Internet Regulation, Two-Sided Pricing, and Sponsored Data *

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Abstract

We consider a network that intermediates traffic between free content providers and consumers. Two-sided pricing of consumers and content providers allows profit extraction by the network and transmission of information on the social value of content. Profit maximizing tariffs give the content providers the option to sponsor the traffic of consumers. We show that a cost-oriented price-cap on the charge to content providers improves social welfare, while banning discrimination or imposing zero price for content providers is not optimal if content is valuable enough.

1 Introduction

The pricing of traffic on Internet is the object of intense debate, with contrasted views on the way the operators of the physical network should treat various contents and on the relationship between content providers and Internet service providers. The economic debate has mostly focused on the opportunity of allowing networks to charge content providers as well as consumers, and on second-degree discrimination. As a practical illustration of recent practices of Internet service providers, consider the AT&T's Sponsored Data program. It involves content providers paying for the data used by their customers, so that it is not be

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counted in their subscriber's monthly data limits, and it induces self-selection among content providers. This paper provides a new perspective on such practices and on price regulation by focusing on the informative role of prices charged to consumers and content providers.

Socially efficient traffic management must ensure that the various actors on the web internalize the cost or benefits they impose on the ecosystem. This is not straightforward in a network like Internet due to the prevalence of free services. When consumers don't pay for the content, there is no price to signal scarcity to consumers and value to producers. To this extent, we may view Internet as an instance of markets with missing prices, implying some misallocation of resources.

When managing traffic, networks face then two types of issues. First, content providers should be allowed to signal the benefits they derive from consumption. A free website deriving high advertising revenue or a blogger attaching a strong value to readers cannot be distinguished from low benefit websites, except through investment in quality or costly signaling. This tends to generate a misallocation of consumers across websites. A standard procedure for commercial services of this nature would be to let the seller adjusts the price charged to consumers according to its costs or benefits. However, with no price charged by the content providers and thus no pass-through, alternative solutions to signal costs or benefits must be considered. Second, the cost of communications results from the interaction between the consumer who receives the traffic and the content provider who sends the traffic. The way consumption is transformed into costs depends on factors that are usually known and controlled by the content provider.[1] Hence consumers can hardly forecast the cost they impose at the time they choose consumption.[2] Moreover, even if the networks were using deep packet inspection and monitoring services, he could not inform consumers at the request level, before they make their consumption choice. As the information can only be sent ex-post, and not ex-ante, the consumers cannot be aware of the full consequences of their choice when making their decisions.

In this paper, we study how tariffs targeted at both consumers and content providers can be designed to address the misallocation problem and provide useful information for consumers to promote efficient network use.

We consider a network that intermediates the traffic between content providers and consumers. Content providers receive a benefit proportional to traffic, such as advertising revenue or direct utility for the producer, but do not charge a retail price for content. This

[1] There is a distinction between what a consumer perceives as content and what a network perceives as a cost. The consumer may care about a video, a voice message or some news article, while the network views bits of information.

[2] The difficulty for consumers to assess correctly their internet consumption is a well document fact (see Strategy Analytics (2013).

benefit is heterogenous and private information of the content providers, who have no direct control on the consumption. In the core of the paper, the cost of traffic generated by consumption of any content (referred to as the load) is known to the network and consumers. But we will analyze in the extension section the case where the load is private information of the content providers and only total traffic is observed ex-post by the network.

Based on the observed cost of traffic, the network can charge a price to one or both of the parties involved in traffic generation. We refer to the pricing of consumers only as one-sided and to the pricing of both sides as two-sided. We assume that the network can charge a hook-up fee to consumers[3] and, in most of the paper, that the network is a monopoly facing inelastic consumers participation. We then extend the analysis to the case of elastic demand and competing networks.

Under laissez-faire, the network would use two-sided pricing and moreover try to discriminate between various types of contents by charging higher prices to high benefit content providers. Discrimination is only possible when the network can personalize the consumption levels for each type of content by proposing different prices to consumers. The network will then offer content providers to choose within a menu of two-sided tariffs, making the tariffs transparent to consumers. We refer to this practice as " sponsored pricing", in the same spirit as the AT&T's Sponsored Data program mentioned earlier, as the price paid by consumers decreases with the price paid by the content provider. Faced to the menu, each content provider must trade-off the volume of consumption with the cost of traffic. The high benefit content providers will then sponsor consumption while the others will prefer to save on their cost by letting consumers pay for traffic.

These different consumers prices allow to transmit a signal to consumers based on the information extracted from the content. The mechanism thus improves efficiency by fostering the transmission of information between content providers and consumers. We nevertheless show that under profit maximizing pricing, the consumption of content is socially suboptimal.

We then discuss some forms of regulatory intervention. First we show that, with an inelastic aggregate participation of consumers, imposing a price-cap at unit cost on the price charged to content providers is always welfare improving. Then, we discuss the effect of preventing discrimination. In this case, the network offers a simple pair of tariffs, one part being paid by consumers and the other by content providers. Uniform two-sided pricing prevents the network from adjusting consumptions to the type of content. As the network cannot extract rents from the most valuable contents without excluding the contents deriving low benefits, there will be more exclusion under uniform pricing than with sponsoring. As a

[3]In most countries, fixed and mobile network operators offer consumers a menu of tariffs with different traffic allowance, thus non-linear tariffs. We abstract from this issue by considering only two-part tariffs.

consequence, it is shown that preventing sponsored pricing reduces welfare if the content is valuable enough.

We show that imposing a one-sided tariff with a sponsoring option is socially dominated by a cost-oriented price-cap, while the same conclusion holds for a one-sided tariff without sponsoring provided that content providers derive large benefits from consumption.

Allowing the load to be private information of the content providers reinforces our conclusions. When high and low benefit contents generate different loads, sponsored pricing allows the content providers to signal their load to consumers. This enables the network to adjust the consumption to the load. By contrast, under uniform prices all contents are pooled so that consumption can only be adjusted to the average load of participating content.

Our work is related to the two-sided market literature (see Armstrong (2006), Caillaud and Jullien (2003), Rochet and Tirole (2003, 2006)), as we try to understand how both sides of the market, the content providers and the consumers, must be priced. Our model is mixed between the participation model of Armstrong (2006) and the usage model of Rochet and Tirole (2003). Indeed, in our basic model, the total number of agents on the consumer's side is fixed and only the consumption is affected by the price and the number (more precisely the types) of content providers on the market. On the content providers' side, the profit depends on the price the platform charges but also on the number of consumers and the price charged to them by the network. And the number of content providers can vary, as too high a price may exclude the high load contents.

In the field of telecommunication, some contributions study the pricing structure between receivers and senders. This literature has emphasized the importance of call externalities, and therefore the social benefit of using positive receiver prices (see Jeon, Laffont and Tirole (2004)). Hermalin and Katz (2004) develop a related idea but focusing on the way to deal with the uncertainty of the private value of exchanging messages and the gaming, i.e. the choice to call or to wait for a call, induced by the tariffs structure. In our paper, the structure of communication is different as it is the receiver (the consumer) who is at the origin of the communication. Moreover, the sender is the only one to know the cost of this communication. The presence of both moral hazard and adverse selection makes our setting quite different from the previous articles studying the pricing of communications.

The literature on the price regulation on Internet has been driven by the debate over net neutrality and the optimal way to price content providers and consumers (see Economides and Hermalin (2012)). One point that emerges from two-sided market models is that, while laissez-faire will not result in efficient pricing, the precise nature of the intervention that would foster efficiency is unclear (see Economides and Tag (2012) for instance). Neglecting the investment question (on this point, see Choi and Kim (2010) or Hermalin and Katz

(2009), and the literature below), we focus on the efficient management of current resources when the real cost of consumption is uncertain. By focusing on the information revelation aspect of prices, we offer a new perspective that complements previous studies on the impact of price discrimination (see Hermalin and Katz (2007)). Several recent contributions discuss the screening of traffic sensitive contents by means of prices and differentiated quality layers, a key aspect of the net-neutrality debate (see Bourreau, Kourandi and Valletti (2014), Krämer and Wiewiorra (2010), Choi, Jeon and Kim (2013), Reggiani and Valletti (2012)). Peitz and Schuett (2015) analyze moral hazard in traffic generation in a model with congestion externalities. Our work departs from these by considering consumption usage and the informational role of consumer prices. A specific contribution is to show that screening among traffic sensitive contents can be achieved with different consumer prices and a single quality layer, and to prove the optimality of a price-cap on the content side.

The next section presents the model. Section 3 discusses the role of pricing both consumers and content providers when the network can propose menus to content providers under laissez-faire. Section 4 focuses on regulation by analyzing standard price-cap regulations (cots orientation and zero price) and no-discrimination rules. Section 5 proposes three extensions, first by considering elastic subscription demand and competition at the network level, second by assuming that the consumers do not know the load ex-ante, and then by characterizing an optimal price-cap. Section 6 concludes.

2 Model

We analyze the tariff charged for traffic by a network (in the case of Internet, an Internet Service Provider) to two sides of the market: consumers and content providers. In practice, some contents are delivered freely while others are paid for. As this is the main originality and the focus of this paper, we simplify the analysis by assuming that all contents are free. We assume for conciseness that contents are non rival so that consumers visit every content. The expected demand for each content when consumers face a price p per unit of content is $q = D(p)$. The representative consumer utility function $U(q)$ is strictly concave with $U'(0) = \bar{p} > 0$ and $U'(\bar{q}) = 0$. Thus the demand $D(.)$ is decreasing, the consumption $D(0) = \bar{q}$ of free goods is positive, and demand vanishes at price $\bar{p}$. We also assume that the demand $D(.)$ is convex[4].

Any transaction between a content provider and a consumer generates a cost for the network. More precisely, for any unit of consumption, each content will generate an expected

[4] The convexity assumption is not necessary for most of the results and our main conclusions can be extended to the case of non-convex demand.

cost θ, referred to as the load, to the network: the consumption of q units of content with a load θ generates a cost θq to the network.[5] The cost may be direct or related to congestion. One view is that the network needs to expand resources to maintain the quality of service and that θ reflects this need.[6] In a set-up with explicit congestion, θ would be interpreted as the shadow cost of congestion. We assume that consumption is positive if priced at the true marginal cost, i.e. $\theta < \bar{p}$.

Each unit of consumption generates a net benefit $a > 0$ for the content provider. This benefit includes the advertising revenue[7] and other gains of the content provider but also the cost of distributing the content if any. Content providers are heterogenous and can be of two types, ℓ and h, which are characterized by benefits $a_\ell < a_h$. In the extension, we will allow θ to depend also on the type. A content is type ℓ with probability $1 - \lambda$ while the content is type h with probability λ. The type of each content is unknown to the consumers and the network, but known to the content provider. We will refer to contents of type h as high benefit (in short HB) contents and to type ℓ as low benefit (in short LB) contents. We denote by $b_\ell = a_\ell/\theta$ and $b_h = a_h/\theta$ the ratio of benefit over load. As this is the most interesting case, we assume that one type of content could not be proposed if the providers had to pay the full cost of traffic:

$$b_\ell < 1 < b_h.$$

This assumption captures the idea that some content providers can afford the cost while others cannot. Thus, it is not possible that all contents are proposed unless part of the cost of traffic is paid by the consumers.[8]

While content providers have some information about the benefit, the level of consumption is determined by consumers. Moreover, the network observes the ex-post realization of cost θq and can charge any side for this cost. We restrict attention to linear traffic prices, that is, prices of the form $s\theta q$ to the content providers and $r\theta q$ to the consumers, where the unit prices s and r are non-negative. However, we allow the network to offer to the content providers the choice between multiple pairs of prices (r, s). We denote by Π the variable

[5] For example, q may be the number of songs downloaded by the consumer while θ is the bandwidth taken by each song. Alternatively one may view q as a number of subscriptions and θ the traffic generated by one subscription.

