General Equilibrium Rebound from Energy Efficiency Policies*

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Energy efficiency policies "rebound" when economic responses undercut their direct energy savings. I develop an analytic general equilibrium framework for investigating supply-side drivers of rebound. I show that supply-side responses always generate positive rebound. Efficiency policies increase energy use ("backfire") under a broader range of conditions than suggested by previous frameworks that ignored energy supply. The presence of taxes on energy resources or emissions can reduce the environmental effectiveness of an efficiency policy (or make it backfire more strongly). The potential for backfire increases as the energy resource becomes more depleted, which suggests that rebound effects may accumulate over time.

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Many governments have adopted energy efficiency policies in order to reduce the greenhouse gas emissions that drive climate change and to reduce dependence on energy resources. The American Recovery and Reinvestment Act of 2009 provided nearly $20 billion for energy efficiency programs (Aldy, 2013). The U.S. Environmental Protection Agency's Clean Power Plan employs energy efficiency as two of the four "building blocks" through which the U.S. electric power sector will reduce its carbon emissions by 30% by 2030. The European Union's 2012 Energy Efficiency Directive mandates a 20% improvement in energy efficiency by 2020. And many countries have programs that require improvements in the efficiency of their vehicles and appliances.

Such large-scale policies are likely to have general equilibrium consequences (compare Acemoglu, 2010). While economists have long recognized the potential for economic responses to undercut efficiency policies' fuel savings (Jevons, 1865), nearly all formal analyses of these "rebound" effects have focused on partial equilibrium settings with an exogenous price of energy resources.[1] At one end, microeconomic analyses have emphasized how income and substitution effects can increase household energy consumption after an improvement in efficiency.[2] At the other end, neoclassical growth settings have emphasized how analogues of these income and substitution effects arise after improving the productivity of energy in the broader economy's production function (Saunders, 1992, 2000). Despite the theoretical literature's focus on partial equilibrium settings, computable general equilibrium models have suggested the potential for strong rebound effects through economy-wide "indirect" channels.[3]

I fill the gap in the theoretical literature by developing an analytically tractable general equilibrium framework for analyzing the implications of efficiency policies for energy resource extraction and emissions. I explicitly model the supply of energy, provide a clear expression for general equilibrium rebound, disentangle the channels through which efficiency policies affect supply, and analyze how other energy policies and advancing resource depletion interact with rebound.[4] In my setting, the energy resource is distributed among pools with different costs of extraction and the resource extraction sector competes with final-good production

[1] Reflecting on the potential importance of economy-wide rebound channels, Dimitropoulos (2007) notes the lack of a theoretical framework for understanding general equilibrium channels. Borenstein (2013) similarly calls for further research on channels for economy-wide rebound.

[2] There are several overviews of this microeconomic, partial equilibrium literature. See Greening et al. (2000), Sorrell and Dimitropoulos (2008), Sorrell et al. (2009), van den Bergh (2011), Borenstein (2013), and Gillingham et al. (2013).

[3] For instance, see the studies mentioned in Allan et al. (2009) and Turner (2013).

[4] Turner (2013) laments the lack of attention given to energy supply in previous analyses of rebound effects. The two other analytic general equilibrium settings are Wei (2007, 2010). Wei (2007) restricts attention to Cobb-Douglas functional forms for all production functions and does not include a physical resource input to energy production. Wei (2010) represents energy supply as a reduced-form, increasing function of its price, which means that energy supply does not compete with other sectors for scarce factors of production. We will see that explicitly modeling resource extraction and supply produces qualitatively new results and insights.

for a scarce factor, interpreted as a labor-capital aggregate. Intermediate firms combine the extracted resource with an energy conversion technology to produce energy services. Final-good producers use energy services and the labor-capital aggregate as inputs. An efficiency policy improves the quality of the energy conversion technology and affects prices and activity throughout the economy.

I show that the effect of an efficiency policy on equilibrium consumption of energy resources is determined by the following forces. If the price of energy services were fixed, then the improvement in efficiency would increase resource extraction by increasing the marginal product of energy resources and thus increasing energy service firms' willingness to pay for resources. However, the equilibrium price of energy services also changes. First, the improvement in efficiency makes energy services more abundant, which reduces their price and disincentivizes resource extraction. Second, the increased quantity of energy services makes the non-energy input to final-good production relatively scarce, which increases the return to employing the labor-capital aggregate in that sector rather than in resource extraction. Third, final-good firms substitute towards the cheaper energy service input, which incentivizes resource extraction. When the elasticity of substitution is sufficiently large, the price of energy services does not decline by much. The improvement in the marginal product of energy resources can then suffice to increase the price of energy resources. In this case, an improvement in efficiency increases resource extraction and pollution.[5]

An engineering estimate of the effects of an efficiency policy would hold energy service production fixed and calculate the energy resources displaced by the improvement in efficiency. "Rebound" is the percentage of these engineering savings lost through economic responses. Contrary to claims in the literature (Turner, 2009; Wei, 2010), I show that general equilibrium responses always produce positive rebound in this setting. I also show that interactions with the tax system are important. Efficiency policies are often implemented in tandem with other policies that aim to reduce resource extraction and pollution, but I show that the presence of these other policies can make efficiency policies less effective at reducing resource extraction. Further, I show that advancing resource depletion can also reduce the environmental effectiveness of efficiency policies, which suggests that efficiency policies may become less effective over time.

When rebound is greater than 100%, an efficiency policy is said to "backfire," actually increasing consumption of energy resources. Neoclassical growth settings have suggested that backfire occurs when the elasticity of substitution between energy and non-energy inputs to

[5]As an example, consider a coal plant producing electricity for final-good production, with capital employed in coal extraction and in final-good production. If we make the coal plant more efficient, then we increase the price that the plant is willing to pay for coal, for a given price of electricity. However, electricity has become more abundant, which reduces the price of electricity and raises the return to capital in final-good production. These price effects are mitigated when the final-good firm has a large elasticity of substitution, because it will then buy much more electricity when the price of electricity falls. Contrary to the suggestion in Gillingham et al. (2013), it is thus possible for an improvement in efficiency to increase the price of coal while decreasing the price of electricity.

final-good production is greater than unity or very close to unity (Saunders, 1992, 2000). Much empirical work has suggested that the elasticity of substitution is less than unity, which would make backfire irrelevant. Nonetheless, computable general equilibrium models have in fact found backfire when analyzing particular policies (Semboja, 1994; Grepperud and Rasmussen, 2004; Glomsrød and Taoyuan, 2005; Hanley et al., 2006; Allan et al., 2009; Hanley et al., 2009). I show that backfire is possible for arbitrarily small elasticities of substitution. The set of elasticities compatible with backfire often grows as the energy resource becomes depleted and as the energy resource is taxed more heavily but often shrinks as the initial quality of energy conversion technology improves. These results suggest that the potential for backfire is indeed an empirically relevant question after all. Analyses of backfire should account for general equilibrium effects, for supply-side responses, and for the presence of other energy policies.

This paper extends several recent literatures exploring the unintended consequences of environmental policies. First, the "green paradox" literature considers how energy policies can backfire by changing resource extractors' incentives to conserve resources for the future (e.g., Sinn, 2008; Gerlagh, 2011). I abstract from dynamic considerations in order to demonstrate static, general equilibrium channels for backfire. Second, several papers have explored how environmental regulations that constrain the energy intensity or emission intensity of production can backfire (e.g., Helfand, 1991; Holland et al., 2009; Fullerton and Heutel, 2010; Lemoine, 2013). These effects arise because an intensity constraint implicitly combines an output subsidy with a tax on energy or emissions. I instead explore the consequences of more common policies that directly incentivize the adoption or development of technologies that reduce the energy intensity of energy service production, without constraining firms' profit maximization problems. Third, other literature on the general equilibrium consequences of environmental policies has explored the potential for leakage between sectors or regions (e.g., Copeland and Taylor, 2004; Baylis et al., 2014). I develop a more textured model of a single energy system that allows me to answer questions about efficiency policies. Finally, the "double dividend" literature has explored how interactions with pre-existing taxes can reduce or reverse the welfare gains from Pigouvian taxes on emissions (e.g., Bovenberg and de Mooij, 1994; Bovenberg and Goulder, 1996). I show that emission taxes can themselves reduce the environmental benefits of a different (and possibly more common) type of policy.

The next section describes the setting. Section 2 derives and analyzes an expression for general equilibrium rebound. Section 3 then describes when efficiency policies backfire and graphically analyzes the efficiency-induced change in extraction. Section 4 analyzes how energy taxes and resource depletion affect general equilibrium rebound. The final section concludes. The first appendix contains the core of the theoretical analysis, and the second appendix contains proofs.

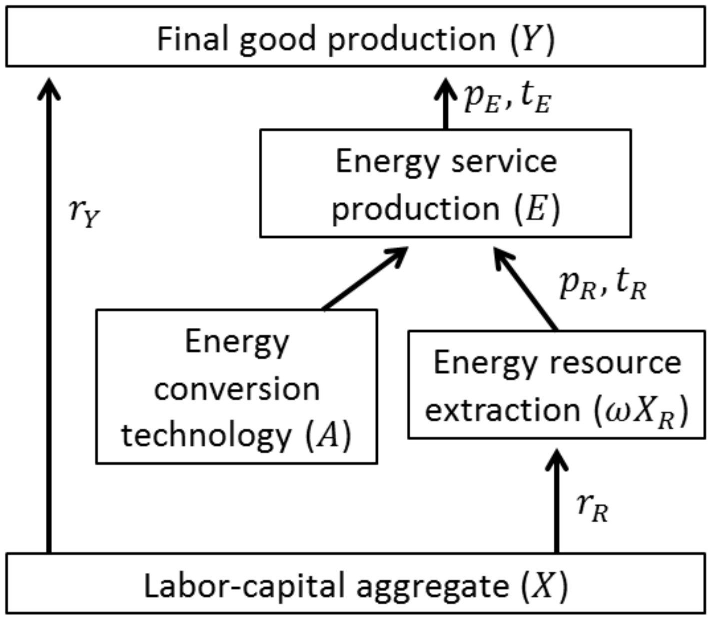


Figure 1: An overview of the theoretical setting. Real variables are in boxes. Prices p, rents r, and taxes t are labeled along the arrows. An efficiency policy improves A, the quality of energy conversion technology.

1 Setting

The economy has a unique final good produced by combining energy services (such as lighting, mobility, and mechanical motion) with a scarce factor of production. Energy services themselves are produced by combining energy resources (such as coal, oil, or gas) with an energy conversion technology. The resources are distributed among pools of declining quality, so that the marginal cost of developing the resources increases in the current level of development and in the history of development. Households earn income by renting out the factor of production and spend their income on a consumption good, over which they have increasing utility. The regulator taxes energy services and resources and returns its revenue to households as a lump-sum transfer. Pollution is an increasing function of energy resource use. We are interested in the implications of improved energy conversion technologies for resource extraction and pollution. Figure 1 summarizes the setting, with real variables in boxes and prices and taxes labeled along the arrows.

Let X be the scarce factor of production, interpreted as a labor-capital aggregate. It is of fixed measure, normalized to unity. Households rent the factor to the highest-paying use, whether producing the final good Y or accessing energy resources R. Let X_Y and X_R indicate the quantity of labor-capital aggregate used in each sector, so that full employment implies $X_Y + X_R = 1$. The rents earned from each sector are r_Y and r_R.

