationary income distributions



## 5 Optimal taxation

In this section, I analyze the effects of taxes on long-run social welfare (comparative statics across steady-states) in order to characterize the optimal tax schedule, i.e., the solution to the government's problem (8,9). Before deriving these formulas, I formally define and characterize two sets of key variables for the analysis of tax policy: the labor income elasticities and the marginal social welfare weights.

### 5.1 Labor income elasticities

I define the *intensive margin labor income elasticity*,  $\eta(y^*)$ , as the elasticity of an individual's *frictionless* (or desired) taxable income  $y^*$  with respect to the net of tax rate. We saw in Section 3 that this elasticity is constant across individuals and is given by the (normalized) structural elasticity parameter  $\varepsilon$ .

**Definition 1.** The (frictionless) intensive margin labor income elasticity is defined as

$$\boldsymbol{\eta}(y^*) \equiv \frac{d \ln y^*}{d \ln (1 - T'(y^*))} = \frac{\varepsilon}{1 + p\varepsilon}, \quad \forall y^* \in \mathbb{R}_+, \quad (43)$$

where the second equality follows from equations (24,25).

In a *frictionless* world ( $\kappa = 0$ ), this variable  $\boldsymbol{\eta}(y)$  would be equal to the response of the individual's true income to a change in the marginal tax rates. It could be estimated empirically by observing the magnitude of the increase in income following an increase in statutory net-of-tax rates (see, e.g., Gruber and Saez (2002)). However, when the adjustment of labor supply in response to changes in the marginal tax rates is *frictional* ( $\kappa > 0$ ), the individual elasticity of *actual* income is either equal to zero, if the agent has not yet adjusted her income in response to the tax change (short-run elasticity), or infinite, since upon adjustment a small tax increase induces a discrete jump in income. In this environment we can nevertheless define and observe empirically the *long-run* elasticity of *aggregate* labor income with respect to a *uniform* change in net-of-tax rates. Formally,

**Definition 2.** The macro labor income elasticity  $\boldsymbol{H}$  is defined as

$$\boldsymbol{H} \equiv \frac{d \ln \left[ \int_0^\infty y f_y(y) dy \right]}{d \ln (1 - \tau)}, \quad (44)$$

where  $f_y(\cdot)$  is the stationary income distribution given the tax schedule  $\{\tau, p\}$ .

The following proposition provides a *neutrality result* that characterizes the relationship between the micro and the macro elasticities in the model:

**Proposition 5.** *The micro (intensive margin) elasticity and the macro elasticity of labor income with respect to the marginal tax rates are equal, that is,*

$$\boldsymbol{H} = \boldsymbol{\eta}(y) = \frac{\varepsilon}{1 + p\varepsilon}, \quad \forall y \in \mathbb{R}_+. \quad (45)$$

*Proof.* See Appendix. □

Proposition 5 shows that in the frictionless model, the *long-run aggregate* labor income elasticity  $\boldsymbol{H}$  is equal to the elasticity of *individual frictionless* income  $y^*$ , even though there is always a non-degenerate distribution of individuals with *actual* incomes  $y$  for a given level of frictionless income  $y^*$  in the steady-state of the model. Intuitively, in the long-run, individuals have had time to fully adjust their behavior to the new tax schedule, and even though in general they do not actually earn their desired income  $y^*$  at any given moment in time, the errors wash out in the aggregate and the magnitude of the aggregate response to the tax change is driven by the structural elasticity parameter  $\varepsilon$ . In other words, in the case of a uniform increase in the net-of-tax rates (i.e., a change in the parameter  $\tau$ ), the economy behaves in the long-run as if there were a representative (frictionless) agent at each income level. Note finally that the result of Proposition 5 allows us to use the effect of



tax changes on long-run aggregate income to estimate empirically the structural individual elasticity parameter  $\varepsilon$  when individual labor supply is lumpy.

Next, I define three *extensive margin labor income elasticity* concepts as the effects on the income distribution at a given level  $y$  of changes in the individual adjustment triggers and target  $\{\underline{\delta}, \delta^*, \bar{\delta}\}$ , keeping the other parameters of the models (in particular,  $\mu_\delta, \sigma_\delta$ ) unchanged:

**Definition 3.** The extensive margin labor income elasticities  $\Xi(y)$ ,  $\Xi^*(y)$ , and  $\bar{\Xi}(y)$  are defined as

$$\Xi(y) \equiv \frac{\partial \ln f_y(y)}{\partial \ln |\underline{\delta}|}, \quad \Xi^*(y) \equiv \frac{\partial \ln f_y(y)}{\partial \ln |\delta^*|}, \quad \text{and} \quad \bar{\Xi}(y) \equiv \frac{\partial \ln f_y(y)}{\partial \ln \bar{\delta}}, \quad (46)$$

where  $f_y(y)$  is the stationary density of incomes given the individual adjustment policy  $\{\underline{\delta}, \delta^*, \bar{\delta}\}$ .

These elasticities capture the effects on the number of employed workers at income  $y$  of percentage variations in the inaction thresholds. While the standard intensive margin elasticity of Definition 1 affects the purely *frictionless* part of labor income, the extensive margin elasticities affect the purely *frictional* part of labor income through the endogenous *option value of adjusting labor supply*. The key difference between these extensive margin elasticities and those typically defined in the literature (e.g., Saez (2002), Jacquet, Lehmann and Van der Linden (2013)) is that in Definition 3 the income thresholds are not exogenously given as in the case of a 0-1 pure participation decision, but instead are endogenously and optimally generated by the individual's frictional labor supply.

Finally, recall that the behavior of frictionless income  $y^*$  is completely characterized by four parameters  $\mathbb{Y} = \{\mu_y, \sigma_y, m_y, s_y\}$ , namely the mean and standard deviation of its distribution at birth, and the drift and the volatility of its process. These four parameters all have the same elasticity with respect to  $p$ , namely,  $-\frac{\varepsilon}{1+p\varepsilon}$ . Suppose that the parameters describing frictionless incomes  $y^*$  are perturbed in the same proportion, i.e.  $d \ln \mu_y = d \ln \sigma_y = d \ln m_y = d \ln s_y \equiv d \ln \mathbb{Y}$ , while the parameters  $\mathbb{D} = \{\mu_\delta, \sigma_\delta, \underline{\delta}, \delta^*, \bar{\delta}\}$  describing the deviations  $\delta$  remain unchanged, that is  $d\mu_\delta = d\sigma_\delta = d\underline{\delta} = d\delta^* = d\bar{\delta} \equiv 0$ . The *relative margin labor income elasticity*  $\xi(y)$  is defined as the effect on the density of income  $y$  of this perturbation:

**Definition 4.** The relative margin labor income elasticity  $\xi(y)$  is defined as

$$\xi(y) \equiv \frac{1 - F_y(y)}{y f_y(y)} \frac{\partial \ln(1 - F_y(y))}{\partial \ln \mathbb{Y}} \Big|_{\mathbb{D}} = - \int_{-\infty}^{\infty} \delta f_{\delta|y}(\delta|y) d\delta, \quad (47)$$

where  $\frac{\partial}{\partial \ln \mathbb{Y}} \Big|_{\mathbb{D}}$  is a shorthand notation to denote the effect of an infinitesimal proportional change in  $\mu_y, \sigma_y, m_y, s_y$ , keeping  $\mu_\delta, \sigma_\delta, \underline{\delta}, \delta^*, \bar{\delta}$  constant, and where the second equality is proved in the Appendix.

Intuitively, we have  $\xi(y) = - \left( \frac{\partial \ln(1 - F_y)}{\partial \ln y} \right)^{-1} \frac{\partial \ln(1 - F_y)}{\partial \ln \mathbb{Y}} = \frac{\partial \ln y}{\partial \ln \mathbb{Y}}$ , which shows that  $\xi(y)$  can be interpreted as the elasticity of income with respect to a change in the parameters driving frictionless income, relative to those driving deviations. The second equality in (47) shows that the elasticity  $\xi(y)$  at income  $y$  is simply given by the average deviation  $\mathbb{E}[\delta|y]$  within the population earning

income  $y$ . Thus, while the intensive and extensive margin elasticities affect the purely frictionless and frictional parts of labor income, the relative margin elasticity affects income through a *relative* change in the processes driving frictionless incomes and deviations (or hours).

