Precautionary Energy Storage

Tunç Durmaz

August 10, 2015

Abstract

Energy storage technologies were primarily used to take advantage of dispatchable sources of energy and demand variability, and the underlying economic analysis focused mainly on pumped hydro storage. Today, increasing shares of renewable energy (RE) have drawn attention to the role of storage technologies in dealing with the vulnerabilities caused by renewables. With the presence of intermittent RE, however, not much consideration has been given to energy storage due to precautionary motives. In our study, we look at to what extent a convex marginal utility (prudence) and a convex marginal cost (frugality) can spur precautionary energy storage. We use a simple theoretical model of energy consumption and production with intermittent renewable sources, dispatchable thermal systems, and energy storage. First, we characterize the optimal solution and demonstrate how prudence and frugality lead to higher levels of energy saving. Furthermore, we demonstrate how the optimal allocation can be decentralized through competitive markets. Our analysis indicates that prudence and frugality exert an upward pressure on spot market energy prices through higher demand for energy storage, higher thermal energy generation, and lower consumption.
Keywords: Precautionary energy storage; Renewable energy; Prudence; Frugality; Rational Expectations Equilibrium

1 Introduction

Renewable energy (RE) is one of the key solutions in combating the climate change problem and is high on the policy agenda, especially in the developed countries. Since the launch of the European Climate Change Policy in 2001, the European Union (EU) has been a leader in the global action to mitigate climate change through its energy policies. Following the proposal for an energy and climate change package in January 2007, the member states agreed on a binding target to raise the share of RE in the EU's total primary energy consumption to 20% by 2020 (IEA, 2008). This agreement was then followed by a directive on the promotion of the use of RE, which endorsed mandatory national targets consistent with the 20% share of the RE target.[1] In 2014, the RE generation from wind, solar, geothermal and other renewable sources accounted for 12% of the electricity generation in Europe (IEA, 2014), which was higher by more than 100% compared to the generation level in 2010.[2] The corresponding figures for the OECD and the world were 6% and 4%, respectively, in 2010. The U.S. Energy Information Administration projects that by 2020 these shares will reach 11% and 8%, respectively.[3]

Nevertheless, RE is inherently variable and uncertain, and can cause vulnerabilities in meeting energy demand. A number of strategies exist to address the challenges created by intermittent RE generation.[4] The use of thermal dispatchable generation is a common way to smooth the operation of existing power grids and enable a balance between demand and supply. However, this comes at a cost because these units are expensive to operate. A demand response is another way to enhance the grid's resilience and enable a greater use of RE. Although the idea of getting consumers to become active in the markets can be seen as a novel solution by many, the culture is changing with smart meters. These devices allow consumers to access real-time knowledge about prices, be more responsive and control their power usage and consumption, which is similar to consumers responding to changing gasoline prices. When active engagement is not practical, consumers can also have access to smart appliances that can react to prices based on criteria set by the consumer (Hamilton et al., 2012). With sustained investments, it is projected that the smart grid will provide a communications network for the energy industry by 2020; that is, a system of interconnected energy networks similar to the Internet in terms of its provisions for business and personal communications (RMI, 2014).

Another way to enhance the reliability of the grid is energy storage, especially in periods of peak demand. Energy storage technologies absorb and store energy for a period of time before releasing it to supply energy or power services. In simple terms, these technologies take excess generation produced on a windy or sunny day, store the power in multiple places (from large hydro stations to home batteries, and everything in between), and supply power when RE is inadequate and thermal energy is expensive to produce. Key benefits include providing balancing services, such as load following, supplying power during brief disturbances, and serving as substitutes for network transmission and distribution upgrades (Wang et al., 2012). Currently, the costs of electricity storage technologies are rather high. However, with the development of better economic storage technologies with larger storage capacities, they will potentially become game changing technologies.[5]

When consumers are responsive, and energy generators –in particular, dispatchable

[1]Directive 2009/28/EC of the European Parliament and Council of April, 23 2009 on the promotion of the use of energy from renewable sources, which amended and subsequently repealed Directives 2001/77/EC and 2003/30/EC, available at http://eur-lex.europa.eu/legal-content/EN/ALL/?uri=CELEX:32009L0028.

[2]In 2010, the share of RE in the total electricity production was 5.5%. In calculating the shares we restricted our attention to the EU member countries except Bulgaria, Croatia, Latvia, Malta and Romania.

[3]The ratios for the OECD and the world were calculated using Tables 13 and 14 in EIA (2013).

[4]Intermittency means that RE generation depends on weather conditions and is non-controllable (Steffen and Weber, 2013).

[5]The energy storage industry is experiencing strong growth and it is expected that the industry will have a global net worth of $10.8 billion in 2018 (RMI, 2014).

thermal energy generators– are responsible to match electricity supply with demand, two precautionary motives can lead to a higher demand for energy storage. One is prudence with respect to electricity consumption, which is formally equivalent to a positive third derivative of the utility function. The other is frugality, which is formally equivalent to a convex marginal cost of thermal energy production function. We refer to the property of a convex marginal cost function as frugality, since, in the presence of uncertainty, it endows a cost minimizing producer with the same motivations as that of a prudent consumer. In Section 1.1, we will motivate the properties of prudence and frugality and give a first intuition as to why they encourage energy storage.

In this study, our aim is to show how prudence and frugality drive the need for precautionary energy storage. We first look at a benevolent planner problem and examine how storage decisions are influenced in the presence of a convex marginal utility (prudence) and a convex marginal cost (frugality). We then demonstrate how the optimal allocation can be decentralized through competitive markets and discuss how current energy prices and the use of energy systems are influenced by prudence, frugality, the degree of intermittency, price elasticities, and RE capacity. Our results indicate that prudence and frugality can cause precautionary energy storage, but to varying degrees. Even in the absence of prudence, we demonstrate that frugality can still allow for precautionary storage and vice versa. Furthermore, a higher degree of intermittency can boost energy storage when prudence, frugality, or both, is present. Higher demand and supply elasticities diminish the effect of prudence and frugality, respectively, on precautionary energy storage. For a highly elastic demand, demand response becomes a good substitute for energy storage and in turn lower the need for precautionary energy storage. When energy supply is more price elastic, dispatchable thermal generation becomes a better substitute for storing energy.

1.1 Motivations for prudence and frugality

Prudence

Let us explain what it means to be prudent in our framework. Consider a consumer with a utility function, $U(q)$, which is increasing, $U' > 0$, and concave, $U'' < 0$, in electricity consumption, q.[6] Suppose that the consumer is exposed to a zero-mean consumption risk, $\tilde{x}$. The difference between certain and expected utility is given by

$$k(q) \equiv U(q) - \mathbb{E}[U(q + \tilde{x})].$$

Due to the Jensen's inequality, $k(q)$ is positive if $U(q)$ is concave. In other words, uncertainty is costly for the consumer when the he/she is risk averse.

A consumer is said to be prudent with respect to electricity consumption if the cost of uncertainty, $k(q)$, decreases as consumption, q, increases. In differential terms, this is

[6] We assume a quasi-linear utility function. Thus, $U(q)$ is the monetary value of utility (or surplus) that is derived from consuming q kilowatt-hour (kWh) of electricity. In economic theory, using such preferences is a standard assumption when discussing issues related to a single market in a general equilibrium framework.

equivalent to $k'(q)$, given by

$$k'(q) = U'(q) - \mathbb{E}[U'(q + \tilde{x})],$$

being negative, which is ensured by the convexity of the marginal utility; that is, $U''' > 0$. Again, this results from the Jensen's inequality. As consuming stored energy is one way to increase q, and thus, to decrease the cost of uncertainty, $U''' > 0$ –that is, prudence– gives a *prima facie* argument for energy storage.

Focusing on income lotteries, the evidence for prudence can be found in the experimental research literature. In line with the prediction of precautionary saving theory, Noussair et al. (2014) indicate that the majority of individual decisions is consistent with prudence.[7] Crainich et al. (2013) provide theoretical arguments to show that prudence is more prevalent than risk aversion, as risk lovers can also demonstrate it. This prediction is shown to hold in Ebert and Wiesen (2014) and Deck and Schlesinger (2014). Accordingly, prudence may be a more universal trait, which suggests that narrowing down risk preferences to the second-order may obscure valuable information. There are also empirical studies such as Chavas and Holt (1996) and Guiso et al. (1996) that support prudence. Carroll and Samwick (1998) indicate that wealth holdings are positively and significantly related to income uncertainty.[8]

Frugality

In this subsection, we shall expound frugality. Consider a producer with a smooth increasing cost function $C(q)$, where q is the level of production. Suppose that the firm faces a zero-mean production risk, $\tilde{x}$. Here, $\tilde{x}$ represents the variation in the residual demand that the firm has to match with its supply. The difference between the expected and the certain cost of production is as follows:

$$\rho(q) \equiv \mathbb{E}[C(q + \tilde{x})] - C(q).$$

Due to the Jensen's inequality, the firm is exposed to a penalty of uncertainty when $C'' > 0$ (i.e., the cost function is convex). In other words, increasing marginal cost implies that uncertainty is costly for the firm: $\rho(q) > 0$.

A producer is said to be frugal with respect to energy generation if the cost of uncertainty, $\rho(q)$, increases as production, q, increases. This is equivalent to $\rho'(q)$, given by

$$\rho'(q) \equiv \mathbb{E}[C'(q + \tilde{x})] - C'(q),$$

being positive, for which the convexity of the marginal cost (i.e., $C''' > 0$) is sufficient. Once

[7]Noussair et al. (2014) also argue that the degree of prudence has implications in a wide range of economic applications such as bargaining, bidding in auctions, rent seeking, discounting, sustainable development and climate change, and tax compliance.

[8]Carroll and Kimball (2008) argue that, although there is evidence for prudence, it is measured differently with different data; that is, the degree of the same motive changes among different data sets.

again, this results from the Jensen's inequality. As using stored energy is one way to decrease q, and thus, to decrease the cost of uncertainty, $C''' > 0$ –i.e., frugality– provides a second *prima facie* reason for energy storage.

By analyzing production and inventory data, Cecchetti et al. (1997) find evidence supporting a positive third derivative of the cost function, and note that, from an operational perspective, a firm is capacity constrained when faced with a convex marginal cost curve. Indeed, a convex marginal (production) cost curve has a transparent economic interpretation, which indicates that it becomes increasingly expensive to make large and positive changes to meet the residual demand.

Now let us explain how the production risk emerges for a fossil fuel power generator. Variations in energy demand are typically limited and more predictable compared with the variations in supply (Nyamdash et al., 2010; Hansen, 2009; Hart et al., 2012; Ummels et al., 2007). However, due to the low operating cost of intermittent RE that leads to its earlier dispatch (Denholm et al., 2010), the residual load is intermittent. Therefore, after accounting for RE, a capacity constrained thermal dispatchable generator that has to supply the residual load can incur high operating costs especially during periods of peak demand and low renewable energy generation. As a result, a frugal firm will intend to balance its limited supply and the residual demand in such a way that it minimizes its expected cost.

As a real world example, consider Figure 1, which illustrates the electricity supply and demand curves for the NordPool Power Exchange.[9] Due to their low marginal costs (zero fuel costs) renewables appear near the bottom of the curve. They are followed by nuclear power, combined heat and power (CHP) plants. Condensing plants and gas turbines have the highest marginal costs for generating energy.[10] In the presence of intermittent RE generation, a low wind power will cause a limited change in the energy price at night due to low generation costs. During peak times, however, the increase in the electricity price will become much more significant, as each additional generating capacity is brought online and the cost of this capacity dramatically increases (Gravelle, 1976).

The sequence of linear cost functions that appears in Figure 1 can be collectively approximated by a smooth curve, which would then represent a convex industry supply schedule. In this case, frugality becomes an industry trait, and in turn, can have economically significant impact on production, consumption and storage decisions.

The remainder of the paper is organized as follows. Section 2 reviews the related literature. Section 3 presents the model and describes the benevolent planner solution given a certain and cheap supply of baseload power, and then given an intermittent RE supply in the energy system. Section 4 demonstrates how the optimal allocation can be decentralized through competitive markets, and discusses the implications of the precautionary motives. Section 5 concludes.

[9]Source: EWEA (2009, p. 18). The Nordic electricity exchange, Nord Pool Spot, is a power market that primarily serves the players in the wholesale market for electricity. It covers Denmark, Finland, Sweden, Norway, Estonia and Lithuania (Nord Pool Spot, 2011).