The composite final good Y is produced competitively using the labor-capital aggregate X_Y and energy services E:

$$Y = \left[(1-\kappa)X_Y^{\frac{\sigma-1}{\sigma}} + \kappa E^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}.$$

The production function is the conventional constant elasticity of substitution (CES) specification. The parameter $\kappa \in (0,1)$ is a distribution parameter that determines the inputs' relative marginal product, and the parameter $\sigma \in (0,\infty)$ is the elasticity of substitution between the two inputs. The two inputs are (gross) substitutes when $\sigma > 1$ and (gross) complements when $\sigma < 1$.[6] I ignore the special Cobb-Douglas case in which $\sigma = 1$.

Energy services E are intermediates like lighting and mechanical motion that directly assist in final-good production. Energy services are produced competitively using energy resources R and an energy conversion technology:

$$E = R^{1-\alpha}A^{\alpha},$$

where $\alpha \in (0,1)$ and A is the quality of energy conversion technology.

Energy resources must be accessed and brought to market, which requires use of the labor-capital aggregate. Resource extraction firms bid competitively for the labor-capital aggregate. A resource extraction firm discovers a pool or deposit of expected size ω for each unit of the labor-capital aggregate employed. The firm extracts this resource upon discovering it and supplies it to energy service producers. The quantity of energy resources produced is thus $\omega\, X_R$. Pollution increases monotonically in resource use and thus in extraction.

The pools or deposits of energy resources differ in their quality. Lower-quality pools have higher costs per unit of energy extracted. In aggregate, firms discover the cheapest pools first, so that the cost of developing the resource rises with contemporary activity (as in Heal, 1976; Solow and Wan, 1976). Q units of the resource have been extracted prior to the period of analysis. The cost of extracting from pool k is $\Psi(k)$, where $\Psi(\cdot)$ is positive, increasing, and differentiable.

A resource extraction firm is equally likely to discover any of the new deposits. An extraction firm's expected per-unit cost of extraction is

$$\psi(X_R; Q) = \frac{1}{\omega X_R}\int_Q^{Q+\omega X_R} \Psi(k)\,\mathrm{d}k.$$

[6] One can interpret X_Y and E as inputs to a physical production function or as inputs to a utility function. In the first case, shifting towards energy services suggests replacing labor via mechanization. Empirical work then points towards the two inputs being complements. In the second case, shifting towards energy services suggests increasing consumption of material goods in place of personal services. Here the two inputs are again plausibly complements. This paper will use the first interpretation, but all results apply under the second.

$\Psi'(k) \geq 0$ implies that $\psi(X_R; Q)$ is increasing in both of its arguments: the expected extraction cost increases in depletion Q and in contemporary extraction effort X_R.

The economy's resource constraint is

$$Y \geq C + \int_{Q}^{Q+\omega X_R} \Psi(k)\, \mathrm{d}k,$$

where $C \geq 0$ is the composite consumption good. Households' monotonically increasing utility for that consumption good leads them to maximize earnings.

The government taxes each unit of energy services at rate t_E and taxes each unit of energy resources at rate t_R. If each resource pool has the same emission intensity, then the resource tax t_R is equivalent to a tax on emissions. The government returns all revenue back to the households as lump-sum transfers, less any revenue spent to implement an efficiency policy.

I undertake a positive analysis of efficiency policies and therefore consider market equilibria rather than a social planner's problem.[7] Denote equilibrium outcomes with an asterisk.

Definition 1. *An* equilibrium *is given by rents* (r_Y^*, r_R^*)*, prices for energy services* (p_E^*)*, prices for energy resources* (p_R^*)*, demands for inputs* (X_Y^*, E^*, R^*, X_R^*)*, and factor allocations* (X_Y^*, X_R^*) *such that: (i)* R^* *maximizes profits of energy service producers, (ii)* (X_Y^*, E^*) *maximizes profits of final-good producers, (iii)* (X_Y^*, X_R^*) *maximizes households' earnings, and (iv) the rents* (r_Y^*, r_R^*) *and prices* (p_E^*, p_R^*) *clear the factor and input markets.*

The equilibrium prices clear all factor markets, all firms maximize profits, and households receive the same income from renting the labor-capital aggregate to either sector. Throughout, I use the price of the final good as the numeraire and assume the existence of an equilibrium with $X_Y, X_R > 0$. The appendix shows that any interior equilibrium is locally asymptotically stable in a tâtonnement sense.

2 Rebound

I now theoretically analyze the effects of introducing a policy to improve the efficiency of energy conversion technology. I assume that the government can procure the improved technology by spending tax revenue, without using the scarce factor X.[8] An *efficiency policy*

[7] Much literature has explored the reasons why households and firms appear, by some calculations, to underinvest in efficiency. The present setting does not require optimal investment in efficiency technologies. Instead, it only requires that, conditional on using some particular technology, households and firms maximize income and profits.

[8] I have also developed an extension to include an innovation sector that works to improve the quality of energy conversion technology and competes for use of X. An efficiency policy can then be represented as a subsidy to innovative activity. This extension complicates the analysis but does not change the major insights.

corresponds to a marginal increase in A, the quality of energy conversion technology. I focus on the implications of a successful policy rather than on any distortions involved in achieving the policy outcome.[9]

The conventional engineering calculation of the energy savings from an improvement in efficiency holds the quantity of energy services constant. For instance, if you upgrade to a more fuel-efficient car, the engineering estimate of your savings holds your mileage fixed and calculates your avoided gasoline consumption. Rebound occurs if using less fuel per mile leads you to drive more, reducing your energy savings from the engineering estimate. Totally differentiating energy service production and setting $\mathrm{d}E = 0$, we have the "engineering" or "no-rebound" change in resource consumption from a marginal improvement in the energy conversion technology:

$$\frac{\mathrm{d}R}{\mathrm{d}A} = -\frac{\alpha}{1-\alpha}\frac{R}{A}. \tag{1}$$

For each 1% improvement in energy conversion technology A, energy resource consumption falls by $\alpha/(1-\alpha)\%$.

"Rebound" refers to how economic responses alter these engineering calculations. If economic responses further increase the energy resource savings, then we have negative rebound, known as "super-conservation." If economic responses reduce the resource savings, then we have the standard case of positive rebound, expressed as the percentage of the engineering calculation's estimated savings that are lost through economic responses. If economic responses are so strong as to end up increasing resource consumption after an improvement in efficiency, then we have a case of >100% rebound, known as "backfire."

Formally, the actual change in resource consumption due to a marginal improvement in energy conversion technology is $\mathrm{d}X_R^*/\mathrm{d}A$. Rebound, as a percentage of the "engineering" savings, is

$$rebound \triangleq 100 * \left(1 - \frac{\mathrm{d}X_R^*/\mathrm{d}A}{\frac{-\alpha}{1-\alpha}\frac{X_R^*}{A}}\right).$$

When $\mathrm{d}X_R^*/\mathrm{d}A$ is strictly positive, rebound is >100% and we have backfire. When this derivative is negative but greater than $-[\alpha/(1-\alpha)]\, X_R^*/A$, rebound is positive but less than 100%.

The appendix solves for $\mathrm{d}X_R^*/\mathrm{d}A$ by constructing a system of equations that defines the equilibrium allocation and then applying the Implicit Function Theorem. It shows that we

[9] I also ignore the welfare impacts of the policy in favor of focusing on its energetic and environmental consequences, which are likely to be crucial components of any welfare calculation. Welfare calculations would require specifying the utility function for final good consumption, the cost of the policy, and the damages from pollution. Further, to the extent that most efficiency policies are implemented to achieve particular energy or environmental goals, rebound is often more pertinent to their objectives than is a full welfare calculation.

can express rebound as

$$rebound = 100 * \left(1 + \frac{B}{\frac{\alpha}{1-\alpha}B + \frac{1}{1-\alpha}C}\right), \tag{2}$$

$$\text{where } sign(B) = sign(\mathrm{d}X_R^*/\mathrm{d}A), \quad \frac{\alpha}{1-\alpha}B + \frac{1}{1-\alpha}C > 0,$$

$$B = -\frac{p_E}{p_E - t_E}\Pi_{R,Rev} + \sigma(1+\Theta)\Pi_{R,Rev} - \Theta\Pi_R, \text{ and}$$

$$C = \frac{1}{X_Y}\left(\frac{p_E}{p_E - t_E}\Pi_{R,Rev} + \Theta\Pi_R\right) + \omega\frac{\partial\psi(X_R;Q)}{\partial X_R}\sigma\left[1+\Theta\right] X_R.$$

$\Pi_{R,Rev} > 0$ is the revenue earned from developing a pool of the energy resource, and $\Pi_R > 0$ is the expected profit from developing a pool of the resource. $\Theta > 0$ derives from the final-good firm's first-order conditions for profit-maximization and from its zero-profit condition. It gives the (negative of the) elasticity of the equilibrium rent r_Y with respect to the price p_E of energy services: if the price of energy services falls by 1%, the final-good firm's willingness to pay for its non-energy input increases by Θ%.

The term labeled B controls the degree of rebound, and its sign determines whether backfire occurs. B tells us how the relative rent r_R/r_Y changes in response to an improvement in energy conversion technology. The negative first term in B recognizes that an increase in the quality of energy conversion technology acts like an increase in the supply of energy services, which reduces their price. This reduction in the price of energy services reduces the price of energy resources and so the revenue from resource extraction. The fraction $p_E/[p_E - t_E]$ gives the percentage change in the producer's price of energy services from a 1% change in the purchaser's price of energy services. The higher the tax t_E, the greater the percentage decline in the producer's revenue from a given percentage reduction in the sale price p_E.

The positive second term in B recognizes that the final-good firm adjusts its input mix in response to a decline in the price of energy services. The greater is the elasticity of substitution σ, the greater the increase in the final-good firm's relative demand for energy services when their relative price declines. Further, from the final-good firm's first-order conditions and zero-profit condition, Θ gives the percentage increase in the price of the non-energy input in response to a 1% decrease in the price of energy services. This increase in the price of the non-energy input also increases the final-good firm's relative demand for energy services via the elasticity of substitution σ. These two effects work to increase the price of the energy resource and so to attract more of the scarce labor-capital aggregate to the resource extraction sector.

Both the first and the second term in B are proportional to the revenue from developing a resource pool because they describe effects on the price of energy resources. In contrast, the negative third term in B is proportional to Π_R, the expected profit in the extraction sector. As before, Θ gives the percentage increase in r_Y, the rent to employing the scarce labor-capital aggregate in the non-energy sector, from a 1% decline in p_E, the price of energy services. This increase in r_Y raises households' opportunity cost of supplying the labor-capital aggregate to the extraction sector rather than to the final-good sector. Due to competition among resource firms for the scarce resource, the equilibrium rent r_R in the resource sector is equal to expected profit Π_R. Expected profit must rise to restore equality between r_Y and r_R, which means, for a given price of energy services, that the quantity of labor-capital aggregate employed in resource extraction must fall.

Proposition 1 establishes the sign of rebound:

Proposition 1. *Reboundis strictly positive.*

Proof. See appendix. □

Economic responses always undercut "engineering" savings in this general equilibrium setting. Contrary to claims in the literature (Turner, 2009; Wei, 2010), negative rebound or "super-conservation" does not occur. The denominator in expression (2) drives positive rebound. Whenever an efficiency policy successfully reduces extraction ($B < 0$), the denominator is larger in magnitude than the numerator. The denominator describes how an efficiency-induced reduction in resource extraction alters the incentives for further extraction. Whereas B described the effects of increasing energy service production on the incentive to extract energy resources, the denominator describes the effects of a reduction in extraction on the incentives to extract resources.

