Averting Catastrophes: The Strange Economics of Scylla and Charybdis

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  - These may be as likely as a climate catastrophe, and could occur sooner and with less warning (so less time to adapt).
  - Can we ignore them and just focus on climate?
  - No. WTPs are not independent and not additive. Impacts and costs are not "marginal."

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  - If only one is to be eliminated, we should indeed choose the first; and we do even better by eliminating all three.
  - But we do best of all by eliminating the second and third and *not* the first: the presence of the second and third catastrophes makes it suboptimal to eliminate the first.

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- Main result: A rule for determining the set of catastrophes that should be averted.
- Time permitting: Examples, partial reduction in likelihood, rough numbers for some key catastrophes.

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- Catastrophe is Poisson arrival, mean arrival rate λ, can occur repeatedly.
- When it occurs, catastrophe permanently reduces consumption by a random fraction ϕ.
- CRRA utility function used to measure welfare, with RRA $=$ η and rate of time preference $= \delta$.

Event Characteristics and WTP

- Assume impact of nth arrival, ϕ_n, is i.i.d. across realizations n. So process for consumption is:

$$c_t = \log C_t = g_0 t - \sum_{n=1}^{N(t)} \phi_n \tag{1}$$

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- Encode the effects of risk using the *cumulant-generating function* (CGF),

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- c_t is a Lévy process, so $\boldsymbol{\kappa}_t(\theta) = \boldsymbol{\kappa}(\theta)t$, where $\boldsymbol{\kappa}(\theta)$ means $\boldsymbol{\kappa}_1(\theta)$,

$$\boldsymbol{\kappa}(\theta) = g_0\theta + \lambda \left(\mathbb{E}\, e^{-\theta\phi_1} - 1 \right) \tag{2}$$

Event Characteristics and WTP (Continued)

- With CRRA utility, welfare is:

$$\mathbb{E}\int_0^\infty \frac{1}{1-\eta} e^{-\delta t} C_t^{1-\eta}\, dt = \frac{1}{1-\eta}\int_0^\infty e^{-\delta t} e^{\boldsymbol{\kappa}(1-\eta)t}\, dt = \frac{1}{1-\eta}\frac{1}{\delta - \boldsymbol{\kappa}(1-\eta)}$$

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- This equation applies to *any* distribution for impact ϕ. We sometimes assume $z = e^{-\phi}$ follows a power distribution with parameter $\beta > 0$:

$$b(z) = \beta z^{\beta-1}, \quad 0 \le z \le 1. \tag{3}$$

(A large value of β implies a small expected impact.) Then,

$$\boldsymbol{\kappa}(\theta) = g_0\theta - \frac{\lambda\theta}{\beta+\theta}. \tag{4}$$

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- WTP to avert catastrophe is value of w that solves

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- Benefit is w and cost is permanent tax, τ, needed to eliminate the risk. Avoid the catastrophe as long as $w > \tau$.
- So far, nothing new. But now let's introduce multiple catastrophes. Start with two.

Two Types of Catastrophes

- Two independent types of catastrophes, arrival rates λ_1 and λ_2 and impact distributions ϕ_1 and ϕ_2. So

$$c_t = \log C_t = g_0 t - \sum_{n=1}^{N_1(t)} \phi_{1,n} - \sum_{n=1}^{N_2(t)} \phi_{2,n} \tag{7}$$

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- If neither catastrophe has been eliminated, welfare is

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- This implies $w_{1,2} < w_1 + w_2$. WTPs are not additive.

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  - Also applies if $B_1 = B_2$ and $K_1 = K_2$; might be we should only avert one of the two.

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  - $B_2/K_2 > 1 + B_1$ is equivalent to $(w_{1,2} - w_1)/(1 - w_1) > \tau_2$.
  - Can have $w_2 > \tau_2$ but $(w_{1,2} - w_1)/(1 - w_1) < \tau_2$. Why? These are not marginal projects, so $w_{1,2} < w_1 + w_2$.