[6] When the cost is only related to congestion, one may view θq as a cost that the network will bear ex-post to maintain the traffic.

[7] Advertising revenue increases with consumption if the time spent on the page by consumers increases with consumption, or if advertising is tied to consumption.

[8] The key assumption is $b_\ell < 1$. Otherwise, as it will appear latter on, imposing a price for content equal to 1 would maximize total welfare.

profit of the network, defined as the difference between of the traffic revenues and the costs.

We assume also that the network is able to charge a hook-up fee F ex-ante to consumers for subscription. The extent to which this hook-up fee allows the network to capture an increase in consumer surplus that traffic management generates depends on various factors, in particular the elasticity of participation to the network and the competition at the network level. In the main part of this analysis, we focus on the case of inelastic participation and discuss more general settings later.

In most of our analysis, we will consider the timing below.

1. The network proposes the prices r and s, and the hook-up fee F.

2. Consumers decide to affiliate or not.

3. Each content provider observes his type and decides to be active or not.

4. Consumers observe the choice of each content provider and choose how much to consume of each content.

5. Traffic is observed, payments to the network take place.

Let us denote by CS the consumer expected surplus from usage (gross of the hook-up fee). We assume for the moment that consumers are ex-ante identical and risk neutral, so that the consumers' subscription decision is based solely on $CS - F$. In this case, the network can extract the full expected surplus with the hook-up fee $F = CS$. It is then optimal for the network to maximize the joint expected surplus with consumers and to use the hook-up fee to share this surplus with the consumers.

As a consequence, the network's objective fully internalizes the surplus of consumers. We therefore ignore the fee F and assume for now on that the network maximizes the sum of its variable profit Π and consumer expected surplus CS, denote $V = CS + \Pi$ and refer to it as the *network value*.[9] As we will show later, the behavior of the network will lead to the maximization of V even under competition between networks or elastic demand, whenever consumers do not have private information about their expected surplus before joining the network.

As a benchmark, we consider the socially optimal prices in the case of full information on the content type and no direct transfer between content providers and consumers. This would correspond to the situation of a regulated network maximizing social welfare. For a content

[9] We use the term network value because, although it coincides with the network total profit in our basic model, this will not be the case with elastic demand and with competition.

with benefit $a = b\theta$, the price perceived by consumers is given by $r\theta$. The consumption is thus $q = D(r\theta)$. The content generates a monetary gain $(b - s)\theta = a - s\theta$ per unit of consumption and thus is proposed only if $s \leq b$. Social welfare then writes as

$$U(q) + (b\theta - \theta)q \text{ s.t. } s \leq b,\ U'(q) = r\theta.$$

Ignoring feasibility constraint $r \geq 0$, this leads to prices $r = 1 - b$ and $s \leq b$. The difficulty with the above solution is that it may involve negative prices when the benefit is high. When negative prices cannot be used, we obtain directly:

Lemma 1 *Under full information, the socially (constrained) optimal allocation is obtained by charging* $r = \max\{1 - b, 0\}$ *and* $s \leq b$.

When the content price is $s = b$, the content providers receive zero surplus. Thus for $s = b$, the network value $V = CS + \Pi$ is equal to the maximal total welfare. This implies that a network maximizing V implements the social optimum under full information about θ. We now investigate the case of imperfect information.

3 Network pricing under *laissez-faire*

Exclusive access to the consumers gives market power to the network over the content providers. We thus expect that excessive content prices by the network will result in excessive exclusion of contents. Moreover, under laissez-faire, the network would like to discriminate between the two types of content. This leads us to consider two types of price strategy for the network, uniform pricing and sponsored pricing.

Uniform two-sided pricing: The network offers a single pair (s, r).

Under uniform prices, the content providers decide only to participate or not. As a content provider does not charge for its product, it generates its profit only through the benefit a. The price s charged by the network to content providers is not reflected in an equivalent increase in the cost borne by consumers. Thus a content provider participates only if $s \leq b$. Facing a price s,a content provider of type t stays on the market if it anticipates a non-negative profit, hence if $s \leq b_t$ for $t = \ell, h$. In particular, if s lies between b_ℓ and b_h, only the HB content providers propose their content.

With uniform pricing, the network faces a trade-off between capturing the rent of HB content providers (with high s) and avoiding the exclusion of the LB content providers (with low s). One way to alleviate this trade-off, is to allow the network to propose more complex tariffs. To this end, we consider now the possibility for the network to achieve second-degree discrimination between the different types of content providers by offering a menu of linear tariffs.

Sponsored pricing: The network proposes the content provider two tariffs (s_ℓ, r_ℓ) and (s_h, r_h). The consumer is informed about the tariff chosen.

Sponsored pricing thus amounts to define several tariffs, a base tariff ℓ and and a sponsored tariff h. Content providers choose which tariff applies and this information is transmitted to the consumers.[10] Note that there is no possibility to discriminate between different content providers without inducing some differential consumptions. Indeed, if consumers were not affected by the choices of the content providers, the consumption would be the same for all contents and all content providers would always opt for the smallest price s.[11] However, the network may try to raise its profits and the value offered to consumers by combining a higher price for the contents with a lower price for consumers. Content providers eager to generate traffic (due to high benefit a) may then opt for this option. If the network succeeds in inducing the LB and the HB content providers to choose different tariffs, then consumers should adapt their behavior to their price of the tariff. We thus define sponsored pricing as a menu $\{(s_\ell, r_\ell), (s_h, r_h)\}$ and consumption levels $\{q_\ell = D(\theta r_\ell), q_h = D(\theta r_h)\}$ such that the LB (resp. HB) content providers are willing to participate and choose tariff ℓ (resp. h), given consumption in each tariff.

Our definition of sponsored pricing encompasses situations where the network chooses to exclude the LB content. Indeed, such an exclusion happens when $q_\ell = 0$ (with a price $r_\ell \geq \bar{p}/\theta$) and $b_\ell < s_h \leq b_h$, as in this case the LB content providers obtain zero demand and therefore zero profit.[12] Sponsored pricing with $q_\ell = 0$ is equivalent to a uniform tariff with $(s, r) = (s_h, r_h)$, with $s_h > b_\ell$ because the LB content providers would not participate under such tariff. Similarly a uniform tariff $s \leq b_\ell$ corresponds to sponsored pricing with $(s_\ell, r_\ell) = (s_h, r_h)$.[13]

Hence, under laissez-faire, we can focus without loss of generality on sponsoring. For each type t of content, the consumer surplus is $CS_t = U(q_t) - r_t\theta q_t$ and the network profit is

[10] This require ex-ante communication. The content providers can directly inform the consumer of the tariff upon visit and a standard unravelling argument shows that it will do so if the network doesn't.

[11] Thus keeping consumers uninformed is not compatible with price-discrimination, because consumption would not react to the content providers' choice.

[12] Thus, under category pricing, there is no loss of generality in assuming that all content providers participate to the mechanism.

[13] This will not be true when different types of content have different loads (see the extension section).

$\Pi_t = (r_t + s_t - 1)\,\theta q_t$. The network maximizes the joint surplus with consumers written as

$$V = \lambda \left[U(q_h) - (1 - s_h)\theta q_h\right] + (1 - \lambda)\left[U(q_\ell) - (1 - s_\ell)\theta q_\ell\right].$$

Then we have a tariff that induces participation of both types if

$$b_\ell \geq s_\ell \text{ and } b_h \geq s_h. \tag{1}$$

As we already argued, $q_t = 0$ is equivalent to no supply of type t content, so we can impose without loss of generality that condition (1) holds even when one type is not active. Then, the following incentive compatibility conditions ensure that the content providers reveal their types:

$$\begin{aligned} (b_\ell - s_\ell)\, q_\ell &\geq (b_\ell - s_h)\, q_h \\ (b_h - s_h)\, q_h &\geq (b_h - s_\ell)\, q_\ell \end{aligned} \tag{2}$$

Sponsored tariffs can therefore be summarized by an allocation (q_l, q_h, s_l, s_h) such that conditions (1) and (2) are satisfied. The program of the network is then to maximize V under these two constraints. Such a program departs from the classical textbook cases because the transfer $s\theta q$ depends on the quantity. Nevertheless, one can follow the usual procedure for solving the program.

First, as it is optimal to raise the content prices as long as they are compatible with the constraints, the participation constraint of the LB content providers and the incentives constraints of the HB content providers will be binding, i.e.

$$s_\ell = b_\ell \text{ and } (b_h - s_h)q_h = (b_h - s_\ell)q_\ell. \tag{3}$$

Second, we remark that under (3), all constraints are satisfied provided that $s_h \geq b_\ell$ or equivalently that $q_\ell \leq q_h$. Replacing the prices by the values given by condition (3), we solve a reduced program written in terms of the quantities:

$$\max_{q_h \geq q_\ell} \lambda \left[U(q_h) - (1 - b_h)\theta q_h - (b_h - b_\ell)\theta q_\ell\right] + (1 - \lambda)\left[U(q_\ell) - (1 - b_\ell)\theta q_\ell\right].$$

This directly leads to the proposition below

Proposition 1 *The optimal sponsored tariffs are such that* $q_t^* = D(r_t^*\theta)$, *for* $t = \ell, h$, *and:*

$$\begin{aligned} s_\ell^* &= b_\ell,\ r_\ell^* = \min\{1 - b_\ell + \frac{\lambda}{1-\lambda}(b_h - b_\ell), \bar{p}\} \\ s_h^* &= b_h\left(1 - \frac{q_\ell}{q_h}\frac{b_h - b_\ell}{b_h}\right),\ r_h^* = 0. \end{aligned}$$

Proof. The solution of the reduced program is obtained at

$$\begin{aligned} U'(q_\ell) &= r_\ell\theta = (1 - b_\ell)\theta + \frac{\lambda}{1-\lambda}(b_h - b_\ell)\theta \text{ if it is less than } \bar{p} \\ q_\ell &= 0 \text{ otherwise} \\ U'(q_h) &= 0 \end{aligned}$$

The condition $q_h \geq q_\ell$ holds. This determines the consumers' usage price. In the case $q_\ell = 0$, we take the convention that $r_\ell\theta = \bar{p}$ for clarity but any larger price would also work. The content prices are then given by condition (3) ■

The menu proposed by the network plays two roles.[14] It allows screening of the different types of content providers and at the same time it improves the efficiency of consumption. Consider first the tariff designed for the HB content providers. As the gains generated by the provider are higher than the cost, the network wants to induce high consumption and therefore sets zero price for receivers. As far as the price paid by the content providers is concerned, the price s_h is strictly smaller than b_h because the platform must leave some profit to the HB content providers to induce truthful revelation. While the price s_ℓ paid by the LB content providers is simply set to minimize their profit, the price r_ℓ paid by consumers to access these contents results from two causes. First, it reflects the net cost of any unit of consumption. Second this price is distorted to minimize the level of profit the network has to leave to the HB content providers to induce truthful revelation. As it is common in the information economics literature, informational asymmetries lead the network to propose higher prices and therefore generate social costs.