## 5.2 Marginal social welfare weights

The social welfare effects of taxation can be characterized using the notion of *marginal social welfare weights* (see, e.g., Saez and Stantcheva (2014) for a recent exposition). Intuitively, in a standard frictionless static model, the welfare weight at income  $y$  is defined as the increase in social welfare, expressed in terms of public revenue, of distributing uniformly among individuals who earn income  $y$  an additional unit of consumption. With a utilitarian social objective, this welfare weight is then equal to the individual's marginal utility of consumption normalized by the shadow value of public funds,  $\lambda^{-1}c_0^{-\gamma}$ .<sup>7</sup> In this section I define formally and generalize the notions of marginal social welfare weights to the *dynamic* and *frictional* model.

First consider, in the frictionless model, the effect of giving an individual with income  $y$  an additional marginal consumption stream  $\{\hat{c}_t\}_{t \geq 0}$  which evolves stochastically over time according to the same process  $(\mu_c, \sigma_c)$  as her frictionless disposable income  $c_t^*$ . The marginal social welfare weight at income  $y$ ,  $\varphi^*(y)$ , is defined as the change in the present discounted value of her utility (and hence, in utilitarian social welfare) due to this additional consumption stream. It is given by:

$$\begin{aligned} \varphi^*(y) &\equiv \frac{1}{\lambda^*} \frac{d}{d\hat{c}_0} \left( \mathbb{E}_0 \left[ \int_0^\infty e^{-(\rho+\beta)t} u \left( c_t^* + \hat{c}_t - \frac{1}{1+1/\varepsilon} (h_t^*)^{1+1/\varepsilon} \right) dt \middle| y_0 = y \right] \right)_{\hat{c}_0=0} \\ &= \frac{1}{\lambda^*} \frac{\left( \frac{1+p\varepsilon}{1+\varepsilon} \right)^{-\gamma} \left( \frac{1-\tau}{1-p} \right)^{-\gamma} y^{-\gamma(1-p)}}{\rho_1 + \rho_2 - (1-\gamma)\mu_c - \frac{1}{2}(1-\gamma)^2\sigma_c^2}, \end{aligned} \quad (48)$$

where  $\lambda^*$  is the marginal value of public funds in the frictionless model. Now, in the *frictional* model, consider the effect on the welfare of an individual with current income  $y$  and current deviation  $\delta$  of giving her an additional marginal consumption stream  $\{\hat{c}_t\}_{t \geq 0}$  as described above. These additional units of consumption do not have the same effect on individuals who earn the same income  $y$  but have different deviations  $\delta$  (i.e., different wages) or employment states  $x \in \{i, s\}$ : the change in the individual  $(y, \delta, x)$ 's utility is the same as if she earned income  $y$  in the frictionless model, times the correction factor  $\tilde{v}_x(\delta)$  defined in (31). Since the income tax system treats all individuals with the same income identically, we define the *static intensive marginal social welfare weight*  $\varphi(y)$  as the long-run social value of distributing the additional stream  $\{\hat{c}_t\}_{t \geq 0}$  uniformly among all the individuals who earn the same income  $y$ , independently of their deviations  $\delta$  or their employment state  $x$ . That is,

**Definition 5.** The static intensive marginal social welfare weight  $\varphi(y)$  is defined as

$$\varphi(y) = \frac{\lambda^* \varphi^*(y)}{\lambda} \times \int_{-\infty}^{\infty} \tilde{v}(y, \delta) f_{\delta|y}(\delta|y) d\delta, \quad (49)$$

<sup>7</sup>In particular, note that these weights are endogenous.

where  $\varphi^*(y)$  is the corresponding frictionless weight defined in equation (48),  $\tilde{v}(y, \delta)$  is defined in equation (32), and  $f_{\delta|y} = f_{\delta|y}^i + f_{\delta|y}^s$  is the total density of deviations conditional on an actual income  $y$ .

Second, in the dynamic model a permanent change in progressivity affects not only the *levels* of current and future consumption, but also the *growth rate* of (the utility from) consumption  $(1 - \gamma)\mu_c + \frac{1}{2}(1 - \gamma)^2\sigma_c^2$  through the drift and volatility of the consumption process, and hence the discount rate  $\rho = \rho + \beta - (1 - \gamma)\mu_c - \frac{1}{2}(1 - \gamma)^2\sigma_c^2$  that individuals use to compute their present discounted value of utility (equation (29)). I thus define the *dynamic intensive marginal social welfare weight*  $\psi(y)$  as the effect of a percentage decrease in the discount rate  $\rho$  on the individual's welfare. In the frictionless model, this is given by

$$\psi^*(y) = -\frac{1}{\lambda^*} \frac{\partial \mathcal{V}^*(y)}{\partial \ln \rho} = \frac{1}{\lambda^*} \frac{\frac{1}{1-\gamma} \left( \frac{1+p\varepsilon}{1+\varepsilon} \frac{1-\tau}{1-p} y^{1-p} \right)^{1-\gamma}}{\rho_1 + \rho_2 - (1-\gamma)\mu_c - \frac{1}{2}(1-\gamma)^2\sigma_c^2}. \quad (50)$$

Similarly, in the frictional model I define:

**Definition 6.** The dynamic intensive marginal social welfare weight  $\psi(y)$  is defined as

$$\psi(y) = \frac{\lambda^* \psi^*(y)}{\lambda} \times \int_{-\infty}^{\infty} \tilde{v}(y, \delta) f_{\delta|y}(\delta|y) d\delta, \quad (51)$$

where  $\psi^*(y)$  is the corresponding frictionless weight given by equation (48).

Third, paralleling our discussion leading to Definition 3, I define the *extensive marginal social welfare weights*  $\Omega_i(y)$  as the effects of changes in the labor supply adjustment policy  $\{\underline{\delta}, \delta^*, \bar{\delta}\}$  on the welfare at the income level  $y$ :

**Definition 7.** The extensive marginal social welfare weights  $\{\Omega_i(y)\}_{1 \leq i \leq 3} = \{\underline{\Omega}(y), \Omega^*(y), \bar{\Omega}(y)\}$  and  $\Omega_{\tilde{v}}(y)$  are defined as

$$\begin{aligned} \Omega_i(y) &\equiv \frac{1}{\lambda} \int_{-\infty}^{\infty} \frac{\partial \ln f_{y,\delta}(y, \delta)}{\partial \ln |\delta_i|} \tilde{\mathcal{V}}(y, \delta) f_{\delta|y}(\delta|y) d\delta, \quad \forall i \in \{1, 2, 3\}, \\ \text{and } \Omega_{\tilde{v}}(y) &\equiv \frac{1}{\lambda} \int_{-\infty}^{\infty} \left( \frac{\partial \ln \tilde{v}(y, \delta)}{\partial p} - \frac{\partial \ln \tilde{v}(y, \delta)}{\partial \ln y} \frac{\partial \ln y}{\partial p} \right) \tilde{\mathcal{V}}(y, \delta) f_{\delta|y}(\delta|y) d\delta, \end{aligned} \quad (52)$$

where  $\{\delta_i\}_{1 \leq i \leq 3} \equiv \{\underline{\delta}, \delta^*, \bar{\delta}\}$ ,  $\tilde{\mathcal{V}}(y, \delta)$  is defined in equation (32), and  $\frac{\partial \ln y}{\partial p}$  is given by (24).

Note that there are four, not three, extensive margin social welfare weights. As in the context of Definition 3, the first three,  $\{\Omega_i(y)\}_{1 \leq i \leq 3}$  capture the effects on welfare of individuals with income  $y$  of perturbing the three adjustment thresholds  $\{\delta_i\}_{1 \leq i \leq 3}$ . The fourth one,  $\Omega_{\tilde{v}}(y)$ , captures the endogenous change in the equilibrium distribution of utilities  $\tilde{v}(y, \delta)$  within the income group  $y$  in response to a change in progressivity. In particular, in the limit as  $q \rightarrow \infty$  with  $\kappa > 0$ , that is, as

the model converges towards the two-sided  $(S, s)$  environment, this welfare weight reduces to

$$\Omega_{\tilde{v}}(y) \equiv \frac{1}{\lambda} \frac{\partial \mathbb{E}[\tilde{v}(\delta) | y]}{\partial p} \mathcal{V}^*(y),$$

and thus computes directly the change in the average welfare within income  $y$  due to a change in progressivity.