First, recall that in the absence of rebound, extraction falls by $\alpha/(1-\alpha)\%$ when the quality of energy conversion technology improves by 1%. This reduction in extraction acts to reduce energy services, which increases the price of energy services and reduces the opportunity cost to supplying the labor-capital aggregate to the resource extraction sector. However, final-good firms substitute towards the non-energy input, which works against these first two effects. The term $[\alpha/(1-\alpha)]\, B$ in the denominator in expression (2) captures the net effect, which increases the incentive to extract resources as long as σ is not too large. In that case, B is negative, which shrinks the positive denominator and so increases rebound.

The terms in C ensure that rebound is always positive. Each term in C is positive, and the formal analysis shows that the denominator in expression (2) is always positive. The first term in C recognizes that any "no-rebound" decline in extraction must imply an increase in the labor-capital aggregate supplied to the non-energy sector. This increase in X_Y increases the price of energy services and decreases the opportunity cost of renting to the extraction sector, both of which strengthen the incentive to rent to the extraction sector. The smaller is X_Y, the more strongly this change in relative supply increases the incentive to rent to the extraction sector. The second term in C recognizes that any "no-rebound" decline in extraction increases the incentive to rent to the extraction sector by reducing the expected cost of extraction. The analysis shows that when $B < 0$, C is sufficiently large that the denominator is greater than $-B$ and the fraction is greater than -1. The terms in C

therefore generate rebound by recognizing that a decline in extraction increases revenue in the extraction sector while decreasing the expected cost of extraction and the opportunity cost of renting to the extraction sector.

In sum, the price effects described in B may support a net decline in extraction, but the terms in C describe general equilibrium forces that work to undercut the "no-rebound" decline in extraction. Rebound is always positive.[10]

3 The Potential for Backfire

I have thus far decomposed the price channels that determine the magnitude of rebound and established that rebound is always positive. The current section describes when rebound is >100%, in which case an increase in energy efficiency actually increases consumption of energy resources and thereby increases pollution. It then analyzes the change in equilibrium resource extraction graphically, decomposing this change into a marginal productivity channel that works to increase extraction and an energy substitution channel that works to decrease extraction. The subsequent section describes how the magnitude of rebound and the likelihood of backfire change with resource depletion and with the level of resource taxes.

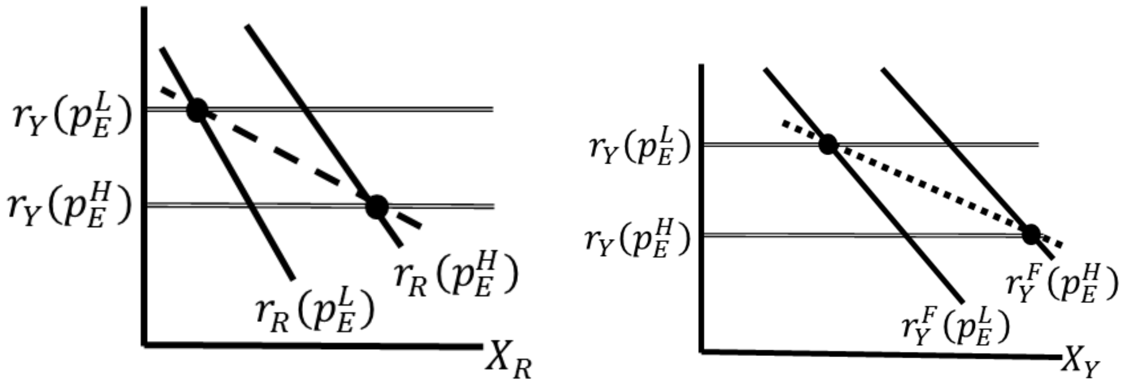
Neoclassical growth models of rebound effects have suggested that backfire can occur if the elasticity of substitution σ is very close to unity or is greater than unity (Saunders, 1992, 2000). Much empirical work has suggested that σ is substantially less than unity, but computable general equilibrium models have nonetheless suggested that backfire is a legitimate possibility. Proposition 2 establishes when backfire may occur in our analytic general equilibrium setting:

Proposition 2. *For any interior equilibrium z, there exists $\hat{\sigma}_z > 0$ such that $\mathrm{d}X_R^*/\mathrm{d}A > 0$ if and only if $\sigma > \hat{\sigma}_z$. If $t_E \leq 0$, then $\hat{\sigma}_z < 1$.*

Proof. See appendix. □

The index z refers to the collection of prices and allocations that define an equilibrium. As in the neoclassical growth settings, backfire occurs for σ sufficiently large and does not occur for σ sufficiently small. If energy services are untaxed or subsidized, then backfire occurs for

[10]Turner (2009) attributes her computable general equilibrium model's cases with negative rebound to "disinvestment" in the energy supply sectors. The present setting includes such disinvestment through a reduction in X_R^*. Something more must be responsible for driving rebound in her numerical simulations. Wei (2010) is also unclear about what drives negative rebound in his analytic setting. However, his setting does not include any of the forces described in C because his extraction sector does not compete with other sectors for scarce inputs. Intuitively, negative rebound may require that expected profit in the extraction sector increase in the level of extractive activity, perhaps because supplying additional energy resources stimulates demand for resources or because a non-homothetic utility (or production) function makes resource demand decrease in income (or in final-good production).



(a) Constructing r_R^* (dashed) (b) Constructing r_Y^* (dotted)

Figure 2: Constructing the equilibrium extraction rent r_R^* (left) and equilibrium non-energy rent r_Y^* (right) curves. For the sake of space, the arguments of each function suppress the dependence on the variable depicted along the horizontal axis.

some σ strictly less than unity. We will later see how the set of σ consistent with backfire changes with resource taxes and resource depletion.

We now analyze the change in extraction graphically. We begin by plotting the rent r_R in the extraction sector as a function of the supply X_R to that sector and plotting the rent r_Y in the non-energy sector as a function of the supply X_Y to that sector. These functions are general equilibrium rent functions. We then find the equilibrium outcome by first imposing the constraint $X_Y + X_R = 1$ and then requiring that households be indifferent between renting to either sector.

Consider the resource extraction sector. The appendix shows that equilibrium rent in this sector is

$$r_R = \omega \underbrace{(1-\alpha)\,(\omega X_R)^{-\alpha}\,[p_E - t_E]\,A^{\alpha}}_{p_R} - \omega\,\psi(X_R;Q) - \omega\,t_R. \tag{3}$$

This expression substitutes for the equilibrium resource price p_R in order to express the rent as a function of the price of energy services p_E. For given p_E, the rent decreases in the supply of X_R because the marginal energetic product of energy resources declines in their quantity and because the expected cost of extracting the resources increases in total extraction. The downward-sloping solid lines in the left panel of Figure 2 give the rent in the resource extraction sector as a function of X_R. Each curve is conditioned on some price of energy services p_E. Because the expected profit from resource extraction increases in the price of energy services (via the price of energy resources), raising the price of energy services from a low value (denoted p_E^L) to a high value (denoted p_E^H) shifts the rent curve outward.

The horizontal lines depict the equilibrium rent r_Y to the non-energy input corresponding to each price of energy services. As proved in the appendix, profit maximization and the final-good firm's zero-profit condition together imply that the equilibrium non-energy rent decreases in the price of energy services.[11] The horizontal line corresponding to a low price of energy services is therefore above the horizontal line corresponding to a high price of energy services.

For a given price of energy services, equilibrium occurs at the intersection of the horizontal non-energy rent line and the downward-sloping extraction rent curve. At this point, final-good firms are maximizing profits while earning zero profits and households are indifferent between renting to either sector. The downward-sloping dashed line connects these potential equilibria. This dashed line describes the combinations of X_R and r_Y that are compatible with the definition of equilibrium for a given price of energy services p_E, ignoring the aggregate resource constraint $X_Y + X_R = 1$.

The right panel of Figure 2 constructs the equilibrium rent curve for the non-energy sector. The solid downward-sloping lines give the final-good firm's demand for X_Y (denoted r_Y^F), conditional on a price of energy services. The appendix shows that equilibrium demand for X_Y is

$$X_Y = \left[\frac{1-\kappa}{\kappa}\right]^{\sigma} [p_E]^{\sigma} \left[r_Y(p_E)\right]^{-\sigma} E, \text{ where } E = (\omega X_R)^{1-\alpha} A^{\alpha}. \tag{4}$$

For a given quantity and price of energy services, demand for the non-energy input decreases in the rent r_Y paid to the non-energy input. This makes each curve r_Y^F slope down, as is common for demand curves. This downward slope reflects the diminishing marginal product of the non-energy input to final-good production. Raising the price of energy services from a low value (p_E^L) to a high value (p_E^H) shifts demand r_Y^F out, reflecting the non-energy input's greater value when its substitute is more expensive.

As in the left panel, the horizontal lines depict the equilibrium rent r_Y corresponding to each depicted price of energy services. For a given price of energy services, equilibrium occurs at the intersection of the horizontal rent line and the downward-sloping demand curve r_Y^F for non-energy inputs. The downward-sloping dotted line connects these intersections. It describes the combinations of X_Y and r_Y that are compatible with the definition of equilibrium given some price of energy services p_E, ignoring the aggregate resource constraint $X_Y + X_R = 1$.

Equilibrium requires that the labor allocation indicated by the left panel's dashed curve and the right panel's dotted curve clears the market for the labor-capital aggregate. We can plot those dashed and dotted curves together by using the aggregate resource constraint $X_Y + X_R = 1$ to express the dotted curve in terms of X_R. When we do this, we obtain the upward-sloping dotted "equilibrium non-energy rent" curve r_Y^* and the downward-sloping dashed "equilibrium extraction rent" curve r_R^* in Figure 3. When σ is small, r_Y^* is nearly

[11] This is why the elasticity Θ in the previous section is positive.

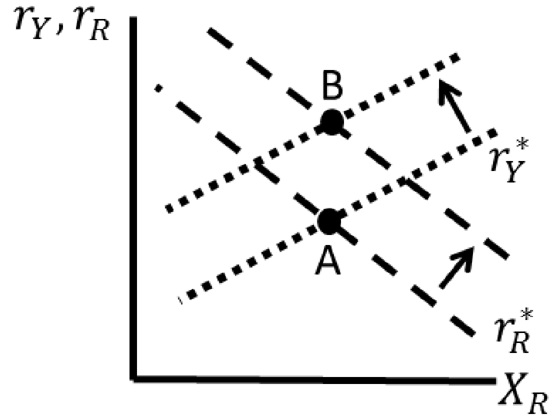


Figure 3: Increasing A shifts the equilibrium from point A to point B. Resource extraction increases if and only if the new equilibrium has greater X_R. The equilibrium price of energy services p_E is inversely related to the rent r_Y, plotted on the vertical axis.

vertical because demand for the non-energy input is price-inelastic, and when σ is large, r_Y^* is nearly horizontal because demand for the non-energy input is price-elastic. The unique equilibrium occurs at the intersection of the dashed and dotted lines, where households are indifferent between supplying the labor-capital aggregate to either sector. This initial equilibrium is labeled as point A.

Figure 3 shows how two competing effects determine whether an efficiency policy backfires. First, improving the efficiency of energy service production shifts the extraction rent curve upward because each unit of extracted resource now produces more energy services. This is a *marginal productivity channel* arising from energy service firms' willingness to pay a higher price for energy resources. The higher resource price increases firms' expected profit in the extraction sector, which increases the rent they pay to the labor-capital aggregate and attracts more labor-capital aggregate to resource extraction.