[10]Hydropower is not identified in the figure (EWEA, 2009).

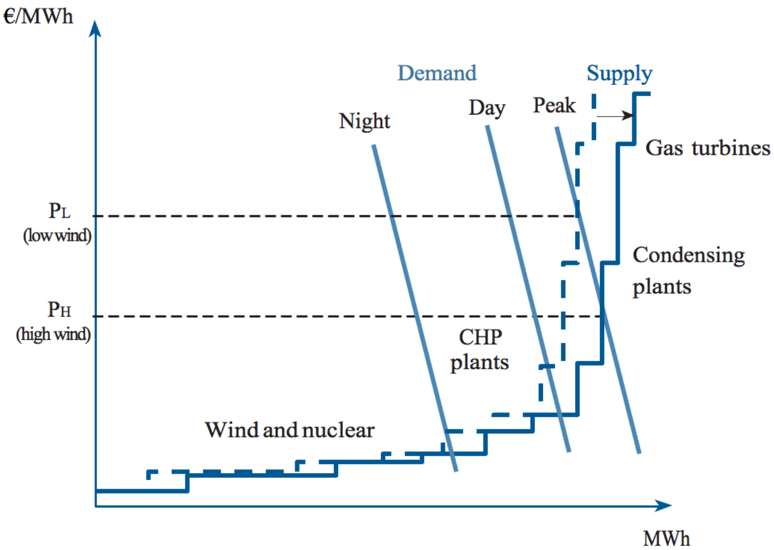

Figure 1: *Supply and Demand Curves for the NordPool Power Exchange*

2 Related literature

The optimal dispatch of energy and energy storage in renewable and thermal energy systems was addressed earlier in the operations research literature. In a model of hydroelectric and thermal systems, Little (1955) studies hydroelectric generation under uncertainty. Disregarding fluctuations in energy demand, the study determines optimal energy dispatch and water storage policies. Borrowing most of his assumptions from Little (1955), Koopmans (1957) calculates the optimal energy generation and storage policies in the presence of complete certainty.[11] He shows how thermal energy generation and storage decisions are related to the energy prices and storage rents.

With a few exceptions, however, the economics of pumped-hydropower (PSH), has not attracted many researchers so far. An early work on the economics of PSH is Jackson (1973), where the motivation to use PSH is due to the flexibility the technology offers as nuclear power plants cannot be efficiently turned on and off. In his analysis, Jackson assumes that storage is always optimal, and hence, the technology always pumps water to an upper reservoir. In contrast, Gravelle (1976) shows the conditions under which storage is efficient. Assuming that demand deterministically varies between off-peak and peak periods, he shows that storage allows the substitution of less costly off-peak production for highly valued peak production. In return, more highly valued peak consumption is substituted for off-peak consumption. Horsley and Wrobel (2002) build on the framework given by Koopmans (1957), and study the optimal operation of existing PSHs and the valuation of energy and storage rents in the presence of uncertain inflows.

Crampes and Moreaux (2010) build their work on Jackson (1973) and Gravelle (1976). Unlike Horsley and Wrobel (2002), who assume an exogenously given demand and

[11]Koopmans (1957) argues that the purpose is to develop concepts and tools that will be useful in a systematic analysis of cases involving uncertainty.

perfectly efficient conversion, they investigate the optimal thermal energy dispatch and PSH when energy demand varies deterministically between peak and off-peak periods and there are losses in converting energy. Assuming a merit order in using thermal generators, the study first calculates a frontier that separates storage and no-storage solutions given technical conditions such as operation cost characteristics and energy losses. The authors then calculate the socially optimal allocation given consumer preferences. When thermal generation is used to pump water to an upper level reservoir, the welfare losses corresponding to this off-peak period is compensated by welfare gains in the peak period when stored water is used. In line with Jackson (1973) and Gravelle (1976), the study then discusses the implementation of an optimal energy dispatch in competitive markets where agents are price takers. The calculations show that the peak and off-peak price differential is reduced when storage is feasible.

The literature on commodity storage has relevant implications for the economics of energy storage. Wright and Williams (1982, 1984) examine the welfare effects of storage in a market with stochastic supply and indicate that the welfare effects of storage depend on the specification of the inverse demand function (i.e., the curvature of the demand curve). Elasticity of supply is another factor that influences welfare. The authors introduce a parameter that is analogous to the coefficient of prudence (cf. Kimball, 1990) and argue that agents will pay for a mean-preserving decrease in the variability of the commodity when relative prudence is bigger than one (Wright and Williams, 1984; Williams and Wright, 1991). Given the storage and current production (i.e., the amount on hand), the authors derive a storage rule numerically. Accordingly, when the stored amount is less than a particular threshold, all of the stored commodity will be consumed, and vice versa. Numerical simulations indicate that storage is more likely and the marginal propensity to store at the threshold increases when there is a higher degree of variability in supply (Wright and Williams, 1982).

Regarding the relationship between the degrees of variability in RE and energy storage, one finds similar results in the operations research and economics literature. Tuohy and O'Malley (2011) argue that intermittency increases the benefit driven from the flexibility offered by PSH and makes energy storage more attractive. Evans et al. (2013) demonstrate that water storage becomes more welfare-enhancing with higher uncertainty. Surely, it is possible to find more discussions in the literature that connect the higher levels of storage to the higher degrees of variation in the RE supply. However, the role that precautionary motives play is not elaborated upon adequately. Evans et al. (2013) assume a linear demand schedule (i.e., $U'' > 0$ and $U''' = 0$) and a convex supply schedule (i.e., $C'' > 0$ and $C''' > 0$) for thermal energy generation. As we will show later in our study frugality will lead to precautionary energy storage, unless capacity constraints are explicitly considered for each thermal unit. Evans et al. (2013) do not address such a relationship. In Bobtcheff (2011), the cost of thermal energy is constant and not subject to any capacity constraints; that is, her model disregards frugality. She numerically shows that a benevolent planner keeps more water in a reservoir when faced with higher uncertainty and explains that this action is due to prudence. However, she does not present a formal analysis. A first step in this regard comes from Hansen (2009). He analyzes the effects of uncertainty on market performance in hydropower systems, and shows that competitive firms decrease hydroelectricity generation and save more water for the future when consumers are prudent (that is, the inverse demand for electricity is convex).

Our work is an extension of Crampes and Moreaux (2010) with intermittent RE. Instead of assuming only a PSH system, we consider a generic storage technology that has the ability to shift energy for long durations when necessary.[12] Thus, we are interested in storage technologies that are more suitable for energy management applications.[13] As these applications include generating power over longer time periods, which require longer discharge times (i.e., continuous discharge rates), we consider storage technologies that have the ability to shift the bulk of energy for a duration of several hours or more (Denholm et al., 2010).[14] This way energy storage technologies can insulate the rest of the power grid from substantial changes in net supply.[15] High energy batteries, pumped hydro (the most widely used form of electrical energy storage), and compressed air are the technologies for this type of applications (Denholm et al., 2010).

Although we focus on uncertainty in RE only, we do not neglect variations in demand and employ a deterministic demand that varies between off-peak and peak periods.[16] Even though we work with a deterministically varying demand function, it can be noted that the residual load is intermittent. This is due to the low operating cost of intermittent RE that leads to its earlier dispatch. After accounting for RE, the net but intermittent load is met by the peaking power plants or "peakers".

Our work discusses storage and no-storage solutions and indicates that intermittency can lead to a higher level of storage when agents are prudent and frugal. With a 100% share of RE sources in the energy system, where thermal systems are obsolete, our results are in line with Hansen (2009). Thus, a higher level of energy that is generated from renewable sources of energy is transfered to the next period when agents are prudent.[17] However, for systems with dispatchable thermal generation, frugality can have a significant impact on precautionary energy storage. We further indicate that higher degree of intermittency leads to higher energy storage, and the effects of prudence and frugality can differ in magnitude due to endogenously

[12]It is possible to assess different types of storage technologies by using different round-trip efficiencies.

[13]Nonetheless, we can always make some inferences for applications that require short discharge times and hydropower systems.

[14]When broadly categorized, one can classify energy storage applications as energy management and power applications (Kim et al., 2012). Power applications correspond to a range of ramping and ancillary services that do not typically require a steady discharge for several hours. Applications such as frequency regulations, voltage stability and contingency spinning reserves enter this category (Kim et al., 2012; Lichtner et al., 2010).

[15]One example of energy management applications is the electric energy time shift, which means charging a storage device when electricity prices are low (e.g., storing excess wind power during periods of low energy demand) and then discharging the device when electricity prices are high (Lichtner et al., 2010; Kim et al., 2012).

[16]Compared with the variations in supply, the variations in demand tend to be limited and more predictable (Nyamdash et al., 2010; Hansen, 2009; Hart et al., 2012; Ummels et al., 2007). In Norway, the annual demand of electricity consumers has not varied by more than 6 terawatt-hours (TWh) between 1999 and 2009. The corresponding number for water inflows is between 85–140 TWh. This is the main reason that numerical models of the Nordic power market, such as the Balmorel model and the EMPS (EFI's Multi-area Power-market Simulator or "Samkjøringsmodellen" in Norwegian), consider inflow variations more than demand variations (Hansen, 2009).

[17]In Hansen (2009), hydroelectric power generation is reduced to transfer more water into the next period.

assigned weights. By applying our findings to perfectly competitive markets, we show that precautionary motives can lead to a higher willingness to pay for electricity today, and in turn, a higher spot market electricity price. While a higher price elasticity of demand decreases the effect of prudence (that is, consumption adjustment becomes a stronger substitute for stored energy), a higher supply elasticity decreases the precautionary storage motive from frugality. This is because the residual demand can be more easily met by dispatchable thermal systems and in turn make the intermittency problem less of an issue.

3 The model

Consider a closed economy in which there is a single-commodity (i.e., energy) that can be supplied using systems of thermal energy, RE and energy storage:

$$q_t = y_t + z_t + s_t - \alpha s_{t+1}. \tag{1}$$

q_t, the energy supply at time t, is composed of dispatchable thermal energy, y_t, RE, z_t, available level of stored energy, s_t, minus the amount of energy transferred to the next period, αs_{t+1}. For $\alpha > 1$, $1/\alpha$ is the round-trip efficiency parameter, which is the ratio of energy recovered to the initially stored energy. Hence, a certain percentage of stored energy is lost with time.

We assume that the power grids are smart in a way that the transmission and distribution systems of electricity are added with digital sensors and remote controls (Ambec and Crampes, 2012; van de Ven et al., 2013; Evans et al., 2013). This assumption instantaneously lets the prices adjust, such that the energy supply meets the demand at all times. Thus, there is no overloading of the power grids.

Once there is an installed capacity, the unit cost of generating RE becomes so low that we consider it as zero (Ambec and Crampes, 2012; Evans et al., 2013; Førsund and Hjalmarsson, 2011). Thus, the renewable system operates at its capacity at all times.[18] Nevertheless, as the weather conditions such as wind, sun or rain are uncertain, so is the RE generation. Accordingly, let z be independently and identically distributed (i.i.d.) with a commonly known cumulative distribution function, $F(z)$, and a compact support $[0, \bar{z}]$, where $\bar{z}$ is the capacity of the RE system. We denote the mean and variance of z by μ and σ^2, respectively. For example, when there is no wind, there is no RE generation; that is, $z = 0$. In contrast, when the wind is sufficiently strong, the level of RE generation amounts to its capacity; that is, $z = \bar{z}$.[19] While z_t is observed prior to making decisions in period t, z_{t+1} is uncertain, and

[18] The only cost of RE generation is the opportunity cost of not generating more than the capacity of the system.

[19] To represent weather conditions such as wind, sun or rain, we could alternatively introduce a random variable with a cumulative distribution function, $F(\nu)$, a compact support, $[0, 1]$, and a mean and variance denoted by μ_ν and σ_ν^2, respectively. Accordingly, $\mu = \mu_\nu \bar{z}$ and $\sigma^2 = \sigma_\nu^2 \bar{z}^2$. When there would be no wind, $\nu_t = 0$, there would be no RE generation; that is, $\nu_t \bar{z} = z_t = 0$ and vice versa. Nonetheless, for simplicity and convenience, we suppress this notation.

thus, is denoted by $\tilde{z}_{t+1}$. In the rest of the analysis, we indicate that a variable is random by placing a tilde over it.