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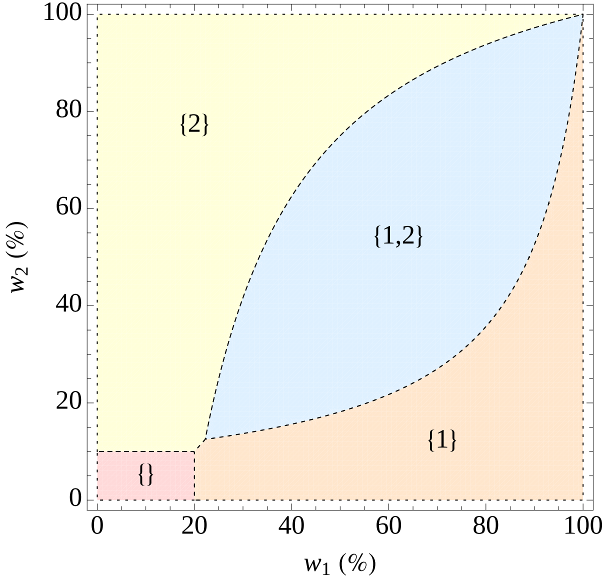
- Numerical example: Suppose $\tau_1 = 20\%$ and $\tau_2 = 10\%$. Figures show, for range of w_1 and w_2, which catastrophes to avert (none, one, or both).

Example: $\tau_1 = .2, \tau_2 = .1, \eta = 2$. What to Do?



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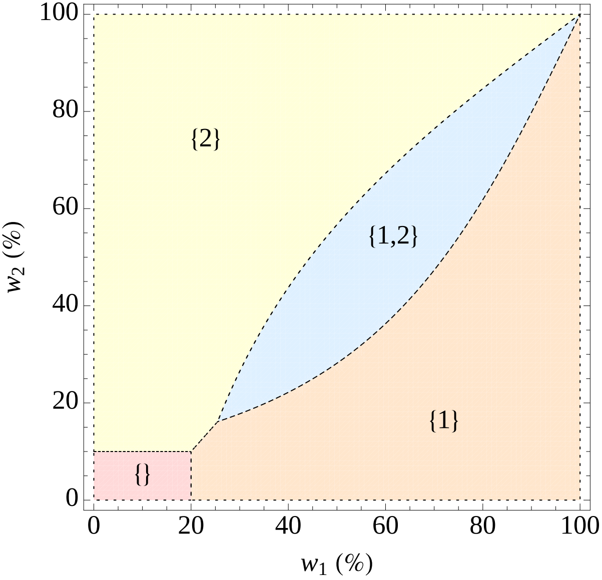

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    - Larger η implies smaller m^* because percentage drop in C, $1 - (1-\tau)^m$, results in larger increase in MU, and thus greater loss of utility from averting one additional catastrophe.

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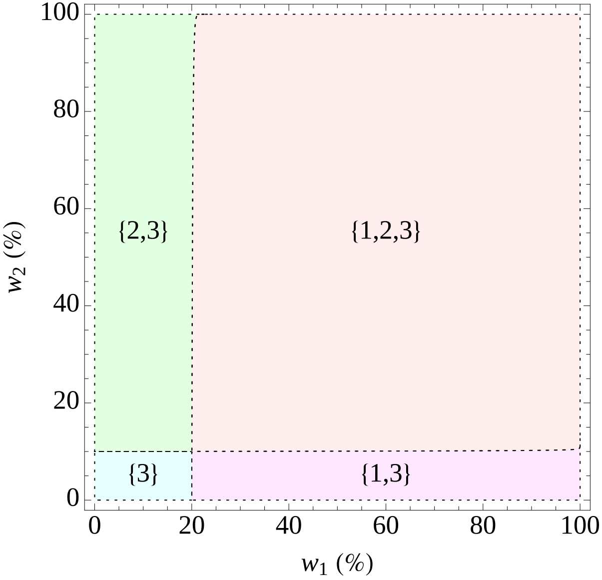
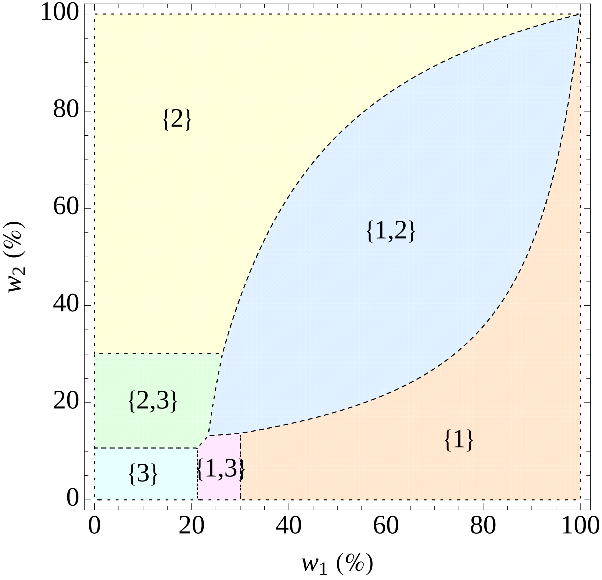
- Example 2: Now three potential catastrophes with $\tau_1 = 20\%$, $\tau_2 = 10\%$, and $\tau_3 = 5\%$. Figures show, for various values of w_3 and η, which ones should be averted as w_1 and w_2 vary between 0 and 1.

