Aversion to risk of regret and preference for positively skewed risks

Christian Gollier
Toulouse School of Economics, University of Toulouse 1

November 19, 2015

Abstract

We fill a gap in the literature by formally defining the notion of aversion to risk of regret. In the spirit of the seminal work by Loomes and Sugden (1982), regret is measured by the distance between the payoff x of the chosen act and the maximum payoff y that could have been obtained if another action would have been selected. An increase in the risk of regret occurs when x and y become statistically less concordant. It is shown that an agent dislikes any such increase in risk of regret iff the utility function $U(x, y)$ is supermodular. We define an index of regret-risk aversion accordingly. When confronted to a one-risky-one-safe-lottery menu, we show that more regret-risk-averse agents are more willing to choose the risky act, and that this bias is increasing in the skewness of the chosen act. Regret-risk aversion also yields a pseudo-RDEU optimistic inverse-S shaped probability weighting function. Moreover, if the index of regret-risk aversion is larger than half the Arrow-Pratt index of risk aversion, the decision maker likes local mean-preserving spreads in the domain of no-regret payoffs.

Keywords: Regret, longshot bias, rank-dependent EU, probability weighting function, risk-seeking, cumulative prospect theory, portfolio problem.

JEL codes: D81

Acknowledgement: We would like to acknowledge funding from the chair SCOR and FDIR at TSE. This draft is still preliminary. Comments welcomed.

1 Introduction

Regret is a negative emotional reaction to personal past acts. It is experienced when realizing that one would have been in a better situation, if only one would have decided differently. It can come from missed opportunities, such as failing to pass a medical test that would have revealed a cancer, or as not buying that asset whose market value has recently surged. It can also come from past actions yielding a bad outcome, such as lost gambles and unsuccessful investments. These emotions are mediated by a cognitive process known as counterfactual thinking involving the orbitofrontal cortex (Camille et al. (2004), Stalnaker et al. (2015)). If regret affects utility ex post, it should not be a surprise that it does influence decisions ex ante. Since Gilovich and Medvec (1995), there is indeed overwhelming evidence in the psychology literature that people alter their choices in response to the anticipation of regret (Zeelenberg and Pieters (2004), Zeelenberg and Pieters (2007)), with applications in marketing (Inman and McAlister (1994)), medicine (Brehaut et al. (2003), Chapman and Coups (2006)), insurance (Braun and Muermann (2004)), and finance (Michenaud and Solnik (2008) and Muermann et al. (2006)) for example.[1]

Since Bell (1982) and Loomes and Sugden (1982), economists have explored the role of the anticipation of regret on optimal choices under uncertainty. Under the modern economic theory of regret that we reexamine in this paper, decision makers (DM) are assumed to maximize expected utility. But, when confronted with a non-trivial menu of lotteries, it is assumed that each DM's statewise utility U depends not just on the state-specific payoff x of the chosen lottery but also on the largest payoff y that could have been obtained within the menu in that state.[2] Observe that the distribution of the forgone best alternative is a function of the menu. This implies that the optimal choice is menu-specific, which implies in turn a potential intransitivity of the preference ordering.[3] Following Loomes and Sugden (1982, 1987a,b), we measure the intensity of regret in any specific state by the difference between y and x that prevails in that state.

How does the anticipation of regret affect choice? The answer to this question obviously depends upon the properties of the bivariate utility function. The key concept here is regret aversion. The problem is that the existing literature has not been very effective to produce a coherent and consensual definition of regret aversion. For example, Sarver (2008) defines regret aversion by the property that adding an ex ante dominated lottery in the choice menu makes the DM worse off. This is because this lottery can yield an ex post payoff that is larger than the payoff of the optimal lottery, thereby raising regret. Obviously, this definition of regret aversion is supported by the assumption that the utility function is decreasing in

[1] Zeelenberg and Pieters (2007) documents an exponentially increasing number of papers published on regret, starting around 1980 and culminating around 100 papers per year at the time of the publication of their paper.

[2] The original theory proposed by Bell (1982, 1983), Loomes and Sugden (1982, 1987a,b) and Loomes (1988)) was limited to menus with only two lotteries, and allowed for rejoice when the chosen lottery generated the largest payoff. In order to generalize the theory to menus containing more than two lotteries, and considering that people focus more on regret than on rejoice, Quiggin (1994) introduced an additional assumption into the model by claiming that statewise dominated alternative should be irrelevant. This supports the theory of regret that has been used by economists since then, and that is used in this paper.

[3] For more on intransitivity with regret-sensitive preferences, see Bikhchandani and Segal (2011).

the forgone best alternative. Gee (2012) refers to this notion of regret aversion as first-order regret aversion. In this paper, we show that the optimal choice within an exogenously given menu of lotteries can be characterized only if one defines a notion of aversion *to risk of* regret that has never been formally defined before.[4] In simple words, a DM is averse to the risk of regret if, everything else unchanged, she dislikes any mean-preserving spread in the distribution of the intensity of regret. As suggested by the psychology and economics literature on the subject,[5] most people prefer a sure regret of 1 than a regret of 100 occurring with probability of 0.01. Savage (1951), who introduced the notion of regret in economics, took the extreme view that a useful decision criterion under uncertainty is the minmax regret in which this aversion to the risk of regret is infinite.

The primitive variables relevant for ex post utility is the actual payoff x and the forgone best alternative y, from which one can derive regret $y-x$. Ex ante, expected utility of the DM depends upon the joint probability distribution of (x, y). It is thus important to define the risk of regret from these primitive variables. In this paper, we define an increase in risk of regret by a reduction in concordance between x and y. The concept of comparative concordance has been introduced in economics by Epstein and Tanny (1980) and Tchen (1980). It is obtained by transferring some probability masses at the corners of any rectangle in the (x, y)-space towards its main diagonal in a way that does not affect the marginal distributions of the two random variables. A reduction in concordance reduces the covariance, but the opposite is not necessarily true. We show that a reduction in concordance between x and y increases the risk of $y - x$ in the sense of Rothschild and Stiglitz (1970). In other words, a reduction in concordance between the actual payoff and the forgone best alternative makes regret riskier without affecting the marginal distributions of the actual payoff and of the forgone best alternative. Our definition of aversion to risk of regret is that the DM dislikes any such reduction in concordance. For example, consider the following two menus described in Table 1. Menu $M_{123} = \{x_1^*, x_2, x_3\}$ has three lotteries whose payoffs all depend upon the same draw of a fair coin. Let us contemplate the possibility to select lottery x_1^* in this menu in which the best forgone alternative is 1 or 2 respectively in state H and T. Because the actual payoff in these states are respectively 0 and 1, regret takes value 1 with certainty. Let us alternatively contemplate the same choice x_1^* in menu $M_{145} = \{x_1^*, x_4, x_5\}$. In that context, the statewise forgone best alternative are reversed, so that regret takes value 0 or 2 with equal probabilities. In short, the two risk contexts yields exactly the same marginal distributions for x and y, but the second context has these variables less concordant. That yields an increase in the risk in regret. Under our definition, any regret-risk-averse DM should prefer the first menu over the second.

We show that a DM is averse to risk of regret if and only if U is supermodular. We define

[4] Gee (2012) proposes a definition of second-order regret aversion that relies on the longshot bias. This is not intuitive. As we show in this paper, this merely substitutes an assumption by a result.

[5] Zeelenberg and Pieters (2004) illustrates the nature of the relationship between the intensity of regret and its emotional impact on utility by the following extreme example: "In April 1995, a man took his own life after missing out on a £2 million price in the British National Lottery. He did so after discovering that the numbers he always selected, 14, 17, 22, 24, 42, and 47 were that week's winning combination. On this particular occurrence, however, he had forgotten to renew his five-week ticket on time. The ticket had expired the previous Saturday."

	H	T
Lottery x_1^*	0	1
Lottery x_2	1	-1
Lottery x_3	-2	2
y^{123}	1	2
Regret	1	1

	H	T
Lottery x_1^*	0	1
Lottery x_4	-1	1
Lottery x_5	2	-2
y^{134}	2	1
Regret	2	0

Table 1: Two menus of choices: $M_{123} = \{x_1^*, x_2, x_3\}$ and $M_{145} = \{x_1^*, x_4, x_5\}$.

a regret-risk premium associated to a marginal-preserving reduction in concordance between the actual payoff and its forgone best alternative as the sure reduction in actual outcome that compensates for it. We show that, in the small, the regret-risk premium equals the product of the increase in the covariance by an index of absolute aversion to risk of regret measured by U_{xy}/U_x. Our approach differs much from the economic literature in which most contributions assumed a specific functional form for the utility function, with $U(x,y) = u(x) - R(u(y) - u(x))$.[6] Although this specification has the advantage to intuitively separate the "choiceless/regretless" utility function u from the penalty R coming from the feeling of regret, we believe that it is too specific, with little benefit associated to this restriction.

An important contribution of this paper is to show that, in spite of the fact that our definition of regret-risk aversion is based on the attitude towards a Rothschild-Stiglitz increase in regret, this concept implies a preference for longshots, i.e., for positively skewed lotteries. Such a preference is well documented in the finance and economics literature.[7] This can be potentially explained by the assumption that people are prudent in the classical expected utility model, or by the assumption that they have an inverse-S shaped probability weighting function in the cumulative prospect theory (CPT) and in the rank-dependent expected utility (RDEU) model, as shown by Tversky and Kahneman (1992). It can alternatively be explained by the aversion to risk of regret. Let us consider a menu that contains a risky binary lottery $x_1 \sim (a, 1-p; A, p)$, with $a < A$, and a sure payoff equaling $\mu = pA + (1-p)a$. We show in this paper that in this context, moving from the safe choice to the risky one entails one mean-preserving spread in the distribution of $x|y = \mu$, one mean-preserving spread in the distribution of $x|y = A$, and a marginal-preserving increase in concordance between x and y. In other words, the risky choice yields a reduction in the risk of regret compared to the safe choice. To illustrate, let us consider the zero-mean symmetric case with $a = -1$, $A = 1$ and $p = 1/2$. The safe choice entails a risk context (x, y) taking value $(0, 0)$ or $(0, 1)$ with equal probabilities. The risky choice entails $(-1, 0)$ or $(1, 1)$ with equal probabilities. Moving from safe to risky is done by a sequence of two manipulations on the joint probability distribution that are described in Figure 1. We first increase the risk on x to generate the intermediary lottery appearing in this figure (all outcomes are equally likely). The second manipulation is

[6]Quiggin (1994) is an exception.

