# CATASTROPHE AVERSION

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# This presentation

- Based on the paper: « ***Catastrophe Aversion and Risk Equity under Dependent Risks*** » by Carole Bernard, Christoph Rheinberger & Nicolas Treich

- Uses results from statistics and actuarial science
  - Carole Bernard (Department of statistics and actuarial science, University of Waterloo)

- Supported by funding from Safera and FonCSI for the SAPHIR project

# SAPHIR research project: BCA of catastrophic accidents



- Example: Evaluation of a public transport infrastructure (e.g., a new railway)
  1. Financial cost
  2. Time savings (and other benefits including pollution reduction)
  3. Life savings
  4. But a more catastrophic risk (e.g., a big train accident vs. many small car accidents)


- In practice, BCA accounts for i), ii) and iii), but usually ignores iv)

# Two risky social situations: *A* and *B*

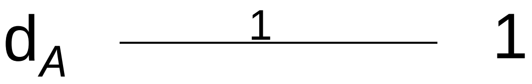
A society with two individuals *i*, and two equiprobable states *s*:

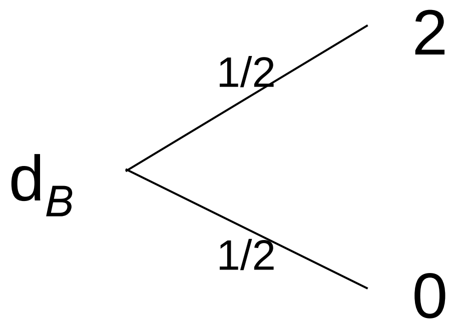
« 1 »: dead
« 0 »: alive

| *A* | i=1 | i=2 |
|---|---|---|
| s=1 | 1 | 0 |
| s=2 | 0 | 1 |
| $p_i$ | 1/2 | 1/2 |

| *B* | i=1 | i=2 |
|---|---|---|
| s=1 | 1 | 1 |
| s=2 | 0 | 0 |
| $p_i$ | 1/2 | 1/2 |

Distribution of fatalities:




Situation B is « more catastrophic » than situation A

# Is BCA catastrophe-averse?

- No, « standard » BCA is catastrophe neutral
  - In the example: Each individual *i* faces a baseline risk $p_i=1/2$ in both situations *A* and *B* => same WTP for risk elimination
- Yet, governmental agencies are catastrophe averse
  - Higher weighting for big accidents using frequency-number lines used in the UK, Norway, Switzerland and the Netherlands (Evans & Verlander 1997, Rheinberger 2010)
  - The disutility of *N* lives lost in a single accident is a function of $N^\alpha$ with $\alpha > 1$ (Slovic et al. 1984, Bedford 2013)

# Intuitive reasoning: context-dependence

- Catastrophe averse
  - Nuclear risk: « *The public appears to accept more readily a much greater social impact from many small accidents than it does from the more severe, less frequent occurrences that have a smaller societal impact*. » (U.S. Nuclear Regulatory Commission 1975, p. 12)
  - Asteroid collision, climate change, strangelet…: In "Catastrophe“, Posner (2004) estimates the cost of human extinction to $336 quadrillion (12 billion people x VSL=$28 million) => VSL is inflated due to the "dreadful" risk

- Catastrophe prone
  - Family risk: « *If a family of four must fly, and has a choice among four aircraft, of which it is known that one is defective but not known which one, it should be possible to persuade them to fly together. The prospects for each individual's survival are the same, no matter how they divide themselves among the aircraft, but the prospect for bereavement are nearly eliminated through the ''correlation'' of their prospects. ''Society's'' interest, in support of the family's interest, should be to see that they are permitted and encouraged to take the same plane together.* » Schelling (1968)

# Are we catastrophe averse?

- Survey studies (i.e., empirical social choice)
  - Neither lay people nor hazard experts display catastrophe aversion (Jones-Lee & Loomes 1995, Rheinberger 2010)

**Which of these two roads would you protect from avalanches?**

| | Road A | Road B |
|---|---|---|
| Expected accidents during 20 years | 4 | 2 |
| Expected number of fatalities per accident | 3 | 6 |
| **I would protect:** | ❑ Road A | ❑ Road B |

**Fig. 1.** Example of one choice task faced by a part of the participants.

Source: Rheinberger (2010)

# Should we be catastrophe averse?

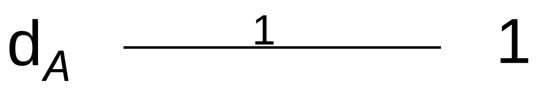
- Theoretical social choice
  - Common (e.g. utilitarian) social welfare functions are catastrophe neutral
  - Bommier & Zuber (2008) characterizes axiomatically a set of preferences which can be catastrophe prone or averse
  - Fleurbaey (2010)’s EDE is catastrophe prone: a more catastrophic situation is more equitable ex post
  - « Ex post transformed prioritarianism » is catastrophe averse iff the transformation is concave (Adler, Hammitt & Treich 2014)