The cost function for thermal energy is assumed to be continuous; that is, there is a continuum of thermal generators. The marginal cost (unit cost) of fossil fuel energy generation is increasing, $C'(y) > 0, C''(y) > 0$ with $C'''(y) \geq 0$, where C', C'' and C''' are the first-, second- and third-order derivatives of the cost function, respectively. When the marginal cost is increasing, where $C''(y) > 0$ with $C'''(y) \geq 0$, one can think of a unique merit order of using individual generators: initially the power plants with the lower marginal costs of energy generation will be brought online (such as a coal-fired power plant), followed by more costlier ones (such as a natural gas power plant with carbon capture and storage). We assume that given the market price for energy, there is no constraint on the availability of y, that is, there is a large existing generating capacity portfolio that can meet the demand when RE is not adequate to supply the total load (Joskow, 2011; Bobtcheff, 2011). However, when $C'''(y) > 0$ (that is, the marginal cost is convex), one can think of an implicitly assigned capacity constraint –an upper bound– on the fossil fuel energy generation that allows for the effect from convexity to dominate when high levels of thermal energy generation are required.

The unit cost of storing energy, $1/\alpha$, which is a constant, is due to energy losses. Given various storage technologies with differing round-trip efficiencies, we could also consider a unique merit order of using storage facilities. Although, such an assumption would diminish the level of energy storage and take our model one step closer to reality, it would not affect our key results.

$U(q)$ is the utility function over the kWh consumption of energy. It is assumed that $U' > 0$, $U'' < 0$ and $U''' \geq 0$, where U', U'' and U''' are the first-, second- and third- order derivatives of the utility function, respectively. Thus, under perfect competition, the inverse demand schedule is downward sloping and convex.

In the following subsection, we solve the model from a benevolent planner perspective. We then turn to a decentralized setting and ask whether competition leads to an optimal solution.

Two-period model

We consider a two-period model. In the first period, $t = 0$, the demand for energy is low. Let's call this the off-peak period. In the final period, $t = 1$, we call it the peak period, the demand is high. For h representing the peak period and l representing the off-peak period, this can be algebraically shown as $U_h'(q) > U_l'(q)$. Hence, for the same level of consumption, q, the benefit from consuming an additional amount of energy is strictly higher in the peak period. We characterize the peak period with $\epsilon > 0$, which is subtracted from q in the utility function. Such a formulation shows a resemblance to the subsistence level of consumption: in the day time, the use of energy becomes so much more valuable that the agents cannot afford to consume less than ϵ. Similar to $U_h'(q) > U_l'(q)$, $U'(q-\epsilon) > U'(q)$.[20]

The planner's problem for the two-period model is as follows:

$$\max_{\{q_0,q_1,y_0,y_1,s_1\}} \quad U(q_0) - C(y_0) + \mathbb{E}\left[U(q_1-\epsilon) - C(y_1)\right] \tag{2a}$$

$$\text{subject to} \quad q_0 = y_0 + z_0 + s_0 - \alpha s_1, \tag{2b}$$

$$q_1 = y_1 + \tilde{z}_1 + s_1, \tag{2c}$$

$$q_0 \geq 0, q_1 - \epsilon \geq 0, \ y_j \geq 0, \ j = 0,1 \tag{2d}$$

$$\bar{s} \geq s_1, \ s_1 \geq 0 \ \text{ and } \ s_0 \geq 0 \ \text{ given.} \tag{2e}$$

As the weather for next period is uncertain, we use $\mathbb{E}[\cdot]$ to denote the expected welfare in period 1. Expressions given by Eqs. (2b) and (2c) are due to the fact that energy supply meets the demand instantaneously. Energy consumption (net of ϵ) is positive and thermal energy can equal zero (that is, become idle) when the RE generation is sufficiently high (c.f., Eq.(2d)). $\bar{s}$ is the capacity constraint for energy storage. When the storage capacity is reached, the remaining energy will be consumed. Stored energy cannot be negative; that is, we cannot borrow energy from the future to consume today. Throughout the study, we assume $s_0 = 0$. This assumption does not change the main results of the study, which identify prudence and frugality as the main drivers of precautionary storage. However, we shall comment on the possible effects of $s_0 > 0$ later in the study. For simplicity, we neglect discounting between the first and final periods. Lastly, we assume that there is no correlation between weather conditions and energy demand.

We solve the problem recursively. Given RE generation in the last period, z_1, and the available amount of stored energy, s_1, the problem in period 1 is the following:

$$\max_{\{q_1,y_1\}} U(q_1-\epsilon) - C(y_1)$$
$$\text{subject to} \quad q_1 = y_1 + z_1 + s_1,$$
$$q_1 \geq 0, y_1 \geq 0.$$

The first-order necessary condition for a maximum yields:[21]

$$U'(y_1 + z_1 + s_1 - \epsilon) \leq C'(y_1), \ \text{with equality if } y_1 > 0. \tag{3}$$

If the level of energy supplied by the renewable systems and energy storage is sufficiently high such that the marginal utility will become less than the marginal cost of fossil fuel energy, then no thermal energy will be produced: for $U'(z_1 + s_1 - \epsilon) < C'(0)$, $y_1 = 0$. Otherwise, $U'(y_1 + z_1 + s_1 - \epsilon) = C'(y_1)$ and the thermal systems will be active. As a result, one can calculate a threshold level, τ, such that when $z_1 > \tau$, the thermal systems become

[20] This is always the case when $U'' < 0$.

[21] The second order condition for a maximum is satisfied by $U''(q_1-\epsilon) - C''(y_1) < 0$.

idle, and vice versa:[22]

$$y_1^* \geq 0 \quad \text{if} \quad z_1 \leq \tau, \tag{4a}$$
$$y_1^* = 0 \quad \text{otherwise} \quad (\text{i.e., } z_1 > \tau), \tag{4b}$$

where we denote the optimal thermal energy decision by

$$y_1^* \equiv y(z_1 + s_1 - \epsilon).$$

When the weather conditions are such that the level of RE is lower than τ, Eq. (4a) demonstrates that the thermal systems will be used to meet the residual demand. In contrast, when RE generation is sufficiently high, the thermal systems will be shutdown.

When there is an interior solution for thermal energy, the comparative statics provide the following:

$$\frac{\partial y_1^*}{\partial z_1} = \frac{U_1''}{C_1'' - U_1''} < 0, \qquad \frac{\partial y_1^*}{\partial s_1} = \frac{U_1''}{C_1'' - U_1''} < 0, \qquad \frac{\partial y_1^*}{\partial \epsilon} = -\frac{U_1''}{C_1'' - U_1''} > 0 \tag{5}$$

where $U_1'' = U''(q_1^* - \epsilon)$ and $C_1'' = C''(y_1^*)$. The analysis indicates that a higher (lower) RE decreases (increases) dispatchable thermal generation. In a similar way, a higher (lower) level of stored enegy decreases (increases) y_1^*. Furthermore, a higher ϵ (that is, energy is valued even more in the peak period), increases the thermal energy generation.

In contrast, when $z_1 > \tau$, the thermal systems are kept idle. Therefore,

$$\partial y_1^* / \partial z_1 = \partial y_1^* / \partial s_1 = \partial y_1^* / \partial \epsilon = 0.$$

Given y_1^*, the maximum value function for period 1 is

$$W_1(z_1, s_1, \epsilon) = U(y_1^* + z_1 + s_1 - \epsilon) - C(y_1^*). \tag{6}$$

The problem in period 0 is then the following:

$$\begin{aligned} \max_{\{q_0, y_0, s_1\}} \; & U(q_0) - C(y_0) + \mathbb{E}\left[W_1(\tilde{z}_1, s_1, \epsilon)\right] \\ \text{subject to} \;\; & q_0 = y_0 + z_0 - \alpha s_1, \\ & q_0 \geq 0, y_0 \geq 0, \\ & \bar{s} \geq s_1, s_1 \geq 0. \end{aligned}$$

[22] Using $U'(z_1 + s_1 - \epsilon) < C'(0)$, one can calculate τ as follows:

$$z_1 > \tau \equiv U'^{-1}(C'(0)) - s_1 + \epsilon.$$

The first-order necessary condition for thermal energy at a maximum is:[23]

$$U'(y_0^* + z_0 - \alpha s_1) \leq C_d'(y_0^*), \text{ with an equality if } y_0^* > 0. \tag{7}$$

Using the maximum value function in Eq. (6) and the Envelope Theorem, the first-order condition with respect to s_1 is

$$\begin{aligned}
&\text{(8a)} \quad U'(y_0^* + z_0) \geq \phi \mathbb{E}\left[U'(\tilde{y}_1^* + \tilde{z}_1 - \epsilon)\right] && \text{if } s_1^* = 0,\\
&\text{(8b)} \quad U'(y_0^* + z_0 - \alpha s_1^*) = \phi \mathbb{E}\left[U'(\tilde{y}_1^* + \tilde{z}_1 + s_1^* - \epsilon)\right] && \text{if } \bar{s} > s_1^* > 0,\\
&\text{(8c)} \quad U'(y_0^* + z_0 - \alpha \bar{s}) \leq \phi \mathbb{E}\left[U'(\tilde{y}_1^* + \tilde{z}_1 + \bar{s} - \epsilon)\right] && \text{otherwise } \Big(\text{i.e., if } s_1^* = \bar{s}\Big),
\end{aligned}$$

where $\phi \equiv 1/\alpha$ is the round-trip efficiency parameter and $y_0^* \equiv y(z_0 - \alpha s_1^*)$.[24] From the benevolent planner's perspective, the willingness to store energy is determined by the expected marginal utility of energy consumption in the next period. For

$$q_0^* \equiv y_0^* + z_0 - \alpha s_1^* \text{ and } \tilde{q}_1^* \equiv \tilde{y}_1^* + \tilde{z}_1 + s_1^* - \epsilon,$$

if it is not optimal to store energy, that is, $s_1^* = 0$, there is an expected loss from energy storage: $U'(q_0^*) \geq \phi \mathbb{E}\left[U'(\tilde{q}_1^*)\right]$. Otherwise, energy is stored until its current and expected social values are equalized. If, however, $s_1^* = \bar{s}$, the marginal expected benefit from storing energy is at least as high as the marginal cost of energy storage; that is, $U'(q_0^*) \leq \phi \mathbb{E}\left[U'(\tilde{q}_1^*)\right]$. Notice that when $s_0 > 0$, the marginal cost of energy storage becomes lower. This makes it more likely that energy will be stored and transferred to the next period.

3.1 Energy storage in the absence of RE generation

As a special case, suppose that energy can only be produced using dispatchable thermal systems and there is energy storage. From Eq. (8b) one can get the following as an interior solution for thermal energy in both periods; that is, $U'(q_0^*) = C'(y_0^*)$ and $U'(q_1^* - \epsilon) = C'(y_1^*)$:

$$\frac{C'\big(y_0^*\big)}{C'\big(y_1^*\big)} = \phi. \tag{9}$$

Eq. (9) satisfies intertemporal efficiency. There is an equality between the marginal rate of transformation of off-peak energy into peak energy and cost of energy transformation. A similar result can be seen in Crampes and Moreaux (2010), where α ($\equiv 1/\phi$) is the level of energy required to add one unit to the stock of energy in a water reservoir for use in period 1 when the demand is high. If the absolute value of the slope of the isocost curve, $C'(y_0^*)/C'(y_1^*)$, is greater than ϕ, no energy is stored in period 0. This is because the cost of storage on the margin is bigger than its value in the peak period. In contrast, if

[23] Similar to the period 1 problem, the second order condition for a maximum is satisfied: $U''(q_0) - C''(y_0) < 0$.

[24] The second order condition for a maximum gives $\alpha^2 U''(q_0) + U''(q_1 - \epsilon) < 0$.

$C'(y_0^*)/C'(y_1^*) < \phi$, the available storage capacity is completely utilized.