[7]See for example Golec and Tamarkin (1998), Garrett and Sobel (1999), Harvey and Siddique (2000), Bhattacharya and Garrett (2008), and Eichner and Wagener (2011). By showing that adding low-probability macroeconomic catastrophes into the beliefs of the representative agent can explain the equity premium puzzle, Barro (2006, 2009) is in line with the idea that investors particularly dislike negatively skewed returns.

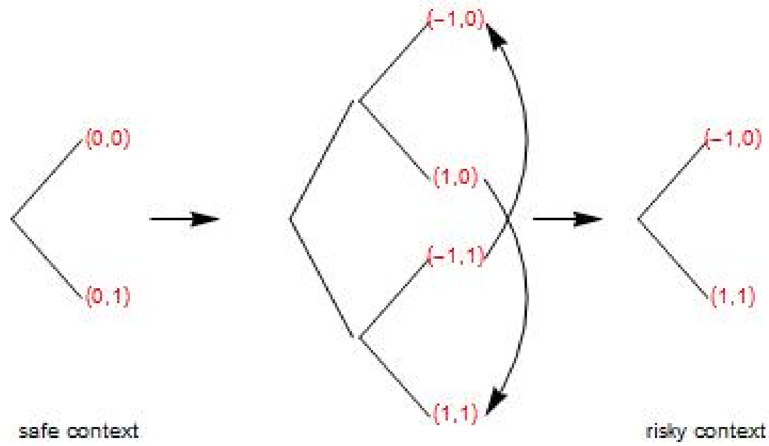

Figure 1: Moving from the safe to the risky context entails an increase in concordance.

performed by the two vertical arrows. In words, one displaces probability masses from atoms where x and y are far apart to atoms where they are closer, without changing the marginal distribution of x and y. This second transformation is an example of increase in concordance. It yields a reduction in the risk of regret, which is desirable for regret-risk-averse agents.

This implies that the optimal choice is ambiguous under the combination of risk aversion and regret-risk aversion. But we also show that their relative role in this comparison of the safe and risky choices is sensitive to the skewness of the risky lottery. More precisely, we show that when moving from the safe to the risky choice, a reduction in the success probability p makes the positive effect of the reduction in the risk of regret stronger relative to the negative effect of the risky payoff. Thus increasing the skewness of the risky lottery raises the plausibility for the DM to prefer the risky lottery over its expectation. This apparent risk-seeking behavior is well documented since Kahneman and Tversky (1979) and Tversky and Kahneman (1992), in particular through the so-called reflection effect, i.e., the tendency to prefer the risky choice in the above menu when p is small either in the loss domain ($a < A = 0$) and in the gain domain ($0 = a < A$).[8] This explains why regret theory can solve the Allais paradox (Bell (1982)).

Tversky and Kahneman (1992), Wu and Gonzalez (1996), Abdellaoui (2000), Abdellaoui et al. (2010) and many others have performed laboratory experiments to elicit the probability weighting function. The typical experiment consists in eliciting the certainty equivalent c of various binary lotteries $x_1 \sim (a, 1-p; A, p)$ with a given pair (a, A) and various success probabilities p. In this context, we show that the regret-sensitive expected-utility-maximizer with a multiplicative bivariate utility function behaves as a RDEU agent with a concave utility function and an inverse-S shaped probability weighting function. Notice however that the weighting function derived from this regret theory always exhibits optimism. This is due

[8]In the wording of Tversky and Kahneman (1992), DMs exhibit "risk-seeking preferences [...] for losses of moderate and large probability [and] for small probabilities of gains."

to the above-mentioned result that moving from safe to risky always yields a combination of more payoff risk and less regret risk, so that the effect of regret-risk aversion is always in favor of the risky choice, i.e., of optimism. Thus, our theory is unable to reproduce the elevation (Abdellaoui et al. (2010)) of the weighting function extracted from these experiments.

Section 2 is devoted to the definitions and the characterization of the risk of regret and of regret-risk aversion. We also show there the link between the risk of regret and the reduction in concordance between the payoff and the forgone best alternative. In Section 3, we define the regret-risk premium and we derive an index of regret-risk aversion that is consistent with these definitions. We derive an approximation à la Arrow-Pratt of the former by using the latter. In Section 4, we examine the role of regret-risk aversion in the risk attitude towards menus that contain a binary lottery and its mean (or certainty equivalent), and we characterize the link between regret-risk aversion and the preference for positively skewed lotteries. We generalize these results in Section 5 for non-binary lotteries, whereas Section 6 is devoted to a short analysis of the two-asset portfolio problem.

2 A definition of aversion to risk of regret

The uncertainty is described by a set of S possible states of nature, indexed $s = 1, ..., S$. There is an objective probability distribution of the states given by vector $(p_1, ..., p_S)$ in the simplex of $\mathbb{R}^S$. A lottery (or an act) is defined by a function from S to $\mathbb{R}$ that specifies the final payoff $x(s)$ in each state s. The choice problem is characterized by a menu $M = \{x_\theta : S \to \mathbb{R} \,|\theta \in \Theta\}$ of lotteries indexed by θ in some index set Θ. In the spirit of Bell (1982, 1983), Loomes and Sugden (1982, 1987a,b) and following Quiggin (1994), we assume that the expected-utility-maximizing agent is sensitive to regret in the sense that her utility U in any state s is a function of two variables: (1) the actual payoff $x(s)$ of the chosen lottery, and (2) the maximal consumption $y^M(s)$ that could have been attained in that state if another feasible choice had been made at the beginning of the period: $y^M(s) = \max_{x_\theta \in M} x_\theta(s)$ for all $s \in S$. Observe that y^M is associated to menu M, but is independent of the lottery selected in that menu. A choice x_θ in menu M is expressed by the joint distribution of (x_θ, y^M), and yields an ex-ante well-being equaling $EU(x_\theta, y^M)$. In this model, preferences over lotteries are menu-specific.

We assume that the decision-maker always prefers to consume more to less, and is averse to risk on actual consumption. More precisely, U is increasing in x, which means that any first-order stochastic improvement in the conditional distributions $x\,|y = b$ increases welfare, for any $b \in \mathbb{R}$. Similarly, U is concave in x, which means that any increase in risk of $x\,|y = b$ reduces welfare. We now define the notion of regret aversion. Sarver (2008) and Gee (2012) define regret aversion as follows:[9] If menu $M_2 = \{x_2\}$ is preferred to menu $M_1 = \{x_1\}$, then adding lottery x_1 in a menu that already contains x_2 cannot make that menu more attractive. In particular, this means that

$$EU(x_2, x_2) \geq EU(x_1, x_1) \Rightarrow EU(x_2, x_2) \geq EU(x_2, y), \tag{1}$$

[9] In fact, Sarver (2008) calls this "regret", but this is clearly a concept of aversion to regret. Gee (2012) refers to this notion as "first-order regret aversion".

Ball number	0	1	...	49	50	...	99
Lottery x_1^*	0	1	...	49	50	...	99
Lottery x_0	100	0	...	0	0	...	0
$y^{01} = max\{x_1^*, x_0\}$	100	1	...	49	50	...	99
Regret	100	0	...	0	0	...	0

Table 2: Menu M_{01}.

Ball number	0	1	...	49	50	...	99
Lottery x_1^*	0	1	...	49	50	...	99
Lottery x_2	1	2	...	50	0	...	0
Lottery x_3	0	0	...	0	51	...	100
$y^{123} = max\{x_1^*, x_2, x_3\}$	1	2	...	50	51	...	100
Regret	1	1	...	1	1	...	1

Table 3: Menu M_{123}.

with $y(s) = \max\{x_1(s), x_2(s)\}$ for all $s \in S$. The intuition is that adding an ex-ante dominated lottery can potentially increase the best alternative outcome in some states, thereby raising the negative feeling of regret in these states. It is obvious that regret aversion holds if and only if U is decreasing in y. It is useful to measure the intensity of regret $r(s)$ in any specific state s by the difference between the forgone best alternative $y(s)$ and the actual payoff $x(s)$: $r(s) = y(s) - x(s)$. It is menu-specific. By definition, the intensity of regret is non-negative. Regret aversion means that the agent dislikes any increase in state regret. Given the fact that y can only increase when enlarging the menu of choices, regret aversion is a also a preference for reducing the choice opportunity set containing the optimal solution (Sarver (2008)).

By contrast, our results rely on another concept that we call Aversion to Risk of Regret (ARR). To illustrate, let us consider an urn that contains 100 balls numbered from 0 to 99. A ball is randomly extracted from the urn. Lottery x_0 yields a payoff of 100 if ball numbered 0 is extracted from the urn, and a payoff of 0 otherwise. Lottery x_1^* yields a payoff equaling the number of the ball extracted from the urn. Suppose that the agent prefers lottery x_1^* in menu $M_{01} = \{x_0, x_1^*\}$, as described in Table 2. Observe that the agent faces regret only if ball 0 is obtained. In other words, the intensity of regret r is 100 with probability 1%, and is 0 otherwise.