In this setting, the network may decide to exclude the LB content providers. This happens when the quantity q_ℓ resulting from the price characterized in proposition 1 is below 0, and thus when λ is large:

$$q_\ell = 0 \Leftrightarrow \lambda \geq \lambda^* = \frac{\bar{p} - (1 - b_\ell)\theta}{\bar{p} - (1 - b_h)\theta}. \tag{4}$$

In this case, the network may simply rely on a uniform tariff $s = b_h$ and $r = 0$. For a low proportion of HB content, the network chooses to induce full participation and screens

[14] The same analysis holds with $b_h < 1$ except that $r_h = 1 - b_h$.

between the two types of content. Thus sponsored pricing is preferred to uniform pricing for $\lambda < \lambda^*$.

Corollary 1 *Under laissez-faire, the network excludes the LB contents for* $\lambda > \lambda^*$, *otherwise it opts for sponsoring. Total exclusion of LB content is all the more likely that* b_h *is high,* θ *high, or that* b_ℓ *is low.*

Proof. Immediate from proposition 1 and formula 4. ■

Therefore the network should never use uniform pricing without exclusion, as this pricing policy is never the solution of the profit maximizing program of the network with full participation. However, in our two-type model, the network will rely on uniform prices in case of exclusion.

From a total welfare perspective, the consumption of HB content is efficient under the constraint that the usage price charged to consumers is non-negative. However efficient consumption of LB content would require that $r_\ell = 1 - b_\ell$ and thus consumption is suboptimal. This provides some rational for regulation. Notice that this rational for regulation doesn't depend on market power on the consumer side. As we shall see later, the analysis relies on the exclusive relationship between the consumers and the network, that induces market power over access to these consumers.[15]

4 Regulation of the price for content providers

In this section we discuss various forms of regulation and their welfare impact. We only discuss the case where this regulation concerns the price for content. Moreover, to be in line with real-world practices, we focus on some standard forms of regulation.[16]

We assume that the regulator maximizes total welfare that can be written as

$$W = \lambda \left[U\left(q_h\right) - (1-b_h)\theta q_h\right] + (1-\lambda)\left[U\left(q_\ell\right) - (1-b_\ell)\theta q_\ell\right].$$

We denote by q_t^{FB} the efficient consumption levels, characterized by

$$q_h^{FB} = D\left(0\right) \text{ and } q_\ell^{FB} = D\left(\left(1-b_\ell\right)\theta\right)$$

As q_h^* is efficient while $q_\ell^* < q_\ell^{FB}$, the main regulatory concern will be to raise consumption of LB contents although avoiding any drop in the consumption of HB contents.

[15] The argument is thus similar to the one justifying the regulation of telecommunication termination charges in many countries.

[16] In the extension, we will discuss the optimal regulatory rules.

4.1 Cost-oriented price-cap

The first regulation considered is a cost-oriented price-cap regulation, a simple and common form of regulation. Formally, regulation at cost amounts to setting a constraint of the form $s \leq 1$.[17]

In the case of sponsoring, the constraints are the same as before except that $1 \geq s_h$ replaces $b_h \geq s_h$ in condition (1). A simple reasoning that we present in Appendix shows that the network will choose $b_\ell = s_\ell$, $q_h \geq q_\ell$. The price for the HB content now accounts for the price-cap and is still given by

$$s_h = \inf\{1, b_h - (b_h - s_\ell)\frac{q_\ell}{q_h}\}.$$

The reduced program then writes as

$$\begin{aligned} \max_{q_h \geq q_\ell} \quad & \lambda \left[U(q_h) - \theta q_h + \inf\{q_h, b_h q_h - (b_h - b_\ell) q_\ell\}\theta\right] \\ & +(1-\lambda)\left[U(q_\ell) - (1 - b_\ell)\theta q_\ell\right] \end{aligned}$$

The difference with the unconstrained sponsored pricing program is that below some level, reducing the consumption of LB content becomes ineffective at reducing the rent of the HB content providers. We can then show the following result.

Proposition 2 *A cost-oriented price-cap leads to* $\bar{s}_\ell = b_\ell,\ \bar{s}_h = 1,\ \bar{q}_h = D(0)$ *and*

$$\begin{cases} \bar{q}_\ell = q_\ell^* \quad if \quad q_\ell^* \geq D(0)\frac{b_h - 1}{b_h - b_\ell}; \\ \bar{q}_\ell = D(0)\frac{b_h - 1}{b_h - b_\ell} \quad if \quad q_\ell^{FB} \geq D(0)\frac{b_h - 1}{b_h - b_\ell} \geq q_\ell^*; \\ \bar{q}_\ell = q_\ell^{FB} \quad if \quad D(0)\frac{b_h - 1}{b_h - b_\ell} \geq q_\ell^{FB}. \end{cases}$$

Proof. See Appendix. ■

It is clear that the price-cap has no effect whenever the original solution involves only content prices below 1. Otherwise, the network has to raise the HB content price to the cost and alongside raises the consumption of LB content.

The main consequence of the price-cap is to deter the network from extracting too much surplus from the HB content providers. This has both a direct effect on the way surplus is shared but also an indirect and positive effect of efficiency. Indeed, as the network cannot extract too much from the HB content providers, it has less incentive than before to

[17] We will discuss the optimal price-cap regulation in the extension section.

distort the consumption for the LB contents. If the efficient consumption of LB content is small enough, then pricing the HB content providers at cost is compatible with efficient consumption of LB content and the allocation will be efficient.

Therefore, the imposition of the price-cap leads to lower distortions in consumption and therefore increases welfare.

Corollary 2 *Under sponsoring, a price-cap at cost,* $s \leq 1$, *raises welfare.*

Proof. Follows from $\bar{q}_\ell \in \left[q_\ell^*, q_\ell^{FB}\right]$. ■

This first conclusion appears to be the main and most robust one of the paper. The question is then whether other forms of regulation, in particular those that have been advocated in the context of the net-neutrality debate, perform as well.

4.2 No discrimination

We consider now a regulation that forces uniform pricing by prohibiting price-discrimination. In the debate on the regulation of network pricing on internet, this corresponds to one form of net-neutrality that has been advocated. As what follows is quite general, we shall be agnostic about the form of price-cap regulation by assuming a price-cap $\sigma \in \{1, b_h\}$. Thus $\sigma = b_h$ corresponds to the case with no price-cap while $\sigma = 1$ corresponds to cost-orientation.

When restricted to uniform pricing, the network can only reduce selectively the consumption of LB contents by excluding them with a price s above b_ℓ. Denoting $M \in \{\lambda, 1\}$ the mass of active content providers, the network maximizes the joint surplus with consumers[18]

$$V = M \times \left[U(q) + (s-1)\theta q\right] \text{ with } q = D(r\theta).$$

The term into bracket captures the incentives to maximize the per content joint surplus of the network and consumers for a given value of s. The net data cost per content is $(1-s)\theta$ and internal efficiency is achieved by setting a consumer price equal to this cost whenever feasible. As content providers' participation is independent of the consumer price, the network chooses:

$$r = \max\{1-s, 0\}. \tag{5}$$

Given (5), the choice of the network boils down to choosing the price s charged to the content providers. Notice that for given participation M, the network value increases with the price s. Therefore, the network chooses s by comparing two prices for content: the maximal price $s = b_\ell$ that maintains full participation with $r = 1 - b_\ell$, and the maximal price $s = \sigma$

[18] Alternatively, we could solve the category pricing program imposing that $q_\ell \in \{q_h, 0\}$.

that preserves participation of the HB contents only with $r = 0$ (as $1-\sigma \leq 0$). The respective consumptions are then $q_\ell^u = D\left(\left(1 - b_\ell\right)\theta\right)$ and $q_h^u = D(0)$ leading to a network value:

$$
\begin{aligned}
V_\ell^u &= U\left(q_\ell^u\right) + \left(b_\ell - 1\right)\theta q_\ell^u \text{ when } s = b_\ell \\
&\quad \text{and} \\
V_h^u &= \lambda\left[U\left(q_h^u\right) + \left(\sigma - 1\right)\theta q_h^u\right] \text{ when } s = b_h.
\end{aligned}
$$

The key difference with discrimination is that, when both types of content providers participate, a network needs to leave a higher rent to the HB content providers because it cannot selectively reduce q_ℓ. As a consequence, there will be more exclusion under uniform pricing than under sponsoring.

Proposition 3 *Under uniform two-sided pricing, the network excludes the LB contents ($s = b_h$) if and only if $\lambda > \lambda^u$. There is more exclusion than with sponsored pricing ($\lambda^u < \lambda^*$ if $\sigma = b_h$, $\lambda^u < 1$ if $\sigma = 1$). The threshold λ^u is decreasing in σ, and increasing in b_ℓ.*

Proof. See Appendix ■

The comparative statics underlying the trade-off is quite simple to analyze in terms of the relative efficiency of contents. The network value under exclusion increases with σ and is independent of b_ℓ. Conversely, the network value when inducing all content providers to participate increases with b_ℓ and is independent of σ. Hence exclusion occurs if σ is large enough and/or b_ℓ is small enough. On the reverse, the network does not exclude the low benefit contents if both b_h and b_ℓ are close to 1.

We now turn to the welfare comparison with sponsoring. As we have seen, there will be more exclusion under uniform pricing as the network cannot accommodate LB content with positive but low consumption. On the other hand, when the network chooses to attract both types of content, it must raise the consumption of LB content and reduce the consumption of HB content. Thus, as often the case, the prohibition of price-discrimination has an overall ambiguous effect.

Proposition 4 *When sponsoring is prohibited:*

- *if $\lambda > \lambda^u$, total welfare decreases (weakly if there is no price-cap): the rent of the HB content providers is lower and the LB contents are excluded from the market;*

- *if $\lambda < \lambda^u$, total welfare decreases if λ is small enough while the effect is ambiguous for intermediate values of λ.*

Proof. See Appendix ■

As it is now standard in the analysis of price-discrimination, sponsored pricing raises welfare compared to uniform pricing if it avoids the exclusion of the LB contents. The new feature is that the HB contents also benefit from the discrimination. The reason here is that only second-degree discrimination is allowed. Thus when allowing the LB contents to stay with a low s (this occurs for $\lambda^* > \lambda > \lambda^u$), the network needs to leave some rents to the HB content providers that were not needed with exclusionary uniform prices.

When there is no exclusion with either regime, the effect of banning sponsoring is more ambiguous. Indeed, the consumption of HB content is too low while the consumption of LB content is higher which may or may not raise welfare. Notice that when λ is small, the distortion of q_ℓ (or equivalently of r_ℓ) is small and the former effect dominates, making sponsored pricing the optimal system. Moreover we also find the result:

Corollary 3 *Under cost-oriented price-cap $s \leq 1$, sponsored pricing dominates uniform pricing if b_h is large.*

Proof. See Appendix. ■

When the benefits generated by the consumption of HB content are large, ensuring efficient consumption level is of primal importance. As uniform pricing tends to reduce the consumption of HB content, it is all the more detrimental that b_h is large. Moreover, increasing b_h tends to increasing the bite of the price-cap, and therefore reduces the extend to which the LB content consumption is distorted under sponsoring. Therefore, looking at the effect on the both types of contents, when b_h is large, sponsored pricing dominates uniform pricing.