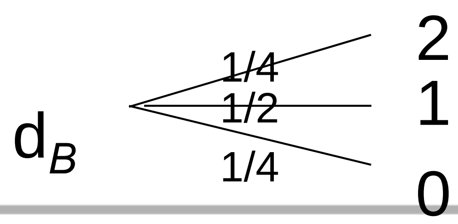
# Risk equity is always in conflict with catastrophe aversion!

- Under independent risks (Keeney 1980)
  - Subsequent literature on ex ante / ex post risk (Keeney & Winkler 1985, Sarin 1985, Fishburn & Sarin 1991, Fleurbaey & Bovens 2012)

| $A$ | i=1 | i=2 |
|---|---|---|
| s=1 | 1 | 0 |
| s=2 | 1 | 0 |
| s=3 | 1 | 0 |
| s=4 | 1 | 0 |
| $p_i$ | 1 | 0 |

| $B$ | i=1 | i=2 |
|---|---|---|
| s=1 | 1 | 1 |
| s=2 | 1 | 0 |
| s=3 | 0 | 1 |
| s=4 | 0 | 0 |
| $p_i$ | 1/2 | 1/2 |



# The motivation for this paper: Dependent risks

Toulouse School of Economics

Leaving aside the positive/normative debate, we address two questions:

1. Does **« more risk dependence »** always induce a « more catastrophic » situation?

2. Allowing for risk dependence, does **« more risk equity »** always induce a « more catastrophic » situation?

# More catastrophic, and more variable

- **Definition 1 (« more catastrophic »)**
  - **A distribution of fatalities $d_A$ is more catastrophic than a distribution $d_B$ iff for any concave function $f(.)$, $Ef(d_A) \leq Ef(d_B)$**

- **Definition 2 (« more variable »)**
  - **A distribution of fatalities $d_A$ is more variable than a distribution $d_B$ iff $var(d_A) \geq var(d_B)$**

- Remark: Def. 1 is simply Rothschild & Stiglitz (1970)'s def. applied to the distribution of fatalities in the population

# A simple result with two individuals

- **Proposition 1: Under N=2, the four following statements are equivalent:**
  - i. **The probability of simultaneous deaths increases**
  - ii. **The correlation between the individual risks increases**
  - iii. **The ditribution of fatalities is more catastrophic (definition 1)**
  - iv. **The distribution of fatalities is more variable (definition 2)**

- Simple proof, using:
  - Proba (simultaneous deaths)= $p_1 p_2 + \rho\, [p_1(1-p_1)p_2(1-p_2)]^{1/2}$
  - $var(d)= p_1(1-p_1)+ p_2(1-p_2) + 2\,\rho\, [p_1(1-p_1)p_2(1-p_2)]^{1/2}$

# Risk equity

- **Definition 3 (« A Pigou-Dalton transfer in risk »):**
  - **Consider two individuals *i* and *j* with probabilities of death: $p_i > p_j$. Then a distribution of fatalities becomes more equitable iff there is a transfer $\delta > 0$ such that $p'_i = p_i - \delta$ and $p'_j = p_j + \delta$ with $\delta \leq (p_i - p_j)/2$ (i.e., non rank-switching). Other individuals' probabilities of death are kept constant.**

- A Pigou-Dalton transfer reduces the « gap » between the level of risk exposure of two individuals

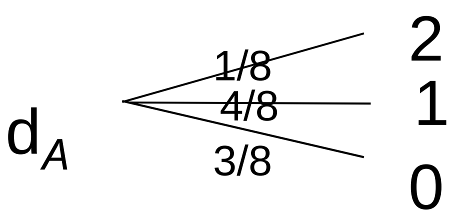
# Under independence (Keeney 1980)

- **Proposition 2:**
  - **Assume that the risks to N individuals are independent. Then, any Pigou-Dalton transfer always leads to a more catastrophic distribution of fatalities.**
- Simple proof:
  - Using Prop. 1: simply compute the variance before and after the Pigou Dalton transfer
- Slightly extends Keeney (1980) (who has a less general definition of a more catastrophic distribution)

# An example where Keeney's result fails

| *A* | i=1 | i=2 |
|---|---|---|
| s=1 | 1 | 1 |
| s=2 | 1 | 0 |
| s=3 | 1 | 0 |
| s=4 | 1 | 0 |
| s=5 | 0 | 1 |
| s=6 | 0 | 0 |
| s=7 | 0 | 0 |
| s=8 | 0 | 0 |
| $p_i$ | 1/2 | 1/4 |