Let us contemplate alternatively menu $M_{123} = \{x_1^*, x_2, x_3\}$ described in Table 3. Lottery x_2 yields a payoff of $s + 1$ if the number of the ball is s if s is less or equal to 49, and 0 otherwise. Lottery x_3 yields a payoff of $s + 1$ if the number of the ball is s if s is larger or equal to 50, and 0 otherwise. Suppose that the agent also prefer lottery x_1^* in menu M_{123}.

But the agent will always face regret from not having selected lottery x_2 (if $s \leq 49$) or lottery x_3 (if $s \geq 50$) ex post, yielding an intensity of regret r equaling unity with certainty.

Let us now compare the risk outcomes of the two menus M_{01} and M_{123} in more details. Because x_1^* is always preferred, the marginal distribution of the final payoff is the same in the two menus. Observe also that the marginal distributions of the best alternative payoff y are also identical in the two menus. More specifically, the marginal distribution of y is uniform over set $\{1, 2, ..., 100\}$ in both menus. Thus, in terms of the marginal distributions of x and y, the two menus are identical. However, the distributions of regret r are different. In menu M_{01}, regret is equal to 1 with certainty, whereas in menu M_{123} it is equal to 100 with probability 0.01. Their expectations are equal, but menu M_{01} generates an increase in the risk of regret compared to menu M_{123}. Thus, a regret-risk-averse agent should prefer menu M_{123} over menu M_{01}.

Because the intensity of regret is measured by the difference between x and y, risk on regret increases when these two random variables are less statistically concordant, a concept developed by Tchen (1980) and Epstein and Tanny (1980). To show this, let us compare two risk contexts characterized respectively by (x_1, y_1) and (x_2, y_2). Let $F_i : R^2 \to \mathbb{R}$ denote the bivariate distribution function associated to context i, $i = 1, 2$. Suppose that F_2 is obtained from F_1 through a Marginal-Preserving Reduction in Concordance (MPRC). A MPRC is based on two transfers of probability masses among four realizations of (x, y), with $x \in \{a, A\}$, $a < A$ and $y \in \{b, B\}$, with $b < B$, as shown in Figure 1. First, probability mass ε in the neighborhood of (a, b) is transferred upwards in the neighborhood of (a, B). Second, probability mass ε in the neighborhood of (A, B) is transferred downwards in the neighborhood of (A, b). Observe that this does not change the marginal distributions of x and y. But it reduces the correlation between x and y, and it yields a mean-preserving spread in the distribution of the intensity $r = y - x$ of regret in the sense of Rothschild and Stiglitz (1970), as claimed in the following proposition.

Proposition 1. *Any marginal-preserving reduction in concordance in (x, y) yields a mean-preserving spread in regret $r = y - x$.*

Proof. A mean-preserving spread in r is obtained by defining an interval I in the support of r from which some probability mass is extracted to be transferred outside I, preserving the mean of r. Define $r_{min} = min\{B - A, b - a\}$ and $r_{max} = max\{B - A, b - a\}$. Let us consider interval $I = [r_{min}, r_{max}]$. The MPRC described above transfers a probability mass ϵ in the distribution of regret $r = y - x$ from I to $B - a > r_{max}$, and another probability mass ϵ from I to $A - b < r_{min}$. Moreover, this change in the distribution of regret preserves the mean. This is because the MPRC preserves the mean of x and y, thereby preserving the mean of $r = y - x$. □ □

Marginal-Preserving Increases in Concordance (MPIC) are defined symmetrically. More generally, F_2 is said to be less concordant than F_1 if and only F_2 is obtained from F_1 through a sequence of MPRCs. Tchen (1980) and Epstein and Tanny (1980) have shown that F_2 is less concordant than F_1 if and only if they have the same marginal distributions and

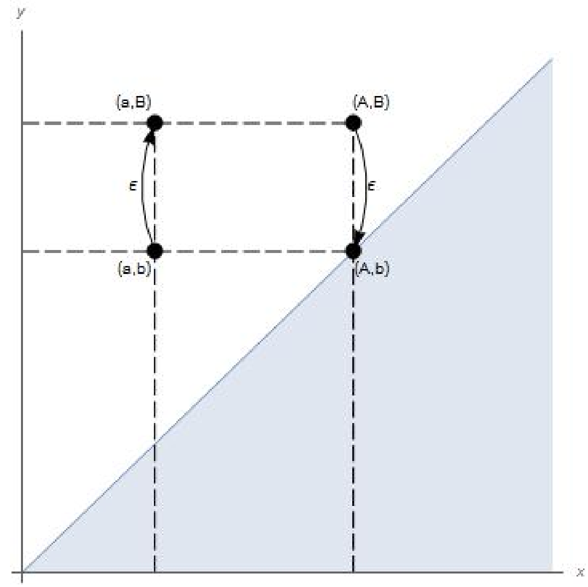

Figure 2: Example of a marginal-preserving reduction in concordance (MPRC).

$$\forall (a,b) \in [\underline{x}, \overline{x}] \times [\underline{y}, \overline{y}] \subset \mathbb{R}^2 : F_2(a,b) \leq F_1(a,b). \tag{2}$$

Remember that because a reduction in concordance has no effect on the marginal distributions of x and y, it does not affect the risk characteristics of the final payoff and of the best alternative payoff. But it increases the risk of regret since the sequence of MPRCs that generates it yields a sequence of mean-preserving spreads in the distribution of regret, i.e., a Rothschild-Stiglitz increase in risk of regret.

Definition 1. *Let F_1 and F_2 be two cumulative distribution functions from $[\underline{x}, \overline{x}] \times [\underline{y}, \overline{y}] \subset \mathbb{R}^2 \to \mathbb{R}$ having the same marginal distributions: For all $x \in [\underline{x}, \overline{x}]$, $F_2(x, \overline{y}) = F_1(x, \overline{y})$, and for all $y \in [\underline{y}, \overline{y}]$, $F_2(\overline{x}, y) = F_1(\overline{x}, y)$. F_2 exhibits more risk of regret than F_1 if and only if F_2 is less concordant than F_1, i.e., if and only if condition (2) is satisfied.*

This justifies the following definition of ARR.

Definition 2. *U is averse to risk of regret if any increase in risk of regret reduces expected utility.*

It is easy to show that in the numerical illustration described in tables 2 and 3, menu M_{01} yields a risk context (x, y) that is less concordant than menu M_{123}, which implies an increase in risk of regret.[10] All agents that are averse to risk of regret should therefore prefer menu M_{123} over menu M_{01}. Observe now that the MPRC described in Figure 2 reduces expected utility $EU(x, y)$ if and only if for all $a \leq A$ and $b \leq B$,

[10]This reduction of concordance can be obtained through a sequence of 100 MPRCs. The first is to move the 0.01 probability mass at $(0, 100)$ downward to $(0, 99)$, and to compensate this by an upward move of the same probability mass from $(99, 99)$ upward to $(99, 100)$.

$$U(a,b)+U(A,B)\geq U(a,B)+U(A,b). \tag{3}$$

By definition, this is true if and only if u is supermodular. When U is twice differentiable, this means that U_x is increasing in y, or $U_{xy}\geq 0$. This yields the following result, which is an application of Epstein and Tanny (1980). This result implies in particular that an increase in risk of regret reduces the covariance between x and y.

Theorem 1. *U is averse to risk of regret if and only if U is supermodular.*

Thus, the aversion to regret corresponds to U being decreasing in y, whereas the aversion to risk of regret corresponds to U_x being increasing in y. Because y is a bad under regret aversion, the aversion to risk of regret can also be interpreted as a preference for mixing good (x) with bad (y), a common property of individual preferences first suggested by Eeckhoudt and Schlesinger (2006).[11]

3 Measure of aversion to risk of regret

It is natural to define the premium associated to an increase in risk of regret by the sure reduction in consumption that has the same effect on expected utility.

Definition 3. *Consider an increase in risk of regret from F_1 to F_2. The associated regret risk premium π is defined as follows:*

$$\iint U(x-\pi,y)dF_1(x,y)=\iint U(x,y)dF_2(x,y) \tag{4}$$

Because U is increasing in its first argument, the regret-risk premium is non-negative under regret-risk aversion. Condition (4) can be rewritten as

$$\iint [U(x,y)-U(x-\pi,y)]dF_1(x,y)=\iint U(x,y)d(F_1(x,y)-F_2(x,y)) \tag{5}$$

Suppose that the increase in risk of regret is limited to a sequence of small MPRCs in the neighborhood of (a,b). The left-hand side of equation (5) can then be approximated by $U_x(a,b)\pi$. Using a second-order Taylor approximation, and taking advantage of the fact that the marginals are unaffected by the change in distribution, we also have that

$$\begin{aligned}\iint & U(x,y)d(F_1(x,y)-F_2(x,y))\\ &\approx U_{xy}(a,b)\iint (x-a)(y-b)d(F_1(x,y)-F_2(x,y))\\ &=U_{xy}(a,b)[cov_1(x,y)-cov_2(x,y)]\\ &=U_{xy}(a,b)\triangle cov(x,y),\end{aligned} \tag{6}$$

[11]These authors examined the special case of a univariate utility function. Eeckhoudt et al. (2007) extended this analysis to the case of a multivariate utility function.

where $cov_i(x,y)$ is the covariance between x and y under distribution F_i, and $\triangle cov(x,y)$ is the reduction in covariance in (x,y) that is associated to this increase in risk of regret. This implies that equation (5) implies that

$$\pi \approx \frac{U_{xy}(a,b)}{U_x(a,b)} \triangle cov(x,y), \tag{7}$$

This justifies the following definition of an index of absolute aversion to risk of regret (AARR), which parallels the standard Arrow-Pratt definition of absolute risk aversion.