A second, competing effect works against backfire and in favor of reducing extraction. Improving the efficiency of energy service production shifts demand for the non-energy input X_Y out because of the greater availability of the other, energy service input. The dotted curve, expressed in terms of X_R rather than X_Y, therefore shifts inward. This *energy substitution channel* tends to decrease extraction. From equation (4), the final-good firm's demand for the non-energy input (as a function of the price of energy services) shifts out when the quantity of energy services E increases. The quantity of energy services E increases in resource extraction (ωX_R) and in the quality of energy conversion technology (A). Improving energy efficiency therefore increases X_Y^* at a given price of energy services, which shifts the dotted curve in Figure 2 outward and shifts the dotted curve in Figure 3 inward.

The net effect of these two channels is to shift the equilibrium to a point such as B in Figure 3. The new equilibrium clearly has higher rents and a lower price of energy services, but the effect on equilibrium extraction is ambiguous. The improvement in efficiency technology could raise the price of energy resources even as it reduces the price of energy services. For large elasticities of substitution, the non-energy demand curve is relatively flat and the shift in the extraction rent curve dominates, leading to an increase in equilibrium extraction (i.e., backfire).[12,13] However, for small elasticities of substitution, the non-energy demand curve is relatively steep, making its inward shift dominate the shift in expected profit. In that case, equilibrium extraction decreases when energy efficiency improves.[14]

Formalizing this intuition, the following proposition describes how prices change when backfire occurs:

Proposition 3. *An efficiency policy increases resource extraction ($\mathrm{d}X_R^*/\mathrm{d}A > 0$) only if it increases the price of energy resources p_R ($\mathrm{d}p_R^*/\mathrm{d}A > 0$). An efficiency policy always decreases the price of energy services p_E^* and increases the rents r_Y^* and r_R^*.*

Proof. See appendix. □

I have already argued that an efficiency policy always reduces the price of energy services. We now see that backfire occurs only if the price of energy resources nonetheless increases.[15] The higher resource price increases the incentive to extract resources. This necessary condition for backfire provides a means of empirically testing for backfire or for market expectations of backfire. However, this condition is only necessary, not sufficient: the price of energy

[12]Indeed, this is quite close to the original channel for backfire proposed by Jevons (1865, p. 141):

> Now, if the quantity of coal used in a blast-furnace, for instance, be diminished in comparison with the yield, the profits of the trade will increase, new capital will be attracted, the price of pig-iron will fall, but the demand for it increase; and eventually the greater number of furnaces will more than make up for the diminished consumption of each.

His increasing "profits of the trade" describes the marginal productivity channel, and his observation that the price of pig iron falls while demand for it increases describes the decline in the price of energy services that is mitigated by final-good firms' substitution patterns.

[13]If we interpret the final-good production function as a utility function, then we may be interested in the implications of non-homothetic preferences. Applying the intuition for the competing effects, non-homotheticity favors backfire if increasing income raises the elasticity of substitution between the non-energy input and energy services.

[14]The tax t_E on energy services creates a tax wedge inside the marginal productivity channel. When part of the additional value from improved technology is lost to taxes, the energy service firm's willingness to pay for resources does not increase by as much. This makes $\hat{\sigma}$ tend to increase in t_E, consistent with Proposition 2.

[15]Borenstein (2013) and Gillingham et al. (2013) informally discuss how an efficiency-induced reduction in energy prices can in turn induce additional energy consumption (though not by enough to produce backfire). Gillingham et al. (2013) further claim that efficiency policies reduce the incentive to extract energy resources. Our formal analysis shows that while the equilibrium price of *energy services* indeed decreases, the equilibrium price of *energy resources* may nonetheless increase and so strengthen incentives to extract energy resources.

resources could increase in the absence of backfire. Because an efficiency policy increases the rent r_Y paid to households who supply the labor-capital aggregate to final-good producers, the opportunity cost of renting the labor-capital aggregate to the resource extraction sector increases. If p_R increases by only a small amount, then it will not be sufficient to offset the greater opportunity cost and X_R^* will fall. In this case, rebound may be quite large, though less than 100%.

4 Comparative Statics of Rebound

We have seen that general equilibrium rebound is positive and that two competing channels determine whether backfire occurs. I now consider how resource taxes and depletion affect rebound and the potential for backfire.

Proposition 4 provides comparative statics for cases in which rebound is near 100%:

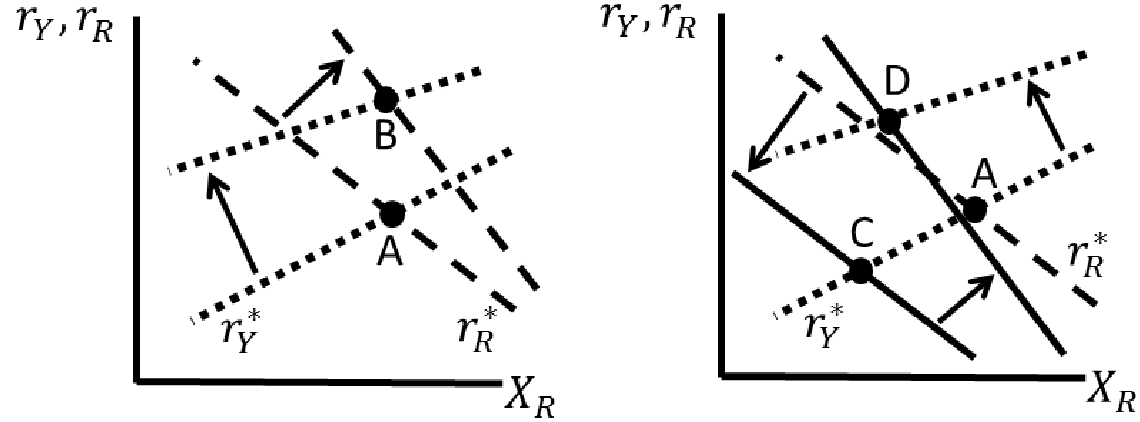
Proposition 4. *Let $\hat{\sigma}$ be defined as in Proposition 2.*

1. $\mathrm{d}\hat{\sigma}/\mathrm{d}Q < 0$ *if* $t_E \geq 0$
2. $\mathrm{d}\hat{\sigma}/\mathrm{d}t_R < 0$ *if* $t_E \geq 0$
3. $\mathrm{d}\hat{\sigma}/\mathrm{d}A > 0$ *if* $t_E \geq 0$

Proof. See appendix. □

Recall from Proposition 2 that backfire occurs if and only if $\sigma > \hat{\sigma}$. If $\hat{\sigma}$ decreases in a parameter, then raising the parameter makes backfire more plausible and increases rebound for σ near $\hat{\sigma}$. The first two results say that if the tax on energy services is weakly positive, then an efficiency policy backfires for a broader range of σ as the resource becomes more depleted and as the resource tax increases. Rebound thus increases as the resource becomes more depleted and as the resource tax increases, at least when rebound is near 100%. The third result says that if the tax on energy services is weakly positive, then an efficiency policy backfires for a smaller range of σ as the initial (pre-policy) quality of energy conversion technology improves. Thus, when rebound is near 100%, it decreases in the initial quality of technology.

To obtain intuition for these results, first consider the marginal productivity channel in more detail. The strength of this channel is determined by $\partial p_R/\partial A$ in equation (3). We see that $\partial p_R/\partial A$ is equal to $\alpha\, p_R/A$, which increases in p_R. Improving the quality of energy conversion technology by 1% increases resources' marginal energetic product by α%. The equilibrium price of energy resources increases by more when that equilibrium price is high to begin with. And we also see from equation (3) that the resource price decreases in the quantity of extraction $\omega\, X_R$. Therefore, the marginal productivity channel is stronger when



(a) Before increasing t_R (b) After increasing t_R

Figure 4: Prior to increasing the resource tax t_R (left), an efficiency policy shifts the equilibrium from point A to point B. Increasing the resource tax t_R (right) shifts the equilibrium from point A to point C, and an efficiency policy then shifts the equilibrium to point D.

X_R is small. Intuitively, making resources more productive is more valuable when resources are scarcer.

Next consider the energy substitution channel in more detail. The strength of this channel is determined by $\partial X_Y / \partial A$ in equation (4). We see that $\partial X_Y / \partial A = \alpha X_Y / A$, which increases in X_Y. Improving the quality of energy conversion technology by 1% increases energy service provision E by α% and so shifts out demand for the non-energy input X_Y by α%. The energy substitution channel is stronger when X_Y is large, which occurs when X_R is small. Intuitively, increasing energy service production more strongly increases the value of non-energy inputs when those inputs are relatively abundant.

The left panel of Figure 4 demonstrates a more precise way of plotting the marginal productivity and energy substitution channels. The marginal productivity channel shifts the equilibrium extraction rent curve out by a larger amount when revenue is greater, which is when X_R is smaller. The energy substitution channel shifts the equilibrium non-energy rent curve in by a larger amount when X_Y is larger, which is when X_R is smaller. Both channels therefore act not just to shift the lines but also to rotate them. The marginal productivity channel makes the equilibrium extraction rent curve steeper, and the energy substitution channel makes the equilibrium non-energy rent curve flatter. An efficiency policy shifts the equilibrium from point A to point B, which in this example leaves equilibrium extraction ωX_R unchanged.

Now consider increasing the resource tax t_R before applying the efficiency policy. The equilibrium extraction rent curve r_R^* shifts down to the solid line in the right panel of Figure 4.

The equilibrium level of X_R falls from point A to point C and the price of energy services rises (i.e., the price r_Y of the non-energy input to final-good production falls).[16] Now consider the effect of an efficiency policy. Because the equilibrium with a higher resource tax (point C) is further to the left than the original equilibrium (point A), the equilibrium resource price is greater at the new equilibrium and the marginal productivity effect is stronger than it was at the original equilibrium. However, the energy substitution channel is now also stronger than it was at the original equilibrium. After the tax increase, an efficiency policy changes the equilibrium from point C to point D. It could now increase or decrease equilibrium extraction, even though an efficiency policy did not affect equilibrium extraction prior to the tax increase.

After the tax increase, both the marginal productivity and energy substitution channels are stronger. Which effect dominates? When σ is relatively large, the final-good firm's demand for X_Y is relatively elastic and the equilibrium non-energy rent curve r_Y^* is relatively flat. The effect of the energy substitution channel at further flattening this curve becomes relatively unimportant. In that case, the stronger marginal productivity channel around the new equilibrium (point C) dominates, so that the efficiency policy now increases extraction. When σ is relatively small, the final-good firm's demand for X_Y is relatively inelastic and the equilibrium non-energy rent curve r_Y^* is relatively steep. The energy substitution channel now has a strong effect as it flattens this curve. In this case, the increase in the energy substitution channel's strength dominates the increase in the marginal productivity channel's strength around the new equilibrium (point C), so that the efficiency policy now decreases equilibrium extraction.

Analytically, a sufficient condition for the strengthening of the marginal productivity channel to dominate the strengthening of the energy substitution channel around $\hat{\sigma}$ is that the tax on energy services be weakly positive. The intuition is that, as discussed previously, larger t_E tends to support larger $\hat{\sigma}$. The only way rebound can decline in the resource tax around $\hat{\sigma}$ is if energy services are subsidized by a sufficiently large amount ($t_E \ll 0$).