Suppose that the energy system is now composed of baseload power plants as well as dispatchable thermal and energy storage systems. Power plants such as nuclear and coal-fired plants that produce at low marginal costs and are devoted to the production of baseload supply, have slow ramp rates and are not flexible to switch on and off. As it is more economical to operate them at constant production levels, these power plants, in general, do not change their production to match changing energy demand. Thus, these plants are rather inflexible in practicing "load following," and electric power companies try to operate them at full output as much as possible (Denholm et al., 2010).[25] Provided that $y_1^* > 0$, in the presence of an inflexible baseload supply, where μ denotes this capacity, we have the following:

$$C'\big(y_0^*\big) = \phi C'\big(y_1^*\big), \tag{10}$$

where

$$y_0^* = y(\mu - \alpha s_1^*) \text{ and } y_1^* = y(\mu + s_1^* - \epsilon).$$

Similarly, energy will not be stored when the storage unit is empty and the unit cost of storage is greater than the net social value of energy storage in the peak demand period. In contrast, the storage capacity is completely utilized if the present cost of storing a unit of energy on the margin is less than its value adjusted for the round-trip efficiency (ϕ) in the next period, that is, $C'(y_0^*) < \phi C'(y_1^*)$. Provided that the marginal rate of transformation of off-peak energy into peak energy is equal to the cost of energy transformation, some energy will be stored and available in the next period.

3.2 Energy storage in the presence of RE generation

For power systems with renewable sources of energy, the availability of energy storage can be crucial in dealing with the uncertain variability of RE (Doherty and O'Malley, 2005; Steffen and Weber, 2013). Therefore, in this subsection, we augment our analysis of energy storage by considering intermittent renewable energy. Considering that the energy generated from RE systems is intermittent, we can write the expected marginal utility as

$$\mathbb{E}\big[U'(\tilde{q}_1^* - \epsilon)\big] = F(\tau)\mathbb{E}[U'(\tilde{q}_1^* - \epsilon)|\tilde{z}_1 \leq \tau] + (1 - F(\tau))\,\mathbb{E}[U'(\tilde{q}_1^* - \epsilon)|\tilde{z}_1 > \tau]. \tag{11}$$

While $\mathbb{E}[U'(\tilde{q}_1^* - \epsilon)|\tilde{z}_1 \leq \tau]$ represents the conditional expected marginal utility from consuming energy supplied by both the thermal and renewable systems, $\mathbb{E}[U'(\tilde{q}_1^* - \epsilon)|\tilde{z}_1 > \tau]$ is the conditional expected marginal utility when consuming energy only from the renewable systems. Thus, the latter corresponds to cases in which thermal systems are kept idle. Moreover, $F(\tau)$ is the probability of $\tilde{z}_1 < \tau$ and vice versa.

[25] Load following describes situations where power plant generation can accommodate changes in energy demand.

Generally, the existing energy systems worldwide can be characterized by rather small shares of RE (Lund et al., 2012). Thus, even with very favorable weather conditions, the RE systems cannot produce enough energy to meet the total energy demand, and thermal dispatchable generation always supplies the residual load. In our model, this translates into $F(\tau) = 1$; thus, $\tilde{z}_1 \leq \tau$. In other words, the energy generated from renewable sources can never be high enough so that thermal systems are taken offline. Yet, in Appendices C and D we consider larger shares of RE. To be specific, we also look at cases where the thermal systems can become or are always idle; that is, $0 < F(\tau) < 1$ and $F(\tau) = 0$, respectively.

In studying the effect of energy storage on welfare, we start from a situation of certainty as presented earlier when we considered baseload power (see Eq. (10)). We then introduce some noise $\tilde{x}$ around μ in period 1 that satisfies $\tilde{z}_1 = \mu + \tilde{x}$, $\mathbb{E}[\tilde{x}] = 0$ and $\mathbb{E}[\tilde{x}^2] = \sigma^2$. Hence, $\tilde{z}_1$ represents the intermittent RE with mean μ and variance σ^2. Our purpose here is to determine whether the optimal level of energy storage under uncertainty (intermittency) is greater than the corresponding level without uncertainty. Let s_1^+ be the optimal level of energy storage when $\tilde{z}_1 = \mu$ in the future with certainty:

$$s_1^+ = \arg\max\ U(y(z_0 - \alpha s_1) + z_0 - \alpha s_1) - C(y(z_0 - \alpha s_1)) + W_1(\mu + s_1 - \epsilon).$$

Without any uncertainty, the only factor that leads to energy storage is the higher valuation of energy in the peak period due to $\epsilon > 0$.

Furthermore, suppose that s_1^* is the optimal level of energy storage when there is uncertainty in RE generation:

$$s_1^* = \arg\max\ U(y(z_0 - \alpha s_1) + z_0 - \alpha s_1) - C(y(z_0 - \alpha s_1)) + \mathbb{E}[W_1(\tilde{z}_1 + s_1 - \epsilon)].$$

Following these definitions, we present our first major result by Theorem 1:

Theorem 1. *If $F(\tau) = 1$, then for every μ and $\tilde{x}$ with $\mathbb{E}[\tilde{x}] = 0$, $s_1^* \geq s_1^+$ if and only if:*

$$\psi_U U''' + \psi_C C''' \geq 0, \tag{12}$$

where $\psi_U \equiv (C''^3)/(C'' - U'')^3$ and $\psi_C \equiv (-U''^3)/(C'' - U'')^3$.

Proof. The proof is provided in Appendix A. □

Therefore, the level of energy storage is larger if RE is intermittent than otherwise. Notice that it is the weighted sum in Eq. (12) that matters for precautionary energy storage, where ψ_U and ψ_C are weights attached to U''' and C''', respectively. Thus, there can be precautionary storage even if $U''' < 0$ and $C''' > 0$ or $C''' < 0$ and $U''' > 0$. However, our main focus is on prudence and frugality. Theorem 1 has a stronger corollary in this regard:

Corollary 1. *$U''' \geq 0$ and $C''' \geq 0$ are sufficient for $s_1^* \geq s_1^+$.*

Hence, if $U''' \geq 0$ and $C''' \geq 0$, then Eq. (12) holds. Therefore, it is optimal to storage a higher level of energy under uncertainty. When there is no prudence ($U''' = 0$), frugality alone leads to precautionary energy storage. The same is true when $C''' = 0$ and $U''' \geq 0$.

As thermal energy is the marginal resource, thermal systems will supply the extra amount of energy for storage. Given $y_0 \equiv y(z_0 - \alpha s_1)$ and $\partial y_0/\partial s_1 > 0$, the economy increases energy storage through further load smoothing in the presence of intermittency.[26] In other words, the economy generates more energy than the demand at the off-peak period, stores the additional amount and uses it in the peak period. A similar analysis for the off-peak energy consumption gives $\partial q_0/\partial s_1 < 0$. Let y_0^+ and q_0^+ represent the fossil fuel energy generation and energy consumption, respectively, in period 0 when there is no uncertainty in period 1. We then have the following corollary:

Corollary 2. $s_1^* \geq s_1^+$ *implies* $y_0^* \geq y_0^+$ *and* $q_0^* \leq q_0^+$.

Higher thermal energy and energy storage result in lower off-peak energy consumption along with a lower welfare in the initial period. However, by transferring the social surplus from the off-peak to peak period, a higher welfare in the future is expected to more than compensate for this loss.

The fact that a convex marginal utility allows for a higher level of energy storage within this framework is referred to as prudence. Although it was Kimball (1990) who coined the term prudence, the analysis of precautionary demand for savings was done earlier by Leland (1968) and Sandmo (1970). Within an expected utility framework, they indicate that a risky future income increases savings only if the third-order derivative of the utility function is positive (that is, the agents are prudent).

Frugality ($C''' > 0$), however, is not fully investigated in the literature. Yet, by analyzing production and inventory data, Cecchetti et al. (1997) find evidence that supports a positive third derivative of the cost function, and note that, from an operational perspective, a firm is capacity constrained when faced with a convex marginal cost curve. Considering the fact that thermal stations are capacity constrained and follow the load when RE and energy storage are not adequate to cover the optimal level of energy demand, it can become increasingly costly to make large and positive changes to meet the residual demand. In this regard, frugality can lead to precautionary energy storage and in turn decrease the risk of such large and costly changes.

Let us take the second-order Taylor approximation of the intertemporal efficiency condition given by Eq. (8b) around the mean-level RE generation, μ. This will allow us to make further use of our main result given by Theorem 1 and see how the degree of intermittency can affect the level of energy storage. To be specific, the second-order approximation of Eq. (8b) is as follows:[27]

$$C'\big(y_0^*\big) \simeq \phi\Big[C'\big(y_1^*\big) + \frac{1}{2}\sigma^2\Big(\psi_{_U} U'''(q_1^* - \epsilon) + \psi_{_C} C'''\big(y_1^*\big)\Big)\Big]. \tag{13}$$

[26]From Eq. (7), comparative statics analysis for the off-peak thermal energy generation and energy consumption provides

$$\frac{\partial y_0^*}{\partial s_1} = \frac{-\alpha U_0''}{C_0'' - U_0''} > 0 \text{ and } \frac{\partial q_0^*}{\partial s_1} = \frac{-\alpha C_0''}{C_0'' - U_0''} < 0.$$

[27]Refer to Appendix B for the calculations. Note that $F(\tau) = 1$.

In Eq. (13) we use the fact that $U' = C'$ for an interior solution of y_1. Further,

$$q_1^* = y_1^* + \mu + s_1^* \text{ and } y_1^* = y(\mu + s_1^* - \epsilon)$$

are peak-period electricity consumption and thermal energy generation, respectively, that are evaluated at the mean-value of RE generation, μ. Eq.(13) yields the intertemporal efficiency condition, which is indeed Eq. (10) augmented with uncertainty and the precautionary motives. In comparison to Eq. (10), we see that the expected marginal value of energy storage in the next period is larger. As a result, when Eq. (12) holds, a higher level of energy will be stored through dispatching more thermal energy in the initial period. For a higher degree of intermittency, σ^2, the level of stored energy will be higher.

On the other hand, it is optimal to store at the capacity $\bar{s}$ if the cost of storing an extra amount of energy in the off-peak period is lower than the net expected marginal cost of producing thermal energy in the on-peak period. In this case, precautionary motives cannot lead to any further energy storage. In contrast, if there is no energy storage when there is no uncertainty (i.e., $s_1^+ = 0$), but Eq. (12) holds, energy can be stored when some uncertainty is introduced. Lastly, notice that for $\psi_U U''' + \psi_C C''' = 0$, or $U''' = 0$ and $C''' = 0$, Eq. (13) boils down to Eq. (10). Thus, even in the presence of intermittency, the level of stored energy will be identical to the one with baseload power generation.

4 Competitive market equilibrium with energy storage

In this section, we want to show that the planner solution can be decentralized through competitive markets. This will enable us to see the role of prices in coordinating the energy market that we have been discussing thus far. This task stipulates a well-defined market equilibrium concept in which agents have rational expectations. As there are no externalities in the model, the planner solution will coincide with the competitive rational expectations equilibrium (REE).

On the demand side, we assume that all consumers have identical preferences. This allows us to model their behavior by a representative consumer. Specifically, the first-order necessary conditions for the consumer problem yield

$$\begin{aligned} U'(q_0^*) &= P_0^*, \\ U'(q_1^* - \epsilon) &= P_1^*, \end{aligned} \tag{14}$$

where $q_t^* \equiv q(P_t^*)$ is the aggregate demand function for energy given the market price $P_t > 0$ for $t = 0, 1$.

Eq. (14) corresponds to the optimization problem of a representative consumer with quasilinear preferences. In economic theory, using such preferences is a standard assumption when discussing issues related to a single market in a general equilibrium framework.[28] Price

[28]This approach can be justified in the absence of income effects (see Mas-Colell et al., 1995, chap. 10), which we do not consider in our study.

taking competitive behavior in Eq. (14) implies an instantaneous adjustment of consumption in response to changes in the market price. In energy markets, this condition typically requires that the consumers are equipped with high-tech equipment that informs them about the spot price or they have access to software that can switch electrical appliances on and off, given the price in the market (Ambec and Crampes, 2012).