Definition 4. *We define the indexes of absolute risk aversion α and of absolute regret-risk aversion ρ as follows:*

$$\alpha(a,b) = -\frac{U_{xx}(a,b)}{U_x(a,b)}, \text{ and } \rho(a,b) = \frac{U_{xy}(a,b)}{U_x(a,b)}. \tag{8}$$

When the increase in risk of regret is localized in the neighborhood of (a,b), $\rho(a,b)$ is the certainty equivalent reduction in consumption corresponding to a unit reduction in the covariance between the actual payoff and the forgone best alternative. For example, if (A,B) is in the neighborhood of (a,b) in Figure 2, the corresponding MPRC has a regret risk premium that can be approximated by this equation:

$$\pi \approx \Delta p \; \Delta x \; \Delta y \; \rho(a,b), \tag{9}$$

where $\Delta p = \varepsilon$, $\Delta x = A - a$ and $\Delta y = B - b$.

Various specifications of the bivariate utility function U exist in the literature. Bell (1982) proposed to use a function $U(x,y) = u(x) - R(u(y) - u(x))$, where u is an increasing and concave "choiceless" utility function, and R is a regret penalty function.[12] Several authors, such as Braun and Muermann (2004), Muermann et al. (2006), Sarver (2008), Michenaud and Solnik (2008), and Gee (2012) for example, have followed this tradition. Under this specification, the index of regret-risk aversion equals $\rho(a,b) = u'(b)R''/(1+R')$ where the derivatives of R are evaluated at $u(b) - u(a)$. Notice that disentangling regret-risk aversion from risk aversion is only partial in this model, since the index of risk aversion α obviously depends upon the shape of R.

Savage (1951) proposed the decision criterion in which the DM minimizes the maximum statewise regret $r(x,y) = y - x$. This can be interpreted as an extreme version of our general model with $U(x,y) = u(r(x,y))$ and $u(r) = -A^{-1}\exp(Ar)$ for $A \in \mathbb{R}^+$. This implies indexes $\alpha(a,b) = \rho(a,b) = A$. Let us define the certainty equivalent regret R as $u(R) = Eu(r(x,y))$. This is equivalent to R equaling $A^{-1}\ln E\exp(Ar)$, which is the cumulant-generating function of random variable r. As is well-known, when A tends to infinity, R tends to the maximum statewise regret. Because u is decreasing in R, maximizing $u(R) = EU(x,y)$ is equivalent to minimizing R. So this specification of our general model leads to the minmax regret criterion that has played an historical role in the development of decision theory during the last century.

[12]Loomes and Sugden (1982, 1987b) considered the special case of this specification with $u(x) = x$.

Finally, in the spirit of this exponential specification, we propose a multiplicative formulation with $U(x, y) = u(x)v(y)$, with $u' \geq 0$, $u'' \leq 0$, $v \geq 0$. Under this specification, the agent is averse to risk of regret if and only if v is increasing: $v' \geq 0$. The index of regret-risk aversion is $\rho(a, b) = v'(y)/v(y)$, whereas the index of risk aversion is $\alpha(a, b) = -u''(a)/u'(a)$. This specification allows for a full separation of risk and regret-risk attitudes in the small.

4 The one-risky-one-safe-lottery menu with two states: Preference for skewness

In this section, we explore the link that exists between the attitude towards skewed risk and the aversion to risk of regret as defined in the previous section. Although we defined ARR strictly in relation to mean-preserving spreads in the intensity of regret, we hereafter show that ARR generates a form of preference in favor of skewed risks in consumption.

To do this, we examine simple menus $M = \{x_1, x_2\}$ that contain two lotteries. We further assume in this section that lottery x_2 is safe as it generates a payoff equaling the mean of x_1 with certainty. Without loss of generality, we assume that $Ex_1 = 0$. We examine the conditions under which, in spite of her risk aversion, the decision-maker wants to select the risky lottery over its mean in this menu. In this problem, the agent faces two risks of regret depending upon her decision. First, if she takes the risky lottery, she will feel regret if she makes a loss on this gamble. Second, if she does not take the risk, she will feel regret if the payoff of the lottery is positive. If the risky lottery yields a large payoff with a small probability, the risk of regret is larger in this second scenario than in the first. If the skewness of the risky lottery large enough, the effect of aversion to risk of regret may dominate the effect of risk aversion to induce the decision-maker to prefer the risky lottery in menu M. In this section, we consider the special case in which the risky lottery x_1 of this menu is binary, with payoffs K and $-k$ respectively with probability p and $1 - p$. We assume that $K = k(1 - p)/p$ in order to have $Ex_1 = 0$. This lottery can be interpreted as betting $k > 0$ on a horse whose probability to win the race is p, under an actuarially fair pricing. Without loss of generality, let's assume at this stage $k = 1$. Obviously, a reduction in p raises the skewness of x_1. Because the alternative choice in menu M is $x_2 =_p 0$, the distribution of forgone best alternative y associated to this menu is characterized by $y = 0$ if $x_1 = -k$ and $y = K$ if $x_1 = K$.

4.1 Selecting the risky lottery yields an increase in outcome-risk and a reduction in regret-risk

In Figure 3, we drew in red the distribution of the risk context (x_2, y^M) if the safe lottery x_2 is selected. It takes value $(0, 0)$ and $(0, K)$ respectively with probability $1 - p$ and p. We also drew in blue the distribution of (x_1, y^M) when the risky lottery is selected.

We hereafter show how to transfer probability masses to transform the risk context where the safe lottery x_2 is selected in menu M into the one in which the risky lottery x_1 is selected, i.e., when moving from red to blue. A sequence of three transfers of probability masses will generate this transformation, two yielding a mean-preserving spread (MPS) in

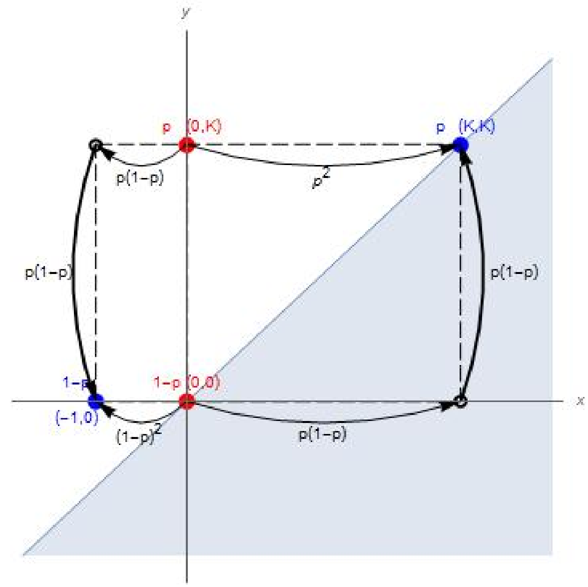

Figure 3: Transfers of probability masses in the context of a menu $M = \{x_1 \sim (K, p; -1, 1-p), x_2 =_p 0\}$.

the distribution of x conditional to y, and one yielding a marginal-preserving increase in concordance (MPIC):

- A MPS in x conditional to $y = 0$: The probability mass $1-p$ at $(0,0)$ is split into $(1-p)^2$ and $p(1-p)$. These masses are transferred horizontally respectively to $(-k, 0)$ and to $(K, 0)$.
- A MPS in x conditional to $y = K$: The probability mass p at $(0, K)$ is split into $p(1-p)$ and p^2. These masses are transferred horizontally respectively to $(-k, K)$ and to (K, K).
- A MPIC: The two probability masses $p(1-p)$ now at $(-k, K)$ and $(0, K)$ are transferred vertically respectively at $(-k, 0)$ and (K, K).

This sequence of three transfers of probability masses are represented in Figure 3. They transform the risk environment (x_1, y^M) into (x_2, y^M). Because the agent is risk-averse in x ($U_{xx} \leq 0$), the two mean-preserving spreads in $x \mid y$ reduce expected utility. But the marginal-preserving increase in concordance reduces the risk of regret. Under the regret risk aversion ($U_{xy} \geq 0$), the MPIC involved in the transformation of the risk context (x, y) from the safe lottery to the risky one raises expected utility, thereby making the optimal choice in M ambiguous.

4.2 The case of small risk

This ambiguity can be removed if we examine the special case of x_1 being a small binary risk, as shown in our next proposition.

Proposition 2. *Consider a menu $M = \{x_1, x_2\}$ with $x_1 \sim (k(1-p)/p, p; -k, 1-p)$ and $x_2 =_p 0$. In this menu, switching from the safe lottery x_2 to the risky one x_1 yields a MPS in $x|y = x_1$, a MPS in $x|y = x_2$, and a MPIC. The corresponding increase in expected utility equals*

$$EU(x_1, y^M) - EU(0, y^M) = k^2 \frac{1-p}{p} U_x(0,0) \Big[(1-p)\rho_0 - 0.5\alpha_0 \Big] + O(k^3). \tag{10}$$

Proof. We rewrite the left-hand side of equation (10) as follows:

$$f(k) = pU\left(k\frac{1-p}{p}, k\frac{1-p}{p}\right) + (1-p)U(-k, 0) - pU\left(0, k\frac{1-p}{p}\right) - (1-p)U(0,0). \tag{11}$$

It is easy to check that $f(0) = f'(0) = 0$ and that

$$f''(0) = U_{xx}(0,0)\frac{1-p}{p} + 2U_{xy}(0,0)\frac{(1-p)^2}{p} = \frac{1-p}{p}U_x(0,0)\left[2(1-p)\rho_0 - \alpha_0\right]. \tag{12}$$

This implies that

$$f(k) = \frac{1}{2}k^2 f''(0) + O(k^3) = k^2 Var(x_1) U_x(0,0) \Big[(1-p)\rho_0 - 0.5\alpha_0 \Big] + O(k^3). \tag{13}$$

This concludes the proof. □

Suppose now that bet k is close to zero. In that context, the risk premium associated to x_1 is approximately to its Arrow-Pratt approximation $0.5\alpha_0 E x_1^2$, with $Ex_1^2 = k^2(1-p)/p$ and $\alpha_0 = \alpha(0,0)$. This measures the certainty equivalent loss in x associated to the sequence of the two MPS described above. Similarly, following equation (9), the (negative) regret risk premium associated to the MPIC can be approximated by $-k^2(1-p)^2\rho_0/p$, with $\rho_0 = \rho(0,0)$, since $\Delta p = -p(1-p)$, $\Delta x = k/p$ and $\Delta y = k(1-p)/p$. This provides an intuition to Proposition 10.