4.3 One-sided pricing regulation

Another possible regulation consists in imposing a zero-price rule for contents ($s = 0$ in our setting), an option supported by some active participants in the debate on Net Neutrality. In this case regulators may allow or disallow sponsored data.

Let us suppose first that this is allowed. This means that the network must first choose a base price r_ℓ for consumer along with a zero-price s_ℓ for the content providers. It may still offer an option to the content providers of sponsoring the consumption in which case the price is $r_h < r_\ell$ for consumers but the content provider pays a price $s_h > 0$. Under such a regulation, it is straightforward to see that the pricing program of the network is the same as in Section 3 except that the constraint $s_\ell \leq b_\ell$ is replaced by the constraint $s_\ell = 0$. The reasoning of proposition 1 applies with this new constraint so that we obtain

$$\begin{aligned} s_\ell &= 0,\ r_\ell = \min\{1 + \frac{\lambda}{1-\lambda} b_h, \bar{p}\} > r_\ell^* \\ s_h &= b_h \left(1 - \frac{q_\ell}{q_h}\right),\ r_h = 0. \end{aligned}$$

Comparing with laissez-faire, we see that the consumption of HB content is unchanged but the consumption of LB content is reduced due to higher consumer prices. It is then immediate that both total welfare and consumer welfare are lower than under laissez-faire, and a fortiori than under a cost-oriented price-cap. The reason is that depriving the network from charging a price to content providers in the base tariff reduces the attractiveness of sponsoring. Thus the network reduces the base consumption even more to raise the value of sponsoring.

Let us now turn to the case of a general ban on any price charged to content providers, which we refer to as strict one-sided pricing. Then discrimination by sponsoring is not possible and the network sets a unique consumer price $r = 1$ from equation (5). The consumption of any content is then given by $D(\theta)$. Thus, under this zero price regulation, the network cannot exclude any content but will choose a high usage price for consumers. When the content is of HB type, it is immediate to see that a price-cap at 1 performs better than the zero price, as this induces efficient consumption. In the case of the LB content, the comparison between a price-cap at 1 and a price-cap at 0 is ambiguous.

Let $W(s)$ denote the total welfare when the sender price is given by s with no sponsoring. The optimality of a strict zero price over exclusion with $s \geq 1$ depends then on the sign of $W(0) - W(1)$. The latter expression is convex in λ, positive when $\lambda = 0$ and negative at $\lambda = 1$. Hence the regulation with $s = 0$ dominates exclusion of the LB contents if λ is below a threshold λ_0. Notice that as $W(0) < W(b_\ell)$, if $\lambda_0 < \lambda^u$, there will be no value of λ for which this policy may be optimal. We then have

Proposition 5 *Welfare is higher under cost-oriented price-cap than under one–sided pricing with sponsoring. It is higher under cost-oriented price-cap than under strict one–sided pricing if b_h is large, if λ is small, or if λ is large.*

Proof. See Appendix. ■

Recall that under cost-orientation allowing sponsored pricing is optimal if b_h is large. Thus we conclude that when the HB content is valuable enough the socially optimal regulation among those considered here is a price-cap on the charge to content providers with sponsored pricing allowed. The reason is that it induces the optimal consumption of HB content while mitigating the network incentives to distort the consumption on the non-sponsored content (as the sponsoring revenue cannot be raised by doing so).

For low values of b_h, a strict zero price regulation reduces welfare whenever cost-orientation leads to almost efficient pricing which is the case when λ is small. When cost-orientation is not efficient and leads to lower consumption of LB content than zero price, the comparison is ambiguous except for large λ as in this case only the HB content matters.

5 Extensions

5.1 Elastic participation and competition between networks

In the main analysis, we considered the case of a monopoly network with inelastic subscription demand. We want to show that introducing demand elasticity does not change the way the variable cost is allocated between consumers and content providers, and thus the main conclusions of our work. The same holds for competition at the network level. To do so, we will give a more detailed description of the participation decision of the consumers.

We consider a model with an initial mass 1 of consumers, a mass 1 of content providers (indexed by x) and $I \geq 1$ networks (indexed by i). Content providers are still divided into a mass λ of type h and a mass $1-\lambda$ of type ℓ. The utility of each consumer subscribing to network i and consuming a consumption profile $\{q_{ih}, q_{i\ell}\}$ is given by

$$\mathbb{E}_t\left(u(q_{it}) - \theta r_{it} q_t\right) + \varepsilon_i - F_i$$

For each network i, F_i represents the hook-up fee, ε_i is an idiosyncratic shock and r_{it} the variable price on this load. The idiosyncratic shock ε_i is a random variable that represents the consumers' heterogeneity relative to the intrinsic taste for network i. We do not put any restriction on the distribution of the preference shocks, except that we implicitly assume that they do not convey any information about the utility derived from consuming contents.

The timing of the game is unchanged and, in case there is competition between networks ($I > 1$), we assume that at stage 1, each network i chooses simultaneously an offer $(F_i, r_{ih}, s_{ih}, r_{i\ell}, s_{i\ell})$. With this slightly modified setting, we assume that the content providers multi-home, paying only a variable price, while the consumers single-home.

If N_i is the mass of consumers subscribing to network i, the profit of each content provider at network i is given by

$$N_i(a_t - s_{it}\theta)q_{it} = N_i\theta(b_t - s_{it})q_{it}$$

A content provider t will choose to participate with network i if $b_t \geq s_{it}$. In this context, the participation of content providers to network i, the choice of tariffs by content providers and the individual consumptions for a given contract are the same as before. What differs is the choice of network by the consumers.

The gross consumer surplus is given by

$$CS_i = \lambda \left(U\left(q_{ih}\right) - r_{ih}\theta_h q_{ih}\right) + (1-\lambda)\left(U\left(q_{i\ell}\right) - r_{i\ell}\theta q_{i\ell}\right)$$

A given consumer joining network i gains $CS_i - \varepsilon_i - F_i$. As there are many potential networks,

$$N_i = \Pr\left[CS_i - F_i + \varepsilon_i \geq \max\{0, \max_{j\neq i} CS_j - F_j + \varepsilon_j\}\right].$$

The total profit of network i is given by

$$N_i.[F_i + \lambda(r_{ih} + s_{ih} - 1)\theta q_{ih} + (1-\lambda)\left(r_{i\ell} + s_{i\ell} - 1\right)\theta q_{i\ell}]$$

For any given strategy of the other networks - denoted z_{-i} - let us define

$$\phi_i\left(R; z_{-i}\right) = \Pr\left[R \geq \max\{0, \max_{j\neq i} CS_j - F_j + \varepsilon_j\} - \varepsilon_i\right].$$

Then we can write the profit of network i as

$$\phi_i\left(CS_i - F_i; z_{-i}\right).[F_i + \lambda(r_{ih} + s_{ih} - 1)\theta q_{ih} + (1-\lambda)\left(r_{i\ell} + s_{i\ell} - 1\right)\theta q_{i\ell}].$$

With this formulation, it is easy to see that the networks best pricing strategy always maximizes the network value per consumer.

Proposition 6 *In any equilibrium of the game with elastic subscription demand and I networks, each network chooses a tariff* (s_{it}, r_{it}), $t \in \{\ell, h\}$ *that maximizes its value per consumer:* $V_i = \lambda\left(U\left(q_{ih}\right) + \left(s_{ih} - 1\right)\theta q_{ih}\right) + (1-\lambda)\left(U\left(q_{i\ell}\right) + \left(s_{i\ell} - 1\right)\theta q_{i\ell}\right).$

Proof. The profit can be written as

$$\phi_i\left(R_i; z_{-i}\right).[V_i - R_i],$$

where $R_i = CS_i - F_i$ is the expected net consumer surplus and V_i is the network value. Notice that V_i is independent of the subscription fee F_i and of other networks strategies z_{-i}, while R_i depends on F_i. This implies that the network will always choose (s_i, r_i) to maximize V_i. ■

The value V_i only depends on the usage prices (r_i, s_i), so there is a natural hierarchy in the pricing strategy. First, the network maximizes the value that can be shared with consumers by setting adequate usage prices. Then, the network decides how much surplus to retain and how much surplus to leave to the consumers. While the surplus R_i left to consumers, and thus the subscription fee F_i, depends on the elasticity of demand and on the way competition is modeled between the networks, the prices $(r_{it}, s_{it})_{t\in\{\ell,h\}}$ do not. So the prices derived in the main model assuming a monopoly network are also the equilibrium prices when there are more than one network competing for consumers.

As far as welfare is concerned, for fixed total demand (inelastic consumers participation), introducing competition at the network level does not alter our results. But when aggregate demand is elastic, it may raise total participation to the market. Notice that, compared to the inelastic demand case, the analysis of the regulation of the traffic price should be more favorable to laissez-faire under competition.

Corollary 4 *If aggregate demand is elastic enough, laissez-faire may dominate cost-oriented price-cap.*

Proof. See Appendix. ■

With elastic aggregate demand, increasing the value V_i also increases subscription demand, an effect that we ignored in the above analysis. Thus the optimal price-cap, if any, will be higher and the case for allowing sponsored pricing would be stronger.

5.2 Heterogenous load

When the type of the content is heterogenous and unknown, the consumers are uninformed about the content's load, which generates consumption inefficiencies for given prices. Thus, the network benefits from informing consumers of the load if it knows it. Therefore, under second-degree price discrimination, it will be optimal for the network to inform consumers about the type of the content.

We assume now that content providers types, ℓ and h, differ by the two components θ and a. A content is type ℓ, i.e. (θ_ℓ, a_ℓ), with probability $1-\lambda$ while the content is type h, i.e. (θ_h, a_h), with probability λ. The type of each content is unknown to consumers and the network, but known to the content provider. We rank the two types of content providers according to the ratio of advertisement revenue over load still assuming that

$$b_\ell = \frac{a_\ell}{\theta_\ell} < 1 < b_h = \frac{a_h}{\theta_h}$$

Note that the model is also compatible with a uniform benefit $a_\ell = a_h$ per unit of consumption across types but different load associated with the consumption, in which case type h is a low cost content.

Consumers rationally anticipate the participation of contents and adapt their expectations about the load consequently. As they do not pay for content, they consume $q = D(r\mathbb{E}(\theta|s))$ for each content. With heterogenous load, a distinction arises between sponsored pricing and a uniform tariff.

Under a uniform tariff, consumers receive no signal of the load. Given the content providers participation decisions, the average data load is given by

$$\mathbb{E}(\theta|s) = \begin{cases} \mathbb{E}(\theta) & \text{if } s \leq b_\ell \\ \theta_h & \text{if } b_\ell < s \leq b_h. \end{cases}$$

Denoting again q_t the consumption of type t content, we conclude that $q_h = q_\ell = D(r\mathbb{E}(\theta))$ if the content price s is below b_ℓ while $q_h = D(r\theta_h)$ and $q_\ell = 0$ if it lies between b_ℓ and b_h.