Regarding the production side of the economy, fossil-fuel and RE generators and energy storage firms are price-taking competitors. There is a continuum of RE generators with measure normalized to one. Given that the unit cost of generating energy is so low –which we consider as zero– the RE generation is price inelastic. Thus, each RE generator operates at its capacity. However, as the weather conditions are uncertain, so is the energy produced by each generator. Therefore, given $P_t > 0$, the profit of each RE generator in both periods is

$$\pi_{it} = P_t^* z_{i_t},$$

where $\bar{z}_i \geq z_{i_t} \geq 0$ and $\bar{z}_i$ is the installed capacity of RE generator i. The total RE generation then satisfies

$$z_t^* \equiv z(P_t^*) = z_t \equiv \int_0^1 z_{i_t} di, \tag{15}$$

where $\bar{z} \geq z_t \geq 0$ and $\bar{z} \equiv \int_0^1 \bar{z}_i di$.[29]

There is a unique merit order of fossil fuel power plants with measure normalized to one. Given P_t, thermal energy generation in the industry extends up to the thermal unit for which the marginal cost of generating thermal energy equals the market price.[30] The profit maximization problem of each thermal energy generator is as follows:

$$\max_{y_{j_t}} \; \pi_{jt} = P_t^* y_{jt} - c_j y_{jt}, \; \text{ subject to } \; \bar{y}_{jt} \geq y_{jt} \geq 0, \tag{16}$$

where y_{j_t} is the energy generation from thermal unit j at time t and $c_j > 0$ is a constant. The

[29] Alternatively, we can formulate the profit maximization of the RE generators as follows. Let ν be an i.i.d. random variable with a compact support $[0, 1]$ and $\bar{z}_i$ be the installed capacity of RE generator i. Given $P_t > 0$ and the fact that the RE generation is price inelastic, the profit maximization problem RE generator i in both periods is given by

$$\max_{z_{it}} \; \pi_{it} = P_t^* \nu_t z_{it} \; \text{ subject to } \; \bar{z}_i \geq z_{it} \geq 0.$$

Obviously, for $t = 0, 1$, the profit maximizing production is $\bar{z}_i$. The total RE generation then satisfies

$$z_t^* \equiv z(P_t^*) = \nu_t \bar{z} = \int_0^1 \nu_t \bar{z}_i di.$$

[30] For the units that are brought online earlier, the individual marginal costs equal the market price minus the shadow prices of the individual capacity constraints.

first order necessary condition of profit maximization for a generator is

$$
\begin{aligned}
P_t^* &\leq c_j \quad \text{if} \quad y_{jt}^* = 0, \\
P_t^* &= c_j \quad \text{if} \quad \bar{y}_j > y_{jt}^* > 0, \\
P_t^* &\geq c_j \quad \text{if} \quad y_{jt}^* = \bar{y}_j.
\end{aligned} \tag{17}
$$

Given that $y_{jt}^* \equiv y_j(P_t^*)$ is the profit maximizing level of energy that thermal generator j is willing to supply at price P_t^*, the thermal industry (aggregate) supply function in both periods is

$$
y_t^* \equiv y(P_t^*) = \int_0^1 y_{jt}^* dj.
$$

We characterize the energy storage sector by a continuum of energy storage firms with identical technologies. When a storage firm decides to store more energy, it rationally anticipates the future energy price based on the available information (that is, the supply schedule of the thermal energy industry, the aggregate demand schedule, the processes that affect the weather, and –in turn– the RE generation). With the ultimate motivation to maximize profits, energy storage firms apply the principle of rational behavior to the acquisition and processing of information and the formation of anticipations. In this sense, they are rational profit maximizers. When storage firms are fully aware of the economic implications of intermittency, they will, for example, change their energy storage levels in anticipation of the effects from intermittent RE rather than wait for these effects to occur in the electricity market. By anticipating the future RE generation, and thus, the future price, the net anticipated profit of energy storage firm ℓ from storing s_{ℓ_1} is

$$
\pi_{\ell_1}^a = \phi P_1^a s_{\ell_1} - P_0^* s_{\ell_1}, \tag{18}
$$

where P_1^a and P_0^* are the anticipated and current equilibrium spot prices, respectively. Each storage firm maximizes its anticipated profits subject to a non-negativity constraint, $s_{\ell_1} \geq 0$, and a capacity constraint, $\bar{s}_\ell \geq s_{\ell_1}$. As each storage firm shares the same rational expectations with every other firms, the anticipated price is not indexed by a particular storage firm. The first-order condition for the maximization problem yields

$$
\begin{aligned}
\frac{\partial \pi_{\ell_1}^a}{\partial s_{\ell_1}} &= \phi P_1^a - P_0^* \leq 0, \text{ with equality if } s_{\ell_1}^* > 0, \\
&= \phi P_1^a - P_0^* \geq 0, \quad \text{otherwise} \quad s_{\ell_1}^* = \bar{s}.
\end{aligned} \tag{19}
$$

The fact that these firms share the same rational expectations, and therefore, anticipate the same market clearing future price indicates that, in equilibrium, the anticipated profit from a marginal unit of energy storage cannot be positive. Otherwise, profit-seeking entrepreneurs would eliminate any type of disequlibria by adjusting the individual levels of energy storage.[31] This allows us to describe Eq. (19) as the condition for market equilibrium rather

[31]This will hold for interior solutions. If the industry storage capacity binds, the storage firms will make positive profits.

than the first-order condition for an energy storage firm's optimization problem. The relationship between the industry level of energy storage and the anticipated profit can then be summarized by

$$
\begin{aligned}
&P_0^* \geq \phi P_1^a, \quad s_1^* = 0, \\
&P_0^* = \phi P_1^a, \quad \bar{s} > s_1^* > 0, \\
&P_0^* \leq \phi P_1^a, \quad s_1^* = \bar{s},
\end{aligned} \tag{20}
$$

where

$$
s_1^* = \int_0^1 s_{\ell_1}^* d\ell \text{ and } s_1^* \equiv s_1(P_0^*, P_1^a).
$$

In the sense that the energy storage firms' expectations are rational and they make informed predictions of future prices, the subsequent market prices that arise from the decisions based on these expectations will confirm their anticipations. Hence, the expected price will be consistent with the level of energy storage that is governed by the anticipated price. Then, in a REE

$$
P_1^a = \mathbb{E}[\tilde{P}_1^*], \tag{21}
$$

that is, the anticipated price will be confirmed in equilibrium.

Having depicted the formation of expectations and the response of competitive energy storage firms to current and anticipated prices and the qualitative relationship between price and profit maximization for each storage firm, we make the following definition:

Definition 1. *REE is a price vector,* $\mathbb{P} = \{P_0^*, \tilde{P}_1^*, P_1^a\}$, *and an allocation vector,* $\mathbb{Q} = \{q_0^*, \tilde{q}_1^*, z_0^*, \tilde{z}_1^*, y_0^*, \tilde{y}_1^*, s_1^*\}$, *that solve Eqs.* (14), (15), (17), (19), *and* (21), *such that markets clear:* $q_0^* = y(P_0^*) + z(P_0^*) - \alpha s_1(P_0^*, P_1^a)$ *and* $\tilde{q}_1^* = y(\tilde{P}_1^*) + z(\tilde{P}_1^*) + s_1(P_0^*, P_1^a)$.

In equilibrium, the prices, P_0^* and $\tilde{P}_1^*$, are implicitly defined by

$$
\begin{aligned}
P_0^* &\equiv P\big(y^*(P_0^*) + z_0 - \alpha s_1^*(P_0^*)\big), \\
\tilde{P}_1^* &\equiv P\big(y^*(\tilde{P}_1^*) + \tilde{z}_1 + s_1^*\big),
\end{aligned}
$$

respectively. In the presence of uncertainty, the storage firms will increase the amount of stored energy until the net expected price, $\phi\mathbb{E}[\tilde{P}_1^*]$, equals the current spot price of energy.[32] This additional demand for energy storage will be met by increased thermal energy and reduced current consumption (Corollary 2).[33]

As the REE quantities in our problem correspond to the allocation dictated by the benevolent social planner, we carry forward our results from the previous section. To

[32] Note also that for $P_0^* > \phi\mathbb{E}[\tilde{P}_1^*],\ s_1^* = 0$, that is, when the net expected price is below the current price, then the energy storage is zero. If the capacity constraint in the energy storage industry is met, then the net expected future price is above the current price at storage capacity: $P_0^* \leq \phi\mathbb{E}[\tilde{P}_1^*],\ s_1^* = \bar{s}$.

[33] This additional demand can also be met by only reducing the current consumption (Corollary 3).

observe the implications of intermittent RE for competitive pricing, we arrange Eq. (13) as in the following:

$$P_0 \simeq \phi \left[1 + \frac{1}{2}\sigma^2 \left(\psi_U \frac{U'''}{U'} + \psi_C \frac{C'''}{C'}\right)\right] P_1, \tag{22}$$

where from Eq. (14) $P_0 = U'(q_0^*)$, and $P_1 = U'(q_1^* - \epsilon)$ is the energy price in period 1 at the mean level of RE generation, $\mu \equiv \mathbb{E}[\tilde{z}_1]$. RHS is the product of ϕ and the expected market price of energy in period 1, which is indeed the market price that corresponds to μ, adjusted for the degree of intermittency, σ^2, and the weighted sum of U'''/U' and C'''/C'. In the absence of intermittency (i.e., $\sigma = 0$) Crampes and Moreaux (2010) indicate that $P_0 = \phi P_1$ and that there is still peak load pricing.[34] In the presence of intermittent RE, the current price, P_0, becomes higher when Eq. (12) holds. Our intuition is that the precautionary motives spur energy storage and increase the spot market electricity price.

It is possible to differentiate the demand- and the supply-side effects in Eq. (22). The demand side refers to the ratio given by U'''/U'. Conversely, the supply side of the market is represented by C'''/C'. Let us first consider the case in which there is no frugality ($C''' = 0$).

When $C''' = 0$, that is, the supply schedule is linear, one can rearrange Eq. (22) to obtain

$$P_0 \simeq \phi \left[1 + \frac{1}{2}\left(\frac{\sigma}{\bar{q}_1}\right)^2 \psi_U \frac{\xi_r^p}{\eta_d}\right] P_1, \tag{23}$$

where $\xi_r^p \equiv -\bar{q}_1 \frac{U'''}{U''}$ is relative prudence, $\bar{q}_1 \equiv y(\mu + s_1^* - \epsilon) + \mu + s_1^*$ and $\eta_d \equiv \left|\frac{d\bar{q}_1/\bar{q}_1}{dP/P}\right|$ are energy consumption and price elasticity of energy demand evaluated at the mean RE generation, respectively.[35] This leads us to the following proposition:

Proposition 1. *If $U''' > 0$ and $C''' = 0$, then in a REE with energy storage, P_0 is augmented by a lower η_d and a higher ϕ, ξ_r^p, σ and ψ_U.*

This proposition indicates that if the consumers are prudent, but the thermal generators are not frugal ($C''' = 0$), then –in a REE with energy storage– a higher round-trip efficiency parameter, ϕ and a higher weight on the demand side, ψ_U, raise the expected price of energy. This is because a more efficient storage technology and a stronger effect from the demand side create arbitrage opportunities that lead to a higher demand for energy storage, and in turn, increase the current price of energy. In addition, $\sigma/\bar{q}_1$ is an indicator that shows the degree of deviations in RE. If the standard deviation in RE generation compared to the level of energy consumption is very small, intermittency is less of a problem for the economy. Accordingly, the precautionary motives will not cause a significant increase the demand for energy storage. If, however, the relative deviations in RE generation are high, energy consumption can also exhibit strong deviations. In this case, the economy will benefit from a higher level of energy

[34]The prices are closer together when energy can be stored, which is a consequence of the constraint that storage must not be negative.

[35]Equivalently, the demand elasticity can be written as $\eta_d \equiv -\frac{U'}{U''\bar{q}_1}$.

storage. A higher demand for energy storage will lead the storage firms to store more energy, which in turn will increase the spot market electricity price.

Given that the agents are prudent ($\xi_r^p > 0$), a higher sensitivity of energy demand to changes in energy price (i.e., a higher η_d) will cause a lower energy price. This is because when the price elasticity of demand is higher, consumption adjustment will become a better substitute to energy storage. Consequently, the higher sensitivity to prices will diminish the impact of prudence on precautionary energy storage. A similar result can be found in the commodity storage literature, where Wright and Williams (1982, 1984) show that higher demand elasticity decreases the scope for commodity storage.