Observe that the MPIC necessary to transform the risk context (x_2, y^M) into (x_1, y^M) reduces the covariance between x and y by $(1-p)^2/p$, whereas the increase in variance in the final payoff equals $(1-p)/p$. This means that

$$\Delta cov(x,y) = (1-p)Var(x_1). \tag{14}$$

Because the negative regret risk premium is approximately proportional to $\Delta cov(x,y)$ whereas the positive risk premium is approximately proportional to $Var(x_1)$, we can conclude that the relative effect of the aversion to risk of regret is decreasing in p. In other words, our definition of ARR is compatible with a preference for longshots. This is formalized in the following corollary, which is a direct consequence of equation (10).

Corollary 1. *Under the assumptions of Proposition 2, and assuming that the size k of the bet is small, then*

- *the safe lottery is always preferred if and only if the regret-risk aversion is smaller than half the risk aversion, i.e., iff* $\rho(0,0) \leq 0.5\alpha(0,0)$*;*

- *a mean-preserving reduction in the probability of success* p *makes the risky choice more desirable. Technically, if the risky lottery* $x_1 \sim (k(1-p)/p, p; -k, 1-p)$ *is preferred to* 0 *in menu* $M = \{x_1, 0\}$*, then for all* $p' \leq p$*, the risky lottery* $x_1' \sim (k(1-p')/p', p'; -k, 1-p')$ *is preferred to* 0 *in menu* $M = \{x_1', 0\}$*.*

The first result in this corollary states that there is a strictly positive lower bound (equaling $0.5\alpha_0$) for the regret risk aversion ρ_0 below which the risky choice in M can never be optimal. This lower bound is obtained from equation (10) by pushing the skewness of the risky lottery to its extreme, with $p \to 0$ and $K/k \to \infty$. The second result illustrates a preference for longshots when $\rho_0 > 0.5\alpha_0$. In fact, under our assumption of an actuarially fair pricing, betting on a horse whose probability to win is p is optimal if and only if p is smaller than $1 - 0.5\alpha_0/\rho_0$.

4.3 Reinterpretation in the RDEU framework

Our results provide an explanation for the standard risk-seeking observations made first by Tversky and Kahneman (1992). First, people often prefer a small probability of winning a large prize over the expected value of that prospect. Second, people also often prefer a large probability of losing a substantial amount of money over the expected loss of that prospect. In both cases, the risky choice is positively skewed. This implies that people who contemplate the safe choice particularly fear the risk of regret when the higher payoff materializes. In the RDEU framework, this is usually explained by the hypothesis that people tend to distort the cumulative distribution function. In this section, we show that some of these distortions can be explained by ARR. The standard method to elicit probability distortion consists in asking respondents to evaluate their certainty equivalent payoff c for various binary lotteries $x_1 \sim (x_+, p_+; x_-, p_-)$, with $x_- < x_+$ and $p_- + p_+ = 1$. Eliciting a certainty equivalent associated to a lottery places the respondent in a situation to recognize that she is indifferent between x_1 and c when confronted to menu $\{x_1, c\}$. In our ARR model, this certainty equivalent payoff is defined as follows

$$p_- U(x_-, c) + p_+ U(x_+, x_+) = p_- U(c, c) + p_+ U(c, x_+). \tag{15}$$

Suppose that the utility function U is multiplicative as described in the previous section: $U(x, y) = u(x)v(y)$. The absolute aversion to risk of regret equals $\rho(x, y) = v'(y)/v(y)$. Under this specification, equation (15) can be rewritten as

$$w(p_-)u(x_-) + (1 - w(p_-))u(x_+) = u(c), \tag{16}$$

with

$$w(p_-) = \frac{p_- v(c)}{p_- v(c) + (1 - p_-)v(x_+)}. \tag{17}$$

Equation (16) is the standard formulation of the rank-dependent expected utility model. In our model, for each value of p_-, the probability-distortion w is jointly determined with the certainty equivalent payoff c by solving system (16)-(17) with respect to these two unknowns.

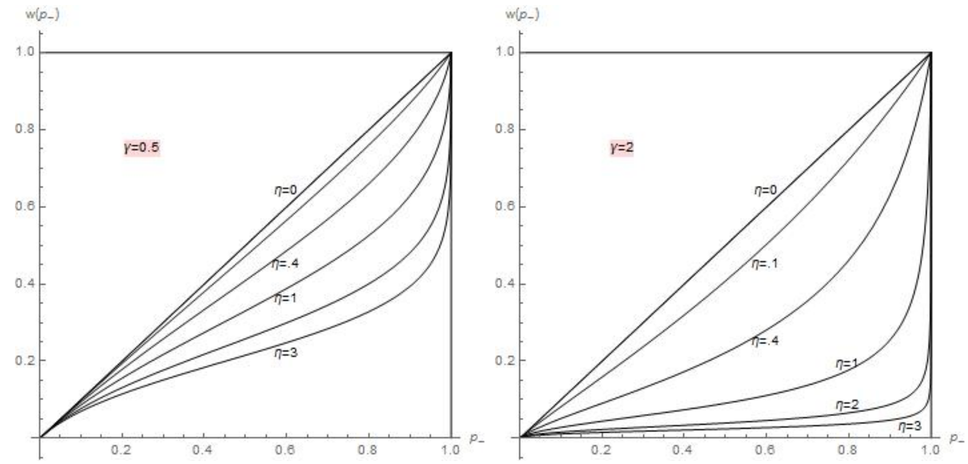

Figure 4: Probability transformation functions induced by the aversion to risk of regret

We can prove that there is an interior solution to this system, as stated in the following proposition, whose proof is relegated to the appendix.

Proposition 3. *Suppose that $U(x, y) = u(x)v(y)$ and that u is increasing. System (16)-(17) has an interior solution with a certainty equivalent c in $[x_-, x_+]$. If v is increasing, i.e., if the DM is regret-risk-averse, then*

- *the probability weighting $w(p_-)$ is smaller than p_- for all $p_- \in [0, 1]$;*
- *the probability weighting function is concave in the neighborhood of $p_- = 0$ and is convex in the neighborhood of $p_- = 1$.*

These results are reversed if the DM is regret-risk-seeking. Moreover, $w'(0) = 1$.

Consider an external observer who knows the agent's utility function u and who tries to elicit the weighting function w by observing a set of certainty equivalent payoffs associated to different binary lotteries with the same support (x_-, x_+). For each possible probability p_- of the low payoff, the observer deduces $w(p_-)$ from the observation of c by solving equation (16). We showed in Proposition 3 that such an observer will deduce from this process that the agent is RDEU with a probability weighting function w that is optimistic ($w(p_-) \leq p_-$) if the agent is averse to risk of regret (and that is pessimistic if the agent is regret-risk-seeking). This optimism may dominate risk aversion ($u'' < 0$), as shown in the previous section. Proposition 3 suggests that this probability weighting function is inverse-S shaped, since it is concave for low probabilities, and convex for large ones. It is noteworthy that this shape is the one that emerges from all empirical studies such as in Tversky and Kahneman (1992), Wu and Gonzalez (1996), Abdellaoui (2000) and Abdellaoui et al. (2010) for example. However, as noticed earlier, this model is unable to generate both optimism for some probabilities and pessimism for others. Using the wording in Abdellaoui et al. (2010), our weighting function has an elevation which is too low.

To illustrate, we have drawn in Figure 4 the probability weighting function that would emerge from this elicitation process when considering lotteries with payoffs 1 and 100, and functions $u(x) = x^{1-\gamma}/(1-\gamma)$, $\gamma \geq 0$, and $v(y) = y^{\eta}$, $\eta \geq 0$. Observe that $\rho(x,y) = \eta/y$, so that η can be interpreted as an index of relative aversion to risk of regret. The left figure corresponds to a relatively low risk aversion of $\gamma = 0.5$, whereas the right one corresponds to a larger risk aversion of $\gamma = 2$. In each case, we have drawn the probability distortion functions associated to different degrees of relative regret-risk aversion $\eta = 0, 0.1, 0.4, 1, 2$ and 3. Observe that an increase in ARR makes the agent more optimistic, with all probability weighting functions exhibiting a typical inverse-S shape. The high discrepancy between the true probability p_- and its distorted value $w(p_-)$ when p_- is close to unity illustrates the longshot bias of ARR agents.

The two figures also illustrate the fact that the theory of regret aversion yields a probability weighting function that cannot be disentangled from the utility function u. They suggest in particular that more risk-averse agents exhibit more optimism. It should also be noticed that the probability weighting function w is sensitive to the context given by (x_-, x_+).

5 The general case of the one-risky-one-safe-lottery menu

In this section, we relax the assumption that the risky lottery has only two atoms. In other words, we characterize the choice of regret-risk-averse agents who face a menu that contains an arbitrary lottery and a safe bet. Since the impact of a change in the payoff of the safe bet has a trivial consequence on the choice, we hereafter assume that the two choices yield the same expected payoff. The generalization presented in this section is derived from the following lemma, whose proof is relegated to the appendix.