Under sponsored pricing however, consumers receive a signal of the load. Indeed if the network succeeds in inducing the LB and the HB content providers to choose different tariffs, then consumers should realize that the average load is different for the two tariffs. They will thus adapt their behavior to the price but also to the load. We thus now define sponsored pricing as a menu of tariffs $\{(s_\ell, r_\ell), (s_h, r_h)\}$ such that the following two properties hold:

1. Consumers anticipate that the load is θ_ℓ for the tariff ℓ and θ_h for the tariff h, and choose consumption accordingly for each tariff;

2. The LB (resp. HB) content providers are willing to participate and choose tariff ℓ (resp. h), given consumption in each tariff.

The first condition imposes that we have consumptions $q_\ell = D(r_\ell\theta_\ell)$ and $q_h = D(r_h\theta_h)$ with tariffs ℓ and h respectively. Hence, the information transmitted to consumers differ between uniform pricing and sponsoring, with more precise signals in the latter case. A remark that will simplify the analysis is that despite the difference of consumer demands, the sponsored pricing strategy encompasses the former.

Lemma 2 *For any uniform tariff (s, r), there exist sponsored tariffs (s_ℓ, r_ℓ) and (s_h, r_h) that result in the same consumption levels $\{q_\ell, q_h\}$ and the same network value V.*

Proof. Consider a uniform tariff with $s \leq b_\ell$ and $q_\ell = q_h = D(r\theta^e)$. The same allocation can be obtained with sponsored pricing by setting $s_h = s_\ell = s$ and $r_h\theta_h = r_\ell\theta_\ell = r\theta^e$. Consumers know the content's type but their consumption is the same for both type. Content providers are indifferent between the two tariffs.

Consider now a uniform tariff $b_\ell < s \le b_h$ and $q_h = D(r\theta_h)$. The same allocation can be obtained with sponsored tariffs $(s_h, r_h) = (s, r)$ and $r_\ell \theta_\ell \ge \bar{p}$. ■

The benefit of "sponsoring" for the network is not only to extract more rent from content providers but also to induce more efficient levels of consumption. This interaction between screening on one side and signalling on the other side is the difference between sponsored pricing and a standard screening model. We show however in Appendix that the participation constraints and the incentive compatibility constraint are unchanged. Hence the optimal sponsored pricing mechanism is obtained as before but with the new objective:

$$\max_{q_h \ge q_\ell} \lambda \left[U(q_h) - (1-b_h)\theta_h q_h - (b_h - b_\ell)\theta q_\ell\right] + (1-\lambda)\left[U(q_\ell) - (1-b_\ell)\theta_\ell q_\ell\right].$$

The characterization then extends as follows:

Proposition 7 *When the type affects the benefit and the load, the profit maximizing sponsored tariffs are such that* $q_t^* = D(r_t^* \theta_t^*)$, *for* $t = \ell, h$, *and:*

$$\begin{aligned} s_\ell^* &= b_\ell,\ r_\ell^* = \min\{1 - b_\ell + \frac{\lambda}{1-\lambda}(b_h - b_\ell)\theta_h/\theta_\ell, \bar{p}/\theta_\ell\} \\ s_h^* &= b_h\left(1 - \frac{q_\ell}{q_h}\frac{b_h - b_\ell}{b_h}\right),\ r_h^* = 0. \end{aligned}$$

Proof. The solution of the reduced program is the same as before except that the relevant cost is θ_h for q_h while the virtual cost is $(1-b_\ell)\theta_\ell + \frac{\lambda}{1-\lambda}(b_h - b_\ell)\theta_h$ for the consumption of LB content. ■

Notice we always have $q_h > q_\ell$ so that under laissez-faire, consumers anticipate correctly the load. The solution is the same as before except that the relevant cost differ across tariffs, allowing a more efficient allocation.

The analysis is then similar. In particular, under laissez-faire the network excludes the LB contents for $\lambda > \lambda^*$. Again exclusion may be achieved with a uniform tariff $s > b_\ell$. When the proportion of HB content is below λ, the network accommodates the LB content with sponsoring. Moreover the critical level for total exclusion of LB content is

$$\lambda^* = \frac{\bar{p} - (1-b_\ell)\theta_\ell}{\bar{p} - (1-b_\ell)\theta_\ell + (b_h - b_\ell)\theta_h}$$

which is decreasing in b_h and θ_ℓ, and increasing in b_ℓ and θ_h.

The analysis of a price-cap is similar. As there is always separation and full participation, the load is always reflected by the consumer price and it is still the case that a cost-oriented price enhances welfare.

The case of uniform tariff differs slightly. The net expected data cost per content is $(1-s)\mathbb{E}(\theta|s)$ and internal efficiency is achieved by setting a consumer price r equal to this cost whenever feasible. As content providers' participation is independent of the consumer price, the network chooses between a tariff $(s=\sigma, r=0)$ with exclusion and a tariff $(s=b_\ell, r=(1-b_\ell)\mathbb{E}(\theta))$ with full participation. Thus the network excludes the LB content if λ is high enough so that

$$\lambda\left[U\left(D\left(0\right)\right)+\left(\sigma-1\right)\theta_h D\left(0\right)\right] > U\left(\left(1-b_\ell\right)\mathbb{E}\left(\theta\right)\right)+\left(b_\ell-1\right)\mathbb{E}\left(\theta\right)D\left(\left(1-b_\ell\right)\mathbb{E}\left(\theta\right)\right).$$

One difficulty with this case is that $\mathbb{E}(\theta)$ depends on λ, which complicates some proofs but is not sufficient to overturn our results.

Proposition 8 *All the conclusions of Section 4 hold when the type affects both the load and the benefit.*

Proof. see Appendix. ■

If we compare with the case where $\theta_h = \theta_\ell = \mathbb{E}(\theta)$, the level of exclusion is the same with heterogenous and homogenous load when $\sigma = 1$. Under laissez-faire there may be more or less exclusion depending on whether θ_h is larger or smaller than θ_ℓ. Recall that the benefit b_t is normalized by the traffic generated, so b_h may be high because a_h is high or because θ_h is low. If the HB contents impose a lower load on the network than the LB contents, then there will be more exclusion with heterogenous load than with equal load. The reverse conclusion holds when $\theta_h > \theta_\ell$.

The welfare comparison is similar with and without heterogenous load. A new effect is that sponsored pricing has an extra social benefit over uniform tariff with full participation in that the consumption level can be adjusted to reflect the true load instead of the mean load. Hence heterogeneity of the load reinforces the conclusion that discrimination has a positive effect once the regulation controls for excessive market power by imposing a price-cap. This is however not sufficient to generate non-ambiguous comparison for intermediate range of λ. But a ban on discrimination reduces welfare if λ is high or low, and if the HB content is very valuable.

Similarly, a zero-price regulation reduces welfare compared to a price-cap at cost if b_h is high, because of an insufficient consumption of HB content and an excessive consumption of LB content.

5.3 Optimal price-cap

We focused in Section 4 on standard regulation methods with little informational requirement. The first best allocation (with non-negative consumer prices) could in principle be implemented with prices $r_h = 0$, $r_\ell = 1 - b_\ell$, $s_\ell = b_\ell$ and $s_h = b_h\left(1 - \frac{q_\ell}{q_h}\frac{b_h - b_\ell}{b_h}\right)$ because then $q_\ell < q_h$. To be able to set those prices, the regulator not only should control content and consumers prices, but also be aware of the precise characteristics of the content providers, i.e. the value of b_h and b_ℓ. In this extension, we want to discuss the optimal price-cap on the content price assuming that the regulator is uncertain over the other parameters. To make the analysis as general as possible, we keep the assumption that the content provider's load is heterogenous.

Following the reasoning of the proof of proposition 2, faced to a price-cap $\sigma \geq s_\ell$, the network will choose $b_\ell = s_\ell$, $q_h \geq q_\ell$ and $s_h = \inf\{\sigma, b_h - (b_h - b_\ell)\frac{q_\ell}{q_h}\}$. The reduced program for the network then writes as

$$\begin{aligned}\max_{q_h, q_\ell} \quad & \lambda\left[U(q_h) - \theta_h q_h + \inf\{\sigma q_h, b_h q_h - (b_h - b_\ell) q_\ell\}\theta_h\right] \\ & +(1-\lambda)\left[U(q_\ell) + (b_\ell - 1)\theta_\ell q_\ell\right]\end{aligned}$$

The choice of the price-cap σ should balance the potential inefficiencies on the HB content with higher consumption of the LB content. Inefficiencies arise because the network will react to a tightening of the price-cap by rebalancing its revenue between content and consumers and raising the consumers price. In our model with inelastic demand, rebalancing is not an issue for welfare as long as it entails only an increase in the fixed fee. Thus only consumption distortions matter.

As shown in the proof of proposition 2, as long as the price-cap is above cost, the consumption of the HB content is efficient and the consumption of the LB content decreases with σ. Thus an optimal price-cap is below the cost. The question is whether it is strictly below or equal to the cost. We first derive the network choice of prices (and therefore the induced consumption levels) for any price-cap σ.

Lemma 3 *For* $\sigma \leq 1$, *let* $q_\ell^+ = D\left((1 - b_\ell)\theta_\ell + \frac{1-\sigma}{b_h - \sigma}\frac{\lambda}{1-\lambda}(b_h - b_\ell)\theta_h\right)$ *and* $q_h^{\sigma*} = D\left((1 - \sigma)\theta_h\right)$.

A price-cap $\sigma \geq b_\ell$ *leads to* $\bar{s}_\ell = b_\ell,\ \bar{s}_h = \sigma$ *and*

$$\begin{cases} (i) & \bar{q}_h = D(0) \text{ and } \bar{q}_\ell = q_\ell^* \text{ if } \frac{q_\ell^*}{D(0)} \geq \frac{b_h-\sigma}{b_h-b_\ell} \\ (ii) & \bar{q}_h = D(0) \text{ and } \bar{q}_\ell = D(0)\frac{b_h-\sigma}{b_h-b_\ell} \text{ if } \frac{q_\ell^+}{D(0)} \geq \frac{b_h-\sigma}{b_h-b_\ell} \geq \frac{q_\ell^*}{D(0)} \\ (iii) & D(0) > \bar{q}_h > q_h^{\sigma*} \text{ and } \bar{q}_\ell = \bar{q}_h\frac{b_h-\sigma}{b_h-b_\ell} \text{ if } \frac{q_\ell^{FB}}{q_h^{\sigma*}} > \frac{b_h-\sigma}{b_h-b_\ell} > \frac{q_\ell^+}{D(0)} \\ (iv) & \bar{q}_h = q_h^{\sigma*} \text{ and } \bar{q}_\ell = q_\ell^{FB} \text{ if } \frac{b_h-\sigma}{b_h-b_\ell} \geq \frac{q_\ell^{FB}}{q_h^{\sigma*}} \end{cases}$$

A price-cap $\sigma < b_\ell$ *induces uniform pricing* $\bar{s} = \sigma$ *and* $\bar{q} = D((1-\sigma)\mathbb{E}(\theta))$.