We have discussed the effects of raising the resource tax t_R. The effects of advancing depletion Q are qualitatively similar. In contrast, improving the initial quality of energy conversion technology A has opposite effects: it tends to reduce the scope for backfire and reduce rebound around $\hat{\sigma}$, assuming energy services are not subsidized. Graphically, improving A before applying an efficiency policy acts like applying an efficiency policy twice. Both curves rotate less the second time around. The flatter is the equilibrium non-energy rent curve to begin with, the more similar is the energy substitution channel from one improvement in A to the next. Around $\hat{\sigma}$, the energy substitution and marginal productivity channels cancel on the first improvement in A. If $\hat{\sigma}$ is relatively large so that the equilibrium non-energy rent curve is relatively flat, then the energy substitution channel dominates the

[16] Increasing a tax on energy services t_E or advancing depletion Q both have similar effects on equilibrium extraction and prices, apart from their interactions with an efficiency policy. Formally, it is easy to adapt the proofs in the appendix to show that $\mathrm{d}X_R^*/\mathrm{d}t_R < 0$, $\mathrm{d}X_R^*/\mathrm{d}t_E < 0$, and $\mathrm{d}X_R^*/\mathrm{d}Q < 0$.

marginal productivity channel when A is improved a second time, leading to a decline in equilibrium extraction.

These comparative static results help us to think about rebound and backfire in a dynamic context. We have seen that if the elasticity of substitution σ is relatively small, then efficiency policies tend to generate less rebound. In this case with small σ, rebound could decrease as resource depletion advances and as resource taxes increase in order to internalize environmental damages. And rebound could increase as the initial quality of efficiency technology improves. We have also seen that efficiency policies tend to generate substantial rebound (or even backfire) when the elasticity of substitution is relatively large. In this case, we expect rebound to increase as resource depletion advances and as resource taxes increase, and we expect rebound to decrease as the initial quality of energy conversion technology improves. Crucially, the proof of Proposition 4 shows that the effect of advancing resource depletion is proportional to $\partial\psi(X_R; Q)/\partial Q$ and the effect of improving initial energy conversion technology is proportional to $1/A$. We should therefore expect the relative importance of the resource depletion channel to grow over time, which will often increase the chance that an efficiency policy backfires.

5 Conclusion

Large-scale efficiency policies have been promoted as one means of meeting the challenge of climate change. We have seen that general equilibrium effects reduce the environmental benefits of these policies. In fact, it is possible that these policies actually increase greenhouse gas emissions by increasing the value created by each unit of energy resources and so increasing the incentive to extract energy resources. Such a perverse outcome becomes more likely as taxes on resource extraction or emissions increase and as the resources become more depleted. Economic assessments of these policy proposals should not ignore general equilibrium channels.

Future theoretical work should extend the present setting to distinguish between different types of efficiency improvements and different types of final goods. Efficiency policies may be more promising when targeted to specific types of sectors or technologies. Future theoretical work should also embed channels for general equilibrium rebound in a dynamic setting. An efficiency policy alters the trajectories of technology and resource depletion, which should affect how rebound evolves over time.

Future empirical work should seek econometric evidence to complement and inform computable general equilibrium models' simulations of economy-wide rebound. We have seen that a necessary condition for backfire is that an efficiency policy increases the price of energy resources. An econometric analysis could study the response of equities or commodity futures markets to news about large-scale efficiency policies. These market movements would tell us about market expectations of rebound and would provide an additional means of disciplining computable general equilibrium models.

First Appendix: Formal Analysis

This appendix derives the system of equations defining the equilibrium. It then analyzes the effect of an efficiency policy on resource extraction and derives the main expression for rebound. The second appendix contains proofs. Because all of the analysis relies on equilibrium relations, I save notation by often omitting the asterisk signifying equilibrium outcomes.

Consider final-good producers. The final good is the numeraire, so they receive one unit of revenue for each unit sold. They rent the non-energy input at rate r_Y and buy energy services at price p_E. The representative final-good firm solves

$$\max_{X_Y,E}\left\{\left[(1-\kappa)X_Y^{\frac{\sigma-1}{\sigma}}+\kappa E^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}-r_YX_Y-p_EE\right\}.$$

The first-order conditions are

$$\begin{aligned}r_Y=&(1-\kappa)Y^{\frac{1}{\sigma}}X_Y^{\frac{-1}{\sigma}},\\ p_E=&\kappa Y^{\frac{1}{\sigma}}E^{\frac{-1}{\sigma}}.\end{aligned}$$

Zero profits and a final-good price of 1 then imply:

$$r_Y=\left(\frac{1-\kappa^{\sigma}[p_E]^{1-\sigma}}{(1-\kappa)^{\sigma}}\right)^{\frac{1}{1-\sigma}}\triangleq r_Y(p_E). \tag{5}$$

Now consider producers of energy services. This sector is also competitive. Each firm buys energy resources at price p_R. It pays a tax t_E on each unit of energy services sold. The representative energy services firm's profit-maximization problem is

$$\max_{R}\left\{[p_E-t_E]R^{1-\alpha}A^{\alpha}-p_RR\right\}.$$

The first-order condition for an interior solution is:

$$p_R=(1-\alpha)\left[p_E-t_E\right]R^{-\alpha}A^{\alpha}. \tag{6}$$

Now consider energy resource extraction. The equilibrium profits from developing resource pool k of unit size are:

$$\begin{aligned}\pi_R^k=&p_R-\Psi(k)-t_R\\ =&(1-\alpha)\left[p_E-t_E\right]R^{-\alpha}A^{\alpha}-\Psi(k)-t_R,\end{aligned}$$

where we substitute in for the equilibrium price of energy resources from the energy service producer's first-order condition. The expected profit from employing a unit of X in resource extraction is thus:

$$\Pi_R = \underbrace{\omega\,(1-\alpha)\,[p_E - t_E]\,R^{-\alpha}A^{\alpha}}_{\Pi_{R,Rev}} - \underbrace{[\omega\,\psi(X_R;Q) + \omega\,t_R]}_{\Pi_{R,Cost}}. \tag{7}$$

Resource firms bid competitively for X, so that the equilibrium rent r_R paid to households equals Π_R.

I now impose market-clearing in the resource extraction, energy service, and labor-capital aggregate markets. By Walras' Law, the market for final goods must also clear. From the final-good good firm's first-order conditions, we have:

$$E = \left[\frac{\kappa}{1-\kappa}\frac{r_Y}{p_E}\right]^{\sigma} X_Y.$$

Recall that equation (5) expresses r_Y in terms of p_E, via the final-good firm's zero-profit condition and a normalized final-good price of unity. Going forward, I write $r_Y(p_E)$ from equation (5). Substitute into energy service demand:

$$E = \left[\frac{\kappa}{1-\kappa}\right]^{\sigma} [p_E]^{-\sigma}\,[r_Y(p_E)]^{\sigma}\,X_Y. \tag{8}$$

Equating energy service demand and supply yields equilibrium demand for the non-energy input to final-good production:

$$X_Y = \left[\frac{1-\kappa}{\kappa}\right]^{\sigma} [p_E]^{\sigma}\,[r_Y(p_E)]^{-\sigma} R^{1-\alpha}A^{\alpha}. \tag{9}$$

There are three endogenous variables in this equation: X_Y, p_E, and X_R (via R). We need two additional relations to close the model. These are the resource constraint ($1 = X_Y + X_R$) and equality between the rent in the resource and non-energy sector ($r_Y = r_R$).

Substituting demand for X_Y from equation (9) into the resource constraint, using the expression for expected profit from resource extraction from equation (7), and recognizing that $R = \omega\,X_R$, we have the following system of equations in p_E and X_R:

$$\begin{aligned}
1 =& X_R + \left[\frac{1-\kappa}{\kappa}\right]^{\sigma} [p_E]^{\sigma}\,[r_Y(p_E)]^{-\sigma}(\omega\,X_R)^{1-\alpha}A^{\alpha} \\
&\triangleq G_1(p_E, X_R), \\
1 =& \omega(1-\alpha)\,[p_E - t_E](\omega X_R)^{-\alpha}A^{\alpha}\,[r_Y(p_E)]^{-1} - \omega\,[\psi(X_R;Q) + t_R]\,[r_Y(p_E)]^{-1} \\
&\triangleq G_2(p_E, X_R).
\end{aligned}$$

The partial derivatives of each equation with respect to the endogenous variables are (where I substitute X_Y^* from equation (9))

$$
\begin{aligned}
\frac{\partial G_1}{\partial p_E} =& \sigma\, X_Y^* \, [p_E]^{-1} \, [1 + \Theta] && > 0, \\
\frac{\partial G_1}{\partial X_R} =& 1 + (1-\alpha) X_R^{-1} X_Y^* && > 0, \\
\frac{\partial G_2}{\partial p_E} =& [r_Y(p_E)]^{-1} p_E^{-1} \left[\Pi_{R,Rev} \left(p_E [p_E - t_E]^{-1} + \Theta \right) - \Theta \, \Pi_{R,Cost} \right] && > 0, \\
\frac{\partial G_2}{\partial X_R} =& -\alpha X_R^{-1} \Pi_{R,Rev} [r_Y(p_E)]^{-1} - \omega \frac{\partial \psi(X_R; Q)}{\partial X_R} \, [r_Y(p_E)]^{-1} && < 0,
\end{aligned}
$$

where Θ is defined as the negative of the elasticity of the non-energy rent with respect to the price of energy services:

$$
\Theta \triangleq - \frac{r_Y'(p_E)}{r_Y(p_E)} \, p_E = \frac{\kappa^\sigma}{[p_E]^{\sigma-1} - \kappa^\sigma}.
$$

Lemma 5 shows that $\Theta > 0$. Define the matrix

$$
G \triangleq \begin{bmatrix} \frac{\partial G_1}{\partial p_E} & \frac{\partial G_1}{\partial X_R} \\ \frac{\partial G_2}{\partial p_E} & \frac{\partial G_2}{\partial X_R} \end{bmatrix}.
$$

$det(G)$ is < 0, where *det* refers to the determinant.

I now demonstrate the tâtonnement stability of the equilibrium. Consider a dynamic, tâtonnement-style process for describing the evolution of the economy in disequilibrium. If there is excess demand for X_Y, then let r_Y increase and p_E decrease. If r_R exceeds r_Y, then let X_R increase. In notation, we are defining the following tâtonnement-style system:

$$
\begin{aligned}
\dot{p}_E =& X_R + X_Y(p_E, X_R) - 1 \triangleq \hat{G}_1(p_E, X_R), \\
\dot{X}_R =& \frac{\Pi_R(p_E, X_R)}{r_Y(p_E)} - 1 \triangleq \hat{G}_2(p_E, X_R),
\end{aligned}
$$

where dots indicate time derivatives. This system's steady state occurs at the equilibrium values, which I denote with stars. Linearizing around the steady state, we have

$$
\begin{bmatrix} \dot{p}_E \\ \dot{X}_R \end{bmatrix} \approx \begin{bmatrix} \frac{\partial \hat{G}_1}{\partial p_E}(p_E^*, X_R^*) & \frac{\partial \hat{G}_1}{\partial X_R}(p_E^*, X_R^*) \\ \frac{\partial \hat{G}_2}{\partial p_E}(p_E^*, X_R^*) & \frac{\partial \hat{G}_2}{\partial X_R}(p_E^*, X_R^*) \end{bmatrix} \begin{bmatrix} p_E - p_E^* \\ X_R - X_R^* \end{bmatrix}.
$$

Label the matrix of partial derivatives as $\hat{G}$. Note that each entry in the top row of $\hat{G}$ is the negative of the corresponding entry in the matrix G, and each entry in the bottom row of $\hat{G}$ is the same as the corresponding entry in G. Therefore, $det(\hat{G}) = -det(G) > 0$ and $tr(\hat{G}) < 0$ (where tr refers to the trace), which implies that the two eigenvalues are negative. The linearized system is globally asymptotically stable, and, by the Hartman-Grobman Theorem, the full nonlinear system is locally asymptotically stable around the equilibrium.