Lemma 1. *Any lottery with mean μ and S possible payoffs can be decomposed into a compound lottery of $S-1$ binary lotteries with mean μ.*

Let x_1 with $Ex_1 = \mu$ have $S = m+n$ possible outcomes in the union of $A^- = \{a_1^-, ..., a_m^-\}$ and $A^+ = \{a_1^+, ..., a_n^+\}$, with $a_1^- < ... < a_m^- < \mu < a_1^+ < ... < a_n^+$. Let us decompose this lottery into $x_1 \sim (x^1, \pi^1; ...; x^{S-1}, \pi^{S-1})$, where $x^k \sim (a^{k-}, p^k, a^{k+}, 1-p^k)$ is a μ-mean binary lottery with $a^{k-} \in A^-$, $a^{k+} \in A^+$, and $p^k \in [0,1]$, for all $k \in \{1, S-1\}$. For each lottery x^k, define y^k that takes value μ if $x^k = a^{k-}$, and a^{k+} otherwise. It is then obvious that

$$EU(x_1, y^M) - EU(\mu, y^M) = \sum_{k=1}^{S-1} \pi^k \left[EU(x^k, y^k) - EU(\mu, y^k) \right]. \tag{18}$$

In other words, comparing risk contexts (x_1, y^M) and (μ, y^M) can be performed by comparing $S-1$ pairs of risk contexts (x^k, y^k) and (μ, y^k). Building on what we know on these comparisons from Section 4.1, we obtain the following results.

Proposition 4. *Consider any menu M containing two lotteries, a risky one x_1 and a safe one x_2 with the same mean. Let random variable y^M denote the forgone best alternatives associated to M. The distribution of (x_1, y^M) can be obtained from the distribution of (x_2, y^M) through a sequence of mean-preserving spreads in $x|y^M$ and of marginal-preserving increases in concordance between x and y.*

Combining this with Proposition 2, this immediately implies the following corollary.

Corollary 2. *Consider menu $M = \{x_1, x_2\}$, where x_2 takes value $Ex_1 = \mu$ with certainty. Risk-averse and regret-risk-seeking agents always prefer the safe choice in this menu. If the riskiness of x_1 is small in the sense of Arrow-Pratt, then the risky choice can potentially be preferred only if the local aversion to risk of regret is larger than half the local aversion to risk, i.e., iff $\rho(\mu,\mu) \geq 0.5\alpha(\mu,\mu)$.*

It is interesting to examine how a local mean-preserving spread in the distribution of the risky lottery x_1 in menu M affects its attractiveness. As we will show, it is important to determine whether this MPS takes place to the regret domain or in the rejoice domain, where regret and rejoice are defined from the point of view of the risk-taker. To examine this, let us formally decompose lottery x_1 into its regret and rejoice domains, where the regret domain corresponds to the state of nature in which the payoff of the risky lottery is less than the payoff of the safe bet: $x_1 \sim (kx_+, p_+; kx_-, p_-)$, with *supp* $x_+ \subset [\mu, +\infty[$, *supp* $x_- \subset]-\infty, \mu]$, $p_+ > 0$, $p_- > 0$, $p_+ + p_- = 1$, and $p_+Ex_+ + p_-Ex_- = \mu$. The risky lottery is preferred iff

$$p_+E\left[U(x_+,x_+) - U(\mu,x_+)\right] + p_-E\left[U(x_-,\mu) - U(\mu,\mu)\right] \geq 0. \tag{19}$$

We can rewrite this condition as follows:

$$p_+Eu_+(x_+) + p_-Eu_-(x_-) \geq 0, \tag{20}$$

where functions u_+ and u_- are defined as

$$u_+(x) = U(x,x) - U(\mu,x) \tag{21}$$

and

$$u_-(x) = U(x,\mu) - U(\mu,\mu). \tag{22}$$

This is a model of state-dependent expected utility. Obviously, the concavity of utility function u_- in the regret domain is the same as the concavity of $U(x,\mu)$ with respect to x. This means that the agent is risk-averse in the regret domain, in the sense that any local MPS in the regret domain reduces the attractiveness of the risky lottery. Its risk aversion in this domain is measured by $\alpha(x,\mu)$. But the analysis is very different in the rejoice domain. Indeed, the local risk aversion of u_+ in this domain of payoffs is measured by

$$-\frac{u_+''(x)}{u_+'(x)} = -\frac{U_{xx}(x,x) + 2U_{xy}(x,x) + U_{yy}(x,x) - U_{yy}(\mu,x)}{U_x(x,x) + U_y(x,x) - U_y(\mu,x)}. \tag{23}$$

The sign of this index of risk aversion is ambiguous. When x is only marginally larger than μ, this can be approximated by

$$-\frac{u_+''(\mu)}{u_+'(\mu)} = \alpha(\mu,\mu) - 2\rho(\mu,\mu). \tag{24}$$

These results are summarized in the following proposition.

Proposition 5. *Consider menu $M = x_1, x_2\}$, where x_2 is degenerated and take value Ex_1 with certainty. Any mean-preserving spread of x_1 in the regret domain $x \leq Ex_1 = \mu$ reduces the attractiveness of the risky lottery. A mean-preserving spread of x_1 in a small neighborhood above μ reduces its attractiveness if and only if $\rho(\mu,\mu) \leq 0.5\alpha(\mu,\mu)$.*

An alternative intuition of this result can be obtained by examing the case of small risks. The following proposition, whose proof is skipped, generalizes Proposition 2.

Proposition 6. *Consider a menu $M = \{x_1, x_2\}$ in which the risky lottery x_1 and the safe bet x_2 have the same mean $k\mu$. Let $x_1 \sim (kx_+, p_+; kx_-, p_-)$ with $x_- \leq_p \mu \leq_p x_+$. In this menu, the increase in expected utility when switching from the safe lottery x_2 to the risky one x_1 equals*

$$\frac{EU(x_1, y^M) - EU(0, y^M)}{U_x(0,0)} = k^2 \left[p_+ (\rho_0 - 0.5\alpha_0) E(x_+ - \mu)^2 - 0.5 p_- \alpha_0 E(x_- - \mu)^2\right] + O(k^3), \tag{25}$$

where $\rho_0 = \rho(\mu, \mu)$ and $\alpha_0 = \alpha(\mu, \mu)$.

The intuition of this result goes as follows. For small risk, the outcome-risk premium is proportional to $Var(x_1)$, with

$$Var(x_1) = k^2 \left[p_+ E(x_+ - \mu)^2 + p_- E(x_- - \mu)^2\right]. \tag{26}$$

As we know from section 3, this outcome-risk premium should be compared to the (negative) regret-risk premium which is proportional to $-\Delta cov(x, y^M)$. The increase in the covariance in (x, y) when shifting risk context from the safe choice yielding $(0, y^M)$ to the risky one (x_1, y^M) equals

$$-\Delta cov(x, y^M) = k^2 p_+ E(x_+ - \mu)^2. \tag{27}$$

The first term of the right-hand side of equality (25) is the difference between these two premia, weighted respectively by $0.5\alpha_0$ and ρ_0.

Proposition 6 confirms our earlier findings. For example, in the small, a regret-sensitive agent will never choose the risky option in her degree of regret-risk aversion is smaller than half her absolute risk aversion. Indeed, in that case, the two terms in the right-hand side of equation (25) are negative. Moreover, the bias in favor of the risky choice is increasing in $p_+ E(x_+ - \mu)^2 / p_- E(x_- - \mu)^2$, which is a measure of positive asymmetry in the distribution of x_1. This generalizes our findings on the positive skewness bias of regret-risk-averse DM when there are more than two possible outcomes in the risky choice.

6 The portfolio problem

In this section, we apply our definition of regret-risk aversion to the static one-risky-one-safe-asset portfolio. Consider an investor with initial wealth z who can invest in a safe asset whose return is normalized to zero and in a risky asset whose return is a random variable x with a known distribution function F. We normalize z to unity. To make the problem interesting, let us suppose that 0 is in the interior of the support of x. If k denotes the share of wealth invested in the risky asset, final wealth is $1 + kx$. Let us assume that the equity share k is restricted to belong to $[\underline{k}, \overline{k}]$ for some arbitrary pair $(\underline{k}, \overline{k}) \in \mathbb{R}^2$ such that $\underline{k} < \overline{k}$. A typical example is $\underline{k}=0$ in which shorting the risky asset is prohibited. Another example is $\overline{k}=1$, in which borrowing at the riskfree rate to invest in the risky asset is prohibited. This analysis generalizes what has been done before in this paper by allowing more than two choices in the menu.[13]

The portfolio menu is $M=\{1+kx|k\in[\underline{k},\overline{k}]\}$. The forgone best alternative y^M associated to this menu is either $1+\underline{k}x$ if x is negative, and $1+\overline{k}x$ if x is positive. In words, the forgone best alternative is always the minimum risk exposure $\underline{k}$ if the return of the risky asset is negative, and the maximum risk exposure $\overline{k}$ otherwise. The decision problem can be written as

$$k^*\in\arg\max_{k\in[\underline{k},\overline{k}]}V(k;\underline{k},\overline{k})=EU(1+kx,y^M). \tag{28}$$

Notice that the objective function V is concave in k, so that the first-order condition is necessary and sufficient. We hereafter suppose that the solution is interior, so that the first-order condition is

$$\frac{\partial V}{\partial k}(k^*;\underline{k},\overline{k})=ExU_x(1+k^*x,y^M)=0. \tag{29}$$

We are interested in determining the impact of a change in the lower and upper constraints on the optimal portfolio allocation k^*.[14] Because V is concave in k, this comparative static analysis is driven by the cross-derivatives of V. We have that

$$\left.\frac{\partial^2 V}{\partial k\partial\underline{k}}\right|_{k=k^*}=\int_{-\infty}^{0}x^2U_{xy}(1+k^*x,1+\underline{k}x)dF(x). \tag{30}$$

This is unambiguously positive under regret-risk aversion, which implies that increasing the minimum risk exposure $\underline{x}$ always raises the optimal risk exposure k^*. Similarly, we have that

$$\left.\frac{\partial^2 V}{\partial k\partial\overline{k}}\right|_{k=k^*}=\int_{0}^{+\infty}x^2U_{xy}(1+k^*x,1+\overline{k}x)dF(x). \tag{31}$$

This is also positive under regret-risk aversion. This yields the following proposition.