Proof. see Appendix. ■

We can interpret the results as follows. In case (i), the price-cap is not binding, i.e. $s_h^* < \sigma$. Starting from this case, let us reduce σ sightly below s_h^*. Then the network sets $s_h = \sigma$. Given this price, the network would like to induce the quantity $q_h^{\sigma*}$ that maximizes the network value $U(q) + (\sigma - 1)\theta_h q$ for HB content at $s = \sigma$. Thus, it would like to raise the consumer price r_h above zero. But to prevent the HB content producers from opting for the LB content tariff, the network must reduce q_ℓ by $\frac{b_h-\sigma}{b_h-b_\ell}$ for each unit of reduction of q_h. As q_ℓ is distorted downward to reduce the HB content rent, the network faces a trade-off between excessive consumption of HB content and insufficient consumption of LB content. When the cost of reducing q_ℓ outweighs the benefit of reducing q_h, the network chooses to keep the HB consumers price at 0. This is the case when $q_\ell^+ \geq q_\ell$ so that the distortion on the LB content is large (case (ii)). The consumption of HB content is only distorted when the efficient consumption $D(0)$ would lead to a consumption of LB content above q_ℓ^+. In this case, the network raises the consumer prices and reduces consumptions below $D(0)$ and $D(0)\frac{b_h-\sigma}{b_h-b_\ell}$ for the HB and LB contents respectively. In case (iii), the internally optimal price $r_h = (1-\sigma)\theta_h$ would require a suboptimal consumption of LB, so that the network prefers to set $r_h < (1-\sigma)\theta_h$. The quantity $q_h = q_\ell(b_h - b_\ell)/(b_h - \sigma)$ is given by the first-order condition

$$\lambda U'(q_h) + (1-\lambda)\left(\frac{b_h-\sigma}{b_h-b_\ell}\right)U'(q_\ell) = \lambda(1-\sigma)\theta_h + (1-\lambda)\left(\frac{b_h-\sigma}{b_h-b_\ell}\right)(1-b_\ell)\theta_\ell$$

as long as the incentive compatibility of the HB content providers is binding. In case (iv), the quantity $q_h^{\sigma*}$ is large enough and the price-cap tight enough so that the "optimal tariff" $\{(\sigma,(1-\sigma)\theta_h);(b_\ell,(1-b_\ell)\theta_\ell)\}$ satisfies the incentive compatibility conditions.

We remark that q_ℓ^+ is increasing in σ. As the price-cap σ decreases, the solution moves continuously from case (i) to case (ii) and then to case (iii).[19] Finally a price-cap strictly below b_ℓ reduces consumption uniformly.

It is interesting to see what happens for σ close to b_ℓ. We may distinguish two cases. Whenever $\theta_h \geq \theta_\ell$, $(b_h - \sigma) / (b_h - b_\ell) < q_\ell^{FB} / q_h^{\sigma *}$ so case (iii) prevails. In this case, the consumptions $\bar{q}_h$ and $\bar{q}_\ell$ converge to $D((1 - b_\ell) \mathbb{E}(\theta))$ and thus quantities evolve continuously with σ. However when $\theta_h < \theta_\ell$, then $(b_h - \sigma) / (b_h - b_\ell) > q_\ell^{FB} / q_h^{\sigma *}$, case (iv) prevails and the consumptions $\bar{q}_h$ and $\bar{q}_\ell$ converge to $D((1 - b_\ell) \theta_h)$ and $D((1 - b_\ell) \theta_\ell)$ respectively. Thus, quantities are discontinuous at $\sigma = b_\ell$. The reason is the following. Suppose that $\sigma = b_\ell$, then the network sets $s_h = s_\ell = b_\ell$. The producers of LB content are indifferent between any quantity because they receive no profit, while the producers of HB content prefer higher quantities. The network can then implement q_ℓ^{FB} for the LB content and the internally efficient quantity $D((1 - b_\ell) \theta_h)$ for the HB content if it is larger than q_ℓ^{FB}, hence if $\theta_h \leq \theta_\ell$. But if σ is reduced below b_ℓ, this becomes unfeasible because then the LB content producers have a margin $b_\ell - \sigma > 0$ and they also prefer higher quantities. Thus as σ falls below b_ℓ, the network has no other choice but to implement uniform prices. From this discussion it appears that:

Proposition 9 *The optimal price-cap belongs to the interval* $[b_\ell, 1]$.

Proof. If $D(0) \frac{b_h - 1}{b_h - b_\ell} \geq q_\ell^{FB}$ then $\sigma = 1$ is optimal because it yields an efficient allocation. Otherwise lowering the price-cap raises the consumption of LB content without affecting the consumption of HB content as long as $q_\ell^{+} \geq D(0) \frac{b_h - \sigma}{b_h - b_\ell}$. It is then optimal to set $\sigma < 1$. Clearly b_ℓ dominates any price-cap below b_ℓ as it yields more efficient consumptions. ■

Thus, an optimal price-cap is always positive and that may or may not be at cost.

6 Conclusion

This paper has investigated the impact of missing content prices for the efficient pricing of transmission network services. It has been shown that, when consumers control their consumption but are not aware of the induced effect, a direct or indirect signal should be sent to them. In the standard setting with paid goods, this signal is sent through the price chosen by the content providers, but when the goods are free, this is not feasible and the network prices must substitute for the missing content price. In this context, our analysis has highlighted some interesting elements.

[19] As $D((1 - \sigma) \theta_h) \frac{b_h - \sigma}{b_h - b_\ell}$ is non monotonic, the solution may alternate between (iii) and (iv).

First, as networks provide a unique access to consumers, even competitive networks will not choose fully efficient tariffs and will induce excessive exclusion of contents. This conclusion is similar to results obtained for telecommunication termination charges[20], although we focus on efficiency of consumption in the presence of adverse selection.

Second, allowing networks to propose a menu of tariffs, among which each content provider must choose, may provide an answer to the excessive exclusion of contents. By letting each content provider choosing not only its own price but also the price paid by its consumers, sponsored pricing avoids the exclusion of the traffic intensive content and raises the volume for the less traffic intensive content.

Our analysis also suggests that some regulation is optimal. In particular, imposing that the tariff proposed to content providers falls below a price-cap reduces excessive exclusion while preserving flexibility in offers and screening possibilities. Therefore, most of the benefits of regulation can be reaped with a price-cap at cost or slightly below cost. Imposing further restrictions such as no-discrimination or zero price is not desirable if the willingness to pay of content providers is high. Notice also that the price-cap doesn't constrain the price charged to consumers, which is not desirable when network uses two-part tariff on the consumer side.

A possible extension of this work would be to discuss the content providers' choice to be free rather than to use prices to mediate their relationship with consumers. Endogenizing this choice would certainly modify the impact of regulation and we plan to investigate this issue in future works.

References

[1] Armstrong, M., 2006. "Competition in Two-Sided Market", *The Rand Journal of Economics*, Vol. 37 (3), 668-691.

[2] Bourreau, M., Kourandi, F. and T. Valletti, 2014. "Net Neutrality with Competing Platforms", CEIS Tor Vergata Research Paper 12(3)-307.

[3] Caillaud, B., and B., Jullien, 2003. "Chicken &Egg: competition among intermediation service providers", *The Rand Journal of Economics*, Vol. 34(2), 309-328.

[4] Choi, J.P., and B.-C., Kim, 2010. "Net Neutrality and Investment Incentives", *The RAND Journal of Economics*, Volume 41, Issue 3, pages 446–471.

[20] Charges for terminating a call initiated by an other network's user toward a client of the network.

[5] Choi, J.P., Jeon D.-S. and B.-C., Kim, 2013. “Asymmetric Neutrality Regulation and Innovation at the Edges: Fixed vs. Mobile Networks”, mimeo, Toulouse School of Economics.

[6] Economides, N. and B., Hermalin, 2012. ”The Economics of Network Neutrality”, New York University Law and Economics Working Papers

[7] Economides N. and J. Tåg, 2012.“Net Neutrality on the Internet : A two-sided Market Analysis”, *Information Economics and Policy*, vol. 24, pp. 91–104

[8] Hermalin, B. and M.L. Katz, 2004. “Sender or Receiver: who should pay to exchange an electronic message”, *The Rand Journal of Economics*, Vol. 35(3), 423-448.

[9] Hermalin, B. and M.L. Katz, 2007. “The Economics of product-line restrictions with an application to the network neutrality debate”, *Information Economics and Policy*, Vol. 38, 215-248.

[10] Hermalin, B. and M.L. Katz, 2009. “Information and the hold-up problem”, *The Rand Journal of Economics*, Vol. 40, 215-248.

[11] Jeon, D.S., Laffont, J.J., and J. Tirole, 2004.“On the Receiver-Pays Principle”, *RAND Journal of Economics*, 35, 85-110.

[12] Krämer, J. and L. Wiewiorra, 2012. “Net neutrality and congestion sensitive content providers: Implications for content variety, broadband investment, and regulation”, *Information Systems Research* 23(4), 1303-1321.

[13] Peitz, M. and F. Schuett, 2015. “Net neutrality and inflation of traffic”, mimeo, University of Mannheim.

[14] Reggiani, C. and T. Valletti, 2012. “Net neutrality and innovation at the core and at the edges”, Economic Discussion Paper EDP-1202, The University of Manchester.

[15] Rochet J.C. and J. Tirole, 2003. “Platform Competition in Two-Sided Markets”, *Journal of the European Economic Association*, Vol. 1, 990-1029.

[16] Rochet J.C. and J. Tirole, 2006. “Two-sided market: A progress Report”, *RAND Journal of Economics*, 37 (3), 645-667.

[17] Strategy Analytics Reports, 2013. *Unlimited to Tiered Data Plans: User Preference and Management Needs.*

A Appendix A

Proof of proposition 2

We consider here the more general case of a price-cap on s_h above or equal to cost $\sigma \in [1, b_h]$. In the case of sponsoring, the constraints can be written as

$$\begin{aligned} &b_\ell \geq s_\ell \quad \text{and } \sigma \geq s_h \\ &(b_\ell - s_\ell)\, q_\ell \geq (b_\ell - s_h)\, q_h \\ &(b_h - s_h)\, q_h \geq (b_h - s_\ell)\, q_\ell \end{aligned}$$

As the network value is decreasing with the prices paid by contents providers, we have

$$\begin{aligned} s_\ell &= \inf\{b_\ell, b_\ell - (b_\ell - s_h)\frac{q_h}{q_\ell}\} \\ s_h &= \inf\{\sigma, b_h - (b_h - s_\ell)\frac{q_\ell}{q_h}\} \end{aligned}$$

Suppose that $s_h \leq b_\ell < \sigma$. Then $s_\ell = b_\ell - (b_\ell - s_h)\frac{q_h}{q_\ell}$. It means that

$$s_h = b_h - (b_h - s_\ell)\frac{q_\ell}{q_h} = (b_h - b_\ell)\left(1 - \frac{q_\ell}{q_h}\right) + s_h,$$

which is only possible if $q_\ell = q_h$ and thus $s_h = s_\ell$.