Using the Implicit Function Theorem, we have:

$$\begin{aligned}\frac{\mathrm{d}X_R^*}{\mathrm{d}A} &= -\frac{det\left(\begin{bmatrix}\frac{\partial G_1}{\partial p_E} & \frac{\partial G_1}{\partial A}\\ \frac{\partial G_2}{\partial p_E} & \frac{\partial G_2}{\partial A}\end{bmatrix}\right)}{det(G)}\\ &= \frac{X_Y\,[p_E]^{-1}\alpha\,A^{-1}[r_Y(p_E)]^{-1}\left[\Pi_{R,Rev}\left[\sigma(1+\Theta)-(p_E[p_E-t_E]^{-1}+\Theta)\right]+\Theta\,\Pi_{R,Cost}\right]}{-det(G)},\end{aligned} \tag{10}$$

with the denominator simplifying to

$$\begin{aligned}-det(G) = &-(1-\alpha)\frac{X_Y}{X_R}p_E^{-1}[r_Y(p_E)]^{-1}\left\{\Pi_{R,Rev}\left[\sigma(1+\Theta)-\left(p_E[p_E-t_E]^{-1}+\Theta\right)\right]+\Theta\,\Pi_{R,Cost}\right\}\\ &+\frac{X_Y}{X_R}p_E^{-1}[r_Y(p_E)]^{-1}\left\{\omega\frac{\partial\psi(X_R;Q)}{\partial X_R}\sigma\,[1+\Theta]\;X_R\right.\\ &\qquad\left.+\Pi_{R,Rev}\left[\sigma(1+\Theta)+\frac{X_R}{X_Y}\left(p_E[p_E-t_E]^{-1}+\Theta\right)\right]-\frac{X_R}{X_Y}\Theta\,\Pi_{R,Cost}\right\},\end{aligned} \tag{11}$$

where I have added and subtracted $[X_Y/X_R]\,\Pi_{R,Rev}\,\sigma\,(1+\Theta)$. Combining, simplifying, and using $X_Y^*+X_R^*=1$ yields the expression for rebound given in the main text:

$$rebound \triangleq 100*\left(1-\frac{\mathrm{d}X_R^*/\mathrm{d}A}{\frac{-\alpha}{1-\alpha}\frac{X_R^*}{A}}\right) = 100*\left(1+\frac{B}{\frac{\alpha}{1-\alpha}B+\frac{1}{1-\alpha}C}\right), \tag{12}$$

where

$$\begin{aligned}B &\triangleq \Pi_{R,Rev}\left[\sigma(1+\Theta)-\left(\frac{p_E}{p_E-t_E}+\Theta\right)\right]+\Theta\,\Pi_{R,Cost},\\ C &\triangleq \frac{1}{X_Y}\left(\frac{p_E}{p_E-t_E}\Pi_{R,Rev}+\Theta\Pi_R\right)+\omega\frac{\partial\psi(X_R;Q)}{\partial X_R}\sigma\,[1+\Theta]\;X_R.\end{aligned}$$

It is clear that $sign(B) = sign(\mathrm{d}X_R^*/\mathrm{d}A)$. Because the denominator is proportional to $-det(G)$, we have that $\frac{\alpha}{1-\alpha}B+\frac{1}{1-\alpha}C > 0$.

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Second Appendix: Proofs

The following lemma establishes signs and relationships useful in the proofs and analysis.

Lemma 5. *Assume that* $X_Y, E > 0$.

1. $r'_Y(p_E) < 0$
2. $\Theta > 0$
3. $\partial\Theta/\partial\sigma = \Theta\ [1+\Theta]\ \ln(\kappa/p_E)$
4. $\partial\Theta/\partial p_E = (1-\sigma)\,[p_E]^{-1}\,\Theta\,[1+\Theta]$

Proof. Using the definition of $r_Y(p_E)$ in equation (5), we have

$$r'_Y(p_E) = -\frac{\kappa^\sigma}{(1-\kappa)^\sigma}[p_E]^{-\sigma}[r_Y(p_E)]^\sigma < 0.$$

This establishes the first part of the lemma. It also implies $\Theta > 0$, which establishes the second part of the lemma.

Differentiate the definition of Θ, using the expression for $r_Y(p_E)$ in equation (5):

$$\frac{\partial\Theta}{\partial\sigma} = \frac{[p_E]^{\sigma-1}\kappa^\sigma \ln\left(\frac{\kappa}{p_E}\right)}{\left([p_E]^{\sigma-1} - \kappa^\sigma\right)^2} = \Theta\ [1+\Theta]\ \ln\left(\frac{\kappa}{p_E}\right).$$

This establishes the third part of the lemma.

Differentiating Θ with respect to p_E and substituting, we have

$$\frac{\partial\Theta}{\partial p_E} = (1-\sigma)\frac{[p_E]^{\sigma-2}\kappa^\sigma}{([p_E]^{\sigma-1} - \kappa^\sigma)} = (1-\sigma)\frac{[p_E]^{\sigma-2}}{[p_E]^{\sigma-1} - \kappa^\sigma}\,\Theta = (1-\sigma)\,[p_E]^{-1}\,\Theta\,[1+\Theta].$$

This establishes the final part of the lemma.

□

Proof of Proposition 1

First, add and subtract B from the denominator in expression (12) to obtain:

$$rebound = 100 * \left(1 - \frac{B}{B-D}\right),$$

where B is as before and

$$D \triangleq \frac{1}{1-\alpha}\Pi_{R,Rev}\left[\sigma(1+\Theta)+\frac{X_R}{X_Y}\left(\frac{p_E}{p_E-t_E}+\Theta\right)\right] + \frac{1}{1-\alpha}\omega\frac{\partial\psi(X_R;Q)}{\partial X_R}\sigma\left[1+\Theta\right]X_R - \frac{1}{1-\alpha}\frac{X_R}{X_Y}\Theta\,\Pi_{R,Cost}.$$

Because $\Pi_{R,Rev}-\Pi_{R,Cost}=\Pi_R>0$, we have that $D>0$. And because $B-D$ is proportional to $det(G)$, we have that $B-D<0$.

Now assume that rebound is weakly negative. In that case, $\mathrm{d}X_R^*/\mathrm{d}A=-\frac{\alpha}{1-\alpha}X_R/A<0$, which implies $B<0$. By the expression for rebound, it also must be true that $B/[B-D]\geq 1$, which implies that $B\leq B-D<0$. But we know that $D>0$, which implies that $B-D<B$. This is a contradiction. Therefore, rebound is strictly positive.

Proof of Proposition 2

From equation (10), we have that

$$\frac{\mathrm{d}X_R^*}{\mathrm{d}A}\propto \Pi_{R,Rev}\left[\sigma(1+\Theta)-\left(p_E[p_E-t_E]^{-1}+\Theta\right)\right]+\Theta\,\Pi_{R,Cost}. \tag{13}$$

This is positive if

$$\sigma \geq \frac{\frac{p_E}{p_E-t_E}+\Theta}{1+\Theta}\triangleq D.$$

The right-hand side is <1 if $t_E\leq 0$, in which case $\mathrm{d}X_R^*/\mathrm{d}A>0$ for all $\sigma\geq 1$. As $\sigma\to 0$, $\mathrm{d}X_R^*/\mathrm{d}A<0$ because $\Pi_{R,Rev}>\Pi_{R,Cost}$. Because the above expression is continuous in σ, some $\sigma>0$ must be a root.

Differentiate equation (13) with respect to σ, holding the initial equilibrium fixed:

$$\left[1+\Theta\right]\Pi_{R,Rev}+\frac{\partial\Theta}{\partial\sigma}\left[\Pi_{R,Rev}\sigma-\Pi_R\right]. \tag{14}$$

Assume that there is more than one σ that is a root of equation (13). Because $\mathrm{d}X_R^*/\mathrm{d}A$ is continuous in σ for $\sigma\in(0,D)$ and switches sign between the interval's endpoints, there must be at least three roots. From equation (13), the following must hold at a root:

$$\Pi_{R,Rev}\,\sigma-\Pi_R=\Pi_{R,Rev}\left(\frac{p_E}{p_E-t_E}-\sigma\right)\Theta^{-1}. \tag{15}$$

Equation (14) must be positive at the largest and smallest roots and negative at the second-smallest root. It is positive at a root if and only if:

$$\begin{aligned}
&[1+\Theta]\ \Pi_{R,Rev} > -\frac{\partial\Theta}{\partial\sigma}\left[\Pi_{R,Rev}\sigma - \Pi_R\right] = -\frac{\partial\Theta}{\partial\sigma}\Pi_{R,Rev}\left(\frac{p_E}{p_E - t_E} - \sigma\right)\Theta^{-1} \\
\Leftrightarrow &[1+\Theta]\ \Pi_{R,Rev} > -\Theta[1+\Theta]\ln\left(\frac{\kappa}{p_E}\right)\Pi_{R,Rev}\left(\frac{p_E}{p_E - t_E} - \sigma\right)\Theta^{-1} \\
\Leftrightarrow &1 > -\ln\left(\frac{\kappa}{p_E}\right)\left(\frac{p_E}{p_E - t_E} - \sigma\right), \qquad (16)
\end{aligned}$$

where the first line's equality substitutes the condition from equation (15) that must hold at a root and where the second line substitutes the explicit expression for $\partial\Theta/\partial\sigma$ from Lemma 5.

If $p_E < \kappa$, then the right-hand side of inequality (16) is monotonically increasing in σ. If inequality (16) does not hold at some root, then it cannot hold at any larger root. The inequality must not hold at the second-smallest root. Therefore equation (14) is negative at the largest root. But this contradicts the requirement that equation (14) be positive at the largest root (because $\mathrm{d}X_R^*/\mathrm{d}A$ is positive for σ sufficiently large). Therefore, the σ that is a root of equation (13) is unique when $p_E < \kappa$.

If $p_E > \kappa$, then the right-hand side of inequality (16) is monotonically decreasing in σ. If inequality (16) holds at some root, then it holds at all larger roots. The inequality must hold at the smallest root, because $\mathrm{d}X_R^*/\mathrm{d}A$ is negative for σ sufficiently small. Therefore equation (14) is positive at all roots. But this contradicts the requirement that equation (14) be negative at the second-smallest root. Therefore, the σ that is a root of equation (13) is unique when $p_E > \kappa$.

If $p_E = \kappa$, then the right-hand side of inequality (16) is zero for all σ. This contradicts the requirement that equation (14) be negative at the second-smallest root. Therefore, the σ that is a root of equation (13) is unique when $p_E = \kappa$.

Combining these results, we have that, for a given equilibrium z, there exists $\hat{\sigma}_z > 0$ such that $\mathrm{d}X_R^*/\mathrm{d}A > 0$ if and only if $\sigma > \hat{\sigma}_z$ and such that $\hat{\sigma}_z < 1$ if $t_E \leq 0$.

Proof of Proposition 3

Using the Implicit Function Theorem, we have:

$$\frac{\mathrm{d}p_E^*}{\mathrm{d}A} = -\frac{det\left(\begin{bmatrix} \frac{\partial G_1}{\partial A} & \frac{\partial G_1}{\partial X_R} \\ \frac{\partial G_2}{\partial A} & \frac{\partial G_2}{\partial X_R} \end{bmatrix}\right)}{det(G)} < 0.$$

The sign follows from previous results and from recognizing that each partial derivative with respect to A is positive. From equation (5) and Lemma 5, we know that r_Y^* must move opposite to p_E^*, which implies that $\mathrm{d}r_Y^*/\mathrm{d}A > 0$. And from household indifference, we know that r_R^* and r_Y^* move together, which implies that $\mathrm{d}r_R^*/\mathrm{d}A > 0$.