Proposition 7. *Consider the portfolio problem in which final wealth is $1+kx$, with $k\in[\underline{k},\overline{k}]$, and suppose that the optimal solution k^* is interior. Raising the lower limit $\underline{k}$ or the upper limit $\overline{k}$ of the risk exposure always raises (resp. reduces) the optimal risk exposure k^* under regret-risk aversion (resp. seeking).*

[13] There exist other interpretations of this model. For example, consider the case of an insurable risk of loss ℓ, which is random. A coinsurance contract can be purchased in which the policyholder with initial wealth z_0 gets indemnity $(1-k)\ell$ ex post against the payment of a premium $(1-k)P$ ex ante, where k is the retention rate, and P is the full insurance premium. Final wealth is thus $z_0-\ell+(1-k)\ell-(1-k)P$, which can be rewritten as $z+kx$, with $z=z_0-P$ and $x=P-\ell$.

[14] The welfare analysis is immediate. As shown by Sarver (2008), if $U_y<0$, i.e., if the investor is averse to regret, any reduction in the choice set (increase in $\underline{k}$, reduction in $\overline{k}$) that does not eliminate the optimal solution $k^*(\underline{k},\overline{k})$ raises welfare ex ante.

This means that regret-risk aversion tends to push the optimal risk exposure to the center of the opportunity set. An extreme illustration of this phenomenon is obtained when assuming that the equity premium is zero, so that $Ex = 0$. As is well-known, in the absence of regret-risk aversion, the optimal portfolio is fully invested in the safe asset in that case. But it is a simple extension of the above proposition that imposing a no-borrowing constraint $\underline{k} = 0$ will induce the regret-risk-averse investor to accept some equity risk in her portfolio.[15] Braun and Muermann (2004), Muermann et al. (2006) and Michenaud and Solnik (2008) derived this result respectively in the context of insurance, portfolio choice and currency hedging decisions, under the Bell's specification $U(x, y) = u(x) - R(u(y) - u(x))$.[16]

Observe also that one can rewrite the first-order condition (29) by using an indirect utility function u:

$$Exu'(1 + k^*x) = 0, \tag{32}$$

with

$$u'(w) = \begin{cases} U_x(w, \underline{y}(w)) & \text{if} \quad w < 1 \\ U_x(w, \overline{y}(w)) & \text{if} \quad w \geq 1, \end{cases} \tag{33}$$

with $\underline{y}(w) = 1 + \underline{k}(w-1)/k^*$ and $\overline{y}(w) = 1 + \overline{k}(w-1)/k^*$. The index of absolute risk aversion of this indirect utility function equals

$$-\frac{u''(w)}{u'(w)} = \begin{cases} \alpha(w, \underline{y}(w)) - \frac{\underline{k}}{k^*}\rho(w, \underline{y}(w)) & \text{if} \quad w < 1 \\ \alpha(w, \overline{y}(w)) - \frac{\overline{k}}{k^*}\rho(w, \overline{y}(w)) & \text{if} \quad w > 1. \end{cases} \tag{34}$$

This shows that ARR plays a more important role to reduce risk aversion in the rejoice domain ($w > 1$) than in the regret domain ($w < 1$). Notice in particular that there is a downward discontinuity in risk aversion around $w = 1$. More specifically, risk aversion goes down from $\alpha(1, 1) - \underline{k}\rho(1, 1)/k^*$ for small negative returns to $\alpha(1, 1) - \overline{k}\rho(1, 1)/k^*$ for small positive returns. This suggests that the impact of a mean-preserving spread in returns reduces the demand for the risky asset less if it concentrated in the domain of positive excess returns than in the domain of negative excess returns. If the ARR is large enough compared to risk aversion, such MPS in the rejoice domain can even raises the demand for the risky asset.[17] This is another illustration of the longshot bias that is generated by regret-risk aversion.

[15] The proof of this result comes from the fact that $ExU_x(1, max(1, 1+\overline{k}x))$ is necessarily positive if $Ex = 0$ and U_{xy} is positive.

[16] These results illustrate again the fact that the optimal choices of regret-sensitive DMs are menu-dependent. Expanding the number of options in the menu changes the nature of the choice problem. In Gollier and Salanié (2012), we explore the portfolio choice problem when the number of assets is large enough to make markets complete.

[17] As shown by equation (32) and discussed in Gollier (1995), what matters to determine the impact of a MPS in the distribution of returns on the asset demand is the concavity of function $f(x) = xu'(1 + k^*x)$. But there is a close relationship between the concavity of f and the concavity of u (Rothschild and Stiglitz (1971)).

7 Conclusion

In spite of its intuitive appeal and the many testable predictions of the theory, regret-risk aversion has received relatively little attention by economists. A possible explanation is the relatively weak theoretical foundation of their regret model. The theory has no clear definition of what actually is regret and regret aversion, or regret-risk and regret-risk aversion. For the sake of comparison, the axiomatic development of the EU theory after WWII has quickly been followed by the building of the crucial tools of an index of risk aversion and of stochastic dominance orders. This has opened the door to a myriad of applications in finance, macroeconomics and IO researches for example. No such evolution has been possible in regret theory. In this paper, we tried to fill this gap by proposing a coherent theory and measurement of regret risk and regret risk aversion. We did that by using a general formulation in which the decision maker maximizes the expectation of a bivariate utility function which is not only sensitive to the actual payoff of the chosen act, but also to the best alternative payoff if another action would have been selected ex ante.

When the decision maker is confronted to a menu of lotteries, we have defined regret in each state as the difference between the best possible payoff and the actual payoff associated to the chosen lottery. We used the concept of comparative concordance, which is a stochastic order useful to measure the degree of dependence between two random variables. We have shown that a reduction in concordance between the forgone best alternative and the actual payoff yields an increase in risk of regret, without affecting the marginal distributions of these two random variables. It is thus natural to define the notion of regret-risk aversion by requiring that ex-ante welfare is reduced by any such reduction in concordance. We have shown that this requires the bivariate utility function to be supermodular. We have defined accordingly the concept of regret-risk premium together with a local index of regret-risk aversion, and we have shown that in the small, the former is proportional to the latter. More importantly, we have shown that regret-risk-averse agents exhibit a natural bias in favor of positively skewed risks, whereas regret-risk-seeking agents would exhibit a bias in favor of negatively skewed risks. If the menu contains a risky lottery and its certainty equivalent, as is the case in most experiments used to elicit preferences under risk, the behavior of regret-risk-averse agents is equivalent to the one of rank-dependent-expected-utility agents who would use a probability distortion function that exhibits both optimism and an inverse-S shape. This observation may lead to a behavioral explanation to the RDEU model.

Appendix 1: Proof of Proposition 3

Eliminating w from (16)-(17) yields

$$F(c)=\frac{p_-v(c)}{p_-v(c)+(1-p_-)v(x_+)}u(x_-)+\frac{(1-p_-)v(x_+)}{p_-v(c)+(1-p_-)v(x_+)}u(x_+)-u(c) \tag{35}$$

We have that

$$F(x_-)=\frac{(1-p_-)v(x_+)}{p_-v(x_-)+(1-p_-)v(x_+)}\left(u(x_+)-u(x_-)\right)\geq 0 \tag{36}$$

and

$$F(x_+)=p_-\left(u(x_-)-u(x_+)\right)\leq 0. \tag{37}$$

Because F is continuous, there must exists a real $c\in[x_-,x_+]$ such that $F(c)=0$. This must be the solution of system (16)-(17). Because c is less than x_+, its is immediate from (17) that $w(p_-)$ is smaller than p_- if v is increasing.

We now examine the shape of the probability weighting function. To do this, we fully differentiate system (16)-(17) with respect to p_-. It yields (we simplify the notation by replacing $p_-=p$)

$$\frac{dc}{dp}=\frac{(u(x_-)-u(x_+))v(x_+)v(c)}{u'(c)(pv(c)+(1-p)v(x_+))^2+p(1-p)v(x_+)(u(x_+)-u(x_-))v'(c)} \tag{38}$$

and

$$\frac{dw}{dp}=\frac{u'(c)v(x_+)v(c)}{u'(c)(pv(c)+(1-p)v(x_+))^2+p(1-p)v(x_+)(u(x_+)-u(x_-))v'(c)}. \tag{39}$$

Let us first examine the case $p=0$. The above equations imply that $c=x_+$ and $w=0$, $c'(0)=(u(x_-)-u(x_+))/u'(x_+)$ and $w'(0)=1$. Moreover differentiating the above equation around $p=0$ yields

$$\left.\frac{d^2w}{dp^2}\right|_{p=0}=-\frac{2v'(x_+)(u(x_+)-u(x_-))}{u'(x_+)v(x_+)}\leq 0. \tag{40}$$

Let us alternatively consider the case $p=1$, which implies that $c=x_-$, $w=1$, $c'(1)=(u(x_-)-u(x_+)v(x_+)/u'(x_-)v(x_-)$ and $w'(1)=v(x_+)/v(x_-)$. Finally, differentiating equation (39) in the neighborhood of $p=1$ yields

$$\left.\frac{d^2w}{dp^2}\right|_{p=0}=\frac{2v(x_+)^2v'(x_-)(u(x_+)-u(x_-))}{u'(x_-)v(x_-)^3}+\frac{2v(x_+)(v(x_+)-v(x_-))}{v(x_-)^2}\geq 0. \tag{41}$$

This concludes the proof of Proposition 3. $\square$

Appendix 2: Proof of Lemma 1

Without loss of generality, let μ be zero. Let $S = m + n$, and let the risky lottery x_1 in menu M be

$$x_1 = x_1^0 \sim (a_1^-, p_1^-; ...; a_m^-, p_m^-; a_1^+, p_1^+; ...; a_n^+, p_n^+), \tag{42}$$

with $a_1^- < ... < a_m^- < 0 < a_1^+ < ... < a_n^+$, $p_i^- > 0$, $i = 1, ..., m$, $p_j^+ > 0$, $j = 1, ..., n$, and $\sum_{i=1}^m p_i^- + \sum_{j=1}^n p_j^+ = 1$. Assume that $Ex_1 = 0$. Define p_{ij} as

$$p_{ij} = \frac{-a_i^-}{a_j^+ - a_i^-} \in [0, 1]. \tag{43}$$

Define lottery $x_{ij} \sim (a_j^+, p_{ij}; a_i^-, 1-p_{ij})$. By construction, $Ex_{ij} = 0$. Initialize the probability vector $Q = (q_1^{0-}, ..., q_m^{0-}, q_1^{0+}, ..., q_n 0+)$ such that for all i , $q_i^{0,-} = p_i^-$ and for all j, $q_j^{0+} = p_j^+$. We also initialize two sets $I = J = \{\emptyset\}$.