The data, nevertheless, indicates that the relative price response is rather low. Accordingly, the short-run (1–5 years) residential own-price elasticity of electricity demand in absolute value is estimated at 0.3 (EPRI, 2008). The same number averaged for potential system peak hours for the summer months is estimated to be 0.15 (Taylor et al., 2005). Surveying the evidence from the recent experiments with dynamic pricing of electricity, Faruqui and Sergici (2010) report that the own price elasticities in peak usage range from 0.02 to 0.10. This in turn can emphasize the role of prudence on precautionary energy storage.[36]

Suppose now that the dispatchable thermal energy industry is characterized by a convex supply schedule, that is, $C''' > 0$, and the demand function is linear, that is, $U''' = 0$. One can rewrite Eq. (22) to obtain

$$P_0 \simeq \phi \left[1 + \frac{1}{2} \left(\frac{\sigma}{\bar{y}_1} \right)^2 \psi_C \frac{\xi_r^f}{\eta_s} \right] P_1, \tag{24}$$

where $\bar{y}_1 \equiv y(\mu + s_1^* - \epsilon)$ is thermal energy generation at the mean level of RE generation and $\xi_r^f \equiv \bar{y}_1 \frac{C'''}{C''}$ and $\eta_s \equiv \frac{d\bar{y}_1/\bar{y}_1}{dP/P}$ are the coefficient of relative frugality and the elasticity of thermal energy supply, respectively.[37] Here C' is the price calculated by the inverse supply curve (industry marginal cost function) for thermal energy. This leads to the following proposition:

Proposition 2. *If $U''' = 0$ and $C''' > 0$, then in a REE with energy storage, P_0 is augmented by a lower η_s, and a higher ϕ, ξ_f^p, σ and ψ_C.*

When the price schedule is linear and the dispatchable thermal energy industry is characterized by a convex supply schedule, Proposition 2 indicates that a higher frugality induces a higher level of energy storage, and therefore, a higher current energy price. A higher elasticity of supply (that is, a more responsive thermal energy generation) causes a lower level of energy storage. Hence, both the supply- and demand-side elasticities have similar effects. If, however, the supply elasticity is low, there will be extra incentives to store

[36] Although electricity consumers exhibit low price elasticities, this may indeed be related to the difficulties in accessing complete information. In other words, the households may be price inelastic because they simply lack complete information. When households are provided with real-time information regarding energy usage, Jessoe and Rapson (2014) demonstrate that they can become more price sensitive.

[37] Equivalently, the elasticity of thermal energy supply can be shown as $\eta_s \equiv \frac{C'}{C''\bar{y}_1}$.

energy. For example, one can think of a baseload power plant, which has low supply elasticity due to its poor flexibility in adjusting its output. For such a plant, it is not very cost effective to vary its supply. Therefore, a higher level of energy storage will be beneficial in the presence of uncertainty.

Let us suppose that the standard deviation in RE generation that is scaled by $\bar{y}_1$, that is, $\sigma/\bar{y}_1$, is high. Thus, the RE energy supply can significantly deviate from $\bar{y}_1$. In this case, there can be costly attempts in the thermal energy industry to supply the desired level of energy when RE is low. This will lead to a higher level of energy storage, and thus, spot market electricity price in equilibrium. In contrast, if σ is small, the deviations in RE generation will become less of an issue and in turn diminish the level of stored energy due to precautionary motives.

Let us now consider that the consumers are prudent and thermal generators are frugal. Eq. (25) shows that there will be a higher demand for storage services and therefore a higher current energy price than those given by Eqs. (23) and (24):

$$P_0 \simeq \phi \left[1 + \frac{1}{2}\Big(\big(\frac{\sigma}{\bar{q}_1}\big)^2 \psi_U \frac{\xi_r^p}{\eta_d} + \big(\frac{\sigma}{\bar{y}_1}\big)^2 \psi_C \frac{\xi_r^f}{\eta_s} \Big) \right] P_1. \tag{25}$$

The interpretation follows from Propositions 1 and 2. An interesting feature of Eq. (25) is related to the weights assigned to the demand- and supply-side effects, ψ_U and ψ_C, respectively. Miranda and Helmberger (1988) show that price variability is more sensitive to the demand elasticity than the supply elasticity. When translated to our case, this can imply a greater weight on the effect of demand elasticity on the current energy price, and thus, ψ_U being greater than ψ_C. For energy markets, measuring this effect can be an interesting problem, and an empirical investigation may supply crucial information when pricing energy to attain the first-best dispatch.[38]

In Appendix E we also look at the implications of larger shares of RE for competitive pricing. In other words, we take into account the possibility that the thermal systems can be taken offline due to high RE generation and examine how the spot market price is affected in this regard.

5 Conclusion

Increased energy generation from renewable sources of energy is one of the key solutions in combating the climate change problem and is high on the policy agenda. Nevertheless, as RE is inherently variable and uncertain, larger shares of RE generation can cause serious vulnerabilities in meeting energy demand and make renewables unreliable baseload

[38] Furthermore, recall the possible relationship between a better access to real-time information and a weaker role for prudence (see our earlier discussion under Proposition 1). When such a relationship becomes more significant, it will place a relatively greater importance on frugality and its impact on the level of energy storage and energy prices.

contributors. In this regard, policymakers and utility planners have demonstrated interest in energy storage technologies as a way to enhance the resilience and reliability of the power grid.

Even though energy storage is addressed in many studies in the literature, the extent to which precautionary motives can spur energy storage is not well known. In designing coherent energy policies and making utility planning decisions, both governments and power utilities can benefit from the knowledge regarding the direct and indirect impacts that the precautionary motives have on electricity prices, and generation and storage decisions. The model we develop in this study provides a simple setup to assess these impacts due to prudence (convex marginal utility) and frugality (convex marginal cost). We first look at a planning problem. We then turn to a decentralized setting and examine how, in the presence of precautionary motives, the use of the energy storage technology affects the spot market electricity price, thermal energy generation, and electricity consumption.

Our analysis indicates that prudence and frugality cause precautionary energy storage. Even in the absence of prudence, frugality can still contribute toward precautionary storage, and vice versa. Decentralization of the optimal allocation through competitive markets allows us to see how prudence and frugality can affect energy prices, which, in turn, help coordinating the energy market. When prudence and frugality are present, there is precautionary energy storage and thus a higher spot market electricity price. The standard deviation in RE boosts energy storage to varying degrees. Considering prudence, this will depend on the ratio of the standard deviation in RE to energy consumption. If this ratio is large, intermittency in RE is less of a concern. Therefore, there will be a relatively lower level of energy storage and energy price. Regarding frugality, what matters is the ratio of the standard deviation in RE to thermal energy generation. If this share is high, the degree of intermittency will increase the current energy price through a higher demand for energy storage.

We find that when agents are prudent and there is intermittent RE, precautionary energy storage and spot market electricity price are negatively related to price elasticity of demand. This is a consequence of the fact that consumption adjustment is a substitute for energy storage. For example, a lower responsiveness of consumers, and thus, a lower price elasticity of demand, will assign a larger role to prudence in augmenting the demand for energy storage. On the other hand, when energy supply is less price elastic, dispatchable thermal energy generation will become a poorer substitute for energy storage. In this case, the intermittent residual load will lead to a higher demand for energy storage due to frugality. Furthermore, both the demand- and supply-side effects, that is, the effects coming from prudence and frugality, respectively, are weighted differently by the endogenously determined weights. This indicates that prudence and frugality cause precautionary energy storage to varying degrees.

One way to extend our study is to incorporate energy generation and storage capacity decisions into the model. Similar to the approach adopted by Ambec and Crampes (2012), a central planner can first determine various capacities and then choose how to dispatch them for each state of supply and demand. While the capacity decisions would be associated with long-term commitments in the economy, the latter, which corresponds to the present study, would be related to short-term decisions constrained by the installed capacities. It would then be interesting to analyze how precautionary motives can impact capacity decisions. Furthermore, considering market imperfections and pollution externalities, it will be worthwhile to investigate the relationship between precautionary motives and energy generation and storage decisions. Lastly, testing the empirical relevance of our model will be interesting.

Appendices

A Proof of Theorem 1

To prove our main result, we will need the following lemma:

Lemma 1. *If $\psi_U U''' + \psi_C C''' \geq 0$, then increase in risk (or, the degree of intermittency) in the sense of Rothschild and Stiglitz (1970) (RS) increases s_1.*

Proof. Let E and F represent the cumulative distribution functions of $\tilde{n}_1$ and $\tilde{z}_1$, respectively. Assume that F is a mean-preserving spread of E in the sense of RS. Thus, although both systems have the same average level of RE generation, the density function $f(\tilde{z}_1)$ has more weight in the tails, and is more risky. Then, for every non-decreasing concave function, υ, we have the following:

$$\int_a^b \upsilon(m)dE(m) \geq \int_a^b \upsilon(m)dF(m).$$

Taking $\upsilon \equiv -U'$ yields the following:

$$\int_a^b U'(m)dE(m) \leq \int_a^b U'(m)dF(m).$$

Then, for $b = \bar{z}$, $a = 0$, and $U' = U'(y(\tilde{j}_1 + s_1 - \epsilon) + \tilde{j} + s_1 - \varepsilon)$ for $\tilde{j} = \tilde{n}_1, \tilde{z}_1$, Theorem 2(A) in RS states that an increase in risk leads to a higher s_1 if U' is convex in $\tilde{j}$; that is,

$$\frac{\partial^2 U'}{\partial \tilde{j}^2} = U'''\Big(\frac{\partial y_1}{\partial \tilde{j}} + 1\Big)^2 + U''\frac{\partial^2 y_1}{\partial \tilde{j}^2} \geq 0. \tag{26}$$

Using Eq. (5) we get

$$\frac{\partial^2 y_1}{\partial \tilde{j}^2} = \frac{C''^2 U''' - U''^2 C'''}{(C'' - U'')^3}. \tag{27}$$

Substituting Eqs. (5) and (27) in Eq. (26) then gives

$$\psi_U U''' + \psi_C C''' \geq 0. \tag{28}$$

□

First, let us prove sufficiency using Lemma 1. Given s_1^+, let $V(s_1^+, \mu)$ be the maximum value function for the intertemporal optimization problem under certainty. Further, let $\mathbb{E}[V(s_1^*, \tilde{z}_1)]$ be the expected value of the maximum value function for the intertemporal optimization problem when RE is uncertain. Given $\tilde{z}_1 = \mu + \tilde{x}$ and $\mathbb{E}[\tilde{x}] = 0$, the first order conditions with respect to s_1 for $V(s_1, \mu)$ and $\mathbb{E}[V(s_1, \tilde{z}_1)]$, that is, $V_s(s_1, \mu) = 0$ and $\mathbb{E}[V_s(s_1, \tilde{z}_1)] = 0$, respectively, yield:

$$-U'(y(z_0 - \alpha s_1^+) + z_0 - \alpha s_1^+) + \phi[U'(y(\mu + s_1^+ - \epsilon) + \mu + s_1^+ - \epsilon)] = 0,$$
$$-U'(y(z_0 - \alpha s_1^*) + z_0 - \alpha s_1^*) + \phi\mathbb{E}[U'(y(\tilde{z}_1 + s_1^* - \epsilon) + \tilde{z}_1 + s_1^* - \epsilon)] = 0.$$

If $V(s_1^+, \tilde{z}_1)$ is convex in $\tilde{z}_1$, then $\mathbb{E}[V_s(s_1^+, \tilde{z}_1)] \geq V_s(s_1^+, \mu) = 0$, or equivalently, $\mathbb{E}[U'(y(\tilde{z}_1 + s_1^+ - \epsilon) + \tilde{z}_1 + s_1^+ - \epsilon] \geq U'(y(\mu + s_1^+ - \epsilon) + \mu + s_1^+ - \epsilon)$. If Lemma 1 holds and we take μ and 0 as the mean and the variance of $\tilde{n}_1$, respectively, then $\mathbb{E}[U'(y(\tilde{z}_1 + s_1^+ - \epsilon) + \tilde{z}_1 + s_1^+ - \epsilon] \geq U'(y(\mu + s_1^+ - \epsilon) + \mu + s_1^+ - \epsilon)$. Hence, the marginal benefit of increasing energy storage is positive when $s_1 = s_1^+$, and thus, $s_1^* \geq s_1^+$. This ends the proof for sufficiency.