Fourth, paralleling our discussion leading to Definition 4, I define the *relative marginal social welfare weights*  $\omega(y)$  as the effect on the individual's welfare of a proportional change in all the variables driving frictionless income,  $\mathbb{Y} = \{\mu_y, \sigma_y, m_y, s_y\}$ , relative to those driving deviations or hours,  $\mathbb{D} = \{\mu_\delta, \sigma_\delta, \underline{\delta}, \delta^*, \bar{\delta}\}$ . That is,

**Definition 8.** The relative marginal social welfare weight  $\omega(y)$  is defined as

$$\omega(y) \equiv -\frac{1}{\lambda} \int_{-\infty}^{\infty} \tilde{\mathcal{V}}(y, \delta) \left. \frac{\partial \ln f_{y,\delta}(y, \delta)}{\partial \ln \mathbb{Y}} \right|_{\mathbb{D}} f_{\delta|y}(\delta | y) d\delta = \frac{1}{\lambda} \int_{-\infty}^{\infty} \delta \frac{\partial \tilde{\mathcal{V}}(y, \delta)}{\partial \ln y} f_{\delta|y}(\delta | y) d\delta,$$

where  $\tilde{\mathcal{V}}(y, \delta)$  is defined in equation (32), the notation  $\left. \frac{\partial}{\partial \mathbb{Y}} \right|_{\mathbb{D}}$  is as in Definition 4, and the second equality is proved in the Appendix.

### 5.3 Optimal tax schedule

I now characterize analytically the optimal tax schedule in terms of the labor income elasticities and social marginal welfare weights defined in Sections 5.1 and 5.2.

**Marginal value of public funds.** I start by deriving the first equation characterizing the optimal tax schedule, which pins down the marginal value of public funds  $\lambda$ , that is, the Lagrange multiplier associated with the constraint (9). The marginal value of public funds  $\lambda$  is found by imposing that a perturbation of the parameter  $\tau$  by  $d\tau$  has no first-order effect on social welfare. Intuitively,  $\lambda$  is equal to the social value of redistributing a dollar of tax revenue through an decrease in  $\tau$  by  $d\tau$ , i.e., through a uniform increase in the net of tax rates.

**Proposition 6.** *The optimal tax schedule  $\{\tau, p\}$  satisfies*

$$0 = 1 - \int_0^{\infty} \mathbf{t}_\tau(y) \varphi(y) \frac{f_y(y)}{\mathbb{E} \mathbf{t}_\tau} dy - \int_0^{\infty} T'(y) \left[ \frac{\mathbf{t}'_\tau(y) y}{1 - T'(y)} \boldsymbol{\eta}(y) \right] \frac{f_y(y)}{\mathbb{E} \mathbf{t}_\tau} dy, \quad (53)$$

where  $\mathbf{t}_\tau(y) = \frac{1}{1-p} y^{1-p}$ ,  $f_y(y)$  is the stationary density of incomes given the tax schedule  $\{\tau, p\}$ , and  $\mathbb{E} \mathbf{t}_\tau = \int_0^{\infty} \mathbf{t}_\tau(y) f_y(y) dy$ . In the frictionless model, the same equation characterizes the optimal tax schedule, where the marginal social welfare weights  $\varphi(y)$  are replaced by their frictionless counterparts  $\varphi^*(y)$ .

*Proof.* See Appendix. □

The interpretation of equation (53) is as follows. It imposes that at the optimum tax schedule, a perturbation  $d\tau$  of the marginal tax rates should have no first-order effects on social welfare. The

first and third terms on the right hand side measure the *actual* change in government tax revenue of a one-dollar *statutory* increase in taxes, that is, taking into account the induced change in individual behavior. The additional tax liability levied at the income level  $y$  after the tax reform is implemented is given, to a first order in  $d\tau \rightarrow 0$ , by  $\mathbf{t}_\tau(y) d\tau$ , and the marginal tax rate changes by  $\mathbf{t}'_\tau(y) d\tau$ . The first term in the right hand side of (53) is the *mechanical effect* of the perturbation, i.e., the statutory increase in government revenue absent behavioral responses. It is equal to  $\mathbb{E}[\mathbf{t}_\tau(y) d\tau]$ , and we normalize the magnitude of the perturbation so that this mechanical effect is equal to one (dollar). The third term in the right hand side of (53) is the *behavioral effect* of the perturbation. The increase  $dT' \equiv \mathbf{t}'_\tau(y) d\tau$  in the marginal tax rate of an individual with income  $y$  induces her to decrease her taxable income by  $\frac{y}{1-T'(y)} \boldsymbol{\eta}(y) dT'$ . This behavioral income response generates a loss in government revenue proportional to the marginal tax rate  $T'(y)$ . Summing over individuals using the density of incomes  $f_y(\cdot)$  yields the third term in equation (53). Finally, the second term in (53) is the *welfare effect*, expressed in monetary units, of the perturbation. An increase in the tax liability of individual  $y$  by  $dT \equiv \mathbf{t}_\tau(y) d\tau$  directly reduces her utility and hence social welfare by  $\boldsymbol{\varphi}(y) \times dT$ , by definition of the marginal social welfare weights (48).

This equation (53) has a structure that is identical to those of the “sufficient statistic formulas” derived by, e.g., Saez (2001), Jacquet and Lehmann (2015) and Golosov, Tsyvinski and Werquin (2014), to characterize the optimal tax systems in frictionless models. Note in particular that all the variables other than the marginal social welfare weights in equation (53) (elasticity, tax schedule, income distribution) are empirically observable.<sup>8</sup>

Proposition 6 extends the result of Proposition 5 by showing that the long-run effects on social welfare (and not only on aggregate income) of a uniform change in marginal tax rates are correctly computed by assuming that the economy is frictionless and thus has a representative agent at each income level  $y$ : in particular, the relevant elasticity is the individual frictionless income elasticity  $\boldsymbol{\eta}(y)$  defined in (43), even though individual labor supply is lumpy. There is one difference, however, between the frictionless and the frictional versions of the optimal tax formula (53): the *frictional* marginal social welfare weights  $\boldsymbol{\varphi}(y)$  must be computed by taking into account the non-degenerate distribution of utilities  $\mathbb{E}[\tilde{v}(y, \delta) | y]$  within income groups. In general this correction term varies with income  $y$ , so that the schedule of frictional welfare weights is not homothetic to the schedule of frictionless weights and the effective redistributive tastes of the government have to be adjusted relative to a model with a representative agent for each income.

**Optimal rate of progressivity.** I now characterize the optimal tax schedule in the frictionless and the frictional models. The optimum is such that a perturbation of the rate of progressivity  $p$  by  $dp \rightarrow 0$  has no first-order effects on social welfare. The next proposition gives the main theoretical result of the paper:

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<sup>8</sup>Note however that these variables are endogenous to the tax system and should be evaluated at the optimum. The values estimated given the current tax code (in particular, the current U.S. income distribution) can be nevertheless used to quantify the welfare effects of *local tax reforms* given by the right hand sides of equations (53) and (54,55). See Golosov, Tsyvinski and Werquin (2014).