Thus we have $s_h \geq s_\ell = b_\ell$ which requires $q_\ell \leq q_h$. The program then writes as

$$\begin{aligned} \max_{q_h \geq q_\ell} \quad & \lambda\left[U(q_h) - \theta q_h + \inf\{\sigma q_h, b_h q_h - (b_h - b_\ell)q_\ell\}\theta\right] \\ & +(1-\lambda)\left[U(q_\ell) + (b_\ell - 1)\theta q_\ell\right] \end{aligned}$$

Given that $b_h \geq \sigma \geq 1$, optimality implies that $q_h = D(0)$. The objective is concave in q_ℓ with a kink at $q_\ell = D(0)\frac{b_h - \sigma}{b_h - b_\ell}$. We thus find:

1. If $q_\ell^* \geq D(0)\frac{b_h - \sigma}{b_h - b_\ell}$, the price-cap is not binding, i.e. that σ is higher than than second-best value s_h^*.

2. If $q_\ell^* \leq D(0)\frac{b_h - \sigma}{b_h - b_\ell} \leq D((1 - b_\ell)\theta)$, then the network will choose $q_\ell = D(0)\frac{b_h - \sigma}{b_h - b_\ell}$ and $s_h = \sigma$..

3. If $D((1 - b_\ell)\theta) \leq D(0)\frac{b_h - \sigma}{b_h - b_\ell}$, then $q_\ell = D((1 - b_\ell)\theta)$ and $s_h = \sigma$.

Setting $\sigma = 1$ completes the proof. ■

Proof of proposition 3

While V_ℓ^u does not depend on λ, V_h^u is linear in λ. At $\lambda = 0$ we have $V_\ell^u > V_h^u = 0$ (because demand is positive at price $(1 - b_\ell)\,\theta$) and at $\lambda = 1$ we have $V_\ell^u < V_h^u$ (because $b_\ell < \sigma$). This implies that $V_\ell^u < V_h^u$ for λ above a threshold λ^u and $0 < \lambda^u < 1$.

We define the surplus $S(p) = \max_{q \geq 0} U(q) - pq$. The threshold λ^u solves

$$\lambda^u = \frac{S\left((1 - b_\ell)\,\theta\right)}{S(0) + (\sigma - 1)\,\theta D(0)}$$

From above, it is direct to see that λ^u is decreasing in σ and increasing in b_ℓ.

For $\sigma = 1$, we have $\lambda^u = S\left((1 - b_\ell)\,\theta\right)/S(0) < 1$.

Suppose that there is no price-cap ($\sigma = b_h$) and let us show that $\lambda^* > \lambda^u$. We have

$$\lambda^* = \frac{\bar{p} - (1 - b_\ell)\theta}{\bar{p} + (b_h - 1)\,\theta}$$

By convexity of $S(.)$ and $S(\bar{p}) = 0$

$$S\left((1 - b_\ell)\,\theta\right) < S(0)\,\frac{\bar{p} - (1 - b_\ell)\theta}{\bar{p}}$$

hence

$$\lambda^u < \frac{S(0)}{S(0) + (b_h - 1)\,\theta D(0)}\frac{\bar{p} - (1 - b_\ell)\theta}{\bar{p}}$$

Using $S(0) < \bar{p}D(0)$ we have (since the RHS increases with $S(0)$)

$$\lambda^u < \frac{\bar{p}D(0)}{\bar{p}D(0) + (b_h - 1)\,\theta D(0)}\frac{\bar{p} - (1 - b_\ell)\theta}{\bar{p}} = \frac{\bar{p} - (1 - b_\ell)\theta}{\bar{p} + (b_h - 1)\,\theta} = \lambda^*$$

■

Proof of proposition 4

If $\lambda > \lambda^u$ it is immediate that price-discrimination is weakly better than uniform pricing as the consumption of HB content is the same, and there is no consumption of LB content under uniform pricing. The superiority is strict except when there is no price-cap and $\lambda > \lambda^*$ (in which case the two regimes are equivalent).

We focus now on the case where $\lambda < \lambda^u$. In the first case expected social welfare under sponsored pricing is given by

$$W^* = \lambda\left[U(q_h^*) + (b_h - 1)\,\theta q_h^*\right] + (1 - \lambda)\left[U(q_\ell^\sigma) + (b_\ell - 1)\,\theta q_\ell^\sigma\right],$$

where q_ℓ^σ is the LB consumption under the relevant price-cap regime. As for the case with uniform pricing, expected social welfare is given by

$$W^u = U(q_\ell^u) + \lambda(b_h - 1)\theta q_\ell^u + (1-\lambda)(b_\ell - 1)\theta q_\ell^u$$

where $q_\ell^u = D((1-b_\ell)\theta)$

At $\lambda = 0$, we have $W^* = W^u$ and $\bar{q}_\ell = q_\ell^* = q_\ell^u = q_\ell^{FB}$. Using the first order conditions above:

$$\frac{\partial(W^* - W^u)}{\partial\lambda}\,|_{\lambda=0} = U(q_h^*) + (b_h - 1)\theta q_h^* - [U(q_\ell^u) + (b_h - 1)\theta q_\ell^u] > 0.$$

Therefore, at least for small values of λ, sponsored pricing dominates uniform pricing. ■

Proof of corollary 3

The welfare differential in favor of sponsored pricing is:

$$\begin{aligned} &\lambda[U(q_h^*) + (b_h - 1)\theta q_h^* - U(q_h^u) - (b_h - 1)\theta q_h^u] \\ &+ (1-\lambda)[U(\bar{q}_\ell) + (b_\ell - 1)\theta\bar{q}_\ell - U(q_\ell^u) - (b_\ell - 1)\theta q_\ell^u], \end{aligned} \tag{6}$$

The first term is always positive as q_h^* is efficient. The second term may be positive or negative.

If $q_\ell^{FB} \leq D(0)\frac{b_h - 1}{b_h - b_\ell}$ then $\bar{q}_\ell = q_\ell^{FB}$ maximizes welfare, implying that the second term is non-negative. This occurs when

$$\frac{D(0) - b_\ell q_\ell^{FB}}{D(0) - q_\ell^{FB}} \leq b_h$$

which holds if b_h is high.

Notice that if $q_\ell^{FB} > D(0)\frac{b_h - 1}{b_h - b_\ell}$, the second term of (6) is negative because

$$\bar{q}_\ell = \max\left\{q_\ell^*, D(0)\frac{b_h - 1}{b_h - b_\ell}\right\} < q_\ell^u = q_\ell^{FB}$$

Thus the comparison is ambiguous. ■

Proof of proposition 5

The welfare differential between cost-orientation price-cap and zero-price regulation is given by

$$\begin{aligned}\Delta &= \lambda\left[U\left(q_h^*\right)+\left(b_h-1\right)\theta q_h^*-U\left(q^0\right)-\left(b_h-1\right)\theta q^0\right] \\ &\quad +\left(1-\lambda\right)\left[U\left(\bar{q}_\ell\right)+\left(b_\ell-1\right)\theta\bar{q}_\ell-U\left(q^0\right)-\left(b_\ell-1\right)\theta q^0\right],\end{aligned} \tag{7}$$

where $q^0 = D(\theta)$. This is strictly positive for λ small because then $\bar{q}_\ell$ is close to the efficient quantity q_ℓ^{FB} and for λ large as q_h^* is efficient.

As in the proof of corollary (3), if

$$\frac{D(0)-b_\ell D((1-b_\ell)\theta)}{D(0)-D((1-b_\ell)\theta)} \leq b_h$$

a price-cap at 1 yields an efficient outcome and dominates zero price.

Suppose that $D((1-b_\ell)\theta) > D(0)\frac{b_h-1}{b_h-b_\ell}$, then

$$\bar{q}_\ell = \max\left\{D\left(\left(1-b_\ell\right)\theta+\frac{\lambda}{1-\lambda}(b_h-b_\ell)\theta\right), D(0)\frac{b_h-1}{b_h-b_\ell}\right\}$$

is suboptimal, decreasing in λ. The second term of (7) is non-negative if $\bar{q}_\ell \geq D(\theta)$. This is the case if

- either $\frac{D(0)-b_\ell D(\theta)}{D(0)-D(\theta)} \leq b_h$
- or $\left(1-b_\ell\right)\theta+\frac{\lambda}{1-\lambda}(b_h-b_\ell)\theta \leq \theta$ which can be written as $\lambda \geq b_\ell/b_h$. ■

Proof of corollary 4

Consider first a monopoly network with elastic demand $\phi\left(CS-F\right)$. Let V^* and W^* be network value and welfare per consumer under laissez-faire. Let $\bar{V}$ and $\bar{W}$ be network value and welfare per consumer under price-cap $\sigma = 1$. When $q_\ell^* < D(0)\frac{b_h-1}{b_h-b_\ell}$, we know that $V^* > \bar{V}$ and $W^* < \bar{W}$.

With network value V, the network choose the fee F by solving

$$\max_F \phi\left(CS-F\right)\left(V+F-CS\right)$$

With elastic participation the consumers' participation is $N\left(V\right)$, increasing with V, given by (under standard concavity conditions):

$$N\left(V\right) = \phi\left(CS-F\right) \text{ where } F = CS-V+\frac{\phi\left(CS-F\right)}{\phi'\left(CS-F\right)}$$

Hence participation N^* under laissez-faire is higher than participation $\bar{N}$ under price-cap. Whenever $N^*/\bar{N} > \bar{W}/W^*$, laissez-faire dominates. Whether this occurs or not depends on the elasticity of demand. Notice that V^*, W^*, $\bar{V}$ and $\bar{W}$ are independent of ϕ, thus the ratio $N^*/\bar{N}$ increases when demand becomes more elastic. Thus laissez-faire will dominate for very elastic demand.

The argument extends to competition provided that higher values of V result into higher aggregate demand. ■

Proof of proposition 8

Proposition 2 and corollary 2 revisited

The proof of proposition 2 is the same replacing the objective with

$$\lambda \left[U(q_h) - \theta_h q_h + \inf\{\sigma q_h, b_h q_h - (b_h - b_\ell) q_\ell\}\theta_h\right] + (1-\lambda)\left[U(q_\ell) + (b_\ell - 1)\theta_\ell q_\ell\right].$$

and $q_\ell^* = D\left((1-b_\ell)\,\theta_\ell + \frac{\lambda}{1-\lambda}(b_h - b_\ell)\theta_h\right)$.

Corollary 2 is then unchanged.

Proposition 3

The proof of proposition 3 has to be adapted as follows. We now have $V_\ell^u = U\left(q_\ell^u\right) + (b_\ell - 1)\,\mathbb{E}\left(\theta\right) q_\ell^u$ with $q_\ell^u = D\left((1-b_\ell)\,\mathbb{E}\left(\theta\right)\right)$ while $V_h^u = \lambda\left[U\left(D\left(0\right)\right) + (b_h - 1)\,\theta_h D\left(0\right)\right]$. It is still the case that V_h^u is linear in λ but V_ℓ^u is not constant. However

$$\frac{\partial^2 V_\ell^u}{\partial \lambda^2} = -\left(b_\ell - 1\right)^2 \left(\theta_h - \theta_\ell\right)^2 D'\left(\left(1-b_\ell\right)\mathbb{E}\left(\theta\right)\right) > 0$$

Hence $V_h^u - V_\ell^u$ is concave, negative at $\lambda = 0$ and positive at $\lambda = 1$. This implies that there exists a unique threshold $\lambda^u < 1$ such that exclusion of the LB content occurs under uniform pricing if and only if $\lambda > \lambda^u$.