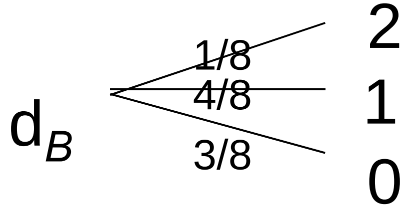
$\rho_A$ **= 0**




Pigou-Dalton transfer $\delta$=1/8

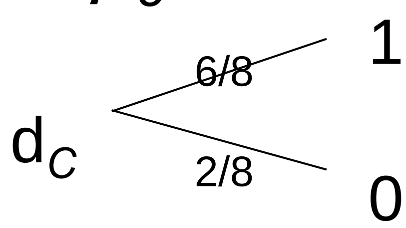
| *B* | i=1 | i=2 |
|---|---|---|
| s=1 | 1 | 0 |
| s=2 | 1 | 0 |
| s=3 | 1 | 1 |
| s=4 | 0 | 1 |
| s=5 | 0 | 1 |
| s=6 | 0 | 0 |
| s=7 | 0 | 0 |
| s=8 | 0 | 0 |
| $p_i$ | 3/8 | 3/8 |

$\rho'_B$ **= -0.06**



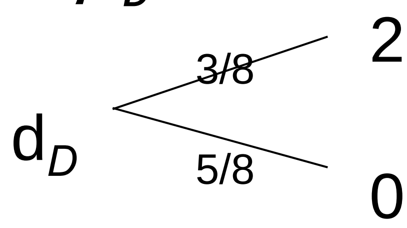

| *C* | i=1 | i=2 |
|---|---|---|
| s=1 | 1 | 0 |
| s=2 | 1 | 0 |
| s=3 | 1 | 0 |
| s=4 | 0 | 1 |
| s=5 | 0 | 1 |
| s=6 | 0 | 1 |
| s=7 | 0 | 0 |
| s=8 | 0 | 0 |
| $p_i$ | 3/8 | 3/8 |

$\rho'_C$ **= -0.6**




| *D* | i=1 | i=2 |
|---|---|---|
| s=1 | 1 | 1 |
| s=2 | 1 | 1 |
| s=3 | 1 | 1 |
| s=4 | 0 | 0 |
| s=5 | 0 | 0 |
| s=6 | 0 | 0 |
| s=7 | 0 | 0 |
| s=8 | 0 | 0 |
| $p_i$ | 3/8 | 3/8 |

$\rho'_D$ **= +1**



# Necessary and sufficient condition

- **Proposition 3:**
  - **Assume N=2 and $p_1>p_2$. Let $\rho$ denote the correlation across the two individual risks before the Pigou-Dalton transfer. Then, the distribution of fatalities becomes more catastrophic iff the correlation after the Pigou-Dalton transfer $\rho$' is larger than a critical level $\rho$***

- Intuition: two effects, the effect i) on the marginal distributions (as in Prop. 1) and ii) on the correlation