We consider the following $n + m - 2$ iterations. At the beginning of iteration k, $I \cup J$ contains the $k-1$ states of nature whose lottery's initial payoff has been replaced by a binary zero-mean lottery.

Iteration k: Take an arbitrary pair (i, j), $i \in \{1, ..., m\}/I$, $j \in \{1, ..., n\}/J$. Consider two cases.

Case 1: Suppose that $q_i^{k-1-} < -q_j^{k-1+} a_j^+/a_i^-$. Then, define $\pi^k = q_i^{k-1-}(a_j^+ - a_i^-)/a_j^+$. Perform the following two operations on lottery x^{k-1}:

- Replace the atom a_i^- by lottery x_{ij}, and raises the associated probability q_i^{k-1-} up to $q_i^{k-} = \pi^k$.
- Reduce the probability associated to a_j^+ from q_j^{k-1+} to $q_j^{k+} = q_j^{k-1+} + (q_i^{k-1-} a_i^-/a_j^+) > 0$.

Moreover, append state i into the set of negative states whose initial payoff a_i^- as been replaced by a binary zero-mean lottery x_{ij}: $I^k = I^{k-1} \cup i$.

Case 2: Suppose that $q_i^{k-1-} \geq -q_j^{k-1+} a_j^+/a_i^-$. Then, define $\pi^k = -q_j^{k-1+}(a_j^+ - a_i^-)/a_i^-$. Perform the following two operations on lottery x^{k-1}:

- Replace the atom a_j^+ by lottery x_{ij}, and raises the associated probability q_j^{k-1+} up to $q_j^{k+} = \pi^k$.
- Reduce the probability associated to a_i^- from q_i^{k-1-} to $q_i^{k-} = q_i^{k-1-} + (q_j^{k-1+} a_j^+/a_i^-) \geq 0$.

Moreover, append state j into the set of positive states whose initial payoff a_j^+ as been replaced by a binary zero-mean lottery x_{ij}: $J^k = J^{k-1} \cup j$.

In both cases, this procedure yields a new lottery x^k that has the same distribution of payoffs, but in which one payoff has been replaced by a binary zero-mean lottery. After

$m + n - 2$ iterations, all payoffs have replaced by such lotteries, expect two of them. By construction, since the $Ex^k = 0$, the remaining two atoms (a_i^-, a_j^+) must be opposite in sign and have remaining probabilities q_i^{m+n-2-} and q_j^{m+n-2+} such that

$$a_i^- q_i^{m+n-2-} + a_j^+ q_j^{m+n-2+} = 0. \tag{44}$$

This concludes the proof of Lemma 1. $\square$

References

Abdellaoui, M. (2000): "Parameter-Free Elicitation of Utility and Probability Weighting Functions," *Management Science*, 46, 1497–1512.

Abdellaoui, M., O. L'Haridon, and H. Zank (2010): "Separating curvature and elevation: A parametric probability weighting function," *Journal of Risk and Uncertainty*, 41, 39–65.

Barro, R. (2006): "Rare disasters and asset markets in the twentieth century," *Quarterly Journal of Economics*, 121, 823–866.

——— (2009): "Rare disasters, asset prices, and welfare costs," *American Economic Review*, 99, 243–264.

Bell, D. E. (1982): "Regret in decision making under uncertainty," *Operations Research*, 30, 961–981.

——— (1983): "Risk premiums for decision regret," *Management Science*, 29, 1156–1166.

Bhattacharya, N. and T. Garrett (2008): "Why people choose negative expected return assets - an empirical examination of a utility theoretic explanation," *Applied Economics*, 40, 27–34.

Bikhchandani, S. and U. Segal (2011): "Transitive regret," *Theoretical Economics*, 6, 95–108.

Braun, M. and A. Muermann (2004): "The impact of regret on the demand for insurance," *Journal of Risk and Insurance*, 71, 737–767.

Brehaut, J. C., A. O'Connor, T. Wood, T. Hack, L. Siminoff, E. Gordon, and D. Feldman-Stewart (2003): "Validation of a decision regret scale," *Medical Decision Making*, 23, 281–292.

Camille, N., G. Coricelli, J. Sallet, P. Pradat-Diehl, J. Duhamel, and A. Sirigu (2004): "The Involvement of the Orbitofrontal Cortex in the Experience of Regret," *Science*, 304, 1167–1170.

Chapman, G. and E. Coups (2006): "Emotions and preventive health behavior: Worry, regret, and influenza vaccination," *Health Psychology*, 25, 82–90.

Eeckhoudt, L., B. Rey, and H. Schlesinger (2007): "A good sign for multivariate risk taking," *Management Science*, 53, 117–124.

Eeckhoudt, L. and H. Schlesinger (2006): "Putting risk in its proper place," *American Economic Review*, 96, 280–289.

Eichner, T. and A. Wagener (2011): "Increases in skewness and three-moment preferences," *Mathematical Social Sciences*, 61, 109–113.

Epstein, L. and S. Tanny (1980): "Increasing generalized correlation: A definition and some economic consequences," *Canadian Journal of Economics*, 13, 16–34.

Garrett, T. and R. Sobel (1999): "Gamblers favor skewness, not risk: Further evidence from United States′ lottery games," *Economics Letters*, 63, 85–90.

Gee, C. (2012): "A behavioral model of regret aversion," Unpublished manuscript.

Gilovich, T. and V. Medvec (1995): "The experience of regret: What, when, and why," *Psychological Review*, 102, 379–395.

Golec, J. and M. Tamarkin (1998): "Bettors love skewness, not risk, at the horse track," *Journal of Political Economy*, 106, 205–225.

Gollier, C. (1995): "The Comparative Statics of Changes in Risk Revisited," *Journal of Economic Theory*, 66, 522–536.

Gollier, C. and B. Salanié (2012): "Individual decisions under risk, risk sharing and asset prices with regret," Toulouse School of Economics.

Harvey, C. and A. Siddique (2000): "Conditional skewness in asset pricing tests," *Journal of Finance*, 55, 1263–1295.

Inman, J. J. and L. McAlister (1994): "Do coupon expiration dates affect consumer behavior?" *Journal of Marketing Research*, 16, 423–428.

Kahneman, D. and A. Tversky (1979): *Prospect theory: An analysis of decision under risk*, vol. 47.

Loomes, G. (1988): "Further evidence of the impact of regret and disappointment in choice under uncertainty," *Economica*, 55.

Loomes, G. and R. Sugden (1982): "Regret theory: An alternative theory of rational choice under uncertainty," *Economic Journal*, 92, 805–824.

——— (1987a): "Some implications of a general form of regret theory," *Journal of Economic Theory*, 270–287.

——— (1987b): "Testing for regret and disappointment in choice under uncertainty," *Economic Journal*, 97, 118–129.

Michenaud, S. and B. Solnik (2008): "Applying regret theory to investment choices: Currency hedging decisions," *Journal of International Money and Finance*, 27, 677–694.

Muermann, A., O. Mitchell, and J. Volkman (2006): "Regret, portfolio choice, and guarantees in defined contribution schemes," *Insurance: Mathematics and Economics*, 39, 219–229.

Quiggin, J. (1994): "Regret theory with general choices," *Journal of Risk and Uncertainty*, 8, 153–165.

Rothschild, M. and J. Stiglitz (1970): "Increasing risk: I. A definition," *Journal of Economic Theory*, 2, 225–243.

——— (1971): "Increasing risk: II. Its economic consequences," *Journal of Economic Theory*, 3, 66–84.

Sarver, T. (2008): "Anticipating regret: Why fewer options may be better," *Econometrica*, 76, 263–305.

Savage, L. (1951): "The theory of statistical decision," *Journal of the American Statistical Association*, 46, 55–67.

Stalnaker, T., N. K. Cooch, and G. Schoenbaum (2015): "What the orbitofrontal cortex does not do," *Nature Neuroscience*, 18, 620–627.

Tchen, A. (1980): "Inequalities for distributions with given marginals," *Annals of Probability*, 8, 814–827.

Tversky, A. and D. Kahneman (1992): "Advances in prospect theory: Cumulative representation of uncertainty," *Journal of Risk and Uncertainty*, 5, 297–323.

Wu, G. and R. Gonzalez (1996): "Curvature of the probability weighting function," *Management Science*, 42, 1676–1690.

Zeelenberg, M. and R. Pieters (2004): "Consequences of regret aversion in real life: The case of the Dutch postcode lottery," *Organizational Behavior and Human Decision Processes*, 93, 155–168.

——— (2007): "A theory of regret regulation 1.0," *Journal of Consumer Psychology*, 17, 3–18.