**Proposition 7.** *In the frictionless model, the optimal tax schedule  $\{\tau, p\}$  is fully characterized by (9), (53), and*

$$0 = 1 - \int_0^\infty \left[ \mathbf{t}_p(y) \boldsymbol{\varphi}^*(y) + \frac{d \ln \boldsymbol{\rho}}{dp} \boldsymbol{\psi}^*(y) \right] \frac{f_y(y)}{\mathbb{E} \mathbf{t}_p} dy - \int_0^\infty T'(y) \left[ \frac{y \mathbf{t}'_p(y)}{1 - T'(y)} \boldsymbol{\eta}(y) \right] \frac{f_y(y)}{\mathbb{E} \mathbf{t}_p} dy, \quad (54)$$

where  $\mathbf{t}_p(y) = \left( \ln y - \frac{1}{1-p} \right) \frac{1-\tau}{1-p} y^{1-p}$ ,  $f_y(y)$  is the stationary density of incomes given the tax schedule  $\{\tau, p\}$ , and  $\mathbb{E} \mathbf{t}_p \equiv \int_0^\infty \mathbf{t}_p(y) f_y(y) dy$ . In the frictional model, the optimal long-run tax schedule is given by (9), (53), and

$$0 = 1 - \int_0^\infty \left[ \mathbf{t}_p \boldsymbol{\varphi} + \frac{d \ln \boldsymbol{\rho}}{dp} \boldsymbol{\psi} + \frac{d \ln \frac{\sigma_y}{|\sigma_\delta|}}{dp} \boldsymbol{\omega} \right] \frac{dF_y(y)}{\mathbb{E} \mathbf{t}_p} + \int_0^\infty \left[ \sum_{i=1}^3 \frac{d \ln \frac{|\delta_i|}{|\sigma_\delta|}}{dp} \boldsymbol{\Omega}_i + \boldsymbol{\Omega}_{\bar{v}} \right] \frac{dF_y(y)}{\mathbb{E} \mathbf{t}_p} - \int_0^\infty T'(y) \left[ \frac{y \mathbf{t}'_p}{1 - T'} \boldsymbol{\eta} + \frac{d \ln \frac{\sigma_y}{|\sigma_\delta|}}{dp} y \boldsymbol{\xi} \right] \frac{dF_y(y)}{\mathbb{E} \mathbf{t}_p} + \int_0^\infty T(y) \left[ \sum_{i=1}^3 \frac{d \ln \frac{|\delta_i|}{|\sigma_\delta|}}{dp} \boldsymbol{\Xi}_i \right] \frac{dF_y(y)}{\mathbb{E} \mathbf{t}_p}, \quad (55)$$

where I denote  $\{\delta_i\}_{1 \leq i \leq 3} = \{\underline{\delta}, \delta^*, \bar{\delta}\}$ ,  $\{\boldsymbol{\Xi}_i\}_{1 \leq i \leq 3} = \{\boldsymbol{\Xi}(y), \boldsymbol{\Xi}^*(y), \bar{\boldsymbol{\Xi}}(y)\}$ , and  $\{\boldsymbol{\Omega}_i\}_{1 \leq i \leq 3} = \{\underline{\boldsymbol{\Omega}}(y), \boldsymbol{\Omega}^*(y), \bar{\boldsymbol{\Omega}}(y)\}$ . In particular, the extensive margin effects due to the option value of adjusting hours can in general not be ignored in the characterization of the optimal tax schedule unless the frequency of individual labor supply adjustments, or the average duration of a job, is unaffected by tax policy.

*Proof.* See Appendix. □

The first result of Proposition 7, equation (54), characterizes the optimal tax schedule in the frictionless model. Its interpretation is similar as that of equation (53), with one important difference. It equates the first-order welfare effects of a perturbation  $dp$  of the optimal tax schedule  $\{\tau, p\}$  to zero. The tax reform induces a change in the tax liability levied at the income level  $y$  given, to a first order in  $dp \rightarrow 0$ , by  $\mathbf{t}_p(y) dp$ , and the marginal tax rate changes by  $\mathbf{t}'_p(y) dp$ . The first term in the right hand side of (54) is the mechanical effect of the perturbation, the third term is the behavioral effect (it measures the actual change in government tax revenue of a one-dollar statutory increase in taxes through an increase in  $p$ ), and the second term is the welfare effect (expressed in monetary units). Importantly, the welfare effect of an increase in progressivity depends on a new welfare weight,  $\boldsymbol{\psi}^*(y)$ , which is absent from expression (53) and of the typical optimal tax formulas derived in static models (Diamond (1998), Saez (2001)). This is because progressivity affects the growth rate of consumption and hence the discount rate  $\boldsymbol{\rho}$  used to compute the present value of utility (hence the term  $\frac{d \ln \boldsymbol{\rho}}{dp}$  multiplying  $\boldsymbol{\psi}^*(y)$ ). This implies that the standard static Mirrlees model, often loosely argued to characterize *long-run* optimal taxes, fails to capture the true welfare effects of progressivity if the “long-run” is not properly modeled as the steady-state of a dynamic economy. I discuss the quantitative importance of this effect in Section 6.

The second result of Proposition 7 is the derivation of equation (55) which characterizes the optimal long-run tax schedule in the frictional model. This expression shows that in theory, the

frictionless formula does *not* correctly account for all of the long-run effects of taxes when individual labor supply is subject to frictions – beyond the use of the frictional rather than frictionless welfare weights  $\varphi(y)$ ,  $\psi(y)$ . There are new long-run effects of progressivity both on welfare and on revenue.

The first and least important difference is the presence of the relative welfare weights  $\omega(y)$  and the relative labor income elasticities  $\xi(y)$  in the standard welfare and behavioral effects of the perturbation. These effects appear because a change in progressivity affects the *relative* values of the frictionless income variables  $\mathbb{Y}$  (elasticity  $\frac{d \ln \sigma_y}{dp}$ ) and the deviation (or hour) variables  $\mathbb{D}$  (elasticity  $\frac{d \ln \sigma_\delta}{dp}$ ). We saw in Definitions 4 and 8 that this decoupling affects the long-run density of incomes.<sup>9</sup>

The second, and most important, difference between the frictionless and frictional formulas (54) and (55) is the presence in the latter of a new welfare effect containing the *extensive margin* social welfare weights  $\underline{\Omega}(y)$ ,  $\Omega^*(y)$ ,  $\bar{\Omega}(y)$ ,  $\Omega_{\bar{v}}(y)$ , and a new behavioral (revenue) effect containing the extensive margin labor income elasticities  $\underline{\Xi}(y)$ ,  $\Xi^*(y)$ ,  $\bar{\Xi}(y)$ . We saw in Definitions 3 and 7 that a change in the optimal individual impulse control policy, i.e., in the *option value of adjusting labor supply*, affects the density of incomes and the welfare at each income level  $y$ .

Specifically, Proposition 7 shows that these extensive margin effects  $\Xi_i$  on revenue cannot be ignored theoretically unless the *frequency* of labor supply adjustments is unaffected by tax policy – that is, unless an increase in the progressivity  $p$  of the tax code induces an *equal* reduction in the volatility of deviations *and* on the size of the individual inaction region, i.e.,

$$\frac{d \ln \{|\underline{\delta}|, |\delta^*|, \bar{\delta}\}}{dp} = \frac{d \ln |\sigma_\delta|}{dp} = -\frac{1}{1-p} \frac{1+\varepsilon}{1+p\varepsilon} dp. \quad (56)$$

This condition is satisfied in particular in the Calvo limit as  $\kappa \rightarrow 0$  and  $q > 0$  (see Appendix for details), where all labor supply adjustments are driven by labor demand: in this case the frequency of adjustment is exogenous to taxes by construction. It is also satisfied in the case of a uniform change in marginal tax rates, because  $\tau$  affects neither the volatility nor the optimal inaction region; this explains why the extensive margin terms do not appear in the formulas of Propositions 5 and 6. In general, however, condition (56) is violated: as we already saw in Section 3, an increase in progressivity both reduces the volatility of deviation and narrows the inaction region, but the two effects do not exactly cancel out: the volatility effect typically dominates the size-of-the-bands effect, so that a higher rate of progressivity strictly reduces the frequency of adjustment. It follows that as soon as the option value of adjusting labor supply is endogenous, the extensive margin terms are non-zero and the effects of taxes on government revenue are not fully described by the intensive margin labor supply elasticity  $\eta(y)$ . Note that while the *intensive margin* elasticities are multiplied by the *marginal* tax rate  $T'(y)$  to obtain the behavioral (revenue) effect of the tax change, because an infinitesimal change in income  $dy$  reduces tax revenue by  $T'(y) dy$ , the three *extensive margin* terms (corresponding to the three variables  $\underline{\delta}, \delta^*, \bar{\delta}$  of discrete adjustment), on the other hand, are multiplied by the *average* tax rate  $T(y)$ .

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<sup>9</sup>Note that if the fixed adjustment cost were on income  $y$  rather than hours  $h$ , the volatility of deviations  $\sigma_\delta$  would always be equal to (minus) that of frictionless incomes  $\sigma_y$ , and these two terms would disappear from the optimal tax formula (55). See the earlier version of this paper (Chapter 1 in Werquin (2015)).