Example: $\eta = 1.01, w_3 = 7\%$



Example: $\eta = 2, w_3 = 7\%$



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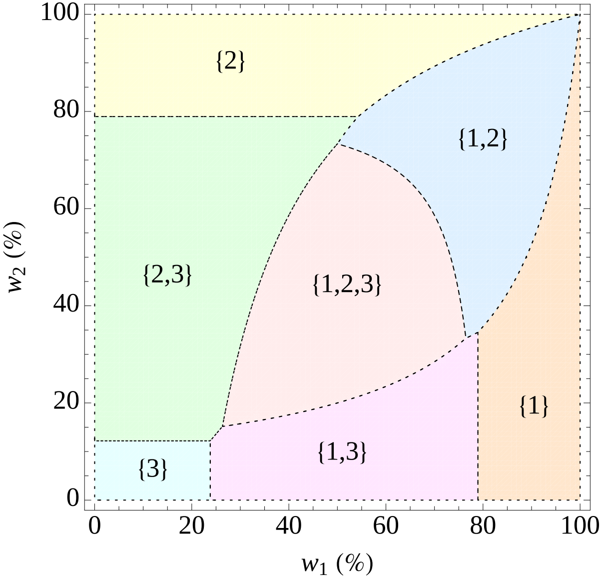
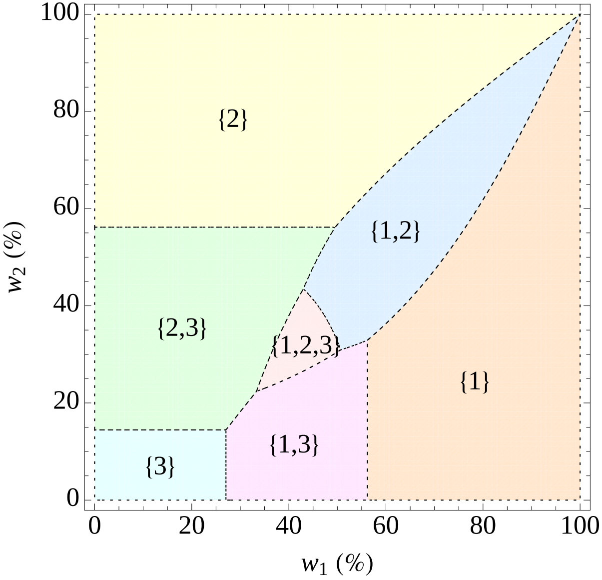


Example: $\eta = 3, w_3 = 20\%$



Example: Many Small, One Large

- Suppose we face many small catastrophes with cost and benefit (k, b), and one large one with (K, B). Must compare

$$\max_m \frac{1 + mb}{(1+k)^m} \qquad \text{with} \qquad \max_m \frac{1 + B + mb}{(1+K)(1+k)^m}.$$

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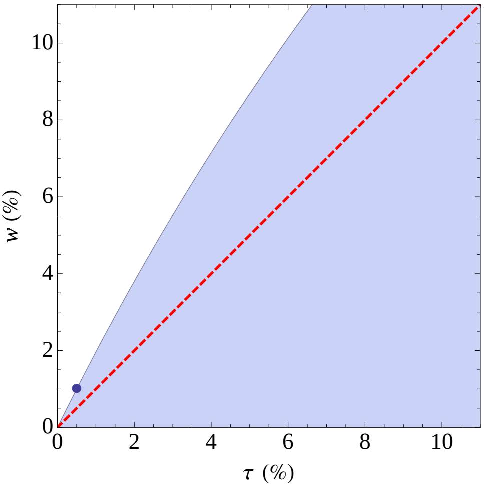
- Thus presence of small catastrophes raises hurdle rate to prevent a large one.