Now consider $\mathrm{d}p_R^*/\mathrm{d}A$. Differentiate equation (6), factor out common positive terms, and simplify (omitting asterisks for equilibrium outcomes):

$$\begin{aligned}\frac{\mathrm{d}p_R}{\mathrm{d}A} =& \alpha\,(1-\alpha)\,[p_E-\tau_E](\omega\,X_R)^{-\alpha}A^{\alpha-1}\\ &+(1-\alpha)\,(\omega\,X_R)^{-\alpha}A^{\alpha}\frac{\mathrm{d}p_E}{\mathrm{d}A}-\alpha\,(1-\alpha)\,[p_E-\tau_E]\omega^{-\alpha}A^{\alpha}\,X_R^{-\alpha-1}\frac{\mathrm{d}X_R}{\mathrm{d}A}\\ &\propto\left(1+\frac{X_R}{X_Y}\right)\frac{p_E-\tau_E}{p_E}\,\Theta\,\Pi_R+X_R\,\omega\,\frac{\partial\psi(X_R;Q)}{\partial X_R}\left(\sigma\,[1+\Theta]\,\frac{p_E-\tau_E}{p_E}-1\right)\triangleq F(\sigma).\end{aligned}\tag{17}$$

From equation (10), $\mathrm{d}X_R^*/\mathrm{d}A \geq 0$ if and only if

$$\sigma \geq \frac{\frac{p_E}{p_E-\tau_E}+\Theta\left(1-\frac{\Pi_{R,Cost}}{\Pi_{R,Rev}}\right)}{1+\Theta}.$$

Substitute into (17) to find that:

$$F(\sigma)|_{\sigma\geq\hat{\sigma}} \geq \frac{p_E-\tau_E}{p_E}\,\Theta\,\left[\left(1+\frac{X_R}{X_Y}\right)\Pi_R+X_R\,\omega\,\frac{\partial\psi(X_R;Q)}{\partial X_R}\left(1-\frac{\Pi_{R,Cost}}{\Pi_{R,Rev}}\right)\right].$$

This is positive, because $\Pi_{R,Rev} > \Pi_{R,Cost}$ at any equilibrium with $X_R > 0$. We therefore have that $\mathrm{d}X_R^*/\mathrm{d}A > 0$ implies $\mathrm{d}p_R^*/\mathrm{d}A > 0$.

Proof of Proposition 4

Rewrite B and C from equation (12) as follows:

$$\begin{aligned}B =& \underbrace{\sigma(1+\Theta)\,\Pi_{R,Rev}}_{B_1}-\underbrace{\left(\frac{p_E}{p_E-t_E}\Pi_{R,Rev}+\Theta\,\Pi_R\right)}_{B_2},\\ C =& \frac{1}{X_Y}B_2+\omega\frac{\partial\psi(X_R;Q)}{\partial X_R}\sigma\,[1+\Theta]\,X_R.\end{aligned}$$

Using these forms, differentiate equation (12) with respect to some parameter z:

$$\frac{\mathrm{d}rebound}{\mathrm{d}z}\propto\left(\frac{\alpha}{1-\alpha}B+\frac{1}{1-\alpha}C\right)\left(\frac{\mathrm{d}B_1}{\mathrm{d}z}-\frac{\mathrm{d}B_2}{\mathrm{d}z}\right)-(B_1-B_2)\left(\frac{\alpha}{1-\alpha}\frac{\mathrm{d}B}{\mathrm{d}z}+\frac{1}{1-\alpha}\frac{\mathrm{d}C}{\mathrm{d}z}\right).$$

At $\sigma = \hat{\sigma}$, we know that $B_1 = B_2$. We also know that $\frac{\alpha}{1-\alpha} B + \frac{1}{1-\alpha} C > 0$. Evaluated at $\sigma = \hat{\sigma}$ and factoring out the positive term, the derivative is proportional to

$$\begin{aligned}
\left.\frac{\mathrm{d}rebound}{\mathrm{d}z}\right|_{\sigma=\hat{\sigma}} \propto & \frac{\mathrm{d}B_1}{\mathrm{d}z} - \frac{\mathrm{d}B_2}{\mathrm{d}z} \\
&= \left[\frac{\partial B_1}{\partial z} + \frac{\partial B_1}{\partial p_E}\frac{\mathrm{d}p_E}{\mathrm{d}z} + \frac{\partial B_1}{\partial X_R}\frac{\mathrm{d}X_R}{\mathrm{d}z}\right] - \left[\frac{\partial B_2}{\partial z} + \frac{\partial B_2}{\partial p_E}\frac{\mathrm{d}p_E}{\mathrm{d}z} + \frac{\partial B_2}{\partial X_R}\frac{\mathrm{d}X_R}{\mathrm{d}z}\right].
\end{aligned}$$

Begin by considering the derivative with respect to Q (i.e., set $z = Q$). Substituting into the previous expression, we have:

$$\begin{aligned}
\left.\frac{\mathrm{d}rebound}{\mathrm{d}Q}\right|_{\sigma=\hat{\sigma}} \propto & \Theta\,\omega\,\frac{\partial\psi(X_R;Q)}{\partial Q} \\
&+ \sigma\,\Pi_{R,Rev}\left[\frac{\partial\Theta}{\partial p_E} + \frac{1+\Theta}{p_E - t_E}\right]\frac{\mathrm{d}p_E}{\mathrm{d}Q} - \left[\frac{1+\Theta}{p_E - t_E}\,\Pi_{R,Rev} + \frac{\partial\Theta}{\partial p_E}\,\Pi_R\right]\frac{\mathrm{d}p_E}{\mathrm{d}Q} \\
&- \frac{\alpha}{X_R}\,\sigma\,(1+\Theta)\,\Pi_{R,Rev}\,\frac{\mathrm{d}X_R}{\mathrm{d}Q} + \frac{\alpha}{X_R}\left(\frac{p_E}{p_E - t_E} + \Theta\right)\Pi_{R,Rev}\,\frac{\mathrm{d}X_R}{\mathrm{d}Q} \\
&+ \Theta\,\omega\,\frac{\partial\psi(X_R;Q)}{\partial X_R}\,\frac{\mathrm{d}X_R}{\mathrm{d}Q}.
\end{aligned}$$

Substitute for $\mathrm{d}p_E/\mathrm{d}Q$ and $\mathrm{d}X_R/\mathrm{d}Q$ using the Implicit Function Theorem and factor $-\frac{\partial G_2}{\partial Q}\frac{1}{-det(G)}$:

$$\begin{aligned}
\left.\frac{\mathrm{d}rebound}{\mathrm{d}Q}\right|_{\sigma=\hat{\sigma}} \propto & \frac{-det(G)}{-\partial G_2/\partial Q}\,\Theta\,\omega\,\frac{\partial\psi(X_R;Q)}{\partial Q} \\
&+ \left\{\sigma\,\Pi_{R,Rev}\left[\frac{\partial\Theta}{\partial p_E} + \frac{1+\Theta}{p_E - t_E}\right] - \left[\frac{1+\Theta}{p_E - t_E}\,\Pi_{R,Rev} + \frac{\partial\Theta}{\partial p_E}\,\Pi_R\right]\right\}\frac{\partial G_1}{\partial X_R} \\
&- \left\{-\frac{\alpha}{X_R}\,\sigma\,(1+\Theta)\,\Pi_{R,Rev} + \frac{\alpha}{X_R}\left(\frac{p_E}{p_E - t_E} + \Theta\right)\Pi_{R,Rev}\right\}\frac{\partial G_1}{\partial p_E} \\
&- \Theta\,\omega\,\frac{\partial\psi(X_R;Q)}{\partial X_R}\frac{\partial G_1}{\partial p_E}.
\end{aligned} \tag{18}$$

Cancel terms on the first line, substitute for $\partial\Theta/\partial p_E$ from Lemma 5, and factor $[1+\Theta]/p_E$

to obtain:

$$
\begin{aligned}
\left.\frac{\mathrm{d}rebound}{\mathrm{d}Q}\right|_{\sigma=\hat{\sigma}} \propto & - det(G)\,\frac{\Theta}{1+\Theta}\,p_E\,r(p_E) \\
& -(\sigma-1)\,\Pi_{R,Rev}\left\{\sigma\,\Theta-\left(\frac{p_E}{p_E-t_E}+\Theta\right)\right\}\left[1+(1-\alpha)X_R^{-1}X_Y\right] \\
& +\Theta\,\Pi_{R,Cost}\left\{1+(1-\alpha)X_R^{-1}X_Y-\sigma\left[1+X_R^{-1}X_Y\right]\right\} \\
& +\sigma\,\alpha\frac{X_Y}{X_R}\left[\Pi_{R,Rev}\left\{\sigma\,(1+\Theta)-\left(\frac{p_E}{p_E-t_E}+\Theta\right)\right\}+\Theta\,\Pi_{R,Cost}\right] \\
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (=0 \text{ at } \hat{\sigma}) \\
& -\Theta\,\omega\,\frac{\partial\psi(X_R;Q)}{\partial X_R}\,\sigma\,X_Y.
\end{aligned}
$$

The second-to-last line is zero at $\sigma=\hat{\sigma}$. Rearrange as

$$
\begin{aligned}
\left.\frac{\mathrm{d}rebound}{\mathrm{d}Q}\right|_{\sigma=\hat{\sigma}} \propto & - det(G)\,\frac{\Theta}{1+\Theta}\,p_E\,r(p_E) \\
& -(\sigma-1)\left[\Pi_{R,Rev}\left\{\sigma\,\Theta-\left(\frac{p_E}{p_E-t_E}+\Theta\right)\right\}+\Theta\,\Pi_{R,Cost}\right]\left[1+X_R^{-1}X_Y\right] \\
& -\alpha X_R^{-1}X_Y\left[\Pi_{R,Rev}\left\{\sigma\,\Theta-\left(\frac{p_E}{p_E-t_E}+\Theta\right)\right\}+\Theta\,\Pi_{R,Cost}\right] \\
& +\alpha X_R^{-1}X_Y\sigma\,\Pi_{R,Rev}\left\{\sigma\,\Theta-\left(\frac{p_E}{p_E-t_E}+\Theta\right)\right\} \\
& -\Theta\,\omega\,\frac{\partial\psi(X_R;Q)}{\partial X_R}\,\sigma\,X_Y.
\end{aligned}
$$

Note that, at $\sigma=\hat{\sigma}$,

$$
\Pi_{R,Rev}\left\{\sigma\,\Theta-\left(\frac{p_E}{p_E-t_E}+\Theta\right)\right\}+\Theta\,\Pi_{R,Cost}=-\sigma\Pi_{R,Rev}.
$$

Substitute in and use $X_Y+X_R=1$ to obtain

$$
\begin{aligned}
\left.\frac{\mathrm{d}rebound}{\mathrm{d}Q}\right|_{\sigma=\hat{\sigma}} \propto & - det(G)\,\frac{\Theta}{1+\Theta}\,p_E\,r(p_E)+(\sigma-1)\,\sigma\,\Pi_{R,Rev}X_R^{-1}\left[1-\alpha X_Y\right] \\
& -\alpha X_R^{-1}X_Y\sigma\,\Theta\,\Pi_{R,Cost}-\Theta\,\omega\,\frac{\partial\psi(X_R;Q)}{\partial X_R}\,\sigma\,X_Y.
\end{aligned}
$$