Note moreover that even when condition (56) holds, so that the extensive margin effects do not matter for efficiency, the fact that labor supply is frictional implies that progressivity has non-zero extensive margin *welfare effects*, captured by the marginal social welfare weights  $\Omega_4(y)$ . From equation (7), we obtain that these weights are in general non-zero, unless in addition to (56) we have

$$\int_{-\infty}^{\infty} \left[ \frac{\partial \tilde{v}(y, \delta)}{\partial p} - \frac{\varepsilon}{1 + p\varepsilon} \ln y \frac{\partial \tilde{v}(y, \delta)}{\partial \ln y} \right] f_{\delta|y}(\delta|y) d\delta = 0.$$

In the limit as  $q \rightarrow \infty$  with  $\kappa > 0$ , i.e. as the model converges towards the two-sided  $(S, s)$  environment, this condition reduces to  $\frac{\partial \mathbb{E}[\tilde{v}(\delta)|y]}{\partial p} = 0$ , and thus holds if the average utility within the population of income  $y$  is exogenous to taxes. If this condition is not satisfied (as is generally the case, see e.g. the right panel of Figure 3), then progressivity has an additional effect on social welfare by affecting the steady-state distribution of heterogeneous utilities within income groups. I discuss the quantitative magnitude of these novel effects in a calibrated version of the model in Section 6.

Extensive margin effects in optimal taxation models are also obtained by Saez (2002) and Jacquet, Lehmann and Van der Linden (2013), who derive optimal tax formulas in static frictionless models with a 0-1 decision whether to participate in the labor force. Here, I generalize their insights by deriving a formula where the extensive margins of adjustment occur *conditional on participation* and are generated *endogenously* by the fixed cost of adjusting labor supply. Proposition 7 shows that these extensive margin effects matter theoretically *even* in the long-run, even though all individuals have by then had time to take into account the new tax schedule in their optimal labor supply choices. Thus, the standard Mirrlees taxation framework with labor supply responses on the intensive margin (which has often been criticized precisely for not modeling convincingly the individual labor supply behavior) does not adequately capture the full effects of taxation, even in the long-run. This is because taxes affect the *option value* of adjusting the *actual* labor supply, and not only the optimal *desired* labor supply. This endogenous extensive margin is important, as the empirical literature repeatedly found that the labor supply adjustments in response to productivity, wage or tax changes are frictional as modeled in this paper, whether the fixed cost represents the search cost of switching jobs (hours constraints within a firm: Altonji and Paxson (1992), Chetty, Friedman, Olsen and Pistaferri (2011)) or cognitive costs (Gelber, Jones and Sacks (2013)). It would be straightforward to add explicitly a participation margin into the model (see for example Alvarez, Borovickova and Shimer (2015)). However, empirically much of the difference in labor supply across countries with different tax regimes is driven by hours worked conditional on employment (see Davis and Henrekson (2005), and Chetty, Guren, Manoli and Weber (2011)). Note moreover that this option value effect would appear in a similar fashion in a labor demand model à la Mortensen and Pissarides (1994) with endogenous job destruction shocks: the job destruction cutoff would similarly respond endogenously to the volatility of idiosyncratic shocks, and hence to the progressivity of taxes. (See Davis, Faberman, Haltiwanger, Jarmin and Miranda (2010) for an empirical evaluation of this mechanism.) Here, I show that optimal taxes depend on the *difference* between these volatility and cutoff effects, i.e. on the frequency of adjustments, and induce further



welfare effects due to the endogenous adjustment of welfare heterogeneity within income groups, in a model where there is in addition an *intensive margin* dimension of labor supply (work intensity or hours).

Finally, the results I derive in this paper apply much more broadly than to this taxation theory framework. The key finding of this paper is that non-linear policies interact with fixed adjustment costs to yield real effects even in the long-run. This is in contrast with most of the literature with fixed costs at the micro level, where typically the aggregate economy behaves in the long-run as a frictionless (representative agent) model. For instance, monetary policy generally is neutral in the long-run, and even in the short-run in specific settings as in Caplin and Spulber (1987). Similarly, in the public finance context, Rogerson (1988) shows in a model of lotteries that the aggregate economy can behave as a representative agent with a large labor supply elasticity even though the individual (micro) elasticity is equal to zero. Chetty, Friedman, Olsen and Pistaferri (2011) reach a similar conclusion in a model with fixed costs. I show in this paper that this insight is correct only if we consider *linear* policies, which do not affect the option value of adjusting labor supply (Propositions 5 and 6). On the other hand, the *curvature* of policy matters in the aggregate (for both efficiency and welfare) even in the long-run when adjustment at the individual level is frictional. This insight should apply generally to models with fixed costs and non-linear policy tools, e.g. to the investment literature.

## 6 Quantitative analysis

In this section I calibrate the model analyzed in the previous sections, and evaluate quantitatively the novel effects highlighted in Proposition 7. I compute the magnitudes of the marginal social welfare weights and labor income elasticities around the current U.S. tax code in a calibrated version of the model, and finally compute the optimal tax schedules.

### 6.1 Calibration

I calibrate the marginal tax rates and the rate of progressivity  $(\tau, p)$  of the tax schedule in the U.S. using the empirical estimates from PSID data of Heathcote, Storesletten, and Violante (2014):  $\tau = -3$  and  $p = 0.151$ . This value of  $p$  implies that earning twice as high an income leads to a 15.1% decrease in the net-of-tax rate (see equation (4)). These parameters yield a value for total U.S. government revenue  $\bar{R} = \$2.33\text{tn}$  (for a population of 320mn) which I keep constant throughout the numerical analysis, so that whenever I vary the progressivity  $p$  I adjust  $\tau$  in order to keep the revenue  $\bar{R}$  unchanged.

The theoretical analysis above requires the coefficient of risk aversion  $\gamma$  to be strictly below 1, I take  $\gamma = 0.9$ . There is substantial controversy in the literature about the value of the taxable income elasticity  $\varepsilon$ . the micro literature typically finds values lower than 0.3, while the macro literature and some structural estimates find it to be closer to 1 (see Saez, Slemrod, and Giertz (2012), and Keane and Rogerson (2012), for an overview of the two strands). In a frictionless environment

Gruber and Saez (2002) find an elasticity between 0.4 and 0.6, while in a context closely related to this paper’s model, Chetty (2012) estimates the structural parameter (Hicksian intensive margin elasticity)  $\varepsilon = 0.33$  using a meta analysis of micro and macro studies and allowing for adjustment frictions to reconcile the wide range of estimates. In my baseline calibration I thus take  $\varepsilon = 0.33$ . I discuss below the effects of varying the preference parameters  $\gamma, \varepsilon$  on the results, in particular, I show how the results are affected for  $\varepsilon = 1$ .

I calibrate the Pareto coefficients of the *observed* U.S. income distribution,  $r_{y,1}^{\rho_2}$  and  $r_{y,2}^{\rho_2}$ , given the parameters of the U.S. tax schedule. The Pareto coefficient of the right tail,  $\left| r_{y,1}^{\rho_2} \right|$ , is well known: it varies around 2 and has been decreasing (the tail of the distribution has become thicker, i.e. more unequal) in the past few decades. The coefficient of the left tail has been estimated by, e.g., Reed (2003), Reed and Jorgensen (2004), Toda (2012). I take  $(r_{y,1}^{\rho_2}, r_{y,2}^{\rho_2}) = (-1.9, 1.4)$ .

The mean  $m_y$  and variance  $s_y^2$  of the lognormal “bulk” of the frictionless income distribution are calibrated using the mean and variance of the observed U.S. distribution of log-incomes.<sup>10</sup> Using  $\mathbb{E}[\ln y] = 10.3$  and  $\mathbb{V}[\ln y] = 1$  I obtain  $(m_y, s_y) = (10.46, 0.43)$ .