$w_i = 1\%, \tau_i = 0.5\%$ (blue dot), $\eta = 4$

If large catastrophe lies in shaded region it should not be averted:



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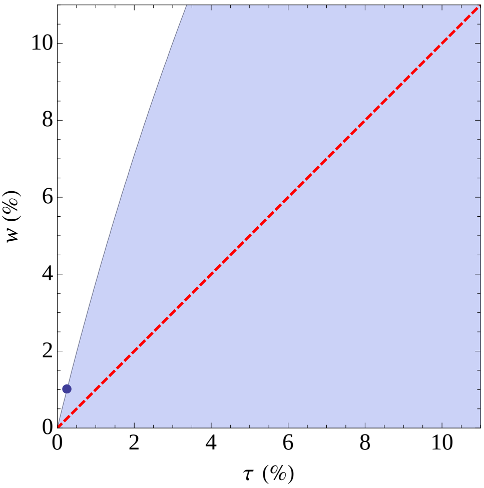
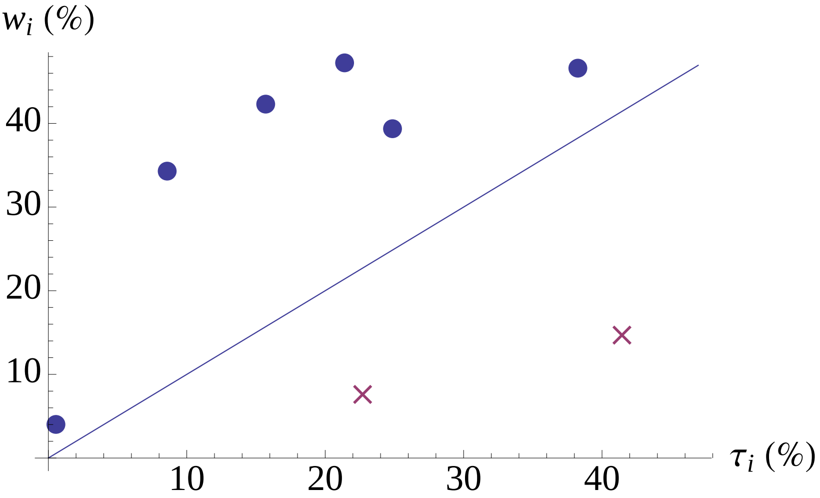
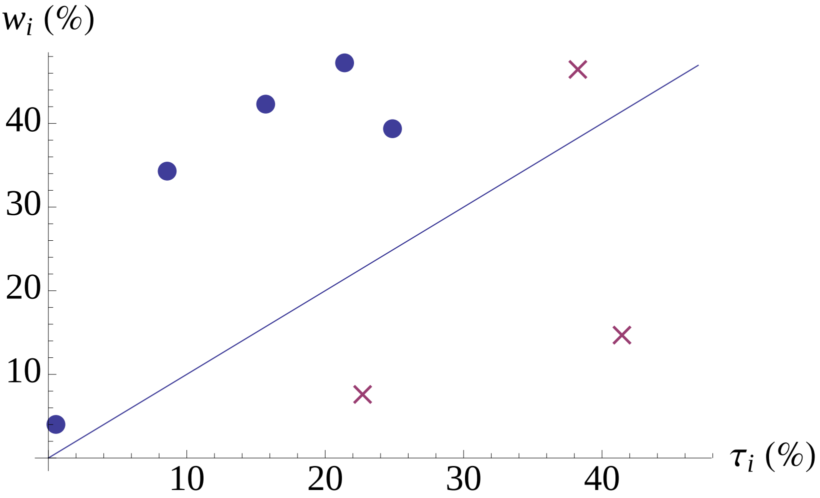