If $s_1^* \geq s_1^+$ for every μ and $\tilde{x}$ with $\mathbb{E}[\tilde{x}] = 0$, then this must also be true for small zero-mean risks. The small risk allows us to focus on 2nd Taylor approximation around μ:

$$V_s(s_1^+, \mu + \tilde{x}) \simeq V_s(s_1^+, \mu) + \tilde{x}V_{sz}(s_1^+, \mu) + \frac{1}{2}\tilde{x}^2 V_{szz}(s_1^+, \mu) + \mathrm{O}\left(\tilde{x}^3\right) \tag{29}$$

where $V_{sz} = \phi\frac{\partial U'}{\partial \tilde{z}_1}$, $V_{szz} = \phi\frac{\partial^2 U'}{\partial \tilde{z}_1^2}$ (see Eqs. (26) and (27)), and $\mathrm{O}\left(\tilde{x}^3\right)$ is the remainder. By assuming that the risk is small, we can ignore the remainder term. From the first order condition, $V_s(s_1^+, \mu) = 0$. Taking the expectation of both sides yields:

$$\mathbb{E}[V_s(s_1^+, \mu + \tilde{x})] \simeq \frac{1}{2}\sigma^2 V_{szz}(s_1^+, \mu) \tag{30}$$

For a small risk, if $s_1^* > s_1^+$, then $\mathbb{E}[V_s(s_1^+, \mu + \tilde{x})] \geq 0$. For $\mathbb{E}[V_s(s_1^+, \mu + \tilde{x})] \geq 0$ to be positive, $V_{szz} \geq 0$ must be positive. One can calculate that $V_{szz} \geq 0$ is equivalent to Eq. (28). This completes the proof for necessity.

B The second-order Taylor approximation of the intertemporal efficiency condition

Let $g(\tilde{z}_1) \overset{\text{def}}{=} C'(\tilde{y}_1^*)$ and $h(\tilde{z}_1) \overset{\text{def}}{=} U'(\tilde{q}_1^* - \epsilon)$, where and $\tilde{y}_1^* = y(\tilde{z}_1 + s_1^* - \epsilon)$ and $\tilde{q}_1^* = \tilde{y}_1^* + \tilde{z}_1 + s_1^*$. Given s_1 and ϵ, a second-order Taylor series expansion around the conditional means $\check{\mu} \equiv \mathbb{E}\left[z_1 | z_1 < \tau\right]$ for $g(\tilde{z}_1)$ and $\hat{\mu} \equiv \mathbb{E}\left[z_1 | z_1 > \tau\right]$ for $h(\tilde{z}_1)$ yield:

$$g(\tilde{z}_1) \simeq g(\check{\mu}) + (\tilde{z}_1 - \check{\mu})g'(\check{\mu}) + (1/2)(\tilde{z}_1 - \check{\mu})^2 g''(\check{\mu}), \tag{31a}$$
$$h(\tilde{z}_1) \simeq h(\hat{\mu}) + (\tilde{z}_1 - \hat{\mu})h'(\hat{\mu}) + (1/2)(\tilde{z}_1 - \hat{\mu})^2 h''(\hat{\mu}). \tag{31b}$$

Here, $g'(\check{\mu}) \equiv \check{C}''\partial y_1^*/\partial\check{\mu}$, $h'(\hat{\mu}) \equiv \hat{U}''$, $g''(\check{\mu}) \equiv \check{C}'''\left(\partial y_1^*/\partial\check{\mu}\right)^2 + \check{C}''\partial^2 y_1^*/\partial^2\check{\mu}$ and $h''(\hat{\mu}) \equiv \hat{U}'''$, where $\check{C}'' \equiv C''(\check{y}_1^*)$, $\hat{U}'' \equiv U''(\hat{q}_1^* - \epsilon)$, $\check{C}''' \equiv C'''(\check{y}_1^*)$ and $\hat{U}''' \equiv U'''(\hat{q}_1^* - \epsilon)$.

For an interior solution for thermal energy, $U' = C'$, one can calculate the second order derivative for the optimal thermal energy decision using Eq. (5):

$$\frac{\partial^2 y_1^*}{\partial^2 \check{\mu}} = \frac{\check{C}''^2 \check{U}''' - \check{U}''^2 \check{C}'''}{(\check{C}'' - \check{U}'')^3}. \tag{32}$$

Hence,

$$g''(\check{\mu}) = \frac{\check{C}''^3}{(\check{C}'' - \check{U}'')^3}\check{U}''' + \frac{-\check{U}''^3}{(\check{C}'' - \check{U}'')^3}\check{C}'''. \tag{33}$$

Calculating the conditional expectations, that is, $\mathbb{E}[g(\tilde{z}_1)|\tilde{z}_1 \leq \tau]$ and $\mathbb{E}[h(\tilde{z}_1)|z_1 > \tau]$, gives the following:

$$\mathbb{E}[g(\tilde{z}_1)|\tilde{z}_1 \leq \tau] \simeq g(\check{\mu}) + \frac{1}{2}\check{\sigma}^2 g''(\check{\mu}), \tag{34a}$$

$$\mathbb{E}[h(\tilde{z}_1)|\tilde{z}_1 > \tau] \simeq h(\hat{\mu}) + \frac{1}{2}\hat{\sigma}^2 h''(\hat{\mu}), \tag{34b}$$

where $\check{\sigma}^2 = \mathbb{E}\left[(\tilde{z}_1 - \check{\mu})^2|\tilde{z}_1 \leq \tau\right]$ and $\hat{\sigma}^2 = \mathbb{E}\left[(\tilde{z}_1 - \hat{\mu})^2|\tilde{z}_1 > \tau\right]$ are conditional variances.

From Eqs. (11), (33), (34a) and (34b), and the fact that $h''(\hat{\mu}) \equiv \hat{U}'''$, one can rewrite Eq. (8b) using a second-order Taylor approximation as in the following:

$$\begin{aligned} U'(q_0^*) &= \phi\left[F(\tau)\mathbb{E}[C'(\tilde{y}_1^*)|\tilde{z}_1 \leq \tau] + (1 - F(\tau))\,\mathbb{E}[U'(\tilde{q}_1^* - \epsilon)|\tilde{z}_1 > \tau]\right] \\ &= \phi\left[F(\tau)\left(\check{C}' + \frac{1}{2}\check{\sigma}^2\left(\psi_U \check{U}''' + \psi_C \check{C}'''\right)\right) + (1 - F(\tau))\left(\hat{U}' + \frac{1}{2}\hat{\sigma}^2 \hat{U}'''\right)\right] \end{aligned} \tag{35}$$

C 100% RE

For our model, a 100% RE system is equivalent to $F(\tau) = 0$. Thus, $\tilde{z}_1 > \tau$ always and there is no thermal energy dispatch. In other words, there is zero dispatchable energy capacity, which implies an infinite marginal cost beyond that state (cf. Eq. (4)). Iceland's electricity sector with minor contributions from thermal system is an example of this extreme case. With hydropower accounting for 74% and geothermal for 26%, electricity generation was produced solely from renewables in 2010 (IEA, 2013). One can also consider Norway's hydropower system that generates almost the whole electricity in the country (Førsund, 2007, p. 95). In 2010, approximately 95% of Norway's electricity was generated from hydropower (IEA, 2013). Although our focus is on variable and intermittent RE, in the absence of thermal systems our model can easily be converted to a model with a reservoir hydroelectric system.[39] For example, suppose z_0 is the current flow of water to a reservoir and $\tilde{z}_1$ is the uncertain flow in the next period. Let s_1 be the water that is stored in the reservoir for use in the peak period. Disregarding losses due to surface evaporation and leakages (i.e., $\alpha = \phi = 1$), $q_0 = z_0 - s_1$ is

[39] Considering a hydropower system with reservoir constraints, and therefore, the allocation of production between seasons (summer and winter), it can be convenient to relax the assumption that there is no discounting between periods.

the consumption of energy that is generated by letting the water flow through water turbines. Considering a 100% RE system where there is zero dispatchable energy capacity leads us to the following theorem:

Theorem 2. *If $F(\tau) = 0$, then for every μ and $\tilde{x}$ with $\mathbb{E}[\tilde{x}] = 0$, $s_1^* \geq s_1^+$ if and only if $U''' \geq 0$.*

Proof. The proof is similar to that of Theorem 1, except that for $t = 0, 1,\ y_t^* = 0$. □

Thus, for an economy with a 100% RE system, prudence is necessary and must be sufficient for precautionary energy storage. The economy increases energy storage by consuming less electricity generated by renewable sources in the initial period. Theorem 2 leads to the following corollary:

Corollary 3. *$s_1^* \geq s_1^+$ implies $q_0^* \leq q_0^+$.*

Consequently, some social surplus will be transferred to the peak period using energy storage technology. In a hydroelectric system with reservoirs, this will result in lower energy generation in the off-peak period, and therefore, a higher level of water transfer to the peak period. Hence, a higher volume of water that is kept in the reservoir due to precautionary motives is equivalent to generating a lower level of electricity from hydropower.

By taking a second-order Taylor approximation of Eq. (8b) the intertemporal efficiency condition for the mean-level RE generation provides the following:[40]

$$U'\big(q_0^*\big) \simeq \phi\Big[U'\big(q_1^* - \epsilon\big) + \frac{1}{2}\sigma^2 U'''\big(q_1^* - \epsilon\big)\Big]. \tag{36}$$

For an interior solution of the energy storage, the marginal (consumption) cost of storing energy (U'/ϕ) equals the expected marginal value of energy. If, however, the marginal cost of storing an extra unit of energy is greater than the expected marginal value of energy in the peak-period, the no energy is stored. It is also possible that when $U''' \geq 0$ and all the storage capacity is completely utilized, the net expected value of energy in the next period is bigger than the unit cost of storing energy in the initial period. Lastly, a higher degree of intermittency increases energy storage whenever $U''' \geq 0$.

D Large share of RE

Translated to our model, large share of RE means that either $z_1 > \tau$ and the thermal systems will become idle or $z_1 \leq \tau$ and the thermal systems will meet the residual load. Such large shares of RE will be observed more frequently in the future. For example, according to the European Commission, it is expected that the RE target of 75%–85% and approximately

[40] Refer to Appendix B for the calculations. Note that $F(\tau) = 0$.

100% will be achieved by 2030 and 2050, respectively.[41] Such large shares of RE sources will most likely meet the energy load under favorable weather conditions and allow the thermal systems to be taken offline for some periods. Considering a large share of RE in an energy system we state the following theorem:

Theorem 3. *For every μ and $\tilde{x}$ with $\mathbb{E}[\tilde{x}] = 0$, $s_1^* \geq s_1^+$ if and only if:*

$$F(\tau)\big(\psi_U U'''(\check{q}_1^* - \epsilon) + \psi_C C'''(\check{y}_1^*)\big) + \big(1 - F(\tau)\big)U'''(\hat{q}_1^* - \epsilon) \geq 0. \tag{37}$$

Proof. Following the proof of Theorem 1, if $V(s_1^+, \tilde{z}_1)$ is convex in $\tilde{z}_1$, $\mathbb{E}[V_s(s_1^+, \tilde{z}_1)] \geq V_s(s_1^+, \mu) = 0$ or equivalently $\mathbb{E}[U'(y(\tilde{z}_1 + s_1^+ - \epsilon) + \tilde{z}_1 + s_1^+ - \epsilon] \geq U'(y(\mu + s_1^+ - \epsilon) + \mu + s_1^+ - \epsilon)$. Hence, the marginal benefit of increasing energy storage is positive when $s_1 = s_1^+$ and there is uncertainty. Conducting a similar analysis will show that:

$$F(\tau)\big(\psi_U U'''(\check{q}_1^* - \epsilon) + \psi_C C'''(\check{y}_1^*)\big) + \big(1 - F(\tau)\big)U'''(\hat{q}_1^* - \epsilon) \geq 0. \tag{38}$$

Therefore, if Eq. (38) holds, $s_1^* \geq s_1^+$.