Now substitute in the expression for $-det(G)$ from equation (11), noting that the first line in equation (11) is zero when $\sigma = \hat{\sigma}$:

$$\frac{\mathrm{d}rebound}{\mathrm{d}Q}\bigg|_{\sigma=\hat{\sigma}} \propto \frac{\Theta}{1+\Theta}\Pi_{R,Rev}\left[\sigma(1+\Theta)\,\frac{X_Y}{X_R} + \left(\frac{p_E}{p_E - t_E} + \Theta\right)\right] - \frac{\Theta}{1+\Theta}\,\Theta\,\Pi_{R,Cost} + (\sigma-1)\,\sigma\,\Pi_{R,Rev}X_R^{-1}\left[1-\alpha X_Y\right] - \alpha X_R^{-1}X_Y\sigma\,\Theta\,\Pi_{R,Cost}.$$

Substitute into the first line for $p_E/[p_E - t_E] + \Theta$, using equation (10) and the condition that $\mathrm{d}X_R/\mathrm{d}A = 0$:

$$\frac{\mathrm{d}rebound}{\mathrm{d}Q}\bigg|_{\sigma=\hat{\sigma}} \propto [\sigma - 1 + \Theta] - \alpha X_Y\left[(\sigma-1) + \Theta\,\frac{\Pi_{R,Cost}}{\Pi_{R,Rev}}\right],$$

where we have used $X_Y + X_R = 1$ and factored $\Pi_{R,Rev}\,\sigma/X_R$. Note that $\alpha\, X_Y < 1$ and $\frac{\Pi_{R,Cost}}{\Pi_{R,Rev}} < 1$. Substitute in $\hat{\sigma}$ for σ:

$$\frac{\mathrm{d}rebound}{\mathrm{d}Q}\bigg|_{\sigma=\hat{\sigma}} \propto \left[\hat{\sigma} - 1 + \Theta\right] - \alpha X_Y\left[\hat{\sigma} - 1 + \Theta\,\frac{\Pi_{R,Cost}}{\Pi_{R,Rev}}\right].$$

This is strictly positive if and only if

$$\hat{\sigma} > \frac{1 - \Theta - \alpha X_Y\left(1 - \Theta\frac{\Pi_{R,Cost}}{\Pi_{R,Rev}}\right)}{1-\alpha X_Y} \triangleq \tilde{\sigma}_Q.$$

Because the cutoff $\hat{\sigma}$ is unique for any given equilibrium, we have that $\hat{\sigma}$ strictly decreases in Q if and only if $\hat{\sigma} > \tilde{\sigma}_Q$.

Note that

$$\tilde{\sigma}_Q < \frac{1 - \Theta - \alpha X_Y\,(1-\Theta)}{1 - \alpha X_Y} = 1 - \Theta.$$

From equation (10),

$$\hat{\sigma} \geq 1 - \frac{\Theta}{1+\Theta}\frac{\Pi_{R,Cost}}{\Pi_{R,Rev}}$$

if $t_E \geq 0$, and it is clear that

$$1 - \frac{\Theta}{1+\Theta}\frac{\Pi_{R,Cost}}{\Pi_{R,Rev}} > 1 - \Theta.$$

Therefore, $\hat{\sigma} > \tilde{\sigma}_Q$ if $t_E \geq 0$.

Now consider how $\hat{\sigma}$ changes in t_R. Adapting the previous derivation for Q, equation (18) becomes:

$$\begin{aligned}\left.\frac{\mathrm{d}rebound}{\mathrm{d}t_R}\right|_{\sigma=\hat{\sigma}} \propto & \frac{-det(G)}{-\partial G_2/\partial t_R}\,\Theta\,\omega \\ & + \left\{\sigma\,\Pi_{R,Rev}\left[\frac{\partial\Theta}{\partial p_E}+\frac{1+\Theta}{p_E-t_E}\right]-\left[\frac{1+\Theta}{p_E-t_E}\,\Pi_{R,Rev}+\frac{\partial\Theta}{\partial p_E}\,\Pi_R\right]\right\}\frac{\partial G_1}{\partial X_R} \\ & - \left\{-\frac{\alpha}{X_R}\,\sigma\,(1+\Theta)\,\Pi_{R,Rev}+\frac{\alpha}{X_R}\left(\frac{p_E}{p_E-t_E}+\Theta\right)\Pi_{R,Rev}\right\}\frac{\partial G_1}{\partial p_E} \\ & - \Theta\,\omega\,\frac{\partial\psi(X_R;Q)}{\partial X_R}\frac{\partial G_1}{\partial p_E}.\end{aligned}$$

After we cancel $\partial G_2/\partial t_R$ with the other terms on that line, we are left with the exact same expression as in the previous derivation. The results for $\mathrm{d}rebound/\mathrm{d}t_R$ are therefore the same as the results for $\mathrm{d}rebound/\mathrm{d}Q$, where both are evaluated at $\sigma=\hat{\sigma}$.

Now consider the derivative of rebound with respect to A (i.e., set $z=A$). At $\sigma=\hat{\sigma}$, $\mathrm{d}X_R/\mathrm{d}A=0$. The derivative is proportional to

$$\begin{aligned}\left.\frac{\mathrm{d}rebound}{\mathrm{d}A}\right|_{\sigma=\hat{\sigma}} \propto & \alpha A^{-1}\Pi_{R,Rev}\left[\sigma(1+\Theta)-\left(\frac{p_E}{p_E-t_E}+\Theta\right)\right] \\ & + \sigma\,\Pi_{R,Rev}\left[\frac{\partial\Theta}{\partial p_E}+\frac{1+\Theta}{p_E-t_E}\right]\frac{\mathrm{d}p_E}{\mathrm{d}A}-\left[\frac{1+\Theta}{p_E-t_E}\,\Pi_{R,Rev}+\frac{\partial\Theta}{\partial p_E}\,\Pi_R\right]\frac{\mathrm{d}p_E}{\mathrm{d}A}.\end{aligned}$$

Using the Implicit Function Theorem and factoring $-\alpha/\left[A\,p_E\,r(p_E)\,det(G)\right]$ yields:

$$\begin{aligned}\left.\frac{\mathrm{d}rebound}{\mathrm{d}A}\right|_{\sigma=\hat{\sigma}} \propto & -det(G)\,p_E\,r(p_E)\,\Pi_{R,Rev}\left[\sigma(1+\Theta)-\left(\frac{p_E}{p_E-t_E}+\Theta\right)\right] \\ & + [1+\Theta]\left\{-(\sigma-1)\left[\Pi_{R,Rev}\left[\sigma\Theta-\left(\frac{p_E}{p_E-t_E}-\Theta\right)\right]+\Theta\,\Pi_{R,Cost}\right]\right\} \\ & \left\{X_Y\left[-\alpha X_R^{-1}\Pi_{R,Rev}-\omega\frac{\partial\psi(X_R;Q)}{\partial X_R}\right]-\Pi_{R,Rev}\left[1+(1-\alpha)X_R^{-1}X_Y\right]\right\}.\end{aligned}$$

At $\sigma=\hat{\sigma}$, we know that $-\left(\frac{p_E}{p_E-t_E}+\Theta\right)=-\sigma(1+\Theta)\Pi_{R,Rev}-\Theta\Pi_{R,Cost}$. Substitute into the second line to obtain:

$$\begin{aligned}\left.\frac{\mathrm{d}rebound}{\mathrm{d}A}\right|_{\sigma=\hat{\sigma}} \propto & -det(G)\,p_E\,r(p_E)\left[\sigma(1+\Theta)-\left(\frac{p_E}{p_E-t_E}+\Theta\right)\right]\frac{X_R}{X_Y} \\ & + [1+\Theta]\,(\sigma-1)\,\sigma \\ & \quad \left\{-\alpha\Pi_{R,Rev}-\omega\frac{\partial\psi(X_R;Q)}{\partial X_R}\,X_R-\Pi_{R,Rev}\left[\frac{X_R}{X_Y}+(1-\alpha)\right]\right\},\end{aligned}$$

where I factor $\Pi_{R,Rev}\, X_Y / X_R$. Now substitute the expression for $-det(G)$ from equation (11), noting that the first line in equation (11) is zero when $\sigma = \hat{\sigma}$ and using $X_Y + X_R = 1$:

$$\begin{aligned}\left.\frac{\mathrm{d}rebound}{\mathrm{d}A}\right|_{\sigma=\hat{\sigma}} \propto & \left[\sigma(1+\Theta) - \left(\frac{p_E}{p_E - t_E} + \Theta\right)\right] \\ & \qquad \left\{\Pi_{R,Rev}\left[\sigma(1+\Theta) + \frac{X_R}{X_Y}\left(\frac{p_E}{p_E - t_E} + \Theta\right)\right] - \frac{X_R}{X_Y}\Theta\,\Pi_{R,Cost}\right\} \\ & - [1+\Theta]\,(\sigma - 1)\,\sigma \Pi_{R,Rev}\frac{1}{X_Y} \\ & + \omega\frac{\partial\psi(X_R;Q)}{\partial X_R}\,X_R\,[1+\Theta]\,\sigma\left\{\left[\sigma(1+\Theta) - \left(\frac{p_E}{p_E - t_E} + \Theta\right)\right] - (\sigma - 1)\right\}.\end{aligned}$$

Substitute from equation (10) (with $\sigma = \hat{\sigma}$) into the last line and into the top line to obtain:

$$\begin{aligned}\left.\frac{\mathrm{d}rebound}{\mathrm{d}A}\right|_{\sigma=\hat{\sigma}} \propto & -\Theta\Pi_{R,Cost}\left\{\sigma(1+\Theta) + \frac{X_R}{X_Y}\left(\frac{p_E}{p_E - t_E} + \Theta\right) - \frac{X_R}{X_Y}\Theta\,\frac{\Pi_{R,Cost}}{\Pi_{R,Rev}}\right\} \\ & - [1+\Theta]\,(\sigma - 1)\,\sigma\Pi_{R,Rev}\frac{1}{X_Y} \\ & + \omega\frac{\partial\psi(X_R;Q)}{\partial X_R}\,X_R\,[1+\Theta]\,\sigma\left\{-\Theta\frac{\Pi_{R,Cost}}{\Pi_{R,Rev}} - (\sigma - 1)\right\}.\end{aligned}$$

Substitute for $\sigma = \hat{\sigma}$ in the top line by replacing the terms on X_R/X_Y with $\sigma(1+\Theta)$:

$$\begin{aligned}\left.\frac{\mathrm{d}rebound}{\mathrm{d}A}\right|_{\sigma=\hat{\sigma}} \propto & \left\{(\sigma - 1)\,\Pi_{R,Rev} + \Theta\Pi_{R,Cost}\right\}\left\{-\frac{1}{X_Y} - \omega\frac{\partial\psi(X_R;Q)}{\partial X_R}\,\frac{X_R}{\Pi_{R,Rev}}\right\} \\ & < 0 \text{ iff } \sigma > 1 - \Theta\frac{\Pi_{R,Cost}}{\Pi_{R,Rev}} \triangleq \tilde{\sigma}_A,\end{aligned}$$

where I use $X_Y + X_R = 1$ and factor $\sigma\,(1+\Theta)$. Because the cutoff $\hat{\sigma}$ is unique for any given equilibrium, we have that $\hat{\sigma}$ strictly increases in A if and only if $\hat{\sigma} > \tilde{\sigma}_A$.

From equation (10),

$$\hat{\sigma} \geq 1 - \frac{\Theta}{1+\Theta}\frac{\Pi_{R,Cost}}{\Pi_{R,Rev}}$$

if $t_E \geq 0$, and it is clear that

$$1 - \frac{\Theta}{1+\Theta}\frac{\Pi_{R,Cost}}{\Pi_{R,Rev}} > 1 - \Theta\,\frac{\Pi_{R,Cost}}{\Pi_{R,Rev}}.$$

Therefore, $\hat{\sigma} > \tilde{\sigma}_A$ if $t_E \geq 0$.