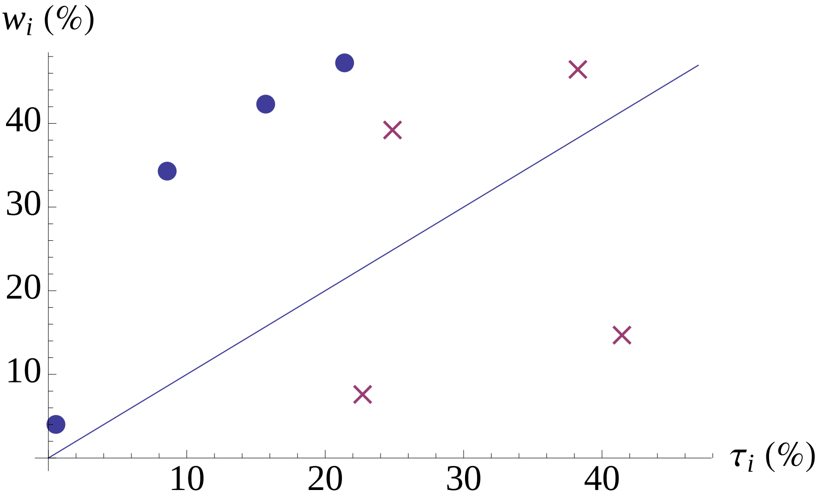
Choosing Among Eight Catastrophes, $\eta \in [1, 1.1]$



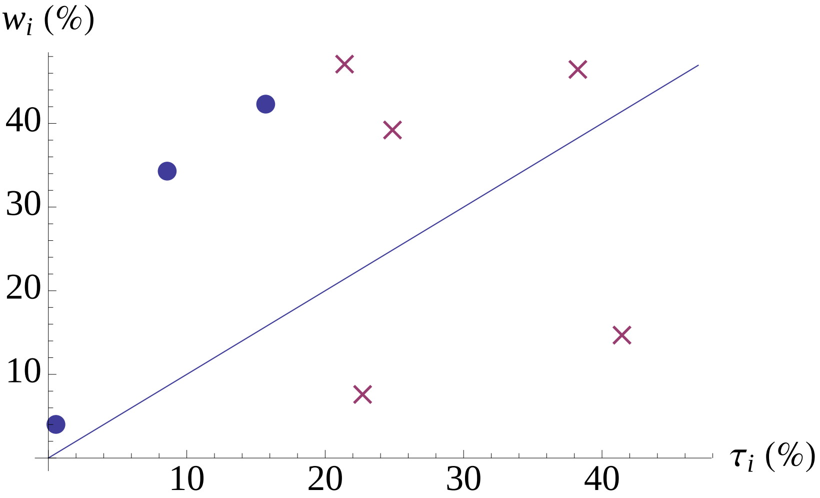
Choosing Among Eight Catastrophes, $\eta \in [1.2, 1.4]$



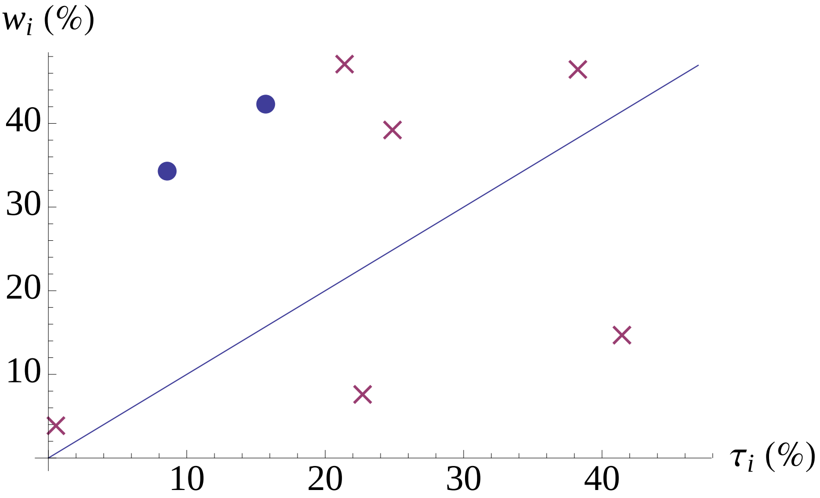
Choosing Among Eight Catastrophes, $\eta \in [1.5, 2.8]$