There is a large literature estimating log-income dynamics that follow a geometric random walk, that is equation (2) without the jumps, see e.g. Meghir and Pistaferri (2004, 2011). The volatility of idiosyncratic wage risk  $\sigma_y^2$  in my model corresponds to the variance of the permanent component of the individual log-income process in this literature. I calibrate  $\sigma_y^2 = 0.01$  (see also Jones and Kim (2014), for an estimate in a model similar to mine and further references to the empirical literature). The jump term in (2) can be calibrated from Guvenen, Karahan, Ozkan and Song (2014), who show that the distribution of earnings growth rates follow double-Pareto distributions  $f_\nu$  with coefficients 2 and 1.4 for the right and left tails, respectively. [**Note:** *the simulations below do not have the jumps, updated computations coming soon.*]

Next, note that the *cross-sectional* income distribution in the economy, specifically the values of the two Pareto coefficients at the tails, allows us to infer information about the *time series* of individual income, since these Pareto tails are generated by the underlying random growth process for income. We have

$$r_{y,1}^{\rho_2} + r_{y,2}^{\rho_2} = \frac{2\mu_y}{\sigma_y^2}, \quad \text{and} \quad r_{y,1}^{\rho_2} r_{y,2}^{\rho_2} = -\frac{2\rho_2}{\sigma_y^2},$$

which pin down the drift  $\mu_y$  and the death rate  $\rho_2$ . Note that this leads to a negative drift of income  $\mu_y$ , but the growth rate  $\mu_y + \frac{1}{2}\sigma_y^2$  is strictly positive. I take a discount rate  $\rho_1$  so that  $(1 + \rho_1 + \rho_2)^{-1} = 0.95$ .

The parameters of the individual wage and consumption processes,  $(\mu_w, \sigma_w)$  and  $(\mu_c, \sigma_c)$ , and those of the wage and consumption distributions,  $(m_w, s_w, r_{1,w}^{\rho_2}, r_{2,w}^{\rho_2})$  and  $(m_c, s_c, r_{1,c}^{\rho_2}, r_{2,c}^{\rho_2})$  are then obtained by equations (11) and (41). In the numerical exercises below, I compute the effects of taxes keeping the parameters of the exogenous *wage* (or productivity) process  $(\mu_w, \sigma_w, m_w, s_w)$

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<sup>10</sup>In the frictionless model, these coefficients are given in closed form by

$$\mathbb{E}[\ln y] = m_y - \frac{1}{r_{y,1}} - \frac{1}{r_{y,2}}, \quad \text{and} \quad \mathbb{V}[\ln y] = s_y^2 + \frac{1}{r_{y,1}^2} + \frac{1}{r_{y,2}^2}.$$

constant, and use equations (11,41) to infer those of the endogenous income distribution.

The fixed adjustment cost  $\kappa$  and the arrival rate of costless adjustment opportunities  $q$  are calibrated as follows. I take the average duration of searching for a new job, equal to  $t_s = 1$  month, and the average duration of a job (with a given amount of hours) equal to  $t_i + t_s = 5$  years. [Note: get better empirical values] Using the explicit expressions

$$t_s = q^{-1}, \text{ and } t_i = \frac{\bar{\delta} - \underline{\delta}}{\mu_y} \left[ \frac{\delta^* - \underline{\delta}}{\bar{\delta} - \underline{\delta}} - \frac{e^{2\delta^*\mu_y/\sigma_y^2} - e^{2\underline{\delta}\mu_y/\sigma_y^2}}{e^{2\bar{\delta}\mu_y/\sigma_y^2} - e^{2\underline{\delta}\mu_y/\sigma_y^2}} \right],$$

I obtain the values of  $\kappa$  and  $q$  that yield these average durations.

For  $\varepsilon = 0.33$ , I get  $\kappa = 0.0038$ . This value implies that the cost of searching for a new job  $\kappa$  is equal to 0.38% of the instantaneous utility  $g(c_0^*)$ , or 1.2% of the average monthly utility  $\mathbb{E} \int_0^T e^{-(\rho_1 + \rho_2)t} g(c_t^*) dt = \rho^{-1} (1 - e^{-\rho T}) \left( \frac{1 + \rho\varepsilon}{1 + \varepsilon} \right)^{1-\gamma} g(c_0^*)$ , that is, 1.2% of the total utility received during the duration of the (one-month long) search. Despite this relatively small value for the fixed cost, the corresponding inaction region is large (because the utility loss from choosing hours suboptimally is second-order, see equation (15)) and given by  $\underline{\delta} = -0.09$  and  $\bar{\delta} = 0.09$ , so that an individual starts searching for a new job when her hours are approximately 9% away from their optimal value (given her wage). Finally, she then adjusts to  $\delta^* = -0.001$ , i.e. 0.1% below her current optimal value (because of the small drift  $\mu_\delta$ ). For  $\varepsilon = 1$ , I get  $\kappa = 0.015$ , which implies that the cost of searching for a new job  $\kappa$  is equal to 5.1% of the average monthly utility. The corresponding inaction region is given by  $\underline{\delta} = -0.19$  and  $\bar{\delta} = 0.19$ , so that an individual starts searching for a new job when her hours are 20% away from their optimal value.

## 6.2 Numerical results

I now evaluate the quantitative magnitude of the effects highlighted in Section 5.

I first compute the extensive margin and relative margin labor income elasticities around the current U.S. tax code, using their Definitions 3 and 4. They are represented in Figure 5 (relative margin elasticities for  $\varepsilon = 0.33$  on the left panel and  $\varepsilon = 1$  on the right panel), and Figure 6 (relative margin elasticities for  $\varepsilon = 0.33$  on the left panel and  $\varepsilon = 1$  on the right panel). Specifically, Figure 5 shows the extensive margin elasticities  $\Xi, \bar{\Xi}$  weighted by the effect of progressivity on the adjustment thresholds, i.e., the tax elasticities

$$\left( \frac{d \ln |\underline{\delta}|}{dp} - \frac{d \ln |\sigma_\delta|}{dp} \right) \Xi(y), \quad \left( \frac{d \ln |\bar{\delta}|}{dp} - \frac{d \ln |\sigma_\delta|}{dp} \right) \bar{\Xi}(y),$$

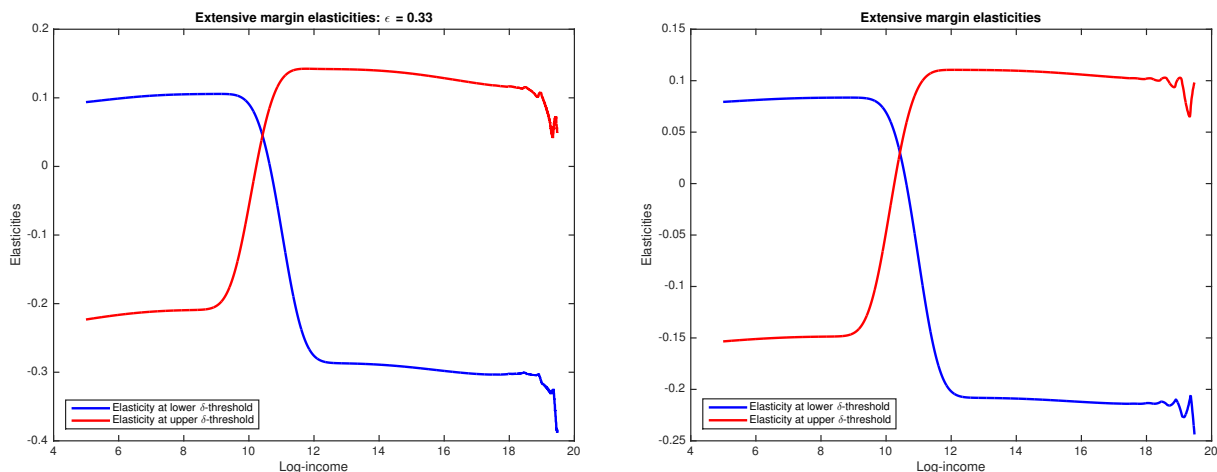
and Figure 6 shows the relative margin elasticity  $\xi$  weighted by the effect of progressivity on the the relative process of frictionless and frictional variables, i.e., the tax elasticities

$$\left( \frac{d \ln \sigma_y}{dp} - \frac{d \ln |\sigma_\delta|}{dp} \right) \xi(y).$$

We first observe that the extensive margin elasticities are far from negligible: of the order of 0.1 to