The proof that $\lambda^u < \lambda^*$ with no price-cap now uses

$$\lambda^u = \frac{S\left(\left(1-b_\ell\right)\left(\lambda^u\theta_h + \left(1-\lambda^u\right)\theta_\ell\right)\right)}{S\left(0\right) + \left(b_h - 1\right)\theta_h D\left(0\right)}$$

$$\lambda^* = \frac{\bar{p} - (1-b_\ell)\theta_\ell}{\bar{p} - (1-b_\ell)\theta_\ell + \left(b_h - b_\ell\right)\theta_h}$$

By convexity of $S(.)$

$$\lambda^u < \frac{\lambda^u S\left(\left(1-b_\ell\right)\theta_h\right) + \left(1-\lambda^u\right) S\left(\left(1-b_\ell\right)\theta_\ell\right)}{S\left(0\right) + \left(b_h - 1\right)\theta_h D\left(0\right)}$$

so that

$$\lambda^u < \frac{S\left(\left(1-b_\ell\right)\theta_\ell\right)}{S\left(0\right)+\left(b_h-1\right)\theta_h D\left(0\right)+S\left(\left(1-b_\ell\right)\theta_\ell\right)-S\left(\left(1-b_\ell\right)\theta_h\right)}$$

By convexity of $S(.)$ and $S\left(\bar{p}\right)=0$,

$$S\left(\left(1-b_\ell\right)\theta_\ell\right)<S\left(0\right)\frac{\bar{p}-(1-b_\ell)\theta_\ell}{\bar{p}} \text{ and } S\left(\left(1-b_\ell\right)\theta_h\right)<S\left(0\right)\frac{\bar{p}-(1-b_\ell)\theta_h}{\bar{p}}$$

We thus have

$$\begin{aligned}\lambda^u &< \frac{S\left(0\right)\frac{\bar{p}-(1-b_\ell)\theta_\ell}{\bar{p}}}{S\left(0\right)+\left(b_h-1\right)\theta_h D\left(0\right)+S\left(0\right)\frac{\bar{p}-(1-b_\ell)\theta_\ell}{\bar{p}}-S\left(0\right)\frac{\bar{p}-(1-b_\ell)\theta_h}{\bar{p}}} \\ &< \frac{S\left(0\right)\frac{\bar{p}-(1-b_\ell)\theta_\ell}{\bar{p}}}{S\left(0\right)\left(1+\frac{(1-b_\ell)(\theta_h-\theta_\ell)}{\bar{p}}\right)+\left(b_h-1\right)\theta_h D\left(0\right)}\end{aligned}$$

Using $S\left(0\right)<\bar{p}D\left(0\right)$, we conclude that

$$\lambda^u < \frac{\bar{p}-(1-b_\ell)\theta_\ell}{\bar{p}+\left(1-b_\ell\right)\left(\theta_h-\theta_\ell\right)+\left(b_h-1\right)\theta_h}=\lambda^*.$$

Proposition 4

The proof is the same, accounting for heterogenous load.

Corollary 3

The proof is the same adjusting for heterogenous load. Notice that it is more likely that banning discrimination reduces welfare because it prevents consumptions to reflect the true load.

Proposition 5

The proof is the same with $q^0=D\left(\mathbb{E}\left(\theta\right)\right)$. Welfare is higher for both types of contents with a cost-orientated price-cap than with zero-price regulation

- if $\frac{D(0)-b_\ell q_\ell^{FB}}{D(0)-q_\ell^{FB}}\leq b_h$
- or if $q_\ell^{FB}>D(0)\frac{b_h-1}{b_h-b_\ell}$ and
  - either $\frac{D(0)-b_\ell D(\mathbb{E}(\theta))}{D(0)-D(\mathbb{E}(\theta))}\leq b_h$
  - or $\lambda\geq\frac{\mathbb{E}(\theta)-(1-b_\ell)\theta_\ell}{\mathbb{E}(\theta)+(b_h-b_\ell)\theta_h-(1-b_\ell)\theta_\ell}$. ■

Proof of lemma 3

We consider first the case of a price-cap at $\sigma \in [b_\ell, 1]$. The constraints can be written as

$$
\begin{aligned}
& b_\ell \geq s_\ell \quad \text{and } \sigma \geq s_h \\
& (b_\ell - s_\ell)\, q_\ell \geq (b_\ell - s_h)\, q_h \\
& (b_h - s_h)\, q_h \geq (b_h - s_\ell)\, q_\ell
\end{aligned}
$$

As the network value is decreasing with the prices paid by contents providers, we have

$$
\begin{aligned}
s_\ell &= \inf\{b_\ell, b_\ell - (b_\ell - s_h)\frac{q_h}{q_\ell}\} \\
s_h &= \inf\{\sigma, b_h - (b_h - s_\ell)\frac{q_\ell}{q_h}\}
\end{aligned}
$$

The reasoning of the proof of proposition 2 shows that $s_\ell = b_\ell$ and $q_\ell \leq q_h$. The program then writes as

$$
\max_{q_h \geq q_\ell} \lambda \left[U(q_h) - \theta_h q_h + \inf\{\sigma q_h, b_h q_h - (b_h - b_\ell) q_\ell\}\theta_h\right] + (1-\lambda)\left[U(q_\ell) + (b_\ell - 1)\theta_\ell q_\ell\right]
$$

- There is a solution $q_h = D(0)$ if $\sigma q_h > b_h q_h - (b_h - b_\ell) q_\ell$: then the price-cap is not biding and $q_\ell = q_\ell^*$ which requires that

$$
q_\ell^* > \frac{b_h - \sigma}{b_h - b_\ell} D(0).
$$

- There is a solution $q_h = D((1-\sigma)\theta_h)$ if $\sigma q_h < b_h q_h - (b_h - b_\ell) q_\ell$: then the price-cap is strictly biding but not the incentive compatibility condition and $q_\ell = q_\ell^{FB}$ which requires that

$$
q_\ell^{FB} < \frac{b_h - \sigma}{b_h - b_\ell} D((1-\sigma)\theta_h).
$$

Suppose now that $q_\ell^{FB} \geq \frac{b_h - \sigma}{b_h - b_\ell} D((1-\sigma)\theta_h)$ and $q_\ell^* \leq \frac{b_h - \sigma}{b_h - b_\ell} D(0)$. Then the solution verifies $\sigma q_h = b_h q_h - (b_h - b_\ell) q_\ell$ or $q_\ell = \frac{b_h - \sigma}{b_h - b_\ell} q_h$. The choice of q_h solves

$$
\max_{q_h} \lambda \left[U(q_h) + (\sigma - 1)\theta_h q_h\right] + (1-\lambda)\left[U(\frac{b_h - \sigma}{b_h - b_\ell} q_h) + (b_\ell - 1)\theta_\ell \frac{b_h - \sigma}{b_h - b_\ell} q_h\right]
$$

The slope is

$$
\lambda \left[U'(q_h) + (\sigma - 1)\theta_h\right] + (1-\lambda)\left[U'(\frac{b_h - \sigma}{b_h - b_\ell} q_h) + (b_\ell - 1)\theta_\ell\right] \frac{b_h - \sigma}{b_h - b_\ell}
$$

Notice that the slope at $q_h = D((1-\sigma)\theta_h)$ is non-negative because $q_\ell^{FB} \geq \frac{b_h - \sigma}{b_h - b_\ell} D((1-\sigma)\theta_h)$, so $q_h \geq D\left(\left(1-\sigma\right)\theta_h\right).$

The slope at $q_h = D\left(0\right)$ is positive if

$$U'(\frac{b_h-\sigma}{b_h-b_\ell}D\left(0\right)) > (1-b_\ell)\theta_\ell + \frac{1-\sigma}{b_h-\sigma}\frac{\lambda}{1-\lambda}\left(b_h-b_\ell\right)\theta_h.$$

which writes as $\frac{b_h-\sigma}{b_h-b_\ell}D\left(0\right) < q_\ell^+$. This doesn't hold if $q_\ell^{FB} = \frac{b_h-\sigma}{b_h-b_\ell}D\left(\left(1-\sigma\right)\theta_h\right)$ because then $\frac{b_h-\sigma}{b_h-b_\ell}D\left(0\right) > q_\ell^{FB} > q_\ell^+$. However this holds if $q_\ell^* = \frac{b_h-\sigma}{b_h-b_\ell}D(0)$ because $q_\ell^+ > q_\ell^*$. Thus we have two regions:

- If $\frac{b_h-\sigma}{b_h-b_\ell}D\left(0\right) \geq q_\ell^+$, then

$$\lambda\left[U'(\bar{q}_h) + \left(\sigma-1\right)\theta_h\right] + \left(1-\lambda\right)\left[U'(\frac{b_h-\sigma}{b_h-b_\ell}\bar{q}_h) + (b_\ell-1)\theta_\ell\right]\frac{b_h-\sigma}{b_h-b_\ell} = 0.$$

- If $q_\ell^+ \geq \frac{b_h-\sigma}{b_h-b_\ell}D\left(0\right)$ then $\bar{q}_h = D\left(0\right).$

We consider here the case of a price-cap at $\sigma \leq b_\ell$. The constraints can be written as $q_h \geq q_\ell$ and

$$s_\ell = \inf\{\sigma, b_\ell - (b_\ell - s_h)\frac{q_h}{q_\ell}\};\ s_h = \inf\{\sigma, b_h - (b_h - s_\ell)\frac{q_\ell}{q_h}\}$$

Let us show that $s_h = s_\ell = \sigma$.

Suppose that $s_h = b_h - (b_h - s_\ell)\frac{q_\ell}{q_h}$ and $s_\ell = b_\ell - (b_\ell - s_h)\frac{q_h}{q_\ell}$. It means that

$$s_h = b_h - (b_h - s_\ell)\frac{q_\ell}{q_h} = \left(b_h - b_\ell\right)\left(1 - \frac{q_\ell}{q_h}\right) + s_h,$$

which is only possible if $q_\ell = q_h$ and thus $s_h = s_\ell$.

Suppose that $s_h = \sigma$ and $s_\ell = b_\ell - (b_\ell - \sigma)\frac{q_h}{q_\ell} \leq \sigma$. It means that

$$\sigma \geq b_h - (b_h - s_\ell)\frac{q_\ell}{q_h} = \left(b_h - b_\ell\right)\left(1 - \frac{q_\ell}{q_h}\right) + \sigma,$$

which is only possible if $q_\ell = q_h$ and thus $\sigma = s_\ell$.

Suppose that $s_h = b_h - (b_h - s_\ell)\frac{q_\ell}{q_h} \leq \sigma$ and $s_\ell = \sigma$. It means that

$$s_h = b_h - (b_h - \sigma)\frac{q_\ell}{q_h} = \left(b_h - \sigma\right)\left(1 - \frac{q_\ell}{q_h}\right) + \sigma,$$

which is only possible if $q_\ell = q_h$ and thus $\sigma = s_h$.

Thus we have $s_h = s_\ell = \sigma$ which requires $q_\ell = q_h = q$. The program then writes as

$$\max_q U(q) - (1-\sigma)\,\mathbb{E}(\theta)\,q$$

which yields $q = D\left((1-\sigma)\,\mathbb{E}(\theta)\right)$. ■