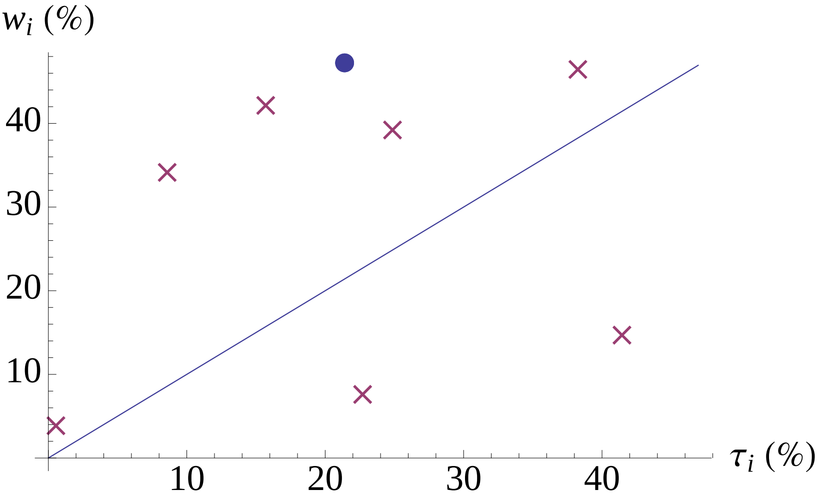
Choosing Among Eight Catastrophes, $\eta \in [2.9, 3.9]$



Choosing Among Eight Catastrophes, $\eta \in [4.0, 4.6]$



Choosing Among Eight Catastrophes, $\eta \in [4.7, \infty)$



Extensions

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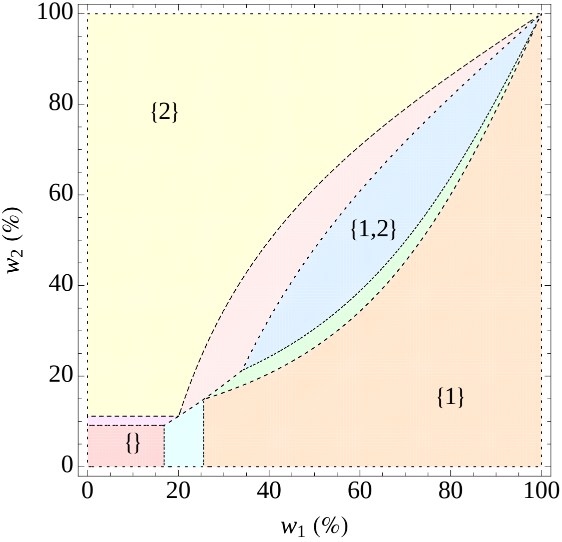
- Partial Alleviation: Results also apply if catastrophe arrival rate can be adjusted on a continuous spectrum.

Partial alleviation



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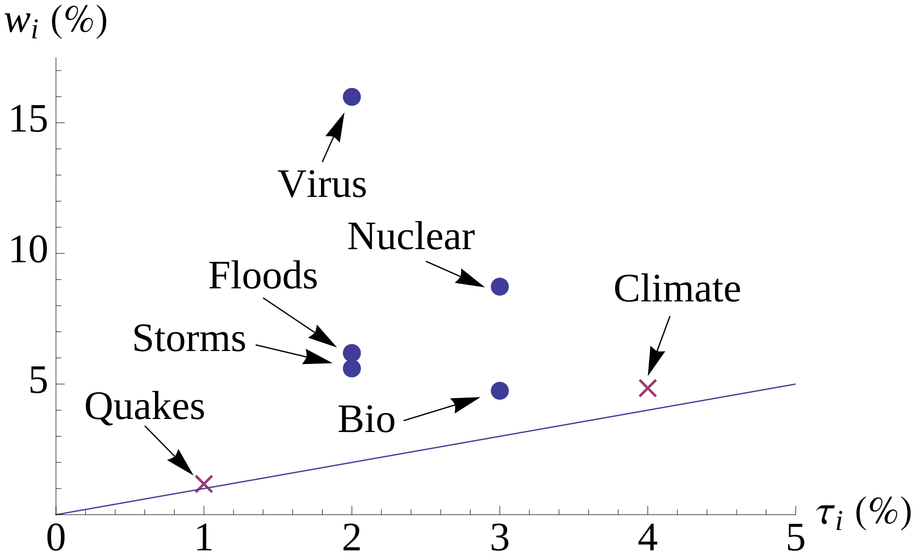
  - For others (climate, nuclear terrorism), little or no data.
- Subjective estimates of likelihood and impact might suffice (and might be the best we can do).