If $s_1^* \geq s_1^+$ for every μ and $\tilde{x}$ with $\mathbb{E}[\tilde{x}] = 0$, then this must also be true for small zero-mean risks. Given that there is a small zero-mean risk allows us to focus only on the second-order Taylor approximation. This yields the following:

$$\mathbb{E}[V_s(s_1^+, \mu + \tilde{x})] \simeq F(\tau)\frac{1}{2}\check{\sigma}^2 V_{szz}(s_1^+, \check{\mu}) + (1 - F(\tau))\frac{1}{2}\hat{\sigma}^2 V_{szz}(s_1^+, \hat{\mu}). \tag{39}$$

For a small risk, if $s_1^* > s_1^+$, then $\mathbb{E}[V_s(s_1^+, \mu + \tilde{x})] \geq 0$. For $\mathbb{E}[V_s(s_1^+, \mu + \tilde{x})] \geq 0$ to be positive, $F(\tau)\frac{1}{2}\check{\sigma}^2 V_{szz}(s_1^+, \check{\mu}) + (1 - F(\tau))\frac{1}{2}\hat{\sigma}^2 V_{szz}(s_1^+, \hat{\mu})$ must be positive. One can calculate that this is equivalent to $\psi_U \check{U}''' + \psi_C \check{C}''' \geq 0$ and $\hat{U}''' \geq 0$. This completes the proof for necessity. $\square$

Note that for a variable (or a function) k, $\check{k}$ and $\hat{k}$ denote that k is evaluated at the conditional means $\mathbb{E}[z_1 | z_1 \leq \tau]$ and $\mathbb{E}[z_1 | z_1 > \tau]$, respectively. Taking a second-order Taylor approximation on the RHS in Eq. (8b) gives the following intertemporal efficiency condition:

$$\begin{aligned} U'(q_0^*) \simeq \phi\Big[&F(\tau)\Big(U'(\check{q}_1^* - \epsilon) + \frac{1}{2}\check{\sigma}^2\big(\psi_U \check{U}''' + \psi_C \check{C}'''\big)\Big) \\ &+ \big(1 - F(\tau)\big)\big(U'(\hat{q}_1^* - \epsilon) + \frac{1}{2}\hat{\sigma}^2 \hat{U}'''\big)\Big]. \end{aligned} \tag{40}$$

In opposition to an economy with a small share of RE, we see that frugality can have a relatively lower impact on precautionary energy storage. This is especially the case when the thermal plants are less frequently brought online; that is, when $F(\tau)$ is small.

[41] EC European Commission (2007) "Communication from the Commission to the European Parliament, the Council, the European Economic and Social Committee and the Committee of the Regions, A roadmap for moving to a competitive low carbon economy in 2050", COM(2011) 112 final, available at: http://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX:52011DC0112.

E Competitive market equilibrium: larger shares of RE

When we consider an electricity market with 100% RE (e.g., an economy with only hydroelectric power generation), then for $\psi_U = 1$, Proposition 1 follows. Hence, for $\psi_U = 1$, the pricing equation is given by Eq. (23). The interpretation regarding how an equilibrium level of energy storage is affected by the degree of intermittency, the price elasticity of demand and the coefficient of relative prudence remains the same. However, note that the energy storage firms obtain the desired level of energy from RE generators instead of purchasing energy from the thermal energy industry.

For the general case when thermal systems are occasionally shut down, we have the following:

$$P_0 \simeq F(\tau)\check{P}_0 + (1 - F(\tau))\hat{P}_0, \tag{41}$$

where $\check{P}_0$ and $\hat{P}_0$ are given by Eqs. (25) and (23), respectively. Note that while $\check{P}_0$ corresponds to the cases when thermal systems are active, $\hat{P}_0$ corresponds to the cases when they are kept idle due to high RE generation. Furthermore, note that the higher the $F(\tau)$ becomes, the lower the impact of frugality on the current spot price of electricity, and vice versa.

References

Ambec, S. and C. Crampes (2012). Electricity provision with intermittent sources of energy. *Resource and Energy Economics 34*(3), 319–336.

Bobtcheff, C. (2011). Optimal dynamic management of a renewable energy source under uncertainty. *Annals of Economics and Statistics/Annales d'Économie et de Statistique* (103-104), 143–172.

Carroll, C. D. and M. S. Kimball (2008). Precautionary saving and precautionary wealth. In S. N. Durlauf and L. E. Blume (Eds.), *The New Palgrave Dictionary of Economics* (Second ed.). Palgrave Macmillan, London, UK.

Carroll, C. D. and A. A. Samwick (1998). How important is precautionary saving? *Review of Economics and Statistics 80*(3), 410–419.

Cecchetti, S. G., A. K. Kashyap, and D. W. Wilcox (1997). Interactions between the seasonal and business cycles in production and inventories. *American Economic Review 87*(5), 884–892.

Chavas, J.-P. and M. T. Holt (1996). Economic behavior under uncertainty: A joint analysis of risk preferences and technology. *The Review of Economics and Statistics 78*(2), 329–335.

Crainich, D., L. Eeckhoudt, and A. Trannoy (2013). Even (mixed) risk lovers are prudent. *The American Economic Review 103*(4), 1529–1535.

Crampes, C. and M. Moreaux (2010). Pumped storage and cost saving. *Energy Economics 32*(2), 325–333.

Deck, C. and H. Schlesinger (2014). Consistency of higher order risk preferences. *Econometrica 82*(5), 1913–1943.

Denholm, P., E. Ela, B. Kirby, and M. Milligan (2010). The role of energy storage with renewable electricity generation. Technical report, National Renewable Energy Laboratory.

Doherty, R. and M. O'Malley (2005). A new approach to quantify reserve demand in systems with significant installed wind capacity. *Power Systems, IEEE Transactions on 20*(2), 587–595.

Ebert, S. and D. Wiesen (2014). Joint measurement of risk aversion, prudence, and temperance. *Journal of Risk and Uncertainty 48*(3), 231–252.

EIA (2013). International Energy Outlook 2013: With Projections to 2040. Technical report, U.S. Energy Information Administration.

EPRI (2008). Price elasticity of demand for electricity: A primer and synthesis. Technical report, Electric Power Research Institute, Palo Alto, CA.

Evans, L., G. Guthrie, and A. Lu (2013). The role of storage in a competitive electricity market and the effects of climate change. *Energy Economics 36*, 405–418.

EWEA (2009). The economics of wind energy. Technical report, European Wind Energy Association.

Faruqui, A. and S. Sergici (2010). Household response to dynamic pricing of electricity: a survey of 15 experiments. *Journal of Regulatory Economics 38*(2), 193–225.

Førsund, F. R. (2007). *Hydropower Economics*. Springer, New York, USA.

Førsund, F. R. and L. Hjalmarsson (2011). Renewable energy expansion and the value of balance regulation power. In P.-O. Johansson and B. Kristrom (Eds.), *Modern Cost-benefit Analysis of Hydropower Conflicts*. Edward Elgar Publishing, MA, USA.

Gravelle, H. (1976). The peak load problem with feasible storage. *The Economic Journal 86*(342), 256–277.

Guiso, L., T. Jappelli, and D. Terlizzese (1996). Income risk, borrowing constraints, and portfolio choice. *The American Economic Review 86*(1), 158–172.

Hamilton, B., C. Thomas, S. J. Park, and J.-G. Choi (2012). The customer side of the meter. In F. P. Sioshansi (Ed.), *Smart grid: integrating renewable, distributed & efficient energy*, Chapter 16. Academic Press, Oxford, UK.

Hansen, P. V. (2009). Inflow uncertainty in hydropower markets. *The Scandinavian Journal of Economics 111*(1), 189–207.

Hart, E. K., E. D. Stoutenburg, and M. Z. Jacobson (2012). The potential of intermittent renewables to meet electric power demand: Current methods and emerging analytical techniques. *Proceedings of the IEEE 100*(2), 322–334.

Horsley, A. and A. J. Wrobel (2002). Efficiency rents of pumped-storage plants and their uses for operation and investment decisions. *Journal of Economic Dynamics and Control 27*(1), 109–142.

IEA (2008). Energy Policies Review: The European Union. Technical report, International Energy Agency.

IEA (2013). Nordic Energy Technology Perspectives: Pathways to a Carbon Neutral Energy Future. Technical report, International Energy Agency.

IEA (2014). Monthly electricity statistics (December 2014). Available at: `http://www.iea.org/statistics/relatedsurveys/monthlyelectricitysurvey`.

Jackson, R. (1973). Peak load pricing model of an electric utility using pumped storage. *Water Resources Research 9*(3), 556–562.

Jessoe, K. and D. Rapson (2014). Knowledge is (less) power: Experimental evidence from residential energy use. *American Economic Review 104*(4), 1417–1438.

Joskow, P. L. (2011). Comparing the costs of intermittent and dispatchable electricity generating technologies. *The American Economic Review 101*(3), 238–241.

Kim, H., D. A. Boysen, J. M. Newhouse, B. L. Spatocco, B. Chung, P. J. Burke, D. J. Bradwell, K. Jiang, A. A. Tomaszowska, K. Wang, et al. (2012). Liquid metal batteries: Past, present, and future. *Chemical reviews 113*(3), 2075–2099.

Kimball, M. S. (1990). Precautionary saving in the small and in the large. *Econometrica 58*(1), 53–73.

Koopmans, T. (1957). Water storage policy in a simplified hydroelectric system. Technical Report 26, Cowles Foundation for Research in Economics, Yale University.

Leland, H. E. (1968). Saving and uncertainty: The precautionary demand for saving. *The Quarterly Journal of Economics 82*(3), 465–473.

Lichtner, S., R. Brindle, and L. Pack (2010). Electric Power Industry Needs for Grid-Scale Storage Applications. Technical report, U.S. Department of Energy.

Little, J. D. (1955). The use of storage water in a hydroelectric system. *Journal of the Operations Research Society of America 3*(2), 187–197.

Lund, H., A. N. Andersen, P. A. Østergaard, B. V. Mathiesen, and D. Connolly (2012). From electricity smart grids to smart energy systems–a market operation based approach and understanding. *Energy 42*(1), 96–102.

Mas-Colell, A., M. D. Whinston, J. R. Green, et al. (1995). *Microeconomic theory*, Volume 1. Oxford university press New York.

Miranda, M. J. and P. G. Helmberger (1988). The effects of commodity price stabilization programs. *The American Economic Review 78*(1), 46–58.

Nord Pool Spot (2011). The nordic electricity exchange and the nordic model for a liberalized electricity market. Available at `http://nordpoolspot.com`.

Noussair, C. N., S. T. Trautmann, and G. Van de Kuilen (2014). Higher order risk attitudes, demographics, and financial decisions. *The Review of Economic Studies 81*(1), 325–355.

Nyamdash, B., E. Denny, and M. O?Malley (2010). The viability of balancing wind generation with large scale energy storage. *Energy Policy 38*(11), 7200–7208.

RMI (2014). Utility energy storage: Batteries, compressed air storage, flywheels, hydrogen, superconducting magnetic energy storage, thermal energy storage, ultracapacitors, and vehicle-to-grid. Technical report, Red Mountain Insights.

Rothschild, M. and J. E. Stiglitz (1970). Increasing risk: I. A definition. *Journal of Economic Theory 2*(3), 225–243.

Sandmo, A. (1970). The effect of uncertainty on saving decisions. *The Review of Economic Studies 37*(3), 353–360.

Steffen, B. and C. Weber (2013). Efficient storage capacity in power systems with thermal and renewable generation. *Energy Economics 36*, 556–567.

Taylor, T. N., P. M. Schwarz, and J. E. Cochell (2005). 24/7 hourly response to electricity real-time pricing with up to eight summers of experience. *Journal of regulatory economics 27*(3), 235–262.

Tuohy, A. and M. O'Malley (2011). Pumped storage in systems with very high wind penetration. *Energy Policy 39*(4), 1965–1974.

Ummels, B. C., M. Gibescu, E. Pelgrum, W. L. Kling, and A. J. Brand (2007). Impacts of wind power on thermal generation unit commitment and dispatch. *Energy Conversion, IEEE Transactions on 22*(1), 44–51.

van de Ven, P. M., N. Hegde, L. Massoulié, and T. Salonidis (2013). Optimal control of end-user energy storage. *IEEE Transactions on Smart Grid 4*(2), 789–797.

Wang, M. W., J. Wang, and D. Ton (2012). Prospects for renewable energy: Meeting the challenges of integration with storage. In F. P. Sioshansi (Ed.), *Smart grid: integrating renewable, distributed & efficient energy*, Chapter 5. Academic Press, Oxford, UK.

Williams, J. C. and B. Wright (1991). *Storage and commodity markets*. Cambridge University Press, NY, USA.

Wright, B. D. and J. C. Williams (1982). The economic role of commodity storage. *The Economic Journal 92*(367), 596–614.

Wright, B. D. and J. C. Williams (1984). The welfare effects of the introduction of storage. *The Quarterly Journal of Economics 99*(1), 169–192.