# Warning: the Pigou-Dalton transfer affects the correlation domain

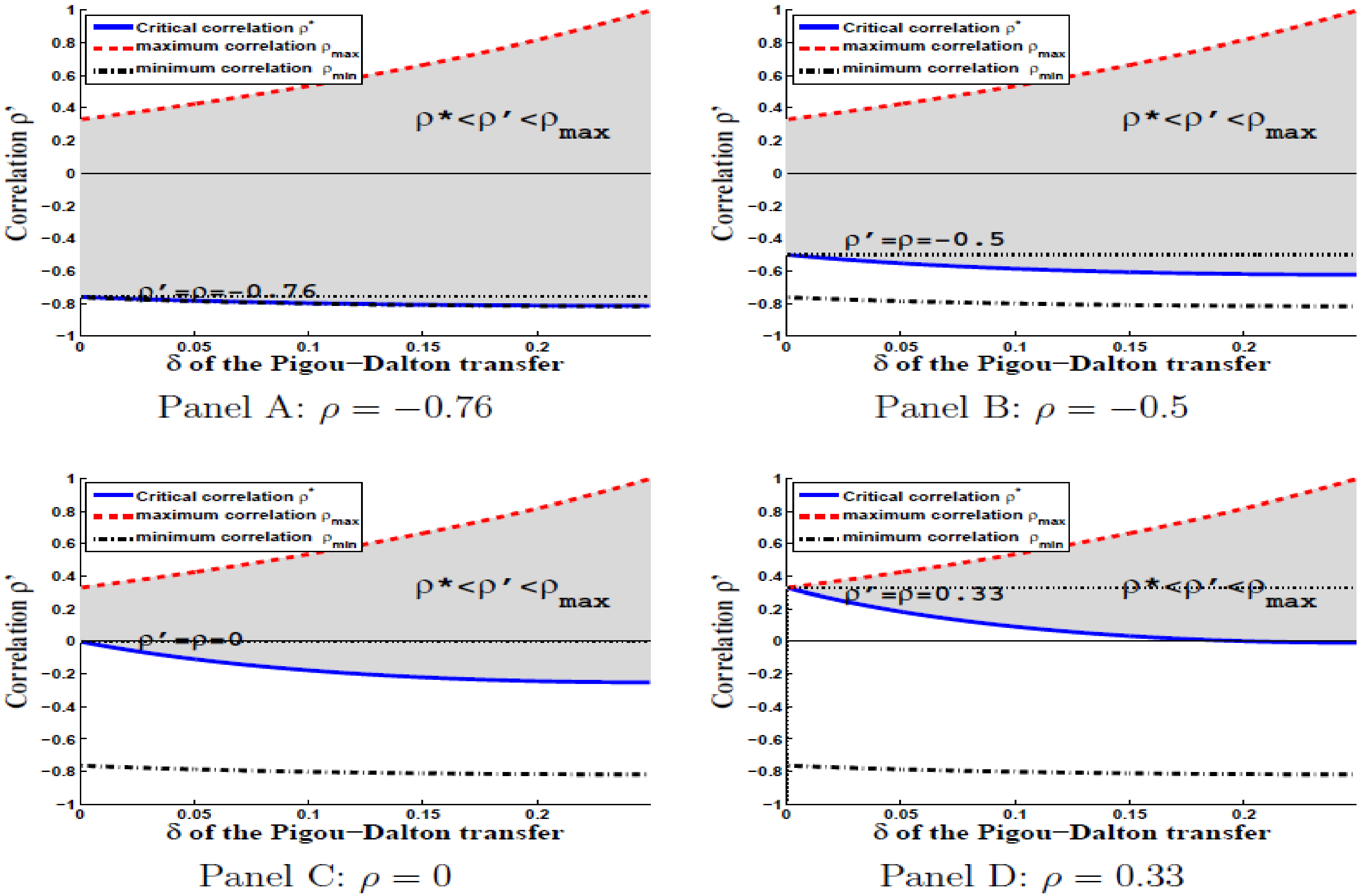



Figure 1: Correlation domains of $\rho'$ for a given value of $\rho$ when $p_1 = 0.8$ and $p_2 = 0.3$ for a range of $\delta \in \left[0, \frac{p_1 - p_2}{2}\right]$.

# Sufficient conditions

- **Proposition 4: The distribution of fatalities is more catastrophic:**

  i. **If $\rho' \geq \rho$**

  ii. **If $\rho$ is equal to the minimum correlation (defined as the countermonotonic dependence structure)**

  iii. **If $\rho'$ is equal to the maximum correlation (defined as the comonotonic dependence structure)**

  *iv.* **$p_1=1$ (individual 1 is certain to die), or $p_2=0$ (individual 2 is certain to survive)**

# The rest of the paper, N>2

- Much more difficult!
  - Not clear how to measure the degree of risk dependence
  - Consider three random variables *x,y* and *z*: if *x* is negatively correlated with *y* and with *z*; then y and z are positively correlated
- Some results
  - Impossibility results
  - Focusing on more variability (no longer on more catastrophic)
  - Assuming a form of independence wrt to individuals unaffected by the Pigou-Dalton transfer

# A result with uncorrelated risks

- **Proposition 6: Assume that all pairwise correlations between individuals *i* and *j*'s risks are zero, i.e. $\rho_{ij} = 0$. Assume that after the Pigou-Dalton transfer all correlations are still equal to zero, i.e. $\rho'_{ij} = 0$. Then:**
  - **The distribution of fatalities after the Pigou-Dalton transfer is more variable**
  - **The distribution of fatalities after the Pigou-Dalton transfer may, or may not, be more catastrophic**

# Conclusion

- Motivation: Catastrophic risks (e.g., storms, terrorist attacks, industrial accidents, climate change) are dependent risks

- This paper
  - Derives statistical conditions under which risk dependence and/or risk equity induce a more catastrophic situation
  - But ignores cost, and expected lives saved

- Next step: Examine optimal safety provision in an economic setting sensitive to risk equity and/or catastrophe aversion
  - Perhaps under the setting of « transformed prioritarianism » (Fleurbaey 2010, Adler, Hammitt & Treich 2014)