Some Rough Numbers

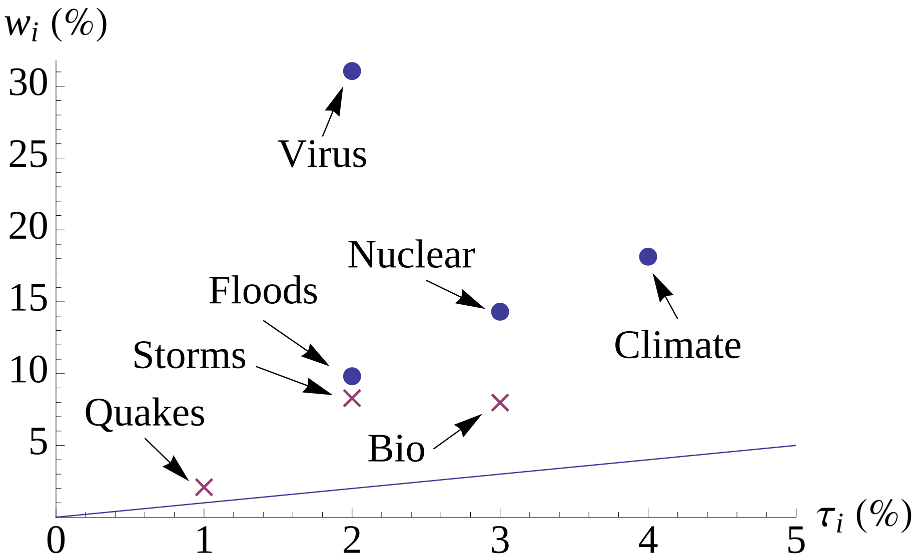
Characteristics of Seven Potential Catastrophes:

Potential Catastrophe	λ_i	β_i	τ_i	$\eta = 2$ w_i	B_i	K_i	$\eta = 4$ w_i	B_i	K_i
Mega-Virus	.02	5	.02	.159	.189	.020	.309	2.030	.062
Climate	.004	4	.04	.048	.050	.042	.180	.812	.130
Nuclear Terrorism	.04	17	.03	.086	.095	.031	.141	.580	.096
Bioterrorism	.04	32	.03	.047	.049	.031	.079	.280	.096
Floods	.17	100	.02	.061	.065	.020	.096	.356	.062
Storms	.14	100	.02	.051	.053	.020	.082	.293	.062
Earthquakes	.03	100	.01	.011	.011	.010	.020	.063	.031
Avert all Seven				.339	.513	.188	.442	4.415	.677

Which to Avert? ($\eta = 2$)

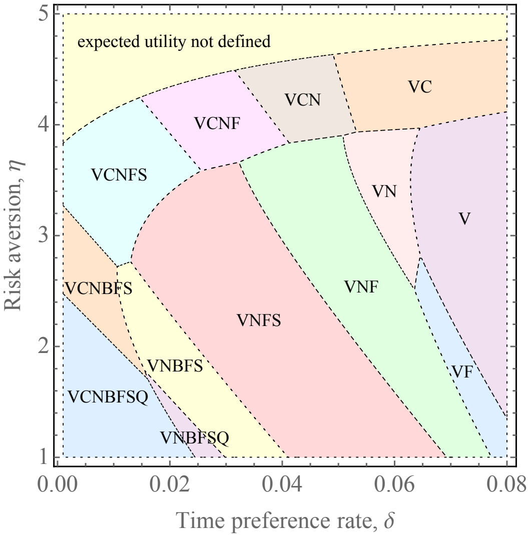


Which to Avert? ($\eta = 4$)



Which to Avert? (a range of δ and η)

Virus; Climate; Nuclear terrorism; Bioterrorism; Floods; Storms; Quakes.



Conclusions

- Studies usually treat catastrophes in isolation.
- This can lead to policies that are far from optimal.
- Projects to avert major catastrophes are not marginal.
  - So they are inherently interdependent, which can lead to optimal policies that are counter-intuitive, even "strange."
  - Even small catastrophes can be non-marginal in aggregate.
- We show how to find the set of catastrophes to be averted.
  - Our framework is quite flexible.
  - Applies to *any* distribution for impact ϕ_i.
  - Can accommodate catastrophes (or Brownian shocks) in the background that cannot be averted, catastrophes that cause death, and catastrophes that can only be partially averted.