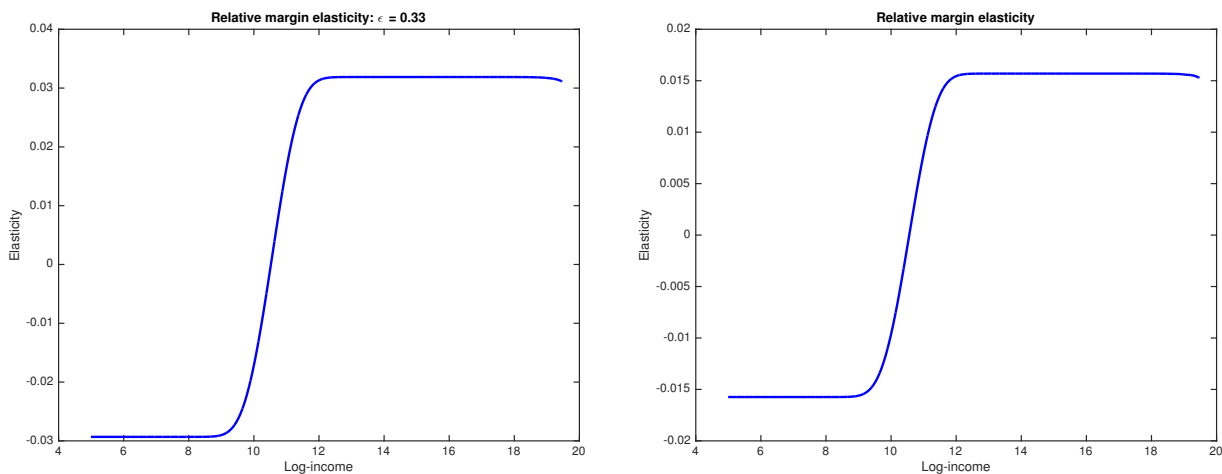
0.3 in absolute value (compare with the range of values obtained in the meta-analysis of Chetty, Guren, Manoli and Weber (2011): 0.17-0.26 for the steady-state extensive margin participation elasticities, and 0.33 for the intensive margin elasticities).

Figure 5: Extensive margin elasticities:  $\varepsilon = 0.33$  and  $\varepsilon = 1$



The relative margin elasticities are an order of magnitude smaller than the extensive margin elasticities.

Figure 6: Relative margin elasticities:  $\varepsilon = 0.33$  and  $\varepsilon = 1$



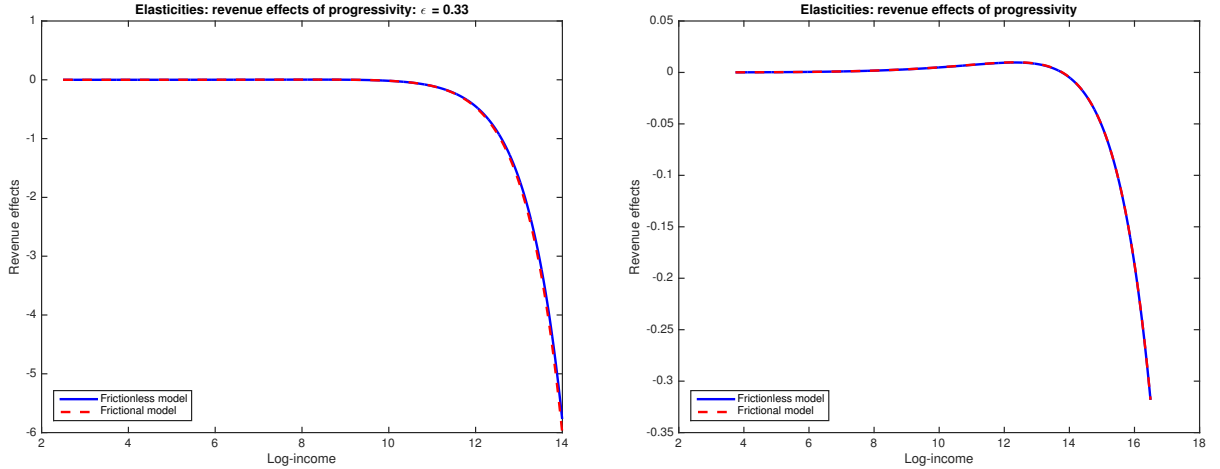
Next, I compute the right hand side of formula (55) where the elasticities, marginal social welfare weights, tax schedule, and income distribution are estimated in the current U.S. economy, and where the marginal value of public funds is calculated by the right hand side of equation (53) evaluated at the current U.S. tax code. I first show in Figure 7 the revenue effects disaggregated by income, that is, I compute the behavioral effect of the right hand side (55) as functions of income (unweighted

by the density of incomes)

$$T'(y) \left[ \frac{y \mathbf{t}'_p(y)}{1 - T'(y)} \boldsymbol{\eta}(y) + \frac{d \ln \frac{\sigma_y}{|\sigma_\delta|}}{dp} y \boldsymbol{\xi}(y) \right] \frac{1}{\mathbb{E} \mathbf{t}_p} + T(y) \left[ \sum_{i=1}^3 \frac{d \ln \frac{|\delta_i|}{|\sigma_\delta|}}{dp} \boldsymbol{\Xi}_i(y) \right] \frac{1}{\mathbb{E} \mathbf{t}_p}.$$

The left and right panels show the revenue effects in the frictionless and the frictional models for  $\varepsilon = 0.33$  and  $\varepsilon = 1$ , respectively. The schedules of revenue effects are nearly identical at every income level. There are two reasons for getting such a small effect even though the extensive margin elasticities are non-negligible. First, note that the elasticities represented in Figure 5 have an opposite sign, so that the extensive margin terms partially cancel each other out in formula (55). Second, and most importantly, these elasticities are bounded, because a given change in progressivity changes the size of every individual's inaction region and the volatility of their income process in the same proportion; in fact, the elasticities are roughly constant at the tails, where the income distribution is approximately Pareto distributed [Note: get a closed form for the elasticities at the tails]. But on the other hand, an increase in progressivity induces a much larger effect on the intensive margin (standard elasticity  $\boldsymbol{\eta}$ ), because it increases the marginal tax rates that individuals face by an amount proportional to the log-income  $\mathbf{t}'_p(y)$ , which is unbounded. Therefore the option value effect is dominated by the standard intensive margin effect in the long-run, more so for larger values of  $\varepsilon$ .

Figure 7: Revenue effects of tax reforms



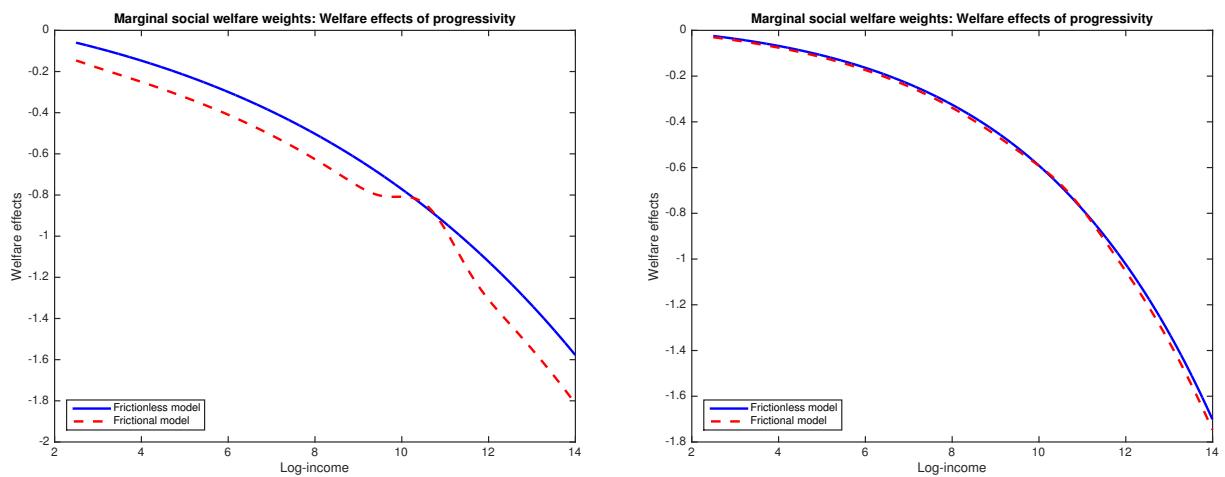
I then show in Figure 8 the welfare effects of increasing progressivity disaggregated by income, that is, I compute

$$- \left[ \mathbf{t}_p(y) \boldsymbol{\varphi}(y) + \frac{d \ln \boldsymbol{\rho}}{dp} \boldsymbol{\psi}(y) + \frac{d \ln \frac{\sigma_y}{|\sigma_\delta|}}{dp} \boldsymbol{\omega}(y) \right] \frac{1}{\mathbb{E} \mathbf{t}_p} + \left[ \sum_{i=1}^3 \frac{d \ln \frac{|\delta_i|}{|\sigma_\delta|}}{dp} \boldsymbol{\Omega}_i(y) + \boldsymbol{\Omega}_{\bar{v}}(y) \right] \frac{1}{\mathbb{E} \mathbf{t}_p}.$$

The left and right panels show the welfare effects in the frictionless and the frictional models for

$\varepsilon = 0.33$  and  $\varepsilon = 1$ , respectively. These effects are non-negligible for the smaller value of the labor income elasticity  $\varepsilon = 0.33$ , and almost zero for the larger value  $\varepsilon = 1$ . The change in the welfare distribution within incomes in response to a change in progressivity (welfare weight  $\Omega_4$ ) plays almost no role in the discrepancy between the two curves in the left panel. This finding implies that for small enough intensive margin labor income elasticities, ignoring the hours restrictions within firms leads to substantially misestimating the welfare costs of raising the progressivity of the tax schedule by not taking into account the extensive margin effects. The effect disappears as the labor income elasticity gets higher, in which case the standard intensive margin welfare costs dominate the extensive margin effects.

Figure 8: Welfare effects of tax reforms



Another way to express these results is by computing the aggregate values of these effects by summing over incomes, to obtain the integrals in the right hand side of (55). This gives the net revenue and welfare effects, expressed in dollars, of a \$1 statutory increase in tax revenue through an increase in the rate of progressivity. It thus provides an estimate of the miscalculations in the welfare cost of taxation when wrongly assuming that the economy is frictionless. For  $\varepsilon = 0.33$ , in the frictionless model the total behavioral (revenue) loss of a \$1 statutory increase in  $p$  is c11.10, and the total welfare loss is c83.12.<sup>11</sup> The frictional effect of perturbing progressivity on *revenue* is 2.25% away from this frictionless effect (the behavioral loss is c0.25 higher), and the frictional effect on *welfare* is 7.3% away from the frictionless effect (the welfare loss is \$6.05 higher). Thus the static model's welfare calculations severely underestimate the welfare costs of raising progressivity, which will translate into a lower optimal rate of progressivity. The reason why the extensive margin effects on welfare tend to reduce the gains of raising progressivity is the following. We saw that the standard marginal social welfare weights capture the true welfare effects of taxes if the decrease in the volatility of incomes due to an increase in progressivity is exactly compensated by the narrowing

<sup>11</sup>Note that the sum of the two is lower than the mechanical effect \$1, implying that the U.S. tax code is too regressive for the parameters of the calibration. The optimum rate of progressivity is increasing the risk aversion  $\gamma$  and decreasing in the elasticity  $\varepsilon$ .

of the inaction region (option value effect). In general, however, the latter effect is dominated by the former, so that an increase in progressivity is equivalent to a higher dispersion of individual incomes around their desired values, which adversely affects welfare, relative to the frictionless benchmark. In contrast, when  $\varepsilon = 1$ , the total behavioral revenue response of an increase in  $p$  is c33.44, and the total welfare loss is c66.80.<sup>12</sup> The frictional effects of perturbing progressivity are 0.46% and 0.75% away from the frictionless effects on revenue and welfare, respectively; therefore the static model’s calculations are extremely accurate in this case. This is because when the labor income elasticity is high, most revenue and welfare effects are those related to the intensive margin of adjustment, which dwarve the extensive margin effects.

I finally compute the optimal tax schedules in the frictionless and the frictional models. The frictionless optimum corresponds to the tax schedule that a planner would compute, wrongly assuming that the observed economy is frictionless; it is thus not given by the optimum that one would compute in a true frictionless economy (with double-Pareto-lognormal income distributions). A frictionless planner would back out the (exogenous) wage or productivity distribution given the observed income distribution using relationship (10) and the observed U.S. economy, and would then infer the income distributions for different values of  $(\tau, p)$  given the relationship (10). The results are shown in Figures 10 and 10. The left panel shows the U.S. tax schedule, the optimal tax schedule that a “static” planner would compute (ignoring the dynamic welfare weights defined in Definition 6), and the optimal tax schedule that a “dynamic” planner would compute wrongly assuming that the economy is frictionless (i.e., using equation (54)). The right panel shows the latter (frictionless) optimal tax schedule, and the full optimum in the frictional economy (given by equation (55)). The results for  $\varepsilon = 1$  are  $(\tau, p) = (xx, xx)$  for the “static” optimum,  $(\tau, p) = (xx, xx)$  for the frictionless optimum, and  $(\tau, p) = (xx, xx)$  for the full optimum. **[Note: The results for  $\varepsilon = 0.33$  and  $\varepsilon = 1$  are coming soon!]**

Figure 9: Optimal tax schedules for  $\varepsilon = 0.33$

[This figure will be added soon]

[This figure will be added soon]

In contrast, the results for  $\varepsilon = 1$  are  $(\tau, p) = (xx, xx)$  for the “static” optimum,  $(\tau, p) = (xx, xx)$  for the frictionless optimum, and  $(\tau, p) = (xx, xx)$  for the full optimum. Thus the optimum tax schedule is only very slightly more progressive than the frictionless optimum when  $\varepsilon = 1$ .

Figure 10: Optimal tax schedules for  $\varepsilon = 1$

[This figure will be added soon]

[This figure will be added soon]

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<sup>12</sup>Note that the sum of the two is slightly larger than the mechanical effect \$1, implying that the U.S. tax code is slightly too progressive for this higher value of the labor income elasticity.

## 7 Conclusion

This paper analyzes a model where individual labor supply is subject to a fixed (search) adjustment cost. The model allows for analytically tractable characterizations of the optimal individual behavior and the long-run aggregate income distributions in the presence of stochastic idiosyncratic wage shocks and a non-linear tax schedule. I derive a theoretical formula for the optimal long-run progressive tax schedule in this frictional economy. I uncover several new effects that are not captured by standard frictionless optimal tax formulas with labor supply responses on the intensive margin. Most importantly, the option value of adjusting hours of work creates an extensive margin of labor supply conditional on participation, which is endogenous to tax rates. The optimal tax schedule must take into account these additional effects on optimal individual behavior, and therefore depends on several new elasticities and marginal social welfare weights. Quantitatively, these novel theoretical effects substantially affect the welfare calculations and optimal tax schedule in the baseline calibration of the model, and decrease as the intensive margin labor income elasticity becomes higher. This implies that the static frictionless optimal tax formula underestimates the optimal long-run rate of progressivity in the frictional model. Finally, I argued that the insights of this paper apply more broadly to environments where fixed costs interact with non-linear policy tools, in which policy instruments have long-run real effects on welfare.

There are several directions for further research that would be interesting to study. First, it would be valuable to estimate numerically the endogenous extensive margin effects of taxes highlighted in the theoretical formulas of this paper in a more realistic structural model. This could allow for such features as savings and borrowings, life-cycle and endogenous participation labor supply choices, transitory as well as permanent wage shocks, income-varying labor supply elasticities, non-proportional fixed adjustment costs, more general non-linear taxes and transfer programs, and other dimensions of labor supply adjustment choices (e.g., job satisfaction). Such a model would also provide realistic estimates of the speed of adjustment of the economy in response to tax changes, and hence a comparison of the short-run versus long-run elasticities. It would also be interesting to add aggregate shocks to the model and study the effects of taxes over the business cycle, if aggregate shocks push the densities of incomes towards or away from the boundaries of the inaction region. On the theoretical side, it would be worthwhile to characterize the optimal tax schedule in environments with fixed costs of adjustment both on the labor supply and the labor demand sides with an endogenous wage, leading to bilateral monopoly situations. I leave these questions for future research.

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